Competing for contracts with buyer uncertainty: Choosing price and quality variables

Edward Anderson and Cheng Qian

Abstract

We model a situation in which a single firm evaluates competing suppliers and selects just one. Suppliers submit bids involving both price and quality variables. The buyer makes a choice which from the supplier's perspective appears to contain a stochastic element - for example the buyer may have information, which is not shared with the suppliers, and that gives one supplier an advantage in the final choice. We use a discrete choice model of buyer choice (e.g. multinomial logit). Our main result is that the supplier's choice of the quality variables is not affected by the competitive environment. Thus the suppliers compete only on price. We compare this with a second model in which the buyer's weighting on different quality variables is uncertain at the time bids are made.

Keywords: Supplier choice, Quality variables, Nash equilibrium, Types of uncertainty

May 2013

BA Working Paper No: 06/2013
http://sydney.edu.au/business/business_analytics/research/working_papers
Competing for contracts with buyer uncertainty: Choosing price and quality variables

Edward Anderson and Cheng Qian
The University of Sydney Business School, Australia
May 3, 2013

Abstract
We model a situation in which a single firm evaluates competing suppliers and selects just one. Suppliers submit bids involving both price and quality variables. The buyer makes a choice which from the supplier’s perspective appears to contain a stochastic element – for example the buyer may have information, which is not shared with the suppliers, and that gives one supplier an advantage in the final choice. We use a discrete choice model of buyer choice (e.g. multinomial logit). Our main result is that the supplier’s choice of the quality variables is not affected by the competitive environment. Thus the suppliers compete only on price. We compare this with a second model in which the buyer’s weighting on different quality variables is uncertain at the time bids are made.

1 Introduction
Selecting the right supplier is a critical strategic decision for many firms. Typically a group of managers will carefully evaluate the merits of competing suppliers, looking at a variety of factors in making their choice. There will inevitably be trade-offs to be made between price and other variables such as reliability, and delivery time. Our aim in this paper is to investigate the way that suppliers compete for contracts when bids are made that involve several factors. An understanding of this will help us to determine the amount of information to be released by the buying firm. Is there an advantage to the buyer in revealing ahead of time the weightings that will be given to different factors or the exact way that scoring rules will operate, or will it be better to keep this information private?

The usual approach when firms select a supplier is some combination of competitive bidding and negotiations (Monczka et al. 2009, p. 55). The first step in a competitive bidding process is for the buyer to send a ‘request for quotation’ (RFQ) to a number of potential suppliers (there may be some pre-qualification stage to determine this set of companies). On receiving these quotations the most common decision method for the buyer is to use a weighted point system that includes a variety of performance categories, a weight
associated with each category, and a scale enabling each supplier to be awarded a score within each category. The category scores are then added up using the predetermined weights and this final score is used to find an overall winner. For example, Chrysler uses cross-functional teams to determine the weights of four performance categories (i.e., cost, quality, technology, and delivery) and to evaluate a supplier’s overall qualification (Trent 2007, p. 60). We have found a similar approach used by a multinational paper tissue and hygiene products manufacturer when selecting a packaging supplier. Essentially the same process can also be used in an online auction. Either as a replacement for competitive bidding or following an initial bidding process, the buyer may enter into negotiation with one or more potential suppliers. Direct negotiation, without a competitive bidding stage, is appropriate when there are very few (if any) alternative suppliers available, or there is great uncertainty associated with some aspects of the bid (such as required performance or technology) and these uncertainties will be resolved as part of the negotiation.

The approaches used in the private sector are matched by those in the public sector. For example in the European Union there are specific requirements for public procurement of large contracts. These must be advertised widely and the announcements must include some specific weighting amongst attributes in any case where a lowest price mechanism is not being used. The principal here is that the selection of a “most economically advantageous tender” should be made on the basis of objective criteria and with a methodology that “is established in an objective and non-discriminatory manner and accessible to all interested parties” (see the European Commission document of December 2011 on a Proposal for a Directive of the European Parliament on Public Procurement, COM/2011/0896 final).

Many empirical studies have examined important factors in supplier selection (see Dickson 1966 for an early example) and researchers have applied different multi-objective decision making approaches. Weber et al. (1991) review 74 articles on supplier selection up to 1991, finding that most supplier selection decisions involve the consideration of multiple objectives (such as net price, delivery and quality). Their work indicates that the most widely used approach in the literature has been linear weighting models, although some researchers have used mathematical programming models and statistical/probabilistic approaches. A more recent survey by Ho et al. (2010) indicates that the most important criterion for evaluating bids is quality, followed by delivery, price/cost, manufacturing capability, service, etc. They note the increased attention paid to Data Envelopment Analysis in the literature, however the use of scoring on multiple criteria and then combining these scores using linear weights remains the primary approach in practice. We base our models on the weighted points system including price and a number of non-price attributes.

A number of researchers have considered the design of auction mechanisms in a multi-attribute supplier bidding environment. The connection between auction theory and supplier selection is intuitive since procurement is a reverse auction process where competitive suppliers bid on multi-attributes (e.g., price, quality, etc.) for a purchase contract from the buyer. Moreover, auction theory can be used to address uncertainty facing both sides (i.e., suppliers and the buyer). Many researchers have worked on multi-dimensional auctions to find optimal auction mechanisms in specific contexts, such as cost-time bidding for highway construction
projects (Herbsman et al. 1995), price-quantity bidding in the electricity market (Parisio and Bosco 2003), and multi-attribute auctions in public procurement (Simon and Melese 2011). In the supplier selection context, Che (1993) and Branco (1997) characterize scoring auctions with two dimensions where suppliers submit offers on price and quality and the buyer evaluates the bids using a scoring rule. The scoring rule could be a) first-score such that the supplier with the highest score wins and his/her offer is finalized as the contract, or b) second-score where the supplier with the highest score wins and is required to match the highest rejected score in the final contract, or c) second-preferred-offer which is the same as second-score auction except that the winning firm has to match not only the score but also the exact quality offered by the highest losing supplier. As an extension of the two-dimension auctions, Chen-Ritzo et al. (2005) experimentally examine three-attribute auctions when suppliers bid on price, quality and delivery time. Although the majority of research in this area assumes one-dimensional private information at suppliers’ side (e.g., suppliers’ marginal costs of quality), Asker and Cantillon (2008) and Asker and Cantillon (2010) model scoring auctions where suppliers have multi-dimensional private information (e.g., marginal cost of quality and fixed cost). We are interested in the suppliers’ decisions when there is uncertainty on the buyer’s behavior.

Other research has looked more closely at the supply chain coordination issues. Some researchers have designed optimal auction mechanism and procurement contract for the buyer in the same context as ours where a number of suppliers compete for the procurement contract from a single buyer. Cachon and Zhang (2006) investigate several bidding mechanisms when the suppliers bid on price and lead time, and the buyer awards the contract to the supplier who minimizes the sum of (the buyer’s) procurement and operation costs based on an estimate of suppliers’ costs. Chen (2007) considers a sole sourcing problem when both purchase quantity and price need to be determined, finding that the buyer’s optimal procurement strategy can be achieved by embedding a supply contract within an auction mechanism, i.e., the buyer first design a supply contract that specifies a payment for each possible purchase quantity and then invites suppliers to bid for this contract on an up-front, lump-sum fee they are willing to pay (for this business opportunity), with the winner being the supplier offering the highest fee; only the winning supplier pays an up-front fee and has the right to decide on the quantity produced and delivered. Note that the focus of this stream of literature is still on the buyer’s decisions.

We will focus on the suppliers’ decisions and our model represents a situation in which the buyer uses a scoring rule evaluating the bid on a number of characteristics and forming a linear weighting of these. We suppose that there is a single (sealed bid) auction for the work and we do not consider any later negotiations with the winning bidder. Suppliers have some uncertainty as to how their bids will be assessed. We consider two different types of uncertainty. First we suppose that in addition to considering the bids themselves the buyer also scores the suppliers as firms independent of the details of the bids. Thus for example the buyer may assess the overall reliability of a firm independent of the information in the bid. We suppose that these assessments are opaque to the supplier. On occasion these type of scores may emerge from discussions within the buying team during the process of evaluating bids, and thus may not be available even to the buyer at
the time that bids are made. The second type of uncertainty we consider relates to the utility function of the buyer as reflected in the weighting on different attributes of the bid. We model a case in which the suppliers have information giving a distribution over weights assigned for quality variables rather than knowing these explicitly.

Notice that the existence of either type of uncertainty will enable equilibria to exist in which the expected profit to the suppliers is much greater than it would be in the case without uncertainty. If suppliers have full information on the way that the winning bid is selected, then the only pure strategy equilibrium will be one in which the cheapest supplier submits a bid just undercutting the second cheapest supplier. (In this paper we will assume that suppliers differ in the cost involved in providing different levels of quality). When suppliers have relatively similar costs then in effect they bid away any profit that might be made on this contract.

There is a close connection between our models and some product differentiation models where two or more firms compete for market share through vertical product differentiation (e.g., qualities). The expected market share of each firm in product differentiation models resembles the probability of a supplier being chosen in our model, which means that we can translate the profit function for a supplier in our model of bidding for a contract directly into the profit function for a firm offering a product into a market place, with different structures for the uncertainty in buyer choice being paralleled by different structures in the heterogeneous preferences amongst consumers. There is a considerable marketing literature that relates to decisions on produce quality by competing firms, see (Moorthy 1988, Vandenbosch and Weinberg 1995, Chambers, Kouvelis, and Semple 2006, Lauga and Ofek 2011). Product differentiation models are particularly concerned with a situation in which consumers place different values on a quality attribute and the distribution of consumer preferences will determine the positioning decisions made by firms. The marketing literature has given attention to the product choices made by identical (or very similar firms): one fundamental question, which goes back to the work of Hotelling, is whether an equilibrium will have firms offering products that are similar to each other or very different. Our interests, however, are more in a situation where firms differ in the costs associated with different levels of quality, and this will drive product differentiation. The most significant difference between the model we give for supplier competition and the usual models for product differentiation is that product differentiation models adopt a two-stage equilibrium framework: in the first stage, each firm makes a product differentiation decision, on say quality, simultaneously with the competitors, and fixes it; in the second stage, each firm, having observed the competitors’ qualities, chooses a price for its product, simultaneously with the competitors. This framework is often appropriate in a marketing context where price decisions are made last but is hardly tenable in a sealed bidding context since suppliers need to bid on all attributes (including qualities and prices) simultaneously and privately. In this paper we explore the connection between equilibrium results in a one-stage and a two-stage framework.

We consider two research questions. Firstly we ask what the equilibrium choices will be for suppliers in competition with each other who each need to choose price and quality variables in their bids, but
are uncertain about the precise weights or scores that will be assigned by the buyer. Secondly we ask how
information on weights and scoring methods revealed by the buyer may affect the outcomes and in particular
the amount of profit made by the suppliers.

Our main result is that in the case where scores for bid attributes are uncertain, it is optimal for each
supplier to bid the non-price variables in a way that is independent of the competitive situation. Non-price
variables should be bid at the values that would be chosen if there were no other bidders involved. This result
holds with remarkable generality (independent of the distribution of the uncertainty and only requiring cost
functions to be convex). However, the same result does not hold for the case where there is no uncertainty
in relation to the scores, but instead the weights on bid attributes are uncertain at the time when bids are
submitted.

For examples with particular cost functions we have explored equilibrium solutions in more detail. We find
that smaller uncertainty leads to lower prices and lower profits for the suppliers. More accurate information
released by the buyer, prior to the suppliers making bids, will increase the competitiveness of the bidding and
lead ultimately to lower expected profits for the suppliers and a more favorable outcome for the buyer. In
the extreme case of perfect information, with each supplier knowing both its own and others' cost functions,
and also knowing the weighting given to different quality variables, then an equilibrium will typically have
the cheapest supplier winning the work at a price just below the price at which any other supplier can make
money.

The paper is essentially divided into two, with different types of uncertainty being investigated in Section
2 and Section 3. Section 2 presents our first model in which scores for certain bid attributes are uncertain,
and in Section 3 we investigate a second model where there is no uncertainty in relation to the scores, but
buyer weights are uncertain at the time when bids are submitted.

## 2 Uncertainty in firm scores

We consider a situation in which a buyer has to select one amongst \( n \) alternative suppliers, (indexed by \( i \)).
Each supplier makes a bid consisting of a price \( y \) and the levels for a set of \( m \) quality variable \( z_1, z_2, \ldots, z_m \).
Our assumption is that the buyer converts the quality level \( z_k \) into a score \( S_k(z_k) \), and then a weight \( \beta_k \)
is applied. If the buyer’s utility is separable and the scoring function \( S_k \) is a multiple of the utility for the
\( k \)’th quality variable, then this can be thought of as a transformation of the original quality measure to get
rid of any nonlinearity from the point of view of the buyer’s utility. Thus we assume that the buyer has a
separable linear utility function, with the utility from a bid \( (y, z_1, z_2, \ldots, z_m) \) given by \( \sum_{k=1}^m \beta_k S_k(z_k) - y \).

We will assume that the suppliers have knowledge of the weights \( \beta_k \) that will be applied to different
quality variables, but there is some uncertainty on the scoring functions \( S_k \). From the supplier’s perspective
they can make a specific bid without being certain of exactly how it will be scored. We call this the score-
uncertainty model and we assume additive uncertainty, so that the supplier sets a quality variable \( z_k \) and
receives a score $S_k(z_k) = \tilde{S}_k(z_k) + \varepsilon_k$ where we use $\tilde{S}_k$ for the scoring function that is the best guess of the supplier. However it is somewhat clumsy to carry around the notation $\tilde{S}_k$, thus instead of explicitly recognizing the estimated scoring functions $\tilde{S}_k$, we will simply write $z_k$ to mean $\tilde{S}_k(z_k)$. Thus $z_k$ is not the actual quality variable (which might be measured in say parts per million defects) but the estimated score associated with that variable (often measured as a number between 1 and 10).

It is convenient to introduce a constant $R$ representing the base utility that the buyer receives from the good or service being supplied. Now if supplier $i$ submits a bid $(y_i, z_{i1}, z_{i2}, ..., z_{im})$ and the (supplier) uncertainty associated with the $k$'th score is $\varepsilon_{ik}$ then the utility obtained by the buyer from accepting this bid is given by

$$U(y_i, z_i) = R + \sum_{k=1}^{m} \beta_k z_{ik} - y_i + \varepsilon_i.$$ 

The ‘noise’ variable $\varepsilon_i$ is a random variable unknown to the supplier at the time bids are made, and is given by

$$\varepsilon_i = \sum_{k=1}^{m} \beta_k \varepsilon_{ik},$$

where $\varepsilon_{ik}$ is the uncertainty on the part of supplier $i$ in respect to the score that the buyer will give for the $k$'th quality variable.

There may well be scores given for variables that do not explicitly form part of the bid. For example the buyer may make an evaluation of the reliability of the supplier on the basis of reputation or previous business dealings, rather than on the basis of statements made within the bid documents. This situation can be handled by forcing the corresponding $z$ value to be zero (or, equivalently, taking the cost of achieving any change in this $z$ value as infinite).

Assuming that the buyer maximizes its utility, the probability of a particular bid being accepted is determined by the bids made by other suppliers together with the distribution of the unobserved portion $\varepsilon_i$ in the buyer’s utility function.

This framework incorporates the standard discrete choice model in which a buyer makes a choice from amongst the choice set of all suppliers’ bids. Discrete choice models have been widely used for product differentiation in marketing research, since the discrete choice approach provides a good framework for describing the demands for differentiated products (Anderson et al. 1992). There is an increasing interest in using discrete choice models to describe supplier selection problems in the management literature (see (Verma and Pullman 1998, Gans 2002) and (Anderson et al. 2011) for examples). The easiest and most widely used discrete choice model is logit which assumes that the unobserved part of the utility has an extreme value distribution, i.e. the density for $\varepsilon_i$ is $f(x) = e^{-x}e^{-e^{-x}}$. With the logit model the choice probabilities have succinct, closed-form expressions (McFadden (1974)): the probability of supplier $i$ winning the contract is

$$P(y_i, z_i) = \frac{\exp(\sum_{k=1}^{m} \beta_k z_{ik} - y_i)}{\sum_{j=1}^{n} \exp(\sum_{k=1}^{m} \beta_k z_{jk} - y_j)}.$$
We suppose that bidders wish to maximize their expected profits. The quality variables are each associated with a cost function, with different suppliers having different costs. Thus the cost for supplier $i$ to achieve a level $z_{ik}$ for quality variable $k$ (strictly for the score for this quality variable) is given by $c_{ik}(z_{ik})$. We assume that each $c_{ik}$ is differentiable convex and increasing. The profit made if supplier $i$ wins the contract with a bid of $(y_i, z_{i1}, z_{i2}, \ldots, z_{im})$ is

$$y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}).$$

The expected profit is obtained by multiplying this by the probability of being selected.

We suppose that the other bids are given and calculate the bid for supplier $i$ which maximizes the expected profit

$$\Pi_i = (y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))P_i(y_i, z_i),$$

where $P_i(y_i, z_i)$ is the probability that the bid $(y_i, z_i)$ is accepted. We can calculate $P_i(y_i, z_i)$ by looking at the probability that this bid has the largest utility for the buyer. Thus

$$P_i(y_i, z_i) = P \left[ \sum_{k=1}^{m} \beta_k z_{ik} - y_i + \varepsilon_i > \max_{j \neq i} \left( \sum_{k=1}^{m} \beta_k z_{jk} - y_j + \varepsilon_j \right) \right].$$

We will assume that each of the random variables $\varepsilon_i$ have a continuous distribution over their support, so that there is a zero probability of utilities being equal.

In this model we assume that each firm has full information regarding its own costs, the cost functions for the other firms, and the noise distributions.

Define a random variable $X_i = \max_{j \neq i} (\sum_{k=1}^{m} \beta_k z_{jk} - y_j + \varepsilon_j) - \varepsilon_i$ and suppose that $X_i$ has a cumulative distribution function $F_i$ and density function $f_i$. Thus

$$P_i(y_i, z_i) = F_i \left( \sum_{k=1}^{m} \beta_k z_{ik} - y_i \right).$$

### 2.1 One-stage equilibrium

We first examine the equilibrium in the one-stage framework where the suppliers make decisions on price and quality simultaneously. We begin by determining the first order conditions for a maximum of the expected profit $\Pi_i$. We obtain

$$-(y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))f_i \left( \sum_{k=1}^{m} \beta_k z_{ik} - y_i \right) + F_i \left( \sum_{k=1}^{m} \beta_k z_{ik} - y_i \right) = 0, \quad (1)$$

$$(y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))(\beta_k f_i \left( \sum_{k=1}^{m} \beta_k z_{ik} - y_i \right) - c_{ik}'(z_{ik})F_i \left( \sum_{k=1}^{m} \beta_k z_{ik} - y_i \right)) = 0, \quad k = 1, 2, \ldots, m. \quad (2)$$
We will assume that the base utility \( R \) is sufficiently large that at the optimum the buyer has positive utility from the offer. Hence, using (1) to substitute for \( F_i \) in (2) we obtain

\[
(y_i - \sum_{k=1}^{m} c_{ik}(z_{ik})) f_i \left( \sum_{k=1}^{m} \beta_k z_{ik} - y_i \right) [\beta_k - c'_{ik}(z_{ik})] = 0, k = 1, 2, ..., m.
\]

We may assume that \( y_i > \sum_{k=1}^{m} c_{ik}(z_{ik}) \) (since otherwise the supplier makes no money) and also we may assume that the bid is placed with values that make it lie within the range in which it may be chosen. This last condition translates into a requirement that \( f_i (\sum_{k=1}^{m} \beta_k z_{ik} - y_i) > 0 \). Hence the first order condition for a maximum is that each \( z_{ik} \) is chosen so that

\[
c'_{ik}(z_{ik}) = \beta_k.
\]

These values of \( z_{ik} \) can then be used in (1) to determine \( y_i \) and we obtain the following result.

**Theorem 1** Suppose that \( c_i(.) \) is strictly convex and let \( z_{ik}^* \) be defined by

\[
c'_{ik}(z_{ik}^*) = \beta_k, k = 1, 2, ..., m. \tag{3}
\]

If the equation

\[
F_i \left( \sum_{k=1}^{m} \beta_k z_{ik}^* - y_i \right) = (y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}^*)) f_i \left( \sum_{k=1}^{m} \beta_k z_{ik}^* - y_i \right) \tag{4}
\]

has a single solution \( y_i^* \), then the optimal bid for firm \( i \) is \((y_i^*, z_{i1}^*, z_{i2}^*, ..., z_{im}^*)\).

**Proof**

We have already shown that the required first-order conditions are satisfied for \( z_{ik} \) at the points \( z_{ik}^* \) defined by (3), and (4) is taken directly from the first order conditions (1). The statement of the theorem with \( c_i \) strictly convex will ensure that there is only one solution to the first order conditions. So it only remains to check the second order conditions.

To shorten the expressions we let \( V_i = \sum_{k=1}^{m} \beta_k z_{ik} - y_i \) be that part of the buyer’s utility known to supplier \( i \) given a bid of \((y_i, z_i)\). For small values of \( y_i \) the left hand side of equation (1) is positive. Given the uniqueness of the zero point (4) we can deduce that the derivative with respect to \( y_i \) at the solution point is negative, i.e.

\[
(y_i - \sum_{k=1}^{m} c_{ik}(z_{ik})) f'_i (V_i) - 2 f_i (V_i) = \frac{\partial^2 \Pi_i}{dy_i^2} < 0.
\]
Now we consider the other second derivatives.

\[
\frac{\partial^2 \Pi_i}{dy_idz_{ik}} = -\beta_k(y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))f'_i(V_i) + c'_{ik}(z_{ik})f_i(V_i) + \beta_k f_i(V_i)
\]

\[
= \beta_k \left[ 2f_i(V_i) - (y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))f'_i(V_i) \right] = -\beta_k \frac{\partial^2 \Pi_i}{dy_i^2}.
\]

\[
\frac{\partial^2 \Pi_i}{dz_{ik}dz_{ih}} = (y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))\beta_k \beta_n f'_i(V_i) - c'_{ih}(z_{ik})\beta_k f_i(V_i) - \beta_n c'_{ik}(z_{ik})f_i(V_i)
\]

\[
= \beta_n \left[ (y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))\beta_k f'_i(V_i) - 2\beta_k f_i(V_i) \right] = \beta_n \beta_k \frac{\partial^2 \Pi_i}{dy_i^2}.
\]

\[
\frac{\partial^2 \Pi_i}{dz_{ik}^2} = (y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}))\beta_k^2 f'_i(V_i) - 2c'_{ik}(z_{ik})\beta_k f_i(V_i) - c''_{ik}(z_{ik})f_i(V_i)
\]

\[
= \beta_k^2 \frac{\partial^2 \Pi_i}{dy_i^2} - c''_{ik}(z_{ik})F_i(V_i) \leq \beta_k^2 \frac{\partial^2 \Pi_i}{dy_i^2}.
\]

In the last inequality we have used the convexity of \(c_i\). To show that the Hessian is negative definite consider the quadratic form \(x^T H x\) where \(x = (x_0, x_1, ..., x_m)^T\) with \(x_0\) corresponding to \(y\). Then

\[
x^T H x \leq \gamma \left( x_0^2 - \sum_{k=1}^{m} \beta_k x_0 x_k + \sum_{h=1}^{m} \sum_{k=1}^{m} \beta_h \beta_k x_h x_k \right) \leq \gamma w^T w < 0,
\]

where \(w = (-x_0, \beta_1 x_1, \beta_2 x_2, ..., \beta_m x_m)^T\) and \(\gamma = \partial^2 \Pi_i/dy_i^2\). Hence the Hessian is negative definite and we have established the required result.

This theorem concerns the optimal choice of quality and price for a supplier knowing the bids of the other suppliers. The result shows that the quality levels chosen are independent of the competitive environment (i.e. the number or aggressiveness of other bids). This independence of quality choice from competitive factors is striking and we will see that it is not a property that holds in our second model of uncertainty in weights. Clearly this property of the best response will carry over into any Nash equilibrium. Notice that \(y_i^*\), implicitly defined by (4), contains dependence on the other bids \(y_j\) and \(z_{jk}\) through the definition of the distribution \(F_i\).

Notice that the quality levels occurring in equilibrium are exactly those that optimize expected supply chain utility in the case of a single supplier. If the buyer ends up using supplier \(i\) then the supply chain utility is

\[
R + \sum_{k=1}^{m} \beta_k z_{ik} + e_i = \sum_{k=1}^{m} c_{ik}(z_{ik}),
\]

and this is maximized by taking the \(z_{ik}^*\) defined by (3).

9
2.2 Two-stage equilibrium

In the two-stage equilibrium framework, the competition between firms occurs in two stages. The extra complications of this model make an analysis more difficult and we will restrict ourselves to a duopoly. In the first stage, each firm chooses a product quality, simultaneously with the other firm, and fixes it. In the second stage, each firm, having observed the other’s quality, chooses a price for its product, simultaneously with the other firm. Given the structure of moves, we define the equilibrium in two steps, starting with the price equilibrium. The Nash equilibrium in prices is simply a price for supplier $i$ and a price for supplier $j$ such that neither firm wishes to choose a different price unilaterally. Necessarily, this price equilibrium will be a function of the qualities chosen in the first stage. The subgame-perfect quality equilibrium is a quality for supplier $i$ and supplier $j$ such that neither supplier would choose a different quality unilaterally, recognizing that the profitability of all quality selections will be determined on the basis of the price equilibrium that follows.

We will show that, subject to a single non-restrictive additional assumption, the two stage equilibrium will match the one stage equilibrium that we have already developed.

Let $y^0_i, z^0_i$ and $y^0_j, z^0_j$ be the one stage equilibrium solution. Clearly given quality choices $z^0_i$ and $z^0_j$ the $y^0$ values will be an equilibrium so we have

$$y^*_i(z^0_i, z^0_j) = y^0_i$$ and $$y^*_j(z^0_i, z^0_j) = y^0_j.$$ 

So in order to show that the equilibria are the same we have to show that varying $z_i$ from this value cannot produce an improvement in the payoff to player $i$. This might happen if varying $z_i$ produced a change in $y^*_j$. (In the case where changing $z_i$ produces no change in $y^*_j$ then the optimality of $y^0_i, z^0_i$ against $y^0_j, z^0_j$ shows the property we need.) Thus we will need to establish that $\frac{\partial y^*_j}{\partial z_i} = 0$.

We let $Y_i$ be the random variable $\varepsilon_j - \varepsilon_i$ and let $G_i$ be the distribution of $Y_i$ and we write $g_i$ for its density. We can reverse the roles of $i$ and $j$ to obtain a random variable $Y_j = -Y_i$, with a distribution function $G_j(z) = 1 - G_i(-z)$. The profit values are given by

$$\Pi_i = \left( y_i - \sum_{k=1}^{m} c_{ik}(z_{ik}) \right) G_i \left( \sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i + y_j \right),$$

$$\Pi_j = \left( y_j - \sum_{k=1}^{m} c_{jk}(z_{jk}) \right) \left( 1 - G_i \left( \sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i + y_j \right) \right).$$

The condition we will need is that the following inequality holds for all choices of $x$ in the support of $G$:

$$2g_i(x)^2 - g'_i(x)G_i(x) > 0. \quad (5)$$

This will be true for most distributions of the error terms. It is easily seen to be true whenever $g'_i(x) \leq 0$. 

so we need only consider $x$ values where $q_i'(x) > 0$. We can check that the condition holds when $\varepsilon_i$ and $\varepsilon_j$ are identically distributed with either a uniform, normal or extreme value distribution.

**Theorem 2** Suppose that $c_i(.)$ is strictly convex, and the condition (5) holds. If there is an equilibrium in the single stage problem then this solution will also be a (subgame perfect) equilibrium for the two stage problem where prices are determined after quality levels are set.

**Proof**

Suppose that quality levels have already been fixed, then the equilibrium prices $y_i^*$ and $y_j^*$ respectively optimize $\Pi_i$ and $\Pi_j$ and are chosen as the solutions to the pair of first order conditions

$$-\left(y_i - \sum_{k=1}^{m} c_{ik}(z_{ik})\right) g_i \left(\sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i + y_j\right) + G_i \left(\sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i + y_j\right) = 0, \tag{6}$$

$$-\left(y_j - \sum_{k=1}^{m} c_{jk}(z_{jk})\right) g_i \left(\sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i + y_j\right) + 1 - G_i \left(\sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i + y_j\right) = 0. \tag{7}$$

Now we consider the best choice of the $z_i$ variables. We look for first order conditions recognizing that $y_i^*$ and $y_j^*$ depend on the $z_{ik}$ values. (we write $y_i^*(z)$ for $y_i^*(z_1, \ldots, z_{im}, z_{j1}, \ldots, z_{jm})$ and similarly for $y_j^*(z)$ Taking derivatives of $\Pi_i$ with respect to $z_{ik}$ we get

$$\left(y_i^*(z) - \sum_{k=1}^{m} c_{ik}(z_{ik})\right) g_i \left(\sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i^*(z) + y_i^*(z)\right) (\beta_k - \partial y_i^*/\partial z_{ik} + \partial y_i^*/\partial z_{ik})$$

$$+ (\partial y_i^*/\partial z_{ik} - c_{ik}'(z_{ik})G_i \left(\sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i^*(z) + y_j^*(z)\right) = 0.$$

Then substituting from (6) and cancelling the common term we obtain

$$\beta_k + \partial y_j^*/\partial z_{ik} - c_{ik}'(z_{ik}) = 0,$$

The next step is to evaluate $\partial y_j^*/\partial z_{ik}$. It will be helpful to write $W = \sum_{k=1}^{m} \beta_k(z_{ik} - z_{jk}) - y_i^*(z) + y_j^*(z)$. We take derivatives with respect to $z_{ik}$ of (6) and this gives (after substituting $c_{ik}'(z_{ik}) = \beta_k + \partial y_j^*/\partial z_{ik}$ )

$$\left(y_i^*(z) - \sum_{k=1}^{m} c_{ik}(z_{ik})\right) g_i'(W) (\beta_k - \partial y_i^*/\partial z_{ik} + \partial y_j^*/\partial z_{ik}) - (\partial y_i^*/\partial z_{ik} - \beta_k - \partial y_j^*/\partial z_{ik}) g_i(W)$$

$$+ g(W) (\beta_k - \partial y_i^*/\partial z_{ik} + \partial y_j^*/\partial z_{ik}) = 0.$$

Thus

$$(\beta_k - \partial y_i^*/\partial z_{ik} + \partial y_j^*/\partial z_{ik}) \left(2g_i(W) - g_i'(W) \left(y_i - \sum_{k=1}^{m} c_{ik}(z_{ik})\right)\right) = 0.$$
Using (6) we see that the second term can be rewritten

\[ 2g_i(W) - g_i'(W) \frac{G_i(W)}{g_i(W)}, \]

and this is positive from our assumption that (5) holds. Thus we have shown that

\[ \beta_k - \partial y_i^* / \partial z_{ik} + \partial y_j^* / \partial z_{ik} = 0. \]  

(8)

Now we take derivatives with respect to \( z_{ik} \) of (7) and this gives

\[ -(y_j - \sum_{k=1}^{m} c_{jk}(z_{jk}))g_i'(W)(\beta_k - \partial y_i^* / \partial z_{ik} + \partial y_j^* / \partial z_{ik}) - (\partial y_i^* / \partial z_{ik})g_i(W) - g_i(W)(\beta_k - \partial y_i^* / \partial z_{ik} + \partial y_j^* / \partial z_{ik}) = 0 \]

Using (8) shows that \( \partial y_j^* / \partial z_{ik} = 0 \) which is the condition we need to show that the \( z_{ik}^* \) is given by the solution to

\[ c_{ik}(z_{ik}^*) = \beta_k. \]

Thus the two stage equilibrium matches the solution we have found for the one stage problem. The normal check that we would make for second derivative conditions is unnecessary here because the required property follows from the fact of this being an equilibrium in the single stage case.

\[ \square \]

2.3 Choosing uncertainty levels: An example

It is often the case that a buyer can choose the level of uncertainty within which supplier bids are made. Now we ask whether some uncertainty is beneficial for the buyer. We cannot give any general results but we can use our characterization of the equilibrium to explore specific cases.

Suppose that there are two firms \( i \) and \( j \) and a single quality variable \( z \). Firm \( i \) has a cost of quality given by \( c_i(z) = 2z^2 \) and firm \( j \) has a cost of quality \( c_j(z) = 3z^2 \). Thus firm \( i \) has a lower cost to achieve a given level of quality. The buyer’s weighting on quality is \( \beta = 2 \), which equates to $2 benefit for each unit increase in \( z \).

Then our result shows that firm \( i \) bids a quality level satisfying \( 4z_i = 2 \), i.e. \( z_i^* = \frac{1}{2} \) with a cost of 1/2. Similarly \( z_j^* = \frac{1}{3} \) with a cost of 1/3. In order to calculate an equilibrium we need to specify the distribution of \( \varepsilon_i \) and \( \varepsilon_j \). For simplicity of exposition we suppose that both \( \varepsilon_i \) and \( \varepsilon_j \) are uniformly distributed over \([−\mu, \mu]\). Then

\[ X_i = \beta z_j^* - y_j + \varepsilon_j - \varepsilon_i \]

and

\[ F_i(x) = \Pr(\frac{2}{3} - y_j + \varepsilon_j - \varepsilon_i < x). \]
Given the distributions of $\varepsilon_i$ and $\varepsilon_j$, the distribution function of $\varepsilon_j - \varepsilon_i$ can be easily derived. It has a triangular density function centred at 0 over the range $(-2\mu, 2\mu)$. Let $\Theta(x)$ and $\theta(x)$ be the cdf and density function for the triangular distribution over this range. Thus $F_i(x) = \Theta(x - \frac{2}{3} + y_i)$ and $f_i(x) = \theta(x - \frac{2}{3} + y_i)$. Similarly $F_j(x) = \Theta(x - 1 + y_i)$. The equations defining $y_i^*$ and $y_j^*$ are, from (4),

$$F_i(1 - y_i) = (y_i - \frac{1}{2}) f_i(1 - y_i),$$
$$F_j\left(\frac{2}{3} - y_j\right) = (y_j - \frac{1}{3}) f_j\left(\frac{2}{3} - y_j\right).$$

These can be rewritten

$$\Theta(\gamma) = \left(y_i - \frac{1}{2}\right) \theta(\gamma),$$
$$\Theta(-\gamma) = \left(y_j - \frac{1}{3}\right) \theta(-\gamma),$$

where $\gamma = (1/3) - y_i + y_j$.

By the symmetry of the triangular distribution we have $\theta(\gamma) = \theta(-\gamma)$. Now suppose $\gamma < 0$. This implies $\Theta(\gamma) < \Theta(-\gamma)$ and hence $y_i - (1/2) < y_j - (1/3)$ i.e. $y_j - y_i > -1/6$ which contradicts $\gamma < 0$. Thus we have shown $\gamma > 0$. Now we can derive from the two equations (9), after some algebra, that

$$\gamma = \frac{36\mu + 1 - \sqrt{(36\mu)^2 - 24\mu + 1}}{24},$$
$$y_i^* = \frac{2}{3} + \mu - \frac{3}{2}\gamma, \quad y_j^* = \frac{1}{3} + \mu - \frac{1}{2}\gamma,$$

Thus

$$y_i^* - y_j^* = \frac{7 - 36\mu + \sqrt{(36\mu - 1/3)^2 + (8/9)}}{24} > 0.$$

so supplier $i$ who has a cost advantage associated with quality always charges a higher price than the competitor.

We can find from this the expected profits at equilibrium for the two suppliers, $\Pi_i^*$ and $\Pi_j^*$, as well as the expected utility for the buyer, which we write as $\Pi_B^*$.

$$\Pi_i^* = \frac{1}{2\mu} \left(y_i^* - \frac{1}{2}\right)^2 \left(1 - \frac{\gamma}{2\mu}\right),$$
$$\Pi_j^* = \frac{1}{2\mu} \left(y_j^* - \frac{1}{3}\right)^2 \left(1 - \frac{\gamma}{2\mu}\right),$$
$$\Pi_B^* = R + \frac{1}{2\mu} \left(y_i^* - \frac{1}{2}\right) \left(1 - \frac{\gamma}{2\mu}\right) \gamma + \left(y_j^* - \frac{2}{3} - y_j\right).$$

The expected profits are plotted in Figure 1 for $R = 0.5$. The most important feature is that reducing the degree of uncertainty makes the equilibrium more competitive in general, leading to a better result for the
buyer. In this case, for small $\mu$, supplier $j$ makes almost no profit. However it is interesting that there is a small increase in profit for supplier $i$ at low values of $\mu$. This is associated with a small loss of utility for the buyer when the uncertainty $\mu$ is set very low. Buyer utility is maximized at $u = 0.0237$. The intuition for this is as follows. Starting from a position of no uncertainty (the buyer reveals everything to the suppliers) a small increase in uncertainty will cause the low cost supplier to drop their bid price a little in order to ensure that there is no chance for the high price supplier to win the bid. This results in a marginally higher profit for the buyer. However this happens for a relatively small range of values. Once uncertainty exceeds this, then the increased uncertainty makes it impossible for either player to be sure of beating the other and bid prices start to increase with uncertainty. We can say that with larger amounts of uncertainty there is less and less point in the suppliers behaving competitively and the price paid by the buyer increases steadily.

A more realistic case has errors following a normal distribution where the degree of uncertainty is determined by the standard deviation $\sigma$. Though it is no longer possible to give closed-form solutions, we can carry out a numerical study and find broadly similar results. Reducing the uncertainty will generally lead to better results for the buyer. There will be a value of the standard deviation, say $\sigma^*$, that maximizes the buyer profit. In the case with normally distributed errors we may find $\sigma^* = 0$, but this only occurs when the two firms have very similar costs, and in most cases $\sigma^*$ is a small positive number.

3 Uncertainty on buyer weights

Now we turn to our second model of uncertainty which reflects a situation in which suppliers are uncertain about the weights that the buyer will assign to different quality variables. We call this the weight-uncertainty model. As before the buyer selects from amongst $n$ alternative suppliers, where supplier $i$ bids a price $y_i$ and $m$ quality variables $z_{i1}, z_{i2}, \ldots, z_{im}$. As in the score-uncertainty model the $z$ values here should be thought of as...
the scores the buyer will assign rather than absolute levels of quality. Since there is no longer any uncertainty in these scores (captured by $\varepsilon_i$ which occurred in the score-uncertainty model), the buyer accepting the bid from supplier $i$ will receive a utility $U(y_i, z_i) = R + \sum_{k=1}^{m} \beta_k z_{ik} - y_i$. In this model, however, we suppose that the exact values of $\beta_k$ are unknown to the suppliers, and instead we assume that the $\beta_k$ are drawn from a known distribution. Other assumptions made in Section 2 continue to hold and in particular we assume that each $x_{ik}$ is differentiable, strictly convex and increasing.

Let $P_i(y_i, z_i)$ be the probability that the bid $(y_i, z_i)$ is accepted. We calculate $P_i(y_i, z_i)$ by looking at the probability that this bid has the largest utility. Thus

$$P_i(y_i, z_i) = \Pr \left[ \sum_{k=1}^{m} \beta_k z_{ik} - y_i > \max_{j \neq i} \left( \sum_{k=1}^{m} \beta_k z_{jk} - y_j \right) \right].$$

We shall assume that the random variables $\beta_k$ have continuous distributions over their support so that there is zero probability of utilities being equal. We will also assume that the constant $R$ is large enough for all the bids to give the buyer a positive utility.

As we will see this model is more complex to analyze and we will explore the behavior of the supplier bids through looking in detail at the case where there is just one quality variable $z$ and two suppliers, $i$ and $j$. Without loss of generality, we assume that $z_i > z_j$. The probability of supplier $i$ being selected is

$$P_i(y_i, z_i) = \Pr(\beta z_i - y_i > \beta z_j - y_j).$$  \hspace{1cm} (10)

We suppose that $\beta$ has a cumulative distribution function $H$ and density function $h$. Thus,

$$P_i(y_i, z_i) = \Pr(\beta > \frac{y_i - y_j}{z_i - z_j}) = 1 - H(w),$$

where we define $w = (y_i - y_j)/(z_i - z_j)$. Also the probability of supplier $j$ being chosen is $P_j(y_j, z_j) = H(w)$.

The expected supplier profits are given by $\Pi_i = (y_i - c_i(z_i))P_i(y_i, z_i)$ and $\Pi_j = (y_j - c_j(z_j))P_j(y_j, z_j)$. The first order conditions at an equilibrium give

$$-(y_i - c_i(z_i))h(w)\frac{1}{z_i - z_j} + 1 - H(w) = 0, \hspace{1cm} (11)$$

$$\frac{y_i - y_j}{(z_i - z_j)^2}h(w)(y_i - c_i(z_i)) - c'_i(z_i)(1 - H(w)) = 0, \hspace{1cm} (12)$$

$$-(y_j - c_j(z_j))h(w)\frac{1}{z_i - z_j} + H(w) = 0, \hspace{1cm} (13)$$

$$\frac{y_i - y_j}{(z_i - z_j)^2}h(w)(y_j - c_j(z_j)) - c'_j(z_j)H(w) = 0. \hspace{1cm} (14)$$

In addition to these first order local conditions we also need to check that neither bidder can do better by undercutting the other bidder at their quality choice. The situation is illustrated in Figure 2 where the
Figure 2: Possibility of undercutting the other supplier at the same quality level may break an equilibrium.

support of $\beta$ is given by $(\beta_0, \beta_1)$, i.e. $\beta_0$ and $\beta_1$ are respectively the lowest and highest weights that may occur. At an equilibrium we may assume that $w \in [\beta_0, \beta_1]$ since otherwise one of the suppliers has no chance of winning the bid, and can only gain by lowering its price $y$ to bring $w$ into the specified range. For an equilibrium however we need to check that supplier $i$ cannot improve by moving its bid to the position $C$ just below supplier $j$’s offer (or similarly supplier $j$ moving to position $D$). The conditions to ensure this are:

\[(y_i - c_i(z_i))(1 - H(w)) > y_j - c_i(z_j),\]  
\[(y_j - c_j(z_j))H(w) > y_i - c_j(z_i).\]  

Substituting (11) into (12) and (13) into (14), we obtain

\[c'_i(z_i) = c'_j(z_j) = \frac{y_i - y_j}{z_i - z_j} = w.\]  

Subtracting (11) from (13) and substitute with (17), we deduce

\[z_i - z_j = \frac{h(w)}{h(w)w - 1 + 2H(w)}(c_i(z_i) - c_j(z_j)).\]  

To solve this we define functions $d_i(\cdot) = (c'_i)^{-1}(\cdot)$ and $d_j(\cdot) = (c'_j)^{-1}(\cdot)$ which give $z_i$ and $z_j$ as functions of $w$. Thus we can rewrite (18) as

\[d_i(w) - d_j(w) = \frac{h(w)}{h(w)w - 1 + 2H(w)}(c_i(d_i(w)) - c_j(d_j(w))).\]
and solve this equation for \( w \) which defines \( z_i \) and \( z_j \). Then \( y_i \) and \( y_j \) can be found from (11) and (13):

\[
\begin{align*}
y_i &= c_i(z_i) + \frac{(1 - H(w))(z_i - z_j)}{h(w)}, \\
y_j &= c_j(z_j) + \frac{H(w)(z_i - z_j)}{h(w)}.
\end{align*}
\] (20, 21)

The following result shows that the quality levels that occur in equilibrium with this model are lower than the supply chain optimal values occurring with the score-uncertainty model.

**Theorem 3** Suppose that the distribution of weights described by \( H \) is symmetric about a value \( \overline{\beta} \). When supplier \( i \) dominates supplier \( j \) from a cost perspective with \( c_i(x) < c_j(x) \) for all \( x \), then in equilibrium supplier \( i \) is selected more often than supplier \( j \) and \( z_i \) and \( z_j \) are both lower than the supply chain optimal values \( z_i^* \) and \( z_j^* \) defined by

\[
c'_i(z_i^*) = c'_j(z_j^*) = \overline{\beta}.
\]

**Proof**

Since \( 0 < c_i(x) < c_j(x) \) for all \( x > 0 \) with both \( c_i \) and \( c_j \) being convex, the tangent line of slope \( \eta \) for \( c_i \) lies to the right of that for \( c_j \) for any \( \eta > 0 \). By considering the point at which these two lines cross the \( x \) axis we have

\[
d_j(\eta) - c_j(d_j(\eta))/\eta < d_i(\eta) - c_i(d_i(\eta))/\eta
\]

for any \( \eta > 0 \). But at equilibrium we know that (19) holds with \( w = (y_i - y_j)/(z_i - z_j) \). Setting \( \eta = w \) shows that

\[
\frac{h(w)}{h(w)w - 1 + 2H(w)} > \frac{1}{w}
\]

and hence that \( H(w) < 1/2 \). In other words \( P_i(y_i, z_i) > 1/2 \) as we require.

Moreover if \( H(w) < 1/2 \) then by symmetry \( w < \overline{\beta} \) and so \( c'_i(z_i) \) and \( c'_j(z_j) \) are both less than \( \overline{\beta} \). Because of convexity, \( c'_i \) and \( c'_j \) are increasing functions, and so \( z_i < z_i^* \) and \( z_j < z_j^* \) as required. \( \blacksquare \)

Now we look in more detail at a specific case. We suppose that costs follow a power law with \( c_i(z) = \rho_i z^k \), \( c_j(z) = \rho_j z^k \), and so we have \( w = k\rho_i z_i^{k-1} = k\rho_j z_j^{k-1} \). Thus,

\[
d_i(w) = \left(\frac{w}{\rho_i k}\right)^{\frac{1}{k-1}}, \quad d_j(w) = \left(\frac{w}{\rho_j k}\right)^{\frac{1}{k-1}}.
\]

So (19) becomes

\[
\left(\frac{w}{\rho_i k}\right)^{\frac{1}{k-1}} - \left(\frac{w}{\rho_j k}\right)^{\frac{1}{k-1}} = \frac{h(w)}{h(w)w - 1 + 2H(w)}[\rho_i \left(\frac{w}{\rho_i k}\right)^{\frac{1}{k-1}} - \rho_j \left(\frac{w}{\rho_j k}\right)^{\frac{1}{k-1}}].
\]

From this we can derive

\[
\frac{h(w)}{h(w)w - 1 + 2H(w)} = \frac{k}{w}.
\]
or equivalently,
\[ H(w) = \frac{1}{2} - (1 - \frac{1}{k})h(w)w. \] (22)

Suppose now that \( \beta \) is uniformly distributed on the range from \( \beta_0 \) to \( \beta_1 \), so \( H(\beta) = (\beta - \beta_0)/(\beta_1 - \beta_0) \) and \( h(\beta) = 1/(\beta_1 - \beta_0) \) within this range. Then (22) gives
\[
w = \frac{k(\beta_0 + \beta_1)}{3k - 1},
\]
provided that \( w > \beta_0 \), i.e. \( \beta_1 > \beta_0 (2k - 1)/k \). Therefore, solving \( w = c'(z_i) = c'_j(z_j) \), we obtain
\[
z_i = \left( \frac{\beta_0 + \beta_1}{\rho_i (3k - 1)} \right)^{\frac{1}{k}}, \quad z_j = \left( \frac{\beta_0 + \beta_1}{\rho_j (3k - 1)} \right)^{\frac{1}{k}},
\]
which shows that in this case the suppliers’ quality decisions are independent of the competition. Furthermore, the quality increases with the mean value of buyer weights and is independent of the uncertainty \( \beta_1 - \beta_0 \).

Substituting \( w, H(w), h(w) \) into (20) and (21), and rewriting we obtain,
\[
y_i = \frac{(3k - 1)(\beta_1 - \beta_0) + (k - 1)(\beta_1 + \beta_0)}{2(3k - 1)}(z_i - z_j) + \rho_i z_i^k, \quad y_j = \frac{(3k - 1)(\beta_1 - \beta_0) - (k - 1)(\beta_1 + \beta_0)}{2(3k - 1)}(z_i - z_j) + \rho_j z_j^k.
\]
Thus, given a fixed value of the average weight, \( (\beta_0 + \beta_1)/2 \), the prices that are offered will decrease as the amount of uncertainty \( (\beta_1 - \beta_0) \) decreases.

In the case that \( k = 2 \) we have
\[
z_i = \left( \frac{\beta_0 + \beta_1}{5 \rho_i} \right), \quad y_i = \frac{3\beta_1 - 2\beta_0}{5}(z_i - z_j) + \rho_i z_i^2, \quad z_j = \left( \frac{\beta_0 + \beta_1}{5 \rho_j} \right), \quad y_j = \frac{2\beta_1 - 3\beta_0}{5}(z_i - z_j) + \rho_j z_j^2.
\]
Note that since we assume \( z_i > z_j \) we must have \( \rho_i < \rho_j \). We have \( w = 2(\beta_0 + \beta_1)/5 \) and
\[
H(w) = \frac{2\beta_1 - 3\beta_0}{5(\beta_1 - \beta_0)}.
\]
The final step is to check the conditions (15) and (16) that we require to ensure this is an equilibrium (and undercutting does not occur). We can rewrite these conditions as
\[
(z_i - z_j) \left( \frac{(3\beta_1 - 2\beta_0)^2}{25(\beta_1 - \beta_0)} - \frac{2\beta_1 - 3\beta_0}{5} \right) > (\rho_j - \rho_i) z_j^2, \quad (z_i - z_j) \left( \frac{(2\beta_1 - 3\beta_0)^2}{25(\beta_1 - \beta_0)} - \frac{3\beta_1 - 2\beta_0}{5} \right) > (\rho_i - \rho_j) z_i^2.
\]
After substituting for \(z_i\) and \(z_j\) and simplifying these inequalities become

\[
\rho_j \left( \frac{\beta_1^2 + (\beta_1 - \beta_0) (11\beta_0 - 2\beta_1)}{5(\beta_1 - \beta_0)} \right) > \rho_i (\beta_0 + \beta_1),
\]

\[
\rho_i \left( \frac{\beta_0^2 + (\beta_1 - \beta_0) (2\beta_0 - 11\beta_1)}{5(\beta_1 - \beta_0)} \right) > -\rho_j (\beta_0 + \beta_1).
\]

Writing the inequalities in this form shows that they will always be satisfied for fixed \(\rho_i, \rho_j\) provided \((\beta_1 - \beta_0)\) is sufficiently small. Moreover for fixed \(\beta_0, \beta_1\) provided \(11\beta_0 > 2\beta_1\) they will be satisfied by making \(\rho_j\) sufficiently large compared with \(\rho_i\).

For example when \(\rho_i = 2, \rho_j = 3\) and we fix the average slope \((\beta_0 + \beta_1)/2 = 2\), we have \(\beta_1 = 4 - \beta_0\) and so we need:

\[
3 \left( (4 - \beta_0)^2 + (4 - 2\beta_0) (15\beta_0 - 8) \right) > 40(4 - 2\beta_0),
\]

\[
2 \left( \beta_0^2 + (4 - 2\beta_0) (24\beta_0 - 44) \right) > -60(4 - 2\beta_0).
\]

These inequalities are both satisfied for \(2 > \beta_0 > 1.1095\)

We can illustrate what happens when the uncertainty in \(\beta\) values is too large by taking \(\beta_0 = 1, \beta_1 = 3\) (retaining the values \(\rho_i = 2, \rho_j = 3\)). Then the solution is

\[
z_i = \frac{2}{5}, y_i = \frac{38}{75}, z_j = \frac{4}{15}, y_j = \frac{22}{75}.
\]

The slope of the line joining these points is \(8/5\) and this gives a probability of \(i\) being chosen of 0.7. In equilibrium the profit made by \(i\) is

\[
\Pi_i = 0.7 \left( y_i - 2z_i^2 \right) = 0.13067.
\]

However if \(i\) undercuts \(j\) at \(y_i = (22/75) - \varepsilon\) and \(z_i = 4/15\) then player \(i\) can obtain a profit of

\[
\Pi_i = \left( \frac{22}{75} - 2 \left( \frac{4}{15} \right)^2 \right) - \varepsilon = 0.15111 - \varepsilon.
\]

So the solution (23) is not a true equilibrium.

4 Conclusion

In this paper, we consider a supplier bidding situation where each supplier bids on the price and a number of non-price variables and the buyer will then choose just one supplier. Assume the buyer has a linear utility function involving scores of each supplier on bidding attributes and weights on these attributes. The suppliers do not know the precise scores they will be given on bidding attributes; they may also not know the
weights that will be used by the buyer. Both types of uncertainty are considered. In our first model, there is uncertainty about how a supplier’s bid will be scored. Under the discrete choice framework, we reveal that it is optimal for each supplier to bid the non-price variables in a way that is independent of the competitive situation. We can say that supplier competition occurs only through prices, and non-price variables should be bid at the values that would be chosen if there were no other bidders involved. This result holds with remarkable generality, independent of the distribution of the uncertainty and only requiring cost functions to be convex.

We have also shown that this result will hold when there is a two stage equilibrium with non-price variables determined first and then prices set in a second stage. This model applies when considering vertical differentiation (on quality variables) in a product market where consumers have the same willingness-to-pay for quality but there is some additive random variation in utility (for example this occurs in a classic multinomial logit model).

In our second model, there is uncertainty in relation to the buyer’s weights for bidding attributes. In this case our analysis is restricted to the case with two suppliers and a single quality variable. We show that there may not exist any equilibria, but that where there is an equilibria the quality levels chosen will be lower than apply in the score-uncertainty model with weights set to their median values. Since the score-uncertainty case gives quality levels that are chain optimal (given the final choice of supplier) this result suggests that there is an additional welfare loss from weight-uncertainty in comparison with score-uncertainty.

Of course we would expect in practice to observe both weight and score uncertainty. From a discrete choice perspective this is equivalent to the use of a Mixed Multinomial Logit (MMNL) model. By pulling apart these two aspects of the uncertainty we have been able to give analytic solutions in important cases. Taking account of both forms of uncertainty in a single model ramps up the complexity very substantially, and it will be necessary to apply numerical techniques to find solutions in specific cases.

For particular examples with quadratic cost functions we have explored equilibrium solutions in more detail. For these examples we find that (with both weight-uncertainty and score-uncertainty) smaller uncertainty generally leads to lower prices; implying lower profits for the suppliers and a higher profit for the buyer. This observation resonates with previous finding that uncertainty (on rivals’ costs) makes the equilibrium (of Bertrand competition) more competitive (Spulber 1995). This provides an additional argument for transparency in the bid process from the buyer’s perspective: increased transparency will reduce the degree of uncertainty experienced by the bidders, which in most cases leads to lower prices and more profit for the buyer.

References


