The Application of Doppler H$_{\alpha}$ Spectroscopy to the Prediction of Experimental Fusion Rates in a deuterium-filled Inertial Electrostatic Confinement device

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Declaration of originality

I certify that the work presented in this thesis was undertaken solely during my PhD candidature, and has not been presented for any other degree.

I certify also that this thesis was written by myself, and that all external contributions and sources have been duly acknowledged.

Signature of candidate:

........................................

John Kipritidis
Acknowledgements

As I sit in this humble student office, it occurs to me that some things in life and physics never turn out quite as you’d expect.

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Abstract

Inertial Electrostatic Confinement (IEC) aims to produce nuclear fusion reactions by accelerating ions of a deuterium plasma through concentric gridded electrodes such that they collide in the centre. Despite the existence of collimated beams of brilliant optical emission emerging from the apertures of the inner electrode (cathode), there has been surprisingly little emphasis on the application of optical diagnostics in characterizing the neutron production in a deuterium-filled device. This work focuses on the development, validation and limitations of such a diagnostic for a steady-state IEC device operating in a gaseous discharge of hydrogen, which is chemically identical to the deuterium discharge and avoids some of the radiation concerns associated with neutron production.

This work began under the auspices of two seemingly opposed models for IEC devices operating as a gaseous discharge. The more conventional model of a 'convergent ion focus' would imply that emission channels arise primarily due to ions undergoing charge-exchange with the gas as they accelerate towards and through the cathode. The other model, based on more recent results using Doppler spectroscopy in hydrogen, suggests that the dominant mass flux inside the beams is that of energetic neutral atoms emerging from the cathode apertures after having been produced by ions accelerating away from a region of positive space charge (or 'virtual anode') within the cathode center. Moreover, earlier observations of electron deflection under a magnetic field suggested that these beams of ions and neutrals exist collinear to a beam of electrons which are highly monoenergetic. For the case of the cylindrically symmetric discharge arising from an open-ended and cylindrical hollow cathode, this alternate model was deemed the 'Abnormal Hollow Cathode' (AHC) discharge in analogy to the hollow cathode effect observed in spectral lamps.

We employ the AHC model to develop an optical diagnostic for the absolute densities of energetic H ion species emerging from the cathode apertures of a cylindrically symmetric, steady-state IEC discharge in H2 for cathode voltages of units and tens of kV and units and tens of mTorr pressures. The method combines results from earlier collisional-radiative (CR) models, comparing intensity of Balmer H_α due to excitation of H_2 by 'fast' (units and tens of keV) electrons to Doppler shifted emission arising from charge exchange of fast ions. The method is advantageous as it can be applied to a single spectrum of H_α using plasma parameters (pressure, current and voltage) which are all readily obtained. Our optical diagnostic is initially applied to a preexisting IEC setup producing a relatively low energy DC discharge of voltage ~5 kV, pressure 20 mTorr and currents 10 – 30 mA. Refitting the electrode arrangement to withstand fusion-relevant voltages (tens of kV), we then correlate predicted and measured neutron rates for separate conditions of constant voltage (~30 kV) and pressure (~ 5 mTorr). Predictions capture the
variation of experimental fusion rates with DC current in the range $10 - 50$ mA and agree with measured values to within an order of magnitude, validating the optical diagnostics.

In the interests of validating the underlying AHC model, the optically determined ion densities are used as inputs for the simulation of electrostatic potential, Doppler spectra and average particle energies and densities along the beam axis for a steady-state discharge in H$_2$. Our semi-analytic, intuitive solution method couples a matrix representation of H$_3^+$ production by electron ionization to that of ion charge exchange with the background gas, and is largely free of the numerical noise associated with more conventional Monte Carlo modeling approaches. The calculated potential profiles support the existence of a positive space charge within the cathode interior, with simulated Doppler spectra replicating the observed increase of fast neutral densities away from the cathode center. For the steady-state discharge we calculate electron energy distributions which support earlier observations of highly energetic electron beams, and we cast new light on the origin of the central peak emission in that the electron contribution is only 10% of the anticipated value. In fact, our simulations suggest that the central peak arises primarily due to excitation of the gas instead. Even more crucially, calculations yield mass fluxes for inflowing and outgoing ions which are approximately equal, but also smaller than those for fast neutrals produced within and emerging from the cathode. This demonstrates an unexpected compatibility between the conventional IEC model of a convergent ion focus, and the AHC model of a ‘divergent neutral flux.’

Finally, we detail the construction of a pulsed power supply and present Doppler H$_\alpha$ spectra measured for the cylindrical discharge operating in a pulsed regime with large peak currents ($0.1 - 10$ A), peak voltages ($32 \pm 1$ kV) and gas pressures ($10 - 150$ mTorr in H$_2$). We analyze the spectrum asymmetries to show that trajectories for fast neutrals in the beam are analogous to those from the low current, steady state discharge. One important difference between the regimes is that the pulsed spectra yield neutral energies which appear smaller than expected, but we demonstrate that this effect can be modeled as an accumulation of energetic, steady-state Doppler spectra over the measured time-variation of current and voltage. Measured neutron production rates for the pulsed system in D$_2$ confirm the existence of highly energetic neutrals, suggesting that between the steady-state and pulsed systems, the same physics ultimately holds. We thus conclude the thesis by discussing the challenges and opportunities for Doppler spectroscopy on pulsed IEC.
Publications by the Author

Refereed articles


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Chapter 1

Introduction

1.1 Inertial Electrostatic Confinement (IEC) Fusion

A nuclear fusion reaction occurs when two light colliding nuclei are energetic enough to overcome their mutual electrostatic repulsion, producing an unstable composite nucleus which decays into smaller fragments and releases radiation by-products such as gamma rays and neutrons. The Farnsworth fusor [1] aims to produce controlled nuclear fusion reactions through confinement of charged particles in a deep electrostatic potential well. A series of external plasma sources or ‘ion guns’ inject positively charged ions into a region of moderate (sub mTorr) vacuum between two spherically concentric gridded electrodes, with the outer electrode a grounded anode and the inner electrode a negatively biased cathode.

The idealized picture of a ‘convergent ion focus’ is represented graphically in Fig. 1.1a and considers fusion reactions between populations of monoenergetic nuclei converging towards the cathode center, with ions presumed to oscillate back and forth throughout the cathode and interelectrode space until a

Figure 1.1: Schematic diagrams for three IEC schemes featuring cathodes for the inner electrodes.
reaction occurs. Ion bombardment of the cathode is also expected to produce a low energy plasma of the background gas through secondary electron emission, acting as an additional fusion target for energetic ions. This general picture has been termed Inertial Electrostatic Confinement (IEC) fusion.

Most IEC research has focused on the deuterium isotope due in part to its seemingly inextinguishable abundance; deuterium gas may be produced readily from heavy water using an electrolytic cell. Two deuterons may undergo nuclear fusion through the reactions,

\[
\begin{align*}
D_{\text{fast}} + D_{\text{slow}} & \rightarrow \text{He} + n \\
D_{\text{fast}} + D_{\text{slow}} & \rightarrow T + p^+
\end{align*}
\]

\(1.1\) \(1.2\)

of which approximately half produce a neutron of energy \(\sim 2.45\) MeV and a helium nucleus, with the other half producing a proton and tritium. The interaction cross section \(\sigma_{D-D}\) for D-D fusion increases like the square of projectile energy in the range \(20 - 100\) keV (see Fig. 1.2), reaching a maximum of order \(10^{-29}\) m\(^2\) for projectiles of energy around \(2\) MeV [2]. For any given cathode voltage, the cross section is thus maximized for collisions which are between two energetic or 'fast' nuclei accelerated to the full applied cathode potential.

Robert Hirsch [3] conducted experiments with the Farnsworth fusor operating at cathode voltages as large as \(-150\) kV, yielding neutron production rates of order \(10^7\) s\(^{-1}\) for a device filled with deuterium gas at the relatively low pressure of \(0.1\) mTorr. Hirsch’s results show that an increase in \(D_2\) pressure from \(0.1 - 10\) mTorr is actually an impediment to neutron production in this device, with the maximum production rates decreasing by an order of magnitude. Given the implicit requirements for low power input and small vacuum systems, as well as the difficulties inherent in maintaining and aligning external ion sources, Hirsch introduced a simplified IEC scheme wherein the primary source of ions was now the gas itself, via a low energy discharge produced outside the anode [4]. The implementation used in most present-day IEC research relies on the formation of a gaseous discharge mode between the anode and cathode, as shown in Fig. 1.1b. Here electrons may ionize molecules of the background gas ‘g’ while accelerating from the cathode to the anode, with the resulting ions accelerating back towards the cathode. Many IEC studies are thus carried out at units and tens of mTorr gas pressures where increasing the pressure is equivalent to increasing the density both of fusion targets, and source ions.

A gridded IEC operating in the gaseous discharge mode at units and tens of mTorr pressures is characterized by collimated beams of brilliant optical emission which arise due to excited particles radiating spontaneously to lower energy levels. These beams ('emission channels') pass through the cathode
apertures, intersecting in the cathode center where the emission is most intense. Figure 1.3a shows emission channels emerging from a gridded cathode for a discharge of hydrogen. Here the strongest optical emission is centered around the Balmer $H_\alpha$ line (which has an unshifted wavelength $\lambda = 6563$ Å) corresponding to the atomic transition $H(n = 3 \rightarrow 2)$ with principal quantum number $n$). A complexity with $H_\alpha$ spectroscopy on gridded cathodes is that analysis of any one emission channel is usually confounded by contributions from nearby channels. In this work the cathode grid is replaced by an open-ended hollow cathode cylinder as shown in schematic form in Fig. 1.1c. Figure 1.3b contains an example of emission channels arising from a cylindrically symmetric, 'bi-conical' hollow cathode used in earlier studies at the School of Physics (see Sec. 1.2.1).

IEC devices are not yet capable of power generation as initially intended by Farnsworth. Nevertheless the controlled neutron fluxes provided by IEC are of interest in commercial applications such as medical isotope production and as a sub-critical neutron source for fission reactors. In particular, there has been a concerted and successful program to implement a compact-IEC fusion discharge as an advanced humanitarian landmine detection system [5]. Accurate determination of neutron production rates, and the associated fast ion and neutral densities is thus desirable.

Historically, the characterization of energetic particle densities in terms of operating parameters and electrode design involved measuring neutron production rates directly (for example, in this work through the scintillation-detection of nuclear fission in a Li-1 crystal placed near the IEC device). Given the propensity towards optical emission in the gaseous discharge mode, much of this thesis focuses either
directly or indirectly on relating the observed emission to absolute particle densities and neutron production rates through a non-perturbing optical diagnostic. To that end, the present study models emission primarily for a hydrogen discharge; the hydrogen atom is chemically equivalent to the deuteron and so we expect a hydrogen-filled IEC to exhibit similar emissive characteristics as for a deuterium filled device but without some of the radiation shielding requirements associated with large neutron fluxes. This may be particularly advantageous for studies of IEC devices operating in a pulsed regime with tens of keV peak voltages and peak currents as large as 100 Amperes [6].

1.2 Conventional IEC dynamics

1.2.1 Typical Doppler spectra

Figure 1.4 depicts a typical $H_{\alpha}$ lineshape viewed parallel to the discharge axis through the center of the cathode of Fig. 1.1c. The spectrum consists of a Lorentzian peak (with an intensity maximum at wavelength $\lambda_0 \sim 6563$ Å) flanked by Doppler-shifted wings. Studies of $H_{\alpha}$ emission in $0.1 - 4$ Torr hollow cathode glow discharges [7, 8] suggest that the central peak is associated with the dissociative excitation of $H_2$ by fast electrons which are thought to travel along the discharge axis from the cathode aperture to the anode wall. This produces excited $H(n, l)$ through the reaction

$$e^- + H_2 \rightarrow e^- + H_2^* \rightarrow e^- + H(1, s) + H(n, l),$$

(1.3)

with $n$ and $l$ denoting principal and orbital quantum numbers. The primary de-excitation mechanism is
spontaneous emission, wherein the kinetic energy $K$ of an emitting neutral is related to the Doppler shift $\Delta \lambda$ through

$$K = \frac{mc^2(\Delta \lambda)^2}{2\lambda^2\cos^2\theta},$$

where $m$ is the particle mass, $c$ is the speed of light and $\theta$ is the angle between the particle trajectory and observation axis. Without applying a de-convolution procedure, the central peak typically has an apparent FWHM (full-width at half-maximum) in the range $0.5 - 1 \text{ Å} (K \leq 10 \text{ eV});$ here we say dissociative excitation produces ‘slow’ H.

The far wings on the other hand generally have Doppler shifts $\Delta \lambda \geq 1 \text{ Å}.$ Investigations of H$_{\alpha}$ emission at $10 - 100 \text{ mTorr}$ for parallel-plate RF [9] and DC [10] discharges suggest that these Gaussian-like peaks arise through the charge exchange of fast ion species H$_x^+$ ($x = 1 - 3$) with the background gas.
\[ \begin{align*}
H^+ + H_2 & \rightarrow H^*(n, l) + H_2^+ \quad \text{(1.5)} \\
H_2^+ + H_2 & \rightarrow H^*(n, l) + H^+ + H_2^+ \quad \text{(1.6)} \\
H_3^+ + H_2 & \rightarrow H^*(n, l) + H_2^+ + H_2^+. \quad \text{(1.7)}
\end{align*} \]

where each fast neutral product (denoted by \( \cdot^\ast \)) possesses same trajectory as the incident ion and a fraction \( 1/x \) of its incident kinetic energy \([11]\). By measuring the linear increase of apparent ion energies with applied cathode voltage, Khachan \textit{et al.} \([12]\) showed that these charge exchange processes also make a significant contribution to the energetic Doppler peaks observed in the emission channels of an IEC device. In fact, it is thought that the Doppler shifted wings are each comprised of three smaller peaks arising from neutrals produced in the three charge exchange processes. It follows that the Doppler peak positions may be used to characterize the energies and trajectories of incident ions, with a decrease in emission wavelength or 'blue shift' arising from ions headed towards the detector, and a 'red-shift' indicating ions traveling away. In terms of the conventional IEC model these Doppler shifted peaks are attributed to radially infalling ions, and this interpretation is supported by the relative lack of energetic Doppler peaks in spectra observed at right angles to the emission channels; this implies that fast ions do not have a significant component of motion in a direction perpendicular to the emission channels.

It is understood that charge exchange, which is the dominant ion-gas interaction for units and tens of keV energies and units and tens of mTorr pressures, will limit both the confinement time and maximum energy attained by accelerating ions. In Ref. \([12]\) as well as other spectroscopic studies at the School of Physics, IEC arrangements of the form in Figs. 1.1b and 1.1c lead to maximum ion energies of about 20 – 50% of the applied cathode potential.

1.2.2 The electron beam

The assumption of an electron-induced central peak (see Fig. 1.4) was bolstered by magnetic deflection of the energetic beam of a similar setup in Ref. \([13]\). Here the energies of particles at the anode were measured directly using a Helmholtz coil providing a magnetic field oriented perpendicular to the undeflected beam axis as in Fig. 1.5.

This work tracked the displacement of the leftmost and rightmost edges of the resulting disk-like glow (of radius 0.5 – 1 cm) on a phosphorous screen and compared this to the deflection for a beam of known energy and particle mass. The deflected beam was identified as an electron beam with kinetic energies of order the applied cathode potential, which to a good approximation have an energy spread smaller
than 10% for cathode voltages in the units of kV range. For fast electrons in the interelectrode space, we therefore tend to assume an electron energy distribution function (EEDF) which is monoenergetic.

1.3 Challenging the established IEC dynamics

1.3.1 Doppler spectroscopy and the divergent neutral flux

The spectroscopic studies of Shrier et al. [14] challenge the notion that energetic nuclei in an IEC device are confined at all. For the same apparatus as in Ref. [12] and with the bi-conical cathode geometry from Fig. 1.3b, Fig. 1.6 shows the emission channel spectra at various distances outside the cathode aperture. Spectra were observed for a cathode voltage of -7 kV and pressure 15 mTorr for H₂, at an angle \( \theta = 30^\circ \) to the beam axis with the detector slit facing towards the cathode.

The spectra are dominated by energetic, blue-shifted emission suggesting neutrals in the emission channels are primarily directed away from the cathode apertures at all points along the beam. The electric field points towards the cathode and so the incident ions cannot have been produced in the nearest interelectrode space. The intensity and positions of the blue shifted peaks are also unaffected by changes to the cathode-anode separation on the opposite side of the cathode, implying that these neutrals were not produced in the opposite interelectrode region. These observations are supported by the behaviour of the red-shifted emission, for which the intensity and peak Doppler position decrease rapidly with increasing distance from the cathode aperture. The authors also considered the possibility that diverging neutrals were produced within the cathode by ions which had accelerated towards this region from outside, but
Figure 1.6: H$_3^-$ spectra observed along the beam axis, at various distances beyond the aperture of the bi-conical cathode operating at -7 kV and 15 mTorr in H$_2$. A hemispherical mesh anode is placed at 16 cm. (Figure reproduced from Ref. [14]).

This argument was eliminated on the grounds that incident energies for charge-exchanging ions appear to increase away from the cathode center (see the data of Fig. 1.7, measured using a similar electrode arrangement in Ref. [15]).

It is worth noting the striking similarity between these observations and those of Adamov et al. [16], who recorded energetic Doppler shifts normal to the interior wall of a 1 – 2 Torr hollow cathode. It was determined that these neutrals arose primarily through the neutralization and fragmentation of incident H ions accelerating towards the wall, but also that this ‘backscattering’ of ions does not contribute significantly to emission observed parallel to the wall surface (parallel to the cylindrical axis of the cathode). It is thus suspected that the ‘divergent’ ion/neutral motion inferred in the IEC experiments of Refs. [14] and [15] must arise through some different mechanism.

Rather, Ref. [15] linked the optically-determined ion energies of Fig. 1.7 with Langmuir probe measurements of a positive space charge or parabolic-like ‘virtual anode’ within the center of a gridded spherical cathode similar to that seen in Fig. 1.3a. These measurements are reproduced in Fig. 1.8. The conclusion from this series (Refs. [12, 14, 15]) of spectroscopic studies was that most ions of a units and tens of mTorr IEC discharge are created predominantly within the cathode, and accelerate away from a region of positive space charge within the cathode center. In fact, the implication here of large diverging neutral mass fluxes has been cited in possible space-thruster applications [17].
Figure 1.7: Ion energies inferred by Doppler spectroscopy (reproduced from Ref. [15]). The vertical line indicates the edge of a two ring cathode.

Figure 1.8: Langmuir probe measurements of a virtual anode inside an IEC cathode (reproduced from Ref. [15]). The vertical line indicates the radial extent of the gridded cathode. The solid curve indicates a vacuum solution of the electrostatic potential.
This is consistent with the analysis of motion of micron-sized impurity or 'dust' particles falling vertically through a similar electrode arrangement [18], as well as collimated proton detection [19] suggesting that the fusion process (1.2) occurs due to neutrals interacting with the background gas at all points along the energetic beams. Moreover, evidence of the virtual anode has been observed using laser-induced fluorescence (LIF) of a gridded IEC discharge operating in a discharge of helium [20].

1.3.2 The ‘Abnormal Hollow Cathode’ (AHC) model for the IEC discharge

The existence of a virtual anode and predominantly ‘divergent’ ion and neutral fluxes are fundamental assumptions for what we have termed the Abnormal Hollow Cathode (AHC) model of the IEC discharge [21]. In the steady-state AHC model the discharge is sustained by ionization of the background gas by electrons,

\[ \text{e}^- + \text{H}_2 \rightarrow \text{e}^- + \text{H}_2^+ + \text{e}_{sec}, \]  

which originate at the cathode interior walls due to secondary electron emission arising from ion and neutral bombardment of the surface. Ions are assumed to accelerate away from the central positive space charge region, undergoing charge exchange and producing fast excited H in the vicinity of the cathode aperture where electric potential is a minimum and ion energies are at a maximum. The assumed axial potential profile is shown in Fig. 1.9 and resembles a double-parabolic well. The inferred energies of incident ions using Doppler spectroscopy are taken to be indicative of the maximum potential height of the virtual anode, of order 50% of the applied cathode potential.

In justifying our terminology, we note that the cathode geometry is reminiscent of hollow cathode spectral lamps, and that the voltage-current relation typically resembles the ‘abnormal glow’ between plane parallel electrodes. Here voltage increases with the logarithm of cathode current, which exceeds the product of cathode surface area and normal glow current density. This is consistent with the observation that it is the intensity of the emission channels (as opposed to their radii) which undergoes the most marked increase over the range of steady state currents (1 - 50 mA in our work). The distinct lack of ion confinement in the AHC model seems at odds with the IEC moniker and so in areas of this Thesis where the AHC model is assumed, we will typically refer to the discharge as an AHC discharge (‘AHCD’).

A difficulty with the model is that unlike the traditional hollow cathode effect, the AHCD operates at pressures at least an order of magnitude lower and so the mean free path for ionization of the gas by tens of keV electrons is generally larger than the length scale of the cathode (units of cm). As such the cathode interior is no longer a region of intrinsically efficient ionization, particularly as any ionization
event outside the cathode will produce a slow 'secondary' electron ($e_{sec}$ in process (1.8)) which itself may lead to further ionization outside the cathode. The mean free path for charge exchanging ions (of order the cathode length for energies $\sim$ 15 keV) is also problematic in that the minimum potential of Fig. 1.9 comprises a comparatively smaller region and so the majority of diverging ions will not undergo charge exchange before decelerating. Finally, it is somewhat paradoxical that the virtual anode is itself thought to arise due to ion bombardment of the cathode wall as this implies a crucial role for in-falling ions also. Indeed, one of the aims in developing an optical diagnostic for fast ion densities was to assist in the numerical modeling of the (possibly complex) interplay between 'converging' and 'diverging' ion species.

1.4 Established models for $H_\alpha$ emission, particle densities and potential profiles in an IEC device

1.4.1 Collisional-Radiative (CR) modeling in IEC and other hydrogen discharges

In conjunction with emission spectroscopy, collisional-radiative (CR) modeling is a useful tool in the development and application of non-perturbing plasma diagnostics which allow characterization of im-
important discharge parameters such as average energies and relative densities for electron, ion and neutral species. CR modeling usually involves the solution of coupled equations governing the population of energy levels dominated by collisional and radiative transitions. The time-dependent equations have the form [22],

$$\frac{dn_i(t)}{dt} = \sum_{j \neq i} (A_{i,j}n_i + B_{j,i}n_j + C_{i,j}n_i n_j), \quad (1.9)$$

where $n_i$ is the population density of a particular energy level $i$ at time $t$, and usually refers to either the ground or excited states of an atomic or molecular species with electron and ion populations often included as separate levels in the equation system. The Einstein coefficients $A_{i,j}$ and $B_{j,i}$ have units of $s^{-1}$ and describe the change in population of an energy-level due to radiative decay from levels $i$ to $j$ and absorption from $j$ to $i$ respectively. Coefficient $C$ has units of $m^3 s^{-1}$ and includes processes such as electron-atom and atom-atom interactions. For a steady-state plasma, the energy level populations are constant and so calculations may be simplified by setting the time-dependent left hand side (LHS) of Eq. (1.9) to zero. Such an equation system relates a population density $n_i$ to the emission intensity per unit volume, $I_{i,j} \propto \nu_{ij} A_{i,j} n_i$, where the transition $i \rightarrow j$ leads to radiation of frequency $\nu_{ij}$ (if the transition is optically allowed). This yields the prediction of either densities or emission intensities, given measurements of intensities or densities respectively.

In Ref. [13] the present author constructed a steady-state CR model for the $H_\alpha$ central peak in the emission channels of a hydrogen-filled IEC device at units and tens of mTorr pressures and assuming an EEDF which is monoenergetic for cathode voltages in the units of keV energy range. The goal of this work was to calculate the energy dependence of $H_\alpha$ and $H_\beta$ central peak intensity ratios towards the development of a diagnostic for electron energies (or equivalently, electric potential) along the beam. Prospects for a diagnostic based solely on the peak ratios are limited however, as the theoretical variation of peak ratios over the units of keV range is small enough ($\sim 20\%$) so as to be indistinguishable from experimental noise.

A more enduring outcome of Ref. [13] was experimental support for the assumption of an electron-induced central peak. Specifically, $H_\alpha$ emission with $\Delta \lambda < 0.3$ Å was reasoned to arise primarily through dissociative excitation of molecular $H_2$ by fast electrons,

$$e^- + H_2 \rightarrow e^- + H_2^+ \rightarrow e^- + H(1s) + H(n = 3), \quad (1.10)$$

in accordance with the aforementioned studies of higher pressure hydrogen discharges [7, 8, 9, 10, 16].
Figure 1.10: Theoretical and measured intensities of $H_\alpha$ and $H_\beta$ observed perpendicular to the emission channel assuming process (1.10) as the only excitation process. Figure appropriated from data collected by the author in Ref. [13].
ergy where the measured values (solid data points) assume a vacuum solution of the electric potential along the electron beam, and theoretical curves (which have both been scaled by the same factor) assume excitation of the H(n = 3) level by process (1.10) only. In general the agreement is quite good, with discrepancies smaller than 50% except for electron energies less than 1 - 2 keV. For these lower energies the discrepancy was explained in terms of the experimental EEDF becoming smeared (no longer monoenergetic) in the region near the cathode apertures where the low energy results were obtained.

The charge exchange process has also been the subject of a CR model for Doppler shifted Hα in a hydrogen-filled IEC. Fitzgerald et al. [23] related the intensities of Doppler peaks arising from fast H\(^+\), H\(_2^+\) and H\(_3^+\) to their relative population densities (pictured in Fig. 1.11). From Fig. 1.11 we generally take that fast H\(_2^+\) and H\(_3^+\) dominate the population of energetic ions and have roughly equal number densities for units and tens of mTorr pressures.

It is worth noting that prior to the work in this thesis, nowhere had a CR model treated the ratio of shifted-to-unshifted emission in a single spectrum of H\(_\alpha\) in terms of the absolute densities of particles responsible for the distinct emission processes. Here we relate the CR models for central and Doppler peak emission in order to develop an optical method for measuring absolute densities of energetic hydrogen ion species emerging from the cathode.
1.4.2 Charge-exchange modeling of the energetic Doppler spectrum

Shrier et al. [24] used a computationally efficient and noise-free matrix representation of the charge exchange process (1.6) as an alternative to a Monte Carlo approach. The method is semi-analytic in that experimental and energy-dependent charge exchange cross sections are used to calculate the probability of ion charge exchange for a source distribution of arbitrary number density as a function of their starting position in a given one dimensional potential field profile. This was used to calculate the average energies of H$_2^+$ accelerating down a linear potential in a 20 mTorr background gas of H$_2$. The average energies of H$_2^+$ versus distance from a structureless anode are shown in Fig. 1.12, given different cathode voltages at the rightmost edge of the system. Results showed good agreement with Doppler shift measurements for ions emerging from the cathode of an AHC discharge under similar conditions, and were taken as further evidence of a divergent ion/neutral flux.

In this Thesis we expand the earlier charge-exchange modeling to include electrons as well, linking a matrix representation of the ionization and total charge exchange processes with optically-determined fast ion densities and the Poisson equation,
\[ \nabla^2 V(z) = -\frac{\rho}{\varepsilon_0}, \quad (1.11) \]

where potential $V$ and total charge density $\rho$ are calculated along the cylindrical beam axis a distance $z$ from the cathode center. As such, this method yields the self-consistent simulation of absolute ion and electron densities, average particle energies, axial potential field, absolute mass fluxes, neutron production rates and Doppler emission spectra.

### 1.4.3 Analytic and Particle-in-Cell (PIC) modeling of the IEC discharge

Most analytic modeling of the IEC discharge has focused on the shape of the potential profile. This is typically achieved by solving Eq. (1.11) in the given form assuming spherical symmetry and collisionless injection of radially moving, monoenergetic charged particle species. See for example the steady-state solutions of Refs. [3] and [25]. Later papers extend this analysis to highlight the importance of finite angular momenta to the stability and shape of calculated potential profiles for the cases of electron-injection only [26], and injection of both ions and electrons [27]. We note that these treatments generally ignore ion, atomic and electron interactions with the background gas, but are nevertheless able to demonstrate the existence of virtual anodes or adjacent virtual anodes and cathodes.

Other analytic treatments [28, 29] critique the power balance between fusion output and input power required to maintain monoenergetic populations of injected charged particles. These papers include gas collisions in the analysis of thermalization of initially monoenergetic particle energy distributions, and discount the possibility for IEC-based power generation due to an array of possible energy losses over ion-ion collision timescales (such as Bremsstrahlung radiation and collisions with the cathode grid). It is striking that these detailed treatments still neglect the role of ion charge exchange, which is likely to remain the foremost impediment to ion confinement in the majority of practical IEC devices, even those operating in an ion injection mode at sub-mTorr pressures.

In fact, aside from the charge exchange modeling of Shrier et al. [24], only Baxter and Stuart [30] present a fully analytic treatment of ion charge exchange as it pertains specifically to IEC. This work portrays accurately the charge exchange of ions accelerating through ion guns, highlighting the importance of the (neutral) beam-gas contribution to measured fusion rates in ion-injected IEC devices. However here the effects of space charge are ignored and so the calculations do not incorporate the effect of charge exchange on the accelerating fields either within the ion guns, or the IEC chamber itself. The work of Sigeneger and Winkler [31] coupled an analytic solution of the Poisson equation with the electron-Boltzmann equation for a high pressure (units of Torr) hollow cathode arrangement in
helium, however this work did not explicitly include ion charge-exchange or model emission from excited energetic atoms.

The most widely employed method for modeling plasmas with the inclusion of atomic processes is called particle-in-cell (PIC) simulation (see, for example, Birdsall’s extensive review [32]). PIC modeling supposes that a plasma may be modeled by a number of particles (electrons, ions and neutrals) which is many orders of magnitude smaller than in a real plasma. Each ‘superparticle’ of this smaller ensemble represents the total charge and mass for as many as $10^6$ real particles. The aim is to solve a finite-element form of Eq. (1.11) through an iterative-process where each iteration represents a given simulation time step (typically of order ns or smaller). By weighting superparticle charges to a spatial grid of sufficient resolution, one can interpolate electric fields and thus the forces acting on superparticles at each grid position. Moreover, in any given timestep, each superparticle has a velocity-dependent probability for undergoing any number of collisions including ionization, excitation, charge exchange or surface interaction. Superparticles will undergo or avoid such interactions through the assignment of a random number (the ‘Monte Carlo’ process).

Clearly PIC models have the advantage of being able to incorporate almost every conceivable atomic interaction which may affect collective behaviour of a plasma. The method can be applied to complicated 3-D geometries which is also advantageous. Where the method typically falters is in accurately reproducing microscopic dynamics of a plasma, particularly small scale electron interactions like Coulomb scattering. Moreover the calculated potential fields and energy distributions can feature noise or irregularities arising from the reduced collision statistics inherent in a superparticle representation. Special care must also be taken so as not to waste computational power on relatively slow moving particles (such as ions) over the small timesteps required to simulate the motion of fast electrons.

PIC methods do exist for spherically symmetric IEC systems operating in the gaseous discharge mode (see for example the modeling of Noborio et al. [33] which is in accordance with neutron production measurements), however up until recently no PIC models have simulated the energetic Doppler emission. As with the work of Masuda et al. [34] we opted to explore a different and novel approach and so make no further comment on traditional PIC methods here. For a more detailed assessment of PIC modeling of the IEC discharge we point the reader towards the PhD thesis of fellow student Michael Fitzgerald [35].

1.5 Aims of this work and structure of the Thesis

This Thesis focuses on the development, validation and limitations for an optical diagnostic for fast H ion species in a steady-state IEC device operating in a gaseous discharge of hydrogen. It is anticipated
that our diagnostic for the hydrogen-filled IEC will provide a useful means for predicting the effects of different electrode geometries or plasma parameters on fusion rates for similar discharges in deuterium.

Our optical diagnostic implicitly calibrates a spectrometer by relating earlier CR models for central and Doppler-shifted $\text{H}_\alpha$ emission to the relative densities of fast and slow excited $\text{H}$ in the energetic beam. These are subsequently related to absolute densities of fast electrons and ions through known rate coefficients for excitation and charge exchange respectively. In the interests of validating the underlying AHC model, the optically determined ion densities are also used as inputs for the simulation of electrostatic potentials, Doppler spectra and average particle energies and densities along the beam axis for the steady-state discharge in $\text{H}_2$. This semi-analytic, intuitive solution method couples a matrix representation of $\text{H}_2^+$ production by electron ionization to that of ion charge exchange with the background gas.

The experimental electrode geometries and apparatus required for optical spectroscopy as well as pulsed and steady-state plasma generation are discussed in Chapter 2, whereas Chapter 3 delves into the detailed theory of our semi-analytic plasma simulation and optical diagnostic. Chapter 4 then describes application of our optical diagnostic to low-current (steady state) and high current (pulsed) discharges of hydrogen and compares predicted and experimental fusion rates for equivalent discharges of deuterium. In each case we also use optically-determined (in some cases, extrapolated) fast ion densities to compare experimental spectra with simulated Doppler lineshapes and potential fields. The aim here being to determine whether the same physics holds for the pulsed and steady-state gaseous discharge IEC modes.

Finally, we conclude the Thesis in Chapter 5 by summarizing our findings and discussing the challenges and opportunities for Doppler spectroscopy on pulsed IEC.
Chapter 2

Experimental Apparatus

Here we discuss the various apparatus employed over the course of the experimental work. This involved operating a gaseous IEC discharge in three different modes. Chronologically, the different modes studied were a low energy discharge operating at both low voltage and current (1–10 kV at 1–50 mA DC, which we artificially term the 'low voltage' mode), a discharge operating at similar currents but at voltages which were larger and more relevant to neutron production (20–30 kV DC or ‘CW’ mode) and finally a ‘pulsed’ mode operating at those same peak voltages and with relatively large peak currents (1–10 A).

For each type of discharge we conducted experiments involving optical spectroscopy in H₂ and neutron detection in D₂, but between the three discharge types there exist some marked differences in electrode arrangements, spectroscopy setups, and plasma generation apparatus (in particular, construction of the pulsed power supply). Where applicable, we describe these variations between apparatus grouped according to function.

2.1 Electrodes and vacuum chamber layout

The one constant for all electrode arrangements was the anode chamber, a vertically-oriented stainless-steel cylindrical vessel of inner radius 24 cm which has a stainless steel floor and is open at the top. For the low voltage mode, a glass bell jar was positioned atop the vacuum chamber, with the cylindrical axis of the inner electrode arrangement raised up above the maximum height of the stainless steel chamber. This allowed direct viewing of the energetic discharge through the glass wall and is shown in the side-on schematic of Fig. 2.1.

The top-down schematic of Fig. 2.2 depicts the electrode arrangement used for studies of the low voltage discharge. The cathode is a segmented stainless steel cylinder of axial length 4.1 cm and major radius 1 cm, and the emission channel of interest travels normal to the cylindrical axis of the chamber,
Figure 2.1: Side-on schematic of the stainless steel vacuum chamber as it relates to the glass bell jar and discharge observation axis for the low voltage mode. The arrangement of hemispherical and cylindrical electrodes inside the glass bell jar is explained in the text.
Figure 2.2: Top-down schematic diagram of the electrode arrangement used for the low voltage discharge. The detector is situated outside the vacuum chamber.

passing through an anode ring of diameter 3 cm and cylindrical length ~ 1 cm, which stands upon a horizontally-oriented anode plate. The axial separation between edges of the cathode and anode ring is 6 cm, and the entire arrangement can rotate 360° about the vertically-oriented, glass-insulated high voltage ('HV') feed which is ~ 60 cm long and connects to the bottom of the cathode.

We extended the discharge to the opposite side by placing an anode mesh (diameter 16 cm) a distance 4 cm from the cathode. The anode mesh was used so as to maintain a similar electrode geometry to Refs. [14, 23]. In contrast, for the emission channel of interest we used an anode ring in hopes of maximizing collimation of the beam in the region where spectroscopy was being performed. A visual inspection suggested that the discharge was in fact largely unaffected by the presence (or absence) of the anodes which is likely because the vacuum chamber itself was grounded and always presents a large anode surface area to the cathode.

The CW and pulsed modes both used the same electrode arrangement, for which a top-down schematic is shown in Fig. 2.3. The cathode here is similar to the low power case, being a horizontally-oriented, open-ended and stainless steel cylinder of length 4 cm, radius 1 cm and steel thickness 3 mm. This was oriented such that the axis of the highly collimated emission channels (labelled 'beam axis' in the figure) would not impinge on the vacuum port or the spare HV feed, the latter of which was not used in the present work.

For these modes the emphasis was on minimizing exposure of the high-voltage connectors to the
Figure 2.3: Top-down schematic of the electrode arrangement and other apparatus components used in the CW and pulsed fusion modes used in this study. See text for explanation.
plasma thereby minimizing arcing. Figure 2.4 shows the cathodes side-by-side with the most important
difference between the two being their high voltage connection. For the earlier cathode the high voltage
connection was covered with ceramic beads, however this still presented gaps in insulation which for the
later cathode were minimized using a solid ceramic sleeve covering the entirety of the HV feedthrough.
To this end the later cathode sat only 10 cm from the floor of the chamber and about 30 cm beneath a
removable stainless steel x-ray shield replacing the glass bell jar. It was important here to encase all sides
of the vacuum chamber in steel, as cathode voltages of order 30 kV would have been large enough to
produce x-rays able to penetrate the thickness of glass used for the bell jar.

We note also that the lack of segmentation in the later cathode allows for easier fabrication and
appears to have little impact upon beam collimation or lineshape.

2.2 Power supplies and environmental sensors

2.2.1 Steady state modes

For the low voltage and CW modes, a Spellman power supply provided a cathode voltage 1 – 30 kV at
1 – 50 mA (DC) current. Voltage, current and gas pressure were all measured directly - voltage using a
HV probe across the electrodes, current using a gauge on the power supply and absolute pressure using
a Pirani gauge accurate to $10^{-4}$ Torr. Current and voltage were typically observed to vary only 5 – 10%
over 10 – 60 minute intervals, and so the discharge was considered effectively steady-state with respect
to the 30 s timescales of our spectroscopic measurements. Fig. 2.5 shows a photograph labeling the
physical placement of the power supply, vacuum chamber and environmental sensors in our lab.

2.2.2 Pulsed mode

For the pulsed mode we constructed a pulsed power supply consisting of a 0.08 $\mu$F capacitor charged to
30 – 35 kV by a DC power supply drawing units of mA currents through a 4 M$\Omega$ charging resistor. The
negative terminal of the charged capacitor was fed into a 1 – 2 cm spark gap filled with compressed air
at a gauge pressure of $\sim$ 14 PSI (about 720 Torr) to prevent spontaneous breakdown across the gap, with
the opposite terminal of the spark gap connected to the cathode through a ceramic-insulated high voltage
feed. The spark gap was triggered at 2 second intervals using the $\pm$20 kV pulse from a low-current spark
generator, initiated via an amplified TTL (‘transistor to transistor logic’) pulse from the spectrometer
setup at the beginning of each scan. The overall arrangement is pictured in schematic form in Fig. 2.6.

In the following subsections, we cover some of the more viscerally interesting aspects of the pulsed
setup in more detail.
Figure 2.4: Photograph showing the two cathodes (left for the low voltage mode, right for the CW and pulsed modes) and the respective ceramic insulators for their connection to the high-voltage feed. See text for explanation.
Figure 2.5: Photograph showing the external apparatus used for the CW mode.
Figure 2.6: Schematic for exterior apparatus in the pulsed mode, featuring a high voltage pulsed power supply triggered by a spectroscopy setup. See text for details.
Pulsed Charging circuit

The charging circuit of Fig. 2.6 was originally constructed as part of the power supply for a decades-old pulsed laser system. It had since been discarded, quite literally in fact - the entire arrangement was found in a School of Physics dumpster - but in this work was restored and refitted by the author to suit the requirements for pulsed IEC. The circuit is represented diagrammatically in Fig. 2.7.

The input charging voltage $V_c \sim -30\, \text{kV}$ is from the same Spellman DC supply used for the steady-state experiments. This provides a peak charging current $I_c = V_c / R_c$ of order 5 mA, where $R_c = 4\, \text{M}\Omega$ is a charging resistor. The charging time of the capacitor is of order $\tau_c = 5R_cC$ or about 0.5 seconds. One could speed up the resulting pulse frequency (of order 2 Hz), however for this work the chosen value of $R_c$ was also the smallest available charging resistance with breakdown voltage larger than 30 kV. The first arm of the circuit connects the negative high voltage input to a capacitor with capacitance $C = 0.08\, \mu\text{F}$ which can withstand a maximum voltage of 45 kV. The other plate of the capacitor becomes positively charged ensuring it cannot discharge back through the input source. The second arm of the circuit features a spark gap and the vacuum chamber in series. The spark gap is able to conduct upon application of a +20 kV input trigger pulse placed units of mm from the output electrode, which itself is produced using a commercial trigger device responding to an external TTL pulse amplified from +1 to +12 V. This trigger creates a localized arc which ionizes the gas in the spark chamber and allows the capacitor to discharge through the vacuum chamber in the form of a beamed IEC plasma.

Fig. 2.7 also contains an open circuit between the terminals of the capacitor. This is actually a safety circuit held open by an electromagnetic switch. In the event of malfunction elsewhere, removing power to the electromagnet closes the switch allowing rapid and safe discharge of the capacitor. In practice,
the capacitor, spark gap and electromagnet switch were all held within a perspex frame where most connections used high-voltage (up to 20 kV) leads placed in rubber tubing and spaced several cm apart. One may glean an impression of the pulsed power supply in situ from the labeled photograph in Fig. 2.8.

One final safety measure involves placing isolating capacitors of small capacitance (of order 0.1 pF) and large breakdown voltage (about 30 kV) on the trigger outputs, which are not shown in the figure. This minimizes the amount of charge that can inadvertently flow from the charging capacitor $C$ back through the sensitive circuitry of the trigger.

**Appearance of, and typical current-voltage traces for the pulsed IEC plasma**

A typical pulsed IEC discharge (peak voltage -30 kV and current 2 A, pressure 50 mTorr in hydrogen) is shown in Fig. 2.9 at the peak of its intensity. This is essentially just the photographic equivalent of the schematic in Fig. 2.3, looking down through the glass bell jar without an x-ray shield. The pulsed discharge is marked by beam-like emission channels emerging from the cathode apertures, just like those observed for the steady state regime. This in itself is highly indicative that similar physics is taking place, however characterization of even the most fundamental plasma parameters (such as cathode voltage and current) now takes on an additional level of complexity as these quantities vary with time.

For pulsed experiments the time-varying cathode voltage was measured using a high frequency probe, with a typical oscilloscope trace shown in Fig. 2.10. This resembles a second order exponential decay with time constants $\tau_{\text{peak}} = 20 - 30 \mu s$ for the 'peak' part of the pulse and $\tau_{\text{tail}} = 1 - 2$ ms for the low voltage trail-off. We define the peak voltage $V_{\text{peak}}$ as the depth of the peak below the offset of the pre-trigger signal (of order -1 kV in Fig. 2.10), with the error in voltage no more than about 10%. Peak current was measured through the cathode input lead using a fast-response magnetic field coil coupled with an RC integrator circuit constructed by the author. The voltage across the capacitor is shown by the dashed curve of Fig. 2.10 - we calibrated the integrator signal using the known peak currents through a dummy load of 2 kΩ. The conversion from peak integrator signal to peak cathode current is $3.7 \pm 0.3$ A/V. We note that the current trace in the figure is due to the integrator response and does not resemble the actual decay of cathode current which, as viewed using a Fluke probe for higher currents (10 - 20 A) through the dummy load, exhibits similar time constants as for cathode voltage.

We may write that the beamed plasma has some resistance $R_p$; for the peak voltages and currents in this work we infer a plasma resistance of order 10 kΩ during the peak part of the IEC pulse. On occasion however, a beamed IEC plasma would not form and an arc discharge was observed instead. The author suspects this was due to sputtering of cathode material onto the ceramic insulators near the chamber floor, in effect reducing the electrode spacing and producing peak currents at least an order of magnitude
Figure 2.8: Photograph labeling various elements of the pulsed power supply as it appears in the lab.
Figure 2.9: Photograph of a pulsed IEC plasma in hydrogen at the peak of its emission intensity.

Figure 2.10: Sample oscilloscope traces for cathode voltage (solid curve, left axis) and signal across a current integrator (dashed curve, right axis). See text for details.
larger than for the beamed plasma mode. This may imply some maximum electric field strength for which beamed mode operation is possible, above which an arc mode dominates.

2.3 Spectroscopy in hydrogen gas

2.3.1 Lens, mirror and fibre-optic arrangements

Optical spectroscopy on the low voltage discharge was performed using an observation axis passing through the anode ring as per Fig. 2.2. A horizontal-periscope arrangement directed light from any point along the observation axis (exiting the glass bell jar) onto the detector slit of a monochromator. These spectra were recorded at an angle \( \theta = 25^\circ \) to the emission channel at the anode, so as to provide a clear line-of-sight through the anode. In general the angle may be chosen arbitrarily, however by Eq. (1.4) a more acute angle ensures a clearer separation between unshifted and Doppler-shifted peaks. Uncertainty in the lens focal lengths introduced an error \( \sim 1 \text{ cm} \) in the sampling position along the beam.

For the CW mode, optical spectroscopy in hydrogen was performed by sampling light at an angle \( \theta \sim 30^\circ \) to the region where the beam struck the chamber wall, with the observation axis represented by the dashed line in Fig. 2.3. These measurements made use of a back-coated mirror which was affixed to the interior (vacuum) side of a quartz optical port so that light from the wall region was channeled by the mirror down through the port. This arrangement is shown in side-on schematic form in Fig. 2.11. The plane of the mirror was rotatable with its normal vector moving through a hemispherical range of azimuthal and inclination angles. The mirror is aligned by first running the IEC discharge at nominal parameters (25 kV and 30 mA at a pressure 5 mTorr in hydrogen) and creating scorch marks of diameter \( \sim 1 \text{ cm} \) through sheets of aluminium affixed to the anode wall. A photograph of one of these scorch marks is shown in Fig. 2.12, with the position of the sheet relative to the cathode and mirror shown in the top-down photograph of Fig. 2.13. From Fig. 2.9 we actually observe a 'fanning out' or spreading of the visible beam beyond the cathode aperture, at an angle about 15 \(^\circ\) to 20\(^\circ\) to the beam axis. As a result, we expect the radius of the burn pattern in Fig. 2.12 to represent an order of magnitude estimate of the beam radius corresponding to the most highly energetic and undeflected neutral particles.

Subsequent to the mirror alignment procedure, light exiting the optical port was focused by a lens of focal length \( \sim 5 \text{ cm} \) onto an optical fibre of effective width 1.25 mm. The total distance of the imaged region to the centre of the converging lens was about 47 cm (see Fig. 2.11). Light exiting the fibre was channeled onto the slit of a 0.5 m focal length Spex 500M monochromator coupled to a Princeton Applied Research 1421G linear diode array (LDA) which was cooled to 0\(^\circ\) C by a Peltier cooler in order to reduce thermal noise. The resolution of the detection setup was \( \sim 0.56 \text{ Å} \), determined by shining a
Figure 2.11: Side-on schematic of the mirror, lens and fibre optic arrangement used to sample light from the beam.
Figure 2.12: Photograph of a scorch mark of $\sim 1$ cm diameter burned into an Al sheet fixed to the anode wall, following exposure to the energetic neutral beam.
cadmium lamp at the wall and observing FWHMs for the three strongest atomic emission lines in the wavelength range 6300 - 6500 Å. We calculated a wavelength separation $0.16 \pm 0.01$ Å between adjacent pixels of the output as viewed on an Optical Multichannel Analyzer (OMA). Optimal balance between signal strength and resolution was obtained for a monochromator slit width 250 μm. The uncertainty associated with a given peak intensity is of order 10%, measured by determining the standard deviation for a series of emission peak intensities for identical plasma parameters.

Optical spectroscopy for the pulsed mode in hydrogen used this same apparatus, but sampled light from two regions of the chamber. The first was at an angle $\theta \approx 31^\circ$ to the beam where it struck the chamber wall as in Fig. 2.3. The second had the cathode rotated about the vertical axis so that the mirror sampled light at an angle $\theta \approx 40^\circ$ to the beam where it exited the nearest aperture. We note that from the geometry of the figure, sampling light near the wall is equivalent to having the monochromator slit face away from the cathode, whereas sampling light from the nearest aperture is akin to having the slit face towards the cathode. Each pulsed spectrum was the accumulation of $10 \times 100$ ms exposures; the implications of having exposure times larger than the pulse length are discussed in Chapter 4.

In Fig. 2.6 for the pulsed mode we note that the pulsed power supply is in fact triggered by the OMA which outputs a TTL (Transistor to Transistor Logic) pulse coinciding with the start of each spectrometer
scan. This is directed into a pulse generator, used to amplify the TTL pulse from 1 to 12 V for subsequent input into the spark trigger. As the spectrometer is thus electrically coupled to the trigger device and the spark gap, we take special care to ensure electrical isolation of the OMA/SPEX/LDA arrangement from unintended current pulses. We achieved this by employing an optical fibre link between the OMA and pulse generator, in conjunction with an Analog-to-Digital Converter (ADC) and a Digital-to-Analog Converter (DAC) as shown in the figure.

2.4 Detection of neutron production rates in deuterium gas

For measurement of neutron production rates in both CW and pulsed fusion modes, the detector setup consisted of a Li-I crystal, highly enriched with $^6$Li and housed within the center of a high density polyethylene sphere (shown by the shaded circle in Fig. 2.3). The 2.45 MeV neutrons produced in process 1.1 were slowed by the polyethylene sphere leading to the nuclear reaction

$$^6\text{Li} + n \rightarrow \alpha + T$$  \hspace{1cm} (2.1)

where product He nuclei scintillated the crystal producing a pulse of current on a photomultiplier. Output was read on a multichannel analyzer (MCA) allowing selection of the peak corresponding to 5 MeV $\alpha$ particles produced through process (2.1). The Li-I detector, MCA and amplifier for the photomultiplier signal are shown in Fig. 2.14.

In CW mode, neutron detection events were recorded for 1 - 2 minutes for each separate instance of pressure, current and voltage; pulsed mode measurements used a similar procedure except that the goal was to determine the average neutron counts per pulse, measured over 90 - 270 pulses depending on the pressure. In each case, the error was taken to be the statistical error (square root of the number of counts). The arbitrary detection rates were then transformed into absolute neutron production rates by comparing measured counts to those observed at distances 20 - 100 cm from an Am-Be source of known activity ($\sim 10^6$ s$^{-1}$). In our calculations we assumed an average separation of 38 ± 4 cm between the detector and the beam source (cf Fig. 2.3), a distance for which the Am-Be source yielded 1100 ± 200 counts in two minutes.

We note that as the mean free path for D-D fusion is very large (in the order $10^{12}$ m for 15 keV deuterons in a 5 mTorr deuterium gas), the density of energetic nuclei would have been largely constant along the beam axis, with neutrons produced at all points along the beam. As such we also expect some systematic error (15 - 20%) in absolute counts due to the variation of detector distance from the extended source, but this is anticipated to be less significant than the statistical error.
2.5 Magnetic deflection of the electron beam

Finally, we describe a supplementary arrangement which, actually, aimed to determine the contribution of electrons towards the L-I matrix and emission. Thus, we used a magnetic field to deflect the electron beam, which impinged on the Li-I detector situated inside the magnet. The magnet was cylindrical in shape and mounted on a motor, allowing it to rotate. The L-I detector was a photomultiplier amplifier with respect to the vacuum chamber exterior.

Figure 2.14: Photograph showing the arrangement of the Li-I detector, MCA and photomultiplier amplifier with respect to the vacuum chamber exterior.
2.5 Magnetic deflection of the electron beam

Finally, we describe a supplementary investigation which, initially, aimed to determine the contribution of electrons towards the H$_\alpha$ central peak emission. Here we used a 0.5 T magnet to deflect the electron beam, allowing a comparison of distinct impact patterns on sheets of stainless steel placed at the anode wall, and the H$_\alpha$ spectra observed at the anode wall in the deflected/non-deflected cases. The magnet was cylindrical with radius $r_{mag} = 4.5$ cm and length 6 cm, with field strength $B = 0.5$ T at each of its circular faces. The magnet was placed about 5 cm beneath the vacuum vessel (north pole pointing upwards) with its cylindrical axis parallel to that of the chamber. The horizontal position of the magnet is shown by the dashed circle in Fig. 2.3.

The maximum magnetic field strength in the horizontal plane of the beam was measured to be 10 mT using a digital fluxmeter (magnetic field probe), however the field was not uniform across this space and remained as large as 3 mT in the vicinity of the nearest cathode edge and 6 mT next to the anode itself. In contrast, the beam on the opposite side experiences a negligible field strength except at the cathode edge itself ($\sim 2$ mT) and so we treat the electron beam on this side as being undeflected.
3.1 Theory: spectroscopic determination of densities of fast ions emerging from the cathode

The earlier CR models for charge exchange and dissociative excitation are described here in detail. We combine these models for the first time, relating central and Doppler-shifted H\textalpha emission a distance several centimetres away from the cathode edge, proposing an optical measurement for densities of fast ions emerging from the cathode.

3.1.1 CR model for fast excited H produced by charge exchange

In Ref. [23], the CR model for charge exchange resulting in fast atomic H\textit{(n, l)} consisted of the following set of rate equations,

\[
\frac{dn^+_x(n, l)}{dt} = n^+_x n^g k_x(n, l) - \sum_{(n', l') < (n, l)} A_{(n, l) \rightarrow (n', l')} n^+_x(n, l) n^g(n, l) - k^Q(n, l) n^g n^+_x(n, l). \tag{3.1}
\]

Here \( n^+_x(n, l) \) is the number density of fast atomic H in the excited state \((n, l)\) produced by energetic ions \( \text{H}^+_x \) with \( x = 1, 2, 3 \) and density \( n^+_x \). Similarly \( n^g \) is the density of background gas. For fast atomic H(3s) we would therefore have three equations, one for each Doppler shifted peak. Background H\textsubscript{2} is assumed to have temperature \( T = 300 \) K, so that H\textsubscript{2} density may be obtained from the ideal gas law.

In our experimental apparatus a steady-state solution is appropriate and so the left hand side of Eq. (3.1) is set to zero. On the right hand side, the first term gives the contribution of charge-exchanging \( \text{H}^+_x \)
to the population of $H_{fast}^+(n, l)$. For charge exchange the rate coefficient is $k_{CX}^{\infty}(n, l)$, which has units $m^3 s^{-1}$. The second and third terms describe the de-excitation processes, respectively these are spontaneous emission and collisional (non-radiative) quenching by the background gas. We include spontaneous emission through optically allowed transitions, characterized by the Einstein emission coefficients $A_{(n, l)\rightarrow(n', l')}$ (units of $s^{-1}$).

Rate coefficients for collisional quenching of an excited state are given by $k^Q(n, l)$ (units $m^3 s^{-1}$), which allow optically forbidden transitions of $H(n, l)$ to the ground state and assume uniform depopulation of the available orbitals. Any energy lost to electronic or vibrational excitation are only a few tens of eV and are negligible compared to the keV range of energies considered here. We neglect linear momentum transfer from energetic H to H$_2$ on the basis that we expect the mean free path for momentum transfer to be larger than the length scale of our discharge. The discharge is taken to be optically thin (emitted light is not reabsorbed).

A rate coefficient $k$ is related to its cross section $\sigma$ ($m^{-2}$) through $k(\bar{v}) = \int \sigma(v) f(v, \bar{v}) dv$ where $v$ is the relative velocity between interacting particles and $f(v, \bar{v})$ is a relative velocity distribution function parameterized by the mean interaction velocity $\bar{v}$. For energetic atoms or ions interacting with the background gas, $v$ is approximately the atom/ion velocity. Assuming a monoenergetic beam then $f(v, \bar{v})$ is approximated by a delta function such that $k(\bar{v}) = \sigma(\bar{v})\bar{v}$.

For charge exchange of monoenergetic $H^+_x$ we may write $k_{CX}^{\infty} = \sigma_{CX}^{\infty} v_x^+$, where experimental cross sections were obtained from Phelps et al. [36]. Cross sections for quenching [37] are approximately constant for interactions in units of keV energy range. Values for $A_{(n, l)\rightarrow(n', l')}$ (averaged over fine transitions and assuming no field-induced mixing effects) were obtained from Wiese et al [38].

### 3.1.2 CR model for slow excited H produced by dissociative excitation

The CR model in Ref. [13] considered central peak emission generated by a beam of units of keV electrons with a monoenergetic electron energy distribution function (EEDF). In this same paper, magnetic deflection of electrons in the inter-electrode emission channel showed they are indeed largely monoenergetic and accelerated to energies in the order of the full applied potential. The model therefore neglects the recombination of low energy secondary electrons as well as any ionizing collisions that would produce them. Rather it was found that the emission is dominated by dissociative excitation of H$_2$ by fast electrons. We neglect loss of low energy ($K < 1$ eV) H due to momentum transfer from atoms with larger kinetic energies. This is based on the assumption that the dissociation fraction $f_{\text{slow}}$ of the gas into low energy excited H is small ($f_{\text{slow}} \ll 1$) and so momentum transfer is more likely to occur only with the low-energy background gas. Given slow H and small $f_{\text{slow}}$ we also neglect atom-atom (de-)excitations,
as well as interactions between electrons and atoms in excited states.

Rate equations take the form,

\[
\frac{dn^0(n,l)}{dt} = k^D(n,l)n^{-}\frac{n^g}{n^s} - \sum_{(n',l')< (n,l)} A_{(n,l)-(n',l')} n^0(n,l).
\] (3.2)

Here \(n^0(n,l)\) is the number density of slow \(H(n,l)\) produced by dissociative excitation of \(H_2\) by fast electrons of density \(n^-\), with the rate coefficient for this process given by \(k^D(n,l)\). For a monoenergetic EEDF we may write \(k^D = \sigma^D n^-\), where experimental cross sections \(\sigma^D\) for low-energy (< 6 keV) dissociative excitation were obtained from Vroom et al. [39]. Experimental cross sections for dissociative excitation in the tens of keV range do not exist, and so we use the high energy Born-fit quoted by Janev et al. [40]; for 30 keV electrons we have \(\sigma^D(n = 3) = 3.0 \times 10^{-24}\) m².

Unlike the model for charge exchange, each level \((n,l)\) is described by a single equation related to the central peak emission. We note also that the original model had each \(n^0(n,l)\) coupled to higher levels through the inclusion of radiative cascades, however the removal of this process has negligible effect on the model output and so is neglected in its present incarnation. We again consider the steady-state case by setting the left hand side to zero.

### 3.1.3 Absolute densities for fast ions diverging from the cathode

We now use the CR models for fast and slow \(H(n = 3)\) to relate densities of fast ions and electrons to the \(H_o\) spectrum obtained using the setup in Sec. 2.3.

In the Abnormal Hollow Cathode model for the discharge (see Sec. 1.3) the Doppler-shifted emission from fast \(H\) is interpreted as arising from ions that are created within the cathode and undergo charge exchange as they accelerate away from a central region of positive space charge ('virtual anode'). For ions accelerating to energies of 2 – 3 kV, the mean free path for charge exchange is in the order 2 – 3 cm and so we expect fast \(H(n = 3)\) to be produced in the vicinity of the cathode edge. This position corresponds to a minimum in electric potential and a maximum in incident ion velocity. Figure 3.1 represents this graphically for the low voltage discharge, with energetic Doppler emission at the anode arising from fast excited \(H\) produced at the cathode. For Doppler emission measured at an angle \(\theta < 90^\circ\) to the discharge passing through the center of the anode ring, we estimate that emitting \(H\) travel a distance \(d_{sep}\) equal to the axial cathode-anode separation. Obtaining the particle energy \(K\) from the observed Doppler shift \(\Delta \lambda\), we can then use Eq. (1.4) to estimate an average time of flight \(t_{sep}\).

For the experiments in this thesis we have \(d_{sep} = 6\) cm for the low power discharge and \(d_{sep} \sim 20\)
Production of fast H(3s) (charge exchange) 

\[ \text{Doppler-emission} \quad H_{fast}(3s \rightarrow 2p) \]

Figure 3.1: Graphical depiction of the mechanism for energetic Doppler emission in the low voltage IEC discharge.

cm for the CW and pulsed modes. As such we are typically concerned with emission through the channel 3s \rightarrow 2p. This has a radiative lifetime \( \tau = 1/A_{3s-2p} = 1.6 \times 10^{-7} \) s. For incident H of energy 1 keV this implies a decay length scale \( v\tau \sim 10 \) cm. In contrast, the channels 3p \rightarrow 2s and 3d \rightarrow 2p have a decay length in the order of 2 and 1 cm respectively, and so we neglect emission from fast H in the 3p or 3d states. We may thus relate the population of fast H(3s) at the anode and cathode,

\[ [\text{anode}] n^*(3s) = [\text{cathode}] n^*(3s) \times \exp(-A_{3s-2p}t_{sep}). \quad (3.3) \]

The intensity \([\text{anode}] J^x_{an} \) of Doppler emission arising from fast H\( ^+ \) and measured at an angle \( \theta \) to the emission channel at the anode may then be written

\[ [a] J^x_{an} \propto [a] n^*_{an}(3s) A_{3s-2p} \times \exp(-A_{3s-2p}t_{sep}), \quad (3.4) \]

where for clarity we have used the shorthand 'a' and 'c' for the anode and cathode, respectively.

Figure 3.2 shows the situation for slow H\( (n = 3) \), where central peak emission at the anode arises through dissociative excitation of H\(_2\) by fast electrons. Since the excited H have negligible kinetic energy \( (K \sim 1 \) eV), we may assume that unshifted emission occurs at the point in space where the gas is dissociated. The intensity \([a] J^0\) for central peak emission at the anode is thus given by
Figure 3.2: Graphical depiction of the model for central peak emission produced by fast monoenergetic electrons in the low voltage discharge.

\[ [a] J^0 \propto [a] n^0 (0.16A_{3s \rightarrow 2p} + 0.44A_{3p \rightarrow 2s} + 0.40A_{3d \rightarrow 2p}), \]  
where it is understood that \([a] n^0\) refers to slow \(H(n = 3)\) at the anode and we have used the dissociative excitation branching ratios \((s : p : d) = (16 : 44 : 40)\) of Fujimoto et al. [41].

As \(I_z^*\) and \(I^0\) are sampled at equal distances from the respective emitters, at approximately the same wavelength and with the same exposure time and detector slit width, we can neglect any spatial or frequency dependence in their ratio,

\[ \frac{[a] J^*_z}{[a] J^0} = \frac{[c] n^*_{3s \rightarrow 2p} \times \exp(-A_{3s \rightarrow 2p} t_{sep})}{[a] n^0 (0.16A_{3s \rightarrow 2p} + 0.44A_{3p \rightarrow 2s} + 0.40A_{3d \rightarrow 2p})}, \]  
where it is understood that \([c] n^*_{3s}\) refers to fast \(H\) at the cathode in the 3s state. Rearranging the steady-state forms of Eqs. (3.1) and (3.2), we can obtain expressions for \([c] n^*\) and \([a] n^0\) respectively. Substituting these into Eq. (3.6), we obtain

\[ \frac{[a] J^*_z}{[a] J^0} = \frac{[c] n^*_{3s} \sigma^{-1}_{3s} (3s) [a] v^*_x \times \exp(-A_{3s \rightarrow 2p} t_{sep})}{[a] n^- \sigma^{-1}_{D}(n = 3) [a] v^- (1 + C_1) C_2}, \]  
where \(C_1\) and \(C_2\) are defined as...
\[ C_1 = \frac{0.44A_{3p \rightarrow 3s}}{(0.16A_{3s \rightarrow 2p} + 0.44A_{3p \rightarrow 2s} + 0.40A_{3d \rightarrow 2p})} \]  
\[ C_2 = \frac{A_{3s \rightarrow 2p}}{(A_{3s \rightarrow 2p} + n_k \sigma_v v_e^+)} \]  

(3.8)  
(3.9)

For additional detail on this derivation, we point the reader to the Appendix. We have thus related the density of fast ions emerging from the cathode to the experimental ratio of \( I_e^+ \) and \( I^0 \) at the anode, and also to the density \( n^- \) of fast, monoenergetic electrons at the anode. This method is convenient as the energies of incident ionic species can be read quickly from a single spectrum of \( \text{H}_\alpha \) using Eq. (1.4), with the ratio between Doppler and unshifted emission computed by comparing areas under the respective peaks.

In this work we estimate electron density by equating the anode current with that measured at the cathode \( I_{\text{anode}} = I_{\text{cathode}} \) by Kirchoff’s current law. At the anode, we assume the current is due to a constant flux of electrons (of charge \( e \) and mass \( m_e \)) streaming from the cathode in a beam of radius \( r \). It follows that,

\[ [\text{a}] n^- = \frac{I_{\text{cathode}}}{e(\pi r^2 v^-)}, \]  

(3.10)

where the velocity \( v^- \) of fast, monoenergetic electrons at the anode is calculated by assuming they are accelerated to the full applied potential as in Ref. [13]. Relativistic effects are neglected. We have assumed also that the electron and neutral beams have equal radii \( r \) at any position along the beam, so that the optical setup samples light from equal volumes of electron and fast neutral emission.

The result of all this is an expression relating intensity ratios to the ratio of ion and electron densities,

\[ [\text{c}] f_{\text{fast}} = \frac{[\text{c}] n_{e}^+}{n_g} = \frac{[\text{a}] I_e^+ [\text{a}] n^- \sigma^D(n = 3) [\text{a}] v^-}{[\text{a}] I^0 \frac{n_e}{\sigma^{\text{CX}}(3s)[\text{c}] v_e^+} \times \left[ 1 + C_1 C_2 \exp(-A_{3s \rightarrow 2p} t_{\text{sep}}) \right]^{-1}}, \]  

(3.11)

where \([\text{c}] f_{\text{fast}}\) is the dissociation fraction of the background gas into energetic nuclei at the cathode aperture, \([\text{c}] n_{e}^+\) is the number density (units m\(^{-3}\)) of fast \( \text{H}_e^+ \) of velocity \( v_e^+ \) emerging from the cathode aperture, and \( n^- \) is the number density of fast, monoenergetic electrons of velocity \( v^- \) at the anode.

It has been shown [14, 24] that for pressures in the tens of mTorr range, the Doppler peaks associated
with fast \( H_2^+ \) and \( H_3^+ \) are not resolved but instead form a merged peak. Moreover these species are expected to be present in approximately equal numbers [23], and so for this peak (referred to in our results section as 'fast \( H_2^+ \)') we assume an average incident ion mass corresponding to \( x = 2.5 \). As a result, we take the cross section for Doppler emission to be the average of total \( \text{H}_\alpha \) production by fast \( H_2^+ \) and \( H_3^+ \) in \( H_2 \) [36]. For \( H_2^+ \) of energy \( 1 - 15 \text{ keV} \) we can estimate that roughly 1/3 of fast neutral \( \text{H}(n = 3) \) will be produced in the 3s state [42], and so we divide the total Doppler \( \text{H}_\alpha \) cross section by three in order to specify emission by fast \( \text{H}(3s) \). For a merged population of 15 keV ions we thus have \( \sigma_{x=2.5}(3s) = 5.0 \times 10^{-22} \text{ m}^2 \). We divide also the total \( n = 3 \) quenching cross section of Ref. [37] in the same way, yielding \( \sigma_{Q}(3s) = 2.5 \times 10^{-19} \text{ m}^2 \). We note that for each set of cross section data used, uncertainties may be as large as 10% and this will contribute to the overall uncertainty associated with the diagnostic method (described more fully in Sec. 3.1.5).

Certain atomic emission processes were neglected in this analysis. One was dissociative ionization of \( H_2 \) by electrons,

\[
e^- + H_2 \rightarrow e^- + H_2^+ \rightarrow 2e^- + H^+ + H(n = 3).
\]  

which from Ref. [9] produces \( \text{H}_\alpha \) with Doppler shifts \( 0.5 \lesssim |\Delta \lambda| \lesssim 3 \text{ Å} \). This is larger than the chosen integration under the central peak \( (\Delta \lambda < 0.3 \text{ Å}) \). Assuming that the density of ground state atomic \( H \) is much smaller than for \( H_2 \), we also neglected electron excitation of atomic \( H \).

3.1.4 Prediction of neutron production rates in deuterium

One of the key claims of our optical diagnostic is that we may use the dissociation fraction at the cathode edge \( (f_{\text{fast}}) \) measured in hydrogen to predict the volumetric neutron production rate \( [\epsilon] R_{\text{vol}} \text{ (m}^{-3} \text{s}^{-1}) \) of process (1.1) at the cathode aperture for a deuterium discharge which has the same current and voltage. We term this an 'equivalent' discharge of \( D_2 \). We anticipate that the values \( f_{\text{fast}} \) and \( R_{\text{vol}} \) are related through

\[
[\epsilon] R_{\text{vol}} = [\epsilon] n_x^+ n e^2 (2n x \sigma_{D-D}^D) v_x^+ = [\epsilon] f_{\text{fast}} n_x^2 (2n x \sigma_{D-D}^D) v_x^+ ,
\]

where now \( [\epsilon] n_x^+ \) is the population density of fast monoenergetic deuteron nuclei at the cathode edge, with mass \( x \) times that of a single deuteron and velocity \( v_x^+ \). The cross sections \( \sigma_{D-D}^D \) for process (1.1) are quoted in Ref. [2], and we multiply these by a factor \( 2x \) to account for the increased density of nuclei.
in the reaction between fast \( \text{D}_2^+ \) and the gas molecule \( \text{D}_2 \). That is, we have assumed each \( \text{D}_2^+ \) dissociates into \( x \) deuterons, and may interact with a background gas molecule consisting of 2 deuterons.

In the AHC model for the discharge, the energetic fast deuteron ions have largely been neutralized beyond the cathode aperture and so we can make the estimate that the density of energetic nuclei (and their energies) remains unchanged along the beam. Assuming the most energetic deuteron nuclei form a collimated channel of radius \( r = 1 \text{ cm} \) and length \( L = 48 \text{ cm} \) from anode to anode, the total absolute neutron production rate \( R_{\text{tot}} \) (s\(^{-1}\)) for the system is thus given by,

\[
R_{\text{tot}} = [c] R_{\text{vol}} \times (L \pi r^2).
\]  
(3.14)

Combining Eqs. (3.10), (3.11), (3.13) and (3.14) we see that the prediction for absolute neutron production rate is, in fact, independent of the beam radius. In contrast, uncertainty in the beam radius dominates the uncertainties for the dissociation fraction \( f_{\text{diss}} \), the absolute density of fast ions and the neutron production rate \( R_{\text{vol}} \).

### 3.1.5 Limitations and uncertainties associated with the optical diagnostic method

Application of the method presented in Sec. 3.1.3 is restricted to regions of the inter-electrode discharge where the electron population is approximately monoenergetic in the units to tens of keV range. This is due to the treatment of central peak emission; from Ref. [13] we expected that the electron-energy dependence of the central peak intensity could be approximated by Eq. (3.2) only for electron energies greater than about 1.5 keV (see Fig. 1.10).

Moreover, some of the experimental and tabulated quantities in Eq. (3.11) are associated with significant uncertainties. For example, populations of fast ions and neutrals are not perfectly monoenergetic as their respective Doppler peaks can be fit by Gaussians [12, 14, 15, 23]. This is due not only to convolution with the spectrometer response function but also to the random nature of the charge exchange process, in which individual ions travel different distances (and accelerate to different energies) before interacting with the gas. We characterize the spread in terms of the Gaussian half-width, which for low power and CW discharges were typically about 3 \( \text{Å} \). Using Eq. (1.4) this suggests a spread of energies \( \pm(100 - 300) \) eV for each peak, or about 10 – 20% for ions and atoms in the units of keV energy range. In fact there exist similar uncertainties for the tabulated cross sections for charge exchange and dissociative excitation. We note also that the underlying AHC model assumes all charge exchange occurs at a single point along the beam (at the cathode aperture). In reality however, charge exchange will occur for a range of positions along and about the discharge axis. For 2 – 3 keV ions traveling through a 20 mTorr
gas, the mean free path length for charge exchange is in the order $2 - 3$ cm (similar to the half-length of the cathode). We therefore assume an axial ‘interaction range’ smaller than $\pm 1$ cm either side of the cathode edge, which would introduce an uncertainty up to 15% in the time of flight $t_{\text{sep}}$.

The assumption of an electron-induced origin for the central peak may also be incorrect. Williams et al. [42] show that like fast electrons, fast H and H$^+_x$ at units and tens of keV energies may produce slow H($n = 3$) by dissociative excitation of hydrogen gas, and that the cross section for these processes are similar to the corresponding production of fast H(3s). For central peak emission at the anode, we may neglect any contribution from H$^+_x$ as ion energies at the anode are low. However we cannot similarly discount the component of central peak emission arising from fast neutral H, as this should exhibit intensities comparable to the Doppler shifted peaks. In fact for H energies of $5 - 15$ keV the ratio of production cross sections of fast H(3s) to slow H($n = 3$) is $1.2 \pm 0.1$ which is about 40 - 50% of the corresponding Doppler to central-peak intensity ratios we measure in Sec. 4.2.1. As such, we already expect that Eq. (3.11) may be overestimating the component of central peak emission due to electron excitation by as much as 50%. In Sec. 4 we use the numerical simulation of the following section to better quantify the contribution to central peak emission by electrons.

Strictly speaking Eq. (3.14) requires a sum over all the fast ion species $x$. However for the measurements in Sec. 4.2.1 we have focused only on the Doppler peak corresponding to fast H$^+_2$ as the fast H$^+$ peaks were not always visible (intensities for H$^+$ Doppler peaks were generally less than 10% of those for H$^+_2$). However, neglecting the contribution of H$^+$ in this way may underestimate the predicted fusion rate by up to an order of magnitude as the fusion cross sections for these more energetic nuclei are about an order of magnitude larger. We have similarly neglected any contribution to fusion rates by fast neutral particles reflecting from the anode surface which, as evidenced by the experimentally recorded Doppler spectra of Sec. 4, may not be insignificant. Equation (3.14) for the total fusion rate also ignores any fusion reactions for fast deuterium nuclei accelerating from the cathode center towards the cathode interior wall; this is justified on the grounds that the cathode interior comprises only a small fraction ($< 10\%$) of the total fusion reaction volume in the system.

We have neglected electric field mixing in the production of fast H(3s) at the cathode and slow H($n = 3$) at the anode. This is justified as we expect the electric field in both of these regions to be small - in our model the cathode edge exhibits a local potential minimum, and we assume the anode to sit sufficiently beyond the cathode fall. We note however that in the cathode fall region we anticipate axial fields exceeding 1 kV/cm, and this may decrease the 3s lifetime by an order of magnitude (see Fig. 9 of Ref. [43]). Taking this effect into account would require careful modification of the assumed H(3s) decay (as given by Eq. (8) of our previous paper), and we acknowledge it as a shortcoming here.
It is also possible that cathode heating may influence the temperature and density profile of the gas, whereas our model assumes uniform profiles for both. Experimentally, we have attempted to minimize this issue by allowing the cathode to cool (switching off the discharge) between individual measurements of H\textsubscript{\alpha} spectra and neutron production rates.

Given all those sources of error in the diagnostic as presented here, we conclude that any single optically-determined prediction of fusion rate is associated with an order-of-magnitude uncertainty at best.

Finally, our treatment of the anode current ignores other parts of the discharge, for example the low intensity emission surrounding the outer surface of the cathode (cf. Fig. 2.9). We suggest that this issue could be countered by insulating the outer surface inside a ceramic sleeve so as to prevent a discharge forming in this region, however this was not implemented in the present work. As a result we may overestimate our experimental anode ring current, and the resulting densities of fast electrons and ions by some small, unknown fraction.

3.2 Semi-analytic simulation of electric potential and Doppler H\textsubscript{\alpha} spectra

We present here our semi-analytic procedure for calculating energies and densities of electrons and ions in the cylindrically symmetric AHC discharge. Section 3.2.1 covers the generalized method, with particulars of the numerical implementation described in Sec. 3.2.2. Advantages and limitations of the method are considered in Sec. 3.2.5.

3.2.1 Iterative relaxation method for steady-state solution of the Poisson equation

The aim here is to calculate the potential profile along the beam axis of a steady-state AHC discharge, and for this purpose we divide the beam into \( m \) segments of equal length \( dz \) from anode to anode. The Poisson equation (1.11) is recast as a finite difference equation in linear cylindrical coordinates,

\[
(V_{j+1} - 2V_j + V_{j-1})/dz^2 = -\rho_j/\epsilon_0, \tag{3.15}
\]

which we solve for the axial potential \( V_j \) a distance \( z \) from the cathode center. We thus require the charge density \( \rho_j = e(n_j^+ - n_j^-) \) where \( e \) is the elementary charge and the number densities of ions and electrons in segment \( j \) are given by \( n_j^+ \) and \( n_j^- \) respectively. Particle densities are calculated through an iterative process depicted by the flowchart in Fig. 3.3.
Figure 3.3: Flowchart depicting stages in our iterative solution for the steady-state electric potential. See text for explanation.
Specifically, the method of Fig. 3.3 is to consider the acceleration of an initial population distribution of electrons,

$$[0]F^- = \left( F_1 \ F_2 \ \ldots \ F_m \right),$$

(3.16)

where $F_j$ represents the fraction of electrons in segment $j$ and where the superscript notation '[$n$]' denotes a value in the $n$th iteration of the solution procedure. The ensemble is accelerated through an initial potential distribution,

$$[0]V = \left( V_1 \ V_2 \ \ldots \ V_m \right),$$

(3.17)

where now the electric potential in segment $j$ is given by $V_j$. These initial vectors are used to calculate the population of $H_2^+$ produced in each segment relative to the electron population. These are then accelerated through the same potential yielding the energies and relative density distribution of energetic neutral $H_2$ through charge exchange with the background gas. As such, the different acceleration timescales for electrons and ions are dealt with implicitly through the dependence of relative densities on ionization and charge exchange cross sections which are energy (velocity) dependent. Relative densities are then scaled using optically determined fast ion densities at the cathode edge for experimental values of voltage and gas pressure; this allows calculation of absolute particle densities and solution of the Poisson equation. The calculated potential becomes the input for the next iteration number $n$ of the process, which in an ideal scenario continues until the potential meets some strict convergence criterion. The initial potential $V_j$ are taken as the Laplace solution corresponding to Eq. (3.15) with initial values of $F_j^-$ set to 1 for $j$ inside the cathode and falling off as $z^{-C}$ along the beam outside the cathode. The choice of constant $C$ can affect the convergence of the solution and is discussed in Sec. 3.2.2.

The first step in the iterative sequence of Fig. 3.3 is to calculate the energy $K_{i,j}^-$ of electrons at the center of segment $i$ having begun its acceleration from the center of segment $j$. To a first approximation, this is given by

$$K_{i,j}^- = V_j - V_i,$$

(3.18)

for all $V_k > V_i$, where $k = i + 1, \ldots, j$. $K_{i,j}^-$ is set to zero otherwise, which signifies that an electron either cannot accelerate from segment $i$ to $j$, or that segment $j$ is at the outer extent of an oscillatory motion. We note that Eq. (3.18) as given does not incorporate energy loss due to ionizing collisions, which must also be included. Appropriating the methods of Shrier et al. [14] as applied to charge exchange, the
average total energy lost by an electron of the ensemble accelerating from segment \(i\) to \(j\) is given by,

\[ E_{i,j}^{\text{loss}} = E_{\text{ioniz}} n_e dz \sum_{k=i+1}^{j} \sigma_{i,k}^{\text{ioniz}} (K_{i,k}^-), \]

where \(E_{\text{ioniz}}\) is the ionization energy of molecular hydrogen (15.4 eV), \(n_e\) is the number density of the background gas and where the ionization cross section \(\sigma_{i,j}^{\text{ioniz}}\) (for an electron of energy \(K_{i,j}^-\)) is taken from Ref. [40]. Since the cross section itself is energy-dependent, the ionization energy losses are incorporated into Eq. (3.18), with the final equation appearing as

\[ [n+1]K_{i,j}^- = V_j - V_i - E_{i,j}^{\text{loss}} ([n]K^-), \]

which is essentially a relaxation procedure in that we find a converging solution by taking a nested iteration loop over \(n\). Following the calculation of \(K_{i,j}^-\) to within some arbitrary convergence criterion, the fraction of the electron ensemble undergoing ionizing collisions in segment \(j\) having started their acceleration in segment \(i\) can be written,

\[ P_{i,j}^- = H(K_{i,j}^-) n_e dz \sum_{k=i+1}^{j} \sigma_{i,k}^{\text{ioniz}}, \tag{3.19} \]

where \(H(x) = 1\) for \(x > 0\) and zero otherwise. \(P_{i,j}^-\) is thus nonzero only if electrons can accelerate from segment \(i\) to \(j\).

For ions, the process (1.6) is but one branch of the total charge exchange process,

\[ \text{H}_2^+ + \text{H}_2 \rightarrow \text{H}_2^+ + \text{H}_2, \]

producing a slow ion and a fast gas molecule \(\text{H}_2^+\) which may dissociate into fast atomic \(\text{H}(n, l)\). Total cross section data was taken from Ref. [36]. In contrast to electrons, which typically lose only a small fraction of their kinetic energy in any one ionizing collision, a charge-exchanging ion is treated as being replaced by an ion at rest. Moreover we are interested in the energies of the resulting energetic neutral \(\text{H}_2\) and so we calculate the energy \(K_{i,j}^+\) of an ion having accelerated without charge exchange,

\[ K_{i,j}^+ = V_i - V_j, \tag{3.20} \]

which is nonzero for all \(V_i > V_k\) with \(k = i + 1, \ldots, j\) and is set to zero otherwise. As in Ref. [14] we can write the fraction of ions starting in segment \(i\) and surviving to segment \((j - 1)\) as,
\[
\exp(-n^e dz \sum_{k=i+1}^{(j-1)} \sigma_{i,k}^{\text{CX}}),
\]
and similarly for ions surviving to segment \((j + 1)\). Thus the probability of a neutral being produced in segment \(j\) with energy \(K_{i,j}^+\) (or equivalently, the probability of an ion of energy \(K_{i,j}^+\) charge exchanging in segment \(j\) and being replaced by a new slow ion) is given by,

\[
P_{i,j}^+ = H(K_{i,j}^+)[\exp(-n^e dz \sum_{k=i+1}^{(j-1)} \sigma_{i,k}^{\text{CX}}) - \exp(-n^e dz \sum_{k=i+1}^{(j+1)} \sigma_{i,k}^{\text{CX}})],
\]

where \(\sigma_{i,k}^{\text{CX}}\) is the energy dependent cross section for total charge exchange for an ion accelerating from segment \(i\) to \(k\).

The third stage of the iterative process is to again use a relaxation procedure, this time to calculate the relative population of electrons (and subsequently, ions) in each segment along the beam. The 'distribution' element \(D_{i,j}\) gives the relative population of electrons in segment \(j\) with energy \(K_{i,j}^-\). The upper-and lower-triangular halves represent the right-and left-going electrons respectively,

\[
[n+1]D_{i,j}^- = H(K_{i,j}^-) \times [F_{i,j}^- + \sum_k ([n]C_{k,i}^- + [n]R_{k,i}^-)],
\]

where the \(F_{i,j}^-\) represent electrons having reached segment \(j\) after being initially injected into segment \(i\) and does not require specification in terms of the present segment \(j\). That is, if \(H(K_{i,j}^-)\) is non-zero then electrons can accelerate from segment \(i\) to \(j\) and are not lost along the acceleration path irrespective of the value of \(j\). New electrons may be created however, and indeed the collision element \(C_{k,i}^-\) represents all slow electrons produced in segment \(i\) as a result of ionizing collisions by electrons accelerating from segment \(k\). Thus the second term on the RHS represents slow electrons produced in segment \(i\) by any other electron in that segment. For iteration \(n = 0\) the collision matrix \([n]C_{i,j}^-\) is set to zero and may be written as the rows of upper-and lower-triangular \([n+1]D_{i,j}^-\) are written (left-to-right and right-to-left, respectively). If the gradient of potential in a segment \(j\) is positive (negative) then ionization produces additional slow electrons moving right (left). We thus write the electron collision term as,

\[
[n+1]C_{i,j}^- = [n] D_{i,j}^- P_{i,j}^-,
\]

where for upper triangular element we must have \(V_{j+1} > V_{j-1}\) (and for lower triangular elements we require \(V_{j+1} < V_{j-1}\), otherwise the element is set to zero. The third term on the RHS of Eq. (3.22) considers electrons which have reached the end of an oscillatory motion and are beginning a return or
'reflected' trajectory in the opposite direction. This is written,

\[ [n+1]R_{i,j}^- = [n]D_{i,j}^- \]  

(3.24)

which is nonzero only when an electron reaches zero velocity along a negative potential gradient, or equivalently where \( K_{i,j}^- > K_{i,(j-1)}^- = 0 \) for left going electrons and \( K_{i,j}^- > K_{i,(j+1)}^- = 0 \) for right-going electrons. The matrices (3.22,3.23 and 3.24) for electrons may be calculated in sequence until fulfillment of some arbitrary convergence criterion in \( D^- \).

The relative density of slow ions produced in segment \( j \) is equal to that of slow electrons produced through ionization. Equivalently, the source vector for ions may be written \( F_j^+ = D_{(j-1),j}^- + D_{(j+1),j}^- \) which is the sum of relative densities of left and right going secondary electrons produced in that segment.

The distribution matrix for ions is given by,

\[ [n+1]D_{i,j}^+ = H(K_{i,j}^+) \times [F_i^+ + \sum_k ([n]C_{k,i}^+ + [n]R_{k,i}^+)] \times \prod_{k=i}^{(j-1)} (1 - P_{i,k}) \]  

(3.25)

where the collision and reflection matrices \( C^+ \) and \( R^+ \) behave similarly to their electron counterparts. In fact \( v^+ \) is conceptually identical to \( v^- \) aside from a final multiplicative factor accounting for the loss of energetic ions to charge exchange as they accelerate from segment \( i \) to \( j \).

For either species in segment \( j \) we can calculate the average energy of right-going particles,

\[ \text{right } K_j = \frac{\sum_{k=1}^j (K_{k,j}D_{k,j})}{\sum_{k=1}^j D_{k,j}}, \]  

(3.26)

which is nonzero for any segment where at least some particles have nonzero energies (the element is set to zero otherwise). We calculate the average energy for left-going particles similarly,

\[ \text{left } K_j = \frac{\sum_{k=j}^m (K_{k,j}D_{k,j})}{\sum_{k=j}^m D_{k,j}}, \]  

(3.27)

where \( m \) was the total number of segments in the beam. Now \( D_{i,j} \) is proportional to the density of particles in segment \( j \) with energy corresponding to particles accelerating from segment \( i \). Taking a sum over the columns of upper-and lower-triangular \( D_{i,j} \) thus gives the relative density \( \text{rel}_n j) \) in each segment for right-and left-going particles respectively, with the average energy in each segment given by Eqs. (3.26) and (3.27). The relative densities are then scaled with distance from cathode edge; visual inspection suggests the beam typically diverges at some angle \( \phi \) outside the cathode and so from trigonometry we expect that for a translation \( dz \) along the beam away from the aperture, the beam radius
will expand by $dz \tan \phi$. The density of particles in each segment is thus reduced by the ratio of the actual beam volume to the volume of a cylindrical segment inside the cathode. The relative densities for electrons and ions are then transformed into absolute densities using the spectroscopically determined densities for energetic ions emerging from the cathode apertures. This is applied in conjunction with experimentally measured values of cathode voltage and gas pressure.

We note that electric potential in a beam segment corresponding to a cathode aperture will not typically sit at the cathode voltage, but rather float above it due to the charge density present in the beam. We expect that the axial charge density $\rho_{r=0}$ calculated for the aperture segments will in fact be the peak of a radial distribution of charge. We calculate the axial potential at the aperture segments by using a relaxation method to solve the radial Poisson equation,

$$\nabla^2 V(r) = -\rho_{r=0} \exp(-r^2/w^2)/\epsilon_0,$$

on a finite grid where the voltage at the cathode radius is the applied potential, and where the RHS indicates a radial charge distribution which is Gaussian and with a width $w$ of order the beam radius [31]. The completion of this step then allows a direct numerical solution of the Poisson equation 3.15.

### 3.2.2 Implementation

The Poisson equation is encoded in the form $AV = -\rho/\epsilon_0$ where $A$ is an $m \times m$ matrix and $V$ and $\rho$ are both column vectors of length $m$. The number of segments $m$ must be large enough to yield good resolution inside the cathode (8 segments long), but also small enough so as to provide speedy solution of $V$ through inversion of $A$. We choose $m = 105$ segments of equal length $dz = 0.46$ cm from anode to anode. The cathode radius (and beam radius inside the cathode) are both set to 1 cm as per the experimental apparatus.

The initial potential (solid curve) and electron population (dashed curve) distributions for our geometry are shown in Fig. 3.4, with the cathode represented by the region between the two vertical (solid) lines. The constant $C$ describes the decay of the initial electron distribution $F^-$ outside the cathode, with a positive value indicating that the initial density of electrons decreases away from the cathode, which is as we expect for a cathode fall. The choice of $C$ can affect the speed (number of iterations) required for convergence as well as the distribution of electron energies far from the cathode. We found that for the plasma parameters of Sec. 4, a value of $C = 0.25$ yields electron energy distribution functions (EEDFs) which are similar to the monoenergetic electron beams inferred from the work in Ref. [13]. The convergence criterion for the energy, probability and distribution matrices is a maximum difference of 1% for
any given element between subsequent iterations of the respective relaxation procedures; the criterion for the potential is less strict (10% maximum difference over an entire iteration of Fig. 3.3). Issues of convergence in the potential, and other limitations with the simulation are discussed in Sec. 3.2.5.

The various matrices and vectors comprising the iterative method of Sec. 3.2.1 were written as a single processor script in the programming language MATLAB [44] and interpreted on a laptop consisting of dual core 2.4 GHz Intel processors and 2 GB of RAM. For the present convergence criteria, a valid set of input parameters (fast ion density, gas pressure and cathode voltage) will typically achieve a converging solution within 2 minutes.

3.2.3 Simulation of energetic Doppler emission

It is possible to use our numerical results to simulate energetic Hα Doppler spectra from fast H atoms produced in the excited 3s state in the process (1.6). We consider an imaginary monochromator directed towards the right side of the chamber, sampling light from segment j at an angle θ to the beam. For an excited neutral H atom produced in segment j the distance to the detector in segment k is given by $d_j = dz|j - k|$, and the velocity of fast H produced in segment j by an incident H$_2^+$ of energy $K_{ij}^+$ is
where the incident energy is partitioned among the fast neutral products of process (1.6) according to their mass [11]. The time of flight of fast H from their production point to the detector with energy $K_{i,j}^+/2$ is thus given by $t_{i,j} = d_j/\nu_{i,j}$ and so for a neutral with initial intensity $I$ at its point of creation, the average intensity following radiative decay along the path to the detector is $I_{i,j}^{\text{decay}} = \exp(-t_{i,j}A_{3s-2p})$ where $A$ is the Einstein emission coefficient for this Balmer transition [38]. We can write the relative density of fast H(3s) produced in segment $j$ and with an energy $K_{i,j}^+/2$ as $n_{i,j}^*(3s) = D_{i,j}^+P_{i,j}^+\sigma_{i,j}^*(3s)$ where the cross section $\sigma_{i,j}^*(3s)$ is for Doppler shifted emission from energetic H(3s) arising from $H_2^+ - H_2$ interactions as given by Ref. [42]. We can thus calculate the (total) relative intensity at the detector of fast H produced in segment $j$ by an ion which begins its acceleration in segment $i$,

$$I_{i,j}^{\text{tot}} = T_{i,j}I_{i,j}^{\text{decay}} \times A_{3s-2p}n_{i,j}^*,$$  \hspace{1cm} (3.29)

where the 'trajectory' function $T_{i,j}$ equals 1 if the neutral approaches the detector position from the left (indicating red shifted emission), -1 if the neutral approaches from the right (blue shift) and 0 if the neutral does not pass through the detection region (no emission recorded). Using Eq. (1.4), the elements of $I_{i,j}^{\text{tot}}$ are each added to one of 100 bins over the range $-70 < \Delta\lambda < 70$ Å, each bin of width $\sim 1.4$ Å.

### 3.2.4 Application of steady-state solutions to the pulsed discharge

From Fig. 2.10, we may treat the dependence of cathode voltage $V_{\text{cathode}}$ on time $t$ as a second order exponential,

$$V_{\text{cathode}}(t) = V_{\text{peak}}\exp(-t/\tau_{\text{peak}}) + V_{\text{trail}}\exp(-t/\tau_{\text{trail}}) + V_{\text{offset}},$$  \hspace{1cm} (3.30)

where $\tau_{\text{peak}}$ and $\tau_{\text{trail}}$ are experimentally determined time constants, with $V_{\text{peak}}$ and $V_{\text{trail}}$ the amplitudes for the fast and slow parts of the pulse, respectively. We also fit a zero offset $V_{\text{offset}}$ which is typically of order $V_{\text{trail}}$. With appropriately scaled amplitudes and offsets, we can assume the same time constants for the time-variation of current.

In Sec. 4 we measure the current-dependence of the fast fraction $f_{\text{fast}}$ for the CW mode in the cases of constant voltage (~30 kV) and pressure (~5 mTorr) in hydrogen. For both conditions, we tend to have $f_{\text{fast}}$ varying proportional to the logarithm of current over the measured current range. Since the pulsed mode also had peak voltages of order -30 kV, the CW measurements allow us to apply the steady state...
solutions to the pulsed discharge. We begin by using the measured peak current to extrapolate beyond the constant-voltage data, yielding the peak value of $f_{\text{fast}}$ (and thus the maximum fast ion density for the pulse). The pulse itself may be described as a decay in voltage and current over time with a constant pressure - we can thus estimate the time decay of $f_{\text{fast}}$ from its peak value by observing its current dependence for the constant pressure measurements in CW mode.

The time-dependent parameter $f_{\text{fast}}(t)$ is then fed into the numerical solution method leading to instantaneous emission intensities $I_{\text{tot}}^{\text{t}}(t)$ and neutron production rates $R_{\text{tot}}^{\text{t}}(t)$. For a small timescale $\delta t$ this yields accumulated intensities and neutrons $I_{\text{t}}^{\text{t}}(t)\delta t$ and $R_{\text{t}}^{\text{t}}(t)\delta t$ respectively. In this work each simulation of Doppler spectra and total neutron production rate is accumulated from 1 $\mu$s to 10 ms using ten values of $t$ (or 'slices') spaced evenly on a logarithmic scale, which assures time steps $\delta t$ larger than the typical times of flight ($< 1 \mu$s) for ~15 keV neutrals. This is necessary in order to treat the emission as time-invariant during each slice.

3.2.5 Advantages and limitations

As with the charge exchange modeling of Ref. [14], the present calculation of average particle energies and densities does not suffer from the numerical noise observed in more conventional particle-in-cell (PIC) models featuring Monte Carlo collisions, and is generally faster than these methods. Unlike some PIC models however, our steady-state equation system features fewer atomic collisions, for example we neglect production and charge exchange of H$^+$ and H$_3^+$. We may justify this on the grounds that H$_2^+$ is the dominant ion species for the units and tens of mTorr pressure range [23]. It is more difficult to justify our neglect of ionization of the gas by fast neutrals, in fact given the results of Chapter 4 we would expect such ionization to add significantly to the ion population within the system; we offer that these calculations are primarily of qualitative interest.

Our method is proficient at the simulation of potential field and momentum fluxes, and this is through reliance on experimentally measured fast ion densities, pressures and cathode voltages. But the calculations cannot show how these ion densities arise over time, as we neglect the production of secondary electrons through collisions of ions with the cathode interior wall - which in Ref. [21] was hypothesized as the mechanism ultimately responsible for sustaining the discharge. In a sense, there is some assumption that low energy electrons have a density maximum within the cathode (see the initial electron distribution $F^-$ imposed in Fig. 3.4), but this initial electron distribution is not explicitly dependent on the calculated ion distribution.

On the issue of time dependence, we note that for each pressure/voltage pair there exists an upper bound to the range of aperture ion densities leading to converging solutions in $V$. Just beyond this,
we tend to observe solutions of $V$ which are physically reasonable (maxima and minima no higher or lower than the anode and cathode, respectively), but do not converge between subsequent iterations of the solution procedure. This suggests certain values of current may not be maintained in the steady-state. Taking the current higher again the potential fails to converge at all; we take this to indicate combinations of parameters which are entirely non-physical. The work of Marocchino et al. [45] in modeling the time-variation of a virtual cathode in an electron-injected system using a PIC method suggests that a finite angular momentum for electrons can lead to a potential profile which is more stable in time; future researchers applying our method to IEC may wish to somehow incorporate finite angular momentum to particles within the cathode.

Finally we note that our neglect of scattering interactions between modeled populations and the background gas may significantly diverge from the real situation as pressure increases. For the present length scales of the anode and cathode, we expect that the generated solutions will be invalid for pressures above $\sim 200$ mTorr.

### 3.2.6 Magnetic deflection of the electron beam

For the ideal case of 30 keV electrons of velocity $v_e$ traveling through a uniform magnetic field $B = 10$ mT, the radius of curvature is $R = m_e v_e / eB \sim 6$ cm. For the electron beam in Fig. 2.3 striking the anode wall after having been deflected by a magnet of radius $r_{\text{mag}} = 4.5$ cm (also placed near the wall), we therefore expect a transverse displacement of about 5 cm relative to the undeflected neutral beam. The experimentally obtained beam deflection was not at all ideal; the results and their implications are discussed in Sec. 4.3.2.
Chapter 4

Results and Analysis

In this Chapter we present measured \( H_\alpha \) spectra for the low voltage, CW and pulsed IEC discharge modes as described in Chapter 2. Where applicable, we apply the optical diagnostic of Sec. 3.1, presenting measured absolute fast ion densities and fast fractions \( f_{\text{fast}} \) for the hydrogen discharge and comparing the predicted and measured neutron production rates for equivalent discharges of deuterium. Finally, we also present results of our numerical simulation for the CW and pulsed modes, gleaning crucial new insight into the potential shape, and dominant trajectories for charged and neutral species.

For purposes of clarity, this results chapter is divided into three sections, each dealing with the separate experiments on low voltage, CW and pulsed mode IEC plasmas.

4.1 Low voltage discharge in hydrogen

4.1.1 Measurement of central-and Doppler-peak intensities

Keeping the cathode bias constant at \(-5\) kV and with cathode currents in the range \(1 - 30\) mA, we recorded several \( H_\alpha \) spectra at an angle \( \theta = 25^\circ \) to the emission channel through the center of the anode ring of Fig. 2.2. Cathode current was varied by adjusting gas pressure in the range \(10 - 30\) mTorr, where the variation may be fit by a power law (Fig. 4.1).

Spectra were recorded for eight values of current across two separate trials (sets 1 and 2 are denoted by filled and hollow data points, respectively). All measurements used identical parameters for the detector setup (\(10 \times 5000\) ms exposures with a constant slit width \(20\mu\text{m}\)). A typical spectrum is shown in Fig. 4.2.

The intensity of the Doppler red-shifted emission is greater than for the blue-shifted wing, with the red peak also extending out to larger Doppler shift magnitudes. Given the line of sight of our detector
Figure 4.1: Cathode current versus pressure (bias = -5 kV). The curve shows a second-order polynomial fit to the data. Filled and hollow points indicate separate experimental runs.

\[ y = 6.6 \times 10^3 (x)^2 - 1.9 \]

\[ R^2 = 0.99 \]

Figure 4.2: Typical spectrum (discrete points) observed at \( \theta = 25^\circ \) to the emission channel through the center of the anode ring. We label the Lorentzian fitted to the central peak as well as Gaussians for red- and blue-shifted Doppler peaks. The dashed curve represents the total fit.
with respect to the electrode arrangement (Fig. 2.2), this suggests the incident fast ions are traveling
towards the anode ring. In Fig. 4.2 the central peak intensity is associated with the area of a Lorentzian
fitted using the commercial software package ORIGIN \[46\]. Following the same procedure as in Refs.
\[12, 15, 23, 14\], emission intensities associated with the energetic red-shifted charge exchange peaks are
measured by fitting three Gaussians, also labeled in the figure. Energies for the incident ionic species are
determined by using Eq. (1.4) with the observed shift $\Delta \lambda$ for a given peak maximum.

Here the red peak closest to the central peak is associated with secondary charge-exchange interac-
tions arising from low energy (hundreds of eV) $H_2^+$ which are produced in the energetic processes of
Eqs. (1.5–1.7) and accelerate back and forth through the potential minimum in the vicinity of the cath-
ode edge. In contrast, we suspect that some of the low energy (10 – 100 eV) blue-shifted emission is
associated with slow $H_{z=2.5}^+$ produced at the anode. That is, the blue shifted Gaussian could be attributed
(in part) to the charge exchange and subsequent Doppler emission by $H_{2.5}^+$ and $H_{2.5}^+$ produced near the an-
ode and beginning their acceleration towards the cathode. Near the anode we expect $H_{2.5}^+$ to be produced
primarily through ionization of the gas by fast electrons.

Figure 4.3 shows the incident ion energies associated with the energetic $H^+$ and $H_{2.5}^+$ Doppler peaks,
as well as the red-shifted secondary peak and low energy blue-shifted peak. We note that the energies
for fast $H^+$ and $H_{2.5}^+$ agree to within 25% across the range of pressures and currents. This suggests the
erenergetic peaks have been identified correctly; all fast species are thought to accelerate away from the
same region of space charge within the cathode, and so should acquire similar energies.

The energies for lower energy blue-and red-shifted peaks have large uncertainties due to their close
proximity to the background noise level (in fact we were not able to identify these peaks in all of the
spectra). Taking into account fitting errors for the peak centers (20% for slow $H_{2.5}^+$, 10% for fast $H^+$
and 5% for fast $H_{2.5}^+$ and red-shifted secondary $H_{2.5}^+$), then the energies for fast $H^+$ and $H_{2.5}^+$ correspond
to about 50% of the applied cathode potential. This is similar to the Doppler and Langmuir probe
measurements performed by Moore et al. \[15\] for (gridded) cylindrical and spherical cathode geometries,
respectively. The only exception is at the lower bound of our current/pressure range, where the fast $H^+$
peak was nearly indistinguishable from the background.

In Fig. 4.4 we plot the intensities for the peaks associated with fast $H_{2.5}^+$ emerging from the cathode,
slow $H_{2.5}^+$ at the anode and central peak emission, where uncertainties are estimated using the fitting
efforts. Intensity for each peak increases roughly linearly with current over the 10 – 30 mA range.
Note that for slow $H_{2.5}^+$ we have halved the raw intensities. Paradoxically, this is because the Gaussian
fitting procedure places almost half of the blue-shifted Gaussian in the red-shifted part of the spectrum.
Nevertheless, the halved-intensities coincide with those for the central peak. In some sense we expect this
Figure 4.3: Incident ion energies for the charge-exchanging species identified in Fig. 4.2. For clarity we do not distinguish between experimental runs.

to be the case as slow ions at the anode are thought to be created by the same fast electrons responsible for unshifted emission.

The ratios of Doppler \( x \sim 2.5 \) to central peak emission versus current are shown in Fig. 4.5 (variation with pressure is similar). Intensity ratios are in the range \( 1 - 5 \) for currents greater than 2.5 mA, however there exists significant variation between trials. In the next subsection we apply the method of Sec. 3.1.3 to extract additional information.

### 4.1.2 Determination of densities for fast ions emerging from the cathode

Using the intensity measurements of Sec. 4.1.1, we apply Eq. (3.11) in order to calculate the ratios of fast \( \text{H}_{2.5}^{+} \) emerging from the cathode to fast electrons at the anode. These are the circular data points in Fig. 4.6. The calculations assume incident ion energies 50% of the applied bias, and approximate a charge exchange cross section for \( x = 2.5 \) by averaging the cross sections for \( x = 2, 3 \). Error bars are not shown as we expect the calculated values to represent order-of-magnitude estimates only (see Sec. 3.1.5). For increasing current the density ratio appears either to approach an asymptote, or decrease following a maximum in the range \( \sim 100 - 200 \).

The ratios \( \frac{[\text{H}_{2.5}^{+}]}{[\text{e}^{-}]_{r}} \) are combined with Eq. (3.10) for electron densities using a beam radius \( r \sim 0.5 \text{ cm} \) as in Fig. 2.12, which appears constant across our current range. The resulting absolute
Figure 4.4: Intensity (arbitrary units) for the various emission peaks identified in Fig. 4.2. For slow infalling H$_{2.5}^+$ the raw intensities have been halved (see text). We do not distinguish between experimental runs.

Figure 4.5: Intensity ratios of energetic Doppler ($x \sim 2.5$) to central peak emission versus current. Filled and hollow points indicate separate experimental runs.
Figure 4.6: Relative population densities of ions to fast electrons at the anode. Circles refer to fast H$_{2.5}^+$ at the cathode. Squares refer to slow H$_{2.5}^+$ at the anode. Filled and hollow data points indicate different experimental runs.

densities for fast H$_{2.5}^+$ emerging from the cathode are shown by the circular data points in Fig. 4.7.

For currents in the range 5 – 25 mA, fast ion densities exhibit a linear-like increase from $10^8 - 10^9$ m$^{-3}$. This is similar to the results obtained using the dusty plasma diagnostic in Ref. [18]. For currents in the range 5 – 25 mA there is also an approximate proportionality between intensity of fast Doppler peaks and the corresponding absolute ion densities (cf. Figs. 4.4 and 4.7). This is in agreement with the model of Ref. [14], where a linear increase of energetic Doppler emission with current would be attributed to a similarly linear increase in the flux of fast neutrals emerging from the cathode.

Finally, in combining the data of Figs. 4.1 and 4.7, we calculate the dissociation fraction $f_{\text{fast}} = [k] n_{x=2.5}^+/n^g$ of the background gas into fast H$_{2.5}^+$ emerging from the cathode (see Fig. 4.8). For currents greater than 10 mA, we find that $f_{\text{fast}} \sim 10^{-6}$ however the current dependence is not clear.

We note that when presenting the data of Figs. 4.7 and 4.8 in Ref. [21], we assumed an equal neutral and electron beam radius of 1 cm. Here however the results have been scaled to account for beam radii 0.5 cm (within the cathode) as per the model of Sec. 3.2.1. From Eq.(3.11), halving the assumed beam radius will increase the calculated values of $f_{\text{fast}}$ and $n_{x=2.5}^+$ by a factor of 4.
Figure 4.7: Absolute densities for fast H$_{2.5}^+$ in the vicinity of the cathode edge (circles). Filled and hollow data points indicate different experimental runs.

4.2 CW mode: Spectroscopy, neutron counting and simulation

This section is comprised of two subsections. The first deals with the measurement of H$_2$, Doppler and associated emission intensity to determine the central density of the UBC operating in CW mode. In the second, the current is corrected to D$_0$, comparing the dissociation fractions obtained from the measurement with similar data and determining the validity of the open-source codes developed in Sec. 3.1.1.

4.2.1 Measurement of central-end Doppler-peak intensities in H$_2$

We next look at the CW discharge with voltages 10 - 30 kV and currents 10 - 50 mA, voltage being observed to be as the highest density at the cathode edge. These results in Fig. 4.10. For these conditions, we carried out three runs, grouped by the three

Figure 4.8: Dissociation fraction $f_{\text{fast}}$ of the background gas into fast H$_{2.5}^+$ at the cathode for the low voltage discharge.
4.2 CW mode: Spectroscopy, neutron counting and simulation

This section is comprised of two subsections. The first details the measurement of $H_\alpha$ Doppler-and central-peak emission intensities for the cases of constant pressure and voltage for the IEC operating in CW mode. In the second, we measure neutron counts in $D_2$, comparing them to predictions obtained from the measurements in hydrogen and demonstrating the validity of the optical diagnostic developed in Sec. 3.1.3.

4.2.1 Measurement of central-and Doppler-peak intensities in $H_2$

**Constant voltage**

We note first that for the CW mode with voltages $10 - 30$ kV and currents $10 - 50$ mA, voltage is observed to rise as the logarithm of current as in Fig. 4.9. This is consistent with the original definition of the AHCD from Sec. 1.3.

Keeping the cathode voltage constant at $-30.0 \pm 0.5$ kV and with cathode currents in the range $1 - 30$ mA, we then recorded several $H_\alpha$ spectra at an angle $\theta = 30^\circ$ to the emission channel. Cathode current was varied by adjusting hydrogen gas pressure in the range $4 - 6$ mTorr, with the variation of pressure with current shown in Fig. 4.10. For this condition we carried out three trials, grouped by the three
We recorded fifteen spectra across the three trials in hydrogen. For each spectrum the total accumulation time was typically 30 seconds, summed over 7 - 15 shorter exposures. Our method does not require a constant exposure time and so this was adjusted to obtain the best signal; for 20 - 30 kV at 20 - 30 mA a maximum accumulation time of 30 seconds was appropriate as it proved difficult to maintain the discharge for longer than this. Fig. 4.11 shows a spectrum recorded at -30 kV and 27 mA.

The total intensity of Doppler red-shifted emission is clearly greater than for the blue-shifted wing, with the red peak also extending out to larger Doppler shift magnitudes, suggesting that fast H(3s) at the imaged region (cf. Fig. 2.3) travel primarily towards the anode wall. The central peak intensity is associated with the area under the Lorentzian fitted using a commercial software package (ORIGIN). The shaded part of the central peak corresponds to the region where $|\Delta \lambda| \leq 0.3$ Å, and corresponds to about 30% of the total central peak intensity. For the central peak this is the region of interest as it corresponds to process (1.10) where the product H($n = 3$) have energies no more than about 1 eV. Following the procedure in Refs. [12, 15, 23, 14, 21], intensities due to charge exchange processes are associated with

Figure 4.10: Pressure versus cathode current (cathode voltage $-30$ kV). Hollow data points represent separate experimental runs in hydrogen, whereas solid points are for deuterium.
Figure 4.11: Typical \(\text{H}_2\) spectrum (discrete points) observed at \(\theta = 30^\circ\) to the emission channel at the anode wall in CW mode (constant voltage of -30 kV). We label the Lorentzian fitted to the central peak as well as Gaussians for red-and blue-shifted Doppler peaks. The dashed curve represents the total fit. Shaded regions are explained in the text.

As explained in Sec. 3.1.1 and in Ref. [21], we are concerned with the red-shifted peak ‘fast \(\text{H}^+_2\),’ (the shaded Gaussian in Fig. 4.11) due to incident fast \(\text{H}^+_2\) of average mass \(2.5 \times m_\text{H}\). We have also labelled red-and blue-shifted peaks which may arise from a combination of lower-energy (tens and hundreds of eV) secondary ions produced in reactions (1.5)-(1.7) as well as projectile excitation of fast H reflected at various angles off the anode wall. We identify also a red-shifted peak for fast \(\text{H}^+\), however this peak was not always visible for the lower-current (noisier) spectra.

Incident \(\text{H}^+_2\) are assumed to be approximately monoenergetic, their energies determined using Eq. (1.4) with the observed Doppler shift \(\Delta \lambda\) for a given peak maximum. For the constant voltage condition, the fast \(\text{H}^+_2\) were found to have incident mean energy 51% of the applied cathode potential (standard deviation 5%), similar to the values found for the low voltage discharge in Sec. 4.1. It is also reminiscent of the ion energies inferred in Refs. [12, 15] featuring a ‘three-ring’ spherical grid (\(\sim 20 - 25\%\)).

Intensities for the central and fast \(\text{H}_2\) peaks at the anode were found to exhibit a linear-like increase with current over the 1 - 30 mA range, however the ratio \(I_x/I^0\) of Doppler-to central-peak emission showed no such systematic variation, with a mean of \(2.5 \pm 0.7\) (where we have defined the uncertainty as one standard deviation across the fifteen spectra). By equating the two RHS lines of Eq. (3.11), the average ratio of densities for fast \(\text{H}^+_2\) (or equivalently, fast \(\text{H}_2\) nuclei) at the cathode edge to fast...
electrons at the anode was $72 \pm 17$, however unlike the corresponding measurements for the low voltage discharge (cf. Fig. 4.6), here the ion-to-electron density ratio showed no clear dependence on current, even at the bottom of the measured current range. This tends to suggest that the ratio does indeed remain unchanging (or at the very least, does not vary systematically with current) across most of the current range associated with beamed operation. We note also that the measured ratio is only 50\% of the maximum recorded for the low voltage discharge (Fig. 4.6). We suspect this may be due to slight differences in the cathode geometries between the low voltage and CW modes; for these higher voltage measurements the cathode had twice the steel thickness of the earlier cathode. This may reduce the heating, secondary electron emission rates and thus ion densities within the cathode.

Dissociation fractions $f_{\text{fast}}$ were calculated using Eq. (3.11) and are shown by the hollow data points in Fig. 4.12. These correspond to the three constant voltage ($-30$ kV) trials in Fig. 4.10, and suggest that $f_{\text{fast}}$ increases from $5 - 30 \times 10^{-7}$ with current over the range $5 - 30$ mA. We note that values of $f_{\text{fast}}$ are associated with an order of magnitude uncertainty due to the range of possible electron and neutral beam radii, with variations up to 50\% between trials also demonstrating the difficulty in replicating particular experimental conditions. Densities for energetic $H_2^+$ at the cathode edge show a similarly linear-like dependence on current over this range, and are of order $10^{14}$ m$^{-3}$ (see Fig. 4.13).
Figure 4.13: Densities of energetic $\mathrm{H}_2^+$ at the cathode edge measured for constant voltage (corresponding to the triangular data points in Figs. 4.10 and 4.12.

We note that when we presented this data in Ref. [47], we assumed separate electron and neutral beam radii of 0.75 and 0.35 cm respectively (see Eq. (9) of that paper), whereas the data of Fig. 4.13 have been scaled to account for equal beam radii (0.5 cm within the cathode) as per the model of Sec. 3.2.1.

**Constant pressure**

The measurements from Sec. 4.2.1 were repeated for a single trial of constant hydrogen gas pressure $\sim 4$ mTorr, with cathode voltage 15 – 25 kV and currents in the range 1 – 45 mA. A plot of voltage versus current is shown by the cross data points in Fig. 4.9. We recorded seven spectra with exposure and accumulation times typical of the constant voltage measurements. Here the fast $\mathrm{H}_2^+$ were found to have incident mean energy $49 \pm 3\%$ of the applied cathode potential, and as with the constant voltage spectra the ratio $I_x^f/I_0^f$ takes a largely unchanging value of $2.6 \pm 0.8$ for the six spectra in the current range 10 – 30 mA. The average ratio of densities for fast $\mathrm{H}_2^+$ nuclei at the cathode edge to fast electrons at the anode was $73 \pm 20$, except for the lowest current spectrum (5 mA) which had a ratio of $\sim 210$.

The dissociation fractions $f_{\text{fast}}$ for the constant pressure condition are shown by the cross data points in Fig. 4.12. We observe an increase from $2 - 6 \times 10^{-6}$ for currents 10 – 45 mA, in fact for currents 10 – 30 mA the dissociation fractions for the constant pressure and voltage conditions are in agreement.

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Figure 4.14: Solid data points represent measured neutron counts for the conditions of constant voltage (-30 kV) in deuterium. Hollow data points show the values predicted using Doppler spectroscopy in hydrogen.

to within about 50%. Corresponding densities for energetic $H_{2+}$ at the cathode edge increase in a similar fashion from $4 - 9 \times 10^{14} \text{ m}^{-3}$ across the 10 – 45 mA range.

4.2.2 Measured and optically-determined neutron production rates in $D_2$

Following the procedure in Sec. 3.1.4, the solid data points in Fig. 4.14 show the total neutron production rates for the case of constant voltage (-30 kV) in the experimental discharge of deuterium, which exhibit a linear-like increase from $1 - 4 \times 10^4 \text{ s}^{-1}$ for currents 10 – 30 mA. The hollow points in the figure show values of $R_{tot}$ predicted using the $H_2$ dissociation fractions obtained in Sec. 4.2.1. Similarly, the solid data points in Fig. 4.15 show the total measured neutron production rates for the case of constant pressure (6 mTorr) in $D_2$, with the $H_2$ predictions (see Sec. 4.2.1) represented by crosses. Here the current dependence is less linear, which is to be expected as the voltage (and thus deuteron energy and fusion cross section) varies with current.

In both experimental conditions, the predicted and measured absolute rates agree to within an order of magnitude, with predicted rates also capturing the current-dependence of the observed neutron production. The agreement between predicted and measured rates is best between 10 – 20 mA for the constant voltage data, and between 15 – 40 mA for constant pressure. For the case of constant pressure,
Figure 4.15: Solid data points represent measured neutron counts for the conditions of constant pressure (6 mTorr) in deuterium. Hollow data points show values predicted using Doppler spectroscopy in hydrogen.

The divergence at lower currents (which is no more than about 50%) may correlate to a change of the discharge mode from an abnormal to normal hollow cathode discharge as current decreases (cf. Fig. 4.9). Similar divergence for the case of constant pressure and larger currents (and also, between separate trials of constant voltage) correlates to increased arcing and larger intensity variations at the upper end of our current range.

The good agreement between predicted and observed neutron production rates in the 10 – 40 mA current range appears to support our present model of the abnormal hollow cathode discharge, where the majority of fast nuclei are neutral particles emerging from the cathode apertures. Nevertheless, in the following subsection dealing with numerical modeling of the CW discharge, the calculation of Eq. (3.13) is superseded by considering the contribution of energetic neutrals and ions (moving either towards or away from the cathode).

4.2.3 Numerically simulated potentials, Doppler spectra and neutron production rate for the CW mode

In the following subsections we discuss several aspects of the numerical simulation of the CW mode, including representative potential profiles, energies and mass fluxes for the modeled species (Sec. 4.2.3),
simulated energetic Doppler spectra (Sec. 4.2.3), as well as discussion of the role of electrons in central peak emission (Sec. 4.2.3) and estimates for the absolute neutron production rate (Sec. 4.2.3).

**Electric potential, energies and mass fluxes**

The experimental fast ion densities of Fig. 4.13 vary from $0.9 - 3.0 \times 10^{14} \text{ m}^{-3}$ in the vicinity of the cathode edge in the tens of mA current range. As the spectroscopic diagnostic method is associated with an order of magnitude error it is instructive to calculate electric potential for values between $1 \times 10^{13} \text{ m}^{-3}$ and $3 \times 10^{15} \text{ m}^{-3}$.

Fig. 4.16 shows four solutions of $V$ with subsequent densities increasing by a factor of 5. The solid curve shows the potential for fast ion densities at the bottom of this range and consists of linear like variation in the interelectrode space, with a region of zero field (flat potential) within the cathode approaching the Laplace solution. This is in stark contrast to the dotted curve corresponding to the upper limit of converging density values ($2.5 \times 10^{15} \text{ m}^{-3}$), which shows a markedly non-linear potential in the interelectrode region and a virtual anode of height 22% of the applied potential. These results support the observation (using Langmuir probe [15]) and inference (from dusty plasma experiments [18]) of virtual anodes within the cathode center of similar electrode arrangements, but suggest also that this effect may become less prominent with decreasing charge density.
Figure 4.17 shows the average energies for right-going neutral H\textsubscript{2} (solid curve) and H\textsuperscript{+} \textsubscript{2} (dashed curve) particles as a function of position, corresponding to the dotted potential \( V \) in Fig. 4.16. Neutrals typically have larger average energies than their parent ions; this is because energetic ions are more likely to undergo charge exchange and so the neutral distributions are skewed towards higher energies. Ion energies are largest in the vicinity of the cathode apertures, but are otherwise symmetric about the cathode center. In contrast, energies for right-going neutrals are highly asymmetric about the cathode center - increasing away from the left anode wall, reaching a maximum in the vicinity of the right cathode aperture and decreasing only slightly beyond this (as the slower ions there produce slower neutrals).

The products \( mn\bar{v} \) of mass, density and average velocity (mass fluxes) for right-going neutrals and ions are shown in Fig. 4.18. Where the mass flux for ions is approximately symmetric about the cathode (that is, approximately equal for right-going and left-going ions at any point along the beam), for right-going neutrals the mass flux is a minimum at the left anode wall and grows to a maximum value within the cathode region.

Figures 4.17 and 4.18 suggest that most energetic neutrals are indeed produced within the cathode region. In fact, the mass flux in the beams is almost entirely dominated by energetic neutral species diverging from the cathode. Both of these results are in agreement with the AHC model. We note however that neutrals of up to 60% of the maximum energy are produced in the interelectrode region.
Figure 4.18: Mass fluxes for right-going neutrals (solid curve) and ions (dashed) versus distance from the left anode wall.

as their parent ions approach the cathode. This suggests that some significant fraction of energetic neutrals are produced within the cathode not because the majority of ions are accelerating away from a virtual anode at the center, but because ions accelerating towards the center are most energetic there. This aspect of the solution is consistent with the conventional picture of a ‘convergent ion focus,’ and is repeated even for lower current solutions where a virtual anode is not visible. Figure 4.17 suggests a small (~10%) increase in average fast neutral energies between the cathode center and cathode aperture. Our calculations were not able to replicate the nearly 100% increase exhibited in Ref. [15], however these were measured for a system featuring a gridded (two-ring) cathode and using an external ion source, so our theoretical picture may not be directly analogous.

The energy distribution function for electrons with energy $K_{i,j}$ in segment $j$ in the beam is,

$$f_{i,j} = D_{i,j}/ \sum_i D_{i,j}^{-1},$$

and in Fig. 4.19 we plot the energy distribution function for right-going electrons at the rightmost anode wall. The calculated distribution does not support the assumption of highly monoenergetic electron beams as inferred for a lower energy discharge [13], nevertheless electron energies at the anode wall are peaked towards the full applied potential and so in a later section we will consider the role of energetic
Simulation of energetic Doppler emission

Here we use the numerical solutions of Sec. 4.2.3 to simulate energetic $\text{H}_\alpha$ Doppler spectra from fast $\text{H}$ atoms produced in the excited $3s$ state in the process (1.6) as described in Sec. 3.2.1. Figure 4.20 shows the energetic Doppler components at $\theta = 31^\circ$ to four points along the beam corresponding to the solution in Figs. 4.18 and 4.19. These are the cathode center, the right cathode aperture, half-way between the aperture and anode, and the right anode wall. The computed spectra have been normalized to the maximum intensity and smoothed using a 5-point averaging method. Note that the unshifted (central peak) emission has not been modeled.

The spectrum at the cathode center is symmetric, which we expect from Figs. 4.17 and 4.18 as the cathode center features equal populations of left-and right-going ions with equal average energies. For this spectrum the centers of the Gaussian-like, blue and red shifted Doppler peaks have the same intensity and Doppler shift magnitude. Moving away from the center towards the right anode wall, the blue shifted peak moves closer towards the unshifted position, and its intensity decreases relative to the red-shifted peak, ultimately decreasing at the wall. This is consistent with the expectation that both densities and energies of left-going ions decrease towards the right anode. On the red shifted side, the...
Figure 4.20: Waterfall plot for the energetic component of Doppler H\(_\alpha\) spectra simulated for an angle \(\theta = 31^\circ\) at different positions along the beam, with the imaginary detector facing towards the right anode wall.

peak position stays relatively constant at a large value (16 ± 2 Å), similar to an experimentally observed peak at ~ 20 Å for the combined H\(_2^+\) and H\(_3^+\) emission (cf. Fig. 6 in Ref. [47]). Our calculations do not predict the blue shifted peaks of the experimental data, however this is likely because our model does not include reflection of energetic neutrals from the anode, which was the suspected cause of those peaks. The simulations agree with some of the findings of Shrier et al. [24] in that Doppler shift emission is dominated by diverging neutrals in the beam. We note that similar behavior is predicted even for the case of lower currents where the potential profile contains no virtual anode.

Central peak emission

For the first time, the solutions of Fig. 4.17 and 4.18 allow us to quantify the role of electrons in the central peak emission. For a similar discharge in Ref. [13], the central Lorentzian peak was found to arise primarily due to dissociative excitation of target gas molecules by electrons,

\[
e^− + H_2 \rightarrow e^− + H_2^+ \rightarrow e^− + H(1s) + H(n = 3),
\]

(4.2)

with a steady-state volumetric production rate (for segment \(j\)) equal to \(n^− n^S \sigma^D v^−\) (with \(\sigma^D\) the cross...
section for electron excitation as given in Ref. [40]). This was in part due to a wealth of literature ascribing the central peak to electron excitation in similarly energetic, parallel-plate and hollow cathode discharges for pressures of 0.1 – 1 Torr in H₂ as described in Sec. 1.2.1. A problem with this explanation was that the rapid decrease of central peak intensity immediately outside the cathode did not conform to collisional-radiative (CR) modeling of the electron excitation process for monoenergetic electrons, but this was thought to arise due to the experimental EEDF being something other than perfectly monoenergetic. The assumption of a primarily electron-induced central peak thus informed the optical diagnostic of Ref. [21].

Given the prevalence here of energetic neutral H₂ in the emission channels, we may now analyze the possibility that the central peak may also be due to excitation of the gas by fast H₂. For this process the production rate (per unit volume) is \( n^* n \sigma \frac{\tau}{v^*} \) (where the cross sections \( \sigma \) for excitation of target H₂ by energetic neutral H₂ of density \( n^* \) were taken from Williams et al. [48]).

Using the calculated densities and velocities for both left-and right-going electrons and fast H₂, their relative contributions to central peak emission intensity (arbitrary units) as a function of distance from the cathode edge are shown in Fig. 4.21. Whereas the electron induced emission (dotted curve) increases from 0 – 2 cm before falling away again, the neutral induced emission (solid curve) shows an exponential-like decrease across the entire range of distances. Whereas the electron excitation drops from its maximum by only \( \sim 50\% \) over the first 8 cm, the decrease in neutral excitation (90%) is more in line with the 80% drop observed in experiment (cf. Fig. 7 of Ref. [13]). Finally, from the figure we see that electrons contribute as little as 1% of central peak emission near the cathode edge, up to a maximum of \( \sim 10\% \) near the anode wall. This suggests that the central peak is primarily due to neutral excitation, as opposed to electron excitation as is the case for less energetic hollow cathode discharges operating at higher pressures (0.1 – 1) Torr [9].

In applying the optical diagnostic of Sec. 3.1.3 we must therefore take care to measure the electron excitation as a value \( \sim 10\% \) of the total central peak intensity. We note that the data of Figs. 4.12-4.15 already incorporate a similar correction as only 30% of the total central peak emission had been attributed to electrons.

**Neutron production rates in deuterium**

We can estimate the contribution of both left-and right-going D₂⁺ and fast neutral D₂ to the volumetric neutron production rate \( R_j^{\text{vol}} \) in segment \( j \) by the beam-background reaction (1.1) in a discharge of deuterium which has the same pressure, cathode current and voltage as for a modeled discharge of hydrogen. For any given fast species we have,
Figure 4.21: Relative central peak emission intensities due to electron (dotted curve) and neutral (solid curve) excitation of the background gas, versus distance from the cathode edge.

\[ R_j^{\text{vol}} = n_j n_c^F (x_0 D_j - D) v_j, \]  

(4.3)

where \( n_j \) is the absolute number density of fast nuclei in segment \( j \) (of mass \( x \) times that of a single deuteron, and velocity \( v_j \)). We can thus obtain the total neutron production rate for this species in any given segment by taking the product of \( R_j^{\text{vol}} \) with the segment volume.

Figure 4.22 shows simulated (circles) and experimental neutron production rates (squares) as a function of cathode current for the case of constant voltage -30 kV and pressure (~5 mTorr) in \( D_2 \). Note that in the AHC model the particle densities increase approximately linearly with cathode current and so we estimate the cathode currents for various simulated fast ion densities using the raw experimental data of Fig. 4.13. The variation of simulated neutron production rates agrees well with the experimental values, suggesting that beam-background reactions do indeed dominate for the present plasma parameters.
4.3 Pulsed IEC: Experiments and simulation

4.3.1 Measurement of voltage-current-pressure relations for the pulsed discharge

Figure 4.23 shows measured curves of peak voltage (25 – 35 kV) versus peak current (1 – 7 A) for pulsed discharges of pressure 17 – 130 mTorr in H₂. The data suggest that for each different pressure, voltage increases proportional to the logarithm of current. We note that this same logarithmic relationship was associated with the CW mode. In conjunction with the visual confirmation of collimated emission channels for the pulsed system (see Fig. 2.9), the recorded V-I curves are highly suggestive of similar kinetics.

We observe current to increase with pressure for any given voltage, and the data of Fig. 4.24 (corresponding to the Doppler measurements in H₂ in the next subsection) clearly show that current asymptotes towards some saturation value (~ 7.5 A) which may indicate the existence of an upper boundary of current for any given voltage. This has important implications for the current-dependence of \( f \) given constant voltage and thus the extrapolation for values of peak \( f_{\text{fast}} \).

In Fig. 4.25 we once again present the values for \( f_{\text{fast}} \) at the cathode edge, measured for constant voltage -30 kV in the CW mode and corresponding to the hollow data points from Fig. 4.12. This time however, we portray possible extrapolations of the fast fraction to larger currents. These allow...
estimates for the peak fast ion density $n^+(I) = f(I)n^G(I)$ at the cathode aperture as a function of the peak cathode current $I$ and the gas density $n^G$, and are used as input for the numerical simulation. From the CW measurements we gleaned that $f_{\text{sat}}$ was proportional to the logarithm of current (dashed line in the figure), or equivalently that $I \propto P \times \exp(f)$. As current approaches its saturation value $I_{\text{sat}}$ however, pressure is observed to increase like $P \propto 1/(I_{\text{sat}} - I)$. For Fig. 4.25 we thus suggest that a more appropriate extrapolation is $f \propto \log[I(I_{\text{sat}} - I)]$. This is shown by the solid curve in the figure, which exhibits a maximum in the units of Amps range before decreasing again.

In Fig. 4.26 we plot measured time constants as a function of current for approximately constant cathode voltage ($33 \pm 1$ kV) in the deuterium discharge. The solid symbols are for the peak part of the pulse $\tau_{\text{peak}}$ and remain constant at about $20 \pm 10 \mu$s for currents above 1 A. In contrast, time constants for the lower energy trail-off $\tau_{\text{trail}}$ (hollow symbols) peak at around $2 \pm 1$ ms and decrease to $30 - 50\%$ of this value as current grows from 1 - 2.5 A. As the time constants are related to the plasma resistance $R$ and charging capacitance $C$ through $\tau = RC$, we infer that the current for the slow part of the pulse is plasma-limited, however the same is not necessarily true for the peak part of the pulse and so beamed operation with higher peak currents may be possible, given larger peak voltages.
Figure 4.24: \( \text{H}_2 \) gas pressure versus cathode current for the -30 kV Doppler measurements of Sec. 4.3.2

Figure 4.25: Extrapolating the constant voltage measurements of \( f \) from Fig. 4.12 to the peak currents measured for the pulsed system at -30 kV and varying pressures. Dashed and solid curve extrapolations are explained in the text.
Figure 4.26: Time constants $\tau_{\text{peak}}$ (solid symbols) and $\tau_{\text{traj}}$ (hollow symbols) versus cathode current for the neutron counting experiments in D$_2$.

4.3.2 Doppler spectroscopy measurements

Constant Voltage

Keeping the peak voltage relatively constant at $-32 \pm 1$ kV and with peak currents in the range $1 - 7$ A (for pressures given in Fig. 4.26), we recorded 10 H$_3$ spectra at an angle $\theta = 31^\circ$ to the beam at the anode, and a single spectrum at $\theta = 40^\circ$ to the beam exiting the cathode. Figure 4.27 compares the (solid) lineshape at the cathode aperture to the (dotted) spectrum at the anode wall, both measured for pressure $70 \pm 5$ mTorr and with a current peak $6.0 \pm 0.5$ A.

The asymmetry of these spectra is consistent with measurements of the low voltage and CW modes where Doppler emission was dominated by diverging neutrals. That is, for the anode spectrum the red-shifted emission is more intense and extends out to larger Doppler shift magnitudes - suggesting the trajectory for fast H(3s) is away from the detector (towards the wall). Conversely, at the cathode the blue shifted emission dominates, as it should for a mass flux moving primarily away from the cathode aperture (toward the detector). As with the steady-state measurements, both central and Doppler peak intensities increase with cathode current, however due to noise the exact relationship was not clear; the ratio of Doppler to central peak intensities is of order 1 across the range of currents.

We used the commercial software package ORIGIN to identify peak positions of energetic Doppler
Figure 4.27: $H_\alpha$ spectra measured at the anode wall (dotted) and cathode aperture (solid) for a voltage of $-32 \pm 1$ kV and current $\sim 6$ A.

Gaussians for the spectrum recorded at the wall. As with our earlier spectroscopy on the steady-state discharges, we are concerned with the combined red-shifted peak arising from $H_2^+$ and $H_3^+$, which we treat as arising due to incident ions of average mass $2.5 \times m_H$. For this peak the average Doppler shift was $10 \pm 1$ Å and using Eq. (1.4) this yields incident ion energies of just $12 \pm 2\%$ of the applied cathode potential. This is significantly smaller than the ratios $\sim 50\%$ observed for the steady-state discharges. From Fig. 4.25, we suspect this is because most of the capacitor charge is expended during the lower voltage part of the pulse (over $0.1 - 10$ ms timescales) where current and voltage are both smaller than $50\%$ of their peak value.

Magnetic deflection

The electron beam in the pulsed $H_2$ discharge was deflected by a $10$ mT magnetic field as described in Sec. 2.5, allowing comparison of typical $H_\alpha$ spectra with those with a lessened contribution of electron excitation in the anode wall region.

In the presence of the $10$ mT field, we observed electrons fluorescing the surface at two regions in the chamber - the anode wall as in the undeflected case, as well as a region of similar area on the chamber floor. This suggests that electrons with energies of order the applied potential were largely undeflected by the field, with lower energy electrons confined by the fringing field lines of the magnet. As the cross
sections for electron dissociative excitation drop exponentially with increasing projectile energy [40], we thus expect that we have removed the primary source of electron excitation of atomic H\((n = 3)\) in the energetic beams.

Figure 4.28 shows two spectra measured for a peak cathode voltage -30 kV, peak current 2 A and a pressure about 50 mTorr in H\(_2\). The solid and dotted lineshapes are for emission channels in the non-deflected, and the deflected cases respectively. We calculate a decrease in the central peak intensity (area) of only \(\sim 10\%\) between the non-deflected and deflected cases, a difference which is about the same as the estimated experimental error for our apparatus. This result supports the calculations of Sec. 4.2.3 suggesting that for these plasma parameters, the electron component to central peak emission is of order \(10\%\) at the anode wall. Moreover we observe no visual excitation of the gas by deflected electrons.

### 4.3.3 Experimental neutron production in \(\text{D}_2\)

The solid data points in Fig. 4.29 show the total number \(N_{\text{pulse}}\) of neutrons produced per pulse for voltages \((32 \pm 1 \text{ kV})\) at pressures \(10 - 60 \text{ mTorr}\) in deuterium. Unlike the results for the CW mode, which demonstrated a linear-like increase in steady-state production rates for cathode current \(10 - 30 \text{ mA}\), here we observe a peak in production around \(1.5 \text{ A (37 mTorr)}\), with the signal dropping to noise level for currents less than 750 mA or larger than about 2 A. The experimental uncertainty for these
Figure 4.29: Experimental (solid) and simulated (hollow) values of neutrons per pulse for voltage $32 \pm 1$ kV in deuterium.

 Measurements is large as the total number of neutrons produced per pulse ($1 - 3 \times 10^3$) equates to just $1 - 10$ neutron counts over $100 - 200$ pulses. The hollow data points represent numerically simulated values and are discussed in Sec 4.3.4.

4.3.4 Numerical simulations of the pulsed discharge

Simulated Doppler spectra

We simulate the observed Doppler shifts of Fig. 4.27 for a spectrum at the cathode aperture using the method of Sec. 3.2.4. Numerical solutions achieved convergence for experimentally-relevant parameters of pressure (65 mTorr), peak voltage and current (-32 kV and 7 A respectively), pulse decay time constants similar to those in Fig. 4.26 and an estimated beam spread of $20^\circ$ outside the cathode apertures. Fig. 4.30 shows the simulated lineshape at time $t$ given accumulations beginning from $t = 3 \mu s$. Output is smoothed using a 5-point averaging method, and clearly demonstrates that the peak Doppler shift magnitudes can indeed decrease over time. For the peak part of the pulse ($t \lesssim 100 \mu s$) we have energetic Doppler emission out to $20 - 25$ Å on the blue shifted side of the spectrum. During the lower energy part of the pulse however, this energetic emission is orders of magnitude less intense than for lower energy emission with Doppler shifts $\Delta \lambda < 15$ Å. Our simulation thus suggests that for experimental spectra
accumulated over the entire timescale of a pulse, the most highly energetic Doppler emission is lost to noise.

We note that these parameters are on the boundary between converging and non-converging numerical solutions; in choosing the initial value for \( f \) only the current-limited extrapolation (solid curve in Fig. 4.25) provides a stable potential. Even then, convergence was obtained only by assuming a slightly larger beam radius (0.75 cm rather than 0.5 cm), which reduces the extrapolated peak value of \( f \) as the optically determined dissociation fraction scales as \( 1/r^2 \). We also had to relax the convergence criterion of Sec. 3.2.1 even further, applying the criterion within the cathode only. And so, while our simulation can model the observed reduction in neutral energies between the pulsed and steady-state cases, these experimental parameters would seem to represent the extreme upper limit of particle densities (given our electrode geometry) for which the method of Sec. 3.2.1 can provide a steady state solution.

Two additional points of interest are that the simulated Doppler accumulations of Fig. 4.30 do not replicate the strong asymmetry of the measured aperture spectrum in Fig. 4.28, and also that the simulated intensity drop between the aperture and the anode wall is about an order of magnitude larger than that observed experimentally. As our numerical method replicates the steady-state Doppler emission quite well, the discrepancy here suggests that Doppler emission for the pulsed discharge (which features lower energy neutral atoms and molecules for the majority of the pulse duration) may actually be
Figure 4.31: Simulated Doppler Hα emission during a -32 kV, 7 A, 70 mTorr pulse assuming H(3s) excitation due to charge exchange only. Spectra are calculated for the cathode aperture (solid lineshape) and the anode wall (dotted).

dominated by some process other than charge exchange.

One such process is (re)excitation of ground state, fast neutral H upon impact with the background gas,

\[ H^*(1s) + H_2 \rightarrow H^*(3s) + H_2, \]  \hspace{1cm} (4.4)

which is distinct to both charge exchange, and fast neutral excitation of the slow gas molecule. In fact for projectile energies in the units and tens of keV range, the cross sections for this process are comparable to those for the equivalent emissive branch of H⁺ charge exchange [42]. We anticipate that this may contribute significantly to the observed Doppler emission for lower energy neutrals far from the cathode, as projectile excitation does not undergo the same exponential decay of emission intensity in space.

We demonstrate the relevance of this process by comparing simulated Doppler spectra near the cathode edge (solid lineshape) and anode wall (dotted) in the case of excitation by charge exchange only (Fig. 4.31) and excitation due to both charge exchange and projectile excitation (Fig. 4.32). The latter figure bears a qualitatively closer resemblance to the experimental spectra of Fig. 4.27, replicating both the asymmetry and intensities of the observed Doppler peaks.
Figure 4.32: Simulated Doppler $H_\alpha$ emission corresponding to Fig. 4.31, with $H(3s)$ excitation due to charge exchange and projectile excitation.

Simulated neutron production

The simulated values of $N_{\text{pulse}}$ are shown by the hollow symbols in Fig. 4.29. These typically differ from the experimental values by less than an order of magnitude, giving experimental confirmation for the existence of energetic nuclei ($\sim 15$ keV) in the beam and demonstrating that the peak part of the pulse is highly analogous to the steady-state system operating at lower currents. In some sense this also lends support to the conclusion of the previous section - that discrepancies between the experimental and simulated pulsed Doppler spectra are most likely due not to incorrect calculation of neutral densities or energies, but rather due to contribution from an unexpected excitation process for projectile $H$.

We note that the simulated values of $N_{\text{pulse}}$ also exhibit a peak in the production rate (albeit less pronounced and occurring for a higher current), however interpretation of this is confounded in that the experimental measurements of pressure did not increase monotonically with peak current. We suspect this was due to incorrect pressure readings arising from occasional cathode arcing and thus temporary heating of the gas. Despite these difficulties, the qualitative similarity between experimental and simulated results is consistent with the idea that $f$ may plateau for some value of cathode current, and then decrease with increasing current and pressure.

These results allow us to make some estimate about the relative efficiency of the pulsed system, where
we define the efficiency of neutron production $\xi$ (units $J^{-1}$) as the total number of neutrons produced per total energy input. For the steady-state system this is just $\xi = N_{\text{tot}} / VI$, where $N_{\text{tot}}$ is the absolute neutron production rate (s$^{-1}$) with $V$ and $I$ the steady state cathode voltage and current. For the CW mode we have $N \sim 3 \times 10^4$ s$^{-1}$ for -30 kV and 25 mA, yielding $\xi = 40$ J$^{-1}$. For the pulsed system we must integrate over changing values of current and voltage,

$$\xi_{\text{pulse}} = \frac{N_{\text{pulse}}}{\int V(t)I(t)dt}$$

which for the voltage traces of Fig. 2.10 suggests a value $\xi_{\text{pulse}} \sim 30$ J$^{-1}$, similar to the steady state case. According to simulation however, the pulsed discharge can provide instantaneous production rates up to $10^7$ s$^{-1}$, which are 1000 times larger than the steady-state value.

In this work we considered two alternate and seemingly opposed models for the cylindrically-symmetric inertial Electrostatic Confinement (IEC) device operating in the plasma discharge mode for runs of kV cathode voltages and units and tens of mTorr gas pressures. In the conventional picture, the discharge is dominated by energetic ions which are produced in the interelectrode region by fast electrons, and undergo charge exchange as they accelerate towards the inner electrode (the cathode). In the alternate model, deemed the Atmospheric Hollow Cathode (AHC) discharge, many flux in the discharge is thought to be dominated by energetic neutrals emerging from the cylindrical cathode apertures and arriving from charge exchanging ions which are produced primarily within the cathode center and undergo charge exchange as they accelerate away from a virtual anode which is formed there. That is, the AHC model challenges the assumption that there is any significant ion confinement in the system.

Using the AHC model we developed an optical diagnostic for fast ion densities emerging from the cathode aperture of a cylindrically symmetric, steady-state IEC discharge in He for cathode voltages of units and tens of kV and units and tens of mTorr pressures. This diagnostic is novel in the sense that it allows measurement of absolute densities (as opposed to relative densities) of fast ions using a single spectrum of $H_\alpha$. Applying this diagnostic to a steady-state hydrogen discharge under conditions of constant voltage (-30 kV) and pressure (~ 5 mTorr) with 10 – 50 mA currents, we were able to make predictions for the neutron production rates for analogous discharges of deuterium. Predictions capture the variation of experimental fusion rates with DC current in the range 10 – 50 mA and agree with measured values to within an order of magnitude, suggesting that the energetic component of the discharge may indeed be treated as being dominated by energetic neutrals of energy $\sim 10^7$ the applied cathode potential as in the AHC model. It is also interesting to note that this work represents the first controlled nuclear fusion experiments carried out at the School of Physics.
Chapter 5

Conclusions and future work

In this work the author considered two alternate and seemingly opposed models for the cylindrically-symmetric Inertial Electrostatic Confinement (IEC) device operating in the gaseous discharge mode for tens of kV cathode voltages and units and tens of mTorr gas pressures. In the conventional picture, the discharge is dominated by energetic ions which are produced in the interelectrode region by fast electrons, and undergo charge exchange as they accelerate towards the inner electrode (the cathode). In the alternate model, deemed the Abnormal Hollow Cathode (AHC) discharge, mass flux in the discharge is thought to be dominated by energetic neutrals emerging from the cylindrical cathode apertures and arising from charge exchanging ions which are produced primarily within the cathode center and undergo charge exchange as they accelerate away from a virtual anode which is formed there. That is, the AHC model challenges the assumption that there is any significant ion confinement in the system.

Using the AHC model we developed an optical diagnostic for fast ion densities emerging from the cathode apertures of a cylindrically symmetric, steady-state IEC discharge in H₂ for cathode voltages of units and tens of kV and units and tens of mTorr pressures. This diagnostic is novel in the sense that it allows measurement of absolute densities (as opposed to relative densities) of fast ions using a single spectrum of H₀. Applying this diagnostic to a steady-state hydrogen discharge under conditions of constant voltage (-30 kV) and pressure (~ 5 mTorr) with 10 – 50 mA currents, we were able to make predictions for the neutron production rates for analogous discharges of deuterium. Predictions capture the variation of experimental fusion rates with DC current in the range 10 – 50 mA and agree with measured values to within an order of magnitude, suggesting that the energetic component of the discharge may indeed be treated as being dominated by energetic neutrals of energy ~ 50% the applied cathode potential as in the AHC model. It is also interesting to note that this work represents the first controlled nuclear fusion experiments carried out at the School of Physics.
The spectroscopically determined fast ion densities were used as inputs for our semi-analytic simulation of the hydrogen discharge, based on matrix representations of the coupled electron ionization and ion charge exchange processes. We thus calculated electrostatic potential, Doppler spectra and average particle energies and densities along the beam axis for the steady-state discharge in H₂. Calculated potential profiles support the existence of a virtual anode as in the AHC model, moreover simulated mass fluxes are dominated by diverging neutrals and simulated Doppler spectra reconstruct many of the asymmetries observed in our steady state experiments, as well as in the earlier work of Shrier et al. [14]. Perhaps a more surprising outcome of our simulations is that the virtual anode, divergent neutral fluxes and Doppler spectra may all occur in conjunction with mass fluxes for inflowing and outgoing ions which are approximately equal. Also, the simulated Doppler asymmetries far from the cathode do not appear to be highly contingent on a virtual anode being present.

It is a contribution of this work, therefore, that we suggest the necessity for a hybrid of the two present models. In the new picture, mass flux in the beam is indeed dominated by energetic neutrals produced within and emerging from the cathode apertures, but this is primarily a result of ions undergoing charge exchange as they accelerate towards and through the cathode center where they are most energetic. It is doubtful that there is any significant confinement of energetic ions due to the units of cm mean free path for charge exchange, but certainly the population of ions which are converging towards the cathode is at least as significant as those that are accelerating away from it. Across the studied range of steady state plasma parameters, a virtual anode seems to be an inevitable consequence of slow ions being produced by charge exchange of fast ions within the cathode. However, it is also possible that the height of this virtual anode is largely just a reflection of the charge density in the system, rather than determining the trajectories of energetic neutrals. Given that our numerical simulations neglected surface interactions, the presence of a virtual anode in the simulation results suggests electron ionization of gas in the interelectrode region may be as important to the final potential profiles as any surface interactions occurring within the cathode. Future modeling of steady state IEC might aim to relate such surface interactions to the energetic electron beams which we claim are responsible for sustaining the gaseous discharge.

Constructing a pulsed power supply, we applied Doppler spectroscopy to a pulsed IEC discharge for the first time. We demonstrate that the pulsed system may operate in a beamed mode as for the steady-state discharge, featuring a logarithmic dependence of peak voltage on peak current with asymmetries in the Doppler spectrum suggesting the discharge is dominated by diverging neutrals. These observations are all highly indicative that similar physics takes place for both the steady state and pulsed regimes. With our cylindrical cathode operating at a constant voltage of -30 kV, increasing the hydrogen gas pressure
in the range $20 - 200$ mTorr reveals a maximum possible peak current of order $7.5$ A for the pulsed beamed mode. Future work may aim to measure peak voltage-current curves for even larger voltages and currents, and determine how the inferred current limit may depend on cathode voltage and geometry. Comparing neutron production output with energy input between the steady state and pulsed systems, we also suggest that for the present electrode geometry and a cathode voltage of $-30$ kV, the pulsed discharge is no more efficient than the steady state system. Given however that each pulse lasts only $\sim 10$ ms we could theoretically sustain up to 100 pulses per second (ignoring the charging time for a single capacitor), which would improve upon the steady-state, per-second production rate without any cost in efficiency.

We anticipate that future researchers may wish to examine the Doppler spectra for the emission channels in pulsed IEC devices operating at higher peak voltages and currents in order to infer the trajectories, energies and densities of fast neutrals. This presents a challenge for practical spectroscopy in that, while suitably energetic neutrals are produced during the peak part of the pulse, it is possible they can only be observed for $\text{H}_\alpha$ exposures over tens of microsecond timescales. Aside from using a spectrometer capable of such short exposures, another option is to measure the time-variation of accumulated intensity at a given wavelength by coupling the output of a photomultiplier with a fast integrator circuit. Future spectroscopic studies may employ one of these approaches. Our results also suggest we can make an order of magnitude prediction for fusion rates in the pulsed system by applying our optical diagnostic to a lower current steady state system operating at the same voltage, and where we have some estimate for the associated current limit. Surprisingly the beam radius has no bearing on the prediction for fusion rates, however uncertainty in this value will dominate the uncertainty in measurements for absolute fast ion densities and dissociation fraction into fast nuclei, $f_{\text{fast}}$. On a related note, we expect that this diagnostic may also have application towards calculating the thrust produced for a similar electrode geometry configured as a thruster [17]. Comparing optically-determined thrust values with those obtained through other means (for example, measuring the force applied to a torsional plate) may yield additional information about the applicability of the underlying plasma model.

This work may have also cast new light on the originating-processes leading to the observed $\text{H}_\alpha$ spectra. We suggest that for Doppler shifted emission in the pulsed system, projectile (re)excitation of fast atomic $\text{H}$ may be as important an excitation process as the initial excitation due to charge exchanging ions. As the cross section for projectile excitation decreases with increasing energy, this process may also render a significant contribution to the lower energy ($< 5$ keV) Doppler peaks we observe in a steady state system.

It is of interest to note that our modeling was not able to replicate the apparent acceleration (from rest) of charge-exchanging ions away from the cathode center as in Ref. [15]. For our simulations the
mass flux (and thus Doppler peak intensities) do indeed increase by as much as 30% across the cathode region, however the mean energy increase for ions away from the cathode center is only of order 10%. We suggest that this isn’t necessarily a contradiction, and that future simulations of the Doppler spectrum may be able to show this by including (for example) additional ion species and excitation processes.

In combination with a magnetic deflection experiment, our numerical simulation also yields one final surprise, which is that central peak emission in the interelectrode beams is dominated by excitation of the gas not by electrons, but rather by fast neutrals. This implies a small correction to our optical diagnostic, as the electron contribution to the total central peak intensity may be as low as 10% for the steady state system. We anticipate that future magnetic deflection experiments may be able to confirm the contribution of fast electrons to the central peak emission as a function of plasma parameters like pressure and cathode voltage, as well as distance along the beam. If the central peak does indeed originate due to fast neutrals more than electrons, it would act to differentiate IEC even further from the traditional hollow cathode effect.
Bibliography


Appendix

In this brief Appendix we show how we derive Eq. (3.7) in Sec. 3.1.3. We begin with Eq. (3.6),

$$\frac{[a] I^*_x}{[a] I^0} = \frac{[c] n^*_x A_{3s \rightarrow 2p} \times \exp(-A_{3s \rightarrow 2p} t_{sep})}{[a] n^0(0.16 A_{3s \rightarrow 2p} + 0.44 A_{3p \rightarrow 2s} + 0.40 A_{3d \rightarrow 2p})},$$

which relates the intensity ratio of Doppler and central peak emission \( \frac{[a] I^*_x}{[a] I^0} \) at the anode to the density of slow \( \text{H}(n = 3) \) at the anode \( [a] n^0 \) and the density of fast \( \text{H}(3s) \) at the cathode \( [c] n^*_x \). We may obtain an expression for \( [c] n^*_x(3s) \) by rearranging the steady-state form of Eq. (3.1) for Doppler emission,

$$[c] n^*_x(3s) = \frac{[c] n^*_{3s} \times k_x(3s)}{(A_{3s \rightarrow 2p} + k_x(3s)n^0)}, \quad (A-1)$$

where the only optically allowed de-excitation transition is from \( \text{H}(3s) \) to \( \text{H}(2p) \). For central peak emission, the steady-state form of Eq. (3.2) considers each sublevel \( (n, l) \),

$$k^D(n, l)n^0 - n^0 = \sum_{(n', l') < (n, l)} A_{(n, l) \rightarrow (n', l')} n^0(n, l), \quad (A-2)$$

and so the aim is to express the RHS in terms of the total density \( n^0(n = 3) \) of slow atomic \( \text{H} \) with principal quantum number \( n = 3 \). According to Ref. [41] the branching ratios for electronic excitation of ground state \( \text{H} \) into the 3s, 3p and 3d levels are 0.16, 0.44 and 0.40 respectively. Eq. (A-2) can thus be rewritten,

$$k^D(n, l)n^0 - n^0 = [0.16 A_{3s \rightarrow 2p} + 0.44(A_{3p \rightarrow 2s} + A_{3p \rightarrow 1s}) + 0.40 A_{3d \rightarrow 2p}] \times n^0(n = 3),$$

with the understanding that the optically allowed de-excitation transitions from \( n = 3 \) are now 3s to 2p, 3p to 2s, 3d to 2p and 2p to 1s. And so our expression for \( [a] n^0(n = 3) \) is given by,

$$[a] n^0 = \frac{k^D(n, l) [a] n^0 - n^0}{[0.16 A_{3s \rightarrow 2p} + 0.44(A_{3p \rightarrow 2s} + A_{3p \rightarrow 1s}) + 0.40 A_{3d \rightarrow 2p}]} \cdot \quad (A-3)$$

Equations (A-1) and (A-3) are then substituted directly into Eq. (3.6). By rewriting any rate coefficients in the monoenergetic form \( k = \sigma v \), we thus obtain Eqs. (3.7-3.9) as given in Sec. 3.1.3.