CHAPTER 8: 3D FINITE ELEMENT MODELLING OF THE BURIED PIPE TESTS

8.1 INTRODUCTION

The modelling in two dimensions of the buried pipe tests with the state parameter constitutive model was described in the previous Chapter. This Chapter deals with the modelling of the buried pipe tests in three dimensions with the same state parameter constitutive model. Prior to discussing the modelling of the buried pipe tests, the numerical simulation of the pipe stiffness tests is presented. The aim of this preliminary analysis was to validate the choice of engineering properties of the pipe.

The various pipe tests are then analysed, ranging from the simulated pavement tests (large plate tests) to the non-paved tests (or rigid plate tests), the latter being conducted chiefly in a soil box in the laboratory, with a further four tests carried out on installations in trenches cut into a stiff clay (field tests). The influence of mesh discretization for all tests, and the sidewall boundary condition (perfectly rough or perfectly smooth), for the tests conducted in the soil box, are examined.

In order to simulate more closely the behaviour of pipes in the soil box tests, the rigidity of the sidewall boundary condition was found to be too severe. More realistic representation of the stiffness of the sidewall plywood and bracing, and relaxation of nodal fixities were applied to the analyses. This finding poses some questions over the validity of the soil box tests and the future of physical modelling of buried pipes.

8.2 FEATURES OF THE 3D ANALYSES

Element type 43 of the AFENA finite element package was devised as the 3D equivalent of the plane strain element type 44, as a consequence of the inadequate performance of plane strain analysis for the modelling of the buried pipes subjected
to a surface patch loading. Both element types incorporate the non-linear, state parameter constitutive model. Input data are identical between the element types.

An eight-noded parallelepiped, or brick, was the only 3D element available for implementation of element type 43. The pipe was also modelled with 8-noded brick elements. Brick elements were applied with eight Gauss points. In all analyses of the buried pipe tests, the pipe was modelled as a single element thickness, elastic material with justification of this choice being provided later in the Chapter. Young’s modulus was therefore a basic material input. In order to balance the values of EA and EI specified for the different pipes in Table 7-I, both the equivalent thickness of the pipe, t, and the equivalent Young’s modulus, E, were manipulated to provide the correct axial and bending stiffnesses. It can be seen in Table 8-I that the resultant values of E are quite low, however the thickness of the pipe adopted in the model is more than twice the actual height of the ribbed profile.

Without any available relevant data on the longitudinal axial and flexural stiffness of these pipes, no attempt was made to apply orthotropic pipe properties in the finite element model. Rather the pipe was assumed to act isotropically, and the longitudinal behaviour was controlled by the circumferential properties of the pipe, an assumption which is in agreement with common pipe design methods based on ring theory.

The performance of the pipe element in bending was enhanced by adoption of non-conforming displacements to model pure bending (after Taylor, Beresford and Wilson, 1976). The modified “non-conforming element” allows curvature of the sides of isoparametric elements, while minimizing the strain energy within the element. Taylor et al. (1976) give numerous examples of the superior performance of the non-conforming element over the conventional finite element for beams and shells subjected to flexure. Dr. Hossein Taiebat, of the Centre for Geotechnical Research at the University of Sydney, implemented the non-conforming element for three-dimensional analysis into AFENA.
Interface or joint elements were not available for three-dimensional analysis, so the simulations of the buried pipe installation were continued with either a perfectly smooth or perfectly rough sidewall.

Only a quarter of the engineering problem needed to be modelled owing to symmetry in two directions.

Initial stresses in the soil mass were taken to be uniform and were calculated based on the effective vertical stress (y plane) at the springline of the pipe. For this purpose, an effective soil unit weight of 16 kN/m$^3$ was adopted for all installation and pipe geometries. The initial horizontal stresses (x and z plane) were based on an arbitrary earth pressure coefficient of 0.4.

**8.3 MODELLING THE PIPE STIFFNESS TESTS**

Before embarking on analyses of buried pipe installations, the performance of the three dimensional elements (type 30 in AFENA) was investigated by simulating the pipe stiffness tests. In this test, a 300 mm length of pipe is laid on a flat surface and vertically loaded with a line load applied to its crown (refer Chapter 4 for further details). Therefore, only half the pipe needed to be modelled due to the central, vertical axis of symmetry.

The number of elements was kept low deliberately (24 around the half cross-section and 6 elements long), commensurate with the expectations of the FEA of the buried pipe installations. Indeed, the pipe cross-section was modelled initially with only one element across the thickness of the pipe. A check on the validity of this discretization was made by comparing the results with the output from analyses with two element thickness. The two meshes can be seen in Figure 8-1. Loading and boundary conditions are indicated on the diagrams where possible. It should be noted that all nodes on the plane of symmetry were constrained in the x direction. Loading was applied by enforcing equal increments of displacement along the crown.
The FEA output was compared with the theoretical solution (Roark and Young, 1975) for the deflections caused by the vertical point loading of a ring. The theoretical vertical and horizontal diametric deflections, $D_v$ and $D_h$, given in Table 8-II are for a unit (1 kN) loading of the ring. The theoretical pipe movements have been based on pipe stiffnesses, which have been adjusted for the finite length of the pipe in the test in the pipe stiffness test. The adjustment of the pipe stiffness is expressed by the equation:

$$ EI = \left(\frac{Et^3}{12(1-v^2)}\right) $$  \hspace{1cm} 8-1

where $t =$ thickness of the pipe

and $v =$ Poisson’s ratio of the pipe material

Poisson’s ratio for uPVC pipe material was taken to be 0.38, as recommended by AS/NZS 2566.1 (1998).

In Table 8-II, the relative differences are given between the displacements under the same loading from FEA and the theoretical displacements, $\delta D_v^{5\%}$ and $\delta D_h^{5\%}$, for the vertical and horizontal directions, respectively. The chosen loading was the force per metre length of pipe predicted by the theory to achieve a 5% vertical diametric strain.

The relative differences of displacement are defined by the equations:

$$ \delta D_v^{5\%} = (100\frac{D_v^{FEA}}{D_v^{R&Y}} - 100)\% $$  \hspace{1cm} 8-2

where $D_v^{FEA} =$ the vertical displacement predicted by the finite element when the pipe is subjected to the theoretical force per unit length to achieve 5% vertical diametric strain

and $D_v^{R&Y} =$ the theoretical vertical displacement (5% of the nominal pipe diameter)

$$ \delta D_h^{5\%} = (100\frac{D_h^{FEA}}{D_h^{R&Y}} - 100)\% $$  \hspace{1cm} 8-3
where $D_{h\text{FEA}}$ = the horizontal displacement predicted by the finite element analysis for the theoretical force per unit length required to achieve 5% vertical diametric strain

and $D_{h\text{R&Y}}$ = the theoretical horizontal displacement at 5% vertical diametric strain

Values of $\delta D_{v5\%}$ and $\delta D_{h5\%}$ are provided in Table 8-II for FEA with either one or two elements across the pipe thickness. The differences between the theoretical and numerical analyses are quite small and ranged between $-1$ to $+4.5\%$, indicating generally that the FEA analyses slightly over-predicted the diametric movements of the pipe under load, when compared with the theoretical solution. The differences were negligible for the vertical movement and were more significant in the horizontally, with a maximum difference of 4.5% recorded for the 450 mm diameter pipe.

It would appear from Table 8-II that the size of the pipe influences the relative difference in movement. This observation is plausible as the number of elements in the cross-section is fixed at 24 for all the pipe models, regardless of pipe diameter. Therefore, the size of the elements increases as the pipe diameter increases. Accordingly, the discretization becomes relatively coarser as the diameter increases. The ratio of the diameter of the pipe to the thickness of the pipe, $D/t$, varied only between 19 and 23 for the four different combinations of pipe diameter and thickness.

The differences in the outputs were insignificant between FEA, with either one or two elements across the pipe thickness. Taylor, Doherty and Ghaboussi (1973) made a similar observation after they had modelled a cantilever beam with the original non-conforming element, with either one or two thickness elements. A slight improvement to the fit with the theoretical prediction was observed for the two larger diameter pipes, when modelled with the two-element thickness.
8.4 MODELLING OF THE BURIED PIPE TESTS

As a consequence of the pipe stiffness test modelling, it was decided to adopt a single element thickness for the pipe discretization, for all future modelling of buried pipe installations.

The soil mesh discretization was initially quite coarse to gain an initial appreciation of the three-dimensional approach. Subsequently the mesh was refined by increasing the number of elements about the edge of the loading plate (subsequently referred to as series 2 analyses). The adopted finer mesh is depicted in Figures 8-2 and 8-3 for the paved and non-paved, finite element analyses. The various materials that comprised the model are shown by colour differences. Materials included the uPVC pipe and the three soils within the trench, namely the sand in the bedding, the surround and the backfill, each possibly at different initial void ratios. For convenience in mesh generation, the boundary between the bedding and the surround was approximated by an inclined line, which ran from the lower quarter point of the pipe to the corner of the soil box.

The flexible loading plate in the pavement tests was incorporated in the mesh and became the fifth material in the mesh. As the mesh generation worked by duplication of sections in the x-y plane throughout the length in the z-direction, surface elements generated outside the area of the loading plate were negated by assigning negligible stiffness for the material for these elements.

Typical boundary conditions of the mesh for FEA of a paved test with a perfectly rough sidewall are illustrated in Figure 8-4. The Figure depicts the cross-section for the vertical \((z = 0)\) plane through the centre of the surface load, which is perpendicular to the longitudinal axis of the pipe. At either ends of the mesh \((z = 0\) and \(z = 1.5\) m), all nodes were fixed in the z-direction. The boundary conditions at the sidewalls continued the complete length of the mesh in the z-direction. The arrows at the top of the mesh represent nodes at which increments of enforced displacement were applied in the simulation of the paved tests. Boundary conditions were the same for the non-paved tests conducted in the laboratory.
8.4.1 Simulated Pavement Tests
With the introduction of 3D finite element analyses, the flexible loading plate could be accurately modelled and indeed the method of loading simulated. The loading ram acted through the rigid loading plate, which was positioned at the centre of the larger plate. The larger flexible plate was modelled with 8-noded brick elements. Nodes of this plate covered by the smaller rigid plate were given enforced, uniform increments of displacement. The increment size was 0.02 mm, which during the early stages of loading was found to correspond to an average applied pressure of 0.4 to 0.5 kPa per increment. As this rate of loading was almost ten times as large as that applied in the 2D FEA, checks were made on the influence of increment size.

The interface between the loading plate and the backfill was assumed to be perfectly rough.

8.4.2 Non-paved Tests
External loading through the rigid plate was simulated with enforced, uniform increments of displacement of the plate nodes. Full lateral and vertical restraint was applied to the nodes of the sidewall boundary, simulating a rigid wall with a perfectly rough interface between soil and wall. When required, a perfectly smooth interface with the sidewall was established by freeing the nodes above the base of the wall. The end faces of the model, comprising the end wall (z = 1.5 m) and the central plane of symmetry (z = 0) were fixed laterally in the z-direction.

8.4.3 Field Tests
The natural soil from which the trench was formed was assumed to be linear elastic and loaded in the undrained condition, so Poisson’s ratio was taken to be 0.49. The isotropic undrained Young’s modulus of the natural soil forming the trench walls was taken to be 30 MPa, based on the unconsolidated undrained triaxial tests, discussed in Chapter 4. The tests were conducted on samples adjacent to the springlines of the installed pipes.
8.5 BOUNDARY CONDITION CHANGES FROM CONSIDERATION OF THE STIFFNESS OF THE SOIL BOX

In the preceding discussion relating to modelling of tests in the laboratory, it was assumed that the soil box provided a perfectly rigid lateral restraint to potential soil movements. Referring back to Chapter 4, and in particular Figure 4-2, it can be seen that in the physical experiments, the box was indeed fixed at the box ends where the side panels and bottom plate intersected the end plates. The bottom end plates consisted of a bolted steel plate, 0.6 m high above the floor. This plate was fitted with a welded circular sleeve to secure the ends of the pipe during testing.

Along the rest of the wall, “fixity” relied upon rows of horizontal I-beams (100 mm deep, with 45 mm wide flanges and 5 mm thick webs and flanges). The centres of these I-beams were located generally at the following heights above the floor of the box: 0.30, 0.66, 0.96, 1.31 and 1.62 mm. The ends of pairs of I-beams on either side of the box at each height were strapped together as shown in Figure 4-2. Some constraint was afforded the centres of the I-beams as they were propped against a substantial structural steel gantry with timber wedges.

In the earliest test (test 375/4), it was observed that the walls of the box were deflecting outwards because of inadequate central support. Subsequently side support was improved. Only one of the later tests was set up to monitor side movements (test 300/6). Dial gauges were set up on the most convenient structure, the central gantry. The dial gauges were positioned at both the height of the pipe springline and mid-height of the 0.8 m deep backfill. Positive outward deflections began to be recorded between pressures of 400 and 600 kPa, applied directly to the surface of the backfill. At the maximum applied surface pressure of 685 kPa, the monitored side of the gantry had deflected 0.4 mm. An I-beam support, 300 mm above the box floor and just 50 mm above the springline was also checked, as it was

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1 Note that the box was not fully assembled when the photographs were taken.

2 The ribbed panels were observed to move outwards approximately 4 mm either side of the pipe springline at an applied force on the plate of 35 kN (350 kPa).
not quite in contact with the gantry. This I-beam deflected 0.8 mm at the maximum applied pressure of 685 kPa. Obviously, the assumption of full lateral fixity in the modelling of the pipes is not justified.

The formwork system, including I-beams, was designed for maximum deflections of span/270 under a maximum hydrostatic load from fluid concrete of 60 kPa. From the theory of plates (Timoshenko and Woinowsky-Krieger 1959, Table 35), the stiffness of the braced formwork may be deduced, at least approximately from plate theory. If it can be assumed that the formwork would be fixed on all four edges during a concrete pour, for a rectangular panel of dimensions a by b (in this case, 1.8 m high x 3 m long) and of thickness, t, uniformly loaded by pressure, q, the central deflection, \( w_{ct} \), is given by:

\[
w_{ct} = \frac{qa^4}{EI} \quad 8-4
\]

for \( \frac{b}{a} = 1.667 \) and Poisson’s ratio = 0.3

In the equation, EI is the flexural stiffness of the plate per metre width.

If the thickness of the formwork, “t”, is arbitrarily chosen to be 20 mm and \( w_{ct} \) is taken as the limiting deflection in the long direction given by the deflection ratio of 1 in 270, then the estimate of the stiffness of the formwork system\(^3\), EI, is 120 kNm\(^2\)/m. Advice from the manufacturers of the panels based on tests in the development of the formwork in the early 1970’s, suggested that EI should range between 60 and 70 kNm\(^2\)/m.

Since fresh, fluid concrete produces a hydrostatic pressure increasing linearly with depth down a vertical formwork panel, the solution for this type of loading should be considered. Timoshenko and Woinowsky-Krieger (1959) provide a solution for the central settlement in this case, again with all edges fixed (Table 36). For an aspect

\(^3\) The “formwork system” consists of the propped, ribbed plywood panels and connectors
ratio for the “plate” of \( \frac{b}{a} \) equal to 1.667 and Poisson’s ratio of 0.3, and a fluid pressure from top to bottom of the wall ranging between zero and \( q \) kPa, the solution approximates to:

\[
\frac{w_{ct}}{qa_0} = 0.0012 \frac{q a^4}{EI}
\]

Applying this equation with a value of \( q \) of 60 kPa yielded an estimate of \( EI \) for the braced formwork of 61 kNm\(^2\)/m, which was closer to the information provided by the manufacturers. It was subsequently decided to adopt a representative value of 60 kNm\(^2\)/m for all subsequent finite element analyses.

The mesh for each laboratory pipe installation test had to be modified to incorporate the finite stiffness of the braced formwork forming the walls of the test soil box (referred to hereafter as “series 5 runs”). An example of the mesh for analysis of reduced sidewall support is provided in Figure 8-5 (test 300/6). Boundary conditions are shown in Figure 8-6 for two cross-sections of the model for test 300/6. To the left is the cross-section at the x-y plane of symmetry. The arrows indicate the direction of enforced displacements of the nodes of the loading plate. The side boundary, that is the outer surface of the plywood wall, was relatively unrestrained. However, the base of the wall was fixed in the x and y directions throughout the length (z) of the model. The same boundary conditions persisted until the final cross-section was reached at \( z_{max} \) of 1.5 m (refer to the right hand side of the Figure). The end of the box was fixed additionally in the x-plane along the outer edge (i.e. at \( x = x_{max} \)).

At values of \( z \) of zero and 1.5 m, all nodes were fixed in the z-plane. The interface between the sidewall and the sand was perfectly rough.
8.6 DISCUSSION OF RESULTS

3D FEA analyses with smooth and rough rigid walls were undertaken. Examples of the deflection and soil pressure data predicted by the FEA are given in this section. All of the pavement tests are reported, however, only selected examples of the other tests are given for each diameter of pipe and pipe profile. Generally, FEA predictions for the tests with the minimum and maximum cover heights have been included.

8.6.1 Comparison of Smooth and Rough Wall Analyses, Simulated Pavement Tests

For the paved tests depicted in Figures 8-7 and 8-8, a perfectly smooth wall provided more vertical deflection and less lateral deflection of the pipe at the same level of applied pressure than FEA with a perfectly rough wall. As expected, the deflection of the soil at the side of the loading plate adjacent to the sidewall was constrained by the rough wall interface. The influence of the interface is illustrated in Figures 8-9 to 8-11, in which comparisons are provided of the FEA data for test P375/2 at increment 400 (or 8 mm of central plate deflection), corresponding to average applied pressures of 189 and 214 kPa for the smooth and rough wall, respectively.

The difference in the mechanism of movement is apparent in the deformed plots in Figure 8-9 and the vector diagrams of Figure 8-10. The vector diagram for the smooth wall analysis indicated the initial formation of a rotational mechanism near the back corner of the large plate. Surface displacements over the last three rows of elements from the end of the laboratory soil box were small, but upwards.

The deflections of the pipe and the sand in the y direction are provided in the contour plots in Figure 8-11. Vertical deflection was more pronounced for the perfectly smooth wall analysis, both along the length of the installation and through the surround soil within the plane of symmetry. The constraint of a perfectly rough sidewall concentrated stresses, and therefore deflections, closer to the crown of the pipe.
It was clear that the observed lateral deflections were significantly underestimated by the FEA described above. The experimental data suggested a similar, but slightly lower level of lateral diametric strain to that of the vertical diametric strain, at any given level of applied pressure. However the numerical analyses predicted that the vertical diametric strain should be at least four and a half times the lateral diametric strain. A perfectly rough wall improved the match with the experimental observations, but fell far short of matching the experimental data.

8.6.2 Comparison of Smooth and Rough Wall Analyses, Laboratory Non-Paved Tests

The selected tests were 300/2, 300/6, 375/1, 375/4, 450/1 and 450/5, which had initial cover heights of 450, 800, 300, 700, 450 and 800 mm, respectively. The diametric strains for the pipe have been plotted against the applied surface pressure for these tests and the FEA simulations in Figures 8-12 to 8-14. As in the FEA simulations of the paved tests, a perfectly smooth wall resulted in greater vertical pipe deflection. However the lateral deflections were also greater than the deflections for the corresponding analysis with the perfectly rough wall condition.

In principle, the absence of vertical shear support on the sidewalls should allow greater pressure to develop above the crown of the pipe at the same level of displacement of the loading plate, most noticeably for installations with greater cover heights and therefore greater shear force development at the sidewall. To investigate this point, the predicted vertical pressures 150 mm above the pipe, at $x = 0 = z$, have been plotted in Figure 8-15 against the applied surface pressure for both tests 450/1 and 450/5. The plotted pressures are pressure increases over and above the initial stress in the soil.

For the analyses with the lower cover height (450/1), the pressures for either side boundary condition were practically identical to the observed pressures, up to a surface pressure of 200 kPa. The vertical soil pressure 150 mm above the pipe in the perfectly rough FEA remained less than that for the perfectly smooth analysis throughout, and accordingly the vertical pipe deflections were less. At the end of the test the average applied surface pressure was 540 kPa and the recorded pressure above the pipe crown was 403 kPa. The FEA estimates at this same surface pressure
as a percentage of the recorded pressure were 87 and 63\% for the perfectly smooth and perfectly rough sidewall analyses, respectively. The corresponding ratios of predicted to the observed vertical diametric strain ratios at the end of the test were 65 and 55\%.

For the simulation of test 450/5, greater pressure was predicted above the pipe crown in the earlier stages of loading for the perfectly smooth FEA, but eventually reached the same level at an applied surface pressure of approximately 800 kPa. This observation matches reasonably well the difference in development of vertical diametric pipe strains predicted by the perfectly smooth and perfectly rough wall analyses. Neither analysis was able to adequately predict the vertical pressure above the pipe crown. The experimental data indicated that the deep soil pressure was insignificant until the surface pressure had exceeded 200 kPa, but then rose sharply to overtake the rough wall FEA prediction by an applied surface pressure of approximately 400 kPa, and the smooth wall FEA prediction by a surface pressure of almost 500 kPa. Again the plot of the development of vertical diametric strain with surface loading (Figure 8-14) adequately reflected the differences in the measured and predicted development of soil pressure above the pipe. In the latter stages of the test in which soil pressures were significantly underestimated, the predicted deflections were correspondingly underestimated.

A summary of the predictions of the applied surface pressure to cause a 3\% vertical diametric strain for the FEA of the six pipe tests discussed in this section is provided in Table 8-III for both the perfectly rough and the perfectly smooth sidewall analyses. The percentages in the table are the percentage difference of each estimate from the observed pressure. Two observations can be made from this Table. Firstly the pressure was underestimated significantly, particularly as the ratio of initial cover height to the diameter of the pipe increased. Test 300/6, a 300 mm diameter pipe with 800 mm of cover was least well modelled; the rough wall analysis predicted a pressure almost three times that observed, while the smooth wall analysis reduced the ratio of predicted to observed pressure to less than two. Only test 300/2 was simulated adequately by the FEA, particularly the smooth wall analysis, and even then, the corresponding rough wall analysis overestimated the surface pressure needed to reach 3\% vertical diametric strain by 30 percent. Finally it should be noted
that this measure of the adequacy of the FEA can be misleading as beyond 3% strain, the divergence from the experimental data invariably became greater.

8.6.3 Finite Element Analyses of the Field Tests
The four field tests were modelled with a perfectly rough interface between the sand backfill and the natural soil. An end view of the mesh for test F450/3 is provided in Figure 8-16. The influence of the adopted value of Young’s modulus of 30 MPa for the elastic and isotropic side soil (shown in green in the Figure) was examined by redoing each analysis as if the test had been conducted in the soil box. The stiff clay soil was replaced by a perfectly rough, and rigid sidewall. A further check was performed on the FEA of the 450 mm diameter pipe tests by changing the Young’s modulus of the side soil to 90 MPa. As noted in Chapter 4, the side soil was considerably stiffer near the 450 mm diameter pipes in the field installations ($E_{\text{min}} = 70$ MPa, $E_{\text{max}} = 95$ MPa).

The plots of predicted and observed diametric pipe strain against applied pressure are provided in Figures 8-17 and 8-18. The match between the experimental behaviour and that estimated by the FEA with the finer mesh was reasonably good, especially when compared with the comparisons given in the previous section for the laboratory tests. The observed lateral deformations were significantly less than the vertical strains and accordingly, the predictions of these lateral strains were generally acceptable, although underestimating the experimental values. The “as if in box” analyses predictions for either lateral or vertical diametric strain were less than the predictions with natural side soil having a Young’s modulus of 30 MPa.

A summary is provided in Table 8-IV of the predictions of the applied surface pressure to cause a 3% vertical diametric strain for the FEA of the four field tests, with either natural side soil support or a rough and rigid sidewall. The predictions of the required pressure were within 50 and 67% of the observed values for the natural side soil analyses and the rough rigid wall analyses, respectively. The predictions based on a natural soil with a Young’s modulus of 30 MPa overestimated the required applied pressure by a maximum of 15% for tests F300/3 and F375/7, while the estimate of the required surface pressure for test F450/4 was 47% above the observed pressure, the least satisfactory matching of the experimental data. For all
four tests, the estimates of pressure at 3% diametric pipe strain were less satisfactory for the rough and rigid wall analyses.

The estimates in Table 8-IV are remarkably good when it is realised that the substantial settlement of the loading plate observed in the tests was unable to be modelled by any of these analyses. In Figures 8-19 and 20, the cover height ratio, defined as the ratio of the current cover height to the initial cover height, has been plotted against the applied pressure for each of the field tests. The reduction in the cover height to the crown of the pipe was adequately matched up to a surface pressure of 300 to 400 kPa. Over this range, settlement was proportional to applied pressure. At greater pressures, dramatic losses of cover height were observed as the plate penetrated into the soil cover above the pipe. The various FEA predicted an almost linear increase of settlement with pressure over the full range of applied pressure.

The FEA predictions of the load-deflection behaviour of the pipe-soil system tended to over-predict the vertical diametric strain of the pipe in the early stages of loading, and then could not match the rate of increase of deflection that was recorded subsequently.

8.6.3.1 Earth pressures

The earth pressures in the backfill predicted by the FEA are shown against applied surface pressure in Figures 8-21 and 8-22, for tests F375/7 and F450/3, respectively. The experimental data have been plotted on these same Figures for purposes of comparison. The plots with the green lines represent FEA with natural soil, while the black lines correspond to FEA with a perfectly rough and rigid sidewall.

The pressure 150 mm above the crown of the pipe at the central cross-section \((z = 0)\) was predicted reasonably well in the case of test F375/7 by the FEA with natural side soil. This pressure was well matched to an applied surface pressure of about 650 kPa and thereafter was underestimated. This observation is reasonably in line with the vertical deflection data; the observed diametric strain became greater than the FEA

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4 Except in the case of F450/4, wherein the deflection was well matched in the early stages of loading
estimate (natural side soil) at approximately 800 kPa. The lower strains produced by the FEA with the rough and rigid wall are also in accordance with the predicted lower level of pressures above the pipe crown with this approach.

The distribution of the vertical pressure about the central cross-section \( z = 0 \) was similar between the two analyses, with the rough rigid wall analysis yielding slightly greater pressures. At 150 mm above the pipe crown, but only 100 mm from the sidewall, both analyses underestimated the pressure. At a surface pressure of 766 kPa, the pressure cell reading peaked at 102 kPa, compared with an estimate of 52 kPa from the analysis with natural side soil. The alternative analysis with the sidewall predicted an earth pressure of 63 kPa.

A comparison of the predicted and measured pressures in the surround at the level of the springline of the pipe is given in Table 8-V. All values correspond to prescribed surface pressures of 766 and 676 kPa for tests F375/7 and F450/3, respectively. Plots of these pressures in the surround soil over the full range of loading are provided in Figures 8-21b and 8-22b. Both FEA significantly underestimated the observed pressures. The estimates of pressure with the FEA incorporating the elastic side soil were close to the estimates from the FEA with a rigid wall, for either the horizontal or vertical pressures. Values ranged between just 48 and 53% of the observed pressures.

The earth pressure data for test F450/3 are summarized in Figure 8-22. A sharp rise in the pressure 150 mm above the crown of the pipe in the central cross-section could not be matched by either analysis, past an applied surface pressure of 350 kPa. Thereafter, this pressure was significantly underestimated, particularly by the rough, rigid wall analysis. Initially, the vertical diametric strains of the pipe were overestimated, until significant underestimates were produced above a surface pressure of 500 kPa. At the final surface pressure of 676 kPa, the vertical soil pressure recorded above the pipe crown was 483 kPa. The corresponding FEA predictions were 316 and 252 kPa for the natural soil and the rigid wall FEA, respectively, representing a maximum of just 65% of the observed pressure.
Good estimates were made by the FEA of the earth pressure 150 mm above the pipe crown and 100 mm from the sidewall, for the first 400 kPa of applied surface pressure. At greater surface pressures, the observed earth pressure was steady, reaching a value of 30 kPa by the conclusion of the test. Both analyses predicted a lessening of the rate of increase of pressure with loading, but could not quite match the experimental data. At the end of the test, the predicted pressures were 44 kPa for the analysis with the natural soil, and 48 kPa for the analysis with the rigid sidewall.

The pressures developed in the surround in test F450/3 were relatively low. Comparative estimates in the surround for test F450/3 are given in Table 8-V for an applied surface pressure of 676 kPa. Although both FEA significantly underestimated the observed pressures, the estimates of pressure were generally better than for test F375/7. The prediction of lateral pressure from the FEA with elastic side soil was 77% of the recorded value. The predictions of the vertical pressures at the springline were less successful, which is evident in Figure 8-22b.

8.6.3.2 Patterns of deflections for the FEA of field test F450/3
To better understand why predicted earth pressure development differed between the FEA models, solution files of nodal displacements were compared for the FEA of test F450/3. Contour plots of y and x displacements on the z = 0 plane are given in Figure 8-23. The comparisons are for similar levels of applied surface pressure and therefore different levels of plate displacement. The rough and rigid wall analysis is for 1000 increments of enforced displacement, or 24 mm of plate displacement, while the natural soil output corresponds to 1200 increments. The average applied pressure in the analyses at the chosen stages were close, being 664 and 671.5 kPa for the natural soil FEA and the rough and rigid wall FEA, respectively (average 668 kPa).

For the full or natural side soil analysis, the transfer of the vertical deflection of the plate to the pipe crown appeared to be greater, when comparing the contour plots of vertical displacements. The deflections above the pipe on the axis of symmetry (x = 0), and along the z = 0 plane, were reviewed and plotted in Figure 8-24 as relative vertical displacements against relative height above the crown. Displacements were normalized by dividing by the plate displacement, while the height above the pipe
was expressed in terms of the pipe radius. It is evident in this Figure that the
distribution of displacements above the pipe are quite similar. However, the vertical
displacements for the full analysis did extend slightly further around the pipe
circumference, as is evident in the contour plots of Figure 8-23.

The distributions of vertical displacements of the pipe crown along the z-axis (i.e.
along the pipe) are presented in Figure 8-25. In order to compare the solutions at
different levels of plate displacement (20 and 24 mm), the crown displacement was
again normalized with respect to the plate displacement. Likewise the distance, z,
was normalized with respect to the full length of the loading plate, i.e., 0.5 m, so that
the end of the plate is half a plate length from the z = 0 plane. The crown of the pipe
was estimated to move by 39% of the displacement of the loading plate, by either
analysis. Moreover the predicted distributions of the pipe deflections along the pipe
crown were quite similar, being within 1% of one another.

The predicted displacements in the x-direction on the z = 0 plane are presented as
contour plots in Figure 8-23. The displacements for the natural soil analysis did
extend across the trench boundary, but were relatively small, being less than 0.5 mm.
The distributions of these displacements with height above the base of the soil box
and through the centre of the surround (x = 0.3 m) for the two solutions are given in
Figure 8-26. The relative location of the pipe is indicated on the diagram. The
maximum displacement was 1.05 mm and occurred 125 mm above the springline of
the pipe, or about half a radius. This estimate was 110% greater than the
corresponding prediction from the “as if in box” analysis. The natural soil analysis
predicted more than twice the lateral deflection in the soil above the level of the
bedding (0.1 m).

The longitudinal distributions of the x-displacements of the pipe are depicted as
contour plots in Figure 8-27 along the x =0 plane. The lateral displacements for the
FEA with the natural side soil analysis were more extensive and reached further
down the pipe.

It may be concluded that the full 3-dimensional model, employing natural soil,
permitted more lateral deflection into the natural soil, which resulted in greater
predicted, lateral deflections of the pipe. The reduced support to the side of the pipe resulted in greater crown deflections for the same level of externally applied load.

The comparative development of plasticity predicted by the two different physical models is presented in Figure 8-28. The patterns of yield development were quite similar and no significant differences could be found between the two models.

8.6.3.3 The influence of Young’s modulus of the natural soil
As stated earlier, the soil proved to be much stiffer near the 450 mm pipe installations and so the analyses were re-run with an increased Young’s modulus of the natural soil of 90 MPa; a Poisson’s ratio of 0.49 was applied again corresponding to undrained loading and the expectation of no volume change. As expected, the three-fold increase of the soil modulus restricted the deflections of the pipes, as is evident in Figure 8-29, which provides load-deflection information for the two field tests on 450 mm diameter pipe. The predicted load-deflection behaviour was further removed from the observed behaviour of the pipe-soil system, with increasing natural soil modulus.

To quantify the reduction in deflections, the pressure required to achieve 3% vertical diametric pipe strain was re-visited (refer Table 8-IV). The predictions of the required surface pressures as a percentage of the observed surface pressure changed from 134 to 153%, and 147 to 172%, for tests F450/3 and F450/4, respectively. The reduction in the lateral deflection of the pipe was just as significant. Indeed the overall predicted load-deflection behaviour of either pipe installation was close to that predicted by the corresponding “as if in box” analysis, which has been included in each of the plots in Figure 8-29.

The development of lateral stress and the patterns of movement in the side soil for the FEA for test F450/3 are provided in Figures 8-30 and 8-31, respectively, for the \( z = 0 \) plane. Both Figures illustrate a partial mesh for clarity, showing only the mesh to the side of the pipe. The solutions are provided for enforced displacement of the loading plate of 24 mm, which generated average surface pressures of 671.5 and 726 kPa for the natural soil analyses with Young’s moduli of 30 and 90 MPa of the sidefill, respectively.
Lateral stresses were concentrated just above the springline of the pipe. Greater stresses were evident in the FEA solution with the higher modulus, for example, at the sidewall opposite the springline, the predicted lateral stresses were 69 and 74.5 kPa, respectively, for the FEA with 30 and 90 MPa. More significantly, a tension zone just below the surface of the natural soil was also apparent in the solution with the higher modulus. Upward vectors, although small, are apparent along this surface in the corresponding displacement vector diagram in Figure 8-31.

The comparison of the vector diagrams indicates that nodal displacements into the natural soil were relatively unrestrained in the solution with the lower soil modulus. In comparison, in the analysis with a Young’s modulus of 90 MPa, the deflections had almost become insignificant after one element into the side soil mesh, or a distance of 50 mm.

The development of plasticity between the two solutions varied slightly, as seen in Figure 8-32. At the same level of plate displacement, less plasticity was evident with the sidefill represented by an elastic material of Young’s modulus of 30 MPa.

In summary, the consequences of increasing the sidefill modulus in the FEA were a stiffer response to external loading, increased lateral support at the springline, greater development of plasticity and a small expansion of the adjacent soil surface. At a Young’s modulus of 90 MPa, the natural soil behaviour approximated that of a rigid soil.

8.6.4 Comparison of Analyses of the Simulated Pavement Tests with a Rigid Sidewall and with a Partly-Supported Wall of Given Stiffness

The influence of the non-compliance of the side boundary conditions assumed in the rigid wall analyses with the finer mesh (series 2) is reviewed in this, and the following section. Comparisons are made with series 5 analyses, which provided the wall with a realistic stiffness and relaxed boundary conditions consisting only of fixity along the base and ends of the wall.

The development of diametric pipe strain with loading for the three simulated pavement tests is shown in Figures 8-33 and 8-34. The performance of the FEA was
substantially improved by providing more realistic side support ("3D FEA, ply"). The match with the observed data was generally good, although the behaviour of test installation P375/1 was unable to be replicated as closely as the other two cases considered. It is of particular interest to note in these Figures the much improved matching of the development of lateral diametric pipe strains with surface pressure, when compared with the previous analyses, which assumed a perfectly rough and rigid wall.

In Table 8-VI, a comparison of the surface pressure needed to reach 2.5% vertical diametric strain is given for the two analyses and the actual test. The series 2 analyses reported in the table are for a perfectly rough sidewall. For the series 5 analyses, the required pressure was overestimated in all cases. The pressure was overestimated by just 16% for test P450/1 and by 20% for test P350/2. However the estimated pressure for test P375/2 was 61% greater than the observed value. In all three cases, the lateral pipe strains were underestimated, however the series 5 analyses were approximately three times or more the estimates from the series 2 analyses. The lateral strain estimate from the series 5 FEA averaged 73% of the test data for tests P375/2 and P450/1, but was only 55% for test P375/1.

It may be concluded that the series 5 analyses better simulated the actual laboratory side boundary conditions.

In Figure 8-33a, the influence of halving the displacement increment size to 0.01 mm is demonstrated for the series 5 analysis of P375/1 (DTx0.5). Over the range of surface pressures encountered in the test, there was no discernible improvement of the analysis; the two plots are inseparable. Subsequently the step size of 0.02 mm could be adopted with some confidence.

8.6.4.1 Lateral displacements
The influence of the sidewall boundary is indicated in Figure 8-35, which provides contours of x-displacements after 400 increments of enforced displacement of the loading plate (8 mm) for the FEA of test P450/1. After 400 increments, the average surface pressure for the series 5 FEA was 112 kPa, while in the series 2 analysis with a perfectly rough sidewall, an applied pressure of 222 kPa was required to reach the
same level of displacement. The general pattern of deflection of the sidewall is indicated in Figure 8-36 and is compared with the series 2 FEA. Figure 8-37 provides a comparison of the lateral deflection contours and displacement vectors for the two analyses on the z = 0 plane.

It can be seen that the predicted lateral deflections were substantial for the series 5 analysis, with the lateral displacements extending along the length of the pipe, almost to the end of the laboratory soil box. The maximum deflection was 3.3 mm. The pattern of displacement of the wall was consistent with that of a vertical plate loaded near its centre and having its top edge free. Lateral displacements along the sidewall at the level of the surface of the backfill were 3.3, 2.9, 2.1 and 0.3 mm, at values of z of 0, 0.3, 0.6, and 1.2 m. Opposite the pipe springline on the z = 0 plane, the lateral displacement varied from 2.0 to 1.5 mm between the pipe and the sidewall.

The marked difference in the development of x-displacements between the two analyses with the different boundary conditions can be seen in Figure 8-36. The deformed meshes for the same solution step are compared in Figure 8-37, which emphasize the relatively large deformations of the sidewall in the series 5 analyses. As was expected, the outward, or positive, lateral displacements were concentrated near the surface of the backfill against the flexible wall. With this additional lateral deflection of the soil, greater lateral deflection at and above the springline of the pipe is evident in the series 5 plots of both the deformed meshes (Figure 8-37) and vectors (Figure 8-36 (c) and (d)).

8.6.4.2 Vertical displacements

Contours of vertical displacement are provided in Figure 8-38 for series 2 and series 5 FEA of test P450/1, again at 400 increments of enforced surface displacement. A comparison of the predicted displacements between the series 2 and series 5 analyses is given on both the x = 0 and z = 0 planes. With a perfectly rough and rigid wall, the vertical displacements were more effectively shed down the pipe, whereas in the alternative analysis with a partly supported wall of finite stiffness, vertical displacements in the vicinity of the pipe crown were more significant. The extent of crown displacement along the pipe length, or the z-axis, predicted by the series 5
analysis was greater also. The distributions of the vertical displacements of the pipe crown along the z-axis are presented in Figure 8-39.

Considering the deformed meshes in Figure 8-37, it would seem that the relaxation of the side boundary condition reduced lateral support and permitted a flattening of the pipe crown rather than “ovalling”. The predicted crown deflections persisted over a greater length of pipe with the reduced side support. Again this difference is in accordance with the earlier observation that the lateral displacements of the wall, and hence reduction in lateral support, persisted almost to the end of the wall.

8.6.4.3 Soil pressures
The vertical pressure 150 mm above the crown of the 450 mm diameter pipe is given in Table 8-VII for 8 mm of enforced displacement (or 400 increments), along with the x and y-displacements of the top quarter point. For the series 5 analysis, the estimated pressure delivered above the crown was 76% of the average pressure applied to the large plate. The output of the series 2 analysis indicated less pressure relative to the surface pressure, being just 62%. Nevertheless, the value of pressure exceeded that estimated in the series 5 FEA (175 c.f. 85.4 kPa). Although the predicted crown displacement was greater for the series 5 analysis, the top quarter point deflected significantly less than that predicted by the series 2 analysis, suggesting relatively greater flattening of the crown. Furthermore the displacement along the x-axis of the top quarter point approached 1 mm, while the corresponding estimate from the series 2 analysis was negligible (< 0.05 mm).

8.6.4.4 Development of plasticity and changes in void ratio
The patterns of plastic Gauss point development for the two analyses are compared in Figure 8-40. All Gauss points above the top quarter of the pipe were plastic for both solutions, however plasticity extended further down the interface with the wall in the series 5 analysis.

How plasticity was achieved appears to be markedly different when the contour plots of void ratios are compared for the solution at 400 increments (Figure 8-41). The initial void ratios for the backfill zone 150 mm above the pipe crown, the surround and the bedding were 0.651, 0.632 and 0.613, respectively, which corresponded to
density indices of 65, 70 and 75%. The maximum void ratio on the \( z = 0 \) plane occurred at the top of the soil mesh in both analyses, but at different locations from the sidewall. For the rigid wall FEA, the maximum void ratio of 0.73, corresponding to a density index of 44%, was generated 55 mm away from the wall, or near the fifth Gauss point from the side boundary. For the FEA with the propped wall of finite stiffness, the maximum void ratio of 0.76 (density index of just 39%) was realized again at the fifth Gauss point from the wall, but in the top line of Gauss points in the second element below the surface of the soil mesh. Volume increase was predicted to occur deeper within the soil, which would be expected with a deflecting sidewall.

With the reduction in wall support afforded by the series 5 analysis, soil volume and hence void ratios were able to increase more readily in the backfill below the edge of the plate. It is also evident from Figure 8-41 that the densification of the soil below the plate and into the surround soil below the side of the perimeter of the loading plate was slightly less for the series 5 FEA.

### 8.6.4.5 Influence of wall stiffness

The influence of the stiffness of the sidewall for the series 5 analyses is indicated in the earlier Figures 8-33 and 8-34. FEA were undertaken for all tests with the original Young’s modulus of the sidewall of 30 MPa either divided, or multiplied by, a factor of two. As would be expected, increasing the sidewall stiffness means less diametric strains either vertically or horizontally, for the same level of applied pressure. Of all the analyses of the three tests, the Young’s modulus of 30 MPa used in the majority of the series 5 analyses produced the best match to the observed lateral diametric strains; the analysis with a wall of twice that stiffness provided arguably the best match with the measured vertical diametric strains for test P450/1, while the vertical pipe deflection data from test P375/1 were best matched using a modulus of just 15 MPa.

### 8.6.4.6 Summary for FEA of simulated pavement tests

It would appear that the approach of the FEA with the stiffened sidewall and edge boundary conditions could not completely simulate the laboratory test set-up and the non-compliant boundary conditions for the simulated pavement tests, which
employed a large, flexible loading plate. However it proved to be a reasonable approximation, as will be demonstrated further in the next section.

8.6.5 Comparison of Analyses of the Laboratory Non-Paved Tests with a Rigid Sidewall and with a Partly-Supported Wall of Given Stiffness

The influence of the non-compliance of the side boundary assumed in the rigid wall analyses (series 2) is discussed in this section in relation to the majority of the laboratory soil box tests, which employed the rigid loading plate of dimensions, 200 mm wide by 500 mm long. The relative performance is presented of the series 5 analyses, in which the rigid side boundary condition was reduced and the sidewall was given a realistic stiffness.

The development of diametric pipe strain with loading is shown in Figures 8-42 to 49, starting with the 300 mm diameter, 110 VX profile pipes, through to the 450 mm diameter pipes. In the plots, series 2 FEA are described as “rough, rigid wall”, while series 5 are described as “ply”.

It is readily seen that, as in the modelling of the paved tests, the series 5 analyses provided a much better simulation of the lateral strain development than did the series 2 analyses. Again, the matching of both vertical and horizontal diametric strain was less successful. More often than not, the vertical diametric strain was overestimated. In most cases, the experimental vertical pipe strain pipe data were bound by the estimates from the series 2 and series 5 analyses.

The best matched data sets with the series 5 FEA appeared to be the tests on the 300 mm diameter pipes, particularly tests 300/1, 300/5, 300/6 (refer Figures 8-42a, 8-44, 8-45). At the end of the FEA analyses, the estimated diametric strains were compared with the experimental data at the same level of applied pressure and it was found on average that, for these three tests, the vertical strains were over-predicted by 9% while the horizontal diametric strain was underestimated by 35%.

A more general view of the performance of the series 5 analyses is provided in Table 8-VIII. In this Table, values are presented of the surface pressures predicted by the series 5 FEA to cause 3% vertical diametric strain. The percentage difference from
the observed pressure for each test is given in the Table. This Table may be compared with Table 8-III, which provides similar information for the series 2 analyses. The vast improvement of the estimates is readily apparent.

All series 5 FEA underestimated the pressure needed to reach 3% strain, except for the FEA of tests 300/5 and 375/1. Therefore it may be said that the series 5 FEA generally predicted less stiff pipe-soil systems than the installations proved to be. However the matching of the load-deflection data was improved considerably.

As stated previously, the six tests on the 300 mm diameter pipes were most successfully modelled, with the pressure required to attain 3% vertical diametric strain being less than 17% on average below the measured value. The comparable performance of the FEA on the four tests on 375 mm diameter pipes with 700 mm of cover was 26% on average below the measured surface pressure. In contrast, the series 5 FEA for test 375/1 with just 300 mm of cover, overestimated the pressure by almost the same amount (25%). The analyses of the two tests on 450 mm diameter pipes with 450 mm of cover were also relatively successful (-20% on average). However the FEA of test 450/5 with the greatest depth of cover of 800 mm was far less successful, with an estimated pressure 42% lower than that observed.

The influence of installation height on the performance of the series 5 FEA was investigated. The installation height is defined as the height of sand in the box at the commencement of testing, measured from the base of the bedding sand to the top of the backfill. In Figure 8-50, the ratio of the pipe deflection data of the series 5 FEA at the end of the analysis\(^5\) to the measured data, have been plotted against the installation height for all the finite element simulations, including the simulated pavement tests (“large plate”).

The ratios of predicted and observed pipe strains for the large plate tests corresponded to the end of each test, as all analyses continued successfully past the test data. The series 5 analyses of the “pavement” tests reasonably predicted the pipe strains at the end of testing, however only 300 mm of cover was employed in these

\(^5\) In most cases, except where test data were exceeded by the FEA data, in which case the end test data were used.
tests. The average ratios of predicted to measured pipe strain for the three tests were 89% and 56% for the vertical and horizontal strains, respectively. The FEA of test P300/1 was least successful, yielding ratios of strain of 71% and 41% for the vertical and horizontal directions (y and x axes), respectively. The remaining two tests were well modelled by the series 5 analyses, particularly with respect to the vertical diametric strain.

In the non-paved tests, it appeared that as the height of the installation increased, the ratio of predicted to observed diametric strain of the pipe also increased. The horizontal diametric strain was similarly influenced by the height of installation, however the influence was less pronounced. The adopted boundary conditions for the series 5 analyses assumed no support of the sidewall from the central gantry column (refer section 8.5). Such an assumption could be expected to be less critical with lower pipe installation heights.

Of all the pipe tests, tests 375/1 and 450/5 were least adequately simulated by the series 5 analyses, the former having the lowest height of 0.775 m, while the latter had the maximum installation height of 1.35 m. The predictions were as low as 44% of the observed vertical deflections for test 375/1 and as high as 317% for test 450/5. FEA solutions for these two tests are provided in Figures 8-51 to 8-54, in which the series 2 analyses (perfectly rough wall) are compared with the series 5 FEA, at similar levels of applied surface pressure for each test. The plots compare the level of plasticity attained and the contours of soil void ratios between the analyses.

Figures 8-51 and 8-52 relate to the analysis of test 375/1 at a predicted surface pressure of approximately 290 kPa, just above the maximum applied test pressure of 250 kPa. This pressure was reached after only 12 mm of plate penetration in the series 2 FEA, compared with 20 mm of settlement for the series 5 FEA. The remaining two Figures are for the FEA of test 450/5 at a predicted surface pressure of 477 ± 20 kPa, which was reached after just 12 mm of plate penetration in the series 2 FEA, and 48 mm in the series 5 analysis. The chosen pressure approximated the maximum pressure reached in the series 5 FEA.
The differences in the development of plasticity between the series 2 and series 5 analyses were most marked for the test on the 450 mm diameter pipe with 800 mm of cover, as seen in Figure 8-53. In this Figure, Gauss points within the body of the mesh, which have become plastic, are also visible. As expected, the series 5 analyses, which included a wall of finite stiffness and more realistic boundary conditions, allowed the development of far greater plasticity. For both tests 375/1 and 450/5, almost all Gauss points above the pipe crown on the z = 0 plane had become plastic and the zone of plasticity extended below the springline of the pipe. In contrast, only about a third of the Gauss points above the crown had reached plasticity in the series 2 FEA of test 450/5. Longitudinally, the plastic zone for the perfectly rough and rigid wall case (series 2) extended just past the end of the loading plate, while in the series 5 analysis, the same zone reached almost to the end of the laboratory soil box, just two element widths short of the end.

The pattern of soil void ratio change was affected by the wall stiffness and assumed side boundary conditions. The series 5 FEA permitted outward deflection of the wall as loading of the plate increased, thereby allowing greater volume, or void ratio, increases (refer Figures 8-52 and 8-54). In reading these two Figures, the initial void ratios adopted in the analyses for the backfill and surrounds should be noted, being 0.669 for test 375/1 and 0.576 for test 450/5. Generally, the series 5 analyses resulted in considerably higher void ratios than the corresponding rigid wall analyses and so when plotted on equal contour intervals, the plotted series 2 FEA output had relatively fewer contours.

Typically the chief zone of volume increase indicated in these contour plots of void ratios was confined to a narrow, vertical band just outside the perimeter of the loading plate. In the series 5 analyses, this zone extended through the backfill to approximately 150 mm above the pipe. In the series 2 analyses, the depth of penetration was significantly less, particularly for the test with the deeper soil cover. It may be concluded from the contours of void ratios that the series 5 FEA supported the formation of a deep punching shear mechanism, which was observed in the test series.
Of significance to the predicted pipe performance, modest soil volume increases were predicted above the top quarter point of the pipe by the series 5 analyses, which were not predicted by the equivalent series 2 analyses. Compression, or loss of soil volume, below the loading plate was less significant in the series 5 analyses as the plate settled into the backfill soil.

In reviewing the FEA of all the laboratory non-paved tests, it was observed that the series 5 analyses resulted generally in pipe-soil systems, which had insufficient stiffness. The inadequacy of the overall system stiffness could be attributed to either (or both) simplification of the sidewall stiffness, or insufficient side restraint through the adopted boundary conditions. The sidewall stiffness was based on concrete industry requirements for formwork, rather than the actual stiffness of the timber and steel composite sections, and so the estimate of sidewall stiffness could be expected to be the least stiffness required. The boundary conditions did not provide any support of the sidewall from the central gantry column, which as previously stated, should be a less critical assumption as the installation height decreased. Partial support along the y-axis at beam supports at \( z = 0 \) would be preferable to either no support or horizontal fixity. However partial point support was considered to be a future investigation and not a part of this thesis.

The remaining parts of this section explore predictions by the series 5 FEA of plate settlement, development of pressures within the soil, deformed pipe shapes and forces and moments in the pipes.

### 8.6.5.1 Settlement of the loading plate

Examples of the predictions of plate settlement with applied surface pressure for each of the pipe diameters are provided in Figures 8-55 to 8-59. These same Figures contain also the series 2 FEA predictions of the change in cover height ratio with loading, which are denoted as “3D FEA, rough, rigid wall”. Unfortunately, there was not any experimental data to judge the predictions for the two examples of the 450 mm diameter pipe tests against (Figure 8-59).

The series 2 analyses with a perfectly rough sidewall generally underestimated the settlement. The settlement of the loading plate with applied pressure was better
predicted by the series 5 analyses, most noticeably as the cover height of the
installation increased. If anything, the settlement tended to be overestimated, in
accordance with the usual overestimation of the vertical diametric strain of the pipe.

The series 5 FEA allowed an acceleration of the reduction of cover height with
increase of surface pressure; in contrast, an almost linear rate of reduction was
predicted by the series 2 FEA. The series 2 analyses continued unrealistically to
higher levels of applied pressure than the equivalent series 5 analyses, before mesh
instability problems intervened.

The plate settlement for test 375/1 (refer Figure 8-57) was overestimated by both
FEA series; very little settlement was recorded in this test. The plate settlement was
estimated by the series 5 FEA to be 4.9 mm at the surface pressure corresponding to
the end of the test, compared with the measured value of 1.9 mm. Nevertheless, the
vertical diametric pipe strain for this same installation was significantly
underestimated. Presumably there was less sidewall shear support for this test than
was provided in the finite element analysis.

8.6.5.2 Earth pressures
The predictions from the series 5 analyses of the development of earth pressure with
applied loading, above and along the pipe crown, as well as within the surround, are
presented in Figures 8-60 to 8-68. Each Figure, except for Figure 8-68, contains two
plots; the first plot shows the distribution of vertical earth pressures along the length
of the pipe at one or two heights above the pipe apex or crown. All plots have data
for vertical pressures at a height of 150 mm above the pipe crown. The second plot
contains the vertical and horizontal earth pressure data in the sidefill beside the pipe
on the z = 0 plane; the data are differentiated with either a V (vertical pressure) or an
H (horizontal or x-direction). The location of the point in the surround, where
pressures were read from the FEA solution files, is provided in each Figure caption
as a pair of x and y coordinates. This point usually corresponded with the centre of
the surround between the pipe springline and the sidewall, depending upon the
approximate location of the pressure cells in the particular test.
There is only one plot for Figure 8-68 (test 450/5) as useful pressure cell data were relatively limited. Pressure cells placed at 150 mm above the pipe crown were placed too far from the $z = 0$ plane ($z = 0.5, 0.75$ and $1.0$ m) to record any useful data.

The first five Figures (8-60 to 8-64) are for 300 mm diameter pipe, all having the 90VX profile except for 300/7 and 300/8, and with cover heights ranging from 450 to 800 mm. Figure 8-65 corresponds to test 375/8, which was an installation with a 375 mm diameter pipe, protected by 700 mm of cover. The last three Figures are for the 450 mm diameter pipe series; two of the tests had a protective backfill cover of 450 mm, while the remaining test had the maximum cover height of 800 mm (test 450/5).

### 300 mm diameter pipe series

The degree of fit with the data varied, as did the degree of fit with the observed data on load-deflection responses of the pipes. For test 300/4, the prediction of diametric strains at a surface pressure of 400 kPa underestimated the lateral pipe strain by approximately 43% and overestimated the vertical strain by 18%. The distributions of earth pressure development generally mirrored the recorded data (refer Figure 8-60).

The vertical earth pressure distribution along the length of the pipe and at a height of 150 mm above the pipe crown for this same level of applied pressure was overestimated by 22% at $z = 0$ and by 5% at $z = 0.15$ m. The corresponding earth pressures in the surround beside the pipe’s springline were significantly underestimated, the observed lateral earth pressure being 117 kPa while the predicted pressure was just 74 kPa. It should be noted that the series 5 FEA overestimated the settlement of the loading plate with increasing surface pressure (refer Figure 8-55b). At the applied pressure of 400 kPa, the FEA indicated the cover to the pipe had been reduced by 17.2 mm, while the observed value was just over one third of this estimate (6 mm).

In summary, for the finite element analysis of test 300/4, pressures above the pipe crown at $z = 0$ m were overestimated up to an applied pressure of 420 kPa and,
accordingly, the vertical strain of the pipe was overestimated. The lateral soil pressure was underestimated, which by itself should have resulted in less restraint to lateral movement, yet the lateral diametric strain of the pipe was underestimated. Obviously the analysis is more complex than this simple approach, as it relies on a complex soil constitutive model, approximations to pipe and soil box properties, and assumptions regarding the boundary conditions.

The finite element analysis was identical for tests 300/7 and 300/8, which were both tests on 110VX profile pipe with the same installation densities and 650 mm of soil cover (refer Figures 8-62 and 63). The comparison of the earth pressure diagrams allows an evaluation of the reliability of the earth pressure data. At a height of 300 mm above the pipe crown, the vertical pressure on the z = 0 plane was reasonably predicted by the FEA, although it was overestimated for test 300/8, for example, by 37% at a surface pressure of 400 kPa, as compared with just 6% for test 300/7.

Closer to the crown but on the same plane, the soil pressure was significantly overestimated by 68% and 84%, again at a surface pressure of 400 kPa, for tests 300/7 and 300/8, respectively. At z = 300 mm, and just 150 mm above the pipe crown, the FEA predicted that the vertical soil pressure would reduce with increasing pressure after approximately 360 kPa, contrary to the steady increase of pressure observed in either test. Consequently the predicted pressure was approximately 55% of the measured pressure at a surface pressure of 400 kPa. The decrease of pressure from the FEA suggests a punching shear failure developing between z = 150 and 300 mm.

Test 300/5 was also conducted on 300 mm diameter pipe with 650 mm of soil cover, but the pipe was wound in the structurally weaker 90VX profile. Two pressure cells were located at a distance of +0.15 m and –0.15 m of the centre of the length of pipe, and 0.15 m above the pipe crown. Consequently an average plot of the values from these two cells is provided in Figure 8-61a, as well as the pressures from one of the cells. Some tilting of the loading plate occurred in this test and so readings varied between the two cells. The average measured pressure was well matched by the FEA predictions above an applied surface pressure of 300 kPa.
The series 5 FEA of all the tests with 650 mm of cover predicted that the horizontal soil pressure in the sidefill would exceed the vertical soil pressure. The test observations were in agreement with this prediction for only test 300/5. A further examination of the sidefill pressures for these three tests on 300 mm diameter pipe is provided in Table 8-IX, in which measured and predicted pressures are given for an applied surface pressure of 400 kPa. The percentage differences of the estimates from the observed values are provided in parentheses.

On inspection of the pressure data for test 300/8 in this Table, it was suspected that the pressure cells may have been labelled wrongly, such that the horizontal pressure was actually the vertical pressure. All sidefill pressures were underestimated, except the horizontal pressure in test 300/8. The estimates of the vertical and horizontal soil pressures were –36% and +112% of the measured pressures, respectively. By switching the measured pressure cell data, the estimates become +38% and –1%, which were much more reasonable, and so supported the suspicion of an error in labelling the pressure cells.

For the deepest installation of the 300 mm diameter test series, test 300/6 with 800 mm of cover, the series 5 analysis could not predict adequately the development of soil pressures either above or to the side of the pipe. Pressures in the soil above the crown on the z = 0 plane were relatively slow to increase, and then increased far too rapidly. The sidefill pressures were underestimated significantly, for example, by 72% and 32% for the vertical and horizontal surround pressures, respectively, at an applied surface pressure of 300 kPa.

Again the series 5 FEA suggested the development of a punching shear failure with loading. After an applied surface pressure of only 335 kPa a slight decrease was predicted of vertical earth pressure just behind the loading plate (z = 0.3 m) and 150 mm above the pipe crown (refer to Figure 8-64). The backfill and surround soil in test 300/6 were relatively weak (estimated I_D about 60%) compared with most other installations, and so punching shear would be more likely to occur. Although the loading plate settlement was well predicted by the series 5 FEA for this test (refer Figure 8-56), the pressure cell at this location recorded a steady increase of vertical
soil pressure with loading, until the second last reading or an applied pressure of 685 kPa.

**375 mm diameter pipe series**

The series 5 FEA of test 375/8 (700 mm of cover) overestimated the deformations of the pipe as well as the settlement of the loading plate. At a surface pressure of 400 kPa, the cover to the pipe, which was seen in the test to be reduced by 13 mm, was predicted to have decreased by 27 mm. The earth pressure predictions in Figure 8-65 for this installation were quite close to the measured pressures in the surround. However, the vertical earth pressure predictions above the pipe were less satisfactory. The predicted pressures 450 and 150 mm above the pipe greatly exceeded the observed pressures; at an applied surface pressure of 400 kPa, these vertical soil pressures were overestimated by between 50 and 170%. The predictions of vertical pressures along the pipe at distance, z, greater than or equal to 0.3 m, were considerably better, although underestimated by 30% at a surface pressure of 400 kPa.

Two interesting observations may be made for the analysis of this test. Firstly, the predicted vertical earth pressure just 0.25 m below the centre of the loading plate was, as expected, greater than the predicted vertical earth pressure 0.3 m deeper into the backfill (or 150 mm above the pipe). Once the applied surface pressure exceeded 306 kPa, it was predicted that the vertical soil pressure closer to the pipe crown would be greater; on average it was predicted to be 18% higher for average plate pressures greater than 360 kPa. However the observed values of vertical soil pressure recorded closer to the plate were greater than the readings from the deeper-seated pressure cell, as would be expected normally.

The second observation concerned the reduction of vertical soil pressure, 150 mm above the crown and at z = 0.3 m, as was seen in the FEA of other tests. A slight decrease of the vertical earth pressure was predicted after the applied surface pressure reached 333 kPa, a similar pressure level at which the same observation was made for the FEA of test 300/6. As previously suggested, this reduction could be associated with the formation of a punching shear failure. Interestingly, the backfill and surround were significantly stronger for this test than for test 300/6, with density
indices ranging between 70 to 80%, as compared with density indices between 60 and 65% for test 300/6.

450 mm diameter pipe series

The predictions of earth pressures for the 450 mm diameter pipes (400/1, 400/2 and 400/5) are provided in Figures 8-66 to 68. Tests 400/1 and 400/2 were identical installations, both with 450 mm of protective cover to the pipe; however the soil densities achieved in the two tests were slightly different\(^6\).

The FEA for test 450/1 overestimated the vertical soil pressure, 150 mm above the crown and at \( z = 0 \), by 46% at a surface pressure of 400 kPa. Interestingly, the predicted pressure by this stage was as high as 98% of the applied surface pressure. The pressures between \( z = 0.25 \) and 0.5 m were underestimated by 26% to 39%. It would seem that the FEA predicted a punching shear failure, which presumably was not realized as quickly in the test. Unfortunately no observations of plate settlements were made in this or any other test reported in this section.

The prediction of a greater concentration of pressure above the pipe crown for a given external loading was clearly largely responsible for the significant overestimates of the vertical pipe deflection with loading. At 400 kPa, the vertical diametric strain had been overestimated by 41%.

A similar pattern was evident in the FEA of test 450/2; at an applied pressure of 400 kPa, the predicted pressure 150 mm above the crown and at \( z = 0 \) m was 99.5% of the applied pressure. However this estimate was 81% higher than the measured pressure. At this same value of applied pressure, the vertical diametric strain was overestimated by the series 5 FEA by 54%. The vertical soil pressure data at \( z = 250 \) mm was adequately modelled.

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\(^6\) The backfill in test 400/2 had 5% more density index; the density of the surround in test 450/1 was not determined, however a value equivalent to the density index of the backfill of 70% was adopted in all the analyses, which was 5% more than the density index of the surround achieved in test 400/2.
The measured pressures in the sidefill appeared to be slightly erratic with the lateral pressure reducing as loading increased in both tests 450/1 and 450/2. This reduction could result from poor or failing restraint of the sidewall. The FEA predicted that the horizontal pressure should be greater than the vertical pressure, which was not observed in either test. The development of vertical pressure in the sidefill with loading was significantly underestimated, while the lateral pressures from the FEA for both tests reasonably represented the experimental data.

Experimental data for test 450/5, the test with the maximum cover height of 800 mm, were limited, as previously explained. Nevertheless, it was clear from the FEA simulation that the predicted vertical soil pressure above the pipe crown was far greater than that measured (almost six times the value at an applied surface pressure of 400 kPa), thus helping to explain the large discrepancy between predicted and measured vertical diametric strains for this test (approximately a factor of three at the surface pressure of 400 kPa). Furthermore the vertical pressure in the surround was underestimated by 56% at the applied surface pressure of 400 kPa.

8.6.5.3 Deformed cross-sections, pipe hoop forces and moments
The deformed shapes of the buried pipes under load on the $z = 0$ plane are provided in Figures 8-69 to 8-72, and contain comparisons with FEA predictions. Generally monitoring during the test provided eight equi-spaced points around the circumference at which deflections were measured. However for test 450/1 (Figure 8-71), fewer displacement transducers were available and so only four data points were able to be plotted at 90° spacing.

For the buried pipe tests in the laboratory, all the FEA data in the Figures correspond to series 5 analyses, i.e. from modelling with a partly supported box wall of given stiffness. The numbers on the vertical axes of the Figures are the height of the pipe in mm. For greater clarity, the deformed shapes have been plotted above a height of 50 mm.

The FEA prediction of the deformed pipe shape at a surface pressure of 400 kPa for the 300 mm, 90VX profile pipes was of a similar standard for both tests 300/4 and 300/8 (refer Figure 8-69). The major difference between the two tests was the depth
of protective backfill cover. In both cases the vertical diametric strains were overestimated slightly, more significantly for the test with the deeper cover (650 mm), test 300/8. In contrast, the lateral diametric pipe strain was well predicted by the FEA of test 300/8, being slightly overestimated, while the FEA of test 300/4 underestimated the lateral pipe strain.

As previously discussed, the behaviour of the 375 mm diameter pipe with only 300 mm of cover (test 375/1) was notably inadequate. The underestimation of the strains in practically all directions is apparent in Figure 8-70. In this Figure, the deformed shapes are given near the end of the test with the surface pressure at 250 kPa. The vertical diametric strain was recorded as –6.8%.

It is evident from Figure 8-71 that the crown deflection was slightly overestimated for test 450/1, in which the pipe was provided with 450 mm of cover. However, the predicted lateral distortion was in agreement with the experimental deflections.

The deflections of the cross-section of field test, F450/3 is given in Figure 8-72. In this plot, the influence of the side soil stiffness on the deformed shape can be seen to be negligible, after comparing the shapes for Young’s modulus of the natural soil of either 30 or 90 MPa. Neither FEA could match the high level of diametric strains achieved by this pipe towards the end of testing (surface pressure of 676 kPa; vertical diametric strain of –3.9%).

Hoop forces and moments at selected cross-sections, as well as down the length of the pipe, have been determined from the FEA predictions of the deformed shapes of the pipes, i.e. from the nodal deflections along the outer wall. The chosen FEA predictions were for tests 300/4 after 1600 increments of enforced displacement (average surface pressure of 422 kPa), 375/1 after 800 increments (pressure of 248 kPa) and F450/3 after 1200 increments (pressure of 664 kPa). The surface pressures achieved at these stages of the respective analyses were within 3% of the previously quoted value, which were adopted in the comparisons of the test and FEA pipe displacement data.
The vertical diametric strains of the pipes predicted by the FEA at the chosen displacement increments were –4.5, –2.95 and –2.0% for tests 300/4, 375/1 and F450/3, respectively. Figures 8-73 to 8-80 provide the three data sets for the tests, beginning with the predicted deformed shape of each pipe, scaled up by a factor of five (Figures 8-73 and 74). The exaggerated displacements highlight the change of curvature that can occur at approximately the quarter point between the crown and the springline. This change of curvature is however not so apparent for the FEA prediction for test F450/3 at 1200 increments of displacement, by which stage the vertical diametric strain had only reached –2.0%. Accordingly, at this same location, the magnitude of negative moment peaked (refer Figure 8-75) and the hoop force was almost zero for all the FEA predictions (Figure 8-76). The maximum positive moment occurred at an angle of 15 to 20° from the pipe crown, near another inflection of the pipe.

All three data sets have been plotted together to compare the distributions of moments and hoop forces between pipes of different cross-section and diameter. Despite these differences and the differences in the chosen stages of loading, the distributions of either moments or hoop forces are remarkably similar. The magnitudes of the moments did not differ significantly, varying generally between ± 0.1 kN.m/m length of pipe. Test 375/1 displayed a greater positive moment near the pipe crown relative to the minimum moment in the cross-section. Nonetheless, the moments were insignificant and could probably be ignored in the design of these very flexible pipes.

The magnitudes of hoop forces were similar for the FEA of tests 300/4 and 375/1. Hoop forces for F450/3 were of significantly smaller magnitude, reflecting the smaller vertical diametric strain of this same pipe. The maximum predicted tensile hoop force (negative hoop force) was located at the pipe crown, while the maximum compressive hoop force (positive hoop force or thrust), which was only approximately a third of the magnitude of the tensile hoop force, occurred about 20° above the pipe springline, but was closer to 30° for test F450/3. Adopting the tensile capacity of the pipe material of 33 MPa as developed in Chapter 4, the hoop forces at the crown represent 55%, 43% and 30% of the tensile capacity for the 300, 375 and
450 mm diameter pipes, respectively. Most notably, the predicted tensile hoop force value for the 300 mm diameter pipe pertains to a vertical diametric strain of -4.5%, almost at the design vertical diametric strain of -5% for these pipes.

A simple approach to checking the vertical hoop force predicted at the springline is to distribute the soil pressure above the pipe to the underlying arch, i.e. the bottom half of the pipe. Using the pressure 150 mm above the pipe and assuming it to be spread evenly over the full pipe diameter, the following estimates were made for tests 300/4, 375/1 and F450/3, respectively: 44 kN/m (cf 58 kN/m from the FEA), 50 kN/m (cf. 59 kN/m) and 77 kN/m (cf 34 kN/m). The estimates for the first two tests are quite reasonable when compared with the more thorough analyses indicated by the corresponding values in the brackets. However the simple estimate for test F450/3 significantly overestimated the hoop force, presumably due to the relatively low deflection of the pipe and the lack of development of arching.

The remaining Figures show the deflections and axial thrusts along the pipe length for two locations about the pipe, namely the crown (Figures 8-77 and 8-78) and the springline (Figures 8-79 and 8-80). The moments were too insignificant to include. The thrusts were greater along the crown, with the maximum compressive thrust peaking at a distance, z, of approximately 0.3 m, just behind the level of the loading plate. The tensile forces along the springline peaked at much the same distance but were only a tenth or less of the values at the crown. The maximum value of compressive force at the crown in the z-direction was relatively small, when compared with the tensile force at the crown, but in the z = 0 plane. For test 375/1, the ratio of the forces at crown level was just 0.27.

8.6.5.4 Influence of side wall stiffness
For a few selected pipe tests, the wall stiffness of the soil box was modified in an attempt to improve the match with the observed pipe deflections. The tests that were re-analyzed were 375/1, 450/1 and 450/5, and the results in terms of the pipe displacements are given in the previous Figure numbers 8-46 and 8-49b, for tests 375/1 and 450/5, respectively. Figure 8-81 provides the load-deflection data for test 450/1.
For test 375/1, a better result was achieved by halving the Young’s modulus of the wall, in terms of the vertical diametric strain of the pipe to 80% of the maximum applied pressure. In contrast, the two analyses of the tests on the 450 mm diameter pipe with 450 and 800 mm of cover showed that at least a doubling of the sidewall modulus was required to better match the experimental data. Even at this level of box wall stiffness, the vertical diametric pipe strain was still overestimated, although the lateral pipe strain was reasonably well matched with the experimental data.

8.7 SUMMARY OF THE CHAPTER

In the beginning of this Chapter, it was demonstrated that pipe ring compression tests could be adequately modelled with a single thickness elastic element, by the adoption of non-conforming displacement finite elements to model pure bending of the pipe section.

Adoption of a perfectly smooth sidewall in the FEA of either the paved or non-paved buried pipe tests was found to produce larger estimates of lateral deflections of the pipe, than if a perfectly rough wall analysis was conducted. Lateral strains however were less for the simulated paved installations with the large flexible loading plate. Generally speaking, although the 3D analyses improved the predictions of behaviour when compared with the 2D analyses, the assumption of a rigid sidewall with either a perfectly rough or perfectly smooth boundary condition could not adequately simulate the observed load-deflection behaviour of the pipes.

Examination of the actual side boundary condition led to incorporating the braced wall in the FEA, as a separate material with a value of Young’s modulus based on both the formwork manufacturer’s specifications and plate theory. The side boundary conditions were accordingly relaxed to allow possible distortion of the wall as loading was increased on the surface of the backfill. This series of finite element analyses were named “series 5” and it was found that the behaviour predicted by the series 5 FEA of the pipe-soil system more closely simulated the observed behaviour. In particular, a better match was seen with measured horizontal diametric pipe strains.
Nonetheless, the assignment of an appropriate stiffness of the sidewall was not obvious, as a matching of both the lateral and vertical diametric strains could not be achieved with a single value of Young’s modulus of the wall, with the adopted boundary conditions. The partly theoretically-derived value of modulus of the sidewall gave reasonable predictions of the pipe behaviour, although it was shown that a modulus of 50 to 200% of this value could provide better estimates of the pipe load-deflection behaviour for any given buried pipe test.

Hoop forces and moments were predicted from the deformed shapes of selected pipes for specific tests and solution stages. With these particularly flexible pipes, the moments that may be developed are relatively insignificant, however the hoop forces developed in the pipe ring can be appreciable, although they were shown to be within safe limits of the capacity of the pipe material.

In the next Chapter, the data from the three-dimensional FEA are further reviewed and approaches to the practical design of flexible buried pipes are proposed.

8.8 REFERENCES TO THE CHAPTER


### TABLE 8-I. Adopted values of pipe properties for 3D FEA

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Section</th>
<th>EA (kPa.m²/m)</th>
<th>EI (kPa.m⁴/m)</th>
<th>t (m)</th>
<th>E (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>90VX</td>
<td>1.52E+03</td>
<td>2.06E-02</td>
<td>0.0128</td>
<td>1.19E+05</td>
</tr>
<tr>
<td>300</td>
<td>110VX</td>
<td>2.55E+03</td>
<td>5.43E-02</td>
<td>0.0160</td>
<td>1.60E+05</td>
</tr>
<tr>
<td>375</td>
<td>110VX</td>
<td>2.55E+03</td>
<td>6.58E-02</td>
<td>0.0176</td>
<td>1.45E+05</td>
</tr>
<tr>
<td>450</td>
<td>125VX</td>
<td>2.94E+03</td>
<td>9.33E-02</td>
<td>0.0195</td>
<td>1.51E+05</td>
</tr>
</tbody>
</table>

### TABLE 8-II. Comparison of FEA analyses of pipe stiffness tests with a theoretical solution

<table>
<thead>
<tr>
<th>Dia. (mm)</th>
<th>Section</th>
<th>EI (kPa.m⁴/m)</th>
<th>Dᵥ7 (m)</th>
<th>Dₜ (m)</th>
<th>One element per width</th>
<th>Two elements per width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>δDᵥ5% (%)</td>
<td>δDₜ5% (%)</td>
</tr>
<tr>
<td>300</td>
<td>90VX</td>
<td>2.43E-02</td>
<td>-0.024</td>
<td>0.022</td>
<td>-0.5</td>
<td>2.4</td>
</tr>
<tr>
<td>300</td>
<td>110VX</td>
<td>6.38E-02</td>
<td>-0.009</td>
<td>0.009</td>
<td>0.2</td>
<td>2.8</td>
</tr>
<tr>
<td>375</td>
<td>110VX</td>
<td>7.70E-02</td>
<td>-0.015</td>
<td>0.014</td>
<td>0.7</td>
<td>3.7</td>
</tr>
<tr>
<td>450</td>
<td>125VX</td>
<td>1.09E-01</td>
<td>-0.018</td>
<td>0.016</td>
<td>1.2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Dᵥ and Dₜ are the vertical and horizontal diametric pipe movements, respectively, for a unit vertical load (1 kN/m length) as predicted by Roark and Young’s (1975) solution, with adjustment of stiffness for a finite length of pipe.
TABLE 8-III. Comparison of surface pressure–strain data from FEA of non-paved tests, with a rigid wall, having either a smooth or rough mesh

<table>
<thead>
<tr>
<th>Test</th>
<th>Ratio of Cover Height to Pipe Diameter</th>
<th>Surface Pressure for 3% Vertical Diametric Strain (kPa)</th>
<th>Test</th>
<th>FEA Smooth wall</th>
<th>FEA Rough wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Smooth wall</td>
<td>Rough wall</td>
</tr>
<tr>
<td>300/2</td>
<td>1.50</td>
<td>445</td>
<td>472.6</td>
<td>+6%</td>
<td>583.3</td>
</tr>
<tr>
<td>300/6</td>
<td>2.67</td>
<td>496</td>
<td>945.1</td>
<td>+90%</td>
<td>1421*</td>
</tr>
<tr>
<td>375/1</td>
<td>0.80</td>
<td>200</td>
<td>317.5</td>
<td>+59%</td>
<td>380.7</td>
</tr>
<tr>
<td>375/4</td>
<td>1.87</td>
<td>475</td>
<td>971.1</td>
<td>+104%</td>
<td>1150</td>
</tr>
<tr>
<td>450/1</td>
<td>1.00</td>
<td>385</td>
<td>580.5</td>
<td>+51%</td>
<td>669.2</td>
</tr>
<tr>
<td>450/5</td>
<td>1.78</td>
<td>795</td>
<td>1412</td>
<td>+78%</td>
<td>1533</td>
</tr>
</tbody>
</table>

*extrapolated

TABLE 8-IV. Comparison of surface pressure – strain data from FEA of field tests, either elastic natural soil (E = 30 MPa), or a perfectly rough and rigid sidewall

<table>
<thead>
<tr>
<th>Test</th>
<th>Ratio of Cover Height to Pipe Diameter</th>
<th>Surface Pressure for 3% Vertical Diametric Strain (kPa)</th>
<th>Test</th>
<th>FEA Elastic soil</th>
<th>FEA Rough, rigid wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Elastic soil</td>
<td>Rough, rigid wall</td>
</tr>
<tr>
<td>F300/3</td>
<td>1.50</td>
<td>455</td>
<td>523.0</td>
<td>+15%</td>
<td>644.7</td>
</tr>
<tr>
<td>F375/7</td>
<td>1.87</td>
<td>936</td>
<td>1004</td>
<td>+7.3%</td>
<td>1180</td>
</tr>
<tr>
<td>F450/3</td>
<td>1.33</td>
<td>630</td>
<td>846.2</td>
<td>+34%</td>
<td>962.0</td>
</tr>
<tr>
<td>F450/4</td>
<td>1.87</td>
<td>740</td>
<td>1090</td>
<td>+47%</td>
<td>1235</td>
</tr>
</tbody>
</table>

* extrapolated value
### TABLE 8-V. Comparison of earth pressure data at springline from FEA of field tests

<table>
<thead>
<tr>
<th>Test Information</th>
<th>Test Soil Pressure (kPa)</th>
<th>FEA Rough, rigid wall</th>
<th>FEA Elastic side soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>Surface Pressure (kPa)</td>
<td>Horiz., against wall</td>
<td>Vertical, surround</td>
</tr>
<tr>
<td>F375/7</td>
<td>766</td>
<td>130</td>
<td>87</td>
</tr>
<tr>
<td>F450/3</td>
<td>676</td>
<td>73</td>
<td>65</td>
</tr>
</tbody>
</table>

### TABLE 8-VI. Comparison of surface pressure – strain data from series 2 and series 5 FEA of simulated pavement tests (values in parentheses are the corresponding lateral diametric strains in %)

<table>
<thead>
<tr>
<th>Test</th>
<th>Surface Pressure for 2.5% Vertical Diametric Strain (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test FEA series 2 Rough, rigid wall FEA series 5 Ply, end supported</td>
</tr>
<tr>
<td>P375/1</td>
<td>102.8 (2.51) 349.3 (0.47)</td>
</tr>
<tr>
<td>P375/2</td>
<td>148 (1.80) 366.8 (0.46)</td>
</tr>
<tr>
<td>P450/1</td>
<td>134.5 (2.07) 353.9 (0.36)</td>
</tr>
</tbody>
</table>

### TABLE 8-VII. Comparison of pressure – displacement data from FEA of simulated pavement tests of test P450/1 at 400 increments

<table>
<thead>
<tr>
<th>FEA</th>
<th>Vertical pressure at 150 mm above crown (kPa)</th>
<th>Proportion of applied pressure</th>
<th>Displacement of pipe at 45° from the crown (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>x direction</td>
</tr>
<tr>
<td>Series 2</td>
<td>136.8</td>
<td>62%</td>
<td>+0.03</td>
</tr>
<tr>
<td>Series 5</td>
<td>85.4</td>
<td>76%</td>
<td>+0.96</td>
</tr>
</tbody>
</table>
### TABLE 8-VIII. Series 5 FEA predictions of the surface pressure required to reach 3% vertical diametric strain for the laboratory pipe tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Surface Pressure for 3% Vertical Diametric Strain (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test Pressure</td>
</tr>
<tr>
<td>300/1</td>
<td>425</td>
</tr>
<tr>
<td>300/2</td>
<td>445</td>
</tr>
<tr>
<td>300/4</td>
<td>365</td>
</tr>
<tr>
<td>300/5</td>
<td>370</td>
</tr>
<tr>
<td>300/6</td>
<td>496</td>
</tr>
<tr>
<td>300/8</td>
<td>625</td>
</tr>
<tr>
<td>375/1</td>
<td>200</td>
</tr>
<tr>
<td>375/4</td>
<td>475</td>
</tr>
<tr>
<td>375/5</td>
<td>516</td>
</tr>
<tr>
<td>375/6</td>
<td>600</td>
</tr>
<tr>
<td>375/8</td>
<td>600</td>
</tr>
<tr>
<td>450/1</td>
<td>385</td>
</tr>
<tr>
<td>450/2</td>
<td>395</td>
</tr>
<tr>
<td>450/5</td>
<td>795</td>
</tr>
</tbody>
</table>

---

<sup>a</sup> Extrapolated value (maximum vertical diametric strain of 2.7% reached)
Table 8-IX. Measured soil pressures and predictions from series 5 FEA for tests on 300 mm diameter pipe with 650 mm of cover

<table>
<thead>
<tr>
<th>Test</th>
<th>300/5</th>
<th>300/7</th>
<th>300/8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Predicted Soil Pressure in Surround at 400 kPa (kPa)</strong></td>
<td>Vertical</td>
<td>37.1 (-17%)</td>
<td>33.9 (-48%)</td>
</tr>
<tr>
<td></td>
<td>Horizontal</td>
<td>56.3 (-53%)</td>
<td>52.1 (-7%)</td>
</tr>
<tr>
<td><strong>Measured Soil Pressure in Surround at 400 kPa (kPa)</strong></td>
<td>Vertical</td>
<td>44.4</td>
<td>64.6</td>
</tr>
<tr>
<td></td>
<td>Horizontal</td>
<td>120.8</td>
<td>56.0</td>
</tr>
</tbody>
</table>

Figure 8-1. Meshes for evaluation of pipe stiffness test (element type 30)
Figure 8-2. Mesh for simulation of paved pipe test P375/1 or 2 (series 2)

Figure 8-3. Mesh for simulation of non-paved pipe test 380/6 (Series 2)
Figure 8-4. Typical boundary conditions for 3D simulation of buried pipe tests
(*paved test illustrated with perfectly rough side wall*)

Figure 8-5. Mesh for simulation of non-paved test 300/6 with reduced sidewall support (Series 5)
Figure 8-6. Boundary conditions for simulation of non-paved test 300/6 with uniformly stiff sidewall (Series 5)
(The left hand diagram is the central cross-section; the right hand diagram is the cross-section at the end of the soil box)
Figure 8-7. Comparison of deflection data for FEA of pavement tests on 375 mm diameter pipe with perfectly rough or smooth sidewall boundary condition
Figure 8-8. Comparison of deflection data for FEA of pavement tests on 450 mm diameter pipe with perfectly rough or smooth sidewall boundary condition.
Figure 8-9. Comparison of exaggerated deformed meshes for FEA of P375/2 with either a perfectly rough or perfectly smooth wall, at 400 increments.

(a) Perfectly smooth wall (189 kPa)

(b) Perfectly rough wall (214 kPa)
Figure 8-10. Comparison of cumulative vectors for FEA of P375/2 with either a perfectly rough or perfectly smooth wall, at 400 increments.
Figure 8-11. Comparison of y displacement contour plots for FEA of P375/2 with either a perfectly rough or perfectly smooth wall, at 400 increments.
Figure 8-12. Comparison of deflection data for FEA of selected non-paved tests on 300 mm diameter, 90VX pipe, with perfectly rough or smooth sidewall
Figure 8-13. Comparison of deflection data for FEA of selected non-paved tests on 375 mm diameter pipe with perfectly rough or smooth sidewall
Figure 8-14. Comparison of deflection data for FEA of selected non-paved tests on 450 mm diameter pipe with perfectly rough or smooth sidewall

a) Test 450/1, 450 mm of cover

b) Test 450/5, 800 mm of cover
Figure 8-15. Development of vertical soil pressure 150 mm above the pipe crown for FEA meshes with perfectly smooth or rough walls, tests 450/1 and 5
Figure 8-16. Mesh for simulation of field tests (material 5 is the side soil)
Figure 8-17. Comparison of deflection data for FEA of the non-paved field tests on 300 mm (90VX) and 375 mm diameter pipe, between mesh with side soil and as if tested in box with perfectly rough sidewall boundary condition.
a) Test F450/3, 450 mm diameter pipe, 600 mm of cover

b) Test F450/4, 450 mm diameter pipe, 700 mm of cover

Figure 8-18. Comparison of deflection data for FEA of the non-paved field tests on 450 mm diameter pipe, between mesh with side soil and as if tested in box with perfectly rough sidewall boundary condition.
a) Test F300/3, 300 mm diameter, 90VX pipe with 450 mm of cover

b) Test F375/7, 375 mm diameter pipe with 700 mm of cover

Figure 8-19. Predicted plate deflection with loading for non-paved, field tests (300, 90VX and 375 mm diameter pipe), compared with observed behaviour
Figure 8-20. Predicted plate deflection with loading for non-paved, field tests (450 mm diameter pipe), compared with observed behaviour
Figure 8-21. Earth pressure data from FEA of non-paved, field test F375/7, compared with observations from testing with pressure cells.
a) Vertical soil pressures, on plane $z = 0$ and 150 mm above the pipe crown

b) Vertical (V) and horizontal (H) soil pressure beside the pipe springline, $z = 0$

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Vertical displacements

Lateral displacements

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b) FEA with natural side soil, 1200 increments

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b) Pressure in the surround (x = 0.23 m, y = 0.25 m, z = 0);
V = vertical, H = horizontal

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b) Pressure in the surround (x = 0.23 m, y = 0.25 m, z = 0);

V = vertical, H = horizontal

Figure 8-61. Predicted earth pressure development above the pipe and in the surround, series 5 FEA, Test 300/5, 650 mm of cover (90VX)
a) Longitudinal distribution of vertical pressure, 150 mm and 300 mm above the pipe crown (x = 0)

b) Pressure in the surround (x = 0.23 m, y = 0.25 m, z = 0);
\[ V = \text{vertical, } H = \text{horizontal} \]

Figure 8-62. Predicted earth pressure development above the pipe and in the surround, series 5 FEA, Test 300/8, 650 mm of cover (110VX)
a) Longitudinal distribution of vertical pressure 150 mm and 300 mm above the pipe crown (x = 0)

Figure 8-63. Predicted earth pressure development above the pipe and in the surround, series 5 FEA, Test 300/7, 650 mm of cover (110VX)
a) Longitudinal distribution of vertical pressure 150 mm above the pipe crown
(x = 0)

b) Pressure in the surround (x = 0.23 m, y = 0.25 m, z = 0)

V = vertical, H = horizontal

Figure 8-64. Predicted earth pressure development above the pipe and in the surround, series 5 FEA, Test 300/6, 800 mm of cover (90VX)
a) Longitudinal distribution of vertical pressure 150 mm and 450 mm above the pipe crown (x = 0)

b) Pressure in the surround (x = 0.28 m, y = 0.29 m, z = 0);

$V = \text{vertical, } H = \text{horizontal}$

Figure 8-65. Predicted earth pressure development above the pipe and in the surround, series 5 FEA, Test 375/8, 700 mm of cover
a) Longitudinal distribution of vertical pressure 150 mm above the pipe crown  
\((x = 0)\)

b) Pressure in the surround \((x = 0.3 \text{ m}, y = 0.325 \text{ m}, z = 0)\);
\(V = \text{vertical}, H = \text{horizontal}\)

Figure 8-66. Predicted earth pressure development above the pipe and in the surround, series 5 FEA, Test 450/1, 450 mm of cover
a) Longitudinal distribution of vertical pressure 150 mm above the pipe crown 

\((x = 0)\)

b) Pressure in the surround \((x = 0.3 \text{ m}, y = 0.325 \text{ m}, z = 0);\)

\(V = \text{vertical}, \ H = \text{horizontal}\)

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b) Test 300/8 (90VX, 650 mm of cover), -1.4% vertical diametric strain

Figure 8-69. Deformed pipe shapes, 300 mm diameter pipes, at an applied surface pressure of 400 kPa
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Figure 8-71. Deformed pipe shape, test 450/1, 450 mm diameter, 450 mm cover, at an applied surface pressure of 400 kPa; -3.2% vertical diametric strain
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Figure 8-73. Deformed pipe shapes, scaled by a factor of 5 from FEA, series 5

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Figure 8-75. Theoretical distribution of moments about pipe cross-section

\( z = 0 \text{ m} \)
Figure 8-76. Theoretical distribution of hoop forces about pipe cross-section

\((z = 0 \, \text{m})\)

Figure 8-77. Predicted crown deflections along \(z\)-axis
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Figure 8-79. Predicted deflections at the level of the springline along z-axis
Figure 8-80. Theoretical distribution of hoop forces along the $z$–axis at the level of the springline

Figure 8-81. The influence of wall stiffness on the deflection of the pipe, Test 450/1, 450 mm of cover