Chapter 1
Introduction

It never ceases to amaze us how biological systems work accurately, efficiently and in a fast manner in such a noisy environment. Even with today’s advanced technology, we haven’t been successful in building a system which, for example, distinguishes one’s voice from someone else’s or understands conversation, as well as we do. It is, therefore, a natural idea that one of the ways to build such a system would be to analyse, understand, and mimic the way biological systems process information they get from their environment.

Hearing is one of the basic and essential abilities that we have, and the mechanism by which sound is processed is relatively simple at the first stage, namely, by the cochlea. The cochlea is a fluid-filled, coiled tube located in the inner ear. It receives input sound signals from the middle ear and produces output signals onto the auditory nerves. Its basic function is to decompose sound waves by frequencies that they contain, and encode the frequency information as spikes which will be transmitted to the brain to be processed further. The purpose of my research is to make a model which represents the main functions of the cochlea, and to possibly in the future construct a system which performs hearing tasks just as human beings do.

Proper modelling is important, because we cannot incorporate every single detail of the cochlea; if we do so, the system would be very complicated, large in size and energy consuming. However, if we make it too simple, it will not perform as well as the real cochlea does. Modelling is also useful to understand what the cochlea is really doing where we cannot see directly; because it is located deep inside the head and the size of its sensory organ is quite small, it has been hard to observe directly what the cochlea does when it is alive. If the model’s response to input signals is similar to that of the real cochlea, the model
can be considered as one of the ways to explain the microscopic operation of the cochlea.

Cochlear modelling can be divided into two categories; passive ones and active ones. Passive models use only input signals as the driving force, whereas active models contain sources in themselves to amplify input signals especially when they are small in amplitude. The mechanism by which the cochlea realises this amplification is yet unclear. However, it is necessary to incorporate this active mechanism if we are to make the system close to the real cochlea, and the main part of my research is to create an appropriate active model based on which the system is implemented.

The ways of implementing the system are limited, as we aim for a system that is small and efficient, just like the real cochlea. Analog VLSI (Very Large Scale Integration) implementation is suitable, because it is simpler and smaller than digital systems in general, and operates in real time. Using the latest CMOS (complementary metal-oxide-semiconductor) technology in weak-inversion, a system can be implemented in the current domain with pseudo-voltage representation. In pseudo-voltage domain, the voltage changes logarithmically as the current changes linearly: this means that while the current represents the signal of importance in full range, the voltage remains lower than it would be in linear voltage domain. Therefore, it allows the voltage supply to be low and leads to low power consumption. Another reason why analog VLSI implementation was chosen is that it is often easier in the analog domain to mimic the function of biological systems, as they work in the analog domain: in this case, as discussed in Chapter 2, there is an analogy between the hydrodynamics for the liquid inside the cochlea and the current through the resistive sheet.

In Chapter 2 we will briefly look at the anatomy of the cochlea, then review previous work on modelling, especially the work of Eric Fragnière, which is a passive model. Chapter 3 presents an analog VLSI implementation of the passive cochlea, which is based on the models in Chapter 2. It utilises log-domain filters to realise filter functions, and its operation and measurement results are described. Chapter 4 introduces an active cochlear model which is motivated by a potential analog VLSI implementation. There have been a number of active models showing similar responses to input stimuli as the real cochlea. However, in order for them to be the basis of an analog VLSI implementation, they have to be modified to be simpler yet still functional enough. The details of the model introduced in this chapter and its simulation results are presented. A summary, a conclusion and directions for further research are presented in Chapter 5.
Chapter 2

Biological Cochlea and Models

In this chapter, the anatomy and the mechanics of the human ear, and the brief history of research on the cochlear mechanisms and modelling, are presented.

Section 2.1 provides basic information on the cochlear anatomy and mechanics. The macromechanics of the cochlea describes the motion of the fluid inside the cochlea and how it interacts with the basilar membrane. The word “macro” is used here as opposed to micromechanics, which focuses on the motions within the structure on the basilar membrane, called the Organ of Corti, in smaller scale. Micromechanics is important for capturing the active mechanism of the cochlea and is presented in chapter 4. A closer look at the transduction mechanism of the sensory cells in the Organ of Corti shows how they successfully convert the motion into auditory nerve signals, which is presented in section 2.1.3.

A number of models for the cochlea have been proposed. A brief summary of the previous works is presented in section 2.2. The model from Eric Fragnière employs hydrodynamics for inviscous incompressible flow, and models the basilar membrane as a bank of resonators with their resonant frequencies changing exponentially along the cochlea. The analog VLSI implementation described in chapter 3 is based on his model. Its details and analysis by simulations are presented in section 2.3.

Information on the anatomy and physiology of the outer and middle ear is taken from [van Schaik, 1997]. Information on the anatomy and physiology of the cochlea is taken from [Harrison and Hunter-Duvar, 1988; Santi, 1988; Dallos, 1978; Hubbard and Mountain, 1996].
2.1 Cochlear Anatomy and Mechanics

2.1.1 Anatomy of the Human Ear

The human ear consists of three parts: the outer ear, the middle ear and the inner ear, as shown in Figure 2.1.

A sound wave enters through the ear canal. When it reaches the eardrum, it is converted to mechanical vibrations and transmitted to the oval window of the cochlea through a series of small bones called the ossicular chain in the middle ear. The movement of the oval window produces fluid pressure waves inside the cochlea. As the wave travels down the cochlea, its wavelength and amplitude change because of the gradient in the stiffness of the basilar membrane, which separates the cochlea into two chambers along its length. The wave reaches its maximum amplitude at a point which is determined by the frequency of the sound wave. This mechanism is further described in section 2.1.2. Vibration of the basilar membrane causes the hair cells of the Organ of Corti to generate nerve signals, which the brain perceives as hearing. This transduction mechanism is further described in section 2.1.3.

Figure 2.1: The human ear, adapted from [van Schaik, 1997]
The Outer Ear

The outer ear consists of the pinna and the ear canal. As the sound wave reaches the pinna and travels through the ear canal, its sound pressure is altered depending on its frequency and the location of the sound source, by being reflected at the pinna and resonated at the ear canal. Its main function is to be an auditory filter to provide cues for localising a sound source, as well as protecting the eardrum from the environment.

The Middle Ear

The middle ear is an air-filled cavity inside the skull. Three small bones called the Malleus, the Incus and the Stapes, which forms a chain called the ossicular chain, are suspended in the cavity by small muscles. The ossicular chain transfers vibrations at the ear drum to the oval window of the cochlea.

The Inner Ear

The inner ear is a fluid-filled bony structure. It consists of the semicircular canals, the vestibule and the cochlea. The semicircular canals and the vestibule are responsible for sensing the orientation and movement of the head, whereas the function of the cochlea is to sense sound travelling through the outer and the middle ear.

The Cochlea

A cross section of the cochlea through its core is shown in Figure 2.2. The cochlea is a snail-shaped, fluid-filled duct, coiled into a little over 2.5 turns. When uncoiled, the total length of the human cochlea is 35mm with a diameter of 10mm at its basal end. The external bony covering is very thin in some animals such as the chinchilla, the guinea pig and the gerbil, which facilitates their use for cochlear research. The central bony core of the cochlea is called the modiolus, through which nerve fibres and blood vessels enter the Organ of Corti.

The cochlea is divided into three longitudinal channels by the basilar membrane, which
continues from a spiralling bony shelf called the spiral lamina, and Reissner's membrane. Reissner’s membrane is very flexible and has little effect on cochlear hydrodynamics. Its main function is to provide an ionic and potential barrier.

The uppermost channel, between the bony wall and Reissner’s membrane, is called the scala vestibuli. The lowermost channel, between the bony wall and the basilar membrane, is called the scala tympani. The channel between Reissner’s membrane and the basilar membrane is called the scala media. The scala media is closed at the apical end of the cochlea, leaving an opening called the helicotrema that allows communication between the two outer scalae, namely, the scala vestibuli and the scala tympani.

The cochlea has two openings to the middle ear cavity. The oval window, located in the wall of the scala vestibuli about 1 – 2 mm from the base, is connected to the stapes. The other opening, called the round window, is a hole in the bony wall over the scala tympani, and is covered by a thin flexible membrane.

The scala vestibuli and the scala tympani are filled with perilymph. Perilymph is a fluid that is high in sodium and low in potassium, and is similar to the composition of extracellular fluid. On the other hand, the scala media is filled with endolymph, which is, in contrast, high in potassium and low in sodium, and resembles intracellular fluid. Also, the potential of the fluid in the scala media is higher by about 90mV than the fluid in the
other two scalae. These ionic and potential differences are maintained by the cells at the outer wall of the scala media called the stria vascularis, shown in the detailed cross section of the cochlea in Figure 2.3. The scala media contains the sensory and supporting cells for converting sound-induced mechanical vibrations to neural responses. The collection of these cells is called the Organ of Corti, which sits upon the basilar membrane.

![Diagram of the cochlea with labels for Scala vestibuli, Stria vascularis, Reissner's membrane, Scala media, Organ of Corti, Scala tympani, Basilar Membrane, Auditory nerve, Spiral ganglion, Spiral lamina, Spiral ligament.]

Figure 2.3: Detailed cross section of the cochlea. The detail of the Organ of Corti (squared) is shown in Figure 2.4. Taken from [Fragnière, 1998]

The Basilar Membrane

The basilar membrane is primarily acellular, and consists of radial fibres and amorphous substance. In humans, its average length is 31.5mm, and its width increases from 150 to 450 μm toward the cochlear apex. The fiber bundles are more densely packed at the cochlear base than the apex. As it is described later in section 2.1.2, this gradual decrease in the density of the fiber bundles along the length of the cochlea, together with the increase
in the width of the basilar membrane, is essential for the cochlea’s main function, namely, to discompose the input sound wave by frequency.

The Organ of Corti

![Detailed cross section of the Organ of Corti](image)

Figure 2.4: Detailed cross section of the Organ of Corti, taken from [Fragnière, 1998]

Figure 2.4 shows the detail of the Organ of Corti. The Organ of Corti consists of sensory cells, their supporting cells and nerves which innervate them. The sensory cells are divided into 2 categories: the inner hair cells, described in section 2.1.3, and the outer hair cells, described in 4.1.1. The supporting cells, most of which are the pillar cells and Deiters’ cells, contain a rigid substructure of protein fibrils including actin. The two pillar cells, the outer pillar cell and the inner pillar cell, form a tunnel of inverted V-shape, providing primary stiffness of the Organ of Corti. The pillar cells, the apical portion of the sensory cells and Deiters’ cells form a hard surface called the reticular lamina. As the fluid inside the Organ of Corti is perilymph, the reticular lamina is supposed to provide the primary ionic barrier between the perilymph and the endolymph.

Innervation

The nerves approach the Organ of Corti through the spiral lamina. The majority (95%) of the afferent nerves (signal to the brain) innervate the nearest inner hair cells from the
position they emerge from the spiral lamina. For this reason, it is naturally considered that the inner hair cells play the key role in sensing input sound waves and generating nerve signals. The rest of the afferent nerves innervate the outer hair cells after travelling along the length of the cochlea toward its base for about 0.6mm [Spoendlin, 1973]. The efferent nerves (feedback from the brain) primarily innervate the outer hair cells with their cell body, and also provide connections with afferent dendrites, which originate from the inner hair cells. The innervations to the outer hair cells are richest at the first turn of the cochlea, then gradually thin out toward the apex.

### 2.1.2 Cochlear Macromechanics

Cochlear macromechanics describes the interaction between the basilar membrane and the fluid in response to input sound waves.

The input to the cochlea is the movement of the stapes caused by a sound wave propagated through the air, the ear canal, the ear drum and the middle ear. The movement of the stapes creates a wave which propagates down the cochlea by combined movement of the fluid and the basilar membrane.

![Figure 2.5](image.png)

Figure 2.5: (a) Illustration of travelling waves along the basilar membrane, adapted from [Fragnière, 1998]. The radial fibres on the basilar membrane are indicated by the red lines. (b) Simulated basilar membrane displacement, exaggerated for better visualisation.

Figure 2.5 shows a simplified, uncoiled cochlea, showing only important components for cochlear mechanics, namely, the scala media and the scala vestibuli (they are shown as one chamber because the Reissner’s membrane between them is very flexible and negligible), the scala tympani and the basilar membrane. It is assumed that coiling does not affect its function, and is only for saving space. An illustration of the wave propagating down the basilar membrane is shown with its amplitude largely exaggerated in order to see it clearly.
As the stapes moves back and forth, the movement is transmitted via fluid throughout the cochlea, causing the “travelling wave”. Note that the energy of the wave is transmitted through the fluid, not the basilar membrane itself. The motion of the basilar membrane is caused by the pressure difference of the fluid at the positions directly above and below the basilar membrane.

The stiffness of the basilar membrane changes exponentially along the cochlea, being stiffest at the basal end and most flexible at the apical end. As a result, the travelling wave’s wavelength becomes shorter and shorter, and its amplitude becomes larger and larger as it travels down the cochlea. It reaches the point where the amplitude becomes maximum, which is called the “best place”, where the energy of the wave is absorbed. After this point, the wave loses its energy and quickly dies away. Conversely, at any given position on the basilar membrane, there is a frequency at which the amplitude at the position becomes maximum. It is called the “best frequency” or the “characteristic frequency” [Hubbard and Mountain, 1996].

![Figure 2.6: Frequency responses at six different points on the cochlear partition, taken from [von Békésy, 1960]]

Figure 2.6 shows the frequency responses at different positions on the basilar membrane. The amplitude of the travelling wave was measured with the input signal of the constant peak stapes displacement. It is seen that the characteristic frequency changes logarithmically along the cochlea, due to the fact that the stiffness of the basilar membrane decreases exponentially. The phase angle changes monotonically from 0 to \(-3\pi\), which indicates travelling waves whose wavelength becomes shorter as the frequency increases [von Békésy, 1960].
2.1.3 Hair Cell Transduction

A closer look at the sensory hair cells in the Organ of Corti shows how the basilar membrane movement is converted into auditory nerve signals.

The key component in converting the mechanical movement of the basilar membrane into nerve signals is the inner hair cell. There is a marked accumulation of mitochondria at the apical pole of the cells, which suggest that this is the location of transducer action.

![Figure 2.7: Detailed view of the inner hair cell, taken from [Pickles, 1988]](image)

The detail of the inner hair cell is shown in Figure 2.7. The inner hair cells are flask shaped, and quite irregular in shape. Each inner hair cell has about 40 – 60 hairs (stereocilia) at its apex, and their length is dependent on the location of the cell in the cochlea, being longer at the apex (about 45 µm) and shorter at the base (about 20 µm). The stereocilia form the shape of a very shallow W, almost in straight rows in the longitudinal direction of the cochlea.

Whether the tallest stereocilia is in contact with the tectorial membrane, which is a gelatinous membrane overlying the Organ of Corti shown in Figure 2.4, is not totally clear. However, the general consensus is that they are not embedded in the tectorial membrane, as opposed to those in the outer hair cells, but fit loosely on the under-surface of the tectorial membrane. As the basilar membrane motion is caused by the input sound pressure waves, the forces exerted on the stereocilia are mainly due to the viscous drag of the endolymph.
Thus, the displacement of the hair bundle of the inner hair cell is proportional to the velocity of their root, namely, the reticular lamina, which moves with the basilar membrane.

![Diagram of the inner hair cell](image)

**Figure 2.8: Schematics of the inner hair cell, taken from [Pickles, 1988]**

The hair cell cilia have tiny filaments which link between the cilia. The link, called a tip-link, connects from the tip of a short stereocilium to the shaft of the tall stereocilium next to it (Figure 2.8). When the stereocilia are displaced toward the tallest cilia, the tip links are stretched and therefore open the ion channels, allowing ion flow. When displaced in the other direction, the channels are closed, stopping the ion flow. The endolymph, which is the fluid in the scala media, has a positive potential of about 80mV relative to the perilymph. The cells within the Organ of Corti have negative resting intracellular potentials, and the inner hair cell has potentials near -45mV. Because of the voltage difference between the endolymph and the inside of the cell, the current flows inwards when the ion channels at the tips of the cilia are open. Since the concentration of the potassium ion is high in both the endolymph and the intracellular fluid of the inner hair cells, the potassium ion is considered to be the physiological charge carrier. Therefore, the displacement of the stereocilia toward the tallest stereocilia results in a depolarisation of the cell’s membrane potential (Figure 12).
At rest, the channels are in their open state about 20\% of the time. It is known that the voltage response for stereociliar displacements in the direction of depolarising is larger than in the hyperpolarising direction, making the cell’s response asymmetric (Figure 2.9). This intracellular voltage change is proportional to the permeability of the hair cell membrane, which affects on how many neurotransmitters are released into the synaptic cleft, i.e., the auditory nerve response [Meddis, 1986].

2.2 Overview of Previous Work on Cochlear Modelling

2.2.1 History of Research on Cochlear Operation

Theoretical modelling of the cochlear mechanisms started as early as the 19th century. Helmholtz suggested that the basilar membrane can be modelled as a parallel bank of resonators whose resonant frequencies change along the length of the cochlea [von Helmholtz, 1954]. In his model, the fluid interaction between the resonators were neglected, and the input signal was assumed to be applied independently to each resonator. In the early 20th century, the dynamic behaviour of the basilar membrane was reported for the first time by von Békésy. He observed “travelling waves” through his stroboscope propagating down the basilar membrane [von Békésy, 1960]. In order to explain this phenomenon, interaction between the fluid and the basilar membrane sections had to be taken into account. Thus

Figure 2.9: Transducer function, taken from [Hudspeth and Corey, 1977]
formed the general idea that the basilar membrane should be modelled as a bank of resonators coupled with each other through the fluid. Most of today’s models employ this structure.

von Békésy’s observed a linear basilar membrane response, which was not sharply tuned. Therefore, when a sharply tuned neural response was found in 1940s, it was assumed that there exists a “second filter” between the basilar membrane vibration and the neural response which realises this sharp tuning. While a physiological search for the second filter failed, and better observation techniques were employed, such as, for instance, the Mössbauer technique, a more sharply tuned response of the basilar membrane itself started to emerge. The basilar membrane response was found to be sharer and sharper as more advanced techniques and animal preparations were used [Hubbard and Mountain, 1996], until it was recognised that the basilar membrane itself is tuned sharply enough to account for the neural response. Since then, the main research focus has shifted to finding the mechanism which realises the sharp tuning, which is impossible to achieve by passive models [de Boer, 1983].

2.2.2 Development of the Electrical and Electronic Cochleae

The electrical analog of the cochlea was first proposed by Peterson and Bogert [Peterson and Bogert, 1950] and Zwislocki [Zwislocki, 1950] as a transmission line. Zwicker later used analog electrical cochlear models to simulate nonlinear phenomena such as two-tone suppression [Zwicker, 1986a,b].

Once VLSI technology started to develop, several cochlear models for silicon implementation were proposed. The first model was by Lyon and Mead using second-order sections to form a one-dimensional delay line [Lyon and Mead, 1988]. Based on their model, several groups have implemented VLSI cochleae. Lyon and Mead implemented the model as a cascaded weak inversion transconductance filters [Lyon and Mead, 1988]. Watts et al. designed an improved version of the first model by using translinear amplifiers with wider linear range and optimising the transistor sizes to minimise mismatch [Watts et al., 1992], and van Schaik et al. [van Schaik et al., 1996] used Compatible Lateral Bipolar Transistors (CLBTs) to create exponentially scaled currents which better matched than using MOS transistors operated in weak inversion. Sarpeshkar et al. [Sarpeshkar et al., 1996]
further developed to achieve nonlinear gain control similar to the nonlinearity seen in live cochleae. Bhadkamkar [Bhadkamkar, 1993] modified the model so that the propagation delay and the cut-off frequency can be controlled by separate variables. Also, Liu et al. [Liu et al., 1992] designed a cascade of first-order sections whose outputs were followed by second-order band-pass filter sections. These cochleae implemented as a cascaded transconductance filters are used for applications such as sound localisers [Bhadkamkar and Fowler, 1993; Bhadkamkar, 1994; Mead et al., 1991; van Schaik and Shamma, 2003].

Several other ways of implementing Lyon and Mead’s model have been developed, including those using switched-capacitor technique [Lin et al., 1992; Cornil and Jespers, 1994] and those using switched-current technique [Machado et al., 1994; Abel et al., 1994]. They are known to be more robust to component mismatch at the cost of larger area and higher power consumption. Hinck et al. [Hinck et al., 1999] designed the cochlea as a travelling wave amplifier using current-mode integrators. Tounazou et al. [Tounazou et al., 1994] suggested using a log-domain filter for building the cochlea. There are also digitally implemented cochleae. Summerfield and Lyon used the bit-serial methodology [Summerfield and Lyon, 1992]. Louiza and Newcomb developed a cochlea in the form of a DSP lattice filters [Sellami and Newcomb, 1997]. More recently, Leong et al. [Leong et al., 2003] implemented the cochlea using FPGA (Field Programmable Gate Array) technology.

As opposed to the one-dimensional models used in the implementations listed above, Watts established the theoretical basis of a two-dimensional model with the height of the fluid being taken into account [Watts, 1993]. It was further developed by Fragnière for analog VLSI implementation [Fragnière, 1998]. In section 2.3, his model is reviewed, as it is relevant to the model presented later in this thesis.

Analog VLSI implementations, especially those using the transconductance filters, are known to be suitable for applications which require the cochlea to be small and low power consuming. Power consumption of the FPGA chip in [Leong et al., 2003] is reported as 5mW for a 119 stage cochlea. Using micro-power switched current circuit blocks reduces power dissipation, which is reported as 254µW for an integrator cell [Jørgensen and Bøgason, 1996]. The power consumption of the transconductance filters used in [van Schaik and Shamma, 2003] was 400µW for 2 32 stage cochleae, and this seems to be the best in efficiency.
2.3 Review of Fragnière’s Model

2.3.1 The Model

Figure 2.10 shows a three-dimensional uncoiled cochlea. The $x,y,z$ axes are defined as shown: taking the origin at the basal end of the basilar membrane, the $x$ axis is along the length of the cochlea, the $y$ axis is in the direction of the height of the cochlea, and the $z$ axis is in the direction of the width of the basilar membrane.

The models described in section 2.2 can be divided into three categories: one-dimensional (1-D), two-dimensional (2-D) and three-dimensional (3-D) models [Hubbard and Mountain, 1996]. 1-D models only take into account the length of the cochlea, namely, along the $x$ axis. 2-D models have both $x$ and $y$ dimensions, and 3-D models incorporate all the dimensions. Fragnière’s model is a 2-D model. It is shown that, a model of at least two dimensions is necessary to obtain a sharp cutoff after the resonant frequency, which is seen in live cochlea, since a 2-D model incorporates vertical fluid motion near the best place [Lighthill, 1981].

The 2-D model is obtained by compressing the 3-D model along the $z$ axis, and as shown in Figure 2.11. This means that the fluid and the basilar membrane movements along the $z$ axis are assumed to be the same. In this way, $p(x,y,z)$, the pressure at position $(x,y,z)$, is represented by $p(x,y)$ and the fluid acceleration by $a(x,y)$. However, the basilar membrane width $w(x)$ is taken into account by expressing the force applied onto the basilar membrane section as $p(x,0)w(x)$. 

Figure 2.10: 3-D uncoiled cochlea, adapted from [Fragnière, 1998]
The Equations

The travelling wave motion is obtained by solving differential equations, which hold for the movement of the liquid according to the hydrodynamics of non-viscous incompressible flow. The Euler equation gives the motion of each element of liquid:

\[-\nabla p(x, y) = \rho \mathbf{a}(x, y)\]  

(2.1)

where \(p(x, y)\), \(\mathbf{a}(x, y)\) are the fluid pressure and the acceleration of the fluid element at position \((x, y)\), respectively, and \(\rho\) is the fluid density. For incompressible flow, the continuity equation gives:

\[\text{div} \cdot \mathbf{v}(x, y) = 0\]  

(2.2)

where \(\mathbf{v}\) is the velocity of the fluid element at position \((x, y)\). By differentiating with respect to time, it is written in the form

\[\text{div} \cdot \mathbf{a}(x, y) = 0\]  

(2.3)

Note that, in order for equation (2.3) to be equivalent to equation (2.2), the initial condition \(\mathbf{a}(x, y) = 0\) has to be met. This means that the which is a reasonable assumption.
The Boundary Conditions

There are boundary conditions that must be met at the bony wall, the oval window, the round window, and the surface of the basilar membrane.

At the bony wall, the acceleration component normal to the bony wall \( a_{n}(x, y)_{BW} \) must be zero, because the wall surface is inflexible and incompressible. Therefore,

\[
a_{n}(x, y)_{BW} = 0
\]

At the oval window, the movement of the fluid must be equal to the movement of the stapes. Thus, the boundary condition at the oval window can be written as

\[
a_{n}(x, y)_{OW} = a_{S}
\]

where \( a_{n}(x, y)_{OW} \) is the acceleration normal to the oval window, and \( a_{S} \) represents the acceleration caused by the movement of the stapes.

The normal component of acceleration at the round window is obtained by integrating equation (2.3) over the cochlea. Since at the points inside the fluid and at the basilar membrane surface the integration cancels out, the acceleration normal to the round window \( a_{n}(x, y)_{RW} \) is determined by the acceleration normal to the oval window \( a_{n}(x, y)_{OW} \) according to the following equation:

\[
a_{n}(x, y)_{RW} = -\frac{S_{OW}}{S_{RW}} a_{n}(x, y)_{OW}
\]

where \( S_{OW} \) and \( S_{RW} \) are the areas of the oval and round windows, respectively.

Lastly, the normal component of acceleration of the fluid at the surface of the basilar membrane is equal to its acceleration due to the pressure difference between the scala media and the scala tympani. As the thickness of the basilar membrane is negligible compared to the size of the cochlear duct [Santi, 1988], the boundary condition at the basilar membrane can be written as

\[
a_{n}(x, y)_{BM} = a_{BM}(x)
\]

where \( a_{BM}(x) \) is the acceleration of the basilar membrane section at \( x \).
The basilar membrane motion is determined by the pressure difference between the points directly above and below in the scala media and the scala tympani.

\[
\{p_{SM}(x, 0) - p_{ST}(x, 0)\}w(x)dx = a_{BM}(x)m(x)dx + v_{BM}(x)h(x)dx + y_{BM}(x)k(x)dx
\]  
(2.8)

where \(w(x)\) is the width of the basilar membrane at position \(x\), \(m(x)\), \(h(x)\), \(k(x)\) are the mass, viscosity and stiffness of the basilar membrane per unit length at position \(x\).

**Simplification**

The shape of the real cochlear duct changes along the cochlea: its cross section is larger at the basal end and gradually gets smaller toward the apical end. However, it has been shown that the actual shape of the cochlear duct has little effect on its dynamics, and can be simplified as a rectangular box with the basilar membrane in the middle, with the same height for both of the scalae, having oval and round window of the same size right at the edge, and filled with the fluid of the same density [Fragnière, 1998]. This is shown in Figure 2.12.

![Figure 2.12: Simplified 2-D uncoiled cochlea, adapted from [Watts, 1993]](image)

In order to facilitate an analog VLSI implementation, it is desirable to assume that the width, the mass and the viscosity of the basilar membrane per unit length are constant along the cochlea, namely, \(w(x) = w\), \(m(x) = m'\), \(h(x) = h'\). Also, the variation in the stiffness of the basilar membrane per unit length is set to be purely exponential by imposing \(k(x) = k'(0)2^{x/b}\), where \(b\) is a parameter which determines the frequency range to which the cochlea senses.
The electrical analogy of the 2D cochlea model described above can be built by using a resistive sheet and a bank of resonators as follows.

In a homogeneous resistive sheet, the following equation holds between the electric field $E(x, y)$ and the voltage $V(x, y)$ at position $(x, y)$:

$$-\nabla V(x, y) = E(x, y) \quad (2.9)$$

The conduction equation states the relationship between the electric field $E(x, y)$ and the sheet current density $J$:

$$J(x, y) = \sigma E(x, y) \quad (2.10)$$

where $\sigma$ is the sheet conductance. Substituting equation (2.10) into (2.9), equation (2.9) is written in the form:

$$-\nabla V(x, y) = \frac{1}{\sigma} J(x, y) \quad (2.11)$$

Also, the continuity equation applies for the sheet current density:

$$\text{div} \cdot J(x, y) = 0 \quad (2.12)$$

Comparing equations (2.1) and (2.11), and (2.3) and (2.12), it can be seen that the electrical cochlea can be built by replacing fluid pressure $p$ with voltage $V$ and fluid acceleration $a$ with sheet current density $J$.

In the Laplace domain, equation (2.8) has the form

$$\{p_{SM}(x, 0) - p_{ST}(x, 0)\} w(x) dx = a_{BM}(x) \{m(x) dx + \frac{h(x)}{s} dx + \frac{k(x)}{s^2} dx\} \quad (2.13)$$

To build the electrical analog of the basilar membrane, from equation (2.13), the model must satisfy the following equation:

$$V_{SM}(x, 0) - V_{ST}(x, 0) = Z_{BM}(x) J_{BM}(x) \quad (2.14)$$
where $Z_{BM}(x)$ and $J_{BM}(x)$ are the impedance and the current through the “electrical basilar membrane” at position $x$ respectively.

Comparing equation (2.13) and (2.14), the electrical basilar membrane impedance $Z_{BM}$ must have the form

$$Z_{BM}(x) = \frac{1}{G_{BM}(x)} + \frac{1}{sC_{BM}(x)} + \frac{1}{s^2S_{BM}(x)}$$

which can be realised by using three circuit elements in series.

A summary of the equations for Fragniére’s 2-D cochlea model and its electrical equivalent is shown in table 2.1. Note that, to match mechanical quantities and electrical elements, two scaling factors are introduced: the voltage scaling factor $F_v$ in [V · s² · kg⁻¹] as

$$V = F_v \cdot pw$$

and the current scaling factor $F_j$ in [A · s² · m⁻²] as

$$J = F_j \cdot a$$

<table>
<thead>
<tr>
<th>Euler equation</th>
<th>(mechanical equation)</th>
<th>$-\nabla w(x)p(x,y) = w(x)\rho a(x,y)$</th>
<th>(electrical equation)</th>
<th>$-\nabla V(x,y) = \frac{1}{\sigma} J(x,y)$</th>
<th>(mapping)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$V = F_v \cdot pw, \quad J = F_j \cdot a, \quad \sigma = \frac{F_j}{F_v} \cdot \frac{1}{w(x)\rho}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Continuity equation</th>
<th>(mechanical equation)</th>
<th>$\text{div} a(x,y) = 0$</th>
<th>(electrical equation)</th>
<th>$\text{div} J(x,y) = 0$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>BM equation</th>
<th>(mechanical equation)</th>
<th>${p_{SM}(x,0) - p_{ST}(x,0)}w(x)dx = a_{BM}(x){m(x)dx + \frac{h(x)}{s}dx + \frac{k(x)}{s^2}dx}$</th>
<th>(electrical equation)</th>
<th>$V_{SM}(x,0) - V_{ST}(x,0) = J_{BM}(x){\frac{1}{G_{BM}(x)} + \frac{1}{sC_{BM}(x)} + \frac{1}{s^2S_{BM}(x)}}$</th>
<th>(mapping)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_{BM}(x) = \frac{F_j}{F_v} \cdot \frac{1}{m(x)}, \quad C_{BM}(x) = \frac{F_j}{F_v} \cdot \frac{1}{h(x)}, \quad S_{BM}(x) = \frac{F_j}{F_v} \cdot \frac{1}{k(x)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Mechanical to electrical mapping, adapted from [Fragniére, 1998]
Longitudinal Symmetry

Now, consider the situation when an input signal is applied at the stapes. In the electrical cochlea, this acceleration of the stapes corresponds to a current input at the end of the resistive sheet. In response to the input signal, current flows through the resistive sheet and the electrical basilar membrane. Suppose the electrical basilar membrane can split into two parts, both of which have the same impedance \( Z'_{BM} = \frac{Z_{BM}}{2} \). Because of the symmetry assumed in Figure 2.12, the voltage difference between the position \((x, y)\) in the resistive sheet and the splitting point is equal to the voltage difference between the position \((x, -y)\) and the splitting point. That is, if we set the voltage at the splitting point as \( V_{sp} \), then \( V(x, y) - V_{sp} = V_{sp} - V(x, -y) \). This is shown in Figure 2.13.

![Longitudinal Symmetry Diagram](attachment:image.png)

This means that the entire cochlea can be modelled by using only one of the scalae. For this reason, in the following sections, the electrical cochlea only models the upper half of the cochlea (scala media and scala vestibuli). This reduces the size of the large number of resistors required to model the fluid by factor of two.

Note that, instead of halving the basilar membrane impedance, the same effect can be obtained by halving the resistive sheet conductance, which is how the simulation in section 2.3.2, was performed. In this case, the sheet conductance \( \sigma \) in table 2.1 has to be corrected as

\[
\sigma = \frac{F_j}{F_v} \cdot \frac{1}{2w(x)\rho} \tag{2.18}
\]
**Spatial Quantisation**

In order to implement the electrical cochlea using existing analog VLSI technology, spatial quantisation of the model described in the previous paragraph is essential.

A natural way of quantising the resistive sheet would be to implement it as a resistive grid of $M$ times $N$. When the length and the height of the resistive sheet are $L$ and $H$ respectively, the position $(x, y)$ in the resistive sheet corresponds to the position $(i \Delta x, j \Delta y)$ in the resistive grid, where $i = 0, 1, ..., M$ and $j = 0, 1, ..., N$, $\Delta x = L/M$ and $\Delta y = H/N$. This is illustrated in Figure 2.14.

![Figure 2.14: Spatial quantisation of the resistive sheet, adapted from [Fragnière, 1998]](image)

Each element in the resistive grid has conductance expressed as follows:

$$G_x(i, j) = \sigma \frac{\Delta x}{\Delta y} \quad (2.19)$$

$$G_y(i, j) = \sigma \frac{\Delta y}{\Delta x} \quad (2.20)$$

The voltages of the quantised resistive grid correspond to those in the resistive sheet are defined accordingly as follows:

$$V(x, y) = V(i \Delta x, j \Delta y) = V_{i,j} \quad (2.21)$$
As shown in Appendix A.2 of [Fragnière, 1998], the current in the resistive sheet
\( J(x, y) = [J_x(x, y), J_y(x, y)]^T \) is quantised as follows:

\[
J_x(i, j) = \frac{1}{2} \{(V_{i-1,j} - V_{i,j})G_x(i, j) + (V_{i,j} - V_{i+1,j})G_x(i + 1, j)\}
\]
\[
J_y(i, j) = \frac{1}{2} \{(V_{i,j-1} - V_{i,j})G_y(i, j) + (V_{i,j} - V_{i,j+1})G_y(i, j + 1)\}
\]

(2.22)

(2.23)

Note that the quantised currents \( J_x(i, j), J_y(i, j) \) are the average of the two currents flowing through the neighbouring resistors.

The boundary conditions at the bony wall in the quantised network are stated as follows:

\[
J_x(M, j) = 0
\]
\[
J_y(i, N) = 0
\]

(2.24)

(2.25)

Equations (2.24) and (2.25) imply that no current is flowing out of the boundary. Figure 2.14 shows this situation. The current flowing between the two nodes at the boundary is half of what it would be if they were inside the resistive sheet. Therefore, the boundary conditions at the bony wall can be incorporated by making the resistances twice as large as those inside the sheet. That is,

\[
G_x(i, N) = \frac{\sigma}{2} \frac{\Delta x}{\Delta y}
\]
\[
G_y(M, j) = \frac{\sigma}{2} \frac{\Delta y}{\Delta x}
\]

(2.26)

(2.27)

The basilar membrane can be quantised by a bank of resonators as follows:

\[
V_{i,0} = Z_{BM}(i)J_{BM}(i)
\]

(2.28)

where \( Z_{BM} \) is the basilar membrane resonator impedance, and \( J_{BM} \) is the current through the basilar membrane resonator. Each resonator has an impedance

\[
Z_{BM}(i) = \frac{1}{G_{BM}(i)} + \frac{1}{sC_{BM}(i)} + \frac{1}{s^2S_{BM}(i)}
\]

(2.29)
where the values of each element in the resonator are obtained as

\[
G_{BM}(i) = G_{BM}(i\Delta x)\Delta x \quad (2.30)
\]

\[
C_{BM}(i) = C_{BM}(i\Delta x)\Delta x \quad (2.31)
\]

\[
S_{BM}(i) = S_{BM}(i\Delta x)\Delta x \quad (2.32)
\]

The quantised cochlea of \( M = 20, \ N = 2 \) is shown in Figure 2.15. As discussed, this electrical equivalent is made of only the upper half of the cochlea, with the ground corresponding to the splitting nodes used to halve the entire cochlea.

The input to the cochlea is the acceleration of the stapes, which corresponds to the current injected at the node mapped to the oval window. However, the same effect can be obtained by applying an appropriate input voltage \( V_{in} \), and is an easier method in terms of implementing in a circuit [Fragnière, 1998]. In the figure, this voltage is shown as the input and in the simulation performed in section 2.3.2, input voltage source is used instead of current source.

Each series of \( G_{BM}, C_{BM} \) and \( S_{BM} \) represents a basilar membrane section. The resistive grid is made of conductors with conductance \( G_x \) and \( G_y \), where the values at the boundaries are doubled to incorporate boundary conditions.

![Figure 2.15: Electrical model of the cochlea, adapted from [Fragnière, 1998]](image)

**2.3.2 Simulation Results**

AC simulation of the electrical cochlea described in section 2.3.1 was performed for \( M=100 \) and \( N=2 \) using Matlab 6.0. The parameter values used in this simulation are shown in table 2.2.
The parameter values for the simulated electrical model are obtained by using table 2.1 and equations (2.19), (2.20), (2.26), (2.27), (2.30), (2.31) and (2.32).

In order to simulate the signals that the auditory nerve receives, it is appropriate to take the basilar membrane velocity as the output, because the inner hair cells respond to the basilar membrane velocity as described in section 2.1.3. The voltage over the capacitors in the basilar membrane resonators are measured, as this corresponds to the temporal integration of the current, which is mapped to the basilar membrane acceleration.

In this simulation, the connection between the ground and the end of the resistive network is replaced with a capacitor of the same capacitance as the ones in the resonators. This is to prevent reflections at the helicotrema, and this can be interpreted as modelling the resistance when the cochlear fluid passes through the little hole.

The frequency response is shown in Figure 2.16. Measured basilar membrane response in squirrel monkeys at two different positions is shown in Figure 2.17 for comparison. The resonant frequency changes logarithmically along the cochlea, and the sharp cutoff after the resonant frequency in the amplitude response is seen in both electrical and live cochleae.

This asymmetry of the tuning curves comes from the interaction between the basilar membrane sections through the fluid, and can be explained as follows. The voltage over the

Table 2.2: Reference parameter values for simulation. BM: the basilar membrane. Taken from [Fragnière, 1998]

<table>
<thead>
<tr>
<th>parameter</th>
<th>meaning</th>
<th>reference value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\rho w$</td>
<td>2 × fluid density × BM width</td>
<td>$u [m] \cdot 2 \cdot 10^3$ [kg m$^{-3}$]</td>
</tr>
<tr>
<td>$m'$</td>
<td>BM mass</td>
<td>$u [m] \cdot 1.5$ [kg m$^{-3}$]</td>
</tr>
<tr>
<td>$h'$</td>
<td>BM viscosity</td>
<td>$u [m] \cdot 2 \cdot 10^3$ [kg m$^{-2}$s$^{-1}$]</td>
</tr>
<tr>
<td>$k'(0)$</td>
<td>BM stiffness at the base</td>
<td>$u [m] \cdot 10 \cdot 10^9$ [kg m$^{-2}$s$^{-2}$]</td>
</tr>
<tr>
<td>$L$</td>
<td>cochlear length</td>
<td>20 [mm]</td>
</tr>
<tr>
<td>$H$</td>
<td>cochlear height</td>
<td>0.40 [mm]</td>
</tr>
<tr>
<td>$M$</td>
<td>no. of grids in $x$</td>
<td>100</td>
</tr>
<tr>
<td>$N$</td>
<td>no. of grids in $y$</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>$L/M$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>$H/N$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$b$</td>
<td>parameter for BM stiffness change</td>
<td>$\frac{\text{Parameter in mm}}{\Delta x \text{ [m]}} \log 2$</td>
</tr>
</tbody>
</table>
The tuning curve of this bandpass filter itself is symmetric.
When a vertical local-force impulse is applied to a basilar membrane section, the fluid pressure spreads from the site and pushes adjacent membrane segments in the direction opposite to the force impulse, as shown in Figure 2.18. Because the basilar membrane stiffness decreases exponentially towards the apex, the response to this diffusing pressure is larger on the side towards the apex. This effect causes the wavelength to decrease as the wave travels down the cochlea. The wave amplitude increases markedly towards the frequency-dependent critical point, beyond which the membrane motion, as well as that of the fluid, undergoes a steep fall [Nobili et al., 1998]. This means that, the wave amplitude at a point slightly off the critical point towards the apex is much smaller than at the point slightly off towards the base, making the tuning curves asymmetric.

Figure 2.18: Coupling between the basilar membrane sections via the fluid, taken from [Nobili et al., 1998]. A force impulse causes a basilar membrane section to move vertically and it pushes adjacent sections in the direction opposite to the force impulse.

The phase changes linearly before the resonant frequency, then the curve becomes steeper in both the simulation and the measurement. The overall change in phase is rather different, and this can be corrected making the shape of the cochlear duct closer to the real cochlea [Fragnière, 1998]. The quality factor decreases in the simulated cochlea as the position shifts basalward along the cochlea. This can be adjusted by using a variable mass scaling [Watts, 1993].

Note that Figure 2.17 shows the basilar membrane displacement, instead of velocity [Rhode, 1971]. However, the amplitude is normalised, being divided by the amplitude of the malleus displacement, which means the sound pressure level at the malleus is larger for higher input frequencies. The response from the electrical cochlea was simulated keeping the amplitude of the input voltage constant, which corresponds to keeping the sound pressure level at the stapes constant. The velocity of the basilar membrane is larger for higher frequencies when the amplitude of the displacement is kept constant. This makes comparing the responses from the electrical cochlea and the data from [Rhode, 1971] reasonable.
2.4 Summary

In this chapter, the anatomy, the macro- and the micromechanism of the human auditory pathway are described. Based on those mechanisms, a 2-D model of the cochlea and its electrical analog is presented. The simulation results of the model show a reasonable match with the measurements from a live cochlea.
Chapter 3

Analog VLSI Implementation of Passive Cochlea

In this chapter, the 2D cochlear model presented in chapter 2 is implemented in analog VLSI.

The circuit proposed in this chapter operates in CMOS weak inversion, and in the current domain rather than the voltage domain. This enables the circuits to be simple and small, and most of all, low-power consuming. This is because the voltage changes in the logarithmic scale whereas the current changes linearly to represent the signals of importance, therefore allowing the circuits to operate with a low power supply. The logarithmic change in the voltage is also useful for realising the exponential stiffness change along the cochlea, as it can be done simply by using a resistive line with linear voltage difference: the linear change in gate voltage corresponds to an exponential change in the current.

The main part of the work presented in this chapter is the realisation of the basilar membrane sections. They have to interact with a resistive network which models the fluid, and at the same time generate outputs which correspond to nerve signals produced by the inner hair cells. Each of the basilar membrane sections is implemented as a second-order filter, and can be realised using two cascaded log-domain filters and feedback through a translinear block. This is shown in section 3.2, together with the schematics of the circuits.

The circuit was fabricated, and the results of the chip measurement and analysis are presented in section 3.3.
3.1 Overview of the Cochlear Circuit Design

3.1.1 The Goal

The aim is to implement the circuit derived in chapter 2 in analog VLSI. The schematics of the circuit and the equations are reproduced below for convenience.

\[ G_x \frac{V_{i,j} - V_{i+1,j}}{2} = J_x(i,j) \]  
\[ G_y \frac{V_{i,j} - V_{i,j+1}}{2} = J_y(i,j) \]

and for the basilar membrane resonators,

\[ V_{i,0} = J_{BM}(i) \cdot \left\{ \frac{1}{G_{BM}(i)} + \frac{1}{sC_{BM}(i)} + \frac{1}{s^2S_{BM}(i)} \right\} \]  
\[ V_{M,0} = J_{BM}(M) \cdot \frac{1}{sC_{BM}(M)} \]

In order to model the living cochlea, it is desirable for each of the basilar membrane sections to produce an output which corresponds to the inner hair cell receptor potential. It is proportional to the basilar membrane velocity as presented in section 2.1.3.

\[ (\text{the output of } i\text{th basilar membrane}) \propto \int J_{BM}(i)dt \]
3.1.2 Current Domain Circuits

For circuits in the current domain, the signals of importance are currents. Using currents, it is simple to realise mathematical functions like summation and subtraction; they can be done using Ohm’s law by summing currents with the appropriate sign on a single node. Duplicating current is also easy using current mirrors, as shown later in section 3.2.2. The voltages in the current domain circuits are in log-domain, or pseudo-voltage domain: they change in the logarithmic scale as the currents change linearly, making it possible to work with lower power supply.

Previously van Schaik and Fragnière [van Schaik and Fragnière, 2001] proposed a silicon cochlea based on the model presented in chapter 2. It was implemented in the pseudo-voltage domain using CMOS technology in weak inversion. The resistive network was realised with pseudo-conductances, and the basilar membrane section was realised by using pseudo-conductances and pseudo-transcapacitances. In this chapter, improving on van Schaik and Fragnière’s idea, a simpler basilar membrane resonator design is proposed using log-domain filters.

3.2 Implementation in CMOS Weak Inversion

3.2.1 Pseudo-voltage, Pseudo-conductance

![Pseudo-conductance](image)

Figure 3.2: Pseudo-conductance

Figure 3.2 shows a PMOS transistor with its source, gate and drain voltage supplied as $V_S, V_D, V_G$ respectively. In weak inversion, the current through this PMOS transistor $I$ is
expressed as

\[ I = I_S e^{-\frac{V_G}{nU_T}} \left( e^{\frac{V_S}{U_T}} - e^{\frac{V_D}{U_T}} \right) \]  
(3.6)

where \( I_S \) is the specific current determined by the physical characteristics, \( U_T \) is the thermal voltage (about 25mV at room temperature) and \( n \) is a slope factor between 1 and 2.

Pseudo-voltage can be defined as

\[ V^* = V_0 e^{\frac{V}{U_T}} \]  
(3.7)

where \( V_0 \) is an arbitrary positive scaling constant. Using this definition, equation (3.6) can be written in the form

\[ I = \frac{I_S}{V_0} e^{-\frac{V_G}{nU_T}} (V_S^* - V_D^*) = G^* (V_S^* - V_D^*) \]  
(3.8)

where \( G^* \) is a pseudo-conductance and \( G^* = \frac{I_S}{V_0} e^{-\frac{V_G}{nU_T}} \).

Equation (3.8) means that in the pseudo-voltage domain, the current is expressed in the form of Ohm’s law, and the pseudo-conductance is determined by the gate voltage. Therefore, a conductor with variable conductance can be implemented, and is used to form a resistive network for the cochlear circuit.

Note that, when \( |V_S - V_D| \) is larger than a few \( U_T \), the pseudo-voltage at one terminal becomes negligible compared with the pseudo-voltage of the other terminal. This means that, when the transistor is saturated, its current is determined by its source and gate voltages, regardless of the drain voltage. In such a situation, the other terminal can be considered as pseudo-grounded (noted as \( V^* = 0 \), also noted as \( 0^* \)).

### 3.2.2 Current Mirrors

In Figure 3.3, a current \( I_1 \) is injected into the NMOS transistor M1. Its gate and drain terminals are connected, and therefore, the current will set the gate voltage so that the current flows through the transistor. The gates of the two NMOS transistors M1 and M2 are connected. Therefore, if the source voltages are the same (in this case they are both at
ground), the current flowing through M2 ($I_2$) should be the same as $I_1$, because the current is determined only by the gate and the source voltages as long as the transistor is saturated.

In this way, a copy of a current can be made, and this circuit is called a current mirror. Using the same setup with PMOS transistors, a copy of a current in the opposite direction can also be made.

Now, suppose a small voltage source $V_{\text{gain}}$ is inserted between the ground and M2. $I_1$ is written in terms of the gate voltage $V_g$.

$$I_1 = I_{S1} e^{\frac{V_g}{VT}} \quad (3.9)$$

where $I_{S1}$ is the specific current for M1. The equation for $I_2$ is

$$I_2 = I_{S2} e^{\frac{V_g}{VT}} e^{\frac{-V_{\text{gain}}}{VT}} \quad (3.10)$$

where $I_{S2}$ is the specific current for M2. From equation (3.9) and (3.10), the relation between $I_1$ and $I_2$ is

$$\frac{I_2}{I_1} = \frac{I_{S2}}{I_{S1}} e^{\frac{-V_{\text{gain}}}{VT}} \quad (3.11)$$

Assuming matched transistors, namely, $I_{S1} = I_{S2}$, the gain is solely an exponential function of $V_{\text{gain}}$.

$$I_2 = I_1 e^{\frac{V_{\text{gain}}}{VT}} \quad (3.12)$$
3.2.3 Translinear Multiplier/Divider

In the circuit shown in Figure 3.5, the currents $I_1, I_2, I_3, I_4$ are flowing into the four NMOS transistors, M1, M2, M3 and M4 respectively. The sources of M1 and M4 are connected to a voltage source $V_{\text{ref}}$, whose value is set near 1V. $V_{\text{ref}}$ has to be high enough for M5 to be saturated, as $V_C$ is expected to be near $V_{\text{ref}}$. M5, M6 and M7 form a feedback loop to make sure $V_C$ is settled at an appropriate value as follows: when $V_C$ is slightly higher than its operating point, M3 conducts less current than $I_3$. The leftover current raises the gate voltage of M7, making the current through M6 and M7 larger. The increased current is copied through the current mirror formed by M5 and M6, drawing current out of node $V_C$ and therefore pulling $V_C$ down to its operating point. The opposite happens when $V_C$ is slightly lower than its operating point.

The following equations can be written for the relationship between the currents, considering the circuit consists of two pairs of variable gain current mirrors described in section

![Figure 3.4: Variable gain current mirror](image)
3.2.2:

\[ I_2 = I_1 e^{\frac{V_{\text{ref}} - V_c}{V_T}} \]  \hspace{1cm} (3.13)
\[ I_3 = I_4 e^{\frac{V_{\text{ref}} - V_c}{V_T}} \]  \hspace{1cm} (3.14)

From Equation (3.13) and (3.14), the relationship between \( I_1, I_2, I_3, I_4 \) is found as:

\[ \frac{I_3}{I_1} = \frac{I_3}{I_4} = e^{\frac{V_{\text{ref}} - V_c}{V_T}} \]  \hspace{1cm} (3.15)

Therefore, when \( I_1, I_3 \) and \( I_4 \) are injected, the measured value \( I_2 \) is expressed in terms of these three currents as:

\[ I_2 = \frac{I_1 I_3}{I_4} \]  \hspace{1cm} (3.16)

This means that, when \( I_1 \) or \( I_3 \) is applied as the input, the output \( I_2 \) is a multiple of the input current and the factor determined by the ratio between \( I_1 \) or \( I_3 \) (depending on which is chosen as the input) and \( I_4 \). When \( I_4 \) is applied as the input, the output \( I_2 \) is a division of a value determined by \( I_1 \) and \( I_3 \) by the input. In this way, multiplication and division can be implemented in the current domain. This is called a “translinear” process, because the voltage changes logarithmic scale as seen in Equation (3.13) and (3.14). The relationships
between the currents, however, remain linear.

### 3.2.4 Log-domain Filter

![Log-domain filter circuit diagram]

Figure 3.6: Log-domain filter

Figure 3.6 shows the log-domain filter used to implement the basilar membrane section as presented in section 3.2.5. The basic structure is the same as the translinear multiplier/divider shown in section 3.2.3. The difference is that the log-domain filter has a capacitor and a current source instead of the feedback loop.

Applying the translinear multiplier/divider principle shown in section 3.2.3, the following equation holds for the current flowing into the node $V_{C2}$:

$$AI_0I_1 = I_{n2}I_2$$  \hspace{1cm} (3.17)

Kirchhoff’s Current Law states the relation between the current flowing through the four transistors:

$$I_{n2} + AI_0 = IC2 + 2AI_0$$  \hspace{1cm} (3.18)

From the definition of the current flowing through a capacitor,

$$IC2 = C \frac{dVc2}{dt}$$  \hspace{1cm} (3.19)
Finally, applying the variable gain current mirror principle to M3 and M4,

$$I_2 = A I_0 e^{\frac{V_{C2} - V_{ref}}{V_T}}$$  \hspace{1cm} (3.20)

By differentiating Equation (3.20) with respect to time,

$$\frac{dI_2}{dt} = \frac{1}{U_T} A I_0 e^{\frac{V_{C2} - V_{ref}}{V_T}} \frac{dV_{C2}}{dt} = I_2 \frac{dV_{C2}}{U_T dt}$$  \hspace{1cm} (3.21)

By substituting Equation (3.19) to Equation (3.18), then using Equation (3.21),

$$I_{n2} = A I_0 + C \frac{dV_{C2}}{dt}$$
$$= A I_0 + \frac{C U_T}{I_2} \frac{dI_2}{dt}$$  \hspace{1cm} (3.22)

From Equation (3.17) and (3.22),

$$A I_0 I_1 = \left( A I_0 + \frac{C U_T}{I_2} \frac{dI_2}{dt} \right) I_2$$
$$= A I_0 I_2 + C U_T \frac{dI_2}{dt}$$  \hspace{1cm} (3.23)

Transforming Equation (3.23) to the Laplace domain, $I_2$ can be expressed in terms of $I_1$ as follows:

$$I_2 = \frac{1}{1 + \frac{1}{A \tau}} I_1$$  \hspace{1cm} (3.24)

where $\tau = C U_T / I_0$.

Equation (3.23) means that this circuit implements a low-pass filter in the current domain. The voltages change in the logarithmic scale since they operate in weak inversion, hence the name log-domain filter. The circuit’s cutoff frequency is $A/\tau = A I_0 / C U_T$, which can be manipulated by changing $A$ and $I_0$.

### 3.2.5 The Basilar Membrane Section

Figure 3.7 shows the implementation of a basilar membrane section. It uses two blocks of the log-domain filter described in section 3.2.4, and the translinear multiplier/divider described in section 3.2.3. The current source $A I_0$ is implemented by applying an appropriate gate voltage to a PMOS transistor as shown in Figure 3.8 and making copies by using cur-
rent mirrors. The current source $I_0$ is implemented by dividing the current $AI_0$ into two currents. The value $A$ is set by the two voltages, $V_{refA}$ and $V_{ctrA}$, since these voltages determine the amount of current flowing through the two PMOS transistors. The relationships between the currents $I_{bm}$, $I_{out}$ and other currents shown in Figure 3.7 are derived in the rest of this section.

The resistive network is implemented using pseudo-conductances, and therefore, the voltages in Equations (3.1) – (3.4) correspond to those in the pseudo-voltage domain. The aim of building the basilar membrane section is to realise the basilar membrane function in
Figure 3.8: \( \text{AI}_0 \) and \( I_0 \) generation

the pseudo-voltage domain, namely,

\[
V_{i,0}^* = J_{BM}(i) \left\{ \frac{1}{G_{BM}^*(i)} + \frac{1}{sC_{BM}^*(i)} + \frac{1}{s^2S_{BM}^*(i)} \right\} \tag{3.25}
\]

In the following discussion, it is shown that this function is realised by using two cascaded log-domain filters in the current domain.

In the same way as discussed in section 3.2.4, the following four equations can be written describing the currents flowing into/out of the node \( V_{C1} \):

\[
I_{in}I_0 = I_{n1}I_1 \tag{3.26}
\]
\[
I_{n1} + A I_0 + I_0 = I_{C1} + 2I_0 + \frac{A I_0 I_2}{I_1} \tag{3.27}
\]
\[
I_{C1} = C \frac{dV_{C1}}{dt} \tag{3.28}
\]
\[
I_1 = I_0 e^{\frac{V_{C1} - V_{\text{ref}}}{U_T}} \tag{3.29}
\]

By differentiating Equation (3.29) with respect to time,

\[
\frac{dI_1}{dt} = \frac{1}{U_T} I_0 e^{\frac{V_{C1} - V_{\text{ref}}}{U_T}} \frac{dV_{C1}}{dt} = \frac{I_1}{U_T} \frac{dV_{C1}}{dt} \tag{3.30}
\]
Substituting Equation (3.28) into (3.27), then using Equation (3.30),

\[ I_{n1} = (1 - A)I_0 + C \frac{dV_{C1}}{dt} + \frac{AI_0 I_2}{I_1} \]
\[ = (1 - A)I_0 + \frac{CU_T dI_1}{I_1} + \frac{AI_0 I_2}{I_1} \]  
(3.31)

From Equation (3.26) and (3.31),

\[ I_{in}I_0 = \left\{ (1 - A)I_0 + \frac{CU_T dI_1}{I_1} + \frac{AI_0 I_2}{I_1} \right\} I_1 \]
\[ = (1 - A)I_0 I_1 + CU_T \frac{dI_1}{dt} + AI_0 I_2 \]  
(3.32)

The relationship between \( I_1 \) and \( I_2 \) has already been derived in section 3.2.4. In the Laplace domain, using Equation (3.24),

\[ I_{in} = (1 - A)I_1 + \tau s I_1 + \frac{A}{1 + \frac{A}{s}} I_1 \]
\[ = \frac{\tau s^2 + \tau s + 1}{1 + \frac{A}{s}} I_1 \]  
(3.33)

Therefore,

\[ I_1 = \frac{1 + \frac{\tau}{s}}{\frac{\tau}{s}^2 + \tau s + 1} I_{in} \]  
(3.34)

\[ I_2 = \frac{1}{1 + \frac{\tau}{s}} I_1 \]
\[ = \frac{1}{\frac{\tau}{s}^2 + \tau s + 1} I_{in} \]  
(3.35)

In order to implement the relationship of the input voltage to the output current in Equation (3.25), the output current is generated by subtracting \( I_1 \) from \( I_{in} \):

\[ I_{bm} = I_{in} - I_1 = \frac{\frac{\tau}{s}^2}{\frac{\tau}{s}^2 + \tau s + 1} I_{in} \]  
(3.36)

The voltage which appears at the interface with the resistive network is created by using a pseudo-conductance \( M_1 \) in Figure 3.7. Applying the gate voltage of \( V_{IG} \), \( V_{bm} \) gives

\[ I_{in} = G_{IG}^* V_{bm}^* \]  
(3.37)
where \( G^*_{IG} = \frac{I_0}{V_0} e^{-\frac{V_{IG}}{\text{ne}}} \), as defined in Equation (3.8). Note that the voltage at the other terminal of the pseudo-conductance M1 is considered to be pseudo-ground (\(0^*\)) by ensuring that it is sufficiently lower than \(V_{bm}\). Substituting Equation (3.37) into (3.36),

\[
V_{bm}^* = I_{bm} \frac{1}{G^*_{IG}} \left( 1 + \frac{1}{\tau s} + \frac{1}{\tau^2 s^2} \right) \quad (3.38)
\]

Comparing the two equations (3.25) and (3.38), \(G^*_{BM}, C^*_{BM}\) and \(S^*_{BM}\) are expressed in terms of \(G^*_{IG}, \tau\) and \(A\) as follows:

\[
\begin{align*}
G^*_{BM} &= G^*_{IG} \quad (3.39) \\
C^*_{BM} &= \tau G^*_{IG} \quad (3.40) \\
S^*_{BM} &= \frac{\tau^2}{A} G^*_{IG} \quad (3.41)
\end{align*}
\]

The current which corresponds to the velocity of the basilar membrane can be generated by subtracting \(I_2\) from \(I_1\):

\[
I_{out} = I_1 - I_2 = \frac{\tau A s}{\tau^2 s^2 + \tau s + 1} I_{in} \quad (3.42)
\]

This has the form of a bandpass filter whose resonant frequency is

\[
\omega = \frac{\sqrt{A}}{\tau} \quad (3.43)
\]

and whose quality factor is

\[
Q = \sqrt{A} \quad (3.44)
\]

### 3.2.6 Generating Input to the Cochlea

To apply an input signal to the cochlea circuit, it is necessary to apply a block of circuitry to the beginning of the cochlea in order to produce the input signal in an appropriate form.

Since the cochlea is designed as a current domain circuit, the input signal should be represented in current as well. However, this current has to be converted into pseudo-voltage...
to be applied to the resistive network. The circuit shown in Figure 3.9 is designed for this purpose.

\[ I_1 = I_0 e^{\frac{V_{pv} - V_{ref}}{U_T}} \]  

(3.45)

Using the definition of pseudo-voltage (Equation (3.7)),

\[ V_{pv}^* = V_0 e^{\frac{V_{pv}}{U_T}} = V_0 \frac{I_1}{I_0} e^{\frac{V_{ref}}{U_T}} \propto I_1 \]  

(3.46)

Thus, the current \( I_1 \) is represented in pseudo-voltage \( V_{pv} \).

M3, M4 and M5 form a feedback loop to make \( V_{pv} \) stable. When \( V_{pv} \) is slightly higher than its operating point, M2 conducts more current than \( I_1 \), causing the gate voltage of M5 to increase. This makes the current through M4 and M5 smaller, which is reflected back through the current mirror of M3 and M4, drawing \( V_{pv} \) back to its operating point. The opposite happens when \( V_{pv} \) is slightly lower than its operating point.

The current flowing through the cochlea circuit is in the order of nanoamperes. It is difficult in reality to generate such a small current externally. Therefore, it is necessary to build a block which converts voltage inputs to current. Transconductance amplifiers can be used for this voltage-to-current conversion. Sarapeshkar proposed a wide linear range transconductance amplifier, which is shown in Figure 3.10 [Sarapeshkar, 1997]. It produces
Figure 3.10: Wide linear range transconductance amplifier, adapted from [Sarpeshkar, 1997] current proportional to the difference in voltage between the two terminals + and -. When a higher voltage is applied to the - terminal than the + terminal, the output current is negative, in that the current flows inwards instead of outwards.

The cochlea input generator block is shown in Figure 3.11. It is built by connecting the wide linear range amplifier and the current-to-pseudo-voltage converter. The voltage $V_{IG}$ in Figure 3.9 is generated elsewhere. An offset current is applied in order to ensure the operation of the current-to-pseudo-voltage converter. A buffer shown in Figure 3.12 is inserted between the cochlea input generator and the resistive network. The pseudo-voltage is successfully copied onto the beginning of the resistive network without drawing extra current from the current-to-pseudo-voltage converter, as the current mirror formed by M1 and M2 ensures the same current, and therefore the same gate voltage for M3 and M4.

### 3.2.7 2D Cochlea Circuit

Figure 3.13 shows the complete circuit which implements the 2D cochlea. Each of the basilar membrane resonators is tuned by applying linearly changing voltages to the terminal
$V_{A_{10}}$. This means that the value $A$ changes exponentially, which determines the resonant frequency of the section. It is achieved by applying different voltages at the beginning ($VAI0_{Start}$) and the end ($VAI0_{End}$), which are connected by a resistive line. This resistive line is realised by using a polysilicon line.

In section 2.3.2, a capacitor was inserted between the end of the cochlea and the ground, to prevent reflection. In Figure 3.13, a basilar membrane terminator block is placed at the end. Its schematics are shown in Figure 3.14.

The basilar membrane terminator block contains a log-domain filter. As shown in
Figure 3.13: 2D cochlea circuit

section 3.2.4, the relationship between $I_{in}$ and $I_1$ is

$$I_1 = \frac{1}{1 + \frac{\tau}{A} s} I_{in} \quad (3.47)$$

The current from the resistive network $I_{bm}$ is obtained by subtracting $I_1$ from $I_{in}$.

$$I_{bm} = I_{in} - I_1 = \frac{\tau s}{1 + \frac{A}{\tau} s} I_{in} \quad (3.48)$$

$I_{in}$ is converted to pseudo-voltage in the same way as the basilar membrane section (section 3.2.5).

$$I_{in} = G_{IG}^* V_{bm}^* \quad (3.49)$$

From Equation (3.48) and (3.49),

$$V_{bm}^* = \frac{1}{G_{IG}^*} \left( 1 + \frac{A}{\tau s} \right) I_{bm} \quad (3.50)$$

which corresponds to a conductor and a capacitor in series.
3.3 Chip Measurement Results

The 2D cochlea circuit described in section 3.2 is fabricated in a 1.5 micron technology. Figure 3.15 shows the setup for testing the chip. One of the basilar membrane sections is selected, and the input is applied directly out of the cochlea input generator, to observe the basilar membrane filter function itself, which is expected to be a bandpass filter function, as discussed in section 2.3.2.

The output from the basilar membrane is the current $I_1 - I_2$. This is converted to a voltage using the wide linear range transconductance amplifier. The - terminal and the output terminal of the wide linear range transconductance amplifier are connected, and $I_1 - I_2$ is injected to the node. As the - terminal of the wide linear range transconductance ampli-
fier is the bulk of the PMOS transistor, the current which flows into/out of the - terminal is negligible. When $I_1 - I_2$ is positive/negative, it is expected to raise/lower the voltage at the - terminal, and at the same time, draw/produce current at the output terminal until it reaches a stable state. Therefore, the current is successfully converted to voltage, which can be measured externally.

![Figure 3.16](image_url)

Figure 3.16: Step responses changing $V_{AI0}$

Figure 3.16 shows a step response with a varying value of $V_{AI0}$, measured in voltage. An extra copy of the input current is made, and measured in voltage in the same way as $I_1 - I_2$. The step size is about 0.6V. As the whole system is designed to be linear, the temporal differentiation of the step response should correspond to the impulse response. Applying the FFT (Fast Fourier Transform) to the impulse response, the frequency response can be obtained, and is shown in Figure 3.17.

From Equation (3.43), the resonant frequency is a function of $A$ and $I_0$ as follows:

$$\omega = \frac{\sqrt{A}}{\tau} = \frac{\sqrt{A}}{CU_T} I_0$$

(3.51)

As shown in Figure 3.8, $I_0$ is a fraction of $AI_0$, and $AI_0$ is determined by the gate voltage $V_{AI0}$ as follows:

$$I_0 \propto e^{-\frac{V_{AI0}}{mU_T}}$$

(3.52)
Therefore, a change in $V_{A0}$ of 0.05 (V) should correspond to a change in $I_0$ and the resonant frequency by a factor of

$$e^{-\frac{0.05}{1.025}} \approx 10^{0.58}$$

(3.53)

which is reasonably close to the value seen in Figure 3.17.

The quality factor seems to decrease slightly as $V_{A0}$ increases (i.e. $I_0$ decreases). What exactly makes the quality factor smaller is yet to be found. However, one of the contributing factors can be the change in the value $A$ as $I_0$ changes, even when $V_{refA}$ and $V_{ctrA}$ are kept constant in the circuit in Figure 3.8. Figure 3.18 shows the the SPICE simulation result of the circuit. The value of $A$ is calculated by dividing simulated $AI_0$ by $I_0$. The decrease in the value $A$ becomes maximum at about $V_{ctrA} = 2.42V$.

Further measurements on the behaviour of the entire cochlea could have been carried out. However, more tests on individual basilar membrane sections revealed that each of the basilar membrane resonators shows completely different response from one another: their resonant frequency is reasonably close to each other under the same input and the bias voltages, but their quality factor seemed to differ drastically. This makes it meaningless to test as a whole cochlea, as the model is build assuming all the basilar membrane sections are identical. In the biological system, the individual difference in the quality factor is thought
to be compensated by the outer hair cells: the outer hair cells are motile, and apply force to enhance softer sound. This mechanism is further analysed in Chapter 4.

### 3.4 Transistor Mismatch Analysis

One likely cause for the difference between the basilar membrane resonators is the transistor mismatch. In this section, the transistor mismatch is taken into account for the basilar membrane resonator output to find out which part of the circuit is critical.

#### 3.4.1 Mismatch in the Translinear Multiplier

In Figure 3.5, when there is a mismatch between the transistors M1 and M2, Equation (3.13) becomes

\[
I_2 = x_1 I_1 e^{\frac{V_{\text{ref}} - V_c}{V_T}}
\]

(3.54)

where \(x_1\) indicates the amount of mismatch. In the same way, Equation (3.14) becomes

\[
I_3 = x_2 I_4 e^{\frac{V_{\text{ref}} - V_c}{V_T}}
\]

(3.55)

From Equation (3.54) and (3.55), the output current \(I_2\) is expressed as

\[
I_2 = \frac{x_1}{x_2} \frac{I_1 I_3}{I_4}
\]

(3.56)
Equation (3.56) indicates that the transistor mismatch in the translinear multiplier changes its overall gain.

### 3.4.2 Mismatch in the Log-domain Filter

The Equations (3.17) ∼ (3.20) for the circuit shown in Figure 3.6 are re-written as follows, when transistor mismatch is taken into account:

\[
I_{n2} = x_1 I_1 e^{\frac{V_{ref} - V_{C2}}{v_T}} \quad (3.57)
\]

\[
I_{n2} + y_1 AI_0 = I_{C2} + 2y_2 AI_0 \quad (3.58)
\]

\[
I_{C2} = C \frac{dV_{C2}}{dt} \quad (3.59)
\]

\[
I_2 = x_2 y_1 AI_0 e^{\frac{V_{ref} - V_{C2}}{v_T}} \quad (3.60)
\]

where \(x_1\) is the transistor mismatch between M1 and M2, \(x_2\) is the mismatch between M3 and M4, \(y_1\) and \(y_2\) are the mismatch between the transistors used in the \(AI_0\) and \(2AI_0\) current sources respectively. From Equations (3.57) and (3.60),

\[
I_{n2} I_2 = x_1 x_2 y_1 AI_1 I_0 \quad (3.61)
\]

By differentiating Equation (3.60) with respect to time,

\[
\frac{dI_2}{dt} = x_2 y_1 \frac{1}{U_T} AI_0 e^{\frac{V_{C2} - V_{ref}}{v_T}} \frac{dV_{C2}}{dt} = \frac{I_2}{U_T} \frac{dV_{C2}}{dt} \quad (3.62)
\]

By substituting Equation (3.59) in Equation (3.58), then using Equation (3.62),

\[
I_{n2} = (2y_2 - y_1) AI_0 + C \frac{dV_{C2}}{dt} = (2y_2 - y_1) AI_0 + \frac{CU_T dI_2}{I_2} \quad (3.63)
\]

From Equation (3.61) and (3.63),

\[
x_1 x_2 y_1 AI_0 I_1 = \left\{ (2y_2 - y_1) AI_0 + \frac{CU_T dI_2}{I_2} \right\} I_2 = (2y_2 - y_1) AI_0 I_2 + CU_T \frac{dI_2}{dt} \quad (3.64)
\]
Transforming Equation (3.64) to the Laplace domain, $I_2$ can be expressed in terms of $I_1$ as follows:

$$I_2 = \frac{x_1 x_2 y_1}{2 y_2 - y_1} \frac{1}{1 + \frac{2}{(2y_2 - y_1)^s}} I_1$$

$$= \frac{x_l}{x_f} \frac{1}{1 + \frac{X_c A s}{x_f A s}}$$

where $\tau = C U_T / I_0$, $x_l = x_1 x_2 y_1$ and $x_f = 2y_2 - y_1$.

Equation (3.65) means that the transistor mismatch affects both on the cutoff frequency and the overall gain. It is also seen that the mismatch in $AI_0$ generations is critical to the filter’s cutoff frequency.

### 3.4.3 Mismatch in the Basilar Membrane Section

Equations (3.26) ~ (3.29) for the circuit shown in Figure 3.7 are re-written as follows, when transistor mismatch is taken into account:

$$I_{n1} = x_1 I_{in} e^{\frac{V_c - V_c}{V_T}}$$

(3.66)

$$I_{n1} + y_1 AI_0 + z_1 I_0 = I_{C1} + 2z_2 I_0 + x_m \cdot \frac{y_3 AI_0 I_2}{I_1}$$

(3.67)

$$I_{C1} = x_C C \frac{dV_{C1}}{dt}$$

(3.68)

$$I_1 = x_2 z_1 I_0 e^{\frac{V_c - V_{ref}}{V_T}}$$

(3.69)

where

- $x_1$ : mismatch between M2 and M3
- $x_2$ : mismatch between M4 and M5
- $y_1$ : mismatch in the $AI_0$ generation in the Log-domain Lowpass Filter 1
- $y_3$ : mismatch in the $AI_0$ generation in the Translinear Multiplier
- $z_1$ : mismatch in the $I_0$ generation in the Log-domain Lowpass Filter 1
- $z_2$ : mismatch in the $2I_0$ generation in the Log-domain Lowpass Filter 1
\( x_c \): mismatch in the size of the capacitor in the Log-domain Lowpass Filter 1, compared to the capacitor in the Log-domain Lowpass Filter 2.

\( x_m \): overall gain produced by the mismatch in the Translinear Multiplier.

Note that there is no mismatch factor for \( I_1 \) and \( I_2 \) in the last term in Equation (3.67), representing the output of the Translinear Multiplier. It is because, when fabricated, the gate of the transistor M5 was directly connected to that of M9, and the gate of the transistor M7 to that of M8 in Figure 3.7, in order to reduce the effect of mismatch.

From Equation (3.66) and (3.69),

\[
I_{n1}I_1 = x_1x_2z_1I_mI_0
\]  
(3.70)

By differentiating Equation (3.69) with respect to time,

\[
\frac{dI_1}{dt} = x_2z_1 \frac{1}{U_T} I_0 e^{\frac{V_{C1}-V_{C0}}{U_T}} \frac{dV_{C1}}{dt} = \frac{I_1}{U_T} \frac{dV_{C1}}{dt}
\]  
(3.71)

Substituting Equation (3.68) into (3.67), then using Equation (3.71),

\[
I_{n1} = (2z_2 - z_1 - y_1A)I_0 + x_cC \frac{dV_{C1}}{dt} + x_my_3 \frac{A_0I_2}{I_1}
\]

\[
= (2z_2 - z_1 - y_1A)I_0 + x_cC \frac{UT}{I_1} \frac{dI_1}{dt} + x_my_3 \frac{A_0I_2}{I_1}
\]  
(3.72)

From Equation (3.70) and (3.72),

\[
x_1x_2z_1I_mI_0 = \left\{ (2z_2 - z_1 - y_1A)I_0 + x_cC \frac{UT}{I_1} \frac{dI_1}{dt} + x_my_3 \frac{A_0I_2}{I_1} \right\} I_1
\]

\[
= (2z_2 - z_1 - y_1A)I_0I_1 + x_cCUT \frac{dI_1}{dt} + x_my_3A_0I_2
\]  
(3.73)

Using the relationship between \( I_1 \) and \( I_2 \) derived in Equation (3.65), the Laplace transform of Equation (3.73) is

\[
x_1x_2z_1I_m = \left( x_{11} - y_1A \right) I_1 + x_c\tau s I_1 + x_my_3 \cdot \frac{x_{12}}{x_f} \frac{A}{1 + \frac{A}{x_fA}} I_1
\]

\[
= \frac{x_c}{x_f} \frac{1}{s^2} + \frac{m}{x_f} A + x_c A \right) \frac{x_{11} - y_1A + x_my_3}{1 + \frac{A}{x_fA}} I_1
\]  
(3.74)
where $x_{l1} = 2z_2 - z_1$, $x_{l2}$ is the overall gain produced by the mismatch in the Log-domain Filter 2. Note that there is no mismatch factor for $I_1$ in the last term in Equation (3.74), representing the output of the Log-domain Lowpass Filter 2. It is because, when fabricated, the gate of the transistor M5 was directly connected to that of M6 in Figure 3.7, in order to reduce the effect of mismatch.

Also note that, when all the mismatch factors are 1 (i.e., no mismatch), Equation (3.75) is the same as Equation (3.33).

The numerator of Equation (3.75) determines the resonant frequency and the quality factor of the basilar membrane section. The quality factor and the resonant frequency are expressed as follows:

\[
Q = \sqrt{A} \frac{x_c \sqrt{x_{l1} x_f - y_1 x_f A + x_m y_3 x_{l2} A}}{x_{l1} - y_1 A + x_c x_f A} \quad (3.76)
\]

\[
\omega = \frac{\sqrt{A}}{\tau} \cdot \frac{1}{\sqrt{x_c}} \sqrt{x_{l1} x_f - y_1 x_f A + x_m y_3 x_{l2} A} \quad (3.77)
\]

The effect of each mismatch can be found by the partial differentiation of Equation (3.76) and (3.77) with respect to each mismatch factor. The sensitivity of a quantity $Q$ with respect to a factor $X$, expressed as $S_X^Q$, is defined as

\[
S_X^Q = \frac{dQ/Q}{dX/X} = \frac{dQ}{dX} \cdot \frac{X}{Q} \quad (3.78)
\]

When all the other mismatch factors are 1, the sensitivities of the quality factor with respect to the mismatch factors are:

\[
S_{x_{l1}}^Q = \frac{1}{\sqrt{A}} \frac{\partial Q}{\partial x_{l1}} \bigg|_{x_{l1}=1} = -\frac{1}{2} \quad (3.79)
\]

\[
S_{y_1}^Q = \frac{1}{\sqrt{A}} \frac{\partial Q}{\partial y_1} \bigg|_{y_1=1} = \frac{A}{2} \quad (3.80)
\]

\[
S_{x_f}^Q = \frac{1}{\sqrt{A}} \frac{\partial Q}{\partial x_f} \bigg|_{x_f=1} = \frac{1 - 3A}{2} \quad (3.81)
\]

\[
S_{x_m}^Q = \frac{1}{\sqrt{A}} \frac{\partial Q}{\partial x_m} \bigg|_{x_m=1} = \frac{A}{2} \quad (3.82)
\]

\[
S_{y_3}^Q = \frac{1}{\sqrt{A}} \frac{\partial Q}{\partial y_3} \bigg|_{y_3=1} = \frac{A}{2} \quad (3.83)
\]
\[ S_{x_{2}}^{Q} = \left. \frac{1}{\sqrt{A}} \frac{\partial Q}{\partial x_{2}} \right|_{x_{2}=1} = \frac{A}{2} \quad (3.84) \]

\[ S_{x_{c}}^{Q} = \left. \frac{1}{\sqrt{A}} \frac{\partial Q}{\partial x_{c}} \right|_{x_{c}=1} = \frac{1 - 2A}{2} \quad (3.85) \]

where

\[ x_{1} = 2z_{2} - z_{1} : \text{mismatch in generating } I_{0} \text{ terms in the Log-domain Lowpass Filter 1} \]

\[ y_{1} : \text{mismatch in the } AI_{0} \text{ generation in the Log-domain Lowpass Filter 1} \]

\[ x_{f} = 2y_{2}' - y_{1}' : \text{mismatch in generating } AI_{0} \text{ terms in the Log-domain Lowpass Filter 2, see Equation (3.65)} \]

\[ x_{m} : \text{overall gain produced by the mismatch in the Translinear Multiplier} \]

\[ y_{3} : \text{mismatch in generating } AI_{0} \text{ terms in the Translinear Multiplier} \]

\[ x_{12} = x_{1}'x_{2}'y_{1}' : \text{gain produced by the mismatch in the Log-domain Lowpass Filter 2, see Equation (3.65)} \]

\[ x_{c} : \text{mismatch in the size of the capacitor in the Log-domain Lowpass Filter 1, compared to the capacitor in the Log-domain Lowpass Filter 2} \]

\( A \) is larger than 1, as \( I_{0} \) is generated as part of \( AI_{0} \), as shown in Figure 3.8. Therefore, from the equations above, it is found that the mismatch factor which affects the quality factor most is \( x_{f} \), which is the mismatch in generating the \( AI_{0} \) terms in Log-domain Filter 2.

When all the other mismatch factors are 1, the sensitivities of the resonant frequency
with respect to the mismatch factors are:

\[
S_{x_1} = \frac{\tau}{\sqrt{A}} \frac{\partial \omega}{\partial x_1} \bigg|_{x_1=1} = \frac{1}{2} \tag{3.86}
\]

\[
S_{y_1} = \frac{\tau}{\sqrt{A}} \frac{\partial \omega}{\partial y_1} \bigg|_{y_1=1} = -\frac{A}{2} \tag{3.87}
\]

\[
S_{x_f} = \frac{\tau}{\sqrt{A}} \frac{\partial \omega}{\partial x_f} \bigg|_{x_f=1} = \frac{1 - A}{2} \tag{3.88}
\]

\[
S_{x_m} = \frac{\tau}{\sqrt{A}} \frac{\partial \omega}{\partial x_m} \bigg|_{x_m=1} = \frac{A}{2} \tag{3.89}
\]

\[
S_{y_3} = \frac{\tau}{\sqrt{A}} \frac{\partial \omega}{\partial y_3} \bigg|_{y_3=1} = \frac{A}{2} \tag{3.90}
\]

\[
S_{x_{12}} = \frac{\tau}{\sqrt{A}} \frac{\partial \omega}{\partial x_{12}} \bigg|_{x_{12}=1} = \frac{A}{2} \tag{3.91}
\]

\[
S_{x_c} = \frac{\tau}{\sqrt{A}} \frac{\partial \omega}{\partial x_c} \bigg|_{x_c=1} = -\frac{1}{2} \tag{3.92}
\]

From the equations above, it is seen that \(x_f\) does not have as much effect as it does on the quality factor. Considering the fact that the difference between the responses from the basilar membrane sections was mainly in the quality factor, the mismatch represented in \(x_f\), which come from the \(A_{I0}\) generations, may have caused the measured individual difference.

### 3.5 Summary

The 2D cochlear model presented in chapter 2 was realised in CMOS weak inversion in the current domain. The fluid is modelled using a resistive network, which is formed by pseudo-conductance. The basilar membrane sections are realised by cascading two log-domain filters to achieve a second-order bandpass filter function. The circuit is fabricated and tested. The frequency response obtained from measured data shows a reasonable match to the theory in terms of the resonant frequency shift by the variation of the parameter \(V_{A_{I0}}\). The complete circuit of the cochlea was not tested because of the extreme difference in the behaviour of the individual basilar membrane sections. The reason for this difference is yet to be found. However, it is theoretically derived that it is important to match the transistors that produces currents for the \(A_{I0}\) terms.
In Chapter 3, the basilar membrane section was successfully implemented using log-domain filters. However, it suffers from one drawback: each section shows a different response to the same input signal, even when the bias voltages are the same. This difference is presumably due to transistor mismatch, and those differences appear to be mostly in the quality factor of the filter.

While research to find out why the mismatch lead to the difference between individual sections is ongoing, another way of overcoming this problem is to make the cochlea equipped with the ability to actively control the quality factor by feedback. This is inspired by the way the living cochlea operates: for softer sounds, due to the effect of this active gain control mechanism, the basilar membrane sections are sharply tuned, whereas for louder sounds the active effect becomes small compared to the amplitude of the basilar membrane motion, making the basilar membrane filters less sharply tuned and close to the behaviour of a basilar membrane without active gain control mechanism.

In this chapter, the biological mechanism of this active gain control is briefly summarised in the first section. A model for analog VLSI implementation that incorporates this mechanism is proposed in the second section, and analysed by simulations in the last section.

When not cited explicitly, information on the anatomy and physiology of the ear in this chapter is taken from [Harrison and Hunter-Duvar, 1988; Santi, 1988; Dallos, 1978].
4.1 Micromechanics in the Organ of Corti

4.1.1 Main Components for Active Gain Control

Chapter 2 showed that the mechanical motion of the basilar membrane (BM) is transduced by the inner hair cells (IHCs) in the Organ of Corti. There are several more components which play an important role in active gain control. Fig. 4.1 shows the cross section of the Organ of Corti, featuring these components and their locations.

![Cross section of the Organ of Corti](image)

Figure 4.1: The cross section of the Organ of Corti, taken from [Nobili et al., 1998]

**Outer Hair Cells (OHCs)** OHCs have a cylindrical shape, are supported at their basal end by Deiters’ Cells (DCs), and are surrounded by the perilymph-filled spaces of Nuel. OHCs are longer at the cochlear apex (50 μm) compared to those at the base (20 μm), but have the same diameter along the cochlea (6 - 7 μm). The fibrous network of material called the cuticular plate at the OHC’s apex end forms the Reticular Lamina (RL) with the phalangeal process of the DCs.

OHCs have hair bundles at their apex, similar to but slightly thinner than those of the IHC. The hair bundles, called stereocilia, contain structural proteins such as actin, which contribute to their stiffness, enabling them to deflect as stiff rods. The bundles form a W-shaped pattern, the open end of the W facing toward the central bony core of the cochlea, with the longest stereocilia on the lateral edge. The number of stereocilia decreases, but
the length of the stereocilia increases, from the cochlear base to the apex. The stereocilia are connected to one another by extracellular cross-linking filaments. Horizontal cross-links, which are within and between the stereocilia rows, appear to help mechanical force to be evenly distributed. Vertical cross-links, which connect between the tip of a short stereocilium to the shaft of a tall stereocilium next to it, are supposed to work as mechanical devices to open ion channels in the tips of the stereocilia.

Large efferent nerve synaptic terminals (feedback from the brain) are present at OHC’s basal end. Afferent synapses (signal to the brain) are also found, but they are small.

OHCs are thought to be responsible for amplifying the basilar membrane motion, as they are known to change cell length depending on their membrane potential. This is further described in section 4.1.2.

**Deiters’ Cells (DCs)** DCs are the supporting structure of the OHCs. They are divided into two parts: the cell body and a slender process called the phalangeal process. DCs are attached to the basilar membrane (BM) at their basal end, and have a cup-like structure to accommodate OHCs at their apical end. The phalangeal process stretches out of the cell body to reach Reticular Lamina (RL), making contact with OHCs at their apical part.

The coupling between the OHCs and the BM by the DCs is considered as viscous, and it makes the force exerted onto the BM by the OHCs increase with frequency. This effect appears to cancel the attenuation of the OHC receptor potential with frequency, and is incorporated in some of the previous active models [Nobili and Mammano, 1996; Nobili et al., 1998].

**Tectorial Membrane (TM)** The TM is an acellular, fibrous structure overlying the Reticular Lamina, and it appears gelatic in its natural state. The TM is divided into 3 regions: the limbal zone is where the TM is attached to the spiral limbus; the middle zone contains a cover net at the TM’s apical surface, a fibrous layer in the middle and Hensen’s stripe on the underside, near the position of the IHCs; the marginal zone contains the marginal net and Hardesty’s membrane, into which the tallest cilia of OHCs are embedded.

By assuming that the TM has the properties of a collagen gel, it is appropriate to
assume that it oscillates like a solid mass attached to the top of the OHC stereocilia bundles [Nobili et al., 1998]. The shearing motion between the TM and the RL causes OHC hair bundle deflection, as described in section 4.1.2

## 4.1.2 The Outer Hair Cell Motility and Its Effect on the Basilar Membrane Motion

From the experimental evidence of sharply tuned basilar membrane vibration observed \textit{in vivo} [Hubbard and Mountain, 1996], as well as the evidence of otoacoustic emissions [Kemp, 1978], it is believed that active elements must be present in the cochlear partition [de Boer, 1983; Neely and Kim, 1986]. These active elements are now believed to be the outer hair cells. Their motile response has been measured and analysed [Ashmore, 1987], and it is summarised in this section for background information on the cochlea’s active mechanism.

Isolated outer hair cells are found to change their length when subjected to voltage steps: when hyperpolarised they elongate, and when depolarised they shorten. The cells are observed to change length up to 4%. It is also observed that the outer hair cells maintain their volume as they change their length, therefore narrowing (bulging) the cells around their middle region when they elongate (shorten). The change in the membrane potential \textit{in vivo} is caused by the deflection of the stereocilia. The conceptual outer hair cell and its motion is shown in Figure 4.2. Displacement of the stereocilia towards the tallest allows the current to flow into the cell body, causing the cell to depolarise. The displacement in the other direction shuts the ion channels at the tips of the stereocilia, causing the cell to recover its original length.

The stereocilia deflection is caused by the interaction between the tectorial membrane and the basilar membrane. Figure 4.3 illustrates the relationship between the tectorial membrane and the basilar membrane when the basilar membrane is vertically displaced. As the tallest stereocilia is embedded in the tectorial membrane, the upward displacement of the basilar membrane causes the stereocilia to deflect towards the tallest. This causes the outer hair cells to shorten, pulling the basilar membrane further up. The opposite happens when the basilar membrane is displaced downwards.
Figure 4.2: The schematic outer hair cell, taken from [Nobili et al., 1998]. When the stereocilia is deflected toward the tallest cilia, the cell shortens. The shortening is reversible.

Figure 4.3: Shearing movement between tectorial membrane and reticular lamina, taken from [Fragnière, 1998], showing the conversion of the vertical movement of the basilar membrane to a shearing movement.

Generally, this would be considered as contributing a negative stiffness. In fact, this is true at very low frequencies [Watts, 1993]. However, the negative stiffness is not instantaneous, and is found to be 90 degrees out of phase with the basilar membrane displacement at higher frequencies. Figure 4.4 shows the relation between the basilar membrane displacement and the outer hair cell undamping effect. The basilar membrane displacement and the deflection of the stereocilia of the outer hair cell, therefore the current flowing into the outer hair cell are in phase. The outer hair cell membrane potential, which changes the outer hair cell length (or the force exerted by the outer hair cell), is 90° out of phase. This effectively counteracts damping on the basilar membrane, as the damping term is proportional to the velocity.
Figure 4.4: Diagram showing the relation between the basilar membrane displacement and the outer hair cell length change. The outer hair cell membrane potential is $90^\circ$ out of phase from the basilar membrane displacement, which is in phase with the velocity but in the opposite direction, counteracting the damping.

It is also found that the outer hair cell length change saturates for larger voltage steps [Ashmore, 1987]. This nonlinear saturation, together with the effect of counteracting the damping of the basilar membrane mentioned in the previous paragraph, explains the following phenomenon seen in live cochleae: for soft sounds the frequency response of a basilar membrane section is more sharply tuned (i.e. the quality factor is higher) than for loud sounds [Ruggero, 1992]. Figure 4.5 clearly shows this phenomenon. When the input sound is soft, the effect of the outer hair cell is comparable to the basilar membrane damping term, therefore it makes the frequency response curve sharp. However, when the input sound is loud, the basilar membrane velocity is large, therefore the effect of the outer hair cell is negligible, making the frequency response curve broad.

Figure 4.5: Frequency response at one point of the basilar membrane of a chinchilla cochlea for 9 different sound pressure levels. The gain is measured as the peak basilar membrane velocity divided by the peak stapes velocity. Adapted from [Ruggero, 1992].
A number of models for the cochlea micromechanics incorporating the effect of the outer hair cells have been proposed [Neely and Kim, 1986; Allen, 1980; Mammano and Nobili, 1993; Nobili and Mammano, 1996], and summarised in [Hubbard and Mountain, 1996]. Most of the models consider that the outer hair cell membrane behaves as a simple RC filter with cutoff frequency below 1kHz [Mammano and Nobili, 1993]. That is, the force generated by the outer hair cells are the low-pass filtered version of the basilar membrane displacement. In the model presented in the next section, this is further simplified and assumes that the force generated by the outer hair cells are proportional to the basilar membrane velocity, in order to be incorporated in the electrical model of the cochlea.

4.2 The Model

4.2.1 Incorporating the Undamping Effect

As described in section 4.1.2, the role of the outer hair cells seems to be undamping. This undamping effect can be incorporated by using a term \( u (0 \leq u < 1) \) in the damping coefficient of Equation (2.8) in chapter 2 as follows:

\[
\{ p_{SM}(x, 0) - p_{ST}(x, 0) \} w(x)dx = a_{BM}(x)m(x)dx + v_{BM}(x)h(x)(1 - u)dx + y_{BM}(x)k(x)dx
\] (4.1)

In the model presented in chapter 2, changing the damping coefficient corresponds to a change in the capacitance of the basilar membrane resonators. Figure 4.6 shows the frequency response of the 50th basilar membrane section in the same model as the one used in chapter 2 (M=100, N=2) for three different values of capacitance. The capacitance is changed to achieve \( u = 0.5 \) and \( 0.9 \).

It can be seen that undamping indeed contributes to an increase in the quality factor. However, as \( u \) increases, it causes oscillation in the frequency domain in the transfer function, distorting the frequency response curve. This is due to the discretisation of the resistive network and the basilar membrane sections, and is explained in the following sections. It is also shown that the effect of the discretisation can be reduced by increasing the number of sections and making a finer resistive grid.
4.2.2 Equations

Figure 4.7 shows the circuit diagram used for the simulation, together with the definitions of the node names and conductances of the circuit elements. $a_i, b_i, c_i$ ($i = 0, 1, \cdots, n$) are defined as the voltages at each node in the resistive grid ($a_0, b_0$ and $c_0$ are all connected together to the input node). For each group of nodes $a_i, b_i, c_i$ ($i = 1, 2, \cdots, n - 1$), the following equations can be written according to Kirchoff’s law.

\[
\frac{G_x}{2} (a_{i-1} - a_i) + G_y (b_i - a_i) + \frac{G_x}{2} (a_{i+1} - a_i) = 0 \quad (4.2)
\]

\[
G_y (a_i - b_i) + G_x (b_{i-1} - b_i) + G_y (c_i - b_i) + G_x (b_{i+1} - b_i) = 0 \quad (4.3)
\]

\[
G_y (b_i - c_i) + \frac{G_x}{2} (c_{i-1} - c_i) + Y_i (0 - c_i) + \frac{G_x}{2} (c_{i+1} - c_i) = 0 \quad (4.4)
\]
where \( Y_i \) represents the admittance of the \( i \)th basilar membrane resonator, which is

\[
Y_i = \left( \frac{1}{G_{bm}(i)} + \frac{1}{sC_{bm}(i)} + \frac{1}{s^2 S_{bm}(i)} \right)^{-1}
\]  

(4.5)

This can be written in the form

\[
A V_{i-1} - B_i V_i + A V_{i+1} = 0
\]  

(4.6)

where

\[
A = \begin{pmatrix}
\frac{G_x}{2} & 0 & 0 \\
0 & G_x & 0 \\
0 & 0 & \frac{G_x}{2}
\end{pmatrix},
\]

(4.7)

\[
B_i = \begin{pmatrix}
G_x + y & -G_y & 0 \\
-G_y & 2(G_x + G_y) & -G_y \\
0 & -G_y & G_x + G_y + Y_i
\end{pmatrix}
\]

(4.8)

and \( V_i = [a_i \ b_i \ c_i]^T \). For each set of nodes \([a_i \ b_i \ c_i]^T\) \((i=1,2,\cdots,n-1)\), the following equations hold:

\[
A V_0 - B_1 V_1 + A V_2 = 0
\]  

(4.9)

\[
A V_1 - B_2 V_2 + A V_3 = 0
\]  

(4.10)

\[
A V_2 - B_3 V_3 + A V_4 = 0
\]  

(4.11)

\[ \vdots \]

\[
A V_{n-2} - B_{n-1} V_{n-1} + A V_n = 0
\]  

(4.12)

\[
A V_{n-1} - B_n V_n = 0
\]  

(4.13)

(4.14)

with the value at the input

\[
V_0 = V_{in} = \begin{pmatrix}
V_{in} \\
V_{in} \\
V_{in}
\end{pmatrix}
\]

(4.15)

and the matrix \( B_n \) of

\[
B_n = \begin{pmatrix}
G_x/2 + G_y/2 & -G_y/2 & 0 \\
-G_y/2 & G_x + G_y & -G_y/2 \\
0 & -G_y/2 & G_x/2 + G_y/2 + Y_n
\end{pmatrix}
\]

(4.16)
and the admittance $Y_n$ at the end of the cochlea

$$Y_n = sC_{bm}(n) \quad (4.17)$$

From Equation (4.9) and (4.15), $V_1$ is expressed in terms of $V_2$ as

$$V_1 = F_1 V_{in} + F_1 V_2 \quad (4.18)$$

where $F_1 = B^{-1}A$. Substituting in Equation (4.10),

$$AF_1 V_{in} + AF_1 V_2 - B_2 V_2 + AV_3 = 0 \quad (4.19)$$

From Equation (4.19), $V_2$ is expressed in terms of $V_3$ as

$$V_2 = F_2 F_1 V_{in} + F_2 V_3 \quad (4.20)$$

where $F_2 = (B_2 - AF_1)^{-1}A$. By the same procedure, $V_i$ is expressed in terms of $V_{i+1}$ as

$$V_i = F_i F_{i-1} \cdots F_1 V_{in} + F_i V_{i+1} \quad (4.21)$$

where $F_i = (B_i - AF_{i-1})^{-1}A$. Using Equation (4.14) and (4.21) for the case of $i=n-1$, $V_n$ is expressed as follows:

$$V_n = F_n F_{n-1} \cdots F_1 V_{in} \quad (4.22)$$

From Equation (4.22) and (4.21) for the case of $i = n - 1$, $V_{n-1}$ is expressed as follows:

$$V_{n-1} = F_{n-1} F_{n-2} \cdots F_1 V_{in} + F_{n-1} F_n F_{n-1} F_{n-2} \cdots F_1 V_{in} \quad (4.23)$$

Using Equation (4.23) and (4.21) for the case of $i=n-2$, $V_{n-2}$ is expressed as follows:

$$V_{n-2} = F_{n-2} F_{n-3} \cdots F_1 V_{in} + F_{n-2} F_{n-1} F_{n-2} F_{n-3} \cdots F_1 V_{in}$$

$$+ F_{n-2} F_{n-1} F_n F_{n-1} F_{n-2} F_{n-3} \cdots F_1 V_{in} \quad (4.24)$$
By the same procedure, $V_i$ is expressed as follows:

$$V_i = G_i V_{in} + \sum_{j=i}^{n-1} H_i^j V_{in}$$

(4.25)

where

$$G_i = \prod_{j=i}^{1} F_j$$

(4.26)

$$H_i^j = \prod_{k=i}^{j} F_k \prod_{l=j+1}^{1} F_l$$

(4.27)

The output signal of the model is the voltage over the capacitor in the basilar membrane resonators, which represents the basilar membrane velocity of a section of the membrane, and is obtained by

$$\int J_{BM}(i) dt \propto c_i \times \frac{1}{sC_{bm}(i)} + \frac{1}{sC_{bm}(i)} + \frac{1}{s^2 S_{bm}(i)} = c_i \times Y(i) \times \frac{1}{sC_{bm}(i)}$$

(4.28)

where $c_i$ is the voltage in the resistive network connected directly to the basilar membrane resonators, as shown in Figure 4.7.

Equation (4.25) can be associated with a physical interpretation of how the voltage $V_i$ is formed. The first term in Equation (4.25), $F_i F_{i-1} \cdots F_1 V_{in}$, can be considered as the input signal $V_{in}$ propagating through all the nodes between the input nodes and $V_i$, if the matrix $F_k$ ($k = 1, 2, \cdots, i$) is interpreted as a matrix stating the change from $V_k$ to $V_{k+1}$ (or from $V_{k-1}$ to $V_k$). In the same way, the terms $H_i^j V_{in}$ can be considered as the input signal propagating to the node $V_j$ ($j = i + 1, \cdots, n$) then reflecting back to $V_i$. Therefore, $V_i$ is considered as the sum of the waves reaching $V_i$ either directly from the input nodes or reflecting at all the nodes beyond $V_i$. This is shown in Figure 4.8.

4.2.3 Simulation Results in Laplace Domain

The frequency responses of the basilar membrane resonators are obtained from Equation (4.28). Figure 4.9 shows the second term $Y(i)/sC_{bm}(i)$ in Equation (4.28) at several values of $i$, changing the capacitance to achieve $u=0.0, 0.5$ and $0.9$ in Equation (4.1). It can be seen that the change in the capacitance increases the quality factor of the curves and
Figure 4.8: An interpretation of Equation (4.25) does cause oscillation in the transfer function $Y(i)/sC_{bm}(i)$. Therefore, the cause for the distortion in the frequency response curve shown in Figure 4.6 is considered to reside in the first term $c_i$.

Figure 4.9: The function $Y(i)/sC_{bm}(i)$ at several positions along the cochlea with different values of $u$. Solid lines: $u=0.0$, dashed lines: $u=0.5$, dotted lines: $u=0.9$. 

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For convenience, each element of the terms in Equation (4.25) are defined as follows:

\[ G_i V_{in} \equiv D_i \equiv \begin{pmatrix} d_i \\ e_i \\ f_i \end{pmatrix} \] (4.29)

\[ \sum_{j=i}^{n-1} H_j V_{in} \equiv \sum_{j=i}^{n-1} P_j \equiv \sum_{j=i}^{n-1} \begin{pmatrix} p_{ij} \\ q_{ij} \\ r_{ij} \end{pmatrix} \] (4.30)

\[ \equiv T_i \equiv \begin{pmatrix} t_i \\ u_i \\ v_i \end{pmatrix} \] (4.31)

Figure 4.10 shows the real and the imaginary part of \( f_i, v_i \) and \( c_i \) for \( i=50 \). As \( u \) is increased, \( v_i \) turns out to cause oscillation, making the curve distorted. On the other hand, \( c_i \) remains smooth with increasing \( u \). Therefore, it is suspected that the distortion in the overall frequency response is caused by \( T_i \) in Equation (4.31).

Figure 4.11 shows the real and the imaginary part of \( r_{ij} \) for \( u=0.0, 0.5 \) and 0.9, \( i=50, j=50, 51, \cdots, 56 \). It is observed that the peaks and notches of the curves are shifted by a fixed amount for increasing \( j \). The distorted curve of \( v_i \) seen in Figure 4.10 is the sum of \( r_{ij} \): summing up \( r_{ij} \), which have the same number of peaks and notches shifted by a fixed amount along increasing \( j \), seems to be causing the distortion. This observation leads to an idea that the distortion in \( v_i \) may be reduced by making the amount of shifting between \( r_{ij} \) smaller. Since each term \( r_{ij} \) is associated with the reflection of a wave at \( V_j \), it is natural to assume that this fixed amount is determined by how fine the grid is, i.e., the number of grid points used to model the entire cochlea.

This idea is shown to be true numerically in Figure 4.12. It shows the real and the imaginary part of \( r_{ij} \) for \( u=0.9, i=50, j=50, 51, \cdots, 56 \). It is shown that increasing the number of grid points does make the amount of shifting smaller. And indeed, it reduces the distortion in \( v_i \), as shown in Figure 4.13. This figure shows the real part and the imaginary part of \( f_i, v_i \) and \( c_i \) for \( i=50, n=100, 200 \) and 400 at fixed value of \( u=0.9 \).

The detailed reason why summing \( r_{ij} \) causes the distortion in \( v_i \) is yet to be found. However, since each \( r_{ij} \) represents a wave reflected at node \( V_j \), it is suspected that when the density of the grid is high enough, the reflected waves reaching the node \( V_i \) cancel out.
Figure 4.10: The real and the imaginary part of $f_i, v_i$ and $c_i$ at $i=50$, $u=0.0$, $0.5$ and $0.9$. Green lines: $f_i$, red lines: $v_i$ and blue lines: $c_i$. It can be seen that $c_i$ starts oscillating as $u$ increases, and it is due to the oscillation in $v_i$. 
Figure 4.11: The real and the imaginary part of $r_i^j$ at $i=50, j=50,51,52,53,55,56$, with $u=0.0, 0.5$ and 0.9. Red lines: $j=50$, green: $j=51$, blue: $j=52$, cyan: $j=53$, magenta: $j=54$, yellow: $j=55$, black: $j=56$. It is observed that $r_i^j$ shifts by a fixed amount as $j$ increases, and the notches appear in the real part as $u$ increases, which looks to be causing the oscillation in Figure 4.10.
Figure 4.12: The real and the imaginary part of $r_j^i$ for $i=50$ with $n=100$, 200 and 400. The red lines are $j=50$ (for $n=100$), $j=100$ (for $n=200$) and $j=200$ (for $n=400$). $j$ increases by 1 as the color turns green, blue, cyan, magenta, yellow, black, then dotted red, dotted green, dotted blue, dotted cyan for $n=400$. It can be seen that the amount of shifting and the notches in the real part get smaller as $n$ increases.
Figure 4.13: The real and the imaginary part of $f_i, v_i$ and $c_i$ at $i=50$, $u=0.9$ with values of $n=100, 200$ and $400$. Green lines: $f_i$, red lines: $v_i$ and blue lines: $c_i$. It can be seen that increasing $n$ removes the oscillation.

Figure 4.14: The frequency response of the 50th basilar membrane resonator for $u=0.9$, $n=100$, 200 and 400. It is seen that increasing $n$ stops oscillation and the smooth frequency response can be obtained.
Figure 4.15: The frequency responses along the cochlea for $n=400$, at $u=0.0$ and 0.9, in terms of the voltage over capacitors. Blue lines: $u=0.0$, red lines: $u=0.9$.

Figure 4.16: The frequency responses along the cochlea for $n=400$, at $u=0.0$ and 0.9, in terms of the temporal integration of current. Blue lines: $u=0.0$, red lines: $u=0.9$. 
making the overall frequency response smooth.

Increasing the number of the filters was a problem for 1D cochleae previously developed [van Schaik, 1997], because it takes more time for the input signal to reach the last filter: theoretically, it is impossible to make a 1D cochlea with infinite number of filters to achieve sharp cutoff, because the sum of the delays caused by each filter would become infinite. For the 2D cochlea presented in this thesis, the input signal travels through the resistive network, instead of a cascade of filters. In this case, increasing the density of the grid and the filter bank would not affect the overall delay. This gives the 2D cochlea a big advantage over 1D cochleae in terms of realising the sharp cutoff, which is seen in live cochleae.

Figure 4.14 shows the frequency responses of the 50th basilar membrane resonator at $u=0.9$, with $n=100$, 200 and 400. This figure shows that the frequency response curve is refined as the number of grid points increases, smoothing the characteristics of the frequency response.

Figure 4.15 shows the frequency responses at several positions along the cochlea with $n=400$ for $u=0.0$ and 0.9. The amplitude is calculated as the ratio of the voltage over the capacitor in the basilar membrane resonators and the input voltage. Although the curves are sharpened, the peak voltage itself does not seem to change much.

Another way of looking at the undamping effect is to plot the temporal integration of the current through the basilar membrane resonators: the current through the basilar membrane corresponds to the acceleration of the section of the membrane, therefore its temporal integration corresponds to the velocity of the section. This is shown in Figure 4.16. Compared to Figure 4.15, the peak value is increased with the shape of the curves remaining the same.

Both in Figure 4.15 and 4.16, steep slopes on the higher frequency side can be seen, which cannot be obtained from passive models. This is one of the important characteristics of the frequency response measured in the living cochlea [Rhode, 1971].
4.2.4 Introducing the Outer Hair Cell Effect

In the biological cochlea, the undamping effect saturates as the input signal gets larger; that is, the larger the input signal, the less effect of the outer hair cells, therefore the more linear the basilar membrane behavior. This nonlinear effect of the outer hair cells is incorporated in the model.

By moving the undamping term to the left side, Equation (4.1) is expressed as follows:

\[
\begin{align*}
\{p_{SM}(x, 0) - p_{ST}(x, 0)\}w(x)dx + u' &= a_{BM}(x)m(x)dx + v_{BM}(x)h(x)dx + y_{BM}(x)k(x)dx \\
(4.32)
\end{align*}
\]

where \( u' = v_{BM}(x)h(x)u \).

\( u' \) can be interpreted as the undamping effect in phase with \( v_{BM}(x) \). When it is provided by the outer hair cells, the saturating nonlinear effect is taken into account by modifying \( u' \) as a function of \( v_{BM} \) as shown below.

\[
\begin{align*}
u' &= f(v_{BM}(x)) \\
(4.33)
\end{align*}
\]

where \( f(\cdot) \) is a saturating function as shown in Figure 4.17. \( u' \) is proportional to \( v_{BM} \) when the signal is small, therefore providing the undamping effect. \( u' \) saturates when the signal is large, making it negligible compared to the term \( v_{BM}(x)h(x)dx \).

In the living cochlea, the force generated by the outer hair cells is asymmetrical with regard to the origin, as shown in Figure 2.9 in chapter 2. However, the function used in this model is symmetrical for simplicity.

\( f(\cdot) \) can be any function as long as it shows the characteristics seen in Figure 4.17. \( \tanh(\cdot) \) is one of the examples. For simplicity, the following function is used for the simulation.

\[
\begin{align*}
f(v_{BM}(x)) &= \begin{cases} 
-uv_{th} & \text{for } v_{BM} < -v_{th} \\
uv_{BM}(x) & \text{for } -v_{th} \leq v_{BM} \leq v_{th} \\
uv_{th} & \text{for } v_{th} < v_{BM}
\end{cases} \\
(4.34)
\end{align*}
\]
4.2.5 Simulating in Time Domain

As discussed in chapter 2, the pressure of the fluid is mapped to voltage in the electrical circuit. Therefore, it is appropriate to incorporate the undamping term as an extra voltage source inserted in series with the basilar membrane resonator, as shown in Figure 4.18 (a).

It is not easy to directly implement a time-varying voltage source in a circuit. One way of producing the same effect as inserting the voltage source would be to add extra current into the node above $G_{BM}$ then subtract it from the node below $G_{BM}$, as shown in Figure 4.18 (b).

Since the model includes nonlinear elements, the frequency response has to be simu-
lated in the time domain. This can be done by solving simultaneous equations, written in accordance with the current law at each node in the circuit, at each time step. However, because of the nonlinear time-varying elements representing the outer hair cell undamping effect, the matrix has to be re-calculated at every time step, which causes the simulation to slow down. This is a serious problem when the cochlea has to be modelled with a fine resistive grid, which increases the number of nodes, as the size of the matrix re-calculated increases as the square of the number of nodes. To avoid this, a new method of time-domain simulation is proposed, and described in the rest of this section.

Method of separating time-varying part

This method utilises the fact that most of the circuit elements don’t change (conductance of the resistive grid), therefore it is possible to calculate the part of the circuit where the value of the elements need to be re-calculated at each time step separately.

Assuming that all the node voltages at time $t = t_j$ are known, and the voltages at time $t = t_{j+1} = t_j + dt$ are about to be calculated in an iterative process. The voltages and currents used below are defined as shown in Figure 4.18.

**Step 1** Assuming values of $V^i_2$ and $V^i_{ohc}$ ($i = 1, 2, \cdots, n$) close enough to the true values (values at $t = t_j$ are used, to start with), find the voltages at the nodes inside the resistive network. $V^i_1$ is calculated as a result.

**Step 2** Calculate the current through the basilar membrane resonators $I^i_{BM}$ using

$$I^i_{BM} = G_{BM}(i) \times (V^i_1 - V^i_2 - V^i_{ohc}) \quad (4.35)$$
Step 3  Calculate $dV^i_3/dt$ which is necessary to find the new value of $V^i_2$ in the next step. This is achieved by

$$
\frac{dV_3}{dt} = \left. \frac{dV_3}{dt} \right|_{t=t_j} + \frac{d^2V_3}{dt^2} \times dt
= \left. \frac{dV_3}{dt} \right|_{t=t_j} + \frac{I_{BM}^i}{S_{BM}(i)}
$$  \hfill (4.36)

Step 4  Calculate the new value of $V^i_2$ by

$$
V_2 = V_2 + \left. \frac{dV_2}{dt} \right|_{t=t_j} \times dt
= V_2 + \left( \left. \frac{dV_3}{dt} \right|_{t=t_j} + \frac{I_{BM}^i}{C_{BM}(i)} \right) \times dt
$$  \hfill (4.37)

As $V^i_{ohc}$ is a function of $V^i_2$ and $V^i_3$, the new value of $V^i_{ohc}$ can also be calculated at this stage as

$$
V^i_{ohc} = f(u(V^i_3 - V^i_2))
$$  \hfill (4.38)

where $u$ is an undamping coefficient, and $f(\cdot)$ is a saturating function introduced in section 4.2.4.

Step 5  Compare the two values of $V^i_2$ from Equation (4.37) and the value used in Step 1. If the difference is smaller than $\varepsilon$, which determines the accuracy of the simulation, $V^i_2$ is considered to be the value at time $t = t_{j+1}$. If not, repeat the procedure through Step 2 to 5, using the new value of $V^i_2$ and $V^i_{ohc}$ in Step 1, until the difference between the original and the new value of $V^i_2$ becomes smaller than $\varepsilon$.

Condition of Convergence

When using the method described in the previous section, it is important to make sure that the value $V^i_2$ converges after a relatively small number of iterations. Therefore, an analysis is necessary to find what value would be obtained at Step 4 when a slightly different value of $V^i_2$ from the final value $V^i_{2\text{true}}$ is used at Step 1.
Suppose a value $V_i^1$ is obtained in Step 1, using a value $V_i^2$. Their true values are $V_i^1_{\text{true}}$ and $V_i^2_{\text{true}}$, and are close enough to $V_i^1$ and $V_i^2$.

Following through Step 1 to Step 4, the new value $V_i^2_{\text{new}}$ is expressed in terms of $V_i^1$ and $V_i^2$.

$$V_i^2_{\text{new}} = V_i^2 |_{t=t_j} + \left( \frac{dV_3}{dt} |_{t=t_j} + \frac{G_{BM}(i)}{S_{BM}(i)} (V_1^i - V_2^i - V_{ohc}^i) dt + \frac{G_{BM}(i)}{C_{BM}(i)} (V_1^i - V_2^i - V_{ohc}^i) dt \right) dt \quad (4.39)$$

On the other hand, by the same procedure, the following equation holds for $V_i^1_{\text{true}}$ and $V_i^2_{\text{true}}$.

$$V_i^2_{\text{true}} = V_i^2_{\text{true}} |_{t=t_j} + \left( \frac{dV_3}{dt} |_{t=t_j} + \frac{G_{BM}(i)}{S_{BM}(i)} (V_{1\text{true}}^i - V_{2\text{true}}^i - V_{oht\text{true}}^i) dt + \frac{G_{BM}(i)}{C_{BM}(i)} (V_{1\text{true}}^i - V_{2\text{true}}^i - V_{oht\text{true}}^i) dt \right) dt \quad (4.40)$$

By subtracting Equation (4.39) from Equation (4.40), the difference between $V_i^2_{\text{new}}$ and $V_i^2_{\text{true}}$ are expressed as follows:

$$V_i^2_{\text{true}} - V_i^2_{\text{new}} = \left( \frac{G_{BM}}{C_{BM}} + \frac{G_{BM}}{S_{BM}} dt \right) \cdot \left\{ (V_{1\text{true}}^i - V_1^i) - (V_{2\text{true}}^i - V_2^i) - (V_{oht\text{true}}^i - V_{ohc}^i) \right\} \quad (4.41)$$

Equation (4.41) indicates that, by making the time step $dt$ sufficiently small, it is possible to make sure that $|V_i^2_{\text{true}} - V_i^2_{\text{new}}|$ is smaller than $|V_i^2_{\text{true}} - V_2^i|$, therefore leading to convergence.

In the simulation shown in the next section, the value of $\varepsilon$ is set to $10^{-9}$, and the average number of the iterations needed for convergence was about 5. From a simple experiment, it was found that the simulation using the method described in this section reduces the simulation time down by about 33% compared to re-calculating the whole matrix.
Simulation Results

Figure 4.19 shows the frequency response of the active cochlea obtained by time domain simulation using the method described above. The simulation used a cochlear model of 100-sections long, and an outer hair cell undamping coefficient of $u=0.5$. A relatively large input value (up to 20.0V) was applied to demonstrate the outer hair cell saturating effect. It is clearly seen that the larger the input signal is, the closer the frequency response to that of the passive cochlear model.

The response was simulated at 50 points between 500Hz and 2000Hz logarithmically spaced, except for the curves for passive cochlea, which were simulated at 200 points between 500Hz and 5000Hz. Because of the amount of time the simulation takes (it took more than 4 days using 2 computers, with Intel Pentium II and III processors respectively), it was not feasible to run any more simulations to obtain smoother curves.

![Figure 4.19: Simulation results](image)

4.3 Summary

The effect of the outer hair cells on the cochlea was analysed. It was discovered that the outer hair cells undamp the basilar membrane motion. This effect was incorporated in the model for an active cochlea using time-varying nonlinear voltage sources.

To simulate the model in the time domain, a new method is proposed to speed up the
simulation time. It makes use of the fact that only the elements in the basilar membrane resonators change at each time step, therefore by separating the resistive network and calculating the parts separately it is possible to avoid re-calculating the whole matrix at each time step, which is the main factor contributing to time-consuming simulations.

The simulation results show a nonlinear phenomenon; the effect of the outer hair cells saturates as the input signal becomes larger, and therefore the response becomes close to that of the passive cochlea model.
5.1 Conclusions

This thesis presents the analysis and development of a passive and an active cochlear model. Simulations and analysis of an existing passive cochlear model were performed. It was implemented in analog VLSI, and a basilar membrane section was tested and found to show a reasonable agreement with the model. However, large variations in quality factor were found between individual resonator sections due to transistor mismatch. A new active cochlear model is proposed, and its development and simulated results are presented. It incorporates the effect of the outer hair cells in the living cochlea, which actively generate force to enhance small input signals. The undamping effect of the outer hair cells was found to cause oscillation in the frequency response of the electrical cochlea. This was further analysed, and it was found that the number of the grid points have to be increased to reduce the oscillation. In order to carry out the time domain simulation to see the nonlinear phenomenon, a new method is proposed to make the simulation much less time-consuming.

5.2 Future Work

A number of things still need to be investigated about the active cochlear model presented in this thesis.

Firstly, the relation between the number of the grid points and the quality factor of the basilar membrane resonators in the cochlea needs to be clarified. It should be possible to
derive a function which states the number of the grid points necessary to achieve a certain value of the quality factor. This is essential for an electronic implementation of the model.

Secondly, the function which represents the outer hair cell (Equation (4.38) in chapter 4) can be reviewed to obtain better performance. It is possibly a function of time, the displacement, the velocity and the acceleration of the basilar membrane. The optimum function may involve all of these factors. However, it needs to remain reasonably simple to be implemented in a circuit. Therefore, the tradeoff between the amount of the details incorporated in the model and the simplicity in terms of the circuit implementation is expected.

Lastly, analog VLSI implementations of the active model are in order. One way of implementing the outer hair cell is to add a current partly through the basilar membrane resonator in order to realise the same effect as adding a voltage source that represents the outer hair cell effect, as shown in Figure 4.18 in chapter 4. When successfully fabricated, it will hopefully overcome the problems that the passive cochlea had, namely, the transistor mismatch, by actively controlling the quality factor.


Curriculum Vitae

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