Noncooperative and Cooperative Transmission Schemes
with Precoding and Beamforming

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Abstract

The next generation mobile networks are expected to provide multimedia applications with a high quality of service. On the other hand, interference among multiple base stations (BS) that co-exist in the same location limits the capacity of wireless networks. In conventional wireless networks, the base stations do not cooperate with each other. The BSs transmit individually to their respective mobile stations (MS) and treat the transmission from other BSs as interference. An alternative to this structure is a network cooperation structure. Here, BSs cooperate with other BSs to simultaneously transmit to their respective MSs using the same frequency band at a given time slot. By doing this, we significantly increase the capacity of the networks. This thesis presents novel research results on a noncooperative transmission scheme and a cooperative transmission scheme for multi-user multiple-input-multiple-output orthogonal frequency division multiplexing (MIMO-OFDM).

We first consider the performance limit of a noncooperative transmission scheme. Here, we propose a method to reduce the interference and increase the throughput of orthogonal frequency division multiplexing (OFDM) systems in co-working wireless local area networks (WLANs) by using joint adaptive multiple antennas (AMA) and adaptive modulation (AM) with acknowledgement (ACK) Eigen-steering. The calculation of AMA and AM are performed at the receiver. The AMA is used to suppress interference and to maximize the signal-to-interference-plus-noise ratio (SINR). The AM scheme is used to allocate OFDM sub-carriers, power, and modulation mode subject to the constraints of power, discrete modulation, and the bit error rate (BER). The transmit weights, the allocation of power, and the allocation of sub-carriers are obtained at the transmitter using ACK Eigen-steering. The derivations of AMA, AM, and ACK Eigen-steering are shown. The performance of joint AMA and AM for various AMA configurations is evaluated through the simulations of BER and spectral efficiency (SE) against SIR.

To improve the performance of the system further, we propose a practical cooperative transmission scheme to mitigate against the interference in co-working WLANs. Here, we consider a network coordination among BSs. We employ Tomlinson Harashima precoding (THP), joint transmit-receive beamforming based on SINR (signal-to-interference-plus-noise-ratio) maximization, and an adaptive precoding order to eliminate co-working interference and achieve bit error rate (BER) fairness among
different users. We also consider the design of the system when partial channel state information (CSI) (where each user only knows its own CSI) and full CSI (where each user knows CSI of all users) are available at the receiver respectively. We prove analytically and by simulation that the performance of our proposed scheme will not be degraded under partial CSI. The simulation results show that the proposed scheme considerably outperforms both the existing non-cooperative and cooperative transmission schemes.

A method to design a spectrally efficient cooperative downlink transmission scheme employing precoding and beamforming is also proposed. The algorithm eliminates the interference and achieves symbol error rate (SER) fairness among different users. To eliminate the interference, Tomlinson Harashima precoding (THP) is used to cancel part of the interference while the transmit-receive antenna weights cancel the remaining one. A new novel iterative method is applied to generate the transmit-receive antenna weights. To achieve SER fairness among different users and further improve the performance of MIMO systems, we develop algorithms that provide equal SINR across all users and order the users so that the minimum SINR for each user is maximized. The simulation results show that the proposed scheme considerably outperforms existing cooperative transmission schemes in terms of the SER performance and complexity and approaches an interference free performance under the same configuration.

We could improve the performance of the proposed interference cancellation further. This is because the proposed interference cancellation does not consider receiver noise when calculating the transmit-receive weight antennas. In addition, the proposed scheme mentioned above is designed specifically for a single-stream multi-user transmission. Here, we employ THP precoding and an iterative method based on the uplink-downlink duality principle to generate the transmit-receive antenna weights. The algorithm provides an equal SINR across all users. A simpler method is then proposed by trading off the complexity with a slight performance degradation. The proposed methods are extended to also work when the receiver does not have complete Channel State Informations (CSIs). A new method of setting the user precoding order, which has a much lower complexity than the VBLAST type ordering scheme but with almost the same performance, is also proposed. The simulation results show that the proposed schemes considerably outperform existing cooperative transmission schemes in terms of SER performance and approach an interference free performance.

In all the cooperative transmission schemes proposed above, we use THP to cancel part of the interference. In this thesis, we also consider an alternative approach that bypasses the use of THP. The task of cancelling the interference from other users now lies solely within the transmit-receive antenna weights. We consider multiuser Gaussian broadcast channels with multiple antennas at both transmitter and receivers. An iterative multiple beamforming (IMB) algorithm is proposed, which is flexible in the antenna configuration and performs well in low to moderate data rates. Its capacity and bit error rate performance are compared with the ones achieved by the traditional zero-forcing method.
Acknowledgements

The production of this thesis would not have been possible without the inspiration, encouragement and support from many people, to whom I am greatly indebted.

My gratitude should first be given to my supervisor, Professor Branka Vucetic, for leading me through this interesting research topic. With great perception, she defined the early directions of the research, and has continuously provided generous support along the way. Although heavily loaded with administrative work and being busy, she manages to make herself available for the meeting with us postgraduate students every week. She is always friendly and approachable even for unscheduled consultations. She has generously provided financial support to me.

I wish to thank my co-supervisor, Dr. Yonghui Li for innumerable and valuable suggestions, comments and discussion. His co-operation as well as his great enthusiasm for work have influenced my research and contributed a lot to my education.

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I am grateful for the financial support of the University of Sydney and the Commonwealth of Australia through the provision of tuition fee and several generous scholarships. Also my great gratitude goes to SingTel Optus Pty Limited for their sponsorship of the projects in which I have participated. The involvement of these industrial projects forms the basis of most of the content in this thesis.

Thanks to all the administrative staff in the department. In particular, I appreciate the efforts of Ping, in handling numerous claim forms promptly. Also a big thank you to Mr. David Brown, for his diligence and enthusiasm in maintaining the school network and solving our
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Thanks to all my colleagues, with whom I shared an office. I would like to thank Yang Tang, Agus Santoso, Zhendong Zhou, Kumudu Munasinghe, Rui Li, Raymond Louie, Kun Pang, Srdjan Vukadinovic and Lixiang Xong for making the office environment friendly and making postgraduate study interesting.

Above all, I would like to express my deepest gratitude for the unreserved love, support and understanding that I received from my parents, my sister and my brother-in-law. Finally, the highest gratitude goes to my wife and daughter, Hazel Tan Meng Jock and Yu-Jynn, for their unwavering support and love.
Statement of Originality

The novel research results reported in this thesis represent an original work by the author. All material presented in the thesis is the original work of the author, unless otherwise stated. Its content has not been previously submitted for examination as part of any academic qualifications. Most of the results contained herein have been published, accepted for publication, or submitted for publication, in journals or conferences of international standing. The author’s contribution in terms of published material is listed in the next section “Related Publications”, and is referenced throughout using alphabetical-numerical citations. Numerical citations refer to the references listed in the bibliography.

The original motivation to pursue research in this field was provided by my thesis supervisor, Professor Branka Vucetic, of the University of Sydney.

The novel ideas and the author’s contributions are described in Chapters 4, 5, 6, 7 and 8. The novel ideas and the author’s contributions are summarized as follows,

1. The original contributions in Chapter 4, “Noncooperative Transmission Scheme Design for Co-working WLANs” consist of a new allocation method for OFDM subcarriers, power and discrete modulation mode (BPSK, 4-QAM, 16-QAM, 64-QAM) and a new transmit-receive antenna weights design that maximizes the signal-interference-plus-noise-ratio (SINR) in co-working MIMO-OFDM WLANs. This work is presented in [C3].

2. The original contribution in Chapter 5, “Cooperative Transmission Scheme Design”, is the application of the transmit-receive antenna weights designed in Chapter 4 to a nonlinear cooperative transmission scheme, employing Tomlinson Harashima Precoding (THP) and transmit-receive beamforming. This work is presented in [C2] and
3. In Chapter 6, “Spectrally Efficient Wireless Communication Systems with Cooperative Precoding and Beamforming”, we present a new downlink cooperative zero forcing transmission scheme employing THP and beamforming. The contributions in this chapter consist of a new novel iterative zero forcing transmit-receive antenna weights design and a new power allocation algorithm, that provides equal SINR across all users. This work is presented in [J1].

4. In Chapter 7, “Cooperative Precoding and Beamforming for Single/Multi-Stream Multi-User MIMO Systems”, we present a new downlink cooperative transmission scheme, based on the uplink-downlink duality concept, employing THP and beamforming. The contributions in this chapter consist of

   (a) the design of transmit-receive antenna weights for multi-stream multi-user MIMO system with multiple transmit and receive antennas that maximize SINR by using the duality concept.

   (b) the extension of the cooperative transmission scheme to work when the receiver does not have complete channel state informations.

   (c) the design of a new method for setting the user precoding order for THP which has a much lower complexity that VBLAST type ordering.

This work is presented in [J2] and [CP1].

5. In Chapter 8, “Iterative Multiple Beamforming Algorithm for MIMO Broadcast Channels”, we present a new linear downlink cooperative transmission scheme, based on a zero forcing method. This is a joint work with Dr. Zhendong Zhou, the University of Sydney. The contribution in this chapter is a new iterative method for finding the optimum transmission space for the zero forcing method. By using the iterative beamforming scheme in this chapter, we outperform the existing linear zero forcing cooperative transmission scheme. This work is presented in [J3] and [P2].
Related Publications

Journal Papers


Conference Papers


Presentation


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<th>Description</th>
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<td>CI</td>
<td>Co-working Interference</td>
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<tr>
<td>CCI</td>
<td>Co-channel Interference</td>
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<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>AP</td>
<td>Access Point</td>
</tr>
<tr>
<td>STA</td>
<td>Station</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>BLAST</td>
<td>Bell Laboratories Layered Space-Time</td>
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<tr>
<td>MIMO</td>
<td>Multiple-Input-Multiple-Output</td>
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<tr>
<td>SISO</td>
<td>Single-Input-Single-Output</td>
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<tr>
<td>MISO</td>
<td>Multiple-Input-Single-Output</td>
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<tr>
<td>SIMO</td>
<td>Single-Input-Multiple-Output</td>
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<td>OFDM</td>
<td>Orthogonal-Frequency-Division-Multiplexing</td>
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<td>AM</td>
<td>Adaptive Modulation</td>
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<td>AMA</td>
<td>Adaptive Multiple Antennas</td>
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<tr>
<td>ACK</td>
<td>ACKnowledgement</td>
</tr>
<tr>
<td>SE</td>
<td>Spectral Efficiency</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Interference-plus-Noise-ratio</td>
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<td>SINRM</td>
<td>Signal-to-Interference-plus-Noise-ratio Maximization</td>
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<td>SIR</td>
<td>Signal-to-Interference-ratio</td>
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<td>THP</td>
<td>Tomlinson Harashima Precoding</td>
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<td>CSI</td>
<td>Channel State Information</td>
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<td>IMB</td>
<td>Iterative Multiple Beamforming</td>
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<td>Acronym</td>
<td>Full Form</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase-Shift Keying</td>
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<td>M-QAM</td>
<td>Multi-Level Quadrature Amplitude Modulation</td>
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<td>S-QAM</td>
<td>Staggered Quadrature Amplitude Modulation</td>
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<tr>
<td>RSSI</td>
<td>Received Signal Strength Indicator</td>
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<td>BT</td>
<td>Bluetooth</td>
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<td>MAC</td>
<td>Medium Access Control</td>
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<td>Clear Channel Assessment</td>
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<td>APO</td>
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<td>APC</td>
<td>Adaptive Precoding Order Maximum Complexity</td>
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<td>DFT</td>
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<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
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<td>SVD</td>
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<tr>
<td>EVD</td>
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<td>MF</td>
<td>Matched Filter</td>
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<td>MVDR</td>
<td>Minimum Variatin Distortionless Response</td>
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<td>Radio-Frequency</td>
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Chapter 1

Introduction

The spectral efficiency of downlink transmission in existing cellular mobile [1] and wireless local area networks (WLAN) [2] is limited by interference. In cellular mobile networks, the dominant interference comes from adjacent cells or other network operators using the same frequency band [1], while in co-working WLANs [2], the interference from other networks, operating in the same area, is a major limiting factor [2]. Due to its high spectral efficiency, multiple-input-multiple-output orthogonal-frequency-division-multiplexing (MIMO-OFDM) is a widely accepted technology for all future wireless standards. Thus, we consider multiple antennas at both the transmitter and receiver and OFDM technology as our transmission scheme.

In conventional wireless networks, the co-existing base stations (BSs) do not cooperate with each other. BSs transmit individually and treat other BSs as interference. Thus, the transmissions from the BSs will interfere with each other. Traditionally, only coexistence mechanisms between different technologies such as Bluetooth (BT) with WLANs have been investigated. Examples of these methods are automatic frequency selection based on the received signal strength indicator (RSSI) [3], sub-carrier symbol erasure [4], and media access control (MAC) level interference avoidance [5, 6]. These solutions work by getting the BSs to avoid interference from other BSs by transmitting in a different frequency band [3, 4] or a different time slot [5, 6].

Applying these approaches to mitigate interference in WLANs or cellular networks will re-
quire extra expensive resources such as additional frequencies or time slots. Therefore, a better solution in noncooperative co-working networks is to suppress the co-working interference. By doing this, we can avoid allocating extra frequency resources. This feature is especially important for co-working WLANs since there are only three non-overlapping channels in the industrial, scientific, and medical (ISM) band available [7, 8]. In addition, all of the interference avoidance schemes mentioned above require IEEE 802.11’s clear channel assessment (CCA) mechanism [7, 8] to work correctly and to be able to detect all nodes. CCA is a process where BSs or MSs sense the wireless channel for a specific time interval and ascertain whether the medium is available for its own transmission. The CCA mechanism, however, will not work if a hidden BS exists in the network [9]. A hidden BS will cause interference to other BSs since its transmission cannot be sensed by the CCA mechanism of other BSs [9].

The interference from other users in noncooperative networks can be suppressed through the use of multiple antennas at BSs and mobile stations (MSs). The performance can then be further improved through the use of adaptive modulation (AM). Adaptive multiple antennas (AMA) with co-channel interference (CCI) and AM have been studied independently in the past. Receivers with multiple antenna configurations are investigated in [10–12]. Joint optimal transmit and receive beamforming to maximize the signal-to-interference-plus-noise-ratio (SINR) in MIMO-OFDM configurations are studied in [13, 14]. AM has also been thoroughly investigated and reported. AM, in the context of single-user and non-interference environments is considered in [15]. In [16] the scheme is extended to multi-user cellular environments. AM is also proposed for WLANs in [17], but without considering co-working interference from other BSs.

An alternative to the above noncooperative transmission from each BS to its respective MS, is a cooperative transmission structure. Here, multiple BSs share information about the transmitted messages to their respective users and wireless channels via a backbone network. Individual BSs are equipped with multiple transmit antennas. Each BS transmitter uses the information of the transmitted signals from other BSs and wireless channel conditions to precode its own signal. The precoded signal for each BS is broadcast through all BS transmit antennas in the same frequency band in a given time slot. The precoding operation and transmit-receive antenna coefficients are chosen in such a way as to minimize the interference coming from other BS transmissions.
Most of the published papers in this area consider either a multi-user MIMO system with a single receive antenna [18–21] or a multi-stream single-user with multiple transmit-receive antennas [18, 22–24]. Obviously the latter case is not applicable to cellular mobile networks or WLANs since there are multiple users transmitting at the same time. In [18, 19, 25, 26], the uplink-downlink duality concept is introduced to find the optimum transmit-receive antenna weights and to allocate downlink power. The authors first show that the downlink SINR can be designed to be equal to the maximum uplink SINR under the same total available power but with a different power allocation in the downlink and uplink. They then propose a method to find the transmit weights and transmission powers that maximize the individual downlink Signal-to-Interference-plus-Noise-Ratio (SINR) by solving its dual uplink equivalence. A different approach is proposed in [21], where a combination of a Zero Forcing (ZF) method, that determines transmit weights by forcing part of the interference to zero, and dirty paper coding (DPC) [27] is used to suppress interference from other users. A more practical approach than [21] is considered in [20] where DPC is replaced with Tomlinson-Harashima-Precoding (THP) [28, 29]. Various approaches in implementing the combination of ZF and DPC are considered in [23, 30–32]. In [31, 32] the pseudo inverse of the channel matrix and sphere encoding method are used to zero force all the interference. Due to the high complexity of sphere encoding, the authors in [23] use the pseudo inverse [33] of the channel matrix and lattice-reduction method [34, 35] to zero force all the interference. In [30] a zero forcing method is initially developed to approximate and simplify for the sphere encoding or vector perturbation [31, 32]. The authors in [30] apply the zero forcing method to cancel interference in a multi-user MIMO system with a single receive antenna. Interestingly the simulation result in [30] shows that its bit-error-rate performance is better than sphere encoding in [31, 32] which is considered to be the best to date. These algorithms however, only consider single receive antenna scenarios, which cannot directly be extended to MIMO systems.

An extension of a multi-user MIMO system with a single receive antenna to a multi-user MIMO system with multiple transmit-receive antennas has been considered by several researchers. In [36], the uplink-downlink duality concept introduced in [18, 19] is used to design the optimum downlink transmit-receive weights and downlink transmission powers in a multi-user MIMO system with multiple transmit-receive antennas. The problem with this method is that the convergence to a solution cannot be guaranteed and the method does not work when more than one symbol stream is transmitted to each user. In [37] and [38],
1.1 Research Problem and Contributions

The research effort in this thesis has been put into designing a downlink transmission scheme for noncooperative and cooperative BSs, operating in overlapping locations and transmitting in the same frequency band at the same slot and time slot, with a good complexity-performance trade-off, for applications in MIMO-OFDM systems. We focus on designing multi-user MIMO-OFDM interference cancellation schemes based on zero-forcing (ZF) and maximum signal-to-interference-plus-noise (SINR) criteria as these two approaches seem to offer the performance-complexity-trade-off required by the present practical systems. The system performance obtained by applying the maximum SINR criterion is proved to be equal to the ones obtained by using the minimum-mean-square-error (MMSE) criterion in [45].

The first research problem considered in the thesis is the co-existence of noncooperative wireless networks in the same location transmitting in the same frequency band. The aim here is to find a new method so that no additional frequency resources need to be allocated...
1.1 Research Problem and Contributions

to these networks. When BSs do not cooperate, the downlink transmission by each BS to its respective mobile station (MS) will interfere with the transmission from the other BSs. The interference from other users is suppressed through the use of adaptive multiple antennas (AMA) at the transmitter and the receiver. Adaptive modulation schemes can then be used to improve the throughput of the system. The problem of joint optimization of AMA, AM and their implementation in co-working WLANs has not been considered in the literature.

The first contribution here is a new method that jointly designs the transmit-receive weights for AMA, AM and acknowledgement (ACK) Eigen-steering. This method improves the downlink performance of MIMO-OFDM in co-working WLANs. The objective of the scheme is, 1) to maximize SINR using AMA, 2) to maximize the data rate using AM, and 3) to eliminate the channel feedback requirement using acknowledgement (ACK) Eigen-steering. AMA weights are computed at the station (STA). The optimum transmit beamforming weights for the access point (AP) and receive beamforming weights for the station (STA) are calculated [13].

The second contribution is an allocation method for OFDM sub-carriers, power, and modulation mode done by AM. Power constraint, discrete modulation constraint (BPSK, 4-QAM, 16-QAM, or 64-QAM as in IEEE 802.11g [7]), and BER constraint are taken into consideration.

The second research problem that we consider is network coordination as a means of providing efficient downlink transmission in co-working WLANs. Here, the performance for each cooperative BS will need to be equal, otherwise the network operators would not want to cooperate. This condition has not been addressed in the literature. The research contribution here is the proposal of a cooperative transmission scheme among co-working BSs, to eliminate co-working interference in MIMO-OFDM WLANs. We develop a practical precoding algorithm combining the Tomlinson-Harashima precoding (THP) scheme [28, 29] with transmit-receive beamforming based on the signal-to-interference-plus-noise-ratio maximization criterion (SINRM) [13, 45, 46]. The transmit-receive beamforming weights are derived by using the SINRM criterion. We show how our proposed cooperative transmission scheme significantly outperforms the performance of the cooperative scheme [41] and conventional non-cooperative schemes [13, 46].

Even though THP-SINRM is better than the noncooperative scheme, the performance de-
1.1 Research Problem and Contributions

grades as the number of users transmitting simultaneously in the co-working WLANs increases. The research contribution here is the design of a new cooperative transmission scheme employing precoding and beamforming for the downlink of a single-stream multi-user MIMO system. The method is applicable for both cellular and WLAN networks. In this algorithm, THP cancels part of the interference while the transmit-receive antenna weights cancel the remaining interference. A new novel iterative method is applied to generate the transmit-receive antenna weights that zero force (ZF) interference from other BSs. These transmit-receive antenna weights are optimized based on the iterative optimization method from [47]. The receive and transmit weights are optimized iteratively until the SINR for each user converges to a fixed value. The convergence behaviour of the proposed method is investigated both analytically and numerically. We also employ SINR equalization, and an adaptive precoding ordering (APO). In SINR equalization [18] power is allocated to users in such a way that all have the same SINRs. This allocation ensures SER fairness. An expression for the power allocation is derived. The APO is used to further improve the performance of MIMO systems, by maximizing the minimum SINR for each user [48]. Here, we apply the VBLAST user detection ordering from [48] for user ordering in the THP. We refer to this method as APO-VBLAST. Simulation results show that our scheme is significantly superior to the existing methods.

The proposed method offers a significant improvement over a nonlinear cooperative precoding algorithm presented in [41], [39], and [42]. The first contribution here is the enhancement of the SER performance due to an iterative transmit-receive weights calculation. The second contribution is the relaxation of the semi zero forcing constraints. Unlike [41] and [42], here we allow transmit signals, intended for different users, to interfere with each other. This interference is forced to zero at the receiver where the signal is multiplied by the receive antenna weights. The next contribution here is in the complexity reduction. The proposed scheme has a much lower computational complexity than the methods in [41], [39], and [42]. The fourth improvement comes from the elimination of the dependency of the number of receive antennas to the number of transmit antennas. In the proposed method, it is not necessary for the number of receive antennas to be at least equal to the number of transmit antennas as required in [41, 42]. This latter feature allows the proposed algorithm to be applied to a wider range of scenarios than the schemes in [41], [39], and [42], while providing a capacity-approaching performance. The proposed method can be used to improve the performance and capacity of co-working WLANs and cellular mobile systems.
1.1 Research Problem and Contributions

It is possible to further improve the performance of the above proposed interference cancellation. This is because the proposed interference cancellation does not consider receiver noise when calculating the transmit-receive antenna weights. In addition, the proposed scheme is designed specifically for a single-stream multi-user transmission. The research contribution here is a new transmission method based on the uplink-downlink duality concept [1], [19], and [18]. The new method is able to handle multi-stream transmission and to take into account the receiver noise when calculating the transmit-receive antenna weights. We design a cooperative transmission scheme employing nonlinear precoding and beamforming for the downlink of a multi-user single/multi-stream multiple-input-multiple-output (MIMO) system. In this algorithm, Tomlinson-Harashima Precoding (THP) [28], [29] cancels part of the interference, while the transmit-receive antenna weights cancel the remaining one. We first propose an iterative method which estimates the transmit-receive antenna weights and allocates the downlink power, such that signal-to-interference-plus-noise-ratios (SINR) for all users are maximized. The transmit-receive antenna weights and the allocation of downlink power are optimized based on the iterative optimization method from [47]. We then show how to simplify this iterative method by eliminating its iteration step which is required to find the transmit-receive antenna weights and power allocation. We then consider a scenario where the receiver does not have complete channel state information (CSI) and the system does not allow BSs to specifically send receive antenna weights information calculated at the transmitter or complete CSI to each receiver. That means that each MS receiver only knows its own CSI, preventing joint design of transmit-receive antenna weights. We refer to this situation as limited CSI. This scenario has not been considered before in open literature. The performance of the proposed algorithms under this condition is shown to have a very small degradation compared to the ideal case where each MS receiver knows the CSI from other users. Finally, we propose a new method of ordering, referred to as low complexity adaptive precoding ordering (APO-LC), that has a much lower complexity than APO-VBLAST. This latter feature is crucial as we want to accommodate a large number of users, transmitting at the same time. Simulation results show that the proposed algorithms are significantly superior to the existing methods.

There are three main contributions in the above proposed scheme. First, the zero forcing and orthogonality constraints for the transmit weights vector are fully relaxed compared to those in [49], [39], [42], and [41]. The relaxation of the zero forcing and orthogonality constraints enable the proposed algorithms to incorporate the effect of the receiver noise and boost the
1.1 Research Problem and Contributions

SINR for each user, leading to a better performance, when the receiver noise is not negligible.

Second, the concept of the uplink-downlink duality is applied to the multi-stream multi-user multi-antenna scenario to obtain a minimum-mean-square-error (MMSE) type of system performance. The main difference here with other duality approaches [26, 50] is in the objective. In [26, 50], the objective was to achieve a maximum sum-rate capacity. That means the achievable SINR or rates for each user or links are not equal. Here, Sato’s sum-capacity upper bound [50] can be used to obtain a closed form solution for optimal weights, since the problem is a convex problem. Note that however, in [50], the Sato upper bound is calculated by using software called SDPSOL [50] which performs an iterative optimization. On the other hand, the objective of the scheme in the thesis is to equalize SINR or rates for each link. This is different from maximizing sum-capacity and this is not a convex problem. As far as we know, there is no closed form for it. That is why we use an iterative method to find the optimal weights. In addition, the published papers, such as [22], [18], [23], [24] consider either a multi-stream single-user or a single-stream multi-user system with a single receive antenna.

However, dirty-paper techniques are largely information-theoretic and worse, the encoding process to achieve the sum-capacity is data dependent. In other words, the cancellation needs to be done independently for every symbol.

A nonlinear precoding scheme is data dependent. In other words, the THP or vector perturbation precoding needs to be done independently for every symbol even though the wireless channel condition does not change. Here, we consider an alternative approach that bypasses the use of THP and thus does not require the system to precode every symbol. The task of THP is to cancel a part of the interference. By not using THP, the task of cancelling all of the interference now lies within the transmit-receive antenna weights. The research contribution here is a new iterative multiple beamforming (IMB) algorithm that extends the method in [30, 39] to a multi-user MIMO system with multiple receive antennas. In addition, unlike most of the previous work, such as [41, 43] which only investigates the system capacity, we also address the error rate performance of realistic systems. Both capacity evaluation and bit error rate (BER) simulations also show that the IMB performs much better than the ZF when the system operates at low to moderate data rates.
1.2 Thesis Outline

Chapter 1 explains the research motivation, states the research problems and presents a brief overview of some promising approaches for increasing the spectral efficiency of noncooperative and cooperative wireless networks.

Chapter 2 introduces the background information to aid the understanding and analysis in subsequent chapters.

Chapter 3 introduces the system model for noncooperative and cooperative transmission. Here, we also show the derivation of the feedback matrix required for THP.

Chapter 4 presents a method to reduce interference and increase downlink throughput for OFDM systems in co-working noncooperative MIMO-OFDM WLANs, by using adaptive modulation and multiple antennas at the transmitter and receiver.

Chapter 5 presents a new nonlinear precoding method, combining Tomlinson Harashima Precoding with transmit-receive beamforming, based on the signal-to-interference-plus-noise-ratio maximization (SINRM) criterion.

Chapter 6 introduces a new downlink cooperative zero forcing transmission scheme employing precoding and beamforming. This scheme is applicable to both cellular and WLAN networks.

Chapter 7 presents a new downlink cooperative transmission scheme based on the uplink-downlink duality concept.

Chapter 8 presents a new linear downlink cooperative transmission scheme based on zero forcing without the use of THP and user ordering.

Chapter 9 summarizes the conclusions.
Chapter 2

Background

2.1 Multiple-Input-Multiple-Output Scheme

In this thesis, we will investigate multiple-input-multiple-output (MIMO) channels. As shown in [51, 52], a MIMO system can provide higher data rates over wireless links at no extra expenditure of power and bandwidth. Compared to the single-input-single-output (SISO) system, MIMO systems provide both spatial diversity and multiplexing gain [53]. First, let us start with a simple MIMO configuration. We consider a single point-to-point MIMO system with arrays of $n_T$ transmit antennas and $n_R$ receive antennas. We focus on a complex baseband linear system model described in discrete time. The system block diagram is shown in Fig. 2.1. In the MIMO system above, the information bits are processed prior to transmission. The processed signal can be represented by an $n_T$ sized column vector $\mathbf{x} = [x_1 ... x_{n_T}]^T$ where $x_{n_t}$ represents the signal transmitted by antenna $n_t$. We assume that the signals transmitted from individual antenna elements have a unit power. The covariance matrix of the transmitted signal $\mathbf{x}$ is given as

$$ R_{xx} = \mathbf{I}_{n_T} \quad (2.1) $$

where $\mathbf{I}_{n_T}$ is a $n_T \times n_T$ identity matrix.

The signal received by the receive antennas 1, ..., $n_R$ is represented as a $n_R$ sized column
vector $\mathbf{y} = [y_1...y_{n_R}]^T$ and can be written mathematically as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2.2}$$

where $\mathbf{H} = \{h_{n_r,n_t}\}$, $n_r = 1, ..., n_R$, $n_t = 1, ..., n_T$ is an $n_R \times n_T$ complex Gaussian matrix. $h_{n_r,n_t}$ represent the channel coefficient between receive antenna $n_r$ and transmit antenna $n_t$ and is a complex Gaussian distributed random variable with zero mean and variance of $\frac{1}{2}$ per dimension; $\mathbf{n}$ denotes an $n_R$-sized additive noise vector, of which the element is a complex Gaussian distributed random variable with zero mean and variance of $\frac{\sigma^2}{2}$ per dimension.

By performing the Singular Value Decomposition (SVD) [54] to $\mathbf{H}$, we can further write (2.2) as

$$\mathbf{y} = \mathbf{U}\Sigma\mathbf{V}^H\mathbf{x} + \mathbf{n} \tag{2.3}$$

where $\mathbf{U} = [\mathbf{u}_1...\mathbf{u}_{n_R}]$, $\mathbf{V} = [\mathbf{v}_1...\mathbf{v}_{n_T}]$ and $\Sigma = \text{diag}(\sqrt{\lambda_1},...,\sqrt{\lambda_m})$, $m = \min(n_R, n_T)$ are $n_R \times n_R$ left eigenvectors, $n_T \times n_T$ right eigenvectors and the diagonal matrix consists of the singular values of $\mathbf{H}$, respectively. Note that here the reason that there are only $m$ singular values for $\mathbf{H}$ is because the rank of $n_R \times n_T$ matrix $\mathbf{H}$ is at most $m = \min(n_R, n_T)$. 
2.1 Multiple-Input-Multiple-Output Scheme

If we use $U$ and $V$ as the receive antenna weight matrix and the transmit weight antenna matrix, respectively, we can further rewrite (2.3) as

$$
y = U^H U \Sigma V^H V x + U^H n
= \Sigma x + \tilde{n}
$$

(2.4)

where $\tilde{n} = [\tilde{n}_1 ... \tilde{n}_m]$ is still an AWGN, since the eigenvectors $U^H$ do not enhance the noise power at the receiver input.

By substituting the entries $\sqrt{\lambda_j}$, $j = 1, ..., m$, we get for the receive signal components

$$
y_j = \sqrt{\lambda_j} x_j + \tilde{n}_j, \ j = 1, ..., m, \ m = \min(n_R, n_T)
= \tilde{n}_j, \ j = m + 1, ..., n_R
$$

(2.5)

From (2.5), we can also see that the use of $U$ and $V$ as the transmit-receive antenna weights transforms the MIMO channel in (2.2) into $m$ uncoupled parallel sub-channels. Each sub-channel is assigned to a singular value of matrix $H$. Since the sub-channels are uncoupled their capacities add up. The overall channel capacity, denoted by $C$, can be estimated by using the Shannon capacity formula

$$
C = W \sum_{i=1}^{l} \log_2(1 + SNR_i)
$$

(2.6)

where $W$ is the bandwidth for each sub-channel and $l$ is the number of available sub-channels. By using (2.6), the channel capacity of the MIMO channel can be written as

$$
C = W \sum_{i=1}^{m} \log_2(1 + \frac{\lambda_i}{\sigma^2})
$$

(2.7)

From (2.7), it can be seen that by using MIMO, we could create $m$ parallel channels, leading to much higher capacity as compared to a system with a single transmit-receive antenna.

If now we assume that the transmitter only transmits one symbol, $x_1$. By using (2.5), we could see that the MIMO system can also provide a spatial diversity for $x_1$. This is so since there are $m$ non-interfering spatial sub-channels available to transmit $x_1$. The gain for these channels depends on $\sqrt{\lambda_j}, j = 1, ..., m$. Thus, to transmit a single symbol $x_1$, we can simply
select the receive and transmit antenna weights that correspond to the largest $\lambda_j, j = 1, ..., m$.

In other words, the best sub-channel from $m$ possible spatial sub-channels is used to transmit $x_1$.

In this thesis, we will extend the point-to-point MIMO concept above to multi-point-to-multi-point MIMO systems. Thus, we have a multiple source transmitting to a multiple destination and consider two cases firstly where these multiple sources do not cooperate and secondly where they do cooperate when they are transmitting to their respective destinations. The system block diagram is shown in Fig. 2.2.

### 2.2 Interference Suppression by Using the ZF Method

As the name implies, a zero-forcing (ZF) method is a method to find a specific vector to separate the desired signal components from their interference, such that the projection of the interference to this vector is zero. We will use an example to illustrate this method in detail. This simple example is adapted from [1]. We consider a simple single-user transmitting $n_T$ symbol streams with $n_T$ transmit and $n_R$ receive antennas as in Section 2.1. We rewrite (2.2) as

$$ y = \begin{bmatrix} h_1 & \ldots & h_{n_T} \end{bmatrix} x + w $$

$$ y = \sum_{i=1}^{n_T} h_i x_i + w $$

(2.8)

where $h_{n_T}$ is a vector consisting of the wireless channel responses from transmit antenna $n_T$ to receive antennas $1, \ldots, n_R$. $x_i$ is the symbol transmitted by transmit antenna $i$. By focusing on the data stream $k$, we can write (2.8) as

$$ y = h_k x_k + \sum_{i=1, i \neq k}^{n_T} h_i x_i + w $$

(2.9)

From (2.9), we could see that stream $k$ faces interference from streams $n_t = 1, \ldots, n_T, n_t \neq k$. Thus, the first term on the right hand side of (2.9) is the desired transmitted signal for stream $k$, while the second term on the right hand side of (2.9) is the interference coming from other streams to stream $k$. 

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2.2 Interference Suppression by Using the ZF Method

Figure 2.2: Block Diagram of a point-to-point MIMO system
To apply the ZF method, we need to force the interference coming from other users to zero for each user $k$. Thus, for each user $k$, we need to find a set of orthonormal basis for a subspace that is perpendicular to vectors $h_1, ..., h_{k-1}, h_{k+1}, ..., h_n$. We denote this set of orthonormal basis as $Q_k$. $Q_k$ exists only if $h_k$ is not a linear combination of $h_1, ..., h_{k-1}, h_{k+1}, ..., h_n$. That means we need to have $n_T$ orthonormal basis for $H$. In other words, we need to ensure that the rank of $H$ is at least $n_T$ to support $n_T$ streams. Since for the $n_R \times n_T$ matrix $H$, the rank is at most $\min(n_R, n_T)$, we need the condition $n_R \geq n_T$ to be satisfied.

The projection operation is shown in Fig. 2.3, where $\text{proj}_{Q_k} y$ denotes the projection of vector $y$ onto row vectors in $Q_k$.

By using the projection matrix $Q_k$, we can write

$$\tilde{y} = Q_k y = Q_k h_k x_k + \tilde{w}, \tag{2.10}$$

where $\tilde{w} = Q_k w$ is the receiver noise, still white, after projection. Now, we perform match...
filtering to make sure we have the maximum SNR possible. The process can be described as

\[ \tilde{y} = (Q_k h_k)^H y = (Q_k h_k)^H Q_k h_k x_k + (Q_k h_k)^H \tilde{w} \]

\[ = r_k^H h_k x_k + r_k w, \quad (2.11) \]

where \( r_k \) is a linear filter applied to the receive signal \( y \). Since the match filter process, described in (2.11) maximizes the output SNR, we can interpret this process as finding a linear filter that maximizes the output SNR subject to the constraint that the filter nulls the interference from all other streams. Intuitively, we are projecting the received signal in the direction that is orthogonal to \( h_1, \ldots, h_{k-1}, h_{k+1}, \ldots, h_n \) and that is closest to \( h_k \). This is shown when we rewrite the linear filter expression in (2.11) as

\[ r_k = ((Q_k h_k)^H Q_k)^H = Q_k^H Q_k h_k. \quad (2.12) \]

Note that here, there is a simple explicit formula for the filter \( r_k \). To show this, we first re-write (2.11) in a matrix format as

\[ y = RHx + Rw \quad (2.13) \]

where \( R \) is the linear filter for stream \( k = 1, \ldots, K \) and \( H = [h_1 \ldots h_n] \) as defined previously. Note that here, to zero force all the interference, we need \( RH \) to be a diagonal matrix. One simple solution is by using the pseudoinverse of \( H \) for \( R \), defined as

\[ R = (H^H H)^{-1} H^H \quad (2.14) \]

### 2.3 Interference Suppression by Maximizing SINR

The ZF method maximizes the output SNR subject to the constraint that the filter nulls the interference from all other streams. The method does not take into account the receiver noise. At a high SNR, the interference from other streams is dominant over the additive Gaussian
2.3 Interference Suppression by Maximizing SINR

receiver noise and the ZF method will perform well. On the other hand, at low SNR, the additive Gaussian receiver noise is dominant over the interference from other streams, in this situation, the ZF method will not perform well, since it does not take into account the receiver noise, when designing the ZF linear filter. The goal of the SINR maximization (SINRM) filter is to maximize SINR, rather than removing interference only.

To illustrate how a SINRM filter can be derived, we consider the same example as in Section 2.2 which is adopted from [1]. Here, we have a simple single-user transmitting $n_T$ symbol streams with $n_T$ transmit and $n_R$ receive antennas. We first rewrite (2.8) as

$$y = h_k x_k + \sum_{i=1,i \neq k}^{n_T} h_i x_i + w$$

$$= h_k x_k + z_k$$

(2.15)

where $z_k$ is the summation of the interference from other streams to stream $k$ and the additive Gaussian receiver noise. We know from Section 2.2 that if $z_k$ is a white noise, it is optimal to project $y$ onto the direction along $h_k$. As $z_k$ is a colored noise, a natural strategy would be to first whiten $z_k$ and then follow up by the match filtering process to maximize signal gain.

The covariance of $z_k$ is given as

$$Z_k = E[z_k z_k^H]$$

$$= U \Lambda U^H$$

$$= U \Lambda_k U^H \underbrace{U \Lambda_k U^H}_{(Z_k^H)^H}$$

(2.16)\hfill (2.17)\hfill (2.18)

where $U$ and $\Lambda$ are the unitary rotation matrix and the diagonal matrix with positive diagonal elements, respectively. Now, we whiten $z_k$ by multiplying $y$ with $Z_k^{-1}$,

$$Z_k^{-1} y = Z_k^{-1} h_k x_k + Z_k^{-1} z_k$$

(2.19)

Note that now $\tilde{z}_k$ is a white noise. The process is shown in Fig. 2.4. To obtain maximum
2.3 Interference Suppression by Maximizing SINR

Figure 2.4: Geometric illustration of applying a SINRM/MMSE method to a single user MIMO system

Interference plus noise Space (whiten)
2.3 Interference Suppression by Maximizing SINR

channel gain, we now project the output in the direction of $Z_k^H h_k$ and obtain

$$
(Z_k^H h_k)^H Z_k^H y = \underbrace{h_k^H Z_k^{-1} h_k}_{r_k^H} x_k + \underbrace{h_k^H Z_k^{-1} z_k}_{r_k^H}.
$$

(2.20)

where $r_k$ is the linear SINRM filter required to decode stream $k$ and to maximize the SINR. The SINR of (2.20) above is given as

$$
SINR = \sigma_{x_k} h_k^H Z_k h_k
$$

(2.21)

where $\sigma_{x_k} = E[x_k x_k^H]$ is the covariance of $x_k$ and $Z_k$ is as defined in (2.18).

It can be shown that the SINRM process above is equal to minimizing the mean square error in estimating $x_k$. To prove this, we first define the estimated transmitted symbol for stream $k$ as

$$
\hat{x}_k = \frac{r_k^H y}{r_k^H h_k}
$$

(2.22)

$$
= x + \frac{r_k^H z_k}{r_k^H h_k}
$$

(2.23)

$$
= x + \frac{h_k^H Z_k^{-1} z_k}{h_k^H Z_k^{-1} h_k}
$$

(2.24)

where $\hat{x}_k$ is the estimated symbol for stream $k$. By using (2.21) and (2.24), the mean square error in estimating $x_k$ can be written as

$$
E[||\hat{x}_k - x_k||^2] = E[\|\frac{h_k^H R_k^{-1} z_k}{h_k^H Z_k^{-1} h_k}\|^2]
$$

(2.25)

$$
= \frac{h_k^H Z_k^{-1} h_k}{h_k^H Z_k^{-1} h_k}
$$

(2.26)

$$
= \frac{\sigma_{x_k}}{SINR}
$$

(2.27)

where $Z_k = E[z_k z_k^H]$ is the covariance matrix of the colored noise $z_k$ consisting of interference from other streams and AWGN receiver noise. It can be seen from (2.27) that there is an inverse relationship between the mean square error and the SINR. Due to this relationship, the SINRM filter in (2.20) that maximizes SINR also minimizes the mean square error in estimating $x_k$. Hence, it is also called the minimum-mean-square-error (MMSE) filter.
2.4 Orthogonal-Frequency-Division-Multiplexing

It is well known that the original OFDM principle was proposed in 1966 in [55]. In OFDM systems, subcarriers overlap with neighbourhood subcarriers, and orthogonality can still be preserved through the staggered QAM (SQAM) technique. As more subcarriers are required, the modulation, synchronization, and coherent demodulation become more complex resulting in additional hardware cost. In 1971, the authors in [56] proposed a modified OFDM system in which the discrete Fourier transform (DFT) was applied to generate the orthogonal subcarriers’ waveforms [56]. Their scheme reduces the implementation complexity significantly, by making use of inverse DFT (IDFT) and the digital-to-analog-converters. In their proposed model, baseband signals are modulated by the IDFT at the transmitter and then demodulated by DFT at the receiver. Therefore, all the subcarriers are overlapped with each other in the frequency domain, while the DFT modulation still assures their orthogonality.

To illustrate an OFDM principle, we consider a simple example. An OFDM symbol consists of a sum of subcarriers that are modulated by using a quadrature amplitude modulation (QAM). The available bandwidth is divided into $N$ sub-channels. We denote the symbol to be transmitted in each frequency sub-channel $k$ as $X(k)$. By denoting $\tilde{x}(n)$ as the transmitted data in $n$ time slot, we could write

$$\tilde{x}(n) = IDFT\{X(k)\} = \sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi nk}{N}}, 0 \leq k \leq N - 1$$ (2.28)

We assume that guard interval $G$ normally implemented in OFDM systems is larger than the maximum expected delay spread, such that multipath components from the previous symbol cannot interfere with the next symbol. The guard interval is chosen as the replica of the data at the end of the OFDM symbol. This ensures that all the OFDM symbols with delayed replicas are always within the DFT interval as long as the delay spread is smaller than the guard time. As a result, multipath signals with a delay smaller than the guard time cannot cause inter-symbol-interference (ISI). The transmitted symbol in the time domain after appending the guard period $\bar{x}$ is given as

$$\bar{x}(n) = \tilde{x}(n - G + N), 0 \leq n \leq G - 1$$
$$\tilde{x}(n - G), G \leq n \leq N + G - 1.$$ (2.29)
2.5 Dirty-paper Coding

Here, the sampling period $T_s$ is defined as $\frac{T}{N+G}$ where $T$ is the time duration of one OFDM symbol after inserting the guard interval. If we assume the symbol is transmitted through a multipath fading channel consisting of $L$ discrete paths where $h(n, l)$ represent the $n^{th}$ sample of the $l^{th}$ channel path, the received signal $y$ can be written in the time domain as

$$y(n) = \sum_{l=0}^{L-1} h(n, l)\bar{x}(n - l) + \omega(n)$$

$$= h(n, 0)\bar{x}(n) + ... + h(n, L-1)\bar{x}(n - L + 1) + \omega(n) \quad (2.30)$$

where $\omega(n)$ represents the additive white Gaussian noise (AWGN) at time $n$. If the channel is constant during the OFDM symbol period, the DFT of the received signal, $y(\hat{n})$, after the removal of the cyclic prefix is given as

$$Y(k) = DFT\{y(n)\}$$

$$= \sum_{n=0}^{N-1} \left( \sum_{l=0}^{L-1} h(l)e^{j\frac{2\pi nk}{N}} + \sum_{n=0}^{N-1} \omega(n)e^{j\frac{2\pi nk}{N}} \right)$$

$$= H(k)X(k) + W(k) \quad (2.31)$$

where $H(k)$ and $W(k)$ denote the DFT of $h(n, l)$ and $\omega(n)$. As can be seen from the equation, the convolution between the transmitted symbol and the channel leads to a simple multiplication relationship when processing the received OFDM symbol due to the cyclic prefix. The effect of the delay spread appears as a multiplication in the frequency domain according to the convolution theorem. This feature is very attractive for high delay spread applications as it removes the need to perform complex time-domain equalization.

2.5 Dirty-paper Coding

Dirty-paper coding (DPC) was first discovered by H.M. Costa and published in his 1983 paper [27]. He equates a transmission problem in a Gaussian channel where the interference is known at the transmitter with the problem of writing on dirty paper. This problem can be modeled as follows

$$y = s + i + w \quad (2.32)$$
2.5 Dirty-paper Coding

Figure 2.5: Extended Constellation for $s \in (-1, 1)$.

where $s \in \{\pm 1, \pm 3, ..., \pm (M - 1)\}$ is the transmitted symbol and $M$ is an even integer. $i$ is the known interference with power $Q$. $y$ is the received signal at the receiver and $w$ is Gaussian noise. Note that the power of the transmitted symbol $s$ is limited to $P$. Costa showed in [27] that the capacity of the system described in (2.32) is the same as the capacity of the system without any interference (e.g., $i = 0$).

For simplicity, we will illustrate the principle in [27] by applying the random coding argument used in [27, 57, 58]. We first assume there are two possible codewords ($M = 2$) for transmission $s \in (-1, 1)$. In [27], each of the possible $s$ is defined as a bin. Now, to find the suitable $s$ to transmit, we replicate each codeword in a bin $K$ times and place the extended constellation of $2K$ points on the real line. Each codeword then corresponds to an equivalent bin of points on this real line. This is shown in Fig. 2.5. In Fig. 2.5, there are 2 bins since there are 2 possible codewords. The numbers on top of the real line indicate the signal amplitude for each point in the extended constellation. The number below the real line indicates the symbol transmitted. Thus for example, when $s = 1$ and it is repeated 2 times, the amplitude for $s$ can be either 1 or $-3$. Here, 1 and $-3$ represent the same codeword $s = 1$.

Now define $q_s(a)$ as the operation to quantize $a$ by finding its equivalent in a bin corresponding to $s$. The transmitted symbol is then modified as

$$y = \left( q_s(i) - i \right) + i + w$$

$$= q_s(i) + w$$

where $x$ is the new precoded transmitted signal. To implement Costa’s method we transmit $x$ instead of $s$. $x$ can be interpreted as a quantization error: the difference between interference and the quantized value [1]. Based on the received signal $y$, the decoder then finds the point in the extended constellation in Fig. 2.5 that is closest to $y$ and decodes the information bits corresponding to their equivalent bin.
Albeit Costa’s idea is a very novel one, there are three problems if we want to directly implement it in a real system. The first problem is related to the number of times we have to replicate the codewords. As explained in [27], we need to have $K = \infty$ to ensure that there is a suitable sequence that satisfies power requirement $P$. The second problem is data storage. Here, at both transmitter and receiver we need to store all $2^K$ points. The third problem is we need to find the quantized value $q_s(i)$ by exhaustive search along the real line.

The difficulty in implementing DPC has motivated progress in the development of a practical DPC algorithm. In [28, 29] a simple modulo operator is used to cancel intersymbol interference. It turns out that we can apply this idea to simplify DPC implementation greatly. First, we define the modulo operator $f$ as

$$f_\tau(y) = y - \left[\frac{y + \frac{\tau}{2}}{\tau}\right]\tau$$  \hspace{1cm} (2.35)

where $\tau = 2M$ and $[x]$ is the greatest integer smaller than $x$. The modulo operation is illustrated in Fig. 2.6. Here, we could see clearly that (2.35) forces $f_\tau(y)$ to lie between $-\frac{\tau}{2}$ and $\frac{\tau}{2}$.

We then precode $s$ by using information about the interference $i$ as follows

$$x = f_\tau(s - i) = s - i - \tau k$$  \hspace{1cm} (2.36)
2.5 Dirty-paper Coding

where \( k = \left\lfloor \frac{s - i + \frac{\tau}{2}}{\tau} \right\rfloor \). By using the modulo operation (2.35) at the receiver, we get

\[
f_\tau(y) = f_\tau(x + i + w) = f_\tau(s - i - \tau k + i + w) = f_\tau(s + w)
\]

(2.37)

Note that here, by using the modulo operator \( i \) is fully cancelled at the receiver as in Costa’s original paper [27].

However, there is a precoding loss when the modulo operator is used to implement DPC. It is proved in [59] that \( x = f_\tau(s - i) \) is uniformly distributed between \( -\frac{\tau}{2} \) and \( \frac{\tau}{2} \) when \( w \) is AWGN noise. Thus, the variance of the channel symbol \( x \) is then given as

\[
E[|x|^2] = \frac{M^2}{3}.
\]

(2.38)

The variance of the channel symbol \( s \) is

\[
E[|s|^2] = \frac{M^2 - 1}{3}.
\]

(2.39)

The precoding loss is then given as

\[
\frac{E[|x|^2]}{E[|s|^2]} = \frac{M^2}{M^2 - 1}.
\]

(2.40)

Thus, there is a trade off between complexity and power. The use of the modulo operation results in a higher power requirement. Another important point from (2.40) is that as \( M \) gets larger the precoding loss disappears. Note that this review is adopted from [1, 60].
Chapter 3

System Model

In this chapter, we describe the wireless channel model and the system model used for multi-user MIMO-OFDM transmission systems. The wireless channel model for noncooperative and cooperative MIMO-OFDM transmission is first explained, followed by a full description of the noncooperative MIMO and cooperative MIMO-OFDM transmission systems model.

3.1 Wireless Channel Model

We adopt the wireless channel model described in [13]. The channel response between antenna $n_t$ and antenna $n_r$ at the receiver of MS $k$ can be written as [13]

$$h_{n_r,n_t}^k(t) = \sum_{l=0}^{L-1} \alpha_{n_r,n_t}^{l,k} \delta(t - l\frac{\Delta_{rms}}{L-1})$$

(3.1)

where $u$ is a scalar and $\Delta_{rms}$ is defined as the ratio of the rms delay spread $\tau_{rms}$ to the OFDM symbol period $\tau_T$. The channel amplitude for each path $l$ and each MS $k$ is modelled as a zero mean complex Gaussian random variable $\alpha_{n_r,n_t}^{l,k}$ and a variance of one. The power exponential delay profile for the above channel is given by

$$\sigma_l^2 = \sigma_0^2 e^{-\frac{l}{\Delta_{rms}}} \text{ subject to } \sum_l \sigma_l^2 = 1.$$ 

(3.2)
3.2 Multi User Noncooperative Transmission Model

For simplification, the MIMO-OFDM systems in this paper are analyzed in the frequency domain, thus bypassing the need of simulating Inverse Discrete Fourier Transform (IDFT) modulators and DFT demodulators required in a real MIMO-OFDM system. The wireless channel described in (3.1), however, is in the time domain. The channel is thus transformed into the frequency domain. The frequency response coefficient matrix for each sub-carrier of MS $k$, $H_k \in \mathbb{C}^{N_{MS} \times K_{BS}}$, can then be obtained by applying the Discrete Fourier Transform operation to $h_{n_r, n_t}^k(t)$ given as

$$H_k(f_c) = \begin{pmatrix} FT(h_{1,1}^k)(f_c) & \cdots & FT(h_{1,N_T}^k)(f_c) \\ \vdots & \ddots & \vdots \\ FT(h_{N_R,1}^k)(f_c) & \cdots & FT(h_{N_R,K_{NS}}^k)(f_c) \end{pmatrix}$$

(3.3)

where $FT(h_{n_r, n_t}^k)(f_c)$ indicates the discrete frequency channel response at $c^{th}$ sub-carrier after the DFT operation of $h_{n_r, n_t}^k(t)$.

$H_k$ can also be thought of as the equivalent channel of the combination of IDFT, the wireless channel in (3.1) and DFT in a real MIMO-OFDM system. We also assume that each OFDM sub-carrier experiences a flat fading and that there is no inter-carrier interference. For that reason, the sub-carrier index is also omitted for simplicity, since the analysis is essentially the same for all sub-carriers. In addition, the wireless channel is also assumed to be a quasi-static channel and the MSs are assumed to have very low mobility. This results in a negligible doppler shift.

### 3.2 Multi User Noncooperative Transmission Model

We first consider a noncooperative transmission model. In particular we consider a non-cooperative downlink transmission in co-working WLANs. We note that in most research work about WLANs such as [2, 8], the terms Access Point and Station are used instead of BS and MS. Thus, in this thesis, we will use these terms interchangeably. Here, each AP treats the transmission from other APs to its respective station (STA) as co-working interference. We consider a simple case where a system consists of a STA and two non-cooperative APs. Our aim here, is to maximize the SNIR and the data rate for AP 1. The frequency domain representation of the MIMO-OFDM system is shown in Fig. 3.1.
3.2 Multi User Noncooperative Transmission Model

Figure 3.1: Noncooperative Transmission System Model

The SINR module at STA extracts the desired channel state information (CSI) denoted by $H_D$ and the interfering CSI, $H_I$. The SNIR module performs the computation of the optimum transmit weights, $W_T$ and the receive weights $W_R$ for each sub-carrier with the objective of maximizing the SNIR. The SNIR module calculates the optimum SNIR for each sub-carrier and passes this information to the AM module at the STA. The AM module performs the power allocation, and determines which OFDM sub-carriers and modulation mode are to be used. The uplink ACK is then eigen-steered (transmitted) using STA's optimum receive weights $W_R$. Allocation of power, sub-carriers, and modulation rate are then extracted from the uplink ACK at AP 1. The desired symbol $I_D$ is then transmitted by the AP 1 and received by the corresponding STA using these parameters. Here, $I_I$ denotes the symbol transmitted by AP 2 to other STA (not shown). This transmission is treated as an interference by AP 1 and its respective STA.

Assuming each STA uses $N_R$ receive antennas and each BS uses $N_T$ transmit antennas, the notations for each sub-carrier are as follows: $I_D \in \mathbb{C}^{x}$, $W_R \in \mathbb{C}^{N_R \times x}$, $W_T \in \mathbb{C}^{N_T \times x}$, $H_D \in \mathbb{C}^{N_R \times N_T}$, $H_I, N \in \mathbb{C}^{N_R \times 1}$, $N$ as the additive white Gaussian noise (AWGN) at the receiver and $I_I \in \mathbb{C}^{1}$ as the interfering symbol. $\mathbb{C}^{a,b}$ indicates the complex matrices with $a$ rows and $b$ columns. Note also that the superscript $x$ indicates the number of spatial channels used in each sub-carrier.
3.3 Multi User Cooperative Transmission Model

In this section, we consider MIMO systems, where \( K \) BSs transmit \( S \) symbol streams to \( K \) MSs using Orthogonal Frequency Division Multiplexing (OFDM) [1]. BSs and MSs are equipped with \( N_{BS_k} \) and \( N_{MS_k} \) antennas, for \( k = 1, ..., K \), respectively. All BSs cooperate with each other to transmit \( S \) symbol streams to their respective MSs via \( N_{BS} = \sum_{k=1}^{K} N_{BS_k} \) antennas. Each of these transmissions is defined as a link.

3.3.1 Transmitter Structure

The transmitter for the proposed cooperative transmission with precoding and beamforming is shown in Fig. 3.2(a). Let \( x_k = [x_{1,k} \cdots x_{s,k} \cdots x_{S,k}]^T \) represent the modulated multiple symbol stream vector, consisting of \( M \)-QAM (\( M \)-ary Quadrature Amplitude Modulation) modulated symbols, where \( x_{s,k} \) is the \( s^{th} \) modulated symbol intended for transmission from BS \( k \) to MS \( k \). Thus, we have a multi-stream transmission where \( S \) symbol streams are transmitted from BS \( k \) to MS \( k \), simultaneously. The modulated symbols for \( K \) MSs can then be written as \( x = [x_1^T \cdots x_k^T \cdots x_K^T]^T \). The transmitted symbols for each user are first permuted by a block diagonal permutation matrix \( M_{perm} = \text{Diag}(m_1, \ldots, m_K) \) where \( m_i = 1, \ldots, K \in 1^S \). \( 1^S \) is a vector, with all its \( S \) elements equal to 1. This permutation operation is referred to as the adaptive precoding order (APO). Note that the APO can be implemented by using the proposed VBLAST user ordering scheme (APO-VLAST) or the proposed low complexity scheme. The methods used to generate APO-VBLAST and APO-LC are discussed in Chapters 5, 6 and 7, respectively.

The APO adaptively selects the precoding order of \( x \) that maximizes the minimum SINR of \( K \) users. It selects a suitable permutation matrix \( M_{perm} \) to permute \( x \). Let

\[
\mathbf{u} = M_{perm} \mathbf{x} = [\mathbf{u}_1 \cdots \mathbf{u}_j \cdots \mathbf{u}_K]^T
\]

be the permuted transmitted symbol vector, where

\[
\mathbf{u}_j = [u_{1,j} \cdots u_{s,j} \cdots u_{S,j}]^T.
\]
Thus, after the APO, $x_k$ for MS $k$ is permuted into $u_j$, which will be transmitted in link $j$.

The symbol vector $u$ is then passed to the THP [28, 29], which performs a precoding operation to create THP precoded symbols arranged in a vector $v = [v_1^T \cdots v_j^T \cdots v_K^T]^T \in \mathbb{C}^{KS \times 1}$ where $v_j = [v_{1,j} \cdots v_{S,j}]^T$. The THP precoding order of link $j$ is assumed to be $j$. In addition, the THP precoding order of symbol $s$ in link $j$ is assumed to be $s$. That is, the first symbol of link 1 is precoded first and the $S^{th}$ of link $K$ is precoded last. In other words, we first precode $u_{1,1}$ to obtain $v_{1,1}$. We then precode $u_{2,1}$ by treating $v_{1,1}$ as known interference. We repeat the process until we precode $u_{S,K}$ by treating $v_{1,1}, \ldots, v_{S-1,K}$ as known interference. Thus, THP needs to perform the precoding $KS$ times in precoding $u$. THP treats the interference from links $1, \ldots, j-1$ and from symbol $1, \ldots, s-1$ in link $j$ to symbol $s$ in link $j$ as known.

To enable the cancellation of the known interference at the THP decoder, THP uses a feedback matrix $M_{THP}$ in the precoding operation. As discussed in [59], $M_{THP}$ is strictly lower triangular to allow data precoding in a recursive fashion. The derivation of the feedback matrix will be explained in Section 3.3.3.

The output of the THP precoder can be obtained as

$$[v]_j = \text{mod}_M([u]_j - \sum_{l=1}^{j-1}[M_{THP}]_{j,l}[v]_l), \ j = 1, \ldots, KS$$  \hspace{1cm} (3.6)$$

where $[M_{THP}]_{j,l}$ denotes the $(j,l)^{th}$ component of $M_{THP}$, $[a]_l$ denotes the $l^{th}$ component of vector $a$ and $\text{mod}_M(u) = [\text{mod}_M([u]_1) \ldots \text{mod}_M([u]_{KS})]^T$ is an element-wise modulo operator [20]

$$\text{mod}_M([u]_j) = [u]_j - \sqrt{M} \left(\left([u]_j + \frac{\sqrt{M}}{2}\right) / \sqrt{M}\right), \ j = 1, \ldots, KS.$$  \hspace{1cm} (3.7)$$

The SINR equalization module then allocates powers to each coded symbol in $v$ in such a way that the received SINRs for all $KS$ symbols are equal. This is done by multiplying $v$ with the matrix $P = \text{Diag}(P_1, \ldots, P_K)$, $P_j = \text{Diag}(\sqrt{p_{1,j}}, \ldots, \sqrt{p_{S,j}})$ where $p_{s,j}$ is the power allocated to the $s^{th}$ THP precoded symbol $v_{s,j}$ in link $j$.

The THP decoder, however, cannot cancel the interference to symbol $s$ in link $j$ coming from links $j + 1, \ldots, K$ and from symbols $s + 1, \ldots, S$ of link $j$ in link $j$, since this interference
3.3 Multi User Cooperative Transmission Model

is unknown to symbol \( s \) in link \( j \). This remaining interference needs to be suppressed by multiplying the transmitted signal from each link by the transmit antenna weight vectors of all BSs, denoted by \( T \), where \( T \in \mathbb{C}^{N_{BS} \times K_S} \) and by the receive antenna weights matrix, denoted by \( R_j \), where \( R_j \in \mathbb{C}^{N_{MSj} \times S} \), at the receiver of link \( j \). The transmitted signal is given as

\[
x_T = TPv. \tag{3.8}
\]

3.3.2 Receiver Structure

The receiver for each link is shown in Fig. 3.2(b). Note that there is no cooperation among the receivers. We first denote the received signal matrix for each link as \( y_j \), where \( y_j \in \mathbb{C}^{N_{MSj} \times 1} \). The received signal matrix for \( K \) links, denoted by \( y, y = [y_1...y_K]^T \), can be written as

\[
y = HTPv + N \tag{3.9}
\]

where \( H = [H_1^T...H_j^T...H_K^T]^T \), \( N = [n_1^T,...,n_K^T]^T \) and \( T = [T_1...T_K] \). \( n_j \in \mathbb{C}^{N_{R_S} \times 1} \) is the noise vector for link \( j \), \( T_j = [t_{1,j}...t_{S,j}] \in \mathbb{C}^{N_{BS} \times S} \) where \( t_{s,j} \) is a transmit weight vector for
symbol \( s \) transmitted in link \( j \) and \( H_j \) is the channels matrix for link \( j \), respectively. After multiplying \( y \) by the receive weights matrix \( R \), the received signal vector becomes

\[
y = RHTPv + RN
\]

where \( R = Diag(R_1^H, ..., R_K^H) \). \( y_j = [y_{j1} ... y_{jS}]^T \) where \( y_{s,j} \) are the receive signal at the input of the THP decoder for symbol \( s \) transmitted in link \( j \). \( R_j = [r_{1,j}...r_{S,j}]^T \) where \( r_{s,j} \) is the receive antenna weight vector for symbol \( s \) transmitted in \( j \). The estimates of the transmitted symbols for link \( j \), denoted by \( \hat{u}_j = [\hat{u}_{1,j}...\hat{u}_{S,j}]^T \) can be recovered from \( y_j \), by applying an element-wise modulo operator in (3.7) to each \( y_{s,j} \), as

\[
\hat{u}_{s,j} = mod_M(y_{s,j}), s = 1, ..., S, j = 1, ..., K.
\]

The received signal \( y \) can be further written as

\[
y = RHTPv + RN = (D + F + B)Pv + RN
\]

where \( D = DiT(RHT) \), \( B = UpT(RHT) \) and \( F = LoT(RHT) \). \( LoT(A) \) is defined as the operation to extract the lower triangular components of \( A \) and to set the other components to zero. \( UpT(A) \) is defined as the operation to extract the upper triangular components of \( A \) and to set the other components to zero. \( DiT(A) \) is defined as the operation to extract the diagonal components of \( A \) and to set the other components to zero. \( DPv \) is a vector of scaled replicas of the transmitted symbols for \( K \) links. \( FP \) is defined as the front-channel interference matrix, since the rows \( j = 1, ...K \) of \( FP \) represent the inter-link interference caused by links \( 1, ..., j - 1 \) and inter-stream interference caused by symbols \( 1, ..., s - 1 \) in link \( j \) to symbol \( s \) in link \( j \). The inter-link interference is the interference between links while the inter-stream interference is the interference between multiple streams in the same link. \( BP \) is defined as the rear-channel interference matrix, since the rows \( j = 1, ..., K \) of \( BP \) represent the inter-link interference caused by \( rear \) links \( j + 1, ..., K \) and the inter-stream interference caused by symbols \( s + 1, ..., S \) in link \( j \) to symbol \( s \) in link \( j \). In the proposed scheme, THP cancels the interference caused by the front-channel interference, while the interference caused by the rear-channel interference is eliminated by the transmit-receive antenna weights optimization process.
3.3 Multi User Cooperative Transmission Model

Figure 3.3: Equivalent Block Diagram for THP process

3.3.3 Tomlinson Harashima Precoding Design at the Transmitter

We assume that the channel state information (CSI) for all users is available at the transmitter. The THP operation in Fig. 3.2(a) aims to cancel the front-channel interference by performing $KS$ successive precoding operations. The operation uses the feedback matrix $M_{THP}$ which is strictly lower triangular and the modulo operator $mod_M(\cdot)$. The THP operation to generate THP precoded symbols $v = [v_1^T \ldots v_K^T]^T$ can then be represented [59] as

$$[v]_j = [u]_j + d_j - \sum_{l=1}^{j-1} [M_{THP}]_{j,l} [v]_l, \ j = 1, \ldots, KS$$  \hspace{1cm} (3.13)

where $[M_{THP}]_{j,l}$ denotes the $(j, l)^{th}$ component of $M_{THP}$ and $[a]_j$ denotes the $j^{th}$ component of vector $a$. $d_j = 2\sqrt{M}\Delta$ and $\Delta$ is a complex number whose real and imaginary parts are suitable integers selected to ensure the real and imaginary parts of $v_j$ are constrained into $(-\sqrt{M}, \sqrt{M})$. Here, the integers for $\Delta$ can be found by an exhaustive search across all integers [59]. Note that if $d_j$ is selected as above, adding $d_j$ to $[u]_j$ is equivalent to performing a modulo operation to $d_j + [u]_j$ [42], [1], [59], [28].

$$[u]_j = mod_M(\tilde{v}_j) = mod_M(d_j + [u]_j), \ j = 1, \ldots, KS.$$  \hspace{1cm} (3.14)

This THP equivalent process is shown in Fig. 3.3. Here, $\tilde{v} = [\tilde{v}_1 \ldots \tilde{v}_j \ldots \tilde{v}_{KS}]^T$, $j = 1, \ldots, KS$ where $\tilde{v}_j$ represents the modified symbol constructed by adding $d_j$ to $u_j$. Using these notations, (3.13) can be written in a matrix format as

$$\tilde{v} = (I + M_{THP})v = Av$$  \hspace{1cm} (3.15)

where $A = I + M_{THP}$. At each receiver, the signal at the input of the receive antennas is the sum of the scaled replicas of the transmitted signals, the rear-channel interference $BPv$.
and the front-channel interference $\text{FP}v$. The rear-channel interference is suppressed by the optimized transmit-receive antenna weights. The received signals at the input of the THP decoder can be represented as $(D + F)PA^{-1}\tilde{v}$. Each desired signal at the output of the THP decoder is given by $DPu$. That is, it consists of the scaled replicas of the transmitted symbols. By equating the actually received signal at the input of the THP decoder with the desired received signal and ignoring AWGN, we have

$$(D + F)PA^{-1}\tilde{v} = DP\tilde{v}. \quad (3.16)$$

The matrix $A = (I + M_{THP})$ is used in the term on the left hand side of (3.16) to cancel the front-channel interference. To derive the feedback matrix $M_{THP}$, we replace $A$ in (3.16) with $I + M_{THP}$, from (3.15), to obtain

$$M_{THP} = (DP)^{-1}\text{FP}. \quad (3.17)$$

By using (3.16), the composite received signal for all $K$ receivers, at the input of the THP decoder, from (3.15) and (3.12), can be represented as

$$\bar{y} = DP(u + d) + BPv + R\!N. \quad (3.18)$$

where $d = [d_1...d_{KS}]^T$. By normalizing the desired component $u$ and applying the modulo operation from (3.14) to remove the effect of the vector $d$ at the receiver, and denoting the transmitted signal estimates of $K$ links by $\hat{u} = [\hat{u}_1...\hat{u}_K]^T$, we get

$$\hat{u} = u + (DP)^{-1}(BPv + R\!N). \quad (3.19)$$

It can be seen that the front-channel interference $\text{FP}v = \text{FP}A^{-1}\tilde{v}$ term no longer exists in (3.18). This is so, because we are using $M_{THP}$ and $A$ as in (3.15), to force the summation of the front-channel interference and the scaled replica of transmitted signals to be equal with the desired receive signal at the receiver end. Thus, the front-channel interference is cancelled at the receiver by the THP precoding/decoding operation.
Chapter 4

A Noncooperative Transmission Scheme for Co-working WLANs

4.1 Adaptive Antenna Array Processing

The system is analysed in the frequency domain for each sub-carrier. In this section, $c^{th}$ sub-carrier notation is omitted for simplicity. At the STA, the OFDM received downlink signal for $c^{th}$ sub-carrier can be represented as

$$Y_{STA} = WHDWTI_D + WHI_I + WN_{STA}$$ (4.1)

where $Y_{STA} \in C^{N_r \times 1}$ is the complex vector of the OFDM received signal at STA. The second term on the right hand side of (4.1) is the co-working interference term (CI). Superscript $H$ denotes transpose conjugate operator. The interference power correlation matrix is defined as

$$R_U = E[H_IH_I^H] + E[N_{STA}N_{STA}^H].$$ (4.2)

In addition, $\sigma_D = E[I_D^HI_D]$ and $\sigma_I = E[I_I^HI_I]$ are normalized to 1 for any modulation scheme used. Note that the chosen modulation scheme can adaptively change.
4.1 Adaptive Antenna Array Processing

To maximize SINR and suppress co-working interference from other APs, we exploit the multiple antennas configuration at the receiver (STA). By using (4.1) and (4.2), the downlink SINR can be written as [13]

\[
SINR_{\text{downlink}} = \frac{W_R^H H_D W_T (H_D W_T)^H W_R}{W_R^H R_U W_R} \tag{4.3}
\]

Now, to maximize (4.3), we need to minimize the denominator of (4.3) while the numerator is maintained unchanged. This formulation can be given in [11] as

\[
\min_{W_R} \quad W_R^H R_U W_R \\
\text{s.t.} \quad W_R^H H_D W_T = 1. \tag{4.4}
\]

(4.4) above is solved using the Lagrange method. The optimum receiver weights are given by

\[
W_R = \frac{R_U^{-1} H_D W_T}{(H_D W_T)^H R_U^{-1} H_D W_T}. \tag{4.5}
\]

and its derivation is shown in Appendix A. \(W_R\) in (4.5) requires knowledge of the transmitter weights \(W_T\). To obtain \(W_T\), we substitute (4.5) in (4.3). After simplification, the downlink SINR \(\gamma\) is given as

\[
\gamma = SINR_{\text{downlink}} = \left\{ \begin{array}{l}
W_T^H H_D R_U^{-1} H_D W_T \\
\eta_{\text{MAX}}.
\end{array} \right. \tag{4.6}
\]

The best spatial channel gain (i.e., the largest eigenvalue) \(\eta_{\text{MAX}}\), and its corresponded eigenvector \(W_T\) can then be found by applying eigenvalue decomposition [33] to \(H_D^H R_U^{-1} H_D\). There are two OFDM spatial channel selection methods described in the open literature. In [13, 14] the eigenvalue for each sub-carrier is selected by finding the best spatial channel within the sub-carrier. In [61], the eigenvalues are selected by finding \(N_c\) best spatial channels from all sub-carriers. That means we can use another eigenvalue in addition to the largest one within each subcarrier. This is provided the non largest eigenvalue used at that particular subcarrier is larger than the largest eigenvalue of other subcarriers.
4.2 Adaptive Modulation

AM uses $\gamma_c$ obtained in the previous section. We let $M = \{0, 2, 4, 16, 64\}$ be the selection of $M$-ary modulation that corresponds to non-transmission, BPSK, 4-QAM, 16-QAM, or 64-QAM. The minimum threshold of SNIR for each $M$ with a given target of BER ($B$) is given in [62] as

$$\gamma_M = \begin{cases} \frac{2(M-1)}{3} \left[ \text{erfc}^{-1} \left\{ \frac{\sqrt{M} \log \sqrt{M}}{\sqrt{M} - 1} \right\} \right]^2 & M \neq 0 \\ 0 & M = 0. \end{cases}$$

The AM independently maximizes the total transmission rate for each modulation $M$. AM selects the modulation mode, sub-carriers, and power that give the maximum data rate. AM is formulated as

$$\max_{P_{c,M}, M} C_M$$

where $C_M$ is the maximum data rate for each $M$. $P_{c,M}$ is power at the $c^{th}$ sub-carrier for each $M$. The allocation of sub-carriers and power in each $C_M$ is optimized and given by

$$C_M = \max \sum_c \log(1 + \gamma_c P_{c,M})$$

s.t.

$$\sum_c P_{c,M} \leq P_T$$
$$P_{c,M} \geq \frac{\gamma_M}{\gamma_c}$$
$$P_{c,M} = 0 \text{ for } P_{c,M} < \frac{\gamma_M}{\gamma_c}. \quad (4.9)$$

$P_T$ represents the total power available for all $N_c$ sub-carriers. The second constraint and the third constraint in (4.9) imply that $P_{c,M}$ is allocated only if it is able to reach $\gamma_M$ that satisfies the BER target. In other words, $P_{c,M}$ will be increased by $\gamma_M$ if $\gamma_c < \gamma_M$, otherwise $P_{c,M}$ will be reduced by $\gamma_M$. $P_{c,M}$ is found using the Lagrange method and given by

$$P_{c,M} = \begin{cases} \frac{1}{\lambda_M} - \frac{1}{\gamma_c} & \frac{1}{\lambda_M} \geq \frac{\gamma_M + 1}{\gamma_c} \\ 0 & \frac{1}{\lambda_M} < \frac{\gamma_M + 1}{\gamma_c} \end{cases}$$

$$P_{c,M}$$

$P_T$ represents the total power available for all $N_c$ sub-carriers. The second constraint and the third constraint in (4.9) imply that $P_{c,M}$ is allocated only if it is able to reach $\gamma_M$ that satisfies the BER target. In other words, $P_{c,M}$ will be increased by $\gamma_M$ if $\gamma_c < \gamma_M$, otherwise $P_{c,M}$ will be reduced by $\gamma_M$. $P_{c,M}$ is found using the Lagrange method and given by

$$P_{c,M} = \begin{cases} \frac{1}{\lambda_M} - \frac{1}{\gamma_c} & \frac{1}{\lambda_M} \geq \frac{\gamma_M + 1}{\gamma_c} \\ 0 & \frac{1}{\lambda_M} < \frac{\gamma_M + 1}{\gamma_c} \end{cases}$$
4.3 ACKnowledgment Eigen-steering

where $\lambda_M$ is the Lagrange multiplier for each $M$-ary modulation. The sub-carriers and power allocation are obtained by plugging (4.10) into the first constraint in (4.9).

This solution can be simplified by replacing $P_{c,M}$ in (4.8) with $PR_{c,M} = \frac{\gamma_M}{\gamma_c}$. We let $\rho_{c,M} \in \{0, 1\}$ be the discrete allocation of $c^{th}$ sub-carrier with $M$-ary modulation. The linear formulation for (4.9) is given by

$$C_M = \max M \sum \rho_{c,M}$$

$$\sum \rho_{c,M} PR_{c,M} \leq P_T \quad \rho_{c,M} \in \{0, 1\}. \quad (4.11)$$

To solve (4.11), first all $PR_{c,M}$ are arranged in ascending order, such as $PR_{i,M} \leq PR_{j,M} \ldots \leq PR_{k,M}$. Starting from the smallest one, they are added one by one until the total power $P_T$ is completely allocated. The power and sub-carriers required are then obtained. The process is repeated for each $M$, and lastly (4.8) is used to select the best modulation mode, power, and sub-carriers. The modulation mode is then sent as a rate request in ACK frame to AP.

4.3 ACKnowledgment Eigen-steering

AP needs $W_T^c$ and $P_{c,M}$ for its downlink transmission. In this section we show how AP can recover this information by exploiting the uplink ACK signal. We first assume the CSI $H_D^T$ is also known at the AP. In a practical system $H_D^T$ can be estimated at the AP by using training symbols transmitted at the beginning of each frame. We then let $W_{\text{opt}}^c = \sqrt{P_{c,M}} W_T^c$ be the optimum transmit vector selected at the STA for AP for the $c^{th}$ sub-carrier and $W_R^c$ in (4.5) be the transmit vector used by the STA to eigen-steer the uplink ACK. At AP, the ZF linear filter is used to detect the uplink ACK. We let $W_T^r$ be the received weights at AP. Omitting the notation $c^{th}$ sub-carrier for simplicity, the received uplink expression for each sub-carrier at AP prior to multiplying with the receive weights $W_T^r$ is then given by

$$Y_{AP, prior} = H_D^T W_R I_D + N_{AP}. \quad (4.12)$$
4.4 Simulation Results

We then apply a linear ZF filter by using the concept described in Section 2.2,\[ H_D^T \dagger Y_{AP,prior} = H_D^T H_D^T W_R I_D + H_D^T N_{AP}. \] (4.13)

where \( H_D^T \dagger \) is the pseudoinverse of matrix \( H_D^T \). By using the training symbols we can then obtain the receive weights,\[ W_R = E[H_D^T Y_{AP,prior} I_p] \] (4.14)

where \( I_p \) is the training symbol.

From (4.13) and (4.14), we can see the quality of the estimation for the receive weights \( W_R \) depends on the magnitude of co-working interference. Thus, to implement ACK Eigen-steering, we need to have a clean uplink channel \( H_D^T \dagger N_{AP} \approx 0 \).

4.4 Simulation Results

The effectiveness of AMA and AM for the co-working WLANs is investigated by computer simulation. The symbol period, guard period and number of sub-carriers are set to 3.2\( \mu s \), 0.8\( \mu s \), and 52 respectively. The roll-off factor of the raised cosine pulse is set to 0.22. The number of paths, RMS delay spread and maximum channel delay are set to 10, 0.16\( \mu s \), and 0.8\( \mu s \) respectively. The signal to noise ratio (SNR) of the system is fixed at 15 dB. The performance of BER and SE (i.e., average transmitted bits per sub-carrier) against the signal to interference ratio (SIR) is simulated to analyze spatial channel allocation, diversity methods, and the proposed joint AMA/AM.

First, the methods of selecting the best eigenvalue from each sub-carrier (C1) and from all sub-carriers available (C2) are analyzed in order to see the effectiveness of the spatial channel method allocation in co-working WLANs. Second, different AMA configurations in co-working WLANs are simulated to see the effectiveness of having multiple antennas at the transmitter and receiver. Finally, joint AMA and AM configurations in co-working WLANs with a BER target of \( 10^{-3} \) are simulated in order to evaluate the effectiveness of our proposed method. To refer to the different antenna configurations, we use the notation K+L (i.e., K transmit antennas and L receive antennas). The conventional standard IEEE 802.11g OFDM transmitter (1+1 OFDM) is used as a benchmark for all our simulations. Here we refer to
4.4 Simulation Results

Figure 4.1: Allocation Methods Comparison

(1+1 OFDM) as an OFDM system with one transmit and one receive antenna.

4.4.1 Spatial Channel Allocation Methods Comparison

The result in Fig. 4.1 (using 4-QAM) shows that the BER performance is practically indifferent to choice of C1 and C2 in co-working WLANs. In Fig. 4.1, we denote (K+L OFDM-C1) and (K+L OFDM-C2) when C1 and C2 are used respectively. This is in contrast to [61] where C2 is proven to be better than C1. This discrepancy is due a smaller number of sub-carriers available in the WLANs and the presence of CI. The presence of CI forces the receiver to suppress the CI instead of creating spatial channels. As a result, the number of spatial channels that can be created per sub-carrier is reduced. Therefore, we conclude that adding complexity into the systems using a method like C2 is not justifiable for co-working OFDM WLANs.
4.4 Simulation Results

4.4.2 A Diversity Gain Comparison

The simulation results with 4-QAM shown in Fig. 4.2 confirm that any AMA configurations outperform 1+1 OFDM in terms of BER performance. The simulation results also confirm that suppressing interference through receive beamforming is much more effective than increasing channel gain through transmit beamforming for getting a maximum SINR. This is clearly seen in Fig. 4.2 where the BER performance of 3+1 OFDM-C1 is much worse than 1+2 OFDM-C1 and 1+3 OFDM-C1 under strong CI (SIR $\leq 0$dB). Fig. 4.2 also shows that combining both transmit and receive beamforming (i.e., 3+2) produces the best BER performance. This configuration, however, requires modification on both the receiver and transmitter sides. Receive beamforming to suppress co-working interference in co-working WLANs might be the better solution, as it requires modification on only one side.
4.4 Simulation Results

4.4.3 Smart Antennas and Adaptive Modulation

The BER target is fixed at $10^{-3}$ to compare SE. The results in Fig. 4.3 clearly show that AM (without AMA) improves SE of fixed modulation transmission (i.e., 1+1 OFDM-16QAM, 1+1 OFDM-4QAM, 1+1 OFDM-BPSK). The results also clearly show that transmit beamforming is practically useless under strong CI (SIR $\leq 0$ dB) since SE $\to 0$. This is in line with findings in the previous section.

The results in Fig. 4.4 show that regardless of whether transmit beamforming is used or not, introducing receive beamforming and combining it with AM, increases SE by $\geq 200\%$. A minimum SE of 2 (i.e., $\geq 4$-QAM) is now achieved under strong CI (SIR $\leq 0$ dB). AMA successfully suppresses CI. AM then uses the improved SNIR and adjusts the transmission mode accordingly. Fig. 4.4 also shows that the benefit of transmit beamforming can only be realised if receive beamforming is also implemented. This is evidenced where 1+2 OFDM-AM has SE $\approx 2.65$ and 2+2 OFDM-AM has SE $\approx 3.75$. We conclude that implementing receive beamforming with AM is the simplest solution to improve performance in the co-working WLANs.
Figure 4.3: Effect of Adaptive Modulation and Transmit Beamforming to Spectral Efficiency
Figure 4.4: Effect of Adaptive Modulation and Transmit-Receive Beamforming to Spectral Efficiency
Chapter 5

A Cooperative Transmission Scheme

In this chapter, we describe how to implement the SINRM method to design the transmit-receive antenna weights required for our cooperative transmission scheme. In this chapter, we consider the simplest design case, where each user receives a single symbol stream. Thus, $S = 1$ and we can omit the stream notation for the cooperative system model in this chapter. As an example, by using the cooperative system model described in Chapter 3, the transmit-receive weight vectors for each link $j$ and THP precoded symbols are written as $t_j$, $r_j$ and $v_j$ instead of $t_{1,j}$, $r_{1,j}$ and $v_{s,j}$, respectively. In addition we also assume there is equal power allocation between links. Thus, each link has an equal power, $p_j = 1$.

5.1 Signal-To-Interference-plus-Noise Ratio Maximization

Beamforming

In this section, we propose a SINRM beamforming scheme aiming to suppress the remaining interference in (3.19), B. The transmitted signal estimate for each link $j$ can be written as

$$
\hat{u}_j = r_j^H (H_j t_j u_j + n_j) + \sum_{i=j+1}^{K} r_j^H H_j t_i v_i.
$$

(5.1)
5.1 Signal-To-Interference-plus-Noise Ratio Maximization Beamforming

The second term in (5.1) represents the front-channel interference caused by link \( j + 1, \ldots, K \) at link \( j \). We will suppress this interference by using the SINRM beamforming method.

It can be noted from (5.1) that there is no front-channel interference at link \( K \). This means that the third term in (5.1) is zero for \( \hat{u}_K \). Thus, we propose to design the transmit-receive beamforming weights in order of link \( K, \ldots, 1 \). After obtaining the transmit-receive beamforming weights for link \( K \), the transmit-receive beamforming weights for link \( l = K, \ldots, 2 \) are determined by treating link \( K, \ldots, l + 1 \) as interference. Therefore, link \( K \) does not have any front-channel interference, and each link \( k \) will need to cancel front-channel interference coming from link \( K, \ldots, k + 1 \). This results in highest and lowest interference in link 1 and \( K \) respectively.

The average SINR for link \( j \) can be calculated as

\[
SINR_j = \frac{r_j^H \mathbf{H}_j \mathbf{t}_j (\mathbf{H}_j \mathbf{t}_j)^H r_j}{r_j^H \mathbf{R}_{N,j} r_j}
\]  

(5.2)

where \( \mathbf{R}_{N,j} \) is the interference correlation matrix defined as

\[
\mathbf{R}_{N,j} = E[\mathbf{n}_j \mathbf{n}_j^H] + \sum_{i=j+1}^{K} \mathbf{H}_j \mathbf{t}_i (\mathbf{H}_j \mathbf{t}_i)^H.
\]  

(5.3)

To maximize SINR for link \( j \), the denominator of (5.2) needs to be minimized while maintaining the unity gain for the numerator,

\[
\min_{r_j} r_j^H \mathbf{R}_{N,j} r_j \\
subject \ to \ r_j^H \mathbf{H}_j \mathbf{t}_j = 1, \| \mathbf{t}_j \| = 1.
\]  

(5.4)

The optimum \( r_j \) can be derived using the standard Lagrange method and is given as

\[
r_j = \frac{\mathbf{R}_{N,j} \mathbf{H}_j \mathbf{t}_j}{(\mathbf{H}_j \mathbf{t}_j)^H \mathbf{R}_{N,j} \mathbf{H}_j \mathbf{t}_j}.
\]  

(5.5)

The SINR [13] can now be written as

\[
SINR_j = \mathbf{t}_j^H \mathbf{H}_j^H \mathbf{R}_{N,j}^{-1} \mathbf{H}_j \mathbf{t}_j
\]  

(5.6)
5.2 Adaptive Precoding Order

with \( \|t_j\| = 1 \). \( SINR_j \) is clearly upper bounded [63] by

\[
t_j^H H_j^H R_{N,j}^{-1} H_j t_j \leq \lambda_{SINR_j} \tag{5.7}
\]

where \( \lambda_{SINR_j} \) is the maximum eigenvalue of \( H_j^H R_{N,j}^{-1} H_j \). The upper bound is achieved by selecting \( t_j \) in the direction of the eigenvector associated with \( \lambda_{SINR_j} \).

5.2 Adaptive Precoding Order

Here in the case of SINRM, we have the highest and lowest SINR in link \( K \) and 1 respectively leading to a different BER performance. In order to maintain the BER fairness across \( K \) links, in this section we proposed an APO scheme. There are two objectives to be achieved by APO; 1) to reduce the variation of the bit-error-rate (BER) performance across \( K \) links, 2) to improve the average BER of \( K \) links. The first objective can be achieved only when SINR for each link is equal. The average BER of the system depends on the SINR of the weakest link. Thus, to achieve our two objectives above, we need to maximize the SINRs of the weakest link by varying the user ordering.

APO arranges the order of \( K \) links by selecting an appropriate permutation matrix. We find a permutation matrix \( \hat{P} \in P \) that maximizes the minimum \( SINR_j \). This optimization process can be formulated as

\[
\hat{P} = \arg_{P} \max_{P} \min_{P} SINR_j(P) \tag{5.8}
\]

where \( SINR_j(P) \) is the SINR of link \( j \) given that the permutation matrix \( P \) is used. Here, we search all the possible \( K! \) permutation matrix to find \( \hat{P} \). This is feasible in co-working WLANs as the number of cooperative APs will not be many. We refer to the APO scheme that search all \( K! \) possible permutation matrix as APC.
5.3 System Design under Limited Channel State Information

By observing the interference correlation matrix in (5.3), we can see that the selection of the optimum transmit weight vector for link $j$ using (5.7) depends on the transmit weight vectors of link $j+1, \ldots, K$. Thus, to generate the optimum receive weight vector each receiver needs either, 1) the channel estimate for other users in addition to its own channel estimate or 2) to obtain the receive weight vector from the BSs. In the previous sections, we implicitly assume the complete knowledge of $H_1, \ldots, H_K$ at the receiver of each link. In reality, the receiver for link $j$ will only know $H_j$, its own CSI. We refer to this situation as limited CSI. To mitigate against this problem, we propose to directly estimate the sub-optimum interference correlation matrix, $\hat{R}_{N,j}$ from the received signal $y$. In a practical system, this can be implemented by using training symbols transmitted using $t_j$ at the beginning of each frame. $\hat{R}_{N,j}$ is given as

$$
\hat{R}_{N,j} = E[y_jy_j^H] = E[n_jn_j^H] + \sum_{i=1}^{K} H_j t_i (H_j t_i)^H
$$

(5.9)

during the training period. (5.7) and (5.5) are then used to find $t_j$ and $r_j$ respectively.

Note that the use of $\hat{R}_{N,j}$ will not degrade the system performance. To prove that, we use the fact that in SINRM

$$
SINR_1 > \ldots > SINR_j > \ldots > SINR_K.
$$

(5.10)

This is so since in our SINRM beamforming design, link 1 and $K$ have the lowest and highest interference respectively. Hence, we only need to prove that the $SINR_K$ calculated using $\hat{R}_{N,K}$ and $R_{N,K}$ are the same. $\hat{R}_{N,K}$ in (5.9) can be rewritten as

$$
\hat{R}_{N,K} = H_K t_K (H_K t_K)^H + E[n_Kn_K^H] + \sum_{i=1, i\neq K}^{K} H_K t_i (H_K t_i)^H
$$

(5.11)

$$
\hat{R}_{N,K} = R_{N,K}
$$
5.4 Simulation Results

It then follows from Woodbury’s identity [64] that

$$\hat{R}_{N,K} = R_{N,K}^{-1} - \frac{R_{N,K}^{-1} H_K t_K (H_K t_K)^H R_{N,K}^{-1}}{1 + H_K t_K R_{N,K}^{-1} (H_K t_K)^H}.$$  (5.12)

A substitution of $R_{N,K}$ with $\hat{R}_{N,K}$ in (5.5) and algebraic simplification leads to the same $r_K$ expression. This concludes the proof.

5.4 Simulation Results

In this section, we study the performance of the proposed cooperative transmission scheme (THP-SINRM-APC) in terms of uncoded BER. Two MIMO-OFDM APs and two stations ($K = 2$) are considered. Each is equipped with two antennas ($N_T = N_R = 2$). APs have full CSIs since each AP can share its CSI with all other APs through backbone networks. Each link is transmitted at equal power. Rectangular 4-QAM (M=4) modulation is used. The symbol period, guard period and number of sub-carriers are set to $3.2\mu s$, $0.8\mu s$, and 48 respectively. The number of paths, RMS delay spread and maximum channel delay are set to 10, $0.16\mu s$, and $0.8\mu s$ respectively. The performance of cooperative APs in an interference-free channel and non-cooperative scheme [46] under the same configuration are used as benchmarks in our simulations. THP-SINRM, with a fixed precoding order (THP-SINRM-FPC) that encodes link 2, then link 1, and a similar cooperative transmission scheme with [41] (THP-ZF-APC1) are also simulated for comparison purposes. In our discussion below, the comparison of the schemes is performed at BER=10^{-4}.

5.4.1 Performance of the Individual Links

The result in Fig. 5.1 shows the BER for individual links. For THP-SINRM-FPC, link 1 has better performance than link 2 since the former has lower interference than the latter. The BER performance difference between link 1 and link 2 exceeds 4 dB. Once APC is incorporated (THP-SINRM-APC), the difference in BER between link 1 and 2 disappears. Here, the performance of link 2 is improved at the expense of link 1. This results in similar
5.4 Simulation Results

BERs across all the links. Note that even though THP-ZF-APC1 \[41\] can properly eliminate the difference in BER, its performance is still worse than the proposed scheme by 3 dB.

5.4.2 Performance of the Overall Symbol Error Rate

Here, we study the performance of the overall SER. Overall SER is defined as the average SER for K links. The overall BER performance is shown in Fig. 5.2. The use of the adaptive precoding order (THP-SINR-APC) results in 3 dB gain over the fixed precoding order (THP-SINR-FPC). This gain is due to an additional degree of freedom provided by APC. THP-SINRM-APC also outperforms the non-cooperative scheme by more than 10 dB and is only 2 dB away from an interference free channel. The large improvement in our proposed scheme over the non-cooperative scheme comes from an increase in transmit diversity (two to four), APC gain, as well as interference cancellation. Finally, Fig. 5.2 also shows the performance of the proposed scheme when only partial CSI is available at the receiver (THP-SINR-APC-PCSI). It can be observed that its performance is very close to
the performance of THP-SINR-APC with full CSI at the receiver. This confirms our earlier analysis in Section 5.3.

We now investigate the performance of (THP-SINR-APC) when we increase the number of APs and stations to three ($K = 3$). Each AP and station is equipped with two antennas. We can see here, from Fig. 5.3 that the BER performances for each individual user are close to each other. This is preferable when each AP is deployed by different operators. Unfortunately here, the overall performance of (THP-SINR-APC) deviates away from an interference free performance. This is shown Fig. 5.4. The performance of (THP-SINR-APC) is 3 dB away from an interference free channel. This gap is much bigger than when $K = 3$. The reason for this wider gap is that, as the number of users increases, there is more interference to be cancelled. Thus, the BER performance depends on the link with the lowest SINR. In this case, we could see from (5.10) that the SINR for link $K$ is the weakest one since we use the beamforming weights to suppress interference from user $1, \ldots, K - 1$ to user $K$.

One interesting point in Fig. 5.4 is that the BER performance of (THP-SINR-APC) starts to
Figure 5.3: BER of individual links for $K = 3$
Figure 5.4: Overall BER for $K = 3$
flatten which results in a BER floor when $SNR = 16dB$. That means no further improvement can be obtained even if we increase the SNR further. The BER floor is caused by very weak $SINR_K$. This is because the beamforming weights for user $K$ is used to suppress interference from $K-1$ users. The reason for $SINR_K$ to be so weak, is because the beamforming design for user $K$ needs to take into consideration the direction of beamforming weights for user $1,...,K-1$. As the number of users increases, the possible direction for beamforming weights of user $K$ is getting more limited. As a result, the SINR for user $K$ degrades rapidly as the number of users increases. Lastly, we observe that the performance of (THP-SINR-APC-PCSI) is very close to the performance of THP-SINR-APC with full CSI at the receiver. This again confirms our earlier analysis in Section 5.3.
In this chapter, we propose a method to design transmit-receive antenna weights for a cooperative single stream downlink transmission scheme. The algorithm eliminates the interference and achieves symbol error rate (SER) fairness among different users. Here, as mentioned in Chapter 3, Tomlinson Harashima precoding (THP) is used to cancel the front-channel interference. Thus, we are left with the rear-channel interference which needs to be cancelled by the transmit-receive antenna weights. A new iterative method is applied to generate the transmit-receive antenna weights. The convergence behaviour of the iterative process is investigated via both analysis and simulations. To achieve SER fairness among different users and further improve the performance of MIMO systems, we develop a power allocation algorithm that provides equal SINR across all users and order users so that the minimum SINR for each user is maximized. The simulation results show that the proposed scheme considerably outperforms existing cooperative transmission schemes in terms of SER performance and complexity and approaches an interference free performance under the same configuration.

In this chapter, as we consider a single stream transmission, $S = 1$, for simplicity, we will
omitting the stream notation $s$ and $S$. That means the modulated symbol, the ordered modulated symbol and the THP precoded symbols are written as $x = [x_{1,1}...x_{1,K}]^T$, $u = [u_{1,1}...u_{1,K}]^T = [u_1...u_K]^T$ and $v = [v_{1,1}...v_{1,K}]^T = [v_1...v_K]^T$ respectively. The transmit-receive weights for each link $j$ are written as $r_j = R_j = [r_{1,j}]$ and $t_j = T_j = [t_{1,j}]$, respectively.

### 6.1 Iterative Joint Transmit-Receive Antenna Weights

In this section, we propose a joint iterative transmit-receive antenna weights optimization method based on ZF to cancel the rear-channel interference, while maximizing the SINR for each link and maintaining the same SER for all links at all times. The received signal at each receiver prior to and after the modulo operation are shown in (3.18) and (3.19), respectively.

We first denote $[(I + M_{THP})^{-1}]_{j,l}$ as the $(j, l)^{th}$ component of $(I + M_{THP})^{-1}$. The transmitted signal estimate of link $j$, $\hat{u}_j$ can be obtained from (3.19) and expressed as

$$\hat{u}_j = \sqrt{p_j r_j^H t_j} u_j + \sum_{i=j+1}^{K} p_i r_j^H t_i [(I + M_{THP})^{-1}]_{j,i} \hat{r}_j + r_j^H n_j. \tag{6.1}$$

The SINR, after the modulo operation for link $j$, can then be written as

$$SINR_j = \frac{p_j r_j^H t_j (H_j t_j)^H r_j}{r_j^H (\sum_{i=j+1}^{K} p_i H_j t_i (H_j t_i)^H [(I + M_{THP})^{-1}]_{j,i} + \sigma^2 I) r_j}. \tag{6.2}$$

Maximizing the minimum SINR for each link, while maintaining it equal for all links, can be formulated as follows

$$\max_{R,T,P} \min_{1 \leq i \leq K} SINR_i \quad \text{subject to} \quad (1) \ T^H T = I, \ (2) \ r_j^H r_j = 1, \ (3) \ T^T p = P_{max}, \ (4) \ r_j^H H_j t_i = 0 \tag{6.3}$$

for $j = 1, ..., K, i = j + 1, ..., K$ where $P_{max}$ and $p = [p_1...p_K]^T = P^2 1$ are the power constraint at the cooperative transmitter and the set of the powers assigned to each link, respectively. Here the objective of (6.3) is to maximize the minimum SINR for each link. The
6.1 Iterative Joint Transmit-Receive Antenna Weights

Figure 6.1: Iterative Joint Transmit-Receive Weights Optimization and SINR Equalization Process

first, second and third constraints in (6.3) ensure that the transmit-receive weight vectors are unitary vectors and the sum of the power allocated to each link does not exceed the maximum power available at the transmitter. These constraints will bound the possible solution for $R$, $T$, and $P$ and ensure the convergence of (6.3) to a solution. Finally, the fourth one is the ZF constraint which ensures the interference from links $j + 1, \ldots, K$ to link $j$ are fully cancelled. Here, to maximize the minimum SINR in (6.3), we reduce the SINR of the best link until the SINRs of all links are equal. Thus, the optimal solution is reached when all links attain an equal SINR [18, 65].

This optimization problem, however, is difficult to solve as it is not jointly convex in variables $R, T$ and $p$. To solve (6.3), we propose a sub-optimal solution that splits the problem into a 2-step optimization.

The first step is to solve $R$ and $T$ iteratively, when $p$ is fixed. Hence in this step we simply ignore the equalization of SINRs among all links. The second step is to solve $p$ in a way that equalizes SINR for all links under fixed $R$ and $T$. The process is described in Fig. 6.1, where $i, f_1(\cdot)$ and $g_j(\cdot)$ are the iteration number, a function to generate transmit antenna weights for $K$ links and a function to generate a receive antenna weights vector for link $j$, respectively.

6.1.1 Transmit-Receive Antenna Weights Design

In the first step, we assume an equal power allocation for each link by setting $P = I$. (6.3) can be simplified as

$$
\max_{R,T} \quad SINR_i \\
\text{subject to} \quad (1) \ T^H T = I, \ (2) \ r_j^H r_j = 1, \ (3) \ r_j^H H_j t_i = 0
$$

(6.4)
6.1 Iterative Joint Transmit-Receive Antenna Weights

for $j = 1, ..., K$, $i = j + 1, ..., K$. To solve (6.4), we propose to alternately optimize $\mathbf{R}$ and $\mathbf{T}$ until they converge, under the ZF constraint in (6.3). We first assign the initial value of the receive antenna weights for $K$ links. The initial receive weights of $K$ links are given as

$$r_j^{(0)} = v_{svd}(\mathbf{H}_j), \quad j = 1, ..., K$$

(6.5)

where $v_{svd}(\cdot)$ is the Singular Value Decomposition operation (SVD) [54] to select a left eigenvector associated with the maximum eigenvalues of $\mathbf{H}_j^H \mathbf{H}_j$. We then transform the system into a downlink multi-link MISO system by fixing

$$\mathbf{R} = Diag(r_1^{(0)}H, ..., r_K^{(0)}H).$$

(6.6)

(3.12) can then be written as

$$\mathbf{y} = \mathbf{R}^H \mathbf{v} + \mathbf{R} \mathbf{N} = \mathbf{H}_e^T \mathbf{v} + \tilde{\mathbf{N}}.$$  

(6.7)

Here, we know from (3.19) that the interference from links $1, ..., j - 1$ to links $j = 1, ..., K$ does not exist at the receiver, after performing decoding, since this front-channel interference is totally cancelled by the THP described in Section 3.3.3. The remaining interference is the rear-channel interference, coming from links $j + 1, ..., K$ to links $j = 1, ..., K$ which needs to be cancelled.

At each iteration, we apply a QR decomposition [54] to $\mathbf{H}_e^H$ to find $\mathbf{T}$ that forces this interference to zero,

$$\mathbf{T} = f_1(\mathbf{R}), \quad f_1(\mathbf{R}) = [Q|QR(\mathbf{H}_e^H)].$$

(6.8)

Here, we choose the unitary matrix $\mathbf{Q}$ obtained from the QR decomposition of $\mathbf{H}_e^H$ in (6.8) as $\mathbf{T}$. We now need to compute $\mathbf{R}$ that gives a maximum SINR for each link for the derived $\mathbf{T}$. This can be calculated as

$$r_j = g_j(\mathbf{T})$$

(6.9)

where $j = 1, ..., K$. $g_j(\mathbf{T})$ is a function that generates receive weight vector $r_j$ for the derived $\mathbf{T}$ such that SINR for each link is maximized.

We now describe how $g_j(\mathbf{T})$ operates. By using (6.2), the SINR maximization for each link...
can be written as
\[
\max_{\mathbf{r}_j} \frac{p_j \mathbf{r}_j^H \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H \mathbf{r}_j}{\mathbf{r}_j^H \mathbf{R}_j \mathbf{r}_j}
\]
where \( \tilde{\mathbf{h}}_j = \mathbf{H}_j \mathbf{t}_j \) and
\[
\mathbf{R}_j = \sum_{i=j+1}^{K} p_i \mathbf{H}_j \mathbf{t}_i (\mathbf{H}_j \mathbf{t}_i)^H + \sigma^2 \mathbf{I}
\]
is the interference in link \( j \). To obtain \( \mathbf{r}_j \) that maximizes SINR in (6.10) we use the Spectral/Eigenvalue Decomposition [54]. Thus the functions to generate \( \mathbf{r}_1, \ldots, \mathbf{r}_K, g_j \) can now be written as
\[
g_j(\mathbf{T}) = v_{EVD}(p_j \mathbf{R}_j^{-1} \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H), \quad j = 1, \ldots, K
\]
where \( v_{EVD} \) is the Spectral/Eigenvalue Decomposition operation [54] to select an eigenvector associated with the maximum eigenvalue of
\[
p_j \mathbf{R}_j^{-1} \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H.
\]
We can obtain a simpler expression for \( \mathbf{r}_j \) that gives the same maximum SINR, as in (6.10), by using the following fact,
\[
\max_{\mathbf{r}_j} \frac{\mathbf{r}_j^H \tilde{\mathbf{h}}_j \tilde{\mathbf{h}}_j^H \mathbf{r}_j}{\mathbf{r}_j^H \mathbf{R}_j \mathbf{r}_j} = \max_{\mathbf{r}_j} \frac{\mathbf{r}_j^H \tilde{\mathbf{h}}_j}{\mathbf{r}_j^H \mathbf{R}_j \mathbf{r}_j}.
\]
Here we state that the optimum \( SINR_j \) obtained by using the term on the left hand side of (6.14) is equal to the optimum \( SINR_j \) obtained by using the term on the right hand side of (6.14). The proof of their equivalence is shown in Appendix B. By solving the term on the right hand side of (6.14), the normalized receive antenna weight vector for link \( j \) can be obtained as [46]
\[
\mathbf{r}_j = g_j(\mathbf{T}) = \frac{\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{t}_j}{\| \mathbf{R}_j \mathbf{H}_j \mathbf{t}_j \|}.
\]
It is straightforward to show that the \( SINR_j \) generated by using the receive antenna weight vector from (6.15) yields the optimum \( SINR_j \) given in (6.14). The proof is shown in Appendix E. We can conclude from this fact and (6.14) that the normalization process of the receive weight vector in (6.15) will not affect the SINR. Note that this receiver design is
6.1 Iterative Joint Transmit-Receive Antenna Weights

also known in the literature as the Minimum Variation Distortionless Response (MVDR) design [11]. The iterative calculations of \( \mathbf{R} \) and \( \mathbf{T} \) continue by fixing one and optimizing the other one, until they converge to a fixed solution. It is proved in Appendix C that the proposed iterative method always converges. This can be summarized in Lemma 1 as follows,

**Lemma 1** The proposed iterative method to solve (6.4) converges to a local maximum and satisfies (6.8) and (6.9) as the number of iterations increases.

6.1.2 Downlink Power Allocation

In the second step, we use \( \mathbf{R} \) and \( \mathbf{T} \) obtained in the first step to find \( \mathbf{p} \). Using the fact that at the optimal solution all links will attain equal SINR and letting \( a_{i,j} = r_i \mathbf{H}_i \mathbf{t}_j \), (6.2) can be written as

\[
\sum_{i=1}^{j-1} |a_{j,i}|^2 p_i + \sigma^2 = \frac{p_j |a_{j,j}|^2}{\text{SINR}}.
\]  
(6.16)

(6.16) can be further represented in a matrix format as

\[
\mathbf{A}^{-1} \mathbf{Bp} + \sigma \mathbf{A}^{-1} \mathbf{1} = \frac{\mathbf{p}}{\text{SINR}}
\]  
(6.17)

where \( \mathbf{A} = D_i T(\mathbf{M}) \), \( \mathbf{B} = U_p T(\mathbf{M}) \) and \( \mathbf{M} \) is a \( K \) by \( K \) matrix with entries \( |a_{i,j}|^2 \) in row \( i \) and column \( j \). By multiplying both sides of (6.17) with \( \mathbf{1}^T \), we obtain [18]

\[
\frac{1}{P_{\text{max}}} (\mathbf{1}^T \mathbf{A}^{-1} \mathbf{Bp} + \sigma \mathbf{1}^T \mathbf{A}^{-1} \mathbf{1}) = \frac{1}{\text{SINR}}.
\]  
(6.18)

By defining the extended power vector \( \mathbf{p}_e = [\mathbf{p} \mathbf{1}]^T \), we can then combine (6.17) and (6.18) to obtain a matrix equation given as [18]

\[
\Psi \mathbf{p}_e = \frac{\mathbf{p}_e}{\text{SINR}}
\]  
(6.19)

where

\[
\Psi = \begin{pmatrix}
\mathbf{A}^{-1} \mathbf{B} & \sigma \mathbf{A}^{-1} \mathbf{1} \\
\mathbf{1}^T \mathbf{A}^{-1} \mathbf{B} / P_{\text{max}} & \sigma \mathbf{1}^T \mathbf{A}^{-1} \mathbf{1} / P_{\text{max}}
\end{pmatrix}.
\]  
(6.20)
6.1 Iterative Joint Transmit-Receive Antenna Weights

Table 6.1: Nonlinear Iterative Precoding Algorithm
1. Initialize receive weights and set Maxiteration
2. For \( i = 2 \) to Maxiteration
3. Find transmit weights using (6.8)
4. Find receive weights using (6.15) or (6.29)
5. end
6. Equalize SINR for all links using (6.19) or (6.22)
7. THP Precoding Operation using (3.6)

Hence the optimum \( p \) can be obtained by selecting \( p_e \) that corresponds to the maximum eigenvalue of \( \Psi \). This is the only possible solution for (6.19) satisfying \( p_j \geq 0 \) for \( j = 1, \ldots, K \) and \( \text{SINR} \geq 0 \). The proof is described in detail in Theorem 1 and 2 of [66]. At a first glance, the power allocation in (6.19) resembles Perron-Frobenius (Eigen-based) power control in [67]. In [67], the aim for the power allocation is to minimize the BSs transmission power and to achieve equal SINR across all links. There is no constraint on BSs transmission power. On the other hand, the power allocation in (6.19) takes into account the power constraint at BSs. Thus, it is more realistic than [67].

The above SINR equalization process can be further simplified, if we assume that after \( i \) iterations, we are very close to the local optima (i.e., \( r_i^H t_j \approx 0 \)). (6.20) then becomes

\[
\tilde{\Psi} = \begin{pmatrix} 0 & \sigma 1 A^{-1} \\ 0 & \sigma 1^T A^{-1} 1 / P_{max} \end{pmatrix}.
\] (6.21)

Substituting (6.21) in (6.19) and after making some simplifications, we have

\[
p_j = \frac{P_{max}}{\sum_{i=1}^{K} \frac{|r_i^H t_j|^2}{|r_i^H t_i|^2}}, \quad j = 1, \ldots, K.
\] (6.22)

The algorithm is tabulated in Table I, where \( \iota \) represents the iteration number and Maxiteration is the maximum number of iterations.
6.1 Iterative Joint Transmit-Receive Antenna Weights

6.1.3 Modification of the Design for Receive Antenna Weights

In this section we modify the receive antenna weights calculations in (6.15) to speed up the convergence to the local maxima and improve the SINR during the iteration process. We define \( r_j^{(i)} \) and \( t_j^{(i)} \) as the receive and transmit antenna weights found in step 1 in Section 6.1.1 at \( i^{th} \) iteration, for each link \( j \). The entries of the front-channel interference matrix \( \mathbf{B}_\mathbf{P} \) at \( i^{th} \) step of the iterative process above, can be written as follows

\[
\varepsilon_{l,j}^{(i)} = \sqrt{p_j} (\mathbf{H}_j^H r_j^{(i)})^H t_l^{(i)}, \ l = j + 1, ..., K, j = 1, ..., K
\]  

(6.23)

where \( \varepsilon_{l,j}^{(i)} \) is the interference from link \( l \) to link \( j \) at the \( i^{th} \) iteration. Note that \( \varepsilon_{l,j}^{(i)} \) also corresponds to the element of the front-channel interference located in row \( l \) and column \( j \).

The diagonal entries of matrix \( \mathbf{D} \) at \( i^{th} \) step of the iterative process, denoted by \( \beta_j^{(i)} \), can be written as

\[
\beta_j^{(i)} = (\mathbf{H}_j^H r_j^{(i)})^H t_j^{(i)}, \ j = 1, ...K
\]  

(6.24)

where \( \beta_j^{(i)} \) is the signal gain for link \( j \) at the \( i^{th} \) iteration. We now formulate a lemma that we are going to use in this section.

**Lemma 2** \( \prod_j \beta_j^{(i)} \leq \text{det}(\mathbf{R}^*\mathbf{H}^*), \) where \( \beta_j^{(i)} = (\mathbf{H}_j^H r_j^{(i)})^H t_j^{(i)} \) and \( (\mathbf{R})^* \) and \( (\mathbf{T})^* \) are the optimal transmit-receive antenna weights vectors for \( K \) links satisfying Lemma 1.

The proof of the lemma is presented in Appendix D. From Lemmas 1 and 2, we know that at the local maximum, 1) \( \prod_j \beta_j^{(i)} \) achieves the maximum value equal to \( \text{det}(\mathbf{R}^*\mathbf{H}^*) \), 2) The front-channel interference \( \mathbf{B}_\mathbf{P} \) converges to 0 since (6.8) forces \( \mathbf{R}^*\mathbf{H}^* \) to have a lower triangular structure. Hence, we could simply maximize \( \beta_j^{(i)} \) to achieve the local maxima. By using the Matrix Inversion Lemma [64] and substituting (6.11) and (6.15) into (6.24), we can rewrite (6.24) for link \( j \) as

\[
c\beta_j^{(i)} = (\mathbf{H}_j t_j^{(i)})^H (\sigma^{-1} \mathbf{I} - (\mathbf{Z}^{-1} + \sigma \mathbf{I})^{-1})\mathbf{H}_j t_j^{(i)}
\]  

(6.25)

where

\[
\mathbf{Z} = \sum_{a=j+1}^{K} \mathbf{H}_a t_a^{(i)} (\mathbf{H}_a t_a^{(i)})^H
\]  

(6.26)
6.2 Adaptive Precoding Order

and \( c = \| \mathbf{R}_j \mathbf{H}_j \mathbf{t}_j \| \), representing a scaling/normalization factor. It is obvious that

\[
(H_j \mathbf{t}_j^{(i)})^H (\mathbf{Z}^{-1} + \sigma \mathbf{I})^{-1} H_j \mathbf{t}_j^{(i)}
\]

in (6.25) reduces the value of \( \beta_j^{(i)} \). Therefore, if we omit this term in calculating the receive antenna weights, we can reach the maximum \( \beta_j^{(i)} \) faster. Thus, we can simply ignore this term to speed up the convergence of the iterative process. Therefore by omitting the term \((H_j \mathbf{t}_j^{(i)})^H (\mathbf{Z}^{-1} + \sigma \mathbf{I})^{-1} H_j \mathbf{t}_j^{(i)}\), we have

\[
\beta_j^{(i)} \sim (H_j \mathbf{t}_j^{(i)})^H (\sigma^{-1} \mathbf{I}) \mathbf{t}_j^{(i)} = \sigma^{-1} (r_j^{(i)})^H H_j \mathbf{t}_j^{(i)}.
\] (6.28)

The maximum \( \beta_j^{(i)} \) can be obtained by aligning \( r_j^{(i)} \) in the direction of \( H_j \mathbf{t}_j^{(i)} \). The total power of the receive weight vector, \( r_j^{(i)} \), is normalized to 1, to ensure it satisfies the second constraint in (6.4),

\[
r_j^{(i)} = \frac{H_j \mathbf{t}_j^{(i)}}{\| H_j \mathbf{t}_j^{(i)} \|}.
\] (6.29)

We refer to this receiver structure as a Matched Filter (MF) design.

6.2 Adaptive Precoding Order

In the THP and the 2-step optimization process described in the previous sections, we fix the order of \( u_j \), resulting in a fixed permutation matrix \( M_{perm} \). The performance of the system, however, differs when a different \( M_{perm} \) is used. In addition, the performance of the system also depends on the weakest link. In this section we propose an APO scheme. APO arranges the order of \( x \) by selecting \( M_{perm} \) that maximizes the minimum SINR for each user.

We formulate the optimization process to find a permutation matrix \( \hat{M}_{perm} \in M_{perm} \) that gives the maximum SINR as

\[
\hat{M}_{perm} = \arg\max_{M_{perm}} \min_{j=1}^{K} (SINR_j(M_{perm}), ..., SINR_K(M_{perm}))
\] (6.30)

where \( SINR_j(M_{perm}) \) is the SINR of link \( j \), given that the permutation matrix \( M_{perm} \) is used. To solve (6.30) without searching \( K! \) possible orderings, we use the idea of the Myopic
Optimization method proposed in [48, 68], which is proven to be optimal. By applying this method to arrange the precoding order for the users, we only need to search \( \sum_{i=0; i \neq 1}^{K-1} K - i \) possible orderings. Here, we refer to the APO generated using the optimization method above as APO-VBLAST.

### 6.3 The Complexity Comparison of the Proposed and Other Known Schemes

In this section, we discuss the advantages of the proposed scheme over other existing schemes. We first compare the proposed method with the scheme in [39], which uses the Coordinated Tx-Rx Algorithm with Block Diagonalization and water-filling power allocation and the scheme in [40], which works by iteratively finding transmit-receive weights that diagonalize the receive signal matrix of \( K \) users without the receiver noise in (3.12).

To have a fair comparison with the proposed method, we replace the water-filling power allocation with (6.22), that equalizes SINR for all links. This change is required as the water-filling power allocation used in [39] tends to assign more power to stronger links and less power to weaker links. Hence, the performance of a weaker link will decrease the overall SINR for all links.

The main differences between the method in [39, 40] and the proposed method are, 1) [39, 40] suppress both the front-channel and rear-channel interference using transmit-receive weights, while the proposed method suppresses the rear-channel and front-channel interference using THP and iterative transmit-receive weights, respectively, 2) unlike [39, 40], the proposed scheme does not calculate null spaces. To compute these null spaces, the iterative scheme in [40] and the non-iterative scheme in [39] perform \( K \) SVD operations per iteration and \( K \) SVD operations, respectively, 3) within a single iteration, a QR decomposition [54] and \( K \) MF receiver calculations are required to find all transmit-receive antenna weights while in [40], \( K \) SVD operations per iteration, are required to find the transmit-receive weights of all links. Note that [39] requires \( K \) SVD operations to find the transmit-receive weights of all links.

The complexity requirements in terms of the number of floating point operations (flops), for
the proposed method and the methods in [39, 40] are listed in Table 6.2, where \( i \) denotes the total number of iterations. Here, we compare the number of computations for the transmit-receive antenna weights and the null spaces for the two methods, since this is the only major difference between the two methods. Hence for \( K = 3 \), \( N_{MS} = 2 \), and \( N_{BS} = 2 \), the proposed method has 339 flops per iteration while the methods in [39] and [40] have 13032 flops and 13032 flops per iteration, respectively.

The second comparison is done with the non-linear precoding methods in [41, 42]. Again to have a fair comparison with the proposed method, after the algorithm in [41, 42], we apply (6.22) to equalize the SINR, instead of using the original power allocation.

Unlike [41, 42], we do not require the constraint of

\[
(K - 1)N_{MS} < KN_{BS}
\]

(6.31)

because we do not create null spaces. Thus, there is no relationship between the required number of transmit antennas and receive antennas. This is a definite advantage, since to support, say five users with \( N_{MS} = 4 \), the proposed method needs five transmit antennas while [39] needs 12 transmit antennas. Another important difference between these methods and the proposed one is in the zero forcing condition definition. In our scheme we have

\[
r_{j}^{H}H_{j}t_{i} = 0 , j = 1, ..., K, i = j + 1, ..., K
\]

(6.32)

while in [41] and [42]

\[
H_{j}t_{i} = 0 , j = 1, ..., K, i = j + 1, ..., K.
\]

(6.33)

Using (6.32), the proposed algorithm allows some inter-link interference to be transmitted (e.g. \( H_{j}t_{i} \neq 0 \)), and cancels the interference by steering \( H_{j}t_{i} \) to be perpendicular with the receive antenna weight vector \( r_{j} \). Hence the receive and the transmit antenna weights jointly cancel the interference. The constraint in (6.33) [41] or [42], on the other hand, does not allow any inter-link interference to be transmitted. Here, the receive antenna weights are not used at all to cancel the interference.

Finally, the computational complexity required to find the null spaces and the transmit-
### Table 6.2: Computational Complexity of Non-Linear Precoding Algorithms (in Flops)

<table>
<thead>
<tr>
<th>Computation</th>
<th>Proposed Method</th>
<th>Scheme in [41, 42]</th>
<th>Scheme in [39]</th>
<th>Scheme in [40]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) THP</td>
<td>((K^2 + 1)(2K - 1) + 0.5K(K - 1)(2KN_MsN_{RS} - 1))</td>
<td>((K^2 + 1)(2K - 1) + 0.5K(K - 1)(2KN_MsN_{RS} - 1))</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(2) SINRE</td>
<td>(2KN_MsN_{RS}(1 + K) + K)</td>
<td>(2KN_MsN_{RS}(1 + K) + K)</td>
<td>(2KN_MsN_{RS}(1 + K) + K)</td>
<td>(2KN_MsN_{RS}(1 + K) + K)</td>
</tr>
<tr>
<td>(3) Tx/Rx Weights</td>
<td>(i{(3K^4(N_{BS} - 1) - KN_{MS}) + 2KN_{BS}(N_{MS} + 1)})</td>
<td>(K^2N_{MS}N_{BS}(4N_{MS} + 8KN_{BS} + 9(KN_{BS})^2)) + (2KN_{BS}(N_{MS} + 1) - KN_{MS})</td>
<td>(K^2N_{MS}N_{BS}(4N_{MS} + 8KN_{BS} + 9(KN_{BS})^2)) + (2KN_{BS}(N_{MS} + 1) - KN_{MS})</td>
<td>(i{4K^3N_{BS} + 8K^4N_{BS}^2}) + (sN_{BS}K^2N_{BS}^2 + 9(KN_{BS})^3)</td>
</tr>
<tr>
<td>(4) Null Space</td>
<td>–</td>
<td>(3(KN_{MS})^2(KN_{MS} - \frac{KN_{MS}}{3}) + \sum_{a=1}^{K-1} KN_{BS}(2KN_{MS} - 1))</td>
<td>(K^2N_{BS}(2KN_{MS} - 1)) + (\sum_{a=1}^{K-1} 4KN_{BS}a^2)</td>
<td>(i{4K^4N_{BS} + 8K^4N_{BS}^2})</td>
</tr>
<tr>
<td>APO-VBLAST</td>
<td>(\sum_{l=0,j\neq1}^{K-1} K - l)</td>
<td>(\sum_{l=0,j\neq1}^{K-1} K - l)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Total</td>
<td>((1) + (2) + (3)) (\sum_{l=0,j\neq1}^{K-1} K - l)</td>
<td>((1) + (2) + (3) + (4)) (\sum_{l=0,j\neq1}^{K-1} K - l)</td>
<td>(2) + (3) + (4))</td>
<td>(2) + (3) + (4))</td>
</tr>
</tbody>
</table>
receive antenna weights for the method in [41] is shown in Table 6.2. The complexity of the method in [41], for a system with $K = 3$, $N_{MS} = 2$, and $N_{BS} = 2$, is 8868 flops.

6.4 Numerical Results and Discussion

Monte Carlo simulations are carried out to assess the performance of the proposed method in a MIMO-OFDM environment. We investigate its performance and compare it with [41], [39] and with an interference free performance.

Here, an interference free performance is defined as the performance of any random single link $i$ assuming there is no interference from any other links. In this case, the received signal of the cooperative transmission system is given as

$$y_i = r_i^H (H_i t_i x_i + n_i) \quad (6.34)$$

where $r_i$ and $t_i$ are the left and right eigenvectors associated with the maximum eigenvalue of $H_i H_i^H$ using SVD.

The comparison of the schemes is performed at $SER=10^{-4}$. We use a fixed permutation matrix that orders MSs $1, \ldots, K$ as links $K, \ldots, 1$, when we are not using APO-VBLAST, for all the simulation results except stated otherwise. For convenience, we will use the notations $(N_{BS}, N_{MS}, K)$ in all figures to denote the number of transmit antennas per BS, the number of receive antennas per MS and the number of BS in the network, respectively. Perfect CSI is assumed to be available at both ends. Rectangular 64-QAM ($M=64$) modulation is used for all transmissions. The OFDM symbol period, guard period and number of sub-carriers are set to $3.2\mu s$, $0.8\mu s$ and 48, respectively. The number of paths, RMS delay spread and maximum channel delay are set to 10, $0.16\mu s$, and $0.8\mu s$, respectively. The channel is assumed to be a quasi-static channel resulting in a negligible doppler shift. In all simulations, we fix the the Signal-to-Noise-Ratio of each THP precoded symbol to be $SNR = \frac{E[v_j^2]}{2\sigma^2}$, where $E[v_j^2]$ is normalized to 1 and $P_{max} = K$. 

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6.4 Numerical Results and Discussion

6.4.1 Convergence Study

Figs. 6.2 and 6.3 show the convergence characteristics of the proposed method for \((2, 2, 3)\) and \((1, 2, 4)\) systems. We plot the number of iterations versus the average error and scaled output SINR (after SINR equalization) while fixing the SNR at 21 dB. The average error is defined as the average of the maximum entries of the front-channel interference BP,

\[
\varepsilon^{(i)} = \max_{j,l} |\varepsilon_{j,l}^{(i)}|, \quad j = 1, \ldots, K, l = j + 1, \ldots, K
\]  

over all channel realizations. The output SINRs for \((1, 2, 4)\) and \((2, 2, 3)\) systems are scaled up by 4 dB and 0 dB to fit in one figure. The scaling does not matter here since we only want to observe the convergence rate. The figures also show the convergence characteristics when the MF receiver design, represented by (6.29), and the MVDR receiver design, represented by (6.15) are used.

An interesting observation here is that, during the first few iterations, the MVDR outperforms MF design. This improvement is due to smaller errors obtained using MVDR and not due to a higher signal gain \(\beta_{j}^{(i)}\). During the first few iterations, the second term of (6.25) for the MF receiver design is larger than for the MVDR receiver design, thus leading to a higher average error for the MF receiver. This happens because MF ignores the interference when calculating the receive antenna weights. However, its average error decreases rapidly and after 5 iterations (for \((1,2,4)\)) and 7 iterations (for \((2,2,3)\)), the average error for \((1,2,4)\)-MF and \((2,2,3)\)-MF in Fig. 6.3 approaches that of the MVDR method. The resulting SINR using the MF method from that point onwards is always greater than the one using the MVDR method. This is shown in the analysis in Section 6.1.3. This analysis is consistent with the results shown in both Figs. 6.2 and 6.3.

In addition, the MF and MVDR methods do not have the same SINR convergence speed. MF converges much faster to the optimal SINR solution than MVDR. This is shown in Fig. 6.2. This in fact confirms Lemma 2 and the previous analysis. The MF’s SINR reaches a plateau after 8 iterations, since it almost converges to the optimal solution while the MVDR’s SINR is still rising. Not much performance improvement can be obtained by increasing the number of iterations further. In all simulations for SER comparison, we set the maximum iteration number for the proposed scheme to 10.
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Figure 6.2: SINR Convergence Comparison of Various Iterative Interference Cancellation Methods
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Figure 6.3: Average Error Convergence Comparison of Various Iterative Interference Cancellation Methods
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Figure 6.4: SER Performance Comparison of Individual Links for a (2,2,3) System when SINRE is not used and when SINRE and APO-VBLAST are used

6.4.2 Performance of the Individual Links

Fig. 6.4 shows the SER of the worst user and the best user versus SNR in a (2, 2, 3) system. In these figures, the proposed method refers to the proposed algorithm with THP, joint iterative transmit-receive weights optimization, SINR equalization (SINRE) and Adaptive Precoding Order (APO-VBLAST). As shown in Fig. 6.4, when the proposed method does not perform (denoted by w/o in the figures) SINR equalization and APO-VBLAST, MS 3 has the best performance while MS 1 has the worst performance. The SER performance difference between link 1 and link $K$ exceeds 3 dB. Once SINR equalization is used, the SER difference between links disappears. This is shown in Fig. 6.4. Here, the performance of link 1 is improved at the expense of links 2 and 3. This results in a similar SER across all the links.
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An interesting point here is that APO-VBLAST tends to equalize the performance of $K$ users even without the use of SINR equalization. This is shown in Fig. 6.4. Hence, it seems sufficient to use APO-VBLAST without SINR equalization to maximize the minimum SINR in the system. Its performance, however, is still worse than when the proposed method does not perform APO-VBLAST. This is denoted as ”Proposed w/o APO-VBLAST” in Fig. 6.4. This suggests that SINR equalization plays a more important role than APO-VBLAST in performance improvement. In other words, using a good power allocation scheme might be more beneficial than searching for the best order of the users to achieve a higher diversity gain.
6.4 Numerical Results and Discussion

6.4.3 Performance of the Overall Symbol Error Rate

Here, we investigate the performance of the overall SER. Overall SER is defined as the average SER of K links. The overall SER performance for the proposed method with or without APO-VBLAST, and [39–41] for a (2,2,3) system is shown in Fig. 6.5.

The proposed method without APO-VBLAST outperforms the methods in [41] and [39] by 5 dB and 3 dB, respectively, and is only 1 dB away from an interference free performance when SER=10^{-4}. The large performance improvement in the proposed scheme with respect to [41] comes from an increase in the degree of freedom and the iteration process used in determining the transmit-receive antenna weights.

In addition, the proposed method without APO-VBLAST is able to achieve a much better performance with much less complexity (we only use 10 iterations). The computational complexity of the proposed method for a (2,2,3) system is on average about 75% less than the complexity of methods in [40–42]. As for a (1,2,4) system, we have on average about 50% complexity reduction. In essence, the proposed method fully utilizes THP, transmit antennas and receive antennas in a more optimal way with much less complexity to create non interfering spatial channels.

Fig. 6.5 also shows the performance of the proposed method. APO-VBLAST moves the SER performance of the proposed method within 0.25 dB from an interference free transmission when SER=10^{-4}. Thus, APO-VBLAST gives about 1 dB gain over the proposed method without APO-VBLAST. This gain however comes at the cost of complexity, since now the proposed method will need to do a search over \( \sum_{l=0}^{K-1} K - l \) possible user orderings. As a result, the complexity of the proposed method is \( \sum_{i=0}^{K-1} K - i \) times more than the proposed method without APO-VBLAST. This is shown in the last column of Table 6.2.

The performance of the iterative scheme in [40] depends on the number of iterations. Therefore, to show that the proposed method performs better than the scheme in [40], we set the iteration number for the scheme in [40] to 5, giving a computational complexity of 65259 flops for a (2,2,3) system. The computational complexity of the proposed method using 10 iterations is 15449 flops while the complexity of the proposed method without APO-VBLAST using 10 iterations is 3433 flops. The performance of [40] is shown in Fig. 6.5. Here, we can see clearly that [40] is worse than the proposed method with or without APO-VBLAST.
Figure 6.6: Average SER Performance Comparison for a (1,2,4) System using Various Non-Linear Precoding Algorithms
6.4 Numerical Results and Discussion

It is also not possible to get much performance improvement in [40] by raising SNR above $21$ dB. We refer to the SNR value, above which there is no further SER decrease, a saturation point. Here, we must stress that the performance of [40] can be further improved by increasing the number of iterations. This is shown in Fig. 6.5 when we increase the number of iterations to 11. However, the performance of the scheme in [40], is worse than the proposed method and is 4 times more complex than the proposed method, making it less desirable for a practical implementation.

In Fig. 6.6, we show how the proposed method performs under a different configuration. We show the performance when the number of users, $K$, the number of transmit antennas per BS, $N_{\text{BS}}$, and the number of receive antennas per MS, $N_{\text{MS}}$, are 4, 1 and 2 respectively. Here, the total number of transmit antennas $KN_{\text{BS}}$ is equal to the number of MSs. Even when the proposed method does not perform APO-VBLAST, it still significantly outperforms the one in [39]. This improvement is even greater than the one in Fig. 6.5 ($>4$ dB). Here, however the performance gap between the proposed method and an interference free condition is 4 dB when \( \text{SER}=10^{-4} \). The reason for this wider gap is the lack of spatial diversity of the transmitter since we have $KN_{\text{BS}} = 4$ transmit antennas broadcasting to $K = 4$ users. In addition, the fact that there are only two antennas at each receiver, also limits the overall spatial diversity of the system.

In Fig. 6.6, we can also see clearly that [40] with 5 iterations is much worse than the proposed method. Note that here the saturation point occurs at 24 dB since no further performance improvement can be obtained [40] by raising SNR above 24 dB. Fig. 6.6 also shows the performance of [40] when the number of iterations is increased to 11. Here, we essentially shift the saturation point further to the right. However, by doing this, the scheme [40] is now 3 times more complex that the proposed method, making it much less desirable for a practical implementation. Note that the computational complexity of the proposed method using 10 iterations is 25132 flops, while the computational complexities of the scheme in [40] using 5 and 11 iterations are 36244 flops and 79636 flops, respectively. In addition, the complexities of the proposed method without APO-VBLAST and with APO-VBLAST are 3442 and 27536 flops, respectively.

As the system has an error performance close to the interference free system, its capacity approaches the sum capacity of individual interference free links. The proposed method can be applied to reduce interference and thus increase the capacity of co-working WLANs.
6.4 Numerical Results and Discussion

and cellular mobile networks. In typical cellular networks or WLANs, there could be only one user transmitting in the same frequency band at a given time slot. The proposed method enables K base stations in the same location to simultaneously transmit to K users in the same frequency band at a given time slot. By using the proposed method, instead of transmitting to one user at one time, we can simultaneously transmit to K users with the performance of each user approaching an interference free performance. As a result, the capacity of both WLANs and cellular mobile networks can be increased by up to K times.
Chapter 7

Cooperative Precoding and Beamforming for Single/Multi-Stream Multi-User Multiple-Input-Multiple-Output Systems

In this chapter, we propose a method to design a single/multi-stream multi-user MIMO cooperative downlink transmission scheme employing precoding and beamforming. Here, as mentioned in Chapter 3, Tomlinson Harashima precoding (THP) is used to cancel the front-channel interference. Thus, we are left with the rear-channel interference which needs to be cancelled by the transmit-receive antenna weights. An iterative method based on the uplink-downlink duality principle [1, 18, 19] is used to generate the transmit-receive antenna weights. The algorithm provides an equal SINR across all users. A simpler method is then proposed by trading off the complexity with a slight performance degradation. The proposed methods are extended to work when the receiver does not have complete Channel State Informations (CSIs).

Finally, we propose a new method of setting the user precoding order, which has a much lower complexity than APO-VBLAST but with almost the same performance. To avoid confusion we refer to the new low complexity APO as APO-LC. The simulation results later show that the proposed schemes considerably outperform existing cooperative transmission schemes in terms of SER performance and approach an interference free performance.
7.1 Iterative Antenna Weights and Power Allocation Optimization

In this section, we propose a joint transmit-receive antenna weights optimization method and power allocation to cancel the rear-channel interference, while maximizing the SINR for each link and maintaining the same SER for all links at all times. To do this, we use the fact that, 1) we can set $E[vv^H] = E[uu^H] = I$ and, 2) the effect of vector $d$ on the received signals is completely removed by the THP decoder modulo operation.

By using (3.19), the received downlink SINR for the $s^{th}$ transmitted symbol in link $j$ can then be written as

$$\text{SINR}_{\text{down}} = \frac{p_{s,j}r_{s,j}^H \tilde{H}_j t_{s,j} (\tilde{H}_j t_{s,j})^H r_{s,j}}{z}$$

where

$$z = \sum_{l=1}^{S} \sum_{i=j+1}^{K} p_{l,i} \| r_{l,i}^H \tilde{H}_j t_{l,i} \|^2 + \sum_{l=s+1}^{S} p_{l,j} \| r_{l,j}^H \tilde{H}_j t_{l,j} \|^2 + 1.$$  

$\tilde{H}_j = \frac{H_j}{\sqrt{\sigma_j}}$ and $\sigma_j$ are the interference term, normalized channel matrix and the MS receiver noise for link $j$, respectively.

The operation of maximizing the minimum SINR for each symbol, while maintaining it equal for all links, can be formulated as follows

$$\max_{r_{s,j}, t_{s,j}, P_{s,j}} \min \\text{SINR}_{\text{down}}$$

subject to

1. $t_{s,j}^H t_{s,j} = 1$
2. $r_{s,j}^H r_{s,j} = 1$
3. $1^T p = P_{\text{max}}$

for $j = 1, ..., K, s = 1, ..., S$. $P_{\text{max}}$ and $p = P^2 1$ are the power constraint at the cooperative transmitter and the set of downlink powers assigned to each symbol stream, respectively. Here, $P = \text{Diag}(P_1, ..., P_K)$, $P_j = \text{Diag}(\sqrt{p_{1,j}}, ..., \sqrt{p_{S,j}})$ where $p_{s,j}$ is the power allocated to the $s^{th}$ symbol $v_{s,j}$ in link $j$. The objective of (7.3) is to maximize the minimum SINR for each stream. The first, second and third constraints in (7.3) ensure that the transmit-receive weights vectors are unitary vectors and the sum of the power allocated to each link does not exceed the maximum power available at the transmitter. These constraints will
bound the possible solution for \( R, T, \) and \( P \) and ensure the convergence of (7.3) to a solution. Here, to maximize the minimum SINR in (7.3), we reduce the SINR of the best link until the SINRs of all links are equal. Thus, the optimal solution is reached when all links attain equal SINR, denoted by \( SINR_{down} \).

This optimization problem, however, is difficult to solve as the transmit weights vectors and the power for each link in (7.1) are entangled with each other. To solve (7.3), we use the uplink-downlink duality concept described in [1, 18, 19, 26]. The authors have shown that the downlink SINR can be designed to be equal to the maximum uplink SINR under the same total available power. Note that the optimum power allocations in the downlink and uplink channels are different.

### 7.1.1 Applying the Duality Concept for Designing Transmit-Receive Antenna Weights and Power Allocation

To apply the duality concept, we create a virtual uplink and swap the role of the transmitter and the receiver. In the virtual uplink, the receiver of a MS acts as a transmitter. The MS previously ordered in link \( j \), now transmits \( S \) virtual symbol streams \( \tilde{u}_j = [u_{1,j}...u_{S,j}]^T \) in link \( j \) using its receive weights vector (e.g., \( R_j \)) to the BS. The BSs then act as a single cooperative receiver and process the signal by using its transmit weights vectors (e.g., \( T \)).

We let the virtual received signal at the BSs be \( \tilde{y}_{up} = [\tilde{y}_1...\tilde{y}_K]^T \), \( \tilde{y}_j = [\tilde{y}_{1,j}...\tilde{y}_{1,S}]^T \) where \( \tilde{y}_{s,j} \) is the \( s^{th} \) virtual uplink received symbol transmitted in link \( j \) and let the transmitted symbols for \( K \) links be \( \tilde{u} = [\tilde{u}_1...\tilde{u}_K]^T \). Here, we want to use the normalized channel matrix \( \tilde{H}_j \) to represent the channel. Thus we need to scale the virtual uplink signal by multiplying each \( j^{th} \) link with the inverse of the receiver noise, \( \sqrt{\sigma_j} \). Note that this scaling will not alter the solution.

By using (3.19), the virtual received uplink signal can then be written as

\[
\tilde{y}_{up} = \text{Diag}(\frac{1}{\sqrt{\sigma_1}}, ..., \frac{1}{\sqrt{\sigma_K}}) \cdot (DQ\tilde{u} + B^HQ\tilde{u} + \text{Diag}(T_H^1, ..., T_K^H)\tilde{N})
\]

(7.4)

, where \( Q = \text{Diag}(Q_1...Q_K) \), \( Q_j = \text{Diag}(q_{1,j}...q_{S,j}) \). \( q_{s,j} \) denotes the virtual uplink
7.1 Iterative Antenna Weights and Power Allocation Optimization

power allocated to the $s^{th}$ virtual uplink symbol in link $j$. $\tilde{N} = [\tilde{n}_1...\tilde{n}_K]^T$, where $\tilde{n}_j \in \mathbb{C}^{N_{BS}}$ is the virtual receiver noise for link $j$ at BSs, modelled as an AWGN with a zero mean and the variance $\sigma_j^2$. The virtual uplink configuration is shown by broken arrows in Fig. 3.2.

By using (7.4), the virtual uplink SINR for each symbol, in each link, can be written as

$$\text{SINR}_{up}^{s,j} = \frac{q_{s,j} t_{s,j}^H \tilde{H}_{s,j} \tilde{H}_{s,j}^H R_{s,j} (\tilde{H}_{s,j}^H R_{s,j})^H t_{s,j}}{z}$$  \hspace{1cm} (7.5)

where

$$z = \sum_{l=1}^{S} \sum_{i=1}^{j-1} q_{l,i} \| t_{s,j}^H \tilde{H}_{j}^H r_{l,i} \|^2 + \sum_{l=1}^{s-1} q_{l,j} \| t_{s,j}^H \tilde{H}_{j}^H r_{l,j} \|^2 + 1$$  \hspace{1cm} (7.6)

for $s = 1,...S, j = 1,..., K$. There are three terms in the denominator of (7.5). The first, second and third terms denote the inter-link interference, the inter-stream interference and the normalized AWGN noise, respectively.

The optimization problem can then be written as

$$\max \min \begin{cases} r_{s,j}, t_{s,j}, q_{s,j} \\ \text{subject to} \end{cases} \text{SINR}_{up}^{s,j}$$

$$\begin{align*}
(1) & \quad t_{s,j}^H t_{s,j} = 1, \\
(2) & \quad r_{s,j}^H r_{s,j} = 1 \\
(3) & \quad 1^T q = P_{max}
\end{align*}$$  \hspace{1cm} (7.7)

for $j = 1,..., K, s = 1,..., S$ and $q = Q^2 1$. Here, the optimal solution is reached when all links attain equal SINR, denoted by $\text{SINR}_{up}^{\text{max}}$.

Here, we propose an iterative solution that splits the problem into a 2-step optimization. Here, the first step is to solve $R$ and $T$, when the inter-link interference power to link $j$, $q_{s,i}\neq j$, $s = 1,..., S, i = 1,..., K$ are fixed to certain values, while setting $q_{s,j} = 1, s = 1,..., S$. For each link, we first find transmission spaces that have the minimum inter-link interference and then use this transmission space to design the transmit-receive weights vectors within each link.

The second step is to solve $q$ in a way that equalizes SINR for all links under fixed $R$ and $T$. The process is then repeated until the SINRs converge to a solution and is summarized in the following lemma as follows,
7.1 Iterative Antenna Weights and Power Allocation Optimization

![Diagram of 2-step Optimization Process](image)

**Figure 7.1: 2-step Optimization Process to find **$\mathbf{R}, \mathbf{T}, \mathbf{q}$**

**Lemma 1** The optimum uplink SINRs, $\text{SINR}_{s,j}^{up}$, $j = 1, ..., K$, $s = 1, ... S$, obtained by solving (7.7), using the proposed 2-Step Optimization, converge to a local maximum of (7.7).

The proof of the 2-step optimization convergence is shown in Appendix F and its operation is described in Fig. 7.1.

### 7.1.1.1 Iterative Transmit-Receive Weights Design

In the first step, we create a transmission space that has the minimum sum of the inter-link interference and receiver noise. We need to find orthonormal vectors that maximize (7.5) without the inter-stream interference term.

By denoting these orthonormal vectors as $\mathbf{T}_j = [\mathbf{t}_{1,j}, ..., \mathbf{t}_{S,j}]$, we can write this problem as

$$
\max_{\mathbf{T}_j} \text{trace} \left( \frac{\mathbf{T}_j^H \mathbf{H}_j^H (\mathbf{H}_j^H \mathbf{R}_j^T \mathbf{H}_j^T) \mathbf{T}_j^H}{\mathbf{T}_j^H \mathbf{R}_j \mathbf{T}_j} \right)
$$

(7.8)

where

$$
\mathbf{R}_j = \sum_{i=1}^{j-1} \mathbf{H}_i^H \mathbf{R}_i \mathbf{Q}_i (\mathbf{H}_i^H \mathbf{R}_i)^H + \sigma^2 \mathbf{I}
$$

(7.9)

is the summation of the inter-link interference and the AWGN in link $j$. Note that since the interference power $\mathbf{Q}_{i\neq j}$ for each link $j$ in the first step of the 2-step optimization is fixed, (7.8) becomes a standard generalized eigenvalue problem and can be solved using standard methods [33, 54],

$$
\mathbf{H}_j^H \mathbf{H}_j \mathbf{T}_j = \Lambda_j \mathbf{R}_j \mathbf{T}_j
$$

(7.10)
where $\Lambda_j = \text{diag}(\lambda_1...\lambda_S)$. $\lambda_s$ is the $s^{th}$ largest eigenvalue of

$$\bar{R}_j^{-1}\bar{H}_j^H\bar{H}_j.$$  \hspace{1cm} (7.11)

$\bar{T}_j$ denotes the $S$ eigenvectors associated with these eigenvalues and represents the solution of the optimization problem solved in (7.8).

We then project channel $\bar{H}_j$ into the transmission space for link $j$, $\bar{T}_j$, to obtain an effective channel, $\hat{H}_j$,

$$\hat{H}_j = \bar{H}_j\bar{T}_j, \quad j = 1, ..., K. \hspace{1cm} (7.12)$$

Here, by transmitting along $\hat{H}_j$ we can ensure that (7.8) is maximized.

To cancel the inter-stream interference while still maintaining an equal SINR across $S$ symbols within each link and the lower triangular structure required by THP [42], we use the Geometric Mean Decomposition (GMD) method from [24, 69]. The GMD is used to decompose $\hat{H}_j$ to a lower triangular matrix with an equal diagonal component. The process is as follows. We first decompose $\hat{H}_j$ by using the Singular Value Decomposition (SVD) [33]. This is given as

$$\hat{H}_j = [U_S \ U_0] \begin{pmatrix} D_S & 0 \\ 0 & D_0 \end{pmatrix} [V_S \ V_0]^T, \hspace{1cm} (7.13)$$

where $U_S$ and $V_S$ consist of the first $S$ left and right singular vectors for link $j$. $D_S$ is a diagonal matrix with entries being the first $S$ non-negative square roots of the eigenvalues of $\hat{H}_j\hat{H}_j^H$.

The GMD takes $U_S$, $V_S$ and $D_S$ as inputs and produces $\tilde{U}_j$, $\tilde{V}_S$ and $\tilde{D}_S$. Here, the GMD transforms $D_S$ into a lower triangular matrix with equal diagonal entries, $\tilde{D}_S$, by rotating $U_S$ and $V_S$. This is given as

$$\tilde{H}_j T_j = \tilde{U}_j \tilde{D}_S \tilde{V}_S^H. \hspace{1cm} (7.14)$$

The goal of our transmit-receive design is to create a lower triangular structure within each link. Thus, by using (7.14), the unitary transmit-receive weights matrix (or a single vector
7.1 Iterative Antenna Weights and Power Allocation Optimization

for a single-stream transmission) can be written as

$$T_j = \begin{cases} \hat{t}_{i,j}, S = 1 \\ T_j \tilde{V}_S D_i T^{-1/2} (\tilde{V}_S^H \bar{T}_j^H \tilde{V}_S) , S > 1 \end{cases}$$

and

$$R_j = \tilde{U}_j , S = 1, ..., K.$$  

(7.16)

Note that it is obvious that if $S = 1$, $[V_S \ V_0]^T$ in (7.13) is a scalar since $\hat{H}_j \in C^{N \times M_S}$. Thus, we can use $\hat{t}_{1,j}$ and $U_1$ directly as the transmit-receive weights in (7.15) and (7.16).

To solve (7.8), however, we also need to know $R_{i=1...,j-1}$. We utilize the fact that link 1 does not have any interference coming from other links. Thus, we start by designing the transmission space and the transmit-receive weights vectors for the first link. We then design the transmission space and the transmit-receive weights vectors for the second link by treating the first link as interference and so on.

7.1.1.2 Power Allocation

In the second step, we use $R$ and $T$ obtained in the first step to find $q$ so that the uplink SINRs for all links are equalized. We let $[\tilde{M}]_{i,j} = \tilde{M}^{i,j}, i = \tilde{i} = 1, ..., KS$ be the $(\tilde{i}, \tilde{i})$th component of matrix $\tilde{M}$ where $\tilde{M} = D_i T(\bar{R}^H \bar{T}) + U p T(\bar{R}^H \bar{T}), \bar{H} = [\bar{H}_1 ... \bar{H}_K]^T$. By using (7.5), the uplink SINRs for all links can then be written as

$$A^{-1}B^H q + A^{-1}1 = \frac{q}{SINR_{up}}$$  

(7.17)

where $A = D_i T(M), B = U p T(M)$ and $M$ is a $KS$ by $KS$ matrix with its component $[M]_{i,j}$ equal to $[\tilde{M}]_{i,j}^2$.

By multiplying both sides of (7.17) with $1^T$, we obtain [18]

$$\frac{1}{P_{max}} (1^T A^{-1}B^H q + 1^T A^{-1}1) = \frac{1}{SINR_{up}}$$  

(7.18)

where $1^T q = P_{max}$. 

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7.1 Iterative Antenna Weights and Power Allocation Optimization

By defining the extended uplink power vector $\mathbf{q}_e = [\mathbf{q}^T 1]^T$, we can then combine (7.17) and (7.18) to obtain an equations matrix given as [18]

$$\Psi \mathbf{q}_e = \frac{\mathbf{q}_e}{\text{SINR}_{\text{up}}}$$  \hspace{1cm} (7.19)

where

$$\Psi = \begin{pmatrix} A^{-1} \mathbf{B}^H & A^{-1} \mathbf{1} \\ 1^T A^{-1} \mathbf{B}^H / P_{\text{max}} & 1^T A^{-1} \mathbf{1} / P_{\text{max}} \end{pmatrix}.$$  \hspace{1cm} (7.20)

Hence, the optimum virtual uplink power $\mathbf{q}$ can be obtained by selecting $\mathbf{q}_e$ that corresponds to the maximum eigenvalue of $\Psi$. This is the only possible solution for (7.19) satisfying $q_{s,j} \geq 0$ for $s = 1, \ldots, S, j = 1, \ldots, K$ and $\text{SINR}_{\text{up}} \geq 0$. The proof is described in detail in Theorem 1 and 2 of [66]. We then repeat the process in the first step by using $\mathbf{q}$ found in the second step. At a first glance, the power allocation in (7.19) resembles Perron-Frobenius (Eigen-based) power control in [67]. In [67], the aim for the power allocation is to minimize the transmission power and to achieve equal SINR across all links. There is no constraint on the transmission power. On the other hand, the power allocation in (7.19) takes into account the power constraint at MSs. Thus, it is more realistic than [67].

Here, we actually apply the uplink-downlink duality concept by stating that the $\text{SINR}_{\text{up}}$, achievable in all virtual uplinks by using the calculated transmit-receive weights in (7.15) and (7.16), are also achievable in the downlink transmission. This leads to the following lemma,

**Lemma 2** The achievable virtual uplink SINR for all users, $\text{SINR}_{\text{up}}$ in (7.7), is always equal to the achievable downlink SINR in (7.3), $\text{SINR}_{\text{down}}$, provided the total power constraints for both the virtual uplink and downlink are equal.

The proof of Lemma 2 is shown in Appendix G. We can conclude from Lemma 1 that the virtual uplink SINR is optimal and that this optimal uplink SINR can also be obtained for the downlink channel. Thus, $\text{SINR}_{\text{down}}$ is optimal.

Since $\text{SINR}_{\text{up}}$ is an optimal solution, we can then use $\mathbf{R}$, $\mathbf{T}$ and $\mathbf{q}$, obtained from the iterative process, to find the optimum downlink power $p$. The optimum downlink power can be written in terms of the transmit-receive weights and the virtual uplink power. It is given
7.1 Iterative Antenna Weights and Power Allocation Optimization

Table 7.1: Algorithm I-APO-LC-Full CSI

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Set the precoding order for users by using APO-LC (7.38)</td>
</tr>
<tr>
<td>1</td>
<td>Initialize $q = 0$</td>
</tr>
<tr>
<td>2</td>
<td>For $i = 1$ to Maxit</td>
</tr>
<tr>
<td>3</td>
<td>For link $j = 1$ to link $KS$</td>
</tr>
<tr>
<td>4</td>
<td>Create transmission space for link $j$ by using (7.8)</td>
</tr>
<tr>
<td>5</td>
<td>Obtain $\hat{H}_j$ by using (7.12)</td>
</tr>
<tr>
<td>6</td>
<td>Obtain $R_j$ and $T_j$ by using (7.15) and (7.16)</td>
</tr>
<tr>
<td>7</td>
<td>end</td>
</tr>
<tr>
<td>8</td>
<td>Obtain $q$ by using (7.19)</td>
</tr>
<tr>
<td>9</td>
<td>end</td>
</tr>
<tr>
<td>10</td>
<td>Obtain $p$ by using (7.21)</td>
</tr>
<tr>
<td>11</td>
<td>THP Precoding Operation by using (3.6)</td>
</tr>
</tbody>
</table>

as

$$p = \hat{P}q$$  \hspace{1cm} (7.21)

where

$$\hat{P} = \left(\frac{A}{\text{SINR}^{\text{down}}} - B\right)^{-1}\left(\frac{A}{\text{SINR}^{\text{up}}} - B^H\right).$$  \hspace{1cm} (7.22)

The proof of (7.21) is shown in Appendix H.

The complete algorithm is tabulated in Table 7.1, where $i$, Maxit represents the iteration number and the maximum number of iterations. We refer to the combination of the described Algorithm I, APO-LC (which will be explained in more detail in a later section), and THP precoding as AI-APO-LC-Full CSI.

7.1.2 Simplification of the Duality Concept Implementation

In this section, we propose a simplification of Algorithm I. We refer to the simplified method as Algorithm II. In Algorithm II, we use the uplink-downlink duality concept to find $R_j$ and $T_j$ while setting the virtual uplink power for the $s^{th}$ symbol in link $j$ as defined in Section 7.1.1, $q_{s,j}$ to be equal. For simplicity, we also assume, $P_{\text{max}} = KS$. Thus we have $Q = \text{Diag}(Q_1...Q_K) = \text{Diag}(q_{1,j}...q_{S,j}) = I$. Here by fixing the virtual uplink power $Q = I$, we do not need to find $Q$ and $T_{j=1,...,K}$ iteratively as in Algorithm I.
Table 7.2: Algorithm II-APO-LC-Full CSI

0. Set the precoding order for users by using APO-LC (7.38)
1. For link $j = 1$ to link $K_S$
2. Create transmission space for link $j$ by using (7.26)
3. Obtain $\bar{H}_j$ by using (7.12)
4. Obtain $R_j$ and $T_j$ by using (7.15) and (7.16)
5. end
6. Obtain $p$ by using (7.21)
7. THP Precoding Operation by using (3.6)

By using the fact above, the uplink SINR in (7.5) can now be expressed as

$$SINR_{s,j}^{up} = \frac{t_{s,j}^H \bar{H}_j^H r_{s,j}(\bar{H}_j^H r_{s,j})^H t_{s,j}}{z}.$$  \hfill (7.23)

where

$$z = \sum_{l=1}^{S} \sum_{i=1}^{j-1} \|t_{s,j}^H \bar{H}_l^H r_{l,i}\|^2 + \sum_{l=1}^{s-1} \|t_{s,j}^H \bar{H}_j^H r_{l,j}\|^2 + 1.$$  \hfill (7.24)

The optimization problem in (7.7) can then be written as

$$\max \min \quad SINR_{s,j}^{up}$$

subject to \hspace{1cm} (1) $t_{s,j}^H t_{s,j} = 1$, \hspace{1cm} (2) $r_{s,j}^H r_{s,j} = 1.$  \hfill (7.25)

The process of constructing transmission space for each link $j$ using $\bar{T}_j$ can then be written as

$$\max \quad \text{trace} \frac{T_j^H \bar{H}_j^H (\bar{H}_j^H)^H T_j}{T_j^H R_j T_j}.$$  \hfill (7.26)

where $\bar{R}_j$ is as defined in (7.9). Once the transmission space is obtained, we can find $R$, $T$, and $P$ by using the same procedure as that described in Section 7.1.1.

The complete algorithm is tabulated in Table 7.2. We refer to the combination of Algorithm II, the APO-LC, and THP precoding as All-APO-LC-Full CSI.
7.2 Limited Channel State Information at the Receiver

Algorithms I and II work by jointly designing \( R \) and \( T \). This joint design process requires MSs to either, 1) know the complete CSI, \( H_{k=1,...,K} \) to be able to design \( R \) or, 2) receive the information about \( R \) from BSs. This condition increases the network cost and reduces its spectral efficiency. In this section, we want to address the limitation of Algorithms I and II by trading off the network complexity/spectral efficiency with a slight performance degradation.

We aim to eliminate the requirement for complete CSI for the algorithms described in Section 7.1 by separating the design of \( R \) and \( T \). We assume that \( N_{MS_k} \geq S, k = 1, ..., K \) and use the following assumptions, 1) if each MS \( k \) knows its own \( H_k \), we can independently design the receive weights vector for each symbol ,2) if BSs know all CSIs, \( H_{k=1,...,K} \), then BSs know the receive weights vectors used by MSs.

First, we describe how to design the receive weights vectors for \( KS \) symbols. The receive weights for a link \( j \) denoted by \( [r_{1,j}...r_{S,j}] \), can be designed as follows

\[
[r_{1,j}...r_{S,j}] = \begin{cases} 
  r_{SVD}(\bar{H}_j, S), S = 1 \\
  r_{GMD}(\bar{H}_j, S), S \neq 1 
\end{cases}
\]  

(7.27)

where \( r_{SVD} \) and \( r_{GMD} \) are the SVD [33] and GMD [24] operations to extract \( S \) left eigenvectors, respectively. Thus, the \( s^{th} \) eigenvector is denoted as \( r_{s,j} \).

In Algorithms I and II, the transmit-receive weights are jointly designed. Thus, we can suppress interference by using (7.14). Here, however, the receive weights vectors are fixed independently in (7.27) and the task of cancelling interference lies with the transmit weights vectors. Now, we create a transmission space for symbol stream \( s \) in link \( j \) that has the minimum inter-link interference under fixed receive weights. This can be represented as

\[
\max_{\bar{t}_{s,j}} \frac{\bar{t}_{s,j}^H \bar{H}_j^H r_{s,j}^H \bar{H}_j \bar{t}_{s,j}}{\bar{t}_{s,j}^H \bar{R}_j \bar{t}_{s,j}}
\]

(7.28)

where \( \bar{R}_j \) is

\[
\bar{R}_j = \sum_{i=1}^{j-1} \bar{H}_i^H R_i Q_i (\bar{H}_i^H R_i)^H + \sum_{l=1}^{s-1} q_{l,j} \bar{H}_j^H r_{l,j} r_{l,j}^H \bar{H}_j + I.
\]

(7.29)
7.2 Limited Channel State Information at the Receiver

(7.28) is a standard generalized eigenvalue problem as in (7.10) and can be solved using standard methods [54],

\[ \bar{H}_{j}^{H} r_{s,j} r_{s,j}^{H} \bar{H}_{j} t_{s,j} = \Lambda_{j} \overline{R}_{j} t_{s,j}. \]  

(7.30)

Once all \( t_{s,j}, s = 1, ..., S, j = 1, ..., K, \) are obtained, we arrange these vectors as \( \bar{T}_{j} = [t_{1,j}...t_{S,j}] \). Here \( \bar{T}_{j} \) consist of the orthonormal vectors defining the transmission space for link \( j \).

The channel \( H_{j} \) is then projected into the transmission space for link \( j, \bar{T}_{j} \), to obtain an effective channel \( \hat{H}_{j} \),

\[ \hat{H}_{j} = \bar{H}_{j} \bar{T}_{j}, \quad j = 1, ..., K. \]  

(7.31)

In the previous algorithms, since the transmit-receive weights are jointly designed, we can use left and right eigenvectors as in (7.15). Here, however, since \( R_{j} \) is fixed, we can only use the right eigenvectors to triangularize the channel \( \bar{H}_{j} \). We use the QR decomposition [33] to find these eigenvectors and arrive at

\[ R_{j}^{H} \bar{H}_{j} \bar{T}_{j} = \hat{D}_{S} \bar{V}_{S}^{H} \]  

(7.32)

where \( \hat{D}_{S} \) and \( \bar{V}_{S}^{H} \) are a lower triangular matrix and a unitary matrix obtained by applying the QR decomposition [54] to \( (R_{j} \bar{H}_{j} \bar{T}_{j})^{H} \). The unitary transmit-receive weight matrix can be written as

\[ T_{j} = \begin{cases} \frac{1}{\| \bar{t}_{s,j} \|}, & S = 1 \\ \bar{T}_{j} \bar{V}_{S} D DiT^{-\frac{1}{2}} (\bar{V}_{S}^{H} \bar{T}_{j}^{H} \bar{T}_{j} \bar{V}_{S}), & S > 1. \end{cases} \]  

(7.33)

Note that the unitary transmit-receive weight matrix is in fact a vector for a single-stream transmission.

For multi-stream transmission, we note that to find the transmit weights vectors for each link, we need to solve (7.28) \( S \) times per link. Here, we propose a simpler method to compute the transmission space in (7.28). We want to compute \( \bar{T}_{j} = [t_{1,j}...t_{S,j}] \) in a single step. To do that, we rewrite (7.28) as

\[ \max_{\bar{T}_{j}} \operatorname{trace} \frac{T_{j}^{H} \bar{H}_{j}^{H} \bar{H}_{j} \bar{T}_{j}}{T_{j}^{H} R_{j} \bar{T}_{j}} \]  

(7.34)

where we use \( \bar{R}_{j} \) defined in (7.9) instead of (7.29).
We also observe that the number of receive weights vectors, available for each link, depends on the rank of $H_j$, $\tilde{S}$. Thus, for a single-stream transmission ($S = 1$), $r_{1,j}$ can be selected from $\tilde{S}$ possible receive weights vectors. We want to utilize this fact to obtain a better performance for a single-stream transmission. We will select the combination of transmit and receive weights that gives the maximum gain

$$\max_{r_{s,j}, t_{s,j}} \| r_{s,j}^H \tilde{H}_{s,j} t_{s,j} \| , \quad s = 1, \ldots, \tilde{S}$$

(7.35)

where each $t_{s,j}$ is computed by using (7.30) for a fixed $r_{s,j}$.

Note that the problem we are trying to address here are identical with the one in Chapter 5. The solutions, however, are fundamentally different. The uplink-downlink duality principle states that a problem can be viewed from two perspectives, the multi-user downlink transmission (e.g., broadcast) or the multi-user uplink transmission (e.g., multiple access). In the uplink transmission, as seen in (7.5), the uplink performance of link $j$ depends only on its own transmit weights. Thus, it is simple to formulate the optimal linear receiver that maximizes the output SINR. Obviously, the solution here is the MMSE receivers. On the other hand, in the downlink transmission, as seen in (7.1), the SINR for link $j$ depends on the transmit weights for other links.

Here, we solve the transmit weights for each link as if they are receive weights for uplink transmission and represent it as an eigenvalue problem. In Chapter 5, however, we directly solve the BSs transmit weights for each link and represent it as an eigenvalue problem. Apart from that, here, we also take into consideration the direction of receive weights at MSs when we calculated the BSs transmit weights as shown in (7.34). This is so since we are treating these receive weights at MSs as if they are transmit weights. In Chapter 5, however, from (5.9) and (5.6) it is obvious that the SINRM solution does not do this. Thus, the solutions in Chapters 7 and 5 are very different.

The procedure described in this section can be implemented iteratively by using Algorithm I and non-iteratively by using Algorithm II. We refer to the former as AI-APO-LC-Limited CSI and the latter as AII-APO-LC-Limited CSI. The complete algorithms are tabulated in Tables 7.3 and 7.4. Note that the proof of the 2-step optimization convergence in AI-APO-LC-Limited CSI can be obtained by using the same procedure shown in Appendix F with $r_{s,j}$ fixed in every iteration.
Table 7.3: Algorithm I-APO-LC-Limited CSI
0 Set the precoding order for users by using APO-LC (7.38)
1 Initialize $q = 0$ and $R$ by using (7.27)
2 **For** $i = 1$ to **Maxit**
3 **For** link $j = 1$ to link $KS$
4 Create transmission space for link $j$ using (7.34)
5 Obtain $\bar{H}_j$ using (7.31)
6 **If** $S = 1$
7 Obtain $r_{s,j}$ that give optimum gain by using (7.35)
8 **End**
9 **End**
10 Obtain $q$ using (7.19)
11 **End**
12 Obtain $p$ by using (7.21)
13 THP Precoding Operation using (3.6)

Table 7.4: Algorithm II-APO-LC-Limited CSI
0 Set precoding order for users by using APO-LC (7.38)
1 $R$ using (7.27)
2 **For** link $j = 1$ to link $KS$
3 Create transmission space for link $j$ by using (7.34)
4 Obtain $\bar{H}_j$ by using (7.31)
5 **If** $S = 1$
6 Obtain $r_{s,j}$ that give optimum gain by using (7.35)
7 **End**
8 Obtain $q$ using (7.19)
9 **End**
12 Obtain $p$ by using (7.21)
13 THP Precoding Operation using (3.6)
In this section, we propose a new user ordering method where the SINR is maximized. In addition, the new user ordering method is much less complex than the adaptive precoding order proposed in Chapter 6.2. Here, we use the concept of the matrix condition number which is defined as [54]

\[ \kappa(U) = \frac{\lambda_{\text{max}}(U)}{\lambda_{\text{min}}(U)} \]  

(7.36)

where \( U \) is an arbitrary matrix. \( \lambda_{\text{max}} \) and \( \lambda_{\text{min}} \) are the maximum and the minimum eigenvalues of \( U \), respectively. Thus, a large \( \kappa(U) \) means the vectors in \( U \) are concentrated in a single direction while a small \( \kappa(U) \) means that the vectors of \( U \) are scattered.

To apply this concept, we first create an interference channel matrix for each link \( j \), denoted by \( \tilde{H}_j \), as follows,

\[ \tilde{H}_j = [\bar{H}_1...\bar{H}_{j-1}\bar{H}_{j+1}...\bar{H}_K]^T. \]  

(7.37)

Let \( (c_1, c_2, ..., c_K) \) be the precoding order for users prior to the THP, where \( c_j \) denotes the \( k^{th} \) user ordered in link \( j \). The precoding is selected by using,

\[ (c_1, c_2, ..., c_K) = \text{sort}(\kappa(\tilde{H}_1), ..., \kappa(\tilde{H}_K)) \]  

(7.38)

where \( \text{sort} \) is used to order the matrix condition in the ascending order. If user \( k \) is ordered as the \( j^{th} \) element of \( (\kappa(\tilde{H}_1), ..., \kappa(\tilde{H}_K)) \), then \( c_j = k \). To update the permutation matrix, \( M_{\text{perm}} \) to reflect the new user ordering, we simply replace the \( k^{th} \) column of the permutation matrix \( M_{\text{perm}} \) with the \( j^{th} \) column of \( M_{\text{perm}} \), for \( k = 1, ..., K \). We refer to this low complexity adaptive precoding order as APO-LC.

7.4 The Advantages of the Proposed Over Other Known Schemes

In this section, we discuss the advantages of the proposed scheme over other existing schemes. We first compare the proposed methods with the linear precoding scheme in [39] and nonlinear THP precoding schemes in [41] and [42]. Here, we modify the original null space in [39]
7.4 The Advantages of the Proposed Over Other Known Schemes

to incorporate the THP precoding. For a fair comparison with the proposed method, a power allocation scheme that allocates power such that SINRs for all links are equalized [45], is applied to these schemes. The water-filling power allocation used in [39, 41, 42] tends to assign more power to stronger links and less power to weaker links. Hence, the performance of a weaker link will decrease the overall SINR for all links.

The main differences between the proposed algorithms with the schemes in [39, 41, 42] are, 1) the relaxation of the zero forcing and orthogonality constraint for the transmit weights vector, 2) the transmit-receive weights can be found by using an iterative process, 3) the effect of the receiver noise is taken into consideration when designing the transmit-receive weights and allocating power, 4) the proposed methods can work when the receiver for link \( j \) only knows its own CSI, \( \mathbf{H}_j \) and 5) unlike [41, 42], we do not require the constraint of \((K - 1)N_{MS} < N_{BS}\) since no null spaces are created. Here, we assume that each MS has the same number of antennas \( N_{MS} = N_{MS_1} = \ldots = N_{MS_K} \). The fifth difference is a definite advantage since to support say 5 users with \( N_{MS} = 4 \), the proposed method only needs 5 transmit antennas, while [41, 42] need 12 transmit antennas.

The computational complexity in terms of the number of floating point operations (flops) for the proposed schemes and the schemes [39, 41, 42] are listed in Table 7.5 where \( \tilde{L} = 1 \) when \( S \neq 1 \) and \( \tilde{L} = N_{MS} \) when \( S = 1 \). To analyze this algorithm, we make a practical assumption that the number of transmit antennas is always greater than the number of receive antennas at each MS, \( N_{BS} > N_{MS} \). This is always true in practical cooperative wireless networks. We then denote the complexity order of the proposed algorithms as \( C_x \), where \( x \) denotes which scheme is used. Now, from Table 7.5, we can obtain the complexity order of our algorithms, \( C_{AI - APO - LC - FullCSI} = O(\text{Maxit} \cdot 30\tilde{L}K\sqrt{N_{BS}}) \), \( C_{AI - APO - LC - LimCSI} = O(30\tilde{L}K\sqrt{N_{BS}}) \), \( C_{AI - APO - LC - FullCSI} = O(\text{Maxit} \cdot 30\tilde{L}K\sqrt{N_{BS}}) \) and \( C_{AI - APO - LC - LimCSI} = O(30\tilde{L}K\sqrt{N_{BS}}) \). By using Table 7.5, the complexity order of methods in [39, 42] is given as \( O(18KN_{BS}^3) \) and \( O(9KN_{BS}^3) \), respectively. In addition, by using Table 6.2, the complexity order for the iterative ZF algorithm discussed in Chapter 6 without APO-LC is given as \( C_{J1 - FullCSI} = O(\text{Maxit} \cdot K\sqrt{N_{BS}N_{MS}}) \). Thus, obviously the proposed algorithms now are more complex than the schemes in [39, 41, 42] and the iterative ZF algorithm discussed in Chapter 6. However, as we will see in a later section, the performance of all proposed algorithms outperforms the schemes in [39, 41, 42] significantly.
### Table 7.5: Computational Complexity of Non-Linear Precoding Algorithms (in Flops)

<table>
<thead>
<tr>
<th>Schemes</th>
<th>Computational Complexity in flops</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm I-APO-LC-Full CSI</td>
<td>$K(K-1)N_{MS}\overline{N}<em>{BS} + L \cdot \text{Maxit} \cdot (30K\overline{N}</em>{BS}^3 + 2KSN_{BS}N_{MS} + 2KS\overline{N}_{BS}$</td>
</tr>
<tr>
<td></td>
<td>$+ 4KN_{MS}^2 + 8KS^2N_{MS} + 9S^3 + N_{MS}S + KS^2 + 30(KS + 1)^3) + 30(KS + 1)^3 + K^2N_{MS}\overline{N}_{BS}$</td>
</tr>
<tr>
<td>Algorithm II-APO-LC-Full CSI</td>
<td>$K(K-1)N_{MS}\overline{N}<em>{BS} + L \cdot (30K\overline{N}</em>{BS} + 2KSN_{BS}N_{MS} + 2KS\overline{N}_{BS}$</td>
</tr>
<tr>
<td></td>
<td>$+ 4KSN_{MS}^2 + 8KS^2N_{MS} + 9S^3 + S_{MS}S + L \cdot \text{Maxit} \cdot (30\overline{N}<em>{BS}^3 + 2KSN</em>{BS}N_{MS}$</td>
</tr>
<tr>
<td></td>
<td>$+ 2KSN_{MS}^2 + 2K^2\overline{N}<em>{BS} + 30(KS + 1)^3) + 30(KS + 1)^3 + K^2N</em>{MS}\overline{N}_{BS}$</td>
</tr>
<tr>
<td>Algorithm I-APO-LC-Limited CSI</td>
<td>$K(K-1)N_{MS}\overline{N}<em>{BS} + 4SN</em>{MS}^2 + 8S^2N_{MS} + 9S^3 + N_{MS}S + S^2 + L \cdot \text{Maxit} \cdot (30\overline{N}<em>{BS}^3 + 2KSN</em>{BS}N_{MS}$</td>
</tr>
<tr>
<td></td>
<td>$+ 2KSN_{MS}^2 + 2K^2\overline{N}<em>{BS} + 30(KS + 1)^3) + 30(KS + 1)^3 + K^2N</em>{MS}\overline{N}_{BS}$</td>
</tr>
<tr>
<td>Spencer et al. (Modified Scheme)</td>
<td>$8KN_{MS}^3\overline{N}<em>{BS} + 16KN</em>{MS}^3\overline{N}<em>{BS} + 18KN</em>{MS}^3\overline{N}<em>{BS} + (2 \sum</em>{i=1}^{K-1} 4i^2S^2\overline{N}<em>{BS} + 8iS\overline{N}</em>{BS}^3 + 9\overline{N}<em>{BS}) + K^2N</em>{MS}\overline{N}_{BS}$</td>
</tr>
<tr>
<td>Liu et al. and Foschini et al.</td>
<td>$4KN_{MS}^3\overline{N}<em>{BS} + 8KN</em>{MS}^3\overline{N}<em>{BS} + 9K\overline{N}</em>{BS} + (N_{MS} + \overline{N}<em>{BS})K + S + (\sum</em>{i=1}^{K-1} 3i^2N_{MS}^2(\overline{N}<em>{BS} - \frac{iN</em>{MS}}{3})) + 2\overline{N}<em>{BS} + 2N</em>{BS}N_{MS} + K^2N_{MS}\overline{N}_{BS}$</td>
</tr>
</tbody>
</table>
7.4 The Advantages of the Proposed Over Other Known Schemes

Now, we compare the proposed ordering method referred to as APO-LC, with APO-VBLAST ordering discussed in Chapter 6 and [48]. In [48], the authors propose the idea of the Myopic Optimization method and prove that this ordering is optimal. With that ordering, to reach the maximum SINR, they only need to search \( \approx \frac{K^2}{2} \) possible orderings. This ordering however, is too complex for the proposed interference cancellation schemes or any other known schemes as shown in Table 7.5. The first term in the computational complexity for AII-APO-LC-Full CSI, AII-APO-LC-Limited CSI, AI-APO-LC-Full CSI, AI-APO-LC-Limited CSI is the computational complexity of the adaptive precoding order.

By letting the complexity order of the proposed algorithms minus the complexity order of APO-LC be \( \hat{C}_x \) where \( x \) again denotes the algorithm name, we could then write the complexity order for the algorithms as

\[
K(K - 1)N_{MS}N_{BS}^2 + \hat{C}_x \quad \text{where we again assume that each user/MS has } N_{MS} \text{ receive antennas. The complexity order of the proposed algorithms when VBLAST ordering is used, is } \frac{K^2}{2} \hat{C}_x. \]

Thus, for the APO to be less complex than the APO-VBLAST ordering, the following condition must be met

\[
K(K - 1)N_{MS}N_{BS}^2 + \hat{C}_x < \frac{K^2}{2} \hat{C}_x \Rightarrow \hat{C}_x > 2N_{MS}N_{BS}^2. \tag{7.39}
\]

Note that however, the complexity for the proposed algorithms \( C_x \), is at least \( C_x = 30KN_{BS}^3 \). We could then further write

\[
30K\overline{N_{BS}^3} > 2N_{MS}N_{BS}^2 \Rightarrow \overline{N_{BS}} > \frac{N_{MS}}{15KS}. \tag{7.40}
\]

This condition in (7.40) is very realistic. As an example, if we have 2 users each receiving 2 symbol streams transmitted from 3 BSs each equipped with 2 antennas, APO-LC will always be less complex than APO-VBLAST as long as the number of receive antennas for each MS is less than 180. Thus, we can conclude that APO-LC is always less complex than APO-VBLAST ordering on any practical condition. In addition to the computational advantage mentioned above, as we will see in a later section, the difference in the performance between APO-LC and APO-VBLAST is at most 1 dB at SER=\(10^{-4}\). Thus, it is not sensible to increase the computational complexity of the transmitter enormously just to achieve a very small gain.
7.5 Numerical Results and Discussion

Monte Carlo simulations have been carried out to assess the performance of the four proposed algorithms. We investigate their performance and compare it with [39, 41, 42] and with an interference free performance.

Here, an interference free performance is defined as the performance of any random single link $i$ assuming there is no interference from other links at all. In this case, the received signal of the cooperative transmission system is given as

$$y_i = R_i^H(H_i^H x_i + n_i)$$  \hspace{1cm} (7.41)

where $R_i$ and $T_i$ are the left and right eigenvectors associated with the $S^{th}$ largest eigenvalues of $H_i H_i^H$ found by using the Singular Value Decomposition (SVD). To generate an interference free performance for multi-stream transmission with $S$ symbols transmitted in each link, we use the left and right eigenvectors associated with the $S$ largest eigenvalues of $H_i H_i^H$ found by using the SVD. We then maximize the minimum SINR of $S$ symbols by applying the power allocation method given in [45].

For convenience, we will use the notations $(N_{BS}, N_{MS}, K, S)$ in all figures to denote the number of transmit antennas of BSs, the number of receive antennas per MS, the number of users, and the number of symbols transmitted per user in the network, respectively. In the simulations, for simplicity, we assume $N_{MS} = 1, ..., K = N_{MS}$ and the receiver noise for $K$ links are equal $\sigma = \sigma_1 = ... = \sigma_K$. A Rectangular 64-QAM ($M=64$) modulation is used for all transmissions. The OFDM symbol period, guard period and number of sub-carriers are set to $3.2\mu s$, $0.8\mu s$ and 48, respectively. The number of paths, RMS delay spread and maximum channel delay are set to 10, 0.16$\mu s$, and 0.8$\mu s$, respectively. The channel is assumed to be a quasi-static channel resulting in a negligible doppler shift. In all simulations, we fix the Signal-to-Noise-Ratio of each THP precoded symbol to $SNR = \frac{2E[v_j^2]}{\sigma^2}$, where $E[v_j^2]$ is normalized to 1 and $P_{max} = KS$. 

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7.5 Numerical Results and Discussion

7.5.1 Convergence Behaviour

Fig. 7.2 shows the convergence characteristics of the proposed method for $(6, 2, 3, 1)$, $(6, 2, 3, 2)$ and $(4, 2, 4, 1)$ systems. We plot the number of iterations versus the SINR down. The channel and the SNRs for the three systems are fixed at a certain realization value and at 27 dB, respectively. Only 3 iterations are required for the SINRs of the three systems to converge. This convergence result confirms Lemma 1.

An interesting observation here is that the SINR convergence seems to be independent of system configurations. The three systems can be seen to converge at the same time after three iterations. As the required number of iterations does not depend on the system configuration, it is possible to fix its value. In all further simulations, we set the maximum number of iterations for the AI-APO-LC-Full CSI and AI-APO-LC-Limited CSI to 3.
7.5 Numerical Results and Discussion

![Graph showing SER comparison](image)

Figure 7.3: Average SER Performance Comparison for (6,2,3,1) System using Various Non-Linear Precoding Algorithms

### 7.5.2 Performance of the Overall Symbol Error Rate

Here, we investigate the performance of the overall SER. The overall SER is defined as the average SER of K links. We first consider a (6, 2, 3, 1) system. In Fig. 7.3, the overall SER performances for the four proposed algorithms are compared with other existing schemes at SER=$10^{-5}$. We compare the proposed algorithms with those in [39, 41, 42]. As [39] is a linear interference cancellation scheme, we modify it to work with THP as described in Section 7.4. Note that all these existing schemes require complete CSIs at both the receiver and transmitter, since the transmit and receive weights are jointly calculated. As can be seen from Fig. 7.3, all four proposed algorithms significantly outperform the existing schemes.

There is no performance loss using All-APO-LC-Full CSI compared to AI-APO-LC-Full CSI. In addition, the performance of these two methods is 0.5 dB away from an interference free performance. These two algorithms outperform the scheme in [39, 41, 42] by more than...
7.5 Numerical Results and Discussion

Figure 7.4: Average SER Performance Comparison for (4,2,4,1) System using Various Non-Linear Precoding Algorithms

4 dB and 2 dB, respectively. The performances of AI-APO-LC-Limited CSI and AII-APO-LC-Limited CSI, designed to work when only partial CSI is available at the receiver, are only 0.3 dB weaker than the performances of AI-APO-LC-Full CSI and AII-APO-LC-Full CSI. It is also only 0.8 dB away from an interference free channel and outperforms the scheme in [39] by 1.5 dB and the scheme in [41, 42] by 3 dB. Note that [39, 41, 42] need complete CSI at the receiver to work.

Here, we also simulate the case where Algorithm I uses VBLAST ordering. This is denoted by AI-APO-VBLAST-Full CSI. AI-APO-VBLAST-Full CSI outperforms AI-APO-LC-Full CSI by 0.2 dB. However as stated earlier, VBLAST ordering increases the system’s complexity significantly in order to obtain this extra 0.2 dB gain. The complexity order of AI-APO-VBLAST-Full CSI and AI-APO-LC-Full CSI is \( \approx 524880 \) flops and \( \approx 117072 \) flops. Thus, Algorithm I with APO-VBLAST is \( \approx 5 \) times more complex than Algorithm I with APO-LC.
Now, we consider the performance of a $(4, 2, 4, 1)$ system at $\text{SER} = 10^{-4}$. In this case, BSs have 4 transmit antennas and BSs transmit a single stream to 4 users. The performances of the proposed algorithms are compared to known schemes. This is shown in Fig. 7.4. All the proposed algorithms outperform [39] significantly. Here, the scheme from [41, 42] cannot work, since the number of transmit antennas, $\overline{N}_{BS} = 4$, is fewer than the total number of receive antennas, $(K - 1)N_{MS} = 6$. Note also that AI-APO-LC-Full CSI is now about 3 dB away from an interference free performance and Algorithm I performs better than Algorithm II for both full CSI and limited CSI scenarios due to the iterative process.

Here, AI-APO-VBLAST-Full CSI also outperforms AI-APO-LC-Full CSI by 1 dB. The complexity order of the former and the latter is $\approx 368640$ flops and $\approx 46848$ flops, respectively. Thus, by using VBLAST ordering, the complexity of Algorithm I is increased by $\approx 8$ times.

Now, we consider the performance of a $(6, 2, 3, 2)$ system when $\text{SER}=10^{-4}$. In this case, BSs
have 6 transmit antennas and BSs transmit 2 symbols each to 3 users. The SER performance is shown in Fig. 7.5. Notice that the performance of AI-APO-LC-Full CSI, AII-APO-LC-Full CSI, AI-APO-LC-Limited CSI in Figs. 7.3, 7.4 and 7.5 differs by less than 1 dB. This is desirable since we only need to give up 1 dB for not requiring full CSI at each receiver. Unfortunately, the performance of Algorithm II under limited CSI in Fig. 7.5 is very far from an interference free performance, even though it still outperforms the schemes in [42]. The reason is that the uplink interference power in link \( j \) \( Q_{i \neq j}, \ i = 1, ..., K \) is fixed when calculating the transmit-receive weights for link \( j \). Thus, we do not search for the optimum \( Q \) as in Algorithm I. In addition, we could not jointly design the transmit-receive weights, which limits the achievable SINR further.

Here, we also simulate the case where Algorithm I uses APO-VBLAST ordering, denoted by AI-APO-VBLAST-Full CSI. This scheme outperforms Algorithm I with APO-LC by 0.25 dB. The complexity order of AI-APO-VBLAST-Full CSI and AI-APO-Full CSI here is \( \approx 262440 \) flops and \( \approx 58536 \) flops, respectively. Thus, Algorithm I with APO-VBLAST is 5 times more complex than Algorithm I with APO.
Chapter 8

Iterative Multiple Beamforming Algorithm for MIMO Broadcast Channels

8.1 Introduction

In all previous chapters, THP is used to cancel the front-channel interference while the transmit-receive antenna weights are used to suppress the rear-channel interference. In this chapter, we take a different approach. Here, we use the transmit-receive antenna weights to cancel both the rear-channel and front-channel interference.

The reason we want to bypass THP is because a nonlinear precoding scheme like THP is data dependent. In other words, the THP precoding needs to be done independently for every symbol even though the wireless channel condition does not change. By bypassing the use of THP, the system does not need to precode every symbol. That means if we have immobile users in the system and a channel that varies slowly, we could potentially save a substantial signal processing cost. This is so since we only need to calculate the transmit-receive weights when the wireless channel condition changes.

In this chapter, we propose a new linear downlink cooperative transmission scheme based on
8.2 System Model and Zero-Forcing

a zero forcing method in a multi-user MIMO system. We consider a MIMO BC equipped with multiple antennas both at the base station and at each mobile terminal. Previous work such as [39,41] where the authors show how a ZF method can be used in a multi-stream multi-user MIMO and [41] does not utilize the iteration process. In [30], it was shown that an iterative method based on the ZF method can perform better than sphere encoding [31]. Unfortunately, the scheme in [30] only works for a single receive antenna.

We extend the concept of ‘coordinated transmit-receive processing’ in [30,39] and propose an iterative multiple beamforming (IMB) algorithm for a multi-user MIMO system, which can be readily deployed in a practical system. Both capacity evaluation and bit error rate (BER) simulations show that the IMB performs much better than the ZF when the system operates at low to moderate data rates.

8.2 System Model and Zero-Forcing

We consider a flat fading MIMO BC with $n_T$ transmit antennas at the base station and $n_R$ receive antennas at each mobile terminal. For frequency selective channels, the proposed algorithm can be used in conjuction with an orthogonal frequency division multiplexing (OFDM) technique and applied to each subband of OFDM symbols. Assuming a total of $K$ users, we use an $n_R \times n_T$ matrix $H_k$ to represent the MIMO channel relating the base station and the $k$-th user. The entries of $H_k$ are assumed to be independent complex Gaussian variables with zero mean and unit variance.

Instead of using time-division (TD) or frequency-division (FD) methods for the transmission to multiple users, we consider space-division (SD) such that the BS transmits all user data simultaneously in the same frequency band at a given time slot. For user $k$, the MIMO channel input and output are represented by a linear model

$$y'_k = H_k x_T + n'_k,$$  \hspace{1cm} (8.1)

where $x_T$ is an $n_T \times 1$ vector representing the input to the channel, while the additive white Gaussian noise (AWGN) and the channel output are denoted as $n_R \times 1$ vectors $n'_k$ and $y'_k$, respectively. The entries of $n'_k$ are also assumed to be i.i.d. complex Gaussian with zero mean and unit variance.
8.2 System Model and Zero-Forcing

mean and variance $\sigma^2$.

Assume $S \leq \min(n_T, n_R)$ parallel data streams are transmitted to each user. For user $k$, we denote the data streams as an $S \times 1$ vector $x_k$, the transmit beamformer as an $n_T \times S$ matrix $W_{Tk}$, and the receive beamformer as an $n_R \times S$ matrix $W_{Rk}$. We can write the equivalent channel model after beamforming as

$$y_k = W_{Rk}^H H_k \sum_{j=1}^{K} W_{Tj} x_j + n_k,$$

(8.2)

where $y_k = W_{Rk}^H y'_k$ and $n_k = W_{Rk}^H n'_k$.

By defining $y = [y_1...y_K]^T$ and $x = [x_1...x_K]^T$, we can write the system model for the MIMO broadcast channel with $K$ users as

$$y = RHTx + Rn,$$

(8.3)

where $R = \text{diag}(W_{R1}^H, ..., W_{RK}^H)$ and $H = [H_1...H_K]^T$, $n = [n_1...n_K]^T$ and $T = [W_{T1}...W_{TK}]$.

The system model of (8.3) is given in Fig. 8.1.

We only consider linear processing techniques for low complexity. To avoid interuser interference, the ZF algorithm restricts the columns of $W_{Tk}$ into the null space of all the other user channels $\{H_j\}_{j \neq k}$, such that

$$H_k W_{Tj} = 0, \forall j \neq k.$$

(8.4)

With this constraint, (8.3) reduces to

$$y_k = W_{Rk}^H H_k W_{Tk} x_k + n_k,$$

(8.5)

which is equivalent to a single user MIMO channel. Under the constraint (8.4), $W_{Rk}$ and $W_{Tk}$ are chosen to maximize the subchannel gains [39]. Eq. (8.4) implies that the dimension of the null space of $\{H_j\}_{j \neq k}$ must be at least $S$ in order to accommodate $S$ data streams for user $k$. This generally translates to $n_T \geq (K - 1)n_R + S$, which approximately means that the number of transmit antennas must be no less than the total number of receive antennas. This is a very stringent requirement and restricts the applicability of the ZF.
8.2 System Model and Zero-Forcing

Figure 8.1: The transmitter and receiver structure for MIMO Broadcast Channel.
8.3 Iterative Multiple Beamforming Algorithm

In fact, to produce the same interference-free equivalent channel expressed in (8.5), we can change the orthogonality requirement to

\[ W_{Rk}^H H_k W_{Tj} = 0, \forall j \neq k. \]

(8.6)

This is the motivation for the proposed IMB algorithm.

In general, \( \text{rank}(W_{Rk}^H H_k) = S \). Therefore, compared to ZF, the dimension of the null space is increased if \( S < n_R \), and hence, the degrees of freedom in choosing the optimum beamforming directions increase, which may possibly result in a better performance. The new requirement on the number of antennas becomes \( n_T \geq KS \), which is much easier to meet when \( S \) is small. When the number of users \( K \) is large such that \( K > n_T \), the SD-based method alone is not enough due to the limitation on the number of transmit antennas. In that case, the IMB can be used in conjunction with TD or FD. A detailed discussion on this can be found in [39].

8.3 Iterative Multiple Beamforming Algorithm

There is no closed-form solution to \( W_{Tk} \) and \( W_{Rk} \) under the new restriction (8.6). Here we propose a suboptimal iterative multiple beamforming (IMB) algorithm based on QR factorization and singular value decomposition (SVD).

1. Initialize \( W_{Rj} = I_{n_R \times S}, A_j = H_j^H W_{Rj}, j = 1, \ldots, K \).
2. Iteration counter itcnt=1, and itmax=the maximum iteration number.
3. User index \( j = 1 \).
4. \( \tilde{A} = [A_1 \ldots A_{j-1}A_{j+1} \ldots A_K] \).
5. QR decompose \( \tilde{A} = QR \).
6. Let \( \tilde{Q} \) be columns \( S(K - 1) + 1 \) to \( n_T \) of \( Q \).
7. \( \tilde{B} = \tilde{Q} \tilde{Q}^H H_j^H \).
8. Singular value decompose \( \tilde{B} = UDV^H \).
9. \( W_{Rj} = V_S \). If itcnt=itmax, \( W_{Tj} = U_S \).
10. Update \( A_j = H_j^H W_{Rj} \).
11. \( j = j + 1 \). If \( j \leq K \), go to step 4, else continue.
12. itcnt=itcnt+1. If itcnt>itmax, terminate. Otherwise, go to step 3.

Note that \( I_{m \times n} \) denotes an \( m \times n \) matrix with 1’s on its main diagonal and 0’s elsewhere; \( U_S \) and \( V_S \) represent the first \( S \) columns of \( U \) and \( V \), respectively.

This algorithm updates \( \{W_{R_k}\} \) user by user. Step 1 calculates the equivalent channel for user \( j \) (Hermitian transposed); Step 4 collects all the other user equivalent channels into \( \hat{A} \); Step 5-6 finds the basis for the null space, i.e. the columns in \( \hat{Q} \); Step 7 projects \( H_j^H \) into the null space, and hence \( \hat{B} \) is the orthogonal component of \( H_j^H \); Step 8 finds the beamformers that maximize the eigenchannel gains of the orthogonal component \( \hat{B} \).

When \( S = n_R \), the IMB is the same as the ZF, because for any full rank \( W_{R_k} \) the null spaces of \( W_{R_k}^H H_k \) and \( H_k \) are exactly the same. In this case, one iteration is enough since the null spaces never change between iterations. When \( S < n_R \), \( \text{rank}(A_k) < \text{rank}(H_k) \). Each iteration will change the null space of \( W_{R_k}^H H_k \), such that the eigenchannel gains of the current processed user (i.e. the singular values in Step 8) are maximized given all the other user beamformers. As iteration continues, the null spaces and the eigenchannel gains will possibly converge to the optimal values, which contributes to the superior performance of the IMB. Although we have not proved the convergency of the IMB, simulation results show that the convergency is always achieved within a small number of iterations. Alternative to stopping after a fixed number of iterations, the iteration process can be terminated adaptively based on the changes of the singular values in Step 8. As iteration continues, the singular values should keep increasing. Once the increment is small enough, the iteration can be terminated early.

8.4 Numerical Results

8.4.1 Capacity Comparison

We compare the sum capacity of the proposed IMB with the ZF and the cooperative-user system. The cooperative-user system allows all receivers to cooperate. Thus, it provides a
capacity upper limit. The sum capacity is given as

$$C = \sum_{k=1}^{K} \log_2 \left| I + \frac{W_{Rk}^H H_k W_{Tk}^H (W_{Rk}^H H_k W_{Tk})^H}{\sigma^2} \right|$$ (8.7)

where $C$ and $W$ denote the overall channel capacity and the bandwidth of each stream. The transmit-receive antenna weights $W_{Rk}$ and $W_{Tk}$ are calculated using the IMB algorithm described in Section 8.3 for a given $H_k$, $k = 1, ..., K$ and a given number of streams. To simulate the wireless channel, each entry of $H_k$ channel matrix is modeled as a complex Gaussian variable with a zero mean and unit variance. With this model, we calculate the sum capacity for every channel realization using (8.7). The sum capacity values are then averaged.

For all cases, we consider $K = 2$ and water-filling power allocation. For the IMB, five iterations are used. In Fig. 8.2, we present the capacity of $4 \times 2$ MIMO BC. For the ZF, we show the cases of $S = 1$ and $S = 2$, while for the IMB, we only show the curve for $S = 1$ since the curve for $S = 2$ is the same as the ZF. The SNR is defined to be the transmit SNR per user, $\gamma_T \triangleq \frac{P_t}{KnR - nR + S}$, where $P_t$ is the total transmit power.

It is shown that for $S = 1$, the proposed IMB outperforms the ZF by almost a constant SNR gap (about 2 dB). When we compare the IMB with $S = 1$ to the ZF with $S = 2$, we can see that the former outperforms the latter at SNRs up to 9 dB or capacity up to 10 bits/s/Hz. This clearly shows one of the advantages of the IMB. At moderate to low SNRs, the proposed algorithm approaches the cooperative-user capacity much closer than the ZF. This observation is true for other channel configurations. For example, for $8 \times 4$ MIMO BC, which we do not show here, the IMB with $S = 3$ outperforms the ZF with $S = 4$ at SNRs up to 20 dB or capacity up to 45 bits/s/Hz.

In Fig. 8.3, we show the $4 \times 4$ MIMO BC with $S = 1$ and 2. Note that, in this case, since $n_T < KnR - nR + S$, the ZF does not work at all. This shows another advantage of the IMB. It suits situations where the number of transmit antennas is relatively small compared to the total number of receive antennas.

From Figs. 8.2 and 8.3, we also note that the capacity curve of IMB with $S = 1$ has a lower slope (multiplexing gain) than the one with $S = 2$. Therefore, IMB with larger $S$ will eventually outperform the one with smaller $S$ as SNR increases. The choice of $S$ behaves as
Figure 8.2: Capacity of $4 \times 2$ channels with IMB, ZF and a cooperative-user
8.4 Numerical Results

Figure 8.3: Capacity of $4 \times 4$ channels with IMB and a cooperative-user
8.4 Numerical Results

a balance between the multiplexing and diversity gains, which is consistent with the case of single-user MIMO systems [53].

8.4.2 Practical Systems

We present the BER simulation results of $4 \times 2$, $8 \times 4$ and $4 \times 4$ BC with the IMB, the ZF and an interference free configuration in Fig. 8.4. The interference free configuration is a virtual/ideal configuration where $K$ non-interfering $n_T \times n_R$ MIMO channels, each using a single beamforming, are simulated. For all cases, we assume $K = 2$, $S = 1$ and the use of QPSK modulation. Again, for the $4 \times 4$ case, the ZF does not work. Please note that the IMB algorithm does not depend on the modulation, $S$, or $K$. The choice of the parameters in this simulation is only for illustration purposes. Other choices of parameters have similar behavior.

It is clear that the IMB outperforms the ZF significantly. In the $4 \times 2$ case, the ZF even fails to achieve full diversity. However, the IMB always achieves the full diversity order of $n_T \times n_R$. Furthermore, as the number of transmit and receive antennas increases, the performance of the IMB scheme approaches the ideal interference free configuration. An intuitive explanation is that the degrees of freedom in choosing the orthogonal subspaces in the IMB algorithm are increased as the number of antennas increases.

8.4.3 Complexity comparison

When we compare the IMB with the ZF algorithm in [39], we find that the complexity of the ZF is approximately equal to that of one iteration of the IMB. Assuming the IMB uses a fixed iteration number of $N$, generally speaking, the IMB has a complexity $N$ times higher than the ZF. This is the price to pay for the superior performance. However, in our simulations, we find that the number of iterations does not need to be large. In particular, when the number of data streams per user ($S$) is small compared to the number of receive antennas ($n_R$), the convergence speed is very fast. For example, in Figs. 8.2-8.4, with two iterations ($N = 2$), the performance degradation is less than 0.3 dB compared to five iterations ($N = 5$). Furthermore, the computation of the IMB only needs to be performed once per channel.
Figure 8.4: BER of IMB, ZF and interference free channels
8.4 Numerical Results

realization. In relatively slow mobile environments, the increase in computation complexity is insignificant because the biggest part of computation is in the signal processing. A further simplification of the IMB is possible when $S$ is small, say $S = 1$. In that case, the QR factorization at Step 5 does not need to be done in each iteration. Instead, a QR update can be used from the second iteration, which will reduce the complexity significantly.
Chapter 9

Conclusions

This thesis has explored various noncooperative and cooperative transmission MIMO-OFDM systems that enable multiple users to simultaneous transmit at the same frequency band at a given time slot. The main challenge here was to find a method that can suppress the interference coming from other users. Our results indicate that network coordination is a promising technique. Large capacity and error rate improvements over the conventional cellular networks can be achieved by getting BSs to cooperate. Without network coordination, the downlink system capacity is limited by the strength of the interference from other users. On the other hand, when network coordination is employed, interference from other users is successfully suppressed and the error rate performance for each user approaches the interference free performance of the multi-user MIMO systems.

In Chapter 4, we present a noncooperative transmission scheme. The scheme exploits multiple transmit-receive antennas and adaptive modulation to reduce interference and to increase downlink throughput for OFDM systems in co-working WLANs. We refer to this scheme as joint AMA and AM with ACK Eigen-steering. AMA is used to suppress the co-working interference and maximizes SINR. AM is then used to maximize the data rate within the specified BER by appropriately allocating the power, sub-carrier and modulation mode. The derivation of AMA transmit-receive antenna weights and the AM scheme are shown. The performance of joint AMA and AM for various system configuration under co-working WLANs is investigated through simulations. We find that receive beamforming is much more effective than transmit beamforming to combat interference. Finally, we show
that by using ACK Eigen-steering, the transmitter can obtain the information about transmit
antenna weights and power allocation by using the uplink ACK signal.

Knowing that we cannot improve the performance of the noncooperative scheme further, we
propose a network coordination so that the base stations can cooperate and simultaneously
transmit the data to its respective users using the same frequency band at the same time slot.
In Chapter 5, we propose a practical cooperative transmission scheme employing precoding,
beamforming and an adaptive precoding order for co-working MIMO OFDM WLANs. The
proposed design eliminates co-working interference (CI) in co-working WLANs with only
partial CSI available at the receiver of each station. The cooperative scheme among APs,
first combines THP with joint transmit-receive beamforming based on SINR maximization.
An adaptive precoding order is then used to further improve overall performance and to
ensure BER fairness among stations served by different APs. We prove analytically and
by simulation that our proposed scheme will not degrade under partial CSI. The simulation
results also show that our proposed scheme (THP-SINRM-APC) gives the optimum overall
BER performance. The performance is only 2 dB away from an interference-free channel, is
3 dB better than the best known cooperative scheme and is 10 dB better than the best known
non-cooperative scheme.

In Chapter 6, we look for a new method so that the performance of the scheme in Chapter
5 can be improved significantly. Here, we propose a method to design a spectrally efficient
cooperative downlink transmission scheme employing precoding and beamforming. THP
and iterative transmit-receive weights optimization are used to cancel interference. A new
method to generate transmit-receive antenna weights is proposed. SINR equalization and
APO are used to achieve symbol error rate (SER) fairness among different users and further
improve the system performance. The error performances for two sets of system parameters
\((N_{BS}, N_{MS}, K)\) are shown. For a \((2, 2, 3)\) cooperative system, the proposed method outper-
forms the existing schemes by at least 3 dB and is only 0.25 dB away from the interference
free performance when SER=10^{-4}. For a \((1, 2, 4)\) system, the proposed method outperforms
the existing schemes by at least 4 dB and is 4 dB away from the interference free performance
for SER of 10^{-4}. In addition, the proposed method eliminates the dependency between the
numbers of transmit and receive antennas. The complexity of the proposed method is also
shown to be much lower than for the existing schemes. The complexities of the proposed
method for \((1, 2, 4)\) and \((2, 2, 3)\) are shown to be 50% and 75% less than the complexities
of the existing schemes with the same configurations, respectively. The proposed method
can be applied to improve the performance and capacity of co-working WLANs and cellular mobile networks. The capacity of these systems can be increased up to $K$ times.

In Chapter 7, we exploit the uplink-downlink duality concept to design a cooperative multi-stream multi-user MIMO downlink transmission scheme employing precoding and beamforming. As in Chapter 6, THP and transmit-receive weights optimization are used to cancel the interference. SINR equalization and APO are applied to achieve symbol error rate (SER) fairness among different users and further improve the system performance. We first propose an iterative method to optimize the transmit-receive weights. Furthermore, we trade off the complexity with a slight performance degradation by eliminating the iteration step needed to find the transmit-receive weights. We then extend these methods to work in situations where the receiver only knows its own CSI. In Chapter 7, the error performance for three sets of system parameters $(N_{BS}, N_{MS}, K, S)$ is shown. For a $(6, 2, 3, 1)$ cooperative system, the proposed method outperforms the existing schemes by at least 2 dB and is only 0.5 dB away from an interference free performance for SER=10^{-5}. For a $(4,2,4,1)$ system, the proposed methods outperform the existing schemes by at least 10 dB and are 3 dB away from an interference free performance for SER= 10^{-4}. For a $(6,2,3,2)$ system, the proposed methods, except AII-APO-LC-Limited CSI, outperform the existing schemes by at least 10 dB and are 3 dB away from an interference free performance for the SER= 10^{-4}. AII-APO-LC-Limited CSI outperforms the existing schemes by at least 4 dB. In addition, the proposed method eliminates the dependency between the numbers of transmit and receive antennas. The application of APO-LC to order users has been shown to degrade the performance of the proposed methods by at most 1 dB compared to APO-VBLAST ordering proposed in Chapter 6. However, the proposed APO-LC is shown to be significantly less complex than VBLAST ordering, when used with the proposed methods. The complexities of the proposed method (Algorithm I) with APO-LC for $(6, 2, 3, 1)$, $(4, 2, 4, 1)$ and $(6, 2, 3, 2)$ are shown to have be at least 80% less complexity than the proposed method with APO-VBLAST.

In the cooperative transmission schemes proposed above, THP cancels a part of the interference. Here, we consider an alternative approach that totally bypasses the use of THP. That means the transmit-receive antenna weights are now tasked to cancel all interference from other users. In Chapter 8, an iterative multiple beamforming algorithm is proposed for MIMO BC. Compared to the linear ZF, it allows more flexible configurations in transmit and receive antenna numbers; has higher capacity at low to moderate SNRs; and has much better BER performance when operated at low to medium data rates. Note that the proposed
methods in Chapters 6, 7 and 8 can all be applied to improve the performance and capacity of co-working WLANs and cellular mobile networks.

As a future research direction, it would be interesting to apply distributed, or local, implementation of network coordination to cellular networks [70]. To achieve this, one has to identify the essential part of the information (channel state, synchronization etc.) that has to be shared among the base stations to realize a significant portion of the promised gains. For example, it might be enough for each base station to have the channel information of a few neighboring base stations.

Also, it is important to quantify the amount of backhaul resources and feedback required to implement network coordination in a practical system. The network coordination in this thesis is an example of cooperation in cellular systems or WLANs. The base stations however, can actually cooperate in other ways. As an example, the base stations can take advantage of the bursty nature of the data transmissions by sharing information to avoid interference during the bursty data arrivals. Base station cooperation can also be in the form of spectrum allocations where each base station transmits in a coordinated way such that overlapping bursty transmissions are avoided. The base station coordination is a futuristic system design that may offer significant capacity and error rate improvement in cellular networks and WLANs. The works in this thesis establishes an initial study for the design of a cooperative system.
Appendix A

The Derivation of Receive Antenna Weights in (4.5)

We first write the Lagrangian function for (4.5) as [71]

\[ \mathcal{L}(\lambda) = W_R^H R_U W_R - \lambda(W_R^H H_D W_T - 1) \]  

(A.1)

where \( \lambda \) is the Lagrange multiplier. The Kuhn-Tucker condition for the minimum value for (4.5) is then given as

\[ \frac{\partial \mathcal{L}(\lambda)}{\partial W_R} = R_U W_R - \lambda H_D W_T = 0. \]  

(A.2)

By using (A.2), the optimum receive weights can be calculated as

\[ W_R = \lambda R_U^{-1} H_D W_T. \]  

(A.3)

where \( \lambda \) is given as

\[ \lambda = \frac{1}{(H_D W_T)^H R_U^{-1} H_D W_T}. \]  

(A.4)

Note that \( \lambda \) is found by inserting (A.3) into the constraint in (4.5).
Appendix B

Proof of SINR Equivalence

We first calculate $\text{SINR}_j$ using the term on the left hand side of (6.14). We first need to prove that $\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{H}_j^H$ only has one eigenvalue. Let us assume that $\mathbf{H}_j^H \neq 0$. We denote $\mathbf{R}_j^{-1} \mathbf{H}_j$ by $\mathbf{a}$ and $\mathbf{H}_j$ by $\mathbf{b}$, where $\mathbf{a} = [a_1...a_{N_MS}]^T \in \mathbb{C}^{N_MS \times 1}$ and $\mathbf{b} = [b_1...b_{N_MS}] \in \mathbb{C}^{1 \times N_MS}$, respectively. We then express $\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{H}_j^H$ as

$$\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{H}_j^H = \mathbf{ab} = [b_1a...b_{N_MS}a]$$

(B.1)

Here, $\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{H}_j^H$ is a matrix that has $N_MS$ columns and rows. We can see from (B.1) that the vectors represented by each column of matrix $\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{H}_j^H$ can be rewritten using vector $\mathbf{a}$ as a basis. This indicates that the rank of this matrix is 1 and as a consequence, there is only one eigenvalue. The receive weight vector computation in (6.12) can be written as

$$\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{H}_j^H \mathbf{r}_j = \lambda_j \mathbf{r}_j$$

(B.2)

where $\lambda_j$ is the eigenvalue for link $j$. By multiplying both sides of this equation by $\mathbf{H}_j^H$, we have

$$(\mathbf{H}_j^H \mathbf{R}_j^{-1} \mathbf{H}_j - \lambda_j) \mathbf{H}_j^H \mathbf{r}_j = 0.$$  (B.3)

The eigenvalue of $\mathbf{H}_j^H \mathbf{R}_j^{-1} \mathbf{H}_j$ is the same as the eigenvalue of $\mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{H}_j^H$. As a consequence, $\mathbf{H}_j^H \mathbf{R}_j^{-1} \mathbf{H}_j$ only has 1 eigenvalue. This eigenvalue is the solution for the term on
the left hand side of (6.14). Thus, $\text{SINR}_j$ for it is given as

$$\text{SINR}_j = \lambda_j = \tilde{H}_j^H \tilde{R}_j^{-1} \tilde{H}_j. \quad (B.4)$$

We now find the $\text{SINR}_j$ using the term on the right hand side of (6.14). The optimum receive weight vector is given by [46] as

$$r_j = \frac{R_j^{-1} \tilde{H}_j}{c} \quad (B.5)$$

where $c = \tilde{H}_j^H \tilde{R}_j^{-1} \tilde{H}_j$. By substituting this receive weight vector into (7.1) and replacing its denominator with $r_j^H R_j r_j$, we obtain the same $\text{SINR}_j$ expression as in (B.4). This concludes the proof.
Appendix C

Proof of Lemma 1 in Chapter 6

First, we note that in order to calculate $SINR_j$ in (7.1), we need to know the receive weights vector for link $j$, $r_j$ and all transmit weights vectors $t_1, \ldots, t_K$, obtained by using (6.9) and (6.8), respectively. Thus, we can write $SINR_j$ as $SINR_j(r_j, T)$ since it is a function of $r_j$ and $T$. Since in the proposed iterative method, we optimize one variable at a time, while fixing the other one, we can write

$$SINR_j(g_j(T), T) = \max_{a \in A_1} SINR_j(a, T), g_j(T) \in A_1$$  \hspace{1cm} (C.1)$$

where $T$ is fixed while the best $r_j = g_j(T)$ in the solution set $A_1$ is searched and

$$SINR_j(r_j, f_1(R)) = \max_{a \in A_2} SINR_j(r_j, a), f_1(R) \in A_2$$  \hspace{1cm} (C.2)$$

where $r_j$ is fixed while the transmit weights vectors for $K$ links, $T = f_1(R)$ in the solution set $A_2$ are searched, respectively. To describe the proposed alternating optimization process, we denote the number of iterations by $i$, the receive weights vector by $r_j^{(i)}$ and transmit weights vectors by $T^{(i)}$. First, $r_j^{(0)}, j = 1, \ldots, K$, are arbitrarily chosen as initial vectors. $T^{(1)}$ is then calculated by using the function in (6.8), $f_1(R^0)$. For $i \geq 1$, we then have,

$$r_j^{(i)} = g_1(T^{(i)}), j = 1, \ldots, K$$  \hspace{1cm} (C.3)$$
where $T^{(i)} = [t^{(i)}_1 \ldots t^{(i)}_K]$ and
\[ T^{(i)} = f_1(R^{(i-1)}) \quad \text{(C.4)} \]

where $R^{(i)} = Diag(r^{H(i)}_1, \ldots, r^{H(i)}_K)$. Here, $r^{(i)}_j$ and $T^{(i)}$ are generated in the order $r^{(0)}_{j=1,\ldots,K}$, $T^{(1)}$, $r^{(1)}_{j=1,\ldots,K}$, $T^{(2)}$ and so on. From (C.1) and (C.2), and by using the fact that $SINR_j(r_j, T^{(i)})$ is non-decreasing and bounded from above by constraints in (6.4), we can write
\[
SINR_j(r^{(i)}_j, T^{(i)}) = SINR_j(r^{(i)}_j, T^{(i)}) \\
\geq SINR_j(r^{(i)}_j, T^{(i-1)}) \\
\geq SINR_j(r^{(i-1)}_j, T^{(i-1)}). \quad \text{(C.5)}
\]

The second and third lines of (C.5) come from the fact that since we are performing an alternate optimization of the transmit-receive weights by using (C.1) and (C.2), the SINR obtained at iteration $i-1$ could only be either equal or less than the SINR obtained at iteration $i$. This shows that as the number of iterations increases, the $SINR_j(r^{(i)}_j, T^{(i)})$ will converge to a local maximum and simultaneously satisfy (6.9) and (6.8). The former will also cause the remaining interference to converge to 0 as the number of iterations goes to $\infty$. This concludes the proof.

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Appendix D

Proof of Lemma 2 in Chapter 6

We know from Convergence Lemma 1 that we can write for the optimal solution,

\[
det(R^*HT^*) = det(Z) = \prod_l |z_{l,l}|
\]  \hspace{1cm} \text{(D.1)}

where \(Z\) and \(z_{l,l}\) are a lower triangular matrix and the entry of the diagonal of \(Z\), respectively. \(R^*\) and \(T^*\) indicate the optimal transmit-receive antenna weights for \(K\) links. We also need (6.8) to be satisfied for the optimal solution for each link \(j\),

\[
(H_j^HT_j^r)^Ht_l^* = 0, \ l = j + 1, ..., K.
\]  \hspace{1cm} \text{(D.2)}

The vector created by multiplying the channel matrix by the receive antenna weights vector is perpendicular to transmit weights for links \(j+1, ..., K\). As a result, there is no interference at all at link \(j\). This is so since the transmission spaces of link \(j+1, ..., K\) do not overlap with the transmission space of link \(j\) and the interference from link 1, ..., \(j-1\) to link \(j\) is cancelled by THP. However, prior to finding the optimal solution, the receiver design from (6.9) destroys the orthogonality created by QR decomposition in (6.8). As a result at the \(i^{th}\) iteration, for link \(j\), we have

\[
(H_j^HT_j^{(i)}t_l^{(i)})^Ht_l^{(i)} \neq 0, \ l = j + 1, ..., K.
\]  \hspace{1cm} \text{(D.3)}
This means the transmission space for link $j$ intersects with the transmission spaces of link $j + 1, ..., K$ prior convergence. In other words, the vector generated by $H_j^H r_j^{(i)}$ also has non-zero components along $t_{j+1}, ..., t_K$, thus reducing the optimal signal gain for link $j$, $z_{j,j}$. By using (D.1), we can then conclude that

$$\prod_j |\beta_j^{(i)}| \leq \prod_j |z_{j,j}|.$$  \hspace{1cm} (D.4)

This concludes the proof.
Appendix E

Proof of Receive Antenna Weights Equivalence

Here we prove that $SINR_j$ calculated by using (6.15) is the same as $SINR_j$ calculated in (6.14) and in Appendix B. (6.15) can be written as

$$r_j = \frac{\mathbf{R}_j^{-1}\mathbf{H}_j}{f} \quad \text{(E.1)}$$

where $f = ||\mathbf{R}_j^{-1}\mathbf{H}_j||$. We then substitute (E.1) into (7.1) and replace its denominator with $r_j^\mu\mathbf{R}_j r_j$ to get

$$SINR_j = \frac{(\mathbf{H}_j^H\mathbf{R}_j^{-1}\mathbf{H}_j/f)^2}{\mathbf{H}_j^H\mathbf{R}_j^{-1}\mathbf{R}_j\mathbf{H}_j/f^2}. \quad \text{(E.2)}$$

After simplifying (E.2), we obtain the same $SINR_j$ expression as in (B.4). This concludes the proof.
Appendix F

Proof for the Convergence of the 2-step Optimization

The iterative solution proposed in Section 7.1.1 essentially splits the optimization problem in (7.7) into a 2-step optimization. The first step is to solve $R$ and $T$, when the inter-link interference power to link $j$ from link $i$, $q_{s,i \neq j}$, $s = 1, ..., S, i = 1, ..., K$ is fixed. Under this condition, (7.7) can be written as

$$f_1(r_{s,j}, t_{s,j}|q_{s,i \neq j}) = \max_{r_{s,j}, t_{s,j}} \min \{\text{SINR}_{s,j}^{up}\}$$

subject to:

1. $t_{s,j}^H t_{s,j} = 1$
2. $r_{s,j}^H r_{s,j} = 1$ \hspace{1cm} (F.1)

for $i = 1, ..., K, j = 1, ..., K, s = 1, ..., S$. Here, the virtual uplink powers for link $i$, $q_{s,i \neq j}$, used to calculate $\text{SINR}_{s,j}^{up}$ in (7.5) are set to fixed values. We perform this optimization process with a function, denoted by, $f_1(r_{s,j}, t_{s,j}|q_{s,i \neq j})$, where $r_{s,j}$ and $t_{s,j}$ are its optimization variables for a given $q_{s,i \neq j}$, $i = k = 1, ..., K, s = 1, ..., S$. The solution of (F.1) is obtained by first finding transmission spaces that have the minimum inter-link interference for each link. These transmission spaces are then used to design the transmit-receive weights vectors, within each link, that give the maximum SINR.

The second step is to solve $q$ in a way that equalizes SINR for all links under fixed $R$ and $T$. 
Under this condition, the optimization problem can be written as

\[
f_2(q_{s,j} | r_{s,j}, t_{s,j}) = \max_{q_{s,j}} \min_{q_{s,j}} SINR_{s,j}^{up}
\]

subject to (1) \(1^T q = P_{max}\) \hspace{1cm} (F.2)

for \(j = 1, \ldots, K, s = 1, \ldots, S\) and \(q = Q^2 1\) as defined in Section 7.1.1. Here, the transmit-receive weights for all users are fixed when solving (F.2). We denote this optimization process with a function, \(f_2(q_{s,j})\) with \(q_{s,j}\) as its optimization variable, optimized for given \(r_{s,j}\) and \(t_{s,j}\).

To describe the iteration process of the 2-Step Optimization, first let us denote the number of iterations as \(a\) and the transmit-receive weights obtained at the \(a^{th}\) iteration as \(r_{s,j}^{(a)}\) and \(t_{s,j}^{(a)}\). The uplink power obtained in \(a^{th}\) iteration is denoted by \(q_{s,j}^{(a)}\). We first initialize \(q_{s,j}^{(0)}\), \(i = j = 1, \ldots, K, s = 1, \ldots, S\). In the first step of the first iteration, we have \(f_1(r_{s,j}^{(1)}, t_{s,j}^{(1)} | q_{s,j}^{(0)}\neq j)\) \(i = 1, \ldots, K\) with outputs \(r_{s,j}^{(1)}\) and \(t_{s,j}^{(1)}\). The uplink SINR in (7.5) can then be calculated using these two variables. We denote this as \(SINR_{s,j}^{up}(t_{s,j}^{(1)}, r_{s,j}^{(1)}, q_{s,j}^{(0)})\). In the second step, we calculate the value of \(f_2(q_{s,j}^{(1)} | r_{s,j}^{(1)}, t_{s,j}^{(1)})\) which is denoted as \(q_{s,j}^{(1)}\). The uplink SINR is then given as \(SINR_{s,j}^{up}(t_{s,j}^{(1)}, r_{s,j}^{(1)}, q_{s,j}^{(1)})\).

At \(a^{th}\) iteration, the uplink SINR in the first step of the optimization is then given as

\[
SINR_{s,j}^{up}(t_{s,j}^{(a)}, r_{s,j}^{(a)}, q_{s,j}^{(a-1)}), \hspace{1cm} (F.3)
\]

while the uplink SINR in the second step is given as \(SINR_{s,j}^{up}(t_{s,j}^{(a)}, r_{s,j}^{(a)}, q_{s,j}^{(a)})\). By using the fact that the uplink SINR in (7.5) is bounded from above by the three constraints in (7.7), we can write the uplink SINR as

\[
SINR_{s,j}^{up}(t_{s,j}^{(a)}, r_{s,j}^{(a)}, q_{s,j}^{(a)}) = SINR_{s,j}^{up}(t_{s,j}^{(a)}, r_{s,j}^{(a)}, q_{s,j}^{(a)})
\]

\[
\geq SINR_{s,j}^{up}(t_{s,j}^{(a)}, r_{s,j}^{(a)}, q_{s,j}^{(a-1)})
\]

\[
\geq SINR_{s,j}^{up}(t_{s,j}^{(a-1)}, r_{s,j}^{(a-1)}, q_{s,j}^{(a-1)}). \hspace{1cm} (F.4)
\]

for \(s = 1, \ldots, S\) and \(j = 1, \ldots, K\). (F.4) shows that as the number of iterations increases, the uplink SINR will converge to a local maximum. This concludes the proof.
Appendix G

Proof for Uplink-Downlink Duality

By using (7.17) the uplink SINRs in (7.5) and downlink SINRs in (7.1) for all links can be rearranged as follows

\[ A^{-1}1 = \left( \frac{I}{\text{SINR}_{\text{up}}} - A^{-1}B^H \right)q \]
\[ A^{-1}1 = \left( \frac{I}{\text{SINR}_{\text{down}}} - A^{-1}B \right)p \]  \hspace{1cm} (G.1)

By simplifying (G.1), the uplink and downlink power \( q \) and \( p \) can be further written as

\[ q = \left( \frac{A}{\text{SINR}_{\text{up}}} - B^H \right)^{-1}1 \]
\[ p = \left( \frac{A}{\text{SINR}_{\text{down}}} - B \right)^{-1}1 \] \hspace{1cm} (G.2)

In Lemma 2, we claim the downlink SINR can be designed to be equal to the virtual uplink SINR under the same total power constraint \( P_{\text{max}} \). Thus, we have \( 1^Tp = 1^Tq = P_{\text{max}} \). In addition, we also claim that this SINR equality exists when the same transmit-receive
weights designed for virtual uplinks are used in computation. Thus, by using (G.2), we have

\[ 1^T \left( \frac{A}{SINR^{up}} - B^H \right)^{-1} 1 = 1^T \left( \frac{A}{SINR^{down}} - B \right)^{-1} 1 \]
\[ = 1^T \left( \frac{A}{SINR^{down}} - B \right)^{-H} 1 \]
\[ = 1^T \left( \frac{A}{SINR^{down}} - B^H \right)^{-1} 1. \]

(G.3)

From (G.3), we can see that as long as the total power constraints are equal, the downlink SINR will always be equal to the virtual uplink SINR. This concludes the proof.
Appendix H

Proof for (7.21)

We now need to find the relationship between the virtual uplink and downlink power. To find this relationship, we use (G.2). By equalizing (G.2), we obtain,

\[
1 = (\frac{A}{SINR_{up}} - B^H)q = (\frac{A}{SINR_{down}} - B)p
\]  

(H.1)

By equating the terms on the right hand side of (H.1), we obtain,

\[
p = (\frac{A}{SINR_{down}} - B)^{-1}(\frac{A}{SINR_{up}} - B^H)q = \hat{P}q.
\]  

(H.2)

Thus, the downlink power can be obtained from (H.2) once the virtual uplink power and the transmit-receive weights are calculated. This concludes the proof.
Bibliography


