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SHAKEDOWN ANALYSIS AND DESIGN OF PAVEMENTS

By

RICHARD WINGRAVE SHARP, B.Sc., B.E.

A Thesis submitted for the Degree of Doctor of Philosophy at the University of Sydney

April, 1983
...and the crooked shall be made straight,
and the rough places made smooth."

TO ADELE

LUKE 3:5
"... and the crooked shall be made straight,
and the rough ways shall be made smooth."

- LUKE 3:5
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SYNOPSIS

Pavements subjected to moving surface loads typically fail in a long-term, incremental mode. It is, therefore, reasonable to assume that a method of pavement analysis should take due account not only of finite material strength and the movement of loads, but also of the gradual accumulation of plastic deformations within a pavement structure. This end may be attained using the theory of shakedown.

The pavement is modelled as a continuum consisting of layers of elastoplastic material, under plane strain conditions of loading. Two methods of solution to the relevant sets of equations are presented, the first using linear programming algorithms to provide basic solutions, and the second incorporating a number of refinements to yield a procedure of much improved accuracy and speed. The results of parametric studies are then set out, and a number of implications for subsequent design procedures are explored.

A more general method of analysis which recognises the three-dimensional nature of pavement loading serves to confirm the plane strain results, and shows them to be slightly conservative in relation to the more general case.

Full-scale road tests enable theories of design to be examined in the light of pavement performance, and the carefully controlled factorial experiments of the AASHO Road Test lend themselves to this purpose. The results confirm that pavement shakedown may be both observed and accurately predicted, and enable a more general relationship between pavement life and shakedown predictions to be formulated. Further field tests in the Sydney region verify these findings.

With its application so demonstrated, shakedown theory is applied to a number of typical design problems, and a generalised design procedure is presented.
PREFACE

The candidate carried out the work described in this thesis during the period from Lent term 1980 to Lent term 1983. With the exception of the materials testing, which was done at the Milsons Point Laboratory of the Department of Main Roads, N.S.W., all work was conducted in the School of Civil and Mining Engineering, The University of Sydney.

Supervision of the study was provided by Professor E.H. Davis, Challis Professor of Civil Engineering and Head of School, until his untimely death in February 1981. Subsequently, the candidate was supervised by Dr. J.R. Booker, Reader in Civil Engineering.

The By-Laws of the University of Sydney require a candidate for the degree of Doctor of Philosophy to indicate which sections of the thesis are original. Due acknowledgement has been made in the text of information or ideas derived from other sources, and accordingly the Author claims originality for the following work:

(a) the stress path approach to pavement response, and (with the exception of those for a uniformly distributed load on a half-space) the numerical results presented in Section 4.5;

(b) the "one-dimensional" analysis of shakedown developed in Chapter 5, the two methods of solution presented, and all results;

(c) the extension of the foregoing analysis to the more general case described in Chapter 6, and subsequent examples of its use;

(d) in Chapter 7, the application of shakedown analysis to the AASHO test data, and the interpretation and discussion of results which follows;
(e) the planning and conduct of the field tests of Chapter 8. The testing work was done by personnel of the Department of Main Roads, N.S.W., and some of the traffic data is taken from Departmental records; however, the remaining traffic data, and the entire analysis and discussion of performance are original;

(f) the complete development of design approaches presented in Chapter 9 is claimed to be original.
ACKNOWLEDGEMENTS

The financial support which made this work possible was provided by the University of Sydney (as a Postgraduate Research Award) and by the Commissioner for Main Roads, N.S.W.

In the development of this thesis, the time and suggestions contributed by many in the School of Civil Engineering was invaluable. Further assistance, particularly in the areas of pavement sampling and materials testing, was freely given by many officers of the Department of Main Roads, N.S.W., and for this I am most grateful. Also appreciated is the work of Ann Baker in producing an excellent typescript.

For their supervision, counsel and encouragement, I am particularly indebted to Professor E.H. Davis - always a man of ideas - and Dr. J.R. Booker - a most perceptive teacher and a generous friend.

R.W.S.
1.1 Background

Man's invention of the wheel has had a dramatic influence upon his own development. Not only has it provided him with ever-increasing mobility, it also enabled him to transport loads of a considerably greater magnitude than previously. As so often occurs, however, improvements in one area of technology served to highlight deficiencies in another, and it soon became apparent that natural soil often required some strengthening where it was expected to endure the passage of many wheel loads.

Among the earliest recorded experience in this area was that accumulated by the Romans over some eighty thousand kilometres of roadworks. Their solution - a heavy stone foundation overlain by lime-bound gravel and surfaced with jointed stone blockwork - was particularly successful, and found application long after their Empire declined. With minor variations, their methods were in use even in the early nineteenth century, at the hands of Trésaguet, Telford and McAdam.

The twentieth century brought with it a rapid rise in both vehicle speeds and the use of the pneumatic tyre. These two influences conspired to accelerate the deterioration of the commonly-used water-bound macadam structures, promoting the move towards asphalt and concrete pavement layers. Even today, however, the principles of pavement construction are little changed from those of two thousand years ago: the techniques have perhaps undergone more change, but in many ways reflect only the growing mechanisation of construction.

The aim of a pavement remains to provide a running surface capable of withstanding the passage of loads and the onslaughts of weather. Recent decades have seen considerable time and energy expended in developing methods to design such pavements, the aim being to determine that combination of layer thicknesses which will most economically meet the required standard of performance, given the constraints of subgrade condition and available pavement materials. Initially, the basis of design methods was purely empirical. There is now, in addition, a growing body of theoretical work aiding our understanding of the behaviour of pavement structures. Indeed, the range of approaches is impressive, and one may choose from models based on linear elasticity, non-linear elasticity, plasticity, visco-elasticity, and many other variations.
With the increasing complexity of theoretical models come, however, a number of difficulties. Not only is analysis rendered more time-consuming, but a considerable increase in the amount of materials testing for characterisation also becomes necessary. Whilst this in itself may not be a bad thing, it is possible that a host of detail may mask other equally important influences on behaviour.

1.2 This Thesis

It may readily be observed that most pavements deteriorate progressively over a large number of cycles of a moving load. It appears to be something of a deficiency, then, that most models consider only static loading - for regardless of the complexity of assumed material behaviour, it may reasonably be expected that the movement of loads should markedly affect pavement response. The use of finite element methods, well established for cases of static loading, imposes further difficulties for moving loads. Particularly is this so for problems involving a large number of cycles of a moving load, and it would appear that at present the costs (time and storage) of obtaining accurate solutions to such questions of progressive deterioration are prohibitive. There exists, therefore, a need for an approach to pavement analysis and design which recognises the influence of moving loads on the response of a structure, whilst taking account of finite material strength and the progressive accumulation of permanent deformations.

This thesis commences by reviewing in some detail the development of techniques of analysis and design. Two shorter chapters follow, the first of which presents some of the models used for material response to load, and discusses the types of material commonly in use and the parameters which define their behaviour. In the second, attention is directed to the way in which a load is applied to the pavement surface, and the characteristics of the pavement's response to that load.

Chapter Five presents an approach to the analysis of pavements under moving surface loads, adapting to continua the shakedown theory already in use for discrete structures. In the following chapter, this analysis is extended to a number of more general cases, and the value of various simplifying assumptions is examined.
This theoretical development leads then to an investigation of its application, and the results of a comprehensive full-scale test are reviewed in Chapter Seven. The analysis of these results serves to demonstrate that pavement shakedown may be both observed and predicted, and leads to a relationship between analytical results and service life being established. The outcome of a field study of pavements in the Sydney region, detailed in the following chapter, lends support to these conclusions while highlighting a number of important considerations for pavement performance studies in general.

Chapter Nine draws together both the theoretical and practical results in developing a number of approaches to pavement design. Suitable procedures for several representative cases are put forward, and on this foundation is established a design technique applicable to a range of pavements commonly in use. The chart accompanying this work is presented as a convenient and compact summary of a large body of data.

The thesis concludes with a summary of the developments of foregoing chapters, and a discussion of their implications for pavement analysis and design.
CHAPTER TWO

A CRITICAL REVIEW OF FLEXIBLE PAVEMENT ANALYSIS AND DESIGN
2.1 Introduction

Man has long sought to predict the performance under load of the structures he uses. From Galileo's work during the 1620s and 1630s on the resistance of solids to rupture, has grown a science of considerable sophistication and, in parts, of substantial accuracy. In the 1820s, Navier and Cauchy drew together the observations and postulates of people such as Hooke (1660), Coulomb (1776) and Young (1807), and the mathematical tools of Newton, to formulate the theory of pure elasticity. This theory has since been shown to be applicable to many fields of structural engineering, and provides the basis for analysis of load-deformation behaviour in numerous applications. Those concerned with the analysis of continua saw the same promise in this theory as did those working with discrete structures. Boussinesq in 1885 presented among other things the solution of the problem of a point load on a homogeneous half-space, and it is to his work that the science of pavement analysis owes much.

2.2 Elastic Analysis of Layered Pavements

With the twentieth century came the motor vehicle and, later, the aeroplane - and as transport technology developed, there emerged a growing need for stronger running surfaces. While the practitioners coped as best they could, the researchers looked to the realm of structural analysis for means of predicting the stresses developed beneath moving vehicles, as a step towards designing pavements capable of satisfactorily carrying those vehicles. The theory of elasticity appeared to offer the potential for such predictions.

Whilst the potential existed, however, the solution was not so readily forthcoming. Boussinesq's theory provided a starting point, but could be regarded as little more than this in view of the general requirement for a strengthening layer to be placed on the natural surface, so that a range of vehicles and loads might be carried in a variety of climatic conditions. A need for an analysis applicable to layered continua clearly existed.
One of the first solutions for a layered structure was presented by Westergaard (1926), who examined the stresses in a plate supported by a subgrade providing vertical reactions only. Some of the approximations involved in this analysis were removed by Marguerre (1931) and Biot (1935), who both developed solutions for the stresses at the base of a single elastic layer underlain by a rough rigid base. Both authors examined the case of plane strain, and Biot also produced solutions for the case of axial symmetry.

Thus, at the end of the 1930s, there was still no solution for a layered continuum available. In 1940, however, an approximate solution was put forward by Palmer and Barber. The case of an elastic layer on an elastic subgrade was examined, and the overlying layer replaced by an equivalent thickness of subgrade, determined as a function of modular ratio. The Boussinesq analysis could then be applied. Whilst deflections could be predicted with satisfactory accuracy, the stresses could not, and the analysis was in any case limited to vertical direct stresses.

The first exact solution for an elastic layer on an elastic subgrade is due to Burmister (1943), who two years later extended his analysis to cover the cases of both continuous and frictionless interfaces. In addition, solutions were presented for two elastic layers on an elastic subgrade (Burmister 1945). A number of workers then proceeded to expand upon the solid foundation thus built, and more complete calculations of various stresses and displacements under a range of loading conditions were subsequently performed by Fox (1948), Hank and Scrivner (1948), Acum and Fox (1951), Schiffman (1957), Lemcoe (1960), Jones (1962), Westmann (1963) and Kirk (1966).

Burmister meanwhile pursued another line of investigation. In 1956 he presented solutions for an elastic layer on a rough rigid base under an applied point load; a decade later Poulos generalised his expressions to derive complete numerical solutions for stresses and displacements under point, line, strip and sector loading (Poulos 1967).
During this process of development and refinement, the search for solutions to the case of three or more layers on an elastic subgrade had of course continued. Such solutions were sought in view of the general construction practice borne of long experience with both successful and unsuccessful pavements. Such experience had seen the widespread adoption of pavements constructed by placing two stronger soil layers upon the subgrade to improve the general bearing capacity, followed by a bituminous layer to add weatherproofing and a component of tensile strength to the structure. A typical pavement cross-section is illustrated by Figure 2.1. In 1959, Mehta and Veletsos published solutions for three and four layers on a subgrade: the solutions were limited, however, to the vertical and radial direct stresses at the interfaces and on the load axis. No more general results were in fact available until 1967, when Verstraeten examined three layers upon a subgrade, for axial symmetry under vertical load, uniform inward shear, and uniform unidirectional shear stresses. Further generalisations of this work were subsequently produced by Gerrard (1968), Charyulu and Sheeler (1968), and Peutz, Van Kempen and Jones (1968).

By the end of the 1960s, then, the analytical techniques had been developed and a wide range of solutions were available for the determination of stresses and displacements within layered elastic pavements. To this was then added an extra dimension by the establishment of finite element and finite layer techniques (see for example, Zienkiewicz (1977), Cheung (1976), Cheung and Fan (1979)) which, subject to constraints of computer speed and storage, make readily available the elastic analysis of stresses and displacements within a pavement of arbitrary design.

2.3 The Development of Design Methods

2.3.1 Early Empirical Methods

Of the many types of pavement design methods currently in use, one of the oldest is that based upon the California Bearing Ratio (CBR) test. A load-penetration test, this was conceived and implemented by Porter (1938), who investigated the behaviour of a range of soils and assigned material classifications on the basis of the shape of the curve resulting from a plot of penetration against applied load.
FIGURE 2.1

TYPICAL PROFILE AND TERMINOLOGY FOR A FLEXIBLE PAVEMENT
Porter showed that, for the samples tested, a correlation of CBR results with pavement performance could be drawn (the performance being assessed by roughness, cracking and moisture in the subgrade). It should be noted, however, that his investigation was limited to two projects only - a total of twenty-one test holes. Further, Porter's aim was simply to differentiate between various qualities of crushed rock, and detect those vulnerable to loss of strength if subject to prolonged soaking.

Porter's test for "bearing ratio", however, found wider application perhaps than he had anticipated. The late 1930s saw the test being extended to subgrade evaluation in California, where an empirical relationship between CBR and required thickness of pavement was established. Other pavement authorities followed suit - for example, the U.S. Corps of Engineers, for airfield pavements - and in 1946, the United Kingdom adopted the method for its own design procedure. Indeed, the test is widely used today as a means of assessing the quality of subgrade material, and is a component in the majority of pavement design methods currently in use around the world. It is subject, though, to several limitations, which will be discussed at a later stage.

While Porter worked on his CBR correlations, similar approaches were under development elsewhere. In the early 1940s, a number of related methods emerged, such as those based on cone penetration tests. Cone bearing pressures were obtained by various means, and results from tests on the subgrades of failed and unfailed roads led to this bearing pressure being related empirically to pavement performance. As with all approaches of this type, however, extreme caution must be exercised in using the correlations outside of the particular geographical, climatic and geological conditions under which they were developed; consequently, their use will always be limited, and disregard for their limitations can be most expensive.

Another approach adopted by several designers around the 1940s was that of examining the soil type, and from this drawing conclusions about its performance in a pavement. The Civil Aeronautics Administration (USA) in its 1944 method, for example, classified the subgrade type using a method similar to Cassagrande. Taking account of frost and drainage conditions, the designer used the classification to
infer the material's strength. With the additional inputs of wheel load, subgrade and sub-base compaction, and grading curves for sub-base and base, the required thickness of each layer was then determined from empirical design curves. In a similar vein, the Group Index method (about 1945) classified the soil type by its Atterberg limits and percentage of particles finer than 75 μm. Making certain assumptions about compaction and moisture conditions, the sub-base and total thicknesses could be deduced from empirical curves. As with those methods based upon CBR, the classification methods are subject to particular limitations also. The major of these is that their empirical relationships are derived for particular soil and climatic conditions, and particular construction techniques. For example, the Group Index method arose from experience with unbound granular road-bases, and so does not cater for any other type of construction. In this way, any empirical approach is restricted to past experience only, and cannot readily be extended to new materials, other geographical regions, or improved construction methods.

2.3.2 AASHO and Other Tests

2.3.2.1 Introduction

During the 1940s, the member states of the American Association of State Highway Officials (AASHO) became increasingly conscious of a need for some form of full-scale research project. The main impetus for this arose from a rapidly growing traffic volume, and a pressing need to establish some policy for vehicle sizes and weights. There was consequently a requirement for data on which to base such a policy, and in 1948 the Association set up an administrative framework for the conduct of suitable research.

Following further confirmation of the need for data, the Highway Research Board in 1950 administered the first test - Road Test 1-MD. An existing concrete pavement in Maryland was used to investigate performance under repeated application of two single- and two tandem-axle loads, and the results presented in HRB Special Report 4. An AASHO recommendation for further projects resulted in the construction of a number of flexible pavement sections by the Western Association of State Highway Officials (WASHO). The location
chosen was in Idaho, the same loadings as used in Maryland were adopted, and testing occurred during 1953 and 1954, with the results appearing in HRB Special Reports 18 and 22.

2.3.2.2 The AASHO Road Test

Concurrently, planning was underway for a third, more comprehensive, test. Following the selection of a site near Ottawa, Illinois, construction of the six test loops commenced in August 1956.

This study sought to investigate "the performance of pavement and bridge structures of known characteristics under moving loads of known magnitude and frequency", and thence to develop suitable design procedures based upon the test results. With this purpose, then, three major experiments were designed - one for asphaltic concrete pavements, one for portland cement concrete pavements, and one dealing with short span bridges. In addition, several secondary experiments were commissioned, relating to base materials, shoulder paving, surface treatments, and subsurface studies.

The pavement facilities comprised six test loops, shown in Figure 2.2. The first of these carried no test traffic, being used for control purposes. The remaining five loops each carried two lanes of traffic, so that ten different load configurations were investigated. Trafficking of the test sections began in October 1958 and continued for 25 months, by which time each pavement had been traversed by 1.1 million axle loads.

The main factorial experiment employed standard asphaltic concrete, base, sub-base and embankment material, and aimed to investigate the effect of load and layer thickness upon pavement performance. For example, loop 3 was used to examine three thicknesses of asphalt, three base thicknesses, and three sub-base thicknesses, giving a total of 27 sections (plus a number of replicate sections constructed for statistical purposes).
FIGURE 2.2

LAYOUT OF A.A.S.H.O. ROAD TEST
The compilation of pavement performance records had effectively begun some years prior to the commencement of trafficking on the test sections. A Present Serviceability Index (PSI) formula was put together using a panel including engineers, representatives of transport firms and motoring organisations, and others, to rate subjectively 138 pavement sections with regard to their ability to serve high speed mixed commercial and passenger traffic. These ratings were then correlated with measurements of transverse and longitudinal profiles, cracking and patching in each section, to produce a relatively objective method of assessing pavement performance by means of a formula using the above measurements. During the actual conduct of the Road Test, each section of pavement was measured at two-weekly intervals, and in this way a complete serviceability record was compiled for the project.

2.3.2.3 Analysis of the Road Test Data

The primary analysis of the performance data for the Road Test was intended to relate the variation of PSI \( p \) with the number of axle applications \( W \). Using an average initial PSI of 4.2, and a terminal serviceability defined by \( p = 1.5 \), the equation used was

\[
p = 4.2 - 2.7 \left( \frac{W}{\rho} \right)^{\beta}
\]

\( W \) is here taken as a function of both load magnitude and distribution. \( \rho \) is a function of the pavement thickness design and \( \beta \) describes the shape of the serviceability trend. Both \( \beta \) and \( \rho \) were formulated for best fit as functions of a thickness index \( D \), where

\[
D = a_1D_1 + a_2D_2 + a_3D_3,
\]

and \( D_1, D_2 \) and \( D_3 \) represent respectively the thickness of the standard asphalt, base and sub-base materials. Thus, \( D \) is simply a linear combination of the individual layer thicknesses, with the coefficients being determined by simple regression analysis.
The Highway Research Board (HRB) Reports on the project point out that the above relationships are not intended as design equations, but rather should be used as a basis for procedures which account for other variables (e.g., subgrade type, environmental conditions, other pavement materials) not investigated during the Road Test. Indeed, the HRB sees the extension of investigations to cover such variables as the most pressing avenue for subsequent pavement research.

Not surprisingly, the Road Test has generated considerable discussion amongst pavement engineers. Some, neglecting the HRB's own reports, criticise the relationships as being particular to the conditions of the test, and, therefore, inadequate as design equations. Others provide somewhat more constructive criticisms. Peattie, for example, in his discussion to Dormon and Metcalf (1965) notes that a disproportionate number of failures of flexible sections occurred during the spring thaw. Consequently, there is not a smooth relationship between load applications and pavement damage, and this may significantly limit the applicability of the Road Test findings. Morgan (1972) notes a similar point, and also finds the standard of embankment compaction rather low. He concludes that the concepts could not be applied to the design of Australian pavements without a rigorous examination.

More recent writers express similar doubts about the validity of the AASHO relationships when applied to current techniques, materials and environments (see for example, Hudson (1981)). Further, Luhr and McCullough (1980) note that the strength coefficients (a₁, a₂, a₃ in equation (2.1)) were empirically determined as independent variables, and subsequently applied to the AASHTO Design Procedure to predict the Structural Number (SN) of a pavement. They point out that these coefficients were in fact dependent on layer thicknesses, and suggest that the averages from all loops, as used by AASHO, are very approximate. As a result, these and other authors do not see the AASHO relationships as being applicable to other than the Road Test without considerable verification or modification.
2.3.2.4 Other Full-Scale Tests

In the United Kingdom, the Road Research Laboratory (RRL) has also sought to evaluate pavement performance by means of full-scale experimental sections. The first series of flexible test pavements was constructed in 1949 as part of the Great North Road in Yorkshire, and the experiences there applied to a more comprehensive set of tests begun in 1957 on the same road at Alconbury Hill, Huntingdonshire. Thirty-three experimental flexible sections were used, involving two different surface treatments and five different base materials. The primary objective - to investigate the relationships between material types, layer thicknesses and pavement life - was attained, as outlined by Croney and Loe (1965). Of perhaps more significance than these results was the conclusion that the predictions of life from the CBR design procedure then in use bore little relationship to the observed performance, and the results were instrumental in formulating a replacement procedure (Road Note 29).

In more recent years, many road authorities have used experimental pavements as a means of testing new forms of material or construction. Lukanen (1979) reports, for example, an evaluation of full-depth asphalt pavements conducted by the Minnesota Department of Highways as a means of determining "gravel equivalents" for full depth AC thicknesses under local conditions. Similarly, the Pennsylvania Transportation Research Facility in the late 1970s constructed 17 test sections using typical local materials, in order to develop a better understanding of the relationship between pavement performance and various modes of distress (see Wang and Larson (1979), Wang and Gramling (1980)).

Understandably, many of these tests go no further than calibrating design procedures for particular climatic and geological regions. Others, by contrast, are used to evaluate the existing design procedures in an attempt to develop methods which more rationally reflect the observed performance of pavements. It is only experiments of the latter type that offer man the opportunity to improve his understanding of pavement mechanics.
2.4 Current Methods of Design

2.4.1 Introduction

As with any process of engineering design, pavement thickness design methods take a series of inputs related to materials, loading and environment, and seek to output suitable dimensions for the structural elements. A subsidiary aim, generally involving design comparisons, is the production of an economical design; the nature of the comparison is related to the type of economy being pursued.

Pavement loading embraces not only the load magnitude, but also the number of repetitions expected and the manner in which the load is distributed over the total tyre contact area. Most authorities simplify this by considering a certain number of repetitions of some "standard" reference load. A wider range of parameters may be dealt with in relation to material properties: stiffness, durability, resistance to fatigue, and the nature of elastic and plastic deformation should ideally all be considered, along with the sensitivity of these to changing temperature and moisture conditions. As might be expected, convenience generally dictates that this list also be simplified.

When in addition the variety of procedures available for analysing pavement behaviour is considered, it is clear that the wide array of parameters which might be included should lead to considerable variety in methods of pavement design. This is indeed the case.

2.4.2 Empirically-Based Methods

2.4.2.1 AASHTO – 1972

The approach formulated by the American Association of State Highway and Transportation Officials (AASHTO) as a consequence of the AASHO Road Test, has been a reference for many organisations. The inputs for the procedure are:

(i) the terminal serviceability of the pavement - a number representing failure of the structure in terms of road user perception ("Present Serviceability Index");
(ii) the loading expected during the pavement's required life - in terms of equivalent 8.2 t axle loads; and

(iii) the subgrade condition ("Soil Support Value") and climatic condition ("Regional Factor").

Using tables, the required Structural Number (SN) of the pavement is determined. This number is a linear combination of the individual layer thicknesses, using coefficients derived from simple regression analysis of the AASHO Road Test data. There is conceivably an infinite number of designs meeting the SN requirement; a number of alternatives may, therefore, be generated in order to select the most economical.

2.4.2.2 Road Note 29 - 1976

In the United Kingdom, the Road Research Laboratory (RRL) has published a series of design guides since 1960, based upon the AASHO Road Test results, experience in RRL full-scale road tests, and various developments in analytical techniques. The current guide assesses pavement loading as an equivalent number of standard 8.2 t single axles over the pavement's life (generally 20 years for flexible, 40 years for concrete pavements). Specific surveys may be undertaken to obtain this data; alternatively Road Note 29 provides tables of commercial vehicles per day and average number of axles per commercial vehicle for several road classes. Similarly, a table setting out average loads per axle on various road types is provided, which with the AASHTO axle-load equivalence relationship allows the equivalent number of standard axles to be computed.

The support offered by the subgrade is determined by its California Bearing Ratio (CBR). Using this value and the load data above, the required sub-base thickness is read from a graph; the one graph applies to all sub-base materials, the only qualifications being grading and sub-base CBR standards which must be met. For the basecourse, more account is taken of the different construction materials. Several types, both unbound and bound, are acceptable, and for each type the required thickness is presented as a function of the number of axles only. A similar treatment applies to surface course thickness, with different materials being recommended for various numbers of axles.
2.4.2.3 Other American Approaches

The California design method introduces another concept in the form of "material equivalency": the relative thicknesses of alternative materials required at a certain depth (e.g., basecourse) to meet the same pavement performance standard. California determines the cover on a given subgrade in terms of a gravel equivalent (GE) - that is, the required thickness of a standard gravel pavement. The value of GE is a function of subgrade strength and expected traffic during the pavement life. The first of these is represented by its R value (stabilometer figure) which bears some relationship to cohesion and friction angle; the second is derived in terms of equivalent 5 kip wheel loads, using traffic survey data and, apparently, the AASHO wheel-load equivalence factors. The manner in which these inputs determine the required cover (GE) is based upon field performance surveys, as also are the material equivalence factors supplied in order to convert gravel thickness to a thickness for materials such as asphaltic concrete or cement-treated base.

There are distinct similarities between the above method and that developed by the Asphalt Institute. In the latter case, the inputs are again subgrade support (by CBR, R-value and/or plate bearing tests) and traffic loading in terms of equivalent 18-kip standard axles. Using these data, the required thickness of asphaltic concrete (AC) is determined, and suggested substitution ratios for other materials (e.g., hot-mix sand asphalt 1.3:1, untreated granular base 2:1, untreated granular sub-base 2.7:1) are offered, along with other provisions concerning cover requirements over various base materials, and so on. The entire design process is heavily based upon the AASHO Road Test data, for both the relationship between the damaging powers of various load-axle combinations, and also the variation of pavement performance with subgrade support and pavement thickness. As a consequence, of course, the method inherits both the advantages and the limitations of the Road Test results.

Some measure of theoretical basis for design may be attached to the procedure of the U.S. Corps of Engineers (and its derivatives, such as that used by the National Crushed Stone Association). With data on subgrade CBR and traffic loading, using AASHO load equivalence factors, these methods seek to provide an adequate pavement
thickness and quality in order to guard against repeated shear
deformations in any layer. The means adopted to this end is the
modelling of the pavement as a homogeneous half-space; each layer of
material is then provided with sufficient cover to ensure that the
greatest theoretical shear stress experienced by the material is less
than its shear strength. Various material specifications are included,
but generally these are designed to be applied to granular courses -
design CBR, grading and Atterberg limits for the subgrade; design
CBR and compaction requirements for pavement courses. A surface
course of asphaltic concrete may also be designed, and its thickness is
presented as a function of the design traffic loading only.

Like many other methods, that of the Corps of Engineers is
subject to several limitations. It is, however, significant in that it was
perhaps the first to amalgamate some theoretical modelling of the
pavement with the results borne of Road Tests and less formal
experience. In this respect, it begins to break the shackles of the
purely empirical methods and provides the engineer with some basis for
assessing the value of previously untried materials and/or designs.

2.4.2.4 Other European Approaches

Like those of the United States, the various European design
approaches based upon correlation may be largely traced back to the
AASHO Road Test results. Since that time, of course, many modifi-
cations related to local experience have been made, but the influence
of the Road Test remains strong. In Poland and Hungary, for example,
the standard axle-load is ten tonnes - the required changes to design
data based originally upon an 8.2 t standard axle, however, were made
using load equivalencies derived from AASHO data. More universally,
it has been necessary to admit a wider range of possible subgrades:
this generalisation was performed independently of the Road Test
results but has been subsequently confirmed (or "fine-tuned") by both
European and U.S. experience. In this regard, the basic approach
seems to be the determination of a suitable pavement design for a
subgrade of CBR = 2.5, followed by an adjustment to account for the
real value of CBR, as follows:

$$\frac{H_{\text{CBR}}}{H_{2.5}} = \left(\frac{2.5}{\text{CBR}}\right)^n,$$

and typically $n \approx 0.4$.

This kind of approach has subsequently been adopted, with modifications where appropriate, by Hungary, Poland, Switzerland, France, and the United Kingdom.

The UK design routine (Road Note 29) has already received attention. In comparison, that used by France (1971) presents a series of design charts heavily based on the Asphalt Institute procedure plus local experience. Seven charts in all (one per base material type) have been assembled, and each contains designs covering four classes of traffic loading and four subgrade bearing capacities. More recently, deflections and stresses for each of the alternatives have been checked by French engineers using the tools of multi-layer elastic analysis.

These tools have been discussed, and their rapid development over the last thirty years noted. Simultaneously, this development has stimulated many changes to procedures of design; some of these will now be considered.

2.4.3 Methods Based Upon Elastic Theory

2.4.3.1 First Developments

Perhaps the first incorporation of elastic analysis into the process of pavement design occurred in the "shear strength methods", such as that of the U.S. Corps of Engineers already discussed. It was noted that whilst this approach has a rational basis, it is subject to several limitations. Principal among these is its use of the Boussinesq stress distribution (appropriate for a point load on a homogeneous half-space). Since a pavement is generally a layered, rather than homogeneous, structure, the method is limited to pavements with materials of low modular ratio, and its applicability is severely curtailed, since most structures containing bound materials are not covered. Other shortcomings include the assessment of the subgrade...
using an unconfined compression test (thereby restricting the approach to purely cohesive subgrades), and the fact that only a single load application and a corresponding material strength are considered. The fatigue nature of pavement failures is thus completely neglected.

2.4.3.2 Shell International Petroleum Company

In 1963, the Shell Corporation produced a design guide offering several improvements. The rationale for the proposed procedure was the need for an approach with sufficient theoretical basis so that extensive full-scale pavement tests would not be required before introducing any new material or construction method. Although this ideal is still being sought, Shell's method put forward several clues to directions which might prove fruitful in the search.

In order to produce the pavement design, two items of data are required. The subgrade strength is assessed by CBR or dynamic modulus (E) at equilibrium moisture. The relationship

\[ E \text{ (MPa)} = 10 \times \text{CBR} \]

is also provided for convenience, but the user should be warned that the relationship is approximate only, and without any real theoretical foundation. The required pavement life is expressed in terms of standard 10 t single axle loads, with a relationship resembling the AASHO fourth power "law" supplied for the conversion of mixed axle loads to a standard form.

Figure 2.3 illustrates the pavement system used for design purposes. A three-layer structure is assumed, and the design procedure is based theoretically upon three-layer elastic analyses, and experimentally upon the work of Dormon and Metcalf (1965) and others on the fatigue behaviour of asphaltic concrete and subgrade materials. Using this basis, along with subgrade and traffic data, the required thicknesses of granular and asphaltic layers can be determined so that strains at the locations shown in Figure 2.3 may be limited to safe values. Indeed, the supplied charts enable several alternative designs to be generated; the design may, therefore, be optimised under constraints such as cost or thickness.
FIGURE 2.3

IDEALISED PAVEMENT - SHELL (1963)
Shell's method, though evidently a marked advance, is not without its problems. The use of CBR to define subgrade properties is subject to severe drawbacks, as previously indicated. Similarly the assumption of a granular base becomes less realistic as the range and use of bound layers increases. Finally, the basis in elastic theory implies an assumption that, prior to failure, a pavement behaves elastically under repeated moving loads. This is demonstrably not so, however, since pavements do deteriorate. The use of limiting strains attempts to introduce the strength properties of the materials into consideration, but is a somewhat piecemeal approach. There is clearly a need for a more unified procedure, taking account of both the stiffness and strength properties of each proposed material.

2.4.3.3 Australian Approaches

Design procedures in Australia remained relatively primitive until 1979, when the National Association of Australian State Road Authorities (NAASRA) produced its Interim Guide to Pavement Thickness Design. Subsequently largely adopted by individual road authorities, the guide's approach bears a resemblance to the Shell method, although a number of significant modifications are included. Once again, subgrade strength is assessed by CBR along with a consideration of moisture conditions. If necessary, this assessment may be by soil classification, though the guide does not recommend this. The required pavement life is represented as the number of standard 8.2 t axles, and considerable data drawn from NAASRA surveys is provided to enable traffic volume and composition to be predicted over a given period. Such data include axle mass and configuration surveys, and various surveys of traffic composition for several different classes of road. Again, the AASHO fourth power relationship is used in converting different axle configurations to an equivalent number of standard axles.

Using the subgrade and traffic parameters, a basic required thickness of "standard" unbound granular pavement may be determined. The guide indicates that the required cover is that necessary to avoid distress in the subgrade, although what type of distress is to be avoided is not clear. An adjustment to the basic thickness may then be made on the basis of the required standard of construction.
As illustrated by Figure 2.4, the standard granular base may then be replaced by other materials, using values of material equivalencies tabulated as a function of material type and depth. For example, a bound dense graded base material (20 mm nominal size) at depth 150 mm below the surface, is given an equivalency of 1.20, meaning that 100 mm of the bound material may be substituted for 120 mm of the standard unbound natural gravel base. It should be noted that specifications are supplied for each acceptable type of material, in order that the equivalencies should have at least some realistic basis.

Once the selected materials have been chosen, and their relative positions and thicknesses determined, the guide prescribes a number of checks. Firstly, for any bound layer, a deflection check is required in order to guard against flexure cracks at the base of the bound layer. Secondly, each type of material has a minimum required cover (the guide does not indicate how these minima were determined), so a check is required here. Finally, notes are included covering suitable types of designs taking into account drainage and cracking considerations. Once a number of satisfactory designs have been generated, a procedure is provided for the economic comparison of alternatives, and the final pavement thickness design may then be selected.

As with the Shell method, the NAASRA guide is based on three-layer elastic analysis and experiments to determine "critical strains" in asphalt and subgrade materials. Likewise, it now has some potential for examining possible new materials or construction techniques without extensive field trials. However, the NAASRA guide in particular appears to forfeit this inherent flexibility by devoting considerable attention to a few materials, with their associated specifications and equivalencies.

2.4.3.4 National Institute for Transport and Road Research, South Africa

In 1977, Walker, Paterson, Freeme and Marais, from South Africa's National Institute for Transport and Road Research (NITRR) presented a paper to the International Conference on the Structural Design of Asphalt Pavements outlining recent developments in their own
### Table: Basic Thickness Required

<table>
<thead>
<tr>
<th>Depth Below Surface (mm)</th>
<th>Asphalt $10^5$ to $10^6$ standard axles</th>
<th>Dense graded base, 20 mm. (DGB 20) Bound</th>
<th>Natural gravel base (NGB) Unbound</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 50</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>50 - 100</td>
<td>1.5</td>
<td>1.2</td>
<td>1.05</td>
</tr>
<tr>
<td>100 - 150</td>
<td>2.0</td>
<td>1.4</td>
<td>1.05</td>
</tr>
<tr>
<td>150 - 200</td>
<td>2.5</td>
<td>1.4</td>
<td>1.10</td>
</tr>
<tr>
<td>200 - 250</td>
<td></td>
<td></td>
<td>1.15</td>
</tr>
<tr>
<td>250 - 300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 2.4

SAMPLE ADJUSTMENT OF PAVEMENT COMPOSITION USING MATERIAL EQUIVALENCIES
studies of flexible pavement behaviour. These developments, forming the basis for South Africa's current design procedures, represent quite a comprehensive body of investigations, and suggest a number of directions worthy of further study in the field of pavement mechanics.

In essence, the design procedure is based upon finite element analyses applied to extensively characterised materials. Table 2.1 sets out the material characteristics used in this process. The suggested approach to design is:

(a) selection of an initial approximate design on the basis of simplified structural analyses and economic comparisons;

(b) refinement, where justified, of the initial design using the more sophisticated processes of characterisation and analysis detailed in Walker et al (1977).

The performance predictions of the method have been initially verified using both experimental sections of normal traffic routes, and accelerated trafficking by the NITRR's Heavy Vehicle Simulator. Results to date have been encouraging, but there remains a need for further work in the areas of:

(a) verification of the pavement performance model;

(b) materials characterisation; and

(c) extension of the model to overlay design procedures.

It should also be noted that the design procedure includes a catalogue of typical designs (in conjunction with materials specifications) for routine use. It might be expected that the more sophisticated procedure, requiring as it does a significant volume of materials testing and traffic study, would be reserved for special applications.

2.4.3.5 Other Approaches

To complete the account of current design methods based on elastic theory, two other approaches will be examined. The USSR, with a method first specified in 1941 and its most recent version produced in 1973, adopts an approximate multi-layer procedure.
<table>
<thead>
<tr>
<th>Type of Material</th>
<th>Idealised Behaviour</th>
<th>Resource Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bituminous</td>
<td>Viscoelastic, anisotropic</td>
<td>Empirical relationships between maximum tensile strain and fatigue life.</td>
</tr>
<tr>
<td>Granular</td>
<td>Non-linear elastic or elastoplastic</td>
<td>$M_R \approx \kappa_1 \theta^{K2}$ where $\theta =$ bulk stress. Strength by repeated load triaxials.</td>
</tr>
<tr>
<td>Cohesive soils</td>
<td>Non-linear elastoplastic</td>
<td>$E \ (\text{MPa}) \approx \kappa .\text{CBR} \ (\kappa \text{ between 5 and 20}).$ Strength by repeated load triaxials.</td>
</tr>
<tr>
<td>Cement-treated soils</td>
<td>Elastic with cracking analysis</td>
<td>Finite element studies of structural effects of cracking. Empirical relationships between strain at break and elastic modulus.</td>
</tr>
</tbody>
</table>
A proposed design is reduced to a two-layer system by means of an "equivalent modulus". From a consideration of the expected traffic load, an allowable pavement settlement is derived. This, in turn, dictates the required flexural strength and complex modulus of elasticity of the pavement materials, against which the proposed construction may be checked and modified if necessary.

The approach adopted in Czechoslovakia is similarly representative of a number of methods. Here, each pavement type is reduced to a three-layer structure (as in the Shell method) and the tables derived by Jones (1962) used for the determination of elastic stresses and surface deflection. Once again, experimental results for the fatigue behaviour of asphalt are used in order to ensure that strains at critical locations are kept at a safe level.

2.4.4 Pavement Design and Management Systems

The process of engineering design is in general begun by assuming a trial structure. Once performance criteria have been constructed, the structure is analysed and the results assessed against the appropriate criteria. On the basis of this comparison, the structure may be rejected, in which case some modification should be introduced and the analysis repeated. Alternatively, if the trial structure is deemed acceptable, its construction should be followed by monitoring in order to determine its performance under service conditions. This provides a feedback to enable "fine-tuning" of both the performance criteria and the method of structural analysis.

In the context of pavement behaviour, there exists a pressing need to bring together the developments in structural analysis and observations of pavement performance, and considerable work has been done in recent years to this end. Figure 2.5 illustrates one such system, due to Monismith (1976). Figure 2.6 shows another formulated by Hadley, Hudson and Kennedy (1972), but despite variations in format the same key features may be observed in both.
* One can use the same system considering only one class of input, holding all others fixed; the objectives would have to be formulated accordingly.

NOTES:— 1. It should be recognized that the pavement system is embedded in a larger system.

2. The formulation of the objective is a difficult and essential first step.

3. It is usually necessary to have more than one measure of performance and the system is required to satisfy them concurrently.

4. It should be recognized that all components of the system can be considered as functions of time.

FIGURE 2-5

SCHEMATIC REPRESENTATION OF PAVEMENT SYSTEM
— after Monismith (1976)
Figure 2.6 Pavement Design System - after Hadley et al. (1972)
The systems approach does not necessarily introduce new techniques of structural analysis or pavement monitoring: its importance lies in that it brings these two facets together, and its emphasis upon the interrelationships is valuable as a means of integrating the pavement design process.

2.5 Conclusions

The foregoing discussion reveals a vast array of approaches to understanding the mechanics and performance of pavement structures. The collection of analytical tools now at our disposal was begun a century ago by Boussinesq, and there have evolved a number of procedures of great sophistication. Yet understanding still seems elusive, and our ability to predict pavement performance could at best be termed "reasonable".

The review has indicated the large body of empirical approaches to pavement design currently in use, and indeed in all but the simplest fields of study empirical investigations are necessary as a first step towards understanding the main influences to be considered. In every case, however, such approaches are difficult to generalise, and the problems which may result are typified by current usage of the California Bearing Ratio and its derivatives. Now applied to materials never intended by its author, the CBR bears only a vague relationship to generally accepted measures of soil stiffness and strength, yet lingers on in various forms whilst contributing little to the understanding of actual subgrades and their behaviour.

The increasing use of elastic theory has been documented, and it is clear that many authorities see promise in this approach. It is, of course, only a theory, and as such still requires verification at the time it is applied to particular pavement structures. The theory does, however, offer much towards the understanding of the behaviour of continua, as more recent work has indicated.
Elastic theory in itself, though, is independent of material strength properties, so that in order to predict pavement life some understanding of failure criteria is required. Most design procedures have employed some of the extensive body of test results now available (e.g., Heukelom and Klomp (1967), Monismith and McLean (1972), Brown and Pell (1972)) to define the point at which permanent deformations begin within a pavement. This permits an assessment of the elastic limit of the body to be made, and empirical relationships between fatigue life and stress or strain level permit the overall pavement life to be estimated.

There is evidently a continuing need for an approach which brings together the two aspects of pavement behaviour: recoverable deformations, and the incremental changes in permanent strains which may eventually lead to failure by fatigue. By this means, it may be possible not only to better predict pavement life, but also to better understand the process of pavement failure.
3.1 Pavement Function

Whilst the structure of a road carrying very light traffic may consist only of the natural soil, increases in the volume or weight of traffic, or requirements for all-weather service, generally call for some improvement to the structure. For this purpose, a pavement may be interposed between the wheel and soil, with the combined objectives of distributing wheel contact pressures in order to reduce stresses on the natural soil or other foundation ("subgrade") to an acceptable level, and of providing a wearing surface to protect the underlying materials from traffic abrasion and moisture penetration.

The two pavement functions are sometimes reflected in the structure, where a thin surfacing is applied to withstand traffic wear and provide waterproofing, while lower layers of soil or other material distribute the load to the subgrade. Commonly, however, the distinction is less clear, as for example with thick asphaltic concrete (AC) surface layers, which serve in both capacities.

This Chapter is presented as a review of the current understanding of materials and their properties.

3.2 Pavement Materials

3.2.1 Introduction

The design of a pavement structure entails the selection of the most economical combination of pavement layers, with respect to both thickness and material type, to suit the soil foundation and the traffic to be carried during the design life. In theory, then, any material satisfying the constraints of economy and performance could be used within a pavement: in practice, certain types of materials tend to be preferred, if only by reason of familiarity. Pavements in recent decades have, therefore, often included granular materials, with or without cement stabilisation, as sub-bases, a variety of materials including bound or unbound granular or macadam type layers as bases, and dense or open-graded bituminous-bound surface layers. Commonly, portland cement concrete, with or without reinforcement, has filled the role of both base and surfacing.
The prediction of pavement behaviour is a necessary prelude to development of a design, and for this an understanding of the behaviour of individual materials, as well as that of the total structure, is essential.

3.2.2 Materials in Design

Early methods of pavement design developed largely from observations of pavement performance. As a consequence, materials testing grew in a piecemeal fashion, with a great variety of tests (e.g. California Bearing Ratio (CBR) for subgrades, Marshall stability for asphalts) whose results bore little relationship to each other. Thus, it was not an easy matter to compare the behaviours of different types of materials, with the result that design processes often tended to develop in a very fragmented manner.

With the evolution of methods of analysis based on elastic theory, the trend towards fragmentation was to some extent stemmed, since the theoretical approach enforced the perception of different materials as having different characteristics, but a basic behaviour that could be described approximately by certain common parameters. The variations in material response could then be viewed as resulting from the dominating effect of particular modes of the same general behaviour.

The present situation is consequently something of a combination of the empirically and the theoretically based approaches, with a clear trend in the direction of the latter. Many correlations between the two sets of parameters have been compiled, but these should be used with great caution, in view of the inevitably limited scope of their application.

3.2.3 Material Behaviour

Even a brief inspection of road pavements is sufficient to show that pavement materials may be stressed beyond their limits of elasticity, and in time most structures under traffic develop permanent deformations and other signs of distress. It is, therefore, desirable in modelling for analysis the behaviour of a general layered pavement, that it be considered as comprising materials of a finite strength.
This may done fairly simply by regarding the component materials as being approximately linear elastic - perfectly plastic. The generally accepted description of material behaviour within the elastic range makes use of the parameters elastic modulus \( E \) and Poisson's ratio \( \nu \). Some diversity of opinion exists with regard to the description of material strength, as outlined in Appendix 3A; the Mohr-Coulomb yield condition (parameters cohesion \( c \) and angle of internal friction \( \phi \)) does, however, appear to offer simplicity along with a realistic modelling of behaviour, and this approach is adopted herein.

The set \( (E, \nu, c, \phi) \) then provides a basis for material characterisation that is consistent for all materials, more descriptive than earlier approaches (e.g. CBR), and appropriate to theoretical methods of analysis.

3.2.3.1 Stiffness Parameters

The primary factor affecting the moduli of cohesive and granular soils alike is the state of stress within the sample; that is, their typical stress-strain relationships are non-linear. In order to use forms of analysis based upon linear elasticity it then becomes necessary to select a representative value of modulus.

Seed, Mitry, Monismith and Chan (1965) have shown that the stiffness of typical cohesive soils tends to decrease with increasing axial stress, and that confining stress exerts a relatively small influence. Increases in moisture content, the other important variable, also tend to reduce the stiffness. Similar results have been reported by Nair and Chang (1973) following an extensive study. The latter authors also examined the behaviour of a range of granular soils, and concluded that stress state (particularly confining stress) appears to be the major influence in this case. Results suggest that increasing the confining stress produces an increase in stiffness, and this is confirmed by Yoder and Witczak (1975) and others. The relationship between modulus and strength has also been formulated in terms of deviatoric stress, or the first stress invariant \( I_1 \).
For many years, cement and lime have been added to natural soils to improve their suitability for pavement construction. The main effect of this addition is undoubtedly attributable to the hydration reaction of both substances in the presence of moisture; in some cases, there is also an improvement in properties due simply to the mechanical effect of the added particles - evidenced as an improvement in the soil grading. Overall, significant gains in stiffness may be achieved, as is clear from Table 3.1. In practice, other factors such as the likelihood of cracking and its effects upon the total structure, have an important influence on the use of both lime and cement.

The limiting case for soil improvement by means of cement is the use of concrete as a pavement layer. Common usage is either as a lean-mix sub-base, or as a structural concrete basecourse. As concrete is not a true "elastic" material, a representative stiffness is often obtained from the secant modulus measured at a stress level equal to 45 percent of the ultimate concrete strength. The empirical relationship of AS1480:

\[ E_c = 0.043 \rho^{1.5} F_c' \] (MPa)

where \( \rho \) = concrete density (kg/m\(^3\))

\( F_c' \) = concrete characteristic strength (MPa)

reflects the dependence of modulus \( E_c \) upon density, and hence upon concrete mix design and aggregate type. The role of \( F_c' \) in the approximation confirms this influence.

The other type of material most commonly used in pavement construction is bitumen (generally in conjunction with aggregates, as for example in asphaltic concrete). By far the most important factor affecting the performance of bituminous materials is temperature. Figure 3.1 shows modulus values determined by several authors, and the influence of temperature is clear. Where a range of values at one temperature is reported by particular authors, the variation as plotted accounts for variables such as stress state, mix design and loading frequency. It is evident that the variation attributable to all these factors is minor in comparison with the effect of temperature.
### TABLE 3.1

**Typical Values of Stiffness Parameters for a Range of Pavement Materials**

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Modulus (MPa)</th>
<th>Poisson's Ratio</th>
<th>Major Influences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesive Soils</td>
<td>20 - 400</td>
<td>0.3 - 0.5+</td>
<td>Stress state, Moisture content</td>
</tr>
<tr>
<td>Granular Soils</td>
<td>100 - 800</td>
<td>0.2 - 0.5+</td>
<td>Stress state</td>
</tr>
<tr>
<td>Lime-stabilised Soils</td>
<td>500 - 10 000</td>
<td>0.1 - 0.3</td>
<td>Stress state, Lime content</td>
</tr>
<tr>
<td>Cement-stabilised Soils</td>
<td>200 - 20 000</td>
<td>0.1 - 0.3</td>
<td>Stress state, Cement content</td>
</tr>
<tr>
<td>Portland Cement Concrete</td>
<td>10 000 - 40 000</td>
<td>0.1 - 0.2</td>
<td>Mix design</td>
</tr>
<tr>
<td>Asphalitic Concrete</td>
<td>200 - 20 000</td>
<td>0.3 - 0.5</td>
<td>Temperature</td>
</tr>
</tbody>
</table>
FIGURE 3.1
MEASURED VALUES OF ASPHALT MODULUS AS A FUNCTION OF ASPHALT TEMPERATURE

KEY
• Epps, Little, O'Neal and Gallaway (1978)
■ Kallas and Puzinauskas (1972)
□ Kennedy and Perez (1978)
◇ Monismith (1978)
○ Nair and Chang (1973)
▲ Seed, Mitry, Monismith and Chan (1965)
■ Sharma and Stubstad (1980)
▼ Yoder and Witczac (1975)
▲ Youdale (1982)
Some authors have presented values of modulus as depending upon the modulus of other layers within the pavement (see, for example, Walker et al (1977)). Whilst modular ratios within the pavement affect the stress response, and this in turn influences modulus values, it is contrary to the concept of elastic modulus to regard it as being primarily dependent on the values of moduli in adjacent pavement layers. The properties set out in Table 3.1 are characteristics of the individual materials determined by common laboratory methods (e.g. repeat load triaxial, diametral resilient modulus, repeated flexural tests).

The Table also presents typical values of Poisson's ratio to be found in the literature. In most instances, this ratio exerts a relatively small influence over the theoretical response of a pavement, and it is most convenient to assume a value based upon typical reported observations.

3.2.3.2 Strength Parameters

The strength properties of natural soils are a function primarily of material grading and moisture content, and this is well reflected in the many methods of characterisation based on sieve analyses and plasticity indices. A considerable range of results is reported by Seed, Chan and Monismith (1955), Seed, Mitry, Monismith and Chan (1965), Ahmed and Larew (1972) and others, which enable typical values of cohesion (c) and friction angle (ϕ) to be deduced. The dependence of measures such as deviator stress at failure upon parameters such as confining stress is quite evident in the reported results, and this (c,ϕ) representation enables a considerable body of data to be formulated in a convenient manner.

The strength of a naturally occurring soil, like its stiffness, is commonly improved by the addition of cement or lime. This imparts a certain cohesion to otherwise cohesionless sands and crushed rock, while with heavy clays the additives utilise the hydration reaction to produce a reduction in void moisture and a substantially increased frictional resistance to shearing. As a result, it is clear that the strength of the "stabilised" soil will be most dependent upon the cement/lime content, the type of natural soil or aggregate employed, and the
conditions of curing. This expectation is confirmed by the results of Moore and Kennedy (1971), and Anagnos, Kennedy and Hudson (1970) following extensive investigations of these and other factors. The typical values presented in the Table which follows are based upon these and similar data, generally obtained by indirect tensile or unconfined compressive strength tests made possible by the existence of both cohesive and frictional strength within the modified soils.

Bituminous materials depend for their strength upon the properties of the bound aggregate and the cohesion of the bituminous binder. Consequently, temperature and mix design exert the major influences over the strength, while of lesser importance is the rate of loading applied to the sample. Once again, properties are most commonly characterised by direct or indirect tensile tests, or flexural (beam) tests, as shown in the work of Epps et al (1978), Kennedy and Perez (1978) and Hadley, Hudson and Kennedy (1971).

Indirect tensile and flexural tests are also used to determine the tensile properties of portland cement concrete, whilst the unconfined compressive strength remains the primary specification and acceptance parameter. A considerable range of strengths may be produced by varying the mix design, and the large degree of control which the producer has over his product is in part responsible for the growing use of cement concrete as a base and surfacing material.

Table 3.2 presents typical values of the strength parameters $c$ and $\phi$ as deduced from the range of results cited above.

3.2.3.3 Properties under Cyclic Loading

Materials within road pavements are subjected to large numbers of stress cycles by traffic loading, and, therefore, should be of such a type that repeated loading does not cause undue deterioration. A number of workers have investigated the effect of load cycling upon the stiffness and strength properties of various materials, and Table 3.3 summarises a considerable number of reported results.
### TABLE 3.2

**Typical Values of Strength Parameters for a Range of Pavement Materials**

<table>
<thead>
<tr>
<th>Material</th>
<th>Cohesion (kPa)</th>
<th>Friction Angle (degrees)</th>
<th>Major Influences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Soils -</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clays</td>
<td>0 - 1 000</td>
<td>0 - 20</td>
<td></td>
</tr>
<tr>
<td>Clayey Soils</td>
<td>700 - 14 000</td>
<td>0 - 20</td>
<td>Material grading</td>
</tr>
<tr>
<td>Well-graded Gravels</td>
<td>0 - 1 000</td>
<td>30 - 50</td>
<td>Moisture content</td>
</tr>
<tr>
<td>Crushed Rock</td>
<td>negligible</td>
<td>40 - 55</td>
<td></td>
</tr>
<tr>
<td>Clean Sands</td>
<td>negligible</td>
<td>30 - 45</td>
<td></td>
</tr>
<tr>
<td>Lime-stabilised Soils</td>
<td>400 - 2 000</td>
<td>30 - 55</td>
<td>Lime content</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Soil type</td>
</tr>
<tr>
<td>Cement-stabilised Soils</td>
<td>500 - 7 000</td>
<td>30 - 55</td>
<td>Cement content</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Soil type</td>
</tr>
<tr>
<td>Asphalitic Concrete</td>
<td>1 000 - 10 000</td>
<td>30 - 45</td>
<td>Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mix design</td>
</tr>
<tr>
<td>Portland Cement Concrete</td>
<td>5 000 - 20 000</td>
<td>35 - 50</td>
<td>Mix design</td>
</tr>
<tr>
<td>Material</td>
<td>Effect on Stiffness Parameters</td>
<td>Effect on Strength Parameters</td>
<td>Reference</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>--------------------------------</td>
<td>-------------------------------</td>
<td>----------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Modulus</td>
<td>Poisson's Ratio</td>
<td>Cohesion</td>
</tr>
<tr>
<td>Soft clay, undrained</td>
<td>Down 50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Highly plastic clay</td>
<td></td>
<td>Down markedly</td>
<td></td>
</tr>
<tr>
<td>Saturated undrained clay</td>
<td></td>
<td>Fall with increasing cycles</td>
<td></td>
</tr>
<tr>
<td>Claysubgrade</td>
<td>Up</td>
<td>Compressive, tensile strength up</td>
<td></td>
</tr>
<tr>
<td>Silty clay, 95% saturated</td>
<td>Down</td>
<td>Up</td>
<td></td>
</tr>
<tr>
<td>Silty clay</td>
<td>Up</td>
<td>Up</td>
<td></td>
</tr>
<tr>
<td>Soil, saturated undrained</td>
<td>Down (maybe)</td>
<td>Little change</td>
<td></td>
</tr>
<tr>
<td>Micaeous silt</td>
<td></td>
<td>Down</td>
<td></td>
</tr>
<tr>
<td>Base, AASHO Road Test</td>
<td>Little change</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand-clay</td>
<td>Down</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand-clay</td>
<td></td>
<td>25% down</td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td>No change</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry Sand</td>
<td>Down</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand, drained</td>
<td>No change</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Granular, dry</td>
<td>No change</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Granular, partially saturated</td>
<td>Up/down 15%</td>
<td>No change</td>
<td></td>
</tr>
<tr>
<td>Granular, partially saturated</td>
<td>Up 20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Granular, saturated</td>
<td>Down 50%</td>
<td>Up 50%</td>
<td></td>
</tr>
<tr>
<td>Granular, saturated</td>
<td>Up 50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Limestone Residual</td>
<td>Down</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lime stabilised laterite</td>
<td>Down 40%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soil cement</td>
<td>Down 50%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
It is evident from the Table that, even for a particular soil type, few conclusions can be drawn regarding the change in either stiffness or strength with number of load cycles. It appears that the way in which these properties may change is influenced not only by soil type (grading or classification), but also by moisture conditions and the level of applied load. Furthermore, in view of the range of roadmaking materials encountered in practice, it is clear that no general conclusions can be drawn at this stage. Should it be necessary to consider the improvement or degradation of properties in any particular case, the individual material should be tested under typical service conditions of moisture content, stress state, load rate and frequency.

3.3 Conclusions

The basic description of material behaviour adopted herein is one which is relatively simple, but retains the key elements of observed pavement behaviour. Elastic behaviour, therefore, is described by parameters $E$ and $v$, appropriate to continua, whilst a finite strength is attributed to all component materials and the onset of yield defined in terms of $c$ and $\phi$.

The available data concerning variations in these parameters with the speed or number of load cycles, however, tend to be patchy and at times contradictory. It is particularly evident from a survey of such data that no general conclusions can be drawn, and for the purposes of the analysis which follows, it will be assumed that load cycling does not markedly affect parameters. More elaborate assumptions about material behaviour should be accompanied by specific tests on particular materials.
FAILURE CRITERIA FOR PAVEMENT MATERIALS

Where a material is loaded to failure in a simple manner, as for example a metal under uniaxial tension, the strength of the material may readily be defined in terms of the single stress variable. As more complex stress states are considered, however, it becomes increasingly difficult to define a "yield function" capable of differentiating between safe stress states and those causing failure.

Coulomb (1773) was among the first to advance a yield function containing more than one stress component, when he considered the failure of soil in shear, and the strengthening effect of a stress normal to the potential plane of shearing. He proposed that a safe state of stress in a plane was one satisfying the inequality

\[ \tau < c + \sigma_n \tan \phi \]

where  
\( \tau \) = shear stress  
\( \sigma_n \) = normal stress  
\( c \) = material cohesion  
\( \phi \) = material angle of internal friction

It was not for some years, however, that Coulomb's hypothesis (Figure 3.A1) was considered more than a theoretical diversion. In the meantime, Tresca (1868) turned his attention to the extrusion of metals, and from measurements of loads required to extrude through dies of various shapes, concluded that yield occurred when the maximum shear stress reached a certain limit. The similarity with Coulomb's work is here evident, but Tresca's expression of his findings was somewhat different. In terms of the three principal stresses within the metal, he concluded that yielding took place when

\[ \left[ (\sigma_1 - \sigma_2)^2 - 4k^2 \right] \left[ (\sigma_2 - \sigma_3)^2 - 4k^2 \right] \left[ (\sigma_3 - \sigma_1)^2 - 4k^2 \right] = 0 \]

where  \( \sigma_1, \sigma_2, \sigma_3 \) = principal stresses  
\( k \) = maximum shear stress.
COULOMB FAILURE THEORY

FIGURE 3·A1

TRESCA YIELD CRITERION - PROJECTED ON DEVIATORIC PLANE

FIGURE 3·A2
In principal stress space as shown in Figure 3.A2, the Tresca yield surface is a regular hexagonal prism, its axis coincident with the hydrostatic axis.

The stress circle representation proposed by Mohr (1900) provided a means of extending the Coulomb criterion to apply to conditions of plane strain, viz:

\[
\left\{ \left[ \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{XY}^2 \right] \sec \phi \right\}^{\frac{1}{2}} < c + \left( \frac{\sigma_x + \sigma_y}{2} \right) \tan \phi
\]

or in terms of principal stresses

\[
\left[ \left( \frac{\sigma_1 - \sigma_2}{2} \right)^2 \sec \phi \right]^{\frac{1}{2}} < c + \left( \frac{\sigma_1 + \sigma_2}{2} \right) \tan \phi
\]

See Figure 3.A3.

The latter formulation also bears a resemblance to the Tresca criterion, particularly if \( \phi = 0 \) whence

\[
(\sigma_1 - \sigma_2)^2 < 4c^2
\]

Subsequent experimental work, again concerned largely with metallic behaviour, gradually led to the yield function of Von Mises (1913) gaining stronger acceptance as a description of material behaviour. His formulation, originally proposed as a convenient approximation to Tresca, but later shown to be an improvement in many respects, was presented as

\[
s_1^2 + s_2^2 + s_3^2 < k^2
\]

Geometrically, the Tresca hexagonal prism has been replaced with a circular cylinder, and this brings with it other advantages for the analytical description of plastic strain behaviour of a yielded material.
MOHR STRESS CIRCLE AND MOHR-COULOMB YIELD CRITERION

FIGURE 3-A3
With the two lines of development - for metals and soils - converging in this manner, the next logical step was to extend the Mohr-Coulomb criterion to three dimensions, or alternatively the Von Mises criterion to frictional materials. The latter approach, taken by Drucker and Prager (1952), resulted in a yield function of the form

$$\alpha J_1 + J_2 < k$$

where

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_X + \sigma_Y + \sigma_Z$$

$$J_2 = \frac{1}{2} S_{ij} S_{ij}$$

$$= \frac{1}{6} \left[ (\sigma_X - \sigma_Y)^2 + (\sigma_Y - \sigma_Z)^2 + (\sigma_Z - \sigma_X)^2 \right] + \tau_{XY}^2 + \tau_{YZ}^2 + \tau_{ZX}^2$$

$$S_{ij} = \sigma_{ij} - \left( \frac{J_1}{3} \right) \delta_{ij}$$

$$\delta_{ij} = \text{Kronecker delta}, \text{ zero for } i \neq j, \text{ unity for } i=j.$$  

For $\alpha=0$, the inequality reduces to that of Von Mises (cylinder), while $\alpha \neq 0$ produces a conical yield surface. In contrast, Shield (1955) generalised the Mohr-Coulomb condition to give

$$\sigma_1 < \sigma_j A^2 + 2cA \quad (i,j=1,2,3)$$

where $A = \tan \left( \frac{\gamma + \phi}{2} \right)$,

and the six planes so generated form a regular hexagonal pyramid in three-dimensional principal stress space.

While the theoreticians have puzzled over ideal soils, a number of investigators - for example Kjellerman (1936), Kirkpatrick (1957), Wu et al (1963) and Bishop (1966) - have examined a range of soils in an effort to determine suitable yield criteria for real materials.
Basically, this work indicates that for sands and clays the Mohr-Coulomb criterion is adequate for predicting failure under plane strain conditions. More complex models of behaviour, such as those of Roscoe and Burland (1968) for clays, and Di Maggio and Sandler (1971) and Lade and Duncan (1975) for granular materials, undoubtedly have application in particular cases. However, with their increasing complexity they are also increasingly specific to particular materials; for the purpose of general analysis this is something of a limitation, and the simpler criteria are more useful in developing broad descriptions of behaviour.

For these reasons, the plane strain analyses of later chapters use as a basis the Mohr-Coulomb description of yield, and subsequent extensions to three-dimensions incorporate the criterion presented by Shield (1955) as a development of the plane strain formulation.
CHAPTER FOUR

PAVEMENT LOADING AND RESPONSE
4.1 Introduction

The structure of a pavement, and the characterisation of its constituent materials, cannot be considered in isolation from the manner in which its behaviour is to be modelled: the assumptions about behaviour directly influence the approach taken to characterisation, and vice versa. In order to identify those idealisations of pavement response which are most appropriate for use in later analyses, this chapter examines the results of a considerable body of experimental work with respect to both the features of pavement loading, and the characteristics of the pavement's response to that load.

4.2 Pavement Loading - Experimental Investigations

The realistic analysis of the behaviour of pavement structures under moving loads presupposes an understanding of the likely form and magnitude of loads, quite apart from assumptions about pavement response. The stress states which arise from the application of load through a flexible pressurised tyre to the pavement surface may to some degree be estimated by analytical means; the primary tool to date, however, has been experimental investigation. In this way, a body of information has been established covering not only load magnitudes, but also the way in which the load is distributed over the area of contact, for changes in this distribution can have a very marked influence on the performance of a pavement.

4.2.1 Load Magnitudes

The simplest measure of pavement loading, and the measure most commonly employed for statutory purposes, is the magnitude of wheel and axle loads applied normal to the surface. This area is one of continuing dispute, as might be expected from the clear conflict of interests between road hauliers and road constructors. Through road authorities, most governments have tended to apply to hauliers limitations upon single tyre loads, and upon the loads which may be carried by axle groups of various layouts, and typical values of such limits for Australian conditions are shown in Table 4.1. It may be noted from the Table that limits are generally also applied to tyre inflation pressure, as a means of directly controlling the normal stresses applied to the pavement.
TABLE 4.1

Typical Australian Tyre and Axle Load Limits

<table>
<thead>
<tr>
<th>Axle Group</th>
<th>Layout</th>
<th>Axles</th>
<th>Tyres</th>
<th>Range of Load Limit (tonnes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Single</td>
<td>Single</td>
<td>4.5 to 4.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Single</td>
<td>Dual</td>
<td>8.1 to 9.0</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>Tandem</td>
<td>Single</td>
<td>9.0 to 9.6</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>Tandem</td>
<td>Single/Dual</td>
<td>11.2 to 11.8</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>Tandem</td>
<td>Dual</td>
<td>13.1 to 16.4</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>Triaxle</td>
<td>Dual</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Maximum gross single tyre load 2.3 to 2.7 t

Maximum tyre pressure - cross ply 700 kPa
- radial 825 kPa
In addition to normal stresses, however, all tyre-pavement systems involve a transfer of shear stress, for without such a transfer vehicular acceleration and deceleration would clearly be impossible. A considerable body of testing exists in connection with determining the friction coefficients \( \mu = \frac{\text{shear force}}{\text{normal force}} \) of various tyre-surface combinations, and results such as those of Moyer (1963) and Department of Main Roads, N.S.W. (1978) show that a wide range of values for \( \mu \) may be encountered. Whilst values are to some degree dependent upon the test method employed, typical values of \( \mu \) may range from about 0.35 for worn asphaltic concrete surfacing (wet) to about 1.0 for dry concrete or asphaltic surfaces in good condition.

The friction coefficient \( \mu \), however, is merely a guide to the shear force imparted by a vehicle to the pavement, since this varies with the condition of vehicular motion. For example, acceleration involves a nett rearward shear on the pavement, whilst for freewheel conditions there is theoretically no nett shear load (in practice, some force will be involved to balance the drag forces on the vehicle). It is then evident that \( \mu \) defines the maximum shear which may exist between tyre and surface; in fact this maximum is seldom attained, and in order to determine actual pavement loadings some further detail is necessary. This shall be dealt with in later discussion of contact stresses.

4.2.2 Contact Areas

Whilst nett wheel and axle loads form the basis of statutory restrictions upon pavement loading, it is evident from both theoretical and practical viewpoints that the actual pavement performance depends primarily upon the way in which the total load is distributed over the contact area.

Marwick and Starks (1941) and Bonse and Kuhn (1959) have shown that at loads and inflation pressures close to those recommended by the manufacturer, typical tyres present an elliptical contact area. This is supported by later findings of Sanborn and Yoder (1967) and Sargious (1975). A more wide-ranging investigation by Lister and Nunn (1968) has contrasted this with the cases of tyres at markedly
lower or higher loads. From a low load (say 30% of manufacturer's recommended maximum) the contact area changes from approximately circular to a shape resembling a "fattened ellipse" or rectangle with rounded corners, at a load of say 150% of the recommendation. Freitag and Green (1962) and Clark (1971) have reached similar conclusions, and Figure 4.1 shows typical forms of contact area.

As a first approximation, the contact area may be computed from the nett tyre load and inflation pressure; due to variations in contact stress across the region, such a calculation acts only as a guide, and it is evident from Lister and Nunn (1968) and Clark (1971) that at low wheel loads, the quotient (load/inflation pressure) underestimates the contact area, whilst at high loads, the area is overestimated. At normal recommended loads, the area may be estimated to within approximately 20% of actual.

4.2.3 Contact Stresses

With regard to normal contact stresses, most authors are agreed that the stresses vary in the longitudinal direction in an approximately trapezoidal form. The peak stress has been found to range from 1.0 to 1.5 times the inflation pressure, and the proportion of the length under maximum pressure varies between 0.6 and 0.9. Figure 4.2 shows typical stress distributions, and Marwick and Starks (1941) have noted that as the tyre load increases to overload edge peaks may appear on the loadform. Similarly, the vehicular motion tends to influence the form, with for example acceleration leading to a slight increase in pressure at the front of the tyre contact (Clark (1971)).

A parabolic or semi-elliptical form appears to characterise the transverse distribution of normal stress, again with a peak of the order of 1.0 p to 1.5 p. At overload, Marwick and Starks (1941), Starks and Whiffin (1959) and Bonse and Kuhn (1959) have shown that the distribution changes to a double peak form, and Freitag and Green (1962) have demonstrated that under extreme conditions of load these peaks may reach a stress of the order of 2.5 p to 3.0 p.
Load (Proportion of Recommended Maximum)

0.3    1.0    1.5

Typical Contact Shape

Circle

Ellipse
$A = 1.6B$

Fattened Ellipse
$A = 1.7B$

FIGURE 4.1

TYPICAL TYRE CONTACT AREAS AT VARYING MAGNITUDES OF LOAD
<table>
<thead>
<tr>
<th>NORMAL TRACTION</th>
<th>LONGITUDINAL DISTRIBUTION</th>
<th>NORMAL LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OVERLOAD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACCELERATION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DECELERATION</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TRANSVERSE DISTRIBUTION</th>
<th>NORMAL LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>OVERLOAD</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SHEAR TRACTION</th>
<th>LONGITUDINAL DISTRIBUTION</th>
<th>CONSTANT VELOCITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACCELERATION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DECELERATION</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.2**

Representative distributions of tyre contact stresses
The shear tractions applied by a pneumatic tyre to a pavement show, as suggested previously, a marked dependence upon the state of motion of the vehicle, and typical longitudinal distributions are illustrated in Figure 4.2. The data on which these diagrams are based is somewhat limited, but it is evident from the papers of Marwick and Starks (1941) and Bonse and Kuhn (1959) that under constant velocity conditions, a nett rearward shear exists and its distribution has a primary peak in the range 0.3 p to 0.5 p. Data concerning the transverse distribution of shear stresses is even more limited, but Clark (1971) has found that an inward shear is present beneath tyres under conditions of constant or zero velocity. Further conclusions are difficult to make at this stage.

None of the above investigations has demonstrated that vehicular speed has any significant effect upon distributions of contact stresses, particularly in comparison to the influence of acceleration and deceleration already noted.

4.3 Pavement Response - Experimental Investigations

Parallel with various authors' work on the loads applied to pavements by pneumatic tyres, there has developed a field of investigation of pavement response to surface loading. Loads have in general been idealised as either typical tyres or flexible circular pressure cells applying normal load only, and pavements have most commonly been represented by single deep layers; nevertheless the understanding of pavement stress and strain response has grown significantly, and now offers some insight into the applicability or otherwise of various pavement design models.

4.3.1 Stresses

The greatest body of work to date in the area of pavement response has been concerned with the stresses within the continuum, and the comparison of these with the predictions of linear elasticity, assuming homogeneous and isotropic materials.
The vertical stresses within a single layer (that is, a modelled half-space) have been measured in granular media by Allwood (1956), Holden (1967), Morgan and Holden (1967), Kolbuszewski and Hu (1961) and Turnbull, Maxwell and Ahlvin (1961). The latter two investigations also covered cohesive materials, as did McMahon and Yoder (1960), Sparrow and Tory (1966) and Brown and Pell (1967a), and it may be concluded that for both types of material a reasonable agreement was found to exist between the linear elastic stress predictions, and experimental measurements.

Data regarding the distribution of horizontal stresses within a half-space has also been presented by all the above authors. The prediction of stresses involves selecting suitable values for Poisson's ratio \(\nu\), yet in most cases this has been done with regard only to "goodness of fit". Using values for \(\nu\) of 0.5 (clayey-silt) and 0.3 (sand), Turnbull, Maxwell and Ahlvin (1961) have obtained a satisfactory agreement between measurements and prediction; on the other hand, even after selecting \(\nu\) for a "best fit" the remaining reports show little apparent agreement in either sands or clays. Beyond suggestions that anisotropy considerations may be useful (Holden (1967)) few reasons for this variation are offered.

Turnbull, Maxwell and Ahlvin (1961) also noted that the vertical-horizontal shear within a half-space may be satisfactorily predicted using isotropic elasticity. The same conclusion is reached by Sparrow and Tory (1966) studying Keuper Marl. Less agreement was observed by Brown and Pell (1967a) even after applying a depth-dependent formulation for Poisson's ratio, but it was observed that shears were lower than predicted, and so to some degree "safe".

A two-layer pavement consisting of 300 mm crushed stone over Keuper Marl was examined by Brown and Pell (1967b). Since the modular ratio \(E_1/E_2\) was found to lie in the range 0.5 to 1.3, it is clear that the predicted values of stress should differ little from those of the single layer case. However, even layered elastic theory was not able to provide satisfactory predictions of response: within the crushed stone layer, vertical and radial stresses provided only fair agreement with theory, whilst tangential and shear stresses differed markedly
from predictions. It was at least found that the (radial) tensile stress at the base of the top layer could be estimated with some success.

A more complex pavement (asphalt/base/subgrade) was investigated in some detail by Sowers and Vesic (1962) in order to assess the effect upon vertical subgrade stresses of different base materials. It is rather unfortunate that the asphalt properties were not recorded, for all comparisons employ only two-layer elastic theory for a pavement clearly consisting of three distinct materials. For the cases of granular and soil-bound macadam bases it was found that the vertical stresses in the subgrade could best be predicted using single layer (half-space) elastic theory, despite the fact that the modular ratio \( E_{\text{base}} / E_{\text{subgrade}} \) lay in the range 3 to 10. This behaviour was attributed by the authors to the relatively low tensile strengths of the base materials, which may have resulted in early yielding and a subsequent redistribution of stresses into the top of the subgrade. Beneath a base of sand asphalt (modular ratio as above), it was similarly found that single layer theory offered the best estimation of subgrade stresses. Near the top of the subgrade, the stresses were higher even than those of one-layer theory, and this variation has been accounted for by postulating a stress dependent modulus for the subgrade (higher modulus at higher confining pressure \( \sigma_3 \)) and a consequent concentration of stress towards the interface. Finally, two-layer theory was found to predict satisfactorily the stresses beneath a soil-cement base (\( E_{\text{base}} / E_{\text{subgrade}} \) in the range 50 to 100). The better agreement in this case may perhaps result from the greatly increased strength and stiffness of the base course, which ensured that the subgrade should be only lightly stressed, and consequently less scope existed for deviations from elastic behaviour. At the same time, the other three tests demonstrate that under normal wheel loads, non-linearity of the subgrade response may be encountered, and further that the possibility of yielding in the base should be considered.

In general terms it is evident that elastic theory can be useful for vertical stress prediction under some circumstances. Horizontal and shear stresses are apparently less easily estimated in this way, and it seems that soil strength parameters (as reflected in both material yield and the onset of non-linearity) should not be neglected in the process of pavement stress analysis.
4.3.2 Strains

Most investigations of strains within pavement structures have suffered from the difficulty of assigning suitable values of elastic modulus to the constituent materials. It is evident that often values obtained from laboratory tests are not closely related to those which would give a "best fit" of measured strains to the predictions of elastic theory, and in consequence many investigations have yielded only qualitative conclusions.

Using single layer pavements, Turnbull, Maxwell and Ahlvin (1961) found only general agreement with linear elastic theory for the vertical strains in both sand and clayey-silt, and concluded that some non-linearity of material response (i.e. lower modulus for more severe stress state) was evidenced by their results. Eggestad (1963) and Holden (1967) similarly concluded that linear elasticity represented a poor approximation in the case of sands, whilst Sparrow and Tory (1966) and Brown and Pell (1967a) reached the same conclusion with regard to clay. The latter two investigations also dealt with horizontal strains, and again it was found that the observed behaviour could be better explained qualitatively using an hypothesis of non-linear elasticity.

The measurement of strains within layered systems has been mainly concerned with pavements topped by a relatively thick asphaltic concrete (AC) layer, in an effort to determine the limiting tensile conditions at the base of this layer. Garrison (1966) and Gusfeldt and Dempwolff (1967) found that whilst the tensile strains at the base of the AC may be satisfactorily estimated (though the scatter of results is large), strains closer to the pavement surface were not well predicted by layered elastic theory. Using three-layer theory and dynamic testing to determine moduli, Klomp and Niesman (1967) concluded that reasonable estimates could be made of strains on the load axis, and longitudinal strains at the base of the AC, while strains elsewhere were more difficult to predict.

It is apparent from these few reports not only that the strain response of layered systems is in need of further experimental investigation, but also that linear elasticity is here quite limited in its application. It is possible that this may be evidence of both material non-linearity, and the variability of material properties within pavement structures. In this regard, more extensive investigation combined with a statistical treatment of the results may be revealing.
4.4 Pavement Idealisations in Analysis and Design

4.4.1 Existing Design Guides

A brief survey of typical design methods is sufficient to establish the way in which the experimental and theoretical work already outlined is translated into engineering practice. From such a survey it is evident that two basic philosophies underlie most design methods currently in use.

The AASHTO Interim Guide, and its many derivatives, continues to be based largely on the results of the AASHO Road Test. With traffic assessed in terms of equivalent standard axles, and subgrade CBR measured, the required value of structural number may be determined. Material thickness equivalence coefficients, also derived empirically from performance testing, are used in order to generate a range of thickness profiles which meet the structural requirement. It is clear that to date the design philosophy has been barely influenced by subsequent theoretical developments.

In the United Kingdom, Road Note 29 adopts a very similar approach. Once again, Road Test results form the basis of layer thickness-performance relationships. Traffic and subgrade are assessed in much the same way as by the AASHTO Guide, and the possible pavement materials are closely specified. The Road Note gives little indication of its underlying philosophy and, as might be expected where the basis is largely empirical, offers little scope for the use of "non-standard" materials in pavement construction.

The design guide prepared by the Shell Corporation also has many derivatives. In many respects it has been a leader in the establishment of a "rational" basis for construction practice. Multi-layer elastic theory has been applied to a model pavement of three layers (AC/base/subgrade), using dynamic moduli which may be measured or otherwise estimated for the specific materials involved. Whilst certain simplifications have been made in the interests of the user, and certain assumptions (such as limiting values of modular ratio) have been included, the Shell procedure does recognise that a range of materials may be employed, and allows the designer a certain freedom with respect to their choice. The criteria against which proposed pavement designs are tested are two values of limiting strains: tensile at the base of the
AC, and compressive at the top of the subgrade. Various workers' results have been used to establish numerical values of these critical strains as functions of the required number of load cycles only, while the load equivalencies of the AASHO Road Test were adapted to permit traffic loading to be expressed in terms of standard axles. In terms of its progress towards a design guide with a basis in both practice and theory, the Shell procedure represents advances on two fronts. Whilst the observations from various full-scale performance tests have been employed, so too have the findings of continuum mechanics through layered elastic analysis. Further, an attempt is made to assess proposed structures in terms of a measure of material strength (here, by limiting strains at particular locations).

Recent trends in Australian practice reflect the general move to use the best of both full-scale tests and theoretical developments. The NAASRA Interim Guide (1979), already discussed in Chapter 2, is constructed primarily upon the CBR approach of the AASHTO Guide and Road Note 29. At the same time, provision is made for the use of elastic theory along the lines of the Shell guide, particularly for pavements containing bound layers in which the tensile strength of the bound material(s) may be critical to the design. Further uses, for example in the prediction of vertical deflections, are also outlined. This type of approach appears to be representative of more recently developed design procedures, and evidences the considerable progress in analysis and materials characterisation as well as the accumulation of performance information for existing pavements.

4.4.2 Present Directions

Monismith (1976) has outlined further developments in pavement analysis and design which have to date exerted little effect upon current design procedures. There exist, for example, analytical tools for determining the elastic response of a pavement of any number of layers. Meanwhile, techniques of material characterisation allow the engineer to determine the influence of stress level, temperature, loading rate and other factors upon the behaviour of pavement materials, and more advanced methods of analysis allow this behaviour to be more closely modelled. Further studies on limiting states (i.e. the onset of failure, by cracking, distortion or other means) continue to broaden our understanding of the materials at our disposal.
While the details continue to be filled in, and a wider range of materials explored, perhaps the greatest challenge at this stage is that of distilling the growing body of information into a form that may be used with convenience at the design stage. One approach to this problem is developed in subsequent chapters.

4.5 Response of Idealised Pavements

To conclude the discussion of pavement response, two idealised continua will be examined. The effect of static loading on linear elastic continua has already been well documented in a wide range of publications; it is intended here to instead examine the stress paths generated at a fixed point within a pavement by a moving surface load. It is envisaged that this approach may shed some light upon appropriate means of analysing a pavement which eventually fails by repeated traffic loading.

The first set of analyses deals with an elastic half-space. In Figure 4.3 are shown the stress paths (in terms of principal stresses) at various depths within the plane strain continuum. The Figures cover both uniform and trapezoidal loadforms, and also illustrate the effect upon stress path of introducing a shear component into the load. It is clear that for a half-space, the stress paths at a point are to some degree influenced by the shape of the loadform: a far more significant influence, however, is the shear component of the load. From Figures 4.3(c) and (d), where the maximum shear stress and maximum normal stress have equal magnitudes, this influence is most evident. The circuitous nature of the stress paths is also of some interest.

By way of comparison, the second set of analyses deals with a representative flexible pavement, as illustrated in Figure 4.4. As many pavement design procedures concentrate primarily on "critical" locations at the base of the asphalt and the top of the subgrade, these locations will also be examined here. The results of a series of analyses (varying shear components of load) are presented in Figure 4.5. Whilst the stress path at the base of the AC may be closely approximated by proportional loading (straight line from the origin), it is again evident that a more circuitous path applies to the top of the subgrade. Indeed, the Figure shows quite clearly the important difference between the stress path at a point under static loading (proportional increase of $\sigma_1$ and $\sigma_2$) and that
STRESS PATHS BENEATH MOVING SURFACE LOADS
(Half-space, Plane strain)
STRESSES BY ANALYTICAL INTERGRATION OF BOUSSINESQ RESPONSE
<table>
<thead>
<tr>
<th>Profile</th>
<th>Thickness (mm)</th>
<th>Material</th>
<th>E  (MPa)</th>
<th>ν</th>
<th>C  (kPa)</th>
<th>ϕ  (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>Asphalitic concrete</td>
<td>5000</td>
<td>0.4</td>
<td>3000</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>Granular base</td>
<td>200</td>
<td>0.3</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>Granular sub-base</td>
<td>150</td>
<td>0.3</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>∞</td>
<td>Clay subgrade</td>
<td>50</td>
<td>0.3</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>

FIGURE 4.4 - Representative Flexible Pavement for Stress Path Analyses
FIGURE 4-5

STRESS PATHS BENEATH MOVING SURFACE LOAD
(TYPICAL LAYERED PAVEMENT, PLANE STRAIN)

STRESSES BY FINITE LAYER ELASTIC ANALYSIS
under moving load conditions. Altogether different stress paths are involved, and where a pavement is to be analysed for inelastic behaviour as well as linear elastic, account needs to be taken of the fact that repeated moving loads may in this way produce quite a different end result from repeated static loads.

4.6 Conclusions

The foregoing discussion has served to clarify a number of points relevant to the analysis of pavement behaviour. The available work suggests firstly, that the load exerted by a pneumatic tyre on a surface may be closely approximated by a fairly simple form, and secondly, that the response of a pavement structure to that load may not be so readily approximated. Variability of both materials and construction appear to render the pavement significantly non-linear in its behaviour, and it appears that linearity must be regarded merely as a guide to the true response.

Despite its shortcomings, however, the linear elastic model is important for its simplicity. It provides a convenient initial model of response, and in the absence of sophisticated computing hardware provides one of the few analytical tools with a basis in both theory and practice. As such, it remains useful.

Studies of stress paths within two idealised continua have demonstrated the significant role of shear loading in the response of pavements. Perhaps even more important, however, is the clear difference in response at a point to a repeated moving load from that of a cycled static load. This serves to emphasise the need for an analysis which recognises this distinction, particularly in view of its implications for inelastic behaviour and the accumulation of permanent deformations.
CHAPTER FIVE

PAVEMENT SHAKEDOWN: A ONE-DIMENSIONAL ANALYSIS
5.1 Introduction

Continua, such as pavement structures, are constrained by economic factors to function at stress levels beyond the elastic limit of the constituent materials. The soil materials which compose such structures cannot, therefore, be regarded as behaving elastically, and some account must be taken of the inelastic strains which may occur under load. Since a typical pavement may undergo $10^6$ or more repetitions of load during its 15 to 20 year life, it is clear that failure will be by a gradual reduction in the serviceability of the structure. Such a mode of failure is associated with the increments of permanent deformation induced by repeated loading. An analysis which gives attention to this long-term, incremental mode of failure is, therefore, both appropriate and highly desirable.

Despite the considerable development of the theories of mechanics prior to this century, little if any attention had been paid to structural behaviour under the repeated application of load. It appears that Grüning (1926) and Kazinczy (1931) were among the first to recognise that structures may fail under variable repeated loading, even though no single load combination within the prescribed limits can, by itself, produce collapse. Bleich (1932) followed with a theory governing this type of failure for simple bar structures, and a more general presentation for trusses is due to Melan (1936). Symonds and Prager (1950) and Neal (1950, 1951) provided simplifications of the theory and an extension of the analysis to rigid frames, and Koiter (1952, 1956) put forward a theory for a three-dimensional continuum.

To date, however, it appears that the general theory has been given little specific application. So, whilst several authors now include discussion of the incremental failure of frames in their texts on structural analysis (Neal (1964, 1977), Horne (1971)), no real applications to continua have been made, even in the light of more recent simplifications and methods of approximation (e.g., Maier (1969), Corradi and Zavelani (1974)).

This chapter develops a method for determining the long-term behaviour of a continuum subjected to variable, repeated moving loads beyond the level at which the structure is able to respond in a perfectly elastic manner.
5.2 Behaviour of Continua under Repeated Loading

Historically, the discipline of mechanics has concerned itself primarily with two limits to loading. Basic structural design has, until recently, sought to limit member stresses to a material yield stress ("elastic limit") by methods such as "working stress design". By contrast, soil mechanics has tended to consider the load required to cause complete collapse of a foundation, and then produce a design having a selected factor of safety against such collapse. Of late, the philosophy of structural design has tended towards such an approach, with various "ultimate strength" methods being adopted.

As many authors have recognised, the repeated application of load can be much more severe on a structure than simple loading to collapse, particularly where a large number of cycles is involved. A single application of a load exceeding the first yield load but below the static collapse load clearly causes yielding at some part of the body, and even complete elastic unloading will not return the body to its original condition - permanent ("plastic") deformation has occurred.

Under certain conditions of load level and sequence, repetition of the load can cause further plastic deformation at some locations, resulting in a gradual accumulation of permanent deformation as load cycling continues. This progression towards eventual failure, as material breakdown occurs, is known as "incremental collapse". Other sequences of load cycling may, instead, cause plastic deformation at some locations alternately in compression and tension, and again, after many cycles, material breakdown may take place. In this case, failure has occurred by "alternating plasticity".

A third possible behaviour may be observed in a structure loaded repeatedly beyond its elastic limit. As mentioned above, unloading following the first load application will generally take place elastically, leaving the body with some permanent deformation - and also with some remaining stresses in equilibrium with zero external loads. Such a set of stresses constitutes a "residual" stress state and in certain circumstances a second application of the original load will see only elastic changes in stress. Where a number of different load distributions are applied, with or without a specific sequence, it is more difficult to assess whether the structure will eventually reach a state in which all subsequent stress responses will take place elastically.
Given appropriate conditions, however, this can occur as the various loadings gradually build up a suitable set of residual stresses. Once this state has been reached, the residual stresses remain constant throughout subsequent loading and the body is said to have undergone "shakedown".

The study of a structure's post-yield behaviour using a step-by-step procedure is, at the very least, tedious, and where unknown sequences of loading are involved this becomes impossible. There exists, therefore, a real need to investigate, generally, the conditions under which a structure may shake down, and a convenient approach is that using Melan's theorem, restated by Koiter (1960) as the first shakedown theorem. The theorem is concerned with the search for a suitable residual stress state matched to the expected loading conditions and is presented below.

For expediency, "elastic" stresses shall be defined as those stresses which would occur within a body under a given loading and assuming a perfectly elastic material of infinite strength. Melan's theorem may then be stated as:-

"If any time-independent distribution of residual stresses can be found such that the sum of these residual stresses and the 'elastic' stresses is a state of stress inside the yield limit, at every point in the body and for all possible load combinations within the prescribed bounds, then the structure will shake down to some time-independent distribution of residual stresses (usually depending on the actual loading program) and the response to subsequent variations of load within the prescribed limits will be elastic."

It is often desirable to be able to define the most severe conditions of loading under which a body will still shake down. Where a single load factor $\lambda$ can be used to characterise the range of loads that may be applied to a structure, then the largest value of $\lambda$ at which shakedown can occur is a convenient measure and is termed the "shakedown limit". In this chapter a method is presented for calculating the shakedown limit of a continuum and the method applied to selected pavement structures. Such an approach would appear to be more closely related to pavement performance than static analyses (either "elastic" or "collapse" calculations) and may provide a convenient measure of the long-term behaviour under traffic of a range of road pavements.
5.3 The Pavement Model

The analysis of a general horizontally-layered pavement structure subjected to wheel loads of varying magnitude, contact area and spatial distribution, as shown in Figure 5.1, represents a problem of considerable complexity. In order to reduce the problem to one of manageable proportions it was found necessary to make a number of simplifications. The first of these arises from the expectation that the region of pavement most heavily stressed by a moving wheel load should be that vertical plane below the centreline of the wheel-track. A plane strain approximation would, therefore, appear to be a suitable first simplification and Figure 5.2 illustrates the result - effectively now the analysis of a roller moving over a layered pavement.

Note that this approximation may well be pessimistic in that plane strain loading probably stresses a pavement more severely than general three-dimensional conditions of load and response. If this is so, then clearly the analysis will provide only a lower bound to the true shakedown limit for the structure.

Physically, it may further be expected that a large number of passes of the roller across the surface will produce a pattern of pavement deformation which is uniform over any horizontal plane. The pattern of permanent deformation should, therefore, also become uniform under the repeated action of load and, consequently, so should the pattern of residual stresses. In this way, a large number of passes of the roller may be expected to leave every point at a given depth in the pavement, at an identical state of permanent deformation and residual stress. A further simplification may, therefore, be introduced into the analysis by noting that the residual stresses within the pavements can be considered as functions of depth $z$ only, and the problem is, thereby, reduced to one involving a single dimension only.

The other variable in the analysis - that of the applied load-form - is similarly a function of a large number of variables. For a plane strain analysis some simplifications occur since it is only necessary to examine the distribution of stresses in the plane perpendicular to the axis of rotation of the roller or tyre. Experimental investigations by Marwick and Starks (1941), Bonse and Kuhn (1959) and Freitag and Green (1962) all suggest that the longitudinal variation of normal stresses may be approximated by a trapezoidal distribution.
FIGURE 5-1

GENERAL THREE-DIMENSIONAL PAVEMENT AND LOADING
FIGURE 5.2

PLANE STRAIN PAVEMENT AND LOADING

FIGURE 5.3

DEFINITION OF PAVEMENT LOADING
Figure 5.3 shows such a distribution with dimensions defined by maximum pressure $V$ and half-length $B$, and shape by $b/B$. Bonse and Kuhn's results suggest a similar variation in longitudinal shear stresses, and whilst other authors' investigations indicate a considerable degree of dependence on the condition of vehicle motion, it is convenient, at this point, to adopt a trapezoidal variation of these stresses also. The maximum stresses $V$ (normal) and $H$ (longitudinal shear) may then be related by the parameter $\mu = H/V$, representing the mobilised coefficient of friction between the pneumatic tyre and the pavement surface.

Using the above notation, it is useful to express the unit load domain as $(V,H) = (1.0, \mu)$, and from this it follows that the shakedown limit can be defined as the largest load factor $\lambda$ for which shakedown occurs under a cycled load of maximum magnitude $(V,H) = (\lambda, \mu \lambda)$.

### 5.4 A One-Dimensional Analysis

#### 5.4.1 General

For the purpose of applying the shakedown theorem already presented, to the plane strain pavement shown in Figure 5.4, a distribution of residual stresses $\sigma_R(x,z) = \begin{bmatrix} \sigma_{XR} \\ \sigma_{ZR} \\ \tau_{XZR} \end{bmatrix}$ is sought. Here the subscript $R$ is taken to indicate a residual stress state, and $x$ and $z$ refer to horizontal and vertical axes respectively, as Figure 5.4 also indicates. For ease of discussion, stress states will be represented by vectors of the type employed above.

At any point $P(x,z)$, equilibrium conditions must be satisfied. For plane strain, this implies

$$\frac{\partial \sigma_{XR}}{\partial x} + \frac{\partial \tau_{XZR}}{\partial z} = 0$$

$$\frac{\partial \tau_{XZR}}{\partial x} + \frac{\partial \sigma_{ZR}}{\partial z} = \gamma(z),$$

the material unit weight at depth $z$ since the residual stress state is in equilibrium with zero external loads.
Maximum applied stresses

\[ H = \mu V \]

Loadform

\[ Z = Z_1 \]
\[ Z = Z_2 \]
\[ Z = Z_b \]

Layer i

\[ Z = Z_i \]

Layer C (\( E_c, \nu_c, C_c, \phi_c \))

Layer B (\( E_b, \nu_b, C_b, \phi_b \))

Layer A (\( E_A, \nu_A, C_A, \phi_A \))

Sub-layer \( i \)

Sub-layer 2

Sub-layer 1

Depth \( Z \)

**FIGURE 5.4**

**PLANE STRAIN PAVEMENT USED IN ANALYSIS**

**FIGURE 5.5**

**PAVEMENT DISCRETISATION FOR FINITE LAYER ANALYSIS**
Since it is reasonable to assume the pavement materials lie above the water table, and are of reasonable permeability, the analysis which follows will neglect pore pressures and assume that effective stresses are identical to total stresses.

Physically, it may further be expected that a large number of passes of the roller across the surface will produce a pattern of pavement deformation which is uniform over any horizontal plane. Thus, both permanent deformations and residual stresses will after many passes also vary with depth only. The residual stress state is, therefore, independent of the horizontal co-ordinate $x$, so that

$$
\tau_{xz} = 0
$$

$$
\sigma_{x} = \int_{0}^{z} \gamma \, dz
$$

The unit weight is a material characteristic, to be determined by appropriate tests. The only stress unknown is, therefore, the residual horizontal direct stress $\sigma_{xr}(z)$. As a consequence of the nature of equilibrium constraints for the one-dimensional formulation, the variation of this stress with depth may be quite arbitrary: whilst a smooth variation might be expected, this need not be so, and discontinuities may occur, for example, at material interfaces.

Prior to determining the shakedown limit, an analysis of the pavement’s elastic response to load must be performed and the most appropriate means is by a conventional finite layer analysis (Cheung (1976)) using a truncated Fourier series representation for tractions and stresses. In order to achieve reasonable accuracy some refinement of the physical pavement layers is necessary. As illustrated by Figure 5.5, each physical layer may be divided into a number of sub-layers - the precise number varies with the problem and the required order of accuracy, and should be investigated independently. An example of such a test appears in subsequent discussion of a computer implementation of the method.
Figure 5.5 also indicates the material properties adopted for computations. Each layer $z_{i-1} \leq z \leq z_i$ is assumed to consist of homogeneous isotropic material, with conventional parameters defining elastic behaviour (Elastic modulus $E_i$, Poisson's ratio $\nu_i$) and soil strength (cohesion $c_i$, friction angle $\phi_i$). Material yield is simply defined using the Mohr-Coulomb criterion. As Figure 5.6 shows, yield within an element of material occurs when the Mohr circle of stresses becomes tangential to the yield surface - that is, when

$$(\sigma_z - \sigma_x)^2 + 4\tau^2 = [\sin \phi (\sigma_z + \sigma_x + 2c \cot \phi)]^2$$

provided $\sigma_z + \sigma_x + 2c \cot \phi > 0$.

A form of this criterion, which is more convenient for subsequent analysis, is produced by a change of axes to the stress space $(\frac{\sigma_z - \sigma_x}{2}, \tau, \frac{\sigma_z + \sigma_x}{2})$. The result is a conical surface, with its apex at the point $(0, 0, -c \cot \phi)$ and axis coincident with the $\frac{\sigma_z + \sigma_x}{2}$ axis. The failure criterion, therefore, intersects the plane

$$\sigma_z + \sigma_x = \text{constant}$$
in a circle of radius

$$R = \left( \frac{\sigma_z + \sigma_x}{2} \right) \sin \phi + c \cos \phi$$
as portrayed in Figure 5.7.

Using the simplifications outlined above, the one-dimensional shakedown problem can be restated as follows:

(a) determine the elastic response of the continuum to a unit applied loadform, giving a vector of stresses $[\sigma_{XE}, \sigma_{ZE}, \tau_E]^T$ for each point $P(x,z)$;

(b) find a distribution of horizontal residual stress $\sigma_{XR}(z)$ and load factor $\lambda$ such that
MOHR-COULOMB FAILURE CRITERION

Mohr-Coulomb failure surface,
\[ R = \left( \frac{\sigma_z + \sigma_x}{2} \right) \sin \phi + C \cos \phi \]
(i) $\lambda$ is maximised, and

(ii) at no point is the yield condition violated by the stress vector $\sigma_s(x,z)$, consisting of the factored elastic response plus the vector of residual stress,

$$\sigma_s(x,z) = \lambda \begin{bmatrix} \sigma_x(x,z) \\ \sigma_z(x,z) \\ \tau(x,z) \end{bmatrix} = \begin{bmatrix} \sigma_{xE}(x,z) \\ \sigma_{ZE}(x,z) \\ \tau_E(x,z) \end{bmatrix} + \begin{bmatrix} \sigma_{xR}(z) \\ \sigma_{zR}(z) \\ 0 \end{bmatrix}$$

(c) the maximised value of load factor $\lambda$ then represents a lower-bound estimate of the shakedown factor $\lambda_{SD}$ for the given structure and unit loadform.

Two approaches to the solution of this problem are developed, the first employing linear programming techniques and the second using a simpler, faster and more accurate method evolved by the author specifically for this application. These approaches are outlined below.

5.4.2 Solution by Linear Programming

5.4.2.1 Background

Methods for the calculation of shakedown limits of trussed and framed structures have been developed by a number of authors, and various techniques and examples are presented by Horne (1971) and Neal (1977). Extensions to continua, however, have not received the same attention, due largely to the additional complexity involved. Maier (1969) first presented an approximate method for application to continua, but without any specific examples or calculations. His subsequent work (e.g., Maier (1970, 1972)) has extended the theoretical aspect to account for work-hardening effects and regions of unstable material, but still presents no numerical results.
The first numerical application of shakedown theory to continua appears to have been made by König (1969), who obtained estimates for the shakedown loads of various plate structures. Belytschko (1972) examined the problem of a thin plate with a circular hole, and a similar analysis performed by Corradi and Zavelani (1974) appears to confirm the theory's applicability. Further results for similar problems are due to Alwis and Grundy (1980) and Aboustit and Reddy (1980). Of perhaps more interest to the present study is the same authors' discussion and analysis of a long uniformly loaded strip footing underlain by a shallow clay stratum. Their solution, for a load of varying eccentricity but fixed inclination, was obtained by finite element methods. A similar method was used by Pande, Abdullah and Davis (1980) to examine the problem of a strip footing subjected to normal and shear loads.

The above two papers present the first numerical results for plane strain conditions, and although their sets of results are not readily comparable, they do indicate that values consistent with known first yield and collapse loads can be obtained.

Apart from these limited results, no application of the shakedown theory has been made to plane strain conditions. This chapter adapts the theory of Maier to the problem of a general loadform moving across the surface of a layered, plane strain continuum. The influence of approximations involved in the formulation is explored in some detail and investigation is made into the effects of structure and load characteristics on the predicted performance under repeated loading. The application of this approach to the extensive, but still largely empirical, field of pavement analysis and design is then examined.

5.4.2.2 Derivation of the method

In order to obtain linear no-yield constraints, the material yield surface must be approximated by a series of planes. Adapting the approach of Lysmer (1970) to the present problem, a regular m-sided pyramid is inscribed within the conical Mohr-Coulomb failure surface. This pyramid then intersects the plane

\[ \sigma_z + \sigma_x = \text{constant} \]

in a regular m-sided polygon, as shown in Figure 5.8.
FIGURE 5.8

LINEARISATION OF MOHR - COULOMB FAILURE SURFACE
The linearised yield surface needs only to be described in sufficient detail to test a point in stress space for yield (which occurs if and only if the point lies on or outside the yield surface). This may be done simply by computing some measure of the distance from the axis of the yield pyramid, to the given point in stress space, and comparing this with a similar measure of the distance to each face of the pyramid. Under some circumstances it may be preferable to compute such distances using the normals to the yield planes; in this case, however, this is unnecessary, and the calculation may be simplified by considering that plane
\[ \sigma_z + \sigma_x = \text{constant} \]
which contains the given stress point.

Consider now a stress state denoted by
\[ \sigma_0 = [\sigma_{x0}, \sigma_{z0}, \tau_0]^T. \]

In the plane
\[ Z = \frac{\sigma_z + \sigma_x}{2} = \frac{\sigma_{z0} + \sigma_{x0}}{2} \text{ (constant)} \]
this stress state appears as a point with co-ordinates
\[ [X_0, Y_0]^T = \left[ \frac{\sigma_{z0} - \sigma_{x0}}{2}, \tau_0 \right]^T. \]

This same plane is intersected by the yield surface in a regular polygon as shown by Figure 5.8. Each of the m sides of the polygon is located at a distance from the Z-axis of
\[ k = R \cos \frac{\pi}{m} \]
where
\[ R = \left( \frac{\sigma_{z0} + \sigma_{x0}}{2} \right) \sin \phi + c \cos \phi \]
and the direction of each plane \( j \) may be denoted by the outward "direction vector"
\[ \tau_{0j} = \left[ \cos \frac{2 \pi j}{m}, \sin \frac{2 \pi j}{m}, 0 \right]^T. \]
A simple test for yielding of \( \sigma_0 \) is, therefore, achieved by calculating the component of the stress vector \([X_0, Y_0, Z_0]^T\) in the direction \( n_{0j} \) and comparing the result with the limit \( k \) calculated above. Thus, the no-yield condition reduces to

\[
n_{0j} \cdot [X_0, Y_0, Z_0]^T \leq k \quad (j=1,2,\ldots,m).
\]

The above constraints apply, however, to the transformed stress space \([X^o, Y^o, Z^o]^T\). A return to the space \([\sigma_x, \sigma_z, \tau]^T\) is effected by setting

\[
n_j \cdot [\sigma_{x0}, \sigma_{z0}, \tau_0]^T = n_{0j} \cdot [X_0, Y_0, Z_0]^T
\]

which gives

\[
n_j = [-\frac{1}{2} \cos \alpha_j, \frac{1}{2} \cos \alpha_j, \sin \alpha_j]^T
\]

where \( \alpha = \frac{2\pi}{m} \).

Suppose now that a unit load is applied to the continuum as illustrated in Figure 5.4. Then a general point \( P(x,z) \) will develop stresses which may be denoted by

\[
\sigma_E(x,z) = [\sigma_{xe}(x,z), \sigma_{ze}(x,z), \tau_E(x,z)]^T.
\]

Therefore, at a load factor \( \lambda \), the requirement of no-yield at \( P \) (where residual stresses \( \sigma_R \) are assumed to have developed) gives the constraint:

\[
n_j \cdot [\lambda \sigma_{x0}(x,z) + \sigma_R(z)] \leq k \quad \text{for all } j=1,2,\ldots,m
\]

where \( \sigma_R(z) = [\sigma_{xR}(z), \sigma_{zR}(z), 0]^T \).

This inequality may be expanded to become

\[
\lambda \left[ \frac{\sigma_{ze} - \sigma_{xe}}{2} \cos \alpha_j + \tau_E \sin \alpha_j \right] + \frac{\sigma_{zR} - \sigma_{xR}}{2} \cos \alpha_j \leq \lambda \left( \frac{\sigma_{ze} + \sigma_{xe}}{2} \sin \phi \cos \frac{\pi}{m} + \frac{\sigma_{zR} - \sigma_{xR}}{2} \sin \phi \cos \frac{\pi}{m} \right)
\]

for all \( j=1,2,\ldots,m \), and all \( x \).
Upon rewriting, this reduces to
\[ \lambda F(j,x,z) + \sigma_{XR}(z) \cdot G(j,z) \leq H(j,z) \]
where
\[ F(j,x,z) = n_j \cdot \sigma_E(x,z) - \left( \frac{\sigma_E + \sigma_X}{2} \right) \sin \phi(z) \cos \frac{\pi}{m} \]
\[ G(j,z) = -\frac{1}{i} \left[ \cos a_j + \sin \phi(z) \cos \frac{\pi}{m} \right] \]
\[ H(j,z) = c(z) \cdot \cos \phi(z) \cdot \cos \frac{\pi}{m} + \frac{1}{i} \sigma_{XR}(z) \left[ \sin \phi(z) \cos \frac{\pi}{m} - \cos a_j \right]. \]

Since \( \lambda \) is a non-negative constant, the number of constraints may be reduced by introducing
\[ M(j,z) = \max_x \{ F(j,x,z) \} \]
so that
\[ \lambda M(j,z) + \sigma_{XR}(z) \cdot G(j,z) \leq H(j,z) \text{ for } j=1,2,\ldots,m, \text{ and all } z. \]

The final discretisation required is the selection of a number of "representative depths" to which the above inequality may be applied. The shakedown problem then becomes:

maximise \( \lambda \),
subject to \( \lambda M(j,p) + \sigma_{XR}(p) \cdot G(j,p) \leq H(j,p) \)
for the yield planes \( j=1,2,\ldots,m \), and the representative depths \( z=z_1,z_2,\ldots,z_p,\ldots,z_q \)
and this constitutes a linear programming problem having \( (mq) \) constraints, and \( (q+1) \) variables, which may be solved by conventional techniques (e.g., Gass (1969), Hadley (1962)).

5.4.2.3 Use of the method

For the purposes of illustration, two problems in continuum shakedown have been considered and their solutions obtained by linear programming using the analysis outlined above.
In the first case, a homogeneous isotropic half-space was examined. The moving load due to the passage of a wheel was modelled by a trapezoidal loadform, and the half-space assumed constrained to a plane strain response. Figure 5.9 shows the idealised pavement and the selection of depth points. In this particular study, only five representative depths were used for the purpose of demonstration.

Finally, the material characteristics were fixed by choosing cohesion \( c \) and angle of friction \( \phi \), and the yield circle, for reasons discussed later, modelled as a nineteen-sided regular polygon. The problem was, then, one of finding the largest load factor \( \lambda \) for which the pavement would shake down.

For the homogeneous half-space, stresses at any point may be determined analytically. The linear constraints were then set up as follows:

(a) for each depth, the 19 values \( M(j, z) \), \( j=1,2,3,...19 \), were calculated;

(b) with \( \phi \) independent of depth, the values \( G(j) \), \( H(j) \), \( j=1,2,3,...19 \), could also be calculated;

(c) the result was the set of 95 constraints

\[
\lambda M(j, p) + \sigma_{\lambda R}(p) G(j) \leq H(j) \text{ with the six variables } \lambda \text{ and } \sigma_{\lambda R}(p), \ p=1,2,...5.
\]

The objective function to be maximised was, then, simply \( \lambda \).

Table 5.1 sets out the results for four values of friction angle, determined using the program INFOPT, developed by the author for half-space applications. In each case, a first solution was obtained by solving the linear programming problem formulated above. Subsequent improvement of this solution was achieved by examining the solutions for residual stress, in order to locate the "critical" depth - that depth at which an increase in \( \lambda \) first produced a vanishing of the range of possible residual stresses. Figure 5.10 illustrates the range of admissible residual stresses at the shakedown limit of each pavement.
FIGURE 5.9

SAMPLE PROBLEM 1 - GEOMETRY AND MATERIALS

Stress depths, \( Z = 0.1, 0.2, 0.3, 0.4, 0.5 \)
Scanning points at each depth, \( X \) from 1.60 to +1.60 by increments of 0.04
TABLE 5.1
Progressive Solutions to Half-Space Problems

<table>
<thead>
<tr>
<th>Angle of Friction (°)</th>
<th>Shakedown Limit by Linear Programming</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Optimisation</td>
</tr>
<tr>
<td>0°</td>
<td>3.761</td>
</tr>
<tr>
<td>10°</td>
<td>5.159</td>
</tr>
<tr>
<td>20°</td>
<td>7.206</td>
</tr>
<tr>
<td>30°</td>
<td>10.398</td>
</tr>
</tbody>
</table>
DISTRIBUTIONS OF PERMISSIBLE RESIDUAL STRESS AT SHAKE-DOWN LIMIT
( Half-space problems )
The second case examined was that of a typical layered pavement. In Figure 5.11, the pavement geometry and materials are shown, along with the idealised loadform used. Prior to analysis, all parameters were to be normalised and this was done with respect to the load characteristics (half-length $B = 100$ mm, maximum contact pressure $S = 700$ kPa). Table 5.2 indicates the normalised parameters used.

Since material characteristics were no longer constant throughout the continuum, the elastic stresses in the system were most conveniently determined using a finite layer analysis (Cheung (1976)). This entailed approximating the given loadform by a number of Fourier terms, and selecting a suitable layering of the pavement. For this problem, the values of parameters found to be necessary for satisfactory convergence were

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of layers</td>
<td>$N_z = 40$</td>
</tr>
<tr>
<td>Depth of base (rough rigid)</td>
<td>$D/B = 18$</td>
</tr>
<tr>
<td>Number of Fourier terms</td>
<td>$N_p = 19$</td>
</tr>
<tr>
<td>Number of sides of yield polygon</td>
<td>$N_y = 10$</td>
</tr>
<tr>
<td>Calculated shakedown limit by linear programming using the program LAYOPT</td>
<td>$1.07$</td>
</tr>
</tbody>
</table>

From this analysis, therefore, we may conclude that the given pavement should exhibit shakedown behaviour under a repeated load of up to $1.07 \times 700$ kPa $= 749$ kPa. For repeated loading of greater magnitude, shakedown would not be expected to occur, and the pavement would continue incrementally to failure.

A number of new variables have been introduced in the second problem above, and their effect upon the calculation of shakedown limits for layered pavements is worthy of some discussion. Figures 5.12 to 5.17 demonstrate, graphically, the influence of several of these parameters. Together, they show that suitable convergence to the true shakedown limit for a pavement is achieved provided:

(a) the sampling region is wide enough;

(b) there are sufficient sampling points per layer;

(c) the yield polygon is sufficiently close to a circle;
### Pavement Data

#### Profile

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Thickness (mm)</th>
<th>Elastic Properties</th>
<th>Failure Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>4500 MPa</td>
<td>Cohesion (kPa)</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>370</td>
<td>270</td>
<td>900 MPa</td>
<td>150 kPa</td>
</tr>
<tr>
<td>450</td>
<td>80</td>
<td>300 MPa</td>
<td>100 kPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50 kPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Load Data

- Normal load only 50 mm
- Maximum pressure 700 kPa
- 100 mm

**Figure 5.11**

Pavement and load details for isotropic layered pavement.
TABLE 5.2
Normalised Parameters for Isotropic Layered Pavement

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness t/B</th>
<th>Elastic Properties</th>
<th>Failure Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>E/S</td>
<td>ν</td>
</tr>
<tr>
<td>Asphaltic concrete</td>
<td>1.0</td>
<td>6400</td>
<td>0.3</td>
</tr>
<tr>
<td>Granular 'A'</td>
<td>2.7</td>
<td>1300</td>
<td>0.3</td>
</tr>
<tr>
<td>Granular 'B'</td>
<td>0.8</td>
<td>430</td>
<td>0.3</td>
</tr>
<tr>
<td>Clay subgrade</td>
<td>∞</td>
<td>140</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Homogeneous isotropic half-space

Length of Sampling Region $\frac{L}{B}$

**FIGURE 5.12**
INFLUENCE OF SAMPLING REGION LENGTH UPON SHAKE-DOWN LIMIT

Homogeneous isotropic half-space

Number of Sampling Points per Layer $N_x$

**FIGURE 5.13**
INFLUENCE OF NUMBER OF SAMPLING POINTS UPON SHAKE-DOWN LIMIT
FIGURE 5·14
INFLUENCE OF YIELD-PLANE LINEARISATION UPON SHAKE\nDOWN LIMIT

Mohr–Coulomb circle approximated by Ny-sided regular polygon.

FIGURE 5·15
INFLUENCE OF LAYER REFINEMENT UPON SHAKE\nDOWN LIMIT
Homogeneous, isotropic soil
\( \phi=0, \gamma=0 \)

FIGURE 5·16
INFLUENCE OF PAVEMENT THICKNESS UPON SHAKEDEOWN LIMIT

Trapezoidal load approximated by Fourier series
\( b/B = 0.5 \)

FIGURE 5·17
INFLUENCE OF FOURIER LOAD APPROXIMATION UPON SHAKEDEOWN LIMIT
(d) adequate layer refinement is used;
(e) the variation of pavement materials with depth is adequately modelled; and
(f) sufficient Fourier terms are used in the load approximation.

Clearly, these factors require consideration for each problem investigated, and some sensitivity analysis of the calculated shakedown limit is most advisable. To neglect one or more of these variables in drawing conclusions can, as the graphs show, result in significant inaccuracies.

5.4.2.4 Other Remarks

The use of the linear programming approach to determine the shakedown limit of a continuum represents a vast improvement over step-by-step methods of elastoplastic analysis. The approach does, however, suffer from a number of deficiencies. Major among these is that the computing effort is approximately proportional to the cube of the number of constraints, so that the program execution time increases dramatically as the yield surface is more accurately approximated. An approach which removes this difficulty might, therefore, be expected to offer advantages in terms of processing time, as well as providing a better model of the assumed yield criterion.

5.4.3 Development of an Alternative Analysis

5.4.3.1 Introduction

The form of curves such as those of Figure 5.10 suggests a slight variation in the approach to the shakedown problem. Tests with solutions by linear programming showed that at a given depth there exists some relationship linking the load factor \( \lambda \) with a range of "safe" values of residual stress \( \sigma_{RR} \), and that the shakedown limit \( \lambda_{SD} \) is simply an extreme value of \( \lambda \), for which the residual stress range reduces to a point. Using these observations as a basis, it is indeed possible to develop a more exact analysis of the shakedown condition.

5.4.3.2 A Method of Conic Sections

Consider a layered continuum under conditions of plane strain, as shown in Figure 5.2. The continuum shall be assumed to be composed of materials whose weight is negligible in comparison with expected surface loadings, and further that both strength and deformation characteristics vary with depth \( z \) only. Under such conditions, the solution to the shakedown problem may be regarded as being "one-dimensional", and the load factor defining the resistance of the continuum to incremental collapse under the given cycled loads as being the "one-dimensional shakedown limit".
The convention of compressive strength positive, and the Mohr-Coulomb failure criterion, shall be adopted. The shakedown condition at any point may then be formulated as

\[ F(\sigma_{XR}, \lambda) = a\sigma_{XR}^2 + 2hc\sigma_{XR} \lambda + b\lambda^2 + 2g\sigma_{XR} + 2f\lambda + k \leq 0 \]  \hspace{1cm} (5.1)

where

\[ \lambda = \text{load factor} \]
\[ \sigma_{XR} = \text{residual horizontal direct stress} \]
\[ a = \cos^2 \phi \]
\[ h = -(\sigma_{ZE} - \sigma_{XE}) - \sin^2 \phi (\sigma_{ZE} + \sigma_{XE}) \]
\[ b = (\sigma_{ZE} - \sigma_{XE})^2 + 4\tau_{E}^2 - \sin^2 \phi (\sigma_{ZE} + \sigma_{XE})^2 \]
\[ g = -2c \sin \phi \cos \phi \]
\[ f = -2c \sin \phi \cos \phi (\sigma_{ZE} + \sigma_{XE}) \]
\[ k = -4c^2 \cos^2 \phi \]

and \((\sigma_{XE}, \sigma_{ZE}, \tau_{E})\) represents the unit elastic stress state (plane strain) at the selected point.

\[ c = \text{material cohesion (} \geq 0 \text{)} \]
\[ \phi = \text{material angle of internal friction (} 0 \leq \phi < \pi/2 \text{)} \]

This condition in terms of the variables \((\sigma_{XR}, \lambda)\) simply represents the "safe domain" for which the combination of load factor \(\lambda\) and residual stress \(\sigma_{XR}\) will not cause failure at the chosen point. The boundary to this domain is given by

\[ F(\sigma_{XR}, \lambda) = 0 \]  \hspace{1cm} (5.2)

and may be seen to represent a general conic section. It is convenient to examine the shakedown condition on the basis of this representation.

A change of origin is first undertaken in order to simplify the form of the equation \(F = 0\).

Let

\[
\begin{bmatrix}
\sigma_{XR} \\
\lambda
\end{bmatrix} = \begin{bmatrix}
x + \sigma_c \\
y + \lambda_c
\end{bmatrix}
\]

as indicated by Figure 5.18.
Figure 5.18

Change of Origin in \((\sigma_{XR}, \lambda)\) Plane
Making this substitution in $F = 0$ and setting the terms linear in $x$ and $y$ to zero, gives

$$
\begin{bmatrix}
\sigma_c \\
\lambda_c
\end{bmatrix} = \frac{1}{ab-h^2} \begin{bmatrix}
hf - bg \\
hg - af
\end{bmatrix} 
$$

(5.3)

Clearly, this requires that $ab-h^2 \neq 0$, so a distinction between various forms of the curve $F = 0$ may be made by

- $ab - h^2 \neq 0$ Central conic (i.e. ellipse or hyperbola)
- $ab - h^2 = 0$ Parabola (no centre).

Expansion yields

$$
ab - h^2 = 4 \left[ \frac{\sigma}{\sigma_E} \cos^2 \phi - \frac{\sigma}{\sigma_E} \sin^2 \phi \right]
$$

(5.4)

from which it is evident that $(ab-h^2)$ may take positive, zero or negative values, yielding curves which are respectively elliptical, parabolic or hyperbolic in form. Figure 5.19 demonstrates typical examples of each type.

The form of the curves may be more readily predicted by noting that

- $(\frac{\sigma}{\sigma_E})^2 > \tan^2 \phi$ leads to an ellipse
- $(\frac{\sigma}{\sigma_E})^2 = \tan^2 \phi$ leads to a parabola
- $(\frac{\sigma}{\sigma_E})^2 < \tan^2 \phi$ leads to a hyperbola,

so that the curves may be classified on the basis of the unit stresses, as shown graphically in Figure 5.20.

The graphs of Figure 5.19 serve to raise a further point - that for a given set of strength parameters $(c, \phi)$, all envelopes have a common pair of intercepts on the $x_R$ axis. This is readily verified by
(a) Elliptical Envelopes

(b) Parabolic Envelopes

(c) Hyperbolic Envelopes

FIGURE 5.19

FORMS OF SHAKEDOWN ENVELOPE
FIGURE 5.20

GEOMETRIC INTERPRETATION OF CURVE PREDICTION BY PLOTTING \((\sigma_{ZE}, \tau_{XE})\) ON THE PLANE \((\sigma_n, \tau)\).

eg.
- Point I — elliptical envelope
- Point J — parabolic envelope
- Point K — hyperbolic envelope
setting $\lambda = 0$ in equation (5.2), and solving the resulting quadratic in $\sigma_{XR}$ to give

$$\sigma_{XR}(\lambda=0) = 2c \left( \frac{\sin \phi + 1}{\cos \phi} \right), \quad (5.5)$$

clearly a function of material characteristics only. Thus, for any given layer of the continuum, all envelopes will have the same residual stress intercepts (exactly two per curve).

### 5.4.3.3 Domains for a Cohesive Frictional Material

Using the approach based on conic sections, the maximum value of $\lambda$ satisfying the shakedown condition (5.1) for a general $(c, \phi)$ material may be determined.

It is convenient to set

$$f(s_X, s_Z, t) = F(\sigma_{XR}, \lambda) = 0$$

where

$$s_X = \lambda \sigma_X + \sigma_{XR}$$

$$s_Z = \lambda \sigma_Z$$

$$t = \lambda \tau$$

Since

$$\frac{\partial F}{\partial \sigma_{XR}} \cdot d\sigma_{XR} + \frac{\partial F}{\partial \lambda} \cdot d\lambda = 0$$

clearly $\frac{d\lambda}{d\sigma_{XR}} = 0$ implies $\frac{\partial F}{\partial \sigma_{XR}} = 0$.

But $\frac{\partial F}{\partial \sigma_{XR}} = \frac{\partial f}{\partial s_X}$, where

$$f = (s_X - s_Z)^2 + 4t^2 - \sin^2 \phi (s_X + s_Z + 2c \cot \phi)^2 = 0 \quad (5.6)$$

and so

$$\frac{\partial f}{\partial s_X} = (s_X - s_Z) - \sin^2 \phi (s_X + s_Z + 2c \cot \phi) = 0. \quad (5.7)$$

Selecting the region of the Mohr-Coulomb failure envelope corresponding to compression failure - that is,

$$s_X + s_Z > -2c \cot \phi$$
it is evident that

\[ s_X - s_Z > 0 \]  \hspace{1cm} (5.8)

\[ 2t = \pm \cot \phi \left( s_X - s_Z \right) . \]  \hspace{1cm} (5.9)

After introducing the Mohr representation illustrated in Figure 5.21,

\[
\begin{align*}
    s_X &= p + R \cos 2\theta \\
    s_Z &= p - R \cos 2\theta \\
    t &= R \sin 2\theta
\end{align*}
\]

where \( R \geq 0 \), (5.8) and (5.9) lead to

\[
\begin{align*}
    s_X &= p + R \sin \phi \\
    s_Z &= p - R \sin \phi \\
    t &= \pm R \cos \phi .
\end{align*}
\]  \hspace{1cm} (5.10)

But, for a stress state at the point of failure,

\[ R = (p + c \cot \phi) \sin \phi \]  \hspace{1cm} (5.11)

which upon substitution in (5.10) yields

\[ s_Z + c \cot \phi = R \cos^2 \phi / \sin \phi . \]

Since

\[ \kappa t = R \cos \phi \hspace{1cm} (\kappa = \pm 1) \]

then

\[
\frac{\lambda \sigma_{ZE} + c \cot \phi}{\kappa \lambda \tau_E} = \cot \phi .
\]

Hence

\[ \lambda = \frac{c}{\kappa \tau_E - \sigma_{ZE} \tan \phi} \]  \hspace{1cm} (5.12)

and

\[ R \cos \phi = \frac{\kappa \tau_E c}{\kappa \tau_E - \sigma_{ZE} \tan \phi} . \]  \hspace{1cm} (5.13)
FIGURE 5.21

MOHR REPRESENTATION OF PLANE STRAIN STRESS STATE
As previously indicated, non-negative values of $R$ are required, and attention is drawn to the conclusions of the previous section. For example, two positive values of $R$ may be associated with an elliptical envelope, for which

\[ \tau_E - \sigma_Z \tan \phi > 0 \]
\[ \tau_E + \sigma_Z \tan \phi > 0. \]

That is, \((\frac{\tau_E}{\sigma_Z})^2 > \tan^2 \phi\).

It is now convenient to return to the graphical approach introduced in Figure 5.20 and adopt the notation

\[ \lambda_{1,2} = \frac{c}{\tau_E - \sigma_Z \tan \phi} \]
\[ R_{1,2} = \frac{c}{\cos \phi (1 + \frac{\sigma_Z}{\tau_E} \tan \phi)} \]

By tabulating the signs of $\lambda$ and $R$ in the $(\sigma_Z, \tau)$ plane, the variation in the form of domain, and its value to the shakedown calculation, may more readily be seen. Note that only positive values of $R$ have meaning in this context, and only positive values of $\lambda$ are of value as maximum values attained by the shakedown envelope. For points lying on the lines

\[ (\frac{\tau_E}{\sigma_Z})^2 = \tan^2 \phi \]

clearly one value of $R$ becomes infinite. Since the remaining value of $R$ is then finite and positive, an envelope of parabolic form results. Figure 5.22 summarises the results.

5.4.3.4 Other Considerations

The foregoing discussion allows the relationship between $\alpha_{XR}$ and $\lambda$ to be simply understood in terms of conic sections. It may readily be determined whether the load factor $\lambda$ attains any stationary values, and if so the value of $\lambda_{\max}$ calculated by the means outlined.
<table>
<thead>
<tr>
<th>Region</th>
<th>$\lambda_1$</th>
<th>$R_1$</th>
<th>$\lambda_2$</th>
<th>$R_2$</th>
<th>Form of envelope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>H</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>-</td>
<td>+</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>EM</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>EM</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>+</td>
<td>+</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>PM</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>HM</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>HM</td>
</tr>
<tr>
<td>$\frac{5}{6}$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>+</td>
<td>+</td>
<td>PM</td>
</tr>
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<td>6</td>
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<td>+</td>
<td>+</td>
<td>+</td>
<td>EM</td>
</tr>
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<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>EM</td>
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<tr>
<td>$\frac{7}{8}$</td>
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<td>+</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>P</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>H</td>
</tr>
</tbody>
</table>

$E$ = ellipse
$P$ = parabola
$H$ = hyperbola
$M$ = maximum value

= regions of practical interest

**FIGURE 5.22**

NATURE OF STATIONARY VALUES OF SHAKEDOWN LIMIT
In several cases, the possibility of \( \lambda_{\text{max}} \) being infinitely large was shown to exist. This may arise when the only stationary values of \( \lambda \) are negative, or where the curve for some reason degenerates to a pair of parallel straight lines. For the purpose of shakedown calculations, it is necessary to determine whether or not \( \lambda \) is unbounded for a given value of residual stress \( \sigma_{XR} \). This possibility is shown graphically in Figure 5.23 (a) – to be contrasted with the relationship of Figure 5.23 (b).

One further calculation should, therefore, be carried out if \( \lambda_{\text{max}} \) is found to be infinite. Since equation (5.3) is always satisfied by the points

\[
(\sigma_{XR}, \lambda) = \left( \frac{2c(\sin \phi + 1)}{\cos \phi}, 0 \right)
\]

and \( F(0,0) \leq 0 \) always, the simplest procedure is to set \( \sigma_{XR} = 0 \) and solve equation (5.3) to obtain up to two corresponding values of \( \lambda \). Clearly then, if no solution for \( \lambda \) exists, or the largest of the solutions is negative, \( \lambda \) is unbounded for any given value of \( \sigma_{XR} \) as shown by Figure 5.23 (a). If, on the other hand, a finite positive solution for \( \lambda \) exists, the load factor is bounded for a given residual stress. In this event, one other calculation may be required at a later stage to determine the maximum permissible value of \( \lambda \) within the "safe" region as previously defined

\[
\sigma_{XR \text{ min}} = \frac{2c(\sin \phi - 1)}{\cos \phi} \leq \sigma_{XR} \leq \frac{2c(\sin \phi + 1)}{\cos \phi} = \sigma_{XR \text{ max}}
\]

By means of conic sections, then, the maximum load factor \( \lambda(x,z) \) for shakedown at any point in the plane can be calculated.

At any depth \( z \) of a given pavement structure, the "safe domain" is clearly that region obtained by overlaying the series of domains associated with various points at the chosen depth, as seen in Figure 5.24.

In simple cases, it may be sufficient to determine the lowest maximum on any curve associated with the given depth. In this case,

\[
\lambda_z = \min_x \{ \lambda_{\text{max}}(x,z) \}.
\]
FIGURE 5-23

ILLUSTRATING FORMS OF HYPERBOLA
WITH TWO NEGATIVE STATIONARY POINTS
Homogeneous isotropic half-space

\[ \frac{\lambda v}{c} = 1.0 \]
\[ \frac{\mu}{c} = 0.5 \]
\[ \frac{fl}{c} = 0 \]
\[ \frac{\phi}{c} = 0 \]
\[ \frac{\gamma}{c} = 0 \]

**FIGURE 5.24**

"SAFE DOMAIN" FOR THE DEPTH \( \frac{Z}{B} = 0.1 \) IN \((\sigma_x, \lambda)\) SPACE
In general, however, this procedure is found to fail, since the lowest $\lambda_{\text{max}}$ and its corresponding $\alpha$ gives a point which is not necessarily contained within all the curves. This difficulty can be more clearly seen by referring to Figure 5.25. A refinement to the method is clearly called for, and it is shown in Appendix 5B that a modification involving the calculation of intersection points of curves, gives a solution converging to the true value of $\lambda_z$ for the depth $z$.

Finally, the shakedown limit $\lambda_{SD}$ for the pavement is calculated. As it is simpler to consider a finite number of representative "depth points" rather than perform a minimisation with respect to depth analytically, the shakedown limit is determined as

$$\lambda_{SD} = \min_{p=1, q} \{ \lambda_z(z_p) \}$$

for the $q$ representative depths $z = z_1, z_2, \ldots, z_p, \ldots z_q$.

5.4.4 Examples and Comparisons

The equipment now exists to determine two sets of solutions to the half-space shakedown problem: one obtained by linear programming (programs INFOPT/LAYOPT), and the other using families of conic sections as set out above (program LAYELLIP).

Table 5.3 presents, for comparison, the solutions to a series of half-space shakedown problems. Identical grids of stress sampling points $(x, z)$ have been used in both sets of analyses, and the cost of each analysis is represented by the machine time, as tabulated. The results of an "accurate" analysis by the method of conics, in which a very fine mesh of points was used, are also presented, along with an "analytical" solution, obtained in Appendix "5C" as an "experiment" rather than a rigorous derivation.

It is clear from the Table firstly that a very close agreement exists between the two sets of results. Indeed the greatest variation is approximately two percent, and this tends to confirm that both are valid approaches to the problem. In addition, there is pleasing agreement between the single analytical solution and the corresponding value obtained by the method of conics, although further conclusions regarding accuracy cannot be drawn at this point. Finally, it is evident that the method of conics offers a major time advantage over solutions by linear programming, with the speed of solution being improved by a factor of six or more.
Homogeneous isotropic half space

FIGURE 5.25

TYPICAL $\lambda - \sigma_{XR}$ DOMAIN FOR DEPTH $Z = 0.1B$
### TABLE 5.3

Solutions and Solution Costs for One-Dimensional Shakedown of Homogeneous Half-Space

<table>
<thead>
<tr>
<th>Angle of Friction $\phi^\circ$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\lambda_{SDV}}{c}$: Analytical</td>
<td>3.789</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Linear Programming (LP)</td>
<td>3.749</td>
<td>5.150</td>
<td>7.191</td>
<td>10.363</td>
</tr>
<tr>
<td>Conics</td>
<td>3.789</td>
<td>5.218</td>
<td>7.327</td>
<td>10.594</td>
</tr>
<tr>
<td>Conics &quot;Accurate&quot;</td>
<td>3.789</td>
<td>5.218</td>
<td>7.324</td>
<td>10.588</td>
</tr>
<tr>
<td>$(\frac{\Delta x}{B} = \frac{\Delta y}{B} = 0.001)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Error: LP</td>
<td>1.1</td>
<td>1.3</td>
<td>1.8</td>
<td>2.1</td>
</tr>
<tr>
<td>Conics</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Time: LP (CPU secs)</td>
<td>32.9</td>
<td>34.4</td>
<td>30.6</td>
<td>30.2</td>
</tr>
<tr>
<td>Conics</td>
<td>4.8</td>
<td>4.6</td>
<td>4.6</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Load:

- $\frac{b}{B} = 0.5$
- $\frac{\Delta x}{B} = 0.04$
- $\frac{\Delta y}{B} = 0.01$
5.5 Shakedown of a Homogeneous Isotropic Half-Space under Moving Surface Loads

In the following pages, the influence of various soil and load parameters upon the shakedown behaviour of a uniform half-space is examined. This serves to establish those parameters of greatest importance, as well as providing a basis for comparison with subsequent analyses of multi-layered pavements.

Figure 5.27 shows the relationship between first yield, shakedown and collapse loads for a range of values of material friction angle. This angle $\phi$, in increasing from 0 to 30°, produces an increase in the shakedown limit of close to 200%, and so is clearly of some importance as a material characteristic. The Figure also serves to demonstrate the significant difference, especially at higher values of $\phi$, between the shakedown and first yield loads. This reveals a reserve of strength within the continuum which is deserving of more detailed investigation.

A further demonstration of the effect of $\phi$ upon the shakedown limit is seen in Figure 5.28. The primary purpose of the diagram, however, is to record the degree to which this limit may be reduced by the addition of a shear component to the wheel load traversing the surface. Since with certain combinations of tyre and pavement surface a friction coefficient $\mu$ of the order of 1.0 may be mobilised, the shear load clearly exerts a considerable influence upon the performance of the pavement.
INFLUENCE OF MATERIAL FRICTION ANGLE UPON FIRST YIELD, SHAKEDOWN AND STATIC COLLAPSE LOADS
INFLUENCE OF MATERIAL FRICTION ANGLE AND COEFFICIENT OF FRICTION UPON ONE DIMENSIONAL SHAKEDOWN LIMIT
Finally, it may also be noted that each curve in Figure 5.28 is in fact a combination of two intersecting curves, associated with the two different failure modes possible for a half-space under a repeated load. Were a normal load to act alone, incremental collapse would be initiated at some finite depth below the surface; by contrast, a shear load alone, if sufficiently large, would generate the same fatigue type of failure, but beginning at the surface of the continuum. For a combined normal and shear load, then, failure may be initiated in either way, depending upon the relative magnitudes of the two component loads. The Figure demonstrates that for low values of friction coefficient $\mu$, normal load effects are of major significance, whilst at higher values of $\mu$, shear load effects dominate the fatigue behaviour.

5.6 Shakedown of a Two-Layer Half-Space under Moving Surface Loads

5.6.1 Introduction

Some aspects of the behaviour of two-layer pavements shall now be examined qualitatively. Throughout the discussion it is assumed that each layer is composed of homogeneous isotropic material, so that the pavement may be characterised by the elastic parameters $(E, E_0, v, v_0)$ and strength properties $(c, c_0, \phi, \phi_0)$ for the two layers. In each case, the subscript is taken to refer to the lower (subgrade) layer. A number of studies have been performed for undrained clay ($\phi = 0$) pavements. Although this idealisation differs somewhat from most practical pavements, the tests are valuable as a qualitative indicator of performance, and are readily extended to frictional materials, as will be shown.

5.6.2 Influence of Layer Stiffness

Figure 5.29 (a) shows a two-layer pavement subjected to normal wheel load only. The ratio of surface layer stiffness to the stiffness of the subgrade $(E/E_0)$ is varied, with the relative strengths of the two layers being held constant. The influence of varying $E/E_0$ is shown in the Figure, where it can be seen that two different curves are
FIGURE 5.29

INFLUENCE OF RELATIVE STIFFNESS ON SHAKEDOWN LIMIT, FOR TWO VALUES OF RELATIVE STRENGTH.
superposed in order to define the effect of stiffness upon the shakedown limit. At any value of \( E/E_0 \), loads exceeding \( \lambda_{SD} \) produce an incremental collapse of the structure, and the two sections of curve have been found to be associated with two distinct mechanisms of progressive collapse, similar to those encountered in the half-space studies. Again, their intersection represents a mode which may be conveniently regarded as a kind of "balanced" failure.

In the case given, then, it is clear that there exists an optimum ratio of stiffnesses \( (E/E_0) \) at which the resistance to incremental collapse is maximised. For higher values of \( E/E_0 \), we find that additional stress is attracted to the surface layer, resulting in a fatigue failure at the base of this layer. Lower values of \( E/E_0 \), by contrast, result in more load being carried by the subgrade, with the consequence that the top of the subgrade fails by fatigue of a compressive nature.

5.6.3 Influence of Layer Strength

Figure 5.29 (b) shows the same type of relationship between stiffness and shakedown limit, but for a different value of relative strength \( c/c_0 \). It can be seen that a third failure mode is introduced by this change, so that for relative stiffness in the range \( 5.8 < E/E_0 < 13 \), incremental collapse first occurs within the surface layer (as opposed to failure at the base of this layer, which occurs for higher values of stiffness). Again, it may also be noted that a maximum value of shakedown limit exists, in this case, when \( E/E_0 = 5.8 \).

At this point, these two curves and others may be conveniently combined in a single diagram, where the effect of varying relative strength and/or relative stiffness is more readily seen. Figure 5.30 shows such a diagram, for a selected value of surface layer thickness. The following features may be noted:

(a) at given values of strength, there exists an optimal value of relative stiffness which maximises the shakedown limit;

(b) at given values of stiffness, increasing the relative strength causes an increase in shakedown limit. This increase ceases when the failure mode changes from surface layer fatigue to incremental collapse at the top of the subgrade,
FIGURE 5.30

INFLUENCE OF STIFFNESS AND STRENGTH ON SHAKEOWN LIMIT OF A TWO-LAYER PAVEMENT
and further increasing the top layer strength (higher $c/c_0$) is clearly to no advantage since this strength no longer governs shakedown of the pavement;

(c) as might be expected, any proportional increase in the strength of both layers produces an increase of the same proportions in the shakedown limit $\lambda_{SD}$.

### 5.6.4 Influence of Layer Thickness

A close examination of Figure 5.30 reveals that the sections of curve associated with surface layer fatigue (large $E/E_0$) are, in fact, all multiples of the same source curve. This arises once attention is restricted to the behaviour of the top layer, for in doing so, an infinite strength is attributed to the subgrade ($c_0 \rightarrow \infty$) to ensure failure is initiated above the interface. Thus, any variation in surface layer strength will be accompanied by a proportional change in the failure load of the continuum.

Effectively then, the information on Figure 5.30 may be simply represented by two curves: the first, for large values of $c/c_0$ and, therefore, associated with subgrade incremental collapse; and the second, for large values of $c_0/c$ and, therefore, associated with surface layer incremental collapse. The composite curve for a selected value of $c/c_0$, such as those shown in Figures 5.29, is then obtained by superposition of the first curve on a suitably scaled form of the second.

This simplified representation permits a further source of variation - layer thickness - to then be explored. In Figure 5.31 are collected the curves governing surface layer fatigue for pavements with a range of surface layer thicknesses. The use of a vertical scale of $\lambda_{SD}/c$ (rather than $\lambda_{SD}/c_0$) enables a single curve to be used for each thickness, as explained above. Similarly, the curves for subgrade fatigue failure (large $c/c_0$) are collected in Figure 5.32.

For a given depth of surface layer $D$, a complete curve is then compiled using the appropriate curve from Figure 5.32, and the curve from Figure 5.31 scaled by the ratio $c/c_0$. The result of this operation, for a range of depths, is shown in Figure 5.33. This diagram confirms that, for given conditions of $c/c_0$ and thickness $D$, there exists an optimal value of relative stiffness $E/E_0$, which maximises the pavement's
FIGURE 5.31

INFLUENCE OF LAYER THICKNESS UPON INCREMENTAL COLLAPSE OF THE SURFACE LAYER
FIGURE 5.32

INFLUENCE OF LAYER THICKNESS UPON INCREMENTAL COLLAPSE OF THE SUBGRADE
FIGURE 5.33
INFLUENCE OF LAYER THICKNESS UPON SHAKEDOWN LIMIT
OF TWO-LAYER PAVEMENT
resistance to incremental collapse. Again, we also see that for lower values of $E/E_0$ the governing failure mode is fatigue in the subgrade, whilst for higher $E/E_0$ the mode changes to one of surface layer fatigue failure.

5.7 Implications for Design

The aim of any design process is the selection of dimensions and properties of components in order that the completed structure should attain a certain standard of performance. In the case of the process of pavement design, the designer often begins with materials whose properties can be defined within practical limits of accuracy, the requirement being to select suitable thicknesses of layers in order that the pavement should meet particular performance requirements.

From the point of view of a pavement's progressive deformation over a period of time, a convenient measure of performance is the shakedown limit - the higher, the better. Having selected a required value of shakedown limit, data such as those shown in Figures 5.31 and 5.32 may be assembled to provide various combinations of relative strength, relative stiffness, and thickness of surface layer, which will enable the performance standard to be attained. One method of presenting these combinations appears in Figure 5.34. In order to demonstrate the approach, the two-layer pavement was assumed to consist of purely cohesive materials and be loaded by a normal traction only. For a typical value of $V$ of 500 kPa, the Figure would be used if a shakedown limit $\lambda_{SD}$ of 1.5 was required and $c_0$ was 50 kPa (yielding a performance standard $\lambda_{SD}V/c_0 = 15$). A separate chart would be required where a different level of performance (say $\lambda_{SD} = 2.0$, $V$ and $c_0$ unchanged), and consequently a different pavement life, was specified.

With an input of material parameters $c$, $c_0$, $E$ and $E_0$, the required thickness of surface layer may be read from the chart. Consider, for example, the case when $E_0 = 50$ MPa, $E = 1500$ MPa, $c_0 = 50$ kPa and $c = 500$ kPa; then the curve for $E/E_0 = 30$ intersects the line $c/c_0 = 10$ at a depth $D/2B$ of 0.73. Typically the load radius takes a value near 100 mm, so that the thickness of surface layer required is $0.73 \times 200 = 146$ mm.
FIGURE 5.34

INFLUENCE OF STRENGTH AND STIFFNESS PARAMETERS UPON REQUIRED THICKNESS OF LAYERS
It is also readily seen that for this strength \((c/c_0 = 10)\), there exists a minimum thickness to meet the required standard \((D/2B \geq 0.55)\) and a corresponding optimal stiffness \((E/E_0 = 22)\); likewise for each stiffness there is a minimum layer thickness (at \(E/E_0 = 30, D/2B \geq 0.52\)), and a minimum strength which will permit this thickness to be used (here, \(c/c_0 \geq 13\)).

Whilst it is clear that the foregoing theory for two-layer continua cannot be applied to all forms of pavement, the principles embodied within the results are general and the data may fairly simply be extended to cover other material properties, and pavements of three or more distinct layers. The results, as presented, demonstrate that the predictions of shakedown analysis are qualitatively consistent with pavement performance in practice, and the accepted or observed modes of pavement deterioration under repeated traffic loading.

5.8 Conclusions

It has been demonstrated that the problem of pavement shakedown may be considered from a "one-dimensional" viewpoint, and that this approach yields a formulation amenable to solution by two different procedures. Comparisons in terms of accuracy and cost clearly demonstrate the superiority of the procedure based on the theory of conic sections.

Solutions presented include those for the shakedown of both one- and two-layer pavement structures, and the means of applying these results to pavement design procedures have been outlined.

Further refinements may readily be incorporated into the analysis. Material degradation, for example, may be accounted for by suitable variations in material parameters: it should be noted, however, that the present analysis does not consider this aspect, confining itself instead to mechanical degradation of the structure.
Throughout the development of the foregoing analysis, it has been clearly evident not only that individual approximations are appropriate to the peculiar case of road pavements, but also that the entire approach using shakedown theory is conceptually simple as well as being consistent with the observed processes of pavement failure.
INFLUENCE OF MATERIAL SELF-WEIGHT UPON PAVEMENT SHAKEDOWN

In the case of road pavements subjected to single wheel loads of the order of 2 - 3 tonnes, it might be expected that the behaviour of the continuum could be fairly well approximated without taking into account the stresses due to material self-weight. To assume this would certainly simplify the analysis, but it is desirable to first check on its validity, and this was done using the method incorporating linear programming.

Two series of tests were performed, each on an isotropic homogeneous half-space model of a continuum. The intention was to consider the cases of clay and granular materials, and the shakedown performance of the two pavements was investigated for a range of the dimensionless parameter for unit weight, $\frac{\gamma B}{c}$. Here, $\gamma$ refers to the material unit weight, $B$ is a characteristic length, and $c$ represents material cohesion. Friction angle $\phi$ was taken to be $0^\circ$ and $30^\circ$ for the cases of clay and granular materials, respectively.

Figure 5A1 illustrates the main points with regard to materials and loading. For this study, a load half-length $B$ of 100 mm was adopted as a representative figure for a range of wheel loads. In order to estimate the expected range of $\frac{\gamma B}{c}$ in practice, it was noted that the maximum unit weight of roadbase (or poorer quality) materials rarely exceeds 25 kN/m$^3$ - for example, see Scott (1975), p. 29. Values of (drained) cohesion are less readily obtained, but for both materials it would be reasonable to assume $c' \geq$ (say) 20 kPa. Hence, the greatest value of $\frac{\gamma B}{c}$ to be examined is of the order of 0.1.

In Figure 5A2, we see the results of the two series of tests, with the shakedown limit being expressed relative to that for $\gamma = 0$ (weightless material). It is evident that for most practical purposes, the variation from the shakedown limit for $\gamma = 0$ is not significant (even for $\phi = 30^\circ$, the variation is still only 7% at $\frac{\gamma B}{c} = 0.3$), and that for analytical purposes pavements may generally be assumed to consist of weightless materials.
FIGURE 5. A1

LOAD AND MATERIALS CHARACTERISTICS FOR SELF-WEIGHT TESTS

FIGURE 5. A2

INFLUENCE OF SELF-WEIGHT UPON SHAKE-DOWN LIMIT FOR TWO TEST PAVEMENTS
MODIFICATION TO PROCEDURE FOR DETERMINING
SHAKEDOWN LIMIT AT A GIVEN DEPTH

The parameter \( \lambda_z \) has been defined as the highest permissible load factor consistent with the yield conditions, at a given depth \( z \). In graphical terms, \( \lambda_z \) may sometimes be determined as the lowest of a series of maxima calculated from curves representing "safe" domains; see Figure 5.24. As is shown by Figure 5.25, however, this approach will not generally succeed, as the lowest maximum does not always lie within all the curves at depth \( z \). The following procedure is, consequently, presented as a means of determining \( \lambda_z \), the shakedown limit at depth \( z \).

1. Given a pavement structure and materials, depth \( z \), and horizontal scanning points \( x_1, x_2, \ldots, x_r \), calculate \( \lambda_{\text{max}}(x_i, z) \) for each scanning point, and then

\[
\lambda_{X_0} = \min_{i=1,r} \left\{ \lambda_{\text{max}}(x_i, z) \right\}
\]

where \( \lambda_{X_0} \) is the corresponding residual stress, as shown in Figures 5.11 and 5.12.

Call the curve from which this point was derived, \( X_0 \).

2. If \((\lambda_{X_0}, \sigma_{X_0})\) satisfies yield condition at \( (x_i, z) \), \( i=1,2,\ldots,r \), optimum \( \lambda_z \) has been obtained.

3. If yield condition has not been satisfied at all points in the row, the optimum value of \( \lambda_z \) has been exceeded, as illustrated by Figure 5.13. Use \( \lambda_{X_0} \) as a starting point to determine \( \lambda_z \).

4. Select that point in the row at which yield is exceeded by the greatest amount, according to some measure such as radii of Mohr circles. Call the associated curve \( X_3 \) - Figure 5.B1.

5. Since \( X_0 \) has a maximum at \( \lambda_{X_0} \), and all curves intersect at common points on the horizontal axis \( (\frac{\sigma_{X_0}}{2C} = \sin \phi \pm \frac{1}{\cos \phi}) \), the curves \( X_0 \) and \( X_3 \) will have gradients of opposite sign at their positive intersection point - Figure 5.B2. This point must exist otherwise yield would not have been exceeded by \((\lambda_{X_0}, \sigma_{X_0})\).

Find this point to give an improved approximation to the shakedown limit, \((\lambda_{X_1}, \sigma_{X_1})\). Rename \( X_0 \) as \( X_1 \), \( X_3 \) as \( X_2 \).
FIGURE 5.81
SELECTION OF ELLIPSE X3, AT WHICH YIELD EXCEEDED BY LARGEST AMOUNT

FIGURE 5.82
IMPROVED APPROXIMATION TO $\lambda_z$
6. Repeat the check on whether the yield condition is now satisfied for all points \((x_i, z), i=1,2,...,r\). If it is, \(\lambda_z = \lambda_{zl}\) and search is terminated.

7. If yield condition not satisfied, select X3 as the curve corresponding to point where yield is most exceeded - Figure 5.B3.

8. Since all curves are conic sections, with two common intersections on the horizontal axis, the sign of the gradient of X3 will be the same at \(\sigma_{XR} = \sigma_{XR1}\), as it is at the intersections of X3 with X1 and X2. As always, for an optimal value of \(\lambda_z\) it is required to find an intersection point where the two curves have opposite gradients. Therefore, select from X1 and X2 that curve with gradient opposite in sign to that of X3 at \(\sigma_{XR} = \sigma_{XR1}\), and call it X0. Calculate the intersection of X0 and X3 to give improved values of \(\lambda_{zl}, \sigma_{XR}\). Rename X0 as X1, X3 as X2, and return to Step (6) - Figure 5.B4.

Finally, one further refinement was found necessary during testing of the algorithm. Whilst \((\lambda_z, \sigma_{XR})\) must satisfy the yield condition for all points \((x_i, z)\) at a given depth \(z\), it is clear that \((\lambda \leq \lambda_z, \sigma_{XR})\) must do so also. In view of the form of the safe \(\lambda - \sigma_{XR}\) domains, it is evident that the value of \(\sigma_{XR}\) corresponding to \(\lambda_z\) must lie between the two pivot points on the residual stress axis - that is, \(\frac{\sin \phi - 1}{\cos \phi} \leq \frac{\sigma_{XR}}{2c} \leq \frac{\sin \phi + 1}{\cos \phi}\). Before each check on yield (Steps (2) and (6)), it should, therefore, be ensured that \(\sigma_{XR}\) lies within this range, and if it does not, a simple adjustment (Figure 5.B5) is made before proceeding with the algorithm above.

In the manner set out, then, the correct value of shakedown limit at a given depth, \(\lambda_z\), is obtained by an iterative process. The procedure terminates in a finite number of steps, controlled by the number of scanning points \((x_i, z)\) and provides a simple refinement to the basic method presented as an efficient alternative to using linear programming.
FIGURE 5.83
INITIAL SHAKEDOWN LIMIT

FIGURE 5.84
IMPROVEMENT OF SHAKEDOWN LIMIT
FIGURE 5.85

ADJUSTMENT OF SHAKEDOWN LIMIT
FOR ALLOWABLE RESIDUAL STRESS RANGE
AN ANALYTICAL EXPERIMENT FOR HALF-SPACE SHAKEDOWN

In deriving the one-dimensional shakedown limit for a homogeneous isotropic half-space, an alternative approach may be explored.

Consider the case of a purely cohesive half-space traversed by a normal trapezoidal load (Figure 5.C1). In order to develop the reasoning which follows, the following assumptions are made:

(a) at any depth, the load factor - residual stress domain varies smoothly with any variation in the horizontal co-ordinate x;

(b) each domain is differentiable for all \( \lambda \geq 0 \);

(c) the highest point (maximum \( \lambda \)) on each domain may be determined, and varies smoothly with variation in either co-ordinate (x or z).

With these assumptions, it is possible to experiment with an "analytical" solution. Using the equation

\[
\lambda_{\text{max}} = \frac{c}{\kappa_T - \sigma_T \tan \phi}
\]

as derived previously (Section 5.4.3.3), and setting \( \phi = 0 \), yields

\[
\lambda_{\text{max}}(x,z) = \frac{c}{\tau_E}
\]

For convenience, now put

\[
B = 1
\]

\[
a = 1 - b
\]

so that without loss of generality, all co-ordinates are considered relative to load half-length.

Then, for the plane strain case being considered, and maximum applied stress \( V \),

\[
\tau_E(x,z) = \frac{-zV}{\pi a} \left[ \tan^{-1}\left(\frac{x+1}{z}\right) - \tan^{-1}\left(\frac{x+b}{z}\right) + \tan^{-1}\left(\frac{x-1}{z}\right) - \tan^{-1}\left(\frac{x-b}{z}\right) \right]
\]
FIGURE 5.C1
DEFINITION OF PAVEMENT LOADING
and the solution then requires

\[ T = \max_{(x,z)} \left\{ \frac{\pi a}{V} | \tau_E(x,z) | \right\} \]

whence

\[ \lambda_{SD} = \frac{c \pi a}{VT}. \]

This maximisation may be performed analytically for the simple case selected, as follows:

(a) obtain \( T_z = \max_x \left\{ \frac{\pi a}{V} | \tau_E | \right\} \)

by \( \frac{\partial T}{\partial x} = 0. \)

Thus,

\[ x_p^2 = \frac{(1+b^2-2z^2) \pm \sqrt{(1+b^2+4z^2)^2 + 12b^2}}{6} \]

and \( \frac{\partial T}{\partial x} = 0 \) at \( x = x_p. \)

Then

\[ T_Z = z \left[ \tan^{-1}\left( \frac{x_p-b}{z} \right) - \tan^{-1}\left( \frac{x_p-1}{z} \right) \right. \]

\[ + \tan^{-1}\left( \frac{x_p+b}{z} \right) - \tan^{-1}\left( \frac{x_p+1}{z} \right) \left. \right] \]

(b) by any convenient (numerical) method, locate \( T = \max_z \{ T_z \} \).

Then \( \lambda_{SD} = \frac{c \pi a}{VT} \) giving the half-space shakedown limit.

For the case \( b = 0.5 \), the above algorithm yields a value

\[ \frac{\lambda_{SD} V}{c} = 3.789. \]
CHAPTER SIX

SHAKEDOWN OF THREE-DIMENSIONAL CONTINUA
6.1 Introduction

The foregoing chapter has dealt in some detail with a "one-dimensional" approach to shakedown, felt to be particularly appropriate to the conditions of loading and response in pavement structures. Indeed, it will be shown in later sections of this thesis that this approach, whilst certainly approximate, has considerable potential for the prediction of pavement life.

At this point, however, it is instructive to examine the behaviour of the more general "three-dimensional" pavement. This serves the dual purposes of verifying the "one-dimensional" assumptions as being appropriate, and establishing the relationship between results obtained by the two approaches. In the sections which follow, the theory presented previously will be extended to the more general case of an axisymmetric load moving across a horizontally-layered continuum. The shakedown of one- and two-layer pavements will then be re-examined, and the results compared with those of the previous chapter.

6.2 Components of Analysis

6.2.1 Loadform and Elastic Response

The experiments of Marwick and Starks (1941), Bonse and Kuhn (1959), Sanborn and Yoder (1967) and Sargious (1975) have demonstrated that the shape of the contact area between a normally loaded tyre and a relatively rigid plane may be approximated by a circular or slightly elliptical shape. Under conditions of overload this changes to a more rectangular form, but this shape is representative of only a small minority of load cases, and it appears that a circle represents a suitable first approximation. The same experiments also showed that the variation of normal pressure across the region of contact could be accurately modelled using a trapezoidal variation, and this form of load distribution was adopted in the previous chapter. In Figure 6.1 is illustrated the axisymmetric loadform to be used in subsequent analyses.
FIGURE 6-1

LOADFORM FOR THREE-DIMENSIONAL ANALYSES
The simplicity of this loadform enables the elastic response of the continuum to be readily determined. For a half-space, the point load (Boussinesq) response was integrated numerically, whilst for layered pavements the elastic stresses at any point could be determined using finite layer techniques (Cheung (1976), Booker and Small (1979), Rowe and Booker (1982)). Figure 6.2 illustrates the accuracy of calculation possible using these methods.

6.2.2 No-Yield Condition

A generalisation of the Mohr-Coulomb yield condition from two- to three-dimensional stress space is attributable to Shield (1955), who has shown that with the assumption of positive compressive stress, the material yield surface may be modelled by the pyramid consisting of the six planes

$$\sigma_i = \sigma_j A^2 + 2cA \quad (i,j=1,2,3)$$

where

$$A = \tan (\frac{\pi + \phi}{4})$$

$$c = \text{material cohesion (}\text{c} \geq 0)$$

$$\phi = \text{material angle of internal friction (}0 \leq \phi < \frac{\pi}{2})$$, and

$$\sigma_1, \sigma_2, \sigma_3$$ are the three principal stresses.

Provided the six inequalities

$$\sigma_1 < \sigma_j A^2 + 2cA \quad (i,j=1,2,3)$$

are all satisfied, the material is then said to be within the elastic range of behaviour.

At a general point \(P(x,y,z)\) it should be noted that stresses will generally be computed within a Cartesian co-ordinate system, as the motion of the wheel load is then simply represented as a changing horizontal co-ordinate. The three principal stresses are then obtained by solving the cubic equation

$$\sigma^3 - J_1 \sigma^2 + J_2 \sigma - J_3 = 0$$
COMPARISON OF STRESSES CALCULATED BY FINITE LAYER METHOD WITH OTHER APPROACHES
where 

\[ J_1 = \sigma_X + \sigma_Y + \sigma_Z \]

\[ J_2 = \sigma_X \sigma_Y + \sigma_Y \sigma_Z + \sigma_Z \sigma_X - \tau_{XY}^2 - \tau_{YZ}^2 - \tau_{ZX}^2 \]

\[ J_3 = \sigma_X \sigma_Y \sigma_Z - \sigma_X \tau_{YZ}^2 - \sigma_Y \tau_{ZX}^2 - \sigma_Z \tau_{XY}^2 + 2 \tau_{XY} \tau_{YZ} \tau_{ZX} \]

In general, the stress vector \( \mathbf{g} \) will consist of both elastic and residual components, viz.

\[ \mathbf{g} = \lambda \mathbf{g}_E + \mathbf{g}_R \]

\[ = \begin{bmatrix}
\lambda \sigma_{XE} + \sigma_{XR} \\
\lambda \sigma_{YE} + \sigma_{YR} \\
\lambda \sigma_{ZE} + \sigma_{ZR} \\
\lambda \tau_{XZE} + \tau_{XZR} \\
\lambda \tau_{YZE} + \tau_{YZR} \\
\lambda \tau_{ZXE} + \tau_{ZXR}
\end{bmatrix} \]

in which event the invariants \( J_1, J_2, J_3 \) may be expressed as functions of \( \sigma_E, \sigma_R \) and the load factor \( \lambda \).

### 6.2.3 Residual Stresses

The residual stresses which are assumed to develop as the pavement materials are stressed beyond their limits of elasticity, constitute a self-equilibrating stress field, and at any point must, therefore, satisfy the equations

\[ \frac{\partial \sigma_{XR}}{\partial x} + \frac{\partial \tau_{XZR}}{\partial y} + \frac{\partial \tau_{XJR}}{\partial z} = 0 \quad (6.1a) \]

\[ \frac{\partial \tau_{XJR}}{\partial x} + \frac{\partial \sigma_{YR}}{\partial y} + \frac{\partial \tau_{YJR}}{\partial z} = 0 \quad (6.1b) \]

\[ \frac{\partial \tau_{XJR}}{\partial x} + \frac{\partial \tau_{YJR}}{\partial y} + \frac{\partial \sigma_{ZR}}{\partial z} = 0 \quad (6.1c) \]
If it is assumed, as in the previous chapter, that the motion of all wheel loads is restricted to, say, the X-direction (see Figure 6.3), then it again appears reasonable to assume that after many passes of the loadform, the distribution of permanent deformations and residual stresses will exhibit a variation that is uniform in the direction X.

That is, \( \frac{\partial}{\partial x} = 0 \) in (6.1), so that

\[
\frac{\partial \tau_{XYR}}{\partial y} + \frac{\partial \tau_{XZR}}{\partial z} = 0 \quad (6.2a)
\]

\[
\frac{\partial \sigma_{YR}}{\partial y} + \frac{\partial \tau_{YZR}}{\partial z} = 0 \quad (6.2b)
\]

\[
\frac{\partial \tau_{YZR}}{\partial y} + \frac{\partial \sigma_{ZR}}{\partial z} = 0 \quad (6.2c)
\]

The stresses \( \sigma_R \) therefore vary with \( y \) and \( z \) only, and whilst the other residual stresses are constrained by equations (6.2) it is evident that \( \sigma_{XR} \) may vary arbitrarily with the variables \( y, z \).

6.3 Method of Analysis

The procedure adopted is merely a generalisation of that outlined in the previous chapter, and is concerned with finding that combination of load factor \( \lambda \) and residual stress distribution \( \sigma_R \) such that

(i) at each point \( P(y,z) \), the stress state

\[
\sigma = \lambda \sigma_{E}(x,y,z) + \sigma_R(y,z)
\]

satisfies the no-yield conditions

\[
\sigma_1 \leq \sigma_j A^2 + 2cA \quad (i,j=1,2,3)
\]

for all values of \( x \);

(ii) the residual stress distribution \( \sigma_R(y,z) \) is self-equilibrating (equations (6.2)); and

(iii) the load factor \( \lambda \) is maximised.
"THREE-DIMENSIONAL" PAVEMENT AND APPLIED LOAD
With the added complexity of the extra co-ordinate direction, and six rather than one yield surfaces, the closed form techniques of the "one-dimensional" approach are no longer applicable. Instead, a method of trial residual stress distributions was employed. This involved:

(i) considering the boundary conditions appropriate to the residual stress distribution. These were taken to be:

(a) \( \sigma_{ZR} = \tau_{YZR} = \tau_{ZXR} = 0 \) at \( z = 0 \)

(b) \( \sigma_R \rightarrow 0 \) as \( y \rightarrow \infty \) or \( z \rightarrow \infty \);

(ii) selecting self-equilibrating distributions of residual stress which satisfied the conditions (i) above, and were functions of \( y \) and \( z \) only - for example, distributions of the form:

\[
\begin{bmatrix}
\sigma_{YR} \\
\sigma_{ZR} \\
\tau_{XYR} \\
\tau_{YZR} \\
\tau_{ZXR}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial^2}{\partial y^2} \\
\frac{\partial^2}{\partial z^2} \\
0 \\
-\frac{\partial^2}{\partial y \partial z} \\
0
\end{bmatrix} t\Theta
\]

where \( \Theta = \frac{z^2 e^{-kz}}{(1+Ay^2)^m} \)

\( t = \) a constant.

With a value selected for \( t \) and the parameters \( A, k, m \) in \( \Theta \), all residual stresses except \( \sigma_{XR} \) were then completely defined;
(iii) at each point $P(x,y,z)$, the elastic response to a load at the origin may be computed, as previously outlined. With the residual stress distribution as determined, there then exists a set of points $(\lambda, \sigma_{xR})$ for which the stress state

$$\sigma = \lambda \sigma_E + \sigma_R$$

lies on the yield surface. An envelope of "safe" values of $(\lambda, \sigma_{xR})$ may thus be drawn for each point $P(x,y,z)$;

(iv) as for the one-dimensional case, the maximum value of $\lambda$ within each envelope is of primary concern, and this value may be computed, numerically or by other means, to yield $\lambda_{\text{max}}(x,y,z)$;

(v) the effect of moving the load (in the $x$-direction) may be mirrored by considering the $(\lambda, \sigma_{xR})$ envelopes for constant $(y,z)$, changing $x$. At the given point $(y,z)$, the safe value of $\lambda$ allowing for motion of the load is then

$$\lambda_{\text{peak}}(y,z) = \min_x \{\lambda_{\text{max}}(x,y,z)\};$$

(vi) the shakedown limit for the continuum is then given by

$$\lambda_{SD} = \min_{(y,z)} \{\lambda_{\text{peak}}(y,z)\}$$

$$= \min_{(x,y,z)} \{\lambda_{\text{max}}(x,y,z)\};$$

(vii) the shakedown limit determined in (vi) above is for the given pavement and the given distribution of residual stress. In general, an optimum distribution of residual stress is sought for which $\lambda_{SD}$ is maximised, and this involves repetition of steps (iii) to (vi) for a range of different distributions $\sigma_{xR}$ satisfying the boundary and equilibrium conditions.
The method is thus seen to be an iterative one, approximate in that the full range of possible residual stress distributions cannot be covered. It was, therefore, conceived primarily as a means of verifying the results obtained by the one-dimensional method, and examining the effect of the additional dimension upon the shakedown limit. In essence, the aim was to determine the suitability of the one-dimensional approximation in calculating shakedown limits of layered continua.

6.4 Results of Analysis

6.4.1 Homogeneous Half-Space

In order to avoid unnecessary complication, a sequential approach was used in the analysis of three-dimensional continua. Whereas the plane strain analyses involved at most three components of residual stress ($\sigma_{XR}, \sigma_{ZR}, \tau_{XZ}$), the addition of a third co-ordinate direction brought with it a further three components ($\sigma_{YR}, \tau_{XY}, \tau_{YZ}$). Corresponding loadforms are illustrated in Figure 6.4. Beginning with the case where all residual stresses were assumed to be zero (that is, the determination of first yield loads), assumptions were relaxed one by one in order to observe the effect on shakedown loads of each new degree of freedom.

6.4.1.1 No Residual Stresses

This first series of tests was conducted with a view to confirming earlier assumptions about the conservatism of the plane strain approximations outlined in the previous chapter. With all residual stresses restricted to zero, the determination of maximum load factor subject to the no-yield constraints produced a value of first yield load under axisymmetric wheel loading. The results, shown in Figure 6.5(a) for various values of friction angle, demonstrate that the three-dimensional loading does indeed stress the pavement less heavily than the previous plane strain case, and the limit loads are consequently higher.
LOADFORMS FOR AXISYMMETRIC AND PLANE STRAIN ANALYSES

**Figure 6.4**

Axially Symmetric and Plane Strain Loadforms

\[
\frac{b}{R} = \frac{1}{2} (\sqrt{3} - 1) = 0.725
\]

Average pressure = \(\frac{3V}{4}\)

\[
\frac{b}{B} = \frac{1}{2}
\]

Average pressure = \(\frac{3V}{4}\)
FIGURE 6.5

COMPARISON OF LOAD LIMITS FOR PLANE STRAIN AND THREE DIMENSIONAL ANALYSES
6.4.1.2 One Residual Stress

The first residual stress added to the calculations was the horizontal direct component \( \sigma_{XR} \). This was expected to exert a major influence upon shakedown behaviour, since it corresponded with the direction of travel of the wheel load.

Upon analysis, however, it was found that this extra degree of freedom made little difference to the ability of the pavement to adjust to repeated passages of a load. This was confirmed by plots of the \( \lambda - \sigma_{XR} \) envelopes for a family of points \( P(x, y, z) \), \( x \) variable and \( y, z \) constant. From Figure 6.6 it is evident that the maximum possible value of \( \lambda \) within the area common to all envelopes is indeed equal to that at \( \sigma_{XR} = 0 \); that is, \( \lambda_{SD} = \lambda_{FY} \) (the first yield load).

In the case of a three-dimensional pavement, then, it appeared that if residual stresses were to be mobilised by the passage of a wheel load, they must include components not parallel to the direction of motion of the load. Preliminary tests, in which \( \sigma_{YR} \) was allowed to vary as a fixed proportion of \( \sigma_{XR} \), showed that the value of shakedown limit \( \lambda_{SD} \) could be increased. This may be clearly seen from the sample envelopes in Figure 6.7. However, with only \( \sigma_{XR} \) and \( \sigma_{YR} \) non-zero, equilibrium of residual stresses no longer applied, so it became necessary to introduce other stress components into the analysis also.

6.4.1.3 Four Residual Stresses

Since it was assumed above that residual stresses would, where possible, attain a distribution which was uniform in the direction of motion of the load \( (x) \), the further assumption that \( \tau_{YZR} = 0 \) leads to

\[
\frac{\partial \sigma_{YR}}{\partial y} = 0.
\]

Thus, if self-equilibrium is to be satisfied using only two residual stresses, the possibility that \( \sigma_{YR} \) should decay towards zero as \( y \to \infty \) is ruled out. This expectation of decay, however, is not unreasonable, and variation in \( \sigma_{YR} \) may be permitted by also allowing some residual shear stresses to develop.
LOAD = Uniform Circular

\[ \frac{Y}{R} = 1, \frac{Z}{R} = 2 \]

\[ \psi = 0.3 \quad \phi = 30^\circ \]

\[ \lambda_{FR}(Y, Z) = \lambda_{SD}(Y, Z) \]

FIGURE 6-6

SAMPLE \( \lambda - \sigma_{XR} \) ENVELOPES, ONE RESIDUAL STRESS ONLY
FIGURE 6.7

SAMPLE \( \lambda - \sigma_{XR} \) ENVELOPES, TWO RESIDUAL STRESSES ONLY \( (\sigma_{yR} = \sigma_{XR}) \)
The equations of equilibrium (6.2) may be satisfied by

\[ \tau_{XYR} = 0 \]

\[ \tau_{XZR} = 0 \]

\[ \sigma_{YR} = \frac{\partial^2 \Theta}{\partial z^2} \]

\[ \tau_{YZR} = -\frac{\partial^2 \Theta}{\partial y \partial z} \]

\[ \sigma_{ZR} = \frac{\partial^2 \Theta}{\partial y^2} \]

where \( \Theta \) represents a "residual stress function" such that the boundary conditions

\[ \sigma_{ZR} = \tau_{YZR} = 0 \quad \text{at} \quad z = 0 \]

\[ \sigma_{ZR}, \sigma_{YR}, \tau_{YZR} \to 0 \quad \text{as} \quad y, z \to \infty \]

are satisfied. Simplicity suggests that the form

\[ \Theta(y,z) = t \cdot P(y) \cdot Q(z), \quad t \text{ a constant} \]

may be used, whence

\[
\begin{bmatrix}
\sigma_{YR} \\
\tau_{YZR} \\
\sigma_{ZR}
\end{bmatrix}
= t
\begin{bmatrix}
PQ'' \\
P'Q' \\
P''Q
\end{bmatrix}
\]

In Figure 6.8 is shown a sample set of \( \lambda - \sigma_{XR} \) envelopes generated using a trial residual stress function

\[ \Theta = \frac{-2z^2}{(1+y^2)(1+z^2)^2} \]

and it is evident from the Figure that it may indeed be possible to improve upon the values of \( \lambda_{SD} \) previously found using a single residual stress.
\[ \theta = \frac{-2z^2}{(1+y^2)(1+z^2)^2} \]

LOAD: Uniform Circular

\[ \frac{y}{R} = 0.5, \quad \frac{z}{R} = 1.0 \]

\[ \nu = 0.5, \phi = 0 \]

FIGURE 6.8

SAMPLE \( \lambda - \sigma_{XR} \) ENVELOPES, FOUR RESIDUAL STRESSES
Using the type of formulation above, it was possible to resume the search for shakedown limits of 3-D continua. For a fixed residual stress function, the method adopted was to scan a grid of points \((y,z)\), at each point determining numerically the value of \(\lambda_{SD}(y,z,\Theta)\) as indicated in Figure 6.8 - that is, the highest value of \(\lambda\) lying within the set of envelopes associated with a range of \(x\) values. Once this was done, the minimum value of \(\lambda_{SD}\) for the chosen \(\Theta\) could be determined: Figure 6.9 illustrates the results of a full analysis of a half-space using one particular \(\Theta\), and the corresponding value of \(\lambda_{SD}\).

This analysis was repeated, first varying \(t\) and later for other functions \(P(y) \cdot Q(z)\), in order to determine an optimum distribution of residual stress, for which \(\lambda_{SD}(\Theta)\) was maximised. After a considerable number of trials, it became evident that \(\lambda_{SD}(\Theta)\). varied with \(\Theta\) in a manner that could be determined using a relatively small number of \(\Theta\) functions. The general forms of \(\Theta\) investigated in detail were

\[
\Theta_1 = \frac{tz^2}{(1+Ay^2)^m (1+Bz^2)^n} \quad (n \geq 2)
\]

\[
\Theta_2 = \frac{tz^2e^{-kz}}{(1+Ay^2)^m}
\]

and Figure 6.10, showing \(\Theta\) for various sets of parameters \((A,B,m,n)\) or \((A,k,m)\), confirms that the variation in \(\Theta\) is limited. Since the critical region for the calculation of \(\lambda_{SD}\) was found to be that with \(\frac{z}{R} \leq 1\), it was further evident that much of the variation for different sets of parameters could be closely accounted for simply by varying \(t\) instead (i.e. the variation was in scale rather than shape). The four distributions finally selected for use in further analyses were

\[
\Theta_1 \quad (A=B=m=1, \ n=5) \\
\Theta_2 \quad (k=0.5, \ A=m=1) \\
\Theta_2 \quad (k=2.0, \ A=m=1) \\
\Theta_2 \quad (k=5.0, \ A=m=1).
\]
Result of analysis: \[ \frac{\lambda_{sdv}}{C} \approx 2.87 \]

\[ \theta = \frac{0.5 Z^2}{(1+Y^2)^2 (1+Z^2)^2} \]

\[ \nu = 0.5, \phi = 0 \]

FIGURE 6.9

TYPICAL RESULTS FROM A SINGLE 3D SHAKE-DOWN ANALYSIS
Key to Parameters $\theta_1 (A, B, m, n)$
$\theta_2 (K, A, m)$

FIGURE 6.10 (a)

VARIATION OF RESIDUAL STRESS FUNCTION
FOR A RANGE OF PARAMETERS
FIGURE 6.10 (b)

VARIATION OF RESIDUAL STRESS FUNCTION FOR A RANGE OF PARAMETERS
With typically four values of $t$ also being used for each $\Theta$, this meant that the determination of the 3D shakedown limit of a pavement of particular geometry and materials required some sixteen analyses—a considerably slower process than the corresponding 1-D analysis previously developed.

In Figure 6.5(b) are recorded the results obtained from this family of analyses. It may again be noted that considering additional components of residual stress permits an increase in $\lambda_{SD}$. Further, it is clear that, as for first yield loads, a typical wheel loading stresses a 3D pavement less severely than that predicted by the earlier plane strain analyses.

### 6.4.1.4 Six Residual Stresses

To complete the derivation of shakedown limits for 3D continua, it remains only to note that two further components of residual stress, $\tau_{XYR}$ and $\tau_{ZXR}$, need to be incorporated into the analysis. In the equilibrium conditions (6.2) it may be seen that these two shears are completely uncoupled from the remaining residual stresses, and the corresponding differential equation (6.2a) may be simply solved by

\[
\begin{bmatrix}
\tau_{XYR} \\
\tau_{ZXR}
\end{bmatrix} = \begin{bmatrix}
\frac{3}{\partial z} \\
-\frac{3}{\partial y}
\end{bmatrix} \phi
\]

where $\phi$ represents a residual stress function similar to, but independent of, $\Theta$ employed above.

The effect of including these two residual shears was examined by repeating a series of earlier analyses (4 residual stresses), with the addition of $\phi$ in various forms similar to those outlined above for $\Theta$. The effect, however, was not found to be significant: the highest values of $\lambda_{SD}$ were found to occur when $\phi \neq 0$, and all the cases tested suggested that in practice the six-stress case would reduce to that of four residual stresses. The results of the analyses for a half-space are shown in Figure 6.5(b) previously considered.
6.4.2 Layered Continua

To confirm that the findings of the previous section could be generalised from a homogeneous half-space to cover layered pavements, a series of numerical studies was conducted for two-layer continua.

As previously outlined, the axisymmetric loadform enabled the elastic response of a layered structure to be readily determined, using the Hankel transform approach developed by Rowe and Booker (1980). The shakedown analysis was then conducted in the manner set out in Section 3 of this chapter, using four residual stresses.

It was indicated in the previous chapter that the shakedown performance of an individual layer within a pavement could be isolated from that of other layers, in order to simplify the presentation of results as well as the understanding of behaviour. For the same reasons, this concept was applied to the present work. This had a further advantage in that it removed the necessity to find a complete distribution of residual stresses satisfying equilibrium, boundary and interface conditions throughout the entire pavement; instead, each layer could be considered separately, since in any given pavement an eventual incremental collapse would originate within one layer only.

In view of this factor, the set of four residual stress functions previously selected for half-space studies was retained for the two-layer analyses. This meant, once again, upwards of sixteen analyses for each pavement geometry and materials set, but it did enable results to be derived with some confidence in their accuracy.

The structure selected for comparison analyses is shown in Figure 6.11, and computations were performed for a range of stiffness ratios $E/E_0$. Figure 6.12 sets out the results of these calculations, with shakedown limits expressed in terms of either surface layer or subgrade cohesion. As before, the curves may be superposed, with appropriate scaling based on strength ratio $c/c_0$, in order to obtain the full relationship between relative stiffness and shakedown limit.
FIGURE 6.11

TWO-LAYER PAVEMENT FOR COMPARISON
OF 3D AND PLANE STRAIN RESULTS
FIGURE 6.12

3D AND PLANE STRAIN RESULTS FOR THE TWO-LAYER PAVEMENT OF FIG. 6.11
6.5 Discussion and Conclusions

An analysis has been developed, and results presented, for the shakedown of three-dimensional continua. The form of analysis has been shown to be applicable to layered as well as uniform continua, and results have been included for both cases.

It is evident, from the sets of results presented in previous sections, that the shakedown limit of a pavement derived by 3-D analysis may be closely related to that obtained using a plane strain approach. In this way, the approximations used in developing the latter have been shown to be appropriate. Further, the plane strain analysis not only provides a suitable approximation to the more elaborate approach, but is also generally conservative in its estimates, yielding a lower bound to the true shakedown limit.

The advantages of the plane strain method are, therefore, considerable. Not only are its estimates safe and reasonably accurate, but its formulation is simpler conceptually, and its execution faster. Indeed, for a typical pavement analysis the time required in computing is of the order of one-hundredth of that consumed in a corresponding 3-D analysis. For parametric studies and the development of design charts, then, the plane strain approximation provides a tool of far greater convenience, and is really the only practical approach. For the case studies and preparation of design tools which follow in later chapters, this is the analytical method which will be employed.
CHAPTER SEVEN

THE AASHO ROAD TEST: A CASE STUDY
7.1 Introduction and Background

The rapid post-World War II growth in traffic volume, and an associated rise in vehicle sizes and weights, led many road authorities to examine more closely the relationship between pavement loading, pavement thickness design, and pavement performance.

The Highway Research Board in the United States conducted a number of experiments with this aim. The largest of these, involving controlled trafficking of six test loops, using 126 vehicles and costing some $27 million, was conducted from November 1958 to November 1960. The large volume of data generated by this test, along with its carefully controlled approach using a statistical basis, makes the test a valuable source for examining procedures of pavement analysis and design.

In this chapter, some of the performance data arising from the Road Test is reviewed using a method of analysis concerned with the incremental deterioration of pavement structures.

7.2 Analysis and Data

7.2.1 Method of Analysis

The elements of shakedown theory deal with the behaviour of a body under repeated loading, and the resistance of a body to a fatigue (incremental) type of failure may be characterised by some measure such as its shakedown limit. This limit defines the most severe loading condition which may be repeatedly applied to the body without inducing an eventual failure by fatigue.

It is, therefore, reasonable to expect that there should be some relationship between a body's resistance to incremental failure, and its life under repeated loading. This study seeks to determine whether the shakedown limit of a pavement may be related to its life under traffic loading, and employs the techniques of one-dimensional shakedown analysis to this end.
7.2.2 Pavement and Loading Parameters

In order to analyse the shakedown behaviour of a pavement, data dealing with its geometry, materials and loading are required.

The range of pavement profiles examined in the Road Test is presented more than adequately in the Highway Research Board's Special Reports 61 (A to G). It should be noted that each test loop comprised a complete factorial experiment on thicknesses of surfacing, base and sub-base material, and therefore represents a fruitful source of performance data.

Material parameters of a type required for the shakedown analysis (i.e. describing both stiffness and strength characteristics) were not so readily available. Shook and Fang (1961) reported the results obtained from a subsequent co-operative materials testing program, in which samples of the Road Test materials were tested by several different highway agencies. These results were supplemented with those from other tests, to obtain an initial set of materials parameters as outlined in the following sections.

7.2.2.1 Elastic Properties

The subgrade material was a yellow-brown clay of AASHO classification A-6, with a maximum dry density of 1.86 t/m³ and optimum moisture content 15%. CBR test values ranged from 2 to 4. Triaxial tests reported by Kansas (Shook and Fang (1961)) suggested modulus values between 4 and 20 MPa, while the very rough approximation of \( E(\text{MPa}) = 10 \times \text{CBR} \) led to values of the order of 20 MPa. Other more general surveys suggested typical values of \( E \) for medium to stiff normally consolidated clays in the range 5 to 20 MPa. An initial value of subgrade modulus was, therefore, taken to be 10 MPa.

The material employed as sub-base was described as a locally available sand-gravel, modified by the addition of small amounts of fine sand and friable fine-grained soil. Its grading and plasticity characteristics, plus its compaction at the time of placement, were such as to suggest a modulus of the order 20 to 100 MPa, and again results from Kansas tests showed values in the range 40 to 60 MPa. A value of 50 MPa was selected as a suitable starting point.
A crushed dolomitic limestone was used as a road base. A range of physical characteristics was reported, but the results of most value again came from Kansas. For a deviator stress up to about 900 kPa (i.e. exceeding typical tyre contact pressures) the drained modulus took values around 85 MPa, and this figure was, therefore, adopted for the analysis.

The courses of asphaltic concrete used dense-graded crushed dolomitic limestone plus natural sand as aggregates, with an asphalt content of 5%. Marshall stability lay in the range 8 to 9 kN, with a flow of 2.5 to 3.0 mm. Total voids comprised between 3.5 and 5.0% of the mix. Numerous investigations have demonstrated the heavily temperature-dependent nature of asphalt moduli, and it was noted that typical temperatures for the site of the Road Test in Illinois lay in the range 10-20°C. From a number of general studies, such as Nair and Chang (1973), it was reasonable to expect that resilient modulus should be about 4000 to 8000 MPa. The value of modulus which could be extracted from the Marshall test results (obtained at 60°C) along with the temperature-dependence data of Keshavan and Nair (1973) confirmed this expectation, suggesting a value for $M_R$ of 4000 to 5000 MPa. Initially, an asphaltic concrete resilient modulus of 5000 MPa was adopted.

It may readily be shown that the shakedown limit of a layer of material within a pavement, depending as it does upon the elastic response of the whole pavement structure, is relatively insensitive to the values of Poisson's ratio applied to each material. The results of Nair and Chang (1973) and others, therefore, provided sufficient information for a first approximation to be made to $\nu$, and the values selected were as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphaltic concrete</td>
<td>0.4</td>
</tr>
<tr>
<td>Base</td>
<td>0.3</td>
</tr>
<tr>
<td>Sub-base</td>
<td>0.3</td>
</tr>
<tr>
<td>Subgrade</td>
<td>0.3</td>
</tr>
</tbody>
</table>
7.2.2.2 Strength Properties

The paucity of information encountered in seeking stiffness parameters for the Road Test materials, was not so acute in the case of strength parameters. It was most convenient for the purposes of analysis to characterise a material's strength in terms of its Mohr-Coulomb failure envelope, using the material properties of cohesion \(c\) and internal angle of friction \(\phi\).

Whilst the reported values of CBR might have been expected to bear some relationship to the required parameters, the exact nature of the relationship is not at all clear and is highly dependent on other material characteristics. It was, therefore, preferable where possible to employ a set of triaxial tests to derive the envelope directly, by a series of tests conducted at different confining stresses.

Shook and Fang (1961) reported a considerable range of such results, covering the materials used as subgrade, sub-base and base. These results are summarised in Figures 7.1 (a) to (c). From the Figures it is clear that, whilst the base and sub-base failure properties may be characterised with some confidence, those of the subgrade may not, and an initial approximation was required. In view of the nature of the material, along with the trend of the envelopes shown in Figure 7.1 (a), it was decided to begin with \((c, \phi) \sim (20\,\text{kPa}, 20^\circ)\) for the subgrade. From the remaining two Figures, the data employed were

- sub-base \((c, \phi) = (30\,\text{kPa}, 45^\circ)\), and
- base \((c, \phi) = (40\,\text{kPa}, 55^\circ)\).

For the asphaltic concrete (AC), some approximation was again necessary. The predominant body of data on AC strength currently available concerns itself with the relationship between repeated flexural strain and number of cycles to failure. While this data was not directly applicable to a shakedown analysis, when modulus values were also provided it enabled an estimate to be made of the flexural strength of the AC. Typical results, from Heukelom and Klomp (1967), Brown and Pell (1972), Monismith and McLean (1972) and others, revealed a flexural strength independent of modulus and of the order of 2000 to 7000 kPa, for \(N = 1\) (i.e. a single load cycle). By comparison ultimate tensile strengths reported by various authors tended to fall
FIGURE 7.1(a)

MOHR RUPTURE ENVELOPES
FOR AASHO ROAD TEST SUBGRADE
Key: 1. Alberta
2. Federal Aviation Agency
3. Missouri
4. Saskatchewan
5. Texas
6. Utah
7. Waterways Experiment Station

FIGURE 7·1(b)

MOHR RUPTURE ENVELOPES
FOR AASHO ROAD TEST SUBBASE
FIGURE 7-1(c)

MOHR RUPTURE ENVELOPES
FOR AASHO ROAD TEST BASE
within the range 2000 to 4000 kPa. Taking a typical value of friction angle of 30°, a consideration of the Mohr circle of stress readily yielded

\[ c = \frac{UTS}{2} \left( \frac{1 + \sin \phi}{\cos \phi} \right) \]

whence c appeared to lie in the range 1700 to 6000 kPa. For a first analysis, the AC cohesion was taken to be 4000 kPa.

Table 7.1 summarises the material properties adopted for the initial analysis of the AASHO Road Test performance data.

7.2.2.3 Loading Parameters

In the one-dimensional model used to examine the shakedown behaviour of the AASHO Road Test pavements, a condition of plane strain load and response was assumed. Thus, the loadform could be considered as a long strip loading, modified so that it had a trapezoidal cross-section. (See section 5.3). Each set of pavements in the Road Test was subjected to repeated loading from a different load, applied by dual tyres on either single or tandem axles. The relevant details are noted in Table 7.2.

The basic tyre pressure adopted for this study was 700 kPa, primarily for the reason that this represents the upper limit to measured values of operating pressures during the Road Test (see Kent (1962)). This value was also convenient in that it is the maximum pressure permitted by Australian state road authorities, and so provides a ready link with regard to performance.

The means adopted for determining load dimensions was quite simple. Given the load on a tyre, and the inflation pressure, the contact area and hence its radius were readily calculated. For the purposes of analysis, this radius was then used as the characteristic linear dimension of the problem. A shear load, defined by \( \mu = 0.4 \), completed the definition of loadform.
### TABLE 7.1

Material Properties for AASHO Road Test Studies

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Parameters</th>
<th>Failure Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E (MPa)</td>
<td>$\nu$</td>
</tr>
<tr>
<td>Asphaltic concrete</td>
<td>5000</td>
<td>0.4</td>
</tr>
<tr>
<td>Base</td>
<td>85</td>
<td>0.3</td>
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<tr>
<td>Sub-base</td>
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</tr>
<tr>
<td>Subgrade</td>
<td>10</td>
<td>0.3</td>
</tr>
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</table>
### TABLE 7.2

Loading Parameters for AASHO Road Test Studies  
(after HRB Special Report 61C)

<table>
<thead>
<tr>
<th>AASHO Vehicle Code</th>
<th>Axle group</th>
<th>Load on axle group (kips)</th>
<th>Tandem axle spacing (av.) (ins)</th>
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</thead>
<tbody>
<tr>
<td>41</td>
<td>Single</td>
<td>18</td>
<td></td>
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<tr>
<td>42</td>
<td>Tandem</td>
<td>32</td>
<td>49</td>
</tr>
<tr>
<td>51</td>
<td>Single</td>
<td>22.4</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>Tandem</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>61</td>
<td>Single</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>Tandem</td>
<td>48</td>
<td>52</td>
</tr>
</tbody>
</table>
7.2.2.4 Performance Data

The performance of a pavement may be simply defined as the variation with time of its condition, or its ability to serve its intended function. Pavement condition was measured at two-weekly intervals throughout the duration of the experiment, and quantified as a "present serviceability index" (p), a function of roughness, rutting, cracking and patching measures. The derivation of the index relationship has been discussed in Chapter 2.

In this way, a performance record for each section of pavement was compiled, and detailed results are contained in HRB Special Report 61E. For the purpose of analysis, some simplification was necessary, and since the parameter of primary interest was pavement life, it was convenient to use as a basic measure the number of load applications required to reduce each pavement to a particular level of serviceability.

The abovementioned report contains several sets of such data. Since it was suggested that a serviceability index of p = 1.5 corresponded to the point at which most heavily-used pavements would be considered "failed" and in need of major maintenance, this value of p was adopted as the "terminal serviceability" for the purposes of shakedown analysis. The number of axle loads applied to a pavement up to the time that serviceability fell to this level, was then taken to be the "life" of that pavement.

The HRB Report on the project presented the performance data in terms of both "unweighted" and "weighted" axle applications. The seasonal weighting function used to make this adjustment was an empirical relationship employed to "allow for changing load effects in a changing environment" - effectively, to smooth the deflection histories of the test pavements. Whilst there can be no doubt that climatic effects exerted an influence upon pavement performance, the "weighting" method of allowance for such effects was purely empirical and without general application. For this reason, the life of pavements to be analysed was considered only in terms of "unweighted" axle applications, and all comparisons of results were made on the same basis.
Figure 7.2 illustrates typical performance records from the AASHO Road Test. It may be observed not only that each pavement exhibits increasing distress as trafficking continues, but also that certain pavements eventually reach a steady state, beyond which little further deterioration occurs. The process by which this steady state is reached is known as shakedown, and it is most significant that this phenomenon has been observed in such a carefully controlled full-scale experiment. On this basis, it is clear that shakedown theory is worthy of serious consideration as an approach to pavement analysis.

7.3 Shakedown Behaviour of AASHO Pavements

7.3.1 Introduction

The details of the one-dimensional shakedown analysis of a pavement have been presented in Chapter 5. It should be recalled that the analysis determines the shakedown limit for the given conditions of load, geometry and material response. This limit is in effect a factor of safety against failure by fatigue, and as such provides a measure of strength of the pavement. In this study, the shakedown limits of a large number of the AASHO test sections have been calculated, and it is shown that these results can be related in a simple manner to the observed pavement life.

7.3.2 Shakedown Analyses

Each loop in the Road Test provided up to 27 different pavement profiles and their corresponding performances, and all pavements were modelled for analysis as shown in Figure 7.3. For each pavement profile, using the material parameters as previously derived, a value of shakedown limit \( \lambda_{SD} \) was computed by means of the program LAYELLIP, which employs the method of conic sections outlined in Chapter 5. The shakedown limit \( \lambda_{SD} \) is defined as the ratio of \( V_{SD} \), the largest load for which the pavement will shake down (i.e. will not fail incrementally) to \( V_0 \), the applied load, and it may, therefore, be regarded as a factor of safety against incremental collapse: the higher the shakedown limit, the stronger the pavement.
Typical Performance Trends - AASHO Road Test

Figure 7.2

Present Serviceability Index (p)

Load Applications (10^3 axles)

End of test

Numerals indicate pavement section numbers

Shakedown

Failed pavements - Performance defined by whether shakedown occurs.

Unfailed at end of test - Performance defined by final p and top life to p = 1.5.
Max contact stress $V_o$

\[ \frac{b}{B} = 0.5 \]
\[ \mu = 0.4 \]

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>$E$</th>
<th>$\nu$</th>
<th>$C$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>Asphalitic Concrete</td>
<td>$E_1$</td>
<td>$\nu_1$</td>
<td>$C_1$</td>
<td>$\phi_1$</td>
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<tr>
<td>$T_2$</td>
<td>Base</td>
<td>$E_2$</td>
<td>$\nu_2$</td>
<td>$C_2$</td>
<td>$\phi_2$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>Subbase</td>
<td>$E_3$</td>
<td>$\nu_3$</td>
<td>$C_3$</td>
<td>$\phi_3$</td>
</tr>
<tr>
<td></td>
<td>Subgrade</td>
<td>$E_4$</td>
<td>$\nu_4$</td>
<td>$C_4$</td>
<td>$\phi_4$</td>
</tr>
</tbody>
</table>

**FIGURE 7.3**

PAVEMENT AND LOADFORM FOR SINGLE-AXLE ANALYSES
For each pavement, then, a measured value of life may be plotted against a calculated value of shakedown limit. In Figure 7.4 is shown such a plot for one set of pavements - the twenty-two loaded with a 30 kip single axle. The Figure clearly illustrates that there exists a trend towards better pavement performance (increased life) with an increase in the calculated shakedown limit. In addition, those pavements which shook down during the Road Test are characterised by shakedown limits over 1.0 - that is, calculation suggests that they should indeed shake down.

These encouraging results are confirmed by subsequent analyses for other axle loads, as detailed in Table 7.3. Figure 7.5 shows collected results for all six test loops, with the line $\lambda_{SD} = 1$ showing a clear division between pavements which shook down, and those which did not. A change of format (Figure 7.6) further illustrates this point, and also demonstrates the similarity of these results to the typical S-N curve characteristic of fatigue studies. Indeed, considering the variation inherent in pavement life and materials, the Figure shows a most pleasing consistency with other studies of life under fatigue loading.

### 7.3.3 Prediction of Pavement Life

The S-N form of presentation used in fatigue studies further enables the relationship between shakedown limit and Road Test performance to be examined statistically. For the data of Figure 7.6, regression analysis was used to fit an S-shaped curve of the form

$$ Y = C + DX $$

where

$$ Y = \log_{10} (\text{pavement life}) $$

$$ X = \tan ( \text{function} ( x = \frac{V}{V_{SD}} )) $$

$C, D$ are constants.

If the dimensionless load $x = \frac{V}{V_{SD}}$ is assumed to range between lower and upper limits $A$ and $B$ respectively, then it is clear that a simple form of the relationship may be modelled by

$$ Y = C + D \tan \left( \frac{\pi}{2} \cdot \frac{B+A-2x}{B-A} \right) $$
PAVEMENTS EXPECTED TO FAIL INCREMENTALLY TO SHAKE DOWN INCREMENTALLY

Solid marks indicate SHAKEDOWN observed

FIGURE 7.4

PAVEMENT SHAKEDOWN LIMITS AND ROAD TEST PERFORMANCE
(30 KIP SINGLE AXLE LOAD SET)
TABLE 7.3

AASHO Road Test - Performance and Shakedown Analysis
(Performance measured by $10^3$ axle applications to $p = 1.5$, or (bracketed) $p$ at end of test. SD Indicates Shakedown observed)

<table>
<thead>
<tr>
<th>Profile (Inches AC-Base-Sub-base)</th>
<th>Axle Load 18S</th>
<th>Axle Load 32T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Road Test Performance</td>
<td>Shakedown Limit</td>
</tr>
<tr>
<td>3-0-4</td>
<td>2</td>
<td>0.41</td>
</tr>
<tr>
<td>3-0-8</td>
<td>72</td>
<td>0.49</td>
</tr>
<tr>
<td>3-0-12</td>
<td>82, 115</td>
<td>0.61</td>
</tr>
<tr>
<td>3-3-4</td>
<td>74</td>
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<td>3-3-8</td>
<td>82</td>
<td>0.59</td>
</tr>
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<td>3-3-12</td>
<td>583</td>
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<tr>
<td>3-6-4</td>
<td>80</td>
<td>0.58</td>
</tr>
<tr>
<td>3-6-8</td>
<td>92</td>
<td>0.71</td>
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<tr>
<td>3-6-12</td>
<td>(1.6)</td>
<td>0.84</td>
</tr>
<tr>
<td>4-0-4</td>
<td>78</td>
<td>0.64</td>
</tr>
<tr>
<td>4-0-8</td>
<td>107</td>
<td>0.70</td>
</tr>
<tr>
<td>4-0-12</td>
<td>426</td>
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</tr>
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<td>4-3-4</td>
<td>87</td>
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<td>100, 111</td>
<td>0.77</td>
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<td>1110</td>
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<td>81</td>
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<tr>
<td>3-9-8</td>
<td>87</td>
<td>0.75</td>
</tr>
<tr>
<td>3-9-12</td>
<td>(SD 2.9)</td>
<td>0.85</td>
</tr>
<tr>
<td>4-3-4</td>
<td>71</td>
<td>0.65</td>
</tr>
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<td>----------------------------------</td>
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<td>----------</td>
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<tr>
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<td>Road Test Performance</td>
<td>Shakedown Limit</td>
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<tr>
<td>4-3-12</td>
<td>373</td>
<td>0.69</td>
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<td>4-6-8</td>
<td>82</td>
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<td>4-6-12</td>
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<td>5-6-12</td>
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<td>6-6-8</td>
<td>106</td>
<td>0.91</td>
</tr>
<tr>
<td>6-6-12</td>
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</tr>
<tr>
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<td>(SD 3.2)</td>
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</tr>
<tr>
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<td>624, (2.1)</td>
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</tr>
<tr>
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<td>(SD 2.8)</td>
<td>1.02</td>
</tr>
<tr>
<td>6-9-16</td>
<td>(SD 2-7)</td>
<td>1.07</td>
</tr>
</tbody>
</table>
Figure 7.5

PAVEMENT SHAKE DOWN LIMITS AND ROAD TEST PERFORMANCE
(all six load sets)
KEY TO LOAD GROUPS

- 18 S, 32 T
- 22 S, 40 T
- 30 S, 48 T

- Solid marks indicate shakedown has occurred

80% confidence limits for prediction of life from shakedown limit

FIGURE 7.6

PAVEMENT PERFORMANCE RELATED TO LOAD RATIO V/V_{sd}
Linear regression studies showed that for limits A = 0.67, B = 2.6 the correlation index $R^2$ was approximately 0.53. This index, given by

$$R^2 = \frac{\frac{1}{n} \sum (x_i y_i) - \left(\frac{1}{n} \sum x_i\right)\left(\frac{1}{n} \sum y_i\right)}{\left[\frac{1}{n} \sum (x_i^2) - \left(\frac{1}{n} \sum x_i\right)^2\right]\left[\frac{1}{n} \sum (y_i^2) - \left(\frac{1}{n} \sum y_i\right)^2\right]}$$

is the square of the coefficient of correlation, and may be taken as a measure of "the ratio of reduction in sum of squares of deviation attributable to taking account of x linearly rather than ignoring it" (Mendenhall (1968)). It, therefore, provides more insight into the strength of the relationship between predicted and actual performance than does the correlation coefficient, and in the present case the computed value indicates that a statistically significant relationship exists between shakedown limit and pavement life.

Of primary interest in a study of this type must be the ability of a procedure to predict the life of a given pavement. It is appropriate, therefore, to examine the confidence limits on pavement life which may be associated with a calculated value of shakedown limit.

Standard statistical methods (e.g. Walpole and Myers (1972), Mendenhall (1968)) may be used to show that, for the linear relationship

$$Y = C + DX + \text{Error}$$

the confidence limits for a predicted value of $Y$ at a given value of $X = X_0$ are:

$$|Y - (C + DX_0)| < t_{\alpha/2} \frac{s}{\sqrt{n}} \sqrt{1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{S_{XX}}}$$

where $t_{\alpha/2}$ is associated with a $(1-\alpha)100\%$ confidence interval and $\nu = n-2$ degrees of freedom

$$s^2 = (S_{YY} - 2C_{XY} + C^2 S_{XX}) / (n-2)$$
\[ S_{XX} = \frac{n}{n} (X_1 - \bar{X})^2 \]
\[ S_{XY} = \frac{n}{n} (X_1 - \bar{X})(Y_1 - \bar{Y}) \]
\[ S_{YY} = \frac{n}{n} (Y_1 - \bar{Y})^2 \]

\[ n = \text{number of observations.} \]

On Figure 7.6 are illustrated the curve for mean pavement life and the 80% confidence interval \((\alpha = 0.2, t = 1.282)\). It is then evident that at a typical value of shakedown limit of, say, \(\lambda_{SD} = 0.75\) \((V_{SD} = 1/\lambda_{SD} = 1.33)\) the performance of an AASHO pavement may be predicted to be between 52 and 337 thousand axle groups. The mean predicted life is 133 thousand axle groups.

7.3.4 Comparisons with Other Approaches to Prediction of Pavement Life

7.3.4.1 AASHO Analysis of Test Data

The nature of the analysis undertaken by the Highway Research Board was considered in Chapter 2. The procedure adopted resulted in a relationship between \(W\), the number of load applications to failure (\(p = 1.5\)), and the thicknesses of surfacing, base and sub-base \((D_1, D_2\) and \(D_3\) inches respectively) of the form

\[ W = \frac{A_2 (D + 1)^{A_1} L_2^{A_3}}{(L_1 + L_2)^{A_2}} \]

where \(D = a_1 D_1 + a_2 D_2 + a_3 D_3\)

\(L_1 = \) nominal axle group load in kips

\(L_2 = 1\) (single axles) or 2 (tandem axles)

and \(A_1, A_2, A_3, a_1, a_2, a_3\) are coefficients to be determined.
A series of regression analyses was then used to determine the values of the various coefficients. The "goodness of fit" of the resulting equation may be measured by the correlation indices, as previously defined. For the full set of pavements, these were

\[
R^2 = \begin{cases} 
0.48 & \text{for unweighted axle applications} \\
0.70 & \text{for weighted axle applications}
\end{cases}
\]

The use of unweighted axle applications for the shakedown analyses has already been noted and discussed, and for these analyses it may be recalled that a correlation index of 0.53 was obtained. That is, an equally significant correlation was observed using either approach to the data.

Two further points are worthy of note. Firstly, the shakedown approach has demonstrated that different wheel loads may readily be accounted for by the one form of analysis, so that no load-equivalence approximations are entailed. Secondly, the foregoing statistical comparison deals only with those pavements which failed: it has already been demonstrated that shakedown analysis is also capable of predicting those pavements which instead attain a steady state.

7.3.4.2 Current Australian Practice

As an example of a recently reformulated design procedure for Australian conditions, the Department of Main Roads, New South Wales (DMR, NSW) Form No. 76 (1980) - Pavement Thickness Design - was considered to be most suitable. Based strongly upon the National Association of Australian State Road Authorities (NAASRA) Interim Guide to Pavement Thickness Design, it is representative of Australian approaches and a convenient guide to current practice. An algorithm was developed using the DMR Form 76 relationships, in order to provide another set of predictions of the life of the AASHO pavements. These predictions could then be compared with measured values of life.
The AASHO Road Test materials data suggested the following classification of materials for the purposes of DMR Form 76:

- **Subgrade** - CBR = 3
- **Sub-base** - Graded Macadam Crushed Rock (GMB 20)
- **Base** - Graded Macadam Crushed Rock (GMB 40)

Figure 7.7 shows the key elements of the DMR design procedure. Essentially, the design process develops a relationship between required pavement life and the thickness design necessary to achieve that life. The data, embodied in various formulae and graphs, was readily used to reverse the procedure, so that knowing the pavement thickness design and materials characteristics the expected life could be calculated. A program DMRLIFE was written to perform these calculations. An adjustment to account for initial and final pavement roughness (values selected as appropriate to AASHO test conditions were $R_1 = 60$, $R_2 = 150$ counts/km) was also included in the program, as was the Form 76 deflection check aimed at ensuring adequate pavement stiffness for the required traffic life.

The resulting procedure was applied to each of the AASHO pavement structures, with an allowance for different axle loading made by means of the "fourth power relationship" employed by Form 76. Figure 7.8 shows the actual pavement life plotted against the predicted value, and it is evident that there is little correlation. This is clearly confirmed by a correlation index ($R^2$) of 0.33, combined with a line of best fit whose slope, at 0.12, differs markedly from the line of equality.

### 7.4 Conclusion

A method of pavement analysis based upon shakedown theory has been presented.

In the interpretation of the results of an extensive Road Test conducted by the Highway Research Board in the late 1950's, the method has been compared with two other approaches, one American and one Australian. The primary basis of comparison - the ability of a procedure to predict the performance of a pavement - has revealed that the shakedown method is at least as effective as other routines.
ASSESSMENT OF DESIGN SUBGRADE C.B.R.  

PREDICTION OF DESIGN TRAFFIC  

DETERMINATION OF BASIC PAVEMENT THICKNESS  

OPTIONAL ADJUSTMENT FOR LEVEL OF PERFORMANCE (ROUGHNESS)  

AVAILABLE PAVEMENT MATERIAL  

SELECTION OF TENTATIVE PAVEMENT STRUCTURE  

COVER MATERIAL FOR EACH PAVEMENT MATERIAL ADEQUATE?  

YES: ENSURE MINIMUM BASE THICKNESS SATISFIED  

ADOPTION OF PAVEMENT DESIGN  

FIGURE 7-7  

BASIC FLEXIBLE PAVEMENT DESIGN PROCEDURE  
D.M.R. Form 76
FIGURE 7-8
PREDICTIONS OF LIFE OF AASHO
PAVEMENTS USING DMR FORM 76
The shakedown procedure is, however, applicable to pavements other than those which reached failure. In the Road Test, shakedown was observed in a number of test sections, and it is clear that this may also be satisfactorily predicted. The procedure, then, is based firmly in the observed long-term behaviour of road pavements, whilst remaining conceptually simple. It would, therefore, appear to deserve further attention.
CHAPTER EIGHT

A FIELD STUDY OF
PAVEMENT PERFORMANCE
8.1 Introduction

Of the considerable resources, both technical and financial, assigned to road construction and maintenance, surprisingly little is generally directed to long-term studies in order to validate procedures of design. Perhaps the long structural lives encountered in the case of pavements, combined with the understandable concern of road authorities with "what has to be built" rather than "what was built", acts to divert attention from long-term performance. In any case, there is a pressing need for studies to provide data by which the prediction of pavement life may be improved.

One of the few comprehensive performance studies, the AASHO Road Test, has been considered in some detail in Chapter 7. The analysis of data from this study indicated that the shakedown limit of a pavement bears a strong relationship to its long-term behaviour, and hence to its life under traffic.

This chapter outlines a further small-scale study, dealing with a number of pavements in the Sydney region. A range of failed pavements (covering a considerable variety of materials and depth profiles) has been sampled, tested and analysed. The study was conceived as a means of validating the use of shakedown methods for the prediction of life under normal service conditions. In its implementation, the study was somewhat restricted by limitations of time and resources (both manpower and equipment); in some respects it also served as a means of discovering how much, or how little, information existed for the determination of the elusive parameter of pavement life.

8.2 Required Data

The traditional and still widely employed tests of material suitability for pavement applications are largely based upon grading analysis and determination of moisture relations (LL/PL/PI). Commonly, this information is supplemented by results of unconfined compressive strength and maximum dry density tests, with subgrade materials generally being assessed by California Bearing Ratio (CBR).
For an analysis which involves the evaluation of stresses within pavements, and the comparison of these stress states with limiting states ("yield conditions"), an approach more closely related to current theories of pavement and material response is called for. The assumptions regarding material behaviour in the study which follows were:

(a) linear-elastic, perfectly plastic material response, defined by modulus (E) and Poisson's ratio (ν), and

(b) a yield criterion of the Mohr-Coulomb type, defined by the constants angle of friction (ϕ) and cohesion (c),

and the Texas Triaxial procedure, outlined in Appendix 8A, was used to estimate values of E, c and ϕ. Typical values of ν were adopted according to material type, as considered in Chapter 3.

It is important to note the nature of the test results, and the relation they bear to the above assumptions of behaviour. In Figure 3.1 are shown typical load-deformation curves obtained from tests of sampled materials. The curves (a) may be considered representative of the great majority of materials tested; curves (b) are of a form which applied at two sites with particularly poor subgrades. From the Figure it appears that in general the assumption of linear elasticity - perfect plasticity is satisfactory, with an "average modulus" computed from the linear portions of the load-deformation traces (as outlined in Appendix 8A). For the isolated cases where subgrade angle of friction was particularly low, the model does not fit quite as well; it has been adopted, however, as a first approximation to the true response.

Also satisfactory for typical materials is the assumption of a linear Mohr-Coulomb yield criterion. This is most clearly evident from the Texas Triaxial procedure, which calls for a linear correlation coefficient r exceeding 0.990 for the sets of measurements

\[
\frac{\sigma_1 + \sigma_3}{2}, \frac{\sigma_1 - \sigma_3}{2}
\]

The line of best fit through these points (representing points of maximum shear stress) would certainly not meet this requirement if any marked deviation from linearity occurred. A typical envelope is shown, with stress circles, in Figure 8.2.
**FIGURE 8-1**

REPRESENTATIVE LOAD DEFLECTION TRACES FROM TEXAS TRIAXIAL TESTS

(a) Left: Fine-crushed rock base, Camden.
(b) Above: Clay subgrade, Kingswood.

Samples cylindrical, height 200 mm, diameter 150 mm.
**Figure 8.2**

Representative Mohr circles at failure, and derived yield criterion

The above assumptions regarding material response, whilst evidently appropriate for most materials used in base and lower courses, were also applied to surface layers of asphaltic concrete (AC). In the determination of values for the parameters, however, difficulties were encountered because of the typically high stiffness and strength of the samples, and some changes of approach were called for. This will be considered in more detail in Section 8.4.3.

The remaining information required in order that a relationship be established between the structural properties of a pavement and its performance is an estimate of its life under traffic. A great many factors exercise an influence over pavement life, and at the least some details of traffic volumes, traffic composition, and traffic distribution (e.g. lane usage) need to be determined. In view of the considerable variability of these items of data, it is also appropriate that some estimates of errors and their effects also be made. The process of traffic life estimation will be treated in detail in Section 8.5.

8.3 Pavement Selection and Sampling

The study was conceived with the object of investigating the validity of shakedown performance as a general measure of pavement life. Consequently, the only restriction placed upon pavement type was that the structure should not include concrete or cement-bound layers. It was felt that the high stiffness and very low "ductility" of these materials under tension introduced a mode of failure which was outside the immediate scope of the shakedown analysis presented in earlier chapters. Beyond this limitation few other requirements existed, and as a result a wide variety of profiles were sampled, in which materials ranged from quite stiff soil-bound macadams placed some 40 or more years ago, to very weak moist subgrade clays.

Nor was the age of the pavement of much concern: the main feature sought was the onset of failure, indicating that the structure's serviceable life was approximately at an end. A "user" definition of failure was employed, whereby the author assessed the pavement "by eye", taking account of such factors as cracking, wheelpath rutting, and the amount of patching evident. In this way, the pavement was deemed to have "failed" when the riding comfort it
afforded the user fell below an acceptable level. Also important was the need to identify cases where failure had occurred as a result of factors outside of simple traffic loading, such as design detailing or later disturbance of the structural integrity due to trenching; such cases were avoided as being again beyond the scope of the study.

Once an area of failed pavement suitable for investigation had been identified, a sample was taken. This generally came from between the wheel paths, where the materials might be expected to be less distressed and so approximate more closely the original materials of construction. In this way it was hoped that a relationship might be developed between the material properties at the time of construction, and the long-term behaviour of the pavement.

Plate 8.1 shows sampling in progress at one site. The procedure adopted began with the marking of the selected location, generally about one metre square. The surfacing of asphaltic concrete was removed using a pneumatic drill, a suitable quantity of AC being kept for testing. The soil layers were then removed one by one, care being taken to avoid mixing of different layers, and about 60 kg of each layer was retained. Once each layer had been sampled, and further exploration (generally another 500 mm or so) suggested no deeper changes of materials, the layer thicknesses were measured at 3 or 4 locations within the hole. Finally, the road was restored by compaction into the hole of any unused pavement material, followed by the addition of cold-mixed asphalt to replace the sampled volume.

8.4 Materials Testing

8.4.1 Strength Parameters

The Texas Triaxial Compression Test, fully detailed in Appendix 8A, was used for the determination of the shearing resistance of samples. The test consists of "applying an axial load to cylindrical specimens supported by various known lateral pressures until failure occurs". For this study, four lateral pressures (0, 45, 90 and 140 kPa) were employed. Testing to failure at each confining pressure produced a set of corresponding corrected maximum axial stresses, from which could be drawn the Mohr circles of stress at failure. The common tangent to these circles was constructed (Figure 8.2), allowing values to be determined for cohesion and friction angle.
8.4.3 Stiffness Parameters

A value of average compressive modulus was also obtained from the equations for the series of samples examined. Preliminary results from shake-down analyses of some of the samples were substantially similar, indicating that the test might be a suitable method for selecting the type of asphalt mixtures. Results for the two samples tested appear to be quite consistent with each other, and with expected values of c and d, when plotted in this manner. The two simple tests, therefore, at this point provide only possible means of estimating the asphalt strength parameters. As a result of the experiment, it was decided to undertake further tests using the apparatus available.

PLATES 8.1

Selection and Sampling of Pavement
Location: Windsor Road, Kellyville
Date: 26 November 1981
3.4.2 Stiffness Parameters

A value of average compressive modulus was also obtained from the stress-strain curves of the Texas tests. For a curve with a clear straight line portion, the compressive modulus was simply computed as the slope of that portion; where no straight line section existed, a secant modulus spanning a strain range of zero to 0.75% strain was generally used. The average modulus was then given by the arithmetic mean of the individual moduli so determined.

8.4.3 Parameters for Asphaltic Concrete (AC)

Particular problems were encountered when the testing procedure above was applied to the samples of AC. Initial tests showed that the 150 mm triaxial apparatus was not strong enough to test the asphalt to failure, even under zero lateral pressure, so a change of approach became necessary.

Preliminary results from shakedown analyses of some of the sampled pavements indicated that the predictions of pavement life were fairly sensitive to AC parameters, and an investigation into suitable ways of estimating these was commenced. It was envisaged that standard tests for unconfined compressive strength (UCS) and indirect tensile strength (ITS) (Appendix 8B) might provide a suitable basis for this estimation.

8.4.3.1 Initial ITS and UCS Tests

A series of tests was first run on a standard mix of AC, comprising both compressive and tensile tests on 10 mm and 20 mm mixes. Table 8.1 sets out the results, and the mean values for each material are plotted in Mohr circle form in Figure 8.3. As the Figure clearly reveals, the two sets of tests - tensile and compressive - appear to be quite consistent with each other, and with expected values of c and $\phi$, when plotted in this manner. The two simple tests, therefore, at this point provided a possible means of estimating the asphalt strength parameters; in an effort to confirm this possibility, it was decided to undertake a series of triaxial tests using the stronger apparatus available at the University of Sydney.
### TABLE 8.1

Results of Initial Tensile and Compressive Strength Tests on Asphaltic Concrete

<table>
<thead>
<tr>
<th>Mix Type</th>
<th>Diameter (mm)</th>
<th>Length (mm)</th>
<th>Density (t/m³)</th>
<th>UCS (MPa)</th>
<th>ITS (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC 10</td>
<td>101</td>
<td>194</td>
<td>2.26</td>
<td>2.30</td>
<td>3.15</td>
</tr>
<tr>
<td></td>
<td>101</td>
<td>194</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>101</td>
<td>62</td>
<td>2.33</td>
<td></td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td>101</td>
<td>63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC 20</td>
<td>101</td>
<td>194</td>
<td>2.32</td>
<td>3.82</td>
<td>3.35</td>
</tr>
</tbody>
</table>
FIGURE 8.3

MOHR CIRCLES (MEAN) FOR INITIAL TENSILE AND COMPRESSIVE TESTS ON AC
8.4.3.2 Triaxial Testing of AC

Using a 100 mm diameter triaxial cell, further samples of both AC 10 and AC 20 were tested to failure under lateral pressures of up to about 1 MPa (a limitation imposed by the machine's maximum axial load of approximately 70 kN). Figure 8.4 records the Mohr circles of stress at failure, along with the corresponding UCS and ITS results, for the two mixes, and a pleasing consistency between confined and unconfined results is clearly evident. Further, it appears that a reasonable approximation to the Mohr–Coulomb failure line may be obtained using the tangent to the failure circles generated by the UCS and ITS tests. Such an approximation is particularly convenient in view of the fact that typically normal stresses within asphaltic layers are unlikely to exceed 1 to 2 MPa.

8.4.3.3 Difficulties with the ITS Tests

Despite the apparent ease with which an approximate AC yield condition has been derived, there do exist certain difficulties of interpretation with respect to the Indirect Tensile Test.

The analysis of Frocht (1948), for a disc under diametrically opposite concentrated loads (Figure 8.5(a)) showed that the loading induced a uniform tension across the line of load, of magnitude

\[ \sigma_X = \frac{-2P}{\pi td} \] (tensile)

where

\( P \) = applied compressive load
\( t \) = thickness of disc
\( d \) = diameter of disc.

The analysis for concentrated loads, however, also yields stresses

\[ \sigma_Y(x=0) = \frac{2P}{\pi t} \left( \frac{2}{d-2y} + \frac{2}{d+2y} - \frac{1}{d} \right) \]

\[ \tau_{XY}(x=0) = 0 \]

from which it is clear that \((\sigma_Y - \sigma_X)\) becomes infinitely large at the points of loading.
RESULTS OF STRENGTH TESTS ON AC
(ITS, UCS AND TRIAXIAL)
FIGURE 8.5
INDIRECT TENSILE TEST
LOADING AND STRESSES ON AXIS
(a) Concentrated Load
(b) Distributed Load
Hondros (1959) attempted to remove this anomaly by considering a load distributed over a small section of the surface, and in doing so produced the analysis appropriate to the conditions of the ITS test. For the case shown in Figure 8.5(b), the stresses along the line of loading are given by

\[
\sigma_x(x=0) = \frac{-2P}{\pi a t} \left[ \frac{(1-\rho^2)\sin 2\alpha}{(1-2\rho^2 \cos 2\alpha + \rho^4)} - \tan^{-1} \left( \frac{1+\rho^2}{1-\rho^2} \tan \alpha \right) \right]
\]

\[
\sigma_y(x=0) = \frac{2P}{\pi a t} \left[ \frac{(1-\rho^2)\sin 2\alpha}{(1-2\rho^2 \cos 2\alpha + \rho^4)} + \tan^{-1} \left( \frac{1+\rho^2}{1-\rho^2} \tan \alpha \right) \right]
\]

\[
\tau_{xy}(x=0) = 0
\]

where \( P \) = applied compressive load
\( a \) = width of load
\( \alpha \) = angle subtended at centre by load
\( t \) = thickness of disc
\( \rho = \frac{r}{R} \)

In Figure 8.6, the variation of stresses \( \sigma_x, \sigma_y \) with \( y \) is presented in dimensionless form, for a typical value of \( \alpha = 0.125 \) radians. In contrast to the case of concentrated loading, it is here found that \( (\sigma_y - \sigma_x) \) is at all times defined. Thus the theoretical first yield load is now non-zero, and it is possible further to derive that envelope containing all the Mohr circles associated with the stress states of Figure 8.6. The envelope of Mohr circles is given in Figure 8.7.

This envelope encloses all the stress states which theoretically occur along the plane of failure of the specimen. It would, therefore, be reasonable to expect that at the time of yielding the envelope should be enclosed by, and tangential to, the Mohr-Coulomb failure line, since one point within the specimen has just reached first yield.
STRESSES ON VERTICAL AXIS IN INDIRECT TENSILE TEST (DISTRIBUTED LOADING, d = 8a)
FIGURE 8.7
ENVOLPE OF MOHR CIRCLES IN INDIRECT TENSILE TEST (DISTRIBUTED LOAD, d = 8a)
The superposition of the envelope upon previous UCS and triaxial results (Figure 8.8), however, highlights the inconsistency: whilst the single circle through the origin and ITS was previously used with satisfactory results, the envelope is considerably larger than would be expected from the compressive test results.

8.4.3.4 Adopted Approach to AC Testing

The difficulty outlined in the previous section was not able to be resolved. Considerations of strain rate effects on apparent strength yielded little improvement, and the problem was eventually put aside, its solution being beyond the scope of the present investigation.

The strength and stiffness parameters for the asphaltic concrete sampled in the field were estimated using the simplified approach derived previously: the material, after remoulding to produce test specimens of suitable dimensions, was tested to determine its unconfined compressive and indirect tensile strengths. Testing is shown in Plates 8.2 and 8.3, while Appendix 8B details the test methods. Using the Mohr circles through the origin and each of these data points, it was possible to obtain the tangent and hence measure cohesion and friction angle. Alternatively, the following formulae may be used:

\[\text{cohesion } c = \frac{1}{2}\sqrt{C_0 T_0}\]

\[\text{friction } \phi = \sin^{-1}\left(\frac{C_0 - T_0}{C_0 + T_0}\right)\]

where \(C_0 = \text{UCS}\)

\(T_0 = \text{ITS}\)

Values of elastic modulus were obtained from the linear portions of the stress-strain curves for the compressive tests, and as shown in Figure 8.9 this proved to be quite reliable as the asphalt exhibited very little non-linearity during the course of testing. For a number of sites, sufficient material was available to perform more than one set of tests, and the repeatable nature of these measurements lent further support to their use.
PLATE 8.2

Unconfined Compression Test

PLATE 8.3

Indirect Tensile Test
PLATE 8.3
Indirect Tensile Test
FIGURE 8.8

SUPERPOSITION OF ITS ENVELOPE
UPON UCS AND TRIAXIAL RESULTS
FIGURE 8.9

REPRESENTATIVE LOAD DEFLECTION TRACES FOR ASPHALT (UNCONFINED COMPRESSION)
8.5 Traffic Life and its Determination

In general, road authorities have chosen to express pavement lives in terms of the number of passes of particular loads. Where sufficient data exists, the loading may be defined by an axle-load spectrum, describing the frequency distribution of axle-loads. More commonly, however, a particular axle configuration and loading may be expressed as an equivalent number of standard axle loads, the equivalence being on the basis of pavement damage, strain at a particular location in a typical pavement, or some other measure. In this State, for example, a standard axle is taken to be a dual-tyred single axle bearing a load of 8.2 tonnes. The basis for equivalence is the "fourth power relation" derived from the AASHO and other road tests, which makes the empirical approximation that the life of a pavement is inversely proportional to the fourth power of the axle load traversing it. Thus, a dual-tyred single axle carrying 9 tonnes would be said to be equivalent to \(\left(\frac{9}{8.2}\right)^4 = 1.45\) standard axles. Also on the basis of road tests, relationships have been established between different axle configurations. In New South Wales, for example, 13.6 t on a dual-tyred tandem axle is regarded as being equivalent in damage to the 8.2 t dual/single axle: with a load of 16 t, then, the dual/tandem group would be equivalent to 1.92 standard axles.

For consistency, traffic lives for the pavements considered in this chapter have been expressed in terms of equivalent standard axles (ESAs). The data used for this purpose were:

(a) Annual Average Daily Traffic (AADT) records held by the Department of Main Roads, N.S.W. These are the results of long-term counts, and represent the total number of vehicles of all types passing a given point during one year, divided by 365 to give the daily average for that year. These counts are conducted on a large number of main roads at intervals of about four years. Figure 8.10 illustrates the information available from such records.
Site sampled: Pennant Hills Rd at Pennant Hills East of Trebor Rd.

Key: ○ East of Boundary Rd.
△ At Railway Bridge

FIGURE 8·10
TRAFFIC VOLUME DATA FOR A TYPICAL SITE
(b) Heavy Vehicle Counts, defined as
\[ C = \frac{\text{Number of commercial vehicles in chosen direction}}{\text{Number of vehicles in chosen direction}}. \]
At some locations, such counts are performed regularly by road authorities, but in most cases the determination of \( C \) has been by the author's manual counts. \( C \) yields the proportion of commercial vehicles (those with dual tyres or more than two axles) in the traffic stream.

(c) Lane distribution counts, defined by
\[ L = \frac{\text{Number of commercial vehicles in chosen lane}}{\text{Number of commercial vehicles in chosen direction}}. \]
\( L \) represents the proportion of heavy vehicle traffic using a particular lane, and has been determined at almost all sampled sites by manual counts by the author.

(d) Traffic spectrum data, defined by
\[ F_1 = \text{Number of ESA's per commercial vehicle}. \]
The value of \( F_1 \) has been presented (Department of Main Roads, N.S.W. (1980)) as a function of road classification, and represents an aggregation of a large amount of traffic spectrum survey data. As such it must be regarded as being only approximately correct for any particular site.

Using these data, the volume of traffic passing a given point during the life of a pavement is given by
\[ V = \int \text{(AADT)} \, dt \]
Then for a two-way road, the life of the sampled pavement (given lane, given direction) may be estimated by
\[ T = \frac{1}{2} F_1 VCL \quad (\text{ESA's}). \]

All these quantities are subject to error, for which some allowance should be made. In Table 8.2 are set out the sources of error, and representative magnitudes of relative error. In this way, upper and lower bounds on the true life of the pavement may be estimated: the value adopted as a reasonable measure of relative error in \( T \) was \( \pm 0.4 \), and the size of error was further reduced at a number of sites by conducting supplementary surveys of traffic characteristics.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Sources of Error</th>
<th>Range of Values in Study</th>
<th>Estimate of Relative Error in Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>V - Traffic Volume in One Direction</td>
<td>Year of construction. Sampling error in AADT data.</td>
<td>10 - 210 (10^6 ESA's)</td>
<td>0.05</td>
</tr>
<tr>
<td>C - Proportion of Commercial Vehicles</td>
<td>Sampling error at site.</td>
<td>0.06 - 0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>L - Lane Usage by Commercial Vehicles</td>
<td>Sampling error at site.</td>
<td>0.20 - 1</td>
<td>0.05</td>
</tr>
<tr>
<td>F - Number of ESA's per Commercial Vehicle</td>
<td>Sampling error in traffic survey data. Classification of road function. Local effects on traffic spectrum.</td>
<td>0.8 - 1.5</td>
<td>0.20</td>
</tr>
<tr>
<td>T - Life in ESA's of Sampled Lane</td>
<td>Above effects aggregated.</td>
<td>0.6 - 13.5 (10^6 ESA's)</td>
<td>0.40</td>
</tr>
</tbody>
</table>
8.6 Test Results and Analysis

The results of tests on the sampled materials, conducted as outlined in Section 8.4, are set out in Table 8.3. The values determined from traffic surveys and available data follow in Table 8.4, along with the calculated pavement lives.

The programme of sampling was undertaken with a view to testing the validity of using pavement shakedown behaviour to estimate life. Using the material properties and profiles already detailed (Table 8.3), a one-dimensional analysis was, therefore, performed for each pavement, and the results presented in Table 8.5. It should be noted that where material properties were estimated only, the variation allowed in the value of parameters was reflected in some variation in shakedown limit.

Figure 8.11 shows the data of Table 8.5 plotted to relate life to shakedown limit. The errors previously determined are represented by error bars or ellipses as appropriate, and the data from the AASHO Road Test (Chapter 7) included for completeness. Whilst the expected magnitudes of some errors are quite large, as might be expected for a field study of this type, the graph serves to demonstrate two important points. Firstly, it is clear that the data of this study are consistent with those of the much more controlled AASHO Test. Secondly, the Figure allows an estimate of the relationship between mean life and shakedown limit to be made, and further permits an approximate design curve (lower bound to life) to be deduced. Despite the spread of data, it appears that this curve may be estimated with some confidence.

8.7 Conclusions

This study has attempted to examine the relationship between the predictions of shakedown theory for pavement structures, and the life of a number of local pavements under normal traffic conditions. In so doing, it has demonstrated that

(a) pavement materials may be tested in the laboratory to yield useful information on both stiffness and strength properties;
<table>
<thead>
<tr>
<th>Site No.</th>
<th>Location</th>
<th>Material Description</th>
<th>Layer Thickness (mm)</th>
<th>Elastic Modulus (kPa)</th>
<th>Poisson's Ratio (assumed)</th>
<th>Cohesion (kPa)</th>
<th>Friction Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F4 Freeway</td>
<td>Open grade asphaltic concrete (AC)</td>
<td>30</td>
<td>839</td>
<td>0.4</td>
<td>1300</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AC, 40 mm aggregate</td>
<td>120</td>
<td>225</td>
<td>0.4</td>
<td>600</td>
<td>25</td>
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### TABLE 8.5

Results of Analysis for Sampled Sites

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<td>9</td>
<td>Pennant Hills</td>
<td>6.7 ± 2.4</td>
<td>1.09</td>
</tr>
<tr>
<td>10</td>
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<td>3.9 ± 1.4</td>
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</tr>
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<td>Villawood</td>
<td>6.9 ± 2.4</td>
<td>1.41</td>
</tr>
<tr>
<td>12</td>
<td>Camden</td>
<td>1.7 ± 0.6</td>
<td>0.56</td>
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<td>Mittagong</td>
<td>6.8 ± 3.7</td>
<td>1.15 ± 0.30</td>
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<tr>
<td>14</td>
<td>Sutton Forest</td>
<td>0.6 ± 0.4</td>
<td>0.92 ± 0.05</td>
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<tr>
<td>15</td>
<td>Kiama</td>
<td>2.3 ± 1.6</td>
<td>1.04 ± 0.12</td>
</tr>
<tr>
<td>16</td>
<td>TR 95</td>
<td>1.7 ± 0.9</td>
<td>1.30 ± 0.05</td>
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<tr>
<td>17</td>
<td>Chullora</td>
<td>5.2 ± 1.8</td>
<td>1.97</td>
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<tr>
<td>18</td>
<td>Casula</td>
<td>8.3 ± 2.9</td>
<td>1.31</td>
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</tbody>
</table>
Approximate mean and confidence limits for prediction of life from shakedown limit

Key:
- AASHO test
- Field tests

FIGURE 8.11

PAVEMENT PERFORMANCE RELATED TO SHAKE DOWN LIMIT
(b) the traffic life of a pavement may be estimated with some accuracy from readily available information. Data on heavy vehicle percentages and lane usage, both very important influences on life, are, however, rather limited and could benefit from further study;

(c) the shakedown limit, obtained by the approximate (one-dimensional) means detailed in Chapter 5, shows considerable promise as a means of estimating pavement life. Not only is the analytical approach consistent with features of normal pavement performance, but this study has shown that within the estimated limits of variation in pavement life and materials it may be used to predict the minimum expected life of a flexible pavement structure.
THE TEXAS TRIAXIAL TEST

The test consists of applying an axial load to cylindrical specimens supported by various known lateral pressures until failure occurs. The combinations of axial and lateral stresses required to induce failure in the specimens may be represented diagramatically as Mohr stress circles, and the common tangent to these circles is taken to define the failure criterion of the subject material. A copy of the method of testing and calculation employed (that used by the New South Wales Department of Main Roads - adapted from the Texas Highway Department's method) appears overleaf.
1. **SCOPE**

This test method describes the procedure for the determination of the shearing resistance of soils, soil aggregate mixtures and fine crushed rocks passing 37.5 mm AS sieve. The test consists of applying an axial load to cylindrical specimens supported by various known lateral pressures until failure occurs.

This method is modified from the Texas Highway Department's test method Tex 117-E-Triaxial Compression Tests for Disturbed Soils and Base Materials.

2. **DEFINITIONS**

(a) Triaxial test is a test in which force is applied in three mutually perpendicular directions.

(b) Axial load is the sum of the applied load and the dead load and is applied along the vertical axis of the test specimen.

(c) Lateral pressure is the force supplied by air in the cell and applied in a radial direction.

(d) Axial Strain is unit deformation and equal to deformation of specimen divided by the original height expressed as a percentage.

(e) Mohr's diagram is a graphical construction used in analysing data from tests on bodies acted on by combined forces in static equilibrium.

(f) Mohr's circle of failure is stress circle constructed from principal stresses acting on the specimen at failure.

(g) Mohr's envelope of failure is the common tangent to a series of failure circles constructed from different pairs of principal stresses required to fail the material.

3. **APPARATUS**

(a) Mixing apparatus such as a steel tray, trowel and scoop.

(b) Metal dishes 350 mm diameter.

(c) A thermostatically controlled oven with good air circulation capable of maintaining a temperature not exceeding 50°C.

(d) A porcelain mortar 180 mm diameter with rubber pestle.

(e) 37.5 mm, 2.36 mm AS sieves.

(f) A thermostatically controlled oven with good air circulation, capable of maintaining a temperature within the range of 105°C to 110°C.

(g) A balance of 20 kg capacity, readable and accurate to 10 g within the operating range.

(h) A balance of 500 g capacity, readable and accurate to 0.01 g within the operating range.

(i) A metal mixing and quartering tray 750 mm by 750 mm with three sides 75 mm high. Quartering metal plates 400 mm by 125 mm and 200 mm by 125 mm, or riffle boxes of appropriate size openings.
(j) Air-tight containers suitable for curing moistened test samples.

(k) A cylindrical metal mould having an internal diameter of 153 mm and internal effective height of 203 mm fitted with a detachable baseplate and removable collar assembly 50 mm high, both of which can be firmly attached to the mould.

(l) A metal rammer with a 50.0 ± 0.5 mm face diameter and a drop mass of 2.7 kg + 10g, - 25 g equipped with a suitable device to control the height of drop to a free fall of 300 ± 2 mm.

(m) A rigid foundation on which to compact the specimen, e.g. a concrete floor or a concrete block of at least 100 kg mass.

(n) A steel, straight-edge, about 300 mm long, 25 mm wide and 3 mm thick, preferably with a bevelled edge.

(o) A 300 mm rule.

(p) A pair of hollow metal end blocks.

(q) A heavy duty extrusion frame complete with hydraulic jack 50 kN capacity.

(r) Tamper-plunger with plate diameter of 147 mm and stem length of 200 mm.

(s) Axial Cells, lightweight stainless steel cylinders; 170 mm inside diameter and 300 mm in height, fitted with standard air valve and tubular rubber membrane 153 mm diameter.

(t) A vacuum pump.

(u) Compressed air cylinder with gas regulator.

(v) Pressure regulating valve.

(w) Pressure dial gauge in the range of 0-160 kPa.

(x) Calibrated proving ring and Avery Universal Machine with 50 kN range.

(y) Micrometer dial gauge, calibrated in 0.1 mm with support to measure deflection of specimen.

(z) Screw jack press and assembly.

4. **MASS REQUIRED FOR A SERIES OF MODIFIED TEXAS TRIAXIAL TESTS**

   For each moisture content tested a minimum of five (10 kg) samples of material passing a 37.5 mm AS sieve are required. Fine grained materials not likely to break down under repeated compaction may be retested at different moisture contents but materials containing a significant proportion of particles likely to break down after repeated compaction will require fresh samples for each test.

5. **PRELIMINARY PREPARATION OF SAMPLE**

   (a) Allow the sample to dry sufficiently to enable it to be crumbled. If necessary, dry the sample in an oven at a temperature not exceeding 50°C.

   (b) Break up any aggregations of particles in such a way as to avoid crushing any discrete particles. All aggregations are to be broken down so that if the sample were screened on a 2.36 mm AS sieve, only discrete, uncrushed particles would be retained. A rubber pestle should be used to avoid breaking down sound pieces of mineral matter. Adhering material should be brushed from the coarse pieces. When in doubt as to whether lumps are to be broken, place some in water and boil. If slaking occurs, the material should be broken further with the rubber pestle.
(c) Dry the sample to substantially constant mass in an oven at a temperature within the range of 105°C to 110°C. Allow the sample to cool to room temperature.

(d) Weigh the cool sample and record the mass \( M_1 \).

(e) Screen the sample on a 37.5 mm AS sieve and so divide the sample into two fractions.

(f) Weigh the fraction retained on the 37.5 mm AS sieve and record the mass \( M_2 \).

(g) Compute the amount retained on 37.5 mm AS sieve \( M_2 \) as a percentage of the total mass of the sample \( M_1 \); i.e. \( \frac{M_2}{M_1} \times 100 \).

(h) Obtain from the fraction passing 37.5 mm AS sieve by quartering or riffling a sufficient amount for the series of Modified Texas Triaxial Tests.

6. PROCEDURE

(a) Determine the maximum dry density and optimum moisture content as outlined in Test Methods T110 or T111 as applicable.

(b) Obtain the moisture content at which the test is to be carried out.

(c) Add the calculated amount of water \( M_3 \) to the oven dry material \( M_4 \) to give the desired moisture content which is equal to \( \frac{M_3}{M_4} \times 100 \).

(d) After thoroughly mixing, place the material in a container and seal. Granular materials may be satisfactorily cured in a few hours but heavy clays may require several days. The next step is compaction. At optimum moisture content, go to Clause (e) A below; at below optimum moisture content, go to Clause (e) B below; at above optimum moisture content, go to Clause (e) C below.

(e) Compaction of specimen.

A. At optimum moisture content.

(i) Weigh the compaction mould and record the mass \( M_5 \).

(ii) Assemble the mould collar and base-plate and place the assembly on the rigid foundation.

(iii) Remove the damp material from the container and mix thoroughly.

(iv) Compact the damp material into the mould in five layers not varying in compacted thickness by more than 5 mm. Subject each layer to 56 uniformly distributed blows of the 2.7 kg hammer falling freely from a height of 300 mm. Use only sufficient material to slightly overfill the mould, leaving not more than 5 mm to be struck off after removing the collar.

(v) Free the material from around the collar and then carefully remove the collar.

(vi) Level the compacted material to the top of the mould by means of the straight-edge. Patch with smaller sized material any holes developed in the surface by removal of coarse material.

(vii) Weigh the mould plus compacted specimen and record the mass \( M_6 \).

(viii) Put a metal end block on top of the mould, invert the mould and eject the compacted specimen from the mould by means of the heavy duty hydraulic extrusion jack. Place the second metal end block on top of the specimen. Go to Clause 7 below.
B. At below optimum moisture content.

(i) Provided the moisture content desired is not less than about 2\% below optimum compaction can be carried out at the desired moisture content by using the procedure given in 6A (i) to (viii). A check should be kept to ensure the maximum dry density is being achieved. If this is not so the following procedure should be adopted.

(ii) Compact the sample as described in Clauses 6 (e) A (i) to (viii) making sure to compact at optimum moisture content.

(iii) Calculate the wet mass of each compacted specimen at the moisture content desired using the following formula:

\[
\text{Wet Mass} = \frac{M_4 (M_6 - M_5) (100 + d)}{100 (M_3 + M_4)}
\]

where \(d\) is the desired moisture content of compacted specimen \(\%\).

(iv) Place impermeable end blocks on each end of the specimens, stand the specimens vertically and allow them to dry back at room temperature.

(v) Check the mass of the compacted specimen frequently. As soon as the mass falls back to the value calculated in B (iii) above, seal the specimens by placing a rubber membrane around them.

(vi) Allow the compacted specimens to cure in the sealed container for 7 days.

(vii) Weigh the dried back specimens and record the masses — (M_7). Go to Clause 7 below.

C. At above optimum moisture content.

(i) Compact the sample as described in Clauses 6 (e) A (i) to 6 (e) A (viii), except that static compaction as described below is used.

(ii) Press the damp material into the mould in five layers not varying in compacted thickness by more than 5 mm by means of the heavy duty hydraulic jack and tamper-plunger similar to the one used in the preparation of Hubbard-Field specimens.

7. TRIAXIAL TEST PROCEDURE

(a) Enclose the specimen in the Texas triaxial cell.

(b) Centre the specimen with upper and lower metal end blocks in place in the press. Put the bearing plate with the steel ball already in the socket on top of the upper metal end block. Raise the press by means of the motor, align and seat the steel ball on the bearing plate into the socket in the proving ring. Then apply just enough pressure to obtain a perceptible reading on the proving ring gauge. Set the proving ring gauge to zero.

(c) Connect the air line to the axial cell and apply the desired lateral pressure, to the specimen. The usual lateral pressures used are 1, 30, 65, 100 and 140 kPa.

(d) Set the axial strain gauge to read zero and record the proving ring at zero deflection. The test is ready to be started.

(e) Turn on the motor and read the proving ring dial at each 0.5 mm deformation of the specimen. Continue readings until 25.0 mm deformation has occurred unless failure occurs earlier. Failure is reached when the proving ring dial readings remain constant or decrease with further increments of deformation. The motorized press travels at 4.1 mm per minute.

(f) Remove the failed specimen from the Texas Triaxial cell.

(g) Break up the material and determine the moisture content of the whole specimen by a procedure such as that described in Test Methods T120, T121, and T122. Then test the next specimen at the next lateral pressure until all five specimens have been tested. Take the average moisture content of the five specimens tested for each test as the moisture content for that test.
8. CALCULATIONS

Calculate the values of stress and strain for each individual specimen from the following relations:

Let $S =$ Percent failure strain.

$d =$ Total vertical deformation at failure, measured in mm by strain gauge.

$h =$ The height of the original specimen in mm.

$6_1 =$ The corrected maximum vertical stress at failure strain in kPa. A correction is necessary because the area of the cross-section increases as the specimen is deformed. The assumption is made that the specimen deforms at constant volume.

$6_3 =$ The lateral pressure in kPa.

$P =$ The total vertical load on the specimen at failure expressed in kN. It is the sum of the applied load measured by the proving ring plus the dead weight of the bearing plate and the upper hollow metal end block.

$A =$ Th end of cylindrical specimen expressed in square metres at the beginning of test.

$S = \frac{d}{h} \times 100\%$

$6_1 = \frac{P}{A} \left(1 - \frac{S}{100}\right) kPa$

9. CALCULATION OF MOHR'S ENVELOPE OF FAILURE

$\tau = C_u + 6_n \tan \phi_u$. 

Let $\tau =$ The shearing stress.

$6_n =$ The normal stress.

$\phi_u =$ The angle of shearing resistance.

$C_u =$ The apparent cohesion of the material in kPa.

$e =$ The normal stress at the centre of a Mohr’s circle in kPa. This is the abscissa of the point of maximum shear stress.

$\tau_{\text{max}} =$ The maximum shearing stress in kPa. This is the ordinate of the point of maximum shear stress.

$n =$ Number of $(e, \tau_{\text{max}})$ pairs (one pair from each test).

$\alpha =$ The angle of slope of the line of best fit through all maximum shear stress points on the Mohr’s diagram.

$a =$ The shear stress intercept of the line of best fit through all maximum shear stress points on the Mohr’s diagram in kPa.

$r =$ The correlation coefficient of the straight line of best fit through all maximum shear stress points.

For each specimen tested there will be one pair of values for $6_1$ and $6_3$. 
Calculate the co-ordinates of the point of maximum shear stress for each Mohr's circle as follows:

\[
\epsilon_i = \frac{6_t + 6_s}{2}
\]

\[
t_i = \frac{6_t + 6_s}{2}
\]

Usually there are five maximum shear stress points \((\epsilon_i, t_i)\) corresponding to the five lateral pressures of 0, 30, 65, 100, 140 kPa.

Calculate the shear stress intercept and angle of slope of the line of best fit for the maximum shear stress points as follows:

\[
a = \frac{\Sigma \epsilon \Sigma r_i - (\Sigma \epsilon_i) (\Sigma r_i)}{n \Sigma \epsilon_i^2 - (\Sigma \epsilon_i)^2}
\]

\[
\alpha = \tan^{-1} \left( \frac{n \Sigma \epsilon r_i - (\Sigma \epsilon_i) (\Sigma r_i)}{n \Sigma \epsilon_i^2 - (\Sigma \epsilon_i)^2} \right)
\]

The correlation coefficient of the line of best fit is also calculated as follows:

\[
r = \frac{\Sigma \epsilon_i r_i - \frac{1}{n} (\Sigma \epsilon_i) (\Sigma r_i)}{\sqrt{\left( \Sigma \epsilon_i^2 - \frac{(\Sigma \epsilon_i)^2}{n} \right) \left( \Sigma r_i^2 - \frac{(\Sigma r_i)^2}{n} \right)}}
\]

\(r\) should exceed 0.990.

\(\phi_u\) and \(C_u\) are then calculated from the equations:

\[
\phi_u = \sin^{-1} (\tan \alpha)
\]

and

\[
C_u = \frac{a}{\cos \phi_u}
\]

If \(r < 0.99\) generally there is one odd result. \(r\) is generally recalculated using 4 specimens at a time until the maximum value of \(r\) is obtained. Alternatively, the Mohr's circles can be plotted to see visually if there is one odd result.

10. CALCULATION OF AVERAGE COMPRESSIVE MODULUS

The compressive modulus for each test carried out is found from the axial stress-axial strain plot. It is defined as the slope of the straight line portion of the stress strain curve. If the stress-strain curve is not straight over an appreciable length the compressive modulus should be taken as the slope of the straight line joining the point of zero strain to the point of 0.75% strain.

The average compressive modulus for each series of tests (in MPa) is equal to the average value of the compressive moduli found for the five individual tests carried out.
11. CLASSIFICATION OF MATERIAL

Plot the envelope of failure on the Texas triaxial classification chart and classify the material to the nearest one-tenth of a class. When the envelope of the failure falls between class limits, select the critical point of weakest condition on the failure envelope. Measure the vertical distance down from the boundary line to the point to obtain the exact classification.

12. REPORTING

(a) Percent retained on 37.5 mm AS sieve.

(b) Angle of shearing resistance $\phi_v$.

(c) Apparent cohesion $C_u$.

(d) Texas classification number.

(e) Moisture content.

(f) Average Compressive Modulus.
THE UNCONFINED COMPRESSIVE STRENGTH AND INDIRECT TENSILE STRENGTH ("BRAZILIAN") TESTS

Both types of test were undertaken using the facilities of the Department of Main Roads, N.S.W. at its Milsons Point Laboratory. The equipment (see Plates 8.2 and 8.3) comprised:

(a) Load frame and motor drive unit -

50 kN capacity, Engineering Laboratory Equipment Ltd., Rickmansworth, England;

(b) Load and displacement transducers -

type D412.01, Schaevitz (England);

(c) Transducer display unit -

TDU-02, C-W Instruments, Clyde-Westeels Ltd., Granville, N.S.W.;

(d) XY Plotter -


Specimens of asphaltic concrete were prepared by remoulding and gyratory compaction to nominal dimensions of \( \phi 100 \times 200 \text{ mm} \) (compressive) or \( \phi 100 \times 62 \text{ mm} \) (tensile), and placed in an oven to bring them to the required test temperature of 25°C. In a couple of isolated cases, the rough ends of compressive specimens required sulphur capping, and this was done in the standard manner used in concrete tests.

Both tensile and compressive tests were conducted at a strain rate equivalent to 4.1 mm/min, this being the rate used for standard Texas Triaxial tests. Each test continued until no further increase in load could be sustained, and the maximum load applied to the specimen was recorded. The strength of the specimen was then calculated by:

(a) compressive strength \( = \frac{4P_c}{\pi d^2} \)
(b) tensile strength = \frac{2P_T}{\pi d t}

where

- \( P_c \) = maximum compressive load
- \( P_T \) = maximum tensile load
- \( d \) = specimen diameter
- \( t \) = specimen thickness.
CHAPTER NINE

APPROACHES TO DESIGN
9.1 Introduction

Engineering design is a process whereby the dimensions and properties of components are selected in such a way that the completed structure shall perform in a specified manner. Where there exists a range of designs, all capable of performing to the required standard, a further stage must be added to the process. This final step involves the selection of an optimum design, for which the criteria may be economic, aesthetic, or other considerations.

The preceding two chapters have explored the performance of a considerable number of pavements, leading to the conclusion that the calculated shakedown limit of a pavement offers a valuable guide to its ultimate performance (expressed in terms of its serviceable life under traffic). The relationship of most use is perhaps that developed between shakedown limit (one-dimensional approximation) and the lower confidence limit on traffic life.

For most practical pavements, the number of parameters which influence performance imposes certain difficulties. Since the behaviour of a particular material may ideally be described by four fundamental parameters \( (E, v, c, \phi) \), and pavements typically consist of three or more distinct materials, the task of formulating design procedures involves determining the influence of perhaps a dozen or more variables. This represents a considerable undertaking.

The manner in which a design system may be developed has been introduced in Chapter 5. Following a review of this approach, this chapter explores a number of possible forms of design procedure appropriate to more complex pavement structures.

9.2 Single Layer on Subgrade

In Figure 9.1 is reproduced the sample design chart formulated in Chapter 5 for the case of a single layer on a subgrade. Whilst fixed values of \( v \) and \( \phi \) were assumed for both materials for the purposes of illustration, the chart serves to clearly demonstrate the following principles:

(a) for fixed material properties, the requisite layer thickness may readily be determined;
INFLUENCE OF STRENGTH AND STIFFNESS PARAMETERS UPON THICKNESS OF LAYERS

FIGURE 9.1

Performance Standard

\[
\frac{\lambda \sigma V}{C_0} = 15
\]

ENVELOPE OF CURVES (Minimum required thickness)

THICKNESS OF SURFACE LAYER \( \frac{D}{2B} \)

\[\begin{align*}
\frac{E}{E_0} & = 10 \\
\frac{E}{E_0} & = 16 \\
\frac{E}{E_0} & = 30 \\
\frac{E}{E_0} & = 100
\end{align*}\]
(b) the influence of relative stiffness on the distribution of stress between layers is reflected graphically;

(c) the influence of relative strength on the mode of failure is clearly portrayed; and

(d) the concept of optimum relative stiffness and strength for minimum layer thickness is substantiated.

It should be noted that in this case the presentation is simplified due to the use of only four variables \( \left( \frac{E/E_0}{c/c_0}, \frac{D/2B}{\lambda_{SD} V/c_0} \right) \). For a fixed value of any one of these, a family of curves is sufficient to describe relationships between the other three, and consequently a set of charts, each one representing a different performance standard, is capable of illustrating the relationship between all four parameters.

9.3 Two Layers on Subgrade (Fixed Properties)

As a first step towards extending the above work to deal with a larger number of pavement layers, it is convenient to examine the influence of layer thicknesses only upon shakedown limit. Using the fixed material properties set out in Figure 9.2, for a representative flush-sealed pavement, a range of thickness designs has been analysed using the one-dimensional shakedown approach developed previously.

With only three variables \( (h_B, h_{SB} \text{ and } \lambda) \) the results lend themselves to graphical presentation using contours of constant shakedown limit \( \lambda_{SD} \) over a range of thickness combinations. Table 9.1 sets out the results of the analyses, and in Figure 9.3 is shown the final product in the form of a design chart. Thus for the given materials, a pavement having a shakedown limit of \( \lambda_{SD} = 0.65 \) may be constructed using any one of the thickness combinations lying on that line.

The role of cost constraints may now be illustrated in a simplified manner. Consider, for example, the case where the particular sub-base material was available at a cost of $30/m³ constructed, with roadbase at $50/m³. With \( R \) being taken as 100 mm, the cost of the pavement in this case may be expressed as

\[
C \left( \$/m^2 \right) = 0.05 h_B \text{ (mm)} + 0.03 h_{SB} \text{ (mm)}
\]

and \( C \) is to be minimised. Figure 9.4 shows the selected \( \lambda_{SD} \) line, and
<table>
<thead>
<tr>
<th>Profile</th>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$\nu$ (kPa)</th>
<th>$c$ (kPa)</th>
<th>$\phi$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>Crushed Rock</td>
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<td>80</td>
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</tbody>
</table>

$R = 0.5, \mu = \frac{H}{\nu} = 0.4$

$V = 700$ kPa

**FIGURE 9.2 - Flush-Sealed Pavement for Development of Design Charts**
TABLE 9.1

Results of Analysis of Representative Flush-Sealed Pavement

Figures represent values of shakedown limit and layer where failure initiated (Base/Sub-base/Subgrade)

<table>
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<th>$\frac{h_B}{R}$</th>
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</tr>
</tbody>
</table>
FIGURE 9.3
SHAKEDOWN LIMIT OF FLUSH-SEALED PAVEMENT (FIXED PROPERTIES) AS FUNCTION OF THICKNESS DESIGN
Figure 9.4

Sample cost functions showing influence on pavement design.
contours of constant $C$, from which it is evident that the optimum thickness design is approximately 75 mm of base over 170 mm of sub-base, with an associated cost of approximately $8.90/m^2$.

In practice, of course, $C$ would tend to be a more complex function of thicknesses. The simplification is used merely to illustrate that at the end of the design process, technical considerations cannot be isolated from economics.

9.4 Three Layers on Subgrade (Fixed Properties)

The foregoing process may readily be extended to cover three layers overlaying a subgrade, as presented in Figure 9.5. If once again material properties are known, then only four variables ($h_{AC}$, $h_B$, $h_{SB}$ and $\lambda$) are to be related. In this case, it is preferable to keep the three thicknesses ($h_{AC}$, $h_B$, $h_{SB}$) together, and prepare a family of charts, each one representing a particular value of $\lambda$ and hence a particular performance standard. Three typical charts are shown in Figures 9.6(a) to (c), and it may be noted that once the required performance is selected, a single chart is capable of presenting the range of suitable thickness designs.

The relevant chart may then be used as before to determine the optimum design. Given the applicable cost function, generally a small number of calculations at various points on the graph will be sufficient to locate the most economical design and its associated cost; if completeness is desired, superimposing contours of constant functional value will readily highlight that point at which cost is minimised.

9.5 Influence of Material Properties

The approaches already outlined have illustrated that if the properties of all constituent materials are known, then the range of pavement designs to reach a particular performance standard may be presented in a simple manner, amenable to subsequent economic analysis.
<table>
<thead>
<tr>
<th>Profile</th>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$v$</th>
<th>$c$ (kPa)</th>
<th>$\phi$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Asphaltic Concrete</td>
<td>2000</td>
<td>0.4</td>
<td>3000</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Crushed rock roadbase</td>
<td>70</td>
<td>0.3</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Gravel/shale sub-base</td>
<td>50</td>
<td>0.3</td>
<td>80</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Clay subgrade</td>
<td>20</td>
<td>0.35</td>
<td>40</td>
<td>15</td>
</tr>
</tbody>
</table>

**LOADFORM**

\[
\frac{b}{R} = 0.5, \quad \mu = \frac{H}{V} = 0.4
\]

\[
V = 700 \text{ kPa}
\]

**FIGURE 9.5** - Representative Three-layer Pavement for Development of Design Charts
FIGURE 9.6(a)

DESIGN CHART - THREE-LAYER PAVEMENT (FIXED PROPERTIES)
PERFORMANCE STANDARD $\lambda_{50} = 0.6$
FIGURE 9.6 (b)
DESIGN CHART - THREE-LAYER PAVEMENT (FIXED PROPERTIES)
PERFORMANCE STANDARD $\lambda_{sd} = 1.0$
FIGURE 9.6(c)

DESIGN CHART - THREE LAYER PAVEMENT (FIXED PROPERTIES)
PERFORMANCE STANDARD $\lambda_{SD} = 1.25$
In the field, however, the engineer encounters a considerable range of material properties, and it is generally inconvenient for him to perform a complete series of pavement analyses for each set of materials which he may contemplate using. The alternative - to provide a design chart of the format presented in Figure 9.6 for each likely combination of pavement layers, subgrade conditions, and required standard of performance - is at this point highly impractical.

One solution to this difficulty has application where standard materials tend to be used often as pavement layers. Under these circumstances, the only variables are the subgrade properties ($E_{SG}$, $c_{SG}$, $\phi_{SG}$ - a value assumed for $\nu_{SG}$) and the layer thicknesses, and of these $c_{SG}$ may be separated out since the shakedown limit of the subgrade is directly proportional to its cohesion. Further, as was discovered in the case of a single layer on a subgrade, the curves representing shakedown limits for the individual layers may be separated (to be scaled according to relative cohesion and superimposed at the time of design). Figure 9.7 is presented as typical of this approach. In this case, fixed values have been assigned to $E_{SG}$ and $\phi_{SG}$, and this permits the method of superposition to again be illustrated. Consider, for example, the case where $\lambda_{SD} = 0.4$ was required, and $E_{SG} = 10$ MPa, $\phi_{SG} = 0$, $c_{SG} = 20$ kPa. Then the appropriate curve from the base/sub-base chart is selected, and from the subgrade chart the required curve is that corresponding to $\frac{\lambda_{SD} c_{SG}}{c_{SG}} = \frac{0.4 \times 50}{20} = 1.0$. The superposition of these two curves yields Figure 9.8, whence further criteria may be used to select an optimum design.

For generality, the two curves of Figure 9.7 must be supplemented by others to account for the likely variation in subgrade properties. Accordingly, one base/sub-base chart is required for each value of $E_{SG}$, and a separate subgrade chart required for each $(E_{SG}, \phi_{SG})$ combination. The total number of charts is then (number of $E$'s) x (1 + number of $\phi$'s) and clearly a dozen or more charts would be involved even for an initial coverage of the likely range of $E$ and $\phi$ in the subgrade.
(a) Base/Subbase Failure  
\( E_{SG} = 10 \text{ MPa} \)

(b) Subgrade Failure  
\( E_{SG} = 10 \text{ MPa} \), \( \phi_{SG} = 0 \)

FIGURE 9·7  
TYPICAL DESIGN CHARTS FOR KNOWN PAVEMENT MATERIAL, VARIABLE SUBGRADE
FIGURE 9.8

SUPERPOSITION OF FAILURE CURVES
for $E=10$ MPa, $\phi_{sg}=0$, $C_{sg}=20$ kPa
Whilst the principle of this method allows further parameters to be considered in design, it appears that its implementation can quickly become unwieldy, and some change in approach is called for.

9.6 Two Layers on Subgrade (General Materials)

9.6.1 Introduction

Many existing pavements may be idealised as two-layer structures overlaying a subgrade. Clearly most flush-sealed designs fall into this category; however, a large proportion of designs using thick asphaltic concrete (i.e. 50 mm or more) are also of this form. The work which follows develops a sample design method for this latter group of structures, and indicates the means by which it may be extended to more general situations.

The restriction of attention to asphalt/granular base/subgrade structures permits certain simplifications to be made. Foremost among these is that the values of friction angle $\phi$ applicable to each layer are likely to vary little from pavement to pavement, and so representative values may be assigned to these angles with very little loss of generality. Similarly, values of Poisson's ratio may be fixed without prejudicing the applicability of the analysis which follows. The variables which remain to be considered are then the modulus, cohesion and thickness of each layer, and Table 9.2 sets out the form of the resulting problem.

A sample thickness design of $h_{AC} = 100$ mm, $h_B = 200$ mm ($h_{AC}/R = 1$, $h_B/R = 2$) serves to illustrate the influence of some of these parameters. In Figure 9.9 are summarised the results of a series of analyses, presented as before in terms of the cohesion of individual layers. Given values of cohesion for each layer, and a required shakedown limit, the curves for each layer may then be superposed in the normal way to obtain a form of design chart.

More importantly, however, the Figure demonstrates firstly that pavement response is a function of modular ratios, rather than absolute values of modulus, since it is the modular ratio which determines the distribution of stress within the structure. Secondly, it is clearly shown that the shakedown limit may be normalised with respect to layer cohesion, and together these two factors allow the
### TABLE 9.2

Parameters (Fixed and Variable) for Two-Layer Pavement Design

<table>
<thead>
<tr>
<th>PAVEMENT</th>
<th>Material</th>
<th>Thickness</th>
<th>Stiffness</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Asphaltic Concrete</td>
<td>$h_{AC}$</td>
<td>$E_{AC}$</td>
<td>$c_{AC}$</td>
</tr>
<tr>
<td></td>
<td>Granular Base</td>
<td>$h_{B}$</td>
<td>$E_{B}$</td>
<td>$c_{B}$</td>
</tr>
<tr>
<td></td>
<td>Subgrade</td>
<td>$\infty$</td>
<td>$E_{SG}$</td>
<td>$c_{SG}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LOAD</th>
<th>$b$ = 0.5</th>
<th>$H = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu = \frac{H}{V} = 0.4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R = 100$mm, $V = 700$kPa</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 9·9

(a) AC failure
Contours of $\frac{\lambda_{SD}}{C_{AC}/1000\text{kPa}}$

(b) Base failure
Contours of $\frac{\lambda_{SD}}{C_{B}/80\text{kPa}}$

(c) Subgrade failure
Contours of $\frac{\lambda_{SD}}{C_{SG}/40\text{kPa}}$

SHAKEDOWN BEHAVIOUR OF
($\frac{h_{AC}}{R} = 1, \frac{h_{B}}{R} = 2$) PAVEMENT
number of variables under consideration to be reduced to five \( \frac{E_{AC}}{E_{SG}}, \frac{E_B}{E_{SG}}, \frac{h_{AC}}{R}, \frac{h_B}{R} \) and \( \lambda \). Further, since the mode of failure most commonly applying to this range of pavements is incremental collapse at the top of the subgrade, attention will be restricted to this: the prediction of failure within AC and base may be approached similarly, but will not be pursued here in the interests of simplicity.

9.6.2 Derivation of Chart

With five variables still to be considered, clearly some approximation will be required if a compact presentation is to be achieved for a design chart. In Table 9.3 are set out the results of a series of analyses, and the form of presentation employed here suggests that there may be some advantage in relating the data of each row (varying \( \frac{E_B}{E_{SG}} \)) to the shakedown limit when \( \frac{E_B}{E_{SG}} = 5 \). Indeed, this simple normalisation proves to be quite effective, as the following table reveals:

<table>
<thead>
<tr>
<th>( \frac{E_B}{E_{SG}} )</th>
<th>Mean ( \lambda ) (15 values)</th>
<th>2.0</th>
<th>3.5</th>
<th>5.0</th>
<th>7.5</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ) (15 values)</td>
<td>0.85 0.93 1.00 1.11 1.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.07 0.03 0.04 0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\sigma}{\text{Mean}} ) (%)</td>
<td>8 3 4 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With a coefficient of variation of no more than 8% for the range of thickness and stiffness ratios tested, this approximation appears to be adequate for the purpose. Further tests using different thicknesses of base confirm this, and without any marked increase in error the approximation may be formalised as

\[
\frac{\lambda_{SD}}{\lambda_5} = 1 + 0.044 \left( \frac{E_B}{E_{SG}} - 5 \right)
\]

where \( \lambda_5 = \lambda_{SD} \left( \frac{E_B}{E_{SG}} = 5 \right) \).

One of the sources of variation in \( \lambda \) is thus isolated.
TABLE 9.3

Shakedown Limits $\lambda_{SD}$ for Subgrade Failure - Various Profiles and Stiffness Ratios

<table>
<thead>
<tr>
<th>$\lambda_{SD}$ ($c_{SG} = 40$ kPa, $V = 700$ kPa)</th>
<th>$E_b / E_{SG}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{AC}$</td>
<td>$h_B$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0.5</td>
<td>2.0</td>
</tr>
<tr>
<td>6.3</td>
<td>0.64</td>
</tr>
<tr>
<td>25</td>
<td>0.67</td>
</tr>
<tr>
<td>50</td>
<td>0.69</td>
</tr>
<tr>
<td>100</td>
<td>0.73</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>6.3</td>
<td>0.74</td>
</tr>
<tr>
<td>25</td>
<td>0.84</td>
</tr>
<tr>
<td>50</td>
<td>0.94</td>
</tr>
<tr>
<td>100</td>
<td>1.12</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>6.3</td>
<td>1.00</td>
</tr>
<tr>
<td>25</td>
<td>1.42</td>
</tr>
<tr>
<td>50</td>
<td>1.83</td>
</tr>
<tr>
<td>100</td>
<td>2.37</td>
</tr>
</tbody>
</table>
Subsequent investigation may now be confined to the influence of $E_{AC}$, $h_{AC}$ and $h_B$ on $\lambda_5$. Figure 9.10 shows a plot of some of the available data, and immediately it is evident that the families of lines associated with a given asphalt thickness $h_{AC}$ are characterised by a nearly constant separation. For the lines shown in the Figure, this separation ($\Delta\lambda_5$) is approximately 0.26. A further formalisation becomes possible, in which the influence of base thickness $h_B$ is isolated:

$$\lambda_5 = \lambda_{5,2} + 0.26 \left( \frac{h_B}{R} - 2 \right)$$

where $\lambda_{5,2} = \lambda_{SD}(E_B/E_{SG} = 5, h_B/R = 2)$.

At this point the original five variables have been reduced to three ($E_{AC}$, $h_{AC}$ and $\lambda_{5,2}$), and the stages in the construction of a design chart begin to emerge when the equations are set out together:

$$\lambda_5 = F_1 (\lambda_{SD}, E_B/E_{SG})$$
$$\lambda_{5,2} = F_2 (\lambda_5, h_B/R)$$
$$\frac{h_{AC}}{R} = F_3 (\lambda_{5,2}, E_{AC}/E_{SG}) .$$

The formulation above, in which the effects of two variables are isolated by simple approximations, lends itself to presentation as a multiple intercept chart occupying three quadrants. Figure 9.11 displays such a chart, with $\lambda$ adjusted to take account of tyre contact stress $V$ and subgrade cohesion $c_{SG}$.

In the final assembly of this chart, certain refinements have been made to the approximations outlined above. In order to better approximate the calculated results, the relationships portrayed graphically are:

(a) $\frac{\lambda_{SD}}{\lambda_5} = 1 + 0.044 \left( \frac{E_B}{E_{SG}} - 5 \right)$ (as before)

for $1 \leq E_B/E_{SG} \leq 15$;
$\left( \frac{h_{AC}}{R}, \frac{h_{G}}{R} \right) = (2, 3)$

**Figure 9.10**

Influence of thickness and asphalt stiffness upon normalised shakedown limit.
Figure 9.11
DESIGN CHART - TWO LAYERS ON SUBGRADE

BASE STIFFNESS $\frac{E_B}{E_{SG}} = 1$

NORMALISED SHAKE DOWN LIMIT

$\lambda_N = \frac{\lambda_{SD} V}{C_{SG}}$

SHAKE DOWN LIMIT $\lambda_{SD}$ for $V = 700$ kPa
$C_{SG} = 40$ kPa

DESIGN CHART - TWO LAYERS ON SUBGRADE
(b) The values of \( \lambda_{5,2} \) used to construct the curves for the third quadrant are set out in Table 9.4.

9.6.3 Suitability and Use of Chart

A primary check of the suitability of the simplifications outlined above may be made by beginning with a profile \( (h_{AC}, h_B, E_{AC}, E_B, E_{SG}) \) and determining \( \lambda \) using the chart. The accuracy of the method is measured by the proximity of this \( \lambda \) to that obtained by complete analysis (program LAYELLIP - method of conics). Figure 9.12 illustrates the outcome of several such comparisons, and it is clear that the vast majority of such tests have resulted in a variation of less than five percent. This is most pleasing, and more than sufficiently accurate for present purposes.

The use of the chart is illustrated in Figure 9.11 itself. Most commonly, the performance standard and material properties \( \lambda, E_{AC}, E_B, E_{SG}, c_{SG} \) will be fixed by external constraints, such as material availability. The aim then is to determine suitable layer thicknesses for the purpose. The process is as follows:

(a) normalise shakedown limit with respect to subgrade cohesion, by

\[
\lambda_N = \frac{\lambda_{SD}V}{c_{SG}} \quad \text{for general } V,
\]

or \( \lambda_N = \frac{700\lambda_{SD}}{c_{SG}(kPa)} \) for \( V = 700 \text{ kPa} \);

(b) proceed to line appropriate to base stiffness (A);

(c) move to quadrant representing base thickness (B);
# Table 9.4

Shakedown Limits $\lambda_{5,2}$ (for $E_B/E_{SG} = 5$, $h_B/R = 2$) - Dependence on Asphalt Thickness and Stiffness

<table>
<thead>
<tr>
<th>$\lambda_{SD} \left( E_{SG} = 40 \text{ kPa} \right)$</th>
<th>$\lambda_{SD} \left( V = 700 \text{ kPa} \right)$</th>
<th>$R$</th>
<th>$h_{AC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.78</td>
<td>0.87</td>
<td>1.09</td>
</tr>
<tr>
<td>10</td>
<td>0.81</td>
<td>0.96</td>
<td>1.32</td>
</tr>
<tr>
<td>25</td>
<td>0.85</td>
<td>1.03</td>
<td>1.56</td>
</tr>
<tr>
<td>50</td>
<td>0.87</td>
<td>1.11</td>
<td>1.88</td>
</tr>
<tr>
<td>100</td>
<td>0.90</td>
<td>1.25</td>
<td>2.33</td>
</tr>
<tr>
<td>200</td>
<td>0.95</td>
<td>1.48</td>
<td>2.77</td>
</tr>
</tbody>
</table>
FIGURE 9.12
TEST OF DESIGN CHART ACCURACY
(d) selecting a possible value for base thickness, proceed through appropriate asphalt stiffness (C) to corresponding asphalt thickness (D);

(e) repeat step (d) for other possible thicknesses of base to obtain a series of thickness designs meeting the required standard.

The lines shown on Figure 9.11 demonstrate the approach for the design of a pavement required to have a shakedown limit $\lambda_{SD}$ of 1.75. The pavement is to carry traffic having a representative tyre load equivalent to 560 kPa on an area of radius 100 mm, and the available materials (AC, roadbase, existing subgrade) have stiffnesses of 1000 MPa, 200 MPa and 20 MPa respectively. The subgrade cohesion is found to be 28 kPa. Using these parameters, the normalised shakedown limit $\lambda_N$ may be calculated as 35, and relative stiffnesses of 50 and 10 apply to asphalt and roadbase respectively. Then, as the Figure shows, a base thickness of 200 mm requires 170 mm of asphalt, while a thicker base (say 400 mm) requires less asphalt - in this case, 90 mm. Here, 80 mm of asphalt has been replaced by 200 mm of roadbase. It needs to be noted, however, that this ratio of replacement (or "material equivalency") depends on both material properties and the form of structure being considered, and should not be blindly applied to other pavement design processes.

Once the relationship between asphalt and base thickness has been determined in this manner, considerations of construction convenience and cost may be superimposed (as previously illustrated) in order to derive the final design of pavement.

9.7 Conclusion

Beginning with the example of a single layer on a subgrade, this chapter has shown how design procedures applying to more general pavements may be developed.
Particular attention has been paid to the case of two layers on a subgrade, as this represents one of the simpler pavement structures of practical interest. The principle of superposition, by scaling based on relative values of cohesion, was shown to simplify the presentation of results, and subsequent analysis was concentrated on subgrade failure: charts for other layers would follow the same pattern as for the subgrade.

With interest focussed on the one layer, then, two approximations were introduced which permitted a large body of results to be presented in a single multiple intercept chart. Not only does this chart accurately reproduce the data, it also is particularly convenient to use, is compact in form, and an interface with criteria for economic evaluation may readily be added. Further, the principles developed in the production of the chart enable it to be extended relatively easily to deal with other pavement layers, and it is envisaged that more complex pavement structures may be approached in a similar manner.
CHAPTER TEN

SUMMARY AND CONCLUSIONS
10.1 Summary

A brief look at the history of methods of pavement analysis and design has revealed a century of work in applying elastic theory to loading on a half-space, and a great many centuries of practical experience in wheel loading on layered continua. Indeed, this long-standing reliance on experience alone has persisted well into the twentieth century, and a range of largely empirical design methods is still in use. Whilst this experience is vital, however, its power is vastly increased when it is united with an appropriate theoretical approach. By this means, findings may be generalised, the behaviour of dissimilar structures related, and the performance of new materials or designs predicted on some rational foundation. In this connection, the rise in the use of the theory of elasticity has been noted.

By itself, though, elastic theory is incapable of explaining why pavements fail, and some consideration needs to be given to defining that point at which a material ceases to behave elastically. Particularly is this so for materials which accumulate permanent deformations incrementally, as their long-term performance then depends heavily upon the way in which residual stresses and strains develop under the passage of a wheel load. The stiffness and strength parameters employed in this study \((E,v,c,\phi)\) provide a model of behaviour suitable for most pavement materials, whilst retaining a valuable simplicity.

Also modelled in an approximate manner are the loading and response of the pavement structure. A considerable quantity of experimental investigation has served to demonstrate that the contact area of the load is generally close to circular, with a variation in contact stress that is approximately trapezoidal. The pavement's response to this loading has been widely examined, and again it may be concluded that linear elasticity combined with some criterion for yield provides an appropriate and convenient guide to the actual response. The results of several studies have also emphasised the dependence of this response upon the shear component of load, and the significant difference between the effects of a cycled static load and those of a repeated moving load. This should not be overlooked in the analysis phase.
An analysis which incorporates these considerations has been developed in Chapter Five. The theory of shakedown, well established in relation to discrete structures, was selected as being appropriate to the service conditions of pavements, and its application extended to layered continua. Two methods of solution to the relevant sets of equations have been presented. The first, using linear programming algorithms, was valuable in providing initial results, while subsequent refinements yielded a procedure (method of conics) of much improved accuracy and speed. The latter approach enabled solutions for the shakedown limits of various simple structures to be determined, and through subsequent exploration of the influence of material and geometric parameters upon this limit, a number of implications for design procedures were noted.

Much of the simplicity of this "one-dimensional" analysis of shakedown must be attributed to the assumptions made in its formulation, and the validity of these assumptions may be tested by more general approaches. Using an axisymmetric loadform on a horizontally-layered pavement, subsequent results lent strong support to the concept of a residual stress response which is uniform in the direction of load movement. The comparison also confirmed that the plane strain approach provides slightly conservative results: the simpler model becomes even more attractive when it is noted that it requires only perhaps one-hundredth of the time to yield comparable results.

The "one-dimensional" model having been developed, refined, and tested against more general procedures, it was then applied to the results of the AASHO tests of pavement performance. These results clearly demonstrate that pavement shakedown may be observed, and analyses confirmed that the phenomenon may also be predicted with some accuracy. Not only is the approach therefore appropriate to the observed performance of pavements, but comparisons also reveal that its ability to predict the life of a pavement is at least as good as more established methods.

A subsequent study of Sydney pavements lent further support to these conclusions, and enabled a relationship between shakedown limit and minimum expected pavement life to be constructed, for use at the design stage.
Finally, the evolution of design procedures using shakedown analysis was presented. The approach of Chapter Five underwent considerable development in its transition through typical pavements with fixed materials, to be eventually applied to the case of two layers overlaying a subgrade (with general material properties). As one of the simpler pavement types in general use, this model permitted the influence of realistic variations in materials and geometry to be explored. From the data a number of approximations were deduced, while the principle of superposition, as previously developed, enabled layers to be considered separately and their design curves later combined by appropriate scaling. Together, these factors permitted an accurate and compact design chart to be formulated in a manner which could be applied without difficulty to other pavement layers. Ready extension to more general pavement configurations is also envisaged. Further, the role of economic criteria in the final design of a pavement was identified, and a simple means of incorporating these into the design process was presented.

10.2 Conclusions

This thesis puts forward the theory of structural shakedown as appropriate to the analysis and design of pavements. The relevance of this theory derives from two considerations: firstly, pavement materials are recognised as having finite strength and being subject to plastic deformation, and secondly, the role of repeated moving loads in the incremental process of pavement failure is in this way given the weight it deserves.

Based upon the assumption that many passes of a load should give rise to permanent deformations which are uniform with respect to the direction of motion, a "one-dimensional" analysis of the shakedown problem may be constructed. More general analysis suggests that such an assumption is valid, and confirms that the "one-dimensional" approximation possesses a considerable speed advantage while providing estimates of more than sufficient accuracy for present purposes.
Pavement shakedown may be not only postulated, but also observed, and this is particularly evident from a study of the AASHO Road Test records. The capacity of the theory to predict pavement shakedown (where it occurs) adds considerably to its strength, and when combined with the results of analyses for failed pavements, this permits a simple relationship between shakedown limit and pavement life to be developed.

Further field studies lend support to this conclusion. A somewhat disturbing by-product of these studies, however, was the discovery that records of original pavement designs and design lives are surprisingly limited. When mated with the often scattered traffic data, this makes the monitoring of a design method particularly intractable. Pavement life becomes a very difficult parameter to assess, and the paucity of original data concerning materials and geometry means that relationships between thickness design and life are often founded on very patchy evidence.

A full shakedown analysis is strictly a computer-based process. For general design purposes, however, a more convenient representation of the influence of various parameters upon shakedown limit is required. For typical pavements this is a major undertaking: by making certain approximations it is nevertheless possible to condense a considerable body of data into a manageable form. The case of a general two-layer pavement (AC/base/subgrade) was selected as representing one of the simpler pavement types in service, and it has been shown that a design chart which is both accurate and convenient may be prepared. Furthermore, the procedure used in developing such an aid is general, and may readily be applied to other pavement configurations.

The field of pavement analysis and design is a broad one, and our understanding of structural behaviour in this area may at best be described as limited. The foregoing work has presented an approach which appears to be particularly appropriate, in view of both the nature of traffic loading and the observed processes of pavement deterioration. Further investigation is certainly warranted.
### ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASCE</td>
<td>American Society of Civil Engineers</td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>HRB</td>
<td>Highway Research Board</td>
</tr>
<tr>
<td>ICE</td>
<td>Institution of Civil Engineers</td>
</tr>
<tr>
<td>ICSDAP</td>
<td>International Conference on the Structural Design of Asphalt Pavements</td>
</tr>
<tr>
<td>ICSMFE</td>
<td>International Conference on Soil Mechanics and Foundation Engineering</td>
</tr>
<tr>
<td>JGED</td>
<td>Journal of the Geotechnical Engineering Division, ASCE</td>
</tr>
<tr>
<td>JSMFD</td>
<td>Journal of the Soil Mechanics and Foundation Engineering Division, ASCE</td>
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<tr>
<td>JSSMFE</td>
<td>Japanese Society for Soil Mechanics and Foundation Engineering</td>
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