1 Mathematics education: The development of principles for classroom learning

A recent emphasis within educational psychology is the development of models of classroom learning incorporating key learning principles (Brown and Campione, 1994; Brown, Ellery and Campione, 1998; Guthrie and Alao, 1997; Scardamalia and Bereiter, 1996a; Penuel and Roschelle, 1999). Principles of learning realised in different models of classroom learning include the value of collaboration, supporting meaningful learning through forming meaningful links with prior knowledge and the importance of communities of discourse. Classroom models that support literacy skills (Palincsar and Brown, 1984; Guthrie and Alao, 1997; Scardamalia, Bereiter and Lamon, 1994), research skills (Brown and Campione, 1994; Brown, Ellery and Campione, 1998; Scardamalia, Bereiter and Lamon, 1994) and the learning of science (Palincsar and Herrenkohl, 1998; Hay and Barab, 2001) are many. Few classroom models incorporating these principles have been developed specifically to support learning within a mathematics classroom apart from the Jasper Woodbury series of videodiscs (Cognition and Technology Group at Vanderbilt, 1996) and to a lesser degree Schoenfeld’s model for developing problem solving skills (Schoenfeld, 1987). However, reform documents such as Professional Standards for Teaching Mathematics (NCTM, 1991), Principles and Standards for School Mathematics (NCTM, 2000), National Statement on Mathematics for Australian Schools (AEC, 1991) and Mathematics Counts: Report of the Committee of Inquiry into the teaching of Mathematics (Cockcroft, 1982) detail several principles of effective mathematics classrooms. While the reform movement has its origins in the United States, often from the state of California (Wilson, 2003), there are similar arguments for reform in Australia (AEC, 1991; DETYA, 2000) and the United Kingdom (Cockroft, 1982). Empirical evidence for the effectiveness of reform classrooms has been obtained in several different studies comparing the results of students in reform classrooms with students in traditional classrooms (Boaler, 1997a; Wenglinsky, 2000; Maher, 1991; Sigurdson and Olson, 1992) all of which found that students who learnt mathematics in reform classrooms achieved higher results on traditional examinations than students in
the development of a model of an effective secondary mathematics classroom that realises these principles that can be adopted by teachers currently teaching in the traditional manner, which is also able to meet the concerns of these teachers.

Calls for the reform of secondary mathematics teaching and learning have focused on enhancing student use of mathematical discourse, developing students’ problem solving skills and mathematical reasoning ability (DETYA, 2000; AEC, 1991; NCTM, 1989; 1991; 2000; Grimison and Pegg, 1995; Fennema and Romberg, 1999). Little has changed, however, in most secondary mathematics classrooms where traditional models of whole-class teaching monologues followed by drill and practice remain. Mathematics teachers continue to place greater emphasis on computational skills rather than developing conceptual understanding, skills and applications (Porter, Floden, Freeman, Schmidt and Schwille, 1988; Fey, 1979; Stigler and Hiebert, 1997; Stodolsky, 1988) even though evidence suggests that methods learnt without understanding are more easily forgotten or used incorrectly (Hiebert and Lindquist, 1990). One such description of mathematical activity by Welch (1978) in the United States aptly describes what happens in many schools, including most Australian schools.

In all math classes I visited, the sequence of activities was the same. First, answers were given for the previous day’s assignment. The more difficult problems were worked through by the teacher or a student at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to working on the homework while the teacher moved about the classroom answering questions. The most noticeable thing about math classes was the repetition of this routine. (p.6)

Clearly, there exist certain limiting factors that constrain secondary teachers’ capacity to change their teaching methods to reflect the characteristics outlined as desirable by such documents as the NCTM document Principles and Standards for School Mathematics (NCTM, 2000). While primary school teachers tend to be more open to adopting some of these approaches, secondary teachers seem less willing to adopt aspects of the reform movement in their teaching. Some of these limiting factors appear to be the existence of
high stakes examinations, inflexible school environments and the emphasis on breadth rather than depth in most mathematics curricula (Addington, Clemens, Howe and Saul, 2000). The purpose of this study is to examine a model of secondary school classroom practice that reflects the concerns of the reform program in mathematics education and also meets the more proximal concerns of classroom teachers within the current educational environment.

1.1 Reform agenda in mathematics education: from individual tuition to communities of practice

Reforming efforts within mathematics have supported changing understandings of what constitutes the domain of mathematics and what constitutes mathematics learning. Conceptions of mathematics as an unchanging body of knowledge are being replaced by conceptions of mathematics as an evolving form of practice conducted by communities of mathematicians. Similarly, conceptions of mathematics learning reflect this shift in perspective. Conceptions of mathematics learning as the acceptance of mathematical truths are also being replaced by conceptions of learning mathematics by doing mathematics, emphasising the collaborative construction of mathematical understanding.

1.1.1 Reform conceptions of what constitutes mathematics: mathematics as a unique form of discourse

Reform documents encourage the adoption of a new approach to the nature of mathematical knowledge (Lampert, 1990). The common view of mathematics within our culture, and subsequently in schools, is that mathematics is associated with certainty, with being able to get the right answer. These assumptions are shaped by school experience in which mathematics is commonly taught as a fixed, static body of knowledge (Romberg and Kaput, 1999). Doing mathematics means following certain rules, applying certain algorithms, and knowing mathematics means remembering and applying appropriate rules to answer certain questions (Lampert, 1990). However, it is the open character of mathematical findings that allows mathematics to grow and develop (Lampert, 1990). Instead of viewing mathematical understanding as a fixed all-or-nothing concept mathematical understanding can be more accurately described as cumulative,
fragmented and incomplete (Lerman, 1998). Mathematical understanding is always open to refutation, potentially advanced through hypothesis testing and strengthened through the development of proofs.

Recent conceptions of mathematics have promoted a “fallibilist” approach to mathematical knowledge (Tymozcko, 1986; Davis and Hersh, 1980; Lakatos, 1976; Ernest, 1991). Instead of viewing mathematics as the gradual accumulation of truth statements which are irrefutable, mathematical knowledge remains uncertain, dependent on axiomatic systems and open to question. Historical examples of mathematical knowledge being reconstructed include the development of imaginary numbers and the development of alternative axioms to Euclid’s five axioms and associated geometries. Accepted proofs remain open to refutation (even if many such proofs have remained unquestioned for centuries). Romberg and Kaput (1999) suggest that school mathematics should reflect the work of mathematicians working with existing concepts and their associated uncertainties. They suggest that students should be encouraged to find out why techniques work, invent new techniques and justify assertions in the same manner that research mathematicians operate.

Paul Kitcher (1984) provides a definition of mathematical knowledge that reflects this fallibilist perspective. Kitcher argues that mathematical knowledge consists of five components – a language, a set of accepted statements (including mathematical statements, diagrams, definitions, theories and proofs), a set of accepted reasonings accepted by the mathematical community, a set of questions selected as important, and a set of meta-mathematical views (including standards for proofs and definition and claims about the scope and structure of mathematics). Standards for proofs are not open for explicit description but rather proof standards are exemplified in texts taken as paradigmatic for proof making. Exemplary problems, solutions, definitions and proofs provide the foundation for accepted norms and criteria that such aspects of mathematical practice are expected to satisfy.
Kitcher’s five components of mathematical knowledge are emergent properties of historical mathematical activity. Mathematical understanding is inextricably linked to the activity of mathematical communities (Toulmin, 1999). This activity can be described as “pattern-seeking” (Schoenfeld, 1992; NCTM, 1989) and the term “mathematics” refers to this form of activity – a science of patterns, rather than a set of statements that are either true or false (Schoenfeld, 1992). Mathematical activity produces “language games” and mathematical “forms of life” (Wittgenstein, 1953) within which mathematical concepts have meaning and make sense to participants of that activity. Classroom communities are contexts in which such activity can be reproduced developing new “language games” and new “forms of life” for students to participate in giving birth to new forms of mathematical understanding.

A feature of mathematical activity often overlooked by theorists working in the field of mathematics education is its collective nature. Doing and thinking mathematics constitute a social practice (Stein, Silver and Smith, 1998) rather than an individual pursuit. Mathematical understanding resides within communities of practice, artefacts and interactions between individuals and their environments (Schoenfeld, 1992). Mental events and activities can be external to the body – suggesting that the concept of the internal sensorium needs to be replaced with the concept of collective knowledge (Toulmin, 1999; Ernest, 1998).

The dialectical relationship between the individual and their cultural heritage also contributes to the distributed nature of mind and cognition. Vygotsky (1978) suggests that wider cultural forces shape intellectual development and are in turn shaped by the products of human activity. Participating in a cultural practice results in the transformation of individual consciousness, just as the individual contributes to the ongoing evolution of the same cultural practice. Mathematical understanding, therefore, exists in Popper’s World 3 knowledge realms (Popper, 1972; Beretier, 1994). Popper’s World 3 knowledge refers to knowledge that exists as a public, collective object outside of an individual’s mind as part of the public domain. Such knowledge structures have histories and are open to criticism and falsification. Effective mathematics classroom
environments support students’ engagement in this process of ongoing mathematical practice.

Mathematical practice within the school context remains distinct from other disciplines studied at school such as science, history and studies of literature. Two distinct features of mathematical practice necessitate a unique approach to the appropriation of these practices. First, more than any other discipline studied at school, mathematical understanding develops principally through a process of deduction. Deduction and induction are necessary for the development of understanding in all domains. Schoenfeld’s approach to developing an effective knowledge base for solving mathematical problems, for example, emphasises the value of both experience at different types of problems (developing expertise through induction) and the development of effective methods (appropriating deductive principles) (Schoenfeld, 1987). The relative significance of these two reasoning processes, however, varies considerably across different domains. Mathematical practice typically relies more heavily on deduction rather than induction. Students learn about trigonometry, for example, by building on other mathematical concepts such as similar triangles, right-angled triangles and algebra.

The value of deduction as a logical procedure, however, is dependent on the viability of the initial assumptions. Included in the category of assumptions are definitions, conventions and mathematical tools such as systems of notation that carry implicit assumptions. In the case of notation, for example, certain systems simplify the practice of mathematics and therefore become accepted as viable systems. For students and professional mathematicians it is the assumptions upon which deduction proceeds that remain open to question rather than the procedures of mathematical logic (Lampert, 1990). Developments in mathematics often occur through the questioning of these “assumptions”. Historical examples of particular note include the development of complex analysis, non-Euclidean geometries and the mathematics of the infinitesimal. The development of symbolic representations is also commonly associated with advancements in the development of mathematical practice. The development of algebra,
the Cartesian plane, the Argand diagram and the Hindu-Arabic number system itself have all resulted in progress in mathematical understanding.

Similarly, within the classroom, quantum leaps in mathematical understanding are associated with the questioning of assumptions (Richards, 1991). The questioning of assumptions represents a significant challenge for most students of mathematics, and students are unlikely to see the need for such revisionist activity without the cultural insights afforded by the teacher into the viability of different assumptions. Students need to appreciate the cultural reasons for such mathematical activities that make use of negative numbers, complex numbers, the replacing of numbers with pronumerals and other accepted mathematical notions. The acceptance of such concepts occurs through the negotiation of mathematical understanding by the community of mathematicians and this same process of negotiation is required in the classroom between members of the classroom community.

Mathematical understanding, therefore, develops through deduction and the reformulation of assumptions. Most mathematical practice in classrooms, however, focuses on induction – developing mathematical understanding through multiple examples and exercises. Practice encourages students to ask “how” questions. Mathematical discussions, however, also have the potential to raise “why” questions about mathematical structures and procedures. Deducing mathematical principles and procedures in traditional classrooms is the activity of the teacher alone who presents certain procedures, including proofs underlying such procedures, for students to learn for later reproduction in examinations.

A further distinctive feature of mathematics compared to other disciplines taught in secondary school is the higher level of abstraction evident within mathematical “objects” such as theories, proofs, procedures and logic compared to “objects” pursued in other disciplines. Mathematical learning activity, therefore, represents a distinct form of learning activity within the classroom in which the appropriating of mathematical abstractions (mathematical tools) represents a meaningful practice within the context of
activities designed to produce culturally valued outcomes. In the context of mathematics, for example, students appropriate trigonometric methods to determine the height of a building or the distance between two objects.

The model of mathematics learning under investigation in this study aims to support student appropriation of cultural mathematical tools as the principal activity within the classroom. Student motivation for participation in the activity of appropriation does not lie within the cultural tools themselves (an intrinsic source of motivation) but from the advantages students gain from using such tools within the context of productive mathematical activity. It is arguable that intrinsic motivation for appropriating processes (in which students are motivated to learn such processes in isolation) represents an inappropriate expectation of students who are purposeful actors in cultural activities. Students may appropriate mathematical tools for several reasons related to the efficacy of these tools for enabling them to achieve personally valued outcomes. For instance, students may be motivated to appropriate mathematical tools to support their overall understanding of mathematics or if they perceive that such tools will be useful in their future workplace (NCTM, 2000). Students may also be motivated to appropriate mathematical tools if they perceive such tools will assist them to achieve more proximate goals such as doing well on external exams. At the next level, students may also be motivated to appropriate mathematical tools to assist them solve quantitative problems of personal interest to them. Finally, students may be motivated to appropriate mathematical tools to support the learning of other members in their collaborative groups. The current model provides a framework within which each of these motivations is made salient for student participants.

1.1.2 Reforming conceptions of what constitutes doing mathematics

The NCTM document *Curriculum and Evaluation Standards for School Mathematics* (1989) outlines five general goals for all students of mathematics that can only be achieved if radical changes in classroom mathematical practice are instituted
that they (students) learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, (5) that they learn to reason mathematically. (NCTM, 1989, p.5)

Each of these concerns has been echoed more recently, encouraging teachers to critically evaluate the model of learning they have inherited from their own school experience (AEC, 1991; NCTM, 2000; Grimison, 1995; Romberg and Kaput, 1999; Stacey, 1997; Boaler and Greeno, 2000; Forman, 1996).

The third goal of elevating problem solving to the centre of the mathematics curriculum is outlined by other theorists eager to reform mathematical practice in schools (Romberg and Kaput, 1999; Stacey, 1995; Forman, 1996). Encouraging students to become mathematical problem solvers requires students to define and organise data for solving open-ended problems and develop multiple solutions for these problems (Forman, 1996). Students develop problem solving skills through experience using processes of mathematics to solve specific types of problems – problems that are quantifiable, embodying change and variation that can be solved using symbolic algorithms and more abstract mathematical structures (Romberg and Kaput, 1999).

Communicating mathematically – the fourth goal outlined by the Curriculum and Evaluation Standards for School Mathematics, represents a significant shift in thinking from traditional approaches to mathematics encouraging students to do mathematics individually and in silence. A growing awareness of the social nature of mathematical knowing within the wider mathematical community (Lerman, 2000; Stein, Silver and Smith, 1998) has raised concerns about the appropriateness of the traditional model of didactic instruction and individual learning (Forman, 1996). Learning to communicate mathematically presumes that students in mathematics classrooms have opportunities to work in small groups negotiating methods and approaches with other students (Boaler, 2000), taking turns explaining ideas (Forman, 1996; Richards, 1991) and engaging in the dialectical process of criticism and justifying other student's mathematical claims (Goos, 2000; 2004). Through engaging in mathematical discursive activities students appropriate
mathematical forms of thinking and become more central members of the mathematics classroom community (Lampert, 1990; Richards, 1991). Providing students with opportunities to work with mathematical ideas in this manner gives students opportunities to learn in a manner consistent with how mathematics is used outside the classroom (Boaler, 2000). Several theorists have suggested that traditional approaches to teaching mathematics teach students how to succeed at school rather than prepare them for engagement in real world problems (Resnick, 1991).

The fifth general goal of learning to reason mathematically demonstrates considerable overlap with the goals of developing students’ ability to solve problems and communicate mathematically. Within mathematical classroom environments students make conjectures (Richards, 1991), work with abstract mathematical activities, engage in developing explanations for their actions and validating their assertions, as well as discussing and questioning their own thinking (Lampert, 1990).

Following the 1989 document, the NCTM developed *Professional Standards for Teaching Mathematics* (NCTM, 1991), identifying five shifts in the teaching of mathematics necessary to achieve these goals in modern mathematics classrooms. First, a shift towards viewing classrooms as mathematical communities instead of a collection of individuals is required. Second, a shift towards logic and mathematical evidence as the process of verification instead of the teacher’s authority is required. Third, teachers need to support a shift in students’ mathematical practice from merely memorising procedures toward appropriating mathematical reasoning. Fourth, a shift is required from mechanistic answer finding towards conjecturing and problem solving as the principal aspects of classroom mathematical practice. Finally, a shift towards connecting mathematics, its ideas and application instead of treating mathematics as a body of isolated concepts and procedures is required.
1.1.3 Gap between this ideal and current practice

Despite the efforts of administrators, teachers, teacher-leaders, teacher educators, mathematicians and policymakers this ideal of mathematics learning is not the reality in most classrooms today (Stigler and Hiebert, 1997; NCTM, 2000; Jaworski, 1994). Traditional pedagogies continue to dominate mathematics classrooms, particularly in senior classes (Rogers, 1995; Boaler, 1997a). Often, attempts to incorporate the principles of the reform documents are unsuccessful due to student expectations about what constitutes appropriate classroom interactions (Forman, 1996). When students are required to work with one another in small groups confusion is evident amongst students about what forms of student-student interaction are allowed. A second difficulty with reform classrooms emerges when students perceive inherent contradictions between the obligatory assignation of grades and student participation in open-ended problem solving activities in which multiple ways of reasoning are acceptable (Forman, 1996). The individual teacher working within systems in which normative grading procedures are mandatory inevitably faces considerable difficulty resolving this contradiction for students (Boaler, 1997b).

Some trenchant obstacles to the realisation of this ideal that are outside the control of individual classroom teachers, however, are likely to remain in the near future. In particular, the requirements of teachers to cover large areas of mathematical content in limited time is likely to place significant pressure on classroom teachers to cover concepts superficially, to ignore requests by policymakers to incorporate extended investigations and to teach to the exam. Furthermore, the existence of high stakes examinations such as the Higher School Certificate (HSC) in New South Wales and the Victorian Certificate of Education (VCE) in Victoria that determine students’ career opportunities place considerable pressure on classroom teachers. Not only are these exams critical to the future success of students, but students’ exam results are often used as the principal indicator of the quality of education being offered by different schools. In the US high stakes assessments used in different states can also have consequences for schools and teachers. In Florida, for example, low performing schools and high performing schools on such assessments are awarded additional funding (Sandham,
1999). Performance on such exams, therefore, remains a highly desired outcome of mathematical classroom activities.

In a study comparing the learning of students in traditional and enquiry-based classrooms, Boaler (1997b) discusses the impact that the General Certificate of Secondary Education (GCSE) examinations had on a school that had incorporated some of these ideals into their classroom practice. At "Phoenix Park" where students learnt mathematics through completing projects rather than textbook exercises and were provided with opportunities for discussion and negotiation of meaning, Boaler reports that this approach was discarded as the time came to prepare for the GCSE examinations. Under pressure to increase GCSE grades, teachers at Phoenix Park were required to teach from the textbook and abandon the project-based approach. Teachers at Phoenix Park who had previously taught using the project-based approach felt demoralised and disempowered as a consequence of this change in policy. One teacher at Phoenix Park left the teaching profession as a consequence of this change.

The nature of external assessment procedures used and the mathematics curriculum are factors outside the control of the classroom teacher and are determined through dialogue between communities outside the classroom including educational administrators, higher education faculties, community members and professional organisations and policymakers (NCTM, 2000). Classroom teachers often feel left with little choice but to persevere with traditional methods of teaching that enable students to fulfil the requirements of these external communities.

Internal forms of assessment are also designed to provide relevant stakeholders with information about the performance of students in mathematics classes. Parents in particular want to know the relative performance of their child compared with other students in their class and teachers typically obtain such information through regular topic tests and yearly examinations. In the US, data collected for the National Assessment of Educational Progress (NAEP) indicated that students in year eight were tested once a
week using traditional paper and pencil examinations (Mitchell, Hawkins, Jakwerth, Stancavage and Dossey, 1999).

However, within most classrooms in Australian schools and schools overseas there are two principal features that may support the incorporation of aspects of the reform program outlined by educational theorists. First, with twenty to thirty students in each classroom there exists within the classroom a rich plurality of student voices for dialogue, critique and the development of a community of mathematicians. Second, the mathematics teacher as the representative of the historically developed international culture of mathematics has a valuable role to play providing students with opportunities to compare their developing understandings with those developed by other members of the international mathematics community. The purpose of this study is to provide a model of mathematics learning that makes the most of these two strengths of traditional mathematics classrooms for realising the ideals of the reform movement. At the same time, the proposed model examined within this study is designed to enable students to perform well on high stakes exams within a classroom environment that incorporates the recommendations of the reform movement.

1.2 Theoretical perspective on learning: the cultural-historical tradition and its emphasis on the concept of activity

The reform movement in mathematics education has theoretical roots in a larger movement within educational psychology in that it describes learning in terms of participation in cultural practices (Lave and Wenger, 1991; Vygotsky, 1978; Greeno 1997). In the domain of mathematics the theoretical approach focusing on the principal theoretical category of “activity” has been influential in the work of Lampert (1990; Lampert and McCombs, 1998), Goodchild (2001), Crawford (1996), Säljö and Wyndhamn (1993) and Gordon (1995). Reform documents also focus on practical activity rather than individual cognition as the fundamental location of mathematical activity. The National Statement on Mathematics for Australian Schools, for example,
sees mathematics as an emergent property of purposeful, student activity (AEC, 1991). While there are different schools within this general movement such as situated learning, social constructivism and sociocultural theory, each one of these approaches draws on the work of Lev Vygotsky whose theories in the Soviet Union were the basis for what Vygotsky himself referred to as cultural-historical theories of learning.

1.2.1 Cultural-historical theory: Vygotsky, Leont’ev, Luria and Engeström

The central theoretical concept within cultural-historical theories is the concept of activity. “Activity”, or deyat’el’nost in Russian, does not refer to individual actions but rather refers to a system that incorporates individual agency, mediation of action through tool use, and the recognition of historically developed rules and practices of divisions of labour.

Activity is a molar, not an additive unit of the life of the physical, material subject. In a narrower sense, that is, at the psychological level, it is a unit of life, mediated by psychic reflection, the real function of which is that it orients the subject in the objective world. In other words, activity is not a reaction and not a totality of reactions but a system that has structure, its own internal transitions and transformations, its own development. (Leont’ev, 1978: Section 3.2)

This system, rather than its individual components, is the unit of analysis within activity theory since activities involve human actors who are motivated towards an object, their actions being mediated by tools and their community (Engeström, 1987). The object (or goal) of an activity system is the defining feature of the system and the notion of goal-directed behaviour is central to the concept of activity (Wertsch, 1981).

Several principles arise from this activity-oriented approach to cognitive processes. The first principle is that consciousness is an emergent property of practical activity (Blanton, Moorman and Trathen, 1998). By engaging in practical activity or labour, and using instrumental as well as psychological tools, both the physical and the mental world are transformed. Secondly, activity theory emphasises the cultural and historical processes that underlie activity systems. Activity theorists often use non-experimental
methodologies to trace the historical roots of different actions, and to tap into the shared understanding of the community that directs the perceptions, attention and interpretations of its members.

1.2.1.1 Vygotsky’s theory of learning

Vygotsky began from the perspective of identifying uniquely human characteristics of thinking. Thus, he made reference to lower mental functions and higher mental functions. Higher mental functions were unique to humans and were formed through the radical transformation of lower mental functions. The constructive principle for these higher mental functions resided in psychological tools and interpersonal relations, both of which are a consequence of cultural-historical activity.

Genetically, they differ in that in their phylogenesis they are the product not of biological evolution but of the historical development of behaviour ... they have a special social history. (Vygotsky and Luria, 1994, p. 137)

In contrast to the reflexologists, who approached the higher psychological functions by examining lower psychological functions such as reflexes and simple motor functions, Vygotsky proposed that higher psychological functions are emergent functions that need to become the focus of psychological research. Vygotsky uses the metaphor of the water molecule to explain the relationship between higher psychological functions and lower psychological functions (Vygotsky, 1986). Hydrogen and oxygen have specific qualities (as do lower psychological functions) that are radically different to the qualities of the water molecule that they form together (as is the case with higher psychological functions). The properties of lower psychological functions are superseded or abolished (Vygotsky borrows Hegel’s concept of *aufgehoben* to describe this process) with the formation of higher psychological functions.

Three main themes can be identified within Vygotsky’s writings (Palincsar, 1998) which have been alluded to in the preceding discussion regarding higher psychological functions. First, Vygotsky emphasised that individual development, including the development of higher mental functions, is social in origin. This is most clearly seen in
Vygotsky’s “genetic law of development” which states that higher mental functions appear first in the social realm as interpsychological categories and secondly in the psychological realm as intrapsychological categories (Vygotsky, 1978).

Learning and development, therefore, contain an essential unity in that they both arise from social processes. Vygotsky’s theory therefore, is in stark contrast to theories of psychological development associated with Piaget and Freud that focus on the individual as an entity that interacts with, but is fundamentally independent of, social processes (Davydov, 1995). Vygotsky also rejected the idea of some generic trajectory of development. Unlike Piaget, who saw development as a linear progression that was universal and teleological, Vygotsky proposed that development arises through social interactions and this development has a specifically historical character (Davydov, 1995). Vygotsky proposed that development was a consequence of cognitive restructuring. Rather than moving in a logical progression, development often occurs in contrary directions to its general trajectory. In *Mind in Society*, Vygotsky criticises Piaget’s approach for suggesting that “maturation is viewed as a precondition of learning but never the result of it” (Vygotsky, 1978, p.80).

The second theme that dominates Vygotsky’s work is the role of tools and artefacts in human activity. In attempting to forge a new path that avoided the pitfalls of reflexology and psychoanalysis, Vygotsky outlined the role of *tool mediation* in the development of higher psychological functioning (see Figure 1).
According to Vygotsky, the relationship between human agents and the environment is mediated by physical tools, symbolic tools (such as language, mnemonic techniques and decision-making procedures) and cultural means (Engeström, 1998). These tools have their origins in social interaction, whereby an “object” becomes identified as a “tool” (Engeström and Escalante, 1996). Vygotsky recognised that through the manipulation of cultural symbols, and in particular language, human thought is radically transformed (Vygotsky, 1978).

The third theme that emerges from Vygotsky’s writings is the need to examine these various functions “historically”. Vygotsky suggested that

To study something historically means to study it in the process of change; that is the dialectical method’s basic demand. (Vygotsky, 1978, p.64)

According to Vygotsky the determinants of human activity, consciousness, and personality, are to be found in the historically developing culture (Davydov, 1995). Vygotsky suggested that psychological development can only be understood as a general historical development (Vygotsky, 1978). This development incorporates phylogenetic,
cultural-historical, ontogenetic and microgenetic history – all of which contribute to the psychological make-up of an individual at any point in time (Cole and Engeström, 1993).

Vygotsky’s methodological approach to consciousness and the themes of his psychological work constitute the building blocks of activity theory. Throughout his writings the emergence of an overarching framework within which to analyse human thought and behaviour focusing on tool mediated activity systems is evident. This concept of activity was developed further by Vygotsky’s colleagues Leont’ev and Luria.

1.2.1.1 Vygotsky’s concept of activity
The concept of activity emerges in Vygotsky’s writings for the first time in “Consciousness as a Problem of Psychology of Behaviour” published in 1925. Prior to the publication of this work, Vygotsky had maintained that “consciousness is a reflex of reflexes” (cited in Davydov and Radzikhovskii, 1985, p.47). In doing so, Vygotsky adopted the theoretical framework that dominated Soviet psychology at the time and directed the concept of the reflex towards the problem of consciousness. Yet Vygotsky became aware of the inherent reductionism of this approach that would lead to errors about the nature of consciousness and his focus shifted towards the study of the structure of behaviour rather than reflexes. From a focus on behaviour, Vygotsky moved towards analysing consciousness as an emergent property of the labour activity of humans (using Marx’s writings on the division of labour) (Cole, 1996). According to Vygotsky and Luria, the means by which we use tools differentiates humans from animals. But it is not merely tool use, for animals too are able to use aspects of their environment to assist them. Humans, on the other hand, use tools to obtain mastery over the environment through goal-directed action (Vygotsky and Luria, 1994).

Concerning consciousness, wherein the work process lies prior to its realisation, Vygotsky suggested that activity is the minimal unit appropriate for understanding consciousness and the higher psychological processes.
The behaviour of a small child … presents a complex skein: it consists of a mixture: direct attempts to attain the goal, the use of tools …. and a direct appeal to the object of attention. This strange alloy becomes meaningless if considered separately from its dynamics. (Vygotsky and Luria, 1994; p.118)

It is this notion of activity that is developed by Leont’ev who emphasised practical activity over symbolic activity.

1.2.1.2 A. N. Leont’ev’s activity theory

Leont’ev saw in activity a means of breaking down the dualism inherent in individualistic psychology (Leont’ev, 1972). According to Leont’ev, all psychological theories present a two-part scheme in which an influence on the subject leads to a response. This finds its clearest presentation in behaviourist stimulus-response theory. This approach is inappropriate according to Leont’ev because it fails to incorporate the processes that active subjects use to form connections with the world – Leont’ev refers to these processes as “objective activity” (Leont’ev, 1972, p.42). For within Leont’ev’s approach activity (which he equates with Marx’s definition of production) objectivises the individual and subjectivises the object (Leont’ev, 1972). This reciprocal transformation occurs as the collective activity of individuals leads to the production of certain outcomes of subjective value to the participants involved. The object of the activity is understood subjectively by each of the individuals involved in the collective activity who appropriate historically developed ideas to make sense of their activity. Furthermore, as each individual participates in the activity they are perceived by other participants as significant components within the production process and are objectified accordingly.

Leont’ev discounts individualistic phenomenological approaches to the interaction between the individual and the external world. This relationship cannot be understood in terms of the individual merely making sense of the external world through some cognitive filter. Such approaches return to the two-part design of conventional psychology plus intervening variables, failing to resolve the traditional dualism between subject and object. Activity-based theories by way of contrast, unite subject and object into an organic whole, the meaning of which can only be obtained by knowing the
phenomenological landscape that each participant appropriates from their cultural-historical context.

Leont’ev’s program for understanding activity focused on three levels or lenses through which activity systems can be examined (Kozulin, 1986; Kuutti, 1996). These different levels are defined according to their functionality rather than their intrinsic properties (Wertsch, 1981). At the uppermost level, collective activity is correspondent with motives towards obtaining a particular object (the word “object” is used here in a similar sense to the word “goal”). The object of a particular activity, for example, may be the building of a house. At the intermediate level within the activity system, individual actions can be discerned related to goals that are more immediate in their execution. In the example of building a house such actions may include fixing the roof, or transporting bricks to the building site.

The concept of action becomes important within activity systems that include some division of labour (Leont’ev, 1972). For while the collective activity is directed towards a certain product or goal, individual action may in fact be directed towards “intermediate” goals that may not satisfy individual needs that are only satisfied by the achievement of collective goals.

The appearance of goal-directed processes or actions in activity came about historically as the result of the transition of man to life in society. The activity of participators in common work is evoked by its product, which initially directly answers the need of each of them. The development, however, of even the simplest technical division of work necessarily leads to isolation of, as it were, intermediate partial results, which are achieved by separate participators of collective work activity, but which in themselves cannot satisfy the workers’ needs. Their needs are satisfied not by these “intermediate” results but by a share of the product of their collective activity, obtained by each of them through forms of the relationships binding them one to another, which develop in the process of work, that is, social relationships. (Leont’ev, 1978: Section 3.5)

Leont’ev explains the alienation that workers feel from their labour within the framework of activity theory. What energises the worker’s actions (the collective goal) and the goal
towards which their activity is directed (the intermediate goal) may not coincide causing psychological conflict.

Finally, Leont’ev’s tripartite theory of activity identifies automatic operations that are determined by the conditions and the tools available (Engeström, 1998). These automatic operations may include hammering nails or changing gears in the truck using the house-building analogy. Western approaches to psychology seem to have focused primarily on the level of operations while ignoring the higher levels of activity (Wertsch, 1981).

In many ways, Leont’ev’s theory is much closer to traditional Marxist theory than Vygotsky’s (Zinchenko, 1995) with regards to the materialist origins of human thought and the impact on human psychology of the alienation of the worker. Leont’ev’s approach to activity theory however was relatively insensitive to cultural diversity and the role that culture and historical context play in the development of higher psychological functions as outlined by Vygotsky (Engeström, 1998). Whereas for Leont’ev consciousness is mediated by tools and objects, consciousness for Vygotsky is also mediated by culture. Through the writings of Alexander Luria however, it is possible to identify the re-instatement of culture as an important contributor to consciousness and activity systems.

1.2.1.3 Luria’s contribution to activity theory

Alexander Luria had many research interests and is best known for his work in neuropsychology. Yet Luria’s contribution to the theoretical ideas of activity theory are significant arising from his involvement with Vygotsky and his subsequent studies examining the role of culture in human thinking. He began his work in the psychology of learning using psychoanalysis as a means of understanding human thinking and it was through this interest in psychoanalysis that Luria became involved with Kornilov at the Moscow Institute of Psychology (Cole, 1978). Luria’s work took a significant turn after meeting Vygotsky in 1924 and as a consequence he became interested in how culture influenced human thought. As early as 1928, Luria undertook studies into how children developed their cultural understandings required by their social context (Luria, 1994;
In doing so, Luria acknowledged the inter-relationship that Vygotsky emphasised between tools, the cultural environment and human thinking. Luria concluded from his studies of reasoning processes amongst Uzbeks that human cognitive activity is a part of the wider process of social history being coded in language (Cole, 1996).

Luria’s contribution to the concept of activity therefore is his re-instatement of the importance of language and culture. From Vygotsky’s theory, which emphasised the interplay between language, culture, history and tool use, Leont’ev focused on the material aspects of activity whereas Luria stressed the significance of cultural and symbolic factors within the activity system. From these two threads of Vygotskian theory modern approaches have developed several conceptions of activity, incorporating these two strands to varying degrees.

1.2.1.4 Engeström’s approach to activity theory

Recently, the work of Engeström (1987) has contributed to the theoretical tools that cultural-historical theorists use to understand practical, object-oriented activity. Engeström outlines three main principles of activity theory drawing on Vygotsky’s cultural-historical theory (Engeström, 1993). The first of these principles is that the collective activity system is the fundamental unit of analysis. As Vygotsky had suggested previously, the unit of analysis cannot be reduced to the individual in isolation but must also incorporate the cultural, social and historical factors that interact with each other to influence behaviour.

Engeström has developed a diagrammatic way of representing the different components that exist within activity systems emphasising the inter-relationships between these different components (see Figures 2 and 3). He begins with the mediating role that community plays between subject and object. “Community” refers to a group of individuals who share a set of social meanings (Holt and Morris, 1993).
From the initial mediated relationship between the subject and object which is common to all species, ruptures emerge in each of the sides of the triangle. Over time humans have developed tools which transform the relationship between themselves and their environment as well. The relationship between the individual and their community is also mediated by the existence of traditions, rules and rituals. Furthermore, the community’s relationship with the object is mediated by the development of divisions of labour. Thus, the diagrammatic representation of activity developed by Engeström has six major components – subject, object, community, tools, rules and divisions of labour (see Figure 3).
Engeström’s second principle of activity theory is that internal contradictions are the driving force behind disturbances, innovations and change (Engeström, 1993). Within each system, paradoxes exist that lead to contradictions within and among the components of the system or between a system and an emerging more advanced system (Holt and Morris, 1983). According to Engeström, school-going as an activity includes contradictions within each of its nodes which are reproduced in Figure 4 (Engeström, 1987). Each of these contradictions is related to the fundamental contradiction of capitalist systems between use value and exchange value. In the case of school-going activity, the transforming of certain texts may have use value becoming an instrument of mastery which can lead to the development of new forms of activity. However, the transforming of texts (normally by reproducing such texts \textit{in toto}) can also primarily have exchange value: that is, it can be exchanged for grades (see Figure 4).
Within the field of mathematics education, activity-based approaches have examined how classroom activity systems encourage and discourage different forms of mathematical practice. Nunes (1995), for example, observed how the symbolic systems used in mathematical classrooms that are provided for students to assist them with calculations (a cultural tool) may in fact become the object of the activity becoming an end in themselves. In an ethnographic study looking at student perceptions of their learning, Goodchild (2001) found that students’ mathematical practice in the classroom did not encourage reflection with most mathematical practice occurring at the level of operations (automatic processes) rather than actions (goal directed processes). Students appeared to be motivated towards producing answers for the teacher’s benefit rather than developing insight or their own understanding.

From both reform documents on mathematics and activity-based sociocultural theories of learning, eight adjectives describing effective mathematics learning emerge – such learning can be described as collaborative, dialogical, dialectical, cultural, self-regulated, meaningful, motivated and developmental. These eight adjectives relate to three principal aspects of mathematical classroom practice – the nature of collaboration within effective classrooms, the development of self-regulated learners, and the degree to which mathematical practice in the classroom resembles real world mathematics.

2 Eight principles of effective mathematics classrooms

The concerted effort to reform mathematics classrooms has focused on developing principles for effective mathematics learning (NCTM, 1991; Cockerst, 1982; NCTM, 2000). The reform movement in mathematics education has evolved within the wider context of educational psychology in which general principles of learning have been identified (Lambert and McCombs, 1998; Brown, Ellery and Campione, 1998). Research
from within this tradition as well as other traditions has contributed to the development of principles for mathematics classrooms.

These principles for effective mathematics learning are consistent with relevant research into teaching and learning mathematics (Hiebert, 1999) drawing on empirical research as well as changing ideas about what constitutes mathematics and what one means by the phrase “doing mathematics”. They also incorporate values about what type of mathematics students should be learning and how mathematics should be encountered in the classroom. For example, whether educators value skills or concepts as the primary subject matter will have a bearing on the principles selected for implementation in the classroom.

Determining the validity of these principles through empirical investigations, however, will always be fraught with difficulties due to the complexity of mathematics classrooms and the multiple factors that influence different outcomes (Hiebert, 1999). For example, while principles might outline key factors for effective learning in mathematics, these principles might not automatically result in higher outcomes for mathematics students. Classroom activity represents the confluence of multiple trajectories of emergent practice that arises from change associated with curriculum reform, changing societal expectations, the interaction of individual histories and changing school policies. The accumulation of research investigating the benefits of different approaches to mathematics can, however, provide a richer picture of how different aspects of classroom practice can be incorporated to enable students to achieve valued outcomes in the classroom. The purpose of the current study is to develop one such approach to incorporating these ideals that can be easily adopted by classroom teachers.

There are eight principles promoted by the reform movement in mathematics education, many of which are evident in Vygotskian approaches to other areas of the curriculum. These eight principles are fostering collaboration, developing a dialogical classroom in which many different voices are heard, providing opportunities for students to become more central participants in communities of practice, embedding learning experiences in
cultural practices, encouraging students to become self-regulated learners, making mathematics meaningful, supporting intentional learning, and developing learning experiences that are appropriate given students' prior experience and development. Each of these principles will be described in more detail in sections 2.1 to 2.8.

2.1 Effective mathematics learning is collaborative

The value of student collaboration for supporting student learning represents an important principle of effective learning in mathematics (NCTM, 1991, Grimison, 1995; Lambert and McCombs, 1986). Supporting collaboration within the classroom is also a key aspect of Vygotskian approaches to learning (Vygotsky, 1978) such as those developed by Brown, Ellery and Campione (1998), Shayer and Adey (2002), and Goos (2004). The 1991 NCTM document *Professional Standards for Teaching Mathematics*, for example, emphasises the changing focus in mathematics education from considering classes as groups of individuals to communities of mathematicians. In Great Britain, the Cockcroft report (1982) encouraged teachers to provide students with opportunities to discuss ideas amongst themselves as well as with their teacher. In Australia, the Mathematics Curriculum and Teaching Project (Lovitt and Clarke, 1988), for example, placed a significant emphasis on students working together. Similarly, Grimison (1995) suggests that developing students’ sharing and communication skills should be a prominent feature of mathematics classrooms while Lambert and McCombs (1998) highlight the significant influence of social interactions, interpersonal relations and communication on learning. Finally, collaboration also represents a necessary aspect of authentic learning environments (Burke, 1994).

The process of collaboration is the process of creating a shared social world amongst students participating in the collaborative activities (Palincsar and Herrenkohl, 1998). Achieving shared understanding involves cognitive stretches from participants that contribute to their ongoing development (Rogoff, 1998; Baker-Sennett, Matusov and Rogoff, 1992). The challenge of learning how to deal with different opinions and achieving consensus represents a significant challenge for students engaged in collaborative activity (Palinscsar and Herrenkohl, 1998; Chang-Wells and Wells, 1993).
2.1.1 Collaborative learning as a strategy for transforming classroom activity

Research investigating the impact of collaborative learning on classroom activity has identified several changes that take place as a result of students working together in small groups. Working in small groups provides students with opportunities to learn mathematics and ways of reasoning in ways that are congruent with real-world tasks (Lampert, 1986; Schoenfeld, 1983). Students also develop problem solving skills, practise meaningful tasks, learn prosocial skills and have opportunities to verbalise thoughts about mathematics (Bossert, 1988; Good, Mulryan and McCaslin, 1992). Constructive controversies can also be beneficial in which heterogeneous groups are forced to accommodate the opinions of others, engage in problem solving and take different perspectives (Bossert, 1988).

Other theorists have suggested that certain classroom environments are more conducive for collaborative learning than others. De Lisi (2002) suggests that collaborative learning is only likely to be successful in classrooms where there exists a mutual respect between teachers and students.

A classroom that is largely teacher-directed and based on obedience and constraint is unlikely to reap the intended benefits from peer learning activities. In such a context, the team is likely to focus on trying to please the teacher and will be less willing to share ideas in an open ended fashion. Neither the academic nor the social benefits of peer experiences are likely to be realized if the larger context in which learning occurs is constraint and unilateral respect.

Conversely, if the general classroom context is one in which the teacher and student have mutual respect for each other, then it is more likely that peer team members will also have mutual respect for each other. In this context, peer team members are more likely to feel comfortable with a free exchange of ideas that can lead to both deeper levels of understanding and an appreciation of another person's individuality. (De Lisi, 2002, p.6)

Encouraging change from existing classroom environments to desirable classroom environments in which students are able to collaborate with each other represents a key
focus of the current study. Classrooms that support collaborative activity amongst students have been shown to have many benefits for students in terms of their learning, their use of thinking skills and their social skills. In a survey of research into group work, Good, Reys, et al. (1989 - 1990) found that most students observed exchanged mathematical ideas in small groups and many students observed developed social and communication skills. Many lessons were also designed to develop higher order thinking skills - involving discovery, problem solving and developing concepts. Work groups frequently provided students with opportunities to explore diverse and sometimes more advanced mathematics.

Work groups also created some difficulties for classroom communities. When teachers developed their own group work activities there was less continuity with the curriculum, and some of the activities developed for groups were inappropriate and were presented to students without a reason for them to work in groups. Tasks for groups to complete should also be varied and at a level of difficulty appropriate for students. Designating roles in groups was rarely beneficial and often seemed artificial, and within each group there were some students who relied on other students to do most of the work. Finally, there was little accountability for each group, or time given to summarizing findings or discussing what different groups had learnt.

For better or worse, organising students to work in groups can have a major impact on the activity that occurs within the classroom. Given the benefits and difficulties associated with group work identified by Good and Reys et al. it is vital that collaborative group work in classrooms is structured to ensure that the benefits of group work are achieved and the difficulties are overcome. Webb (1989; Webb and Farivar, 1994) identifies several key variables for consideration when planning group activities in the classroom. These variables can be categorised as task (nature of learning tasks, subject matter, opportunities for dialogue and inquiry, allocation of time and materials), teacher and instructional variables (teacher's perceptions and expectations, classroom characteristics, teacher role, classroom management, preparation of teachers, assessment and accountability) and individual student and group variables (age/grade, achievement,
gender and SES, previous experience in school, personality variables, socialisation of inner speech, student perceptions, group composition and stability of group membership).

In contrast to this approach, Forman and McPhail (1983) outline a socially situated approach that focuses on the dialectic between the social-instructional environment and the individual student as the unit of analysis rather than specific variables. Within this approach it is not possible to dissect learning situations to identify separate variables, since by doing so, the characteristics of the whole would be lost.

In a socially situated perspective, then, the individual is always engaged in relation to others: learning, motivation and socialisation are not distinct processes ...socialisation encompasses learning, motivation and socialisation processes. A contextual approach to small-group learning rests on a construct of socialisation that involves dynamic change among evolving systems; it is concerned with the emergent interaction of the self, task and other in an historical and ongoing sense. (Forman and McPhail, 1993, p.214)

Vygotskian approaches to collaborative discussion as outlined by Forman and McPhail (1993) view collaborative problem solving as a culturally and historically situated activity embedded in cultural practices and institutions. Cultural and historical contexts influence the psychological meaning of the activity and the actions of the participants transform the institutional context, mediational means, beliefs and values. Theoretical questions asked by this approach focus on the role of cultural mediators, cultural values and beliefs, social roles and the status of individuals involved. Instead of isolating key variables that have an impact on the collaborative dialogue, Vygotskian approaches aim to examine the activity as a “molar unit” (Leont’ev, 1978) and its subsequent emergent properties.

2.1.2 Relationship between collaboration and learning

Significant benefits for learning arise from students collaborating with others. Students learn to take on other people’s perspectives and work with multiple ideas developing their own understanding and the understanding of others in their group. Groups that promote peer teaching have been found to have benefits for both those who do the teaching and those who are taught.
2.1.2.1 Learning through teaching

Groups that provide opportunities for students to teach each other bring benefits both to those who are taught and those who are teaching (Webb, 1989; 1985). Giving students opportunities to present their ideas and provide explanations supports the development of their understanding (Bossert, 1988), so long as students are encouraged to provide explanations rather than just answers (Webb, 1989). The value of giving help for the help-giver is most evident when helpers are encouraged to clarify and organise their thinking, often giving explanations in new or different ways (Webb, 1989). As a consequence, students who provide active instruction and task leadership during group work are most likely to benefit from grouping (Webb, 1989). Furthermore, help is beneficial to the receiver of this help when the help is directly relevant to the student's particular misunderstanding, when the degree and type of elaboration correspond to the help required, when the timing is appropriate in close proximity to student error or questioning, when the student understands the explanation and when the student has the chance to use the explanation to solve the problem (Webb, 1989).

2.1.2.2 Developing empathy with others through collaboration

Collaborative play between children requires participants to encourage each other to be understood and to understand the other (Rogoff, 1990). Similarly, collaborative argumentation between peers provide participants with opportunities for taking others' perspectives and resolving contradictions between different perspectives (Miller, 1987). Peers who are able to discuss their ideas with other peers are more likely to develop their own conceptual understanding (Gauvain and Rogoff, 1989) through the reformulation of these ideas.

2.1.2.3 Learning to incorporate multiple perspectives through collaboration

Approaching the same information from different perspectives through collaboration with others also engenders a flexible approach to learning, enhancing the adaptability of what is learnt to new environments not encountered within the classroom (Newman, 1997). This is demonstrated in Boaler’s study (1997b) in which students who learnt mathematics by solving problems collaboratively instead of listening to teacher expositions followed
by individual practice showed a greater ability to use mathematical concepts to solve new problems not previously encountered.

Students collaborating with others also have their ideas challenged by those of others and are free to critique other conceptions as well as their own. The public declaration of ideas for consideration by members of the group contributes to the development of shared understanding that transcends the intellectual domains of particular individuals. One of the aspects of effective teaching practice identified in the DETYA report *Numeracy, A Priority for All* (DETYA, 2000) is that students are given opportunities to share their thinking with the teacher and other students. "Thinking aloud" (Collins, Brown and Newman, 1989) represents an essential aspect of collaboration through which participants make use of publicly accessible mediational instruments such as talk, gesture and the manipulation of physical tools to contribute to the collaborative conversation. Dialogue within collaborative groups provides students with opportunities to organise their own ideas, gain insight into the topic of discussion through the multiple perspectives offered by the group, and contribute to the development of the group’s shared understanding. As students talk about informal strategies in mathematics for example, teachers are able to assist them to develop an awareness of their implicit informal knowledge (NCTM, 2000).

### 2.1.2.4 The CAME project: an example of a collaborative learning model

Across 17 schools in the United Kingdom, Adhami, Johnson, and Shayer (1998) have investigated the benefits of collaboration in mathematics as part of the Cognitive Acceleration in Mathematics Education (CAME) Project (Adhami, Johnson and Shayer, 1998; Shayer and Adey, 2002). This project was conducted with students from grades 7 and 8 over a period of two years. The teaching style adopted within this project involved breaking the lesson into three parts (Shayer and Adey, 2002). The first of these parts (Act 1) involved the whole class participating in a discussion in which the teacher provided the context for the problem to be attempted including relevant vocabulary. The intention of this stage is to induce the learning behaviour that will be required to solve problems in the second stage. The second stage (Act 2) was focused around small group collaborative work in which the teacher assisted groups by asking questions to focus them on possible
contradictions in their conceptual understanding. The third stage (Act 3) was a session of whole class discussion in which groups of students shared their ideas with the rest of the class. Results from this study found that students working within this project over a period of two years achieved higher results on mathematics tests, and teachers involved in the project reported that students showed improvements in their ability to ask questions, offer explanations, reflect on their learning and explain their reasoning (Adhami, Johnson and Shayer, 1998).

2.1.2.5 Collaboration and learning: a multi-level analysis

Collaboration, however, if defined loosely as students talking with each other about mathematics, need not result in improved learning outcomes. In a wide scale comparison of different classroom environments by Wenglinsky (2000) the results of 7146 mathematics students were analysed looking at the relationship between the different teaching methods used and the results achieved by students who were taught using these different methods. The analysis of teacher quality included variables such as experience, qualification, amount of professional development and teaching strategies. Teaching methods examined by Wenglinsky included placing emphasis on higher order thinking skills, encouraging students to participate in discussions about mathematics and hands-on practical activities. Using multi-level structural equation modeling, Wenglinsky identified a positive relationship between use of hands-on activities and test scores. Students of teachers who provided them with hands-on activities on a weekly basis (only about 8% of teachers in the study reported doing this) achieved results which were 72% of a grade level above students who did not take part in hands-on activities. Students who learnt mathematics in classrooms with an emphasis on higher order thinking skills also achieved higher results than other students. However, Wenglinsky did not find any positive or negative effect of discussing mathematics in small groups. Surprisingly, 86% of teachers reported that students in their class held group discussions once a week and 58% reported that they conducted classroom discussions at least once a week. 86% seems so high that what was being described by classroom teachers as “group discussions” may vary considerably from one class to another. The main difficulty with Wenglinsky’s analysis is that the measure of classroom interactions – teacher reports of what happens in
their classroom, is a relatively blunt instrument bringing together multiple phenomena and describing them as a single phenomenon.

Mathematical understanding, therefore, emerges through articulating what one knows, communicating one’s knowledge and reflecting on one’s conceptual understanding. Articulation of conceptual understandings in collaborative groups represents a public form of reflection (Carpenter and Lehrer, 1999). Through encouraging student discussion and collaboration classroom teachers can develop a supportive environment for students in which serious mathematical thinking is the norm. Collaborative activities provide a forum in which students are expected to justify their thinking, develop conjectures, conduct experimentation with various approaches, and construct and respond to mathematical arguments (NCTM 2000).

Few mathematics teachers engage students in a public analysis of the assumptions behind different explanations that lead to “correct” answers (Lampert, 1990). Typically, students are required to provide answers in classrooms without reasons, developing into effective symbol manipulators without necessarily grasping the semantic meaning of the symbols manipulated. Students of mathematics often resemble the interpreter of Chinese symbols in Searle’s Chinese room thought experiment (Searle, 1980). Many students are able to provide the correct symbolic answer to a question without necessarily understanding the meaning of the question or the symbols used to answer it.

2.1.3 Forms of collaborative dialogue: productive and unproductive argumentation

Different forms of dialogue can lead to significant differences in the success of collaborative learning. Argument, for example, can be an important part of learning. Developing a case for a particular idea or decision and counterarguments against alternatives are valuable learning activities for students (Greeno, Collins and Resnick, 1996). Argumentation between peers is perhaps more valuable than argumentation conducted with the classroom teacher since the opinions of the teacher carry significant authority with students (Mercer, 1995). In contrast, the opinions of peers represent
equally authoritative voices within the classroom that are open to critique, refutation or elaboration.

Different forms of argumentation, however, can be productive or maladaptive for supporting the processes of meaning negotiation and the appropriation of cultural forms of thinking. Mercer and colleagues (Mercer, 1995; Wegerif, Mercer and Dawes, 1999) outline three different forms of collaborative talk that students typically engage in during collaboration with other students. First, disputational talk between students is characterised by disagreement and individualised decision making instead of developing shared understandings. Such talk involves short exchanges of assertions and counter-assertions and rarely leads to the development of new forms of shared understanding (Wegerif, Mercer and Dawes, 1999). The second form of talk identified by Mercer (1995) is cumulative talk that is less confrontational than disputational talk. However, cumulative talk involves students building uncritically on what other students have said towards the construction of common knowledge. Cumulative talk is characterised by repetitions, confirmations and elaboration. The third form of talk identified by Mercer is exploratory talk in which partners engage critically but constructively with each other’s ideas. With exploratory talk, knowledge becomes more publicly accountable, and the reasoning of the participant is more visible in the collaborative dialogue that ensues. Within the context of exploratory talk there is freedom for students to present their ideas and critically assess their own ideas (Goos, 2000) and the ideas of others. In doing so, students are able to formulate conjectures regarding mathematical constructs and refutations in response to the ideas of others.

2.1.3.1 Conjectures and refutations

A key aspect of mathematical discussion is the encouraging of conjectures and refutations (Lampert, 1990; Lakatos, 1976). In her own classroom, for example, Magdalena Lampert (1990) led a classroom discussion designed to support students in their development of conjectures and provide an environment in which students felt comfortable refuting such conjectures where they saw necessary. In a study conducted by Brodie (2000) the classroom teacher would intervene to provide counter-examples without expressing doubt
about any particular method and encourage students to articulate their own ideas and critically evaluate them. The teacher also worked hard to argue that developing methods that did not work was also valuable from two perspectives – finding mistakes can lead to greater mathematical understanding, and secondly, some of the method is probably correct, because it had achieved partial success. In a Lakatosian sense, then, refutation allows for refining of conjectures. However, alternative forms of intervention may involve redirecting students’ attention towards aspects of the problem that may prove to be more profitable. Teachers could also focus students’ attention on what they know from previous problem solving activities. In the study, however, students had difficulty breaking down their method into smaller parts or lemmas. With an understanding of students’ own misunderstandings, a teacher could assist students to examine the different aspects of their method.

Research examining the quality of learning experiences that emerge from discussion of this kind has shown numerous benefits for students who are engaged in critiquing each other’s ideas (Hiebert, 1999; Kamii, Lewis and Livingston, 1993). Students are able to develop their own strategies for solving computational problems, and can assess the efficiency of different strategies and the generalisability of different procedures.

2.1.4 Collaboration and the development of shared values

Students working in collaboration with others also need to reach an explicit agreement about how such joint activity should progress (Chang-Wells and Wells, 1993). Collaboration requires students to adopt shared social values such as valuing the opinions of others, maintaining a modesty regarding personally held opinions (Lampert, 1990), the courage to revise one’s own beliefs when there is good reason to do so and the wise restraint to not change beliefs wantonly (Lampert, 1990). Some collaborative projects are often less successful than others when students feel uncomfortable commenting critically on other people’s ideas (Palincsar and Herrenkohl, 1998). Antagonisms based on culture or gender may also be responsible for group work descending into chaos (Chang-Wells and Wells, 1993). Other interpersonal difficulties that can limit the effectiveness of the
collaborative process include student preferences for working independently or with specific members of the classroom with whom friendships exist.

Students, therefore, need considerable support when collaborating with each other in order to develop appropriate forms of communication and shared values that are upheld by members of the group. Teachers and students need to establish some agreement in the classroom about what ‘talk’ in the classroom is intended to achieve and how it should be conducted (Mercer, 1995). Teaching students collaborative learning strategies, however, need not occur through a process of negotiation separate to the collaborative activity itself. Students also learn how to collaborate through observing collaboration modelled by the teacher and through the feedback and encouragement offered by teachers (Hagan and Weinstein, 1995). Teachers interacting with each of the collaborative groups have a responsibility for modelling effective collaboration with other members of the classroom community. Teachers contribute to students’ development as collaborative learners by providing positive feedback on students’ contributions to the process of developing shared understanding.

2.1.5 Developing collaborative communities of learners: Reciprocal teaching and inquiry mathematics

Across different subject areas, there has been a growing interest in supporting the development of communities of learners with particular characteristics. Brown and colleagues, for example, have developed a significant model of collaborative learning they describe as fostering communities of learners (CoL) in which students are provided with a small number of strategies for collaborating with each other that evolve into routinised activities over the duration of the program (Brown, 1994; 1997; Palincsar and Brown, 1984; Brown and Campione, 1994; Brown, Ellery and Campione, 1998; Palincsar, Brown and Martin, 1987; Brown, Ash, Rutherford, Nakagawa, Gordon and Campione, 1993). Several principles of learning underlie the design of these communities of learners which are summarised by Brown, Ellery and Campione (1998) in five basic principles that guide the design of classroom activities within communities of learners. First, students, teachers and experts are all participants with multiple roles as
researchers, learners and teachers. Usually, however, adult teachers take on the major role of directing the development of public knowledge within the classroom. Within communities of learners, Brown, Ellery and Campione (1998) recommend that students should be taught through guided discovery that requires teachers to be constantly monitoring student understanding and adjusting the level of support appropriately to guide them towards constructing new understandings. The second principle of learning that guides the design of CoL classrooms is that the learning of individuals and groups benefits from collaboration and knowledge sharing with others. Third, two essential ingredients of this collaborative process are distributed expertise as well as shared discourse, or common knowledge that enables participants to benefit from the expertise of others. Fourth, the development of a community of discourse characterises effective learning environments in which discussion, questioning and criticism are fundamental aspects of this discourse. Fifth, multiple zones of proximal development exist within the community of learners through which different learners move individually, legitimising individual differences within the community of learners.

Central to the community of learners approach is the model of collaboration described as reciprocal teaching (Palincsar and Brown, 1984). Reciprocal teaching represents a model of collaborative learning for supporting students’ reading skills. It involves students taking turns as leaders in small groups asking questions, summarising arguments, clarifying any problems, and asking participants for predictions about future content. Such activities are designed to support multiple zones of proximal development (Brown, 1994; Brown, Ellery and Campione, 1998). Through student engagement in such discursive practices, thinking becomes externalised, enabling less able students to benefit from the ideas of more able students.

Mathematical communities of practice, however, may not develop the same discursive patterns as communities of practice focused on developing reading skills. Instead of making predictions about what might happen in different literary genres, students in mathematics form stronger predictions based on quantitative data. While questions related to texts focus on the content of these texts, mathematical questions typically focus
on processes required to achieve certain outcomes. Summaries of arguments derived from
texts are also less important than deductive proofs of mathematical concepts. Effective
mathematics learning environments need to be able to support these and other specifically
mathematical forms of discourse.

Goos (2004) reports on the role of one teacher in transforming a mathematics classroom
from fostering learning mathematics through memorisation and rote learning to one in
which collaboration amongst classroom participants created a “climate of intellectual
challenge” (p.259). Within this emergent mathematical community of practice, students
learnt how to participate in mathematical discussions focused on solving problems, set
forth conjectures, and respond appropriately to the mathematical ideas of others. Both
Goos’ “inquiry mathematics” community of practice and Brown’s community of learners
share a common emphasis on collaboration and discussion amongst students designed to
facilitate students’ participation in a certain cultural practice. Both theorists draw on
Vygotsky’s concept of the zone of proximal development to describe how students are
“pulled forward” (Goos, 2004, p.262) into different communities of inquiry through their
interaction with other peers and the classroom teacher who gradually introduces cultural
forms of knowing. However, in the case of Goos’ inquiry mathematics community of
practice the focus is on the development of mathematical forms of discourse in contrast to
Brown’s work with young readers.

Significant improvements in learning have been observed when students become
members of communities of learners. Students who participated in reading groups using
the model of reciprocal teaching, for example, were more able to solve problems that
required them to use analogies from past examples (Brown, Campione, Reeve, Ferrara
and Palincsar, 1991). Compared to a control class whose performance on such tasks
showed little improvement before and after the intervention, students in the reciprocal
teaching reading groups were able to get twice as many questions correct after the
intervention (from 40% to 80%). Students participating in communities of learners
research teams demonstrated higher levels of knowledge through portfolios and student
work (Brown and Campione, 1994), used more domain-relevant information to answer
novel questions (Brown, Ash, Rutherford, Nakagawa, Gordon and Campione, 1994), showed higher levels of comprehension of texts (Brown and Campione, 1994), and more advanced argumentation skills.

Collaboration represents the fundamental component of successful classrooms in mathematics and other subject areas such as reading and science. As a catalyst for transforming classroom activity, a means of promoting cognitive development in numerous ways, a context for productive dialogue, and the promotion of positive social values, students collaborating with each other represents a goal of every reform document recently produced for mathematics educators. The nature of discourse that occurs within collaborative groups can be dominated by one or two voices or it can promote the interaction of multiple voices, creating a dialogic atmosphere in which new forms of understanding emerge through the bringing together of different, often conflicting points of view. The value of dialogical activity in the mathematical classroom has also been identified clearly in recent reform documents.

2.2 Effective mathematics learning is dialogical

An important learning principle for effective mathematics classrooms is that learning occurs best within the context of dialogue rather than monologue. Shared activity needs to be genuinely discursive activity in which pupils are encouraged to ask questions, seek answers, consider different perspectives, exchange viewpoints, and add their findings to existing understanding (van Oers, 1996). While traditional mathematics classrooms typically present a single approach for solving a certain type of problem, reform documents in Australia and overseas encourage the development of alternative solutions (DETYA, 2000; AEC, 1991; NCTM, 2000). The NCTM document Professional Standards for Teaching Mathematics (NCTM, 1991) also encourages the development of a dialogic classroom. The NCTM recommends that teachers of mathematics need to view the classroom as a community rather than a set of individuals, in which logic and mathematical evidence becomes the basis of verification rather than the teacher’s authority. Brown, Ellery and Campione (1998) describe effective learning environments as “communities of discourse involving negotiated meaning” (p.348) in which students
and teachers are co-learners, co-researchers and co-teachers. Power and authority within such communities of learners is shared amongst participants rather than residing with the teacher or within canonical texts. Students are empowered to contribute to classroom discussions, teach other members of the classroom community and bring their own experience and perspective to bear on concepts developed within the classroom.

According to Lerman (1998), mathematical activity is conversational since it emerges from the dialectic between personal and public knowledge through students presenting assertions and counter-assertions. Central to this dialectic is the importance of conversation and interpersonal negotiation through which conjectures and refutations are developed within the classroom. Over a period of time, as a consequence of participation in social practices individuals develop personal understandings of mathematics, manifested in linguistic behaviours and symbolic productions in “conversations” in a variety of different contexts.

Within the writings of Mikhail Bakhtin and Lev Vygotsky a complex picture emerges of the relationship between dialogue and learning and between communication and understanding which shares much in common with Gergen's theory of social constructionism (Gergen, 1995) and Wittgenstein's later theory of language (Wittgenstein, 1953; 1974) in which the meaning of any communication is found in the way it is used and for what purpose. All language has an underlying "dialogic" nature - every word participates in a history of rich intertextual relations in which it is related to all other utterances (Bakhtin, 1981). Vygotsky proposes a similar notion in his understanding of word meaning.

The word is a thing in our consciousness, as Ludwig Feuerbach put it, that is absolutely impossible for one person, but that becomes a reality for two. The word is a direct expression of the historical nature of human consciousness. (Vygotsky, 1986, p. 256)

Utterances, or speech acts, are also dialogical according to Bakhtin incorporating multiple voices. Wertsch uses the term “voice” to refer to the speaking consciousness rather than auditory sounds (Wertsch, 1991). The concept of “voice” reflects three basic
ideas about the relationship between communication and individual psychological functioning. First, to understand human mental action, it is necessary to understand the semiotic devices used to mediate such action. Second, aspects of mental action are fundamentally tied to communicative processes. Even individual psychological functioning involves processes of a communicative nature. Finally, human communicative practices give rise to mental functioning in a genetic, developmental manner.

Each speech act incorporates at least two “voices” (Resnick, Levine and Teasley, 1991) – that of the speaker, and that of the social language appropriated for a specific purpose. Bakhtin (1986; 1981) rejected the grammatical sentence as the fundamental unit of analysis for understanding language, preferring to focus on utterances. According to Bakhtin, the meaning of utterances does not reside in the semantic content of a particular sentence alone, but derives its meaning from its instantiation within a dialogue between two people.

...speech can exist in reality only in the form of concrete utterances of individual speaking people, speech subjects. Speech is always cast in the form of an utterance belonging to a particular speaking subject, and outside this form it cannot exist. (Bakhtin, 1986, p.71)

In his commentary on Dostoyevsky’s novels he goes even further to suggest that truth as a concept can only exist within the sphere of dialogical relations.

Truth is not born nor is it to be found inside the head of an individual person, it is born between people collectively searching for truth, in the process of their dialogic interaction. (Bakhtin, 1984, p.110)

All utterances make use of accepted national languages to communicate ideas. However, there are two other aspects of language identified by Wertsch that are tools which can be used to form utterances. These are social languages in which different members of the community develop novel language tools for their purposes and genres or scripts within which linguistic exchanges occur although individuals are still free to be original in the
way they work within these genres and scripts (Wertsch, 1991). Each of these has a sociohistorical origin that needs to be understood in order to make sense of the utterance. However, many more than two voices are present in most utterances that occur within the classroom. For example, the mathematics classroom teacher appropriates (or “rents”) several different voices while presenting mathematical concepts to a group of students. These voices include the individual voice of the teacher, voices of textbooks, the social languages of teachers in general, the social languages of the broader mathematics community and voices arising from the teacher’s own educational experiences.

Utterances of individuals are constructed from other people’s thoughts and speech and are therefore never wholly original, but involve retelling the original speaker’s intent (Harden, 2000). Part of learning mathematics, therefore, involves the appropriating of other people’s voices, be they the voices evident in the utterances of the teacher, the utterances of other students, or the voices evident in texts. Appropriating multiple voices results in greater depth of understanding as individuals structure their own personal interpretations. In mathematics, for example, students who work with multiple representations of functions which include numeric, graphic and symbolic representations develop a more thorough understanding of the concept of “function” (Leinhardt, Zaslavsky and Stein, 1990).

Traditional mathematics teaching, however, often involves using texts or the teacher’s utterances to perform a univocal rather than a dialogic function (Kozulin, 1990). Texts and utterances are univocal when presented as the final authority for acceptance by students in toto. The potential for dialogue is minimised and the development of personal understanding is limited to the reproduction of the authoritative text or utterance. This model of teaching reflects a widespread perception of mathematics as a domain of unchanging truth in which there are right and wrong answers. However, mathematicians and mathematics educators concerned with reforming mathematics education do not share this perception of mathematics. Contrary to popular perceptions, the truth value of a solution in mathematics is always dependent upon underlying assumptions. Algebraic theory, for example, does not accept that $2 + 2 = 4$ unless one specifies the ring (more
commonly, the field) over which such a calculation is conducted. Student understanding of basic arithmetic such as that incorporated in the expression $2 + 2 = 4$ normally takes place after the community of learners accepts numerous (often implicit) assumptions. Typical assumptions include the field over which students perform such a calculation, the meaning of the mathematical symbols $+$ and $=$ and the meaning of concepts of “two” and “four” derived from the cardinal and ordinal nature of numerals (Piaget, 1952). These assumptions reflect the voices of different members of the mathematical community, typically evident in the utterances of the teacher.

Understanding mathematical discourse involves more than just learning how to speak the language of the mathematical community. It also involves learning about the beliefs and values of that community which may be in conflict with the discourses children participate in at home or in their respective communities (Lemke, 1990; Gee, 1990). Discourses embody ideologies (Hicks, 1996) and monologic classrooms in which classroom teachers present mathematics as a body of knowledge to be appropriated can alienate many students whose discursive histories are in conflict with this way of thinking. Students from non-Western backgrounds, for example, often find traditional pedagogies more alienating than students from Western backgrounds (Banks, 1993).

In contrast, texts and utterances can serve a dialogic function by promoting discourse within the classroom and supporting the development of personal interpretations. The dialogic potential of the teacher’s utterances and texts in the classroom represents a valuable untapped resource in most traditional classrooms. Students wrestling with problems such as $2 + 2$ develop personal interpretations of what such a statement means, the properties of the field of real numbers and the cardinal and ordinal nature of numbers. The development of personal interpretations is supported by multiple voices in dialogue rather than univocal monologue presented either by the teacher or in text form. Dialogic classrooms have the potential to empower students from different social and cultural backgrounds to participate more fully in mathematical practices as they contribute to the development of shared values and beliefs within the classroom.
Dialogue in the classroom is closely linked to the negotiation of meaning (van Oers, 1996; Brown, Ellery and Campione, 1998). As outlined previously, mathematics is a practice as well as a knowledge domain (Stein, Silver and Smith, 1998), and is created using socially appropriate tools and conventions (Tymoczko, 1986). The practice of mathematics is a social process of negotiation, of agreeing upon what constitutes an acceptable answer or proof. Mathematical practice is only possible if certain meanings and definitions are shared within a community of mathematicians. Classroom mathematics, therefore, is equally dependent on students developing common understandings of such mathematical entities as proofs, solutions and arguments.

The negotiation of meaning, however, is rare in mathematics classrooms, replaced with statements of “fact” offered by the teacher or the textbook for acceptance by the student and replicated under exam conditions. Negotiation requires dialogue in the classroom, something that is often difficult to manage, and viewed by many teachers as an inefficient use of limited classroom time. Supporting student negotiation through dialogue within the classroom in a manner that alleviates these concerns, however, remains an important goal of the current study.

Classroom environments that foster the production of multiple voices provide a rich learning environment for students and teachers. Carpay and van Oers’ theory of polyphony (1999) suggests that learning in the classroom depends strongly on intertextuality – the development of personal texts from the synthesis of available texts. Mathematical discussions within the classroom that engage multiple voices, including those of teachers, students and other texts support the constant negotiation and recreation of cultural meanings (Bruner, 1986). Lampert (1986) argues that discourse and social interaction in the classroom promote the recognition of connections among ideas and the restructuring of individual understandings. Engaging in mathematical talk - practising using the mathematical register or genre, however, is rare in traditional classrooms (Forman, 1996).
Lampert (1990) provides a detailed description of a discussion within a mathematical classroom related to the concept of exponents. The classroom teacher (Lampert) provides a mathematical problem – what is the last digit in $5^4$, $6^4$, $7^4$? From this initial mathematical problem, the class developed general conjectures about exponents. The discussion progressed through three stages – clarification of terms, symbols and definitions, consideration of the special properties of powers of 5 and 6 and developing hypotheses about powers of 7, and finally, developing general hypotheses about how exponents could be used as a short cut in arithmetic operations. The character of this discussion involved the teacher posing questions to which individual students responded.

Classroom discussion focused around such learning activities represents a significant ideal for reformers of mathematics learning. In many classrooms, however, where there is a typical pattern of teacher-class monologue instead of dialogue, students often remain disengaged from such classroom conversations. Often, classroom discussions deteriorate into a discussion between a select few in the classroom with the inclination to pursue a particular issue further with the teacher. In Lampert’s fifth grade classroom discussion, 14 of the 18 students made mathematically substantial contributions – a significant achievement, however such success may be more difficult to achieve in classrooms with a larger number of students. Supporting whole-class discussions with secondary students may also prove more difficult due to changing social interactions amongst students compared to the relative ease with which Lampert’s fifth grade students offered comments relevant to the classroom discussion.

Dialogue between teachers and students need not be limited to whole class discussions. Typically, teachers spend much of their time answering individual questions once classroom presentations are completed. However, each of these forms of dialogue has difficulties, limiting the potential of classroom dialogue to maximise the learning of members of the mathematical community. In both cases, the opportunities for individual students to participate in a balanced dialogue with teachers as cultural representatives are limited.
The model of teacher-student dialogue adopted in the current study is that of a small group conversation, described in more detail later, in which the teacher participates as a member. The teacher clearly has a special role within the conversation as a representative of the culture of mathematicians (Lampert, 1990; van Oers, 1996; Forman, 1996) providing students with cultural tools for solving different problems. However, the students within that group set the agenda for such conversations. Students bring to such conversations their own opinions formed through discussion amongst themselves for comparison with existing cultural modes of thinking (van Oers, 1996). Within such a context, the teacher is more able to ensure the involvement of all students in the process of negotiating mathematical meaning by monitoring the public articulations of each participant. Within whole-class discussions, the potential for monitoring the understanding of each student is limited to the infrequent contributions each student is likely to make to the class discussion. Furthermore, students who fail to grasp certain ideas are even less likely to contribute to such discussions in secondary classes for fear of public disapproval and rejection by peers.

Dialogue between the teacher and small groups of students provides a context of polylogue - the interaction of multiple voices in which new forms of mathematical understanding can emerge that are beyond the individual understanding of participants in the dialogue (including the teacher). It provides a context within which students are able to develop their own dialogical thought processes in which competing voices are internalised. Vygotsky’s theory regarding the relationship between thinking and speech focuses specifically on the incorporated ideas of dialogism in thought – taking as the prototype for logic the dialogic consciousness rather than the monologic perspective of the natural sciences (Kozulin, 1990). Thinking is fundamentally dialogic, according to Vygotsky. The development of thinking involves the transformation of dialogic social interactions into dialogic intrapsychological processes (Vygotsky, 1978).

2.2.1 The emergence of individual thinking from social interaction

Social relations, according to Vygotsky, are genetically prior to individual understanding. Vygotsky’s genetic law of psychological development emphasises the existence of
mental functions within the social plane preceding their incorporation into the psychological plane.

Any function on the child’s cultural development appears twice, or on two planes. First it appears on the social plane and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category. .... It goes without saying that internalisation transforms the process itself and changes its structure and functions. ....Social relations or relations among people genetically underlie all higher functions and their relationships (italics added). (Vygotsky ,1978, p.57)

The process of internalisation represents a complex process of transforming social phenomena into intrapersonal mental functioning (Cole, 1985). Internalisation involves the construction rather than the copying of interpsychological functions and the mastery of cultural systems of symbolic representations (Wertsch, 1985).

The dimensions of internal conversations, therefore, originate in recurrent prior social interactions. Vygotsky argues that ‘relations among higher mental functions were at some earlier time actual relations among people’ (Vygotsky, 1978, p.57). To understand how different words and semiotic systems are used by the individual, for example, it is necessary to investigate how such words and semiotic systems are used in social behaviour (Vygotsky, 1981b). Internal activity, therefore, arises from external, practical activity – it is not separate from it, nor does it rise above it, but retains its fundamental and two-way connection with it (Leont’ev, 1974). This involves the dialectical transformation of the internal and the external into something new (Tobach, 1995).

Developing specific mathematical forms of discourse that can be internalised by individual students, therefore, represents an important aspect of effective mathematics classroom environments. Part of learning through participation is learning to speak the social language of that community (Lemke, 1985; Resnick, Levine and Teasley, 1991). For example, Lampert (1990) encouraged certain patterns of social interaction using particular forms of mathematical reasoning within a classroom environment designed to support the development of individual reasoning patterns. Students were taught language forms appropriate for mathematical reasoning such as “I want to question ***’s
hypothesis” and social norms were established within the classroom encouraging mathematical reasoning as the appropriate means for dealing with differences of opinion rather than personal attacks.

Mathematical language is not an abstract linguistic system with no connection to mathematical practice – it is chronotopic, to borrow Bakhtin’s term, meaning that it is an historical system that acquires its forms in concrete verbal communication or dialogue (Harden, 2000). Hence, it is essential to the development of individual mathematical understanding that the language of the classroom reflects the language of mathematical practice. The teacher as a representative of the historical practice of mathematics is of utmost importance in structuring the discourse of the classroom to reflect the discourse of communities of mathematicians. Lampert (1998) provides an example of how teachers can introduce cultural forms of discourse by reinterpreting student utterances to present societal perspectives on particular mathematical constructs. By expanding on student utterances in this way, mathematical utterances in the classroom come closer to what would be heard outside the classroom.

Different communities of practice are characterised by the use of different semiotic systems that include, but are not limited to, language (Lemke, 1985; 1990). Semiotic systems in mathematics might also include algebraic systems and the development of specific symbolic means for representing different mathematical relationships. Lemke (1985) argues that learning within different disciplines consists of learning to speak, interpret, write and read using the semiotic systems of that discipline and thereby participating in practices of meaning-making.

Creating dialogical classroom environments presents a considerable challenge for secondary mathematics teachers. Such classroom environments can be more difficult to manage and challenging in terms of the concepts that are raised. Furthermore, these classrooms have the potential to promote radical changes in students’ understanding, their roles in the classroom and their participation in mathematical activities. Each of these changes are described in the next section as dialectic changes – emergent properties of
classrooms that lead to new forms of understanding and participation in classroom activity.

2.3 Effective mathematics learning is dialectical

The third principle of effective mathematical learning is that such learning is fundamentally dialectical. That is, learning is a transformative process through which new cultural ways of knowing and doing are developed that emerge from the synthesis of individuals’ current forms of understanding and historically developed forms of understanding. Accompanying the development of cultural ways of knowing and practice are associated transformations in the participation of learners in cultural practices from peripheral to central participants (Lave and Wenger, 1991). Traditional learning environments, however, are unable to accommodate the transformative nature of this dialectic that characterises learning, focusing primarily on replacing students’ existing knowledge with new understandings.

Within the domain of mathematics, the synthesis of existing and new understandings is often brought about by perceived breakdowns in understanding that require the learner to reorganise their mathematical understanding to overcome such breakdowns. Learning environments can either encourage students to explore such breakdowns or assist students to avoid having to ever confront them by giving them pre-packaged methods and solutions that have high levels of efficacy. However, developments in mathematical understanding often occur in times of crisis when existing conceptions are challenged. Three different ways in which this can occur in the classroom are described in the document *A National Statement on Mathematics for Australian Schools* (AEC, 1991). First, students might conduct experiments which provide unexpected results. Students might also be challenged by a peer who disagrees with their interpretation of a certain situation. Finally, in the process of doing mathematics students might discover that a certain rule which they had believed to be true doesn’t work in every single instance. In each case, learning takes place when students are able to accommodate these unexpected findings or different perspectives and develop a new level of understanding that supersedes the old.
2.3.1 Dialectic vs mechanistic explanations of learning

Learning is a process of change – of change in behaviour, thought or patterns of participation (Rogoff, 1998). Mechanistic and dialectic explanations for change in nature, alterations in phenomena, succession of forms and motion represent competing theories of change that impact upon psychological theories of learning. Conceptual change models of learning, for example, represent mechanistic conceptions of learning in which existing understandings are transformed to incorporate logically superior understandings. New knowledge and existing understandings do not represent equal moments participating in the dialectic; existing knowledge is “assimilated” while new knowledge is “accommodated” in a linear, additive fashion through the processes of reason and logic. Dialectic conceptions of learning, in contrast to mechanistic conceptions, emphasise the revolutionary nature of learning, the negation of old forms of thinking, and the negation of the negation resulting in qualitative changes in conceptual understanding. Change in conceptual understanding is not necessarily linear, nor is it orderly. Instead, change is viewed as the outcome of struggle and contradiction.

Theories of conceptual change provide examples of mechanistic models of learning. Conceptual change is closely associated with research conducted by Posner and colleagues representing an organismic metatheoretical position (Pepper, 1942). Standard models of conceptual change describe learning as the interaction that takes place between an individual’s experience and current conceptions and ideas (Pintrich, Marx and Boyle, 1993). Current conceptions provide a framework for interpreting information gathered through experience either creating a problem resulting from the discrepancy between experience and current beliefs or by providing a framework for judging the validity and adequacy of solutions to these problems. Processes of assimilation and accommodation are guided by a principle of equilibration whereby individuals seek a stable equilibrium between internal conceptions and new information available in the environment (Pintrich, Marx and Boyle, 1993).
Conceptual change theories of learning, however, do not reflect a dialectical model of learning in which cultural concepts taught to students are in the process of evolving as well as individual understandings. Instead, such cultural concepts are assumed to be incorporated into individual conceptual frameworks unchanged. In the same manner as stimulus-response theories of behavioural change, models of conceptual change assume that such changes are efficient, caused by external factors rather than changes driven by contradictions emerging within the context of practical activity.

Models of conceptual change incorporate an implicit assumption about the nature of learning as a process of reproducing existing cultural conceptual frameworks in individual minds. The reproductive aspect of educational practice remains a significant goal of education, maintaining cultural forms of knowing from one generation to another. However, education that is solely reproductive fails to provide students with the critical skills for questioning existing cultural practices, modes of understanding and use of cultural tools.

Mechanistic explanations account for qualitative change as the outcome of quantitative combinatorial relations between simple elements (including ideas and the contents of consciousness) (Wozniak, 1998). If the elements and their combination are known, the phenomenon is fully understood. The whole, therefore, is only the sum of its parts. Change is purely efficient, accounted for by external forces and quantitative changes in matter or motion. Dialectical explanations view change as necessary rather than efficient. According to dialectical theories of change, everything is in a state of change proceeding through struggle. Development is not necessarily linear, but revolutionary. Learning, according to Vygotsky, is a dynamic process of upheaval, sudden changes and reversals (Kozulin, 1986). In mathematics, for example, developing an understanding of the statement $y = 3x - 5$ involves such processes. This equation can be represented both algebraically and graphically – in some instances an algebraic representation is most beneficial, in other cases a graphical representation is most beneficial. Working mathematically with the statement $y = 3x - 5$ involves moving between these two different approaches, revisiting existing understandings of how pronumerals can be used
(most students take considerable time to grasp the concept that pronumerals can represent specific numbers or can represent any number) and developing further concepts such as linearity, gradient, slope and function.

The implications for theories of learning of a dialectic theory of change are many. First, the dialectic theory of change requires that phenomena must be understood in the process of continual change and development (Wozniak, 1998) rather than as static entities. Individual and cultural forms of understanding, for example, are continually evolving entities, interacting with each other in the context of practical activity.

Second, psychological dialectic theories view human beings as active constructors rather than passive receivers, capable of directing their own actions. In contrast, behavioural analyses of human psychology within a stimulus-response framework are unidirectionally reactive (Vygotsky, 1978); change in human behaviour is associated with external environmental forces. Instead, Vygotsky (1978) argues that human behaviour transforms the external environment as well as being transformed by it. Individuals and their environment do not represent ontologically independent entities but mutually constitutive elements of practical activity within which incompatible aspects that presuppose each other create conflict and force change. Participation in practical activity inevitably leads to changes in behaviour, thought processes and patterns of participation – changes that sociocultural theorists describe as “learning” (Lave, 1996; Rogoff, 1998; Lave and Wenger, 1991).

Third, dialectical theories reject the reduction of psychological properties into constituent elements. At each level of psychological activity, functional reorganisation occurs such that the whole is always greater than the sum of its parts. Consciousness, therefore, becomes an appropriate focus for psychological study. Learning in the classroom represents an even higher level of psychological reorganisation developing its own system of laws that are not reducible to the psychological properties of individuals. Dialectical theories of collaboration, for example, view sociocultural activity as an ontologically distinct level of psychological activity. In contrast, “social influence”
models of collaboration that begin with the individual as the unit of analysis (Rogoff, 1998) view social interaction as an external efficient cause of individual behavioural change. Sociocultural approaches reject the mechanistic interpretation of social interaction for the dialectical interpretation viewing the individual, the interpersonal and the cultural as interdependent entities (Rogoff, 1998).

Finally, dialectic theories of learning see learning and development occurring through a series of non-linear leaps (Wozniak, 1998). The tendency of psychological theories towards determinism is rejected by dialectical theories of learning. Development does not occur from the sum of elementary processes but the emergence of qualitatively new psychological forms of activity (Vygotsky, 1978) that negate previous activities, merging existing practices into an ontologically distinct form of practice.

Certain similarities are evident between the dialectical psychology outlined by theorists such as Vygotsky, Leont’ev and Luria and current cognitive psychology (Wozniak, 1998). In particular, the adoption of consciousness as a proper subject matter for psychology, and the development of theories of mind from observations of practical activity are evident within contemporary cognitive psychology. However, Vygotsky’s emphasis on the cultural development of individuals dialectically interacting with their natural development distinguishes Vygotsky’s theory of learning and development from modern cognitive conceptions of learning.

2.3.2 Distinction between productive and reproductive practice

One of the criticisms of much educational practice today focuses on the predominantly reproductive nature of educational activity whose emphasis is on the reproduction of existing ways of thinking rather than the development of new ways of solving problems. Reproductive educational practices serve a valuable societal function. Societal practices survive by being passed on from one generation to the next. This aspect of educational practice leads to students reproducing the practices of their teachers and, more generally, reflecting the understanding of previous generations in their own participation in societal
practice. However, societies stagnate and decay if each generation merely reproduces the practices and understandings of the generation before. The ongoing changes in the wider physical and social world require new practices and new ways of thinking that enable societies to progress towards more effectively satisfying the needs of its members. Hence, reproductive educational practices alone are inadequate as the principal focus of educational institutions. Effective educational practices also incorporate generative practices, critical evaluations of existing practices, and the production of new solutions to existing societal problems. Bowen (1975) describes the purpose of education in these words.

Today, however, more than ever before we expect not only the sustaining of our cultural traditions but also their critical revision and development. We demand of education that it provide a means to ever greater cultural vitality (p.xvii).

Transmission-based approaches focusing on the reproduction of existing understandings remain entrenched within schools. This model of imparting information for later retrieval reflects the model of education prevalent in the Middle Ages. Engeström (1987) suggests that the purpose of this model of education was the reproduction of existing texts rather than the transformation of these texts towards the production of solutions for existing societal problems. This emphasis on reproduction rather than production reflected the practices of monasteries intent on the preservation of canonical texts from generation to generation (Miettinen, 1999).

Dialectical approaches to teaching provide for the productive restructuring of existing ways of knowing, thinking and doing. Vygotsky argues that the dialectic of natural and cultural development results in the development of new forms of knowing that are neither cultural nor natural, but ontologically new, distinct forms of understanding. This dialectic between natural and cultural forms of understanding will be elaborated in the next section of this thesis.
2.3.3 Dialectic between cultural and natural understanding

As Vygotsky outlines in his analysis of the development of attentional capacity, development involves both a natural and a cultural component (Vygotsky, 1981c). The interplay between everyday (spontaneous) and cultural (sometimes described as scientific) concepts represents a central aspect of Vygotsky’s theory of learning. Vygotsky differentiates between scientific and spontaneous concepts suggesting that they are characteristically different, yet dependent on each other. Everyday or spontaneous concepts clear the path for the scientific concept as it develops “downward” while scientific concepts provide structures for the “upward” development of the child’s spontaneous concepts (Harvey and Charnitski, 1998). The dialectical relationship between these two forms of concepts represents the mechanism through which learning occurs.

According to Vygotsky, learning is a dialectic process in which an individual tests a personally held concept against those of other members of the culture (Vygotsky, 1978) leading to the development of public knowledge, the synthetic residue of this dialectic process (Pask, 1975). Mathematical understanding in the classroom is not merely achieved through consensus between social partners. Students must also incorporate the historically gathered insights of a domain of culture (van Oers, 1996). Learning involves two types of discussion. First, dialogue focused on the negotiation of meaning with other people is motivated by a desire to determine what can be said about a certain subject. Second, there is need for “discussion” with historical agents to determine the value of their approaches to the ideas under investigation. Both kinds of discourse are necessary – described by van Oers (1996) as polylogue. Humans only understand themselves when they test their meanings on all possible others, contemporary and historical (van Oers, 1996).

Effective classroom activities support students’ appropriation of conventional definitions and ways of knowing, however, this process of appropriation does not imply a reproduction of existing cultural definitions. Students use conventional forms of knowing to make sense of their own experience, reconstructing existing discourses and hence,
going beyond the conventional assumptions of how mathematical concepts are used (Lampert, 1998). The appropriation and transformation of cultural forms of understanding occurs through multiperspective activities (dialogical activities) that involve discourse with a wide range voices representing the cultural heritage and scientific ideas which interact with personal texts to bring about a new “text”. Instead of replicating existing texts, the ultimate goal of classroom learning activities, according to Carpay and van Oers (1999), is the establishment of a new personalised mode of speaking about the world. The responsibility for introducing cultural forms of understanding is shared amongst members of the classroom, although the classroom teacher remains the principal representative of the existing mathematical culture.

2.3.4 Teacher as cultural representative/instigator of joint productive practice

Personalised modes of speaking about the world emerge from the interaction between canonical texts (some verbally presented by the teacher) and subjective texts that are available within the classroom (Carpay and van Oers, 1999). The role of the teacher, therefore is to insert traditionally approved insights to encourage students to discursively reconstruct culturally shared meanings (Lampert, 1990; Mercer, 1995; van Oers, 1996). The role of the teacher is not merely to explain things, leading to the reduction of the collective aspect of the discussion, but to incorporate into the class discussion the language and frames of reference of expert discourse.

Teachers act as the purveyors of cultural understandings in many different ways within the classroom. Lampert (1998) describes an example of how teachers can make available culturally valued mathematical understandings by reinterpreting student utterances to present societal perspectives on mathematical concepts. By expanding on student utterances, the mathematical utterances evident in the classroom approach the form of utterances developed outside the classroom. Teachers can also facilitate the incorporation of cultural perspectives into the classroom, working with students and providing them with cultural tools (such as language and symbols) to assist them in their own thinking
Teachers are partners in classroom conversations with a special responsibility of ensuring that the classroom culture is rich in challenges, alternatives and models of mathematical language amongst other cultural artefacts (Lampert, 1998).

### 2.3.5 Dialectical outcome of learning activities

The classroom community, therefore, is engaged in developing a new form of understanding that transcends both existing and cultural forms. Similarly, each participant within this community is developing their own forms of understanding that will be related to, but not the same as, those forms of understanding accepted within the public sphere of the classroom. As students participate in this dialectic of understanding, there is another aspect of classroom activity that is also being transformed – that of individuals’ participation in cultural practices. Changes in understanding are accompanied by changes in participation. For some theorists, changes in participation are ontologically prior to changes in understanding. Rogoff (1998), for example, defines learning as a transformation of participation in sociocultural activity (Rogoff, 1998; 1990; Lave and Wenger, 1991) rather than the acquisition of knowledge. Changes in participation are a fundamental aspect of activities that evolve over time and are as ubiquitous as participation in cultural practices (Lave, 1996). Lave and Wenger (1991) reconceptualise the notion of learning to describe the process of moving from positions on the periphery of communities of practice to becoming central members of these communities of practice. They define a *community of practice* as a set of relations among persons, activities and the world over time united by a single purpose (Lave and Wenger, 1991). Mathematical communities, for example, represent particular communities of practice united by a common purpose achieved through collective activity. Mathematical communities make use of historically emerging cultural artefacts such as mathematical theories, methodologies, technologies and activity structures. Through a social process of increasing “centripetal participation” (Lave and Wenger, 1991, p.68) newcomers become “oldtimers”. Developing knowledge skills and identifying oneself as a practitioner, becoming a full participant in the community of practice are co-existent realities.
Sociocultural approaches to learning as changing patterns of participation (Rogoff, 1998; Lave, 1988; Lave and Wenger, 1991; Beach, 1999) have examined how the relative position of learners within communities of practice changes over time. Learning environments that support this movement of novices towards the centre of different communities of practice allow for transformations in identity, and changes in participation over time. School-based learning environments often obstruct this movement by presenting students with the principles of different communities of practice in the abstract, removed from actual practice such that student participation in these practices is non-existent.

For Lave and her colleagues (Lave and Wenger, 1991; Lave, 1996), learning does not involve merely replicating existing patterns of thought and behaviour, but leads to the dialectical emergence of new forms of activity. Learning leads to the transformation of practical activity. Apprenticeship integrated within communities of practice represents a model that supports the legitimate peripheral participation of newcomers as they develop into more central members of particular communities of practice. Over time, novices become more like experts, making increasingly significant contributions to the productive activity of the community of practice.

In contrast, traditional classroom-based learning environments do not support this process of transforming learners and learning environments. Students and teachers are not interpenetrating moments dialectically transformed through learning activities. At the completion of most classes, students do not move to more central positions of participation within the classroom (or any other community of practice) but remain subordinate to their teacher whose status as the expert remains intact. Instead of supporting the development of new forms of thinking and doing, classroom-based learning supports the reproduction of existing forms of thinking and doing fossilised in canonical texts (Miettinen, 1999; Engeström, 1987).

Effective mathematics learning, therefore, is dialectical promoting the synthesis of personal and culturally accepted forms of knowing within classroom communities as well
as the gradual shift in students’ participation from peripheral to central members of the mathematical community. For the classroom teacher, developing a dialectical classroom will involve challenging misconceptions, presenting cultural forms for critical assessment, and providing opportunities for students to become more central participants in the cultural practice of mathematics. The end result of providing such opportunities, therefore, is the enculturation of students into cultural practices that are valued by mathematicians.

2.4 Effective mathematics learning is embedded in cultural practices

The fourth principle of effective mathematics learning is that such learning should be cultural. By cultural, the intention of this principle is to emphasise the importance of student engagement with cultural tools, cultural modes of thinking and cultural practices. Instead of classroom mathematics being ontologically distinct from “everyday” mathematics (Lave, 1988) or the activities of mathematicians, classroom practices should overlap with cultural ways of doing in terms of the physical and semiotic tools that students use and the mathematical objects that students pursue in the classroom.

2.4.1 Definition of culture

Within the mathematics education literature there exist several problems associated with the use of the terms “culture” and “classroom culture”. The first of these problems is whether or not the term “culture” is a general term referring to widely held beliefs and practices, or whether it refers to more specific phenomena such as what happens within the classroom (Seeger, Waschescio and Voigt, 1998). What type of cultural influences impact upon the development of activity systems within the classroom? If the current study were to adopt this general definition of culture, it would examine the impact of ethnicity, race, economic systems and religious background on the activities of classrooms under investigation. In the introduction to his book Cultural Perspectives in the Mathematics Classroom (1994) Stephen Lerman adopts such a definition of culture:
Culture is invasive, both outwards around the globe and inwards into the construction of individual subjectivities. The apparently irresistible appetite of late twentieth century capitalism creates demands for its products, rather than merely seeking markets, and as a consequence we become the consumers that multi-national companies need to feed upon in order to continue to grow. (p.1)

However, the more specific definition of culture, which is associated with the term “classroom culture”, suggests that different classrooms develop specific cultural practices, beliefs and artefacts which are, to some extent, unique to each classroom. The current study works with such a definition of culture – that culture is shared by a group of individuals and is characterised by specific practices, beliefs and artefacts. While each of the classrooms investigated in the current study were studying similar areas of mathematics, and were therefore drawing upon a common mathematical heritage, the practices which emerged in each of these classrooms differed greatly from classroom to classroom.

A second difficulty with the concept of culture is whether cultural systems should be considered as products of action or as conditioning influences upon further action. Does culture provide a frame within which teaching-learning processes occur or is "classroom culture" something to be constructed? Culture as something which is appropriated is emphasised by definitions of culture as an entity transmitted from one generation to the next. Miraglia, Law and Collins (1996) suggest that there exists a “cultural template” before a member of that culture is born which shapes the actions of individuals within that culture. They believe that the essence of culture is that it is learned – a set of behaviours defined before a member of the group is born which are learned by those members as they become participants in the culture. Geertz’s definition of culture as a system of meanings (1973) suggests that culture presents a context within which certain ways of understanding the world are possible.

However, other definitions of culture, while acknowledging the historical nature of cultural systems, describe cultures as adaptive (Bodley, 1994), being in a permanent state of flux and ongoing historical change. Trilling (1965) describes cultural practices as the
means of establishing coherence, of developing ritual and art - the pieties and duties
which make possible the life of the group and the individual (p.92). Coherence once
gained needs to be maintained in the face of changes in membership of cultural groups
over time, changes in environmental constraints on a culture’s survival and changes in
neighbouring cultures with which different cultures may interact. Classroom culture
according to this second view represents not so much a cultural template restricting the
activity within the classroom but rather something which is constructed through the
interaction between participants in classroom activities.

Cultural phenomena such as shared ways of thinking and doing, systems of meaning and
cultural artefacts are more than “cultural templates” that exist prior to the birth of human
beings whose lives are impacted by these phenomena – such phenomena must be
grounded within practical activity to be understood fully. Cultural phenomena emerge out
of practical activity. They are transformed over historical time through practical activity –
practical activity being the means by which collective ways of understanding the world
develop. This dialectical view of culture as a dynamic entity is promoted by Cobb (1995)
who argues that the individual student’s mathematical activity and the classroom
microculture are reflexively related.

Within the mathematics classroom, there are multiple shared systems of meaning, ways
of doing and cultural artefacts. For example, in the area of deductive geometry there
exists a complex system of meaning which includes linguistic tools of geometry,
axiomatic systems, types of proof, processes of deduction, definitions of geometrical
entities, and implicit assumptions about the reality or otherwise of these entities. Ways of
doing include producing proofs, giving evidence, developing arguments and using
different strategies such as drawing diagrams to assist in answering geometrical
questions. Physical artefacts that support such cultural practices and beliefs include
compasses, paper, pens, tables, computers and calculators.

Cobb (1995) identifies three different levels at which norms exist within the classroom
(focusing on shared ways of thinking and doing) – social norms, sociomathematical
norms and classroom norms. Cobb views the culture of the mathematics classroom as analogous to a scientific research tradition. Both are created by a particular community, and both influence the individual’s construction of mathematical or scientific ways of knowing by supporting and constraining what counts as a problem, solution, explanation and justification.

Amongst other features of cultural practices, physical and symbolic tools make certain cultural practices possible and lead to the subjective re-interpretation of these tools. Vygotsky’s theory of tool and symbol development places specific emphasis on the role that tools play in transforming human activity.

2.4.2 Use of and familiarity with cultural tools

Cultural artefacts in the form of psychological tools transform the activity of cognition (Vygotsky, 1978; 1981a). Vygotsky’s account of human psychology identified cultural mediation as the unique aspect of human psychology that leads to the development of uniquely human characteristics. He outlined several examples of mediational means that transform human thinking ranging from simple external psychological tools (such as a knot in a handkerchief to remind one of something) through to complex patterns of inner speech. Cultural artefacts present individuals with a range of tools for interpreting reality rather than being a determiner of activity, and as such artefacts provide certain boundaries to the range of possibilities that can be enacted (Wertsch, 1998).

All mathematical activity involves the appropriation of cultural tools and methods, and it is meaningless to describe mathematical activity by focusing on either the individual in isolation or the mathematical tool. For example, most school students are able to multiply two two-digit numbers by hand setting up two rows of working, one for the units multiplication and one for the tens multiplication, then adding these two rows together. This method is cultural, taught to students by members of the mathematics community, developed historically before students contemplated appropriating this method. When students are asked to perform the multiplication without using this cultural method,
however, few students are able to perform such multiplications successfully. Wertsch (1998) argues that neither the individual nor the method in isolation performs the multiplication task: both represent fundamental elements of the action of multiplication.

Artefacts, both physical and psychological, transform mental functioning in fundamental ways. Hence, all psychological functions are originally, and generally remain, culturally, historically, and institutionally situated (Cole and Wertsch, 1996). Mathematical functions that are able to be performed using particular cultural tools and methods (such as multiplying two-digit numbers) are often unable to be performed using different cultural tools and methods. Research into the performance of individuals in everyday contexts using specific cultural tools and methods are often confined to specific contexts (Scribner, 1984; Lave, Murtaugh and de la Rocha, 1984; Nunes, Schliemann and Carraher, 1983). This research clearly suggests that practices themselves including the intellectual tasks the practices pose, the knowledge these tasks require, and the intellectual operations involved in the accomplishment of different tasks are principal determinants of cognitive activity (Scribner, 1984). Human mental reflection, therefore, cannot be separated from the activities that engender it (Crawford, 1996). Mathematical calculations performed by workers at a dairy farm loading and unloading crates, for example, were closely tied to the intellectual requirements of the practices in which they were embedded (Scribner, 1984). Lave, Murtaugh and de la Rocha (1984) found that mathematical calculations performed in the supermarket when comparing items for best buying did not reflect practices taught in schools. Nunes, Schliemann and Carraher (1983) found similar differences in mathematical activity across different forms of practical activity. In their study, street vendors who were adept at calculating the cost of different orders tended to use less effective algorithms for completing mathematical problems within a classroom context.

As outlined in the NCTM document Principles and Standards for School Mathematics (NCTM, 2000), preparing students to engage in mathematical practices in the workplace is an important goal of mathematics education. However, research into mathematical practices suggests that mathematical practices are closely linked to specific cultural tools
and methods. The methods taught in schools for calculating “best buys”, for example, share little in common with the methods used in the supermarket (Lave, 1988).

The introduction of new forms of technology to support learning in mathematics is also likely to have a significant impact on the nature of the learning that takes place within the mathematics classroom according to Vygotsky’s theory of cultural mediation. Technological tools have the potential to enhance learning (NCTM, 2000). For example, the widespread use of calculators in schools has resulted in teachers and students spending less time working through basic calculations and more time undertaking more complex forms of mathematical reasoning.

### 2.4.3 Cultural modes of thinking

Vygotsky’s theory of psychological functioning was primarily concerned with interpsychological functioning. Leont’ev’s theory of activity structures, however, extends Vygotsky’s theory to be a full account of mind in society instead of mind within the microsociological interpsychological plane (Wertsch, 1998). According to Leont’ev’s theory of activity, cognitive activity is but one aspect of wider cultural and social systems of activity, and cannot be analysed in isolation from the material sphere of activity. Vygotsky and Leont’ev adopted the Marxist concept of dialectical materialism to emphasise the coexistence of the material and cognitive (Leont’ev, 1978; Pea, 1996). Marx and Engels (1996) argue that

> men (sic), developing their material production and their material intercourse, alter along with this … their thinking and the products of their thinking. (p.38)

The genetic origins of cognitive activity emerge from the cultural historical activity within which such cognitive activity arises. Transforming the cognitive activity of individuals, therefore, begins with the transformation of socially organised activity (Ratner, 1996). The interconnectedness of the social and cultural environment and the cognitive activity of individuals is also evident in Dewey’s work *Democracy and Education*. 

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The social environment...is truly educative in its effects in the degree in which an individual shares or participates in some conjoint activity. By doing his share in the associated activity, the individual appropriates the purpose which actuates it, becomes familiar with its methods and subject matters, acquires needed skill, and is saturated with its emotional spirit. (Dewey, 1916, p. 26)

Cultural historical activity represents an emergent entity, ontologically distinct from its components with its own structure, internal transformations and development (Leont’ev, 1974). Practical activity represents an amalgam of subjects and objects in which there occurs a process of reciprocal transformation between subject and object moments (Leont’ev, 1974) superseding these moments and creating a new emergent reality. Objects become “subjectified” as objects with certain value, purposes and roles, while subjects become “objectified” as labour producing entities, performing certain tasks that contribute to the productive enterprise that is the practical activity. The epistemological separation of the subject (the knower) from the object (the known) becomes unsustainable when considered within the context of practical activity in which each is transformed by the other.

As outlined earlier, activity systems are shaped by the object and motive of the activity (Leont’ev, 1978; 1981). Mathematical activities in the classroom, therefore, represent fundamentally distinct mathematical activities from those that develop within other sociocultural contexts such as the workplace or in research institutions since different motives for engaging in mathematical activity exist in both contexts. Within the school environment, students engage in mathematical activity to transform the object of their own psychological states to achieve desirable outcomes including success in the subject of mathematics, preparedness for engaging in other mathematical activities, peer acceptance, and advancing their own mathematical understanding. Participants in mathematical activities outside schools do so transforming mathematical objects to achieve some outcome separate to the mathematical activity itself. For example, in the case of calculating best buys, in the classroom this activity is completed to develop one’s own understanding and to achieve success in the subject of mathematics. Customers in supermarkets participate in this activity for the purposes of saving money. The
fundamental difference between school activity systems and external systems appears almost impossible to overcome. However, instead of school activities replicating activities that develop outside the classroom, several theorists argue that effective classroom activities act as mediating activities (Beach, 1999; Wilson, Teslow and Taylor, 1993) between student activity and the activities evolving outside the classroom, preparing students for participation in broader social forms of activity. Thus, it is possible for authentic learning environments to incorporate aspects of external practice, within the context of an activity energised by specifically educational outcomes.

2.4.4 Participation in cultural practices

Participation in cultural activities is an essential aspect of learning (Greeno, Collins and Resnick 1996). Different types of cultural activities support learning in the classroom such as problem solving activities, apprenticeships and projects (Greeno, Collins and Resnick, 1996). Effective learning environments engage students in domain-related activities (Barab and Duffy, 2000) or real world practices (Guthrie and Alao, 1997) that are intrinsically motivating, support conceptual understanding and enhance the quality of questions students pose. According to Lave (1996), effective learning practices allow peripheral participants to become more central participants by breaking down distinctions between learning and doing, and between education and occupation. This conceptual unity of learning and doing is evident in apprenticeships where learning and practice are interwove such that the process of learning is an integral component of the practice as a whole.

Lave (1996) presents an example of this interpenetration of learning and doing within Muslim law schools in Cairo. First, within these schools the process of learning was shaped by the logic of interpretation that characterises Muslim legal practice, beginning with the Koran, followed by the hadith (the sayings of Muhammad), and then to commentaries on the Koran. The sequence of learning and the form that the process of learning took was determined by this practice of interpreting sacred texts. Second, patterns of learning were repeated in different forms of legal practice conducted by
experienced lawyers, for whom the activity of learning was simply one aspect of their daily practice of the law.

Attempts to align classroom mathematical activity with mathematical activity occurring outside the classroom are often motivated by a desire to make learning in the classroom more authentic. Dewey’s model of progressive education, for example, supported the establishment of rich, authentic learning environments (Wilson, Teslow and Taylor, 1993). Within educational psychology, teaching authentically involves meeting at least three criteria – disciplined inquiry, integration of knowledge and value beyond evaluation (Archbald and Newmann, 1988). Disciplined inquiry moves beyond knowledge produced by others to a formulation of new ideas. Through disciplined inquiry, students can sometimes even reject the public knowledge base. Integration of knowledge involves considering the whole rather than fragments. Archbald and Newmann (1988) argue that “students must also be involved in the production, not simply the reproduction of new knowledge” (p.3). Finally authentic achievements have value beyond evaluation – rather than using tests or quizzes students could write letters, news articles, or speak a foreign language – practices that have value beyond merely being assessed (Burke, 1994).

Rather than maximise the realism of authentic learning environments, however, Lebow and Wagner (1994) suggest that authentic learning environments should instead support the learner in establishing a learning enterprise within the larger global task environment. Furthermore, they argue that the learning situation should afford the kinds of activities that are necessary for success in the transfer environment - the environment for which students are being prepared. What is most essential is not the fidelity of the learning situation but whether or not students practise what is essential for the transfer situation. Beach (1999) describes classroom learning activities as mediational activities designed to simulate activities yet to be experienced. They vary along a continuum from “as if” activities that simulate relations in the world beyond the school to partial or peripheral participation in the activities themselves. However, they maintain a mediational status as a “third object” between where the participants are and where they are going developmentally. When considering the manner in which cultural practices are made
available to classroom students, teachers need to consider how best to support students’
movement from their current participation in mathematical practices to their future
participation.

2.5 **Effective mathematics learning is self-regulated**

The fifth principle of effective mathematics learning is that students develop skills that
enable them to regulate their own learning. Teaching students to be autonomous, self-
regulated learners has long been a desired outcome for educators. Lambert and McCombs
(1998) argue that students’ ability to select and monitor mental operations facilitates
creative and critical thinking. Self-directed learning, or autonomous learning, refers to
activities that are wholly or partly under the control of the learner. There is evidence that
giving students greater control over their own learning and improving their capabilities
for self-directed learning results in increases in personal efficacy, increased motivation to
learn and increased effort expended on learning tasks (Thomas, Strage and Curley, 1988).

The NCTM document (2000) suggests that students’ experience in school mathematics
should support the development of student autonomy. Mathematics learning is enhanced
when students take control of their own learning, defining their own goals and monitoring
their own progress, reflecting on their thinking and learning from mistakes (NCTM,
2000). McGaw (1996) suggests that the perception of students as active constructors of
their own learning provides a rationale for viewing learners as responsible for managing
much of their own learning. Schools need to be encouraging students to learn how to
learn. Norman (1994) argues that students should be encouraged to understand their own
learning, moving towards becoming autonomous learners.

2.5.1 **Definitions of self-regulation**

Self-regulated learning involves two aspects of learning – the active control of resources,
motivational beliefs and emotions, and the adoption of various cognitive strategies
(Pintrich, 1995; Thomas, Strage and Curley, 1988). Self-regulated learners are
“metacognitively, motivationally, and behaviourally active participants in their own
learning process” (Zimmerman, 1986, p.307). Although interest in self-regulated learning
has diverse theoretical origins, recent theoretical approaches attempt to identify specific learning strategies that students employ to improve their academic achievement (Zimmerman, 1989a). Metacognitive approaches focus on information processing characteristics as strategies for self-regulation (Borkowski and Thorpe, 1994) while social cognitive theories include help seeking and environmental restructuring (Zimmerman, 1989b). Research has also attempted to identify self-evaluative subprocesses through which students monitor their performance (Bandura, 1986). Vygotskian theorists, for example, emphasise the role of private speech in self-reflective thought (Rohrkemper, 1989) and the relationship between other-person regulation and self-regulation (Rogoff, Ellis and Gardner, 1984; Wood, Bruner and Ross, 1976; Wertsch, McNamee, McLane and Budwig, 1980). Finally, research into processes of self-regulation has also attempted to identify motivating factors behind why students adopt such strategies (Zimmerman, 1994). Social cognitive theorists stress the importance of self efficacy and personal goals as significant factors in determining whether students will adopt self-regulatory strategies (Schunk, 1989; Malpass, O’Neil and Hocevar, 1996). Volitional theorists suggest that students must also be able to protect their intentions from distractions if they are to be self-regulated learners (Corno, 1989; 1986).

Self-management activities that self-regulated learners engage in include time management, effort management, and volitional management (Thomas, Strage and Curley, 1988). Bandura (1986) suggests that before an individual can successfully use appropriate cognitive strategies of self-regulation they must first be able to monitor their own learning and be aware of how effective or ineffective their current strategies are. This requires students to be aware of several subprocesses of self-evaluation. Self-observation involves deliberate attention to one’s behaviour. Self-regulated learners keep records, regularly evaluate their progress and seek out areas for improvement. Self-judgment involves setting personal standards and evaluating one’s progress in the light of these standards. Self-reactions to their own progress may be tangible or evaluative. Tangible reactions may involve rewarding or punishing as a consequence of their progress. Evaluative reactions are personal assessments of their progress. By successfully regulating their own learning through each of these subprocesses students are then able to
choose appropriate cognitive and behavioural strategies to enhance their ability to learn and to achieve their self-set goals.

In the category of cognitive strategies, Thomas, Strage and Curley (1988) identify five distinct strategies students may adopt. First, students engage in a process of selection - focusing selectively on important material. Second, students engage in strategies designed to enhance comprehension. Such activities may include previewing material, noting points of difficulty and consulting relevant references. Third, students may appropriate strategies for memory enhancement including rehearsal, elaboration and the use of mnemonics. Fourth, self-regulated students integrate their learning. Finally, students engage in cognitive monitoring, monitoring their own learning and evaluating their progress.

Five principles are outlined by Pintrich (1995) for encouraging self-regulated learning in the classroom. First, students develop greater awareness of their own behaviour, motivation and cognition gaining self-awareness through self-reflection. Carpenter and Lehrer (1999) argue that reflecting on one’s own experiences and learning results in the development of mathematical understanding. Second, students need to have positive motivational beliefs including mastery goal orientations and positive self-efficacy for learning. Third, teachers have a responsibility to model self-regulated learning. Fourth, students benefit from opportunities to practise these strategies. Fifth, classroom tasks provide opportunities for self-regulated learning through the provision of choice and control.

Vygotsky also emphasised the importance of understanding metacognitive processes of planning and self-control (Scribner, 1985). For Vygotsky, processes of self-regulation are initially processes of other-regulation internalised (Vygotsky, 1978). In a study conducted by Brown (1987), for example, students showed significant improvement over a period of ten sessions in which an adult working with the child demonstrated metacognitive strategies for the students to appropriate.
2.5.2 Metacognition and self-regulation

Schoenfeld (1987; 1992) has placed considerable emphasis on self-regulation as an important aspect of mathematics learning. He describes the construct “metacognition” as a tripartite construct comprised of beliefs and intuitions about mathematics, knowledge of one’s own thought processes and self-awareness (Schoenfeld, 1987). Teaching students how to participate in mathematical practice also involves teaching students how to think about their own beliefs, intuitions and thought processes. Good problem solvers are able to reflect on the strategies and methods they have adopted, monitoring their progress and adjusting their approach where necessary (Bransford, Brown and Cocking, 1999). When teaching mathematics, Schoenfeld works with students solving unfamiliar problems, thereby enabling them to gain insight into the way that he and other students solve problems. Often when solving problems, mathematicians make mistakes or choose incorrect strategies. Effective problem solving requires students to monitor their thinking, to recognise when strategies are inappropriate, and be prepared to change direction.

2.6 Effective mathematics learning is meaningful

Lambert and McCombs (1998) identify the goal of learning in general as the development of meaningful, coherent representations of knowledge, constructed through the linking of new information with existing knowledge in meaningful ways. Within the field of mathematics education, the issue of learning mathematics “meaningfully” represents a central issue for teachers of mathematics. Sigurdson and Olson (1992) compared the examination results of students who were taught by teachers who emphasised meaning and teachers who emphasised procedural algorithms. While both groups achieved higher results than “control” classrooms, above average students in the “meaning” classrooms achieved higher results than students in the “algorithmic” classrooms while for other ability levels there was no difference between the “meaning” and “algorithmic” classrooms. In another comparison between students who were encouraged to construct their own meaning of mathematics, students who focused on developing their own meaning displayed higher test scores than students taught in the traditional manner (Maher, 1991). Over a period of nine years the Kenilworth Project
involved changing teaching practices away from traditional approaches to meaning-based approaches in which students were observed

...inventing their own methods, searching for meaning in their carrying out of mathematical activities, and discussing and arguing about strategies as they work (often collaboratively) to construct solutions to problems. (Maher, 1991, p. 227)

Learning mathematics should be meaningful, providing students with a rich understanding of the linkages between different mathematical concepts, the logical consistency of mathematical structures and the knowledge to continue developing their mathematical understanding. Meaningful mathematics learning provides students with a unified conceptual understanding of the domain of mathematics (Romberg and Kaput, 1999).

Sociocultural theorists offer a second interpretation of what constitutes “meaningful” learning – learning that is culturally relevant, that is purposeful and contextual. According to this second definition, mathematics is meaningful for students when it enables them to accomplish desirable outcomes. Thus, there exist two distinct definitions of meaningful mathematics which shall be discussed in turn – the development of meaningful structures and the development of meaningful practices in the form of problem solving.

2.6.1 The development of meaningful structures

According to reformers of mathematical education, learning mathematics in a meaningful way requires focusing on important mathematical ideas and assisting students to organise these ideas into a coherent whole (Romberg and Kaput, 1999; NCTM, 2000). Supporting students’ evaluation of their own thinking and that of other students, developing mathematical reasoning, and proposing new ideas and conjectures enhance the meaningfulness of the mathematics students engage in (Yackel and Hanna, 2003).

Enhancing the meaningfulness of mathematics requires that students develop a rich understanding of the mathematics they perform in the classroom. The 1991 NCTM
document calls for a shift from memorising procedures and mechanistic answer finding towards conjecturing and mathematical reasoning. Stigler and Hiebert (1997) describe learning mathematics in the United States as the process of learning terms and practising procedures. Research suggests, however, that conceptual understanding is an important component of proficiency, alongside factual knowledge and procedural facility (Bransford, Brown and Cocking, 1999). Mathematics makes more sense to students and is easier to remember and apply when students are able to connect new knowledge to existing knowledge in meaningful ways (Schoenfeld, 1988). Furthermore, learning with understanding supports the development of autonomous learners (NCTM 2000).

Traditional teaching of mathematics often focuses on supporting students’ manipulation of symbols without an understanding of the underlying mathematical constructs. Introductory calculus often progresses in such a manner. Schoenfeld (1991) argues that many students develop mathematical "nonreason" - engaging in activities that don't make sense due to the lack of any conceptual understanding. For example, many students have a tendency to use all available data in problem statements regardless of relevance. However, this strategy is particularly effective during typical classroom activities working on sets of isomorphic problems. Students can successfully participate in classroom activities focused on completing isomorphic textbook problems with little or no understanding of the underlying mathematical concept (Wilson, Teslow and Taylor, 1993).

An example of student “nonreason” in mathematics was exhibited by students answering problems in the National Assessment of Education Progress secondary mathematics exam (Schoenfeld, 1987). Students were required to determine how many buses were required to transport 1128 soldiers if each bus was able to carry 36 soldiers. Seventy percent of students were able to perform the long division required correctly. However, 29% gave fractional answers to describe the number of buses - 31 remainder 12, while another 18% answered 31. Only 23% gave the correct answer of 32.
Critics of traditional mathematics instruction have argued that traditional instruction encourages mechanistic manipulation of numbers, symbols and deductive proofs (Romberg and Kaput, 1999). Students learn to master specific techniques, equated with knowledge, instead of linking such practices to the purposes for which people use mathematics. Student activity resembles that of the Chinese interpreter in Searle’s Chinese room thought experiment (1980) outlined earlier in this thesis. This person has no understanding of Chinese, but is able to respond to written requests in Chinese using a book of procedures that describes what to write in response to different sets of characters. Students in mathematics sometimes appear to perform symbolic manipulations in much the same manner without any understanding of what they are doing or why they are doing it.

Mathematical understanding, however, is cumulative (Lerman, 1998), developing over extended periods such that a student of mathematics rarely fully understands a mathematical concept. Mathematical understanding is also contextual, with different criteria for understanding existing within different developmental contexts. For example, in NSW schools students manipulate percentages and fractions from an early age (by the end of primary school) but it is unlikely that such students fully understand the concept of fractions at this early age. As Wittgenstein (1974) states about linguistic understanding "I understand the word" perhaps comes to the same as "I know how it is used, etc.": not that I try hard to call to mind its entire application in order to answer the question whether I understand the word. (p.64)

Similarly with mathematical concepts, understanding of concepts such as percentages and fractions depends upon the context within which percentages and fractions are used. Students in primary school who can equate percentages such as 50%, 30% and 25% with \( \frac{1}{2}, \frac{3}{10} \) and \( \frac{1}{4} \) have an understanding of fractions appropriate for their developmental context. These same students, however, may not fully grasp the meaning of the symbol\% (few students recognise that 100\% = 1) and yet may still successfully solve problems involving percentages. Symbolic manipulation without a conceptual understanding of the
symbols involved may be indicative of a certain level of understanding prior to
developing a deeper understanding later in one’s mathematical career.

2.6.2 Problem solving as a meaningful mathematical practice

Reform documents are unanimous in their recommendation that students need to engage
in problem solving activities within the classroom (DEYTA, 2000; Romberg and Kaput,
1999; Cocker, 1982; Lovitt and Clarke, 1988; NCTM, 1991; NCTM, 2000). Since the
1960's there has been a shift in thinking about mathematics teaching from an emphasis on
analysis of mathematical structure and rigour towards the use of problem contexts to
develop meaning (Nickson, 1992; Schoenfeld, 1992). Other mathematical activities
proposed as meaningful activities include mathematical modelling (Romberg and Kaput,
1999), mathematical investigations (Cockcroft, 1982) and developing mathematical
conjectures (NCTM, 1991; Milton, 1995). Problem solving as a meaningful mathematical
practice, however, has received the most attention in reform documents. Participation in
meaningful activities such as problem solving represents valuable mathematical activities
supporting students’ understanding of concepts as well as forming links between school
curriculum and the external world (Cooper, 1994).

What, then, can be described as a problem? Good, Mulryan and McCaslin (1992) suggest
that problem solving involves having to choose from a range of alternative strategies
when it is uncertain which strategy will prove to be most efficacious. Classroom activities
that support problem solving in the classroom encourage students to take a flexible
approach, setting tasks that are unpredictable, and providing students with opportunities
to take control of their own learning. Problems are “open” (Lubienski, 2000) having no
obvious solutions. Problems are also presented as motivating situations often based in the
real world (NCTM, 1989). Problem solving, therefore, has the capacity to bring together
the affective and the cognitive, merging together motivation, emotion and cognitive
strategies (Good, Mulryan and McCaslin, 1992).

Working in collaborative groups has the potential to help students develop problem
solving skills and dispositions (Good, Mulryan and McCaslin, 1992). Socially shared
problem solving (Resnick, 1989) sets up several conditions that support the development of problem solving skills. First, it provides opportunities for students and teachers to model effective thinking strategies – strategies that would normally remain implicit (Schoenfeld, 1985). Second, “thinking aloud” provides opportunities for others to critique and shape one’s performance. Third, social settings provide opportunities for scaffolding of performances of students by other participants that leads to students developing these skills over time. Fourth, social settings perform a motivational function, and fifth, such settings lend status to the disposition towards meaning construction activities.

The Cognition and Technology Group from Vanderbilt University (CTGV) (1990; 1994; 1996; Goldman, et. al. 1996) have developed a substantial body of research into the use of videodiscs as a problem solving context within which students are able to engage in practical mathematical activities. Their work arises from a concern with the lack of mathematical problem solving skills displayed by high school graduates. Students do not appear to approach mathematical knowledge as a tool that can be used to solve problems (CTGV, 1990) but rather as difficult concepts to be endured. Little positive evidence currently exists to suggest that individuals are able to transfer what they have learnt in schools to real world problems that require mathematics. In contrast to these negative findings, several researchers have documented examples of sophisticated problem solving outside the classroom (Lave, 1988; Lave and Wenger, 1991; Carraher, Carraher and Schliemann, 1983; Nunes, 1995) however these abilities seem inextricably linked to situations that have practical significance for the individuals involved. The CTGV group, therefore, set out to provide multimedia software that would allow students to attempt problems with greater practical relevance. They refer to their framework as one of anchored instruction (CTGV, 1990; Goldman, et. al. 1996). In particular they designed a series of videos that featured a character Jasper Woodbury observed in situations that required some application of mathematics. The videos afforded a range of learning activities such as generating sub-goals, identifying relevant information, and cooperating with others to solve problems.
Campbell (1996) reports on a separate project that involved introducing teachers of primary school students to a problem solving, collaborative approach to mathematics based on constructivist principles. Teachers from the schools involved attended a 22 day training program over the summer break before teaching students using a problem solving based approach. Students who were taught in these classrooms scored more highly on mathematics exams and were more able to perform tasks that required higher levels of abstraction. Similarly, Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti and Perlwitz (1991) compared the results of students in Grade 2 classrooms, some of whom were working in problem solving classrooms and some working in traditional classrooms. In their study which ran for a year students in the problem solving classrooms achieved higher results than those learning primarily through instruction followed by practice.

Another study conducted by Ginsburg-Block and Fantuzzo (1998) examined the potential benefits of problem solving based approaches and collaboration. They compared four different groups of students in 3rd and 4th class who attended two 30 minute sessions each week. The first group learnt through problem solving, the second group learnt through collaborating with others, the third through problem solving and collaborating with others, and the fourth group was a control group. Ginsburg-Block and Fantuzzo found that students who learnt through problem solving or through problem solving and collaboration were more able to complete word problems and perform computations. These students also displayed higher levels of motivation and perceived competence.

Each of these studies suggests that students who learn mathematics by focusing on problems and learn through collaborating with others rather than merely learning computation skills achieve higher results than students who are instructed in computation skills and then given time to practise these skills. However, in each of the studies cited, students from the early years of primary school participated in the studies rather than secondary students. Studies investigating the benefits for secondary students are rare. Perhaps one of the reasons for this scarcity of studies with secondary students is the difficulty of covering the secondary curriculum with a problem solving approach which requires considerable time and greater flexibility than is available to most secondary teachers who are conscious of preparing students for external examinations.
Within the mathematics classroom, the issue of making mathematics meaningful remains an important issue for teachers faced with teaching a curriculum that may bear little relation to the mathematics students could make use of outside the classroom. The cultural significance of such tasks may not be evident to students, nor their relation to other forms of knowing. Many mathematics teachers ignore the applications of what they are teaching, while many others struggle to force applications from topics that are unlikely to be of practical relevance outside the classroom. Making mathematics topics such as geometry, calculus and advanced algebra meaningful within a cultural context represents an important goal of the current study.

2.7 Effective mathematics learning is motivated

The affective aspect of classroom environments represents an important dimension of effective classroom practice. Lambert and McCombs (1998) outline several learner-centred psychological principles relating to the motivational and affective factors that impact upon student learning in the classroom. Specifically, emotional states, beliefs, interests, goals, and habits of thinking influence motivational states of students, as well as the learner’s creativity, higher order thinking and natural curiosity. Several issues relating to the motivational states of learners have received considerable attention from researchers including learning goals (Hagan and Weinstein, 1995), intentionality (Scardamalia and Bereiter, 1996a; 1996b; 1996c; Scardamalia, Bereiter and Lamon, 1994), agency of students in the classroom (Boaler and Greeno, 2000), developing goals for classrooms (Newman, 1997), making links with student interests and providing tasks with clear purposes (Mercer, 1995).

2.7.1 Learning goals

Much research on the importance of students adopting learning goals has occurred within the theoretical tradition of social cognition. A goal is defined as a cognitive representation of the different purposes students may adopt in different achievement situations (Pintrich, Marx and Boyle, 1993). The two main distinctions within this tradition are between intrinsic, mastery and task-involved orientation and extrinsic,
performance and ego-involved orientation resulting in different patterns of cognitive engagement (Dweck and Leggett, 1988; Nolen, 1988; Lepper, 1988; Pintrich and Schrauben, 1992). Intrinsic, mastery and task-involved orientations result in students using more elaboration procedures as well as appropriating metacognitive and self-regulatory strategies such as planning and comprehension monitoring (Pintrich, Marx and Boyle, 1993). Classroom environments can support the adoption of such goals. Tasks that are challenging, meaningful and authentic encourage the adoption of mastery goals. Teachers need to emphasise the meaningful aspects of the learning task rather than the trivial aspects, design tasks that are interesting and challenging to students, encourage students to set short-term goals, and support the use of meaningful learning strategies (Ames, 1992). Choice and control also support the adoption of mastery goals. Teachers need to provide students with opportunities to make choices about course assignments, and support students’ self-monitoring by assisting them to solve problems on their own (Hagan and Weinstein, 1995). Finally, evaluation procedures that focus on competition, social comparison and external rewards discourage mastery goals (Pintrich, Marx and Boyle, 1993). Ames (1992) suggests that instructors focus on each student’s improvement and mastery of material, emphasising effort rather than ability, evaluate students privately rather than publicly, and assist students to see errors as part of the process of learning rather than an indication of poor ability. Evaluations of progress also need to emphasise individual accomplishment, specific comparisons to criteria and cooperation (Hagan and Weinstein, 1995).

Other research conducted by activity theorists has produced similar recommendations using Leont’ev’s theoretical approach to understanding human activity. Results obtained from a study into involuntary memory indicated that students who performed tasks automatically had difficulty remembering what they had actually done one or two minutes later while students who had been required to develop the same type of tasks themselves were more able to recall them (Zinchenko, 1981). Zinchenko suggests that more effective learning occurs at the level of action than at the level of operation – that is, goal-directed behaviour results in greater recall of what the task involved when compared with the amount that people could recall from tasks that were automatic.
2.7.2 Co-construction of classroom goals

An activity-based approach raises questions about the intentions of participants in cultural-historical activity (Crawford, 1996). Within the classroom, both the teacher(s) and the students are participants in classroom activity, however, the goals of these different participants may exist in opposition to each other. These opposite goals may cancel each other out, or lead to the goals of one participant directing classroom activity at the expense of the other participants’ goals. A well-designed functional learning environment is one that coordinates the children’s and teacher’s points of view so that both the children and teacher can achieve meaningful goals (Newman, 1997). Typically, teachers aim to achieve learning outcomes while students may pursue such goals as well as other goals associated with personal interests, social acceptance and performance goals.

Newman (1997) has documented several models of learning using technology and the effectiveness of coordinating the goals of students and teachers. One such approach involved students writing notes to each other from different places in the US to develop a monthly newspaper for students in another area of the country. The production of the monthly newspaper not only encouraged students to write but also provided a context for students to develop their editing and revision skills on their work and the work of others. Within this project a coordination of teacher and student goals is evident, partly due to students finding the task interesting because they were able to communicate with people in “exotic” places. The teachers also found the tasks worthwhile because they provided a context for students to practise their writing and their editing and revising of work. Within the classroom, as such an example indicates, it is possible for students and teachers to appropriate goals that are complementary rather than in conflict with each other.

2.7.3 Intentionality

Students also need to be motivated to pursue learning outcomes intentionally in a similar manner to researchers who are motivated by the desire for recognition from peers, the desire to have an impact, and the desire to participate in significant discourses.
define intentional learning as synonymous with self-regulated learning and autonomous learning. Intentional learning involves actively and strategically pursuing learning as a goal. Intentionality is not a property of individual actors within the classrooms but is a property of the classroom community. Scardamalia, Bereiter and Lamon (1994) suggest that classes adopt collective goals for understanding different concepts.

Through the interactions of a community of learners a domain of public knowledge is developed in line with Popper’s concept of World 3 knowledge (Bereiter, 1994). Popper’s World 3 knowledge refers to knowledge that exists as an immaterial object outside of the mind existing within the public domain. Such knowledge structures have origins, histories and are open to criticism and falsification. Much of Scardamalia and Bereiter’s research has focused on the opportunities for learning that are provided by the Computer Supported Intentional Learning Environment (CSILE) database. The CSILE database provides a medium within which individuals are able to negotiate meaning within their particular learning communities leading to the development of socially constructed understandings.

2.7.4 Perceptions of agency within the classroom

Developing student agency represents a central aspect of effective learning environments across the curriculum (Holland, Lachicotte, Skinner and Cain, 1998). The concept of agency is closely associated with the notion of intentionality. Bandura (1997) defines agency as acting intentionally in different situations. Boaler and Greeno (2000) describe students acting actively or passively within different mathematics classrooms reflecting their own perceptions of agency within these classrooms. They describe an active role within the classroom as one in which students are actively engaging with the procedures encountered in the classroom interpreting the meaning of these procedures. Students who adopt a more passive role receive predetermined procedures methods and approaches and reproduce these procedures when answering questions. Boaler and Greeno (2000) describe how students who take active and passive roles within the classroom develop different perceptions of what constitutes mathematics. In their study, students
participating in classrooms that encouraged them to take an active role viewed mathematics as a subject in which they could explore and make sense of different mathematical problems and ideas. Students who took on a more passive role viewed mathematics as a set of procedures to be learnt and memorised. Several other studies have found similar results within traditional classrooms (Boaler, 1999; Schoenfeld, 1988).

Personal agency is a key idea within activity theory. Leont'ev (1978) asserts that all activity is object-oriented, being motivated towards the transformation of a particular object to achieve a certain outcome. The character of mathematical activity within the classroom, therefore, is closely related to the role which students adopt and their perceptions of what constitutes mathematics. Mathematical activity may be exploratory, engaging and discussion-based with students taking an active role within the classroom activity or it may be fundamentally reproductive, ritualistic and individualistic as students adopt a more passive role. Unfortunately, most mathematics classrooms encourage students to adopt a passive role reproducing teacher methods as they complete exercises designed to provide practice at reproducing these procedures (Schoenfeld, 1988). By encouraging this passive role, students are focused on procedures rather than concepts underlying these procedures (Romberg and Kaput, 1999; Mason, 1989). In such classrooms, perceptions of agency are likely to be low compared with classrooms in which students are provided with opportunities to participate in the knowledge-building process.

2.7.5 Student interests

Effective learning environments also make links between classroom practice and existing student interests (Lambert and McCombs, 1998; Dewey, 1902; 1938; NCTM, 1991; 2000). Well-chosen tasks can pique students’ curiosity drawing them into mathematics – by connecting with the real-world experiences of students. Worthwhile tasks, however, should be intriguing, providing a level of challenge that encourages speculation and effort (NCTM, 2000).
Dewey (1902; 1938) suggested that classroom activities need to be related to the child’s experiences, interests and goals. Achieving this goal, however, has proved difficult with the restrictions of limited resources, limited time, materials and training. Part of the difficulty for classroom teachers rests in the heterogeneity of student interests evident within classrooms of approximately twenty-five students. As children develop into adolescents the heterogeneity of student interests increases in the classroom. Developing classroom activities related to the multiple cultural, sub-cultural and individual interests that are apparent in secondary classrooms represents a significant challenge for classroom teachers. Classroom environments that engage students in similar activities are unlikely to relate to the interests of all students within the classroom. The ideal of making connections between student interests and curriculum activities and the ideal of providing a common curriculum for all students represent contradictory ideals for classroom teachers.

Forming links between student interests and the curriculum covered in the classroom has also proved difficult due to the cultural content of the curriculum being remote from most spontaneous student interests. The mathematics curriculum, for example, covers aspects of mathematics that bear little or no resemblance to the everyday interests of students such as trigonometry, deductive proofs in geometry and calculus. However, such topics represent valuable aspects of the mathematics curriculum designed to achieve the outcomes outlined by the document *Principles and Standards for School Mathematics* (NCTM, 2000). The practice of performing geometrical proofs is personally satisfying and empowering for students, providing students with an insight into the role that mathematics has played in the development of Western culture. Trigonometry represents a valuable cultural tool in many fields of endeavour that require an intensive knowledge of mathematics such as engineering, surveying, building and design to name but a few. Calculus represents one of the most significant intellectual developments in Western culture that is also fundamental to a variety of disciplines in the physical, biological and social sciences. Developing a model of mathematics classroom learning that forms links, where possible, between student interests and the many different aspects of the mathematics curriculum represents a significant goal of the current study.
2.7.6 Clear rationale for learning

Effective learning environments are functional learning environments in which the learning activities have a function or purpose that is clearly identifiable by students (Newman, 1997). Learners need access to the rationale for classroom activities and learners need to find this rationale convincing (Mercer, 1995). One of the problems for many activities in schools is that students are not aware of the rationale for participating in the activity.

All mathematical learning should take place in the context of an activity that students want to participate in and are able to participate in given their current level of development (van Oers, 1996). The meaningfulness of the activity needs to be made clear to students at all times so that the mathematical activity is recognised as “real” by the mathematical community and is also recognised as “real” by students within the context of their activity (van Oers, 1996). Furthermore, the teacher should make sure that the mathematical activity is consistent with the methods (but not necessarily the results) of present-day mathematics. Mathematical activity should be embedded within a sensible activity for students. Providing students with a clear rationale involves providing students with access to cultural practices, engaging students in tasks that are extrinsically meaningful as well as intrinsically meaningful such that the purpose for participating in such activities is apparent.

2.8 Effective mathematics learning is developmental

The use of the term “development”, after Piaget, is often synonymous with biological or cognitive development that proceeds through a set of universal stages of development towards the endpoint of being able to form hypotheses and deduce relationships. Piaget’s universal approach to development, with its fundamentally Western telos, has received criticism from a number of theorists who see development being linked more closely with the child’s interaction in cultural practices (Matusov and Hayes, 2000; Rogoff, 1990). Vygotsky’s approach to development maintained an ethnocentric telos, although instead of viewing the end result of development as the emergence of scientific thinking, Vygotsky saw the endpoint of development as the enculturation into western “high”
culture (Matusov and Hayes, 2000). Within Vygotsky’s theory, however, the interplay between cultural and biological development represents the fundamental dynamic leading to individual development (Scribner, 1985). According to Vygotsky, ontogeny does not replicate phylogeny – a position that Piaget also eventually rejected. Whereas in general history the cultural displaces the natural, in the development of the child there remains a fusion of the two and both lines of development co-occur.

Effective mathematical learning is developmental in the sense that such learning occurs in a structured manner taking into account the developmental history of students. An awareness of the principles of the practice of mathematics and the development of children’s thinking in mathematics represent the fundamental foundations upon which the development of a program of learning proceeds (Shayer and Adey, 2002; Brown, Ellery and Campione, 1998).

2.8.1 Vygotskian approach to development

According to Vygotsky (1978) development proceeds through the process of social interaction with more able others. Vygotsky suggested that there exists a zone or region of sensitivity for every learner that extends beyond their current capacity within which they are able to perform tasks with assistance.

This . . . is what we call the zone of proximal development. It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. (Vygotsky, 1978. p. 86)

The zone of proximal development represents a dynamic region of sensitivity in which interpsychological functioning can be internalised as intrapsychological functioning (Wertsch, 1985). It refers to the distance between the level of actual development and the more advanced level of potential development that comes into existence through interactions between more and less capable participants (Vygotsky, 1978; Cole and Wertsch, 1996). An essential aspect of these interactions is that less capable participants are able to participate in practices that are beyond their competence when acting alone.
Reform documents also recognise the need to provide mathematical activities for students that are challenging (DETYA, 2000; NCTM, 2000; AEC, 1991).

Typically, North American interpretations of Vygotsky’s theory of development focus on the provision of scaffolding for student learning (Bruner, 1975, Wood, Bruner and Ross, 1976, Clay and Cazden, 1990; Pressley and McCormick, 1995) in domains such as reading and mathematics. Scaffolding learning involves an adult (normally) providing hints or prompts that enable a child to achieve learning outcomes such as reading a certain text. However, equating Vygotsky’s theory with the method of scaffolding represents an inadequate exposition of the applications of Vygotsky’s theory. Scaffolding focuses on transforming student behaviour so that students are able to produce correct answers, rather than transforming students’ participation in communities of practice. Pressley and McCormick (1995) argue that scaffolding seems similar to the behaviourist concept of “fading”, implicitly acknowledging that this focus on scaffolding represents a distinctly behaviourist interpretation of Vygotsky’s sociocultural theory.

Moll and Whitmore (1993; Moll, 1990) suggest that such conceptions of the zone of proximal development are too narrow, focusing only on the assistance that more capable others provide and failing to incorporate the socially provided resources for development. The zone of proximal development is evident when the child is engaged in collaborative activity within particular social discursive environments (Moll and Whitmore, 1993) – a collective zone of proximal development that includes social and cultural resources for supporting individual activity. Teachers, therefore, need to provide students with opportunities to participate in different forms of social discourse, supporting students’ use of social and cultural resources such as concepts, language and tools that students may only partially understand how to use effectively (Palincsar, Brown and Campione, 1993; Chang-Wells and Wells, 1993).

Often development is compared with embryological development in which development is slow and smooth – and refers to the unfolding of possibilities contained in the embryo. Vygotsky (1981a: 1981b) suggests that a more appropriate comparison is with the
evolutionary development of species. Child development is not a smooth process, but involves rapid change as the child adapts to the external world. In this process new forms of thought develop which were not present embryonically. The history of cultural development, according to Vygotsky occurs through the collision of the organism with the environment and subsequent adaptation (Vygotsky, 1981b). Development, or learning, for Vygotsky is a process of upheaval, sudden changes and revolution (Kozulin, 1986).

The implications for mathematics learning of Vygotsky’s theory of development are many, not least of which is the importance of developing a clear picture of students’ existing level of understanding. Teachers need to be fully cognisant of the current level of students’ understanding so that the lower limit of the zone of proximal development is apparent and the level of appropriate challenge identified.

Providing opportunities for students to work with other students who are able to support them through their zone of proximal development is another important implication of Vygotsky’s theory. Collaborative activities involving peers provide activity settings within which students are able to participate in cultural practices beyond their current capabilities. Collaborative activities provide multiple zones of proximal development within which individuals are able to progress with the support of other participants in the collaborative activity (Brown, 1994; Eraut, 1994). Collaborative activities in the classroom, therefore, can be structured to enable students to participate in discussions that are beyond their individual capability, with individuals being supported by other participants in the activity.

Students also learn through engaging with more capable others. Within collaborative activities, different students demonstrate advanced understandings of different aspects of mathematical practice. Thus, it is possible that within collaborative groups students will lead each other through different zones of proximal development.
Vygotsky’s theory of the zone of proximal development supports the notion of joint participation in cultural activities by students and more capable others. Capable peers can support the development of other students by building bridges from current understandings to new understandings and structuring participation, with shifts in the responsibilities given to learners as they assume increasingly skilled roles in the activities of the community (Rogoff, 1990). As children begin to participate in certain activities, they perform only a minor part of the activity. Over time, however, learners take responsibility for more aspects of the activity.

2.8.2 The CAME project: Piagetian and Vygotskian perspectives brought together

The Cognitive Acceleration approach underlying the CAME project takes into account developmental considerations such as those proposed by Piaget and Vygotsky (Adhami, 2002; Shayer and Adey, 2002). The content of each lesson and its structure is developed using Piagetian stages of development while the model of collaboration adopted in each classroom supports students’ development through individual zones of proximal development. Prior to the introduction of the Cognitive Acceleration approach at a primary school, for example, the first year of the project involved designing specific lessons for implementation in the following year. Each lesson involved identifying Piagetian levels of development through which students were intended to progress and strategies for inducing such learning were developed by teachers and teacher researchers at the school participating in the study. The substantial investment of time and resources required to prepare teachers for involvement in such projects represents a significant barrier to more widespread appropriation of the Cognitive Acceleration approach. Furthermore, planning lessons around a singular Piagetian trajectory of development can constrain the learning that may occur in the classroom. Critics of Piaget have claimed that he underestimated the abilities of children (Donaldson, 1978). For example, algebra is not normally introduced in mathematics classrooms until students have reached the age at which most students can understand abstract thought of this kind – around twelve years old, according to Piaget. Within Davydov’s approach to teaching mathematics, however,
students are introduced to algebraic approaches to solving word problems between the ages of 7 and 10 (Davydov, 1990).

A second problem with the Piagetian framework is the inability of this framework to make sense of the diverse ways in which individuals can develop through their involvement in different cultural practices. Piaget’s model of development originated from observations of children from middle class Switzerland who had been educated in Swiss schools. While Australian classrooms a generation ago would have been populated almost exclusively by students from Anglo Saxon backgrounds whose cultural heritage would have shared considerable overlap to those students Piaget observed, this is no longer the case with many cultures being represented in different classrooms. Apart from European cultures, Asian cultures, African cultures, Middle Eastern cultures and (in Australia) Pacific Islander cultures are all more likely to have representatives present in mathematical classrooms in Australia. The confluence of multiple cultural trajectories within the classroom provides opportunities for development amongst members of the classroom that are outside the prescribed Piagetian trajectory. Experience interacting with the social and cultural resources of different cultural activities provides students with the opportunity to progress with other students through their individual zones of proximal development, engaging in activities that would be beyond their individual competence level. Social resources that exist as part of the interpsychological plane of different activity settings include cultural artefacts, experiences of other peers and dialogue structures that support expansive learning (Mercer, 1995).

Vygotsky’s concept of the zone of proximal development provides a theoretical foundation for introducing collaborative groups into the mathematics classroom. Furthermore, for these groups to be most effective, they need access to a wide range of cultural artefacts and dialogue structures that support the incorporation of pre-existing cultural ways of doing mathematics. The current project examines a model of collaborative learning that attempts to give students access to a wide range of cultural artefacts. This model will be described in detail in the next chapter.
3 A collaborative learning model of mathematics learning

Reform documents in mathematics education, consistent with the recommendations from sociocultural theorists, have argued that mathematics classrooms should encourage collaborative, dialogical, dialectical, cultural, self-regulatory, meaningful, intentional and developmentally appropriate activity. Transforming secondary mathematics classrooms into such learning environments, however, has proved difficult to achieve within the current educational context. The pressures on teachers to prepare students for external examinations are significant within Australian schools, overriding any concerns to reform the classroom in the manner outlined above. The intention of such examinations to select students for tertiary positions, to provide a comparative indication of students’ performance is unlikely to change in the near future. Selection of students for allocation of restricted opportunities requires a format that allows for individual comparison among students as competing candidates. The purpose of such assessment is to differentiate students on one or more dimensions, and so the purpose of the measurement is normative (McGaw, 1996). Hence, an assumption underlying the current project is that the reality of such examinations will remain a significant limiting factor on efforts to reform mathematical learning environments.

Classroom teachers are also constrained in their efforts to reform mathematical classrooms by external policies, the structure of the curriculum, school timetables, the organisation of school environments, and the requirements of external assessment procedures. Pressures to complete the syllabus that emphasises breadth rather than depth (Addington, Clemens, Howe and Saul, 2000) make it almost impossible for teachers to encourage alternative approaches to mathematics that focus on more meaningful practices such as problem solving, investigations and modelling activities (Grimison, 1995). Given these restrictions on the activities of teachers in the classroom, the intention of this study is to provide classroom teachers with a model of mathematics classroom learning that is
consistent with the concerns of reformers as well as the proximal concerns of classroom teachers.

3.1 The Collaborative Learning Model: transforming traditional classroom practice

The following model of classroom mathematics learning represents a synthetic model incorporating the concerns of the reform movement and the concerns of classroom teachers faced with the practical reality of preparing students for external examinations. It assumes that existing practice within schools involves teaching mathematical concepts according to topic rather than the reform model of teaching themes or within the context of projects. A second assumption is that the content contained in the curriculum includes a significant number of concepts for coverage within a brief period of time (approximately three or four weeks). A third assumption is that teachers assess students across their grade with traditional pen and paper tests on a regular basis. The final assumption is that teachers cover topics in class that demonstrate few obvious links with the intrinsic interests of students.

3.1.1 Engagement in authentic mathematical activity

Criticisms of traditional mathematics teaching have focused on the inauthentic nature of much of the mathematics that takes place in the classroom (Romberg and Kaput, 1999; Resnick, 1991; Lave, 1988). Authentic mathematical activities are clearly evident within the context of particular communities of practice such as running street stalls, calculating best buys and deriving formulae for engineering applications. Such activities involve complex mathematical applications, integrating different aspects of mathematical practice and have value beyond evaluation (Archbald and Newman, 1988). Mathematics activities in the classroom typically reflect rarefied examples of mathematics outside the classroom. Worked examples that focus on singular processes followed by textbook exercises designed to develop this particular skill are simplified mathematical applications, abstracted mathematical practices rather than concrete, integrated mathematical practice. Furthermore, such practices prepare students for an evaluation process without necessarily having intrinsic value for students. The purpose of
mathematical activities, the benefits for students and the intrinsic value of mathematical tools should be evident to students participating in such activities.

The NCTM document *Principles and Standards for School Mathematics* (NCTM, 2000) recognises that students will require mathematical forms of understanding in a range of occupations as well as in the workplace in general. The difficulty facing mathematics teachers (and teachers in other disciplines) relates to the multiple cultural practices that require mathematical forms of understanding. Classrooms of twenty-five students include students from many backgrounds and interests, each of which are likely to engage in unique forms of cultural practice, each of which require different forms of mathematical understanding. Providing authentic activities appropriate for all students represents a significant challenge for classroom teachers. Covering the content of the secondary curriculum represents another formidable challenge for teachers at the same time providing students with authentic activities that reflect classroom mathematical practice.

However, there exists a cultural practice that requires all forms of mathematical understanding covered in the secondary curriculum – the practice of teaching secondary mathematics. While few students are likely to train as teachers of secondary mathematics, all secondary students can engage in the practice of teaching secondary mathematics. One of the tasks that secondary mathematics teachers perform is the construction of different forms of assessment tasks for assessing performance in different topics. This is a task that students can undertake in the classroom and, by doing so, develop their understanding of different mathematical concepts. To develop such tasks which effectively assess other students’ understanding of the topic students need to develop an understanding of the mathematical concepts contained within each topic, an understanding of student understanding and potential difficulties students may experience with the topic. The development of assessment tasks that incorporate a wide range of problems that can be solved using specified mathematical techniques also requires students to think creatively about how they might use these concepts in different contexts.
Developing assessment tasks advantages a student in several ways, not least of which is the increased familiarity the student gains with such tasks. Creating topic tests, for example, provides the author with valuable insights into the nature of mathematical examinations, including how to set out answers, how to answer questions correctly to maximise marks and how exam questions are typically structured. Depending on the quality of the assessment tasks developed, teachers could collate the tasks as a resource for use in other mathematics classrooms. Assessment tasks developed by students provide other teachers with a valuable resource, as well as providing students themselves with valuable tools for revision.

The current model of authentic learning meets the three criteria outlined by Archbald and Newman (1988) of disciplined inquiry, integration of knowledge and value beyond evaluation. Developing assessment tasks requires students to produce new ways of assessing the topic in question, and to form connections within the topic under investigation and with related topics within mathematics and from other related disciplines. Furthermore, developing assessment tasks has value beyond evaluation. For students, such assessment tasks provide valuable resources for future revision. For the classroom teacher, these assessment tasks can become valuable resources for use in other classroom contexts. Students may also benefit indirectly by producing assessment tasks for publication and use by other teachers and, perhaps, even other schools.

Within this study, the purpose of classroom activity, therefore, over the duration of time normally devoted to a particular topic, is the collaborative development of assessment tasks (with worked solutions) for that particular topic. As part of this activity, students work in collaborative groups to work on developing a common understanding of the concepts underlying the topic, using the cultural resources available within the classroom (including the expertise of the teacher). Students are encouraged to incorporate within their assessment tasks items relating to individual interests and experiences from outside the classroom, and are assessed on the quality of their final product rather than their performance on these assessment tasks.
Authentic, collaborative classroom activities motivated towards the production of assessment tasks in mathematics are not typical of current classroom practice, but represent a model of classroom learning designed to satisfy some of the proximal concerns of classroom teachers. Furthermore, the model is also portable in the sense that it can be implemented within schools with restrictive timetables, limited resources and traditional mathematics curricula – factors that are typically outside the control of the classroom teacher.

3.1.2 Detailing pre-requisite knowledge

At the commencement of the new topic, all students complete an assessment of pre-requisite skills identified by the teacher from previous topics. For each pre-requisite skill, three questions are offered of varying difficulty so that all students develop a clear picture of their current level of understanding of different pre-requisite skills. The purpose of this aspect of the model is twofold – to provide the classroom teacher with an indication of the existing understandings of students in the class, and to provide students with feedback regarding their own understanding.

3.1.3 Division of class community into collaborative groups

Based on previous performances in mathematics, the classroom teacher divides the class into quartiles. One student from the first, second, third and fourth quartile joins each of the collaborative groups. Students are placed in groups with students who display different levels of understanding so that collaborative groups provide multiple, overlapping zones of proximal development (Brown, Ellery and Campione, 1998). Evidence from research into co-operative learning also recommends that students work with students of differing levels of understanding (Webb, 1980; Mugny and Doise, 1978). Rather than a random assignment of students into different groups for each topic, the classroom teacher assigns students to different groups. The classroom teacher uses their own experience with students to assign students avoiding gender or ethnic imbalances in different groups as well as potential personality clashes between students.
3.1.4 Provision of outcomes to be covered

Students receive the list of outcomes contained in the program as well as the prerequisite skills for each outcome including example questions. Once the classroom teacher has taken the time to assess their performances on the example questions for each pre-requisite skill, students receive individual comments on their understanding of each of the pre-requisites (see Appendix Five for sample feedback sheets). Collaborative groups also receive comments on which pre-requisites someone in the group has demonstrated a good understanding. Identifying prior understanding of individual students and members of different groups as well as the outcomes contained within each topic provides students with initial support for forming links between their current understanding and culturally shared understandings contained within particular topics.

3.1.5 Nature of collaboration

From this information, groups negotiate their understanding of the pre-requisite skills and the accompanying outcomes of the topic under investigation. Groups are encouraged to begin by negotiating understanding of aspects of the topic that they feel they already “know” and then making use of the cultural resources and artefacts available within the classroom to extend their understanding to cover all of the outcomes contained in the program. Students with deeper understandings articulate these understandings for other students to compare and contrast with their own understanding. Sharing one’s understanding with other students represents a valuable process for all students involved. Students teaching others are encouraged to elaborate on their understanding providing multiple representations of different concepts and to co-ordinate and translate these representations. Student teachers are also encouraged to give specific examples, translate unusual vocabulary into familiar terms, create analogies, describe processes, and develop detailed justifications, supporting one’s opinions and beliefs with evidence and making links between real-world experiences and theoretical models (Webb and Palincsar, 1996). Supporting student teaching is an important responsibility of the classroom teacher as they interact with different groups.
Classroom discussion may take part at the end of each topic to develop links between aspects of this topic and other topics as a classroom community, depending on the topic of study. However, interaction between the teacher and members of the classroom community primarily occurs at the group level rather than the class level. Collaborating with members of different groups, the classroom teacher models the social and ethical values that support effective collaboration. The classroom teacher also supports student collaboration by encouraging students to make their thoughts and ideas public, externalising reflections, and offering ideas for discussion by other members of their group. Within each group, the development of a shared understanding necessitates students providing reasons for their opinions, developing mathematically valid arguments for different conceptions, and framing questions to ask other members of the group about their own understanding. The teacher encourages students to develop such reasons, arguments and questions as well as modelling such practices for students. Further to their own modelling students receive rubrics and key phrases that might assist them to discuss mathematical ideas with others (Goos, 2000).

Webb and Palincsar (1996) suggest that students require preparation for participation in group work. In particular, classroom communities need to develop norms for cooperation and prosocial behaviour and for instructing students how to use specific helping skills known to influence learning. Developing prosocial values such as kindness, fairness, consideration, responsibility and honesty in the classroom for example, promoting the understanding of different people groups, and encouraging helping behaviours in the classroom all represent valuable norms for supporting group work in the classroom. In a study by Webb and Farivar (1994) investigating the effect of instruction on group interaction in the classroom, students received instruction in communication skills, developing norms for group behaviour and helping skills. Students’ performance in mathematics improved as a result of this instruction. Teachers received five full days of training in this instruction and these teachers then took five lessons to give students practice helping each other using the strategies outlined by Webb and Farivar such as providing detailed explanations rather than just answers. However, Webb and Farivar’s description of classroom interaction within this study indicated that
classroom interactions were still based upon the teacher providing a thorough explanation of the concept to be learnt at the beginning of the lesson followed by students practising using the concept by doing exercises from a textbook. Most classroom teachers allow students to discuss how they might solve different problems - Wenglinsky’s study (2000) suggests that as many as 86% of mathematics teachers provide opportunities for group work during their lessons. The difference between classrooms in Webb and Farivar’s study and traditional classrooms was the training provided to teachers and students in how to foster positive conversations in the classroom.

The current model of classroom practice supports the teacher’s role as a facilitator of collaboration in different groups through discussion, demonstration, practice and feedback (Swing and Peterson, 1982). Teachers may require training in identifying and developing such prosocial norms in the classrooms. Training for teachers precedes their participation in this classroom model of collaboration in which they are provided with information on basic communication skills, norms for group behaviour and helping skills (Webb and Palincsar, 1996). Basic communication skills include checking for understanding, sharing ideas and information, encouraging and checking for agreement. Norms for group behaviour include attentive listening, positive reinforcement, moderate voice level, equal participation of group members and conflict resolution skills. Helping behaviours include providing other students with elaborated answers rather than just answers and asking clear questions (Webb and Farivar, 1994).

3.1.6 Resources for developing understanding

Vygotsky’s theory of learning suggests that learning involves a continuous dialectic between cultural and everyday understandings. Hence, effective learning environments encourage students to engage with a range of cultural voices. Collaboration amongst students represents the initial interaction of individual understandings with cultural understandings. According to Vygotsky, learning is a dialectic process in which an individual tests a personally held concept against those of other members of the culture (van Oers, 1996). The testing of individual ideas begins in the context of the classroom collaborative groups. However, it is expected that such collaboration is limited in terms
of developing cultural forms of knowing relating to the practice of mathematics. Collaborative groups eventually reach a point where they are unable to progress any further without interacting with other cultural voices. Within the classroom several resources are available to assist collaborative groups to further their collective understanding.

Groups have access to mathematical technologies that can assist in developing understanding such as calculators, graphical calculators, graphical programs, blackboard and chalk, geometrical instruments and spreadsheets (where available). Collaborative groups make use of available cultural tools and engage in cultural activities associated with such tools providing a societal zone of proximal development for the group to progress through (Engeström, 1987).

Within any classroom, there exists a wide range of cultural experience, expertise and interests. Students in one collaborative group may turn to students in another collaborative group for assistance with particular concepts or explanations. Within dialogic classroom environments, the sharing of perspectives, multiple voices and texts supports the development of deeper forms of personal understanding as students appropriate the different mathematical voices of the teacher, other students, and participants in mathematical activity outside the classroom.

3.1.6.1 Texts

Traditionally, textbooks represent repositories of exercises rather than teaching tools. In the current model, students are encouraged to use their textbooks as resources to achieve the learning outcomes of their groups. Groups also have access to several other textbooks in the classroom for researching mathematical domains. As well as textbooks, students have access to other mathematical texts which may be relevant to the topic such as newspapers, magazines, historical documents and commercial catalogues. Students with access to the Internet also have access to many valuable hypertext documents of relevance to different mathematical topics.
3.1.6.2 Discussion with the teacher

The teacher acts as a seeding agent within the classroom, bringing to the collaborative discussions aspects of the broadly accepted historical culture of mathematics. Teachers, therefore, are required to know and understand the mathematics they are teaching, the potential difficulties students might face in developing student understanding and to draw on this understanding flexibly when teaching students (NCTM 2000; Milton, 1995). Furthermore, research into the effective teaching of different subdomains within the domain of mathematics (such as Pegg, 1995; Booth, 1995) provides teachers with valuable ideas regarding the discourse with students on different topics. The most valuable resource in the classroom is still the classroom teacher as they facilitate, encourage and support positive collaboration and learning amongst students. While this will often be through indirectly supporting students’ teaching of each other, teachers also take direct responsibility at times for teaching other members of the classroom community.

The teacher represents one of many options for developing cultural understandings within the classroom. Students are encouraged to develop cultural understandings through interaction with other students, mathematical tools and artefacts such as textbooks as well as discussions with the teacher. However, some areas of mathematics require significant conceptual revisions that necessitate substantial leaps in student understanding best supported through discussion with classroom teachers.

Interaction between teachers and students occurs between the teacher and individual groups. In this manner, the teacher becomes a participant in the discussion, albeit one with significant authority and expertise. In this manner, the teacher presents existing cultural understandings for discussion by the group. The intention of this model of classroom learning, however, is that for most topics this will be a relatively rare occurrence. When discussing concepts with students, the teacher begins by making sure that students have mastered the pre-requisite skills for a particular outcome before explaining the new concept. Once the teacher has discussed with a group a particular conceptual understanding, it becomes the responsibility of that group to share that
conceptual understanding with other groups through the class notice board. Over the
duration of the topic, therefore, the teacher engages in discussing each concept only once
to different groups within the classroom. Further requests regarding that particular
concept are then directed to the group who participated in discussions with the teacher
regarding that particular concept. Each group, therefore, may engage with the teacher for
direct assistance with only a small proportion of the total subject matter. Having
explained a particular concept once to a certain group, it should not be necessary for the
teacher to explain the same concept to another group in the classroom.

The principal role of the teacher, then, is to support student participation in the cultural
practice of teaching and learning mathematics. There are several ways that teachers can
support student mathematical activity. First, the teacher can make suggestions about what
resources groups might employ to develop their understanding of different concepts.
Second, the role of the teacher may be to demonstrate how individual groups might use
different resources to develop their understanding. Third, the teacher could assist them to
make sense of mathematical texts (being the mediator between the discourse of historical
mathematics and classroom understanding). Finally, the teacher could act as a mediator in
discussions in which students do not demonstrate the principles of effective
communication for collaboration such as respect for other opinions, and courage to
change one’s own opinions when required.

3.1.7 Hierarchical model of mathematical investigation

Students are provided with a pattern for mathematical investigations which outlines the
order which they are encouraged to appropriate to progress towards developing their
understanding. First, individual students are able to discuss any difficulties with other
members of their collaborative group. Second, students have the opportunity of
conducting research using textbooks and other mathematical technologies. Third, the
groups have the option of placing questions on the classroom notice board. Finally,
students may decide to ask the classroom teacher for assistance.
3.1.8 Development of assessment task

Students work together to develop an assessment task that effectively examines understanding of each of the outcomes in the topic including worked solutions. At the completion of the topic, each student is “assessed” using two assessment tasks developed by members of the classroom – the task developed by their own group and the task completed by another group. The classroom teacher may use a student’s performance on his or her own task as evidence of effective group work. Similar results for group members would provide an indication of effective collaboration, while a wide range of responses would provide an indication of ineffective collaboration.

3.1.9 Formative and summative assessment

Assessment for each topic is primarily based on the quality of students’ assessment tasks rather than their performance on such tasks. The purpose of such assessment is primarily formative rather than summative, providing students with feedback on their understanding of the topic. Thus, the marking scale for such assessment tasks should reflect this emphasis by grading highly assessment tasks that demonstrate a thorough understanding of the topic. Secondary aspects of the marking scale should reflect the concerns of the teacher for encouraging students to develop links between their mathematics learning in the classroom and contexts outside the classroom, and their ability to think creatively about how mathematics can be applied in different contexts. Students’ ability to collaborate with each other represents another secondary aspect of interest to the classroom teacher. An example marking scale is presented in Appendix One.

Students receive a copy of the marking scale so that they are fully aware of what is expected of them and how the assessment tasks will be graded. Students from each group receive a single mark based on the quality of their assessment task according to the above criteria. The purpose of this method of assessment is not to provide an extrinsic reward for participation in classroom activities, however, but to provide feedback to students on the quality of their work. In particular, students’ ability to work with other members of
their group to achieve collective goals is one of the core criteria for assessment using this marking scale.

Often teachers may “feel trapped between the goals they have for their students’ mathematical learning and those of high-stakes tests” (NCTM 2000, p.374). The multiple purposes of secondary schooling drive multiple assessment practices (McGaw, 1996) such as classroom tests, assessments of different classroom performances, teacher observations and external examinations. School assessment often represents the criterion for selection into mathematics classes or mathematics courses. Assessment at the completion of the final year of school represents the principal criterion for entry into different tertiary courses. The assessment procedure integrated within the collaborative learning model of the classroom is not designed to be used for such purposes.

An assumption of this classroom model is that within the mathematics courses, assessment procedures exist in which students’ mathematical understanding is tested using a format more suitable for student selection. The assessment procedure contained within the current model supplements existing assessment procedures that exist within the school. Student performance on traditional examinations, however, remains an indicator of the effectiveness of this model.

3.1.10 The Collaborative Learning Model – a summary

In summary, the model provides groups of students with the goal of constructing assessment tasks with worked solutions that effectively assess specific topics in mathematics. Student understanding of outcomes and pre-requisite skills is “tested” before students begin working on this task so that each student, as well as each group of students, receives feedback regarding their individual and collective understanding. In conjunction with this feedback, students also receive a list of the outcomes they are required to assess in their assessment tasks. Thus, what students know, and what they need to know to complete the task are evident to students from the outset.
A pattern of collaboration and learning activity is established within the classroom so that each individual student works primarily with other members of their group to develop their collective understanding of the necessary material. Once a group of students has identified what they need to learn, they are then encouraged to develop their collective understanding through using available cultural tools such as calculators, software packages, and other concrete resources available in the classroom and cultural artefacts such as textbooks and other mathematical texts. Other valuable resources in the classroom, in the form of other students and the teacher, are also made available to groups when their collective efforts do not yield results. For example, collaborative groups have the option of engaging with the teacher in discussion about different areas of the topic that they may find difficult. However, once a group has consulted with a teacher regarding an aspect of the topic, they are given the responsibility of teaching other groups this particular component.

An important aspect of the classroom model is that dialogue rather than monologue becomes the defining feature of classroom interaction. To support dialogue, the interaction between the teacher and students occurs primarily through the teacher working with small groups of students instead of the whole class. Hence, students learn from each other in collaborative groups, sometimes with the added input of the teacher or other groups within the classroom, so that they are equipped for producing an assessment task that assesses a student’s understanding of a certain topic.

Drawing on Vygotsky’s theory of cognition and learning and Leont’ev’s cultural-historical theory of activity, collaboration within the classroom can be depicted pictorially in the following manner. The focus of the classroom model is on the collective understanding of collaborative groups in which the significant role of teachers, cultural tools and artefacts and interaction with other groups in developing this understanding is clearly identified. This is portrayed diagrammatically in Figure 5.
Figure 5: Dialogical relationships within the Collaborative Learning Model
The dual purposes of the current model – to reflect the concerns of both the reform movement and the proximate concerns of classroom teachers whose performance is closely tied to student performance on external examinations, provide the rationale for answering the following research questions.

3.2 Research Questions: How are the eight principles of effective mathematics learning realised in collaborative classrooms?

Four research questions are developed drawing on the eight principles for effective mathematics learning identified in the literature. Research Question One draws together the principles relating to collaboration, dialogism, the movement of students from peripheral to central participants (that learning should be dialectic) and the opportunities for development within the context of collaborative dialogue. This first question also looks at how collaboration as a central aspect of classroom practice can transform the classroom practice as a whole. Research Question Two focuses on the motivational aspects of classroom practice – whether or not students are motivated to learn and whether or not students are motivated to regulate their own learning. Research Question Three focuses on the meaningfulness of students’ learning experience and the degree to which learning mathematics is embedded in cultural practices. Research Question Four reflects the more immediate concerns of classroom teachers relating to whether or not students learning in a collaborative classroom perform better or worse on traditional exams.

3.2.1 Research Question One: How do students work together in the collaborative classroom and how does the collaborative learning model transform the culture of classrooms in each school?

In the collaborative classrooms, the current study will explore the nature of collaboration that takes place within the collaborative classrooms and the nature of collaboration that occurs within the traditional classrooms. In particular, the current study will explore whether or not there is evidence of dialogism within the classroom – to what degree do students interact with multiple voices in contrast with the monologic nature of traditional
classrooms. Some of the voices that students might hear could be those of other texts, including conventional textbooks, the voices of other students, as well as the voice of the teacher. The way that these voices interact within the classroom will also be explored.

Collaboration also has the potential to enable learning to take on more of a dialectical nature – through the interaction that collaboration involves, students may come to radically new understandings that supercede previous understandings. Furthermore, participants in the classroom may take on new roles, whereby students move from being peripheral participants to central participants within the classroom activity. For example, students become teachers, teachers become students and new relationships develop amongst classroom participants that continue to evolve over time.

Finally, collaboration provides a context within which students are able to move through their individual zones of proximal development with the assistance of other students.

To develop a picture of the type of collaboration that occurs within the different classrooms, the current study will examine the scores obtained on various sub-scales related to peer learning from a revised version of the Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich, Smith, Garcia and McKeachie, 1991), as well as interview data collected during the study and at the completion of the study, and data collected during classroom observations. The development of the Revised MSLQ is described in Section 4.5.5.

3.2.2 Research Question Two: What evidence is there that students develop into motivated, self-regulated learners of mathematics?

The second research question relates to the development of self-regulatory skills and students being motivated to achieve personal academic goals within the classroom – or what Scardamalia and Bereiter (1996a; 1996b; 1996c) describe as intentional learning. Several sub-scales of the MSLQ relate to self-regulation and motivation and these will be examined to determine whether or not students display higher levels of motivation and use more self-regulatory strategies. Furthermore, students’ responses to interview
questions and observations within the classroom will be used to develop a picture of students’ self-regulation and levels of motivation during the collaborative classrooms.

3.2.3 Research Question Three: How closely does collaborative classroom mathematics resemble real world mathematics?

From classroom observations and interviews the match between the mathematical activities in the collaborative classrooms and mathematical activities that occur outside the classroom will also be considered to ascertain in what ways classroom activities resemble mathematical activities outside the classroom. In particular, to what degree are the activities in the classroom embedded in existing cultural practices.

3.2.4 Research Question Four: How effectively are the proximal concerns of teachers to prepare students for traditional exams met when students work within the collaborative classroom?

Students working together in collaborative classrooms will be required to complete a traditional exam at the conclusion of each topic which will also be completed by students working in a traditional classroom. Comparisons between different classes looking at their examination results as well as interviews with students and teachers will be used together to develop a picture of whether or not students were sufficiently prepared for traditional examinations within the collaborative classrooms.

The nature of these research questions and the general complexity of mathematics classrooms necessitate the adoption of a number of methods for collecting and analysing data. These methods as well as the approach to implementing this model in secondary mathematics classrooms are outlined in the next chapter.
4 Developing an activity-based grounded approach for evaluating the collaborative learning model

In this chapter the methodology adopted for use in the current study will be outlined beginning with the general strategy adopted for answering the research questions presented at the end of Chapter Three.

4.1 Research strategy

The methodological paradigm within which the current study is framed is that of multiple embedded case studies (Scholz and Tietje, 2002) where each of the schools involved represent separate cases. The purpose of an embedded case study is to comprehend each case as a whole in its real-world context (Scholz and Tietje, 2002). Yet embedded case studies are more complex in their unit of analysis than case studies designed to describe a single case. Beginning with the “case”, which may be an organisation, for example, embedded case studies also involve identifying smaller sub-units embedded within the case so that the object of analysis may vary at different stages in the analysis. Not only are embedded case studies more complex in terms of the levels of analysis, but they also suit a wider range of methodologies than holistic case studies that focus on qualitative description. Embedded case studies collect data from a wide variety of sources (Yin, 2003; 1994) which are then integrated to make inferences about the case or cases in question.

Both embedded and holistic case studies are used when the complexity of the phenomena under investigation makes identifying each of the key variables for isolation practically impossible (Yin, 2003; 1994). Case studies are an ideal approach, therefore, to studying cultural-historical phenomena such as cognition, learning and development that are emergent properties of participation in social interaction rather than the sum total outcome of the combining of component variables. Vygotsky proposes that psychological
phenomena are the dialectical outworking of the individual’s participation in cultural historical activity and represent qualitatively different phenomena from their constituent parts (Vygotsky, 1986). Understanding such phenomena, therefore, necessitates a holistic approach that analyses them embedded in their cultural historical context.

Two schools participated in the current study on the condition that the schools remain anonymous. Hence, each school and the names of teachers at each school have been replaced with pseudonyms. The two schools, Brindale Christian School and Southwestern High School, will be described in more detail in sections 5.1 and 5.2. For the purposes of this study, the implementation of the collaborative learning model at each of these schools represents a case for analysis within which the implementation of the collaborative learning model took place in a number of different classes. Within each school, however, identifying characteristics of different classes, different groups and different individuals within each school are a necessary step towards analysing each school as a separate case.

Case studies are useful for answering “how” and “why” research questions (Yin, 2003) such as research questions one, two and three in the current study referring to how learning collaboratively has an impact on the classroom culture. Yin (1994) suggests that case studies are also particularly useful for studying phenomena where the “boundaries between the phenomena and the context are not clearly distinguished” (p.13), where there is little control over the events of interest, and the focus is on contemporary phenomena occurring within a real world context (Yin, 2003). From a cultural-historical perspective, however, context cannot be treated as an additional variable, but as the substratum without which psychological phenomena would not exist. Case studies, therefore, are an ideal strategy for understanding psychological phenomena that emphasises the interconnectedness of such phenomena with their cultural and historical contexts.

Case studies often use a number of different approaches to understanding phenomena associated with each case. Much of the data in the current study was collected through a process of naturalistic inquiry observing and describing phenomena within their natural
setting. Using the theoretical framework of activity theory, phenomena in each classroom are described in terms of activities, actions and operations taking place within the wider context of schools, educational systems and multicultural communities. A quasi-experimental design was also used to evaluate different aspects of the collaborative learning model as it was implemented at the two schools using the seven different classrooms as control and experimental groups.

The current study also adopted a wide range of data collection methods to obtain an understanding of what happened in each classroom during the collaborative lessons. These methods included classroom observations, interviews with teachers and students at the completion of the topics taught in a collaborative manner and questionnaires asking respondents to provide quantitative and qualitative information about their experiences, their beliefs and their perceptions of classroom activities. Documents developed by students were also analysed to understand classroom activity. At the conclusion of each lesson each group filled in a learning log for that lesson outlining what they learnt, and how they developed their collective understanding during each lesson. At the conclusion of the topics, students also developed assessment tasks for each of the topics they had been learning about.

The adoption of mixed methods in this study is fundamental to the research strategy focused on understanding a particular intervention and the rationale for using mixed methods is provided in section 4.9. Concurrent qualitative and quantitative approaches were used to provide multiple perspectives on classroom activity with a greater weight being given to qualitative approaches rather than quantitative approaches since these methods provide scope for the analysis of individual differences, and reveal the meanings and interpretations of participants within the different activities. Qualitative and quantitative approaches were combined at the stage of interpreting the data bringing together the findings of each type of approach to develop a more complete understanding of the different phenomena within each classroom.
4.2 Research design

The data was collected over a period of six months enabling the researcher to study the changing dynamics of the classroom over time. Due to time constraints a cross-sectional design rather than a longitudinal design was adopted to investigate the impact that learning collaboratively might have on the classroom experiences of students and teachers. The study examined the implementation of the collaborative learning model in each classroom for a period of one term.

Two schools agreed to be involved in this study. Section 4.5.2 describes in more detail the experience of recruiting schools and the number of schools that were approached to take part. The two schools who were prepared to be involved in this study are described in more detail in the following section.

4.3 Cases for analysis: Brindale Christian School and Southwest High School

The two schools who participated in the study were radically different in terms of their purpose, the school population, the communities they serve and the school environment. Both of them placed a high emphasis on academic performance, however, as a result of being a selective school the academic standard maintained within the selective classes at Southwest High School was very high with most students receiving help from tutors and other teachers outside the school.

4.3.1 Brindale Christian School

The first school involved in this study was Brindale Christian School - a small independent school with approximately 400 students from Kindergarten through to year 12. Brindale Christian School is a school that uses for its buildings a group of old homes donated by benefactors originally for the purposes of housing male wards of the state. All of the mathematics classes took place in three rooms which were all located in the same building which was an old house within which rooms were used as teaching spaces. The kitchen, which was situated on the ground floor, was the staffroom. Off the kitchen there
were two classrooms on the ground floor, and one classroom upstairs. One of the classrooms on the ground floor was very small and was referred to as the “Long Room” – it was approximately six metres wide and fifteen metres long. The other two classrooms were typical square shaped classrooms being approximately fifteen metres wide and long.

4.3.1.1 School ethos

Brindale Christian School is an independent school run by a small Baptist community strongly committed to upholding Christian values. It is a church school that sets aside approximately half an hour each morning for doctrine lessons that place great emphasis on biblical studies. The school is one of a growing number of independent schools that have emerged in the past twenty years in New South Wales. Within these schools there exists more than the occasional chapel service in which Christian values and beliefs are taught. Instead, Christian values and beliefs are evident across the curriculum as well as in the classes specifically set aside for Christian studies.

Unlike other church schools, however, all of the teachers at Brindale Christian School are members of the same church community. In all of my conversations with members of this church community, however, the emphasis was always on “community” rather than “church”. They are a close knit community consisting of a small number of families who live together and meet to pray and worship together. This emphasis on community is also evident in the classroom as well. Teachers are referred to by students as Aunt X or Uncle Y, often using family nicknames to describe their teachers rather than their Christian names or surnames.

The emphasis of Brindale Christian School is inclusion rather than elitism. In his address to parents at the 2002 end of year function, the principal stated

Throughout my career, I have been implacably opposed to any attempt made to garner high performance students together in a ghetto situation.
All classes are mixed ability, partly due to the small size of the school, but also as a result of this non-elitist policy. The whole school population, for example, rather than the school choir sang at the end of year presentation of awards and speeches (referred to as the “Annual Service of Worship”).

Brindale Christian School is described as “a ministry of the Brindale Church”. As such, it has many people willing to be involved in this “ministry” who help out in the classroom. Often there would be two adults in the room – the teacher and an assistant who was training to be a teacher who would assist in organising different aspects of the lesson. Teachers would often wander in and out of each other’s classrooms to talk to each other or to talk to students in the other class.

The school places great emphasis on academic progress, particularly in relation to literacy. Given the large number of students in the school from non-English speaking backgrounds there exists within the school an ongoing need to provide additional support for these students in the area of literacy. However, the principal focus of the school community is on the development of students’ moral, social and ethical values. This is evident in the plethora of quotes that are provided in the program for the “Annual Service of Worship”. One such quote from Stanley Hauerwas states that “The true teacher takes responsibility for shaping students morally”.

Because of their interest in community and the moral development of students, the teachers at Brindale Christian School were keen to put into practice a model of teaching that encouraged students to work together, to be other-person centred and that focused on looking after each other in the classroom. One of the teachers commented that

I have always been keen to implement collaborative learning in my classroom, I just wasn’t sure how to do it in mathematics. (Ms Black)

Across the school, the focus on community and inclusion meant that students were often required to work in groups in several other subject areas. Ms Black commented that
I think we give them lots of opportunities to work in groups in lots of different subject areas, so actually working in a group wasn’t a problem, it wasn’t a big problem, they’re used to that kind of thing and it does work … (Ms Black)

The school is viewed by the church community as the principal ministry focus of the community and many of the two hundred members of this community have some connection with the school. All of the children from this community attend the school. All of the teachers and staff at the school are members of this church community and as a consequence the teachers at the school know each other very well outside the school environment. Two of the teachers who took part in this study had also been students at the school.

4.3.1.2 Community from which teachers are drawn

All the teachers teaching at the school are members of the Brindale Christian School church community. There is a high level of commitment to the school and to teaching from all the teaching staff who see their work as a “ministry of Brindale Christian School”. Teachers spend a considerable amount of the holiday periods meeting up with other teachers teaching parallel classes. Interactions between myself and the staff took place during the school holidays and during recess and lunch breaks before and after lessons. Teachers at Brindale Christian School were prepared to put in a lot of time towards preparing resources using the guidelines I had provided and thinking about what other resources might be suitable for use in their classrooms.

4.3.1.3 Ethnic mix of students

A wide range of ethnic groups were represented at Brindale Christian School. Of the 72 students from this school who participated in this study, fourteen came from homes that spoken Middle Eastern languages (Arabic, Lebanese and Persian), thirteen came from homes in which Korean was spoken at home, nine spoke European languages (Greek, German, Hungarian, Russian and Turkish), eight came from homes in which a Chinese language was spoken (Mandarin, Cantonese or Teo Chiew) and five students spoke other Asian languages (Japanese, Urdu and Phillipino). Many of these students had only recently arrived in Australia, and their English, therefore, was relatively weak. These
students receive extra care alongside their participation in the non-streamed classes. Most of these students, however, were from homes in which the parents were at least sympathetic to the Christian worldview.

However, within the school students referred to their teachers as “Aunt X” or “Uncle Y” and so it was normal for students to refer to their teachers by their first name. Thus Ms Black was referred to as “Aunt Barbara” or simply “Barbara”, Mr Grey was called “Steven” most typically, Mr Smith was referred to as “Kevin” and Ms Gold was referred to as “Anthea”. Typical mathematics lessons at Brindale Christian School were teacher-directed with little discussion or comment from students - the majority of the lessons was taken up by the teacher explanation given from the front of the classroom. Students were shown how to answer a certain type of question on the board and were then given an exercise to complete from the textbook. Teachers at Brindale Christian School were aware of the large number of Non-English Speaking Background (NESB) students and required students to copy from textbooks lists of terms and definitions as well as notes presented throughout the text.

4.3.2 Southwest High School – Extending students in mathematics

Southwest High School is a government co-educational high school with a mix of selective and non-selective students. Southwest High School has been a partially selective high school since 1988. Prior to that it was a comprehensive high school for about twenty years. It is a multicultural school with an enrolment of 1100 students and has a reputation for high academic standards.

The introduction of selective high schools and partially selective high schools in Sydney has been a contentious issue with much controversy surrounding the process of determining which students receive places at selective high schools. In June of each year, students in New South Wales sit for three examinations to gain entry into one of 27 selective high schools – a mathematics test, an English test and a general abilities test, all of which are multiple choice. Each year approximately ten thousand students sit for these exams for one of 2000 places.
At Southwest High School there are approximately 1100 students, half of whom have come through the Selective Schools Unit testing procedure described above. In year eight mathematics, there are three selective mathematics classes all of whom participated in the current study. Of these three classes, there was an overwhelming majority of students from non-English speaking backgrounds (NESB). 86 students participated in this study – 80 of them were NESB students. Of the total 86 students, 49 (57%) came from households in which a Chinese language was spoken (Mandarin, Cantonese or Teo Chiew) and 28 (33%) came from Vietnamese speaking households.

The school was built in 1962 and the architecture of the school reflects the austerity of the early sixties. As with most public schools in Sydney, some of the classrooms at the school are portable classrooms that were brought in to accommodate a growth in numbers for a limited period of time, but are now a part of the ongoing life of the school as regular classrooms. In each classroom there are sufficient seats for thirty students. Desks are individual desks which are typically arranged in rows of pairs at the beginning of each lesson. Few resources apart from the blackboard and the textbook are available to students in ordinary lessons. One of the non-portable classrooms had posters around the walls of students’ work while the two portable classrooms did not have any work displayed around the room.

4.3.2.1 School ethos

As a partially selective high school, Southwest High School has two very different communities within the school. Ethnically, the selective community is predominantly from south east Asian backgrounds, while the comprehensive community of local students is drawn from south east Asian backgrounds and Middle Eastern backgrounds. As I walked through the playground it was evident that students from the classes with whom I was working would spend their time with other members of the selective classes. As the classes participating in the study were all selective, the following comments about the school environment refer to the selective community rather than the non-selective community.
The school describes one of its aims as promoting active involvement in the learning process to encourage students to see learning as a lifelong process. This aim was picked up by Ms Diamond who described the school’s vision as the promotion of life long learners.

Ms Diamond: For kids - in terms of the kids of the calibre in my class, it's learning for the sake of learning.

Interviewer: - -is sufficient and a good enough reason - -

Ms Diamond: It's okay, but we know they’re life-long learners, that's one of our aims with our vision statement for the school is to promote life-long learning, so knowledge for the sake of knowledge and knowing for the sake of knowing isn't a bad thing. I mean that's not to say they're not focused on, is it going to affect the reports, our HSC you know, that kind of thing.

Within Ms Diamond’s comments on students becoming life long learners there is also evidence of another aspect of the school ethos that exists within the student population – students being “focused” on examinations and preparing for upcoming assessments. This is not surprising given that admission to the selective school system is dependent upon performing well on an examination. Students may be motivated to do well in examinations for a number of reasons – to please their parents, to achieve their own personal goals, to obtain feedback on their progress and other such reasons. There are indications that at least some of the students at Southwest High School are motivated to do well in examinations to successfully compete with other students in their class. One student in Ms White’s class for example stated this most explicitly.

John: Well competition is the thing to motivate you the best. Who gets to be the class best. Like when you’re in a selective school everybody’s basically against each other. If you have a mark more, a mark less, it really counts, so that really brings the best out of you, to try to be the best of your class. (Ms White’s class)

A student in Ms Martin’s class commented that working in groups was a positive experience because it “helped (him) to compete”. Another student in Ms White’s class
described the school as “competitive”. There was a sense in which coming assessment tasks had a tendency to overshadow what was happening in the classroom. A student from Ms Martin’s class commented that she would enjoy working in groups more often so long as she didn’t have to prepare for so many assessment tasks. In her opinion working in groups was less effective for preparing her for these assessment tasks.

4.3.2.2 Ethnic background of students

As mentioned earlier, the vast majority of students at Southwest High School were from Asian cultural backgrounds. In the top class, every single student in the class was from an Asian background. Asian families in Australia tend to uphold the Confucian presumption that everyone is educable and view success as a consequence of hard work rather than ability (On, 1996). While some theorists suggest that the preferred learning style of students with a Confucian heritage is rote learning, Biggs (1996) argues that in fact such students are more likely to be repetitive in their continual effort rather than their approach to answering certain questions.

In New South Wales, students from Asian cultural backgrounds typically do better in mathematics than other cultural groups. The focus on repetitive effort as a means towards developing understanding promoted in countries that have a Confucian heritage is one of the reasons why students from these backgrounds do well in mathematics. As well as the high level of effort students display, families from Confucian backgrounds hold high expectations of their children in the subject of mathematics. This is evident in the large number of Asian families that send their children to school on Saturday to learn mathematics. For many students from Asian cultural backgrounds there is an expectation that they will go to school six days a week. There was a general understanding amongst the teachers at Southwest High School that every student in these three classes was receiving extra tuition outside school. One student who was not from a south-east Asian Confucian background commented that “Most of my peers get tutoring” and many students when asked how they learnt a particular outcome at the end of the study wrote down that they learnt everything from their tutor outside school. The teachers felt
frustrated by the perception amongst the students that they had already covered the work being done in class with their tutor.

4.4 Participants

The teachers and students who participated in this study are described in sections 4.4.1 and 4.4.2. At each school participation in the study was optional and those who did participate were given the option of withdrawing from the project if at any time they did not wish to continue.

4.4.1 Teachers

Seven teachers participated in the current study from two schools – Brindale Christian School and Southwest High School. The four teachers from Brindale Christian School who participated in the current study were Ms Black, Mr Grey, Mr Smith and Ms Gold. Ms Black was the eldest of the teachers involved and had been teaching for around twenty years. Mr Grey and Mr Smith had been teaching for about fifteen years, and Ms Gold had been teaching for about five years.

At Southwest High School three teachers participated in the current study – Ms Diamond, Ms White and Ms Martin. Many teachers at the school had not taught at any other school (Ms Martin had only taught at this school) and because the school was a selective high school there was agreement amongst the staff that Southwest High School was one of the “better” high schools in the area at which to teach. The three teachers teaching at this school who took part in this study were all very experienced. Ms Diamond, the head teacher for mathematics had been teaching for over twenty years as had Ms White. Ms Martin had been teaching for twelve years.

4.4.2 Students

At Brindale Christian School, a total of 72 students participated in the study. Each classroom was mixed (although at Brindale Christian School typically students are
separated into male and female classes in Years 7 and 8) with the year seven classes being smaller than the year eight classes. The number of students in each class and their gender is presented in Table 1.

Table 1: Student numbers at Brindale Christian School

<table>
<thead>
<tr>
<th>Class</th>
<th>Teacher</th>
<th>Male students</th>
<th>Female students</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>7B</td>
<td>Ms Black</td>
<td>6</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>7S</td>
<td>Mr Smith</td>
<td>9</td>
<td>7</td>
<td>16</td>
</tr>
<tr>
<td>8Gr</td>
<td>Mr Grey</td>
<td>16</td>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>8Go</td>
<td>Ms Gold</td>
<td>13</td>
<td>7</td>
<td>20</td>
</tr>
</tbody>
</table>

Classes at Southwest High School were much larger having close to 30 students in each class. 86 students from Southwest High School participated in the study. The number of students in each class and their gender is presented in Table 2. Students in 8M1 were the top 29 performing students at Southwest High School and the remaining selective students were in 8M2 and 8M3.

Table 2: Student numbers at Southwest High School

<table>
<thead>
<tr>
<th>Class</th>
<th>Teacher</th>
<th>Male students</th>
<th>Female students</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>8M1</td>
<td>Ms Diamond</td>
<td>10</td>
<td>19</td>
<td>29</td>
</tr>
<tr>
<td>8M2</td>
<td>Ms White</td>
<td>12</td>
<td>16</td>
<td>28</td>
</tr>
<tr>
<td>8M3</td>
<td>Ms Martin</td>
<td>16</td>
<td>13</td>
<td>29</td>
</tr>
</tbody>
</table>

4.5 Procedure

The project began in November 2001 and involvement with the schools continued through until the end of 2002. Figure 6 gives a timeline of key points over the duration of the project.
4.5.1 Ethical considerations

Specific ethical considerations relevant to this study were informed participation, confidentiality and ensuring that students were not disadvantaged as a result of their involvement in the study.

Teacher and student involvement in the project was voluntary and parents of all students who participated in the study returned a consent form giving permission for their child to be involved in the study. The consent form outlined the purpose of the study and its duration so that parents, students and teachers were aware of what the study would involve. A copy of the consent forms and information statements used is provided in Appendix Two. Throughout the study, teachers and students were reminded of their voluntary involvement and were free to choose not to be involved in the study. During classroom observations, student discussions were recorded using audio tape recorders. Groups were given the opportunity to refuse to be recorded and three groups over the duration of the project asked not to be recorded during the lessons observed.
The confidentiality of participants was ensured through each student participating in the study being given an identification number which was used on all forms and written work collected over the duration of the project. Pseudonyms are also used in this thesis to refer to teachers, students and schools.

Finally, the issue of students being disadvantaged through their involvement in this study was addressed through teachers consenting to participate in the study and, by doing so, taking responsibility for the learning of students in their class. Teachers, like students, were free to decide not to continue with the project if they were concerned about the quality of the learning opportunities of their students.

Each of these issues was considered by two bodies that were approached to obtain ethical approval for this study. Approval was sought to conduct this study with the University of Sydney Human Research Ethics Committee in November 2001 and was approved in April 2002 after minor emendations were requested in February 2002. Information regarding the University of Sydney Human Research Ethics Committee can be found at http://www.usyd.edu.au/ethics/human/. Approval was also sought from the Department of Education in December 2001 and was approved in March 2002.

4.5.2 Recruitment of schools

In December 2001 schools were approached to participate in this case study. Initially, large schools with at least eight classes in the same year were approached so that the study could be conducted at a single school. Three different schools were approached of this size, all three deciding that they could not participate due to their involvement in other projects or because they were facing staffing changes that would make it difficult for them to consider involvement in such a project. A further seven smaller schools were approached to be involved, all of whom were unable to take part due to similar reasons. In one school the deputy principal had just passed away, at another school there were already interruptions to the normal schedule due to the performance of a musical. In all, ten schools were approached who were unable to take part. In each case, most of the
teachers were keen to attempt to implement a collaborative model, but felt that it would be too difficult to do it during 2002.

Two schools agreed to be involved in the study – Brindale Christian School and Southwest High School. While these two schools were very different to the schools originally approached to participate in the study, the purpose of this case study was not to develop general statements about the collaborative learning model in a particular type of school environment. Like all case studies, the findings can contribute to the development of general theoretical propositions rather than general statements about particular populations (Yin, 2003).

Meetings with teachers at each school had to be conducted in the teacher’s own time either after school or during lunchtimes since no professional development time was provided by the schools for this project. At Brindale Christian School, all four of the year seven and year eight teachers were encouraged to take part, even though three of the teachers were uncertain about how they might teach using a collaborative approach given that they were quite comfortable teaching in the traditional manner of exposition followed by time for practice. At Southwest High School the mathematics staff were given the option of participating by the head teacher who was interested in “trying something new”. Of the seven teachers who were interested, three teachers, one of whom was the head teacher, took part since they were responsible for teaching the three year eight classes that were selective. All the teachers who decided to participate in the study were concerned about the amount of extra time their involvement might mean for them.

4.5.3 Prior meetings with teachers involved in the study

Prior to the beginning of the study two meetings were arranged with each set of teachers at both schools. Each meeting lasted for about an hour. The first meeting involved explaining how the model might look in their classrooms and answering teachers’ questions. Further to this, teachers from both schools made suggestions about how the model could be implemented more effectively. At Brindale Christian School the teachers decided to provide their students with information about what exercises they could
attempt to assist them to learn each of the outcomes. At Southwest High School the teachers suggested that each group had to check with their teacher during each lesson what they were planning to do for homework to ensure they were completing a sufficient range of questions. Over the duration of the project, I met up with each teacher at the end of each classroom observation and talked to them about how they were finding the project and whether they were having any difficulties.

Prolonged engagement was maintained over a period of twelve months working with teachers from both schools. At both schools there probably existed a certain level of suspicion about my intentions as a researcher, particularly in relation to my own beliefs about what mathematics classrooms should be like. One of the purposes of these meetings was to reassure teachers that I had realistic expectations of them and came from a similar background to them as a mathematics teacher who struggled to incorporate collaborative learning into my own classrooms.

Given the time constraints that each of the teachers faced in preparing other lessons, I offered to prepare several resources for them so that they were not unnecessarily burdened by their involvement in the project. For each topic I developed lists of outcomes based on the programs provided by both schools. Included with each outcome were sample questions relating to that outcome (see Appendix Three for outcome sheets). I also offered to mark all pre-tests and return them with feedback sheets completed for each student.

4.5.4 Implementation of Collaborative Learning Model

At each of the two schools who participated in the study, for each topic at least one class completed the topic using the collaborative learning approach and at least one class completed the topic in the same manner as they had covered topics in the past with an emphasis on exposition and individual practice. Table 3 provides information on topics completed by each class collaboratively and topics completed using the traditional approach.
Each teacher introduced the model and explained to their class what would be expected of them over the duration of the project. The teacher explained that each group would be required to design an assessment task for that topic including worked solutions and that they would be assessed on the quality of their assessment task. Prior to beginning each topic, teachers assessed students’ existing understanding of the topic by asking all students to complete a pre-test. At the beginning of the next lesson, each student received feedback on their performance on the assessment task as well as the performance of other members of their group. Sample pre-tests and feedback sheets are provided in Appendix Five.

Table 3: Participating classes and Topics covered (Note: Collaborative topics are italicised)

<table>
<thead>
<tr>
<th>Classroom Teacher</th>
<th>School</th>
<th>Class</th>
<th>Term Two (Weeks 1 – 5)</th>
<th>Term Two (Weeks 6 – 10)</th>
<th>Term Three (Weeks 1 – 5)</th>
<th>Term Three (Weeks 6 – 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms Black</td>
<td>Brindale Christian School</td>
<td>7B</td>
<td>Properties of Solids</td>
<td>Data Representation</td>
<td>Fractions and Decimals</td>
<td></td>
</tr>
<tr>
<td>Mr Smith</td>
<td>Brindale Christian School</td>
<td>7S</td>
<td>Properties of Solids</td>
<td>Data Representation</td>
<td>Fractions and Decimals</td>
<td></td>
</tr>
<tr>
<td>Ms Gold</td>
<td>Brindale Christian School</td>
<td>8Go</td>
<td>Measurement</td>
<td>Algebra</td>
<td>Ratio and Rates</td>
<td>Data Representation</td>
</tr>
<tr>
<td>Mr Grey</td>
<td>Brindale Christian School</td>
<td>8Ga</td>
<td>Measurement</td>
<td>Algebra</td>
<td>Ratio and Rates</td>
<td>Data Representation</td>
</tr>
<tr>
<td>Ms Diamond</td>
<td>Southwest High School</td>
<td>8M1</td>
<td>Equations</td>
<td>Measurement</td>
<td>Ratio and Rates</td>
<td>Pythagoras’ Theorem</td>
</tr>
<tr>
<td>Ms White</td>
<td>Southwest High School</td>
<td>8M2</td>
<td>Equations</td>
<td>Measurement</td>
<td>Ratio and Rates</td>
<td>Pythagoras’ Theorem</td>
</tr>
<tr>
<td>Ms Martin</td>
<td>Southwest High School</td>
<td>8M3</td>
<td>Equations</td>
<td>Measurement</td>
<td>Ratio and Rates</td>
<td>Pythagoras’ Theorem</td>
</tr>
</tbody>
</table>

All students were then provided with a list of the outcomes to be included in the assessment task which were appropriated from the programs being used at each school.
As well as the list of outcomes, sample questions assessing each of the outcomes were also provided to each student. Examples of such lists of outcomes and questions are provided in Appendix Three. Over the duration of each topic, students were then required to work together to ensure that every member of their group understood each of the outcomes and to develop sample items for an assessment task.

4.5.5 Data collection procedures

Observations of classrooms at both schools took place over the duration of the project. Classroom lessons were recorded on audio tape and during the collaborative lessons each group was recorded separately on audio tape. From these recordings, selected student discussions and classroom lessons were transcribed for later analysis.

At the conclusion of each topic, both the collaborative and the non-collaborative classes completed assessment tasks written by the teachers involved in the project. Scores on these assessment tasks were then used to compare the understanding of students in the collaborative and non-collaborative classrooms. The procedure for analysing these scores is described in more detail in section 7.2.

At the end of each term (T2 and T3) students in each of the seven classes were interviewed in groups. Classroom teachers selected students to take part in the group interviews. I asked each teacher to select four students with different levels of understanding to take part in the group interviews. The interview protocol can be found in Appendix Six. Each of the interviews were taped and transcribed for further analysis.

At the end of each term students also completed questionnaires which included revised MSLQ items and open ended questions relating to their experiences in the collaborative and non-collaborative classrooms. A copy of the questionnaire completed at the end of each topic is in Appendix Seven. The original MSLQ used to develop the revised MSLQ was designed as a general tool which could be administered to tertiary students doing a wide range of courses (Pintrich, Smith, Garcia and McKeachie, 1991). For example, item 1 of the MSLQ reads “In a class like this, I prefer course material that really challenges
me so I can learn new things”. This item was revised to read “In mathematics, I prefer course material that really challenges me so I can learn new things”. For each item the words “class like this”, “course” or “this class” were replaced with “mathematics” to develop the revised MSLQ. Furthermore, references to lectures and tutorials were replaced with references to classes. All of the original 81 items from the MSLQ were used to develop new items in this manner except for item 73 of the MSLQ which is “I attend classes frequently”. This was removed because secondary students do not attend classes on a voluntary basis.

All of the classes (except for 8M1 at Southwest High School) completed the revised MSLQ at three different time points (T1, T2 and T3).

Finally, teachers involved in the study were interviewed in two groups – the teachers at Brindale Christian School as one group and the teachers at Southwest High School as one group. During these group interviews, teachers were asked about their perceptions pertaining to the process of implementation of the model in each of the different classes.

4.6 Developing a mixed methods approach

In the current study both quantitative and qualitative approaches were used to answer each of the four research questions. Yet there continues to be much debate over the merits of using qualitative and quantitative methods in educational research and whether or not it is appropriate to combine such methods. Qualitative methods, including interviews, observations and open-ended questionnaires provide rich descriptions of phenomena, while quantitative methods such as factor analysis, multiple regression and analysis of variance provide quantities associated with different groups normally consisting of large numbers of participants. However, it is at the level of assumptions about ontology and epistemology that the battle lines have been drawn between the two different approaches. Lincoln and Guba (2000; Guba and Lincoln, 1989; 1985) argue for the incommensurability of qualitative and quantitative approaches. Because of the different ontological and epistemological assumptions underlying qualitative and quantitative
methods, which they label interpretivist and positivist, they argue that it would be inconsistent to adopt positivist methods within an interpretivist framework or vice versa.

Other theorists have argued that using both qualitative and quantitative approaches is preferable to obtain a more complete picture of different phenomena (Scholz and Tietje, 2002, Miles and Huberman, 1994; Reichardt and Rallis, 1994). The current study adopts this view using more than one method to develop an understanding of phenomena under investigation. By doing so, biases inherent in particular methods or instruments are reduced and the validity of the results increased when both qualitative and quantitative evidence can be obtained.

While some theorists have suggested that the role of the researcher differentiates qualitative and quantitative approaches (Merriam, 1988; Miles and Huberman, 1994), others have suggested that within both paradigms beliefs about the role of the researcher have converged to be an area of similarity rather than difference (Reichardt and Rallis (1994). Fundamental values shared by both qualitative and quantitative researchers include the value-laden nature of enquiry and the theory-laden nature of “facts”. Throughout the duration of any investigation, therefore, the researcher develops conceptual frameworks within which the phenomena under investigation are understood. Within the hermeneutic circle of research the researcher begins with certain ideas about the phenomena that direct the process of observation and which are, in turn transformed as a consequence of these observations. The process of idea formation, therefore, begins with the researcher’s presuppositions, background and experience. The preexisting ideas of the researcher in this study prior to engaging with teachers and students at the two schools are presented in section 4.7.1. Both the quantitative and qualitative analyses conducted as part of this study have been shaped by these ideas and both forms of analysis have subsequently informed them.

The battles between adherents of both qualitative and quantitative approaches are referred to by Tashakkori and Teddlie (1998) as “paradigm wars” (p.3) in which members of both camps have argued for the superiority of the constructivist/phenomenological
(qualitative) paradigm or the positivist/postpositivist (quantitative) paradigm. In contrast to the either/or position proposed by theorists such as Guba and Lincoln (1994; Lincoln and Guba, 1985), a pragmatic paradigm is proposed by Tashakkori and Teddlie (2003; 1998) that draws on the pragmatic perspective of theorists such as Peirce, James and Dewey whose philosophy focused on “what works” rather than defining concepts such as truth and reality. This approach is referred to by several theorists as a “mixed methods” approach (Tashakkori and Teddlie, 1998; Moghaddam, Walker and Harré, 2003; Cresswell, 2003).

Mixed methods approaches focus on the research problem and then develop pluralistic approaches for making sense of this problem (Cresswell, 2003). Instead of developing a single approach to understanding the problem using either quantitative or qualitative methods alone, mixed methods approaches draw on multiple methods that provide the best understanding of the problem in question. Knowledge claims that underlie mixed methods approaches are described by Cresswell (2003) as “pragmatic” claims – rather than defining the nature of reality and truth, pragmatic approaches view the “truth” value of different methodologies and epistemologies in terms of their usefulness for answering particular research questions. Hence, mixed methods approaches draw on both quantitative and qualitative approaches where appropriate for answering specific types of research questions.

From a cultural-historical perspective, using multiple methods of analysis provides a more detailed picture of the activity-based phenomena under investigation. Methodologies reflect cultural values and biases (Moghaddam, Walker and Harré, 2003) and by adopting a diversity of methodologies the risk of relying on one cultural perspective to make sense of different phenomena is reduced. Moghaddam, Walker and Harré (2003) invoke the concept of complementarity from quantum physics in which the apparently incompatible wave properties and particle properties of light are brought together to obtain a deeper understanding of the phenomenon of light as a metaphor for mixed methods approaches to studying psychological phenomena. At different levels of abstraction it is common for similarly incompatible explanations to be put forward.
regarding sociocultural phenomenon. Higher levels of abstraction (typically adopting quantitative approaches) sacrifice complexity for generality, identifying patterns and relations between a wide range of phenomena while lower levels of abstraction (qualitative approaches) provide insights into processes underlying such patterns and relations. Developing a more complete understanding of the phenomena in question requires methodologies at multiple levels of abstraction.

Within the current study, research questions one, two and three focus on understanding phenomena while the fourth question represents a hypothesis about the performance of students within the collaborative classrooms compared with students in traditional classrooms. To answer each of these questions, as well as questions that emerge throughout the process of engagement with participants, both qualitative and quantitative methods will be used. For the first three questions, qualitative methods such as interviews, observations and questionnaires will be used, although some of the data obtained using these methods will be collapsed into categories for the purposes of quantitative comparisons. Other quantitative comparisons using factor analysis and analysis of variance will also be conducted to support the qualitative analysis. Qualitative approaches to data collection and analysis, however, will be the primary methods used for answering these three questions. For the fourth research question investigating the test scores and level of understanding of students in the collaborative and traditional classrooms, the primary data sources will be quantitative such as test scores and questionnaire responses, supported by qualitative data collected from interviews.

4.7 Data collection and analysis

Case studies typically draw on multiple sources of information. These include documentation, archival records, interviews, participant observations, direct observations and physical artifacts (Yin, 1994). The final four sources are drawn upon in the current study as well as documents developed by students such as learning logs. Data was developed through observations by the researcher in the classroom by asking questions of the participants in interviews, collecting samples of students’ work and by analysing questionnaire responses and test scores.
Data analysis, therefore, involved using mixed methods including quantitative approaches such as factor analysis and Analysis of Variance, as well as qualitative methods used to obtain the different perspectives of participants in each of the classrooms within the theoretical framework of activity theory. Using qualitative procedures such as interviews and observations requires the researcher to take on the role of principal instrument for data collection. Methods of data collection can vary along the two dimensions of amount of structure and degree of researcher’s participation. One of the strengths of the current study is the appropriation of multiple methods that vary along both of these dimensions. Figure 7 provides a diagrammatic way of classifying each of the different methods adopted in the current study.

Figure 7: Participation level of researcher and level of structure in each approach to data collection

Some of these methods have a high level of researcher involvement and the data collected using these methods could be affected by the researcher’s own biases. Pre-existing biases and perspectives of the researcher, therefore, can influence the data collection and analysis and are set out in the next section.
4.7.1 The researcher as instrument: preexisting ideas

One of the criteria for trustworthiness of qualitative studies identified by Lincoln and Guba (1985) is prolonged engagement of the researcher with the phenomena being investigated. Strauss and Corbin (1998; 1990) also discuss the importance of the researcher’s theoretical sensitivity regarding the phenomena being investigated. In relation to the current study, my engagement with cultural phenomena associated with mathematics classrooms began prior to my involvement with the schools in question. As a teacher of mathematics in New South Wales schools over a period of seven years, I have become familiar with how ways of doing develop in mathematics classrooms within the same sociocultural context as the students and teachers who participated in the study. During my time as a mathematics teacher, I faced similar struggles and challenges faced by the teachers whom I worked with as part of the study – the challenges of teaching mixed ability classes, teaching students with very low levels of English proficiency, and working with students from low socioeconomic backgrounds who are the children of new Australians.

I taught mathematics in secondary high schools full time for five years and part-time for three years. Over that time, the majority of the lessons I taught followed the model of teaching described by Welch (1978) in which the focus of the lesson was my exposition of certain mathematical concepts followed by time for students to practise using the concepts by individually doing questions from a textbook. This was the model of mathematics learning I was familiar with from my time at school and as I entered the teaching profession I became aware that nearly every lesson taught by other members of staff followed this same pattern. While I have been dissatisfied with this approach to teaching mathematics for some time, given the course work I was required to “teach” there seemed to be very few other approaches to teaching mathematics that would provide students with sufficient grounding in the whole course as prescribed by the syllabus.
There were a small number of lessons in which I used co-operative learning strategies, however these were always the exception rather than the rule. More recently, however, since starting to formulate an idea about how the principles of the reform movement could be incorporated into the classroom, I have been using a method similar to the one described in this study with senior students for over a year. During that time, I have enjoyed working with students collaboratively and was motivated to examine whether this model of teaching could be adopted by other mathematics teachers working in different contexts. Throughout the process of examining how the model was appropriated within different contexts, I was aware that my own bias was to see how the model might have a positive impact rather than a negative one in each of the cases in which the model was implemented. This bias is unavoidable given that the impetus for the research project arises from an interest in how a certain approach to mathematics education could benefit students and teachers. Similar biases are evident in the work of other researchers investigating the efficacy of different approaches to teaching and learning. In Boaler’s study (1997a; 1997b), for example, she indirectly expresses her wish to find positive outcomes arising for students at Phoenix Park who were being taught using a collaborative problem solving based approach. At one stage, while observing students at Phoenix Park, Boaler began to doubt whether any learning was taking place and if the whole project might have been a waste of time (Boaler, 1997a). As I collected data from students and teachers about their experiences, I needed to ensure that this bias did not distort my own perceptions of what took place in the different classrooms. In every interview, for example, participants had opportunities to describe negative aspects of their experiences and were in fact explicitly encouraged to do so.

4.7.2 Classroom observations

Observations of student activity in the classroom took place over a period of six months with each of the seven classrooms being observed on at least four separate occasions. For each class at least three observations took place while the students and teachers were adopting the collaborative model of learning and at least one observation took place in which the teacher taught the class using the traditional approach of providing exposition from the front of the class before setting exercises for the students to complete. A total of
thirty three lessons were observed. The total amount of time observed within each classroom varied, however, due to the different lengths of lessons observed. All lessons at Southwest High School run for 100 minutes. Most lessons at Brindale Christian School run for one hour although some of the lessons observed were only forty minutes long. A total of 33 hours of class time was observed over a time period of six months. The number of observations for each class was not as high as I would have preferred, however the number was limited due to classroom teachers having to change their plans on several occasions as a consequence of out-of-routine activities within the different schools.

During these observations, I sat up the back of the classroom and took field notes regarding the structure of the lesson and observations of the interactions between the teacher and the students and between different students. As the observer in each class I did not participate in the classroom activity, walking around at times to observe different groups more closely, but not to participate in the group discussions. At times I would speak with students in the class in response to their questions about what I was doing. I would provide them with answers giving them a brief description of the study and that I was hoping they would help me to find out whether or not working together in classrooms was a good or a bad thing.

For the teacher-directed classrooms a single tape recorder was used to record the teacher’s explanations and the interaction between the teacher and students within the class. For the collaborative classrooms a separate tape recorder was placed with each of the groups. During the observations I made notes on the character of the learning experiences of students in different classrooms – whether or not students were working collaboratively or independently, whether they were focused on mathematical goals or non-mathematical goals, and identifying different activities occurring within the classroom.

Progressive subjectivity checks represent one way of enhancing the credibility of research findings (Guba and Lincoln, 1989). This involves detailing the a priori and en route constructions or interpretations of what is happening at different stages throughout
the research. Over the course of the current study changes in direction, new emergent themes and constructions about what was happening in each of the different classrooms were recorded in field notes taken during observations and throughout the course of the analysis at the completion of the involvement with each of the schools.

4.7.2.1 Analysing observation data and interview data

The procedure for analysing observation data and interview data was a grounded approach beginning with the data and developing categories which were sufficient to account for all of the variation in the data collected. This approach draws on the work of Glaser and Strauss (1967) and Strauss and Corbin (1990) beginning with the identification of emergent categories (open coding) that are then examined for inter-relationships indicative of theoretical links between different phenomena (axial coding). The final stage in this process as outlined by Strauss and Corbin involves identifying the key category or theme that emerges from the data (selective coding).

Rich descriptions of classroom phenomena were developed in the current study using data collected from interviews, observations, learning logs and student work samples. Data was sorted into provisional categories until a sufficient amount of data was accrued in each category. The rationale for including or excluding new data from each of these categories was then used as a source of information about the qualities of each category that was formed – the relationships between different categories and the conditions under which the qualities of each category are minimized or enhanced (Glaser and Strauss, 1967; Glaser, 1998). With the emergence of every new coding category the whole data set was reviewed to determine whether the coding structure needed to be changed with the introduction of this new coding category. These codes were then used as the basis for analysing questionnaire data again using an exhaustive revisiting of the data each time a new coding category was developed to account for variations in the data.

As each category emerges, the conditions for the emergence of each category, the actions and interactions between different categories and within different categories, as well as the consequences of different interactions are identified (Strauss and Corbin, 1998) and
used to explicate inter-relationships between different categories of phenomena. Unlike most applications of grounded theory, however, the current approach to understanding classroom phenomena is not purely “emergent” (Glaser, 1992), but draws on the theoretical perspective provided by Leont’ev’s activity theory. The condition of primary importance in the emergence of different categories is the form of practical activity within which each phenomena emerges. Actions and interactions associated with each of the phenomena in question represent the “actions” and “operations” of different forms of practical activity and the consequences of different interactions are the outcomes of different forms of practical activity.

Using the theoretical framework of activity theory and the categories developed from the data, theories regarding the impact of the collaborative learning model on classroom activity were developed. These theories were then tested using negative case analysis. Negative case analysis involves testing theories about the data by checking for contradictory instances in the data. In the current study, each conclusion was tested by searching through the data for contradictory cases. Where such cases were found the conclusions were modified to incorporate the occurrence of contradictory findings and further hypotheses were developed to account for the variability in students’ and teachers’ experiences.

4.7.2.2 Making sense of classroom phenomena

Data from observations were coded to identify recurring phenomena within both the conventional and collaborative classrooms. These phenomena were then grouped according to the form of practical activity within which they emerged. Initially, “forms of practical activity” were the phenomena observed within classrooms that involved a group of people working towards a particular goal.

Data obtained from interviews, observations and questionnaires, once categorised into “forms of practical activity”, were then analysed through the three “lenses” proposed by Leont’ev – identifying activities, actions and operations. For a group of phenomena to be classified as an “activity” the researcher attempted to identify the collective body
performing the activity and the collective need which this activity meets. Each of these different phenomena is located within the wider matrix of cultural historical activities. What happens in individual classrooms can be understood as being part of a larger activity (such as education or work) that is reproduced in multiple contexts. However, each classroom provides a setting for different forms of these more general activities to come into existence.

Since all “knowing” is grounded in practical activity, the role of the researcher is not to develop an independent perspective on classroom phenomena since such a perspective is developed from within the cultural practice of research rather than the cultural practice of learning mathematics. Instead, the voices and perspectives of those who participate in classroom activity – the voices of students and teachers recorded in interviews, questionnaires and learning logs represent the primary data for consideration. The purpose of the analysis is to obtain collective representations through interviewing groups of students and teachers to identify the shared phenomena that exist within each classroom. Participating in activity involves participating in a collective practice designed to achieve certain objects. Identifying these practices and objects represents the principal focus of the current study.

According to Leont’ev and Engeström, each activity has an object or motive driving the production of a certain outcome. The transforming of an object to produce this outcome fulfils a need of the participants within the activity. Actions, which take place within activities, are performed by individuals towards achieving a goal. A goal achieved may not in and of itself meet a certain need, but may contribute towards the participants in the activity fulfilling a certain need. Operations refer to automatic processes that occur in response to situations and are not motivated by a certain goal or need state.

Engeström’s theoretical framework for describing activity systems begins with the identification of the object of the activity system and the desired outcome obtained from transforming this object. Further to identifying the object and outcome, Engeström argues
that the subjects, tools, rules and divisions of labour constitute the molar whole that is an activity. Activities identified in the analysis are described using this approach.

Peer debriefing involves discussing findings with disinterested peers within the research community. Within the current study, few disinterested peers were consulted throughout the duration of the project except for participants at conferences who provided feedback on the analysis conducted in this study reported in Pietsch (2004). Interactions with two supervisors contributed to the formulation of perspectives on what happened in each classroom, although such peers could not be regarded as disinterested in the outcomes of the study as a whole.

4.7.3 Interviews

At each of the schools, the collaborative model was adopted by each classroom for two topics. At the conclusion of these two topics, focus groups of students from each class were interviewed about their experiences. Students were interviewed about the teacher-led topics as well so that comparisons might be drawn between the two learning experiences. At the conclusion of the whole project, teachers from each school were interviewed in focus groups. For each interview a relatively structured format was adopted (see Appendix Six). Each interview was taped and the interviews transcribed for analysis using open ended coding followed by an activity theory analysis.

The credibility of a study can also be enhanced by highlighting the multiple perspectives of participants rather than the singular perspective of the researcher. The voices of participants should be evident within the analysis as well as that of the researcher whose own perspective contributes to the interpretation of these perspectives. In the current study, where possible, the voices of participants are recorded in the results section to place the perspectives of participants at the foreground of the analysis.

4.7.4 Questionnaires

Questionnaires were also completed by students at the completion of the topics taught/learnt collaboratively (see Appendix Seven). Questionnaires included items from
the Revised Motivated Strategies for Learning Questionnaire (revised MSLQ), students’ self-assessment of their understanding, student opinions and attitudes about collaborative learning, the roles of students and teachers in their classrooms and student reports of how they learnt different outcomes.

4.7.4.1 Analysis of revised MSLQ scores

Students’ responses on the revised MSLQ were first analysed to assess whether the revised MSLQ demonstrated the same factor structure identified by Pintrich, Smith, Garcia and McKeachie (1991) with the original MSLQ. Confirmatory factor analyses were conducted using LISREL 8.0 (Jöreskog and Sörbom, 1998) in conjunction with exploratory factor analyses using maximum likelihood factor analysis conducted using SPSS 12.0 (SPSS Inc, 2003). Factor loadings were then used to obtain scores for each of these general factors for each person who participated in the study. A repeated measures Analysis of Variance using SPSS 12.0 for Windows (SPSS Inc, 2003) was conducted on the factor scores obtained by students on the revised MSLQ scores prior to students’ participation in the collaborative classrooms and afterwards.

Exploratory data analysis was conducted to check for any violations in the assumptions underlying confirmatory factor analysis, exploratory factor analysis and analysis of variance and to gain insight into general patterns of relationships between variables. These assumptions are multivariate normality, independence of variables, and homogeneity of error variances. Normality was examined using graphs of distributions of variables and skewness and kurtosis statistics for each variable. While graphs of distributions for variables indicated that there were some variables for which the distribution was not normal these deviations from normality were not regarded as problematic given the size of the skew and kurtosis. Each of the skewness and kurtosis statistics were within reasonable bounds with skewness statistics ranging from -1 to 1 and kurtosis statistics ranging from -1.5 to 1.5. The independence of variables was examined looking at correlations between different variables. No correlations existed between variables with absolute values greater than 0.65 and correlations between variables suggested that factor analysis would reveal commonalities in line with previous research.
conducted using the MSLQ. The assumption of homogeneity of variance was tested using Levene’s test when conducting Analyses of Variance.

4.7.4.2 Analysis of students’ self-assessment of their understanding
Students’ assessment of their own understanding of each topic was obtained by calculating the mean of their self-assessment of their understanding of each learning outcome. These means were then used to compare students’ self-assessment of their understanding in the collaborative and non-collaborative classrooms.

4.7.4.3 Analysis of students’ opinions about collaborative learning and expectations of other members of their class
Student opinions about collaborative learning were obtained during the interviews at the end of each topic as well as through items on the questionnaire which asked students for the advantages and disadvantages of working in groups and their preferred way of learning. These opinions were used to develop an understanding of the cultural values and expectations that existed within each of the classrooms. Different types of responses were coded and used to develop a better understanding of the cultural values that existed within each school. Expectations of other members of different classrooms also contributed to the developing picture of cultural values that were evident in each of the different classrooms.

4.8 Strategies for answering research questions
Four research questions were presented at the end of Chapter Three and different data collection methods and analytic procedures were used to develop answers to each of these different questions. Multiple methods for analysing practice and shared ways of doing and thinking within different classrooms were used to provide corroborative evidence of different aspects of classroom activity. To answer each of the research questions, a number of different methods were used to develop a more credible picture of what actually happened within each of the classrooms. To investigate the nature of collaboration in each classroom, classroom observations, audio recordings of classroom discussions, student and teacher interviews, student learning logs and questionnaire responses were used to obtain evidence about different forms of collaboration in the
classroom. To examine students’ motivations and levels of self-regulation, classroom observations, audio recordings of classroom discussions, student and teacher interviews and questionnaire responses were used. The relationship between classroom mathematics and real world mathematics was examined using classroom observations, audio recordings of classroom discussions, student interviews and questionnaire responses. Finally, students’ performance on traditional exams were analysed as well as the responses of students and teachers about their level of understandings.

These methods and procedures are presented in Table 4. The approaches used to address each of the four foci of the research project are then discussed in sections 4.8.1, 4.8.2, 4.8.3 and 4.8.4.

Table 4: Research questions, data collection methods and analytic procedures

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data collection methods</th>
<th>Analytic procedures</th>
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</table>
| 1. What is the nature of collaboration evident in collaborative classrooms and what are the resultant changes in the classroom culture? | * Observations  
* Student group interviews  
* Teacher group interviews  
* Questionnaire responses  
* Learning Logs | * Grounded theory analysis  
* Coding using lenses of activity, action and operation |
| 2. What evidence is there that students develop into motivated, self-regulated learners? | * Observations  
* Student group interviews  
* Teacher group interviews  
* Questionnaire responses (revised MSLQ) | * Grounded theory analysis  
* Coding using lenses of activity, action and operation  
* Factor analysis of revised MSLQ  
* Analysis of Variance of revised MSLQ |
* Student group interviews  
* Student work samples | * Grounded theory analysis  
* Coding using lenses of activity, action and operation |
4. How well are the proximal concerns of teachers met when students work within collaborative classrooms?

<table>
<thead>
<tr>
<th>Method</th>
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<tbody>
<tr>
<td>Teacher group interviews</td>
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<tr>
<td>Student group interviews</td>
</tr>
<tr>
<td>Student work samples</td>
</tr>
<tr>
<td>Test scores</td>
</tr>
<tr>
<td>Questionnaire responses</td>
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</table>

* Grounded theory analysis
* Multiple regression using test scores
* Analysis of variance using test scores
### 4.8.1 Approach to describing collaboration amongst students

The potential for collaboration between students and teachers to act as a catalyst for transforming classroom activity represents the principal focus of this study. Describing the type of collaboration within the classroom and the impact that such collaborative activity has on the cultural practices of the classroom are tasks requiring a range of different data collection techniques and analyses. During classroom observations, different types of collaboration were categorised accounting for all of the variation occurring within each classroom. Classroom discussions recorded on audio tape were also used to categorise different forms of collaboration. These categories were then revised using data collected during interviews with students and teachers.

The impact of the different collaborative activities on the cultural practices of the classroom – the shared ways of doing, knowing and understanding the classroom environment, was examined using interview data and questionnaire data. Leont’ev’s three lenses of activity, action and operation (Leont’ev, 1978) were used to interpret phenomena observed in different classrooms. The fundamental phenomenon, according to cultural-historical theorists, is collective, object-oriented activity and it is within the context of such activity that the meaning of different phenomena needs to be considered. What can be known is never the thing-in-itself, but rather the subjectified objects or objectified subjects (to use Leont’ev’s terminology) as they are understood by participants in the activity. The process of analysis, therefore, begins with the identification and description of collective, object-oriented activities.

Questions for each interview were set out prior to the interview in a semi-structured format (see Appendix Six for a copy of the interview questions). As students and teachers provided different responses to these questions, the emphasis shifted away from each person providing responses towards the promoting of discussion amongst participants. Interviews with groups, therefore, incorporated some of the features of “focus groups” as defined by Powell and Single (1996) who describe a focus group as
…a group of individuals selected and assembled by researchers to discuss and comment on, from personal experience, the topic that is the subject of the research. (p. 499)

While group interviews are designed to elicit responses to certain questions from a group of individuals, focus groups are designed to encourage interaction amongst participants about a certain topic. Group interviews conducted in the current study were primarily designed to elicit responses to a set of questions, however, discussion amongst participants was a feature of most interviews conducted with teachers and students.

Group interviews that provide opportunities for general discussion amongst participants as well as individual responses to questions have the potential to create a dynamic within which new ideas are developed by participants. In the case of secondary school students who are more likely to be reserved or intimidated by the presence of the researcher in the context of a one to one interview, group interviews also have a greater capacity to elicit discussion from participants who might otherwise be reluctant to offer their opinion. There is greater scope for students to feel comfortable being critical in a group, and to respond to the responses of others in the group either in agreement or disagreement. Situations of disagreement represent opportunities for multiple perspectives to be presented and the reasons for the emergence of multiple perspectives.

During the group interviews with students and teachers several questions were presented to students and teachers relating to the nature of the collaboration in the classroom. These are provided in Appendix Six. Ensuing discussion also revealed differences in opinion amongst members of each group and these discussions provided valuable insights into the experiences of students and teachers in the collaborative classrooms.

Several questionnaire items were related to the nature of collaboration that took place in the classrooms. A copy of the questionnaire completed at the end of the study is provided in Appendix Seven. For each topic students were presented with statements about how they learnt a particular topic, were asked to write down the advantages and disadvantages of working in groups, to describe what happens in collaborative and non-collaborative
classrooms, to write down how they prefer to learn and to write down their expectations of other students in their class and their classroom teacher.

Learning logs were also a source of information about the nature of collaboration occurring in each class. Students were asked to write down at the end of each lesson how they developed their understanding during that lesson and to provide examples of the methods and approaches they had been developing during that lesson. Students took turns filling in the learning logs at the end of each lesson on behalf of their group.

4.8.2 Obtaining evidence for increased motivation to learn mathematics and developing into self-regulated learners

Motivating students to learn mathematics and the associated goal of assisting students to become self-regulated learners are valued outcomes for mathematics educators. Several different data collection techniques and methods of analysis were used to determine if there was any evidence of increased motivation or higher levels of self-regulation demonstrated by students in the collaborative classrooms. Both qualitative and quantitative approaches were used to obtain such evidence.

4.8.2.1 Qualitative data collection and analysis

During classroom observations students’ levels of interest were scored by myself as I walked around the different groups and developed general impressions of the level of interest displayed by students in each of the different classes. At the end of each class, I spoke with the classroom teacher and asked them to score the level of interest of students in their classrooms.

As part of the classroom observations, recordings of classroom discussions were also analysed to determine whether or not students showed high levels of interest or self-regulation. These discussions were used to provide a better picture of the level of interest of students in each of the classes. Classroom discussions were also scanned for evidence of self-regulation amongst students in both the collaborative and non-collaborative classrooms. The current study used the broad definition of self-regulation provided in the
Principles and Standards for School Mathematics (NCTM, 2000). Classroom discussions were examined for evidence of students taking control of their own learning, setting their own learning goals, monitoring their progress, reflecting on their own thinking and learning from their mistakes.

At the end of each term, students were also asked during group interviews to score their level of interest in each topic that they were learning about. Other questions relating to the level of motivation were also included in the group interviews with students and teachers. These questions are provided in Appendix Six.

Data obtained from classroom observations and student and teacher interviews were coded using three codes for level of motivation (high, medium and low). When scores were provided out of ten, scores of 8, 9 or 10 were coded as high, scores of 6 or 7 as medium and scores of 5 or below were coded as low. Student comments relating to their level of motivation were also coded using these three categories. Using the qualitative software program NVivo 2.0 (QSR International, 2002) counts on the number of times each code was used to describe comments or incidents occurring in both collaborative and non-collaborative classrooms were used to provide evidence of the level of motivation evident within each type of classroom. The node tree developed using NVivo outlining the different categories used for coding the data is provided in Appendix Twenty.

Similarly, comments and discussions amongst students were coded as evidence of self-regulated learning when students displayed initiative pursuing their own learning objectives. Searches for evidence of self-regulation were performed using NVivo on comments relating to collaborative and non-collaborative classrooms.

4.8.2.2 Quantitative data collection and analysis

Measures of levels of motivation and self-regulation were also obtained using the revised MSLQ. Once factor analyses had been conducted using the items from the revised MSLQ
to identify the underlying factor structure these factor scores were then used to compare the levels of motivation and self-regulation at times T1, T2 and T3. Depending on the time each classroom worked collaboratively, these scores provided either two pre-test scores and one posttest score or one pre-test score and two posttest scores. These scores were then compared with each other using a repeated measures design for all classes who participated in the collaborative learning model.

4.8.3 The relationship between collaborative classroom mathematics and real world mathematics

Observation data and interview data were then examined to look for evidence that learning mathematics collaboratively resembled “real world” mathematics. During observations, the use of cultural mathematics tools – language forms, physical artifacts and ways of doing mathematics was examined in each of the different classrooms. Student discussions were analysed for evidence of setting out proofs, proposing conjectures and refutations, and developing models – examples of mathematical activity occurring amongst mathematicians, and for evidence of students applying mathematical ideas to solve everyday problems. During observations students’ use of physical artifacts such as calculators, mathematical instruments and computers were noted in both the collaborative and non-collaborative classrooms. Students’ involvement in mathematical discussions, argument and testing of hypotheses in the different classrooms was also recorded during classroom observations. Interview questions were also designed to provide an indication of the ways in which activity within the collaborative classrooms was related to “real world” mathematics.

4.8.4 Determining how well the proximal concerns of teachers are met in collaborative classrooms

For each topic that classes at both schools learnt collaboratively, there was at least one other class who learnt the same topic in a teacher-directed classroom to enable comparisons to be made between collaborative and non-collaborative classrooms. However, for most classes the relatively small number of students (between 13 and 29) and the variance within each class (particularly at Brindale Christian School where
classes were not streamed) reduced the power of such an analysis considerably so that the probability of a Type II error was very high. A final comparison comparing the results obtained in the collaborative classrooms with results obtained in the traditional classrooms for all students participating in the study was also conducted. The approach to analysing the test scores factoring out prior performance will be described in section 7.2.

Analysis of test scores was also complemented by other measures of student understanding including student self-assessments of their understanding and perceptions of the classroom teachers. During the interviews with students and teachers questions relating to student understanding were included and the questionnaire that students completed at the end of the topic required students to rate their understanding of each outcome for each topic (see Appendix Seven).

Results arising from these different forms of analysis are reported in the following three chapters. Chapter Five describes the conventional classrooms at both schools and presents an activity-based analysis of these classrooms. Chapter Six describes how the collaborative learning model at each of the two schools was implemented and presents a parallel activity-based analysis to that in Chapter Five focused on the collaborative classrooms. Chapter Seven presents qualitative and quantitative evidence for changes apparent at both schools as a consequence of introducing the collaborative model.
5 Forms of activity in conventional classrooms at Brindale Christian School and Southwest High School

At each of the schools there existed a pre-existing culture and way of teaching and learning mathematics that will be described in sections 5.1 and 5.2. Significant differences were observed between the two schools in terms of the way that mathematics is presented, the interactions between the teacher and the students, and the interactions between students. As a precursor to analysing what happened in classrooms during the collaborative lessons the following chapter provides a description of each school context, the culture of the classroom, school ethos and patterns of classroom interaction.

At each school, the collaborative learning model also took on its own character reflecting the dialectical interaction between the existing cultural practices of the classroom and the suggested new model presented to the teachers at both schools. The collaborative learning model at both schools is described in terms of the types of collaboration observed and the aspects of each model that emerged within each school. The resultant classroom practices represent an amalgam of several different trajectories that come together to form unique classroom environments including the existing cultural practices at each school, the contextual environment of each school, the knowledge base existing at each school amongst students and the principles and ideas behind the collaborative learning model.

5.1 Traditional classroom activity at Brindale Christian School – making mathematics easier

When approached to be part of this study teachers at Brindale Christian School were very enthusiastic about the project for several reasons. In other subjects within the school, teachers had used collaborative teaching strategies successfully – in particular, subjects with a more practical emphasis such as Design and Technology. One of the mathematics teachers in particular was very keen to use collaborative methods but had not worked out
a means of doing so which enabled her to cover the program each term. Furthermore, the collaborative approach appealed to them as something that reflected the focus of the church community on building relationships and working with one another rather than competing with each other.

While the teachers varied in age and experience, there was a greater degree of similarity amongst the teaching styles of teachers from Brindale Christian School compared with teachers at Southwest High School. Unlike the teachers at Southwest High School whose intention was to extend their students, the teachers at Brindale Christian School adopted a different goal of assisting students to be able to answer typical questions from each topic area. In each of the classes observed there was considerable variation in the existing understanding of different students. The mixed nature of these classes led to teachers pitching the lessons towards the middle of the class – and it was evident in some classes that this produced difficulties at both ends of the classroom ability-wise. Some students completed the set work in class, while other students struggled to get started on the set work and were then required to complete a great deal more for homework.

5.1.1 Patterns of classroom interaction

At Brindale Christian School typical classrooms were focused on providing students with methods that could be used to solve different types of problems. Teachers went to considerable efforts to simplify procedures and ideas so that even the very weakest students in the class would be equipped with approaches that they could use to answer future questions. An example lesson is described in detail in Appendix Eight.

The lesson described in Appendix Eight ran for fifty five minutes from 11:50 am to 12:45 pm. Most of the lessons at Brindale Christian School run for fifty five minutes, except for lessons on Friday which run for 40 minutes. This lesson was presented by Mr Grey in the Long Room. Students were seated in pairs – students chose where they sat for that lesson; males sat next to males, and females sat next to females. There were in fact only four female students in this class. There were two rows of desks in this room – in one row
there were six pairs, and in the other row there were five pairs. All of the seats in the classroom were full – there were a total of twenty two students in this class.

At the conclusion of the lesson Mr Grey spoke about the lesson and how successful it was in terms of achieving his goals. He also mentioned that he had not expected to be observed for that lesson and therefore the lesson was in many ways “a natural lesson, a normal lesson without extra preparation”

My main emphasis in surface area is the setting out, in fact the whole topic of measurement is just setting out, …. I feel about sixty percent confident that it went well, I would have liked to have given them examples in their hands (in the form of a worksheet)... mildly confident that they understood.

Just going around (the class) I was very pleased with the general setting out …. I was a bit surprised when everyone in the room when they were asked to draw a pyramid did one … that was so small, to do anything with, that's a problem to get right for year 12. From that point on everyone did do good-sized diagrams.

Mr Grey’s class on surface area provides an example in several different ways of the mathematics lessons which occurred at Brindale Christian School. The basic intent of the teachers at Brindale Christian School appeared to be to assist students to learn how to do certain questions – to prepare them for answering similar questions in exams. Instead of focusing on developing a conceptual understanding, Mr Grey chose to focus on a method which students could apply to answer questions relating to surface area. In fact, the method was taught and set up in such a way as to be relatively automatic; the successful application of the method did not necessarily require students to develop an understanding about what surface area was. In many ways Mr Grey’s method represented a scaffolding technique – a means by which students could answer the question in his absence. Students could safely rely on this scaffolding to work their way towards obtaining the correct answer without having to necessarily understand the work involved.

Hence, Mr Grey took considerable time to set out a method that they could follow that would enable them to answer questions on surface area in the future. Students were not
required to develop their own strategies or to explore how surface area could be
calculated – instead, the “appropriate” method was made available by the teacher and
students were provided with opportunities to practise using this method. Furthermore, Mr
Grey encouraged students not to diverge from this method at any point but to follow the
method in a somewhat rigid fashion to ensure that they did not make a mistake.

Student input was called upon by Mr Grey to support different steps in the method rather
than provide input towards developing such a method. For example, students were called
upon to do simple calculations, to provide the formulae for the area of different plane
shapes such as rectangles and triangles and to identify possible pitfalls in undertaking
different steps within the method. However, the general strategy for answering questions
to do with surface area was provided by Mr Grey.

Students at Brindale Christian School were also discouraged from using calculators
before year nine, although students in year eight did use them for specific activities.
Emphasis is placed upon mental arithmetic and basic skills. Mr Grey wanted the students
to be doing multiplication questions “in their head”, and mentioned during the lesson the
student’s homework – exercises from a book called “Mathsmate” which places great
emphasis on practising basic procedures. Again, the emphasis on procedures rather than
concepts is evident in this approach.

The nature of the subject matter may also have contributed to this emphasis on
procedures rather than concepts. However, this lesson on surface area may be compared
to a lesson at Southwest High School discussing the notion of measurement and issues
relating to accuracy which will be discussed in section 5.2.

The typical mathematics classroom lesson at Brindale Christian School, therefore, is
tightly controlled by the teacher who provides students with a particular approach to
solving certain types of problems. Where students’ contributions are constrained to
supporting the explication of the teacher’s method and they are discouraged from moving
too far from the approach provided, some theorists have suggested that the benefits of
collaborative learning are unlikely to be evident (De Lisi, 2002). De Lisi (2002) proposes that classroom cultures characterized by obedience and constraint rather than mutual respect are classrooms in which collaborative learning is unlikely to be very productive.

The lesson observed had aspects of both of De Lisi’s alternatives. Mr Grey directed what happened in the classroom, including the type of headings students used, the size of their diagrams, the method they should adopt and the work they should attempt. To a certain degree, obedience was expected and constraint was placed upon students – at regular intervals the class was asked to stop talking and listen to what Mr Grey had to say, sometimes with the threat of students being kept in at lunch time if they did not stop talking. While there was no need to do so, Mr Grey would probably have used similar approaches to ensure that students copied down the work on the board and made progress on their homework.

However, at the same time a mutual respect was evident in the classroom between Mr Grey and the members of the class in terms of the relational climate of the classroom. Mr Grey was clearly shown respect by the students when he started to talk as students would generally stop talking when Mr Grey started to explain something, and Mr Grey incorporated the comments of students into the general flow of the lesson. During the lesson there was also an ongoing secondary conversation which occurred between students and Mr Grey about the arrangements for sport that afternoon which were planned for immediately after the lesson. Mr Grey responded to students’ comments in a manner which demonstrated respect and concern for their well being.

Mutuality did not exist, however, in terms of the exchange of ideas within the classroom. Lampert (1990) suggests that encouraging conjectures and refutations within the classroom is an important aspect of good classroom practice. The open sharing of ideas by students and teachers within the classroom and the evaluation of these ideas using shared ways of evaluating the mathematical quality of different statements was not evident in this class. Instead of a dialogue between mutual members of a mathematical community, the classroom lesson was more akin to a monologue in which the opinions of
the teacher were passed from the teacher to the students for appropriation. The tendency towards monologue was evident in most of the classes observed at Brindale Christian School.

Within the class there was also little room for students making decisions about their learning, for directing their learning or contributing to the learning of other students. The highly structured nature of Mr Grey’s presentation even pre-empted potential errors that students might make, such as errors in the identification of the units for such questions, and at one stage in the presentation Mr Grey made a deliberate mistake to assist students to work out how to solve problems in which there are different rectangles. Students asked very few questions during the lesson, and such questions typically focused on the details of the method rather than about the concept of surface area. As is common in most mathematics classes, the process of practising different skills is structured by Mr Grey to provide them with specific questions on which they should practise. Opportunities for exploration of the different concepts, therefore, were limited.

5.2 Traditional classrooms at Southwest High School – extending gifted students

Each of the teachers at Southwest High School was very experienced and demonstrated a sound and thorough knowledge of the curriculum. Unlike the teachers at Brindale Christian School, however, there was greater variation between the different teachers and their classes in terms of the teaching/learning activities evident in each classroom. During the classes observed, Ms Diamond was meticulous in her setting out on the board, made no errors in her working out and was particular about every aspect of her students’ work including using the appropriate abbreviations for different units of measurement. In her normal classes, she would often extend her class (the top class) and go beyond what was written in the syllabus in terms of the concepts and knowledge areas that she covered.

Ms White was also very experienced having taught for about twenty years. Ms White was very competent as a mathematician and was keen to use different approaches to teaching topics she felt her students had a good understanding of already. Her class was by far the
most talkative, however, on every occasion that she started to explain something to the whole class, the class would settle down immediately and listen to what she had to say. Her style was less structured than Ms Diamond; however, the notes she provided to her students on the board were accurate and complete.

Ms Martin was the least experienced of the teachers taking part in this study having taught for only twelve years. Ms Martin was also a very competent teacher, having a good understanding of the subject matter and a capacity to present the material in a manner which the students found interesting. Ms Martin had a strong rapport with her students who clearly respected what she had to say.

All three teachers had little or no experience with “group work”. All of the normal lessons I observed, except two, were traditional in structure – an explanation by the teacher from the front of the classroom was followed by some examples which were worked through on the board, followed by the teacher setting an exercise from the textbook for the students to complete. There were two exceptions to this – one class taught by Ms Diamond incorporated an activity which required the students to estimate the lengths of different parts of the classroom and then get them to measure these different parts. One class taught by Ms White also included an activity in which students “discovered” Pythagoras’ theorem using pieces of string with knots in them.

In each of the three classes the students were very attentive when the teacher was talking. When the teacher was writing on the board, students would often talk to their neighbours about non-mathematical subjects. When students were working through the exercises they would also talk to each other about what they did on the weekend while making steady progress through the work. In Ms Diamond’s class (the top class) the talking with neighbours was done in a manner which did not distract other students working in the class. In Ms White’s class, students spoke with each other at a higher volume and could be very disruptive to other students. Ms Martin’s class was also prone to talk with each other at a level that could distract other groups of students in the class. In all classes, students sat in rows of two and spoke most of the time to the student sitting next to them.
and less often to students sitting either immediately in front of them or behind them. Rarely did students attempt to communicate with students on the other side of the room.

On pre-tests students demonstrated a high level of understanding of many of the outcomes prior to starting each of the topics. Many students felt that they already knew all of the work and as a consequence, when they were working in groups and were given the responsibility of teaching each other, their participation in mathematical activity declined. There was little evidence of students taking responsibility for the learning of other members of their group. In one group there were two students who clearly understood most of the work who chose not to assist the other students in their groups who were struggling with the work they had to learn in that topic.

Students in Ms Diamond’s class (the top class) were highly motivated during the lessons observed. They worked at a rapid pace to complete the set work and responded positively to Ms Diamond’s efforts to extend them by encouraging them to think through deeper issues that would not necessarily be covered in examinations. For example, in one lesson observed, Ms Diamond discussed with the class issues surrounding accuracy using different measurement techniques. Samples of the board work presented by Ms Diamond in this lesson can be found in Appendix Nine. During this lesson Ms Diamond asked the class to give her examples of different instruments which measure things – the responses include such instruments as seismographs and computers measuring heartbeats. Ms Diamond outlined how one can never measure the exact measurement, as there will always be error associated with measurement.

Ms Diamond spent considerable time with her class, being particular about the terms and language associated with mathematics. During the lesson she clarified the understanding of the word “scale” as “gradations of measurement”, distinguished between “detailed” and “coarse” gradations, distinguished between error and mistake as outlined above and used the correct term “rule” rather than the more conventional word “ruler”. She also ensured that students in the classroom had used the correct abbreviations for units such as hour (h) and minute (min). Given the high standard of these students, Ms Diamond had
time during each lesson to ensure that students are using correct terminology and abbreviations rather than spending an inordinate amount of time developing students’ understanding of mathematical concepts with which they were familiar.

Given the high standard of students in the selective classes as a whole many of the lessons observed were designed to extend the students beyond the stated curriculum in an attempt to maintain the interest of students who had already learnt much of the course in external tutorial classes. One of Ms White’s lessons on Pythagoras’ theorem, for example, focused on generating Pythagorean triads. After discussing multiplying existing triads by a common factor, Ms White provided the class with two approaches to developing triads – the first of these is fairly straightforward and the second one involves using a generalized approach to developing Pythagorean triads which involved writing on the board the following steps.

1. Write down any odd number $n$
2. Square it $n^2$
3. Divide by 2 and ignore the fraction
4. Write next whole number after $\frac{n^2 - 1}{2} = \frac{n^2 + 1}{2}$

It was clear that the interest level of the students was very high during this explanation. When Ms White was going over one of the questions from the homework about eight students in the class were not paying attention to her explanation. During the above explanation of a generalized approach to developing Pythagorean triads the whole class paid attention. Teachers at Southwest High School teaching the selective classes tended to look for different ways of extending students in their class with work they had not actually seen before as a result of their involvement in tutorial classes on the weekend.

Students came to expect the teacher to provide them with a clear indication of the work required of them each lesson. During the collaborative classes this was most evident in
the comments of several students who felt frustrated that they weren’t able to find out exactly what they needed to know about a particular topic. One student in Ms Martin’s class commented that

I didn’t really like working in groups because I thought there was too much freedom and you wouldn’t know what to do.

Another student in Ms Martin’s class commented that they “… would like for the teacher to just write down in a book what you have to finish for the lesson on the board.” A third student from Ms Martin’s class discussed the difference between collaborative learning and teacher-directed learning in the following way

Gary: Just to raise a point. I don’t find asking a friend or asking a teacher any easier than just asking the teacher.

Interviewer: You don’t find asking a friend to be easier?

Gary: No, because that can place more pressure on some people, and your friends think of you as very poor, unable to do simple questions probably. (Ms Martin’s class)

The concern for this student is clearly the possibility of appearing less bright than other students in the class. This is borne out by the questionnaire students completed at the end of their involvement in the collaborative model of learning. Students were asked what their expectations of other students were and 79 students responded to this question with some of them writing more than one expectation. Overall, students recorded 96 expectations. Of these 96, 41 comments (43%) referred to the level of understanding that other students had – either that their expectations of other students were that they were smart (6), that they understood the work (20) or that they were as smart or smarter than the respondent (15). 10 of these (10%) stated that they expected other students to be smarter than them. By way of contrast, only 4 of the 57 responses (7%) from Brindale Christian School indicated that students expected other students to be smarter and only 18 of the 57 responses (32%) referred to the level of understanding of other students in their class.
At Southwest High School, therefore, a significant shared understanding amongst students is that comparisons between students and competition are important aspects of classroom activity.

5.2.1 Patterns of classroom interaction

At Southwest High School students within each of the classes were keen to be challenged and to do well in mathematics. When the teacher provided an explanation from the front of the classroom, the students in each of the selective classrooms would cease their talking and pay full attention to what the teacher was saying. However, there was a different dynamic between the students and the teacher compared with that which existed at Brindale Christian School. While students at Brindale Christian School were more likely to interact with the teachers on a number of different levels discussing upcoming sporting activities, or coming to talk to them at lunchtime about issues in the playground, the relationship between teachers and students at Southwest High School was more clearly limited to discussions about mathematics.

Discussions amongst students were often about issues unrelated to mathematics. The content of the curriculum presented few difficulties to these students and as the teacher covered the content required by the school program by writing notes on the board, students would copy them down while conducting a discussion about video games they were playing on the weekend. This predilection for non-mathematical discussion was also evident when students were completing set work which they did efficiently while discussing matters totally unrelated to their work. When the teacher began to explain something or conduct a discussion about a particular mathematical idea, though, these conversations would cease for the duration of the teacher’s explanation or the classroom discussion.
There was very little discussion amongst students about mathematics at Southwest High School. Most students needed little help and were able to complete the set work without discussing it with their neighbour.

5.3 Similarities and differences between the two schools

Through my interactions with teachers and students at both schools several pre-existing similarities and differences were evident. While both schools had a standard approach to teaching mathematics that was similar, there were significant differences in the school cultures, the relationships that were formed within the classrooms and the interactions between students.

5.3.1 Approach to teaching mathematics at Brindale Christian School and Southwest High School

The majority of non-collaborative lessons observed at both schools followed a similar pattern of exposition followed by time for individual practice. At both schools, the teacher would provide an explanation of the mathematical problem to be solved and the appropriate method to solve this problem. Included in this explanation would be definitions of relevant terminology, worked examples outlining particular methods, and relevant historical and contextual information related to these different methods. At both schools, the teacher was viewed by the students as being an authority on matters mathematical. Students listened intently to teacher explanations and copied down information provided by the teacher on the board as examples or other information relevant to the problem being discussed.

At the conclusion of the exposition, students would then be given time to practise using the designated method by completing a textbook exercises consisting of a large number of problems that required this same method. Unfinished work would be set for homework and checked the next day.

The content of this exposition and the teacher’s purpose during each mathematics lesson, however, differed across the two schools. At Brindale Christian School there was a
greater emphasis on simplifying mathematical procedures, breaking them down into steps that could be followed to answer similar questions to those presented as examples. At Brindale Christian School, teachers’ primary role was providing students with efficacious procedures that would help them to develop their conceptual understanding. In comparison, at Southwest High School, the classes I observed were focused on extending students by providing them with more detailed expositions of mathematical methods. The examples presented by the teacher were more likely to include extension questions that would stretch the students in the class beyond the level expected of students at Brindale Christian School.

5.3.2 Interaction between the teacher and the students

At both schools, students were well-behaved, listening to the teacher’s expositions and completing the set work as quickly as they could. However, the authority that teachers at both schools held was quite different. At Brindale Christian School, this authority extended beyond mathematics to other domains. Teachers were more like parents who had moral authority as well as intellectual authority. Each of the teachers was referred to as an aunt or uncle and perceived their role at the school to be a component of their overall mission as a church community. Students would come to show teachers at Brindale Christian School their work from other lessons, or talk to them about issues that arose in the playground. At Southwest High School, the teachers’ authority rested with their knowledge they possessed about mathematics. The students at Southwest High School acknowledged the greater mathematical understanding of their teachers by listening intently to their explanations and copying down the methods provided by teachers on the board.

5.3.3 Student interactions

At both schools interaction between students was limited to same sex interactions. Male students would only talk to male students and female students would only talk to female students. At both schools males were more likely to be talking about non-mathematical topics than female students, however, this difference was more prominent at Southwest High School than at Brindale Christian School where most students would be discussing
mathematical concepts rather than non-mathematical concepts. Other significant differences in student interactions were also observed at both schools. At Brindale Christian School, within the non-collaborative classrooms, students would talk to each other during the time allocated for individual practice to receive help from one another. While some of the students would discuss other matters, this was rare. This was much more unlikely at Southwest High School where students would either work in silence on their own or talk with the person next to them about some matter unrelated to their mathematics work. There seemed to be a reluctance, particularly among some of the male students at Southwest High School, to seek help from other students as this might be perceived as an indication that their level of understanding was low.

5.4 Identification of recurring phenomena within conventional classrooms

Using Strauss and Corbin’s grounded theory approach to understanding phenomena, the initial stage of open coding was conducted first on the classroom observations and the accompanying interview transcripts. Thirteen categories emerged from the process of open coding when describing what was happening in the conventional classrooms. These categories are recorded in Table 5. These were then grouped into three larger categories of student learning, teacher activity and student/teacher interactions.

Over the duration of two terms (twenty weeks) a total of fourteen conventional classroom lessons were observed – nine at Brindale Christian School and five at Southwest High School. A total of twelve hours and twenty minutes of conventional classroom time was observed (see Appendix Ten for a log of classroom observations). Although this was a relatively small number of observations, there was a remarkable degree of similarity in the phenomena observable within each of the classrooms observed. Some of these phenomena were recurring phenomena that were characteristic of almost every classroom observed and these were coded with an ‘R’.

The second stage of analysis involved drawing together different phenomena that constituted specific instantiations of the more general collective activities. This stage is similar to Strauss and Corbin’s stage of axial coding in which the relationships between...
different phenomena are drawn out. In the current study, related phenomena are those that occur within the same type of activity observable in the classroom. Three different types of activity were identified during the classroom observations within which each of the above phenomena were observed. In teacher-directed conventional classrooms, the most dominant form of activity observed was “educational activity” associated with the learning and teaching of mathematics (within which phenomena B2, RB3, RB4, RC2, A1, A3, A5 were evident). This was the first form of activity evident in the classroom. This activity is described in more detail in Section 5.4.1.

Two other separate forms of activities were also present – those of practising mathematics (phenomena B1, C1, RA2, RA4, RA6) and non-mathematical discourse (phenomenon A7) which are both described in Sections 5.4.2 and 5.4.3. In conventional classrooms, these activities shared few common elements although non-mathematical discourse would often occur at the same time as students’ participation in one of the other forms of activity. Describing forms of activity begins with an identification of the object that motivates the activity and proceeds from the object to the participants and their patterns of interaction (Engeström, 1987; 1993). Using this approach, the different forms of activity evident at both schools are described in the next section.
Table 5: Recurring phenomena identified in the traditional classrooms

<table>
<thead>
<tr>
<th>Student learning</th>
<th>Teacher activity</th>
<th>Student/teacher interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RA1</strong>: Students appropriate teacher’s demonstrated method for answering a question</td>
<td><strong>B1</strong>: Teacher checks that students have completed their homework</td>
<td><strong>RC1</strong>: Students ask teacher for help as soon as difficulty is encountered</td>
</tr>
<tr>
<td><strong>RA2</strong>: Students work through exercises set by teacher</td>
<td><strong>B2</strong>: Teacher insists on the appropriation of their methods as set out on the board (Brindale Christian School only)</td>
<td><strong>RC2</strong>: Pattern of interaction between teachers and students follows the pattern of interaction identified by Mehan (1979) as IRE (Initiation by the teacher, Response by the student and Evaluation by the teacher)</td>
</tr>
<tr>
<td><strong>RA3</strong>: Students copy down teacher’s examples from the board</td>
<td><strong>RB3</strong>: Teacher sets work from textbook</td>
<td></td>
</tr>
<tr>
<td><strong>A4</strong>: Students ask teacher for help as soon as difficulty is encountered</td>
<td><strong>RB4</strong>: Teacher presents students with a range of methods they can adopt</td>
<td></td>
</tr>
<tr>
<td><strong>RA5</strong>: Students ask person next to them for help</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A6</strong>: Student interest highest when being extended (Southwest High School only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A7</strong>: Students work through the set work at different rates during class time</td>
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Three distinct components of classroom practice were evident in traditional classrooms. Beginning with the teacher’s exposition which formed the focus of the first component,
students would then begin practising mathematical methods outlined in the exposition, normally by completing exercises in a textbook. The last component of classroom practice evident in traditional classrooms was the discussion amongst students relating to a wide range of topics unrelated to the mathematical concepts presented and practised in the classroom.

5.4.1 “Learning mathematics” activity – interacting with new mathematical ideas

In teacher-directed conventional classrooms, the most dominant activity observed was that of teaching/learning mathematics – instantiation of a wider activity in Australian societies of mathematics educational activity which has as its object the development of students’ understanding. The principal focus of most mathematics classrooms is this activity of “teaching/learning mathematics”. The central actors within this activity system as it is played out within the classroom are classroom teachers. Several researchers have documented how similar mathematics classroom activity is across different classrooms (Welch, 1978; Porter, Floden, Freeman, Schmidt and Schwille, 1988; Fey, 1979; Stigler and Hiebert, 1997; Stodolsky, 1988). While each classroom has the potential to promote the emergence of multiple ways of teaching/learning mathematics, there remains a consistent pattern of classroom activity evident across most mathematics classrooms which was also evident in the classrooms observed at Brindale Christian School and Southwest High School.

In each of the fourteen conventional lessons observed, the lesson would start with a “teaching/learning mathematics” component once the teacher had dealt with a small number of administrative details. It is quite likely that educational activity instantiated in this manner is not evident in every classroom, however, when teachers are aware that their class is going to be observed they will typically make an effort to provide an example of such practices. Sometimes this aspect of classroom activity might take up the whole of a lesson, although more frequently it would extend for no longer than twenty minutes with time provided for students to practise certain methods and applications until the conclusion of the lesson.
The first stage in describing object-oriented, collective activity is to identify the object of the activity. The object is the phenomenon which the collective activity is directed towards transforming to achieve a certain outcome. Objects of activities, however, are subjective entities (Leont’ev, 1978) which can only be identified by asking participants to share their perspectives on what they perceive to be happening within the activity. Student descriptions of classroom activity obtained from interviews at the completion of the topics covered, therefore, were a significant resource for analysing different classroom activities.

Educational activity has as its object the transforming of students understanding to enable students to more effectively participate in societal practices. The object of “teaching/learning mathematics” activities, therefore, is student understanding of mathematics that is transformed so that students are more able to participate in mathematical practices.

The participants involved in this activity initially appear to be both teachers and students. However, from the student reports of what happens in such classrooms, their participation in such activities appears to be relatively passive. The only active agents within such activities identified by the students are the teachers themselves.

We normally have heaps of examples – the teacher explains what everything means and how the scale works and everything and then we get exercises. (Mr Smith's class)

James: Usually if we were starting a new topic, Steven – or if we were starting something new inside the topic, Steven will show us how to do it and explain it and everything and then we'll go and do the exercises and if we have questions you can ask him while everyone else is doing the exercises.

Connor: Yeah, we do examples, he'll keep doing them until you, like, get it, but if you still don't get it you ask him for help. (Mr Grey's class)

Ruby: We just sat down, the teacher just taught us different ways. (Ms Diamond's class)

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Alana: We just go and sit down and the teacher will start writing notes on the board and then after she’s written them down, she explains them and then we copy it. (Ms Diamond’s class)

Andrew: We’d come in and Miss ........... the formula and the formula is on the board and we copy down and then she gives us lots of examples on the board and we do examples along with her until we all understand what she says and she gives us the exercises and we all kind of do it and when we finish, she gives us another one or her own. (Ms White’s class)

Adrian: Well, the teacher would just come in, tell us to sit down, just start writing on the black board and tell us to work. (Ms White’s class)

Ruby: Because usually if we just do it as a class, she can control us more. She kinds of controls us, but when we’re in a group, we’re actually doing what we desire, not what the teacher wants us to do and that changed. (Ms Diamond’s class)

Students in these different classes do not appear to perceive themselves as actors within this particular form of activity, but rather acted upon by the teachers involved. This is borne out by the phenomena identified in each of the classrooms – students appropriate methods, copy down examples, ask the teacher for help, and work through set work provided by the teacher. As with texts in general, mathematical methods have become “dead objects to be reproduced for the purpose of gaining grades” (Engeström, 1987, p.102). Traditional schooling, according to Miettinen (1999), is characterised by memorisation and reproduction of school texts – and in mathematical classrooms students reproduce methods they have memorised for the purposes of solving particular types of problems.

In each of the traditional classes observed, the classroom environment was monological rather than dialogical. Bakhtin argues in his commentary on Dostoyevsky’s writings that monologism results in the objectification of the other rather than the interaction with a partner in dialogue.

Monologism, at its extreme, denies the existence outside itself of another consciousness with equal rights and equal responsibilities, another I with equal rights (thou). With a monologic approach (in its extreme pure form) another person remains wholly and merely an object of consciousness, and not another consciousness. No response is expected from it that could change anything in the world of my consciousness. Monologue is finalized and deaf to other’s
response, does not expect it and does not acknowledge it any force. Monologue manages without the other, and therefore to some degree materializes all reality. Monologue pretends to be the ultimate word. It closes down the represented world and represented persons. (Bakhtin, 1984, pp.292-293)

While the monologism evident in these classrooms would be described as moderate rather than extreme, the monological tendency of these classrooms promotes the objectification of students and the classroom activity being built around the exposition of the teacher’s perspective with little reference to the perspectives of students. As a consequence, students did not perceive that their particular needs were being met by classroom activities. Instead, there existed the perception amongst students that teachers had a certain goal they were trying to accomplish - to teach them certain strategies, which they do in a typical manner by demonstrating certain techniques for students to appropriate. The “subjects”, therefore, within this activity system were the teachers rather than the teachers and students. As Bakhtin suggests, within this monologic framework, students are objectified in such activity systems as that which is acted upon rather than being members of a collective body that includes the teacher working towards a collective goal.

The “teaching/learning mathematics” activity within the traditional classroom is best described as a “teaching mathematics action” – that is, the single participant in this activity is the teacher, whose goal is to cover certain parts of the curriculum by acting upon students. This action is one of many within the broader activity of the teachers at each of the two schools directed towards preparing students to participate in a wide range of activities. In doing so, the needs of the teacher to fulfil their professional responsibilities are met resulting in numerous other benefits to the teachers, some of which would be job satisfaction, income and enjoyment. This broader activity can be described as the provision of a service for parents – that of educating their children in a certain way. The object of this activity at both schools is slightly different – at Southwest High School the aim of the school is to develop “lifelong learners”, while at Brindale Christian School the aim is to develop students’ moral, social and ethical values as well as their academic ability. Within each of these schools there are existing divisions of
labour, rules, communities, and tools that are used for achieving the different outcomes, however describing these activity systems is not the focus of the current study.

5.4.1.1 Teacher/student interaction during “learning mathematics” activities: Models of discourse and patterns of behaviour

Different models of discourse were evident within these “teaching mathematics actions”. In particular, many interactions followed the Initiation, Response, Evaluation (IRE) model of classroom interaction (Mehan, 1979) in which the teacher initiates an interaction, and asks for a response from the students which is then evaluated. For example, in Mr Grey’s class, Mr Grey wrote on the board a sentence “This 3-D solid has _______ faces” and then asks students to write this down and then fill in the gap. Mr Grey then asks the students how many faces a square pyramid has and waits for a student to give a response. This response is then evaluated as right or wrong. In each of the classes it was critical to the teacher’s perception of success that they managed to maintain the focus on the topic at hand, and that their agenda was being followed by members of the classroom such that they were able to achieve their goals.

Unlike discourse occurring within other subjects that might focus on explanation of certain concepts, much of the communication between the mathematics teacher in these classrooms was focused around providing examples.

Andrew. We’d come in and Miss .......... the formula and the formula is on the board and we copy down and then she gives us lots of examples on the board and we do examples along with her until we all understand what she says and she gives us the exercises and we all kind of do it and when we finish, she gives us another one or her own. (Ms White’s class)

Sherlyne: She is the one who explains and she gives us more examples and she wrote out a whole thing with me - I understand it. (Ms White class)

David: The teacher just stands up at the board and explains stuff and makes you do work from the textbooks, that’s all. (Ms White’s class)

Anthony: She gives us like, some examples.

Clare: And then that just repeats. (Ms Martin’s class)
Karoline: she’d give us some examples and then she asks us if we have any trouble with anything and then we start to do work. (Ms Black’s class)

Karl: She went through it with me on the board and she set examples of it and like all the different questions and how to do them. (Ms Gold’s class)

The most common way in which mathematical content is presented, therefore, is through examples. This reinforces the idea that mathematics is about learning “how” to use certain mathematical concepts rather than “why” certain mathematical concepts are used. Students’ understanding could be said to be syntactical rather than semantic – that is, they know processes and how to apply them rather than knowing why certain processes are used and the meaning of these processes.

5.4.1.2 Student/student interaction during “teaching mathematics actions” activities: the dialogic reshaping of teacher’s expectations

As teachers performed the action of presenting certain mathematical procedures by giving examples, students would also discuss what the teacher was saying amongst themselves. One student in Mr Grey’s class, for example, when describing what would happen in normal classrooms, said that she would “usually discuss things with my friend Cheryl, she usually gets a high mark in the test, I ask her how she did (it)”. Furthermore, in the classrooms that had worked collaboratively for one term and returned to being taught without an emphasis on collaboration it was evident that the students found it easier to discuss mathematics with other students having spent some time collaborating with them in groups. One teacher described how one of her students had moved from being an independent worker without consideration for others to someone who was prepared to assist other students, although this change didn’t take place until after they had finished working collaboratively.

It took a lot of work, like I didn’t see a change in his attitude until we’d finished collaborative learning and then … when we started doing it in the classroom, he sat next to someone who was really struggling again and was really helping. In first semester I wouldn’t have let him sit next to him, because he would have mucked around with this kid, but then he was starting to
help this kid now and I thought, okay well you can stay then and help him out. That was a complete change. (Ms Gold)

Several of the students also reported that when they went back to teacher-directed learning there was a tendency for them to discuss problems they may have been facing with other students.

Anthony: Even when we were doing Pythagoras theorem (a non-collaborative topic), we didn’t learn it in a different way, we just consulted with other people in our group and reinforced what we learnt with the textbook. (Ms White’s class)

….. and after we’d finished doing the group work - I think we were doing ratio and rates - there were like four girls and two turned around to work with the others even after I’d explained it, they still wanted to work together and then I thought, well okay. (Ms Martin)

5.4.2 “Practising mathematics” activity – repeatedly reproducing teacher’s methods to solve textbook problems

The second main component observable within each of the conventional classrooms was the practising of mathematics at the conclusion of the teacher’s exposition of the main ideas to be covered during that lesson. Once students had learnt a particular method, students were required to practise this method by completing set exercises. The time allocated for this activity would vary depending on the length of the exposition. Questions were taken from the class textbook in most classrooms observed, with some teachers also providing practical activities to help students make sense of the mathematical concepts being investigated. In Ms Diamond’s class on the 18th June, for example, once the students had completed an exercise provided by Ms Diamond on measures, students were then given the task of estimating and then measuring the length of different objects in the classroom. Ms White’s class on 18th September looking at Pythagorean triads also included a practical activity using string to create a 3,4,5 right-angled triangle. The remaining twelve lessons observed provided time remaining at the end of the lesson for students to practise certain techniques for solving particular types of problems.
Ms Gold’s class on the 2nd of August, for example, consisted of an introduction to the concept of percentages. As part of this lesson, students were provided with a definition of percentages and were then given examples of converting percentages to fractions. The length of the lesson was 40 minutes and it was not until there was only five minutes to go that the students started working on the exercises looking at converting percentages to fractions, the remainder of which was to be finished for homework.

Rather than being one activity system within the classroom incorporating the student body and the teachers “practising mathematics” together, in these classrooms of twenty students there was little evidence of students working towards a collective goal that would fulfil their individual needs. Instead, each student worked towards fulfilling the teacher’s requirements. By doing so, students could maximise the time spend on other activities unrelated to their schoolwork either during the class lesson or after school by minimising the amount of homework they had to complete. The goals of these twenty actors happen to be the same goal, in that each of these actors were working towards completing the set work. This goal, however, is not a shared goal – each person is working towards completing their own work rather than completing a joint task, or participating in a collective enterprise. As was evident in some of the classes, particularly at Brindale Christian School where there were mixed ability classes, students progressed through this work at varying rates – some students completed the work in the time allocated while others were faced with the prospect of completing most of it for homework. At times during the performing of these actions, there was evidence of interaction between different students when students asked each other for help, however, there was no necessity for the actions to coincide at this point, and, in the case of a more able student helping a less able student, the more able student would have to stop working towards their own goal to assist the less able student to work towards their goal.

One student who was struggling in Mr Grey’s class at Brindale Christian School responded to a question about how she developed her understanding of a certain topic by asking a friend of hers. Other students would typically ask the teacher for help when difficulties were encountered as they worked on the method provided by the teacher.
However, the involvement of the teacher was typically limited to answering the particular question presented by the student so that they were able to continue with their task of completing the set work.

Hence, the object students were working towards transforming was not necessarily their understanding, but rather completing the set work in the minimum time. Whether or not students understood the work was a secondary consideration for many students whose principal goal was to complete the set work in minimum time. This indicates that the fundamental contradiction within activity systems – the tension between exchange value and use value, is evident within these mathematics classrooms with an emphasis on exchange value rather than use value. Students complete their work so that they can fulfil the work requirements presented by the teacher and, by doing so, have extra time to pursue their own interests. Instead of viewing mathematical practice as something that can be used to achieve certain outcomes (use value), practising mathematics represents a means of fulfilling the teacher’s expectations (exchange value).

Student goals driving their actions in conventional classrooms, therefore, are not necessarily consistent with the goals of the teacher or the motives of the broader activity of “educational activity”. Instead, students have appropriated goals that reflect a different form of activity – a form that Engeström describes as “school-going” (Engeström, 1987). This form of activity approaches mathematical methods as objects to be reproduced to obtain grades rather than developed to make sense of mathematical ideas. Exchange value of mathematical methods for students is dominant rather than the use value and students’ actions are motivated by short-term goals of fulfilling requirements rather than transforming their own level of understanding.

5.4.3 “Developing social relationships” activity – emergence of other forms of discourse within the classroom

The third component evident in the classroom could be described as “developing social relationships” or “non-mathematical discourse”. Within some classrooms, small groups of students would engage in discussions unrelated to the mathematical content of the
lesson or the practising of using different mathematical techniques. Sometimes these activities would occur simultaneously with the actions of teaching mathematics and practising mathematics. Students at Southwest High School in particular demonstrated a remarkable capacity to participate in the activity of discussing unrelated matters at the same time as listening to an explanation, copying down an example, or completing set work from the textbook. For example, without any obvious reduction in their attention to the work presented by the teacher or the work in the textbook students would participate in an ongoing complex discussion about a certain computer game that they had been playing on the weekend.

This particular type of activity was evident at Southwest High School where the teachers were much more prepared to allow discussions of a non-mathematical nature to take place when they were not in the process of providing a verbal explanation from the front of the class. In most cases where the teacher would start to provide an explanation of a particular idea from the front of the class, students would cease their engagement in other conversations and listen to what the teacher had to say. This was particularly evident in Ms White’s class in which students were the loudest and most socially gregarious. Students would be very active in developing these social relationships with each other while working on the set exercises from the textbook, however, as soon as Ms White called the classroom back to the front to listen to an explanation there was an immediate cessation in the non-mathematical discussions.

In summary, there exist three distinct aspects of classroom practice that represent very different forms of activity. The three general forms of activity evident within the classroom can be described as “educational activity”, “school-going activity” and “social activity”. While there would appear to be significant overlap between these activities, in particular educational activity and school going activity, the different motivations for these activities evidenced in the goals and associated actions performed by teachers and students would suggest that three distinct forms of activity are apparent. These activities are summarised in Figure 8.
<table>
<thead>
<tr>
<th>Educational activity</th>
<th>School-going activity</th>
<th>Social activity</th>
</tr>
</thead>
</table>

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instantiated in the classroom in the following components

Teaching actions
Practising mathematics
Non-mathematical discourse

which have the following objects

Student understanding
Completing set exercises
Social relationships

transformed to achieve following outcomes

Students prepared to participate in mathematical activities
Students fulfil teachers’ expectations
Closer social relationships

Individual Teachers
Individual students
Groups of students

who achieve this outcome by

Providing monologues focused on examples
Working through set work individually
Discussing topics of common interest
being transformed by
Figure 8: Forms of activity evident in conventional classrooms
6 Forms of activity in collaborative classrooms at Brindale Christian School and Southwest High School

As outlined in the previous chapter, significant differences between the two schools were evident prior to the introduction of the collaborative learning model in terms of students’ level of understanding, the relationships between students, the relationships between the teacher and students and the general ethos of each school. The existing cultural patterns at each school were not entirely dissimilar, however, with common forms of activity evident at both Brindale Christian School and Southwest High School, although the instantiation of these forms of activity at each school revealed aspects unique to each school.

The introduction of the collaborative learning model at each school introduced new cultural patterns at both schools, although the specific features of classroom activity were markedly different within different classrooms. In this chapter, descriptions of the collaborative learning model as it was realised at each school are provided detailing the different tools and artefacts introduced by teachers and the different forms of collaboration in which students and teachers were engaged. Teacher and student comments describing what happened in their classroom during interviews conducted at the end of each term were also used to develop a picture of different forms of collaboration evident in each classroom.

6.1 Collaborative classroom activity at Brindale Christian School

A diverse range of strategies and forms of collaboration were evident in classrooms at Brindale Christian School. Section 6.1.1 will outline the strategies that teachers adopted to support learning within the collaborative classroom while Section 6.1.2 will describe the different forms of collaboration observed at Brindale Christian School. These are described in more detail in Appendix Eleven.
6.1.1 Strategies for facilitating collaboration and tools adopted by classroom teachers

During the planning phase, I introduced the teachers at Brindale Christian School to the principal components of the model – that is, that students would work in groups to teach each other and prepare assessment tasks for other students in the classroom to complete. As part of the preparation, teachers were introduced to some of the types of questioning strategies suggested by Brown, Ellery and Campione (1998) to support students’ learning together (see Appendix Thirteen) and teachers were also introduced to the main ideas that mathematics educators have been suggesting for effective mathematics teaching. However, students in each class were given freedom to determine how they would teach each other the different outcomes.

To assist the collaborative teaching groups, each student was provided with a list of the outcomes to be taught and assessed, sample questions for each of these outcomes (see Appendix Three for examples of these documents), and a path diagram outlining a suggested order in which each of these outcomes might be covered (see Appendix Five for path diagrams used in this study). Each of the teachers took these ideas and transformed them to suit the needs of their classroom, providing students in their classrooms with additional tools to support their collaboration. Across the two separate cases, the implementation of the model resulted in different classroom practices and tool use. For each case, Brindale Christian School and Southwest High School, the tools incorporated at each school and the forms of collaboration evident are described in detail below.

6.1.1.1 Chalkboards

One innovation adopted by Mr Grey in his class which assisted the process of collaboration for many of the students was the introduction of small chalkboards on each table.

I noticed at the start that most groups had difficulty logistically, trying to - one person at the table showing the other person at the table how to do things, they needed to go then and get lots of paper, so that’s when I felt why don’t we get little blackboards and found out we’ve got
little blackboards down in our little school and I found - 'cause I had some people, I had lots of groups wanting to use the blackboard at the start, the big one and it felt like they were coming out to the blackboard, and I found that once they started doing that, that I could, in terms of my interaction, I could stand back more .... (Mr Grey)

During the lessons observed, students in every group made use of the chalkboards. The chalkboards were approximately thirty centimetres by thirty centimetres and were not much larger than a blank sheet of paper. However, students found them to be a useful artefact for “teaching” – in some sense the chalkboards were appropriated as a smaller version of the chalkboard at the front of the classroom. In many ways the chalkboards allowed students to more effectively take on the role of teacher in a “micro” classroom that emerged within their group. Micro classrooms were one form of collaboration observed in the different classrooms and are described in more detail in Appendix Eleven.

6.1.1.2 Path diagrams

The path diagrams were an integral part of the model at Brindale Christian School where teachers felt that the students needed them to understand how to progress through the topic.

I think if you didn't have a pathways diagram thing, the learning would not be as - the learning wouldn't be efficient because they would be trying to chart that for themselves and they honestly do not have the ability, it doesn't matter how clever they are, they don't have a view of the curriculum and you've got to give that to them and I think that - I don't think - whilst my kids related to it and we sort of ticked it off, I think Ken stuck it on the wall and that worked really well too. (Ms Black)

One teacher, however, decided not to focus students’ attention on the path diagrams preferring to guide them through the outcomes as they were set out in the outcomes sheets.

I didn’t really use the path diagrams very much at all. My kids basically followed through the outcome sheet, A,B,C. I don’t know whether it was the topics that I was doing or I didn’t particularly draw their attention to it, so that’s why they didn’t focus on it, but I just thought that
the outcomes for the topics that we were doing, followed fairly sequentially so they just went through and did them as is. (Ms Gold)

As outlined earlier, the cultural pattern of teaching at Brindale Christian School was one of scaffolding student calculations – providing a method or structure, which they could appropriate to answer future questions. The path diagrams presented a similar tool for students to use. For the teachers it represented something they felt comfortable with as a means of supporting students’ learning. Students also felt comfortable using the diagrams and crossing off sections of the diagram as they completed them. In so doing, students appeared to get a sense of accomplishment as they progressed through the outcomes. It also represented a way of keeping track of other members of their group to ensure that all members of the group had covered each of the outcomes.

6.1.1.3 Outcomes sheets

The outcomes sheets were developed in collaboration with the teachers of each of the classes. The outcomes plus sample questions were provided for each of the topics to which the classroom teachers added details of relevant exercises and page numbers from different textbooks which might assist them to understand the different outcomes. Ms Black also added a list of suggested practical activities that students could do to develop their understanding of each of the outcomes in the topic Properties of Solids.

Students would start each outcome by attempting to do the sample questions. Some students interpreted the difficulty or complexity of the outcomes by the number of sample questions provided. One student commented to the other members of her group “Come on, we’ve nearly finished – there are only two questions!”. However, the vast majority of the students took very seriously the task of learning the outcomes through practice and set considerable amounts of homework for themselves and the rest of the group. Often this would be through the efforts of the most conscientious student requiring other students in the group to complete the same amount of homework as they were prepared to complete. In general, teachers at Brindale Christian School felt that students did much more homework when they were responsible for setting it themselves.
6.1.1.4 Sample questions from different texts

Accompanying each of the outcomes and sample questions, teachers from Brindale Christian School also provided their students with details of exercises from their textbook as well as the other textbooks available in the classroom (see Appendix Three). By doing so, time spent searching for specific details was reduced during class time.

6.1.1.5 Availability of different texts

Within each of the classrooms a significant number of other texts were made available. These texts supplemented the standard texts that all students were supplied with often providing extension questions on top of those available in the textbook. In Mr Grey’s class all students received photocopied pages from relevant textbooks and references to these texts to assist them with the different outcomes. This was particularly relevant in their class, as many of the outcomes were not covered in their textbook.

Karoline: We got a fair bit of information from just asking the teacher, but I think most of it we – someone in the group would have known it already – it was just one person, someone would have had the best idea of it and for anything else which we didn’t understand which the teacher wanted us to find out we had – we’d go to encyclopaedias or other textbooks. (Ms Black’s class)

In Ms Black’s class for Properties of Solids encyclopedias were also available to assist students to learn about different types of solids, mathematicians such as Plato and Pythagoras and the religious beliefs that were associated with the Platonic solids. Students such as Karoline made use of these throughout the topic and during the classes observed. However, Sarah, who became the expert on Plato explored further afield for homework on Plato to find some information that no one else in the class had managed to find. There were several comments made by students about Sarah’s contribution to the rest of the class’s understanding.

Karoline: In properties of solids, probably for Plato, got some - Sarah, she got all this stuff off the Net.

John: Generally we received the work, asked each other questions - mainly asked the group, except for Plato, Sarah brang in some stuff on Plato, mainly it was good.
Nerida: Yes, and Sarah from group A she gave, she - I listened to her for Plato - we needed some information on Plato.

6.1.1.6 Posters of path diagrams

In one of the classes (Mr Smith’s class), the path diagrams were both private and public tools to guide students in terms of the outcomes they would cover each lesson. Each student received a copy of the path diagram for the topic, and a group copy was also placed on the classroom wall. As a group completed an outcome and had also written assessment questions for that outcome, Mr Smith would colour in the outcome so that each group could see exactly where they were up to and where each of the other groups was up to as well. In doing so, the posters of the path diagrams acted as a motivational tool for motivating groups to make sure they weren’t falling too far behind the other groups.

Yes, I blew it up and had one for each room. If they’d finished it, indicated - shown to me that they’d learnt this outcome, then follow up the outcome. So they could all see how each other was going and how they were going, knew where they were heading and I think it worked quite well…. (Mr Smith)

6.1.1.7 Practical activities

During some of the topics teachers made available different practical activities to support their understanding of different topics. In particular, the topics *Properties of Solids* and *Introduction to algebra* included a number of different activities students could pursue during the lessons.

But in that first topic (*Properties of Solids*) I thought that they did much more hands-on type work because we had done it collaboratively. In class it would have been more - you know, you would have shown them the shapes or maybe given it to them, but the length of time that they actually contacted those things, it would not have been the same. (Ms Black)

The fact that they’d done *Properties of Solids* hands-on, meant that when they came to writing a test, their questions were really - were quite creative I thought, in comparison to the way we might have set a test, because they now had an understanding and so therefore when they were coming to test something, it opened up more possibilities to them other than just to identify this or whatever. (Ms Black)
Copies of the tests students wrote for *Properties of Solids* are available in Appendix Fourteen.

6.1.1.8 Direct instruction

It was explained to all of the teachers that there would still be a place for direct instruction within this model – when the classroom teacher was of the opinion that all of the groups did not possess the pre-requisite knowledge for a certain outcome, nor were they likely to be able to bridge the conceptual distance between their current level of understanding and the new concept that they were required to master. Webb and Farivar (1994) suggest that collaborative learning is unlikely to be successful if students do not have the necessary prerequisite knowledge, and it remains the responsibility of teachers to monitor the level of understanding evident in each group to identify areas of difficulty that may arise. Mr Grey found it necessary to teach directly a considerable amount while his class was learning about *Ratio and Rates* to ensure that all members of his class had a good understanding of a certain topic.

I loved just to say whatever, “Well stop, everyone to watch the front.” I did it a little time, but would have liked to - there were some sets on ratios, the unitary method and I have a very particular way of teaching ratios and I just - I have a worry that they will - if they do it, they don’t know if they’re doing it the right way or the wrong way and they may have bad procedures that they will keep with them for the rest of their lives sort of thing. (Mr Grey)

Other teachers spent some time teaching the whole class, however it was rare for teachers apart from Mr Grey to teach the whole class in the traditional manner of direct instruction. One of the students from Mr Smith’s class commented on Mr Smith’s involvement with each of the groups in the following way.

Ryan: He’d probably just ask us to work and he’d just sit there in case someone needed help and to come out to him and then he’d just explain it on the blackboard or sometimes when more than one person came out to him with the same question, he’d stop the whole class and he’d show them this question so no-one needed to come out anymore. Just sometime during the lesson, if he sees that this might be a different question, he’d stop the class and just
explain that and let us get back to work, but it may just be that occasionally you’d go up to him and ask. (Mr Smith’s class)

Ms Black also made ongoing judgments about the likelihood that students in her class would be able to teach each other different concepts within the syllabus.

Where you didn’t think they were going to get some concept, you could actually teach them and I felt that was important and in the end I became used to saying, "I can see up ahead, that’s going to be a trouble point, so I’ll make sure that I teach them that particular thing, and then let them go away and continue working … I think if you have the sense that it doesn’t actually mean that you can’t use direct instruction; you can still use direct instruction within the context of collaborative learning at the point where you feel that they’re going to need that direction. If it’s a new idea or you know it’s a problem then you have responsibility of actually identifying it and make sure that they encounter it in the way you want it. (Ms Black)

6.1.1.9 Assessment task development

In each class, students were required to develop assessment tasks for the particular topic that they were studying. The intention of this aspect of the model was to provide students with a reason for teaching each other the particular concepts and for learning about the different concepts. Working on the assessment tasks of other students would also provide them with some practice at completing assessment tasks in that particular topic.

Each of the teachers at Brindale Christian School required the groups in their class to develop a joint assessment task. Some of the classes, however did not get a chance to practise using another group’s assessment task, and the development and implementation of the assessment task was hindered by interruptions to ordinary classes.

I found the - I would like to have done better, but I found that for some reason the end of every topic coincided with either some sporting thing where I never actually had in the last two lessons or three lessons of any topic, I never actually had the whole group there. There was always something on where I only had half the class here, and so it was those who were left did the test, so I never really felt that it was like the whole group and it really brought it to my attention. So the second time round, the second subject, I got people on a daily basis - I said, "Make sure there’s a member of every group," who had to write up test questions on that day when they did that outcome and I found that was the second time around was much more valuable. (Mr Grey)
Ms Gold required her groups to develop assessment items as they covered each of the outcomes. This approach of completing assessment items as the outcomes were completed was also adopted by Mr Smith and Mr Grey with their classes.

I found something that worked for us, at the end of every outcome and before they’d moved on to another outcome, I got them to write examples of test questions. So, we had a sheet designed and they’d finish the outcome and then they had to write on this sheet and explain what they were testing and this is the question. So, when it came to writing the test, they just pulled all those sheets out and they used those to write the test. (Mr Smith)

6.1.1.10 Learning journals

To assist students with their planning and reflection students were also required to complete a learning journal each lesson. The teachers at Brindale Christian School decided to develop their own learning journal based on models that the students were familiar with from other classes. The journal also provided additional information for the purposes of research about the nature of the learning taking place within each classroom. Some copies of journal entries from students at Brindale Christian School are in Appendix Fifteen.

6.1.2 Different types of collaboration observed

Collaborative groups at Brindale Christian School took on a number of different forms during the lessons observed. Interviews with students and teachers also provided evidence of different forms of collaboration. These included groups with minimal collaboration (Parallel workers), groups that did not work well together (Minimum work), groups where different members took on different roles (Co-operative groups), groups where individuals taught other individuals (Peer tutoring), groups that worked separately and then compared their answers (Parallel/Collaborative groups) groups where one person taught the rest of the group (Micro-classrooms), groups where each person contributed to the developing understanding of the group (Collaborative groups) and groups that sought help from other groups (Cross-group collaboration). Each of these forms of collaboration is described in more detail in Appendix Eleven.
While there were many different forms of collaboration evident at Brindale Christian School, the most common forms of collaboration were categorised as “micro-classrooms” or collaborative groups who would receive input from the teacher during each lesson when difficulties arose. Both forms of collaboration represent the reshaping of prior ways of learning that existed in the classrooms at Brindale Christian School where the teacher would provide students with a method that could be used to solve particular types of questions. At Brindale Christian School, students would often ask each other “How do you do this type of question?” referring to an identified group of questions that they did not as yet have a sufficient method for solving. Sometimes individuals within their group would teach the rest of the group, at other times the classroom teacher was called upon for help, and less frequently groups of students would work together to develop their own method.

At Southwest High School, a different form of collaboration was most evident reflecting a different set of expectations about the role of the teacher in the classroom and expectations of other students.

6.2 Collaborative classrooms at Southwest High School

Classrooms at Southwest High School displayed greater variation across classrooms and within in terms of the strategies teachers used and the forms of collaboration students appropriated. Section 6.2.1 outlines the strategies teachers adopted and section 6.2.2 describes the different forms of collaboration observed.

6.2.1 Strategies for facilitating collaboration and tools adopted by classroom teachers

During the planning phase, I introduced the participating teachers at Southwest High School to the principal components of the model over the course of three meetings that took place after school. To assist the collaborative teaching groups, each student was provided with a list of the outcomes to be taught and assessed and sample questions for each of these outcomes (see Appendix Three for examples of these documents). Each
student completed a pre-test for the topic they were studying and received feedback about their demonstrated level of understanding of each of the outcomes. Each of the teachers took these ideas and transformed them in some manner or other to suit the needs of their classroom, providing students in their classrooms with additional tools for supporting their collaboration. These additional tools are described below.

6.2.1.1 Outcomes sheets
Students at Southwest High School received outcomes sheets similar to those provided at Brindale Christian School (see Appendix Three), however the Southwest High School program did not adopt the structure of the new syllabus which provides a list of what students learn to do and what they learn about. At Southwest High School the outcomes sheets did not provide students with suggested activities and exercises for each outcome, providing them with sample questions only.

6.2.1.2 Availability of different texts
Within each classroom students at Southwest High School made use of many different textbooks. At the beginning of each lesson someone from each group would choose some of the textbooks from the front of the class to assist that group to learn about different outcomes. A wider range of textbooks was made available to students at Southwest High School, however, students were not provided with suggestions about which textbooks or sections to look at to develop an understanding of particular outcomes. During the lessons observed a number of students made use of the different textbooks. Several groups chose a number of textbooks at the beginning of each lesson almost out of habit.

6.2.1.3 Direct instruction
At different points during each of the collaborative topics, teachers would provide students with direct instruction about particular aspects of the topic that they felt students would not be able to work out in their groups. The frequency of such instruction was relatively low, however, given that within each class there were a number of students who had a very good understanding of the work already from their experiences outside school with tutors. When students described what happened during the collaborative classes, none of the students from Southwest High School mentioned instances of direct
instruction, although I was aware that Ms White had taught one aspect of the first topic on measurement from the front of the classroom.

6.2.1.4 Assessment task development

In each class, students were required to develop assessment tasks for the particular topic that they were studying. The intention of this aspect of the model was to provide students with a reason for teaching each other the particular concepts and for learning about the different concepts. Working on the assessment tasks of other students would also provide them with some practice at completing assessment tasks in that particular topic.

Each of the teachers at Southwest High School required the groups in their class to develop a joint assessment task. The benefits of this aspect of the program were questioned by teachers at Southwest High School who couldn’t see much benefit from preparing the assessment tasks.

Ms Diamond: I started to wonder at the end actually why get them to write their test. The first time, yes, so they understood, they learnt something I think about writing tests and outcomes and how to look at the topic, but after that, after you’ve done that for once, I thought well, couldn’t they just have the outcomes and then the test. Then you say you have to work through that, make notes from your book, work it and that sort of thing and I will set the topic test.

Ms Martin: Yes, I think so too, that’s why I ended up doing the measurement, I didn’t intend doing it from the outset, but that’s why I had to do it in the end, and I found it was a lot better. It means I’ve got a mark that I can actually use towards reports because it was a kind of test. I could still look at in the group and see the marks that they came close to or not to judge that as well. Yes but I think they sort of get hung up with all this time and energy spent on doing or not doing this test -

Ms Diamond: While typing it up beautifully, yes.

Ms Martin: - - and blaming each other for not doing it.

As Ms Martin reported, the assessment tasks developed by students for the second topic were so poor that she decided to write her own. The other issue with the assessment tasks
was the amount of time that students spent on presentation rather than content. The teachers themselves also tended to provide feedback relating to the marking scales more than the content itself.

Ms Diamond: The whole concept of a mark getting scaled is beyond them. I said it was out of 3, but to mark the scale.

Ms White: Most of them were out of 1.

The process of putting together the assessment tasks did not seem to provide students at Southwest High School with additional learning opportunities. This may have been because of the high level of understanding some students had prior to starting each topic which led to them approaching the topic with the perspective of completing set work rather than developing their own understanding.

6.2.1.5 Learning journals

The teachers at Southwest High School made use of learning logs developed for the purposes of this study. Sample learning logs can be found in Appendix Seventeen. The journal also provided additional information for the purposes of research about the nature of the learning taking place within each classroom. At the end of each lesson a member of each group was required to fill in the journal providing information about their level of understanding and the means by which they developed their understanding during that lesson.

6.2.1.6 Assessing quality of collaborative work on assessment tasks rather than performance

Teachers at Southwest High School appropriated the assessment scheme in Appendix One so that each student could be allocated a class mark. At Brindale Christian School, by the end of each topic teachers often found that they were unable to give their class an opportunity to complete another group’s assessment task and class marks were not collected for each student.
6.2.2 Different types of collaboration observed

The same general categories of collaboration developed at Brindale Christian School were evident at Southwest High School. These included groups with minimal collaboration (Parallel workers), groups that did not work well together (Minimum work), groups where different members took on different roles (Co-operative groups), groups where individuals taught other individuals (Peer tutoring), groups that worked separately and then compared their answers (Parallel/Collaborative groups), groups where one person taught the rest of the group (Micro-classrooms), groups where each person contributed to the developing understanding of the group (Collaborative groups) and groups that sought help from other groups (Cross-group collaboration). These are described in more detail in Appendix Sixteen.

While the dominant form of collaboration evident at Brindale Christian School was the micro-classroom, at Southwest High School the most common form of collaboration was the collaborative group in which each member took on responsibility for teaching other members of the group. In both cases, these forms of collaborations reflect aspects of the pre-existing culture at both schools which were transformed within the context of the collaborative classrooms.

Interactions between the classroom teacher and students at Southwest High School were less frequent than interactions between the teacher and groups of students at Brindale Christian School and focused on assisting students with extension questions rather than with the questions comprising the focus of each topic.

6.3 Identification of recurring phenomena within collaborative classrooms

Twenty four categories emerged from the process of open coding when describing what was happening in the collaborative classrooms. These were then categorised into three larger categories of student learning, teacher activity and student/teacher interaction. These categories are presented in Table 6.
Table 6: Categories of recurring phenomena evident within collaborative classrooms

<table>
<thead>
<tr>
<th>Student learning</th>
<th>Teacher activity</th>
<th>Student/teacher interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1: Students self-motivated to start working</td>
<td>B1: Teachers answer questions posed by different groups</td>
<td>C1: Multiple interactions occurring within the class at the same time</td>
</tr>
<tr>
<td>A2: Students approach teachers for help with their particular difficulties</td>
<td>B2: Teachers provide explanations for whole class</td>
<td>C2: Basic interaction within “learning mathematics” activities between teacher and students is teacher to group and consists of three, sometimes four identifiable stages (Inquiry into progress (teacher) – sometimes) -&gt; Query (student) -&gt; Guidance (teacher) -&gt; Further Collaborative Investigation (students)</td>
</tr>
<tr>
<td>A3: Students verbalise their understanding to each other</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4: Students develop understanding by relating what they know to current outcomes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A5: Students appropriate other students’ methods for answering questions</td>
<td>B3: Teachers beliefs about students change</td>
<td></td>
</tr>
<tr>
<td>A6: Students demonstrate interest in learning mathematical methods</td>
<td>B4: Teachers provide a “system” that assists in organising the work for students so that learning could occur</td>
<td></td>
</tr>
<tr>
<td>A7: Students take responsibility for own learning and learning of others</td>
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<tr>
<td>A8: Students appropriate goals as a group</td>
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<td></td>
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<tr>
<td>A9: Students ask questions about how mathematics works</td>
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<td></td>
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<tr>
<td>A10: Students focus on working through syllabus outcomes rather than completing textbook exercises</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A11: Students develop understanding collaboratively</td>
<td></td>
<td></td>
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<tr>
<td>A12: Students enjoy mathematics</td>
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</tr>
</tbody>
</table>
The same three general forms of activity – teaching/learning activity, practising mathematics and developing social relationships, were evident in the collaborative classrooms as were evident in the conventional classrooms with some of the phenomena identified occurring within more than one form of activity. Compared to the traditional classrooms, however, the phenomena related to each of these forms of activity in the collaborative classrooms were quite different. Again, the dominant component was teaching/learning activity (phenomena A1 – A9, A12 – A14, A17, B1 – 4, C1-C3). As well as this activity there was also the component of practising mathematics (phenomena A10, A15, B4, C3) and developing social relationships (phenomena A16, A17).

As outlined previously, the same three components were evident in the collaborative classrooms as were evident in the traditional classrooms representing three separate forms of activity. These components were teaching/learning mathematics, practising mathematics and developing social relationships. Again, to develop a more accurate understanding of the activity systems within each classroom it was necessary to incorporate into the analysis student and teacher perceptions of classroom activity obtained during interviews conducted at the end of each term.

Data collected from interviews conducted with students and teachers at the end of each term were categorised thematically to develop a picture of classroom activity in the
collaborative classrooms. Phenomenological data relating to the perspectives of participants within each of the different classrooms was used to make sense of the phenomenological world of actors in each activity. Each of the interview transcripts were scanned for recurring themes relating to the changing beliefs and practices of students and teachers as they compared their experiences in the collaborative classrooms to their experiences of conventional mathematics classes. As new thematic categories emerged, transcripts were revisited to ascertain whether the data needed to be recoded using the emergent categories. The process of coding continued until no new thematic categories emerged.

The following thematic codes emerged from student and teacher responses.

A: Self-directed learning
B: Level of interest (high/medium/low)
C: Students having difficulties working with other students
D: Students working well with other students
E: Evidence of developing a collaborative perspective
F: Teacher involvement in different groups
G: Students struggling with new concepts
H: Teacher struggling with aspects of program
I: Teacher acceptance of aspects of program
J: Evidence of students sharing ideas
K: Development of student understanding
L: Social development

Comments coded in each of these different categories were used to inform the analysis of related phenomena observed within the different classrooms and this analysis is presented in sections 6.3.1 to 6.3.3. As well as identifying the activity structures within the collaborative classrooms, comments in each of these thematic areas shed light on the emergence of other properties within the collaborative classrooms and the existence of different reactions to the collaborative model amongst the students and teachers who
participated in this study. These emergent properties and students’ perceptions of their own levels of motivation and learning are discussed in Chapter 7.

6.3.1 “Teaching/learning mathematics”

The principal component evident within the collaborative classrooms which took up most of the time was the “teaching/learning mathematics” component which represented an instantiation of the broader activity of “education”. Students and teachers within each class would work together to teach each other and learn from each other about different mathematical ideas. In interviews at the end of the different topics students were asked to describe what happened in the collaborative classrooms. Most of these comments were coded as indications of self-directed learning (category A).

John: We’d all sit down in our groups and just start and if anyone had any questions, we’d either go ask ... or go to another group, yeah done that one really well… (Ms Black’s class)

Nerida: For Properties of Solids, what we first did was ummm, we decided to do, just saying we were doing Outcome A and then we would try and work as hard as we could on that topic for that time, the time we had, if we don’t finish it though we finish off we try and finish off the outcome. (Ms Black’s class)

Warren: Basically we just decided what work tasks we wanted to learn and then we go through the exercises explaining to whoever didn’t understand explaining the basics and what you had to do in each question and how to do it and then you just worked through the exercises. (Ms Gold’s class)

Irene: When we were working in groups we wouldn’t ask the teacher how to do it, we’d actually go to the textbook where it referred in our outcomes sheet and we’d look how to do it - have a look how to do it and we’d do one question and see how it’s done properly and if we got it right we knew how to do it then. (Mr Grey’s class)

Daniel: Well we’d mark our homework and we did an exercise every single day, sometimes like two exercises and we’d revise what we did the last day to see if anyone had problems or anything. Then we’d mark homework together and we did some examples if we had difficulties. (Mr Smith’s class)

Karl: When we worked as a group, basically when we started the class - when the class started, we would go through what we’d done previous last lesson and then work out what outcomes we could build onto that. Work through the formulae, make sure everyone
understood that, then work on the exercises and work on that. If anyone had any problems we would just stop working, go through that, make sure everything was solid, get their minds to try to figure that out and then complete the exercise. Then once - we’d complete the exercise we ......... a few revision questions and then, that was it. (Ms Gold class)

Gary: We would have probably, I would say, doing our own exercise, set a group exercise that everyone would do – the outcomes question paper, we would do the questions related to that topic, if anyone needs help we’d help them and then afterwards we’d get an exercise for ourselves to for them to and then another exercise for homework – that’s basically it. We usually have a lot of arguments in class especially about hard questions, do it this way or do it that way, which answer is right which answer is wrong – that’s the way we have arguments. (Ms Martin’s class)

In each of these comments the students perceived themselves to be the principal actors within the learning/teaching component. Compared to the descriptions of what happens in conventional classes, the most obvious difference is the introduction of the pronoun “we” replacing “the teacher”. Furthermore, the nature of the tasks they were performing differs from the collaborative to the conventional classrooms. Whereas in the conventional classrooms the tasks students performed included “copy(ing) down”, “do examples along with her (the teacher)”, “we just sat down”, “then we get exercises”, in the collaborative classrooms students “decided what work tasks we wanted to learn”, “would try and work as hard as we could”, “explaining to whoever didn’t understand”, “we wouldn’t ask the teacher how to do it, we’d actually go to the textbook … and we’d look how to do it”. Students demonstrate a much greater sense of control over the learning experience, making choices and pursuing what they wanted to learn during the lesson.

Within each classroom, numerous teaching/learning activities could be observed with each group pursuing their own goals and objectives during the lesson. At the beginning of the lesson, students started discussing what they were going to do this lesson. Often this would involve going over the homework from the previous lesson. Once they had determined which outcomes they were going to look at during that lesson, students would then work on developing their understanding of these outcomes. The first step towards developing this understanding involved working out who already understood the outcome and who was having difficulty understanding the outcome. Students would refer to the
feedback from the pre-tests (see Appendix Five) or the most recent homework to see who was struggling with the outcome in question. They would then work together to teach those who did not understand the outcome. Once they were sure that all of the people in their group understood the outcome in question, they would then consider how they might assess the outcome (by developing assessment items) and what exercises they might work on for homework to practise what they had been talking about.

With the introduction of the collaborative model many of the primary contradictions identified by Engeström (1987) resolve themselves away from exchange value towards use value creating new contradictions that drive a new form of classroom activity. Classrooms focused on the exchange value of reproducing texts, methods and algorithms position the students as producers rather than consumers, fulfilling the desires of the teacher rather than their own. Instead, within the collaborative classrooms, students focused on the use value of different texts, treating these texts as tools or resources for developing their own understanding and the understanding of others.

Drawing on Engeström’s identification of use value/exchange value contradictions evident within each node of the school-going activity system a strong case for the general shift from exchange value to use value can be made as students move from traditional classrooms to collaborative classrooms. In collaborative classrooms, students view themselves as co-learners trying to make sense of mathematical ideas rather than students performing tasks to achieve grades. Mathematical tools – methods, algorithms, techniques and symbols are instruments which can assist in the teaching of others rather than rules to be learnt to achieve certain grades through the reproduction of these rules. The class community has moved from being a class of separate individuals to being teams of inquiry, working individually is replaced with cooperation between different members of the class, and the competitive “rules” are replaced by a sense of inventiveness and responsibility for other members of the group. Instead of reproducing various texts or mathematical techniques, the object that students pursue is the collective understanding of what different mathematical ideas mean.
The object of these activities for the students is the collective understanding of the group – not only was it evident that students were concerned for developing their own understanding, but they were also concerned for assisting in the development of other students’ understanding. Many of the comments students made concerning their activity in the collaborative classrooms were coded as evidence of developing a collaborative perspective (Category E). Once students had received feedback from the pre-tests for example they could work together to identify what they would need to teach to other members of the group.

Nerida: Well when we got the test back, we knew what we had to learn, so then we knew what maybe things we had to teach the other people in the groups from that test. (Ms Black’s class)

Interviewer: Now just finally, would you like to work in groups again in the future in mathematics, Daniel?

Daniel: Yeah.

Interviewer: Yeah, why?

Daniel: Because it’s faster and you learn all at the same time. Whatever you don’t know, other people know, they become the teacher and they can teach you. (Mr Smith’s class)

Warren: … in collaborative learning, I had to totally change everything I did because I had to teach all the other people and to me that was - it was hard at first but then after that I thought it was a really, really good experience because it also made me see that I can’t just not - I’m not the only one in the class here, I’ve got other people I could help. (Ms Gold’s class)

Carina: Yeah that’s what we did. We got the best person and they taught the way they got that question right and that person went over the steps to it and we learnt it that way. (Mr Grey’s class)

Lyly: Well we got to show each other, like different varieties of ways we can do the same question whereas if we did it with Miss Diamond giving us the solution, she would show us her way and that wouldn’t be necessarily easier. (Ms Diamond’s class)

Some of the students at Southwest High School in particular, however, did not appear to have the same object of transforming the collective understanding. These students
remained primarily focused on their own understanding and were frustrated with the collaborative approach as it seemed to obstruct their progress towards achieving their individual goal. These comments were coded as “Students having difficulty working with other students” (category C) and comments included in this category were obtained almost entirely from students at Southwest High School. Several of these comments were similar to those made by “Lucy” in Mary Barnes’ study (Barnes, 2001). Barnes records the following comment from Lucy about collaborative learning.

It gets too confusing with everyone putting in totally different ideas and sometimes a few people decide to do one thing and the others want to do something else. I also find it easier just to listen to the teacher, make notes, complete questions by myself etc. … (and I find it difficult because) … it takes longer to do things, because you have to discuss it in your group and then all the groups have to discuss what they did, and I just find it takes a lot longer. (Barnes, 2001, p.83)

Similarly, Jin at Southwest High School, when asked how he preferred to learn, said

Jin: I prefer the teacher to explaining the whole thing to me, and then for me to practise until I can comprehend. (Ms White’s class)

Other students felt that group work did not provide them with the opportunity to go away and sufficiently prepare for examinations.

Sherlyne: I prefer the teacher’s teaching because they make us take down notes, and when it comes to the exams you can help yourself and self-learn by rereading over the notes. (Ms Martin’s class)

Students at Southwest High School wanted to have the information that they needed to prepare for exams made available to them so that they could make their own progress towards learning the material. This was in contrast to students at Brindale Christian School who had appropriated the goal of developing the understanding of other members of their group as well as their own. This focus on individual learning was also evident in the comments from Southwest High School students about the disadvantages of working in groups. Students were asked to complete as part of a questionnaire at the completion of
the project what they thought were the advantages and disadvantages of working in
groups. At Southwest High School, 42% (36 out of 86) wrote that they didn’t learn as
much, got less work done or felt that their learning was ineffective. At Brindale Christian
School, only 22% of students provided similar comments.

One ethnic group at Brindale Christian School, however, consistently responded to items
in the questionnaire in a manner which suggested that their goal remained their own
individual progress rather than that of the group as a whole. Students who spoke Korean
at home wrote down several disadvantages of group work related to their own progress.
For example, the problems with group work are that “you get held back trying to help
someone, they bring you down”, “if someone who doesn’t understand (then) people of
the group have to stay together (rather than move on at their own pace)”, “someone
knows everything, but someone doesn't know everything”. Each of these comments
expresses a frustration that some members of the group felt with others who were going
slower or who were slowing them down. Nearly all of the Korean students, when asked
how they preferred to learn, wrote down that they preferred to learn from the teacher
rather than working in groups.

Some of the students at Southwest High School also felt that there was insufficient
guidance from the teacher about what was required of them during each lesson and did
not find other members of their group particularly helpful (category C). Students in Ms
Diamond’s class, for example, responded with the following when asked whether they
learnt more or less working in groups.

   Ruby: I think I learnt more in .......... (the class taught by the) teacher.

   Lyly: Yeah, I think more than group though.

   Eveline: Yeah, because group work you just like muck but you still learn.

   Andrea: But then sometimes you don't work well together.

   Interviewer: So sometimes you don't - -
Ruby: Work well together.

Lyly: Sometimes you don’t know what to do.

Interviewer: So that wasn’t clear?

Ruby: Yeah, the teacher’s already organised things, then teaches us.

Another student in Ms Martin’s class commented that working in groups was more difficult because you didn’t have a clear idea of what was expected.

Ronald: It’s kind of hard, because I would like for the teacher to just write down in a book what you have to finish for the lesson on the board or if you don’t finish you have it for homework, and that is the best way because you have to finish it.

Fulfilling the requirements of the teacher each lesson is something that students at Southwest High School are very good at and are very comfortable working this way in mathematics. When this structure and organisation is taken away several of them expressed their concerns about the learning that took place.

In Engeström’s model of school-going activity, the emergence of the use value/exchange value contradiction is evident in students determining whether or not the purpose of the activity is the obtaining of grades or the development of understanding. In the classes observed, and in the interviews conducted with students at the conclusion of the intervention, this fundamental contradiction between grades (exchange value) and understanding (use value) is replaced by a new contradiction between teaching other students what one has learnt (exchange value) and making sense of the mathematical concepts as individuals (use value). Instead of referring to each activity as a teaching or learning activity, therefore, the term “teaching/learning” (or the Russian word obuchenie) is used to describe more accurately the activity occurring in the collaborative classrooms.

At Brindale Christian School in particular there was a widely held perception that it was appropriate to expect other students to help you as well. As part of the questionnaires completed at the end of the topic, students were asked to write down their expectations of
other students and of their teacher. 57 students from Brindale Christian School wrote down their expectations of other students. 22 of these wrote down that they expected other students to help them or help others. Some of the comments were “to help others if they needed the help”, “to work together and help each other understand what we have to learn”, “I expect students to help me when I’m having troubles”, “I expect that if I ask them a question they will help me”, “help me with questions”. At Brindale Christian School, Daniel was very keen to work in groups in the future because, as he put it

Daniel: ….. it’s faster and you learn all at the same time. Whatever you don’t know, other people know, they become the teacher and they can teach you. (Mr Smith’s class)

At Southwest High School, out of 82 responses only 17 referred to expectations that other students would help.

Because this activity was the most common activity within the collaborative classrooms, a more detailed description of the activity is provided in the following sections 6.3.1.1 to 6.3.1.4 using Engeström’s general model of activity systems. 6.3.1.1 describes the subjects acting within this activity while 6.3.1.2 describes aspects of the different communities operating within this form of activity including discussion of divisions of labour evident within these communities. 6.3.1.3 and 6.3.1.4 provide additional information about this form of activity identifying rules and tools evident within this activity.

6.3.1.1 Subjects of collaborative teaching/learning activities

The subjects within this activity system were the members of the group who developed their own form of community which mediated the relationship between the members of the group and the collective understanding of the group. Each group developed their own sense of community, responsibility and purpose – certain similarities were evident at the different schools between the different groups since the emergent properties of each group have their genetic roots in the same classroom culture. At Brindale Christian School most groups were highly motivated towards teaching/learning the different outcomes and developing assessment task items for each outcome.
6.3.1.2 Features of communities participating in collaborative teaching/learning activities

Within some of the groups at Brindale Christian School it was evident that the community developed positive norms regarding the value of completing their work. Peer pressure was often used to encourage some members of the group who were struggling to complete their homework. When asked about how successful the groups approach was, a teacher at Brindale Christian School made the following comment.

Ms Gold: I think a lot would depend on the attitude of the individuals in the group. If they - which generally they were, or mostly were, if they were happy to work with others, happy to push each other along or help those who needed the help, then it was fine.

Another teacher said

Mr Smith: I felt that there were some really exciting lessons in some of my groups when some - three people in the group would all get behind the other person who was having trouble. They'd all sit there and make sure they'd set outlines and make sure they were doing it and that was really good because they were all focussing on this one person to make sure they understood it. Whereas that person usually in class, would sort of follow along but would often times fall behind a little bit and they were all making sure that that person understood it. That was really good, that was a positive thing they'd done.

Each of these comments was coded as evidence of category I – teacher acceptance of aspects of the program. During Ms Gold’s class on the 21st of June one of the groups was recorded discussing the homework at the beginning of the lesson.

Colleen: Did everyone do their homework last night?

Jason: What'd you give us?

Colleen: Ex 4E LHS and 4F

Jason: You didn't give me that

Colleen and Denise: Yes we did.
Jason: You didn’t tell us!

Karl: Hey, I didn't understand some of it.

Colleen: Okay we'll go through it then. Open it up and we'll go through it.

Karl: I don't understand all this stuff.

Colleen: Okay, Denise, Denise, check under my folder.

In this group Colleen and Denise took responsibility for setting homework in the class, and going through the homework with the other two members of the group who didn’t understand. Karl commented at the conclusion of the topic how much he enjoyed learning from the other students in his group

Karl: Yeah, I found it easier to work because ummm… of the interaction with people our own age, like, the teacher sometimes put it in like scientific things that you don’t really understand whereas like people our own age just can explain it a bit easy like in our sort of language, basically what you want to call it. So it was a bit, it was better I think. (Ms Gold’s class)

Karl: …. in our group if someone doesn’t understand it like I would call Alina that’s the person that helps us all she says "Does everyone understand it?" and if someone says “no” she would come over and help also we used the board a lot to ummm write like instead of just using paper if we happened to rub it out we use the board so our whole group could see it. (Ms Gold’s class)

The multiple communities which developed within each of the classrooms at Southwest High School had much in common with the groups that emerged within classrooms at Brindale Christian School as well as a number of significant differences. Groups at Southwest High School were considerably larger in size, with most groups having five people, and some having six. As a result there was a tendency for some of these groups to splinter into two smaller groups – one student said that his group “disintegrated … the girls separated, the boys separated”, while other groups were more like separate individuals who did their own work.

Eveline: Yeah, but for Pythagoras we really didn’t work well together.
Interviewer: You didn’t work well together?

Eveline: Yeah, we hardly talked to each other at all. (Ms Diamond’s class)

Many of the groups spent considerable time pursuing other outcomes apart from developing the collective understanding of the group – sometimes this would be developing their own individual understanding. Comments from the interviews with students suggest that there was considerable variety between the different groups. Two students from Ms Martin’s class suggested that the shared understanding of the group was their primary goal.

Ronald: I think I worked in groups in theory, maybe faster. You’re making it faster. You make everyone the same as you are, by helping them, so you get evenly.

Interviewer: Okay, good. David?

David: Working in teams you find you can co-operate and help each other.

A student from Ms White’s class rated the two groups he worked in out of ten commenting on them in the following way.

Terrance: The first one about a 6. There were some minor arguments and I wasn’t with that, but the second one I’d give them about a 9 because we worked really well together, all sort of at the same vibe about mathematics. (Ms White’s class)

In Ms Diamond’s class students also reported that their groups worked well together.

Ruby: I think when I was working in groups we talked more about maths, because we have different problem students, so we had to discuss it with each other, we had to work out – (Ms Diamond’s class)

Other students in Ms Diamond’s class described their group experiences very differently though. In some of the groups at Southwest High School there was a problem with more able students either choosing to do their own work or being asked by other members of the group to do all of the work. In Ms Diamond’s class one of the groups split into two
subgroups – one of whom worked towards developing the assessment task while the other group did very little work.

Lyly: Yeah, me and Eveline, well our group - -

Eveline: Sort of split into two.

Lyly: We worked all together and other people they didn't do anything and they expected a lot from us, 'cause, they're like you know, "You are more advanced than me in this subject, in maths, so therefore you have to do the work", so they didn't do anything. But they just sat there and talked about some anniversary present which is really - - (Ms Diamond's class)

Other students from Ms Martin’s class also took a very different perspective on group work

Gary: Because I don't find this topic particularly interesting, and yet as Thomas says our group we had a section of smart people and some not so smart but it proved that the smart people were helping themselves and not ...

Interviewer: You weren't very happy.

Gary: They did not help us much. (Ms Martin's class)

A capable student from Ms White’s class, when asked whether he learnt more or less working in groups, responded in the following way.

Jin: Less. Because we just - I don't know, I wasn't really caring, we had a pretty good understanding of stuff of everything, so we'd just start talking, while in a normal class, you just have to learn it, you have to do the work. (Ms White’s class)

While there appears to have been considerable collaboration amongst students at Southwest High School and that this was productive for students working through different outcomes, there is also evidence to suggest that different groups incorporated dysfunctional elements such as sub groups working independently of each other and inequitable divisions of labour. At Brindale Christian School, divisions of labour reflected the teacher-directed lessons that students were used to prior to working in
groups. One or two students were identified by the other students as the capable students who could be relied upon to explain the work to the rest of the class.

6.3.1.3 Rules of collaborative teaching/learning activities

Rules about how they would develop their understanding were also apparent in students’ descriptions of what happened during these lessons. Some of the groups would start by revising the outcomes covered during the previous lesson, others would start by marking the homework together, or discussing which outcome they would pursue during that lesson. From class to class there were differences in the amount of structure provided by the classroom teacher regarding these rules. At Brindale Christian School teachers were very keen to give as much structure as they could to provide students with a general strategy for each lesson.

Mr Grey: - I just find this whole approach - it just focuses them on the actual maths work and you are - you've got their attention in what’s being done much more than what would occur if there was no structure, there was just like, “What do we do today sir?” What did you decide we have to tick off?” They just see that maths is not just a series of getting a question right or wrong, you actually, they actually learn the subject and I find it’s great for that reason.

Mr Smith: And that yeah, that saves 10 minutes of your lesson really, just having that structure there, so there’s so much work you can get through.

Ms Black: In a sense it’s to do with too, what you focus your attention on, but that was - my kids found that that was the way to - knowing that they’d covered everything and knowing what they needed to cover and knowing the directions that they should take. I think if you didn’t have a pathways diagram thing, the learning would not be as - the learning wouldn't be efficient because they would be trying to chart that for themselves and they honestly do not have the ability, it doesn’t matter how clever they are, they don’t have a view of the curriculum and you’ve got to give that to them and I think that - I don’t think - whilst my kids related to it and we sort of ticked it off, I think Kevin stuck it on the wall and that worked really well too.

At Southwest High School there was a greater degree of freedom offered to the students – the path diagrams were not developed for subjects at Southwest High School after the first topic when students showed little interest in using them and the teachers provided less direction at the beginning of each lesson about what students should be covering that
lesson (although near the end of each topic teachers at Southwest High School provided students with a timetable of lessons and where they should be up to). Because of the higher degree of prior knowledge amongst students from Southwest High School (most students would have managed to get over 80% of the pre-test correct), students chose very different strategies for teaching/learning the different outcomes and developing an assessment task. The most common approach at Southwest High School was to look at the feedback from the pre-tests and ascertain which outcomes people did not understand and concentrate on these.

6.3.1.4 Tools used in collaborative teaching/learning activities

The tools that students used at both schools to transform the collective understanding of the group varied considerably as well. At Brindale Christian School, students in one class made use of small chalkboards shared by each group, while in another class, students would walk out to the front of the classroom and use the blackboard. Providing students with an opportunity to make public their private conceptual understanding is a key aspect of collaborative learning that was evident in many of the classes. Students also made use of the path diagrams in three of the four classes at Brindale Christian School to structure their approach to working through each of the different outcomes, while students at Southwest High School made more use of the feedback provided from the pre-tests to decide which outcomes were deserving of their collective efforts.

Other resources were also used to develop the collective understanding of students in each group. In particular, students at both schools made use of the assigned textbook to follow examples and to practise different skills. Students at Southwest High School were also provided with a box of other textbooks that they could make use of for alternative explanations. During the classes observed students would often take two or three textbooks from the box for their group to use during the lesson. One class at Brindale Christian School benefited from the research conducted by one of the students in the class who looked for further information about Plato. Students from Ms Black’s class were asked who they listened to about mathematics.

Interviewer: And you listen to them about mathematics?
Nerida: Yes, and Sarah from group A she gave, she – I listened to her for Plato – we needed some information on Plato.

Interviewer: Good. Karoline?

Karoline: Mostly the two teachers, but there's also people in my group which is John, Leslie and Sophia, and at one point Sarah did sort of explain a bit of Plato to us but not heaps.

John: Mainly the teachers, probably Sarah because of Plato and probably Karoline because she knows more and the rest of our group. (Ms Black's class)

These tools also had the effect of incorporating other subjects, or other voices into the community of discourse emerging from the different activities in classrooms at each of the schools. In conventional classrooms the voice of the teacher supported by the voice of the textbook provides a monological discourse within which mathematics is appropriated by students. Apart from the occasional instances whereby students appropriate this voice when attempting to explain what the teacher has said to the person next to them, the discursive outcome within the classroom is the transfer of the teacher’s monologue to the page. Students refer to “copying down” what the teacher writes on the board.

Irene: The teacher looks at you and tells the answer to the question and you wrote it down, he asks you another one. (Mr Grey’s class)

Gary: I prefer the teacher teaching us, because it’s more professional and you have notes and everything you have, and basically you have everything incorporated properly, everything such as notation, formula, everything's written down in your books. (Ms Martin's class)

Teresa: In a normal class, ... the teacher sit us individually and (we) just copying down teacher’s notes. (Ms Martin’s class)

6.3.1.5 The development of dialogical patterns of discourse within collaborative teaching/learning activities

Evidence from the classroom discussions suggests that there was considerable discussion amongst the students in explaining to each other how to do different types of questions. The following example relates to a question about two painters painting a fence.
**Question:** If it takes one person two hours to paint a fence and it takes another person three hours to paint the same fence, how long will it take them to paint the fence together?

Andrew: Why is it one and a half hours?

Byron: That's what I keep trying to reason with you!

Andrew: I don't know. Oh look so when he takes two hours to paint half the fence in one hour it takes him a third so half plus a third is ….

Cynthia: See! Now you're being sexist! How do you know it's a he, it could be a she!

Andrew: No, in one hour a half and the guy a third.

David: If the dude takes three hours painting and the one that paints faster also …

Andrew: No, in one hour the girl paints half ….

David: Interruption! (laughs) Go on whatever!

Andrew: And then, so… wait hold on …

Cynthia: It will take 72 minutes.

David: Whoa!

Cynthia: I say its 72 minutes.

Andrew: Okay, no, no, no. Oh yeah that's right, that's right it takes one hour to paint five sixths. One over sixth times ….

David: Isn't there a more simpler way?

Cynthia: There is. See how Alice takes two hours and Benny can take three hours. So we make it a simple hour so that's six hours. Alice can paint three fences in six hours and Benny can paint two fences in six hours. So both of them will paint five fences in six hours, so both of them will paint one fence in 72 minutes.

(Appplause from other members of the group)
Cynthia: That's how I look at it.

Andrew: …. Smart in maths

Cynthia: That's not me (laughs) (Ms Diamond’s class)

Students’ learning logs also indicated the ways in which students developed their understanding of different outcomes. A total of 137 learning logs were collected from teachers at Southwest High School. As part of these learning logs, students were asked how they developed their understanding. While many of the learning logs left this section and other sections blank (35 in all) the most common responses to this question and their frequencies are given below (some wrote more than one method). Table 7 provides the different categories and number of comments pertaining to each of the different methods identified.

Table 7: Student comments on methods used to develop individual/collective understanding

<table>
<thead>
<tr>
<th>Category</th>
<th>Number of comments (Total number = 102)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples and exercises</td>
<td>22</td>
</tr>
<tr>
<td>Team work/explaining to each other/sharing methods</td>
<td>63</td>
</tr>
<tr>
<td>Asking the teacher</td>
<td>7</td>
</tr>
<tr>
<td>Using the text book</td>
<td>27</td>
</tr>
<tr>
<td>Research on Internet/Other texts</td>
<td>10</td>
</tr>
</tbody>
</table>

Evidence from the learning logs, interviews and classroom observations points towards a large number of students developing their understanding through discussion with other students.
In the collaborative classrooms, a multitude of voices were evident in the discussions apart from just that of the teacher. For example, Sarah in Ms Black’s class brought to each of the groups the voice of other texts relating to Plato. As well as the voices from outside the classroom, each of the members of the group were able to provide their perspective towards the developing collective understanding. In the following snippet of discussion recorded in Ms Black’s class on 25th May, there are four students Alex, George, Mark and Dana who work together towards understanding how to use a distance ready reckoner (see Appendix Eighteen).

Alex: How much would it cost to send a parcel from Taree to Sydney? (B: $12) (Mark: Answer was 7).

George: No it weighs 35 kg.

Mark: 400, this is 750.

George: Its over 400 km then you go here.

Mark: Its 720 km.

George: its over 400 km isn’t it? So you go here and the cost for between 31 and 35 kilos.

Mark: How far is it from Moree to Sydney?

Dana: Do you understand that? Moree is here, Sydney is here.

Mark: How much would it cost to post a parcel 55 kg from Newcastle to Taree?

George and Dana: 55 kg.

George: Newcastle is there, Taree is there.

Alex: How far are we sending it? 647 km right? So how much does it weigh? 4.5 kg which is here. OK its going to cost …… (Ms Black’s class)

Within this discussion it is clear that some of the students involved, George and Dana, appear to have a greater confidence in being able to solve these types of questions.
However, over the duration of the conversation the voices of the other students are given an opportunity to participate in this discussion and Alex makes use of some of the utterances expressed to develop a means of answering the question about the cost of sending a package to Moree. Interestingly, there are three separate questions being discussed simultaneously by the group of four students – the three questions relating to the cost of sending various packages from Taree to Sydney, from Moree to Sydney and from Taree to Newcastle. Two of these questions provide examples from which Alex is able to develop a solution to his third question. In the final line Alex appropriates the method which has been discussed up until that point and applies that method to a separate question (Sydney to Moree) using the methods demonstrated.

As part of this same lesson, students were also developing questions for their assessment task. This same group developed an assessment item in the following manner

Alex: How much would it cost to spend, to send a parcel weighing....

George: Don't worry about a parcel, just say (Alex: a parcel) it says a mass of kilograms, so you can deliver anything let's say a pencil case, or a maths books.

Mark: Let's say "how much would it cost to deliver a computer ...."

George: A computer usually weighs around 20 kg.

Mark: Yeah, so ....(Alex: A laptop.)

George: A laptop weighs about 3 kg.

Mark: "How much would it cost to send a computer to ..."weighing 20 kilos from what do you call it, what did you do? From Newcastle to Sydney? From Newcastle to Sydney - weighing 20 kg. (Ms Black's class)

There are at least three voices echoed in the final item developed by this group – the voices of Alex, George and Mark. As well as these three voices there is also an echo of the type of language apparent in questions from textbooks. Instead of asking a question stripped of context as George suggests (“George: Don’t worry about a parcel, just say it says a mass of kilograms”) Mark proposes developing the question to be a practical
application relating to a computer. Furthermore, the type of question they have chosen to write echoes the examples provided to them with the original outcome sheet (see Appendix Three).

The dialogism that emerges from collaborations within the mathematics classroom is evident in numerous aspects of classroom practice. In most of the collaborative classrooms observed, for example, there was at least one student who went to obtain assistance from someone in another group adding to the number of voices participating in their original activity. Many of the comments from the Learning Logs also refer to the multiple voices at play in the group discussions. In response to the question “How did you develop your understanding of these outcomes?” students from Southwest High School wrote down the following responses.

- Learning other methods of working it out. (Gp E, Ms Diamond, 6th August)
- Trialling different methods. (Gp A, Ms Diamond, 20th August)
- Discussing different approaches in answering a question. (Gp C, Ms Diamond, 18th September)
- We used some different textbooks and worked out some sheets. (Gp A, Ms Martin, 27th July)
- The use of textbooks and sharing ideas. (Gp D, Ms Martin, 1st August)
- We looked in several books. (Gp D, Ms Martin, 3rd June)
- Everyone giving their idea. (Gp B, Ms Martin, date unknown)
- Had a talk about all of the tasks, showed our group different kinds of graphs. (Gp C, Ms Black, 26th June)

Sometimes the discussions incorporate the voice of the teacher who provides them with a particular method or strategy, the voices of other textbooks, or the voices of other students from other groups in the classroom. In the following example, the students discuss how to interpret the symbol °.
Sam: (commenting on the degrees symbol) No, that’s degrees.

Nick: Isn't it 1?

Jenna: Yeah, that’s to the power of zero.

Sam: That’s the degrees. This chapter’s not all about …

Jenna: That’s like when a triangle has 180 degrees or something?

Sam: Yeah, that’s it. (Ms White’s class)

Bringing the three voices of these students together enables the group as a whole to develop a shared understanding of what the symbol ° stands for in this particular question. The symbol ° is one of many mathematical symbols that can have multiple meanings depending on the context. The meaning that students attribute to different symbols depends on the dialogical context within which the symbol is encountered. In this example, the group of students came to the conclusion that the symbol represents degrees rather than the power of zero using the context provided by the exercise within which they encountered the symbol. Not only are the three voices of the students, therefore, evident in this conversation, but also the voice of the textbook as well as the voice of the teacher/textbook (“…to the power of zero”).

6.3.2 “Practising mathematics” activity

The second main component evident in the collaborative classrooms could be described as the “practising mathematics” component. Within this component, there was considerable similarity to the “practising mathematics” component evident in the conventional classrooms. The main difference, however, was that students set their own practice exercises rather than the teacher. As students practised mathematics, however, it was obvious that these students were directing their own learning rather than following an acceptable path towards greater understanding.

George: We've got to make sure we've done everything on the homework sheet, we'll make sure we've done everything on the homework list, and anything we haven't done from the rest
of the lesson, work on that, make sure it’s finished. (Alex: Will we do question one, two three, four or five?) Anything we haven’t done make sure you’ve done, that’s for homework, and start working on next one. (Ms Black’s class)

Penny: Okay we’re going to do dividing a quantity into a given ratio

James: Exercise what?

Penny: 10:04

James: "Dividing a quantity into a given ratio" So we’ll read this okay?

Penny: Okay, that’s for homework maybe, and on this page its Q4 its Q4 and 5. Okay we’ll do 4 and 5 today and we’ll do 6 and 7 tomorrow. And we’ll do 8 last ‘cause its hardest. (Ms Diamond)

Carl: So we get an understanding of the thing … what we’re doing

James: Do one more.

Penny: Let’s do number four as well another three question that’s all.

Carl: We have to do one with the triangle - number seven.

Penny: But if we do them all won’t we get a better understanding?

Carl: Let’s do number seven, because we’ve already done, we’ve done trapezium, now let’s do rhombus and kite. Just so that we know that … we know it.

Penny: OK, let me check it out for several questions.

Carl: let’s do number seven and that’s it.

Andrew: What are we doing now?

Penny: Number seven.

James: Why?

Penny: Why? To get a better understanding of kites and rhombus.
Andrew: Why aren't we doing this one?

Penny: Because Carl thinks that ….

Andrew: Why aren't we doing the whole thing?

Penny: I don't know ask James. James, are we doing seven or the whole thing?

James: Number seven.

Penny: Very well

James: Seven a, b, c. (Ms White's class)

During each lesson observed there were some groups who worked through exercises from the textbook, other textbooks or sheets provided by the teacher to practise doing different types of questions. These groups would sometimes be working in silence for the duration of the lesson, talking to other students only when they had difficulties with certain questions. However, the majority of groups observed did not use the time in class for practising, but instead used the time in class for talking to each other about the different outcomes to be learnt. Table 8 provides the number of groups working collaboratively or independently for the bulk of the lessons observed.
Table 8: Number of groups working collaboratively or individually in each class

<table>
<thead>
<tr>
<th>Class</th>
<th>Collaborative groups</th>
<th>Individual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms Martin 21st May</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Ms Black 31st May</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Ms Martin 4th June</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Ms Gold 7th June</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Ms White 12th June</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Ms Gold 21st June</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Ms White 26th June</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Ms Black 26th July</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Ms Diamond 7th August</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Mr Grey 10th August</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Ms Martin 13th August</td>
<td>4</td>
<td>2</td>
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<tr>
<td>Mr Smith 13th August</td>
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<td>Mr Smith 30th August</td>
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<td>Mr Grey 10th September</td>
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<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Ms Diamond 24th September</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>77</strong></td>
<td><strong>18</strong></td>
</tr>
</tbody>
</table>

As can be seen above 77 of the 95 groups observed (81%) worked collaboratively rather than independently for most of the lessons observed. The vast majority of the groups observed took the opportunity to discuss ideas with other members of the class and complete practice exercises outside class.

At Brindale Christian School students were given the opportunity to choose what work they would complete to practise answering different types of questions, however, because the teachers were concerned about some students not choosing suitable questions to
attempt as groups, each class was provided with a list of the exercises that needed to be completed over the duration of that topic (see Appendix Three).

6.3.3 “Developing social relationships” activity – emergence of other forms of discourse within the classroom

The third form of activity evident in the collaborative classrooms was that of “developing social relationships”. Sometimes groups would lose focus on the mathematics outcomes they were discussing during that lesson and take part in a totally independent activity – that of discussing other matters unrelated to their mathematics work. While the frequency of this activity in the observed classes was relatively low (at Brindale Christian School there would be typically one group at any one time which was engaged in “other talk”, while at Southwest High School the number of groups engaging in other talk could be as high as three groups at the same time) it is clear that for some of the students they felt that many people in the class wasted time. This was particularly true at Southwest High School where many students reported that they felt like they knew the work already and so spent most of their time talking about other issues.

The three different phenomena which have been described as teaching/learning mathematics, practising mathematics and non-mathematical discourse are related to different activities outside the class and are displayed in Figure 9.
The fundamental difference between these activities and the corresponding activities in the traditional classrooms was the central role of students in the activities within the collaborative classrooms. Students within the classroom are central participants in each of
these activities rather than being acted upon by the teacher. Within the teaching/learning activity which was the dominant activity in the collaborative classes students contributed to the learning experiences of other students rather than the teacher alone having sole responsibility for the learning experiences of students. By doing so, student discussions expanded the dialogical sphere of the classroom within which students could hear many different voices rather than the single authoritative voice of the classroom teacher. Practising mathematics, typically indicative of the traditional activity of school-going in which students work towards achieving certain grades, remains as a phenomenon evident in the collaborative classroom, although the time allocated to this activity is much lower. Furthermore, the object motivating this phenomenon has shifted from that of obtaining grades to developing individual and, in many cases, group understanding. Instead of completing work set by the teacher, students set their own work to transform their own understanding and that of other group members, thereby achieving the desired outcome of greater mathematical understanding.

6.4 Implementation of the Collaborative Learning Model at Brindale Christian School and Southwest High School

The dialectic between the existing cultural practices at the two schools and the collaborative learning model produced two similar yet different results in terms of the new cultural practices that emerged within the different classrooms. The form that the collaborative learning model took at each school reflected some of these pre-existing differences between the two schools. At Brindale Christian School, for example, teachers went to great pains to support students’ progress through each topic. Three of the four teachers placed great emphasis on the path diagrams with one teacher displaying a path diagram for each group on the wall. Each of the teachers at Brindale Christian School provided information to students in their classes about how they could develop their understanding of different outcomes. Two teachers provided a list of suggested practical activities, while all of the teachers provided directions for where to look in different textbooks to find out about different outcomes. At Southwest High School, teachers took a more passive role providing fewer structures to support students’ progress and making themselves available to assist groups should such assistance be requested by that group.
The amount of interaction between the teachers and the different groups varied across the different classrooms, although more obvious differences were evident across the two schools. At Brindale Christian School, teachers were more likely to provide instruction to the whole class and were more likely to approach different groups to ask them how they were going. While teachers at Southwest High School would also walk around and check on different groups, teachers at Southwest High School would spend more of their time waiting for different groups to approach them. One of the reasons for this was the different length of lessons at each school. At Brindale Christian School, where lessons were much shorter, the lesson would be almost entirely taken up with the teacher checking in on each of the different groups. At Southwest High School, where lessons were 80 minutes long, there was considerable additional time remaining once the teacher had spent time with each group to wait for groups to approach them. Furthermore, students at Southwest High School were much less likely to invite the teacher to join their group as they often had fewer perceived areas of difficulty for which they were looking for help. During the observations of classes at Southwest High School, I only observed groups of students approaching the teacher for assistance with the extension questions that could be found at the end of each topic.

At both schools there were examples of each of the types of collaboration identified, however groups at Brindale Christian School were more likely to have designated “teachers” and “students”. The teachers in each group were not necessarily the students who were identified by the teacher as the highest achiever in that group, and these roles were not necessarily fixed over the duration of the collaborative topics or even over the duration of individual lessons. At Southwest High School, the most common form of collaboration was collaborative argumentation in which members of each group would argue with each other about the best way to answer different questions. For some groups this was a positive experience where members of the group felt that they were able to work together well, supporting each other’s learning, while others expressed frustration about the inability of their group to work together.
While there remain significant differences between the two schools in terms of how the collaborative learning model was realised in each context, certain changes were evident at both schools as students were given responsibility for their own learning and the learning of other members of their group. These changes represent the principal focus of the current study and are outlined in the following chapter.
7 Moving from conventional to collaborative classrooms

By making a fundamental change to the teaching/learning structure in the classroom, consequent changes to the classroom culture within each school were observed. Collaboration as a model for teaching/learning acted as a catalyst bringing about profound changes in each of the classrooms observed. Quantitative and qualitative data analyses indicated several changes occurring within the classroom as a consequence of students working together in the manner described above although several of these changes were not uniform across the whole population of students who participated in the study. Student responses on some of the MSLQ sub-scales differed markedly before and after their class had worked collaboratively and student examination results were also compared between collaborative and non-collaborative classes. Qualitative procedures were also used to examine the ethos of each classroom, student and teacher perceptions, the roles they adopted in the classroom and the relationships between different members of the classrooms.

In sections 7.1 and 7.2 quantitative approaches and the results obtained using these approaches will be described and section 7.3 will describe qualitative approaches used to develop theories about changes in the collaborative classrooms. Chapter 8 brings the different findings from the quantitative analyses, qualitative analyses and additional qualitative evidence obtained through interviews, questionnaires and classroom observations together to develop answers to each of the research questions.

7.1 Responses to the MSLQ

Preliminary analysis of student responses to the MSLQ was conducted using confirmatory factor analysis. Parallel analyses to the original analysis conducted by Pintrich, Smith, Garcia and McKeachie (1991) were performed using the preliminary responses to the MSLQ collected prior to students working collaboratively.
7.1.1 Factor analyses

Prior to further analyses relevant to the current study using the MSLQ confirmatory factor analyses were conducted on the 80 MSLQ items to examine the factor validity of the MSLQ given that each item had been modified to refer specifically to mathematics. Factor analyses were conducted using the responses collected prior to students’ involvement in the collaborative learning project. A total of 156 students’ responses were included in the factor analyses.

Factor analyses, both confirmatory and exploratory, typically use a larger number of responses, particularly when examining a large number of items. Gorsuch (1983) and Hatcher (1994) for example argue that the number of subjects (respondents) to items ratio should be higher than 5:1. Two separate analyses were conducted in the current analysis – one using 31 items (Motivational factors) and one using 49 items (Learning Strategies factors). Using this general rule, only the first analysis would provide valid results. However, other theorists suggest that the stability of an analysis of data for underlying factors depends on a wide range of factors such as correlations between components, standard error of different items and the number of items underlying the factors. Arrindell and van der Ende (1985) found that variations in the ratio of subjects to items made little difference to the stability of the factor solutions. Guadagnoli and Velicer (1988) and MacCallum, Widaman, Zhang and Hong (1999) suggest that the factor loadings reflect an adequate analysis rather than sample size. Analyses in which factor loadings are above 0.6 represent stable factor structures even with sample sizes as low as 100 and analyses in which factor loadings are around 0.5 with sample sizes between 100 and 200 are adequate. Pattern matrices for each of the exploratory factor analyses are reported in Tables 11 and 16 demonstrating that the vast majority of factor loadings are above 0.5.

7.1.1.1 Overall model fit

As is customary with confirmatory factor analyses, several different indices of model fit were employed in the current study – Goodness of Fit Index (GFI), Critical N, Root Mean Residual and $\chi^2/df$. A summary of the different indices is provided in Appendix Nineteen.
The confirmatory factor analyses conducted replicated the analyses conducted by Pintrich, Smith, Garcia and McKeachie (1991) in their original analyses to determine the factor validity of the MSLQ scales. Correlation matrices for confirmatory factor analyses are included in Appendix Twelve. Two separate sets of factors were tested using confirmatory factor analysis. The first contained the 31 items theoretically loading on six motivation factors and the second contained the 49 items theoretically loading on nine cognitive strategy items.

7.1.2 Rationale for use of both confirmatory and exploratory approaches

Confirmatory factor analyses hold several advantages over exploratory procedures that have led to a general preference for confirmatory procedures within the area of self-perceptions. First, confirmatory factor analysis allows for several models to be compared for their goodness of fit rather than a single model proposed by a particular statistical procedure as optimal (Thompson, 1997). Thus confirmatory factor analysis provides a means of falsifying a proposed model in line with Popper’s guidelines for scientific research (Popper, 1959). Confirmatory factor analysis is theory driven rather than data driven and requires the researcher to specify a priori the model’s specifications in accord with theoretical considerations (Cuttance, 1987).

Second, confirmatory procedures allow for the specification of individual variance and covariance paths. Exploratory techniques do not allow for covariances between error terms and assume either that all latent variables are correlated (in the case of common factor analysis) or that all latent variables are uncorrelated (in the case of principal components analysis) (Daniel, 1989). Confirmatory factor analysis allows the researcher to specify error terms that are likely to be correlated as well as which latent variables are correlated with other latent variables. Finally, results using confirmatory procedures are more interpretable than those obtained using exploratory procedures (MacCallum, 1995). Exploratory procedures may offer an optimal model that includes significant cross-loadings with commonalities between items loading on single factors difficult to identify.
Several theorists have argued that theory testing is the activity of science and so confirmatory procedures should replace exploratory procedures (Cuttance, 1987). However, as Velicer and Jackson (1990) argue, most investigations incorporate exploratory and confirmatory aspects. Very rarely is the underlying structure of latent constructs fully understood and thus research is necessarily a mixture of theory testing and theory building. Furthermore, exploratory procedures can be viewed as an unrestricted procedure (i.e. the bias of the researcher cannot affect the end result), whereas confirmatory approaches are an attempt to fit the data to a preconceived model, which may result in a confirmatory bias.

In the context of confirmatory factor analysis, several models may fit the data equally well, however these models may not be distinguishable by mathematical procedures (MacCallum, 1995). Constant theory testing is therefore required once a model has received empirical support to ascertain whether alternative models fit the data equally well that provide a better theoretical understanding of the underlying relationships. Unrestricted procedures using exploratory techniques can be viewed as a more conservative confirmatory test. The conjunctive use of both techniques can therefore provide some indication of the robustness of the solutions obtained across different extraction procedures.

Exploratory and confirmatory procedures should therefore be viewed as complementary rather than competing (Velicer and Jackson, 1990). The exploratory factor analysis outcomes provide a somewhat more stringent test than those of the confirmatory factor analysis, and allow for a direct examination of cross-loadings across extracted factors. Exploratory procedures were therefore used in a confirmatory manner to examine the factor structure underlying the 80 items of the Revised MSLQ used in the current study.

7.1.3 Exploratory factor analysis procedures

Maximum Likelihood was used as the estimation method in exploratory factor analyses. Common factor analysis rather than principal components analysis was used given that some measurement error was expected to be present within individual scale items. An
oblique procedure (promax) was used to rotate the solution to approximate simple factor structures as inter-correlations between the factors were expected.

Given that the widely employed eigenvalue greater than one rule typically leads to over extraction (Velicer and Jackson, 1990; Browne, 1968; Cattell and Jaspers, 1967; Linn, 1968), both the Kaiser criterion and the scree test were used to determine the number of factors to retain. Thus, for eigenvalues that marginally approached or exceeded one, the outcomes of the scree test and theoretical considerations about the factor structures extracted were used to form conclusions about the appropriate number of factors to retain.

7.1.4 Motivational factors

A six factor model was tested first incorporating the six factors related to students’ level of motivation (Intrinsic goal orientation, Extrinsic goal orientation, Test anxiety, Self-efficacy for learning and performance, Control beliefs about learning and Task value). Fit indices were slightly lower than those obtained by Pintrich et al. (1991) indicating that the MSLQ rewritten specifically for mathematics did not display the same level of fit as the original MSLQ. This is not surprising given the changed wording of the items and the fact that the original MSLQ was designed using North American tertiary students instead of Australian secondary students. Fit indices for both the original MSLQ and the revised MSLQ used in the current study are presented in Table 9.
Table 9: Fit indices for six motivational factors on the MSLQ and revised MSLQ

Path coefficients were comparable and these are provided in Table 10.
Table 10: Path coefficients for six factor models on MSLQ and Revised MSLQ

<table>
<thead>
<tr>
<th>Latent variable</th>
<th>Indicator</th>
<th>Lambda-Ksi estimate for original MSLQ</th>
<th>Lambda-Ksi estimate for revised MSLQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic Goal</td>
<td>Q1</td>
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<td>Orientation</td>
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<td>Q22</td>
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<td>Q24</td>
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<td>Extrinsic Goal</td>
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<td>Orientation</td>
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<td>Q28</td>
<td>.76</td>
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</table>
The poor fit of the confirmatory factor analyses conducted on the revised MSLQ motivational scales indicate that the items were not loading cleanly on six separate factors. To explore the factor structure of the revised MSLQ factors in more detail, exploratory factor analyses using the maximum likelihood extraction method were conducted using these items from the revised MSLQ scales.

Using the Kaiser criterion in tandem with the scree plot (Figure 10), six factors were initially extracted accounting for 60.92% of the variance (Extraction Sums of Squares loadings = 52.53 %, \( \chi^2 = 402.923 \, df = 294 \)). The initial eigenvalues for these six factors ranged from 9.406 to 1.071. On examination of the pattern matrix (Table 11), however, it was clear that the fifth and sixth factors were not highly related to any of the factors identified in the revised MSLQ. Only four items had weightings above 0.1 on the fifth factor (weightings of .451 for Q9, .116 for Q13, .208 for Q18 and .978 for Q25 which are theoretically loading on three separate factors according to the original MSLQ) and only one item had a weighting above 0.2 on the sixth factor (Q29 with a weighting of 0.845). It is known that the Kaiser criterion has a tendency to over extract factors (Velicer and Jackson, 1990; Browne, 1968; Cattell and Jaspers, 1967; Linn, 1968) and from the pattern matrix a model with only four factors would appear to be more parsimonious.
Figure 10: Scree plot for factor analysis conducted on motivation items of the revised MSLQ

A four factor model was then examined to determine whether such a model produced four factors that were theoretically reasonable. The pattern matrix obtained from this analysis is displayed in Table 11 showing which items are most heavily loading on each factor.
<table>
<thead>
<tr>
<th>Item</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
<th>Factor 4</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Self-Efficacy for Learning and Performance</td>
<td>Intrinsic value of tasks</td>
<td>Test anxiety</td>
<td>Extrinsic value of completing tasks</td>
</tr>
<tr>
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<tr>
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</tr>
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<td></td>
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<td>.697</td>
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<td>Q26</td>
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<td>Q4</td>
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<tr>
<td>Q16</td>
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<tr>
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<td>Q11</td>
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<td>.408</td>
</tr>
<tr>
<td>Q2</td>
<td></td>
<td></td>
<td></td>
<td>.354</td>
</tr>
<tr>
<td>Q22</td>
<td></td>
<td></td>
<td></td>
<td>.346</td>
</tr>
</tbody>
</table>

Each of the items loading on Factor 1 are identified in the original MSLQ as items measuring Self-efficacy for Learning and Performance. Hence, Factor 1 was identified as the latent construct Self-efficacy for Learning and Performance. The five items loading most on the second factor are identified in the original MSLQ as items measuring task...
value (Q17, Q23, Q27, Q26 and Q4) while the next three items are identified as items measuring Intrinsic Goal Orientation (Q24, Q16 and Q1). Thus, the second factor is measuring a latent construct which might be referred to as the Intrinsic Value students assign to tasks. The five items loading most heavily on the third factor are all identified as items measuring test anxiety (Q28, Q19, Q3, Q8 and Q14). The weightings of the final two items are considerably smaller (.301 and .200) suggesting that this third factor is primarily associated with test anxiety. When comparing pre-test and posttest scores for this latent variable only the five items most closely related to this latent construct were used. Finally, the fourth factor has six items loading on it, three of which relate to Extrinsic Goal Orientation in the original MSLQ (Q11, Q30 and Q7), one which relates to Task Value (Q10), one to control beliefs about learning (Q2) and one to Intrinsic Goal Orientation (Q22). Only the three items relating to Extrinsic Goal Orientation were used to obtain a factor score for this latent construct since the other three items did not appear to be theoretically related to the three Extrinsic Goal Orientation items.

Factor scores for each of these four latent variables were obtained by using the loadings in the Factor Score Coefficient Matrix. The formulae for each factor score are presented in Table 12.

Table 12: Formulae for obtaining factor scores for each latent motivation variable

<table>
<thead>
<tr>
<th>Latent variable</th>
<th>Formulae for obtaining factor scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-efficacy for Learning and</td>
<td>((0.084 \times Q5) + (0.211 \times Q6) + (0.068 \times Q12) + \ldots)</td>
</tr>
<tr>
<td>Performance</td>
<td>((0.155 \times Q15) + (0.119 \times Q20) + (0.171 \times Q21) + \ldots)</td>
</tr>
<tr>
<td>Intrinsic value assigned to tasks</td>
<td>((0.082 \times Q1) + (0.092 \times Q4) + (0.073 \times Q16) + \ldots)</td>
</tr>
<tr>
<td>Test anxiety</td>
<td>((0.230 \times Q3) + (0.124 \times Q8) + (0.135 \times Q14) + \ldots)</td>
</tr>
<tr>
<td>Extrinsic Goal Orientation</td>
<td>((0.098 \times Q7) + (0.371 \times Q11) + (0.207 \times Q30))</td>
</tr>
</tbody>
</table>
7.1.5 Learning strategy factors

A nine factor model was then tested incorporating items from the original MSLQ designed to measure nine factors related to students’ level of motivation in mathematics (Rehearsal, Elaboration, Organisation, Critical Thinking, Metacognitive Self-Regulation, Time and Study Environment, Effort Regulation, Peer Learning and Help Seeking). Fit indices were considerably lower than those obtained by Pintrich et al. (1991) indicating that the MSLQ rewritten specifically for mathematics did not display the same level of fit as the original MSLQ. Fit indices for both the original MSLQ and the revised MSLQ used in the current study are presented in Table 13.

Table 13: Fit indices for nine cognitive strategy factors on MSLQ and revised MSLQ

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>( \chi^2 )</th>
<th>df</th>
<th>( \chi^2/df )</th>
<th>CN</th>
<th>GFI</th>
<th>RMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive strategy scales from MSLQ</td>
<td>Nine cognitive strategy factors with no items loaded on two factors as tested by Pintrich, Smith, Garcia and McKeachie (1991)</td>
<td>2465.6</td>
<td>1091</td>
<td>2.26</td>
<td>180</td>
<td>0.78</td>
<td>0.08</td>
</tr>
<tr>
<td>Revised Motivational scales from MSLQ</td>
<td>Nine cognitive strategy factors related to mathematics with no items loaded on two factors tested in current study</td>
<td>2891.9</td>
<td>1091</td>
<td>2.75</td>
<td>67</td>
<td>0.58</td>
<td>0.11</td>
</tr>
</tbody>
</table>

The model testing the revised MSLQ factor structure did not show adequate fit according to any of the fit indices. Furthermore, the model required 68 iterations to converge with positive definite phi and ksi matrices. This model was obtained by turning the admissibility criterion in LISREL off which normally restricts the number of iterations to 20 suggesting that model fit was less than adequate.
There are several different explanations for the lack of adequate fit in the factor structure for the revised MSLQ. First, the original MSLQ was developed with tertiary students rather than secondary students. Second, the original MSLQ was not designed to measure cognitive strategies specific to mathematics. Some of the latent variables measured by the revised MSLQ may not be well formed in the domain of mathematics. Latent constructs such as rehearsal, elaboration and organisation as measured by the MSLQ, for example, may not have parallel constructs in the domain of mathematics. However, the fit indices for the original MSLQ are also less than adequate compared to criteria widely used to determine goodness of fit.

The path coefficients for the revised MSLQ closely reflected those achieved for the original MSLQ. These are presented in Tables 14 and 15. A second analysis was conducted removing the factors rehearsal, elaboration and organisation which may be less likely to be strategies employed in mathematics. Furthermore, some of the items within the current MSLQ appear to be poor indicators of different latent variables. Items with loadings less than 0.4 were removed from the secondary analysis to provide a model which fit the data more adequately.

This second model demonstrated a more adequate fit than the first model; however, it was clear that the fit remained less than adequate overall. The number of iterations was 23, and the factor loadings were slightly higher than those obtained for the first model. Goodness of fit indices were also slightly higher but still less than adequate for such models. However, the Lagrange Multiplier modification indices indicated that the fit of the model could be improved if several items were allowed to load on more than one factor. In particular, the vast majority of these items were indicators for factors that were closely related to the general factor of self-regulation. Effort regulation and time and study management factors appeared as factors with considerable overlap with the metacognitive self-regulation factor. This would suggest that for the cohort taking part in the current study, these different factors were all closely related.
Table 14: Path coefficients for cognitive strategy factors rehearsal, elaboration, organisation and critical thinking on MSLQ and Revised MSLQ

<table>
<thead>
<tr>
<th>Latent Variable</th>
<th>Indicator</th>
<th>Lambda-Ksi estimate for original MSLQ</th>
<th>Lambda-Ksi estimate for revised MSLQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rehearsal</td>
<td>Q39</td>
<td>.62</td>
<td>.66</td>
</tr>
<tr>
<td></td>
<td>Q46</td>
<td>.63</td>
<td>.70</td>
</tr>
<tr>
<td></td>
<td>Q59</td>
<td>.56</td>
<td>.73</td>
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<tr>
<td></td>
<td>Q72</td>
<td>.58</td>
<td>.68</td>
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<tr>
<td>Elaboration</td>
<td>Q53</td>
<td>.60</td>
<td>.65</td>
</tr>
<tr>
<td></td>
<td>Q62</td>
<td>.60</td>
<td>.64</td>
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<tr>
<td></td>
<td>Q64</td>
<td>.74</td>
<td>.69</td>
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<tr>
<td></td>
<td>Q67</td>
<td>.42</td>
<td>.70</td>
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<tr>
<td></td>
<td>Q69</td>
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<td>.75</td>
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<tr>
<td></td>
<td>Q81</td>
<td>.65</td>
<td>.68</td>
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<tr>
<td>Organisation</td>
<td>Q32</td>
<td>.57</td>
<td>.75</td>
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<tr>
<td></td>
<td>Q42</td>
<td>.55</td>
<td>.57</td>
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<tr>
<td></td>
<td>Q49</td>
<td>.45</td>
<td>.49</td>
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<tr>
<td></td>
<td>Q63</td>
<td>.75</td>
<td>.74</td>
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<tr>
<td>Critical Thinking</td>
<td>Q38</td>
<td>.49</td>
<td>.55</td>
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<tr>
<td></td>
<td>Q47</td>
<td>.76</td>
<td>.59</td>
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<tr>
<td></td>
<td>Q51</td>
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<td>.66</td>
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<tr>
<td></td>
<td>Q66</td>
<td>.74</td>
<td>.67</td>
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<tr>
<td></td>
<td>Q71</td>
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<td>.75</td>
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<tr>
<td></td>
<td>Q33</td>
<td>.40</td>
<td>.32</td>
</tr>
</tbody>
</table>

There are clear indications from the confirmatory factor analyses conducted on the revised MSLQ that the items were not loading cleanly on six separate factors. To explore the factor structure of the revised MSLQ factors in more detail, exploratory factor analyses using the maximum likelihood extraction method were conducted incorporating these items from the revised MSLQ scales after removing items pertaining to rehearsal, elaboration and organisation.
Table 15: Path coefficients for cognitive strategy factors related to metacognition and help seeking on MSLQ and Revised MSLQ

<table>
<thead>
<tr>
<th>Latent Variable</th>
<th>Indicator</th>
<th>Lambda-Ksi estimate for original MSLQ</th>
<th>Lambda-Ksi estimate for revised MSLQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metacognitive self-regulation</td>
<td>Q36</td>
<td>.44</td>
<td>.53</td>
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<tr>
<td></td>
<td>Q41</td>
<td>.47</td>
<td>.66</td>
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<td></td>
<td>Q44</td>
<td>.54</td>
<td>.47</td>
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<tr>
<td></td>
<td>Q54</td>
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<tr>
<td></td>
<td>Q55</td>
<td>.58</td>
<td>.64</td>
</tr>
<tr>
<td></td>
<td>Q56</td>
<td>.43</td>
<td>.66</td>
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<tr>
<td></td>
<td>Q57</td>
<td>.35</td>
<td>.27</td>
</tr>
<tr>
<td></td>
<td>Q61</td>
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<td>.53</td>
</tr>
<tr>
<td></td>
<td>Q76</td>
<td>.61</td>
<td>.55</td>
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<tr>
<td></td>
<td>Q78</td>
<td>.55</td>
<td>.68</td>
</tr>
<tr>
<td></td>
<td>Q79</td>
<td>.50</td>
<td>.57</td>
</tr>
<tr>
<td>Time and study</td>
<td>Q35</td>
<td>.52</td>
<td>.63</td>
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<tr>
<td>Environment</td>
<td>Q43</td>
<td>.81</td>
<td>.80</td>
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<td></td>
<td>Q52</td>
<td>.52</td>
<td>.42</td>
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<td></td>
<td>Q65</td>
<td>.56</td>
<td>.47</td>
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<tr>
<td></td>
<td>Q70</td>
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<td>.62</td>
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<tr>
<td></td>
<td>Q73</td>
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<td></td>
<td>Q80</td>
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<td>.46</td>
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<td>Effort regulation</td>
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<td>Q48</td>
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<td></td>
<td>Q74</td>
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<tr>
<td>Peer learning</td>
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<td>Q45</td>
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<td></td>
<td>Q58</td>
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<td></td>
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<tr>
<td></td>
<td>Q75</td>
<td>.79</td>
<td>.68</td>
</tr>
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</table>

Using the Kaiser criterion in tandem with the scree plot (Figure 11), eight factors were extracted accounting for 61.47% of the variance (Extraction Sums of Squares loadings = 50.92 %, $\chi^2 = 369.259 \ df = 316$). The initial eigenvalues for these eight factors ranged from 8.668 to 1.025. A ninth factor had an eigenvalue of 1.001, however the scree plot
indicated that eight factors would be more suitable to extract. A second analysis restricting the number of factors to eight was then conducted.

Figure 11: Scree plot for factor analysis conducted on cognitive strategy items of the revised MSLQ

The second analysis indicated several problems with identifying eight factors suggesting that a model with fewer factors would be more appropriate. First, one or more communality estimates were greater than one indicating that results needed to be interpreted with caution. Second, 23 iterations were required to extract eight factors. Inspection of the factor correlation matrix also revealed high correlations between several factors. Furthermore, only six factors are theoretically hypothesized to be related to these 36 items.

Using the scree plot again to guide the choice of number of factors to extract, the slope of the scree plot drops off dramatically after the fourth factor is extracted suggesting that a more suitable analysis would involve the extraction of four factors instead of eight. An analysis restricting the number of factors to four was then conducted. Using the pattern matrix from this analysis 30 of the 36 items loaded heavily on one of these four factors.
That is, for each of these items the difference between the weighting on one factor and the weighting on every other factor was greater than 0.1. These loadings are presented in Table 16.

**Table 16: Pattern matrix for cognitive strategy factors on Revised MSLQ**

<table>
<thead>
<tr>
<th>Item</th>
<th>Factor 1 Critical/Self - critical thinking</th>
<th>Factor 2 Use of time</th>
<th>Factor 3 Ability to work with others</th>
<th>Factor 4 Planning for study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q61</td>
<td>0.755</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q44</td>
<td>0.729</td>
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<td>Q54</td>
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<td>0.402</td>
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</tbody>
</table>
To make sense of each of these items the original MSLQ scales were used to determine the nature of each of these latent factors. Items loading on Factor 1 included four of the five Critical Thinking items (Q47, Q51, Q66 and Q71), one of the Help Seeking items (Q40) and seven of the Metacognitive Self-Regulation items (Q41, Q44, Q54, Q55, Q56, Q61 and Q75). The seven metacognitive items related to critically assessing one’s own understanding and study strategies (“When I get confused about something I’m studying for mathematics, I go back and try to figure it out” – Q41, “If what I am reading in the textbook is hard, I change the way I read the textbook” – Q44, “I ask myself questions to make sure I understand the material I have been studying in mathematics” – Q55). Factor 1 was therefore labeled critical/self-critical thinking. Q40 was removed from subsequent analyses since it was not theoretically related to this construct and, despite this item being reversed in the revised MSLQ, it had a positive loading with Factor 1. Factor 2 included three of the five items related to time and study environment (Q52, Q76 and Q79), two of the four items relating to effort regulation (Q37 and Q60) and two items measuring metacognitive self-regulation (Q33 and Q57). Each of these items refers to use of time – either the ability to concentrate or put in effort for periods of time or setting aside sufficient amounts of time for study. Factor 2, therefore refers to efficient use of time. Each of these items, however, were reversed items in the original MSLQ – a high score on this factor, therefore, indicates a low level of efficient use of time. Factor 3 includes all three items measuring peer learning (Q34, Q45 and Q50) and three of the four items measuring help seeking (Q58, Q68 and Q74). Factor 3, therefore, refers to the ability to work with others efficiently. Loading on Factor 4 were two of the eight time and study environment items (Q35 and Q65) one metacognitive self-regulation item (Q77) and one of the critical thinking items (Q40). Each of these items refers to setting up a study environment or strategy. Factor 4, therefore, refers to efficient planning for study.

The theoretical construct of self-regulation within the current study was measured using four sub-scales derived from the original MSLQ. These four distinct factors were critical/self-critical thinking, efficient use of time, help-seeking and planning for study. There exist a variety of definitions in the literature, some of which are outlined in Section
2.5 of this thesis, that relate to these four sub-scales. The concept of self-monitoring common to each of the theoretical approaches shares considerable overlap with the factor labelled critical/self-critical thinking. Corno’s focus on students’ ability to avoid distractions overlaps with the planning for study factor, while one of the key cognitive strategies self-regulated students adopt is seeking help from others when they identify problems. Other cognitive strategies are closely related to the factor labeled efficient use of time.

The factor score coefficient matrix was then used to obtain loadings for each of these four factors using only the items that loaded cleanly on one of these four factors. The formulae for each factor score are presented in Table 17.

Table 17: Formulae for obtaining factor scores on latent cognitive strategy variables on revised MSLQ

<table>
<thead>
<tr>
<th>Latent variable</th>
<th>Formulae for obtaining factor scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical/Self-critical thinking</td>
<td>(0.043 _ Q36) + (0.044 _ Q40) + (0.090 _ Q41) + (0.107 _ Q44) + (0.108 _ Q47) + (0.082 _ Q51) + (0.079 _ Q54) + (0.074 _ Q55) + (0.142 _ Q56) + (0.116 _ Q61) + (0.093 _ Q66) + (0.118 _ Q71) + (0.069 _ Q76)</td>
</tr>
<tr>
<td>Use of time</td>
<td>(0.120 _ Q33) + (0.172 _ Q37) + (0.069 _ Q52) + (0.171 _ Q57) + (0.226 _ Q60) + (0.110 _ Q77) + (0.125 _ Q80)</td>
</tr>
<tr>
<td>Ability to work with others</td>
<td>(0.129 _ Q34) + (0.169 _ Q45) + (0.114 _ Q50) + (0.078 _ Q58) + (0.249 _ Q68) + (0.194 _ Q75)</td>
</tr>
<tr>
<td>Planning for study</td>
<td>(0.145 _ Q35) + (0.107 _ Q38) + (0.267 _ Q65) + (0.158 _ Q78)</td>
</tr>
</tbody>
</table>

Path coefficients obtained for both the motivational and the cognitive factors were then used to obtain weighted averages for each of the eight latent variables obtained through the joint processes of confirmatory and exploratory factor analysis.
7.1.6 Repeated measures analysis of the eight factor scores

Each of the eight factors were analysed in separate repeated measures analyses. Results for 114 students were included in this analysis. One of the classes at Southwest High School did not complete the revised MSLQ questionnaire at one of the time points and so this group of 29 students was omitted from the analysis. Other students who were absent from school and subsequently did not complete the questionnaire at any of the three time points were also omitted.

Half of the students were in the collaborative classes in term two and in the conventional classes in term three and the other half were in conventional classrooms in term two and in the collaborative classrooms in term three. The analyses, therefore, were conducted on both halves separately using repeated measures contrasts. Means and standard deviations for these two groups are presented in Tables 18 and 19.

Table 18
Group One: Mean scores of students who participated in collaborative classrooms in term two and conventional classrooms in term three

<table>
<thead>
<tr>
<th>Factor</th>
<th>Time 1 (pre-test)</th>
<th>Time 2 (posttest)</th>
<th>Time 3 (second posttest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-efficacy</td>
<td>5.28 (1.38)</td>
<td>5.22 (1.32)</td>
<td>5.36 (1.58)</td>
</tr>
<tr>
<td>Intrinsic value</td>
<td>4.87 (1.14)</td>
<td>4.79 (1.12)</td>
<td>4.86 (1.25)</td>
</tr>
<tr>
<td>Test anxiety</td>
<td>4.00 (1.45)</td>
<td>4.14 (1.44)</td>
<td>4.37 (1.46)</td>
</tr>
<tr>
<td>Extrinsic value</td>
<td>3.71 (0.93)</td>
<td>3.66 (0.79)</td>
<td>3.70 (0.87)</td>
</tr>
<tr>
<td>Critical/self-critical thinking</td>
<td>4.82 (1.10)</td>
<td>4.96 (1.11)</td>
<td>5.16 (1.20)</td>
</tr>
<tr>
<td>Use of time</td>
<td>3.64 (0.94)</td>
<td>3.86 (1.00)</td>
<td>4.12 (1.01)</td>
</tr>
<tr>
<td>Help seeking</td>
<td>4.15 (1.06)</td>
<td>4.13 (0.96)</td>
<td>4.43 (0.83)</td>
</tr>
<tr>
<td>Study</td>
<td>3.34 (0.85)</td>
<td>3.12 (0.84)</td>
<td>3.30 (0.82)</td>
</tr>
</tbody>
</table>
Table 19
Group Two: Mean scores of students who participated in conventional classrooms in term two and collaborative classrooms in term three

<table>
<thead>
<tr>
<th>Factor</th>
<th>Time 1 (first pretest)</th>
<th>Time 2 (second pretest)</th>
<th>Time 3 (posttest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-efficacy</td>
<td>5.89 (0.86)</td>
<td>5.79 (1.05)</td>
<td>6.11 (1.17)</td>
</tr>
<tr>
<td>Intrinsic value</td>
<td>5.51 (0.84)</td>
<td>5.28 (0.97)</td>
<td>5.46 (0.96)</td>
</tr>
<tr>
<td>Test anxiety</td>
<td>3.67 (1.41)</td>
<td>3.95 (1.46)</td>
<td>4.07 (1.77)</td>
</tr>
<tr>
<td>Extrinsic value</td>
<td>3.91 (0.66)</td>
<td>3.91 (0.65)</td>
<td>3.90 (0.67)</td>
</tr>
<tr>
<td>Critical/self-critical thinking</td>
<td>4.92 (1.41)</td>
<td>5.31 (1.07)</td>
<td>5.76 (1.01)</td>
</tr>
<tr>
<td>Use of time</td>
<td>3.55 (1.04)</td>
<td>4.06 (0.90)</td>
<td>4.24 (1.01)</td>
</tr>
<tr>
<td>Help seeking</td>
<td>4.00 (1.31)</td>
<td>4.41 (1.01)</td>
<td>4.76 (1.04)</td>
</tr>
<tr>
<td>Study</td>
<td>3.17 (0.99)</td>
<td>3.32 (0.70)</td>
<td>3.56 (0.83)</td>
</tr>
</tbody>
</table>

The 20 different analyses of variance were conducted with time of measurement as a within subjects factor. Repeated measures contrasts were tested with SPSS. These contrasts compared the scores obtained at the first time period with the scores obtained at the second time period ($C_1$), scores obtained at the second time period with scores obtained at the third time period ($C_2$) and scores obtained at the first time period with scores obtained at the third time period ($C_3$). For Group 1, therefore, $C_1$ is a comparison between pretest and posttest scores, $C_2$ is a comparison between the two different posttest scores and $C_3$ is a comparison between the pretest score and the second posttest score. For Group 2, $C_2$ is a comparison between pretest and posttest scores and $C_1$ is a comparison between pretest scores. Comparisons between the first set of pretest scores and the posttest scores ($C_3$) were not conducted for Group 2.
7.1.6.1 Critical/Self-critical thinking factor

Main effects for time of measurement were analysed by a single degree of freedom, "repeated" contrasts. There were no significant main effects for contrasts $C_1$, $C_2$ and $C_3$ for self-efficacy, intrinsic task value, anxiety, or extrinsic value in either group. A significant main effect for critical thinking/self-critical thinking was found for contrast $C_2$ but not $C_1$ for Group 2 but no significant differences were found for Group 1. The contrasts indicate that there was a significant increase in critical/self-critical thinking scores from the pretest ($M = 5.31, SD = 1.07$) to the posttest ($M = 5.76, SD = 1.01$). This difference was significant ($F(1, 29) = 4.337, p < .05$). There was no significant change from the first pretest to the second pretest suggesting that for this group there was an increase in critical thinking/self-critical thinking associated with participating in the collaborative classrooms.

7.1.6.2 Use of time factor

For Group 1 there were also significant differences related to use of time. Both $C_1$ and $C_2$ were significant. There was a significant increase in use of time scores from the pretest ($M = 3.64, SD = 0.94$) to the posttest ($M = 3.86, SD = 1.00$) and from the posttest to the second posttest ($M = 4.12, SD = 1.01$). For Group 2 only $C_1$ was significant – the change from the first pretest ($M = 3.55, SD = 1.04$) to the second pretest ($M = 4.06, SD = 0.90$). Each of these significant results indicated that students’ perception of their efficient use of time decreased over the duration of their time in the collaborative and conventional classrooms since scores on this factor were reversed.

7.1.6.3 Help seeking factor

For Group 1, there was a significant difference for $C_2$ but not for $C_1$. There was a significant increase in help seeking from the first posttest ($M = 4.13, SD = 0.96$) to the second posttest ($M = 4.43, SD = 0.83$) but not between the pretest and the posttest. For Group 2, there was a significant difference on help seeking scores for contrast $C_2$ but not for $C_1$. There was a significant increase in help seeking from the pretest ($M = 4.41, SD = 1.01$) to the posttest ($M = 4.76, SD = 1.04$) but no significant difference between the two pretests.
7.1.6.4 Study factor

For Group 1, $C_1$, $C_2$ and $C_3$ contrasts were significant. There was a significant increase from the pretest to the posttest and from the first posttest to the second posttest. Neither $C_1$ nor $C_2$ were significant for Group 2.

The results for each of the repeated measures analyses on each of the different factors are summarised in Table 20. Bold results indicate pretest-posttest contrasts.

Table 20: Summary of repeated measures analyses on each of the four factors with significant findings (Sig = Significant difference, NS = non-significant difference)

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th></th>
<th>Group 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_1$</td>
<td>$C_2$</td>
<td>$C_3$</td>
<td>$C_1$</td>
</tr>
<tr>
<td>Critical/self-critical thinking</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>Use of time</td>
<td>Sig</td>
<td>Sig</td>
<td>Sig</td>
<td>Sig</td>
</tr>
<tr>
<td>Help seeking</td>
<td>NS</td>
<td>Sig</td>
<td>Sig</td>
<td>NS</td>
</tr>
<tr>
<td>Study</td>
<td>Sig</td>
<td>Sig</td>
<td>Sig</td>
<td>NS</td>
</tr>
</tbody>
</table>

7.1.7 Qualitative evidence of students developing as self-regulated learners

Several different changes evident in the collaborative classrooms related to the development of students’ ability to regulate their own learning. Students reported a greater sense of personal agency within the collaborative classrooms making decisions about their own learning. Students also worked collaboratively to monitor their own learning and made decisions about how much practice they would need to do to develop their understanding of different outcomes. Students also reported how working collaboratively had supported their critical and self-critical thinking.
7.1.7.1 Shifts in perceptions of personal agency

A significant change in the collaborative classrooms was the shift in perceptions of agency in the classroom. Students described what happened in the collaborative classrooms with reference to themselves rather than referring to the actions of the teacher. During the interviews with students who had just completed topics covered in the collaborative classrooms 206 different statements by students were coded as indicating active involvement of students in the classroom activity while describing collaborative classroom activity while only 11 statements describing conventional classroom activity described students’ active participation in the classroom activity. In terms of passive activity, the results were reversed with 16 passages coded as passive student activity in the conventional classrooms while only 1 passage coded student activity as passive in the collaborative classroom. Students identified themselves as the central actors in the classroom who called on the teacher for help where necessary rather than merely responding to the teacher’s requests.

Ms Black: You know, they were responsible for directing what they were doing, they felt they had more, I suppose, freedom to direct their learning…

7.1.7.2 Development of self-regulatory skills

Student and teacher responses as well as classroom interactions in both the collaborative and non-collaborative classrooms indicating or referring to examples of students displaying self-regulatory skills were coded as such using NVivo. Six different sub-categories of self-regulation were identified using NVivo. These were being self-motivated (1728 characters), teaching one’s self (2857 characters), learning from others (3513 characters), self-correcting (570 characters), self-assessment of ability (811 characters) and deciding what to learn (1820 characters). Evidence from the coding counts conducted using NVivo indicated that every instance of students displaying self-regulated learning occurred within the collaborative classrooms. 11299 characters were coded as evidence of “self-regulation” (33 separate passages), all of which were referring to phenomena occurring within the collaborative classrooms.
As well as regulating their own learning, students were also engaged in regulating each other’s learning. There were several different ways in which students and teachers were observed assisting other students to regulate their learning – students directly teaching other students (17181 characters), setting homework for others (2017 characters), taking responsibility for the learning of others (2537 characters), planning work together (2585 characters) and marking homework together (2469 characters). 93 separate passages (27429 characters – 15.5% of characters referring to collaborative classrooms) were coded as evidence of other-regulation using NVivo – all of which occurred within the collaborative classrooms. Many of the discussions that students had with other members of their groups focused on how different members of the group could organise themselves, monitor their own progress and develop their own understanding.

Students from these classes who were interviewed at the conclusion of the study discussed the benefits of working collaboratively for supporting their critical and self-critical thinking.

Interviewer: When you were working in groups did you yourself find that you were talking about mathematics more than in a normal classroom?

Karl: Yes.

Interviewer: Yes, and why do you say that?

Karl. Because we had to make sure everyone understood that, and that meant talking it out more than just thinking it and that to me made it like much faster learning … (Ms Gold’s class)

These classes also seemed to continue to work collaboratively even within the conventional classrooms.

Ms Martin: I think a lot of the kids enjoyed working in groups. I’m not too sure, I think it was effective in a lot of the ways, but they seemed to enjoy it and after we’d finished doing the group work - I think we were doing ratio and rates - there were like four girls and two turned around to work with the others even after I’d explained it, they still wanted to work together and then I thought, well okay.
Some of the benefits of working together collaboratively, however, only appeared once the students had finished working in the collaborative groups. The long term benefits of working in this manner may only reveal themselves over a considerably longer period. Certainly this was the impression of some of the teachers who participated in the study.

Mr Grey: You feel like they've actually come away and they actually have a greater depth and knowledge of the topic, subject called mathematics and how it all works. It sets them up, I find more for Year 11 and 12 and it’s great for that. They will learn skills now which will set them up for problem solving - like Year 11 and 12 questions are totally different from 7 to 10, for everyone, there’s all the twists and by discussing it - I’m kind of off the question a bit - but that will help Year 11 and 12 by a long way.

Mr Grey’s comments also relate to his own perception of the critical thinking or problem solving skills that he saw students develop during the collaborative learning experience.

While the phenomena identified by Mr Grey and other teachers as well as those described by students indicate that there were considerable benefits for students from working collaboratively in terms of their critical thinking/self-critical thinking skills, these phenomena are not sufficiently widespread to appear as consistent significant differences in quantitative comparisons of revised MSLQ scores for mathematics. There was considerable diversity amongst students in their questionnaire responses with groups displaying high levels of variability. Qualitative data obtained from interviews and from classroom observations, however, identified a significant group of students for whom working collaboratively resulted in the development of self-regulatory skills and critical thinking about their learning.

7.1.7.3 Motivating students to learn mathematics

One of the significant outcomes from the collaborative learning topics was the greater degree of enjoyment students reported about working in groups. NVivo comments by students indicating high levels of enjoyment were far more common discussing collaborative lessons. 30 different passages (3529 characters – 2% of characters coded referring to collaborative classrooms) were coded as indicating a high level of enjoyment in the collaborative classes compared with only 13 (741 characters – 0.06% of characters
coded referring to conventional classes) in the conventional classes. All of the comments about enjoyment from the conventional classrooms referred to individual topics that students had completed in their conventional classrooms. Students did not make general comments indicating a high level of enjoyment of mathematics in the conventional classrooms. In contrast, many students commented on how much they enjoyed working in the collaborative classrooms. Students enjoyed having opportunities to communicate about mathematics with each other and displayed higher levels of interest as a result. One student in Ms Martin’s class commented that collaborative learning was

Sherlyne: …really cool, it’s out of the ordinary. You get a - it can actually become fun, maths
(Ms Martin’s class)

A student in Ms Black’s class described her experience in terms of a change in identity as well as being fun.

Nerida: In my group and before this collaborative learning, I wasn’t really a maths person, but after this, working with people made me realise that you know, it doesn’t matter what you work with or what you do in subjects, in this case maths, can still be fun. (Ms Black’s class)

Another student in Ms Gold’s class at Brindale Christian School, commented on finding mathematics more enjoyable when working in groups.

Karl: I think we got to like it more. Yeah, we were starting to like it a bit more, but like I said I like it heaps more in groups.

Interviewer: You would have liked it to have kept going?

Karl: Yes, definitely. (Ms Gold’s class)

Students in groups were more highly motivated compared with conventional classrooms in which the teacher felt it necessary to “get the big stick out” to get students to work. One teacher at Brindale Christian School described his experiences in the following way.
Mr Grey: - - “open it up to page this,” and then you get the big stick out and start with them. I mean that’s 12, 13-year-old boys, but here one of the things I did find, they’d come in and they knew what they had to do, so they’d set their desks up for the room and they’d get going themselves.

Students in Mr Grey’s class showed particularly high levels of interest in each lesson.

Mr Grey: So the interest level of every lesson was higher than I would say would be in a normal situation. …Generally I found the level of interest was greatly heightened to them, even topics like data analysis which would normally be - some people would get excited by it, but generally others wouldn’t. Yes, that was a very enjoyable topic.

Another student in Ms Gold’s class described the higher level of interest that he had for mathematics when learning collaboratively.

Warren: Well I think that it’s really good. I mean collaborative learning for me is a good experience, because it makes maths more interesting. I know a lot of people who didn’t like maths at the start ’cause they thought, “The teacher’s boring,” or whatever. (Ms Gold’s class)

Teachers at Brindale Christian School reported that students were so enthusiastic about what they were doing in mathematics lessons that conversations about mathematics continued outside the classroom.

Mr Grey: I liked the fact that sometimes at recess you hear them saying, “Now okay so if you get that done then you’ll be right and we’ll be doing this and okay, so tomorrow we each do this.” Maths is a sort of topic that they don’t talk about any other time, it’s a very - don’t talk about maths, any other time, you do your homework at home - -

Ms Black: They talk about how much you hate it!

Mr Grey: Yeah, exactly and I think it’s a really good thing for anything to be making maths a positive thing that we can talk about.

Such enthusiasm was not identified at Southwest High School. Students at Southwest High School displayed a certain reluctance to enjoy mathematics having fairly strong opinions about mathematics being something to be endured rather than enjoyed. One
student during the collaborative class brought a halt to the ensuing conversation by saying

Andrea: Stop! We're talking about maths! It's a maths lesson! (Ms Diamond’s class)

For this student, what you talk about in mathematics lessons is anything but mathematics!

While students in different classes reported enjoying mathematics more and teachers at Brindale Christian School reported on students’ higher level of interest and motivation in mathematics, students at Southwest High School, while describing mathematics as “fun” when working collaboratively, were less likely to describe collaborative learning as being beneficial for learning about mathematics. When asked for the advantages of working collaboratively, 77% of Brindale Christian School students wrote down that you could learn effectively working with others while only 58% at Southwest High School thought that you could learn effectively working with others. 13% of Brindale Christian School students described it as fun while 55% of Southwest High School students wrote down that working collaboratively was fun. When asked for the disadvantages of working collaboratively, at both schools about 35% wrote that learning could be more ineffective when working in groups.

7.1.7.4 Evidence of non-mathematical discourse in collaborative classrooms

At Brindale Christian School 27% of students also wrote down that one of the disadvantages of working in groups was that you were easily distracted. At Southwest High School 38% felt easily distracted. Similarly, 42% at Brindale Christian School wrote that the group might be uncooperative, while 52% at Southwest High School wrote that a group being uncooperative was a disadvantage of working in groups.

These results would suggest that at Southwest High School there was a greater frequency of non-mathematical discourse during the collaborative lessons where students chose not to pursue mathematical goals, instead opting to pursue social goals. One teacher at Brindale Christian School commented on how students in her class were focused on their work rather than on their social life.
Ms Black: Doing it (mathematics) collaboratively, probably meant that their focus wasn't so much on their social life, it was more on their work. I guess that would be one way of putting it.

Comments from students at Southwest High School, however indicated that some of the groups were dysfunctional for much of the time leaving a negative impression of their experiences of working in groups.

Ruby: ....they expected a lot from us, 'cause, they're like you know, “You are more advanced than me in this subject, in maths, so therefore you have to do the work”, so they didn’t do anything. But they just sat there and talked about some anniversary present which is really - -

Lyly: Yeah, because group work you just like muck but you still learn.

Eveline: Yes, it was like when we were in groups not a lot of people participated in the class, like what we were doing - - (Ms Diamond’s class)

Ronald: Some people would just - they didn’t really care, just as long as we had fun and some of us care more about work. (Ms Martin’s class)

7.1.7.5 Intentionality of students

Within the collaborative classrooms, it was also evident that students had appropriated learning goals for much of the time they were working together. This was evident in comments such as "we've gotta help each other in order to get a group understanding of 3 3 3 3 (referring to three ticks)" (Ms White's class) or " C: Let's do number seven … just so that we know that … we know it." (Ms White's class). In Ms Gold's class the intention of students in the classroom was to ensure that their group had a common understanding about different outcomes and methods.

Brett: Hey, I didn't understand some of it.

David: Okay we'll go through it then. Open it up and we'll go through it.
These examples indicate that the principal motivation for these students was the development of understanding within their group. Unlike the comments from students in the conventional classrooms indicating that the classroom activity was focused on fulfilling the requirements of the teacher, students in the collaborative classrooms were focused on learning goals, seeking to perform certain tasks in the class with the intention of developing their own understanding and the shared understanding of the group.

7.2 Comparisons between collaborative and non-collaborative classroom test results

The second set of quantitative techniques were used to compare test scores for students in the collaborative and non-collaborative classrooms. Adjusted scores for each student were obtained using multiple regression (the procedure for adjusting scores is described in Section 7.2.1) and comparisons were conducted on these adjusted scores using analyses of variance and independent samples t-tests. The methods for adjusting test scores and comparing them are described in the next two sections.

7.2.1 Obtaining adjusted scores

For each of the topic tests adjusted scores were obtained using multiple regression. For each pair of classes, the actual test score was identified as the dependent variable and several variables were entered as potential predictors of this score. These variables were age, sex and pretest scores. Once significant predictors had been included in the regression model standardised residual scores were then saved as adjusted scores for each topic for each person in the collaborative and conventional classrooms. For every set of test scores, the pretest scores available (normally end of term results from previous term) were significant predictors. Pretest scores, therefore, are treated as covariates with the test scores for each topic and the impact of these covariates is removed using this method.
7.2.2 Independent samples t-test scores.

A total of eight different comparisons were conducted. Because each set of data was independent of the other sets of data no adjustment was made for multiple comparisons using Bonferroni corrections. For each comparison the significance level was \( p < 0.05 \). A summary of these comparisons is provided in Table 21.

<table>
<thead>
<tr>
<th>Year</th>
<th>School</th>
<th>Topic</th>
<th>Adjusted mean (collaborative)</th>
<th>Adjusted mean (conventional)</th>
<th>( p ) value (2-tailed two sample unequal variance t-tests)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Brindale Christian School</td>
<td>Properties of solids</td>
<td>0.09</td>
<td>-0.08</td>
<td>( p = 0.648 )</td>
</tr>
<tr>
<td>7</td>
<td>Brindale Christian School</td>
<td>Data representation</td>
<td>-0.21</td>
<td>0.20</td>
<td>( P = 0.281 )</td>
</tr>
<tr>
<td>7</td>
<td>Brindale Christian School</td>
<td>Fractions</td>
<td>0.19</td>
<td>-0.20</td>
<td>( P = 0.311 )</td>
</tr>
<tr>
<td>7</td>
<td>Brindale Christian School</td>
<td>Decimals</td>
<td>-0.05</td>
<td>0.05</td>
<td>( p = 0.807 )</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Measurement</td>
<td>-0.32</td>
<td>0.29</td>
<td>( p = 0.045^* )</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Algebra</td>
<td>-0.32</td>
<td>0.29</td>
<td>( p = 0.045^* )</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Percentages</td>
<td>-0.27</td>
<td>0.30</td>
<td>( p = 0.058 )</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Data representation</td>
<td>-0.13</td>
<td>0.14</td>
<td>( p = 0.367 )</td>
</tr>
</tbody>
</table>

Only two of these comparisons indicated any significant differences between the two groups and in both cases the conventional classrooms demonstrated higher adjusted scores than the collaborative classrooms.
At Southwest High School for each topic there were three classes completing each of the assessment tasks. To compare collaborative and conventional classrooms, one way Analyses of Variance were conducted using contrasts which compared the collaborative classrooms with the non-collaborative classrooms. Table 22 provides means and standard deviations of adjusted scores and whether or not the class was working collaboratively and Table 23 reports on the contrasts tested and the $F$-values and $p$-values for each of these contrasts. Only one of the contrasts indicated a significant difference between the two groups suggesting that students in the non-collaborative classes did significantly better than students in the collaborative classes in the topic of Measurement.

Box plots for each of these comparisons are presented as Figures 12, 13 and 14. Many of these indicated that non-parametric tests would be more appropriate since the variances for each class were dissimilar for many of the comparisons. The Mann Whitney test using combinatoric approaches for obtaining exact probabilities of two groups coming from the same population were also used to ascertain whether there were any significant differences between the collaborative and the conventional classroom test scores for each of the different topics.

Table 22: Adjusted test scores for classes at Southwest High School and the model of classroom learning adopted for each topic

<table>
<thead>
<tr>
<th>Topic</th>
<th>Class 1 (Ms Diamond)</th>
<th>Class 2 (Ms Martin)</th>
<th>Class 3 (Ms White)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equations</td>
<td>0.20 (0.73)</td>
<td>-0.11 (1.15)</td>
<td>-0.09 (1.02)</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>Collaborative</td>
<td>Conventional</td>
</tr>
<tr>
<td>Measurement</td>
<td>0.44 (0.67)</td>
<td>-0.25 (1.14)</td>
<td>-0.20 (0.95)</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>Collaborative</td>
<td>Collaborative</td>
</tr>
<tr>
<td>Ratios and Rates</td>
<td>0.25 (0.85)</td>
<td>0.13 (0.87)</td>
<td>-0.39 (1.12)</td>
</tr>
<tr>
<td></td>
<td>Collaborative</td>
<td>Conventional</td>
<td>Collaborative</td>
</tr>
</tbody>
</table>
Table 23: Contrasts tested for each topic

<table>
<thead>
<tr>
<th>Topic</th>
<th>Contrast Coefficients</th>
<th>$F$-value</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equations</td>
<td>[1 -2 1]</td>
<td>0.748</td>
<td>0.457</td>
</tr>
<tr>
<td>Measurement</td>
<td>[2 -1 -1]</td>
<td>3.089</td>
<td>0.003*</td>
</tr>
<tr>
<td>Ratios and Rates</td>
<td>[1 -2 1]</td>
<td>-0.940</td>
<td>0.350</td>
</tr>
</tbody>
</table>

For each of the comparisons, therefore, the collaborative and the non-collaborative classes were compared using the Mann Whitney $U$ test which does not assume normality. While it does assume that both samples are similar in their distribution, and although such non-parametric tests are not necessarily robust when there is more than one violation regarding similar variances and similar skewness (Zimmerman, 1998), the Mann Whitney results are included to examine the reliability of the $t$-test results. These are reported in Table 24. These indicated that out of ten topics there were five topics for which students in the collaborative classes and the non-collaborative classes achieved significantly different results. For four of these topics students in the non-collaborative classes achieved better results on topic tests compared with students in the collaborative classes ($Measurement$, $Algebra$ and $Percentages$ for year 8 at Brindale Christian School and $Ratio and Rates$ for year 8 at Southwest High School) and for one of these topics students in the collaborative class achieved better results on topic tests than students in the non-collaborative class ($Fractions and Decimals$ for year 7 at Brindale Christian School).
Table 24: Mann Whitney results for comparisons between collaborative and non-collaborative classes

<table>
<thead>
<tr>
<th>Year</th>
<th>School</th>
<th>Class topic</th>
<th>Number of students in each group</th>
<th>U-statistic</th>
<th>Exact significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Brindale Christian School</td>
<td>Properties of Solids</td>
<td>$N_1 = 12, N_2 = 15$</td>
<td>84</td>
<td>0.792</td>
</tr>
<tr>
<td>7</td>
<td>Brindale Christian School</td>
<td>Data representation</td>
<td>$N_1 = 14, N_2 = 15$</td>
<td>70.5</td>
<td>0.137</td>
</tr>
<tr>
<td>7</td>
<td>Brindale Christian School</td>
<td>Fractions and Decimals</td>
<td>$N_1 = 13, N_2 = 15$</td>
<td>29</td>
<td>0.001*</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Measurement</td>
<td>$N_1 = 19, N_2 = 21$</td>
<td>122</td>
<td>0.036*</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Algebra</td>
<td>$N_1 = 19, N_2 = 21$</td>
<td>123</td>
<td>0.039*</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Percentages and ratios</td>
<td>$N_1 = 19, N_2 = 21$</td>
<td>119</td>
<td>0.029*</td>
</tr>
<tr>
<td>8</td>
<td>Brindale Christian School</td>
<td>Data representation</td>
<td>$N_1 = 19, N_2 = 21$</td>
<td>159</td>
<td>0.282</td>
</tr>
<tr>
<td>8</td>
<td>Southwest High School</td>
<td>Equations</td>
<td>$N_1 = 28, N_2 = 57$</td>
<td>776</td>
<td>0.842</td>
</tr>
<tr>
<td>8</td>
<td>Southwest High School</td>
<td>Measurement</td>
<td>$N_1 = 56, N_2 = 29$</td>
<td>674</td>
<td>0.204</td>
</tr>
<tr>
<td>8</td>
<td>Southwest High School</td>
<td>Ratios and Rates</td>
<td>$N_1 = 56, N_2 = 29$</td>
<td>509</td>
<td>0.005*</td>
</tr>
</tbody>
</table>
An overall comparison between the collaborative and the non-collaborative classes was conducted using the adjusted scores for each student and obtaining an average score for each student for each classroom type. A paired samples $t$-test was conducted on these average scores comparing students’ scores in the collaborative and the non-collaborative classrooms. For the collaborative classrooms the average $z$-score was -0.108 and for the non-collaborative classrooms the average $z$-score was 0.136. A two-tailed paired samples $t$-test was conducted on these scores which was significant ($t = -3.006, p < 0.05$) indicating that students performed significantly better on examinations when working in the non-collaborative classrooms compared to when working in the collaborative classrooms- the difference between the two groups being approximately 0.2 standard deviations.

Figure 12: Adjusted test scores for Year 8 students at Brindale Christian School
Figure 13: Adjusted test scores for Year 7 students at Brindale Christian School

Figure 14: Adjusted test scores for Year 8 students at Southwest High School
7.2.3 Self-reports of understanding

Students reported on their understanding as part of the interviews conducted at the end of the study and by rating their understanding of each outcome covered in each of the topics. Several comments during the interviews indicated that students felt that they had a deeper understanding of the work as a result of discussing the work with other people which was also corroborated by the teachers. In fact, 44 separate passages (5981 characters – 3.4%) were coded as students or teachers indicating a high level of student understanding when referring to what they learnt during collaborative classrooms compared with only 7 passages (429 characters – 1.9%) associated with the non-collaborative classrooms.

John: In group work, you went a bit slower and you understood it, ‘cause it stuck in your head, but in individual work, you went through it faster. (Ms Black’s class)

Nerida: Yeah, we would have understood more - -

Interviewer: Working in groups?

Nerida: Yeah, working in groups we would have understood, just had it in your head, you can understand it (Ms Black’s class)

Daniel: It took longer because you had to explain and its better that everyone knows what they’re doing, instead of taking less time and getting through it and everyone understanding what’s happening. (Mr Smith’s class)

Addressing myself as the interviewer, one student said at the conclusion of the interview

Karl: And the thing that you’ve done has helped us understand a lot more about mathematics. (Ms Gold’s class)

Of the students interviewed at Brindale Christian School, eleven out of twelve felt that they learnt more working in collaborative groups. At Southwest High School, only four out of the ten students felt that they learnt more in collaborative groups. The reasons given by those who felt that they learnt less in collaborative groups varied between different classrooms. In Ms Diamond’s class, the main reason was the inability of the groups to work well together.
Eveline: I think I learnt more in ........ teacher.

Ruby: Yeah, I think more than group though.

Leanne: Yeah, because group work you just like muck but you still learn.

Lyly: But then sometimes you don’t work well together.

In Ms White’s class, the main reason given by one student was that he felt that they already understood the work and decided not to do anything when they weren’t required to do so.

Jin: (I learnt ) Less (in collaborative groups). Because we just - I don’t know, I wasn’t really caring, we had a pretty good understanding of stuff of everything, so we’d just start talking, while in a normal class, you just have to learn it, you have to do the work.

Students were also asked to rate their understanding of each of the outcomes covered in the different topics. Appendix Seven includes a sample questionnaire in which Part B requires students to rate their understanding of each outcome. For each outcome, students wrote down whether their understanding was Very good, Good, OK or whether they thought they had little understanding of each outcome. The analysis of student responses began by assigning a numerical value to each of these responses so that students received a number indicating their level of understanding from 1 to 4 with 4 indicating a very good understanding. These values were then averaged for each student and the average for each student was used to conduct t-tests comparing the level of perceived understanding of students in each of the different classes. For some topics, these results were not available due to the teachers at Southwest High School deciding to omit this section in the questionnaire provided to students at the end of term two. At Brindale Christian School, the questionnaires were completed by students before they had completed the second topic at the end of term two and so their perceptions of their understanding of these topics were not collected. The available results are presented in Table 25.
Table 25: Student perceptions of level of understanding of different topics

<table>
<thead>
<tr>
<th>School</th>
<th>Topic</th>
<th>Level of understanding in collaborative classrooms</th>
<th>Level of understanding in non-collaborative classrooms</th>
<th>p-values for 2-tailed two sample unequal variance t-tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brindale Christian School (Year 7)</td>
<td>Properties of Solids</td>
<td>2.32</td>
<td>2.01</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>Data representation</td>
<td>Not available</td>
<td>Not available</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fractions and Decimals</td>
<td>2.09</td>
<td>2.26</td>
<td>0.43</td>
</tr>
<tr>
<td>Brindale Christian School (Year 8)</td>
<td>Measurement</td>
<td>2.24</td>
<td>2.15</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>Algebra</td>
<td>Not available</td>
<td>Not available</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Percentages, ratios and rates</td>
<td>2.00</td>
<td>2.22</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>Data interpretation</td>
<td>2.20</td>
<td>2.34</td>
<td>0.42</td>
</tr>
<tr>
<td>Southwest High School (Year 8)</td>
<td>Equations</td>
<td>Not available</td>
<td>Not available</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Measurement</td>
<td>Not available</td>
<td>Not available</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ratios and Rates</td>
<td>2.16</td>
<td>2.04</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>Pythagoras’ Theorem</td>
<td>2.49</td>
<td>2.20</td>
<td>0.002*</td>
</tr>
</tbody>
</table>

For the seven topics where comparisons were available between student perceptions of their own understanding in the collaborative and the non-collaborative topics, only one comparison revealed a significant difference. For the topic *Pythagoras’ Theorem* completed at Southwest High School students in the collaborative classroom reported higher levels of understanding than students in the non-collaborative classroom.

In general, however, there were no significant differences in the level of perceived understanding students developed in the collaborative and the non-collaborative classrooms.
7.2.4 Teacher reports of understanding

Teachers at both schools were asked questions about the level of understanding that students in their class demonstrated during the collaborative topics compared with normal lessons. Teachers at Brindale Christian School were enthusiastic about the prospect of students developing an understanding of how mathematics works rather than merely obtaining the right answer.

Mr Grey: I just find this whole approach - it just focuses them on the actual maths work and you are - you've got their attention in what's being done much more than what would occur if there was no structure, there was just like, “What do we do today sir?” What did you decide we have to tick off?” They just see that maths is not just a series of getting a question right or wrong, you actually, they actually learn the subject and I find it's great for that reason.

Mr Grey: I found that this is excellent in actually taking them, they're looking at a subject called mathematics and how maths work, rather than just turning up to a class and just answering questions, where they're worried about how many ticks or crosses they get. You feel like they've actually come away and they actually have a greater depth and knowledge of the topic, subject called mathematics and how it all works. It sets them up, I find more for Year 11 and 12 and it's great for that.

Ms Black: But they (students doing home schooling) don't have that bigger interaction of the classroom environment or bigger interaction of the group which then is bringing them to a far bigger understanding than they would have come if they'd just learnt it and then went on.

Ms Black: I think that their understanding would have to be better, I mean against the theory, if they have engaged in the content.

Ms Black: … it really benefits the more able student and the weaker student and that the more able student in the longer term will learn more by instructing the weaker…. when they get their exam results and they see they've gone really well, you know that they have a much more in-depth understanding of that.

Mr Smith: And it wasn't necessarily even just written down work, that was one thing that - I like to have their books nice and everything written down and they did write down a lot, but a lot of it was also talking about it or maybe writing it on a piece of paper, or writing it in one person's book and talking about the examples, so they didn’t necessarily produce a thick exercise book of written examples by the end, but they've produced a test and they've talked about it a lot and they've still learnt a lot.
Developing this deeper understanding was attributed to the fact that students had had opportunities to discuss their ideas and to teach each other what they understood. Teachers at Brindale Christian School also felt that students had more time to learn mathematics in the collaborative classes compared with conventional classes.

Mr Smith: Yes, you get much more work done. Although you've got to stop 10 minutes from the end, you do get more time actually doing maths.

However, one aspect of the collaborative classes was that students sometimes displayed deficiencies in their ability to assess their own level of understanding. Teachers identified some outcomes that students had assumed they knew and dealt with hastily in class which had not been covered in sufficient depth to prepare them for the end of topic assessment.

Interviewer: …do you feel like students managed to learn all of the outcomes when they were working collaboratively?

Ms Black: No, I would say no. Some less well than others. Because when you mark their test and when you mark their treatment of another person's test, 'cause we had the class test sort of as a measure and also the students' test, it became very evident what they did and didn't know, very quickly. So when I came to marking their test they did for another group, I'd say you know, “You need to attend your outcome A, D and F,” or - because it was really evident they either did know it or they didn't know it, which is good for the teacher to understand, but you'd like to think that they knew it all at the end. That's the thing which was difficult.

Ms Gold: And they'd say that they understand it, “Oh yeah ma'am we've done that.”

Ms Black: “Yeah, we've done it.”

Ms Gold: “We don't need to be guided.”

Teachers at Brindale Christian School raised two other issues related to the level of understanding that students were able to develop in the different topics. The first of these issues was the notion of “conceptual distance” between the topic being examined collaboratively and prior topics studied by students in that class. Collaborative learning was more “successful” in terms of providing a forum within which discussion could facilitate students learning when the distance between the new topic and previous topics
was small. In the case of topics such as percentages where the distance between this treatment of percentages and previous topics on percentages is quite low, students were able to make the necessary conceptual leaps towards the next level of understanding as a group. Topics such as Introductory Algebra which introduce numerous new concepts such as variable, equations, factorizing and expanding were more difficult for students who were working in groups without much input from the teacher.

Ms Gold: I think it also might depend on what are the topics, because, say fractions and decimals, they're things that are spirals, so that they've learnt those along the way, but when you come to something like algebra, it might be something totally new and foreign, so when they see that as an expression, that doesn't look like anything they've ever seen before, so that may also be a factor.

The second issue upon which success depended was the level of language students had to communicate their ideas. On one level, several students at Brindale Christian School had very poor English having only recently arrived in Australia. On another level, students also needed the requisite mathematical language to communicate certain mathematical concepts.

At Southwest High School, there was more concern expressed about the level of understanding that different groups were able to develop within the groups. Teachers at Southwest High School were aware that nearly all of the students in their classes received extra tuition outside class and that their understanding of each topic was considerable before the classroom teacher began to teach the topic. This was borne out by their performances on pre-tests in which most students demonstrated a very good understanding of nearly all of the outcomes. They were concerned about how much was actually being added to their understanding through their interactions in groups given this high level of prior understanding. There were similar concerns to those expressed by teachers at Brindale Christian School about students assuming that they understood certain outcomes and not giving them sufficient attention to ensure that every member of their group had a good understanding of the topic.
7.3 Beliefs and practices in collaborative classrooms

Using the interview transcripts and the twelve emergent categories identified in student and teacher responses to questions, a picture of the ethos of each classroom and the changing relationships and roles within each classroom was developed. These are described in Sections 7.3.1 through to 7.3.5.

7.3.1 Ethos of the collaborative classroom

When describing the ethos of the classroom, the intention is to describe what is the primary motivation driving classroom interactions and activity. Some of the students interviewed described a change in the ethos of the classroom, particularly at Southwest High School suggesting that collaborating together moved the fundamental motivation for participation in classroom activities from competition to collaboration. Several comments by students from Southwest High School indicated the development of a collaborative perspective (category E).

Jin: Well competition is the thing to motivate you the best. Who gets to be the class best. Like when you're in a selective school everybody's basically against each other. If you have a mark more, a mark less, it really counts, so that really brings the best out of you, to try to be the best of your class. (Ms White's class)

Terrance: What you're really saying about the competitiveness, I think the group sort of cancelled that out, because you didn't have an individual mark so you didn't feel the need to compete against your team mates and I think that's really, really important for the - for the better of your team, because if you're constantly trying to impress each other then you make a mistake. (Ms White’s class)

Ronald: Working in teams you find you can co-operate and help each other. (Ms Martin’s class)

Lyly: Pythagoras theorem, in my group, we worked well together and there's this guy called Charles, he didn't know much, yeah and we taught him about Pythagoras theorems and things, but ratios and rates, no. (Ms Diamond’s class)

Eveline: We started working, like, totally individually and then our group kind of got together. (Ms Diamond’s class)
Sherlyne: The teacher helped a lot. With collaborative learning with your friends we would talk to them and help each other out. (Ms Martin’s class)

At Brindale Christian School, this change was described by students in terms of looking out for the needs of others.

Warren: I thought it was a really, really good experience because it also made me see that I can’t just not - I’m not the only one in the class here, I’ve got other people I could help. I have to make sure that I cater to their needs so that I - like, to be less selfish. (Ms Gold’s class)

Karl: Well, the smart person has to consider the other students in the class - like ‘cause they got to know - they can’t go at a speed that they can’t go on, ‘cause then they won’t learn anything, so they’ve got to work slowly even though they understand it properly, they’ve just got to have patience and just work with the students. (Ms Gold’s class)

Daniel: At first we had difficulties because like Richard, we had to help him the most, ‘cause he couldn’t understand English and it was a bit hard, had to explain to him how to do the questions and how to get the right answer, but as soon as like he got the concept, the rest of us, we pretty much worked together well. As soon as we helped him get to the level that he needed to get to, it worked well from there. (Mr Smith’s class)

John: We didn’t work too fast so we just rushed through it, but we had to work together as a group, so we had a question at a time with each other and made sure they understood it, so it wasn’t quickly, but it was a lot quicker than you usually go in a normal class. (Ms Black’s class)

Interviewer: So explaining things took longer?

Connie: Yeah.

Daniel: It took longer, but as long as we knew that the aim was - we’d do as much portion as we could each day and that everyone understood. Like we don’t really care if we didn’t finish, so long as we - some lessons we’d do more homework because we spent more time explaining it, so long as everyone understood. (Mr Smith’s class)

Teachers at Brindale Christian School also commented on this change as well when asked whether or not the culture of the classroom changed with the introduction of collaborative learning.
Ms Black: I just found the culture developed in a group situation is everyone has the same - generally has the same endpoint goal that they don’t have when they’re working individually. When they’re working individually they don’t really care about what they’re doing, the only thing they worry about is getting you know, the best mark they can, but in a group situation, I noticed that the whole way of approaching any work is totally different.

Mr Smith: They’d all sit there and make sure they’d set outlines and make sure they were doing it and that was really good because they were all focussing on this one person to make sure they understood it.

Mr Smith: I was surprised that they consistently made sure that that person (who had missed a particular lesson) caught up and they helped that person catch up.

Ms Black: … they weren’t afraid to contribute to someone else’s learning and the other students weren’t intimidated by somebody else knowing better.

Mr Smith: In terms of that, I like the group concept where the stronger student helps the weaker student and they really did that. I had - there was one group there, they got the whip out to this weaker student and they set extra work, “So you get this, we’re going to give you extra questions to do,” and they weren’t being cruel. This kid wanted it anyway, he wanted to get it and in that sense it worked really well.

Ms Gold: If they - which generally they were, or mostly were, if they were happy to work with others, happy to push each other along or help those who needed the help, then it was fine. I think I had one student who didn’t like that concept and so he would just race ahead and leave the others behind. He soon realised that that was not going to work and it wasn’t going to work for his group and his group in the end, got it together.

Ms Gold: When we went back to normal classroom, direct learning, he then - I spoke to him lots of times about it - he then was actually much more able to help the person who was sitting next to him, who really struggles and made big improvements for that other person in terms of their learning this last term, just by sitting there and actually helping him all the time, so I think he got a lot from that in terms of being able to help. It doesn’t change his outcomes, his end result drastically, he was always getting a really good mark anyway, but it just means that he’s actually able to focus on helping someone else at the same time now, though he struggled with that at the beginning.

Comments from teachers and students at Brindale Christian School both point to a significant change in perspective shared by many of the students in these classes away
from the pursuit of individual learning goals towards the pursuit of collective learning goals. Instead of students viewing themselves as separate individuals, there is a shared perception that the classroom represents a community of learners working towards achieving collective learning goals.

This outcome was less evident at Southwest High School where a stronger competitive culture existed prior to the introduction of collaborative learning as a teaching/learning strategy. While there were students who recognised that the collaborative groups encouraged working together rather than competing with each other there was considerable resistance to this approach evident in the above quote from Ms White’s class. However, on inspection of transcripts of group discussions collected during classroom observations several examples of groups appropriating collective goals were obtained such as the one recorded below. This particular passage was coded in three separate categories – categories A (Self-directed learning), D (Students working well with other students) and E (Evidence of developing a collaborative perspective).

Edward: OK, ummm, Might I say we have to welcome Ian (who has been absent the past few lessons). Alright we gotta help you estimating lengths and area and calculate the perimeter of shapes given the measurements - we've gotta help each other in order to get a group understanding of 3 3 3 3 3 (referring to three ticks)

Francine: I realise that we're all bad at it, we all not good at it.

Edward: OK, did anyone get one tick for number nine?

Ian: No

David: No

Jenny: I have to help you with number nine.

Edward: We're all gonna have to help you. Alright let's work on number nine - I didn't understand that …. 

Jenny: Do you have the sheets?
Edward: Let's work on how to calculate the circumference of a circle.

Francine: You answered that.

Edward: Yes, I know how to do that but I don't know why I got the answer.

Other groups disintegrated as those who were more able worked at their own pace ignoring other members of the group (category C).

Gary: …our group we had a section of smart people and some not so smart but it proved that the smart people were helping themselves and not …

Interviewer: You weren't very happy.

Gary: They did not help us much. (Ms Martin’s class)

Evidence from interviews and the learning logs suggests that students at Southwest High School were able to learn through discussing their ideas with others. When asked how they understood the work covered in class one student commented that they “…just discussed it with the others - that’s the way how I learnt”.

Out of the 102 responses recorded to the question in the Learning Logs asking how they developed their understanding, 63 provided responses that were categorised as “working together”. Appendix Seventeen provides three examples of students’ learning logs from Southwest High School. For each of these examples, the way that students developed their understanding was categorised as “working together”.

Lots of explanation and doing example questions (Ms White’s class)

Everyone helped each other (Ms Martin’s class)

Our group discussed the questions and worked them out together (Ms Diamond’s class)

However, when asked about their expectations of other students in the questionnaire at the conclusion of the study, these were more often framed in terms of comparative ability
rather than other students being willing to help. At Brindale Christian School (57 respondents), 32 students (56%) wrote down that their expectations of other students were that they would work with others, help them or respect others. 18 (32%) had expectations of other students related to their level of understanding. At Southwest High School (82 respondents), 28 (34%) students had expectations relating to students working together while 43 (53%) had expectations about the level of ability of other students in their class.

While the ethos of collaboration appeared to be dominant in both schools during the collaborative lessons, competing cultural values, norms and perceptions were also apparent. Cultures are rarely homogeneous in their beliefs, values or practices and the heterogeneity of these different cultural groups was evident in perceptions, relationships and roles. As classes moved from conventional teaching methods to collaborative learning approaches, however, different participants in these cultures were able to identify emergent cultural beliefs and practices that were unique to the collaborative classroom. These beliefs and practices were related to teacher and student perceptions, relationships and roles.

7.3.1 Changing expectations of teachers about different students

Teachers in the collaborative classrooms discussed how their perceptions of students in their classes changed while working in collaborative groups. At both schools teachers’ perceptions of students’ ability to work in groups (categories C and D), their level of maturity (categories E and L) and self-regulation (category A), changed during the collaborative classrooms. At Southwest High School, teachers commented on the level of organisation that students demonstrated during the collaborative learning lessons and their ability to negotiate what work they would complete during that lesson.

Ms Diamond: I was pleasantly surprised sometimes, at the organisation that came from one kid or a few kids in the group and even the way they expressed, like one kid didn’t sit there right, “you do this and you do that and you do that,” and they were saying, “how about - why don’t we organise it like this,” you know, very diplomatically and you know, splitting the work load and stuff like that.
Ms Martin: Yes, I agree. A few times I thought, wow that's pretty good - high level organising.

The level of control that one teacher at Southwest High School felt was necessary in the classroom to ensure that work was completed also changed as a result of participating in the collaborative classroom.

Ms Martin: I think a lot of the kids enjoyed working in groups. I’m not too sure, I think it was effective in a lot of the ways, but they seemed to enjoy it and after we’d finished doing the group work - I think we were doing ratio and rates - there were like four girls and two turned around to work with the others even after I’d explained it, they still wanted to work together and then I thought, well okay.

This was also reflected in one of the students’ comments from another class.

Terrance: I think she lets us be more independent now, because she’s seen how we give out in a group and most of us - most of us, act in a pretty responsible way, so she’s sort of letting us off, being more independent now. (Ms White’s class)

At Brindale Christian School, teachers’ changing perceptions of students were related to the level of interest (category B) that students demonstrated in mathematics classrooms and the level of willingness of some of the more able students to assist others.

Ms Black: … generally I found the level of interest was greatly heightened to them, even topics like data analysis which would normally be - some people would get excited by it, but generally others wouldn’t. Yes, that was a very enjoyable topic.

Ms Gold: In first semester I wouldn’t have let him (Warren) sit next to him (Karl), because he would have mucked around with this kid, but then he was starting to help this kid now and I thought, okay well you can stay then and help him out. That was a complete change.

General comments about the level of interest in mathematics were very positive when students and teachers were discussing the collaborative lessons.
7.3.2 Relationships within the collaborative classroom

In some classrooms there was a change in the relationships between the teacher and students in the classroom (category F). As mentioned earlier, in one classroom at Southwest High School there was a perceived change in the degree of trust which existed between the students and the teacher in that particular class. Whereas there is a sense in which students perceive teachers to be taking “control” of the classroom, this sense of being controlled was considerably reduced in the collaborative classrooms.

The most significant change within classrooms at Brindale Christian School, however, was the transition from viewing the teacher as the principal source of information in the classroom to the teacher being one possible source of information.

    John: We got work done, a lot more work done (in collaborative classrooms) and if you needed something you could discuss it with the teacher. (Ms Black’s class)

One of the teachers at Brindale Christian School described how they were viewed as a “resource” by students in the class rather than the focus of the classroom activity. Typically, interactions between the teacher and students in the classroom were initiated by students who would approach the teacher with a specific question that they wanted answered.

    Warren: I already knew some of the stuff from beforehand, but if I didn’t know it, I’d ask someone else or a teacher or just look in a textbook. (Ms Gold)

One aspect of the classroom activity in which the teacher displayed more proactive participation was supporting groups or individuals to organise and plan what they were going to do that lesson or how they might approach learning or teaching a particular concept. In the vignette from Ms Black’s class provided in Appendix Eighteen, for example, students were working together to learn about different graphs and received considerable assistance from Ms Black in planning how they would collect graphs for their assessment tasks.
7.3.3 Student/student relationships

As a result of teachers allocating students to different groups, there were resultant changes in the relationships between students as well. Students were required to interact with students who they wouldn’t normally interact with and for the majority of students and teachers this had a positive impact on the classroom community. At both schools teachers described the students in the different classes as being relatively friendly prior to working together collaboratively, however working collaboratively gave them opportunities to meet other people in their class (category L).

Mr Grey: So, I had kids who normally wouldn’t give each other the time of day, not that they’re enemies or anything like that, they’re not friends, they don’t knock around together, but when they sat down and were in this situation, they started to relate to each other in that working environment which was really good.

Terrance: I was in a group of people I hadn’t really associated with before and we sort of formed a bond sort of thing. (Ms White’s class)

For one class, however, the fact that students had to work with other students that they did not normally relate with was identified as a negative aspect of working in groups. Students in Ms Diamond’s class, the top class at Southwest High School, tended to equate the effectiveness of their groups with the level of cooperation that existed amongst the members of that group and this level of cooperation in turn was dependent upon the personalities in each group. One group was described as dysfunctional by one student because “some of us don’t like the other people”. Another student commented that “we have to work with the right people to be happy”. Students from this class had mixed opinions about their experience in groups depending on the ability of that group to work together.

7.3.4 Development of social skills, social awareness and collective responsibility

One particular aspect of the changing student/student relationships was the developing awareness amongst students of the importance of helping others and working with others towards obtaining a common goal (category E). Students were aware that they needed to
develop a positive working relationship with other members of their group if they were going to achieve their goals.

Nerida: If you didn’t like sort of - you’d have to work with them anyway, so you’d have to be able to get on with people or you couldn’t get anything done. (Ms Black’s class)

As part of learning how to work with other students, several students commented on the value of supporting other people towards achieving the common goals of the group.

Terrance: I think the group class thing is really good, because instead of just working for yourself and saying, “I don’t really care if I fail, I’m only letting myself down,” you’re letting the team down if you don’t do anything, so that’s better motivation. (Ms White’s class)

Ms Gold: I think that one of the things that it gave me the opportunity to do, was talk to them about their contributions to someone else’s learning and it wasn’t so in every case, but where it was like that, they were often fiercely just doing their own thing and to bring them back to help the other people, in a sense they began to understand the value for themselves, as the process went on that they were not losing out in that process in contributing to someone else’s learning.

Ms Gold: When we went back to normal classroom, direct learning, he (Warren) then - I spoke to him lots of times about it - he then was actually much more able to help the person who was sitting next to him, who really struggles and made big improvements for that other person in terms of their learning this last term, just by sitting there and actually helping him all the time, so I think he got a lot from that in terms of being able to help. It doesn’t change his outcomes, his end result drastically, he was always getting a really good mark anyway, but it just means that he’s actually able to focus on helping someone else at the same time now, though he struggled with that at the beginning.

7.3.5 Roles within the collaborative classroom

The roles that students appropriated within the collaborative classroom were radically different from the roles they appropriated in conventional classrooms. Instead of being passive recipients of the methods and techniques offered by the teacher, students adopted more active roles teaching each other and pursuing their own learning goals.

What was evident in these classrooms, however, was the type of learning dialectic described by Lave and Wenger (1991) in which legitimate peripheral participants move
towards becoming more central members of particular communities. Within traditional classroom communities, the teacher remains the solitary central member of this community. There are few opportunities for students to move from being peripheral members towards being central members with equal status and role. Within traditional classrooms the apprentice is always the apprentice and the master is always the master.

Collaborative classrooms provide opportunities for apprentices to become masters – for students to develop expertise in certain areas and take on the role of master. This was particularly evident in Ms Black’s class where one of the students became an expert on Plato and was given responsibility for teaching other members of the classroom. In many classes at Brindale Christian School, the students who initially took on the role of teaching others maintained that role for much of the topics observed. At the same time, however, other students reported that there were a wide range of students who taught other members of the class.

    Karoline: Yeah it was good, ‘cause you know who’s going to be the group expert and how they have to help everyone and if the group expert doesn’t know one thing but you know somebody else does, you can just ask them or you can go to a different group and ask them if you don’t know it. (Ms Black’s class)

Other groups provided opportunities for each student to provide their opinion towards assisting others to develop their understanding.

    Rachel: When you're with a group you can get different opinions and say, “What do you think, what about if you did this and that,” and then explain. (Mr Smith’s class)

As the culture of the classroom changed, so too did the nature of collaboration occurring within the classroom, the level of motivation students reported and the degree to which classroom activity displayed similarities with activities occurring outside the classroom. The initial research questions proposed for this study relate to these three areas as well as the degree to which the concerns of teachers for preparing students for examinations were also met. Each of the research questions and associated findings and conclusions will be revisited in Chapter 8.
8 Discussion and conclusions

In this final chapter, evidence obtained relevant to each of the research questions will be discussed and used to draw conclusions about the nature of the changes evident at each school brought about by the introduction of the collaborative learning model. Discussion and conclusions relevant to the nature of collaboration evident within each classroom will be presented followed by discussion and conclusions about the levels of motivation and self-regulation evident within the collaborative classrooms. The relationship between the mathematical activity within the collaborative classrooms and “real world” mathematics will also be discussed as well as the degree to which teachers’ concerns for preparing students for examinations were met. Strengths and weakness of the current study will be discussed followed by implications for policy and practice and directions for future research. Summary conclusions will then be presented.

8.1 Research Question One: Nature of collaboration developed within each classroom

Findings associated with the first research question will be presented beginning with some discussion regarding the modes of collaboration evident in the collaborative classrooms, the expanding dialogic sphere within each classroom, the availability of multiple perspectives, and the relationship between understanding and the ability to explain something. In this section discussing the first research question, collaboration as a catalyst for cultural change will also be discussed.

The first principle of effective classrooms identified in the literature was that students have opportunities to collaborate with each other. Collaborating with other students provides students with opportunities to reformulate their ideas by comparing their perspectives with the multiple perspectives of others (Gauvain and Rogoff, 1989; Rogoff, 1990; Miller, 1987). Collaborative groups also produce multiple overlapping zones of proximal development (Brown, 1994; Brown, Ellery and Campione, 1998) through which individuals can make progress within the region of sensitivity beyond their current level
of understanding. Given that students were required to work in groups it is not surprising that “collaboration” was evident in these classrooms. However, the nature of this collaboration varied across different groups with some forms of collaboration encouraging the exchanging of multiple perspectives and supporting the learning of other members more than others. Some groups split into dyads and conducted peer tutoring, while other groups allocated tasks to different members of the groups in the manner suggested by co-operative learning theorists. The majority, however, engaged in whole group discussions to develop a common understanding in which students had opportunities to exchange ideas and support other members who they identified as needing assistance.

Many comments by students demonstrated a shift in their thinking towards the development of a sense of responsibility for other members of the group. Students who were more advanced in their understanding showed signs of an awareness of the needs of others and greater social responsibility. This greater degree of responsibility for the learning of others extended into the non-collaborative classrooms at the conclusion of the collaborative phase and was also identified by teachers as a key positive outcome of the collaborative classrooms.

Within each of the classrooms observed most students in the class spent the majority of the class time collaborating with other students. Throughout the lesson, this collaboration would be evident in their discussions about what they would do that lesson, which outcomes they would concentrate on, and what exercises they would look at to develop their understanding. Once students had determined which outcomes they were going to do they would then discuss what the outcome meant and how each member of the group would approach different types of questions. Groups of students would then spend time attempting different questions and sharing their approaches as they completed the agreed upon exercise.
8.1.1 Modes of collaboration identified in different classrooms

Numerous different forms of collaboration were identified in each of the classrooms – peer tutoring, group collaboration, cooperative groups, cross-group collaboration, mini-classroom groups and parallel workers. At Southwest High School, the dominant forms of collaboration observed were groups of parallel workers and whole group collaboration. There was also a high incidence of groups at Southwest High School that disintegrated into separate groups, some which worked together learning mathematics and others which tended to be easily distracted from their focus on learning mathematics. Groups which disintegrated in this manner did not provide students with opportunities to share different ideas, nor did they support the development of individual members. Some of the students at Southwest High School felt considerably frustrated as a result and did not perceive there to be much benefit for them from working together.

At Brindale Christian School, the dominant form of collaboration could be described as a mini-classroom in which one of the students took on the role of the teacher explaining to the rest of the group how to do a particular question. In these collaborative groups there was evidence of weaker students being “pulled through” their individual zones of proximal development (Goos, 2004) through the concerted efforts of other members of their group. The other principal form of collaboration was whole group collaboration in which each member of the group contributed to the emergent collective understanding.

The experiences of students at both schools point to the existence of residual components from previous ways of doing that existed in each school. At Southwest High School, the ethos of competition discouraged some group members from working with other group members. At Brindale Christian School, the emergence of mini-classrooms within each group reflected the pre-existing pattern of classroom activity in which the teacher took on sole responsibility for providing others in the class with mathematical methods and approaches.

Collaboration within each classroom provided not only opportunities for learning but also resulted in the development of new relationships within the classrooms. Students enjoyed
getting to know students outside their immediate peer group, to work with both male and female students and to develop a common purpose with other students with whom they were working. Other naturally occurring divisions in the class such as ethnicity and level of understanding were also broken down as students worked with other students from a range of different backgrounds.

Within each classroom the introduction of mixed-ability collaborative groups also provided an impetus for a shift in perspective amongst students and teachers from the classroom consisting of individuals to the classroom as a whole being a community of mathematicians. This community engaged in debate, arguments, and the development of assessment tasks. Individuals moved between different groups to develop their understanding, shared resources such as textbooks and knowledge obtained from the internet or through private study at home. They provided assessment tasks for each other to use for revision and discussed their ideas within groups and between groups. Where groups took on positive models of collaboration that facilitated communication between all members of the group there were greater learning opportunities for members of these groups and additional social benefits identified by both staff and students.

8.1.2 The expanding dialogic sphere of the classroom

According to Bakhtin (1984; 1986) it is truly dialogical environments that promote the development of new ideas rather than the reproduction of existing patterns of thinking and doing. One of the aims of the current study was to shift classroom environments away from monological learning environments to dialogical learning environments by increasing the number of mathematical voices interacting in the classroom.

The differences between the collaborative and the conventional classrooms in terms of the interaction of multiple voices were easy to observe in the different classrooms. Collaboration in the collaborative classrooms facilitated the emergence of multiple voices that could be identified in classroom discussions and in the way that students made use of different texts and resources unlike conventional classrooms in which classroom discussion involved reflecting different aspects of the teacher’s monologue. Some of
these voices evident in the collaborative classrooms were traditional voices such as the teacher and textbooks, however, other voices were able to be heard as well. Instead of the teacher being the sole authority, the teacher’s approach was one considered alongside many. Most evident were the voices of students forming utterances alongside the voices of teachers or textbooks. Voices arising from other textbooks, newspapers and magazines were also evident in each of the different classrooms.

In the conventional classrooms all external voices including the class text were filtered through the voice of the teacher. The teacher chose which aspects of the textbook were relevant for students, which parts of the school program and syllabus students would be required to concentrate on and how these different voices were to be modified for the purposes of instruction. In the collaborative classrooms, these voices remained a part of the dialogic sphere of the classrooms, however the scope for students to interact directly with these voices was increased considerably.

When asked how students developed their understanding of different outcomes it was evident in the collaborative classrooms that many students were able to develop their understanding from discussing their ideas with other members of their group, using textbooks, teaching themselves or using other texts apart from their textbook. Through each of these methods, students were able to engage in dialogue with many different mathematical partners including the authors of textbooks, mathematicians whose methods, proofs and symbolic systems are recorded in textbooks, other students in the classroom and last, but certainly not least, the voice of the classroom teacher who is also a participant in group discussions.

Furthermore, students were provided with more opportunities to “speak” mathematics rather than just “do” mathematics. Students in conventional classrooms spent most of their time listening to the teacher or practising mathematics. Instead, in the collaborative classrooms, students had opportunities to discuss their ideas, to make public their understanding of different concepts, to ask questions of peers as well as teachers, and to contribute to the development of the collective understanding of the group.
8.1.3 Availability of multiple perspectives

In the interviews conducted at the completion of the collaborative learning topics, students commented on the opportunities to engage with multiple ideas. Comments from students and teachers categorised as “students sharing ideas with each other” (category J) often included comments about the opportunities to learn different methods. Some students suggested that this made learning mathematics more difficult.

Irene: It was harder when you were working with your other friends because they had other ways of working that out. I enjoyed you know learning the new ways and everything. (Mr Grey’s class)

Bakhtin (1984) uses the metaphor of the carnival to describe what can be possible when dialogism emerges within a community. He describes the atmosphere within a carnival as the "temporary suspension of all hierarchic distinctions and barriers among men … and of the prohibitions of usual life." (Bakhtin 1984, p. 15). Within the carnival atmosphere, there can be a certain level of discomfort as the certainties associated with monologic interaction are removed, and many students commented on how they wanted the teacher to tell them how to do each question as they had in previous lessons. Most students, however, recognised that within the collaborative classrooms they were exposed to a wider variety of methods, ideas and approaches to doing different questions.

Lyly: Well we got to show each other, like different varieties of ways we can do the same question whereas if we did it with Miss Diamond giving us the solution, she would show us her way and that wouldn’t be necessarily easier …. (Ms Diamond’s class)

Terrance: We talked a bit more. Instead of just being with the person you’re always with, you change into our groups and you become more, you know, share ideas more, just share things and interests. (Ms Martin’s class)

Ruby: I think when I was working in groups we talked more about maths, because we have different problem students, so we had to discuss it with each other, we had to work out (how to answer each question). (Ms Diamond’s class)
Some students commented on the benefits that they received from working with others and being exposed to many different ideas. They commented on the opportunities to be challenged, to be open to new ways of doing things. Daniel’s comments recorded here provide an example of this openness.

Daniel: The thing I found enjoyable was, when you were learning by yourself it’s a bit hard because you just thinking by yourself, it’s all going in one direction. When you’re with a group you can get different opinions and say, “What do you think, what about if you did this and that,” and then explain. There’s probably some disadvantages as well, like if one in your group didn't understand it or doesn’t speak English, you have to work on that but otherwise I found it a good experience. (Mr Smith’s class)

Daniel: There’s nothing really wrong with working independently, but usually to work independently you just go with your own ideas and you’d have one direction of thinking that you would be able to pass, but you won’t be seeing that, ’cause you’re just stuck on one thing. If you have people to refer to, you find different ways of doing this and make it easier for you … (Mr Smith’s class)

From a Vygotskian perspective, collaboration between students provides the context within which individuals can appropriate the ideas of other members of their group possessing a more advanced conceptual understanding. When the advanced conceptual understanding falls within the individuals’ zone of proximal development they are able to use this conceptual understanding with assistance initially (in the collaborative discussion) and then individually as they come to master the concept. Furthermore, the introduction of multiple ideas into the collaborative dialogue has the potential to spark multiple dialectics between an individuals’ understanding and a more advanced culturally developed understanding as presented by another person in the group. Exposure to different ways of solving the same problems (in the context of collaborative discussion) provides students with multiple opportunities for appropriating new forms of mathematical understanding.
8.1.4 Relationship between understanding and the ability to explain something

One of the significant features of groups that functioned well together according to the teachers at Brindale Christian School was the ability and desire of students who had a good understanding to explain it to other members of the group. At both schools, however, some of the students with a high level of understanding were unwilling to assist those in their group who did not have such a good understanding.

In other groups at Southwest High School in particular, there was a tendency for some groups to split into two with those who already knew the work and those who did not. Comments categorised as students having difficulties with each other (Category C) at Southwest High School referred to their groups splintering into smaller groups of those who knew how to do the work and those who did not. Other groups observed in Ms White’s class developed a similar split, however, it was the students who felt that they already knew the work who did little work while those who felt less confident struggled through the outcomes together. One student from Ms Martin’s class commented on a similar pattern in which he suggested his group “disintegrated” and that the “smart people were only helping themselves”.

In both cases, an animosity developed between those who could have been more actively contributing to the understanding of other students in the group and those who could have benefited from their involvement. In each of the classes at Southwest High School there was at least one group which “disintegrated” in this particular manner. At Brindale Christian School, however, groups that had the potential to disintegrate in this manner due to the unwillingness of the more capable students developed into effective groups where these students could see the benefits for them of working with other students.

Warren: You can’t be selfish. That’s the point. If you’re selfish then it’s the end of the group basically because you’re all off somewhere and they’re all stuck somewhere else, it’s impossible to work like that, and also you have to be able to figure out how to do something and you also have to be able to know exactly how to do what you’re doing because you can’t
guess how to do it. You have to know that because if you have to teach someone else, you have to be able to do that.

On one level, the experience of working in groups for this student who would consistently get close to full marks in mathematics exams was an opportunity for him to learn to become more aware of other students and appreciate their needs. But there was also a developing awareness that teaching others requires the teacher to develop a more complete understanding of the mathematical concepts being taught. Several students commented on the benefits of collaborative learning for those who teach as well as those who learn.

8.1.5 Collaboration as a catalyst for cultural change

Using the theoretical lenses offered by Leont’ev’s activity theory initially, concomitant changes in the classroom with the introduction of the collaborative learning model were also observed at both schools. Fundamental to this change was the shift in focus from the exchange value of performing mathematical tasks (fulfilling the teacher’s requirements) to the use value of performing mathematical tasks (to develop an understanding of different mathematical concepts). Students made choices about what mathematical tasks they engaged in to develop their understanding rather than completing set tasks out of a sense of obligation to the teacher.

Furthermore, developing this understanding was not just an individual pursuit for most students but a collective activity that students worked on together. Particularly at Brindale Christian School where an emphasis on community rather than competition was more evident prior to the introduction of the collaborative learning model, students took responsibility for developing the understanding of other members of their group rather than just their own. While there were students at Brindale Christian School and at Southwest High School who were frustrated when other students in their group did not understand and as a consequence slowed down their own progress, there were also a significant number of students who discussed how their perceptions of their role in the classroom changed as a result of their involvement in the collaborative classrooms.
8.1.5.1 Dialectical collaboration

One of the main outcomes of the collaboration amongst students was the emergence of new roles within the classroom as the dialectical moments of “teacher” and “student” interpenetrated each other to produce a new role of “teacher/student”. This was evident in the students appropriating the role of the teacher within their group, between groups and teaching the teacher within their class. Developing a picture of the emergent practical activity within the classroom revealed that the defining feature of the activity within collaborative classrooms was the teaching/learning dialectic instead of the learning/grade producing dialectic more commonly seen in conventional classrooms.

This dialectic, resulting in the development of a new role, had ramifications for relationships within the classroom between students and the classroom teacher. Teachers’ expectations of students changed, and teachers displayed more trust in the students at the completion of the collaborative lessons. Students also developed as teachers, gradually developing a confidence in their own ability to teach and the ability of other students to teach them. In one classroom, one student became the resident expert on one aspect of the curriculum teaching students and the teacher. Other students reported a change in their attitudes towards other students and a growing awareness of the importance of taking responsibility for the learning of others. Furthermore, several students reported finding the explanations of other students easier to understand, providing further evidence of the dialectical transformation of student participation in the mathematics activity within the classroom.

8.1.5.2 Collaboration that supports development

Student comments and teacher comments both emphasise the benefits of learning in collaborative groups of this kind. Instead of the whole class working through the same material at the same rate, opportunities existed for more individual trajectories of learning. Students who typically struggle in mathematics found themselves surrounded by many teachers willing to assist them develop their understanding.
Personal assessments of the learning of each student varied, however, with some finding that they were able to learn more through discussing their ideas with others, while others were frustrated by the process of having to discuss mathematical ideas. Many of these felt that working in groups did not force people to do enough work, nor did it necessarily provide students with the “correct” answer.

Other changes were also evident in the classrooms with the introduction of the collaborative learning model. These changes relate to the other two research questions which are outlined in sections 8.2 and 8.3.

8.2 Research Question Two: Motivation and self-regulation

Students’ level of motivation and self-regulation were assessed in the different classes using scores obtained from a revised version of the MSLQ, comments from interviews, classroom discussions and classroom observations. Within the collaborative classrooms several changes in students’ beliefs and attitudes were evident in the comments students made in response to various questions posed during interviews and in questionnaires. Changes to students’ perceptions of agency, level of self-regulation, motivation and intentionality were observed during the collaborative lessons. Each of these different areas will be discussed in turn.

8.2.1 Transfer of agency from teacher to student

As indicated previously, a significant change was the shift in perceptions of agency in the classroom. Students were more likely to make decisions related to their own learning in the collaborative classrooms and reported a greater sense of agency in these classrooms. Descriptions of classroom activity focused on the actions of students rather than the actions of teachers and the activities described incorporated students as the principal actors rather than objects to be acted upon. Teachers also reported on the greater freedom that students enjoyed when directing their own learning.
Some students were uncomfortable with the perceived “freedom to direct their learning”, particularly at Southwest High School. Some students preferred to be told the correct method without further discussion by the teacher, and to know exactly what was expected of them in terms of workload. Students at Brindale Christian School, however, with the guidance provided by teachers in the form of flow charts, suggested activities and exercises were less threatened by the perceived freedom they had when working in collaborative groups.

With the transfer of agency from teachers to students, the role of the teacher as the regulator and motivator of students and the teacher’s responsibility for setting student goals were also transferred to students. Students had opportunities to become self-regulated learners, to become self-motivated and to set their own goals.

8.2.2 Developing self-regulated learners

In the collaborative classrooms, students had to work together to determine how they would teach and learn each outcome and how they would develop an assessment task for each topic. Only in the collaborative classrooms were students observed making decisions about their own learning. Instead of the teacher setting homework, each group set their own homework. Instead of the teacher providing a particular method, it was the responsibility of each group to learn and teach certain methods. To successfully manage the teaching/learning environment, students had to develop into self-regulated learners instead of mechanically completing the set work provided by the teacher.

Evidence obtained using the revised MSLQ also pointed to the development of self-regulation amongst students in the collaborative classrooms. Students who were in the conventional classrooms initially and then the collaborative classrooms during the second term of the study indicated on the revised MSLQ an increase in their ability to think critically about their mathematics. This same group also displayed higher levels of help seeking at the conclusion of the collaborative lessons compared with the levels of help seeking evident beforehand.
Students in the collaborative classrooms initially also displayed higher levels of self-regulation while working in the collaborative classrooms. Students also showed higher levels of help seeking at the completion of the project compared with the scores obtained before the collaborative learning project and just after. This suggests that the potential benefits for this particular group were not evident until some time after the collaborative learning model was implemented. This group also showed improvements in their study strategies over the duration of the intervention and also showed additional improvement after their involvement in the collaborative learning model.

Both the quantitative evidence and the qualitative evidence indicated that working collaboratively has a positive effect on students’ level of self-regulation. Students have more opportunities to direct their own learning, make decisions about their own learning and observe other students making similar choices and decisions. Each group represents an arena for discussions about regulating one’s own learning and the learning of other members of the group. A general finding emerging from the qualitative analysis of interview transcripts and classroom observations was that within the collaborative classrooms in which there were opportunities for students to engage in collaborative decision making about their own learning and observe other students engaged in this process, individuals developed self-regulatory skills that they could use on their own or in the context of collaborative groups.

Teachers participated in these discussions during each lesson at Brindale Christian School and Southwest High School. At Southwest High School discussions about how each group would approach teaching/learning certain outcomes was the principal type of discussion between students and the teacher. Many of the discussions between students were about learning rather than actually teaching or learning. As a result of these discussions, through which both teachers and students provided other students with strategies for organising their work, students’ ability to regulate their own learning improved. Scores on the MSLQ sub-scale for critical self-reflection were higher at the completion of the collaborative topics and teachers also reported that students’ ability to organise their work, to be self-motivated and regulate their progress improved.
The collaborative classrooms provided many opportunities for students to develop their ability to regulate their own learning. According to Pintrich (1995) and Thomas, Strage and Curley (1988), self-regulated learning involves the active control of resources and the adoption of cognitive strategies. Within the collaborative classrooms in which decision making about one’s own learning became the responsibility of students rather than the teacher, there existed greater opportunities for students to attempt to regulate their own learning and to observe other students regulating their learning as well. The use of cognitive strategies by students in the collaborative classrooms was also made possible within these less structured lessons in which students had greater freedom to pursue individual approaches and explore different strategies. Suggesting strategies and modelling such strategies remains a responsibility of the classroom teacher whose involvement with each group included discussing how each group might approach learning about particular aspects of the syllabus. Furthermore, students have opportunities to learn such strategies from other members of their collaborative group.

8.2.3 Relationship to other-person regulation

Within the collaborative classroom, the collaborative decision making of groups provided a context within which individuals could develop their own skills for self-regulation. Supporting students’ self-regulation was also a feature of the teachers’ interaction with different groups. At Brindale Christian School, teachers supported students’ planning and organisation through the use of flow charts and a list of suggested sections from the textbook and questions for each outcome. At Southwest High School, teachers provided students with questions associated with each outcome. However, it was in the discussions with other students and the teacher that other-regulation took place on a daily basis. According to Vygotsky (1978), other-regulation becomes internalised as self-regulation within the context of collaboration with others and the development of self-regulation in the form of critical thinking and help seeking in the collaborative classrooms may have its origins in the other-regulation that students in these classes observed. To establish this connection would require further research examining the development of self-regulatory
processes in individuals within different groups that either encouraged or discouraged other-regulation.

8.2.4 Development of critical thinking skills

One of the factors measured by the MSLQ items in the current study was the factor critical thinking/self-critical thinking. For students who took part in the collaborative classrooms in Term Three there was a significant increase in the ability of students to think critically about their own learning over the duration of this term. For students who took part in the collaborative classrooms in Term Two there was no significant difference at the end of the term compared to the beginning, however by the end of term three, scores on the Revised MSLQ indicated that there was a gradual increase in the level of critical/self-critical thinking.

Analyses of the revised MSLQ for mathematics revealed, however, that the development of these skills was a gradual process that continued beyond students’ experience in the collaborative classrooms. In classes that worked collaboratively during the first term of the study and worked in conventional classrooms in the second term of the study, scores for critical thinking/self-critical thinking increased significantly during the second term after students had participated in the collaborative classrooms.

Given the relatively short duration of the current intervention it is possible that evidence for the link between working collaboratively and improvements in critical/self-critical thinking may be stronger in studies of longer duration. Such studies might involve students working in collaborative groups over a longer period and having a longer lead in time as well for them to develop the capacity to work together more effectively. One of the difficulties with the current study identified previously was the small but significant number of groups that disintegrated at Southwest High School and the number of dysfunctional groups (most of which were at Southwest High School) in which opportunities for the sharing of multiple perspectives were limited. The variation in the quality and effectiveness of the collaborative groups may have been directly related to the variation in the changing levels of critical/self-critical thinking. Future research could
investigate the effectiveness of this collaborative learning model over a longer period of time incorporating more of an emphasis on preparing students for their participation in collaborative groups.

8.2.5 Motivating students to learn mathematics

One of the significant outcomes from the collaborative learning classrooms was the greater degree of enjoyment students reported about working in groups. Students made general comments about their enjoyment of mathematics only when referring to their experiences in the collaborative classrooms. Students enjoyed having opportunities to communicate about mathematics with each other and displayed higher levels of interest as a result. They would discuss mathematics outside the mathematics classroom, describe mathematics as fun and interesting, and no longer required instructions from the teacher prior to starting their work each lesson.

One of the aspects of the collaborative learning model designed to make mathematics more interesting was providing students with freedom regarding the content of their assessment tasks. Following from Dewey’s suggestion that classroom activities be related to students’ experiences and interests (1902; 1938), within the collaborative learning model students were able to relate what they were learning to areas of specific interest to them through the process of designing assessment tasks.

However, mathematics learnt in the classroom cannot be constrained by student interests and there remains a need for teachers to provide students with opportunities to engage in mathematical practices in which they have not yet developed an interest. By maintaining a strong link between the existing syllabi and programs and content covered in the collaborative classrooms, students in the collaborative classrooms are still able to participate in a wide variety of mathematical practices. As students came across areas of interest, however, their general sense of mathematics being useful, interesting and worthwhile was gradually transformed resulting in more positive attitudes towards mathematics as a field of study.
Positive attitudes about mathematics are not necessarily related to higher levels of understanding (Pietsch, Walker and Chapman, 2003) however, for many students negative attitudes towards mathematics represent a barrier to them more fully participating in mathematical activities. Moving students from peripheral participants to more central participants is almost impossible if students themselves see little benefit for them personally arising from further engagement in the practice of mathematics. If mathematics classrooms are going to be effective in supporting the dialectical transformation of student roles, teachers need to be promoting positive attitudes to mathematics and a willingness on the part of students to develop identities as “maths people”.

8.2.6 Intentionality of students

When students describe what happened in the classroom the activities they describe are directed towards achieving a certain outcome; that outcome being the collective understanding of people in their group. Whereas in conventional classrooms the motivation is to complete the set work as quickly as possible to fulfil the requirements of the teacher, this situation is radically different in the collaborative classroom. The learning within the collaborative classrooms reflects the intentional learning that Scardamalia and Bereiter (1996a; 1996b; 1996c) aimed to achieve with the CSILE computer program - students seeking out answers to questions of interest to them with the intention of developing their understanding.

Several actions performed in the collaborative classroom are similar to those performed in the conventional classroom. However, the motivation behind such activities is quite different in the collaborative classrooms. For example, students completed exercises in their textbook so that they might improve their understanding rather than merely fulfilling the requirements of the teacher for that lesson. Students would ask each other for help so that they understood how to work with different mathematical concepts instead of asking for help in clarifying the teacher’s method or instructions. Students also made use of the same resources in both classroom models. In the collaborative classrooms, students would use examples in text books to get a better understanding of different concepts or to
teach these concepts to another member of the group. In conventional classrooms, examples from the textbook were copied down so that each student had a complete set of notes. In each case the goal driving the action was the teaching/learning of mathematical ideas and the development of new ways of understanding.

8.3 Research Question Three: Relationship to real world mathematics

The relationship between what happens inside and outside the classroom was explored in the current study through interviews, classroom observations and recording classroom discussions. Several aspects of real word mathematics were apparent in these classrooms which will each be discussed in turn. These are the embedding of classroom practice in cultural practices, learning independently, developing social skills that enable individuals to work with others and working towards the production of a collaborative outcome.

8.3.1 Embedding classroom practice in cultural practices

The principal concern for many sociocultural theorists regarding traditional approaches to teaching and learning is the dissimilarity between activities and practices in the classroom and activities and practices that students participate in outside the classroom (Lave, 1988; Griffin, 1995; Green, Smith and Moore, 1993). Sociocultural approaches to school learning have attempted to implement changes in the classroom so that classroom activities and practices reflect the activities and practices occurring outside the classroom. Collins, Brown and Newman (1989), for example, have developed the notion of “cognitive apprenticeships” whereby classroom learning is designed using the model of apprenticeship learning in which skills and techniques are modeled by an expert. Some have argued for apprenticeship models of learning (Lave, 1996; Rogoff, 1990; 1995) in which learning takes place as individuals engage in guided participation. Other approaches emphasise the need for learning tasks to be authentic (Billett, 1996) by bringing real world problems into the classrooms for students to solve. However, the classroom practice described in this study does not reflect an “authentic” activity which necessarily focuses on a real world problem. While many of the items for assessment tasks that students developed could be described as real world problems, the fundamental
activity of teaching/learning is not necessarily driven by the desire to solve real world problems.

Other theorists have suggested that there exist more significant discontinuities between real world practice and classroom practice than the content of the problem being solved and that is the structure of the learning activity when compared with the structure of the real world mathematical activity (Griffin, 1995). Some of the discontinuities identified by Griffin include the focus on individual learning rather than working together as a group, an emphasis on working without tools rather than working with tools, and a focus on symbolic manipulation rather than learning how to operate on the world.

Within the collaborative classrooms, the structure of the activities observed reflected the structure of activities more typically observed in workplaces. First and foremost, these activities were collaborative activities. Students collaborated together, planning what they were going to do that lesson, identifying what their goals were for each lesson and how they were going to achieve these goals. Students shared their understanding of different mathematical concepts with each other, discussed the relative merits of different ideas and used their collective understanding to develop assessment task items. Second, activity observed in each classroom involved using mathematical ideas for the development of an assessment task. Mathematical concepts were not learnt for the purposes of reproducing methods or algorithms to earn grades or marks, but were used to develop a resource which could be used by other members of the classroom community, or even future classroom communities for facilitating the development of their understanding.

School activity, work activity and the activities of everyday life will always remain distinct from each other to some degree given the purposes behind each of these activities. However, within different forms of classroom practice, the similarities with work activity and everyday activity can assist students to make the transition from one form of activity to another (Beach, 1999). Emergent activities within the collaborative classrooms demonstrated a number of similarities with work and everyday activity in
terms of their collaborative structure. As well as focusing participants on collaborative discourse and practice, teaching/learning activities in the collaborative classrooms were similar to workplace activities in terms of the level of independence students enjoyed, the development of participants’ social skills and in the appropriating of the object of developing a resource using their mathematical understanding.

8.3.2 Becoming independent learners

Students in collaborative classrooms were responsible for many of the organisational tasks that teachers perform in conventional classrooms. They had to decide which outcomes they would look at each lesson, how they would develop their understanding of each outcome, and what they would do for homework. For many of the students this was a positive aspect of the program, giving them opportunities to work out strategies that would suit the members of their group best. At Brindale Christian School, students felt that this was a positive experience, although sometimes they found it more difficult working with other people independently of the teacher’s help.

John: You had to like - it was basically what Karoline just said, you can’t always have the teacher in collaborative learning because at one stage ..... was with different groups, so you’ve got to actually just read from a textbook and/or anything and figure it out for yourself and that later on it will help you in life. (Ms Black’s class)

Daniel: It is good to work independently because you can learn by yourself and you know you don’t need someone else to help you, but if you do need something you will have to ask or look at the textbook. (Mr Smith’s class)

Some students at Southwest High School, however, felt frustrated having to work independently suggesting that there was “… too much freedom and you wouldn’t know what to do” (Ms Martin class). There was a significant number of students who felt frustrated by the teacher not intervening and setting out exactly what they had to do each lesson.

Gary: It’s (group work) kind of hard, because I would like for the teacher to just write down in a book what you have to finish for the lesson on the board or if you don’t finish you have it for homework, and that is the best way because you have to finish it. (Ms Martin’s class)
Eveline: Yeah, well in my group for Pythagoras, everyone just started .... the teacher wasn’t controlling them and the teacher wasn’t making them work, so they just .... all they cared about was finishing the test .... (Ms Diamond’s class)

The division of labour typical of conventional classrooms in which students work under the close supervision of the teacher differentiates such classroom activities from activities occurring in the workplace, in supermarkets or other contexts outside the classroom. In the conventional classrooms, students are not required to make decisions about what they know and do not know or what they need to do to achieve a certain goal. Furthermore, when working through mathematical exercises in the classroom, students don’t even have to think about what strategies to use to answer different exercises, since mathematics textbooks typically set out exercises which require the same strategy to be used over and over again.

Outside the classroom, students are required to make such decisions – they need to determine their current level of preparedness for performing a certain task, decide which options to pursue to achieve certain goals and determine the best strategies for solving certain problems. Students working within the collaborative classrooms were required to perform all of these tasks. However, the classroom activity remains distinct from activities which occur outside the classroom, due to the presence of the teacher who is able to assist student involvement in these tasks. During each of the lessons observed, the classroom teacher would monitor how each group was progressing in their decision making and towards understanding each of the outcomes. Sometimes students would also approach the teacher when they were having difficulties working out what they should do. While students were involved in collaborative, productive activity involving mathematical concepts similar to activities that may be found in workplaces, the most significant difference was the presence of the teacher and their role in supporting students’ engagement in these activities.
8.3.3 Development of social skills and capacity to work with others

A third aspect of the collaborative classrooms was the opportunities that students had to work with other students. Students had to work with students from outside their own social group and work together to achieve a common goal. This presented several challenges to different groups at both schools. One student at Brindale Christian School spoke about the struggles she faced encouraging the other members of her group to contribute to the learning of other people in the group. Students at Southwest High School commented on how some students expected them to do all the work, while other students who felt confident with the work decided not to assist other people in their group. Again, the role of the teacher was to facilitate these group interactions and to ensure that each group was able to work together in a productive manner to achieve their goals each lesson.

8.3.4 Working towards the production of a collaborative outcome

Students also had to work together to produce an outcome which incorporated the collective understanding of the group of which they were members. Students participated in the teaching/learning activity with other students to enable the group as a whole to develop an assessment task which incorporated their collective understanding of the topic.

Teachers at Southwest High School were not convinced that writing up the assessment task was a valuable aspect of the program and felt that the activity of writing up the assessment task the second time did not contribute much to the students’ learning. Students became caught up with matters such as marking guidelines for their assessment tasks and how to format assessment tasks rather than concentrating on the content.
8.4 Research Question Four: Teachers’ concerns to prepare students for exams and developing student understanding of mathematics

Some of the teachers at both schools were initially skeptical about the benefits of collaborative learning with regards to their performance on traditional exams. One teacher commented on her relief once the class started working collaboratively within the model proposed and realising that the students were actually working through the different outcomes.

Ms Black: But I think once we got going, this is all right, this will work. But I think just the fact that it was unknown and the thing that scared me was that they would not work within these - so I think setting up the system was crucial to that, not only for them but for me, so I knew what was happening. Putting myself at ease to say, “Hey, they’re learning that,” and, “I think that they might be working over there.”

Three main sources of evidence were collected pertaining to the level of understanding students were able to develop working in collaborative groups – exam results, student self-reports and teacher reports of student understanding. These sources of evidence are discussed in sections 8.4.1, 8.4.2 and 8.4.3.

8.4.1 Exam results for collaborative and non-collaborative classrooms

Student performance on examinations was the primary source of information about the level of student understanding. For each topic, a class at each school worked collaboratively and the other class worked within a conventional classroom context. These classes were then compared with each other to determine whether or not students performed better or worse in collaborative classrooms when compared with conventional classrooms. For most of the class comparisons using the Mann Whitney test, there were no significant differences between the collaborative and the non-collaborative classrooms. For four of the comparisons, the students in the non-collaborative classrooms scored significantly higher marks on the end of topic assessment tasks, while students in the collaborative classrooms scored higher on one of the end of topic assessment tasks.
Comparisons between different classes using end of term examination results (at Southwest High School) or end of topic topic tests (at Brindale Christian School) did not provide a clear picture of the level of understanding developed in the different classrooms. While it was clear that such examinations did not necessarily test understanding, they remain a concern for teachers who have responsibility for preparing students for external examinations. In contrast to previous research, there was evidence to suggest that for some classes students were better prepared for traditional examinations in the non-collaborative classrooms.

Results obtained from examination results, however, need to be interpreted with caution given that a number of irregularities were evident in the different classrooms regarding the testing procedure. First, there were reports from teachers at both schools that such irregularities in the assessment tasks were evident which may have contributed to this result. At Brindale Christian School, for example, on one end-of-topic exam there was a question worth 10% of the final mark which was covered in the conventional classroom, but was not part of the set work for the collaborative classroom. At Southwest High School, a teacher reported that students in her class who were working collaboratively failed to answer a question on simplifying ratios correctly. Instead of simplifying 6:8 to 3:4, students in her class wrote down 1:4/3 indicating that they understood the concept of simplifying but were confused about which type of simplifying the question required. Finally, at Brindale Christian School, the teacher teaching the topic *Fractions and Decimals* in the conventional format commented that she was absent for many of the lessons at the end of the topic which may have affected the results.

Second, the level of understanding typically assessed by such examinations is at a fairly superficial level, not requiring students to demonstrate a deeper understanding of the mathematical concepts involved. For many of the items tested, students are required to either reproduce particular methods or provide definitions of different mathematical terms.
Student reports and teacher reports suggest that the level of understanding achieved when working in collaborative groups may be considerably deeper than that achieved through conventional instruction. Reports from teachers at Brindale Christian School indicated that they believed this to be the case. A limitation of the current study, however, is that no quantitative or more direct qualitative measures of deeper understanding were used which may have provided evidence to support these reports. In general, individual class tests were used because of their primary importance to mathematics teachers and did not match the intervention which was designed to facilitate understanding through the process of collaborating with others. Assessment tasks that included a collaborative component would have been more suited to assessing the quality of this intervention. Such assessment tasks were not currently in use at either of the schools that participated in the study.

Third, some of the classes worked collaboratively first, then went back to conventional lessons. The teachers of these classes reported that several students in their class preferred teaching others and continued to do so as part of the conventional classroom lesson. From the “experimental” condition to the “control” condition, therefore, there was evidence of contamination from one condition to the other.

Some students described the experience of working in groups as beneficial regarding the development of their understanding of mathematical concepts and ideas. Other students, however, were frustrated working in groups and found it difficult to work collaboratively towards developing a collective understanding. Qualitative evidence from both teachers and students, however, suggested that there were significant benefits for many students, particularly for students at Brindale Christian School, where the collaborative groups formed were more likely to promote the exchange of ideas and the sharing of multiple perspectives. Such groups were more likely to provide students with opportunities to challenge their own ideas, and pull students through their individual zones of proximal development. Future research into this collaborative learning model could look at ways of encouraging more effective collaborative groups. As with the findings relating to self-
regulated learning and critical/self-critical thinking, some of these benefits may in fact be
more long term benefits that would only be evident in a study of longer duration.

8.5 Strengths and weaknesses

The study as a whole had a number of strengths associated with its mixed methods
approach as well as some weaknesses related to the limited generalisability of the current
study. As a mixed methods case study the current study was able to investigate the
effectiveness of the intervention using naturalistic approaches such as observation and
interviews within ordinary classrooms across two schools and year levels. However, as
with all case studies, there were also certain weaknesses that limit the generalisability of
the findings.

8.5.1 Strengths of the current study

The most significant strength of the study is that within this model of collaborative
learning, various theoretical and policy statements regarding mathematics education were
realised within traditional mathematics classrooms. Within the intervention classrooms
students collaborated together, promoting a dialogism within the classroom that led to
multiple ideas being exchanged within different groups and transformed the
teacher/student dialectic so that students were able to take on the role of teacher and the
classroom teacher was able to participate in student discussions. Furthermore, evidence
of self-regulated learning in the collaborative classrooms was plentiful while such
learning was not obvious in the traditional classrooms. For many students, they
experienced higher levels of motivation tackling problems that were meaningful to them
and students had opportunities to engage in cultural practices that are valued within the
field of mathematics. Finally, students had opportunities to develop through their
individual zones of proximal development within their respective collaborative groups.

The current study is also exploratory in nature providing numerous avenues for future
research. The exploratory nature of the study is evident in the mixed methods used to
develop a multi-faceted picture of classroom activity. Arising from this exploratory study
are many different avenues for future research that will be outlined in section 8.6: looking
at different ways to assess mathematical understanding, ways of increasing the amount of
time students spend engaging in mathematical discussion in collaborative classrooms,
and further applications of activity theory in mathematics education.

As part of the mixed methods approach multiple assessments of students’ learning were
incorporated in this study including measures of students’ levels of motivation and self-
regulated learning. Student reports were also used in conjunction with results obtained
using the Revised MSLQ to determine levels of motivation and self-regulation while
working collaboratively compared with levels of motivation in traditional classes. Given
that students’ motivation levels and ability to self-regulate their own learning are related
to their performance in mathematics, these findings provide additional insights into the
benefits of collaborative learning as experienced within the model under investigation.

A further strength of the current study was the fully collaborative nature of classroom
activity under investigation. Unlike previous studies into collaborative learning in
mathematics (Webb and Farivar, 1985), students were engaged in learning new material
as well as practising it in groups. Explanations, examples and strategies for answering
different types of questions were developed and presented by students rather than the
teacher whose role was to assist students in planning how they would learn different
concepts rather than providing them with a pre-packaged method for solving certain types
of questions.

The current study is also unique in that it introduces an intervention in line with the
principles outlined in the literature and policy documents to a set of traditional
mathematics classrooms. Rather than examining pre-existing classroom communities in
which collaborative learning was well established as the principal classroom activity,
classroom teachers with no experience teaching mathematics in a non-traditional manner
were given a model of teaching consistent with the reform movement in mathematics
education that enabled teachers to cover the curriculum in the same amount of time as
teachers teaching in the traditional manner. Studies looking at schools where problem-
based collaborative learning is the norm such as those conducted by Boaler (1997a;
have demonstrated that over time such approaches have many benefits for students including higher examination results and higher levels of motivation for engaging in mathematical activity. In the current study, the success of the transition from a traditional format to a collaborative learning format was the principal focus, revealing that while students may not perform as well on conventional examinations, many students report higher levels of understanding, higher levels of motivation and within each classroom higher levels of self-regulation were also evident.

Finally, the current study contributes to the growing interest in how activity theory can be applied to educational practice. The current study represents a unique approach to applying activity theory within the classroom identifying separate activity systems that interact with each other within the classroom and comparing these different systems of activity in traditional and collaborative classrooms. In describing classroom activity, the current study uses shared perspectives of students to develop a picture of the different activity systems present in each classroom that begins by identifying the shared, collective objects of different activities and then attempts to make sense of individual actions within the context of these different activities.

8.5.2 Weaknesses of the current study

While the current study has several strengths, a number of weaknesses should also be outlined. These weaknesses are associated with the limited methods used to assess understanding, and the low generalisability common to many case studies.

First and foremost, the current study was dependent primarily upon existing pen and paper assessment tasks as the principal measure of student understanding. A significant reason for using these tasks was to determine whether students’ participation in the collaborative learning model would adequately prepare them for traditional forms of assessment. However, more appropriate measures could have been used to assess students’ understanding such as collaborative tasks, tasks that required extended responses, and novel problem solving tasks such as those used by Boaler (1997a; 1997b). Such tasks would have assessed learning using tasks that were more closely aligned with
the classroom activities within which such learning took place and would also have
provided a more accurate picture of students’ ability to work flexibly with mathematical
corcepts rather than their learning of definitions and specific techniques for specific types
of questions. As a consequence of time constraints on the research project (access to each
classroom was limited to a small number of lessons each week) and the desire not to add
any additional burden to classroom teachers participating in the study these alternative
forms of assessment were not used. Future research conducted over an extended period of
time at different schools could investigate using such forms of assessment within this
collaborative learning model.

Second, there exists a sampling issue with the current study limiting the generalisability
of the results. As outlined in section 4.5.2, a total of twelve schools were approached to
take part in this study, only two of them being willing to participate in the study. Thus,
there remains the possibility that the two schools who agreed to take part in the current
study were already comfortable with the notion of collaborative learning in mathematics
and were therefore more adept at implementing the model than teachers at other schools.
However, when speaking to each of the teachers at the different schools involved in the
project there appeared to be a general level of skepticism about collaborative learning
amongst the teachers rather than an enthusiasm to incorporate the principles of this model
in their classroom. None of the teachers involved in the study had any experience
providing collaborative activities in mathematics classes, although some of the teachers at
Brindale Christian School had used similar approaches in other subject areas. In fact, the
general level of inexperience in teaching using the collaborative model was one of the
possible reasons for teachers struggling to know how best to present information to
students where necessary. Teachers at both schools commented on the uncertainty they
experienced about whether or not they should provide the class with whole-class
instruction at different points throughout the two collaborative topics. Decisions about
how to ensure that the classroom community has the necessary resources for each group
to develop a viable, shared understanding of different concepts could be the focus of
future research in which teachers receive additional training regarding these issues.

Again, because of the limited time available in the current study as a consequence of the
busyness of school activities occurring at each school and the high level of commitments teachers at both schools took on outside classroom time, the training of teachers at both schools was limited to two or three sessions of approximately one hour in which teachers were familiarised with the main features of the collaborative learning model.

A third weakness of the current study was the possible “researcher effect” evident in classes that were observed throughout the research project. Teachers were always given forewarning when their classes were going to be observed and this knowledge could have influenced their decisions about how they would approach those particular lessons. Furthermore, while students were being observed, it is quite likely that they acted in certain ways knowing that they were being observed and, in the case of the collaborative classrooms, being recorded as well. This sensitivity to my presence in the classroom may have had several effects on student behaviour. First, students may have been less inclined to discuss ideas freely and more inclined to work individually given that this is typically the form of behaviour required of them by classroom teachers. Second, students would sometimes make specific comments regarding the recording of their conversations such as “…this is off the record…” or “…just for the tape…”. Third, for some students the novelty of an additional person in the classroom proved to be a considerable distraction as they sought to ask numerous questions regarding the project as well as questions of a personal nature.

As was alluded to earlier, a fourth weakness of the current study was the limited amount of time available at both schools for training teachers and preparing students for participation in the collaborative learning model. As a consequence of the fact that students in each class received minimal instruction in how to collaborate in groups, some of the groups did not provide students in these groups with opportunities to collaborate effectively with each other. While the model was successful in terms of promoting collaboration at both schools and realising principles of the reform documents outlined previously, there existed at both schools, though more so at Southwest High School, dysfunctional groups that did not provide their members with the same benefits enjoyed by other students who participated in the study.
Finally, as with all case studies, the findings of this study have limited generalisability since the two schools and seven classes that participated in this study do not provide a representative sample from which conclusions about all mathematics classrooms might be made.

8.6 Directions for future research

The current study represents an exploratory study investigating how a certain approach to collaborative learning might be implemented in traditional mathematics classrooms. As such, it suggests numerous avenues for future research arising from the findings of this current study. A more prolonged investigation of this form of collaborative learning would have the potential to incorporate additional forms of assessment to supplement the traditional forms of assessment overcoming one of the weaknesses of the current study. Furthermore, additional training could be given to teachers who plan to use this approach focusing on the specific content material to be covered and providing assistance determining how and when the classroom teacher might choose to present certain ideas to the whole class, how different groups might develop their understanding through discussions with other groups, and how and when to participate in group discussions.

Furthermore, while students reported that they engaged in discussions about mathematics for the bulk of the lesson time, in most classes observed there remained a tendency for some groups to concentrate on their own individual work during the lesson and for others to use the lesson time to engage in non-mathematical discussions while completing their individual work. Future research could look at additional strategies for motivating students to use the time during each lesson to discuss their ideas with others rather than using this time for individual practice.

Future research could also consider the relationship between different forms of collaboration and levels of mathematical understanding within such classrooms. Within the current study there were many different patterns of collaboration evident at both schools, some providing more opportunities for engaging with different ideas than others.
An analysis identifying the different preferred forms of collaboration adopted by groups and their associated development of mathematical understanding would provide additional insights into the relationship between collaboration and mathematics learning.

Future analyses using the theoretical perspective provided by Vygotsky, Leont’ev and Engeström would provide a different perspective to that associated with cognitive psychology. Rather than an individualistic picture of classroom interaction, activity theory focuses on collective, object-oriented activity that is motivated towards transforming these objects to meet certain needs. Within the mathematics classroom the current analysis has identified three separate, yet overlapping, activity systems in both traditional and collaborative classrooms. The interaction between these different activities determines the success or otherwise of learning activities and future research could explore how this interaction can be more effectively managed in different types of mathematics classrooms.

8.7 Implications for policy and practice

One of the key aspects of the current study was the implementation of policy statements as contained in policy documents such as Numeracy, A Priority for All (DETYA, 2000), National Statement on Mathematics for Australian Schools (AEC, 1991), Principles and Standards for School Mathematics (NCTM, 2000), Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and Professional Standards for Teaching Mathematics (NCTM, 1991). As a consequence of this research it is clear that it is possible to put these principles into practice within classrooms in which traditional methods of drill and practice have been the principal features of classroom activity.

Central to the model developed as part of this research is the role that collaboration plays in transforming classroom activity. Policy documents relating to mathematics education could place greater emphasis on collaboration as the central aspect of calls for reform. The emphasis on problem solving in some documents, while admirable, has perhaps obscured the significance of students talking with each other about mathematics. Furthermore, this emphasis has not always been easily transferred to most classrooms in
which programs and syllabi run to tight schedules which do not allow students time for extended investigations.

The difficulty associated with using traditional assessment procedures also highlights the need for alternative forms of assessment to be introduced into the classroom. While current policies endorse the use of alternative forms of assessment, they have proved difficult to implement within the context of most secondary mathematics classrooms. In conjunction with research efforts making use of alternative forms of assessment within the context of the collaborative learning model, policy documents could provide guidelines for the cohesive introduction of such forms of assessment alongside guidelines for the introduction of new ways of learning mathematics such as the approach outlined in this study that is grounded in collaboration.

Clear implications for practice emerge from this research. Although most schools face the daunting task of teaching a wide array of mathematical concepts that for many precludes the possibility of exploring alternative approaches to teaching mathematics, the current study provides an approach that can be incorporated within classrooms by teachers with little or no experience using collaborative learning approaches. It does not depend upon revisions to the syllabus or additional resources yet can still result in changes to the culture of the mathematics classroom thus achieving many of the desired outcomes of reform documents. Whether a teacher has had some experience or no experience teaching using collaborative learning approaches, it is possible for them to consider teaching in this manner without taking up additional lesson time or cutting significant portions out of the syllabus.

**8.8 Summary conclusions**

The two schools involved in the current study reported different experiences with the collaborative model of learning although across both schools there was evidence of higher levels of self-regulation, enjoyment, collaborative discussion, seeking help from others and the development of social skills within the collaborative classrooms. Evidence pertaining to the development of students’ mathematical understanding was mixed with
some collaborative classes displaying lower test scores although reports from students and teachers indicated that there was a perception amongst classroom participants that significant learning had occurred in the collaborative classrooms of a different character to that occurring in the traditional mathematics classroom.

Differences between the two schools, however, were evident in their reaction to the collaborative learning model which could be due to either the differences in how the model was implemented, or the dramatically different school cultures. The number of students who preferred learning in the collaborative classrooms was much higher at Brindale Christian School compared with Southwest High School and these same students displayed more enthusiasm for working in groups. Teachers at both schools, however, expressed a desire to use this model in the future and felt that students benefited from participating in the program learning about how to work with other people and developing positive social relationships within the classroom.

The level of engagement of teachers and the provision of supporting tools and materials were both crucial to the success of the collaborative model of teaching/learning. Close monitoring by the teacher of each group was also necessary to ensure that different groups were focused on achieving learning goals rather than other goals and it was evident at both schools that teachers were concerned about how they would go about monitoring the progress of different groups.

Despite the differences observed between the two schools in terms of the implementation of the model and its impact, teachers in each of the seven classes observed were able to facilitate the emergence of student-driven, collective, mathematics-oriented activity in their classrooms. Prior to the introduction of this collaborative approach, what happened in mathematics classrooms at each of these schools could be more accurately described as teacher-driven actions towards achieving goals that served the wider collective activity of the schools within which they worked. Collaboration amongst students acted as a catalyst for change in these classrooms, empowering students to take responsibility for their own learning and, by doing so, transforming the activity structure of these classrooms. From
classrooms in which the students were objectified and acted upon by the teachers to classrooms in which students took it upon themselves to learn and teach each other, the collaborative classrooms encouraged the emergence of a fundamental contradiction between teaching and learning replacing the contradiction between learning for grades or learning for understanding.

In doing so, many of the ideals of the reform movement in mathematics were realised within these collaborative classrooms – students displayed higher levels of self-regulation, shared ideas with each other, engaged in dialogue about mathematical problems and displayed higher levels of motivation. Furthermore, teachers’ concerns about covering the set work in a short period of time and preparing students for examinations were also met even though students in some of the collaborative classrooms did not perform as well on classroom tests. Thus, teachers were able to reach their goals and thereby contribute to achieving the outcomes desired by each school while facilitating students’ involvement in mathematics classrooms that displayed many of the principles outlined by the reform movement.

The current study represents a substantive theoretical contribution to the growing interest in applying activity theory to educational contexts. However, it goes beyond the theoretical, developing a practical way of transforming mathematics classrooms from teacher-directed classrooms focused on drill and practice to student-centred classrooms focused on mathematical discussion. By doing so, this study presents new possibilities for theorists and practitioners eager to more fully realise the reform agenda in mathematics classrooms.
References


QSR International (2002). *NVivo 2.0.* Qualitative Solutions and Research Pty Ltd.


SPSS, Incorporation (2003). SPSS 12.0. SPSS, Inc.


**Appendix One: Sample marking scale**

- Depth of coverage – 25%
- Accuracy of worked solutions – 25%
- Quality of application questions relevant to situations outside the classroom – 10%
- Group work – 10% (partially determined by individual performances on the test as well as teacher observations of how well the group works together)
- Forming links between current topic with other areas of mathematics – 10%
- Originality of questions – 5%
- Layout – 5%

Each student also completes another assessment task developed by another collaborative group and provides their evaluation of the quality of the task using the same criteria outlined above. This evaluation contributes the remaining 10% of the mark for each group’s assessment task.
Appendix Two: Consent forms and information statements for students and teachers participating in study

Consent form for Students Participating in Research into Collaborative Learning in Mathematics

I have read (or have had explained) and have understood the Subject Information Statement and Consent Form and understand the purpose and risks of the study. I understand that during classroom observations some audio recording may take place.

YES I wish to participate in this study

______________________________
(Name of Student)

Signature: ____________________

YES I grant permission for my child _________________________ to participate in this study.

______________________________
(Child’s name)

Page 354
Signature: ________________________________

Name of parent or guardian: ________________________________

If you would like to receive a written report of the results of this study, please provide details of your mailing address below

_______________________________
_______________________________
_______________________________
_______________________________

Page 355
Dear Parent/Caregiver,

Beginning in Term Two 2002, students and staff at Brindale Christian School will be working with the Faculty of Education at the University of Sydney to undertake a study into collaborative learning in mathematics. In essence, the study will evaluate a model that has emerged from recent research on effective teaching and learning in Mathematics education. The research is being conducted by Mr James Pietsch (PhD student) under the supervision of Dr Elaine Chapman.

All students in year 8 will be invited to participate in the study. Initially, participating students will complete a questionnaire (which will take approximately 30 minutes). The questionnaire will assess students’ attitudes to mathematics, the strategies students use in learning mathematics, and their perceptions of their classroom environment. The results will not be recorded or interpreted on an individual basis. All students will receive an identification number to facilitate the analysis. The teachers at the school will not be provided with individual results from these questionnaires – they will, however, receive a summary of the outcomes for their own classes. This information will provide very useful feedback to the teacher on ways to improve student outcomes in mathematics. Students who wish to receive individual results will be able to do so.
Following this, teachers of four classes in year 8 will use a model of collaborative learning that encourages students to teach each other and work together to develop their understanding of mathematics. In the intervention, the students will work together to develop assessment tasks and will be assessed on the quality of the tasks designed. Students in these classes will cover exactly the same material in class as other students in their year at the same time. Their normal classroom teacher will teach them for the full duration of the project. Researchers from the University of Sydney, however, will observe some of these lessons over the duration of the project.

A small number of students from each class will also be invited to participate in interviews with the researchers. Responses to questionnaires, interview questions and end-of-term exam results will be used to evaluate the effectiveness of the model for supporting mathematics learning. Interviews will be recorded on audiotape to assist with transcribing student responses. However, as indicated, the results will not be interpreted on an individual basis, and will not be recorded or reported against students’ names or any other identifying criteria. The results will be analysed and used to complete a PhD thesis. All records will be stored at the University of Sydney using identification numbers only.

As this study is being conducted in collaboration with the University of Sydney, only students who give consent and receive the consent of their parents will be able to participate in the study (however, all students are free to withdraw from the study at any time after giving consent).

Thus, if you would like your child to participate in the collaborative mathematics learning study, please sign the form below and return it to your child’s classroom teacher. If you have any questions please do not hesitate to contact either James Pietsch (9796 3391) or Dr Elaine Chapman (9351 6238). Any person with concerns or complaints about the conduct of this research study can contact the Manager of Ethics and Biosafety Administration, University of Sydney, on (02) 9351 4811.
Should you wish to receive a copy of the results, please leave your address on the consent form enabling us to send you a summary of the results obtained in this study.
Consent form for Teachers Participating in Research into Collaborative Learning in Mathematics

I have read (or have had explained) and have understood the Subject Information Statement and Consent Form and understand the purpose and risks of the study. I understand that during classroom observations some audio recording may take place.

YES I wish to participate in this study

__________________________________________

(Name of Teacher)

Signature: __________________________
Research into Collaborative Learning in Mathematics

Mathematics Teachers of Brindale Christian School,

Beginning in Term Two 2002, students and staff at Brindale Christian School will be working with the Faculty of Education at the University of Sydney to undertake a study into collaborative learning in mathematics. In essence, the study will evaluate a model that has emerged from recent research on effective teaching and learning in Mathematics education. The research is being conducted by Mr James Pietsch (PhD student) under the supervision of Dr Elaine Chapman.

All students in year 8 will be invited to participate in the study. Initially, participating students will complete a questionnaire (which will take approximately 30 minutes). The questionnaire will assess students’ attitudes to mathematics, the strategies students use in learning mathematics, and their perceptions of their classroom environment. All students will receive an identification number to facilitate the analysis. You will not be provided with individual results from these questionnaires – you will, however, receive a summary of the outcomes for their own classes. This information will provide very useful feedback to you as the teacher on ways to improve student outcomes in mathematics.

Following this, teachers of three classes in year 8 will use a model of collaborative learning that encourages students to teach each other and work together to develop their understanding of mathematics. In the intervention, the students will work together to
develop assessment tasks and will be assessed on the quality of the tasks designed.

Students in these classes will cover exactly the same material in class as other students in their year at the same time. As the classroom teacher, you will be responsible for teaching your class for the full duration of the project implementing the model for one term. Researchers from the University of Sydney, however, will support your teaching of the model and observe some of these lessons over the duration of the project.

A small number of students from each class will also be invited to participate in interviews with the researchers. Responses to questionnaires, interview questions and end-of-term exam results will be used to evaluate the effectiveness of the model for supporting mathematics learning. Interviews will be recorded on audiotape to assist with transcribing student responses. However, as indicated, the results will not be interpreted on an individual basis, and will not be recorded or reported against students’ names or any other identifying criteria. The results will be analysed and used to complete a PhD thesis. All records will be stored at the University of Sydney using identification numbers only.

As this study is being conducted in collaboration with the University of Sydney, only students who give consent and receive the consent of their parents will be able to participate in the study (however, all students are free to withdraw from the study at any time after giving consent). Only teachers who give their consent will participate in this study and you are also free to withdraw from the study at any time after giving consent.

As a teacher participating in this research project, we would appreciate you signing the accompanying consent form to indicate your willingness to participate. If you have any questions please do not hesitate to contact either James Pietsch (9351 6388), Dr Elaine Chapman (9351 6238). Any person with concerns or complaints about the conduct of this research study can contact the Manager of Ethics and Biosafety Administration, University of Sydney, on (02) 9351 4811.
Appendix Three: Outcome sheets used at each school

Part A – Outcome sheets used at Brindale Christian School

Contents

Properties of Solids Outcomes (Year Seven)
Outcomes for Data Representation and Interpretation (Year Seven)
Outcomes for Area, Surface Area and Volume (Year Eight)
Outcomes for Algebraic Techniques (Year Eight)
Fractions and Decimals Outcomes (Year Seven)
Outcomes for Ratios and Rates (Year Eight)
Outcomes for Data Representation, Data analysis and Evaluation (Year Eight)
Properties of Solids Outcomes

Students learn about

A. Describing solids in terms of their geometric properties

- number of faces
- shape of faces (including for pentagon, hexagon and octagon)
- number of flat and/or curved surfaces
- number of equal faces
- uniformity of cross-section

1) Students learn to recognise solids with uniform and non-uniform cross-sections
2) Students learn to analyse three-dimensional structures in the environment to explain why they may be particular shapes eg buildings, packaging

i) For the following eight solids fill in the following table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
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<td><img src="https://example.com/image2.png" alt="Image" /></td>
<td><img src="https://example.com/image3.png" alt="Image" /></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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<td><img src="https://example.com/image6.png" alt="Image" /></td>
<td><img src="https://example.com/image7.png" alt="Image" /></td>
<td><img src="https://example.com/image8.png" alt="Image" /></td>
</tr>
<tr>
<td>Name of shape</td>
<td>Number of faces</td>
<td>Shape of faces</td>
<td>Number of flat faces</td>
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<td>---------------</td>
<td>----------------</td>
<td>---------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>Solid A</td>
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<td></td>
<td></td>
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<tr>
<td>Solid B</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Solid C</td>
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<td></td>
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<tr>
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<td></td>
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<tr>
<td>Solid G</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid H</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Suggested resources and learning activities**

7.2.1 Labelling plane figures and lines p.317

7.2 Plane figures - polygons p.312 - 317

B.6 Polygons Ex B.G. p.41

Visit to building site on (date)

Addison and Wesley 7 p.260 - 261

Maths World 7 p.27 - 43, 44 - 45

Signpost 7

**You might like to try**

- Writing a description for each solid which defines the shape eg. A sphere is a round solid with no faces, or flat surfaces. It has a curved surface and a uniform cross-section - see if you can improve this definition/description and write one for each solid you have explored. (Verbal/Linguistic)
• Using a stamp pad and solids kit, identify cross sections for each solid. You may explore cross-sections further with everyday objects that have been cut through at one point and an impression recorded on paper (Visual/Spatial)

• On our visit to the building site see if you can identify each of these solids in the construction. Sketch and label the solid in the particular site in the building. In a group compare your notes and discuss the possible reasons for using each shape in the construction (Interpersonal and Visual/Spatial)

B. Determining if two edges are parallel, skew or intersecting
i) Draw a rectangular prism. Find two parallel edges and colour them blue. Find two skew edges and colour them red. Find two intersecting edges and colour them green.

Suggested resources and learning activities
7.3 Parallel lines

Draw a picture of the inside of the classroom identifying parallel, skew and intersecting lines
Addison and Wesley 7 p.108 - 110, 152 - 153
Mathsworld 7 p.291 - 307

You might like to try
• On your site visit choose three sections of the building. Sketch these section and identify parallel, skew or intersecting lines. (Visual/Spatial)

• Ask some questions of the architect about the effect of the use of these types of lines on the strength and rigidity of structures. List the reasons for their use. (Logical/Mathematical)

• Based on your understanding of strong constructions, create a group design for a climbing tower for young children that will be safe. Explain how you have designed for safety in your construction. (Interpersonal/Intrapersonal)
C. Classifying solids on the basis of their properties (including cross-sections),

eg prisms (triangular, rectangular, etc)

pyramids (triangular, square, etc)

cones, cylinders, spheres

right or oblique prisms and pyramids

polyhedra

1) Students learn to recognise solids with uniform and non-uniform cross-sections
2) Students learn to analyse three-dimensional structures in the environment to explain why they may be particular shapes eg buildings, packaging

For the following solids determine whether they are prisms (right or oblique), pyramids, cones, cylinders or spheres. Which of these shapes are also polyhedra?

A

B

C

D

E

F

Suggested resources and learning activities

7.1.2 Prisms
Addison and Wesley 7 p.96 - 97

7 Plus Maths p.92

You might like to try
• As a group select two shapes. Construct a table with column headings in order to make a comparison between two shapes. Write up your results as a discussion on the essential features of each shape. Draw up a table which identifies the shape according the their features. (Verbal/Linguistic and Visual/Spatial)

• Having explored solid shapes, write a test to help a student decide what sort of solid shape an object is. You might like to use a flow chart for your test. (Verbal/Linguistic. Visual/Spatial, Logical/Mathematical)

D. Sketching on isometric grid paper shapes built with cubes

1) Students learn to interpret and make models from isometric drawings

Using isometric grid paper draw sketches of the following shapes

Suggested resources and learning activities

7.1.1 7.2

You might like to try

• Using MAB blocks, create shapes for your friends to draw on isometric grid paper. (Visual/Spatial)

• Try writing a description of a shape you have created and see if your friends can draw the shape from your description. (Visual/Spatial and Intepersonal)
E. Making sketches of simple solids and their cross-sections
i) Draw a sketch of a rectangular prism shading in its cross-section
ii) Draw a sketch of a cone and shade its cross-section
iii) Draw a sketch of a square pyramid and shade its cross-section

Suggested resources and learning activities
Exp 1. Ex 7.2 p.300
7.2
Addison and Wesley 6 p.250 - 251
Addison and Wesley 7 p.96- 97, p.122
HBJ 7 p.204 - 207

You might like to try
- Using an overhead projector, MAB blocks and a thin straw, cut 3D shapes at certain points and draw what you imagine them to be. Test your diagram by using the overhead projector to demonstrate what the cross-section should look like.
  (Visual/Spatial and Interpersonal)

F. Representing three-dimensional objects in two dimensions from different views using the appropriate conventions
  1) Students learn to visualise and name a common solid given its net
  2) Students learn to recognise whether a diagram is a net of a solid
i) Given the following views of a solid draw the three dimensional solid
ii) Draw a top, front and side view of this solid

![Image of a solid]

iii) The following solids are referred to as Platonic solids. For each solid, choose from the possibilities which net would make each solid

![Images of Platonic solids with nets]

Suggested resources and learning activities
7.1.3  7B  7C  7D
Mathsworld 7 p.48 - 57

You might like to try
• Collect unusual 3D objects from home and see if you can represent different views of these objects. Test your friends to see if they can identify the real object from your sketch. (You may also use the 3D shapes kit for this activity).

• Discuss as a group the difference between all 3D objects and the Platonic solids. Prove the difference by writing a discussion that demonstrates the difference using the nets of each solid explored. (Visual/Spatial, Verbal/Linguistic and Interpersonal).

• See also outcome H

G. Determining Euler’s relationship for polyhedra, linking number of faces, vertices and edges

\[ F + V = E + 2 \]

i) For each of the following polyhedra determine the number of faces (F), edges (E) and vertices (V)

\begin{align*}
F &= \\
E &= \\
V &= \\
F + V - E &=
\end{align*}

\begin{align*}
F &= \\
E &= \\
V &= \\
F + V - E &=
\end{align*}

\begin{align*}
F &= \\
E &= \\
V &= \\
F + V - E &=
\end{align*}

\begin{align*}
F &= \\
E &= \\
V &= \\
F + V - E &=
\end{align*}

Suggested resources and learning activities

7.1.4 Corners, Edges and Surfaces p.310 - 312

Exp1 Ex 7.4

Addison and Wesley 6

Mathsworld 7 p.45 - 48

You might like to try
• Find out about Euler's life and how he developed mathematical ideas and understandings. See if you can find out the specific details associated with polyhedra.

• Write an explanation for one polyhedron according to Euler's relationship

H. Exploring the history of Platonic solids and how to make them
i) What was so special about the five Platonic solids for Plato and his followers?
ii) How were these solids used in Greek mathematics?

Suggested resources and learning activities
World Book Encyclopaedia

You might like to try
• Write a biographical account of the life of Plato and identify the significance of these five polyhedra or Platonic solids to those who were his followers (Verbal/Linguistic, Spiritual)
• Design the packaging for a particular item using one of the Platonic solids. Write an explanation for your choice of solid for this job. (Visual/Spatial, Verbal/Linguistic, Intrapersonal)

I. Making models of polyhedra

Suggested resources and learning activities
7.1.3 Introduction to nets and prisms
7C Models of polyhedra p.308 - 309

You might like to try
• Use circles from lines in your text and see if you can create models of polyhedra from the shapes you create. (Visual/Spatial)
• Name the polyhedra you have created according to the accepted labelling system for such shapes. (Visual/Spatial, Verbal/Linguistic)
• Create an interesting display of models of polyhedra for your friends (Interpersonal, Visual/Spatial)
Outcomes for Data Representation and Interpretation

Students learn about

A. using the term ‘mean’ for average and finding the mean for a small set of data

1) Students learn to explain information presented in the media that uses the term ‘average’ eg the average temperature for the month of December was 24 degrees (Communicating)

i) Use your own words to re-write the statement "Don Bradman's average score was 99.94"

ii) Greta is a goal shooter for her netball team. For the first five games of the season she managed to shoot 6, 5, 9, 7, and 8 goals. What was the mean number of goals Greta scored each game?

iii) "The average number of homes sold each month in Sydney in the year 2001 was 3050." If this was true, how many homes do you think were sold over the whole year?

Suggested resources and learning activities

13.1 Collecting Information
13.6 Information in Numbers
13.1.1, 13.1.2, 13.1.3
Addison and Wesley 6 p.134 - 135, p.146 - 147, p.150 - 151

B. Picture Graphs

a) naming a picture graph and labelling the axis
b) determining a suitable scale for data and recording the scale in a key
c) drawing picture graphs where one picture or symbol represents more than one item
d) interpreting a given picture graph using the key

i) Charlotte observes that there are five different breeds of dogs on her street - German shepherds, corgis, silky terriers, golden retrievers and labradors. She counts the number of each dog and finds that there are 20 German shepherds, 10 corgis, 5 silky terriers, 5 golden retrievers and 30 labradors. Draw a picture graph to represent this information giving your graph a suitable name and label for the horizontal axis. Choose a suitable picture or symbol that represents five dogs.

ii) The pictograph below shows the number of different types of trees Sarah plants on her land. Each tree represents 4 trees.

- Gum
- Eucalyptus
- Birch
- Oak
- Wattle

a) If there were 2 wattle trees, how would this be represented on the above graph?

b) Given that Birch and Oak trees are imported, are there more imported trees or native trees on Sarah's land?

c) How many wattle trees are there?

d) Which tree is the least common on Sarah's land?

iii) What are the advantages of picture graphs and what are the disadvantages of picture graphs?

Suggested resources and learning activities
13.2 Picture graphs
13.2.1, 13A, 13.2.2, 13B

Page 374
C. Column Graphs

a) naming a column graph and labelling the vertical and horizontal axes
b) determining a suitable scale for data and recording the scale on the vertical axis
c) drawing column graphs using a scale to determine the height of each column
d) interpreting a given column graph using the scale on the vertical axis

i) Use the same data from part i) for picture graphs to draw a column graph to represents that information. Label the vertical and horizontal axes, and use a suitable scale to determine the height of the column (1 cm = 5 dogs). Which graph do you think best represents this data? Why?

ii) Use this column graph to answer the following questions

![Bar Graph]

Cars bought in West Wyalong

0 10 20 30 40 50 60

Toyota Mitsubishi Holden Ford Nissan

a) How many cars were sold in West Wyalong?
b) If you saw a car on the street in West Wyalong what do you think it is most likely to be?

Suggested resources and learning activities

13.3 Information in Bars and Columns
13.3.1
Exploratory Exercise 13.5
13C
13.3.2
Maths Net 7 - p.29, p.37

D. Line Graphs

a) naming a line graph and labelling the vertical and horizontal axes

b) drawing a line graph to represent any data which demonstrates a continuous change eg hourly temperature

c) determining a suitable scale for the data and recording the scale on the vertical axis

d) using the scale to determine the placement of each point when drawing a line graph

e) interpreting a given line graph using the scale on the axes

i) Use the following graph to answer the following questions
a) What was the profit of this company in August?
b) What is the scale on the vertical axis?
c) What is the title of this graph?
d) Do you think this company will continue to grow in 2002? Why? Why not?

ii) Consider the following graph depicting the profit a company made in 2001

Profit for the year 2001 of Dodgy Brothers Used Cars

a) Joe Dodgy claims that sales are have almost doubled over the year. Do you agree or disagree?

b) What is misleading about this graph?

Suggested resources and learning activities
13.4 Information in Tables and Line Graphs
13.4.1, 13F, 13.4.2, 13G
Addison and Wesley 6 p.136 - 137, p.148 - 149
E. Divided Bar Graphs / Sector (Pie) Graphs

a) naming a divided bar graph or sector (pie) graph
b) naming the category represented by each section
c) interpreting divided bar graphs
d) interpreting sector (pie) graphs

i) At a playgroup there are 100 children who come each week. 30 of these children are 4 years old, 25 of them are 3 years old, 35 are 2 years old and 10 are less than two years old. Draw a divided bar graph to represent this data. Use a bar 10 cm long.
   a) How many children are there altogether?
   b) What will be the scale of this divided bar graph?

ii) Use this pie graph to answer the following questions

Languages spoken at home

- English: 34%
- Arabic: 20%
- Vietnamese: 15%
- Macedonian: 13%
- Spanish: 8%
- Tagalog: 10%

a) Apart from English, which language is spoken most at home at this school?
b) If there were 200 students at this school, how many speak Vietnamese at home?
c) What fraction of the school population speaks English?
d) What fraction of the school population speaks English, Tagalog or Spanish?

Suggested resources and learning activities
13.5 Information in Circle: Pie Charts

Exploratory Exercise 13.7, 13H, 13.5.1, 13I

Signpost Year 8

Addison and Wesley p.138 - 139, p.471

For each of Outcomes B to E, students learn to

1) collect, represent and evaluate a set of data as part of an investigation including data collected using the Internet (Applying Strategies)

2) determine what type of graph is best to display a given set of data (Reflecting)

3) discuss and interpret graphs found in the media and in factual texts (Communicating, Reflecting)

4) identify misleading representations of data in the media (Reflecting)

5) discuss the advantages and disadvantages of different representations of the same data (Communicating, Reflecting)

6) pose questions that can be answered using the information from a given table (Questioning)

7) enter collected data into a given spreadsheet (Applying Strategies)

8) generate a simple graph, giving it a title and labelling the axes, using a computer application (Applying Strategies)

i) Consider the following data sources and determine what type of graph you would use to represent each data set. Give reasons for your answers including some discussion of the type of data being represented as discrete, continuous or categorical.

   a) Company profits over a period of ten years
   b) Percentage of people who listen to different radio stations
   c) Scores of students on an English exam
   d) Scores of a certain student achieved on their yearly English exams from year seven to year twelve
   e) Number of different types of cars passing a school during the day

F. Pose questions that can be answered using the information from a given table
a) Answer the following questions using these two tables

**Distance Ready Reckoner**

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<td>494</td>
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</table>

**Sending Parcels by Rail**

<table>
<thead>
<tr>
<th>Kilometres</th>
<th>Mass in Kilograms</th>
<th>Each additional 10 kg or part thereof</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 - 5</td>
<td>6 - 10</td>
</tr>
<tr>
<td>0 - 75</td>
<td>$1.90</td>
<td>$2.50</td>
</tr>
<tr>
<td>76 - 200</td>
<td>$2.10</td>
<td>$2.70</td>
</tr>
<tr>
<td>210 - 400</td>
<td>$2.40</td>
<td>$3.10</td>
</tr>
<tr>
<td>Over 400</td>
<td>$2.60</td>
<td>$3.50</td>
</tr>
</tbody>
</table>

i) How far is it from Orange to Sydney?
ii) How much would it cost to send a parcel weighing 35 kg from Orange to Sydney?
Outcomes for Area, Surface Area and Volume

Part A - Area

Students learn about

A. Developing the formula for the area of a rectangle, and using the formula
1) Students learn to make reasonable estimates involving length and area and check by measuring (Applying Strategies)
2) Students learn to compare shapes with the same perimeter and ask questions relating to their area such as whether they have the same area (Questioning)
3) Students learn to solve problems relating to area (Applying Strategies)

i) Find the area of a rectangle with sides 5 cm and 9 cm.

ii) Find the area of these two rectangles, both of which have a perimeter of 24 cm.

\[
\begin{array}{cc}
9 \text{ cm} & 7 \text{ cm} \\
3 \text{ cm} & 5 \text{ cm}
\end{array}
\]

iii) A farmer wants to fence off a paddock with areas of 56 m\(^2\). If the length of the paddock is 14 m, find the length of the paddock.

iv) Estimate the area of this rectangle in cm\(^2\)

\[
\text{9 cm} \quad \text{3 cm}
\]

v) A second farmer has a paddock which is 40 m by 30 m. Find the area of this paddock.
Suggested resources and learning activities

12.3.2

B. Using the abbreviations for square units eg \( \text{mm}^2 \), \( \text{cm}^2 \), \( \text{m}^2 \)

i) What units would be most appropriate to describe the area of
   a) a postage stamp
   b) a football field
   c) a blank page in an exercise book

ii) How many \( \text{mm}^2 \) in 36 \( \text{cm}^2 \)?

iii) How many \( \text{mm}^2 \) in 1 \( \text{m}^2 \)?

Suggested resources and learning activities

12F 12.3.1

C. Developing and using the formula for the area of a triangle by forming a rectangle

1) Students learn to make reasonable estimates involving length and area and check by measuring (Applying Strategies)

2) Students learn to identify the perpendicular height of triangles and parallelograms in different orientations (Communicating)

3) Students learn to solve problems relating to area (Applying Strategies)

i) Find the area of each of these triangles

A       B       C
Suggested resources and learning activities
12.3.3 12G

D. Finding the areas of simple composite figures which are reducible to rectangles and triangles
1) Students learn to solve problems relating to area (Applying Strategies)

i) Find the area of these shapes

A

B

C

Suggested resources and learning activities
12F 12G

E. Developing the formula for finding the area of a parallelogram by practical means eg by forming a rectangle using cutting and folding techniques
1) Students learn to identify the perpendicular height of parallelograms in different orientations (Communicating)
i) Find the area of these parallelograms

A

B

C

ii) Find the area of these trapezia

A

B

Suggested resources and learning activities

12.2 12H

F. Converting between units of area

i) \(1 \text{ cm}^2 = \ldots \ldots \text{ mm}^2\)

\(4.5 \text{ m}^2 = \ldots \ldots \text{ cm}^2\)

\(900 \text{ cm}^2 = \ldots \ldots \text{ mm}^2\)

\(8 \text{ cm}^2 = \ldots \ldots \ldots \text{ m}^2\)

Suggested resources and learning activities
12.3.1

**G. Using the formula to calculate the area of circles \( A = \pi r^2 \)**

1) Students learn to find the area of quadrants and semi-circles (Applying Strategies)
2) Students learn to solve problems relating to area (Applying Strategies)
   
i) Find the area of a circle with radius 8 cm (use \( \pi = \frac{22}{7} \))
   
   ii) Find the area of a circle with diameter 14 cm (use \( \pi = 3.14 \))
   
   iii) Find the area of these circles (use \( \pi = 3.14 \))

\[ \text{A} \quad 5 \text{ cm} \quad \text{B} \quad 40 \text{ m} \]

Suggested resources and learning activities
12.3 12I 12J

**Part B - Surface Area and Volume**

Students learn about

**H. Identifying and drawing the cross-section of a prism**

Suggested resources and learning activities

**I. Developing the formula for volume of rectangular prisms by considering the number and volume of layers of identical shapes**

Volume = base area \( \times \) height

Suggested resources and learning activities
12K 12.12 12L 12M 12O
J. Using the abbreviations for cubic units eg cm$^3$, m$^3$

Suggested resources and learning activities
12.4.1

K. Finding the volume of a right prism given the area of its cross-section
1) Students learn to solve problems involving volume and capacity of right prisms and cylinders (Applying Strategies)

i) Find the volume of these prisms

A

\[ A = 18 \text{ m}^2 \]

B

\[ A = 14 \text{ cm}^2 \]

Suggested resources and learning activities
12O 12P 12Q

L. Calculating the volume of right prisms with cross-sections that are rectangular and triangular
1) Students learn to solve problems involving volume and capacity of right prisms and cylinders (Applying Strategies)

i) Find the volume of these right prisms and the capacity of each prism

A

\[ 8 \text{ cm} \]

\[ 20 \text{ cm} \]

\[ 15 \text{ cm} \]

B

\[ 6 \text{ cm} \]

\[ 10 \text{ cm} \]

\[ 6 \text{ cm} \]

\[ 8 \text{ cm} \]

C

\[ 11 \text{ mm} \]

\[ 17 \text{ mm} \]

\[ 8 \text{ mm} \]

\[ 15 \text{ mm} \]
M. Using the formula to find the volume of cylinders

\[ V = \pi r^2 h \]

1) Students learn to solve problems involving volume and capacity of right prisms and cylinders (Applying Strategies)

i) Find the volume of these cylinders (use \( \pi = 3.14 \))

A

\[
\begin{align*}
\text{20 cm} \\
\text{15 cm}
\end{align*}
\]

B

\[
\begin{align*}
\text{12 cm} \\
\text{9 cm}
\end{align*}
\]

C

\[
\begin{align*}
\text{9 m} \\
\text{10.5 m}
\end{align*}
\]

ii) Find the capacity of each of these cylinders

Suggested resources and learning activities

12P 12N 12Q

N. Using the kilolitre as a unit in measuring large volumes

1) Students learn to recognise that the prefix ‘kilo’ means one thousand times the base unit (Communicating)

i) \( 1 \text{ kL} = \ldots \ldots \text{ m}^3 \)

ii) \( 1 \text{ kL} = \ldots \ldots \text{ L} \)

Suggested resources and learning activities

O. Converting between \( \text{m}^3 \) and \( \text{Kl} \)

1) Students learn to solve problems involving volume and capacity of right prisms and cylinders (Applying Strategies)
Suggested resources and learning activities

**P. Students also learn to**
1) Recognise, and give reasons why, prisms with the same volume may have different surface areas (Reasoning, Applying Strategies)
2) Solve problems involving surface area of rectangular and triangular prisms (Applying Strategies)

Find the surface area of these shapes

A

B

C

Suggested resources and learning activities
Outcomes for Algebraic Techniques

By the end of this topic students learn about

A. Using letters to represent numbers and developing the notion that a letter is used to represent a variable

i) Gemma decides to use the symbol ♣ to stand for a number. How would you represent two times this number?
ii) Yvette sets up an expression in the following way
   ♠ + 8
   What would this be equal to if ♠ = 8? What would it be equal to if ♠ = 25?
iii) Gemma then decides to use the letter x to stand for a number. What would three times this number be?
iv) Yvette uses the letter w. What would be three less than this number?

B. Using concrete materials such as cups and counters to model expressions that involve a variable and a variable plus a constant
   eg  a, a + 1

i) If a cup of counters is called x, use cups and counters to represent x + 4
ii) If a cup of counters is called m, use cups and counters to represent m + 11

C. Using concrete materials such as cups and counters to model expressions that involve a variable multiplied by a constant
   eg  2a, 3a

i) If a cup of counters represents d, use cups and counters to represent 2d. Does it matter how many counters are in each of the cups for your representation to be correct?
ii) Carla sets up the following equation
\[2x + 3 = x + 2\]
Use cups and counters to represent this equation if a cup of counters is equal to \(x\).

D. Using concrete materials such as cups and counters to model sums and products
eg \(2a + 1, 2(a + 1)\)

i) Place 6 counters in a cup and label this cup as \(x\). Remembering that we have said that \(x\) is equal to six counters in a cup, develop a model of \(2x + 1\) and \(2(x + 1)\). Are these the same? How are they different?

E. Using concrete materials such as cups and counters to model equivalent expressions
such as
\[x + x + y + y + y = 2x + 2y + y = 2(x + y) + y\]

i) Use cups with a certain number of counters in them to represent \(x\) (you choose the number!), and use cups with a different number of counters in them to represent \(y\). Determine whether or not the following expressions are equivalent or not

a) \(x + x + y + y = 2x + 2y\)  
b) \(x + x + y + y = 2(x + y)\)  
c) \(x + y + y = x + 2y\)  
d) \(x + x + y + y + y = 3(x + y)\)

F. Using concrete materials such as cups and counters to assist with simplifying expressions, such as
\[(a + 2) + (2a + 3) = (a + 2a) + (2 + 3)\]  
\[= 3a + 5\]

i) Let a cup with a certain number of counters (you choose the number!) be equal to \(x\). Develop a model of \(x + 3\) and \(2x + 1\). Show that when these are added together they are equal to \(3x + 3\)
ii) Let a cup with a certain number of counters (you choose the number!) be equal to $y$.
Develop a model of $2y + 1$ and $2y + 6$. Add these together and write an expression for the
sum of these two expressions

G. Recognising and using equivalent algebraic expressions

eg

$y + y + y + y = 4y$
$5 \times n = n \times 5 = 5n$
$w \times w = w^2$
$a \times a \times a \times a \times a = a^5$

$a \times b = ab$
$a + b = \frac{a}{b}$

i) Simplify these expression
   a) $x + x + x + x$
   b) $y \times y \times y$

ii) For each of these algebraic expression, write these expressions in another form
   a) $a + f$
   b) $g \times 8$
   c) $b \times d$
   d) $md$

H. Recognising like terms and combining like terms to simplify algebraic expressions

eg

$2n + 4m + n = 4m + 3n$

i) Simplify by collecting like terms
   a) $3x + 4y + 2x$
   b) $4y - 3x + 2x$
   c) $4x^2 + 6x + 5x - 3x^2$
   d) $6x + 8 - 3x + 2y$
I. Recognising the role of grouping symbols and the different meanings of expressions, such as

\[ 2a + 1 \text{ and } 2(a + 1) \]

i) Explain in your own words the difference between \( 2x + 3 \) and \( 2(x + 3) \)

J. Simplifying algebraic expressions involving multiplication and division

\[ \text{eg} \]

\[ 12a + 3 \]
\[ 4x \times 3 \]
\[ 2ab \times 3a \]
\[ 15x + 5x \]

i) Simplify these expression

\[ a) 3a \times 4b \]
\[ b) 5x \times 3x \]
\[ c) 6 \times 3d \]
\[ d) 15a + 3 \]
\[ e) 30xy + 5y \]
\[ f) \frac{5xy}{3} \]
\[ g) \frac{15y}{16x^2y} \]
\[ \frac{42xy^2}{42xy^2} \]

K. Expanding algebraic expressions by removing grouping symbols (the distributive property)

\[ \text{eg} \]

\[ 3(a + 2) = 3a + 6 \]
\[ 2(a - b) = 2a - 2b \]
\[ -5(x + 2) = -5x - 10 \]
\[ a(a + b) = a^2 + ab \]

i) Simplify these expressions

\[ a) 5(c + 3) \]
\[ b) 4x(x - 7y) \]
\[ c) -3(x - 3) \]
d) \(-(x + 4)\)

e) \(x(x + y)\)

L. Factorising a single term

**eg**

\[6ab = 3 \times 2 \times a \times b\]

i) Write these terms in expanded form as in the above example

a) \(42xy\)

b) \(52x^2y^3\)

c) \(36xy^2z^2\)

M. Factorising algebraic expressions by finding a common factor

**eg**

\[6a + 12 = 6(a + 2)\]

\[x^2 - 5x = x(x - 5)\]

\[5ab + 10a = 5a(b + 2)\]

\[-4t - 12 = -4(t + 3)\]

i) Factorise these expressions

a) \(5y + 20\)

b) \(x^2 + 4x\)

c) \(14ab + 12b\)

d) \(-6y - 12\)

e) \(5t^2 + 10t - 15t^3\)

*By the end of this topic students will learn to*

N. Generate a variety of equivalent expressions that represent a particular situation or problem

i) Determine expressions for each of these problems

   a) I take 7 away from a number. How much do I have left?

   b) If I add 12 to twice a certain number, how much do I have?
O. Determine and justify whether a simplified expression is correct by substituting numbers for letters

i) Determine whether or not the following expressions are equivalent by substituting numbers for $x$ and $y$ in both sides

   a) $3x + 4y = x + 2x + 2y + 2y$
   b) $x^2y = \frac{x^3y^2}{xy}$
   c) $3(x + 2) = 3x + 2$
   d) $5(x + 7) - 3(x + 2) = 2x + 41$

P. Describe relationships between the algebraic symbol system and number properties

i) Explain why $a \times b = ab$ but $7 \times 5 = 35$

ii) Apart from the difference in the way that multiplication can be expressed using algebra and normal numbers, are there any other major differences you can think of?

Q. Translate between words and algebraic symbols and between algebraic symbols and words

i) Write the following sentences using algebraic expressions

   a) Five times $x$ plus 17
   b) Subtract 13 from 4 times $x$
   c) Divide 3 times $x$ by 4 times $y$
   d) $x$ squared multiplied by two times $y$ and then subtract 4 times $z$

ii) Interpret the following algebraic expressions using English

   a) $x + y - z$
   b) $3x - 4y$
   c) $x^3y^2$

R. Link algebra with generalised arithmetic eg for the commutative property, determine that

\[ a + b = b + a \]
i) Show by substituting in numbers for \( x \) and \( y \) that

a) \( xy = yx \)

b) \( x + y = y + x \)

c) \( x(y + z) = xy + xz \)

S. Distinguishing between algebraic expressions where letters are used as variables and equations where letters are used as unknowns

i) In each of the following expressions determine whether \( x \) is a variable or an unknown value. In cases where \( x \) is an unknown value, find the value of \( x \)

a) \( x - 4 \)

b) \( 3x - 4 \)

c) \( x - 4 = x + 2 \)

d) \( 2x - 3 = 13 \)

e) \( 7 - 5x \)

T. Substituting into algebraic expressions

i) If \( x = 3 \), \( y = 4 \) and \( z = 5 \), evaluate

a) \( 3x + y \)

b) \( x - y - z \)

c) \( 3x^2y^3 + 2x^2z \)

d) \( \frac{2x}{3yz} \)

U. Generating a number pattern from an algebraic expression

i) Substitute the numbers from 1 to 10 in the following expressions to obtain a certain number pattern

a) \( x + 5 \)

b) \( 2x \)

c) \( x^2 \)

d) \( x^2 - 2x \)
V. Replacing written sentences describing patterns with equations written in algebraic symbols

eg you add five to the first number to get the second number, could be replaced with

\[ y = x + 5 \]

i) Replace these sentences with equations written in algebraic symbols

a) you subtract seven from the first number to get the second number

b) you multiply the first number by three then add seven to get the second number
Fractions and Decimals Outcomes

*By the end of this topic students will learn about*

a) finding equivalent fractions using a variety of methods eg paper folding, diagrams, number lines and mental computation

i) Use a rectangular paper and fold it into quarters. Use a second piece of paper and fold it into eighths. What do you notice about

a) \( \frac{1}{4} \) and \( \frac{2}{8} \)

b) \( \frac{3}{4} \) and \( \frac{4}{8} \)

ii) Draw two circles. Divide one circle into thirds (that is, three equal pieces) and the other circle into sixths (that is, six equal pieces). Shade in one of the pieces on the first circle and two of the pieces on the second circle. What do you notice?

b) expressing two fractions as fractions with the same denominator to enable addition and subtraction

i) Write these pairs of fractions with the same denominator

\[
\begin{align*}
\frac{1}{4} & \quad \frac{3}{8} \\
\frac{1}{5} & \quad \frac{4}{3}
\end{align*}
\]

c) reducing a fraction to its lowest equivalent form

i) Simplify these fractions to their lowest equivalent form

\[
\begin{align*}
\frac{10}{20} & \quad \frac{10}{15} & \quad \frac{7}{14} & \quad \frac{12}{14} & \quad \frac{120}{110}
\end{align*}
\]

d) adding and subtracting fractions eg \( \frac{1}{3} + \frac{2}{5} \)
Students will learn to recognise and explain incorrect operations with fractions eg

explain why $\frac{2}{3} + \frac{1}{4} \neq \frac{3}{4}$

i) Perform these additions and subtractions without a calculator. Check your answer with a calculator if you have one available. Reduce your answers to their lowest equivalent form

\[
\begin{align*}
1) \quad \dfrac{1}{4} + \dfrac{1}{4} &= \\
2) \quad \dfrac{3}{8} + \dfrac{2}{8} &= \\
3) \quad \dfrac{3}{4} + \dfrac{1}{5} &= \\
4) \quad \dfrac{7}{10} + \dfrac{1}{5} &= \\
5) \quad \dfrac{5}{8} - \dfrac{2}{8} &= \\
6) \quad \dfrac{7}{8} - \dfrac{1}{3} &=
\end{align*}
\]

e) expressing improper fractions as mixed numbers and vice versa

i) Write these improper fractions as mixed numerals

\[
\dfrac{5}{4}, \dfrac{10}{3}, \dfrac{100}{1}, \dfrac{25}{4}
\]

\[
3\dfrac{1}{4}, 1\dfrac{1}{2}, 5\dfrac{1}{3}
\]

ii) Write these mixed numerals as improper fractions

f) adding mixed numbers

i) Add these mixed numerals together

\[
\begin{align*}
1) \quad \dfrac{1}{3} + \dfrac{2}{3} &= \\
2) \quad \dfrac{1}{2} + \dfrac{3}{4} &= \\
3) \quad \dfrac{3}{4} + \dfrac{1}{5} &=
\end{align*}
\]
g) subtracting a fraction from a whole number  
   \[ 3 - \frac{2}{3} = 2 - \frac{2}{3} = 2 \frac{1}{3} \]

i) Perform these subtractions

1) \(5 - \frac{1}{4} = \)
2) \(3 - \frac{2}{3} = \)
3) \(2 + 2 - \frac{1}{3} = \)

h) multiplying and dividing fractions through various concrete methods leading to written processes

Students will learn to demonstrate multiplication of a fraction by a fraction using a diagram to illustrate the process

i) Take an egg carton and cut it into two halves. Now take one of the halves and tear off one third of what you have left. What do you have left? How could you write what you have just done using fractions and multiplication?

ii) Use a calculator to perform these multiplications and divisions. Can you work out what might be an easier way to perform multiplication and division of fractions without a calculator from these results?

\[
\begin{align*}
1) \frac{1}{3} \times \frac{1}{4} &= \frac{2}{5} \times \frac{3}{7} = \\
3) \frac{1}{6} \times \frac{5}{4} &= \frac{3}{4} \times \frac{5}{7} = \\
\end{align*}
\]

Use your rule to predict what the answer to \(\frac{3}{5} \times \frac{1}{2} = \) will be. Check your prediction on the calculator

Students will learn to explain division by a fraction is equivalent to multiplication by its reciprocal

\[
\begin{align*}
1) \frac{1}{3} + \frac{1}{2} = \frac{2}{8} + \frac{5}{7} = \\
\end{align*}
\]
Use your rule to predict what the answer to $\frac{1}{5} + \frac{1}{3}$ will be. Check your prediction on the calculator.

iii) How many 5's in 35? How would you write this sentence as a number sentence? How many halves in 5? How would you write this sentence as a number sentence? Does this help you to work out how to divide fractions?

iv) A cook has one half of a pizza. He then divides this half into eighths. How many eighths will there be? Write a number sentence to represent this problem.

i) multiplying and dividing fractions and mixed numbers

i) Perform these multiplications and divisions without using a calculator

\[
\begin{align*}
1) \quad & \frac{3}{4} \times \frac{5}{6} = \\
2) \quad & \frac{1}{4} \times \frac{7}{8} = \\
3) \quad & \frac{1}{2} \times \frac{3}{4} = \\
4) \quad & \frac{1}{5} \times \frac{4}{2} = \\
5) \quad & \frac{3}{4} + \frac{1}{3} = \\
6) \quad & \frac{4}{8} + \frac{3}{4} = \\
7) \quad & 1\frac{1}{3} + \frac{1}{2} = \\
8) \quad & 5\frac{1}{2} + 2\frac{1}{2} =
\end{align*}
\]

j) adding, subtracting, multiplying and dividing decimals

i) Perform these calculations involving decimals

\[
\begin{align*}
1) \quad & 0.6 + 0.3 + 0.1 = \\
2) \quad & 0.5 + 0.34 = \\
3) \quad & 0.001 + 0.53 = \\
4) \quad & 0.532 - 0.13 = \\
5) \quad & 0.9 - 0.5 = \\
6) \quad & 0.6 \times 0.3 = \\
7) \quad & 0.4 \times 0.02 = \\
8) \quad & 0.03 \times 1.4 = \\
9) \quad & 0.85 \div 0.05 =
\end{align*}
\]
10) 1.35 ÷ 0.9 =

k) rounding decimals to a given number of places
i) Round off these decimals to two decimal places
1) 0.5483
2) 0.768
3) 3.8732
4) 2.9872

Students will learn to interpret a calculator display in formulating a solution to a problem, by appropriately rounding a decimal

ii) Paul calculates the cost of buying 15.45 litres of petrol if petrol costs 89.9 cents per litre. His calculator gives the answer 1388.955. How much will it cost in dollars and cents?

l) using the notation for recurring (repeating) decimals
eg 0.333 33…=0.3̇, 0.345 345 345… = 0.345̇

i) Use appropriate notation to write these repeating decimals
1) 0.5555555…..
2) 0.77777…
3) 0.3535353535 …. 
4) 0.146371463714637….

m) converting fractions to decimals (terminating and recurring) and percentages
i) Convert these fractions into decimals and then into percentages

\[
\frac{3}{4} = 0.75\quad \frac{15}{8} = 1.875\quad \frac{8}{9} = 0.8888888...
\]
n) converting terminating decimals to fractions and percentages
i) Convert these decimals into fractions and then into percentages
1) 0.\(\frac{1}{9}\)  2) 0.\(\frac{2}{3}\)  3) 0.\(\frac{5}{6}\)

o) converting percentages to fractions and decimals
i) Convert these percentages to fractions and decimals
1) 15%  
2) 30%  
3) 55%

Students will learn to question the reasonableness of statements in the media that quote fractions, decimals or percentages eg ‘the number of children in the average family is 2.3’

p) calculating fractions, decimals and percentages of quantities
Students will learn to choose the appropriate equivalent form for mental computation eg 10% of $40 is \(\frac{1}{10}\) of $40

i) Find one-fifth of 35  
ii) Find three-fifths of 95  
iii) Find 0.6 of 40  
v) Find 60% of 700
Outcomes for Ratios and Rates

Students learn about

1) converting percentages to fractions and decimals
   a) Change these percentages into fractions and then change them into decimals
      i) 60%
      ii) 35%
      iii) 12.5%
      iv) 22.5%
      v) 0.1%

   Students learn to interpret descriptions of products that involve fractions, decimals, percentages or ratios eg on labels of packages

2) calculating fractions, decimals and percentages of quantities
   a) Find
      i) 50% of 200
      ii) 19% of $360
      iii) 0.1% of 3000

   b) Find
      i) \(\frac{3}{5}\) of $48
      ii) \(\frac{2}{9}\) of $720
      iii) \(3\frac{1}{5}\) of 680

   c) Find
      i) 0.35 of 80
Students learn to interpret media and sport reports involving percentages

d) In the NRL in a certain year, 55% of all tries sent to the video referee were allowed and the rest were disallowed. If 65 tries were sent to the video referee, how many of these were disallowed?

e) From a survey of voters before the last federal election, 47.1% were intending to vote for the Coalition and 43.2% were intending to vote for the Labor party. If 1143 people were surveyed, how many were intending to vote for someone else?

3) increasing and decreasing a quantity by a given percentage

a) Increase $450 by 10%

b) Decrease 605 by 20%

Students learn to evaluate best buys and special offers eg discounts

c) Find the discounted price on an item if it was originally $40 and it was discounted by 20%

d) Two items are on sale at a shop. One is originally priced at $55 and has a discount of 15% advertised. The other item is originally priced at $70 and has a discount of 25%. Which item will be cheaper?

Students learn to recognise equivalences when calculating eg multiplication by 1.05 will increase a number/quantity by 5%, multiplication by 0.87 will decrease a number/quantity by 13%

4) interpreting and calculating percentages greater than 100% eg an increase from 6 to 18 is an increase of 200%; 150% of $2 is $3

a) Increase 45 by 250%

b) Find 145% of $12
c) Can you decrease 45 by 250%?

Students learn to solve a variety of real-life problems including problems involving fractions, decimals and percentages

d) A house bought in 1980 for $60 000 was sold last year for $270 000. What was the percentage increase?

5) expressing profit and/or loss as a percentage of cost price or selling price
a) A car was bought for $36 000 and sold for $40 000. What was the profit as a percentage of the cost price.
b) A house bought for $250 000 is sold for $320 000. Find the profit as percentage of the cost price.
c) Hermione buys shares valued at $4 000 and sells them for $9 000. Find the profit as a percentage of the cost price.
d) Raymond buys these shares for $9 000 and sells them for $6 000. Find the loss as percentage of the cost price.

Students learn to solve a variety of real-life problems including problems involving fractions, decimals and percentages

6) ordering fractions, decimals and percentages
a) Place these in ascending order
\[ \frac{3}{8}, 0.35, 30\%, 0.41, \frac{3}{7} \]

7) expressing one quantity as a fraction or a percentage of another
eg 15 minutes is \( \frac{1}{4} \), or 25%, of an hour
a) In a test Sarah scored 36 out of 60. What was her score as a percentage?
b) Greta covers 30km of a 40km journey. What percentage of her journey has she completed?

c) Karen is eating a 500gm chocolate block, of which she has eaten 300gm. Mark has eaten 50gm of his 200gm block of chocolate. Which person has eaten the largest percentage of their chocolate?

d) What fraction of 3 km of a marathon if a marathon is 42.2 km long?

e) What fraction of an hour is 90 seconds?

Students learn to solve a variety of real-life problems including problems involving fractions, decimals and percentages

8) using ratio to compare quantities of the same type

Students learn to interpret descriptions of products that involve fractions, decimals, percentages or ratios eg on labels of packages

a) A car operates with a gear ratio of 1 : 1.5. If the first gear rotates 60 times, how many times does the second gear rotate?

b) On a packet of cake mix the ratio of sugar to flour is 3 : 14. If there are 42 grams of sugar in the packet, how much flour is in the packet?

Students learn to solve a variety of real-life problems involving ratios eg scales on map, mixes for fuels or concrete, gear ratios

a) In a certain class there are 16 boys and 10 girls. What is the ratio of boys to girls in simplest form?

b) In a certain car engine 8 litres of petrol requires 56 litres of air to combust effectively. What is the ratio of petrol to air required for this engine to run efficiently?
c) A concrete mix consists of sand, cement and gravel in the ratio 3 : 2 : 8. Find the amount of water required if there is 5 kg of cement available.

9) writing ratios in various forms eg. 4/6, 5:3, 4 to 1, and 3:2:1
(see sample questions for other outcomes for examples of different notations)

10) simplifying ratios
    eg 4:6 = 2:3 , 4:2 = 1:4, 0.3:1 = 3:10

Simplify these ratios
6:8
ii) 15:40
iii) 4:8:14
iv) 14/8
v) 2/3 : 5 _
vii) 0.65 : 2.15
Identify which of these ratios is equivalent to 3:5

30 : 50  4 : 6.25  6 : 14  1.5 : 2.5  1 : 5/3

11) applying the unitary method to ratio problems
Write the following ratios in the form 1 : X
5 : 8
6 : 18
14 : 10
0.4 : 0.15

Write the following ratios in the form Y : 1
6 : 8
3.5 : 9
2/3 : 5/8

c) A certain cake recipe suggests using 150 gms of sugar for every 800 gms of flour. Garry only has 40 gms of sugar. How much flour will Garry need to make his cake?

12) dividing a quantity in a given ratio
Divide 600 into the ratio 6 : 9
Frances and Laura set up a business together in which Frances invests $4000 and Laura invests $3000. If their business makes a profit of $35 000 how much should each person receive?
Three friends decide to divide their marbles between them in the ratio 1 : 2 : 5. If they have 96 marbles, how much will each of the friends receive?
Rachel’s salary is divided each week between her savings, household expenditure and rent in the ratio 4 : 7 : 4. If in one week she earns $500 how much will she save?

The ratio of sand to water in a certain mixture is 7 : 10. If there are 56 kg of sand how much water is there?
At a certain school there is a policy of having three teachers for every twenty eight students on an excursion. If a whole year of 160 students goes on an excursion how many teachers must also go to meet this requirement?
In a certain town more baby girls are born than baby boys. In fact, the ratio of girls to boys born in the town is 8 : 7. In 2001 there were 91 boys born. How many girls were born in 2001?

13) calculating rates from given information eg 150 kilometres travelled in 2 hours
Simplify these rates
240 km in 5 hours
5 litres in 4 minutes
540 wheelbarrows in 9 hours
Simplify these rates

100 km/hour = ___________ km/min
6 kg/day = ___________ gm/day
5 L/hr = __________ mL/s
60 km/hour = _______ m/s
50 cm/min = __________ km/hr

Students learn to solve a variety of real-life problems involving rates eg batting and bowling strike rates, telephone rates, speed, fuel consumption

Sarah drives 650 km in 7 hours. Find her average speed over her journey.

Greta and her family are planning a trip to Queensland. If they are planning to travel 1200 km at an average speed of 90 km per hour, how long will it take her family to drive to Queensland if they do not take a break?

The distance from the earth to the sun is approximately 150 million kilometres. How long will it take light to reach the earth from the sun if light travels at 386 000 km per second?

A certain cat can catch 5 mice every hour. How many mice can this cat catch in 24 hours?

Two people, Alice and Benny are painting a fence together. If Alice can paint the fence in two hours on her own and Benny can paint the fence in three hours on his own, how long will it take both of them to paint the fence?
Outcomes for Data Representation, Data Analysis and Evaluation

Note: Please check the pre-test for further examples of data representation questions

Students learn about

a) formulating key questions to provide data for a problem of interest

i) Jan wanted to know what type of music people listened to at her school. What questions could she ask people at her school?

ii) Graeme was interested in how often different authors use the word 'I' in their books. What questions could he formulate to determine how often authors use the word 'I'?

iii) Greta was set an assignment in which she had to find out how much time students at her school watched television during the week. Design a questionnaire Greta could use to collect data to answer this question.

b) refining key questions after a trial

i) Greta found that students at her school were confused about whether watching videos, playing computer games and surfing the net were counted as watching television or not. How could Greta refine her questionnaire to accurately answer the questions about how much television people watch?

ii) She also found that the viewing habits of students of different ages varied greatly. How could her questionnaire help her to identify the viewing habits of students of different ages?
c) recognising the differences between a census and a sample
Determine if these methods of data collection are examples of censuses or samples
i) 20 students from Sarah's school are given a questionnaire to ask them for their opinions on the school playground

ii) King David determines the number of fighting men in his army.

iii) 1000 people are polled on their voting intentions for the next election

iv) Talia wants to know how many children are in the families of people in her class. She asks each member of her class how many children there are in their families.

d) distinguishing continuous from discrete data
Determine if the following are examples of continuous or discrete data
i) Heights of plants

ii) Daily temperatures

iii) Age of plants

iv) Goals scored in a football match

v) how many people attend school each day

e) finding measures of location (mean, mode and median) for small sets of data
i) For the following set of data determine the mean, mode and median
5, 8, 8, 10, 12, 13, 21
Mean =
Mode =
Median =
ii) Find the mean, median and mode of this set of data
1, 8, 3, 2, 2, 9
Mean = 
Mode = 
Median = 

f) using a scientific or graphics calculator to determine the mean of a set of scores

i) Using a calculator, determine the mean of the following data set
3, 6, 4, 7, 3, 8, 2, 9, 5, 7, 4, 8, 3, 7, 1, 6, 4, 6, 8, 9, 7, 4, 6
Mean = 

ii) Determine the mean of the following set of data

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

g) using measures of location (mean, mode, median) and the range to analyse data that is displayed in a frequency distribution table, stem-and-leaf plot, or dot plot

Students learn to
draw conclusions based on the analysis of data (eg a survey of the school canteen food) using the mean, mode and/or median, and range
interpret media reports and advertising that quote various statistics eg media ratings
question when it is more appropriate to use the mode or median, rather than the mean,
when analysing data
compare two sets of data by finding the mean, mode and/or median, and range of both
sets

i) A group of 10 students completed their yearly exam in mathematics and their results
are represented in the following stem and leaf plot

<table>
<thead>
<tr>
<th>Class scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 5 8</td>
</tr>
<tr>
<td>4 0 3</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6 8</td>
</tr>
<tr>
<td>7 1 3</td>
</tr>
<tr>
<td>8 3</td>
</tr>
<tr>
<td>9 1 8</td>
</tr>
</tbody>
</table>

What is the mean for this group of students?

What is the median?

ii) The following dot plot provides the number of goals a hockey team scores in each of
twenty games.

What is the modal number of goals?
What is the mean number of goals scored?

iii) Two classes scored the following scores on a test. Which class do you think did better? Why? Why not?

Class A - 37 59 48 79 89 98 46 27 58

Class B - 67 68 47 78 75 68 71 73 74

h) collecting data using a random process eg numbers from a page in a phone book, or from a random number function on a calculator
i) Work out some other methods for determining random numbers

i) making predictions from a sample that may apply to the whole population
Students learn to
consider the size of the sample when making predictions about the population
predict the reliability of the sample
detect bias in the selection of a sample

i) A newspaper reports that in a survey of people's drinking habits, 40% of people surveyed said that the drink they had most often was coffee, and 30% said their preferred drink was tea. In a group of 30 people, how many would you expect to drink coffee? Is there any other information you would like about this survey?

ii) A survey of 10 students from a Jewish school found that 60% of students eat corn beef sandwiches for lunch. Do you think that 60% of students across Australia eat corn beef? Why? Why not?
j) making predictions from a scatter diagram or graph

i) Use this graph to answer the following questions

The average amount of homework students do at this particular level is thought to be six hours a week. Is this group above or below average? Why? Why not?

What do you think would be the most likely number of hours spent on homework for a student from this sample?

k) using spreadsheets to tabulate and graph data

i) Use a spreadsheet to develop a frequency histogram and polygon for the data in the frequency distribution table on page 3

l) analysing categorical data eg a survey of car colours

i) George collects the following data on the books different people read in his class

Novels - 14
Plays - 5
Textbooks - 7
Dictionaries - 13

What is the modal category?

Which were more popular, fiction or non-fiction?
By the end of this topic students will learn to

work in a group to design and conduct an investigation
eg
decide on an issue
decide whether to use a census or sample
choose appropriate methods of presenting questions (yes/no, tick a box, a scale of 1 to 5, open-ended etc)
analyse and present the data
draw conclusions
work cooperatively in small groups to analyse and represent data eg preparing and entering data into a spreadsheet and using the graphing function
Part B– Outcome sheets used at Southwest High School

Contents (all year eight classes)
Outcomes for Equations topic
Outcomes for Measurement
Outcomes for Ratio and Rates
Outcomes for Pythagoras’ Theorem
Outcomes for Equations topic

Outcomes covered A4.5, WM4.1 – WM4.5

At the end of this topic, students will be able to

1. Solve simple one-step equations using appropriate methods
   Solve these equations
   a + 3 = 11
   b – 4 = -1
   3f = 12
   \[ x = 3 \]

2. Solve two-step equations using appropriate methods
   2d + 5 = 17
   4w – 7 = 9
   5r + 11 = -1
   \[ \frac{t}{3} + 6 = 7 \]

3. Solve equations using grouping symbols
   3(c – 2) = 10
   -4(x + 2) = 16
   -3(x + 4) = 12
   -3(x + 4) = -12

4. Solve equations with pronumerals on both sides
2d + 1 = d – 4

5 – 3x = 2x + 15

4(w + 1) = 3(2 – w)

x + 4 = 4x + 11

5. Solve equations including one fractional term

\[ \frac{t}{3} + 6 = 8 \]

\[ \frac{x + 3}{2} = 7 \]

\[ 5 + x = \frac{x}{2} + 7 \]

6. Solve equations involving more than one fractional term

\[ \frac{2a}{3} + \frac{a}{2} = 12 \]

\[ \frac{x + 4}{3} \]

\[ \frac{a}{3} + \frac{a}{2} = 13 – \frac{a}{4} \]

7. Solve simple inequalities, graphing solutions on a number line

Find all the solutions of \( x + 3 < 6 \) and represent these solutions on a number line

Find all the solutions of \( 3x + 5 \leq 2x + 8 \) and represent these solutions on a number line

(Requires reversing the sign) Find all the solutions of \(-x + 5 < 8\) and represent these solutions on a number line

8. Solve problems using equations including applications to geometric diagrams

a) 

\[ \begin{array}{c}
\text{x} \\
\text{x + 2}
\end{array} \]
The perimeter of this block of land is 72 m. Find the length of the block and the area of the block.

b) In the diagram below, find the size of each angle

9. Translate word problems into equations and solve them

When 7 is subtracted from four times a certain number, the answer is 37. What is the number?

A man is twice as old as his daughter. Ten years ago he was three times as old. How old is the daughter now?

Dara bought four ice creams and received $2.80 change from her $10 note. How much did each ice cream cost?

A container of water is 80% full. When ten litres are taken out of the container the container is only 2/3 full. How many litres does the container hold?

The number 57 is divided into two parts so that the sum of $\frac{1}{5}$ of the first part and $\frac{1}{6}$ of the second part is 10. What are the two parts?
Outcomes for Measurement

At the end of this topic students will be able to

1. Recognise appropriate units for measuring length, time, mass and capacity

a) What units would be most appropriate to describe the area of
   i) a postage stamp
   ii) a football field
   iii) a blank page in an exercise book

b) What units would be most appropriate to describe the time it takes for
   i) a car to travel around Australia
   ii) Cathy Freeman to run 400 metres
   iii) a television program to run
   iv) a tree to grow

c) What units would be most appropriate to describe
   i) the distance from Sydney to Perth
   ii) the length of a hair from your head
   iii) the width of a match stick

d) What units would be most appropriate to measure
   the mass of a textbook
   a person’s weight
   the amount of water someone drinks in a day
   the amount of sodium chloride in a bottle of mineral water
2. Perform conversions between different units of measurement of length, time, mass and capacity

a) Complete this table of conversions

1 cm\(^2\) = .............. mm\(^2\)

4.5 m\(^2\) = .............. cm\(^2\)

900 cm\(^2\) = .............. mm\(^2\)

8 cm\(^2\) = .............. m\(^2\)

4.2 ha = .............. m\(^2\)

b) Complete this table of conversions

7.6 km = .............. m

8.3 g = .............. Mg

5430 mL = .............. L

c) Express 4.25 hours in hours and minutes

d) Express 3h 48 min in hours

3. Estimate lengths and areas

a) Estimate the area of this rectangle in cm\(^2\) and the perimeter of this shape

b) Estimate the area that you could cover with 1000 $100 notes.
4. Find the perimeter of a shape by measurement

a) Use your ruler to find the perimeter of these shapes in mm.

A

B

C

5. Calculate the perimeter of a shape given the measurements for different sides

a) Find the perimeter of these shapes

A

B

C

6. Calculate the circumference of a circle

a) Find the circumference of these circles

A

B

C
b) Find the perimeter of this shape

![Diagram of a shape with sides labeled 20 cm and 12 cm.]

c) Find the perimeter of this shape given that the part of this circle removed had an angle at the center of 90 degrees and the radius of this circle is 12 cm.

![Diagram of a circle with a sector removed.]

7. Read and interpret scale drawings

a) Use the following scale drawing to answer the following questions
i) How far is it, as the crow flies, from Paynesville to Bruben.

ii) Cara walks from Lakes Entrance. She walks 20 km north and then travels 15 km east. How far is she from her starting point as the crow flies?

iii) Which town is north west of Metang?

iv) Which town is due west of Lakes Entrance?

b) What does a scale of 1 mm: 5 km mean?

c) What does a scale of 1:1000 mean?

8. Construct scale drawings

a) Draw a floor plan for your classroom showing the location of your teacher’s desk, the blackboard and any other major pieces of furniture. Use a scale of 1 cm : 40 cm.
b) Draw a scale drawing of a car using a scale of 1cm : 20 cm which has wheels with circumference 40 cm, and length of 3.5 m.

c) Use the grid below to fill in the following details

T represents the town hall.

i) Draw in Red Street which runs for 800 m due east of the town hall.
   ii) Draw in North Road which runs 400 m due north of the town hall.
   iii) The intersection of Red Street and Green Street is 400 m east of the town hall.

Draw in Green St if it runs 600 m due north of the intersection of Red Street and Green Street.

9. Work with simple and more complex forms of compass bearings

a) Use the grid above to fill in the following details.
i) The post office is 1000 m from the town hall and is north west of the town hall. Locate the post office on the grid

ii) Graeme walks from the town hall on a bearing of N40°E. He travels 600 m then turns and walks 500 m on a bearing of S60°E. Find how far he is from the town hall.

10. Determine the area of a shape by counting the number of square units inside
Find the area of these shapes

a) 

b) 

11. Calculate the area of a rectangle

a) Find the area of these rectangles, both of which have the same perimeter

9 cm

3 cm

7 cm

5 cm
b) Find the area of a rectangle with sides 5 cm and 9 cm.

c) A farmer wants to fence off a paddock with area 56 m$^2$. If the length of the paddock is 14 m, find the width of the paddock.

d) A second farmer has a paddock which is 40 m by 30 m. Find the area of this paddock.

12. Calculate the area of a triangle

a) Find the area of these triangles

A

\[
\text{Base} = 8 \text{ cm}, \quad \text{Height} = 6 \text{ cm}, \quad \text{Area} = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2
\]

B

\[
\text{Base} = 7 \text{ mm}, \quad \text{Height} = 10 \text{ mm}, \quad \text{Area} = \frac{1}{2} \times 7 \times 10 = 35 \text{ mm}^2
\]

C

\[
\text{Base} = 9 \text{ cm}, \quad \text{Height} = 15 \text{ cm}, \quad \text{Area} = \frac{1}{2} \times 9 \times 15 = 67.5 \text{ cm}^2
\]

D

\[
\text{Base} = 14 \text{ m}, \quad \text{Height} = 6 \text{ m}, \quad \text{Area} = \frac{1}{2} \times 14 \times 6 = 42 \text{ m}^2
\]

13. Calculate the area of composite figures including rectangles and triangles
a) Find the area of these composite figures

A. 

B. 

C. 

14. Calculate the area of a circle

a) Find the area of these circles

A

B

b) Find the area of these shapes

A

B
c)

i) Find the area of a circle with radius 8 cm (use $\pi = \frac{22}{7}$)

ii) Find the area of a circle with diameter 14 cm (use $\pi = 3.14$)

15. Calculate the area of a parallelogram

a) Find the area of these parallelograms
16. Calculate the area of a trapezium

a) Calculate the area of these trapeziums

b) Find the area of a trapezium with height of 12 cm and parallel sides that are 5 cm and 8 cm in length.
17. Calculate the area of a rhombus and the area of a kite

a) Calculate the area of these rhombuses

![Rhombus A with diagonals 12 cm and 8 cm]

![Rhombus B with diagonals 20 cm and 14 cm]

b) Calculate the area of this kite

![Kite with diagonals DC and BC, AC = 15 cm, DB = 12 cm]
Outcomes for Ratios and Rates

At the end of this topic students will be able to

Manipulate ratios written using different notations

(see sample questions for other outcomes for examples of different notations such as $4 : 9$, 4 to 9, $1 : \frac{9}{4} = \frac{4}{9} : 1$, and three-part ratios such as $3 : 2 : 5$)

Simplify ratios

What does it mean to simplify a ratio?

Simplify these ratios

6:8

ii) 15:40

iii) 4:8:14

iv) 14/8

v) $2/3 : 5$ 

vi) 0.65 : 2.15

vii) $4x^2 : 8x$

viii) $8xy : 14xyz$

What are equivalent ratios?

Identify which of these ratios is equivalent to 3:5

<table>
<thead>
<tr>
<th>30 : 50</th>
<th>$3xy : 5xz$</th>
<th>4 : 6.25</th>
<th>6 : 14</th>
<th>1.5 : 2.5</th>
<th>1 : 5/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15xz : 25xz$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In a certain class there are 16 boys and 10 girls. What is the ratio of boys to girls in simplest form?
In a certain car engine 8 litres of petrol requires 56 litres of air to combust effectively. What is the ratio of petrol to air required for this engine to run efficiently?

Write ratios in the form 1 : X and Y : 1 and use these ratios to solve problems
Write the following ratios in the form 1 : X
5 : 8
6 : 18
14 : 10
0.4 : 0.15

Write the following ratios in the form Y : 1
6 : 8
3.5 : 9
2/3 : 5/8

A certain cake recipe suggests using 150 g of sugar for every 800 g of flour. Garry only has 40 g of sugar. How much flour will Garry need to make his cake?

Four numbers \(a, b, c, d\) (in that order) are in proportion if \(a : b = c : d\). Find the value of \(k\) if 24, 8, \(k\) and 2 are in proportion.

Dividing quantities into a given ratio

Divide 600 into the ratio 6 : 9

Frances and Laura set up a business together in which Frances invests $4000 and Laura invests $3000. If their business makes a profit of $35 000 how much should each person receive?

Three friends decide to divide their marbles between them in the ratio 1 : 2 : 5. If they have 96 marbles, how much will each of the friends receive?

Rachel’s salary is divided each week between her savings, household expenditure and rent in the ratio 4 : 7 : 4. If in one week she earns $500 how much will she save?
5. Solve problems involving quantities in a given ratio

The ratio of sand to water in a certain mixture is 7 : 10. If there are 56 kg of sand how much water is there?

At a certain school there is a policy of having three teachers for every twenty eight students on an excursion. If a whole year of 160 students goes on an excursion how many teachers must also go to meet this requirement?

In a certain town more baby girls are born than baby boys. In fact, the ratio of girls to boys born in the town is 8 : 7. In 2001 there were 91 boys born. How many girls were born in 2001?

Simplify rates

Simplify these rates
240 km in 5 hours
5 litres in 4 minutes
540 wheelbarrows in 9 hours

Simplify these rates
100 km/h = ___________ km/min
6 kg/day = ___________ g/day
5 L/h = ________ mL/s
60 km/ h = _______ m/s
50 cm/min = _________ km/h

Solve problems involving rates

Sarah drives 650 km in 7 hours. Find her average speed over her journey.

Greta and her family are planning a trip to Queensland. If they are planning to travel 1200 km at an average speed of 90 km per hour, how long will it take her family to drive to Queensland if they do not take a break?
The distance from the earth to the sun is approximately 150 million kilometres. How long will it take light to reach the earth from the sun if light travels at 386 000 km per second?

A certain cat can catch 5 mice every hour. How many mice can this cat catch in 24 hours?

Two people, Alice and Benny are painting a fence together. If Alice can paint the fence in two hours on her own and Benny can paint the fence in three hours on his own, how long will it take both of them to paint the fence?

Solve harder problems involving ratio and rates

The ratio of men to women is 3 to 2. The ratio of women to children is 5 to 8.

Find the ratio of men to women to children

ii) If there are 820 people altogether, how many are men, women and children?

i) What is the angular velocity (rate of turning) of the minute hand of a clock in degrees per minute?

What is the angular velocity of the hour hand?

Find the first time after minute when the minute hand is perpendicular to the hour hand.
Outcomes for Pythagoras’ Theorem

At the end of this topic students will be able to

Establish the relationship between the sides of a right angled triangle by practical means
a) Draw a right triangle with sides 3cm, 4cm and 5cm and a right triangle with sides 5cm, 12cm and 13cm. Square the lengths of each of the sides in both triangles and determine the relationship between the squared lengths of these sides in each of these triangles.

Locating the hypotenuse when triangles are drawn in different ways
a) In each of these triangles (all of which are right triangles) locate the side which is the hypotenuse by going over this side with a red pen.
Use a calculator or a table to determine the square roots of numbers and distinguish between exact and approximate answers

a) Use a calculator to determine the square roots of these numbers to two decimal places
   i) 524   ii) 354   iii) 129   iv) 434

b) Simplify the following expressions writing your answer as an exact answer and as an approximate answer to two decimal places
   i) $\sqrt{40 + 30}$  ii) $\sqrt{13^2 + 15^2}$  iii) $\sqrt{(0.6)^2 - (0.1)^2}$

c) Use you calculator to determine whether the following statements are true or false
   i) $\sqrt{36} + \sqrt{64} = \sqrt{100}$
   ii) $\sqrt{9} + 16 = \sqrt{25}$
   iii) $\sqrt{289} - 225 = \sqrt{64}$
   iv) $\sqrt{169} - \sqrt{144} = \sqrt{25}$

Use Pythagoras’ theorem to find the length of the hypotenuse and the length of a shorter side in a right-angled triangle.

a) Find the length of the unknown side in each of the following right-angled triangles.
   Write your answers in exact form.
Solve simple problems using Pythagoras’ theorem

a) A rectangular soccer field is 110 m long and 70 m wide. How long is the diagonal from one corner post to the opposite corner post?

b) A ladder is leaning up against a wall. Find how long the ladder is if the base of the ladder is 3 m from the base of the wall and the ladder reaches 4 m up the wall.

c) Two ships A and B leave from port P at the same time. Ship A travels 12 nautical miles due north and ship B travels 5 nautical miles due east. How far apart are they?

d) A playground slide is made up of two right triangles as in the diagram. Giving your answer to the nearest centimetre, find:

i) the height of the slide
ii) the length of the slide

e) The walking track to a lookout rises in three stages. Find:
i) the distance walked uphill at each stage
ii) the total distance of the walk (correct to the nearest 0.1 km)

6. Use Pythagoras’ theorem to determine whether a triangle is right angled
a) Determine which of the following triangles are right-angled.
7. Outline the basic activities and beliefs of the Pythagoreans
   a) Use the Internet to research the practices of the Pythagoreans.

   b) What did they believe about the relationship between numbers and the universe?

   c) Why was Pythagoras' theorem so revolutionary?

   d) What were the religious beliefs of the Pythagoreans?

   e) How were the religious beliefs of the Pythagoreans related to their ideas about mathematics?

Solve more complex problems using Pythagoras’ theorem

   a) A block of land is in the shape of a quadrilateral. Giving your answer correct to 0.1 m, find:

      i) the distance AB
      ii) the distance BC

      ![Diagram of a quadrilateral with distances]
b) In the following diagram, OX and XP are each 1 unit long. The dotted lines are arcs of circles each with centre O.

Find the distances
i) OP 
ii) OA 
iii) OQ 
iv) OB 
v) OR 
vi) OC 
vii) OS 
viii) OD

c) A box used to store wood has the shape of a cube with side lengths 1 m. Giving your answer to the nearest cm, find the length of the longest stick that can:
   i) lie along the base of the box
   ii) fit inside the box

d) In a rectangular prism, PS = 260 mm, QS = 240 mm and SR = 144 mm. Calculate the length of
   i) PQ 
   ii) RQ
Appendix Four: Sample path diagrams used in the current study

Contents
Equations path diagram
Properties of Solids path diagram
Data representation and interpretation path diagram
Fractions, Decimals and Percentages path diagram (Note: Students did not cover the percentages outcomes)
Area, Surface area and Volume path diagram
Introduction to Algebra path diagram
Percentages, ratios and rates path diagram
Data representation and anaysis path diagram
Equations Path Diagram

1. Solve simple one-step equations
2. Solve two-step equations
3. Solve equations using grouping symbols
4. Solve equations with pronumerals on both sides
5. Solve equations including one fractional term
6. Solve equations involving more than one fractional term
7. Solve simple inequalities, graphing solutions on a number line
8. Solve problems using equations including applications to geometric diagrams
9. Translate word problems into equations and solve them
Path Diagram for Properties of Solids

A. Describing solids in terms of their geometric properties

B. Determining if two edges are parallel, skew or intersecting

C. Classifying solids on the basis of their properties

D. Sketching on isometric grid paper shapes built with cubes

E. Making sketches of simple solids and their cross-sections

F. Representing 3D objects in two dimensions

G. Determining Euler's relationship for polyhedra

H. Exploring the history of Platonic solids and how to make them

I. Making models of polyhedra
Path Diagram for Area, Surface Area and Volume

A. Developing the formula for the area of a rectangle and using the formula

B. Using the abbreviations for square units

C. Developing and using the formula for a triangle

D. Finding the area of simple composite figures

E. Developing the formula for finding the area of a parallelogram

F. Converting between units of area

G. Using the formula to calculate the area of circles

H. Identifying and drawing the cross-section of a prism

I. Developing the formula for volume of rectangular prisms $V = l \times w \times h$

J. Using the abbreviations for cubic units

K. Finding the volume of a prism given the area of a cross-section

L. Calculating the volume of right prisms with cross-sections that are rectangular and triangular

M. Using the formula to find the volume of cylinders

N. Using the kilolitre as a unit for measuring large volumes

O. Converting between metres cubed and kilolitres

Page 447
Path Diagram for Data Representation and Interpretation

A. Using the term 'mean' for average and finding the mean for a small set of data

B. Interpret and create picture graphs
C. Interpret and create column graphs
D. Interpret and create line graphs
E. Interpret and create divided bar graphs / sector (pie) graphs

You need to understand A, B, C, D, and E before doing 1, 2, 3, and 4

1) Collect, represent and evaluate a set of data as part of an investigation including data collected using the Internet
2) Determine what type of graph is best to display a given set of data
3) Discuss and interpret graphs found in the media and in factual texts
4) Identify misleading representations of data in the media
Path Diagram for Fractions, Decimals and Percentages

Proceed through the fractions outcomes first before beginning on the decimals outcomes
Path Diagram for Area, Surface Area and Volume

A. Developing the formula for the area of a rectangle and using the formula
B. Using the abbreviations for square units
C. Developing and using the formula for a triangle
D. Finding the area of simple composite figures
E. Developing the formula for finding the area of a parallelogram
F. Converting between units of area
G. Using the formula to calculate the area of circles
H. Identifying and drawing the cross-section of a prism
I. Developing the formula for volume of rectangular prisms $V = l \times w \times h$
J. Using the abbreviations for cubic units
K. Finding the volume of a prism given the area of a cross-section
L. Calculating the volume of right prisms with cross-sections that are rectangular and triangular
M. Using the formula to find the volume of cylinders
N. Using the kilolitre as a unit for measuring large volumes
O. Converting between metres cubed and kilolitres
Path diagram for Data representation and Analysis

a) Formulating key questions to provide data for a problem of interest

b) Refining key questions after a trial

d) Distinguishing continuous from discrete data

e) Finding measures of location (mean, mode and median) for small sets of data

f) Using a scientific or graphics calculator to determine the mean of a set of scores

g) Using measures of location (mean, mode, median) and the range to analyse data that is displayed in a frequency distribution table, stem-and-leaf plot, or dot plot

h) Collecting data using a random process

i) Making predictions from a sample that may apply to the whole population

j) Making predictions from a scatter diagram or graph

k) Using spreadsheets to tabulate and graph data

l) Analysing categorical data e.g. a survey of car colours
Appendix Five: Sample pre-tests and feedback sheets

Contents
Equations Pre-Test (Southwest High School, Year Eight)
Feedback on Equations Pre-Test
Fractions Pre-Test (Brindale Christian School, Year Seven)
Feedback on Fractions Pre-Test
Ratio and Rates Pre-Test (Southwest High School, Year Eight)
Feedback on Ratio and Rates Pre-Test
1. Determine if the following activities could be described as a census or a sample.
   a) Sarah wants to know what television shows people watch in her class. She asks ten people what they watch.
   b) The Australian government asks all households in Australia to declare their living arrangements.
   c) Therese wants to know what Year 8 students at her school do in her spare time. She develops a questionnaire which all students in Year 8 complete.

2. Determine if the following data sets are continuous or discrete
   a) Heights of students in a class
   b) Number of cans of soft drink students drank last weekend
   c) Age of plants
   d) Number of books students bring to school

3. For the following set of data determine the mean, mode and median
   5, 8, 8, 10, 12, 13, 21
   Mean =
   Mode =
   Median =

4. Find the mean, median and mode of this set of data
   1, 8, 3, 2, 2, 9
   Mean =
   Mode =
Median =

5. Using a calculator, determine the mean of the following data set
3, 6, 4, 7, 3, 8, 2, 9, 5, 7, 4, 8, 3, 7, 1, 6, 4, 6, 8, 9, 7, 4, 6
Mean =

6. Samuel wants to answer the following questions. Choose from the mean, mode or median (or more than one) as the best statistic to answer these questions. Give a reason for your choice.

a) How many legs do human beings have?
b) How much do houses cost in Sydney?
c) How tall are most 13 year olds?
d) How many children are there in Australian families?

7. Use the following frequency distribution table to answer the following questions

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

a) What is the mode of this set of data?
b) What is the range?
c) What is the median?
d) What is the mean?

8. Use the following frequency distribution table to answer the following questions

<table>
<thead>
<tr>
<th>Number of hours</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>I I I</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>I I</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>I I I</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>I I I</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>I I</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>I I I</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>I I I</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>I I</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>45</td>
</tr>
</tbody>
</table>

a) What was the most common number of hours?
b) What was the range in the number of hours?

9. Two classes complete the same Maths Quiz which was marked out of 5. The scores for each class are contained in the following frequency distribution tables.

<table>
<thead>
<tr>
<th>Mrs Doubtfire's class</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>Frequency</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mr Polly's class</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>Frequency</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

a) Which class did better on the test? Why? Refer to either the mean, mode or median in your answer.

10. The frequency histogram below represents the number of hours different students spent doing their homework.

<table>
<thead>
<tr>
<th>Number of hours spent on homework</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

a) How many students spent more than five hours a week doing their homework?
b) What was the average number of hours students spent doing homework?

a) How many students passed (received a mark higher than 50%)

11. The dot plot below shows the number of goals scored by a soccer team during the season.

b) What was the median score?

c) What was the range of scores?

da) What was the mean?

a) What is the most common number of goals scored in a match? (The mode)

b) What is the mean number of goals scored?

12. A group of 10 students completed their yearly exam in mathematics and their results are represented in the following stem and leaf plot

a) How many students passed (received a mark higher than 50%)

b) What was the median score?

c) What was the range of scores?

d) What was the mean?

13. Two cricket players both went in to bat ten times in one season and their scores are presented in the following stem-and-leaf plot

a) Who had the lowest score?

b) Who scored more often over 30?
c) What was the range of scores for June and Ivana? Who was the more consistent player?

b) If 10 people were asked, would this change your prediction of who would win the election? Why? Why not?

c) Determine the mean for June and Ivana during this season. Who do you think was more successful during this season? Why?

c) If all of the people who were surveyed were from Baulkham Hills, would this have an impact on how you interpreted this result?

14. Denny needs a set of random three-digit numbers to assist her to choose a random sample. How could she generate a list of random numbers?

15. Tracy reads in the paper that a survey which asked a random sample of people who they would vote for in the next federal election. The survey reported that 60% of people surveyed said they would vote for the Liberal party and 30% said they would vote for the Labor party.

a) If 1000 people were asked, who do you think would win the election?

b) If 10 people were asked, would this change your prediction of who would win the election? Why? Why not?

c) If all of the people who were surveyed were from Baulkham Hills, would this have an impact on how you interpreted this result?

16. Greg conducts a survey of favourite television shows amongst his classmates. He finds that 40% like the *Teletubbies*, 30% like the *Hoobs*, 25% like *Bob the Builder* and 5% like *Hi-5*. If you were to ask 10 people from Greg's class what was their favourite television show, how many do you think would say they like the *Hoobs*?

17. Carmen draws a graph representing the frequency of calls received at her work each minute.
a) What is the most likely number of calls you might hear in the next minute?

18. Use the following graph to answer these questions

![Cars bought in West Wyalong graph]

- What is the most common car sold in West Wyalong?
- Nathan analyses this data and suggests that the mean car is Toyota. What is wrong with this statement?
- Charlotte suggests that 30% of people in West Wyalong buy Holdens. Is this correct?

19. Graham wants to find out how people get to school each day. He is interested in the type of transport people use, who takes them to school, how far people live from school and whether they get to school with other students or arrive on their own. Write some trial questions which Graham could ask students at his school to obtain some answers to these questions.

20. After running a trial, Graham finds that a large number of people catch the bus, and that the distances people lived from school differed greatly and Graham had difficulty making sense of the answers people gave him.
a) How else could he ask his question about the type of transport people take if he is only interested in whether they catch a bus or not?

b) How could he ask the question about distance so that he gets fewer answers?
Equations Pre-Test

Time: 30 minutes

Part A
Solve these equations. Show all working

1. \( b - 3 = -1 \)
2. \( \frac{x}{5} = 5 \)
3. \( 2d + 15 = 17 \)
4. \( 4w - 17 = 9 \)
5. \( \frac{t}{4} + 6 = 8 \)
6. \( 4(c - 2) = 12 \)
7. \( -3(x + 5) = 15 \)
8. \( 5 - 3x = 2x + 15 \)
9. \( x + 4 = 4x + 11 \)
10. \( \frac{t}{3} + 6 = 18 \)
11. \( 5 + x = \frac{x}{2} + 7 \)
12. \( \frac{2a}{3} + \frac{a}{2} = 12 \)
13. \( \frac{x + 4}{3} = \frac{x + 1}{2} \)

Part B
14. Find all the solutions of \( x - 3 < 6 \) and represent these solutions on a number line

15. Find all the solutions of \( 2x + 5 \leq x + 8 \) and represent these solutions on a number line
16. In the diagram below, find the size of each angle

17. A certain number, when added to 12 is 3 less than 4 times the same number. What is the number?

18. A father is 20 years older than his daughter. In four years he will be three times her age. How old is the father now?
# Feedback on Equations Pre-Test

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Relevant Questions</th>
<th>Personal Understanding</th>
<th>Group Understanding</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Solve simple one-step equations using appropriate methods</td>
<td>Q1, Q2</td>
<td></td>
<td></td>
<td>Make sure you use <em>appropriate methods</em>, even when doing easy questions</td>
</tr>
<tr>
<td>2. Solve two-step equations using appropriate methods</td>
<td>Q3, Q4, Q5, Q10</td>
<td></td>
<td></td>
<td>Reduce to a one-step equation then solve</td>
</tr>
<tr>
<td>3. Solve equations using grouping symbols</td>
<td>Q6, Q7</td>
<td></td>
<td></td>
<td>Review grouping symbols, including questions with negative numbers</td>
</tr>
<tr>
<td>4. Solve equations with pronumerals on both sides</td>
<td>Q8, Q9</td>
<td></td>
<td></td>
<td>For any equation, each step should be given its own line of working</td>
</tr>
<tr>
<td>5. Solve equations including one fractional term</td>
<td>Q5, Q10, Q11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Solve equations involving more than one fractional term</td>
<td>Q12, Q13</td>
<td></td>
<td></td>
<td>Find the lowest common multiple or cross-multiply</td>
</tr>
<tr>
<td>7. Solve simple inequalities, graphing solutions on a number line</td>
<td>Q14, Q15</td>
<td></td>
<td></td>
<td>You may need to look at a Year Nine textbook for examples of these</td>
</tr>
<tr>
<td>8. Solve problems using equations including applications to geometric diagrams</td>
<td>Q16</td>
<td></td>
<td></td>
<td>Give reasons when using geometric properties</td>
</tr>
<tr>
<td>9. Translate word problems into equations and solve them</td>
<td>Q17, Q18</td>
<td></td>
<td></td>
<td>First step should be to work out an appropriate equation</td>
</tr>
</tbody>
</table>

**Star Rating**

* - Developing Understanding
** - Good Understanding

*** - Excellent Understanding
Pythagoras’ Theorem Pre-Test

Time 1 hour

Name: _______________

1. In each of these triangles (all of which are right triangles) locate the side which is the hypotenuse by going over this side with a red pen.
2. Use a calculator to determine the square roots of these numbers to two decimal places

i) 224

ii) 384

3. Simplify the following expressions writing your answer as an exact answer and as an approximate answer to two decimal places

i) \( \sqrt{13^2 + 25^2} \)

ii) \( \sqrt{40 + 20} \)

iii) \( \sqrt{(0.6)^2 - (0.1)^2} \)

4. Use you calculator to determine whether the following statements are true or false

i) \( \sqrt{36} + \sqrt{64} = \sqrt{100} \)

ii) \( \sqrt{9} + 16 = \sqrt{25} \)

iii) \( \sqrt{289} - 225 = \sqrt{64} \)

iv) \( \sqrt{169} - \sqrt{144} = \sqrt{25} \)

5. Find the length of the unknown side in each of the following right-angled triangles. Write your answers in exact form.

\[
\begin{align*}
4m & \quad x \, \text{m} \\
\end{align*}
\]
6. A rectangular soccer field is 110 m long and 70 m wide. How long is the diagonal from one corner post to the opposite corner post?

7. Two ships A and B leave from port P at the same time. Ship A travels 8 nautical miles due north and ship B travels 15 nautical miles due east. How far apart are they?
8. Spaghetti is stored in a circular container of height 23 cm and diameter 9 cm. Find (to the nearest mm) the length of the longest piece of spaghetti that can fit inside the storage container.

9. In this triangular prism, find the length of BE and EC.

10. Determine which of the following triangles are right-angled.

   - Triangle with sides 4.5 m, 7.5 m, 6 m
   - Triangle with sides 85 m, 40 m, 55 m
11. Why was Pythagoras' theorem such a revolutionary idea to Greek mathematics?

12. When and where did Pythagoras and his followers live?
13. A block of land is in the shape of a quadrilateral. Giving your answer correct to 0.1 m, find:

i) the distance AB
ii) the distance BC

![Diagram of a quadrilateral block of land]

14.

A 20 m conveyor belt is set up in a factory to transfer goods from the bottom corner of one level in a factory to the top corner as shown. Calculate the height of the factory wall correct to 0.1 m.

![Diagram of a conveyor belt in a factory]

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15. A sports drink comes in circular cans each with diameter of 6 cm and a height of 16 cm. A 20 cm straw is placed in the can with one end touching the base. Giving your answer to the nearest mm, find

i) the maximum length that the straw can extend outside a can

ii) the minimum length that the straw can extend outside a can.
### Feedback on Pythagoras’ Theorem Pre-Test

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Relevant Questions</th>
<th>Personal Understanding</th>
<th>Group Understanding</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Establish the relationship between the sides of a right angled triangle by practical means</td>
<td>Not tested</td>
<td></td>
<td></td>
<td>See the outcomes for a sample question</td>
</tr>
<tr>
<td>Locating the hypotenuse when triangles are drawn in different ways</td>
<td>Q1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use a calculator or a table to determine the square roots of numbers and distinguish between exact and approximate answers</td>
<td>Q2, Q3, Q4</td>
<td></td>
<td></td>
<td>If a question requires an exact answer you need to leave your answer in surd form (for example, write $\sqrt{2}$ instead of 1.414)</td>
</tr>
<tr>
<td>Use Pythagoras’ theorem to find the length of the hypotenuse and the length of a shorter side in a right-angled triangle.</td>
<td>Q5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve simple problems using Pythagoras’ theorem</td>
<td>Q6, Q7, Q8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Use Pythagoras’ theorem to determine whether a triangle is right angled</td>
<td>Q10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Outline the basic activities and beliefs of the Pythagoreans</td>
<td>Q11, Q12</td>
<td></td>
<td></td>
<td>The complete answer to Q11 requires some understanding of the nature of numbers and how the Greeks' understanding of numbers changed with Pythagoras' Theorem.</td>
</tr>
</tbody>
</table>
8. Solve more complex problems using Pythagoras’ theorem

<table>
<thead>
<tr>
<th>Star Rating</th>
<th>√ Developing Understanding</th>
<th>√√ Good Understanding</th>
<th>√√√ Excellent Understanding</th>
</tr>
</thead>
</table>

Q9, Q13, Q14, Q15

A general comment - you should always show working in an exam in such questions!
1. Write down three fractions equivalent to the given fraction.

2. Place the following fractions on this number line: \(\frac{1}{4}, \frac{3}{4}, \frac{5}{8}, \frac{7}{8}, \frac{9}{8}, \frac{11}{8}, \frac{13}{8}, \frac{15}{8}, \frac{17}{8}, \frac{19}{8}\).

Which fractions are equivalent to \(\frac{1}{2}\)?

3. Change this pair of fractions into equivalent fractions so that both fractions have the same denominator.

\(\frac{3}{4}\) and \(\frac{1}{2}\)

Simplify fully the following fractions:

\(\frac{8}{24}\)

\(\frac{20}{100}\)

c) \(\frac{5}{12}\)

4. Simplify fully the following fractions:

\(\frac{24}{8}\)

\(\frac{100}{20}\)

c) \(\frac{3}{4}\)

5. Simplify fully the following fractions:

\(\frac{1}{4} + \frac{1}{4}\)

\(\frac{3}{8} + \frac{2}{8}\)

\(\frac{1}{2} + \frac{1}{3}\)

\(\frac{3}{5} + \frac{1}{4}\)
\[
\frac{7}{8} - \frac{1}{3} = \frac{13}{5} + 2\frac{1}{5} = \\
\]

9. Write these improper fractions as mixed numerals

a) \(\frac{12}{7}\)

b) \(\frac{35}{3}\)

c) \(\frac{56}{7}\)

Write these mixed numerals as improper fractions

a) \(3\frac{1}{4}\)

b) \(5\frac{1}{3}\)

c) \(3\frac{2}{7}\)

\[
\frac{1}{2} \times \frac{1}{3} = \\
\frac{2}{3} \times \frac{3}{4} = \\
\]

Page 476
\[ \frac{1}{2} + \frac{1}{3} = \]
0.85 + 0.05 =

\[ \frac{1}{3} + \frac{1}{4} = \]
0.42 + 0.8 =

\[ \frac{3}{2} \times \frac{2}{3} = \]
Round off these decimals to two decimal places
0.7483

0.6 + 0.4 =
0.764

0.3 + 0.21 =
3.4732

0.5 + 0.32 + 0.21 =
2.9972

0.532 – 0.23 =

0.164 – 0.064 =
Paul calculates the cost of buying 15.45 litres of petrol if petrol costs 89.9 cents per litre. His calculator gives the answer 1388.955. How much will it cost in dollars and cents?

0.5 \times 0.2 =

0.15 \times 0.8 =

0.04 \times 0.2 =
33. Use appropriate notation to write these repeating decimals
0.5555556.....

Page 477
0.77777… b) \(0.\overline{7}\)

0.3535353535 …. c) \(0.\overline{35}\)

0.1463714637….

36. Convert these percentages to fractions and decimals
15%

34. Convert these fractions into decimals and then into percentages
\(\frac{3}{8}\) 30%

\(\frac{5}{8}\) c) 55%

\(\frac{8}{9}\)

37. Find three-fifths of 95

38. Find 0.6 of 40

39. Find 60% of 700

35. Convert these decimals into fractions and then into percentages
a) \(0.\overline{8}\)
## Feedback for Fractions and Decimals Pre-Test

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Relevant Questions</th>
<th>Personal Understanding</th>
<th>Group Understanding</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Finding equivalent fractions using a variety of methods</td>
<td>Q1, Q2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Expressing two fractions as fractions with the same denominator to enable addition and subtraction</td>
<td>Q3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Reducing a fraction to its lowest equivalent form</td>
<td>Q4</td>
<td></td>
<td></td>
<td>Q5 needs to be looked at by most people. What is the area 1 textbook covers? Then multiply by 100!</td>
</tr>
<tr>
<td>D. Adding and subtracting fractions</td>
<td>Q5, Q6, Q7, Q8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. Expressing improper fractions as mixed numbers and vice versa</td>
<td>Q9 (both of them!)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F. Adding mixed numbers</td>
<td>Q10, Q11, Q12, Q13, Q14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G. Subtracting a fraction from a whole number</td>
<td>Q15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H. Multiplying and dividing fractions through various concrete methods leading to written processes</td>
<td>Not tested</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. Multiplying and dividing fractions and mixed numbers</td>
<td>Q16, Q17, Q18, Q19, Q20</td>
<td></td>
<td></td>
<td>You may need to look at a Year Nine textbook for more examples of these</td>
</tr>
<tr>
<td>J. Adding, subtracting, multiplying and dividing decimals</td>
<td>Q21 - Q30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K. Rounding decimals to a given number of places</td>
<td>Q31, Q32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L. Using the notation for recurring (repeating) decimals</td>
<td>Q33</td>
<td></td>
<td></td>
<td>Q15 H - Many wrote the wrong answer as 49 cm². How did they get this mistake?</td>
</tr>
<tr>
<td>M. Converting fractions to decimals (terminating and recurring) and percentages</td>
<td>Q34</td>
<td></td>
<td></td>
<td>Make sure you use the value of π given in the question. Also, be careful rounding off.</td>
</tr>
<tr>
<td>Percentages</td>
<td>Q35</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>-----------------------------</td>
<td>------------------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. Converting terminating decimals to fractions and percentages</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O. Converting percentages to fractions and decimals</td>
<td>Q36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P. Calculating fractions, decimals and percentages of quantities</td>
<td>Q37, Q38, Q39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did you recognise that 15D and 15H are trapeziums as well as composite shapes?</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Star Rating</th>
<th>Developing Understanding</th>
<th>Good Understanding</th>
<th>Excellent Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

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1. Change these percentages into fractions
   a) 70%
   b) 37.5%
   c) 24.5%
   d) 0.2%

2. Change these percentages into decimals
   a) 54%
   b) 0.3%
   c) 145%

3. Find
   a) 50% of 300
   b) 0.3% of 3000

4. Find
   a) \( \frac{3}{5} \) of $60
   b) \( 3 \frac{1}{5} \) of 480

5. Find
   a) 0.35 of 80

6. During the Commonwealth Games, Australia won 15% of all the gold medals on offer. If there were 600 medals available to be won, how many did Australians win?

7. From a survey of voters before the last federal election, 47.1% were intending to vote for the Coalition and 43.2% were intending to vote for the Labor party. If 1143 people were surveyed, how many were intending to vote for someone else?

8. Increase $450 by 10%

9. Decrease 605 by 20%

10. Two items are on sale at a shop. One is originally priced at $55 and has a discount of 15% advertised. The other item is originally priced at $70 and has a discount of 25%. Which item will be cheaper?
11. Increase 56 by 250% 

12. Find 145% of $60 

13. A house bought in 1980 for $60 000 was sold last year for $270 000. What was the percentage increase? 

14. A car was bought for $32 000 and sold for $40 000. What was the profit as a percentage of the cost price? 

15. A house bought for $250 000 is sold for $420 000. Find the profit as percentage of the cost price. 

16. Raymond buys shares for $9 000 and sells them for $6 000. Find the loss as percentage of the cost price. 

17. Place these in ascending order 
   \[
   \frac{3}{8}, 0.35, 30\%, 0.41, \frac{3}{7}
   \]

18. In a test Sarah scored 36 out of 80. What was her score as a percentage? 

19. What fraction is 3 km of a marathon if a marathon is 42.2 km long? 

20. Simplify these ratios 
   a) 6:8 
   b) 15:40 
   c) 4:8:14 
   c) 14/8 
   e) 2/3 : 5 
   f) 0.65 : 2.15
21. Identify which of these ratios is equivalent to 3:5
   30 : 50
   6 : 14
   1 : 5/3

22. Write the following ratios in the form 1 : X
   a) 5 : 8
   b) 0.4 : 0.15

23. Write the following ratios in the form Y : 1
   a) 6 : 8
   b) 2/3 : 5/8

24. A car operates with a gear ratio of 1 : 1.8. If the first gear rotates 60 times, how many times does the second gear rotate?

26. In a certain class there are 12 boys and 10 girls. What is the ratio of boys to girls in simplest form?

27. A concrete mix consists of sand, cement and gravel in the ratio 3 : 2 : 8. Find the amount of water required if there is 5 kg of cement available.

28. Divide 600 into the ratio 6 : 9

29. Frances and Laura set up a business together in which Frances invests $4000 and Laura invests $3000. If their business makes a profit of $42 735 how much should each person receive?

30. Simplify these rates
   a) 280 km in 5 hours
   b) 585 wheelbarrows in 9 hours

31. Simplify these rates
   a) 60 km/ hour = _______ m/s
   b) 50 L/min = _________ L/hr
32. Sarah drives 630 km in 7 hours. Find her average speed over her journey.

33. Greta and her family are planning a trip to Queensland. If they are planning to travel 1200 km at an average speed of 80 km per hour, how long will it take her family to drive to Queensland if they do not take a break?

34. Two people, Alice and Benny are painting a fence together. If Alice can paint the fence in two hours on her own and Benny can paint the fence in three hours on his own, how long will it take both of them to paint the fence?
# Feedback for Percentages, Ratios and Rates Pre-Test

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Relevant Questions</th>
<th>Personal Understanding</th>
<th>Group Understanding</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. converting percentages to fractions and decimals</td>
<td>Q1, Q2</td>
<td></td>
<td></td>
<td>Fractions should have whole numbers as the numerator and denominator</td>
</tr>
<tr>
<td>B. Calculating fractions, decimals and percentages of quantities</td>
<td>Q3, Q4, Q5, Q6, Q7</td>
<td></td>
<td></td>
<td>The word ‘of’ in mathematics normally means ‘×’. Replace ‘of’ with ‘×’ and calculate.</td>
</tr>
<tr>
<td>C. Increasing and decreasing a quantity by a given percentage</td>
<td>Q8, Q9, Q10</td>
<td></td>
<td></td>
<td>When increasing by 10% for example, find 110%. When decreasing by 10% find 90% of the original amount</td>
</tr>
<tr>
<td>D. Interpreting and calculating percentages greater than 100% eg an increase from 6 to 18 is an increase of 200%; 150% of $2 is $3</td>
<td>Q11, Q12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. Expressing profit and/or loss as a percentage of cost price or selling price</td>
<td>Q13, Q14, Q15, Q16</td>
<td></td>
<td></td>
<td>Always use the cost price as the denominator unless otherwise specified.</td>
</tr>
<tr>
<td>F. Ordering fractions, decimals and percentages</td>
<td>Q17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G. Expressing one quantity as a fraction or a percentage of another eg 15 minutes is ( \frac{1}{4} ) or 25%, of an hour</td>
<td>Q18, Q19</td>
<td></td>
<td></td>
<td>Change the whole number into an improper fraction first</td>
</tr>
<tr>
<td>H. Using ratio to compare quantities of the same type</td>
<td>Q24, Q26, Q27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. Writing ratios in various forms eg. 4/6, 5:3, 4 to 1, and 3:2:1</td>
<td>Q20, Q21, Q22</td>
<td></td>
<td></td>
<td>When writing ratios, they should be simplified so that both numbers are whole numbers, unless a ratio fo the form X : 1 or Y:1 is required.</td>
</tr>
<tr>
<td>J. Simplifying ratios</td>
<td>Q20, Q21</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>K. Applying the unitary method to ratio problems</td>
<td>Q22, Q23</td>
<td></td>
<td></td>
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<tr>
<td>L. Dividing a quantity in a given ratio</td>
<td>Q28, Q29,</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>M. Calculating rates from given information eg 150 kilometres travelled in 2 hours</td>
<td>Q30 - Q34</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Star Rating**  
- ✓ Developing Understanding  
- ✓ ✓- Good Understanding  
- ✓ ✓ ✓ Excellent Understanding
Appendix Six: Interview protocol for teachers and students

Protocol for Interviews with Teachers

1. What aspects of the model do you think worked well and what aspects did not work so well?

2. How do you feel about your role as the teacher when teaching in this way?

3. Can you describe how you interacted with the students in your class while teaching this way?

4. Do you think that your students managed to learn all of the outcomes through this approach?

5. Some topics may appear to be better taught this way while other topics might be better taught directly. What do you think were the main factors that determined whether or not it was appropriate to teach it in a certain way?

6. How do you think students benefited from the experience?

7. Were there any ways in which they were disadvantaged?

8. Did you notice any changes in the culture of the classroom? Changes in teacher student relationships, student/student relationships, approaches to mathematics, student expectations about what happens in the classroom, etc.

9. Were there any other changes in the classroom that you noticed which may not be revealed in a change to their test scores?

10. Do you think you might make use of any of these methods again in the future?
Protocol for Interview with Students

What do you remember learning in each topic?

Describe a typical lesson when doing each topic.
What normally happened during one of those lessons?
What did the students do?
What did the teacher do?
Who took on the role of the teacher?
How effective do you think your learning and the learning of the class was during those lessons?
Why do you think students were learning or were not learning during those lessons?

How did you yourself come to understand the work in each of the topics?

On a scale of 1 to 10 rate your understanding of each topic.
Why did you choose that number?

On a scale of 1 to 10 rate your interest in each topic.
Why did you choose that number?

On a scale of 1 to 10 rate how much you enjoyed each topic.
Why did you choose that number?

When working in groups did you find you spent more or less time talking about mathematics?
Why do you think that was the case?
In the case of groups spending less time talking about mathematics, how could this problem have been overcome?

When working in groups did you find that you were able to cover the work more quickly or not?
Why do you think this was the case?
If your group experienced any problems how do you think these might have been overcome?

When working in groups did you learn more or less than in the normal classes? Why do you think this was the case? If your group experienced any problems how do you think these might have been overcome?

Do you think your group worked well together? If so, why? If not, why not?

When you were working in groups, what was your teacher doing in the classroom to assist you in your learning?

Do you think the activity of designing the topic test and practising using other students’ topic tests helped you to understand this topic? Why/Why not?

Do you think your classroom changed as a result of working in groups? (Why?) For example, Did relationships between students change? Did relationship between the teacher and the class change? Do you think the atmosphere of your classroom changed? Do you think the way that you went about doing mathematics changed? Do you think the attitudes of your class towards mathematics changed? Do you think the way you learnt mathematics changed? Do you think your understanding of mathematics changed?

Would you like to work in groups again in the future? Why/Why not?

Several surveys of people who employ school leavers say that one of the things they are looking in people they employ is that their employees are able to work as part of a team.
What did you learn about working as part of a team when working in small groups in mathematics?

Surveys also suggest that employers are looking for people who are self-motivated, and able to work independently. What did you learn about working independently and being self-motivated when working in small groups in mathematics?

Different people like to learn in different ways, some like to work with other people, some like to hear explanations from a teacher, while others like to work independently. How would you describe your preferred way of learning? Did you find that there were opportunities during your time working in groups to learn in this way?

At the beginning of each topic done in groups you completed a pre-test and received feedback on your understanding before even starting the topic. How did completing the pre-test and receiving feedback contribute to your learning during these topics?

How did these things help your group to work together and plan how you might develop your understanding for each topic?
Appendix Seven: Student questionnaire

Student Questionnaire

Personal Information

ID Number: __________

School: _________________

Age (in years and months): __________

Male/Female (Circle One)

Languages spoken at home other than English:

____________________

____________________

____________________

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Part A
Choose from the following list which statement best describes your current understanding of each of the outcomes listed below. Place the appropriate letter in the box next to each statement.

A  Very good
B  Good
C  OK
D  Little understanding of this idea

Percentages, Ratios and Rates
A. Converting percentages to fractions and decimals
   _____
B. Calculating fractions, decimals and percentages of quantities
   _____
C. Increasing and decreasing a quantity by a given percentage
   _____
D. Interpreting and calculating percentages greater than 100%
   _____
E. Expressing profit and/or loss as a percentage of cost price or selling price
   _____
F. Ordering fractions, decimals and percentages
   _____
G. Expressing one quantity as a fraction or a percentage of another
   _____
H. Using ratio to compare quantities of the same type
   _____
I. Writing ratios in various forms eg. 4/6, 5:3, 4 to 1, and 3:2:1

J. Simplifying ratios

K. Applying the unitary method to ratio problems

L. Dividing a quantity in a given ratio

M. Calculating rates from given information eg 150 kilometres travelled in 2 hours

Data Representation, Data Analysis and Evaluation
A. Reading and interpreting tables, charts and graphs

B. Organising data into a frequency distribution table (class intervals to be given for grouped data)

C. Drawing and using dot plots

D. Drawing and using stem-and-leaf plots

E. Using the terms 'cluster' and outlier' when describing data

F. Formulating key questions to provide data for a problem of interest

G. Refining key questions after a trial

H. Recognising the differences between a census and a sample

I. Distinguishing continuous from discrete data

J. Finding measures of location (mean, mode and median) for small sets of data
K. Using a scientific or graphics calculator to determine the mean of a set of scores

L. Using measures of location (mean, mode, median) and the range to analyse data that is displayed in a frequency distribution table, stem-and-leaf plot, or dot plot

M. Collecting data using a random process

N. Making predictions from a sample that may apply to the whole population

O. Making predictions from a scatter diagram or graph

P. Using spreadsheets to tabulate and graph data

Q. Analysing categorical data eg a survey of car colours

Part B
1) Choose from the following list which statement best describes the way you came to understand each of the outcomes listed below. Place the appropriate letter in the box next to each statement. If you need to provide more details, feel free to use the space available after each outcome.

A  I taught myself

B  I understood this idea before doing this topic

C  Another student in my group explained this idea to me

D  Another group explained this idea to our group

E  I understood this idea from the teacher’s explanation
F I used a textbook to understand this idea

G A person from outside our classroom explained this idea to me

H I used a book other than a textbook to work out what this idea meant

I I still do not understand this idea

J Other (give details)

Percentages, Ratios and Rates
A. Converting percentages to fractions and decimals

B. Calculating fractions, decimals and percentages of quantities

C. Increasing and decreasing a quantity by a given percentage

D. Interpreting and calculating percentages greater than 100%

E. Expressing profit and/or loss as a percentage of cost price or selling price

F. Ordering fractions, decimals and percentages

G. Expressing one quantity as a fraction or a percentage of another

H. Using ratio to compare quantities of the same type

I. Writing ratios in various forms eg. 4/6, 5:3, 4 to 1, and 3:2:1

J. Simplifying ratios

K. Applying the unitary method to ratio problems
L. Dividing a quantity in a given ratio

M. Calculating rates from given information eg 150 kilometres travelled in 2 hours

Data Representation, Data Analysis and Evaluation
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E. Using the terms 'cluster' and outlier' when describing data

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G. Refining key questions after a trial

H. Recognising the differences between a census and a sample

I. Distinguishing continuous from discrete data

J. Finding measures of location (mean, mode and median) for small sets of data

K. Using a scientific or graphics calculator to determine the mean of a set of scores

L. Using measures of location (mean, mode, median) and the range to analyse data that is displayed in a frequency distribution table, stem-and-leaf plot, or dot plot
M. Collecting data using a random process

N. Making predictions from a sample that may apply to the whole population

O. Making predictions from a scatter diagram or graph

P. Using spreadsheets to tabulate and graph data

Q. Analysing categorical data eg a survey of car colours

In the space provided, write down all of the areas outside the classroom where you think what you have learnt in the topic Data Representation, Data Analysis and Evaluation might be applicable.

In the space provided, write down all of the areas outside the classroom where you think what you have learnt in the topic Percentages, Ratios and Rates might be applicable.
Part C
Rate how appropriate each of these statements are for describing the way that you
developed your understanding of Data Representation, Data Analysis and Evaluation.

I learnt about *Data Representation, Data Analysis and Evaluation* by

- listening to explanations provided by the classroom teacher
  
  False  Partly false  More false than true  More true than false  Partly true True

- working with other students to develop a shared understanding
  
  False  Partly false  More false than true  More true than false  Partly true True

- listening to explanations provided by other students
  
  False  Partly false  More false than true  More true than false  Partly true True

- discussing ideas with the teacher
  
  False  Partly false  More false than true  More true than false  Partly true True

- practising mathematics questions on my own
  
  False  Partly false  More false than true  More true than false  Partly true True

- discussing ideas with other students
  
  False  Partly false  More false than true  More true than false  Partly true True

- reading a textbook
  
  False  Partly false  More false than true  More true than false  Partly true True
working on practise questions with other students

Rate how appropriate each of these statements are for describing the way that you developed your understanding of *Percentages, Ratios and Rates*.

I learnt about *Percentages, Ratios and Rates* by

listening to explanations provided by the classroom teacher

working with other students to develop a shared understanding

listening to explanations provided by other students

discussing ideas with the teacher

practising mathematics questions on my own

discussing ideas with other students

reading a textbook
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<th>False</th>
<th>Partly false</th>
<th>More false than true</th>
<th>More true than false</th>
<th>Partly true</th>
<th>True</th>
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working on practise questions with other students

<table>
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<tr>
<th>False</th>
<th>Partly false</th>
<th>More false than true</th>
<th>More true than false</th>
<th>Partly true</th>
<th>True</th>
</tr>
</thead>
</table>
Part D

The following questions relate to your ideas about what happens in mathematics classrooms.

In the space below describe what normally happened in your classroom when you were studying the topics *Percentages, Ratios and Rates* and *Data Representation, Data Analysis and Evaluation*

In the space below describe what normally happened in your classroom when you were studying the topic *Area, Surface Area and Volume and Algebraic Techniques* in Term 2

What do you expect of other students in your mathematics class?

How do you think your teacher should teach your class?
We know that different people prefer to learn in different ways. How do you prefer to learn about mathematics?

Some students described what they were doing in class one day as “doing mathematics”. What comes into your head when you think about the phrase “doing mathematics”?

What do you think are the advantages and disadvantages of working in groups?

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
</table>
Rate how important each of these activities is to you in your mathematics class by circling one of the numbers on this scale

a) Listening to other students

<table>
<thead>
<tr>
<th>Not very important</th>
<th>Very important</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

b) Talking about mathematics

<table>
<thead>
<tr>
<th>Not very important</th>
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c) Doing practise questions

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d) Reading the textbook

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e) Teacher explaining something

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f) Helping other students

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g) Thinking of different ways to do a question

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h) Knowing how to get the right answer

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Knowing why mathematicians have developed certain methods for solving problems

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j) Asking the teacher questions

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k) Asking other students questions

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Part E
For each item below circle one of the options on the scale provided

1. In mathematics, I prefer course material that really challenges me so I can learn new things.

Not at all true of me
1 2 3 4 5 6 7

Very true of me

2. If I study in appropriate ways, then I will be able to learn the material in mathematics.

Not at all true of me
1 2 3 4 5 6 7

Very true of me

3. When I take a test I think about how poorly I am doing compared with other students.

Not at all true of me
1 2 3 4 5 6 7

Very true of me

4. I think I will be able to use what I learn in mathematics in other subjects.

Not at all true of me
1 2 3 4 5 6 7

Very true of me

5. I believe I will receive an excellent grade in mathematics.

Not at all true of me
1 2 3 4 5 6 7

Very true of me
6. I’m certain I can understand the most difficult material presented in mathematics.

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7. Getting a good grade in mathematics is the most satisfying thing for me right now.

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8. When I take a test I think about the items on other parts of the test I can’t answer.

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9. It is my own fault if I don’t learn the material in mathematics.

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10. It is important for me to learn about mathematics in this class.

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11. The most important thing for me right now is improving my overall results at school, so my main concern in mathematics is getting a good grade.

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12. I’m confident I can understand the basic concepts taught in mathematics.

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13. If I can, I want to get better grades in mathematics than most of the other students.

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14. When I take tests I think of the consequences of failing.

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15. I’m confident I can understand the most complex material presented by my mathematics teacher.

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16. In mathematics, I prefer course material that arouses my curiosity, even if it is difficult to learn.

Not at all true of me
1  2  3  4  5  6  7
Very true of me

17. I am very interested in the content area of mathematics.

Not at all true of me
1  2  3  4  5  6  7
Very true of me

18. If I try hard enough, then I will understand the course material in mathematics.

Not at all true of me
1  2  3  4  5  6  7
Very true of me

19. I have an uneasy, upset feeling when I take an exam.

Not at all true of me
1  2  3  4  5  6  7
Very true of me

20. I’m confident I can do an excellent job on the assignments and tests in mathematics.

Not at all true of me
1  2  3  4  5  6  7
Very true of me
21. I expect to do well in mathematics.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

22. The most satisfying thing for me in mathematics is trying to understand the content as thoroughly as possible.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

23. I think the course material in mathematics is useful for me to learn.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

24. When I have the opportunity in mathematics, I choose tasks that I can learn from even if they don’t guarantee a good grade.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

25. If I don’t understand the course material in mathematics, it is because I didn’t try hard enough.

Not at all true of me

Very true of me

1 2 3 4 5 6 7
26. I like the subject matter of mathematics.

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27. Understanding the subject matter in mathematics is very important to me.

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28. I feel my heart beating fast when I take an exam.

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29. I’m certain I can master the skills being taught in mathematics.

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30. I want to do well in mathematics because it is important to show my ability to my family, friends, employer or others.

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31. Considering the difficulty of this level of mathematics, the teacher, and my skills, I think I will do well in mathematics.

Not at all true of me  Very true of me
1    2    3    4    5    6    7

32. When I study classroom notes and relevant chapters of the textbook for mathematics, I outline the material to help me organise my thoughts.

Not at all true of me  Very true of me
1    2    3    4    5    6    7

33. During class time I often miss important points because I’m thinking of other things.

Not at all true of me  Very true of me
1    2    3    4    5    6    7

34. When studying for mathematics, I often try to explain the material to a classmate or a friend.

Not at all true of me  Very true of me
1    2    3    4    5    6    7

35. I usually study in a place where I can concentrate on my mathematics.

Not at all true of me  Very true of me
1    2    3    4    5    6    7
36. When studying for mathematics, I make up questions to help focus my studying.

Not at all true of me  
1 2 3 4 5 6 7  
Very true of me

37. I often feel so lazy or bored when I study for mathematics that I quit before I finish what I planned to do.

Not at all true of me  
1 2 3 4 5 6 7  
Very true of me

38. I often find myself questioning things I hear or read in mathematics to decide if I find them convincing.

Not at all true of me  
1 2 3 4 5 6 7  
Very true of me

39. When I study for mathematics, I practise saying material to myself over and over.

Not at all true of me  
1 2 3 4 5 6 7  
Very true of me

40. Even if I have trouble learning the material in mathematics classes, I try to do the work on my own, without help from anyone.

Not at all true of me  
1 2 3 4 5 6 7  
Very true of me

41. When I become confused about something I’m studying for mathematics, I go back and try to figure it out.

Not at all true of me  
1 2 3 4 5 6 7  
Very true of me
42. When I study for mathematics, I go through relevant chapters of the textbook and class notes and try to find the most important ideas.

Not at all true of me 1 2 3 4 5 6 7
Very true of me

43. I make good use of my study time for mathematics.

Not at all true of me 1 2 3 4 5 6 7
Very true of me

44. If what I am reading in the textbook is hard, I change the way I read the textbook.

Not at all true of me 1 2 3 4 5 6 7
Very true of me

45. I try to work with other students from this class to complete my homework.

Not at all true of me 1 2 3 4 5 6 7
Very true of me
46. When studying for mathematics, I read my notes and relevant chapters of the textbook over and over again.

Not at all true of me  Very true of me
1 2 3 4 5 6 7

47. When a theory, interpretation or conclusion is presented in class or in a textbook, I try to decide if there is good supporting evidence.

Not at all true of me  Very true of me
1 2 3 4 5 6 7

48. I work hard to do well in mathematics even if I don’t like what we are doing.

Not at all true of me  Very true of me
1 2 3 4 5 6 7

49. I make simple charts, diagrams, or tables to help me organise course material.

Not at all true of me  Very true of me
1 2 3 4 5 6 7

50. When studying for mathematics, I often set aside time to discuss the course material with a group of students from the class.

Not at all true of me  Very true of me
1 2 3 4 5 6 7
51. I treat the course material as a starting point and try to develop my own ideas about it.

Not at all true of me | Very true of me
---|---
1 | 7
2 | 6
3 | 5
4 | 4
5 | 3
6 | 2
7 | 1

52. I find it hard to stick to a study schedule.

Not at all true of me | Very true of me
---|---
1 | 7
2 | 6
3 | 5
4 | 4
5 | 3
6 | 2
7 | 1

53. When I study for mathematics, I pull together information from different sources, such as classroom discussions, textbooks and discussions with other students.

Not at all true of me | Very true of me
---|---
1 | 7
2 | 6
3 | 5
4 | 4
5 | 3
6 | 2
7 | 1

54. Before I study new course material thoroughly, I often skim it to see how it is organised.

Not at all true of me | Very true of me
---|---
1 | 7
2 | 6
3 | 5
4 | 4
5 | 3
6 | 2
7 | 1

55. I ask myself questions to make sure I understand the material I have been studying in mathematics.

Not at all true of me | Very true of me
---|---
1 | 7
2 | 6
3 | 5
4 | 4
5 | 3
6 | 2
7 | 1

56. I try to change the way I study in order to fit the requirements of different topics and the teacher’s approach.

Not at all true of me | Very true of me
---|---
1 | 7
2 | 6
3 | 5
4 | 4
5 | 3
6 | 2
7 | 1
57. I often find that I have been studying for mathematics but don’t know what it was all about.

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58. I ask the teacher to clarify concepts I don’t understand well.

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59. I memorise key words and formulae to remind me of important concepts in this class.

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60. When course work is difficult, I give up or only study the easy parts.

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61. I try to think through a topic and decide what I am supposed to learn from it rather than just reading about it when studying.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

62. I try to relate ideas in mathematics to those in other subjects whenever possible.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

63. When I study for mathematics, I go over my class notes and make an outline of important concepts.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

64. When studying mathematics, I try to relate the material to what I already know.

Not at all true of me

Very true of me

1 2 3 4 5 6 7

65. I have a regular place set aside for studying.

Not at all true of me

Very true of me

1 2 3 4 5 6 7
66. I try to play around with ideas of my own related to what I am learning in mathematics.

67. When I study for mathematics, I write brief summaries of the main ideas from textbooks and classroom discussions.

68. When I can’t understand the material in mathematics, I ask another student in my mathematics class for help.

69. I try to understand the material in mathematics by making connections between the textbook and the concepts taught in class.

70. I make sure I keep up with homework and assignments for mathematics.

71. Whenever I read or hear about an idea in mathematics, I think about possible alternatives.
72. I make lists of important formulae for mathematics and memorise the lists.

Not at all true of me  Very true of me
1 2 3 4 5 6 7

73. Even when class work is dull and boring, I manage to keep working until I finish.

Not at all true of me  Very true of me
1 2 3 4 5 6 7

74. I try to find students in my mathematics class whom I can ask for help if necessary.

Not at all true of me  Very true of me
1 2 3 4 5 6 7

75. When I study for mathematics I try to determine which ideas I don’t understand well.

Not at all true of me  Very true of me
1 2 3 4 5 6 7
76. I often find that I don’t spend very much time on mathematics because of other activities.

Not at all true of me  2  3  4  5  6  Very true of me
1                        7

77. When studying for mathematics, I set goals for myself in order to direct my activities.

Not at all true of me  2  3  4  5  6  Very true of me
1                        7

78. If I get confused taking notes in mathematics, I make sure I sort it out afterwards.

Not at all true of me  2  3  4  5  6  Very true of me
1                        7

79. I rarely find time to review my notes or relevant chapters of the textbook before an exam.

Not at all true of me  2  3  4  5  6  Very true of me
1                        7

80. I try to apply ideas from textbooks in other class activities.

Not at all true of me  2  3  4  5  6  Very true of me
1                        7
Appendix Eight: Sample lesson taught by Mr Grey at Brindale Christian School

Prior to the teacher entering the room, students sat chatting with each other. Mr Grey entered the room and began by announcing that the topic for the lesson was Surface Area. Students immediately stopped talking to each other and listened to the instructions Mr Grey presented from the front of the class. He then proceeded to tell them that this work was not in the textbook, so he would be providing them with extra notes on the topic. From a previous lesson, Mr Grey had provided some notes on Surface Area which he asked a student to dictate as he wrote on the board:

Surface Area

An extension of area is finding the total outside area of a 3-D solid. To do this, you need to consider the area of each face

Mr Grey then said “What I am going to do is start with an example which should make it a bit clearer.”

He started by drawing a diagram of a square pyramid on the board which he waited for them to copy down and then chose five books around the classroom to make comments about their diagrams which were too small - “I want all your diagrams to be at least four lines down”.

The dimensions of the square pyramid were then added in – including the altitude of one of the triangles.

Underneath the pyramid Mr Grey wrote

This 3-D solid has ____ faces

and asked all students to write this down. He then asked a student to give him the number of faces. The first student (Andrew) answered “5”.
Mr Grey then asked what type of shape each face was. The question was directed at Andrew, but another student called out the correct answer which he then wrote on the board

1 square and 4 triangles

He then showed them how to set out the working for this question. As he wrote he explained what each of the symbols he was writing actually meant.

\[ S.A. = A(square \text{ subscript}) + 4 \times A(triangle \text{ subscript}) \]

Mr Grey then explained that he expected this working out all the way to Year 12.

He then asked what the next step would be. Students offered different formulas such as \( l \times h \) or \( lh \). Mr Grey then said “You put the formulas down. You always put the formulas down for every area question”.

Even though some students argued that they could put the numbers down, Mr Grey insisted that even though this is a relatively easy question to work out, it is best to always write the formulae. As he wrote the formulae on the board he asked the class for the formulae for a square and a triangle. Students were able to give him the formulae for these.

The next line of working that Mr Grey asked of the students was the numbers. At this point he told the students that he did not want them to use calculators that day. Together the class worked through the calculations involved. Mr Grey encouraged the class to look for short cuts for calculating 4 times a half times 10 times 8 - “I want you to do these types of calculations in your head”.

Once the students had completed this calculation, Mr Grey focused their attention on the correct units. He identified a common mistake which was to think that because such questions involved 3-D shapes the units must be cubed.
Mr Grey then turned to the textbook for further examples using volume questions as sources of possible shapes of which students could calculate the surface area.

The second example was a rectangular prism with dimensions 6cm, 7cm and 4cm.

To do this question, Mr Grey referred the students back to the previous example; “How did we start the pyramid off?” He then directed his question to one of the students in the class – Stuart.

Stuart started to outline the step “\(SA = A_{\text{rectangle}} + \)” before Mr Grey suggested that he start by identifying the number of different sides - “That’s what goes on in your brain – how many faces are there?”

Mr Grey then led them through the process of working out what he outlined in the first example. The only difference in this example was that the method outlined did not distinguish between different types of rectangular faces and Mr Grey alerted them to this fact by writing down the working incorrectly and asking students to comment on what he had written.

At this point in the lesson, some of the students called out that it was a mistake and started to claim a bottle of Coke. Mr Grey explained to me that there was an arrangement between himself and the students that if they spotted a mistake of Mr Grey’s he bought them a bottle of Coke.

Some of them raised the problem of writing \(6A_{\text{rectangle}}\) and Mr Grey asked them about whether or not it was possible to write algebraic statements leaving out the multiplication sign. The class agreed that it was OK to write it as \(6A_{\text{rectangle}}\).

Mr Grey then asked them what the problem was with writing \(6A_{\text{rectangle}}\). One student answered correctly that the rectangles have different dimensions and are therefore different rectangles. He then asked them how many different types of rectangles there are, before using a textbook to demonstrate which rectangles are the same and which ones are different.
We're going to call them the top, and the top refers to the bottom as well, the side and the end, I don't mind you leaving that line there (the line about the 6 rectangles) but then do this;

\[= 2 \times A_{\text{rectangle \ "top"}} + 2 \times A_{\text{rectangle \ "side"}} + 2 \times A_{\text{rectangle \ "end"}}\]

As Mr Grey wrote this on the board he elaborated on the meaning of each symbol.

Mr Grey then asked the class to give him the formulae and he proceeded to set out the working on the board in a similar manner to the previous question.

\[= 2lb + 2lb + 2lb\]

I want you to keep it directly below the description of the shape, so that all down is the top, all down is the side and all down is the ends.

\[= 2 \times 6 \times 7 + 2 \times 6 \times 4 + 2 \times 4 \times 7\]

Mr Grey then asks “How do you multiply 2 times 6 times 7 in your head – Sean”

Sean’s method was first to do 6 times 7 to get 42, then he multiplied it by 2 to get 84. Mr Grey asked for other methods and mentioned that there are six different ways of performing these calculations.

Mr Grey then moved on and performed the other calculations, writing down the numbers

\[= 84 + 48 + 56\]

Students were then asked to perform this addition in their heads. Different students started to give answers and various suggestions were offered. Mr Grey then provided them with his approach which was to add up the units and then add up the tens. “You just do it like you are doing a normal addition”. Students, therefore, were encouraged to mentally perform a task using a method they had seen before.

\[= 188cm^2\]
Mr Grey then gave them a third example taken from the textbook which was also a rectangular prism – with different dimensions of 10, 8 and 5. Specific instructions were given this time about the number of lines they should use for their diagram.

The previous method was left on the board for students to follow.

Mr Grey then provided the same working out for the next question. “I want it just like that.”

The maths of this topic is not hard, it’s your setting out – because you’re going to get hard ones next year and if you can’t do the easy ones properly, you’ll have problems with the later problems.

To assist students with this particular question, Mr Grey helped them to identify the top, side and ends of each shape. Students were given a few minutes to complete this question for themselves.

He then wrote out the working for this question in exactly the same format as the previous question.

I want to see the pluses below the pluses, the equals below the equals all the way down.

Mr Grey then went carefully through finding the numbers associated with each of the different steps.

\[ SA = 2 \times A_{\text{rectangle “top”}} + 2 \times A_{\text{rectangle “side”}} + 2 \times A_{\text{rectangle “ends”}} \]
\[ \quad = 2 \times lb + 2 \times lb + 2 \times lb \]
\[ \quad = 2 \times 10 \times 5 + 2 \times 8 \times 10 + 2 \times 5 \times 8 \]
\[ \quad = 100 + 160 + 80 \]
\[ \quad = 340 \text{ cm}^2 \]

Students were then given an exercise to do from the textbook – the exercise was originally designed as an exercise on volume, however Mr Grey asked them to find the surface area of the first five shapes in this exercise. Students were given eight minutes to do these five
questions, with the expectation that students would only get three done during the lesson, and the remaining two would be done for homework.

Instructions were given to students, including what the heading should be and how the answers to each question should be set out.

Mr Grey wrote this work on the board, including some homework from another book which provided students with practice at basic skills in mathematics.

For the next few minutes the class worked quietly on this exercise. There was no talk amongst students as they worked through these questions. One student asked Mr Grey about the third question – working out what is the top, sides and ends. Mr Grey made a suggestion about what should be regarded as the top, sides and ends.

I know it makes no difference, but if we all do the same, then we will know what each other are talking about.

After this Mr Grey worked with one student who was experiencing difficulty with the work.

The final question in this exercise required students to use Pythagoras’ theorem to find the edge of a side on a triangular prism. This problem was identified by Mr Grey before the end of the lesson and he pointed the problem out to the class. The depth of the triangular prism was 12, and the base and height of the triangular base were 4 and 5 respectively. He asked students for a comment on question (e)

Can you do question (e)? What is the problem with (e)?

Mr Grey worked through the question on the board as he did with each of the other questions. He got to the point of using the unknown edge in the calculation for the area of the rectangle and stopped here, showing the class that it can’t be done using their current understanding. By this stage it was time for students to go to their next class, and he told the students to omit question (e).
Appendix Nine: Board work provided by Ms Diamond for classroom lesson on measurement and error

On the board during this lesson, Ms Diamond wrote

The error in a measurement is the limitation in accuracy due to the scale used.

Example

Measure the above line with a rule marked in cm.

Ms Diamond then measures this line on the board using a metre rule and then wrote down

41 cm to the nearest cm.
The actual length is between 40.5 cm and 41.5 cm.
The error in measurement is 0.5 cm.

Ms Diamond then read through these notes on the board before giving a second example.

Measure the line with a rule marked in mm.

(Answer) 41.2 cm to the nearest mm.
The actual length is between 41.15 cm and 41.25 cm
The error in measurement is 0.5 mm

Underneath, Ms Diamond wrote

Note: Error does not mean mistake
The greatest possible error (absolute error) is half the smallest unit.
Appendix Ten: Log of classroom lessons observed

Log of Observations and Visits to Classrooms (Note: coll = collaborative classroom, non = non-collaborative classroom)

21\textsuperscript{st} May – Ms Martin (coll) (80 minutes)

24\textsuperscript{th} May – Mr Grey (non) (40 minutes)

31\textsuperscript{st} May – Ms Black (coll) (40 minutes)

4\textsuperscript{th} June – Ms Martin (coll) (80 minutes)

4\textsuperscript{th} June – Mr Smith (non) (45 minutes)

4\textsuperscript{th} June – Mr Grey (non) (55 minutes)

7\textsuperscript{th} June – Ms Gold (coll) (40 minutes)

12\textsuperscript{th} June – Mr Smith (non) (55 minutes)

12\textsuperscript{th} June – Ms White (coll) (80 minutes)

18\textsuperscript{th} June – Ms Diamond (non) (80 minutes)

21\textsuperscript{st} June – Ms Gold (coll) (40 minutes)

25\textsuperscript{th} June – Ms Black (coll) (1 hour)

26\textsuperscript{th} June – Ms White (coll) (80 minutes)
26th July – Ms Black (coll) (40 minutes)

2nd August – Ms Gold (non) (40 minutes)

7th August – Ms Diamond (coll) (80 minutes)

9th August – Mr Grey (coll) (40 minutes)

13th August – Ms Martin (non) (80 minutes)

13th August – Mr Smith (coll) (1 hour)

13th August – Ms Gold (non) (55 minutes)

16th August – Ms Black (non) (40 minutes)

21st August – Ms White (coll) (80 minutes)

30th August – Mr Smith (coll) (40 minutes)

3rd September – Ms White (non) (80 minutes)

10th September – Mr Smith (coll) (1 hour)

10th September – Mr Grey (coll) (1 hour)

13th September – Ms Gold (non) (40 minutes)

17th September – Mr Smith (coll) (1 hour)

17th September – Mr Grey (coll) (45 minutes)

18th September – Ms White (non) (80 minutes)
20th September – Ms Gold (non) (40 minutes)

24th September – Ms Diamond (coll) (80 minutes)

24th September – Ms Black (non) (55 minutes)

33 classes observed. Total classroom time observed of 32 hours.

Ms Black –
  coll – 2h 20 min
  non – 1 h 30 min

Mr Grey -
  coll – 2h 25 min
  non – 1 h 35 min

Mr Smith -
  coll – 3 h 40 min
  non – 1 h 40 min

Ms Gold -
  coll – 1 h 20 min
  non – 2 h 15 min

Ms Diamond –
  coll - 2 h 40 min
  non – 1 h 20 min

Ms White –
  coll – 4 h
  non – 2 h 40 min

Ms Martin –
  coll – 2 h 40 min
  non – 1 h 20 min
Appendix Eleven: Forms of collaboration evident at Brindale Christian School

Parallel workers

At Brindale Christian School there were very few groups who did not collaborate at all. From all of the observed lessons, only one of the groups (in Ms Gold’s class) displayed this general pattern of interaction. Within this group, students would work quietly on their own work rarely asking the person next to them for help. During the lessons observed, Ms Gold would often check the progress of each of the members of this group to ensure that they were working through relevant exercises for each of the outcomes.

When asked, very few students commented on having worked individually rather than collaboratively during the collaborative lessons. In Ms Gold’s class a student referred to a pair of students who separated themselves from the other two students in the group and concentrated on completing their homework during the class. Another student in Ms Black’s class mentioned that she worked on her own at times when other members of her group were not cooperating. These instances of students working individually were rare compared to the number of comments students made relating to their group’s collaborative efforts. A total of 147 different sections were coded as students working collaboratively together with 44374 characters compared with only 6 comments with 610 characters relating to students working individually.

Minimum work

Similarly, there were only a small number of groups observed who spent the majority of the time during class talking about issues not related to mathematics. One of the groups in Ms Gold’s class and one of the groups in Mr Grey’s class could be categorised as “minimal work” groups with members of this group spending the bulk of their time discussing unrelated issues after being distracted from the discussions they were initially having about a mathematical problem.

Co-operative groups

For some of the groups, the principal activity category was co-operation rather than collaboration. At the beginning of the lesson, one of the members of the group who took on
the role of “leader” would allocate tasks to each member of the group. These tasks normally involved each group member taking a certain outcome or section of an exercise and either developing an understanding of the outcome or completing the set portion of the exercise. Many of these groups worked quietly throughout the lessons observed with students working individually on their allocated outcome or exercise. Again, however, this was a relatively rare occurrence in the classrooms observed.

**Peer Tutoring**

Within several groups, there was the clear emergence of two separate groups consisting of two pairs rather than a collaborative group of four. Often the pairing would involve the student from the first quartile joining up with the student from the fourth quartile, and the students from the middle two quartiles forming a second pair.

> Yes, I found that within a group there would obviously be one person who is probably the quickest at catching a concept and they’d often have to work with a person specifically who has the most trouble and then the other two who were pretty much middle of the range would keep working together. (Mr Grey)

Peer tutoring of this form was particularly evident in groups where there was one member of the group whose English was relatively weak. In such groups the student who had the most complete understanding of the topic would often take on the responsibility for teaching this weaker student using paper and pen to demonstrate slowly how to answer different questions.

> Well sometimes - well the people in my group, they're really bad at English and I tried to help them with everything. Sometimes they asked some more questions to me and everything and I tried to answer them. (Mr Smith’s class)

In Ms Gold’s class this model of a group splitting into two was evident in one of the groups in particular which included the student who had come first in the year in the most recent exam. Within this group, the following discussion was conducted by the two middle quartile students. One of the students is trying to teach the other one, although it the process of doing so he is clearly trying to understand the question himself.
Ryan: OK, if you get \( n \) packets, that will give you 5 times \( n \). If a car travels 40 km each hour, how far does it go in two hours.

David: 80.

Ryan: OK, in three hours?

David: 240 (Ryan: 3 times 4. What's three times 4?) 120. Okay, if it travels \( t \) - you get 40 times \( t \). 40\( t \). Okay 3h hours. 40 times 3 is …..

Ryan: Warren, is 4 times 2\( a \) eight \( a \)?

Warren: Yes

Ryan: Good. Okay write those letters up. (Ms Gold's class)

**Parallel/Collaborative groups**

Some of the groups observed spent similar amounts of time working individually and together as a group. They would set exercises for themselves to complete which would take up to half of the lesson time interspersed with discussions about questions that presented difficulties to particular members of the group. For example, when students were describing what typically happened in their classrooms, one student in Ms Gold’s class described classroom activity in the following manner.

If anyone had any problems we would just stop working, go through that, make sure everything was solid, get their minds to try to figure that out and then complete the exercise. Then once - we'd complete the exercise we .......... a few revision questions and then, that was it.

In Mr Smith’s class one student described his group’s interaction emphasising a similar pattern

Well we'd mark our homework and we did an exercise every single day, sometimes like two exercises and we'd revise what we did the last day to see if anyone had problems or anything. Then we'd mark homework together and we did some examples if we had difficulties.

**Micro-classrooms**

Many of the groups formed themselves into “micro-classrooms” – that is, an individual within the group, normally the student from the first quartile though not always, would take
responsibility for teaching the other members of the group what they either already knew or had worked out on their own. In Mr Grey’s class for example, this would often involve the student writing on the chalkboard in the same manner that Mr Grey might when explaining how to do a question.

In Mr Smith’s classroom several students took responsibility for teaching other members of their group. David from one of the groups reported that the way he learnt best was if someone else from his group explained it to him.

Andrew: I found it probably the best way for understanding anything, is if you don’t know it is getting it to be said, explained by someone who does know it. It will come out easier and it did happen a lot, because Sue is probably the smartest one in our group. Everyone turned to her for an explanation and we kind of got through it after she explained it, so it made it easier. (Mr Smith’s class)

What is interesting about David’s group is that he was the student from the first quartile and Sue was the student from the second quartile. However, during the collaborative learning lessons observed, it was clear that Sue was the leader of the group to whom the other members of the group looked for assistance.

In Ms Black’s class, this model was also evident as in the following example.

Cheryl: Answer the following questions using these two tables (distance ready reckoner and a table of postage charges). OK, you know the tables that look like this.

Britney: With steps.

Karoline: Yeah, steps.

Cheryl: A step graph!

Britney: How come Orange goes to …..

Karoline: I don’t even know how to use them.

Britney: Don’t you know how to use them? I do.

Karoline: How do you use them?
Britney: It says "how far it is from Orange to Sydney?", and then you just look at the first graph it says Orange (Karoline: Do you go down like this?) It says Orange there (Karoline: and you go down to Sydney) and it says Sydney, (Karoline: so you go all the way along there)

Karoline: Isn't it, isn't it 197

Britney: No, no, no, its 261 because look - its that, and that's how far to Sydney..

Karoline: So that's how you use them.

Britney: And the next graph.

Karoline: And the next graph.

Britney: It says how much would it cost to send 35 kg from Orange to Sydney. So Orange to Sydney, you've already worked out is 261 km. It says here 200 km to 300 (on the table) so you look here it says 35 kg so you go here so its $18

Karoline: Ohhh, Is that how you use them?

Britney: Yeah

Karoline: That's pretty easy. Thank you.

Britney: Pleasure!

The activity category of micro-classrooms was the most common activity observed within Brindale Christian School classrooms. Furthermore, teachers commented that students would attempt to teach other students using much the same methods that they had used as teachers with the whole class.

For some of the groups, however, it was apparent that there was a reciprocal relationship between many of the students who taught each other and learnt from each other. A student in Ms Gold’s class, for example, described what happened in the classroom as a complex web of “teachers” and “students” working together.

Karl: I reckon our group was - they were able to help themselves, but a lot of the time, Nathan was helping Collette and Collette was helping Nathan. Nina didn't really understand any of that so I went to Wesley's group a lot of the time and got help from him and taught Joshua, 'cause Wesley was helping Stephen a lot of the time, so our group was capable of doing it but I got help from other groups.
**Collaborative groups**

During the classes observed and the classroom discussions transcribed there was evidence that groups were able to work together to develop their understanding of different outcomes. During Ms Black’s class, for example, the students would often work together to understand how to use different graphs. In the following example, all three of the participants actively contributed to the understanding of the other group members.

Alex: Now we're doing F. Hurry up there's only two questions for the whole thing!

Sam: So how much would it cost to ship 35kg from Orange to Sydney.

Alex: Well, $8.70

Sam: How far is it from Orange to Sydney? We've got that …. Orange to Sydney, so it weighs 35 kg. So its that one …here

Alex: No, its over 400.

Sam: Ohh that's distance, sorry. Over 400 and its (Alex: $8.70). Its over … What did you say? $8.70 (Alex: $8.70)

Alex: No the second one, its $8.70.

Sam: A1 its 497 km, for A2 its 800 its $8.70.

Alex: OK now we're after the numbers, what are the numbers.

David: What's two? $8.70.

When students were interviewed at the conclusion of the program there were also indications that such collaboration had taken place. In Mr Smith’s class, for example, David reported that his group had worked together to teach one of the other students in his group.

….at first we had difficulties because like Ricky, we had to help him the most, 'cause he couldn’t understand English and it was a bit hard, had to explain to him how to do the questions and how to get the right answer, but as soon as like he got the concept, the rest of us, we pretty much worked together well. As soon as we helped him get to the level that he needed to get to, it worked well from there. (Mr Smith’s class)
When students were asked at the conclusion of the topic whether or not they spent more time talking about mathematics in collaborative classrooms or in teacher-led classrooms, students from Brindale Christian School suggested that they spent more time talking about mathematics in collaborative classrooms than in normal classrooms. Not only did they spend more time talking about mathematics, though, but the nature of this talk was often of a collaborative nature.

David: I think we talked more, because usually whenever we come to check the homework and stuff, we'd explain, like we got round to each person and they'd say, “How did you get that?” And so we’d explain it for them before we went to the new subjects. So we did that for every time and we just - it helped everyone because it helped Ricky, because if he did a mistake you’d see, “Oh you didn’t do this,” and he’d correct himself and for the lessons he just showed that he - like he got better at it. (Mr Smith’s class)

Susan: Everyone was teaching Ricky how to do divisions or fractions. The teacher made ........ the second fraction and multiply - he’d understand it at first but we all got to the blackboard and explained each way at the end and it kind of got to him, ‘cause we were explaining it more than once. Different examples and how if this happens if that occurs, you do this. (Mr Smith’s class)

In fact, for this group, the focus of teaching the weakest member of the group became the defining feature of their group’s interaction. Mr Smith commented on this particular group himself as being relatively successful in terms of their interaction with each other.

A2 I felt that there were some really exciting lessons in some of my groups when some - three people in the group would all get behind the other person who was having trouble. They’d all sit there and make sure they’d set an outline and make sure they were doing it and that was really good because they were all focussing on this one person to make sure they understood it. Whereas that person usually in class, would sort of follow along but would often times fall behind a little bit and they were all making sure that that person understood it. That was really good, that was a positive thing they’d done. (Mr Smith)

When asked how they developed their understanding of different topics, some of the students responded that “mainly we just talked about it and tried to figure it out” and that they “worked it out with each other” (Mr Smith’s class). In Ms Black’s class one of the students commented that “Yeah, it’s complicated to work by yourself but when you get all together you figure it out”.

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Cross-group collaboration

Students in different groups appeared to move freely between their group and other groups to find out what they could about different outcomes. More often than not, however, this involved an individual from one group being sent by the rest of the group, or choosing for their own reasons to visit another group to learn from them about a particular outcome.

Irene. Sometimes I would ask people in my group or one - I think the division and multiplication of fractions together, I asked my brother, and then sometimes we’d go to David’s group. (Mr Smith’s class)

David: We were able to - say like Anthony wanted to get help from our group and like ‘cause his group didn’t understand it properly, they’d have to come over and ask us. We’d explain it to him and one lesson when we’d be all finished and all the people except his group, we went over and helped his group and so it just helps them to get to know how - get to know each other a bit better. (Mr Smith’s class)

Karl: I reckon our group was - they were able to help themselves, but a lot of the time, Nathan was helping Collette and Collette was helping Nathan. Nina didn’t really understand any of that so I went to Wesley’s group a lot of the time and got help from him and taught Joshua, ‘cause Wesley was helping Stephen a lot of the time, so our group was capable of doing it but I got help from other groups. (Ms Gold’s class)

These quotes indicate the reciprocal nature of this interaction that often occurred. In Mr Smith’s class in particular, groups often helped each other and were in turn helped by other students in the class. In Ms Black’s class, there was one student who became the class expert on Plato and all of the other groups in the class would approach her to find out about Plato.

Interviewer: And you listen to them about mathematics?

Nerida: Yes, and Sarah from group A she gave, she - I listened to her for Plato – we needed some information on Plato.

Interviewer: Good. Karoline?

Karoline: Mostly the two teachers, but there’s also people in my group which is James, Laura and Skye, and at one point Sarah did sort of explain a bit of Plato to us but not heaps.

John: Mainly the teachers, probably Sarah because of Plato and probably Karoline because she knows more and the rest of our group.
Interviewer: List the people who other students listen to.

Nerida: I think that most of the students would listen to the teachers for advice but ummm,

Interviewer: You’ve all mentioned Sarah.

Nerida: Yes, because she went and researched Plato one night. I think most people went to her for information for Plato. (Ms Black’s class)

Sarah was identified by all of the students interviewed as the resident expert on Plato. However, it became clear that Sarah was not one of the students identified as a “bright” student by other students. During the collaborative learning time there were opportunities for many different students to be approached as “experts” on different subjects.

Students and teachers were both conscious of the change in the classroom environment brought about by students helping each other. One student in Mr Smith’s class stated that “everyone had to come to someone for help”. For Ms Black, the manner in which students would help each other made a significant impression upon her

..they weren’t afraid to contribute to someone else’s learning and the other students weren’t intimidated by somebody else knowing better. You’d often hear them say, “Ask so and so, they’ll tell you.” Like because kids always know who is supposedly dumb in their terms, or who is supposedly capable in their terms and you’re having a lend of yourself if you don’t think that that exists within a classroom. But to have them interact in a positive way for each other, is I think one of the biggest benefits of working this way

Teacher-group collaboration

Within each classroom interaction between the teacher and students in the class would typically involve the teacher talking with a group of students instead of to individuals or the whole class. Students would either ask the teacher for help on a specific type of question or the teacher would initiate a discussion as part of their monitoring of progress.

Carla: Yeah, he would come round, have a look at our work and we weren’t trying it all right, he would just tell us the way it should sort of be. Yeah, he would just go around the classroom and if someone needed help, he’d be there to help us. (Mr Grey’s class)

Corinne: He’d probably just ask us to work and he’d just sit there in case someone needed help and to come out to him and then he’d just explain it on the blackboard or sometimes when more
than one person came out to him with the same question, he’d stop the whole class and he’d show them this question so no-one needed to come out anymore. Just sometime during the lesson, if he sees that this might be a different question, he’d stop the class and just explain that and let us get back to work, but it may just be that occasionally you’d go up to him and ask. (Mr Smith’s class)

In the following excerpt from a class discussion, the teacher supported the collaboration of the group

Ms Black: Now what sorts of places would you look for such things (data)?

Nerida: In ads.

Ms Black: Don’t go to making them yet, what are you being asked to do initially? OK, what you’re being asked to initially is what?

Sonia: Find graphs.

Ms Black: Find graphs.

Nerida: We could look for inflation and farming industry and like that.

Ms Black: Good, good - anything else?

Nerida: Inflation and farming industry you could look at how people have designed the how many of this and that that you’ve put into a garden you could find a picture graph.

Ms Black: Picture graphs that’s a good idea. What other sort of graphs, what other places?

Sonia: Business things.

Nerida: Line graphs.

Ms Black: Business things what sort of places would you find those sorts of graphs?

Nerida: Banks and like shares.

Ms Black: Shares, OK. What about … Hmm?

Andrew: Profit and loss.

Ms Black: Profit and loss is another thing. When you have a line graph, when you see the line going up what does it mean?
Nerida: Profits going up.

Ms Black: OK, and if you see it going down?

Sonia: They're going down.

Nerida: It depends on what you are graphing actually. It could be inflation going up is bad, cause that's I mean if it's a graph of profit its good.

Ms Black: Good, so what you are talking about is its not good enough to see a line going up and saying oh this is good.

Nerida: You've got to work out what it really means.

Ms Black: You've got to understand it. So you need to find examples of as many different types of graphs as you can a range of sources you have to be able to bring them here to be studied together, tomorrow, weather section of the paper is good to look as well.

Sonia: Oh rainfall.

Andrew: Yeah umm pie graphs sometimes

Ms Black: OK

This questioning approach reflects the Socratic method detailed in Plato’s *Meno* in which the teacher attempts to elicit the correct answers from the student. While this method focuses on students providing specific answers, the teacher also tries to encourage students to develop their understanding further by focusing the attention of the group on a contradiction that emerges from their current approach to a particular problem. The teacher in one class, working with a group of students attempting to use a distance ready reckoner (see Appendix 3), makes explicit two separate answers that could be obtained to the same question.

Ms Black: For the question on the distance from Orange to Sydney, Kevin says its 261 km. Why can’t you go from Orange to Sydney - 480?

By focusing on contradictions in this manner the teacher’s interaction is similar to the Lakatosian approach described by Brodie (2000) in which the teacher attempts to support the development of ideas around conjectures and refutations. This method can be
problematic if the students are unable to break down their approach into smaller lemmas that reveal the exceptions which result in a reformulation of the method (Brodie, 2000). In the current example, however, the students were well equipped to discuss their interpretations of the ready reckoner to develop their own methods for reading the table.

Teacher interaction with groups of students often focused on supporting students’ learning through providing suggestions about how they might develop their understanding. This included directing them to a particular part of the textbook, helping them plan what they might do that lesson or encouraging one student to assist other students in the group as in the following example.

Connie: I'm not quite sure how to read this table?

Ms Black: Who thinks they know? What do you think, Martin? Try and explain it

Martin: Well, if it's like from Moree to Wollongong (Ms Black: Yep), you go down here and its Wollongong and you go down here and they meet there (Ms Black: Yep).

Connie: What I don't get is see, you've got this whole bottom line, what does that mean?

M: Well it's …

Ms Black: Show her what you are doing there.

M: So from Newcastle goes down to the end, so where it's Newcastle from Sydney and you go down there.

C: Ohhhhh.

Ms Black: So you understand how it works now? Good.

While there were many different forms of collaboration evident at Brindale Christian School, the most common forms of collaboration could be categorised as “micro-classrooms” or collaborative groups who would receive input from the teacher during each lesson when difficulties arose. Both forms of collaboration represent the reshaping of prior ways of learning that existed in the classrooms at Brindale Christian School. This included reshaping the existing pattern of learning where the teacher would provide students with a method that could be used to solve particular types of questions. Students would often ask each other “How do you do this type of question?” referring to an identified group of
questions that they did not as yet have a sufficient method for solving. Sometimes individuals within their group would teach the rest of the group, at other times the classroom teacher was called upon for help, and less frequently groups of students would work together to develop their own method.
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Table 2: Correlation matrix for Learning Strategies Factors

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<td>Q74</td>
<td>.15</td>
<td>.18</td>
<td>.13</td>
<td>.16</td>
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<td>.03</td>
<td>.06</td>
<td>.52</td>
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<td>.61</td>
<td>.11</td>
<td>.24</td>
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<td>.52</td>
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<td>Q76</td>
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<td>-.19</td>
<td>-.31</td>
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<td>Q77</td>
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<td>.45</td>
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<td>.45</td>
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</tbody>
</table>
Appendix Thirteen: Sample questions for students to use in discussions

Suggestions for Collaborative Discussions

To learn from other people in your group choose an outcome and begin by asking each person to explain their own ideas about that outcome. Once they have given their explanation, the other members of the group take turns questioning, clarifying, summarising and predicting. Here are some example questions and statements that you could use in your discussions.

Explaining

I think we do questions like this by ….

I think that …… means ……..

I think that we need to do …….. to get the right answer because ….

Remember: You should always be prepared to give reasons and ask others to give reasons for their work

Questioning

How would you solve this type of question?
What is your method for solving these questions?

Do you know any other ways to do these types of questions?

Could I use your method to solve a different type of question?

Clarifying
How did you get that answer?

Why did you choose that method?

Summarising
What I think you are saying is …..

If I put what you said into my own words I would describe it like this …..

So the method suggested by the textbook is to …..
Predicting

Using you method on this question I would get ……

So if the question said …… then I would do it using your method by ……
Appendix Fourteen: Assessment task developed by students whilst studying Properties of Solids

<table>
<thead>
<tr>
<th>Outcome A</th>
<th>Name of shape</th>
<th>Number of faces</th>
<th>Number of flat faces</th>
<th>Number of curved surfaces</th>
<th>Shape of top base</th>
<th>Shape of bottom base</th>
<th>Has a vertex?</th>
<th>Is it a pyramid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square pyramid</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>Triangle</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Cone</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>Circle</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Rectangular prism</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>Square</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Parallelogram prism</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>Rectangle</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Cylinder</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Circle</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Triangular pyramid</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>Triangle</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Rectangular pyramid</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>Rectangle</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>X</td>
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<tr>
<td>Sphere</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>Circle</td>
<td>X</td>
<td>no</td>
<td>yes</td>
<td>X</td>
</tr>
</tbody>
</table>

What is the name of this shape?

Spherical pyramid

Write a description of this shape identifying all its features.

- It has an octagon on each end of the shape. It has eight vertices.
- It can be sold with difficulty.
- It has uniform construction.
- It has 10 sides altogether.
- The angle of the faces are octagons and they angles vary.
- Long ones to.
Outcome B

Draw a pair of parallel lines.

Draw a skew line.

Draw three intersecting lines.

Mark the parallel edges with arrows, colour two intersecting edges red & colour two skew edges blue.
Outcome C

Label the following solids as: prisms-right or oblique, pyramids, cones, cylinders or spheres.

Draw an oblique cube.

Draw a right square pyramid.
Outcome D

Sketch the following solid shapes on the provided isometric grid paper.
Outcome E

Draw & name the shape of the cross section if the prism below was cut where shown.

Cylinder

Sketch a square pyramid.

Sketch a cone.

Sketch a triangular pyramid.
Outcome F

Draw a top front & side view in two dimensions of this solid.

Given the following two dimensional views of a solid, draw the three dimensional solid

Match the following nets to their solids.
**Outcome C**

For each of the following solids determine the number of faces, edges & vertices

- **Tetrahedron**
  - $F = 3$
  - $E = 6$
  - $V = 4$
  - $F + V - E = 0$

- **Icosahedron**
  - $F = 20$
  - $E = 30$
  - $V = 12$
  - $F + V - E = 9$

- **Dodecahedron**
  - $F = 12$
  - $E = 30$
  - $V = 20$
  - $F + V - E = 0$

- **Octahedron**
  - $F = 8$
  - $E = 12$
  - $V = 6$
  - $F + V - E = 4$

- **Hexagonal prism**
  - $F = 7$
  - $E = 15$
  - $V = 8$
  - $F + V - E = 0$

- **Icosahedral prism**
  - $F = 9$
  - $E = 20$
  - $V = 12$
  - $F + 2E - V = 28$
Outcome H

Name the 5 Platonic solids.

I. Hexagon
II. Pentagram
III. Octagon
IV. Heptagon
V. Octahedron

Circle which of the following nets are nets of the Platonic solids.

Plato was born into the aristocracy of the city of Greece. What was Plato seeking after which was impossible to find in this world? Perfection. Why were the five Platonic solids so important to Plato & his followers? Because they were perfect solids and that is what he was seeking.
Outcome 1

Trace three of the following nets & construct their solids using the sheets of blank paper at the back of this test

- Triangular Prism
- Triangular Pyramid
- Pentagonal Prism
- Octagonal Prism
- Heptagonal Prism
- Octahedron
### Topic 1 - Area, Surface Area, Volume

#### Daily Learning Journal for Maths Group E

<table>
<thead>
<tr>
<th>Outcome for This Lesson:</th>
<th>Part A - Area</th>
<th>Date: 21/5/21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our Group's Aim for This Lesson:</td>
<td>To find the area of a rectangle and squares. Do 12 E. 2 and Exercise 12 F LHS.</td>
<td>Complete/Incomplete</td>
</tr>
</tbody>
</table>

**Today… How did you work towards developing your understanding and what did you learn?**

Include some examples to demonstrate your understanding and show that you can use the appropriate mathematical language needed to express your ideas.

- We all learned how to find the area of rectangles and squares.

  1. **E.g.:**
     - We times the length and width to get the area.
     - 2.6 cm × 2 cm = 5.2 cm²
     - 12.5 cm × 0.7 cm = 8.75 cm²

- We times 15.5 by 7 to get 108.5.

- Then we get the area for the rectangle and square in the middle. We take away the area in the rectangle and square by the whole area (108.5 cm²).

- First we divide the shape. We get the first part (8 cm) and add to the other rectangle (15.5 cm).

**Did Your Group Have Any Problems and How Did You Overcome Them?**

- We didn’t have any problems finding the area of the rectangles and squares.

**Tomorrow’s Outcome Will Be:**

- B - Using the abbreviations for square units

**This Outcome Will Assist Our Group Understanding Because…**

- We need to find our about the units to get the correct answer.

**Our Homework for To-Night Is:**

- Finish 12 F and put in questions for assessment.
<table>
<thead>
<tr>
<th>Exercise 1.12 LHS of 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>This outcome will assist our group in understanding in the classroom.</td>
</tr>
<tr>
<td>This outcome will be</td>
</tr>
<tr>
<td>developing the formula for finding the area of a parallelogram.</td>
</tr>
<tr>
<td>The odd bordered numbers like the length of the parallelogram.</td>
</tr>
<tr>
<td>Yes, we always forget that we don't add</td>
</tr>
</tbody>
</table>

**Today---How did you work towards developing your understanding and what did you learn about the mathematical principles needed to express your ideas?**

- Developing the formula for finding the area of a parallelogram. The area by |
  - Understanding formulas of the parallelogram. The area by |
  - Developing the formula for finding the area of a parallelogram. |

**Diagram: 2.18.3.2**

**Question:** Don't you group have any problems and how do you overcome these?

**Parallelogram:**

- Accuracy in solving the area of a parallelogram.
- Understanding formulas of the parallelogram.
- The parallel lines are parallel.

- Many diagrams and many different examples.
- No less for everyone to understand.
- We had to go through many examples and understand.
**Did your groups have any problems and how did you overcome them?**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>Solution 1</td>
</tr>
<tr>
<td>Problem 2</td>
<td>Solution 2</td>
</tr>
<tr>
<td>Problem 3</td>
<td>Solution 3</td>
</tr>
</tbody>
</table>

**Things to Remember**

- \( y + 3 = z \)
- \( 3x + 2 = 5 \)
- \( 4y - 3 = 6 \)

**How did you work towards developing your understanding of what you learned?**

- I worked independently.
- I worked with a partner.
- I worked in a small group.

**One group's name for this lesson?**

- Group A
- Group B

**Date:** 3/17/02
Appendix Sixteen: Forms of collaboration evident at Southwest High School

Parallel workers

By far the majority of groups observed displayed some form of collaboration during the lesson. There was only one group observed who worked for the majority of the lesson individually. In Ms Martin’s class, this particular group worked quietly for the duration of the lesson on their own work without any form of discussion between members of the group unless there was some question with which one of the members was struggling. Only six comments indicated that some of the students worked separately for brief periods of time during the collaborative lessons – in Ms Martin’s class one of the three students interviewed reported that their group would sometimes work individually and sometimes as a group.

In Ms White’s class, however, there was one group in which a single member of the group found it impossible to work with the other members of the group during the first topic and spent the whole of each lesson doing his own work.

Minimum work

In two of the classes, it was evident that one or two groups decided not to pursue the goals of teaching each other and, instead, spent most of the lessons observed discussing non-mathematical matters with each other. The main reason given by one of the students for not doing any work was that several different members of the groups in question felt that they already understood all of the work from their tutoring outside class.

Jin: I wasn’t really caring, we had a pretty good understanding of stuff of everything, so we’d just start talking (Ms White class)

Connie: Them two knew too much, they didn’t do anything. (Ms White class)

Terrance: Most of my peers get tutoring, so they could talk, so then they - they know all this (Ms White class)
At Southwest High School this was an issue for the teachers who would often express their frustration with the high level of tutoring that students received outside school and the concomitant difficulties they faced when teaching students who felt like they already knew the work being covered.

**Parallel/Collaborative groups**

For several groups, this was the principal form of collaboration pursued. Students would decide together what they would do as a group that lesson and then they would come back together to discuss how they were going. The following example from Ms Diamond’s class demonstrates how the group decided together what they would do for that lesson, and then worked individually on completing the set work.

Edward: Did everyone ...... (John: Yes, Jane: Yes) do your homework properly? Did you understand everything?

John: Yes, Jane: Yes

Edward: Did you have anything wrong?

John: Yes, uhhh no

Jane: Yes, I did have some wrong

Edward: Okay we're going to do dividing a quantity into a given ratio

John: Exercise what?

Edward: 10:04

John: "Dividing a quantity into a given ratio" So we'll read this okay?

Edward: okay, that's for homework maybe, and on this page its Q4 its Q4 and 5. Okay we'll do 4 and 5 today and we'll do 6 and 7 tomorrow. And we'll do 8 last 'cause its hardest.

(group works quietly attempting these two questions)

**Micro-classrooms**

This form of collaboration was less evident at Southwest High School compared with Brindale Christian School at which it was the most common form of collaboration. However, some groups described what they were doing in this way and evidence of one
person teaching the rest of the group emerged from the classroom discussions and interviews.

Jeffrey: How did we develop our understanding of these outcomes?

Nathan: (Inaudible) hard work and labour.

Jeffrey: How about the person who knew most went over them continually until we all knew them.

(Ms White’s class)

Students at Southwest High School were much more similar to each other in terms of level of understanding due to the classes being streamed and selective. Within each group, therefore, there was not necessarily an obvious person who would take on responsibility for teaching other members of the class.

**Collaborative groups**
The most common form of collaboration between groups observed could be described as collaborative argumentation. Many students at Southwest High School described what happened in their groups as arguing and some found this frustrating when their group failed to provide them with the correct method straight away. Other groups, however, discussed ideas with each other more effectively, supporting the learning of other members of their group.

Edward: Let's work on how to calculate the circumference of a circle.

Francine: You answered that.

Edward: Yes, I know how to do that but I don’t know why I got the answer.

John: How do you calculate the circumference of a circle?

Jane: Isn't it calculate the area of a circle?

Francine and Bella laugh

Francine and Bella: diameter times pi.
Edward: Its that easy? I never knew that! OK, so what's our aim for this lesson to find the circumference (misemphasises syllable - emphasises third syllable) of a circle.

Francine and Bella: Circumference! OK!

John: You need help with your words! (Ms White's class)

Lyly: Pythagoras theorem, in my group, we worked well together and there's this guy called Charles, he didn't know much, yeah and we taught him about Pythagoras theorems and things” (Ms Diamond's class)

Ruby. I think when I was working in groups we talked more about maths, because we have different problem students, so we had to discuss it with each other, we had to work out - - (Ms Diamond's class)

Gary: We usually have a lot of arguments in class especially about hard questions, do it this way or do it that way, which answer is right which answer is wrong – that's the way we have arguments. (Ms Martin's class)

Interviewer: All right. Just think about each of those topics. How did you yourself think you came to understand the work? What was the basic way that you managed to understand the work on each of those topics?

Lyly: With the group, I just discussed it with the others - -.

Ruby: Yeah, me too.

Lyly: That's the way - how I learnt.

Interviewer: What about you guys, so you just discussed with each other, Eveline and Ashley?

Eveline. Yeah, yeah.

Interviewer: The same, just discussed it with each other? That was the main way you learnt? You didn't go off on your own to check through it or?

Eveline: Yeah, we used textbooks too.

Interviewer: But the main way was talking about it in groups?

Eveline: Yeah. (Ms Diamond's class)
Cross-group collaboration

In each of the classes, several students were identified moving between groups to obtain different ways of solving problems. This was encouraged by the teachers who supported students making enquiries of different groups. However, students would only leave their group if there was a particular problem that they couldn’t solve rather than to receive assistance with learning about a particular outcome. In Ms Diamond’s class some students showed a preference for working with groups apart from their own, and had to be encouraged to return to their groups after receiving assistance from another group.

Ms Diamond: have you joined this group now?

Andrew: I like working with this group cause I can relate to them more.

Ms Diamond: That's okay for visiting, but just remember that the assessment is with this group. It's fine to visit, but perhaps not permanent.

Teacher-group collaboration

Alternatively, the teacher would take a more proactive role in providing members of the group with ideas about how to answer a particular question.

Graeme: You know for this one, say when it says it takes 2 and a half hours the water is still right?

Berenice: It doesn't say it, it just says upstream, I reckon. It doesn't mean if it's still or not.

Graeme: Yeah but then how do we know, because like …

Berenice: The water is flowing. There. Two and a half km per hour.

Ms Diamond: Let me try and give you some hints on how to get started rather than just tell you how to do it. Think of it as the two halves of the journey, going with the current and going against the current as two completely separate problems. So try and separate them, so think about the first half of the journey first of all and write down what you know everything you know about that journey and everything you need to work out, and write down what am I trying to find out, and do that for the first part of the journey and see if any connections come out, and then try that for the second part of the journey.

Graeme: Uhhhh?
Berenice: It took two and a half hours .... And the water was flowing at two and a half kilometres .... umm okay ....

Whenever the teacher was interacting with one of the groups, the form of help was indirect, designed to support student discovery rather than present them with the final answer. In the above example, the teacher explicitly details her intention to provide some guidance in how to set up the problem in a manner that may be of use to the student rather than provide students with the method in full.

While the teachers at Brindale Christian School would often provide students with assistance in planning and working out how to develop their understanding, there was less evidence of teachers providing this form of support at Southwest High School. One exception to this was in the following discussion where Ms Diamond assisted the group to use the feedback sheets to make informed choices about which areas would require additional effort.

Ms Diamond: You know you have lots of ticks on your outcomes sheets. So when you say you want to do the screening test, what's your aim on that?

Lyly: To see how we're going.

Ms Diamond: To see how you're going, right, are you going to do the whole screening test, or just the things you've covered so far?

Ruby: The whole screening test because we want to do, like, all the topics by then.

Ms Diamond: But wasn't your pre-test like that to tell you which bits you needed to do and which bits you already knew?

Ruby: We just did, ... ummm, we are just doing it all again to see how we're going and to develop our understanding.

Ms Diamond: All right, fair enough, sounds like a good plan. So, today's plan where are you up to.

Lyly: Ummm, we've done quite a bit.

Ms Diamond: Okay (leaves)

Interactions between the classroom teacher and students at Southwest High School were less frequent than interactions between the teacher and groups of students at Brindale Christian
School and focused on assisting students with extension questions rather than with the questions comprising the focus of each topic.
Appendix Seventeen: Sample learning logs from Southwest High School

Sample 1: From Ms White’s class (date uncertain)

Learning Log For Group E

Aim for this Lesson
Solve problems involving quantities in a given ratio
Simplify rates

What did you work on this lesson? (which outcomes)

What problems did you encounter?
Some lengthy discussion and explanation was required on some questions

How did you develop your understanding of these outcomes?
Lots of explanation & doing example questions

What did you learn this lesson?
How to simplify rates & solve problems involving quantities in a given ratio

Write out an explanation in the space below describing the methods you developed during this lesson for answering different questions or a summary of the mathematical concepts covered.

3 : 2 = 6 : 4 unitary method
7 books cost £360
1 book costs \( \frac{630}{7} \approx 90 \)

Provide a worked solution in the space below of a question related to the outcome(s) you covered during this lesson

\[
\frac{4}{5} \times 1\frac{3}{4} = \frac{4}{5} \times \frac{7}{4} = \frac{4 \times 7}{5 \times 4} = \frac{7}{5} = 1\frac{2}{5}
\]

\[
= \frac{5}{5} + \frac{7}{5} = \frac{12}{5}
\]
### Learning Log For Group D

**Aim for this Lesson**
Make sure everyone knows how to do q.5.

**What did you work on this lesson? (which outcomes)**
equations with brackets (algebra)

**What problems did you encounter?**
some people didn't know it at all, some people were really good.

**How did you develop your understanding of these outcomes?**
everyone helped each other

**What did you learn this lesson?**
we learnt how to do q.5. How to explain and how to do equations with grouping symbols.

Write out an explanation in the space below describing the methods you developed during this lesson for answering different questions or a summary of the mathematical concepts covered.

- signpost 8 mathematics
- and our own.

<table>
<thead>
<tr>
<th>Provide a worked solution in the space below of a question related to the outcome(s) you covered during this lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2(a - 2) = a + 2)</td>
</tr>
<tr>
<td>(3a - 6 = a + 2)</td>
</tr>
<tr>
<td>(+ 6 + 6)</td>
</tr>
<tr>
<td>(3a = a + 8)</td>
</tr>
<tr>
<td>(- q - q)</td>
</tr>
<tr>
<td>(2a = 8)</td>
</tr>
<tr>
<td>(a = 4)</td>
</tr>
</tbody>
</table>
Sample 3: From Ms Diamond’s class (date 13\textsuperscript{th} August 2002)

Date: 13.8.02  Learning Log For Group 0

Aim for this Lesson
- outcome 8, assessment task

What did you work on this lesson? (which outcomes)
- outcome 8

What problems did you encounter?
- question 8, a. i)
  - b. iii)

How did you develop your understanding of these outcomes?
- our group discussed the questions and worked them out together

What did you learn this lesson?
- how to solve harder problems involving ratio and rates.

Write out an explanation in the space below describing the methods you developed during this lesson for answering different questions or a summary of the mathematical concepts covered.
- I learnt how to put two ratios into one.
  - ex. the ratio of men to women is 3 to 2. Ratio of women to children is 5 to 8.
  - methods: \[
  \frac{M}{W} \quad \frac{W}{C} = \frac{3}{2} \quad \frac{2}{5} \quad \frac{5}{8}
  \]

Provide a worked solution in the space below of a question related to the outcome(s) you covered during this lesson
- a. \[
  \frac{M}{W} \quad \frac{W}{C} = \frac{3}{2} \quad \frac{2}{5} \quad \frac{5}{8} \times 2
  \]
  - make the ratio with a common number for women.
  - \[\text{the ratio: } 15:10:16\]

On the back of this sheet record the group’s homework for next lesson.
Appendix Eighteen: Vignette from Ms Black’s class

Ms Black: OK where could you collect data from?

Nerida: Internet

Ms Black: where else?

Nerida: Encyclopaedias

Ms Black: Encyclopaedias. Where else? If you're doing data representation, where could you find lots of representations of data?

Nerida: In our textbook

Ms Black: Other than, other sources. Have you ever read the newspaper?

Clare: I was going to say the newspaper?

Ms Black: The newspaper, right. Where would you look for those sorts of....

Clare: In the herald.

Ms Black: Sydney Morning Herald! Well, you may find it sometimes there.

Clare: In the financial section.

Ms Black: Yeah, anywhere else?

Nerida: Magazines?

Ms Black: uh huh

Byron: TV?

Ms Black: Yeah, but you want examples you can bring to look at - newspapers, magazines. And find good examples. Sometimes when you have a graph it can be a really good representation of something, sometimes it can be sensational so its trying to say something or read something more into something that's been said. So see if you can find an example of data representation that may not necessarily be straightforward - a good time to look for those sorts of graphs is in an election time, and they have you know polls? Where whose going to win, somebody's behind, and yeah and also in the paper you might find things like that - you might find something like that now?
By the way, there is an outcome F down here, have you done it?

Nerida: Yeah we've looked at it.

Byron: We've finished doing the questions, but we haven't done questions yet.

Ms Black: Do you think that this is linked in some way to what we have to do here?

Nerida: Yeah

Ms Black: Then why don't you explore those two things then. You need to read those questions there, and why don't you go back and look at F and see if you can pose some of those questions yourself. Remember you are working towards an assessment task, so if you are practising writing your own questions now that will help you in the end.

Byron: OK, we'll try and write a question going on that graph. We have to write questions for this outcome. Each write one question for F and then we'll compare what we've got and write down what everyone else's got,

Clare: In your book?

Byron: in your exercise book?

Clare: What are we doing - on the first table?

Byron: Both, they're related. This one's about how much it will cost to carry something over a certain distance and …..

(they go to writing their own questions)
Appendix Nineteen: Fit indices used in current analyses

In structural equation modelling, models are tested using two general criteria: overall model fit and relative fit in comparison to conceptually plausible alternative models. Overall fit indices provide an indication of the extent to which the specified model accounts for patterns of variability within the data set.

There is a range of indices available for assessing the overall fit of a particular model (Marsh, Balla and Hau 1996; Fan, Wang and Thompson, 1997). Given that different indices assess fit using different criteria it is common practice to report several of these rather than relying on a single index of fit. In general, the fit indices available can be categorised into two classes: indices of absolute fit that assess how well the model reproduces the covariance or correlational data, and indices of incremental fit that assess how much better the model fits the data when compared with the null model.

Some indices measure absolute fit by assessing how well a particular model reproduces the sample data (Hu and Bentler, 1995). Such indices are equivalent to the $R^2$ measure used in regression analysis to ascertain how much of the variance has been accounted for by the model. Examples of such absolute fit indices are the Goodness of Fit Index ($GFI$) and the Adjusted Goodness of Fit Index ($AGFI$) developed by Jöreskog and Sorbom for use with the LISREL program (1998). Unlike the $GFI$, the $AGFI$ takes into account the degrees of freedom ($df$) and the number of parameters ($p$) in the model and so penalises less parsimonious models that incorporate additional parameters:

Values of 0.8 or higher for the $AGFI$ are regarded as indicative of adequate model fit (Reed and MacCallum, 1995) although this index has not received much attention in the area of self-perceptions. Pintrich et. al. (1990) only report the $GFI$ for their confirmatory factor analyses. $GFI$ values of 0.9 or above are regarded as indicative of adequate model fit (Reed and MacCallum, 1995).
A second measure of absolute fit used by other researchers in the field of self-perceptions is the Standardised Root Mean Residual (SRMR) (Pietsch, Walker and Chapman, 2003; Bong, 1997). The SRMR gives a measure of how much of the variance is unexplained by the model and is calculated by dividing the fitted residuals (the difference between the sample and the estimated variances) by the standard error of the residual. Models with adequate fit normally demonstrate SRMR values less than 0.05 (Byrne, 1998). In the current study the Root Mean Residual (RMR) was used since this was the statistic reported by Pintrich et al. (1990) which does not divide the fitted residuals by the standard errors and has similar properties to the SRMR.

Other indices measure incremental fit by comparing the adequacy of the model in relation to a baseline model (normally the null model). The Non-Normed Fit Index (NNFI) or Tucker Lewis Index (TLI) is an example of such an index whereby the target test statistic $T_T$ is compared with the test statistic for the null model as the baseline model $T_B$. The test statistic $T$ for a particular model is determined by minimising the discrepancy between the hypothesised model structure and the sample data and is therefore an indication of the absolute fit of the model. It is incorporated into the NNFI in the following way (from Hu and Bentler, 1995).

\[
NNFI = \frac{\left[ \frac{T_B}{df_B} - \frac{T_T}{df_T} \right]}{\left[ \frac{T_B}{df_B} - 1 \right]}
\]

The NNFI index is widely used in research into self-perceptions (Bong, 1997; Bong, 1998; Marsh and Yeung, 1997) and a value greater than 0.9 is normally indicative of adequate model fit. The NNFI index can be interpreted as the amount of improvement the target model provides over the null model (Hu and Bentler, 1995). However, the NNFI was not used by Pintrich et al. (1990) and so the NNFI indexes are not reported in this study.

Incremental fit indices may also make use of the non-centrality statistic from the non-central chi-squared ($\chi^2$) distribution. The non-central $\chi^2$ is more appropriate for models that are assumed to be minimally mis-specified. Given the degree to which the model is assumed to be mis-specified, a non-central chi-squared distribution can be referred to instead of the central chi-squared distribution (Fan, Wang and Thompson, 1997). An example of a fit
index that uses the sample non-centrality statistic is the Comparative Fit Index defined in the following way (Hu and Bentler, 1995).

$$CFI = 1 - \frac{\max[T_r - df_r, 0]}{\max[(T_r - df_r), (T_a - df_a), 0]}$$

The criterion for acceptable model fit with the $CFI$ is normally 0.9 with values greater than 0.9 indicating adequate model fit. The $AGFI$ and the $NNFI$ incorporate a penalty for loss of parsimony gained by adding extra parameters into the model by including the degrees of freedom in their calculation whereas the $CFI$ does not penalise less parsimonious models.

The chi-squared statistic ($\chi^2$) is the most widely used index even though problems inherent with the chi-squared statistic are widely acknowledged (Hu and Bentler, 1995; Marsh, Balla and Hau, 1996; Bollen, 1989). Since the $\chi^2$ statistic is known to be biased for large samples it has been suggested that a more appropriate use of the $\chi^2$ statistic would be to consider the ratio of the $\chi^2$ value and the degrees of freedom. Several theorists have proposed that a $\chi^2/df$ ratio no greater than two represents adequate model fit (Byrne, 1989; 1988) however Hayduk (1987) has argued that models displaying a $\chi^2/df$ no greater than five should be regarded as displaying adequate fit. In the current study, while $\chi^2$ has been reported, only $\chi^2/df$ ratios that excessively exceed 5 will be considered to be suspect consisten with previous research in self-perceptions (Bong, 1998; Pietsch, Walker and Chapman, 2003).

For sample sizes larger than 100, it is known that $\chi^2$ will always indicate a significant difference between the observed and expected values. Hence, when using $\chi^2$ where there is a significant difference between the observed and expected for sample sizes larger than 100 it is advisable to consider the value of Hoelter’s Critical $N$ statistic. This statistic gives the sample size which would be required to give a non-significant result. Values greater than 200 are indicative of adequate model fit.
Appendix Twenty: Node tree used to code data with NVivo

NVivo revision 1.2.142  Licensee: Faculty of Education
Project: Interview data 1st July  User: Administrator  Date: 7/8/04 - 2:21:43 PM

NODE LISTING

Nodes in Set:  All Nodes
Created:  7/1/04 - 3:34:47 PM
Modified:  7/8/04 - 2:13:13 PM
Number of Nodes:  169

1 (1) /other-regulation
2 (1 1) /other-regulation/Teacher regulation
3 (1 2) /other-regulation/Student regulation
4 (1 3) /other-regulation/Teach other students
5 (1 8) /other-regulation/Set homework
6 (1 9) /other-regulation/Responsibility for others' learning
7 (1 10) /other-regulation/Plan work together
8 (1 14) /other-regulation/Mark homework
9 (2) /Level of understanding
10 (2 1) /Level of understanding/High
11 (2 2) /Level of understanding/Medium
12 (2 3) /Level of understanding/Low
13 (2 4) /Level of understanding/High level of prior understanding
14 (3) /Role of teacher structuring learning
15 (3 1) /Role of teacher structuring learning/Prescribed tasks
16 (3 1 1) /Role of teacher structuring learning/Prescribed tasks/Writing assessment tasks
17 (3 2) /Role of teacher structuring learning/Providing resources
18 (4) /Self-regulation
19 (4 1) /Self-regulation/Becoming self-motivated
20 (4 2) /Self-regulation/Teaching oneself
21 (4 3) /Self-regulation/Learning from others
22 (4 4) /Self-regulation/Self-correcting
23 (4 5) /Self-regulation/Self-assessment of ability
24 (4 6) /Self-regulation/Deciding what to learn
25 (5) /Student activity
26 (5 1) /Student activity/Active student activity
27 (5 1 1) /Student activity/Active student activity/Seek help
28 (5 1 1 1) /Student activity/Active student activity/Seek help/Seek help from teacher
29 (5 1 1 2) /Student activity/Active student activity/Seek help/Seek help from other students
30 (5 1 1 3) /Student activity/Active student activity/Seek help/Seek help from someone else
31 (5 1 2) /Student activity/Active student activity/Use texts to develop understanding
32 (5 1 2 1) /Student activity/Active student activity/Use texts to develop understanding/Use class text book
33 (5 1 2 2) /Student activity/Active student activity/Use texts to develop understanding/Use outcomes provided
34 (5 1 2 3) /Student activity/Active student activity/Use texts to develop understanding/Use other texts
35 (5 1 3) /Student activity/Active student activity/Work collaboratively together
36 (5 1 4) /Student activity/Active student activity/Work separately
37 (5 1 5) /Student activity/Active student activity/teach other students
38 (5 1 6) /Student activity/Active student activity/Complete exercises
39 (5 1 7) /Student activity/Active student activity/Non-mathematical discussion
40 (5 1 8) /Student activity/Active student activity/Set homework
41 (5 1 9) /Student activity/Active student activity/Do examples together
(5 1 10) /Student activity/Active student activity/Plan work together
(5 1 11) /Student activity/Active student activity/Mathematical discussion
(5 1 12) /Student activity/Active student activity/Share ideas
(5 1 13) /Student activity/Active student activity/Presenting assessment task
(5 1 14) /Student activity/Active student activity/Mark homework
(5 1 15) /Student activity/Active student activity/Write assessment task items
(5 2) /Student activity/Passive student activity

(6) /Group interaction
(6 1) /Group interaction/Positive
(6 1 1) /Group interaction/Positive/Collaboration
(6 1 2) /Group interaction/Positive/Peer teaching
(6 1 3) /Group interaction/Positive/Enjoyment
(6 1 4) /Group interaction/Positive/Mutual encouragement
(6 1 5) /Group interaction/Positive/Receive help from others
(6 1 6) /Group interaction/Positive/Learning new methods
(6 2) /Group interaction/Negative
(6 2 1) /Group interaction/Negative/Arguments
(6 2 2) /Group interaction/Negative/Lazy group members
(6 2 3) /Group interaction/Negative/Language difficulties
(6 2 4) /Group interaction/Negative/Other talk
(6 2 5) /Group interaction/Negative/Having new ways presented
(6 2 6) /Group interaction/Negative/People at different levels
(6 2 7) /Group interaction/Negative/Lack of understanding in group
(6 2 8) /Group interaction/Negative/Group malfunction
(6 2 8 1) /Group interaction/Negative/Group malfunction/Disintegration
(6 2 8 2) /Group interaction/Negative/Group malfunction/Social malfunction
(6 2 9) /Group interaction/Negative/Lack of direction
(6 2 10) /Group interaction/Negative/Deterioration from first to second

(7) /Talking about mathematics
(7 1) /Talking about mathematics/More opportunities
(7 2) /Talking about mathematics/Less opportunities
(7 3) /Talking about mathematics/Varied

(8) /Doing mathematics
(8 1) /Doing mathematics/More work completed
(8 1 1) /Doing mathematics/More work completed/Worked quickly
(8 2) /Doing mathematics/Less work completed
(8 2 1) /Doing mathematics/Less work completed/Working slowly

(9) /Responsibility for others' learning

(10) /Development of social skills
(10 1) /Development of social skills/Learning to get along with people
(10 2) /Development of social skills/Working as a team
(10 3) /Development of social skills/Getting to know other students
(10 4) /Development of social skills/Difficulties working with others
(10 5) /Development of social skills/Value of helping others

(11) /Motivational states
(11 4) /Motivational states/Level of interest
(11 4 1) /Motivational states/Level of interest/High
(11 4 2) /Motivational states/Level of interest/Medium
(11 4 3) /Motivational states/Level of interest/Low
(11 5) /Motivational states/Level of enjoyment
(11 5 1) /Motivational states/Level of enjoyment/High
(11 5 2) /Motivational states/Level of enjoyment/Medium
(11 5 6) /Motivational states/Level of enjoyment/Low

(12) /Teacher activity
(12 7) /Teacher activity/Active participation
(12 7 1) /Teacher activity/Active participation/Help individuals
(12 7 2) /Teacher activity/Active participation/Set learning activity
(12 7 3) /Teacher activity/Active participation/Set work to be completed
(12 7 4) /Teacher activity/Active participation/Provide explanation
(12 7 4 1) /Teacher activity/Active participation/Provide explanation/Give examples
.Student comments.RBS students.FB Class
.Student comments.RBS students.KS class
.Student comments.RBS students.SG class
.Student comments.SHS students
.Student comments.SHS students:BD class
.Student comments.SHS students:KM class
.Student comments.SHS students:MW class
.Teacher Comments
.Teacher Comments.RBS teachers
.Teacher Comments.SHS teachers