Collaborative learning in mathematics: 
Realising reform principles in 
traditional classrooms

Submitted by
James Pietsch
B. Sc. (Hons) (University of NSW)
Dip. Ed. (University of Sydney)
M.Phil. Ed. (University of Sydney)

Faculty of Education
University of Sydney

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The University of Sydney
Author’s Declaration

This is to certify that:

I. This thesis comprises only my original work towards the Doctor of Philosophy in Education Degree.

II. Due acknowledgement has been made in the text to all other material used.

III. The thesis does not exceed the word length for this degree.

IV. No part of this work has been used for the award of another degree.

V. This thesis meets the University of Sydney’s Human Research Ethics Committee (HREC) requirements for the conduct of research.

Signature(s):

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Abstract

This study looked at the implementation of a collaborative learning model at two schools in Sydney designed to realise the principles recommended by reform documents such as the Principles and Standards for School Mathematics (NCTM, 2000) and policy documents including Numeracy, A Priority for All (DETYA, 2000). A total of 158 year seven and year eight students ranging in age from 12 to 15 years old from two schools participated in the study. In all, seven classroom teachers participated in the study each completing two topics using the collaborative learning model.

Four research questions were the focus of the current study. Three research questions were drawn from eight principles identified in the literature regarding what constitutes effective mathematics learning. These questions related to the nature of collaboration evident in each classroom, the level of motivation and self-regulation displayed by students in the different types of classrooms and the relationship between learning mathematics within the collaborative learning model and real-world mathematics. A final research question examined the degree to which the concerns of teachers relating to preparing students for examinations are met within the collaborative learning model.

Several different data collection strategies were adopted to develop a picture of the different forms of activity evident in each classroom and the changes that took place in each classroom during and after the implementation of the collaborative learning model. These included classroom observations, interviews with student and teacher participants, questionnaires and obtaining test results. Both exploratory and confirmatory factor analysis were used to reduce the data collected. Factor scores and test results were compared using t-tests, ANOVAs and Mann Whitney nonparametric tests. Data collected from interviews and classroom observations were analysed using a grounded approach beginning with the open coding of phenomena. Leont’ev’s theoretical approach to activity systems (1972; 1978) was then used to describe the changing nature of classroom activity with the introduction of the collaborative learning model.
Within the collaborative classrooms there were a greater number of mathematical voices participating in classroom discussions, a breaking down of traditional roles held by teachers and students, and dominant patterns of collaboration evident in each classroom reflecting pre-existing cultural ways of doing. Furthermore, there was some quantitative evidence suggesting that student levels of critical thinking, self-regulation and help seeking increased and students were also observed regulating their own learning as well as the learning of others. Classroom practice was also embedded in the cultural practice of preparing topic tests, enabling students to use mathematics within the context of a work group producing a shared outcome.

Finally, there was quantitative evidence that students in some of the collaborative classes did not perform as well as students in traditional classrooms on topic tests. Comments from students and teachers, however, suggested that for some students the collaborative learning model enabled them to learn more effectively, although other students were frustrated by the greater freedom and lack of direction. Future research could investigate the effectiveness of strategies to overcome this frustration and the relationship between different types of collaboration and developing mathematical understanding.
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