SINGLE SIDEBAND MODULATION
ASSIGNMENT 1

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ABSTRACT

This report investigates single sideband modulation theory. The content in this report comprises three parts： the principles and characteristics of single sideband modulation, the definition of equations used, and a conclusion on single sideband modulation. Some additional mathematical theorems used in single sideband modulation will also be introduced briefly. Single sideband modulation Matlab code will be presented in laboratory report 1.

1. INTRODUCTION

It is essential to modulate signals in radio transmission. This process reduces the wavelength of signals in order to permit the use of much shorter transmission aerials. Single sideband modulation is a widely applied method of amplitude modulation which uses bandwidth and electrical power more efficiently than the transmission of an original signal.

Amplitude modulation is a method of converting data into an alternating-current carrier waveform. The modulated data generated, appears as a form of signal component with a frequency slightly higher and lower than the carrier frequency. These signal components are called sidebands. Lower sideband signals (LSB) appear at frequencies below the carrier frequency; upper sideband signals (USB) appear at frequencies above the carrier frequency. Upper and lower sideband signals are mirror images in the graph of signal amplitude versus frequency as shown in Figure 1, and this indicates that these sidebands carry the same information. Demodulation of a modulated amplitude synthesizes the upper and lower sidebands, and produces an output signal. This signal has twice the bandwidth of the original signal. Thus single sideband modulation requires only one sideband which is then demodulated into a full output signal whilst reducing the power demand of transferring a full signal [1].

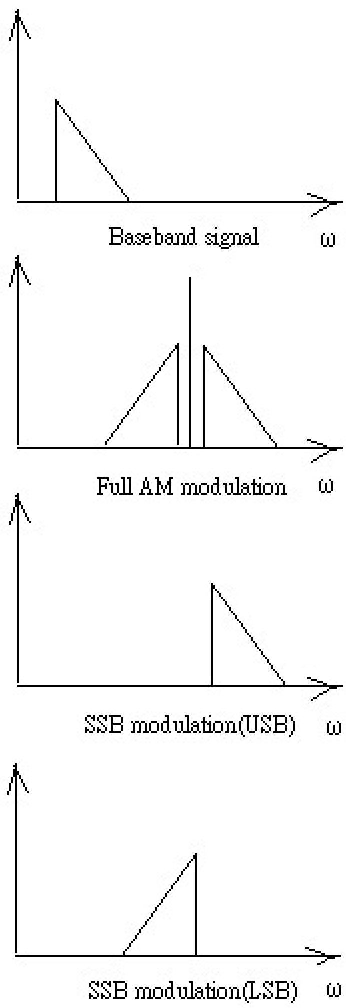

Figure1. *The graphs above show the baseband signal, the signal of full amplitude modulation, the upper sideband and the lower sideband signals of single sideband modulation of signal frequency versus amplitude on the X-Y axis.*

The Hilbert Transform and the Fourier Transform are two mathematical operations applied to single sideband modulation. These operations are explained below.

1.1 The Hilbert Transform

The Hilbert Transform on an amplitude modulated signal changes its phase by 90°. The Hilbert Transform of a signal h(t) is represented as $\hat{h}$(t).

$$\hat{h}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{h(\tau)}{t-\tau} d\tau \quad \textbf{(1)}$$

$$h(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\hat{h}(\tau)}{t-\tau} d\tau \quad \textbf{(2)}$$

h(t) and $\hat{h}(t)$constitute a Hilbert Transform pair. It is clear that the Hilbert Transform convolutes g(t) with $\frac{1}{\pi t}$.

The properties of the Hilbert Transform are listed below:
1. h(t) and $\hat{h}(t)$ have the same magnitude spectrum.
2. If $\hat{h}(t)$ is the Hilbert Transform of h(t), then the Hilbert Transform of $\hat{h}(t)$ is –h(t).
3. h(t) and $\hat{h}(t)$ are orthogonal over the entire interval -∞ to ∞.

1.2 The Fourier Transform

The Fourier Transform is a mathematical operation used to decompose a signal into sine and cosine components. The output of the transformation represents the signal as a function of frequency, while the input signal is a function of time. The Fourier Transform can refer to four categories of operation: aperiodic-continuous(also called the Fourier Transform), periodic-continuous(also called the Fourier Series), aperiodic-discrete(also called the Discrete-time Fourier Transform), and periodic-discrete(also called the Discrete Fourier Transform). Digital computers can only handle the information that is discrete and finite in length, so the only type of the Fourier Transform that can be used in computer algorithms is the Discrete Fourier Transform. [2]

The formula for the Discrete Fourier Transform is

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi\frac{k}{N}n} \quad \textbf{(3)}$$

The transform is sometimes denoted by the symbol *F*, as X=F{x}, F(x), or F_x. This formula shows that the Discrete Fourier Transform comprises N discrete frequency components. The N-periodic signal also provides the Discrete Fourier Transform with a finite length signal.

The properties of the Discrete Fourier Transform are:
1. The Discrete Fourier transform is an invertible, linear transformation.
2. The infinite signal produced by the Discrete Fourier Transform is periodic with period N.

3. EQUATIONS

The equations relevant to single sideband modulation are described below:

Where s(t) is the real-valued baseband signal to be transmitted, the Discrete Fourier Transform of s(t) is S(*f*). S(*f*) is a signal which is symmetrical about the frequency axis where $f = 0$. Modulating s(t) to a carrier frequency f_c moves the axis of symmetry to $f = f_c$ and the two symmetrical sides of the axis are called sidebands.

So $\hat{s}(t)$ is the Hilbert Transform of s(t). An analytic signal $s_a(t)$ is useful to demonstrate the mathematical concept as in:

$$s_a(t) = s(t) + i \cdot \hat{s}(t) \quad \textbf{(4)}$$

The Discrete Fourier Transform of $s_a(t)$ is equal to $2 \cdot \mathrm{S}(f)$. This signal has no negative components, so it can be modulated to a radio frequency and produce a single sideband signal.

A representative analytic signal $s_a(t)$ for this is:

$$\cos(2\pi f_c t) + i \cdot \sin(2\pi f_c t) = e^{i2\pi f_c t} \quad \textbf{(5)}$$

The Discrete Fourier Transform of $s_a(t)$ is $\delta(f - f_c)$. Modulating $s_a(t)$ by $e^{i2\pi f_c t}$, all frequency components are shifted by $+f_c$. This produces the upper sideband signal $s_{SBU}(t)$ which is a real-valued signal.

$$s_a(t) \cdot e^{i2\pi f_c t} = s_{SBU}(t) + i s_{SBU}(t) \quad \textbf{(6)}$$

$$s_{SBU}(t) = Re\{s_a(t) \cdot e^{i2\pi f_c t}\} \quad \textbf{(7)}$$

$$s_{SBU}(t) = Re\{[\mathrm{s}(t) + i \cdot \mathrm{s}(t)] \cdot [\cos(2\pi f_c t) + i \cdot \sin(2\pi f_c t)]\}$$
$$= \mathrm{s}(t) \cdot \cos(2\pi f_c t) - \hat{s}(t) \cdot \sin(2\pi f_c t)$$

The lower sideband signal $s_{LSB}(t)$ is a mirror signal that is symmetrical to $s_{SBU}(t)$ about the axis $f = f_c$. Due to the Discrete Fourier Transform characteristics, this symmetry means the signal $s_{LSB}(t)$ is a complex conjugate of the signal $s_{SBU}(t)$.

$$s_{LSB}(t) = s(t) - i \cdot \hat{s}(t) \quad \textbf{(8)}$$
$$= \mathrm{s}(t) \cdot \cos(2\pi f_c t) + \hat{s}(t) \cdot \sin(2\pi f_c t)$$

Note that:

$$s_{SBU}(t) + s_{LSB}(t) = 2s(t) \cdot \cos(2\pi f_c t) \quad \textbf{(9)}$$

Equation (9) demonstrates why the product of amplitude modulation has a doubled signal compared to the baseband signal.

The demodulation formula of single sideband modulation is:

$$\mathrm{p(t)} = s_{SBU}(t) \cdot \cos(2\pi f_c t) \quad \textbf{(10)}$$

To recover the original signals from the upper sideband signal, the upper sideband signal must be shifted down to its original range on the baseband frequency. This can be done by using a product detector demodulator which mixes the upper sideband signal with the output of a beat frequency oscillator (BFO). The BFO output waveform is $\cos(2\pi f_c t)$.

When the upper sideband signal is multiplied by the BFO waveform, this signal is shifted into two frequencies, one of which is unwanted. The unwanted signal can be removed by a low-pass filter.

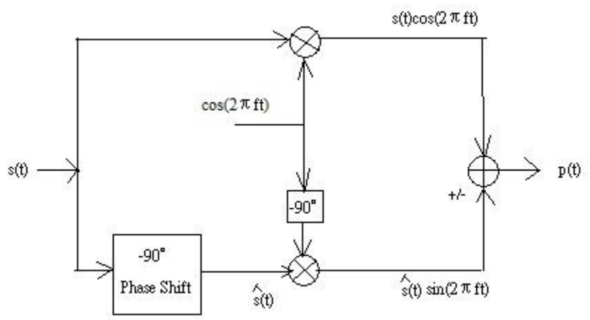

Figure2. *The schematic representation of single sideband modulation.*

4. CONCLUSION

Although single sideband modulation was the basis for long distance telephone communications up to the last decade, there are some difficulties in its application. For instance, designing and implementing a sharp low-pass filter is not an easy task for a circuit designer. Furthermore, single sideband modulation cannot be applied to message signals which contain significant energy around zero frequency, like video signals and computer data signals. However, if the message signal does not contain significant energy, like a speech signal for example, the signal can be modulated by single sideband modulation.

5. REFERENCES

[1] Udo Zolzer, Ltd, *Digital Audio Effects*, John Wiley& Sons, 2002, pp. 77-79.

[2] S. W. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing,* California Technical Publishing, 2002, Chapter 8.