Trend Inflation and Inflation Persistence in Australia: A New Keynesian Perspective

Anirudh S. Yadav*

University of Sydney, School of Economics

Supervisor: Denny Lie

Abstract

The empirical shortcomings of the purely forward-looking New Keynesian Phillips curve (NKPC) have generally been attributed to its inability to generate sufficient persistence in inflation. While the literature moved towards incorporating backward-looking terms into the NKPC, their somewhat ad-hoc rationales have been an ongoing source of criticism. This thesis attempts to ascertain the extent to which inflation dynamics in Australia can be explained by the NKPC without having to rely on such arbitrary backward-looking terms. Specifically, this analysis considers whether an adapted version of Cogely and Sbordone's (2008) model of time-varying trend inflation is sufficient to explain the presence of the backward-looking term in Australia's NKPC. The results show that even when time-varying trend inflation is taken into account, there remains a role for a backward-looking indexation. Moreover, when one considers the closed form estimates, lagged inflation and expected future inflation enter the NKPC with near equal weights. These results imply a considerably greater role for backward-looking behaviour when compared to typical GMM estimates of the Australian NKPC, which suggest that inflation is predominantly forward-looking.

*I would firstly like to thank my supervisor, Denny Lie, for his expert suggestions in forming the concept for this analysis and also for his guidance and unwavering assistance during the writing of this thesis. I am also grateful for his diligence in responding to my countless emails and queries. I also thank Tim Robinson at the Reserve Bank of Australia for generously providing the raw data used in this thesis. The skills I have acquired in macroeconomic analysis during the course of this year have also been greatly beneficial in writing this thesis. As such, I am indebted to my lecturers Dr. Aarti Singh, Dr. Yunjong Eo and Professor Adrian Pagan. Finally, I also thank Diane Hutchinson and Russell Ross for their advice throughout the year.

This thesis is submitted in partial fulfilment of the requirements for the Bachelor of Economics (Honours).
October, 2011.
Email address: anirudh.yadav@sydney.edu.au

Statement of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any other degree or diploma at University of Sydney or at any other educational institution, except where due acknowledgement is made in the thesis.

Any contribution made to the research by others, with whom I have worked at University of Sydney or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation and linguistic expression is acknowledged.

Signed: . . . .

Date: . . . .

Contents

1 Introduction

The true nature of inflation dynamics is an ongoing matter of debate and investigation in modern macroeconomics. That such attention is devoted to the dynamics of inflation is due to its importance, not only for understanding the nature of business cycles, but also for determining the appropriate path for monetary policy. Modern models of inflation are typically derived from the seminal contributions of Calvo (1983) and Taylor (1980) which imply a purely forward-looking New Keynesian Phillips curve (NKPC) where inflation depends on its future expectation and the level of real marginal costs. Despite its theoretical elegance, the purely forward-looking incarnation of the NKPC has been shown to perform poorly against the data. The empirical shortcomings of the NKPC are generally attributed to its inability to replicate the innate persistence which is present in inflation (see, for example: Fuhrer and Moore, 1995). In order to enhance the degree of persistence within the model several authors have proposed somewhat ad-hoc rationales for the inclusion of lagged inflation terms in the NKPC (see, for example: Gali and Gertler, 1999 and Christiano, Eichenbaum and Evans, 2005). While these 'hybrid Phillips curves' do indeed improve the fit of the model, their questionable microfoundations are an obvious source of criticism. This thesis attempts to ascertain the extent to which inflation dynamics in Australia can be explained by the NKPC without having to rely on arbitrary backward-looking terms that have limited structural meaning. In particular, this analysis considers whether an adapted version of Cogley and Sbordone's (2008) extended model of time-varying trend inflation is sufficient to explain inflation persistence in Australia without the need for a backward-looking term in the NKPC.

1.1 The NKPC and the necessity of lagged inflation

To provide some context consider the following simple comparison of a purely forward-looking NKPC and a hybrid specification. The traditional NKPC derived from the Calvo model implies an Euler equation for inflation, π_t, that takes the form[1]

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \tilde{y}_t,$$

[1] For a complete derivation see Gali (2008). Note that for the purpose of this simple illustration the output gap is used as the driving process, rather than real marginal costs.

where β denotes the discount factor, $\tilde{y}_t$ is the output gap and the coefficient γ is a function of the Calvo model's structural parameters[2]. Thus, according to the traditional NKPC, inflation inherits its persistence entirely from the persistence in the driving process, which in this case is the output gap. Now, consider a hybrid Phillips curve that allows for the inclusion of lagged inflation

$$\pi_t = \mu\pi_{t-1} + (1-\mu)E_t\pi_{t+1} + \gamma\tilde{y}_t.$$

Accordingly, the hybrid specification allows for both forward and backward-looking elements, the latter of which provides a channel for the "intrinsic" persistence in inflation to enter the model.

To explore the dynamic implications of the two specifications, and without loss of generality, they can be embedded in a basic small-scale macroeconomic model. The model is completed with the inclusion of an IS relation and a Taylor-type policy rule, which take the form

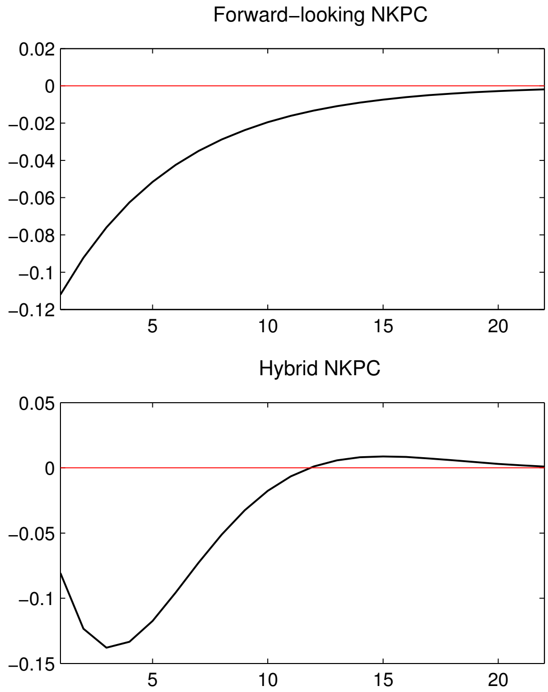
$$\tilde{y}_t = \alpha_1\tilde{y}_{t-1} - \alpha_2(i_t - E_t\pi_{t+1}) + \epsilon_{yt},$$
$$i_t = \phi_y E_t\tilde{y}_{t+1} + \phi_\pi E_t\pi_{t+1} + \epsilon_{it},$$

where i_t is the short-term interest rate controlled by the monetary authority. For the purposes of this illustrative example the discount factor, β, in the forward-looking NKPC is fixed to equal 1. In the hybrid specification $\mu = 0.5$, giving equal weight to lagged and future expectations of inflation. The coefficient α_1 is set to equal 0.9, such that inflation inherits some persistence from the output gap. Finally, the policy parameters, ϕ_π and ϕ_y, are calibrated as 2 and 0.3 respectively.

Figure 1 displays the impulse responses of inflation to a monetary policy shock. Given the forward-looking formulation, inflation falls immediately in response to the policy shock. Without any intrinsic persistence, inflation must immediately deviate downwards, and then rise to its steady-state value from below. However, in the hybrid specification inflation declines more gradually, exhibiting the typical "hump-shaped" impulse response associated with monetary policy transmission.

[2]Gali and Gertler (1999) demonstrate that $\gamma = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}$, where $(1-\alpha)$ is the probability that a firm may re-optimise its price in any period.

Figure 1
Impulse responses of inflation to a monetary policy shock



To illustrate the implications that the two contrasting specifications have for policy, consider a shock to the driving process, $\tilde{y}_t$. The impulse responses of inflation to such a shock are displayed in Figure 2 below. Again it is evident that with no inherent persistence, the forward-looking NKPC implies that inflation must "jump" immediately in response to the shock. On the other hand, given the hybrid form, inflation displays a more gradual response. Such divergence in inflation dynamics has important implications for the appropriate path of monetary policy. Figure 3 displays the policy response to the output gap shock. It is unsurprising, given the specification of the Taylor-rule, that the central bank's response is dictated by the path of inflation itself. In the case where inflation is determined by the forward-looking NKPC, the central bank responds by increasing its short-term interest rate immediately. In the hybrid case, the interest rate is raised more gradually in comparison, but is lowered more rapidly so as to match the path of inflation. Thus, the disparity in the dynamics of inflation implied by the two specifications can have significant implications for the optimal design of monetary policy.

Although the above example is highly stylised, it is clear that the dynamics of inflation implied by the traditional forward-looking NKPC are markedly at odds with the empirical

Figure 2
Impulse responses of inflation to an output gap shock

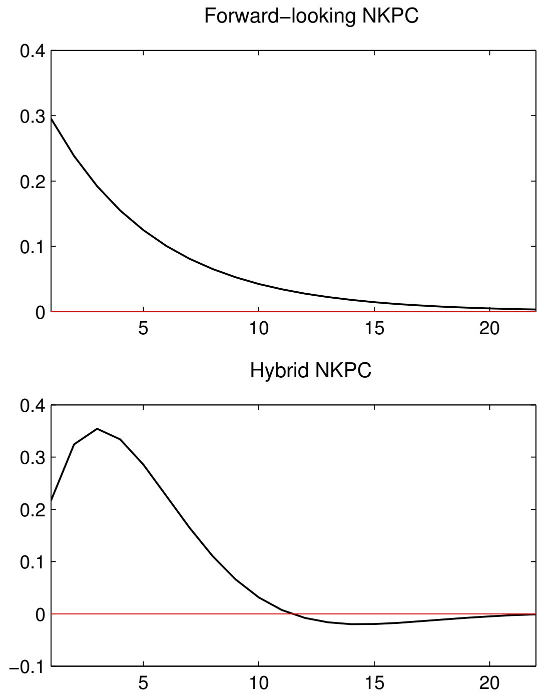
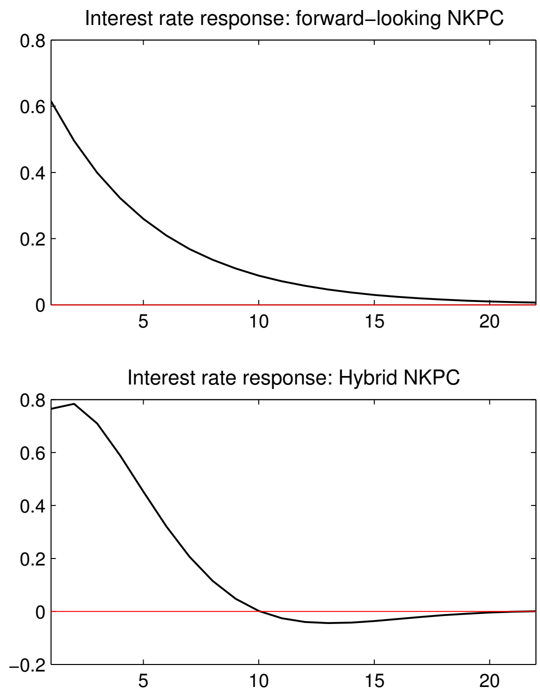


Figure 3
Policy responses to an output gap shock



evidence (see, for example the VAR analysis in Christiano, Eichenbaum and Evans, 1999). Thus, the inclusion of a backward-looking component in the NKPC has become a ubiquitous feature in modern macroeconomic models. However, a central bank needs to understand the sources of inflation dynamics in order to act appropriately. As alluded to above, the limited structural interpretation attached to the backward-looking term in the hybrid NKPC gives rise to the question of whether the apparent persistence in inflation arises from persistence intrinsic to the price-setting process, or another source. Is there some other mechanism that can capture the inertia in inflation without depending on ad-hoc backward-looking terms? This thesis attempts to provide some answers to this question by examining Australia's inflation experience with particular reference to Cogley and Sbordone's (2008) recently developed model based on time-varying trend inflation.

1.2 A non-technical summary

Cogley and Sbordone (2008) propose that the apparent structural persistence present in inflation is derived from a source distinct from the dynamics of price adjustment itself. They stress that to understand inflation persistence it is paramount to model variation in the slow moving *trend* in inflation. In general equilibrium, trend inflation is pinned down by the long-run target in the central bank's policy rule, and therefore contributes a highly persistent component to actual inflation. Given that shifts in trend inflation are attributable to movements in the policy target, it follows that this source of persistence is independent of any intrinsic persistence derived from the price-setting process. Accordingly, Cogley and Sbordone (2008) hypothesise that the apparent structural persistence arises due to the interaction between trend inflation and the non-linearities of the Calvo model. Conventional versions of the NKPC described earlier abstract from the variation in trend inflation and model persistence as a pure consequence of the dynamics of price adjustment.

Cogley and Sbordone (2008) extend the traditional Calvo model to incorporate drifting trend inflation. In particular, they log-linearise the equilibrium conditions of the model around a shifting stead-state associated with time-varying trend inflation. Their model gives rise to an extended NKPC with time-varying coefficients. This analysis adopts the two-stage minimum distance procedure used by Cogley and Sbordone (2008) to estimate the structural parameters of the model using Australian data. The first stage involves estimating an unrestricted reduced-form VAR, with drifting parameters, for inflation, real marginal costs and other variables. In the second stage the procedure exploits the cross-equation restrictions implied by the VAR forecasts and the theoretical model in order to estimate the structural parameters of interest.

During the course of the analysis, Cogley and Sbordone's (2008) model is appropriately adapted so as to capture the small open economy effects of Australia. Furthermore, this thesis extends on Cogley and Sbordone's (2008) analysis by providing the parameter estimates implied by the NKPC, not only in its difference (Euler) equation form, but also in its closed form. The closed form parameter estimates are of particular interest due the additional restrictions imposed on the evolution of inflation expectations, which improve efficiency.

The documented results point to three main conclusions. First, although the estimated role for backward-looking indexation is near zero in some difference equation specifications, when one considers the closed form specifications of the NKPC, the parameter estimate increases dramatically, implying a high degree of indexation to past inflation. Thus, the estimates suggest that accounting for time-varying trend inflation in the NKPC cannot explain away the inertia in Australian inflation. Second, despite the variation in trend inflation, which alters the relative magnitude of the NKPC coefficients, the NKPC assigns near equal weights to lagged inflation and to expected future inflation. The enhanced magnitude of the backward-looking coefficient is in contrast to conventional GMM estimates of the Australian NKPC, which suggest a predominant role for forward-looking behaviour. Finally, and notwithstanding the aforementioned conclusions, the reduced-form evidence and the structural analysis reveals a marked decline in the persistence of inflation since the Reserve Bank's implementation of an inflation targeting regime in 1993.

The remainder of the thesis proceeds as follows. Section 2 reviews the relevant literature and in particular documents the theoretical evolution of the NKPC. Section 3 presents reduced-form evidence of inflation persistence in Australia. In Section 4 the difference equation and closed form specifications are described in the context of a simple NKPC for Australia and the two-step estimation procedure is introduced. Section 5 documents the primary structural analysis and results, incorporating Cogley and Sbordone's (2008) model of inflation dynamics with time-varying trend inflation. A robustness analysis is conducted in Section 6. Finally, Section 7 concludes.

2 Literature Review

A central theme in the New Keynesian literature, and indeed macroeconomics, has been the quest to better understand the dynamics of inflation. The short-run dynamics of inflation not only affects the nature of business cycles but also has obvious implications for the appropriate course of monetary policy. A string of papers emerged in the 1990s which aimed to provide a new theoretical approach to the modelling of inflation dynamics. These papers expanded on the earlier work of Calvo (1983) and Taylor (1980) that highlighted the role of staggered contracts, price stickiness and the resulting monetary non-neutralities. In particular, the literature extended the contributions of Calvo and Taylor by embedding the price setting decision within a firm optimisation problem[3]. The resulting theoretical abstraction lead to a formulation that links short-run inflation to expected future inflation and a measure of real activity (typically, real marginal costs or the output gap). This formulation of inflation dynamics has become known as the New Keynesian Phillips curve (NKPC) and is a central feature of the canonical New Keynesian macroeconomic theory.

Estimation of the NKPC in its original formulation has had limited success at best. Despite its theoretical virtue and derivation from microfoundations the empirical literature has raised doubts over the validity of the NKPC. In particular a number of studies in the U.S have concluded that the purely forward-looking incarnation of the NKPC generates too little *inflation persistence* and consequently does not provide a good match to the data. According to the New Keynesian formulation inflation is completely forward-looking, having no intrinsic persistence. Instead any persistence in inflation is inherited entirely from the persistence in the driving process (i.e. real marginal costs or output). As evidenced by the illustrative examples in the preceding section, following a shock to monetary policy or, indeed, any shock which affects the driving process, inflation can jump immediately in response. However, a key feature of monetary policy transmission is that inflation demonstrates a gradual response to policy actions over several periods – a characteristic that the traditional NKPC cannot replicate[4]. Furthermore, Fuhrer and Moore (1995) showed that the basic NKPC derived from Calvo-type staggered contracting

[3]Kimball (1995) and Yun (1996) pioneered the use of Calvo-type price contracts within stochastic, optimising-agent models.

[4]Evidence of such a gradual inflation response has been well documented in the vector autoregression (VAR) analysis of Christiano, Eichenbaum and Evans (1999). Other references include Sims (1992), Gali (1992) and Bernanke and Mihov (1998).

models implied a degree of inflation persistence that was far lower than was apparent in the U.S. inflation data for the post-war period.

In order to address the issue of inflation persistence the empirical literature moved towards adding ad-hoc backward-looking terms to the NKPC, resulting in the 'hybrid Phillips curve'. The hybrid curve typically contains a lag of inflation as an explanatory variable and accordingly this formulation has been shown to enhance the degree of inflation persistence within the model. A seminal formulation of the hybrid Phillips curve was introduced by Gali and Gertler (1999). Essentially, the authors postulated that there exists a proportion of backward-looking firms who, rather than behaving like firms in Calvo's model, follow a simple rule-of-thumb that is based on the recent history of the aggregate price level. Gali and Gertler's (1999) influential results suggested that, while the purely forward-looking version of the NKPC is rejected by the data, the hybrid specification performs reasonably well and moreover it continued to indicate a predominant role for forward-looking behaviour. They interpreted the dominant role for forward-looking behaviour as giving credence to theoretical underpinnings of the NKPC and in this respect they suggested that the NKPC does indeed provide useful insights into the nature of inflation dynamics. The authors also demonstrated that measures of real marginal costs are better suited as the relevant determinant of inflation (which is indeed what the theory suggests), as opposed to an ad-hoc measure of the output gap. Gali and Gertler's (1999) findings established the empirical virtue of the hybrid specification, which has since become commonplace in the new generation of macroeconomic models used in central banks throughout the world.

Several authors, however, have since suggested that the seminal results of Gali and Gertler (1999) were the product of specification bias or questionable estimation methods. Rudd and Whelan (2005) assert that Gali and Gertler's (1999) finding of a significant role for expected future inflation in determining current inflation may, in fact, reflect the effects of the variables included in their instrument set, which were omitted from the hybrid model specification. If these instrument variables directly cause inflation then the estimation results may be biased in finding a role for forward-looking behaviour even if that role is truly absent or negligible. They also suggested that Gali and Gertler's (1999) results were likely a product of misspecification given that the estimates of the closed form specification were significantly different from those obtained from estimating the structural form directly. In their detailed response, Gali, Gertler and Lopez-Salido (2005; henceforth GGLS) were able to refute many of the criticisms levelled by Rudd and Whelan (2005). However, Gali and Gertler's (2008) original formulation of the hybrid Phillips curve remains the subject of criticism largely due to its somewhat ad-hoc nature and lack of any convincing microfoudations. As clarified by Rudd and Whelan (2005), Gali and Gertler's (2008) specification involving a fixed fraction of backward-looking price-setters each using an arbitrary and fixed rule-of-thumb can hardly be considered structural in any meaningful sense. GGLS (2005) conceded that a more coherent rationale for the role of lagged inflation in the hybrid NKPC was needed.

The theoretical literature has proposed several other rationales for the presence of lagged inflation terms in the NKPC. Christiano, Eichenbaum and Evans (2005) introduced a lagged inflation term by imposing a form of price indexation whereby firms who cannot optimally reset their price under the pretences of the Calvo model instead index their price to a weighted average of past inflation. Fuhrer and Moore (1995) proposed an alternative contracting model which differed from the typical Taylor/Calvo-type specification in that it focused on a two-sided average of the *inflation rate* rather than the *price level*. Unfortunately such models which incorporate backward indexation have also suffered criticism, despite their structural superiority to models which employ simple rule-of-thumb behaviour. In particular, backward indexation of prices implies that *all* prices in the economy change every period. As detailed by Chari, Kehoe and McGrattan (2008), such a model of price dynamics is strongly at odds with evidence from microdata. For example, Nakamura and Stiensson (2008) show that consumer prices in the U.S are fixed for many periods at a time, with an implied median duration of $4-5$ months (including sales). Thus, although backwardly indexed models can mechanically generate inflation persistence in the NKPC the mechanism itself is inconsistent with the behaviour of prices at the micro level.

The debate over the empirical success of the basic forward-looking NKPC specification continues today, with the ability of the specification to replicate the persistence of inflation an important focus (Fuhrer, 2009). More recently the theoretical literature has diverged from hybrid abstractions with the aim of providing new approaches to modelling the NKPC while capturing the apparent inertia in inflation. One such approach has been documented in a series of papers, beginning with Cogley, Primiceri and Sargent (2010) and Cogley and Sbordone (2008), which emphasise the importance of accounting for "trend inflation" in a model of inflation dynamics. Cogley and Sbordone (2008) assert that the slow-moving trend in inflation contributes a highly persistent component to the actual inflation series. However, this source of persistence is obviously different from any intrinsic persistence derived from the dynamics of price adjustment itself. In general equilibrium, it is the long-run target in the central bank's policy rule that determines trend inflation, and any drift in trend inflation should ultimately be attributed to shifts in the target. The authors hypothesise that the structural persistence evident in inflation, and documented in the empirical literature discussed above, arises due to the fact that conventional models neglect the interaction between time-varying trend inflation and the non-linearities of a more exact version of the Calvo model. Motivated by their hypothesis, Cogley and Sbordone (2008) derive an innovative model where, unlike in the typical New Keynesian paradigm, steady-state inflation is *not* constant. Thus, the traditional log-linearised version of the NKPC, and its hybrid counterpart, no longer apply. The authors extend the original Calvo model to incorporate trend inflation. Specifically, they log-linearise the equilibrium conditions of the model around a shifting steady-state associated with a time-varying inflation trend. Cogley and Sbordone's (2008) model culminates in an extended NKPC in which the coefficients are functions not only of the model's structural parameters, but also trend inflation.

Cogley and Sbordone's (2008) empirical findings were profound and somewhat controversial. The authors found that once the model incorporates time-varying trend inflation, there is no need for including a lag of inflation to account for its persistence. Accordingly, their specification suggests the NKPC is purely forward looking, and provides a possible alternative to the heavily criticised hybrid Phillips curve. As stipulated in Fuhrer (2009) such a formulation of inflation dynamics is an important contribution to the literature.

It should be noted that a recent study by Barnes, Gumbau-Brisa, Lie and Olivei (2011; henceforth BGLO) examines the robustness of the findings of Cogley and Sbordone (2008). One of their main results indicates that simply changing the form of the augmented NKPC – from a difference equation to a closed form equation – completely reverses the findings of Cogley and Sbordone (2008). The disparity in the parameter estimates arises due to the way inflation expectations enter the model. In the difference equation specification, as estimated by Cogley and Sbordone (2008), inflation expectations are left unconstrained. However, due to the nature of the closed form solution, inflation expectations are themselves restricted so as to be model-consistent. In the context of minimum distance estimation, such additional discipline on inflation expectations may be particularly relevant in determining an appropriate model of inflation dynamics. BGLO (2011) conclude that time-varying trend inflation is, in fact, insufficient to explain inflation persistence and as a result the NKPC is not purely forward-looking.

Australian Studies

Documentation of inflation persistence in Australia has been limited with the bulk of the research having been conducted in the U.S. and Europe[5]. Benati (2008) documents empirical evidence on changes in the reduced-form persistence of inflation for a broad array of developed countries, including Canada, New Zealand and the United Kingdom. His analysis draws evidence from long samples and focuses on differences in estimated inflation persistence across different monetary policy regimes. While the analysis does not focus specifically on Australia's inflation experience, a key finding of the paper is that inflation persistence has declined dramatically in recent years for all countries that have adopted an inflation targeting monetary policy. In Australia the inflation targeting framework was first adopted by the Reserve Bank in 1993 as an operational interpretation of its price stability mandate[6]. Not only has inflation fallen since the adoption of an explicit target, but Benati's (2008) results also suggest that inflation in Australia should exhibit a marked decline in persistence post-inflation targeting[7].

Discussion of structural sources of inflation persistence in Australia is relatively sparse. Gruen et al. (1999) discusses the development of the Phillips curve in Australia from the 1950s – 1990s. Although their paper does not focus on the issue of inflation persistence, the authors' preliminary results indicate that inflation expectations are predominantly backward-looking. Nimark (2009) estimates a structural model of the Australian economy and employs an ad-hoc hybrid Phillips curve, similar to the version discussed above, with one lag of inflation. Similarly, Jaaskela and Nimark (2011) employ a hybrid Phillips curve with an indexation parameter in their medium-scale New Keynesian model of the Australian economy. A recent study by Kuttner and Robinson (2010) discusses the flattening of the Phillips curve with particular reference to the experiences in the U.S and Australia. The authors follow a rule-of-thumb price-setting model as

[5] For a discussion of inflation persistence in Europe see, for example: Altissimo, et al. (2006) and Angeloni, et al. (2006).

[6] International institutions, such as the OECD and IMF, have accepted the above dating. The regime was not formally endorsed until 1996, when a new government signed a letter of agreement with a new Governor, Ian Macfarlane, upon his appointment.

[7] A reduced-form analysis of inflation persistence in Australia is conducted in Section 3 of this thesis.

stipulated in Gali and Gertler (1999) and estimate the resulting hybrid NKPC for both the U.S and Australia. The authors' results for Australia indicate an economically sizeable flattening of the Phillips curve since 1960. More importantly, for this context, their reduced-form estimates of the Australian NKPC show that while current inflation does have a positive and significant backward-looking component, it is predominantly dependent on future expectations of inflation.

The empirical literature on the Australian NKPC emphasises that the open economy aspects of the inflationary process demand greater attention than in structural models of the U.S economy. An innovative approach to embedding open economy aspects within a New Keynesian framework was developed by Monacelli (2005)[8]. In essence, Monacelli (2005) postulates that the domestic economy is populated by two types of firms: domestic producers and importers. Prices of domestically produced goods follow Calvo dynamics and may be adjusted for backward-looking behaviour as in Nimark (2009) and Kuttner and Robinson (2010). The Australian models employed by Nimark (2009), Jaaskela and Nimark (2011) and Kuttner and Robinson (2010) simplify Monacelli's (2005) specification by assuming the law of one price holds for import prices "at the docks". The two-sector generalisation yields two Phillips curves where domestic CPI inflation is simply the weighted average of inflation in the two sectors. Kuttner and Robinson (2010) make the simplifying assumption that the Calvo parameter governing price-stickiness is constant across both the domestic and importing sector – resulting in a single equation aggregate Phillips curve, which is the form used in the subsequent empirical analysis.

It is evident from the above survey that although the literature recognises the presence of a backward-looking component in the Australian NKPC, there has been no extensive discussion as to its source and whether there are possible mechanisms which do not rely on an ad-hoc backward-looking inflation term to capture its apparent inertia. This thesis attempts to ascertain whether a model which exploits the interaction between price adjustment and drifting trend inflation is adequate to explain the apparent structural inflation persistence that is evident in the Australian data.

[8] And its closely related precursor, Gali and Monacelli (2005).

3 Reduced-Form Inflation Persistence in Australia

In order to contextualise the structural investigation that is pursued in the following sections, it is appropriate to first establish reduced-form evidence of inflation persistence in Australia. There is no single definitive measure of reduced-form persistence. While the empirical literature has established a broad array of measures to capture the persistence in inflation, most methods tend to be based on the extent of its serial correlation. In this section a truncated version of Fuhrer's (2009) analysis of reduced-form inflation persistence is adapted to Australian data. Specifically, three measures of reduced-form persistence are presented:

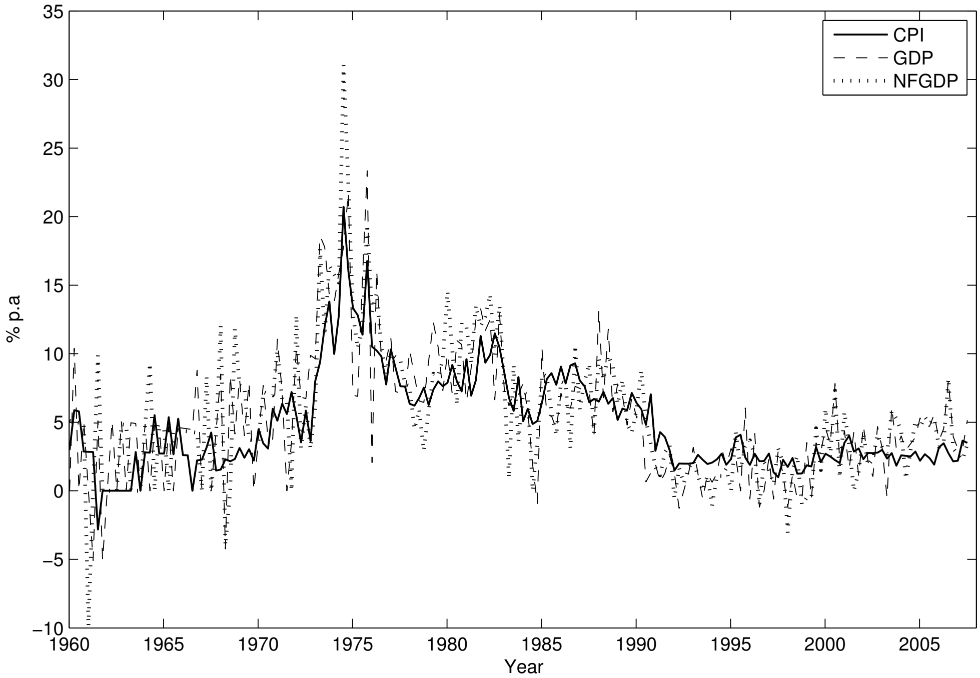
- Unit root tests;
- First-order autocorrelations of the inflation series;
- Autocorrelation functions of the inflation series.

3.1 Inflation data

Three key measures of inflation are used for the reduced-form analysis. The relevant price indexes are all quarterly measures and include the GDP deflator, trimmed-mean consumer price index (CPI) and the non-farm GDP deflator (NF GDP). Each inflation series is defined as 400 times the first difference of the logged price index. Figure 4 presents the three inflation series for the sample period 1960:Q1 - 2007:Q2. The figure displays well-known historical trends in Australian inflation. Inflation was relatively low throughout the 1960s averaging approximately 3% per annum. With the onset of the oil crisis and the corresponding cost-push inflation episode, inflation rose dramatically during the 1970s, with CPI peaking at 21% in 1974. It then remained high, with CPI peaking again at 11.5% in 1982. Despite the fact that inflation in most advanced countries fell sharply in the early and mid 1980s and remained low thereafter Australia continued to run fairly high inflation with CPI averaging 7.7% for the remainder of the decade. The recession of the early 1990s lead to a significant reduction in inflation, reaching levels as low as 1.5% in 1992. Low inflation was maintained during the remainder of the 1990s, coinciding with the Reserve Bank's adoption of an inflation targeting regime in 1993[9]. In addition, fluctuations in inflation have been considerably smaller than in the pre-inflation targeting years. With

[9] The inflation target is characterised as maintaining consumer price inflation between 2 and 3 percent, on average, over the cycle.

Figure 4
Inflation in Australia



the Australian economy near full employment and experiencing strong growth, inflation rose slightly during the 2000s, with an average of 2.71%, nearing the upper bound of the target[10]. The figure also captures the high correlation amongst the three inflation series, however, the pursuant statistical tests reveal notable variations across the different measures of inflation and, in particular, in their degrees of persistence.

3.2 Unit root tests

The preliminary gauge of persistence is a conventional unit root test. If inflation contains a unit root its persistence is infinite. In fact, if a series contains a unit root a shock in period t has an influence in all subsequent periods. Early US studies seemed to suggest the presence of a unit root in inflation[11]. However, more recent studies indicate that US inflation, while persistent, is stationary. Most authors attribute this stationarity to the more rigorous monetary regime adopted in the US post-Volcker and in particular, the Federal Reserve's low inflation goal. Table

[10]Note that the sample period used throughout this analysis ends at 2007:Q2. The subsequent GFC was an extraordinary event that perhaps requires deeper thinking about inflation dynamics.

[11]See, for example: Barsky (1987) and Ball and Cecchetti (1990).

1 presents the results of conventional unit root tests for Australian inflation, testing the null hypothesis that the series contains a unit root. Two samples are tested: a long sample (1960:Q1 - 2007:Q2), and a shorter sample covering the inflation targeting period (1993:Q1 - 2007:Q2). The results for Australian inflation are definitive – for all measures of inflation and for both samples the augmented Dicky-Fuller test (ADF) and the Phillips Perron test reject the null of a unit root. It should be noted, however, that the CPI measure only weakly rejects the null for both tests in the longer sample. In the inflation targeting period one can strongly reject the possibility of a unit root in Australian inflation, which is consistent with the fact that inflation has been well anchored around its 2-3% target level.

Table 1
Unit Root Tests for Inflation
p-values, H_0: series has a unit root

	1960:Q1 - 2007:Q2		1993:Q1 - 2007:Q2	
	ADF	Phillips Perron	ADF	Phillips Perron
CPI	0.0096	0.0448	0.00	0.00
GDP	0.00	0.00	0.00	0.00
NF GDP	0.00	0.00	0.00	0.00

3.3 First-order autocorrelations

The previous tests irrefutably rejected the possibility that Australian inflation has a unit root. Proceeding naturally from this point, the next simple measure of persistence involves an examination of the first-order autocorrelation coefficient for the inflation series. Table 2 presents the first-order autocorrelations for the long sample and for the inflation targeting period. The results are largely in line with the hypothesis that the Reserve Bank's adoption of an explicit inflation target has lead to a reduction in the persistence of inflation. For the long sample, persistence is relatively high according to all three measures, while there is a marked decline after the introduction of the target. It is worthy to note that for both sample periods CPI displays significantly higher serial correlation than the two price deflators.

Table 2
First-order Autocorrelations of Inflation

	1960:Q1 - 2007:Q2	1993:Q1 - 2007:Q2
CPI	0.8821	0.3386
GDP	0.5224	0.1257
NF GDP	0.6084	0.2598

3.4 Autocorrelation functions

Extending on the previous results this section presents the full autocorrelation functions for the three measures of inflation. The i^{th} autocorrelation, ρ_i, of some stationary variable x_t is defined as:

$$\rho_i = \frac{E(x_t x_{t-i})}{V(x)},$$

where $V(x)$ is the variance of x. The variable's corresponding autocorrelation function is expressed as the vector of autocorrelations of current period x with each of its own lags x_{t-i} from $i = 1$ to k:

$$A = [\rho_1, ..., \rho_k].$$

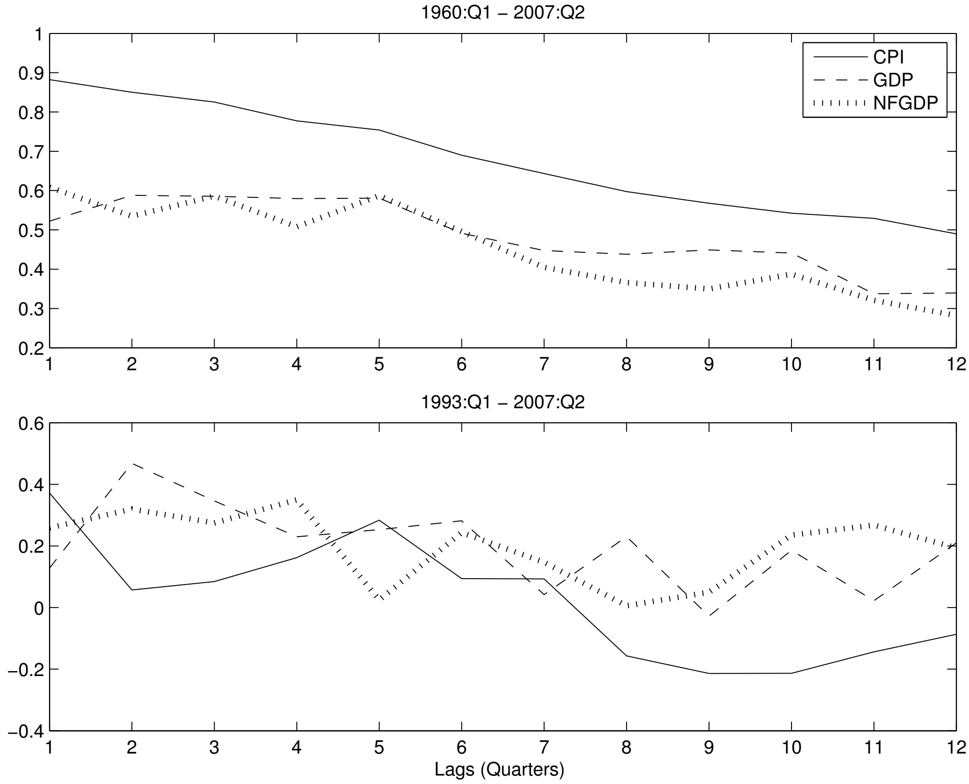
The autocorrelation function captures much of the information in a time series variable, and thus it may be regarded as the most complete measure of persistence. Figure 5 displays the full autocorrelation functions for the two relevant sample periods[12]. The figure expands on the results in Table 2. All three measures of inflation display relatively high reduced-form persistence for the longer sample. Again, it is evident that the CPI measure is considerably more persistent that both the GDP and NF GDP measures. Since 1993 it is clear that all three inflation measures have exhibited a lesser degree of persistence, giving credence to the notion that the Reserve Bank's inflation target has lead to a change in inflation behaviour.

3.5 Conclusions from the reduced-form evidence

From the above analysis one can strongly dismiss the possibility that Australian inflation possesses a unit root component. While, at face value, this initial result may seem trivial, it has

[12]The correlation of inflation with its first 12 lags are presented, that is $k = 12$.

Figure 5
Autocorrelations of Inflation data



important practical implications as discussed in Fuhrer (2009). In particular, the absence of a unit root implies that inflation may be regarded as a stationary process, which will eventually return to the Reserve Bank's inflation target in finite time.

The examination of the autocorrelation properties of inflation yielded interesting results. As with most other developed countries, inflation in Australia exhibited relatively high persistence, in the reduced-form sense, from the 1960s through to the mid-to-late 1980s. After the introduction of the inflation target in 1993 the evidence tends to suggest a significant decrease in the persistence in inflation, which is in line with Benati's (2008) results regarding the international experience under inflation targeting regimes. As noted above, it is evident that the trimmed-mean CPI inflation series exhibited a markedly higher degree of autocorrelation than both the GDP deflator and non-farm GDP deflator. These results are indicative of the fact that the trimmed-mean CPI is an underlying measure that abstracts from much of the noise and transitory fluctuations captured in headline CPI. While these transitory fluctuations affect Australian consumers, from a modelling perspective they are difficult to reconcile structurally, thus it is important to analyse such underlying measures. In the following sections, a structural analysis of inflation persistence will be undertaken, which will assist in identifying the economic sources of the aforementioned reduced-form persistence.

4 A Baseline New Keynesian Phillips Curve for Australia

This section considers a variant of the Calvo (1983) pricing model augmented to include an indexation mechanism in the vein of Christiano, Eichenbaum and Evans (2005). In each period t, every firm faces a constant probability, $(1-\alpha)$, of being able to adjust its nominal price optimally. The firms' ability to re-optimise its price is independent across firms and time. Firms that cannot re-optimise their price can still update their current price according to an indexation mechanism based on lagged aggregate inflation. In line with BGLO (2011) the indexation mechanism is adapted to allow for two lags of inflation:

$$P_t(i) = (\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho} P_{t-1}(i).$$

Here $\Pi_{t-1} = P_t/P_{t-1}$ is the aggregate gross inflation rate. The parameter $\rho \in [0,1]$ governs the degree of indexation, with $\rho = 0$ stipulating the absence of indexation (thus, there is no mechanical updating of prices for firms who cannot re-optimise, resulting in a purely forward-looking NKPC), and $\rho = 1$ denotes full indexation to weighted average of lagged inflation. The weight parameter, given by $\tau \in [0,1]$, represents the importance given to $t-1$ aggregate inflation relative to $t-2$ aggregate inflation. In this framework, where trend inflation is zero, the NKPC takes the form[13]

$$\pi_t = \left[\frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-1} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-2} + \left[\frac{\beta}{1+\beta\rho\tau}\right]E_t\pi_{t+1} + \left[\frac{\lambda}{1+\beta\rho\tau}\right]mc_t + u_t, \quad (1)$$

where π is inflation, mc is real marginal costs and $0 < \beta < 1$ is a discount factor. The coefficient λ is a function of the model's deep parameters, where $\lambda = (1-\alpha)(1-\alpha\beta)/(\alpha + \alpha\theta\omega)$. The parameter $\theta > 1$ is the Dixit-Stiglitz elasticity of substitution across differentiated goods, and ω is the elasticity of firms' marginal cost to their own output. The above expression of the NKPC also nests more familiar formulations. When $\tau = 1$ only $t-1$ aggregate inflation is relevant, and as such, the specification may be collapsed into a familiar reduced-form:

$$\pi_t = \gamma_b\pi_{t-1} + \gamma_f E_t\pi_{t+1} + \tilde{\lambda} mc_t, \quad (2)$$

[13] See Christiano, Eichenbaum and Evans (2005) for a complete derivation.

where $\gamma_b = \rho/(1+\beta\rho)$, $\gamma_f = \beta/(1+\beta\rho)$ and $\tilde{\lambda} = \lambda/(1+\beta\rho)$. This generalisation is commonly referred to as the hybrid NKPC and is of the same form as the reduced-form NKPC derived in Gali and Gertler (1999) who introduce lagged inflation through rule-of-thumb pricing, rather than an indexation mechanism[14]. The coefficients γ_b and γ_f measure the backward-looking and forward-looking components of inflation respectively. Nonetheless, the inclusion of τ in the subsequent estimations is significant from an empirical perspective as it may act to reduce the effect of misspecification bias in the structural estimates.

From (1) it is possible to solve the NKPC forward so as to obtain a closed form expression of inflation. Since (1) holds in every period, future expectations of inflation are constrained so as to follow the structure implied by the NKPC. Iterating forwards yields the closed form expression of the NKPC

$$\pi_t = \rho\tau\pi_{t-1} + \rho(1-\tau)\pi_{t-2} + \lambda\sum_{k=0}^{\infty}\beta^k E_t mc_{t+k} + u_t. \tag{3}$$

4.1 Open Economy Dimensions

The empirical literature on the Australian NKPC emphasises that the open economy aspects of the inflationary process demand greater attention than in structural models of the U.S economy. An innovative approach to embedding open economy aspects within a New Keynesian framework was developed by Monacelli (2005). In essence, Monacelli (2005) postulates that the domestic economy is populated by two types of firms: domestic producers and importers. Both domestic producers and importers set prices according to the augmented Calvo mechanism detailed above, where a fraction α^d of firms producing domestically and a fraction α^m of importing firms cannot adjust their prices optimally in a given period. As described in Nimark (2009) the two-sector generalisation yields two Phillips curves of the form

$$\begin{aligned}\pi_t^d &= \left[\frac{\rho^d\tau^d - \beta\rho^d(1-\tau^d)}{1+\beta\rho^d\tau^d}\right]\pi_{t-1}^d + \left[\frac{\rho^d(1-\tau^d)}{1+\beta\rho^d\tau^d}\right]\pi_{t-2}^d + \left[\frac{\beta}{1+\beta\rho^d\tau^d}\right]E_t\pi_{t+1}^d \\ &\quad + \left[\frac{\lambda^d}{1+\beta\rho^d\tau^d}\right]mc_t^d + \upsilon_{t,d}, \end{aligned} \tag{4}$$

[14] Although the reduced-form NKPC in (2) is of the same form as that in Gali and Gertler (1999), the coefficients represent different functions of the models' structutral parameters.

and

$$\pi_t^m = \left[\frac{\rho^m\tau^m - \beta\rho^m(1-\tau^m)}{1+\beta\rho^m\tau^m}\right]\pi_{t-1}^m + \left[\frac{\rho^m(1-\tau^m)}{1+\beta\rho^m\tau^m}\right]\pi_{t-2}^m + \left[\frac{\beta}{1+\beta\rho^m\tau^m}\right]E_t\pi_{t+1}^m + \left[\frac{\lambda^m}{1+\beta\rho^m\tau^m}\right]mc_t^m + \upsilon_{t,m}. \tag{5}$$

Domestic CPI inflation is simply the weighted average of inflation in both sectors:

$$\pi_t = (1-\phi)\pi_t^d + \phi\pi_t^m, \tag{6}$$

where ϕ is the share of imports in consumption, and the superscripts d and m denote domestically produced goods and imports respectively[15].

The Australian models employed by Justiniano and Preston (2010) and Jaaskela and Nimark (2011) measure the marginal cost of importers as the relative price of imported goods 'at the dock' (where the law of one price holds) to the retail price of imported goods[16]. In accordance with Monacelli (2005), these models assume that in setting the retail price of their goods, the importers solve a dynamic markup problem (à la Calvo), thus providing a short-run channel for deviations from the law of one price[17]. However, in estimating such models difficulties arise due to the fact that there is no direct measure of the retail price of imported goods. Accordingly, Justiniano and Preston (2010) and Jaaskela and Nimark (2011) treat these prices as an unobserved variable and estimates are achieved using the Kalman filter. This thesis follows the derivation in Kuttner and Robinson (2010), who make the simplifying assumption that the Calvo parameter governing price-stickiness is constant across both the domestic and importing sector, that is $\alpha^d = \alpha^m = \alpha$. This is a strong assumption that is not supported in either Nimark (2009) or Justiniano and Preston (2010), however, given the mixed evidence on the relative duration of prices it is an appropriate compromise for this exercise[18]. The simplifying assumptions allow for the derivation of a single equation aggregate Phillips curve[19].

[15]Note that in the above Phillips curves the slope coefficients, λ^d and λ^m, are functions of their respective sector-specific structural parameters.

[16]The analysis of Justiniano and Preston (2010) and Jaaskela and Nimark (2011) are pursued using generalisations of the small open-economy framework proposed by Monacelli (2005) and its closely related precursor Gali and Monacelli (2005).

[17]Complete exchange rate pass-through is attained only asymptotically, implying a long-run holding of the law of one price.

[18]Nimark (2009) finds the duration of prices to be less for domestic goods relative to imported goods, whereas Justiniano and Preston (2010) find the opposite.

[19]It is also assumed that the degree of indexation is common across both sectors, i.e. $\rho^d = \rho^m = \rho$. This

Combining (4), (5) and (6) yields

$$\pi_t = \left[\frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}\right]\left[(1-\phi)\pi^d_{t-1} + \phi\pi^m_{t-1}\right] + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right]\left[(1-\phi)\pi^d_{t-2} + \phi\pi^m_{t-2}\right]$$
$$+ \left[\frac{\beta}{1+\beta\rho\tau}\right]\left[(1-\phi)E_t\pi^d_{t+1} + \phi E_t\pi^m_{t+1}\right] + \left[\frac{\lambda}{1+\beta\rho\tau}\right]\left[(1-\phi)mc^d_t + \phi mc^m_t\right].$$

Thus, it follows from (6) that the Australian NKPC in its difference equation form can be written as

$$\pi_t = \left[\frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-1} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-2} + \left[\frac{\beta}{1+\beta\rho\tau}\right]E_t\pi_{t+1}$$
$$+ \left[\frac{\lambda}{1+\beta\rho\tau}\right]\left[(1-\phi)mc^d_t + \phi mc^m_t\right]. \tag{7}$$

In a similar vein to that described previously, (7) can be solved forwards to obtain the closed form representation of the Australian NKPC:

$$\pi_t = \rho\tau\pi_{t-1} + \rho(1-\tau)\pi_{t-2} + \lambda\sum_{k=0}^{\infty}\beta^k E_t\left[(1-\phi)mc^d_{t+k} + \phi mc^m_{t+k}\right] + u_t. \tag{8}$$

4.2 Econometric Methodology

This thesis follows the approach in Cogley and Sbordone (2008) in estimating the deep structural parameters of the augmented Calvo model: α, θ, ρ and τ. Since the main interest is assessing the importance of the inertial component of inflation the parameters β and ω are pinned down for ease of estimation. The strategic complementarity parameter ω is calibrated at a value of 0.429, while β is constrained to equal 0.99 in all estimations[20]. Thus, the focus of the estimation procedure is to provide inferences about the elasticity of substitution across differentiated goods, θ, the frequency of optimal price readjustment, reflected in the estimates of α, and of most relevance, the extent of indexation to past inflation, ρ.

The estimation procedure employs the two-stage minimum distance framework as detailed in Cogley and Sbordone (2008)[21]. Such a procedure exploits cross-equation restrictions between the

assumption is similarly imposed in Kuttner and Robinson (2010) and Nimark (2009) who assume that the share of firms that use rule-of-thumb pricing is the same for domestic producers and importers. As a corollary it is also assumed that $\tau^d = \tau^m = \tau$.

[20] The strategic complementarity parameter is defined as $\omega = 1/(1-\delta)$, where $1-\delta$ is the Cobb-Douglas labour elasticity. Thus, $\omega = 0.429$ is consistent with a value of $\delta = 0.3$.

[21] The estimation procedure follows Cogley and Sbordone (2008) and its subsequent adaptation in BGLO (2011). Their notation is retained wherever possible.

structural parameters of the Calvo model and those of an unrestricted reduced-form VAR. The VAR estimated in the first stage of the procedure is also used to represent agents' expectations about future inflation and real marginal costs.

Consider a time series vector $\mathbf{x}_t$ that includes n variables. For now, $\mathbf{x}_t$ is constrained to include period t inflation and real marginal costs of both domestic producers and importers[22], so that $n = 3$. It is assumed that the law of motion of $\mathbf{x}_t$ can be expressed as a reduced-form VAR(p). Then, defining a vector $\mathbf{z}_t = (\mathbf{x}'_t, \mathbf{x}'_{t-1}, ..., \mathbf{x}'_{t-p+1})'$, the VAR(p) may be expressed as

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \epsilon_{z,t}, \tag{9}$$

where $\mathbf{A}$ is a square matrix containing the VAR coefficients in the first n rows[23]. In order to exploit the cross-equation restrictions detailed below it is assumed that the solution to the structural NKPC model for the variables in $\mathbf{x}_t$ coincides with the reduced-form representation captured by the VAR in (9).

As mentioned above, the conditional expectation of a variable $y_{t+j} \in \mathbf{x}_{t+j}$ at time t can be obtained from the first-stage VAR, so that

$$E_t y_{t+j} = \mathbf{e}'_y \mathbf{A}^j \mathbf{z}_t. \tag{10}$$

The vector $\mathbf{e}'_y$ is a selection vector that isolates y_t in $\mathbf{z}_t$. For example, if y_t is the second of three variables in $\mathbf{x}_t$ and the VAR is of order 2, then:

$$\mathbf{e}'_y = [\ 0\ 1\ 0\ 0\ 0\ 0\ ].$$

[22]In subsequent sections with time-varying trend inflation $\mathbf{x}_t$ will be extended to include other variables, such as output growth and a nominal discount factor.

[23]The coefficient matrix $\mathbf{A}$ has all roots inside the unit circle; $\epsilon_{z,t}$ is a vector of i.i.d. residuals. For simplicity the regression intercepts are omitted here, however, they play an essential role in the NKPC with time-varying trend inflation and will be appropriately introduced in the pursuant sections.

4.2.1 The Difference Equation Specification

Taking expectations as at $t-2$ of the Australian NKPC as expressed in (7) and using the conditional expectation rule (10) yields

$$\mathbf{e}_\pi'\mathbf{A}^2\mathbf{z}_{t-2} = \left[\frac{\rho\tau-\beta\rho(1-\tau)}{1+\beta\rho\tau}\right]\mathbf{e}_\pi'\mathbf{A}\mathbf{z}_{t-2} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right]\mathbf{e}_\pi'\mathbf{I}\mathbf{z}_{t-2} + \left[\frac{\beta}{1+\beta\rho\tau}\right]\mathbf{e}_\pi'\mathbf{A}^3\mathbf{z}_{t-2}$$
$$+ (1-\phi)\left[\frac{\lambda}{1+\beta\rho\tau}\right]\mathbf{e}_{mcdom}'\mathbf{A}^2\mathbf{z}_{t-2} + \phi\left[\frac{\lambda}{1+\beta\rho\tau}\right]\mathbf{e}_{mcim}'\mathbf{A}^2\mathbf{z}_{t-2}, \tag{11}$$

where $\mathbf{I}$ denotes an identity matrix of the same dimensions as $\mathbf{A}$[24]. The above restriction captures the essence of the minimum distance problem, where the left-hand side of (11) represents the conditional expectation of inflation obtained from the reduced-form VAR, and the right-hand side represents inflation expectations as derived from the structural NKPC. If inflation is truly determined according to the NKPC then the reduced-form VAR forecast of inflation and the forecast according to the NKPC must be equivalent. Thus, imposing that (11) holds for all realisations of $\mathbf{z}_t$ yields a vector of non-linear cross-equation restrictions involving the VAR coefficient matrix $\mathbf{A}$ and the structural parameters of the NKPC, which are collected in the vector $\boldsymbol{\psi} = [\alpha, \theta, \rho, \tau]$. From (11) it follows that

$$\mathbf{e}_\pi'\mathbf{A}^2 = \left[\frac{\rho\tau-\beta\rho(1-\tau)}{1+\beta\rho\tau}\right]\mathbf{e}_\pi'\mathbf{A} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right]\mathbf{e}_\pi'\mathbf{I} + \left[\frac{\beta}{1+\beta\rho\tau}\right]\mathbf{e}_\pi'\mathbf{A}^3$$
$$+ (1-\phi)\left[\frac{\lambda}{1+\beta\rho\tau}\right]\mathbf{e}_{mcdom}'\mathbf{A}^2 + \phi\left[\frac{\lambda}{1+\beta\rho\tau}\right]\mathbf{e}_{mcim}'\mathbf{A}^2$$
$$\equiv \mathbf{g}^{DE}(\mathbf{A}, \boldsymbol{\psi}). \tag{12}$$

Or, equivalently

$$\mathbf{F}^{DE}(\mathbf{A}, \boldsymbol{\psi}) \equiv \mathbf{e}_\pi'\mathbf{A}^2 - \mathbf{g}^{DE}(\mathbf{A}, \boldsymbol{\psi}) = \underline{0}', \tag{13}$$

where $\underline{0}$ is a vector of zeroes, the same size as $\mathbf{e}_\pi'$ and the superscript DE indicates that the expression applies to the NKPC in its difference equation form.

The constrained case, imposing $\tau = 1$, yields the following cross-equation restrictions, where

[24] The subscripts *mcdom* and *mcim* denote real marginal costs corresponding to domestic producers and importers respectively.

expectations are taken at $t-1$:

$$\begin{aligned}\mathbf{e}'_\pi\mathbf{A} &= \left[\frac{\rho}{1+\beta\rho}\right]\mathbf{e}'_\pi\mathbf{I}+\left[\frac{\beta}{1+\beta\rho}\right]\mathbf{e}'_\pi\mathbf{A}^2 \\ &\quad +(1-\phi)\left[\frac{\lambda}{1+\beta\rho}\right]\mathbf{e}'_{mcdom}\mathbf{A}+\phi\left[\frac{\lambda}{1+\beta\rho}\right]\mathbf{e}'_{mcim}\mathbf{A} \\ &\equiv \mathbf{g}^{DE}_{con}(\mathbf{A},\boldsymbol{\psi}). \end{aligned} \tag{14}$$

Equivalently[25],

$$\mathbf{F}^{DE}_{con}(\mathbf{A},\boldsymbol{\psi}) \equiv \mathbf{e}'_\pi\mathbf{A}-\mathbf{g}^{DE}_{con}(\mathbf{A},\boldsymbol{\psi}) = \underline{0}'. \tag{15}$$

Thus, the two-stage minimum distance estimation procedure may be summarised as follows. In the first stage the data, contained in the vector $\mathbf{x}_t$, is fitted to an unrestricted reduced-form VAR as specified in (9). This step yields an estimated coefficient matrix $\hat{\mathbf{A}}$. The second stage of the estimation procedure exploits the cross-equation restrictions described above and involves searching for values of the parameters in $\boldsymbol{\psi}$ that constrain $\mathbf{F}^{DE}(\hat{\mathbf{A}},\boldsymbol{\psi})$ as being close to zero. Specifically, when τ is constrained to equal 1 the parameters $\boldsymbol{\psi}$ are estimated so as to minimise the squared deviation of $\mathbf{g}^{DE}_{con}(\hat{\mathbf{A}},\boldsymbol{\psi})$ from $\mathbf{e}'_\pi\hat{\mathbf{A}}$:

$$\widehat{\boldsymbol{\psi}}^{DE,con} \equiv \arg\min_{\psi}\mathbf{F}^{DE}_{con}(\hat{\mathbf{A}},\boldsymbol{\psi})\cdot\mathbf{F}^{DE}_{con}(\hat{\mathbf{A}},\boldsymbol{\psi})' \tag{16}$$

When τ is unconstrained then the estimated parameters $\boldsymbol{\psi}$ are those that minimise the squared deviation of $\mathbf{g}^{DE}(\hat{\mathbf{A}},\boldsymbol{\psi})$ from $\mathbf{e}'_\pi\hat{\mathbf{A}}^2$:

$$\widehat{\boldsymbol{\psi}}^{DE} \equiv \arg\min_{\psi}\mathbf{F}^{DE}(\hat{\mathbf{A}},\boldsymbol{\psi})\cdot\mathbf{F}^{DE}(\hat{\mathbf{A}},\boldsymbol{\psi})' \tag{17}$$

4.2.2 The Closed Form Specification

Estimation of the closed form formulation of the Australian NKPC, represented in equation (8), is achieved in the same manner as described above. Again, taking expectations at time $t-2$

[25] The subscript *con* indicates that the expression corresponds to the specification where τ is constrained so as to equal 1.

conditional on the forecasting rule (10) yields

$$\begin{aligned}\mathbf{e}_\pi'\mathbf{A}^2\mathbf{z}_{t-2} =& \rho\tau\mathbf{e}_\pi'\mathbf{A}\mathbf{z}_{t-2} + \rho(1-\tau)\mathbf{e}_\pi'\mathbf{I}\mathbf{z}_{t-2} + (1-\phi)\lambda\mathbf{e}_{mcdom}'(\mathbf{I}-\beta\mathbf{A})^{-1}\mathbf{A}^2\mathbf{z}_{t-2} \\ &+ \phi\lambda\mathbf{e}_{mcim}'(\mathbf{I}-\beta\mathbf{A})^{-1}\mathbf{A}^2\mathbf{z}_{t-2}. \end{aligned} \tag{18}$$

The corresponding vector of non-linear cross equation restrictions is given by

$$\begin{aligned}\mathbf{e}_\pi'\mathbf{A}^2 &= \rho\tau\mathbf{e}_\pi'\mathbf{A} + \rho(1-\tau)\mathbf{e}_\pi'\mathbf{I} + (1-\phi)\lambda\mathbf{e}_{mcdom}'(\mathbf{I}-\beta\mathbf{A})^{-1}\mathbf{A}^2 \\ &\quad + \phi\lambda\mathbf{e}_{mcim}'(\mathbf{I}-\beta\mathbf{A})^{-1}\mathbf{A}^2 \\ &\equiv \mathbf{g}^{CF}(\mathbf{A},\boldsymbol{\psi}), \end{aligned} \tag{19}$$

which may be equivalently expressed as

$$\mathbf{F}^{CF}(\mathbf{A},\boldsymbol{\psi}) \equiv \mathbf{e}_\pi'\mathbf{A}^2 - \mathbf{g}^{CF}(\mathbf{A},\boldsymbol{\psi}) = \underline{0}'. \tag{20}$$

The superscripts CF denote expressions which correspond to the closed form NKPC. The first stage VAR is identical to that described for the difference equation specification. The second stage of the estimation is also analogous and involves determining the values of the structural parameters in $\boldsymbol{\psi}$ that minimise the squared deviation of $\mathbf{g}^{CF}(\mathbf{A},\boldsymbol{\psi})$ from $\mathbf{e}_\pi'\mathbf{A}^2$. That is,

$$\widehat{\boldsymbol{\psi}}^{CF} \equiv \arg\min_{\psi} \mathbf{F}^{CF}(\hat{\mathbf{A}},\boldsymbol{\psi}) \cdot \mathbf{F}^{CF}(\hat{\mathbf{A}},\boldsymbol{\psi})'. \tag{21}$$

4.3 Difference Equation vs. Closed Form Estimation

Estimation of the NKPC is typically carried out in its difference equation (Euler equation) form as specified in eqn. (1) (or its open economy equivalent, eqn. (7)). Indeed Cogley and Sbordone's (2008) results focus purely on the estimation of the NKPC with time-varying trend inflation expressed in its difference equation form.

The relationship between the difference equation estimates and those implied by the closed form NKPC is an issue explored deeply in BGLO (2011)[26]. As is evident from BGLO's (2011) analysis, the estimation of the difference equation and the closed from NKPC will yield signifi-

[26] Sbordone (2005) originally estimated the closed form of a hybrid model similar to eqn. (3) using the two-step minimum distance procedure.

cantly different results when using minimum distance estimation. To understand the differences in the structural estimates, recall that inflation expectations are generated by the forecasting rule (10), estimated from the first-stage VAR. Conditional on the estimated inflation forecasts, the second stage uses the minimum distance methods described above to estimate the parameters $\boldsymbol{\psi}$. Since the first-stage VAR estimates are the same across both specifications (DE and CF), the disparity in the parameter estimates arise from the way inflation expectations enter the NKPC. In the difference equation specification, inflation expectations are left unconstrained. In particular, the VAR forecasts of inflation are taken as the only available information on inflation expectations and the structural relationship implied by the difference equation is estimated directly. On the other hand, in the closed form representation of the NKPC the expectations of inflation themselves are constrained so as to follow the structure implied by the difference equation NKPC in every period. In this way the recursive closed form solution adds model-consistent discipline to the evolution of inflation expectations (BGLO, 2011).

Given that the structural relationship implied by the NKPC can only be considered as an approximation of the "true" data generating process for inflation, the DE and CF formulations will not be equivalent[27]. Thus, it will be of interest to examine how the parameter estimates, $\boldsymbol{\psi}$, given $\hat{\mathbf{A}}$, are effected when one compare the estimates based on the difference equation NKPC to the estimates implied by the closed form specification, with its additional model-consistent discipline on inflation expectations.

4.4 Data

In line with much of the Australian literature concerned with modelling inflation, an underlying measure is used, as opposed to headline CPI[28]. Specifically, inflation is measured as the quarterly percentage change in the trimmed-mean CPI adjusted for the introduction of the GST[29]. Although empirical papers on inflation modelling in different countries generally use headline measures of inflation (typically based on the consumer price index), an underlying measure such as trimmed-mean CPI may be preferable as it precludes much of the noise and variation in the CPI. Headline CPI encompasses several components (such as food and energy prices) that

[27]For a more comprehensive discussion of the differences between the DE and CF estimates see BGLO (2011).

[28]See, for example: de Brouwer and Ericsson (1998), Norman and Richards (2010) and Kuttner and Robinson (2010). This thesis uses data from Kuttner and Robinson (2010).

[29]This is the same CPI series as used in the reduced-form analysis in Section 3. Note that prior to March 1982 the trimmed-mean CPI is not official data and have been constructed by the RBA for research purposes.

are subject to large transitory fluctuations. Such transitory fluctuations may have significant ramifications for consumers, however, they are not easily captured in a structural model.

When modelling CPI inflation the labour share (which is often used as a proxy for real marginal costs in the US literature) will be an inappropriate measure of real marginal costs as it deflates nominal marginal costs by the GDP deflator (rather than the CPI). Accordingly, a measure of nominal unit labour costs deflated by the CPI is used as a proxy for real marginal costs of domestic producers[30]. As noted by Norman and Richards (2010), deflating nominal unit labour costs by trimmed-mean CPI has the added benefit of abstracting from the substantial influence that commodity prices have on the GDP deflator (and hence labour's share) in Australia. As discussed above, marginal costs for the import sector are measured as import prices relative to consumer prices[31]:

$$mc_t^m = p_t^m - p_t.$$

For the constant trend inflation case the reduced-form time-varying VAR is of order 2, with the ordering of the variables given by: mc_t^d, π_t and mc_t^m. In line with small open economy models, foreign real marginal costs , mc_t^m (as proxied by real import prices), are regarded as exogenous and it is therefore the last variable in the VAR(2) ordering.

4.5 Estimation Results

Table 3 presents median estimates of the deep structural parameters $\boldsymbol{\psi} = [\alpha, \rho, \tau]$ for the full sample period: 1960:Q1-2007:Q2[32]. The results are presented for four specifications of the NKPC: the difference equation (DE) form and the closed form (CF) representation, with τ constrained to equal one, and unconstrained τ. Given identification issues, θ was calibrated as 5 during all estimations, implying a desired steady-state price markup of 25%[33]. In addition, given that the weight of imported goods in consumption, ϕ, has been a difficult parameter to

[30]Nominal unit labour costs are for the non-farm sector. The series is calculated as Compensation of Employees (ABS Table 41 National Accounts) + Payroll Tax (Table 39) less Subsidies (Table 39) divided by seasonally adjusted real non-farm GDP (Table 41).

[31]Import prices are measured as the implicit price deflator (ABS Table 5 National Accounts) adjusted for the declining rate of tariff protection on imports into Australia over much of the sample period. Following Beechy, Bharucha, Cagliarini, Gruen and Thompson (2000) log import prices are defined as: $p_t^{m,tar} = p_t^m + (1 + \mathit{tariff}_t)$, where tariff_t is the average tariff rate on Australian imports (not its log).

[32]The 90% confidence intervals presented in Table 3 were derived using bootstrapping procedures.

[33]This is the desired steady-state markup in the flexible-price equilibrium. The calibrated value is consistent with the subsequent estimates of θ in the model with time-varying trend (see Table 6 in Section 5.4), which does not suffer from the same identification problem.

Table 3
Structural parameter estimates (baseline NKPC with zero trend inflation)
Sample period: 1960:Q1 - 2007:Q2
$\phi = 0.2$; $\theta = 5$

	ρ	α	τ
DE_{con}	0.005	0.999	1
	(0, 0.317)	(0.678, 1)	-
DE_{uncon}	0.874	0.771	0.706
	(0.401, 1)	(0.623, 1)	(0.385, 0.990)
CF_{con}	0.396	0.896	1
	(0.150, 0.562)	(0.824, 1)	-
CF_{uncon}	0.795	0.943	0.801
	(0.373, 0.861)	(0.854, 1)	(0.310, 0.992)

Notes: numbers in parentheses are 90% confidence intervals; DE_{con} and CF_{con} correspond to the difference equation and closed form specifications respectively, with $\tau = 1$; DE_{uncon} and CF_{uncon} correspond to the difference equation and closed form specifications with unconstrained τ, respectively.

estimate (see Kuttner and Robinson, 2010), the estimates presented in Table 3 correspond to the case where ϕ is calibrated to 0.2. This calibrated value is approximately equal to the average share of imports in Australian GDP since 1993 and is also used by Kuttner and Robinson (2010) in their analysis of the Australian NKPC[34].

The first row of Table 3 reports the DE parameter estimates with τ constrained to equal 1. For this specification (denoted DE_{con}) the indexation parameter, ρ, is estimated at 0.0051 indicating that the role of lagged inflation in determining current inflation is negligible, which of course implies a purely forward-looking NKPC. However, when the DE specification is considered without constraining τ to unity (i.e. DE_{uncon}), the median estimate for ρ increases dramatically to 0.8739, with its confidence interval bounded well above zero. Thus, allowing for two lags of inflation in the indexation mechanism produces a much higher (and significant) estimate for ρ, suggesting the possibility of misspecification bias in the DE_{con} estimates. Furthermore, τ is estimated at 0.706, with the upper bound of its confidence interval lying below 1. In all, the DE_{uncon} estimates indicate that the indexation to the last two lags of inflation is more appropriate for characterising inflation persistence in Australia.

Focusing on the CF estimates it is clear that in the constrained case (denoted CF_{con}) the me-

[34]This value is also consistent with Nimark (2009), who calibrates the import share as 0.18.

dian estimate for ρ is noticeably higher than its DE counterpart. However, for the unconstrained specification (denoted CF_{uncon}) the estimated value for ρ is lightly lower than in the corresponding DE case. Nevertheless, the estimates for ρ are clearly indicative of a backward-looking component in Australia's NKPC given that the (seemingly misspecified) DE_{con} specification is the only case where ρ is statistically and economically insignificant.

4.5.1 Implied NKPC coefficients

From (7), the Australian NKPC may be expressed in its reduced-form

$$\pi_t = \gamma_{b,1}\pi_{t-1} + \gamma_{b,2}\pi_{t-2} + \gamma_f E_t\pi_{t+1} + \tilde{\lambda} mc_t,$$

where

$$\gamma_{b,1} = \frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}$$
$$\gamma_{b,2} = \frac{\rho(1-\tau)}{1+\beta\rho\tau}$$
$$\gamma_f = \frac{\beta}{1+\beta\rho\tau} \ .$$

In the constrained cases, where $\tau = 1$, the coefficients collapse such that $\gamma_{b,1} = \rho/(1+\beta\rho)$, $\gamma_f = \beta/(1+\beta\rho)$ and $t-2$ inflation does not enter the specification.

Conditioning on the point estimates of the structural parameters, Table 4 displays the implied

Table 4
Implied NKPC coefficients
Sample: 1960:Q1 - 2007:Q2

	$\gamma_{b,1}$	$\gamma_{b,2}$	γ_f
DE_{con}	0.005	-	0.985
DE_{uncon}	0.225	0.155	0.615
CF_{con}	0.284	-	0.711
CF_{uncon}	0.294	0.097	0.607

Note: the implied NKPC coefficients are conditional on the point estimates of the parameters presented in Table 3.

NKPC coefficients for the full sample period. Unsurprisingly the DE_{con} specification implies essentially a purely-forward looking NKPC, with the coefficient on expected future inflation near unity. However, all three remaining specifications suggest that there is an important backward-looking component in Australia's NKPC. The results seem to indicate an enhanced role for backward-looking behaviour in comparison to the conventional GMM estimates of the Australian NKPC presented in Kuttner and Robinson (2010). Kuttner and Robinson (2010) estimate $\gamma_f = 0.806$ and $\gamma_{b,1} = 0.166$ for the same sample period. Comparing the CF estimates to the DE estimates seems to suggest that when all model-consistent restrictions are placed on the evolution of inflation expectations the weight given to lagged inflation tends to increase. Despite the apparent increased role for lagged inflation, the estimates in Table 4 continue to suggest that the Australian NKPC is predominantly forward-looking, even when one considers the closed form coefficients.

The aforementioned results are largely illustrative but nonetheless display the clear disparity between the DE and CF estimates. The estimates reported for the DE_{uncon} specification and the closed form suggest the presence of an important backward-looking component in the Australian NKPC. In the next section these issues are explored in greater detail in the context of a refined Calvo model[35]. Trend inflation is incorporated into the Calvo model yielding an extended NKPC with time-varying coefficients.

[35] Discussion of the estimates for α and θ are reserved until Section 5.4.

5 The Australian NKPC with Time-Varying Trend Inflation

The traditional NKPC framework, as analysed in the previous section, relies on the assumption that inflation in the steady-state is zero. This section introduces an adapted version of Cogley and Sbordone's (2008) refined Calvo model, which divorces itself from this conventional assumption, and documents its performance against the Australian inflation data.

In essence, Cogley and Sbordone (2008) develop a dynamic version of the NKPC characterised by the inclusion of trend inflation, which is subject to variation over time. It is their contention that variation in the long-run trend component of inflation is the source of the apparent persistence in US inflation. In contrast to the simple NKPC framework, the Cogley and Sbordone's (2008) model takes the log-linearisation of the equilibrium conditions of the refined Calvo model around a steady-state associated with *drifting* trend inflation. In the baseline NKPC, the log-linearisation is taken around a constant trend ($\overline{\pi} = 0$), which is the same in every period. In the context of this framework trend inflation is assumed to be an exogenous process that is modelled as a random walk. Unlike the traditional framework, with zero trend inflation, the NKPC coefficients in this setup will evolve over time according to the level of trend inflation.

First, the equilibrium conditions of Cogley and Sbordone's (2008) generalised Calvo model are described. Their model is augmented slightly to capture open economy dimensions and, in the same vein as described in Section 4, also allows for two lags of inflation in the indexation mechanism[36].

The primary equilibrium relationship is the restriction between trend inflation and steadstate real marginal costs, which is of the form

$$\left(1 - \alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}\right)^{\frac{1+\theta\omega}{1-\theta}} \left[\frac{1 - \alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{\theta(1+\omega)(1-\rho)}}{1 - \alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{(1-\rho)(\theta-1)}}\right] = (1-\alpha)^{\frac{1+\theta\omega}{1-\theta}} \frac{\theta}{\theta-1} (\overline{mc}_t^d)^{1-\phi} (\overline{mc}_t^m)^{\phi}, \quad (22)$$

where $\bar{q}$ denotes the steady-state real discount factor, $\bar{g}^y$ is steady-state output growth, $\overline{\Pi}_t$ is gross trend inflation at time t. The structural parameters, $\alpha, \rho, \theta, \omega$ and ϕ, retain their definitions from the previous section. The above restriction differs in an important respect from the equivalent expression in Cogley and Sbordone (2008)[37]. In this case, the right hand side includes both

[36] For the full details of the derivation see Appendix A. Note that this paper follows the version in BGLO (2011), which allows for two lags of inflation in the indexation mechanism.

[37] See equation (7) in Cogley and Sbordone (2008).

$\overline{mc}_t^d$ and $\overline{mc}_t^m$, which represent trend real marginal costs of domestic producers and importers, respectively. Both terms enter the restriction according to the respective weights of domestically produced goods and imports in domestic consumption. In this way (22) provides a simple mechanism for capturing the open economy aspects of the inflationary process in Australia. Following Cogley and Sbordone (2008) it is assumed that the following two inequalities hold

$$\varphi_{1,t} = \alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{(1-\rho)(\theta-1)} < 1 \tag{23}$$

and

$$\varphi_{2,t} = \alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{\theta(1-\omega)(1-\rho)} < 1. \tag{24}$$

These inequalities ensure that the steady-state relationship (22) is well defined. As in the constant trend case, it is assumed that the structural parameters are equivalent across both the domestic and foreign sectors. This simplifying assumption allows for the derivation of a single equation extended NKPC, which is obtained by log-linearising (22) around a steady-state with drifting trend inflation

$$\begin{aligned}\widehat{\pi}_t \quad = \quad & \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^\pi) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^\pi - \widehat{g}_t^\pi) + \Omega_t E_t[\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}_t^\pi)] \\ & + \lambda_t\left[(1-\phi)\widehat{mc}_t^d + \phi\,\widehat{mc}_t^m\right] + \gamma_t\widehat{D}_t + u_{\pi,t}\,, \end{aligned} \tag{25}$$

where hatted variables represent log-deviations of stationary variables from their steady-state values[38]. $\widehat{D}$ is defined recursively as[39]

$$\widehat{D}_t = \varphi_{1,t}E_t(\widehat{q}_{t,t+1} + \widehat{g}_{t+1}^y) + \varphi_{1,t}(\theta-1)E_t\{\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}_t^\pi)\} + \varphi_{1,t}E_t\widehat{D}_{t+1}. \tag{26}$$

For ease of comparison to Cogley and Sbordone (2008), the extended NKPC in its DE form (as

[38]In particular, $\widehat{\pi}_t = \ln(\Pi_t/\overline{\Pi}_t)$, $\widehat{mc}_t^d = \ln(mc_t^d/\overline{mc}_t^d)$, $\widehat{mc}_t^m = \ln(mc_t^m/\overline{mc}_t^m)$ and $\widehat{g}_t^\pi = \ln(\overline{\Pi}_t/\overline{\Pi}_{t-1})$ is the growth rate of trend inflation; $u_{\pi,t}$ is a structural shock.

[39]Here $\widehat{g}_t^y = \ln(g_t^y/\overline{g}^y)$ and $\widehat{q}_{t,t+1} = \ln(q_{t,t+1}/\overline{q}_{t,t+1})$. Also note that $\varphi_{0,t} = \left[\frac{1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}{\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}\right]$, and $\Omega_t = \varphi_{2,t}(1+\varphi_{0,t})$.

described by equations (25) and (26)) may be expressed as[40]:

$$
\begin{aligned}
\widehat{\pi}_t \;=\; & \widetilde{\rho}^{DE}_{1,t}(\widehat{\pi}_{t-1} - \widehat{g}^{\pi}_t) + (1-\tau)\widetilde{\rho}^{DE}_{2,t}(\widehat{\pi}_{t-2} - \widehat{g}^{\pi}_{t-1} - \widehat{g}^{\pi}_t) \\
& + \widetilde{\lambda}^{DE}_t \left[(1-\phi)\widehat{mc}^d_t + \phi\, \widehat{mc}^m_t\right] \\
& + b^{DE}_{1,t} E_t \widehat{\pi}_{t+1} \\
& + b^{DE}_{2,t} E_t \sum_{j=2}^{\infty} \varphi^{j-1}_{1,t} \widehat{\pi}_{t+j} \\
& + b^{DE}_{3,t} E_t \sum_{j=0}^{\infty} \varphi^{j}_{1,t} \left[\widehat{Q}_{t+j,t+j+1} + \widehat{g}^{y}_{t+j+1}\right] + \tilde{u}_{\pi,t}. \qquad (27)
\end{aligned}
$$

While the extended NKPC in (27) retains a similar form to the traditional NKPC, the two variations differ in some important respects. Firstly, the extended NKPC contains additional variables, such as the growth rate in trend inflation ($\widehat{g}^{\pi}_t$), as well as terms involving the nominal discount factor ($\widehat{Q}_t$), real output growth ($\widehat{g}^{y}_t$) and higher-order leads of inflation. While these additional terms may suggest the presence of omitted variable bias in the traditional NKPC, perhaps a more important distinguishing feature of the extended NKPC is the fact that the NKPC coefficients in (27) are non-linear functions of trend inflation and the structural parameters of the model. Consequently these coefficients, which are of interest to policy makers, are subject to variation over time and evolve according to the drift in trend inflation[41].

It is possible to obtain a closed form representation of the NKPC by iterating equations (25) and (26) forward[42]:

$$
\begin{aligned}
\widehat{\pi}_t \;=\; & \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}^{\pi}_t) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}^{\pi}_{t-1} - \widehat{g}^{\pi}_t) + \lambda_t \sum_{k=0}^{\infty} \Omega^k_t E_t \left[(1-\phi)\widehat{mc}^d_{t+k} + \phi\, \widehat{mc}^m_{t+k}\right] \\
& + \gamma_t \sum_{k=0}^{\infty} \Omega^k_t E_t \widehat{D}_{t+k} + u_{\pi,t}\,. \qquad (28)
\end{aligned}
$$

To maintain consistency with the baseline case in Section 4 in the pursuant analysis the same four specifications of the extended NKPC are estimated. In particular, the DE specification (27)

[40] See Appendix A for complete derivation and definition of coefficients.

[41] A discussion of the implied NKPC coefficients is undertaken in Section 5.5.

[42] For a complete derivation of the closed form specification see Appendix B. As stipulated in BGLO (2011) equation (28) is more appropriately referred to as the "quasi-closed form" NKPC as $\widehat{\pi}_t$ remains a function of higher-order leads of inflation. It is possible to obtain an exact closed form solution, however, BGLO (2011) show that the US estimates of the exact closed form are very similar to the quasi-closed form. Their results indicate that the additional restrictions that the exact closed form solution imposes over (28) are not critical for estimating the structural parameters of the model.

is estimated for the case where τ is constrained to equal one (in this case the NKPC collapses to the corresponding version originally estimated by Cogley and Sbordone, 2008), and for the case where τ is left unconstrained. Similarly, the NKPC in its CF representation (28) is estimated for both the constrained and unconstrained cases.

5.1 Estimating the NKPC Structural Parameters

As in Section 4 the objective is estimate the structural parameters of the NKPC model, $\boldsymbol{\psi} = [\alpha, \rho, \theta, \tau]$. The same two-step estimation approach, as described in Section 4.2, is adopted here. In the present context the time series vector $\mathbf{x}_t$ is extended so as to include not only inflation and real marginal costs, but also a nominal discount factor and output growth, so that $n = 5$. The reduced-form VAR is now written as

$$\mathbf{z}_t = \boldsymbol{\mu}_t + \mathbf{A}_t \mathbf{z}_{t-1} + \epsilon_{z,t}, \tag{29}$$

where $\epsilon_{z,t}$ is a serially uncorrelated error vector. In contrast to the estimation of the baseline NKPC, the reduced-form VAR now has drifting coefficients, captured in $\boldsymbol{\mu}_t$ and $\mathbf{A}_t$. The drift in the VAR coefficients follows from the assumption that trend inflation is time-varying.

As detailed previously, if the extended NKPC is the true data generating process for inflation, then the forecasts from the reduced-form VAR in (29) and the structural forecasts from (27) should be equivalent. Accordingly, the forecasting rule in equation (10) is augmented such that the conditional expectation of a variable $\widehat{y}_{t+j} \in \mathbf{x}_{t+j}$ at time t is now written as

$$E_t \widehat{y}_{t+j} = \mathbf{e}_y' \mathbf{A}_t^j \widehat{\mathbf{z}}_t, \tag{30}$$

where $\mathbf{e}_y'$ is the selection vector as defined previously. The above condition differs from the original forecasting rule (10) as it defines expectations of $\widehat{y}_{t+j}$, rather than y_{t+j}. As such $\widehat{\mathbf{z}}_t$ represents the vector of variables expressed in deviations from their time-varying trend levels at time t

$$\widehat{\mathbf{z}}_t \equiv \mathbf{z}_t - (\mathbf{I} - \mathbf{A}_t)^{-1} \boldsymbol{\mu}_t. \tag{31}$$

5.1.1 The Difference Equation Specification (*DE*)

Given the structural representations of the NKPC in its difference equation form (27), and the above forecasting rule (30), the conditional expectation of inflation is written as

$$\begin{aligned}
\mathbf{e}'_\pi \mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} &= \widetilde{\rho}^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}_{t-2}\widehat{\mathbf{z}}_{t-2} + (1-\tau)\widetilde{\rho}^{DE}_{2,t-2}\mathbf{e}'_\pi \widehat{\mathbf{z}}_{t-2} \\
&+ (1-\phi)\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcdom}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} + \phi\,\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcim}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&+ b^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}^3_{t-2}\widehat{\mathbf{z}}_{t-2} + b^{DE}_{2,t-2}\varphi_{1,t}\mathbf{e}'_\pi \mathbf{M}_{t-2}\mathbf{A}^4_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&+ b^{DE}_{3,t-2}(\mathbf{e}'_q \mathbf{M}_{t-2}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} + \mathbf{e}'_{g^y}\mathbf{M}_{t-2}\mathbf{A}^3_{t-2}\widehat{\mathbf{z}}_{t-2}), \qquad (32)
\end{aligned}$$

where

$$\mathbf{M}_t \equiv (\mathbf{I} - \varphi_{1,t}\,\mathbf{A}_t)^{-1}. \qquad (33)$$

That expectations are taken as at $t-2$ follows from the fact that the indexation mechanism allows for two lags of inflation. The assumption that trend inflation follows a driftless random walk implies that the expected future growth rate of trend inflation is zero, thus all terms involving $\widehat{g}^\pi_t$ are ignored[43]. The vector of non-linear cross-equation restrictions derived from the conditional expectation in (32) is given by

$$\begin{aligned}
\mathbf{e}'_\pi \mathbf{A}^2_{t-2} &= \widetilde{\rho}^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}_{t-2} + (1-\tau)\widetilde{\rho}^{DE}_{2,t-2}\mathbf{e}'_\pi \mathbf{I} \\
&+ (1-\phi)\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcdom}\mathbf{A}^2_{t-2} + \phi\,\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcim}\mathbf{A}^2_{t-2} \\
&+ b^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}^3_{t-2} + b^{DE}_{2,t-2}\varphi_{1,t}\mathbf{e}'_\pi \mathbf{M}_{t-2}\mathbf{A}^4_{t-2} \\
&+ b^{DE}_{3,t-2}(\mathbf{e}'_q \mathbf{M}_{t-2}\mathbf{A}^2_{t-2} + \mathbf{e}'_{g^y}\mathbf{M}_{t-2}\mathbf{A}^3_{t-2}) \\
&\equiv \mathbf{g}^{DE}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}). \qquad (34)
\end{aligned}$$

Equivalently

$$\mathbf{F}^{DE}_1(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \equiv \mathbf{e}'_\pi \mathbf{A}^2_{t-2} - \mathbf{g}^{DE}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) = \underline{0}', \;\; \forall t\ . \qquad (35)$$

[43] The assumption, in equation (23), that $|\varphi_{1,t}| < 1$, combined with the fact that the coefficient matrix $\mathbf{A}_t$ is constrained such that the roots of $\mathbf{A}_t$ lie inside the unit circle at each period in time, implies that the series $\mathbf{I} + \varphi_{1,t}\,\mathbf{A}_t + \varphi^2_{1,t}\,\mathbf{A}^2_t + ...$, converges and can be expressed as in (33).

In contrast to the baseline NKPC case, when inflation has a time-varying trend the parameters must also satisfy the steady-state restriction between trend inflation and marginal costs given by (22). The condition may be re-written as

$$\begin{aligned} \mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \quad &\equiv \quad \left(1 - \alpha \overline{\Pi}_t^{(1-\rho)(\theta-1)}\right)^{\frac{1+\theta\omega}{1-\theta}} \left[\frac{1 - \alpha \bar{q} \bar{g}^y \overline{\Pi}_t^{\theta(1+\omega)(1-\rho)}}{1 - \alpha \bar{q} \bar{g}^y \overline{\Pi}_t^{(1-\rho)(\theta-1)}} \right] \\ &\quad -(1-\alpha)^{\frac{1+\theta\omega}{1-\theta}} \frac{\theta}{\theta - 1} (\overline{mc}_t^d)^{1-\phi} (\overline{mc}_t^m)^{\phi} \\ &= \quad \underline{0}' \end{aligned} \tag{36}$$

Thus, $\mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi})$ must also be minimised at each period of the estimation sample. The two moment conditions (35) and (36) are combined by defining the vector

$$\mathcal{F}_t^{DE} = [\mathbf{F}_1^{DE}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \; \mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi})].$$

The complete set of cross-equation restrictions that must be satisfied is represented as the long vector,

$$\mathcal{F}^{DE}(\boldsymbol{\Theta}) = \left[\mathcal{F}_1^{DE}, \mathcal{F}_2^{DE}, ..., \mathcal{F}_T^{DE}\right], \tag{37}$$

where

$$\boldsymbol{\Theta} \equiv \{\boldsymbol{\mu}_t, \mathbf{A}_t\}_{t=1}^{T}. \tag{38}$$

As in the baseline NKPC case the first stage of the estimation procedure involves fitting the data to an unrestricted reduced-form VAR. However, in the presence of time-varying trend inflation, estimation of the first-stage VAR is achieved by using the Bayesian methods as detailed in Cogley and Sargent (2005) to deliver the posterior distribution of $\boldsymbol{\Theta}$ from a set of N estimates $\{\widehat{\boldsymbol{\Theta}}_i\}_{i=1}^{N}$[44]. Conditional on the estimates $\{\widehat{\boldsymbol{\Theta}}_i\}$, the structural parameters $\widehat{\boldsymbol{\psi}}_i^D$ are estimated in

[44] Due to memory constraints the number of estimates in this analysis is set to $N = 1,000$. Both Cogley and Sbordone (2011) and BGLO (2011) use 5,000 ensembles. This discrepancy is explored in Section 6. For a complete description of VAR specification, see Section III B in Cogley and Sbordone (2008).

similar fashion to the baseline NKPC case

$$\widehat{\boldsymbol{\psi}}_i^D = \arg\min_{\psi} \mathcal{F}^D(\widehat{\boldsymbol{\Theta}}_i) \cdot \mathcal{F}^D(\widehat{\boldsymbol{\Theta}}_i)' \quad \text{for } i = 1, ..., N. \tag{39}$$

5.1.2 The Closed Form Specification (*CF*)

Given the closed form specification of the NKPC in (28) and the forecasting rule (30), the conditional expectation of inflation as at $t-2$ is expressed as[45]

$$\begin{aligned}
\mathbf{e}_\pi' \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} \quad = \quad & \widetilde{\rho}_{2,t-2}^{CF} \mathbf{e}_\pi' \mathbf{A}_{t-2} \widehat{\mathbf{z}}_{t-2} + (1-\tau) \widetilde{\rho}_{2,t-2}^{CF} \mathbf{e}_\pi' \widehat{\mathbf{z}}_{t-2} + (1-\phi) \widetilde{\lambda}_{t-2}^{CF} \mathbf{e}_{mcdom}' \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} \\
& + \phi\, \widetilde{\lambda}_{t-2}^{CF} \mathbf{e}_{mcim}' \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} + b_{0,t-2}^{CF} \mathbf{e}_\pi' \mathbf{K}_{t-2} \mathbf{A}_{t-2} \widehat{\mathbf{z}}_{t-2} \\
& + b_{1,t-2}^{CF} \mathbf{e}_\pi' \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} + b_{2,t-2}^{CF} \mathbf{e}_\pi' \mathbf{K}_{t-2} \mathbf{A}_{t-2}^3 \widehat{\mathbf{z}}_{t-2} \\
& + b_{2,t-2}^{CF} \varphi_{1,t-2} \mathbf{e}_\pi' \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^4 \widehat{\mathbf{z}}_{t-2} \\
& + b_{3,t-2}^{CF} (\mathbf{e}_q' \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} + \mathbf{e}_{g^y}' \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^3 \widehat{\mathbf{z}}_{t-2}),
\end{aligned} \tag{40}$$

where

$$\mathbf{K}_t \equiv (\mathbf{I} - \Omega_t \mathbf{A}_t)^{-1}. \tag{41}$$

In order for the series $\mathbf{I} + \Omega_t \mathbf{A}_t + \Omega_t^2 \mathbf{A}_t^2 + ...$, to converge and be represented as in (41) the roots of $\Omega_t \mathbf{A}_t$ need to lie inside the unit circle, so that $|\Omega_t \mathbf{A}_t| < 1$. As stressed in BGLO (2011) this condition is not guaranteed by the conditions of the model, and thus is an important empirical issue. Essentially, the estimation procedure (in compliance with BGLO, 2011 and Cogley and Sbordone, 2008) assumes that the NKPC has a reduced-form VAR representation as in (22), which implies $|\Omega_t \mathbf{A}_t| < 1$. This issue is explored further in the robustness analysis conducted in Section 6.

The vector of non-linear cross-equation restrictions derived from the conditional expectation

[45] For a complete derivation see Appendix B.

in (40) is now given by

$$\begin{aligned}
\mathbf{e}_\pi'\mathbf{A}_{t-2}^2 &= \widetilde{\rho}_{2,t-2}^{CF}\mathbf{e}_\pi'\mathbf{A}_{t-2} + (1-\tau)\widetilde{\rho}_{2,t-2}^{CF}\mathbf{e}_\pi'\mathbf{I} + (1-\phi)\widetilde{\lambda}_{t-2}^{CF}\mathbf{e}_{mcdom}'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^2 \\
&\quad + \phi\,\widetilde{\lambda}_{t-2}^{CF}\mathbf{e}_{mcim}'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^2 + b_{0,t-2}^{CF}\mathbf{e}_\pi'\mathbf{K}_{t-2}\mathbf{A}_{t-2} \\
&\quad + b_{1,t-2}^{CF}\mathbf{e}_\pi'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^2 + b_{2,t-2}^{CF}\mathbf{e}_\pi'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^3 \\
&\quad + b_{2,t-2}^{CF}\varphi_{1,t-2}\mathbf{e}_\pi'\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}_{t-2}^4 \\
&\quad + b_{3,t-2}^{CF}(\mathbf{e}_q'\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}_{t-2}^2 + \mathbf{e}_{g^y}'\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}_{t-2}^3) \\
&\equiv \mathbf{g}^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}). \qquad (42)
\end{aligned}$$

The corresponding minimum distance problem is given by

$$\mathbf{F}_1^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \equiv \mathbf{e}_\pi'\mathbf{A}_{t-2}^2 - \mathbf{g}^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) = \underline{0}', \ \forall t \ . \qquad (43)$$

As in the difference equation specification the steady-state restriction (22) must be satisfied at all stages of the estimation. Thus, the moment condition given by (36) also applies for the closed form specification. The steady-state condition (36) is now combined with the closed form minimum distance condition (43) by defining the vector $\mathcal{F}_t^{CF} = [\mathbf{F}_1^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi})\mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi})]$. The complete set of cross-equation restrictions that must be satisfied is given by

$$\mathcal{F}^{CF}(\boldsymbol{\Theta}) = \left[\mathcal{F}_1^{CF}, \mathcal{F}_2^{CF}, ..., \mathcal{F}_T^{CF}\right]. \qquad (44)$$

The first stage VAR that provides the Bayesian posterior for $\boldsymbol{\Theta}$ is identical to the VAR estimated for the difference equation specification. The estimates of the model's deep parameters are obtained according to

$$\widehat{\boldsymbol{\psi}}_i^{CF} = \arg\min_{\psi} \mathcal{F}^{CF}(\widehat{\boldsymbol{\Theta}}_i) \cdot \mathcal{F}^{CF}(\widehat{\boldsymbol{\Theta}}_i)' \quad \text{for } i = 1, ..., N. \qquad (45)$$

5.2 Data

The data for inflation and real marginal costs used in the estimation of the first-stage VAR is the same as in the baseline case, detailed in Section 4.4. However, as mentioned above, in the time-varying trend case the time series vector $\mathbf{x}_t$ is extended to include measures of output growth and a nominal discount factor, in compliance with the steady-state condition (22). Output growth is calculated using quarterly non-farm real GDP, seasonally adjusted at an annual rate[46]. The nominal discount factor, Q_t, is constructed by expressing the Reserve Bank's "cash rate" on a quarterly discount basis [47]:

$$Q_t = (1 + r_t)^{-\frac{1}{4}},$$

where r_t is the RBA's cash rate expressed as a decimal. The monthly cash rate data, as published by the RBA, was converted to quarterly values by point-sampling the first month of each quarter.

As in Cogley and Sbordone (2008) the reduced-form time-varying VAR is of order 2. However, in order to capture open economy effects, this analysis requires the inclusion of real marginal costs of importers, resulting in a five-variable VAR as opposed to Cogley and Sbordone's (2008) four. The ordering of the variables given by: g_t^y, mc_t^d, π_t, Q_t and mc_t^m. Again, given the exogeneity of foreign real marginal costs, mc_t^m is the last variable in the VAR(2) ordering. The sample encompasses the period 1960:Q1 - 2007:Q2. However, the time-varying trend estimation requires that data from 1960:Q1 - 1965:Q4 be used to initialise the prior, thus the model is estimated for the sample period 1966:Q1 - 2007:Q2.

5.3 Trend Inflation and Persistence

Prior to the exposition of the estimation results in the next section, it is worthwhile to present some preliminary evidence of the importance of trend inflation and in particular its relevance as a possible explanation of persistence. As in Cogley and Sbordone (2008), a measure for trend inflation is approximated from the first-stage VAR by estimating mean inflation at time t according to:

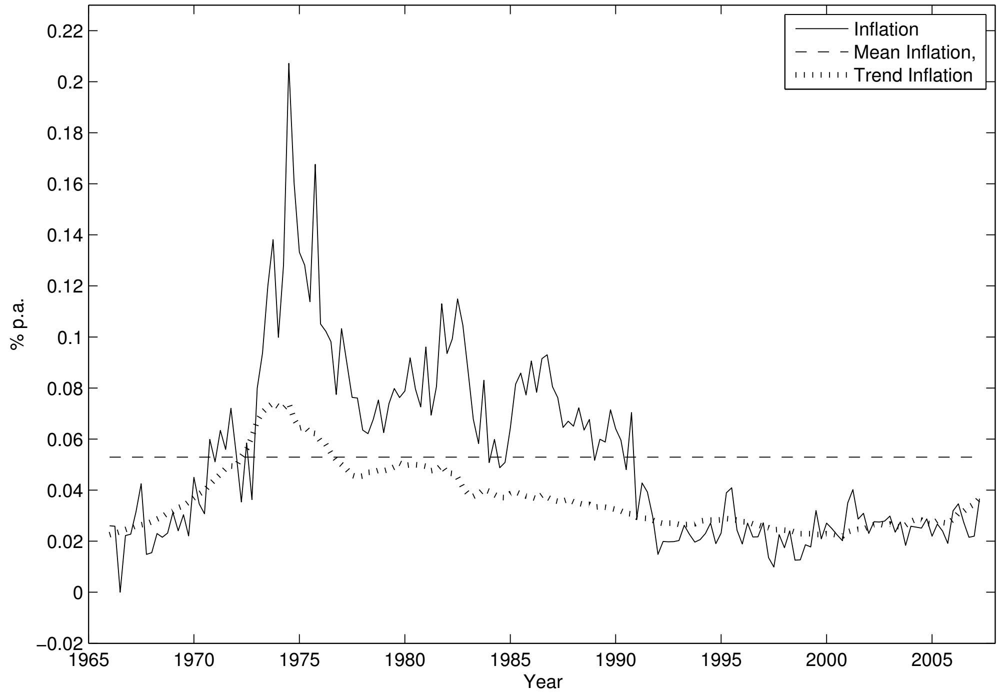
$$\overline{\pi}_t = \mathbf{e}'_\pi (\mathbf{I} - \mathbf{A}_t)^{-1} \boldsymbol{\mu}_t. \tag{46}$$

[46]Real non-farm GDP is the same series as used in Kuttner and Robinson (2010) (ABS National Accounts, Table 41).

[47]Prior to 1990 the Reserve Bank did not publish a cash rate target, thus the cash rate measures prior to 1990 are proxies of the current measure, and were obtained upon request from the Reserve Bank. The Reserve Bank previously used other measures of the short-term interest rates, such as the unofficial 11am call rate and the official (authorised dealers') rate, as its tool of conducting monetary policy.

Figure 6 displays the median estimate of trend inflation at each period for the sample 1966:Q1 - 2007:Q2. For comparison, actual inflation, as measured by the trimmed-mean CPI, and mean inflation for the full sample are also presented.

Figure 6
Trend Inflation in Australia, 1966:Q1 - 2007:Q2



The graph provides a sound depiction of some of the more important features of trend inflation, and its significant role in explaining the persistence of inflation. The first notable feature of the plot is, of course, the fact that the estimated trend in inflation exhibits variation over time. The estimates indicate that trend inflation rose from 2.3% per annum in the late 1960s to approximately 7.8% in the 1970s, and then fell back to levels below 3% during the 1990s. The latter part of the sample is perhaps of most interest in terms of its relevance to policy measures enacted by the Reserve Bank of Australia. As stipulated by Cogley and Sbordone (2008), in general equilibrium, trend inflation is determined by the long-run target in the central bank's policy rule[48]. Accordingly, their methodology allows for the interpretation of movements in $\overline{\pi}_t$ as ultimately reflecting shifts in that target. Figure 6 clearly depicts a reduction in the level of trend inflation during the 1990s. Furthermore, trend inflation from 1993 - 2007 hovers between

[48]Or, peoples' expectations of that target.

2-3%, which coincides with the Reserve Bank's targeted inflation level.

The time-varying nature of trend inflation also has significant implications for the evolution of the inflation gap. Conventional NKPCs model inflation in terms of deviations from a constant mean or a zero inflation steady-state. In the present analysis, however, trend inflation exhibits drift over time, and accordingly the inflation gap is measured as the deviation of inflation from its time-varying trend. Recall:

$$\widehat{\pi}_t = \pi_t - \overline{\pi}_t. \tag{47}$$

Cogley and Sbordone (2008) argue that the measurement of the inflation gap is an important consideration as it affects the degree of persistence within the model. It is their contention that mean-based measures of the inflation gap will be display greater persistence than their trend-based counterparts. Examination of Figure 6 seems to loosely support such a conclusion, as there are long periods – in particular, during the 1960s, then from the 1970s through to the late 1980s and during the 1990s – where inflation does not cross its sample mean. In contrast, inflation intersects its time-varying trend more frequently, especially during the inflation targeting period. Cogley and Sbordone (2008) go further by conducting an autocorrelation analysis of the two different measures of the inflation gap. Indeed, their results for the US support their assertion that the trend-based inflation gap displays lower persistence than the mean-based measure. These results lead the authors to question whether mean-based measures, in fact, reflect an exaggeration of persistence, rather than there being a lack of persistence in forward-looking models. Examination of the corresponding autocorrelations, however, reveals a somewhat murkier picture for inflation persistence in Australia. Table 5 presents the autocorrelations of the two measures of the inflation gap.

Table 5
First-order Autocorrelations of the Inflation Gap

	1966:Q1 - 2007:Q2	1993:Q1 - 2007:Q2
Inflation	0.8935	0.3386
Trend-Based Gap	0.8088	0.3627

The first row refers to the deviation of inflation from its constant mean[49]. The second row displays the autocorrelations of the trend-based gap as described by equation (47). For the long sample both measures display high degrees of persistence, albeit, with the trend-based measure slightly less persistent. During the inflation targeting period the autocorrelation of the inflation gap is reduced significantly according to both measures. However, contrary to the findings of Cogley and Sbordone (2008), the trend-based inflation gap for Australia displays slightly higher persistence than the mean-based gap after the onset of an inflation target. According to both measures the Australian NKPC only needs to account for a relatively modest degree of persistence post-inflation targeting, indicating that a purely forward-looking NKPC may be sufficient to explain inflation dynamics in more recent years.

5.4 Estimation Results

Table 6 displays the benchmark estimates of the structural parameters $\boldsymbol{\psi} = [\alpha, \rho, \theta, \tau]$ for the full-sample (1966:Q1 - 2007:Q2). As in the constant trend case, the discount factor, β, and the strategic complementary parameter, ω, are pinned down to values of 0.99 and 0.429 respectively. Again, given the difficulty in estimating the weight of imported goods in consumption, ϕ, the estimates reported in Table 6 correspond to the case where ϕ is calibrated to 0.2. For robustness, the model was also estimated for the case where ϕ is left unconstrained. As expected the parameter estimates are left largely unaffected (see Table 8, in Section 6), however, due to the imprecision of the estimates of ϕ itself, the results presented in Table 6 are preferred as a benchmark.

In Table 6 the estimates implied by the DE and CF specifications are compared.[50]. The first row of the table corresponds to the estimates that are presented by Cogley and Sbordone (2008) (denoted as DE_{con}). These parameter values correspond to the DE estimates $\widehat{\boldsymbol{\psi}}^{DE}$ with τ constrained to equal 1. Interestingly, the Australian data implies a non-zero median estimate for the indexation parameter ρ. This result is in contrast to Cogley and Sbordone (2008), who estimated ρ at zero – implying a purely forward-looking NKPC for the US. However, as is evident from the confidence interval (which includes zero), ρ is not significant at the 10% level,

[49]The results for the mean-based gap are reflective of the reduced-form evidence presented in Section 3 of this thesis.

[50]Due to the fact that the distributions of the parameter estimates are non-normal, the table presents the median estimates and their 90% confidence intervals.

Table 6
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
$\phi = 0.2$

	ρ	α	θ	τ
DE_{con}	0.158	0.654	5.150	1
	(0, 0.784)	(0.509, 0.765)	(4.614, 6.502)	-
DE_{uncon}	0.828	0.635	5.002	0.597
	(0.390, 1)	(0.421, 0.734)	(4.550, 7.899)	(0.256, 0.873)
CF_{con}	0.794	0.892	6.015	1
	(0.480, 1)	(0.255, 0.927)	(5.011, 9.782)	-
CF_{uncon}	0.890	0.889	4.644	0.709
	(0.646, 1)	(0.157, 0.925)	(4.238, 5.923)	(0.378, 0.955)

Notes: numbers in parentheses are 90% trust regions; DE_{con} and CF_{con} correspond to the difference equation and closed form specifications respectively, with $\tau = 1$; DE_{uncon} and CF_{uncon} correspond to the difference equation and closed form specifications with unconstrained τ, respectively.

suggesting that a purely forward-looking NKPC may also be applicable for Australia. The Calvo parameter, α, governing the degree of price-stickiness implies a median duration of prices of 1.63 quarters – a value very similar to that implied by Cogley and Sbordone's (2008) estimates and the US micro evidence presented in Nakamura and Stiensson (2007)[51].

The second row of Table 6 considers the DE specification adjusted so that τ is no longer constrained to equal 1. As stipulated by BGLO (2011), this is a minor but important refinement, as the misspecification bias that arises from constraining τ to equal unity can be very large. The estimation results (denoted as DE_{uncon}) indicate that this modification produces a large disparity in the estimation of ρ – indeed suggesting the presence of misspecification bias in the DE_{con} case. The median estimate of ρ increases dramatically from a value of 0.158 in the DE_{con} case to 0.828, implying that the Australian NKPC includes a backward-looking component. Although, the estimate of ρ is relatively imprecise, the 90% confidence interval does not include zero. Furthermore, τ is estimated as 0.597, with the upper bound of its confidence interval below 1. This result is in line with the estimates presented in BGLO (2011) and implies that average inflation over the previous six months is more relevant in determining current inflation

[51]For a purely forward-looking Calvo model, the time between re-optimisation can be approximated as α^t. Thus, the median duration of prices is given by $-\ln(2)/\ln(\alpha)$.

than merely the inflation rate in the most recent quarter.

The final two rows in Table 6 present the estimates implied by the CF specification of the NKPC. In the constrained case (denoted CF_{con}) the median estimate of ρ is 0.794, and retains a 90% confidence interval bounded well away from zero. The unconstrained case (denoted CF_{uncon}) implies a median estimate of ρ equal to 0.890. Thus, the CF estimates of ρ are larger than their DE counterparts – implying a greater degree of backward-looking behaviour – and are also estimated somewhat more precisely. The estimated value of τ in the CF_{uncon} case, 0.709, implies that last quarter's inflation rate is given a greater weight in the indexation mechanism, compared to its DE counterpart. However, the confidence interval is still bounded away from 1, suggesting that $t-2$ inflation is still relevant in explaining the current inflation rate and consequently should be included in the model.

In both CF specifications α is estimated at approximately 0.89, suggesting that prices are re-optimised every 5 quarters, on average. However, as stressed by BGLO (2011), in the presence of backward-indexation (implied by a non-zero estimate for ρ) α^t now refers to the approximate time elapsed between price *re-optimisation*, rather than a price change. Backward-indexation implies that prices are changed much more frequently (in fact, prices are changed every period) than they are actually being re-optimised. That the frequency of price adjustment is relatively greater than the frequency of optimal price resets may be reflective of the micro evidence which suggests that the information required to set the optimal markup is costly to obtain (Zbaracki, et al. 2004).

The median estimates of θ are fairly consistent across all four specifications and imply a steady-state markup price of 20% - 27%[52]. It is worth noting that the above estimates for θ are markedly lower than those found by both Cogley and Sbordone (2008) and BGLO (2011), implying that the desired steady-state markup in Australia is approximately double that suggested by the US data. In the context of general equilibrium models, US estimates of the steady-state markup range from approximately 6 to 23 percent[53]. Therefore, one can conclude that the estimates for θ presented in Table 6 imply that the steady-state markup in Australia lies at least towards the upper limit of the US estimates. Estimation of θ in the Australian empirical

[52]In a Calvo setting the desired markup is given by $\frac{\theta}{\theta-1}$ in a flexible price environment.

[53]See, for example: Christiano, Eichenbaum and Evans (2005) whose estimates range from 6.35 to 20 percent depending on the specification of their model; Rotemberg and Woodford (1997) estimate a steady-state markup of 15 percent ($\theta \approx 7.8$).

literature is relatively sparse. In their medium-scale DSGE model for Australia, Jaaksela and Nimark (2011) estimate $\theta \approx 1.10$, implying an implausibly high domestic markup. However, the authors also consider a model specification in which the domestic markup calibrated at 20 percent – a value which is consistent with the estimates presented in Table 6. Adolfson, *et al.* (2007) estimate a steady-state markup of 17.4 percent in their open economy DSGE model for Europe, suggesting that domestic price markups in Australia are more comparable to those estimated for the Euro area, rather than in the US.

What can one conclude from the estimates reported in Table 6? When one compares the results presented in Table 6 to those for the baseline NKPC, with zero trend inflation, reported in Table 3, it is evident that the estimates of ρ have not been greatly affected. Thus, it would seem that even once time-varying trend inflation is incorporated into the Calvo model, there is no meaningful change in the degree of autonomous inflation required to match the Australian data. This conclusion defies the original hypothesis of Cogley and Sbordone (2008) and suggests that their model is inadequate for explaining inflation persistence in Australia. However, given the reduced-form evidence presented in Section 3, and the examination of the persistence of the inflation gap in Table 5, analysis of the NKPC's performance during the inflation targeting period is of particular interest.

5.4.1 Parameter estimates during the inflation targeting period

As evidenced in Section 3 and by Figure 6, the inflation targeting period has been characterised by low levels of inflation and little volatility. It is therefore relevant to ask whether such an environment of low and stable inflation has lead to any changes in the structural parameters of the extended Calvo model. This subsample analysis considers the same specifications of the NKPC examined in Table 6, with the only change being the chosen sample period. Table 7 presents the structural parameter estimates for the inflation targeting period (1993:Q1 - 2007:Q2).

The estimates of α and τ are left largely unchanged when compared to the longer sample – suggesting that the time between optimal price readjustment and the weight given to the first lag of inflation in the indexation mechanism, have not changed in any meaningful sense since the introduction of an inflation targeting regime. The median estimates of θ are marginally smaller than those presented in Table 6 indicating that steady-state price markups may have increased slightly during the inflation targeting period. Perhaps of the greatest relevance in the present

Table 7
Structural parameter estimates (median and 90% trust region)
Sample period: 1993:Q1 - 2007:Q2
$\phi = 0.2$

	ρ	α	θ	τ
DE_{con}	0.00	0.702	4.268	1
	(0, 0.427)	(0.457, 0.820)	(3.796, 29.290)	-
DE_{uncon}	0.740	0.698	4.171	0.598
	(0.213, 1)	(0.132, 0.829)	(3.775, 4.705)	(0.232, 1)
CF_{con}	0.424	0.882	5.291	1
	(0.180, 0.863)	(0.789, 0.921)	(4.140, 20.052)	-
CF_{uncon}	0.789	0.850	4.899	0.706
	(0.350, 1)	(0.162, 0.924)	(4.137, 25.702)	(0.174, 1)

Notes: numbers in parentheses are 90% trust regions; DE_{con} and CF_{con} correspond to the difference equation and closed form specifications respectively, with $\tau = 1$; DE_{uncon} and CF_{uncon} correspond to the difference equation and closed form specifications with unconstrained τ, respectively.

context are the changes exhibited in the estimates of ρ. Although the pattern of the findings is largely the same as in Table 6, for all four specifications the median estimates of ρ are lower than their corresponding values in the full sample. Indeed, the DE_{con} specification now yields an estimated ρ centred at zero. That the median estimated values for ρ are lower during the inflation targeting period reflects the decrease in the reduced-form persistence of inflation found in Section 3 and presented in Table 5. The decline in backward-looking behaviour in price setting may be explained by the fact that in the presence of low and stable inflation firms give greater weight to real marginal costs (an issue which is explored further in Section 5.5). Nonetheless, given that the DE_{con} case is the only specification that implies a zero value for ρ seems to suggest that, despite the decline in backward-looking behaviour, a purely forward-looking NKPC is still insufficient to explain inflation dynamics in Australia.

At this point, it should be noted that conclusions about the changes in the structural parameters post-inflation targeting are tentative at best. It is clear from Table 7 that for the shorter sample period the parameters have been estimated with low precision. Although the median estimates of α and τ have been seemingly unaffected during the inflation targeting period, the wide confidence intervals are of some concern. Indeed, for both the DE_{uncon} and CF_{uncon} cases the upper bounds for τ include 1. While the median estimates of ρ have decreased, their confidence intervals overlap with the corresponding confidence intervals in the full sample. Thus, any inferences drawn from Table 7 about the changes in the structural parameters since the adoption of an inflation targeting regime should be made with caution.

5.5 NKPC Coefficients

Recall the expression of the extended NKPC in its DE form (27)[54]:

$$
\begin{aligned}
\widehat{\pi}_t = {} & \widetilde{\rho}_{1,t}^{DE}(\widehat{\pi}_{t-1} - \widehat{g}_t^{\pi}) + (1-\tau)\widetilde{\rho}_{2,t}^{DE}(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\pi} - \widehat{g}_t^{\pi}) \\
& + \widetilde{\lambda}_t^{DE}\left[(1-\phi)\widehat{mc}_t^{d} + \phi\,\widehat{mc}_t^{m}\right] \\
& + b_{1,t}^{DE} E_t \widehat{\pi}_{t+1} \\
& + b_{2,t}^{DE} E_t \sum_{j=2}^{\infty} \varphi_{1,t}^{j-1} \widehat{\pi}_{t+j} \\
& + b_{3,t}^{DE} E_t \sum_{j=0}^{\infty} \varphi_{1,t}^{j}\left[\widehat{Q}_{t+j,t+j+1} + \widehat{g}_{t+j+1}^{y}\right] + \tilde{u}_{\pi,t}.
\end{aligned}
$$

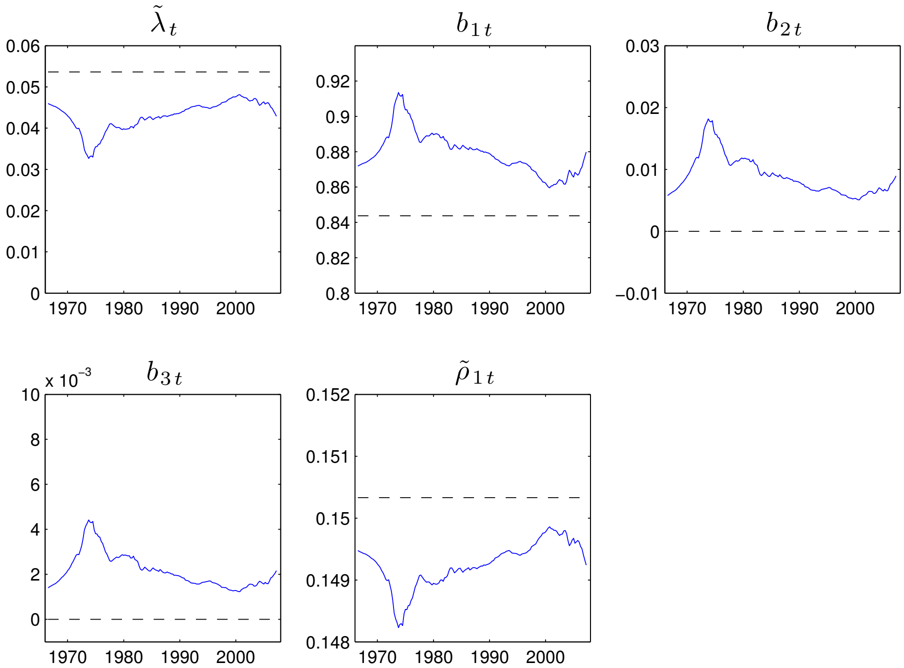
Of interest to policy makers is how trend inflation, through its interaction with the structural parameters of the Calvo model, affects the NKPC coefficients $\widetilde{\rho}_{1,t}$, $\widetilde{\lambda}_t$, $b_{1,t}$, $b_{2,t}$ and $b_{3,t}$. Conditioning on median estimates of the VAR and the DE_{con} Calvo parameters, Figure 7 displays the NKPC coefficients, as defined by (66) in Appendix A. Dashed lines represent conventional approximations, which assume zero trend inflation, while the solid lines represent approximations based on the extended model with time-varying trend inflation.

The evolution of the NKPC coefficients are clearly contingent on the level of trend inflation, $\overline{\pi}_t$, and are very similar to the corresponding time paths presented in Cogley and Sbordone (2008)[55]. The coefficient $\widetilde{\lambda}_t$, which represents the weight given to current marginal costs of domestic producers and importers, varies inversely with the level of trend inflation. Similarly, the backward-looking coefficient, $\widetilde{\rho}_{1,t}$, moves in the opposite direction to trend inflation. In contrast, the three forward-looking coefficients – $b_{1,t}$, $b_{2,t}$ and $b_{3,t}$ – evolve directly with the level of trend inflation. As described by Cogley and Sbordone (2008), this variation in price-setting dynamics follows from the fact that a high level of trend inflation accelerates the rate at which a firm's relative price is eroded when it lacks the opportunity to re-optimise its price. Accordingly,

[54]See Appendix A for definitions of the NKPC coefficients.

[55]See Figure 4 in Cogley and Sbordone (2008).

Figure 7
NKPC Coefficients – DE_{con}



when trend inflation is high firms become more vulnerable to contingencies that may prevail in the future if their price remains fixed for some period of time. As such, when trend inflation rises, the backward-looking component and the weight given to current marginal costs both decrease, and the relative influence of the forward-looking terms are enhanced.

The coefficients implied by the conventional approximations (dashed lines) accord well with the reduced-form GMM estimates of Kuttner and Robinson (2010). Interestingly, when one compares the coefficients implied by the extended model (solid lines) to those implied by the conventional approximation, current costs and lagged inflation matter less and future expectations matter more. Focusing on the forward-looking coefficients, it is evident that the coefficient $b_{3,t}$ is always close to zero. Thus, terms involving forecasts of output growth and the discount factor do not contribute to inflation in any meaningful sense.

The above analysis draws on the evolution of the NKPC coefficients as derived from the DE_{con} specification. As a point of comparison, Figure 8 portrays the NKPC coefficients in (27), this time contingent on the median estimates of the CF_{con} Calvo parameters. Although the shape of the coefficients' time paths are nearly identical to those derived from the DE_{con} case, it is clear that their relative magnitudes are significantly different.

Figure 8
NKPC Coefficients – CF_{con}

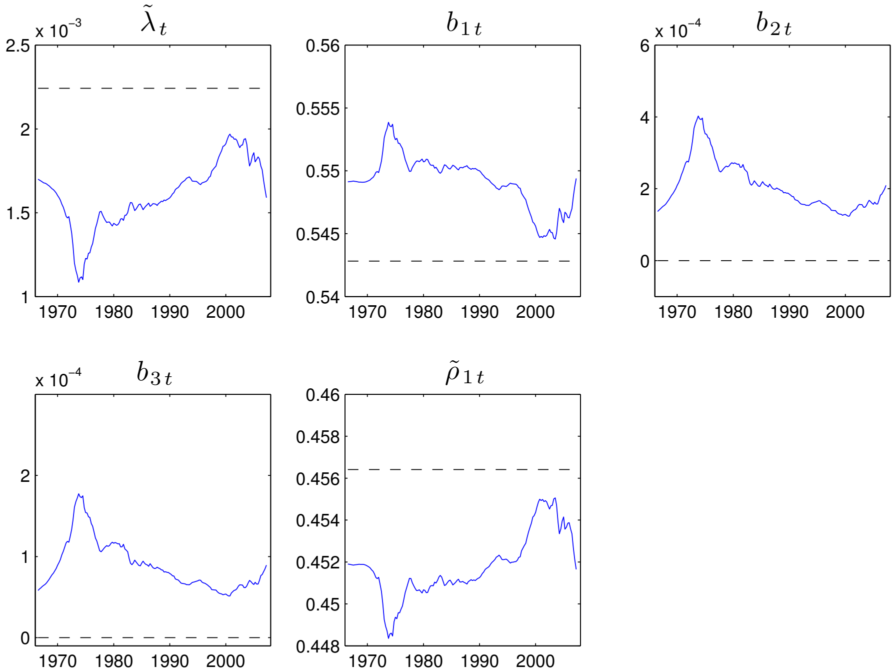


Figure 8 shows that when model-consistent restrictions are placed on the evolution of inflation expectations, the backward-looking component, $\widetilde{\rho}_{1,t}$, increases dramatically (reflecting the increased value of the indexation parameter ρ). Indeed, according to the CF_{con} specification lagged inflation and future expectations of inflation enter the NKPC with almost equal weights, despite the time-variance in trend inflation. Furthermore, the CF_{con} approximations suggest that the link between marginal costs and current inflation, captured by $\widetilde{\lambda}_t$, is significantly weaker than traditional DE estimates seem to predict. The fact that $\widetilde{\lambda}_t$ is always close to zero brings into question the relevance of marginal costs as the driving process for inflation. These results could be of particular relevance to policy makers and imply that when all model restrictions on expectations are taken into account, the Australian NKPC is *not* predominantly forward-looking in the sense implied by the theory and the previous empirical literature. As discussed in the introduction, the relative magnitude of the backward-looking component has important implications not only for the dynamics of inflation but also in determining the appropriate inflation management policies to be employed by the central bank.

6 Robustness Analysis

This section reports the results from four robustness exercises. The first involves estimation of the second stage with an unconstrained import share parameter, ϕ. The second considers an alternative specification of the first-stage VAR, with four variables. Thirdly, the parameter estimates are presented for the case where an extended number of ensembles are used in the Bayesian estimation of the first stage VAR. Finally, the validity of the parameter estimates are considered, with reference to the necessary condition $|\Omega_t \mathbf{A}_t| < 1$.

As alluded to in Section 5.4, given the difficulty in estimating the import share parameter, ϕ, the benchmark estimates in Table 6 correspond to the case where ϕ is calibrated as 0.2. Table 8 presents the median parameter estimates when ϕ is left unconstrained.

Table 8
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
Unconstrained ϕ (import share).

	ρ	α	θ	τ	ϕ
DE_{con}	0.108	0.635	4.756	1	0.00
	(0, 0.992)	(0.018, 0.759)	(4.177, 12.277)	-	(0, 1)
DE_{uncon}	0.858	0.654	4.768	0.585	0.100
	(0.405, 1)	(0.418, 0.762)	(4.1317, 13.330)	(0.266, 0.865)	(0, 1)
CF_{con}	0.80	0.888	6.289	1	0.259
	(0.441, 0.983)	(0.836, 0.923)	(4.555, 18.750)	-	(0, 1)
CF_{uncon}	0.934	0.886	4.990	0.701	0.478
	(0.625, 0.996)	(0.800, 0.925)	(4.291, 20.609)	(0.423, 0.964)	(0, 1)

Notes: This table presents the structural parameter estimates of the extended Calvo model for the case where the import share ϕ is also estimated.

The overall effect of leaving ϕ unconstrained has a seemingly negligible effect on the estimates of the other parameters. In fact, the inclusion of ϕ in the estimation seems to improve the precision of the CF estimates (with the exception of θ). However, that the estimates of ϕ itself are extremely imprecise in all specifications of the NKPC is an indication that the parameter is weakly identified, and is perhaps better suited for calibration.

The next robustness exercise also follows from the difficulty in capturing the effects of marginal costs in the importing sector. An alternative four-variable VAR is considered so as to mechanically capture the inflationary effect derived from the importing sector. Specifically, data on the marginal costs of domestic producers, mc_t^d, and importers, mc_t^m, are aggregated to create a single marginal cost variable:

$$mc_{t,total} = (1 - 0.2) \times mc_t^d + 0.2 \times mc_t^m.$$

Thus, the first stage VAR collapses to the 4-variable case (with their ordering given by: g_t^y, $mc_{t,total}$, π_t, Q_t), as opposed the 5-variable VAR detailed in Section 5. Table 9 reports the resulting parameter estimates.

Table 9
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
4-variable VAR

	ρ	α	θ	τ
DE_{con}	0.00	0.649	7.041	1
	(0, 0.265)	(0.510, 0.755)	(6.021, 8.642)	-
DE_{uncon}	0.834	0.634	6.982	0.584
	(0.471, 1)	(0.485, 0.728)	(5.840, 11.904)	(0.383, 0.797)
CF_{con}	0.869	0.919	9.539	1
	(0.645, 1)	(0.470, 0.944)	(6.850, 19.623)	-
CF_{uncon}	0.892	0.917	8.005	0.646
	(0.763, 1)	(0.269, 0.940)	(6.156, 15.265)	(0.474, 0.860)

Notes: This table presents estimates based on a different VAR specification used in the first stage. Here the sector-specific marginal cost data was combined to yield a single variable, such that the first stage VAR comprised of four variables as opposed to five. Since there is only a single marginal cost variable, ϕ is set as 0.

The parameter estimates are marginally affected by the alternative VAR specification. Most notably, the median estimate of ρ for the DE_{con} case is now zero, in line with Cogley and Sbordone's (2008) results using US data. The median estimates of θ are also somewhat higher than their corresponding values in Table 6[56]. Nonetheless, the conclusions drawn from Table 6

[56] However, the median estimates for θ presented in Table 9 are still markedly lower than those reported in BGLO (2011), supporting the conclusion that the desired steady-state price markup is higher in Australia compared to the US.

remain robust to the alternative VAR specification. Namely, the estimates of ρ in Table 9 (with the exception of the DE_{con} specification) continue to suggest an important role for backward-looking indexation.

The third robustness exercise extends the number of ensembles used to deliver the Bayesian posterior for the VAR parameters $\{\boldsymbol{\mu}_t, \mathbf{A}_t\}$. Cogley and Sbordone (2008) and BGLO (2011) characterise the distribution of the posterior for $\{\boldsymbol{\mu}_t, \mathbf{A}_t\}$ from a set of $N = 5000$ estimates. However, due to computational limitations, N is set to 1000 in order to deliver the estimates presented in Table 6. Table 10 reports the results with N extended to equal 3000, thereby providing a larger distribution of parameter estimates in the second stage[57]. The inclusion of greater number of ensembles does not affect the parameter estimates in any meaningful sense. Thus, the benchmark estimates presented in Table 6 are robust despite the use of 1000 ensembles instead of 5000.

Table 10
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
$\phi = 0.2$, $N = 3000$ ensembles

	ρ	α	θ	τ
DE_{con}	0.165	0.663	5.156	1
	(0, 0.785)	(0.492, 0.764)	(4.604, 6.581)	-
DE_{uncon}	0.843	0.636	4.997	0.613
	(0.396, 1)	(0.137, 0.736)	(4.545, 6.233)	(0.286, 0.873)
CF_{con}	0.799	0.892	5.978	1
	(0.486, 1)	(0.270, 0.927)	(5.020, 9.350)	-
CF_{uncon}	0.890	0.894	5.653	0.707
	(0.623, 1)	(0.177, 0.927)	(4.877, 7.655)	(0.419, 0.968)

Notes: This table presents the parameter estimates when $N = 3000$ ensembles are used in the Bayesian estimation of the first stage VAR, as opposed to $N = 1000$ in Table 6.

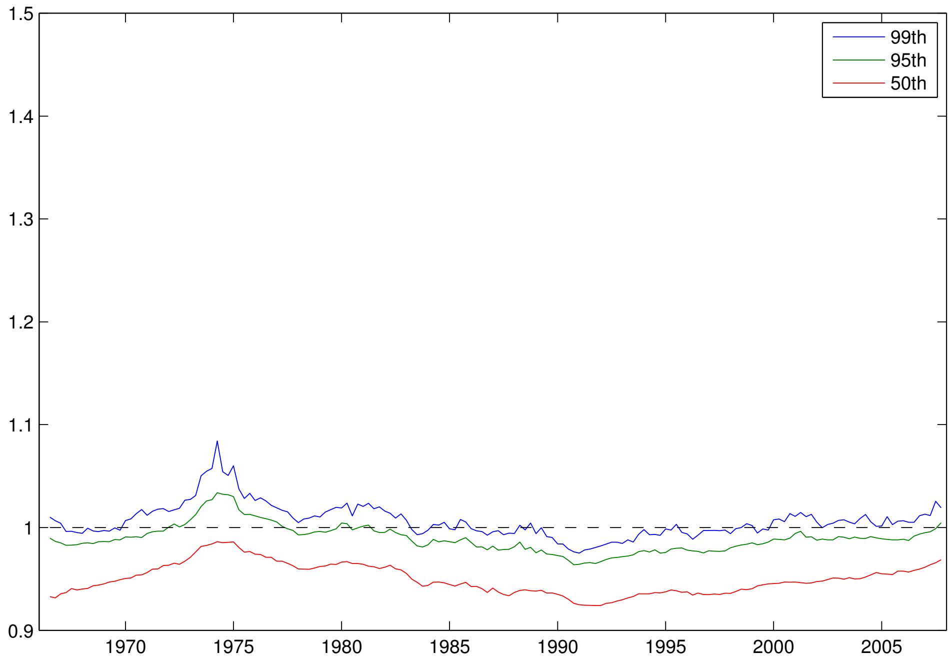
Finally, compliance with the necessary condition $|\Omega_t \mathbf{A}_t| < 1$ is analysed. As emphasised in BGLO (2011), violation of this condition would render the estimates presented in Table 6 invalid as this condition is necessary for the first stage and second stage estimates to be compatible

[57] Again, due to computational restrictions we were unable to achieve parameter estimates for $N = 5000$. However, the results for $N = 3000$ should be virtually identical to those with $N = 5000$.

with each other[58]. Panel A in Figure 9 displays the distribution of the largest estimated root of $\widehat{\Omega}_t \cdot \hat{\mathbf{A}}_t$ for the CF_{uncon} specification, while Panel B displays the corresponding distribution for the DE_{con} case (note that $\hat{\mathbf{A}}_t$ is the same for both specifications). The figure captures the sharp contrast between the compatibility of two specifications with the first stage VAR. Although the 99th and 95th percentiles for the CF_{uncon} specification do occasionally rise above unity, it is strikingly clear that $|\widehat{\Omega}_t^{CF} \hat{\mathbf{A}}_t| < 1$ is satisfied much more readily than in the DE_{con} case. Indeed, in the DE_{con} case, the 99th and 95th percentiles lie almost completely above unity. These findings strongly suggest an inconsistency between the assumption of a VAR representation in the first stage, and the DE estimates obtained in the second stage. Figure 9 unequivocally shows that the minimum-distance estimation procedure is better suited for the CF specification.

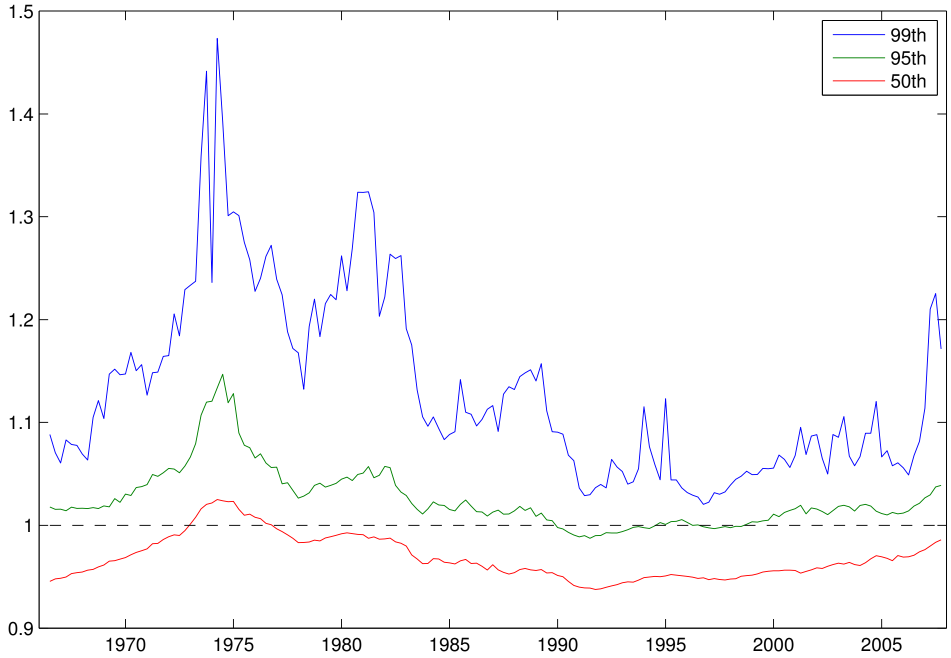
Figure 9
Distribution of the largest root of $\widehat{\Omega}_t \cdot \hat{\mathbf{A}}_t$ in absolute value,
Median, 95th and 99th percentiles

Panel A: Closed Form, distribution of $|\widehat{\Omega}_t^{CF} \hat{\mathbf{A}}_t|$



[58] When the condition $|\Omega_t \mathbf{A}_t| < 1$ is violated, the NKPC solution is indeterminate. See BGLO (2011) for an in depth discussion.

Panel B: Difference Equation Form, distribution of $|\,\widehat{\Omega}_t^{DE}\hat{\mathbf{A}}_t|$



7 Conclusion

This thesis has highlighted the nature of inflation in Australia as being a highly persistent series, at least historically. In essence, the structural analysis undertaken attempts to address whether a more exact version of the Calvo model, based on Cogley and Sbordone's (2008) recent model of time-varying trend inflation, can explain inflation dynamics in Australia without having to rely on ad-hoc backward-looking terms. The results illustrate the substantial difference in estimates when the model is expressed in its typical difference equation (DE) form rather than its closed form (CF). While estimation of all NKPC specifications seem to suggest at least some role for backward-looking indexation, its estimated importance is contingent on the particular specification. In line with Cogley and Sbordone's (2008) US results, the backward-looking indexation parameter is negligible in some DE specifications. In contrast, when a more plausible indexation mechanism is imposed on the DE, and when using the CF specification, the backward-looking parameter is estimated at values close to one. Thus, in light of the evidence presented, and especially when one considers estimation of the closed form, the results suggest that accounting for time-varying trend inflation in the NKPC cannot explain the apparent inertia present in the Australian inflation data.

What can one make of the disparity between the DE and CF estimates? It is arguable that explicitly imposing rational expectations ad infinitum, as the nature of the closed form dictates, is an overly restrictive empirical test of a model. However, Cogley and Sbordone (2008) themselves concede that, in the context of minimum distance estimation, identification of forward and backward-looking terms in the NKPC depends on assumptions about other structural equations in a DSGE model. When such equations are left unspecified — as is the case in the present analysis — their identification is reliant on supplementary assumptions about features of the model, such as the VAR lag length. Thus, by imposing further discipline in terms of economic restrictions on the evolution of the model, the CF specification may provide more convincing estimates. Moreover, BGLO (2011) have shown that for the US, estimates that discipline inflation expectations for only a few periods ahead converge quickly to the CF estimates.

Analysis of the NKPC coefficients implied by the closed form parameter estimates has shown that lagged inflation and future expectations of inflation enter the NKPC with almost equal weights. Such a finding is, of course, in stark contrast to the results of Cogley and Sbordone (2008) who find in favour of a purely forward-looking NKPC. In the context of Australia, the traditionally accepted GMM estimates of the NKPC suggest that inflation is predominantly forward-looking (see Kuttner and Robinson, 2010). However, as emphasised in BGLO (2011), an almost equal split between past and future inflation when analysing inflation dynamics according to the NKPC is common when estimation procedures take into account the model-consistent constraints placed by the NKPC on all future expectations of inflation (Fuhrer and Moore, 1995; Linde, 2005; BGLO, 2011). Notwithstanding the historical performance of the NKPC in Australia, this thesis has also documented a significant decline in both reduced-form and structural persistence in inflation since the adoption of an inflation targeting regime by the Reserve Bank in 1993. Accordingly, it is possible that the characterisation of the Australian NKPC as being predominantly forward-looking is more applicable in recent years.

The structural analysis in this thesis has shown that in order to explain inflation dynamics in Australia using the NKPC, ad-hoc backward-looking terms are required even when shifts in trend inflation are accounted for. Thus, when using the NKPC, a specification that has become commonplace in modern macroeconomic analysis, there is a need to assume some degree of autonomous inertia in inflation as there are no existing microfounded mechanisms which can adequately capture such persistence. While backward-indexation or rule-of-thumb behaviour may help, in a limited sense, to capture the persistence in inflation, there is a need to develop more comprehensive mechanisms which can explain the complex structural behaviours underlying inflation dynamics. Whether such mechanisms as time-variance in the Calvo pricing parameters, the impact of learning on pricing, sticky-information models or state-dependent menu-cost models provide avenues to stronger microfoundations remains to be seen. Nonetheless, the quest to uncover the true nature of inflation dynamics is an important exercise that requires ongoing research and investigation.

Appendix A: Derivation of the NKPC in difference equation (DE) form

This appendix details the derivation of the extended NKPC with time-varying trend inflation in its difference-equation (DE) form as described in (25) and (26). The NKPC derivation is almost identical to that in BGLO (2011), however, the model is augmented slightly to capture open economy effects.

First, the log-linear approximation of the evolution of aggregate prices is derived. Let X_t be the optimal nominal price at time t chosen by firms that are allowed to adjust their prices, which occurs with probability probability $(1-\alpha)$. Based on the indexation mechanism, the price of an individual firm i that is unable to adjust its price (with probability α) is determined according to

$$P_t(i) = (\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho} P_{t-1}(i) \ .$$

Hence, the aggregate price based on the CES aggregator is given by

$$P_t = \left[(1-\alpha)X_t^{1-\theta} + \alpha\left\{(\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho} P_{t-1}\right\}^{1-\theta}\right]^{\frac{1}{1-\theta}} \ .$$

Dividing by the price level P_t, yields

$$1 = (1-\alpha)x_t^{1-\theta} + \alpha\left\{(\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho}\Pi_t^{-1}\right\}^{1-\theta} \ , \tag{48}$$

where x_t is the optimal relative price at time t. Next define stationary variables $\widetilde{\Pi}_t = \Pi_t/\overline{\Pi}_t$, $g_t^{\bar{\pi}} = \overline{\Pi}_t/\overline{\Pi}_{t-1}$, $g_t^y = Y_t/Y_{t-1}$, and $\widetilde{x}_t = x_t/\overline{x}_t$. Here, for any variable k_t, $\overline{k}_t$ is its time-varying trend. Equation (48) can then be transformed in terms of these stationary variables to yield:

$$\begin{aligned} 1 \;&=\; (1-\alpha)\widetilde{x}_t^{1-\theta}\overline{x}_t^{1-\theta} \\ &\quad +\alpha\begin{bmatrix} \widetilde{\Pi}_{t-2}^{\rho(1-\tau)(1-\theta)}\widetilde{\Pi}_{t-1}^{\rho\tau(1-\theta)}\widetilde{\Pi}_t^{-(1-\theta)}\overline{\Pi}_t^{(1-\rho)(\theta-1)}\cdot \\ (g_{t-1}^{\bar{\pi}})^{-\rho(1-\tau)(1-\theta)}(g_t^{\bar{\pi}})^{-\rho(1-\tau)(1-\theta)}(g_t^{\bar{\pi}})^{-\rho\tau(1-\theta)} \end{bmatrix} \ . \end{aligned} \tag{49}$$

In the steady state where $\widetilde{x}_t = \widetilde{\Pi}_t = g_t^{\bar{\pi}} = 1$, (49) can be solved for $\overline{x}_t$ as a function of $\overline{\Pi}_t$:

$$\overline{x}_t = \left[\frac{1 - \alpha \overline{\Pi}_t^{(1-\rho)(\theta-1)}}{1-\alpha} \right]^{\frac{1}{1-\theta}} . \tag{50}$$

Defining $\widehat{\pi}_t \equiv \ln \widetilde{\Pi}_t \equiv \ln(\Pi_t / \overline{\Pi}_t)$ and $\widehat{x}_t \equiv \ln \widetilde{x}_t$, imposing (50), and rearranging gives the log-linear approximation of (49) around the steady state, which can be expressed as

$$\begin{aligned} \widehat{x}_t \quad = \quad & -\frac{1}{\varphi_{0,t}} \rho (1-\tau) \left(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}} \right) \\ & -\frac{1}{\varphi_{0,t}} \rho \tau \left(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}} \right) \\ & +\frac{1}{\varphi_{0,t}} \widehat{\pi}_t \ , \end{aligned} \tag{51}$$

where $\varphi_{0,t} = \frac{1 - \alpha \overline{\Pi}_t^{(1-\rho)(\theta-1)}}{\alpha \overline{\Pi}_t^{(1-\rho)(\theta-1)}}$.

Next, take the log-linear approximation to the first-order condition (FOC) of firms' pricing problem. The firms' FOC can be expressed as

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} Y_{t+j} P_{t+j} \Psi_{tj}^{1-\theta} \left(X_t^{(1+\theta\omega)} - \frac{\theta}{\theta - 1} MC_{t+j} \Psi_{tj}^{-(1+\theta\omega)} P_{t+j}^{\theta\omega} \right) = 0 \ , \tag{52}$$

where $Q_{t,t+j}$ and MC_{t+j} are the nominal discount factor and average marginal cost at $t+j$, respectively. The variable Ψ_{tj} enters in the CES demand function for any good i, $Y_{t+j}(i) = Y_{t+j} \left(\frac{P_{t+j}(i) \Psi_{tj}}{P_{t+j}} \right)$, with

$$\Psi_{tj} = \begin{cases} 1 & j = 0 \\ \prod_{k=0}^{j-1} \left(\Pi_{t+k}^{\tau} \Pi_{t+k-1}^{1-\tau} \right)^{\rho} & j \geq 1 \end{cases} \tag{53}$$

Combining (52) and (53) and rearranging leads to

$$X_t^{1+\theta\omega} = \frac{C_t}{D_t} \ ,$$

where C_t and D_t are recursively defined by

$$\begin{aligned} C_t &= \frac{\theta}{\theta-1} Y_t P_t^{\theta(1+\omega)-1} MC_t \\ &+E_t \left[\alpha q_{t,t+1} \Pi_t^{-\rho\tau\theta(1+\omega)} \Pi_{t-1}^{-\rho(1-\tau)\theta(1+\omega)} C_{t+1} \right] \end{aligned} \tag{54}$$

$$\begin{aligned} D_t &= Y_t P_t^{\theta-1} \\ &+E_t \left[\alpha q_{t,t+1} \Pi_t^{\rho\tau(1-\theta)} \Pi_{t-1}^{\rho(1-\tau)(1-\theta)} D_{t+1} \right] \ , \end{aligned} \tag{55}$$

where $q_{t,t+1}$ now is the real discount factor. Defining the stationary variables $\widetilde{C}_t = \frac{C_t}{Y_t P_t^{\theta(1+\omega)}}$ and $\widetilde{D}_t = \frac{D_t}{Y_t P_t^{\theta-1}}$, yields the following two expressions, based on (54) and (55):

$$\begin{aligned} \widetilde{C}_t &= \frac{\theta}{\theta-1} mc_t \\ &+E_t \left[\alpha q_{t,t+1} g_{t+1}^{y} \Pi_{t+1}^{\theta(1+\omega)} \Pi_t^{-\rho\tau\theta(1+\omega)} \Pi_{t-1}^{-\rho(1-\tau)\theta(1+\omega)} \widetilde{C}_{t+1} \right] \end{aligned} \tag{56}$$

$$\widetilde{D}_t = 1 + E_t \left[\alpha q_{t,t+1} g_{t+1}^{y} \Pi_{t+1}^{(\theta-1)} \Pi_t^{\rho\tau(1-\theta)} \Pi_{t-1}^{\rho(1-\tau)(1-\theta)} \widetilde{D}_{t+1} \right] \ . \tag{57}$$

Also note that

$$\frac{\widetilde{C}_t}{\widetilde{D}_t} = \frac{C_t}{D_t} \frac{1}{P_t^{(1+\theta\omega)}} = x_t^{1+\theta\omega} \ , \tag{58}$$

where $x_t \equiv X_t/P_t$. Evaluating (56) and (57) at the steady state leads to

$$\overline{C}_t = \frac{\frac{\theta}{\theta-1}\overline{mc}_t}{1-\alpha\overline{q}\overline{g}^{y}\overline{\Pi}_t^{\theta(1+\omega)(1-\rho)}}$$

$$\overline{D}_t = \frac{1}{1-\alpha\overline{q}\overline{g}^{y}\overline{\Pi}_t^{(\theta-1)(1-\rho)}}$$

Imposing the assumption that $MC_{t+j} = (MC_{t+j}^{d})^{1-\phi}(MC_{t+j}^{m})^{\phi}$ and combining the two expressions above with (50) while using (58) leads to the steady-state restriction (22). This restriction does not depend on τ and hence is identical to the case in Cogley and Sbordone with $\tau = 1$. Next, define $\widehat{C}_t = \ln \frac{\widetilde{C}_t}{\overline{C}_t}$, $\widehat{D}_t = \ln \frac{\widetilde{D}_t}{\overline{D}_t}$, and $\widehat{mc}_t = \ln \frac{mc_t}{\overline{mc}_t}$. Log-linearizing (58) yields

$$(1+\theta\omega)\widehat{x}_t = (\widehat{C}_t - \widehat{D}_t) \ . \tag{59}$$

Combining (59) with (51) and rearranging leads to an intermediate expression for $\widehat{\pi}_t$:

$$\begin{aligned}\widehat{\pi}_t &= \rho\tau\left[\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}\right] \\ &\quad + \rho(1-\tau)\left[\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}\right] \\ &\quad + \frac{\varphi_{0,t}}{(1+\theta\omega)}(\widehat{C}_t - \widehat{D}_t)\ . \end{aligned} \tag{60}$$

The expressions for $\widehat{C}_t$ and $\widehat{D}_t$ are obtained by log-linearizing (56) and (57). Combining the resulting expressions with (59) leads to an expression for $\widehat{\pi}_t$ similar to that in the main text

$$\begin{aligned}\widehat{\pi}_t &= \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) \\ &\quad + \Omega_t E_t(\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}})) + \lambda_t \widehat{mc}_t + \gamma_t \widehat{D}_t + u_{\pi,t}\end{aligned} \tag{61}$$

$$\begin{aligned}\widehat{D}_t &= \varphi_{1,t} E_t(\widehat{q}_{t,t+1} + \widehat{g}_{t+1}^{y}) \\ &\quad + \varphi_{1,t}(\theta-1)E_t\left\{\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}})\right\} + \varphi_{1,t}E_t\widehat{D}_{t+1}\ .\end{aligned} \tag{62}$$

To derive the final NKPC as expressed in (25) and (26) recall that the in order to capture open economy effects of the inflationary process it is assumed

$$MC_{t+j} = (MC_{t+j}^{d})^{1-\phi}(MC_{t+j}^{m})^{\phi},$$

where average aggregate marginal costs, MC_{t+j}, is a combination of average marginal costs of domestic producers, MC_{t+j}^{d} and importers, MC_{t+j}^{m}. As mentioned in the main text, it is assumed that the structural parameters of the Calvo model (α, ρ, θ, ω and τ) are identical across both sectors. Thus, given (61) and (62), imposing such assumptions leads to the formulation of an open-economy NKPC as expressed in (25) and (26)

$$\begin{aligned}\widehat{\pi}_t &= \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) + \Omega_t E_t(\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}})) \\ &\quad + \lambda_t\left[(1-\phi)\widehat{mc}_t^{d} + \phi\,\widehat{mc}_t^{m}\right] + \gamma_t\widehat{D}_t + u_{\pi,t}\end{aligned} \tag{63}$$

$$\begin{aligned}\widehat{D}_t &= \varphi_{1,t} E_t(\widehat{q}_{t,t+1} + \widehat{g}^y_{t+1}) \\ &\quad + \varphi_{1,t}(\theta - 1) E_t \left\{ \widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}^{\bar{\pi}}_t) \right\} + \varphi_{1,t} E_t \widehat{D}_{t+1} \ .\end{aligned} \tag{64}$$

with the time-varying coefficients given by

$$\begin{aligned}\lambda_t &= \chi_t \varphi_{3,t} \\ \Omega_t &= \varphi_{2,t}(1 + \varphi_{0,t}) \\ \gamma_t &= \frac{\chi_t(\varphi_{2,t} - \varphi_{1,t})}{\varphi_{1,t}} \\ \chi_t &= \frac{\varphi_{0,t}}{1 + \theta\omega} \\ \varphi_{1,t} &= \alpha \overline{q}\overline{g}^y \overline{\Pi}_t^{(\theta-1)(1-\rho)} \\ \varphi_{2,t} &= \alpha \overline{q}\overline{g}^y \overline{\Pi}_t^{\theta(1+\omega)(1-\rho)} \\ \varphi_{3,t} &= 1 - \varphi_{2,t}.\end{aligned}$$

Finally, iterating $\widehat{D}_t$ in (64) forward, substituting the resulting expression for $\widehat{D}_t$ into (63), converting real discount factors $\widehat{q}_{t+j,t+j+1}$ into nominal discount factors $\widehat{Q}_{t+j,t+j+1}$ and rearranging terms yields the NKPC in DE form (as expressed in equation (27) in the main text):

$$\begin{aligned}\widehat{\pi}_t &= \widetilde{\rho}^{DE}_{1,t} \left(\widehat{\pi}_{t-1} - \widehat{g}^{\bar{\pi}}_t\right) + (1-\tau)\widetilde{\rho}^{DE}_{2,t} \left(\widehat{\pi}_{t-2} - \widehat{g}^{\bar{\pi}}_{t-1} - \widehat{g}^{\bar{\pi}}_t\right) \\ &\quad + \widetilde{\lambda}^{DE}_t \left[(1-\phi)\widehat{mc}^d_t + \phi\, \widehat{mc}^m_t\right] \\ &\quad + b^{DE}_{1,t} E_t \widehat{\pi}_{t+1} \\ &\quad + b^{DE}_{2,t} E_t \sum_{j=2}^{\infty} \varphi_{1,t}^{j-1} \widehat{\pi}_{t+j} \\ &\quad + b^{DE}_{3,t} E_t \sum_{j=0}^{\infty} \varphi_{1,t}^{j} \left[\widehat{Q}_{t+j,t+j+1} + \widehat{g}^y_{t+j+1}\right] \ + \widetilde{u}_{\pi,t},\end{aligned} \tag{65}$$

where the coefficients are defined by

$$
\begin{aligned}
\widetilde{\rho}_{1,t}^{DE} &= \left[\rho\tau - \lambda_t \rho(1-\tau) - \gamma_t(\theta-1)\rho(1-\tau)\varphi_{1,t}\right]/\Delta_t \\
\widetilde{\rho}_{2,t}^{DE} &= \rho/\Delta_t \\
b_{1,t}^{DE} &= \widetilde{b}_{1,t}^{D} + b_{3,t}^{D} \\
b_{2,t}^{DE} &= \widetilde{b}_{2,t}^{D} + b_{3,t}^{D} \\
b_{3,t}^{DE} &= [\gamma_t \varphi_{1,t}]/\Delta_t \\
\widetilde{\lambda}_t^{DE} &= \lambda_t/\Delta_t \\
\Delta_t &= 1 + \rho\tau\Omega_t + \gamma_t(\theta-1)\rho\varphi_{1,t}\left\{\tau + (1-\tau)\varphi_{1,t}\right\} \\
\widetilde{b}_{1,t}^{DE} &= \left[\Omega_t + \gamma_t(\theta-1)\varphi_{1,t}\left\{1 - \rho\tau\varphi_{1,t} - \rho(1-\tau)\varphi_{1,t}^2\right\}\right]/\Delta_t \\
\widetilde{b}_{2,t}^{DE} &= \left[\gamma_t(\theta-1)\varphi_{1,t}\left\{1 - \rho\tau\varphi_{1,t} - \rho(1-\tau)\varphi_{1,t}^2\right\}\right]/\Delta_t
\end{aligned} \tag{66}
$$

Note that as in Cogley and Sbordone (2008) and BGLO (2011), the “anticipated utility” assumption (Kreps, 1998) is used in deriving the NKPC in (65) such that $E_t \prod_{k=0}^{i} \varphi_{1,t+k}\widehat{h}_{t+i} = \varphi_{1,t}^{i+1} E_t \widehat{h}_{t+i}$ for any variable $\widehat{h}_{t+i}$.

Appendix B: Derivation of the CF specification

This appendix describes in detail the derivation of the CF representation of the NKPC. The derivation closely follows that in BGLO (2011) and is based on solving (63) and (64) forward. First, define the variable

$$\widehat{B}_t = \widehat{\pi}_t - \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) - \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) \ ,$$

so that

$$E_t\widehat{B}_{t+1} = \widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) \ .$$

Note that the expectation above reflects the fact that $\widehat{g}_t^{\bar{\pi}}$ is an innovation process so that $E_t\widehat{g}_{t+j}^{\bar{\pi}} = 0$ for $j \geq 1$. Using this definition, we can rewrite (63) as

$$\widehat{B}_t = \Omega_t E_t \widehat{B}_{t+1} + \lambda_t \left[(1-\phi)\widehat{mc}_t^d + \phi\, \widehat{mc}_t^m\right] + \gamma_t \widehat{D}_t + u_{\pi,t}. \tag{67}$$

Solving (67) forwards

$$\widehat{B}_t = \lambda_t E_t \sum_{j=0}^{\infty} \Omega_t^j \left[(1-\phi)\widehat{mc}_{t+j}^d + \phi\, \widehat{mc}_{t+j}^m\right] + \gamma_t E_t \sum_{j=0}^{\infty} \Omega_t^j \widehat{D}_{t+j} + u_{\pi,t}. \tag{68}$$

In deriving (68) (and (69) below), the "anticipated utility" assumption is used so that

$$\begin{aligned} E_t\lambda_{t+j}\prod_{k=0}^{j}\Omega_{t+k}\left[(1-\phi)\widehat{mc}_{t+j}^d + \phi\, \widehat{mc}_{t+j}^m\right] &= \lambda_t\Omega_t^{j+1}E_t\left[(1-\phi)\widehat{mc}_{t+j}^d + \phi\, \widehat{mc}_{t+j}^m\right] \\ E_t\gamma_{t+j}\prod_{k=0}^{j}\Omega_{t+k}\widehat{D}_{t+j} &= \gamma_t\Omega_t^{j+1}E_t\widehat{D}_{t+j} \end{aligned}$$

for any $j > 0$. Next, solving forward (64), converting real discount factors into nominal ones, and rearranging leads to

$$\begin{aligned} \widehat{D}_t &= \varphi_{1,t}E_t\sum_{j=0}^{\infty}\varphi_{1,t}^j\left[\widehat{Q}_{t+j,t+j+1} + \widehat{g}_{t+j+1}^y\right] \\ &\quad -\kappa_{1,t}\left[\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}\right] + \kappa_{2,t}\widehat{\pi}_t + \kappa_{3,t}\widehat{\pi}_{t+1} \\ &\quad +\kappa_{3,t}E_t\sum_{j=2}^{\infty}\varphi_{1,t}^{j-1}\widehat{\pi}_{t+j} \ , \end{aligned} \tag{69}$$

with the new coefficients defined by

$$
\begin{aligned}
\kappa_{1,t} &= (\theta-1)\rho(1-\tau)\varphi_{1,t} \\
\kappa_{2,t} &= (\theta-1)\rho\tau\varphi_{1,t} + (\theta-1)\rho(1-\tau)\varphi_{1,t}^2 \\
\kappa_{3,t} &= \theta\varphi_{1,t} - (\theta-1)\rho\tau\varphi_{1,t}^2 - (\theta-1)\rho(1-\tau)\varphi_{1,t}^3
\end{aligned}
$$

Next, the auxiliary variables $\widehat{B}_t$ and $\widehat{D}_t$ are removed and the NKPC is derived. Using the definition of $\widehat{B}_t$, and sustituting into (68)

$$
\begin{aligned}
\widehat{\pi}_t &= \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) \\
&\quad + \lambda_t E_t \sum_{j=0}^{\infty} \Omega_t^j \left[(1-\phi)\widehat{mc}_{t+j}^d + \phi\, \widehat{mc}_{t+j}^m\right] + \gamma_t E_t \sum_{k=0}^{\infty} \Omega_t^j \widehat{D}_{t+j} \ . \qquad (70)
\end{aligned}
$$

Finally, substitute for $\widehat{D}_{t+j}$ terms in (70) using (69) and rearrange the resulting expression to obtain the CF representation of NKPC:

$$
\begin{aligned}
\widehat{\pi}_t &= \widetilde{\rho}_{1,t}^{CF}(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) + (1-\tau)\widetilde{\rho}_{2,t}^{CF}(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) \\
&\quad + \widetilde{\lambda}_t^{CF} E_t \sum_{j=0}^{\infty} \Omega_t^j \left[(1-\phi)\widehat{mc}_{t+j}^d + \phi\, \widehat{mc}_{t+j}^m\right] \\
&\quad + b_{0,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \left[\widehat{\pi}_{t+k-1} - \widehat{g}_{t+k}^{\bar{\pi}}\right] \\
&\quad + b_{1,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \widehat{\pi}_{t+k} \\
&\quad + b_{2,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \widehat{\pi}_{t+k+1} \qquad (71) \\
&\quad + b_{2,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \sum_{j=2}^{\infty} \varphi_{1,t}^{j-1} \widehat{\pi}_{t+j+k} \\
&\quad + b_{3,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \sum_{j=0}^{\infty} \varphi_{1,t}^{j} \left[\widehat{Q}_{t+j+k,t+j+k+1} + \widehat{g}_{t+j+k+1}^{y}\right] + \ u_{\pi,t},
\end{aligned}
$$

with the new coefficients defined as

$$
\begin{aligned}
\widetilde{\rho}_{1,t}^{CF} &= \rho\tau \\
\widetilde{\rho}_{2,t}^{CF} &= \rho \\
\widetilde{\lambda}_{t}^{CF} &= \lambda_t \\
b_{0,t}^{CF} &= -\gamma_t \kappa_{1,t} \\
b_{1,t}^{CF} &= -\gamma_t \kappa_{2,t} \\
b_{2,t}^{CF} &= \gamma_t \kappa_{3,t} \\
b_{3,t}^{CF} &= \gamma_t \varphi_{1,t}
\end{aligned}
$$

Cross-equation restrictions

Using the forecasting rule (30), the $t-2$ conditional expectation of the CF (71) takes the form

$$
\begin{aligned}
\mathbf{e}_{\pi}'\mathbf{A}_{t-2}^{2}\widehat{\mathbf{z}}_{t-2} = {} & \widetilde{\rho}_{2,t-2}^{CF}\mathbf{e}_{\pi}'\mathbf{A}_{t-2}\widehat{\mathbf{z}}_{t-2} + (1-\tau)\widetilde{\rho}_{2,t-2}^{CF}\mathbf{e}_{\pi}'\widehat{\mathbf{z}}_{t-2} + (1-\phi)\widetilde{\lambda}_{t-2}^{CF}\mathbf{e}_{mcdom}'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^{2}\widehat{\mathbf{z}}_{t-2} \\
& + \phi\,\widetilde{\lambda}_{t-2}^{CF}\mathbf{e}_{mcim}'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^{2}\widehat{\mathbf{z}}_{t-2} + b_{0,t-2}^{CF}\mathbf{e}_{\pi}'\mathbf{K}_{t-2}\mathbf{A}_{t-2}\widehat{\mathbf{z}}_{t-2} \\
& + b_{1,t-2}^{CF}\mathbf{e}_{\pi}'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^{2}\widehat{\mathbf{z}}_{t-2} + b_{2,t-2}^{CF}\mathbf{e}_{\pi}'\mathbf{K}_{t-2}\mathbf{A}_{t-2}^{3}\widehat{\mathbf{z}}_{t-2} \\
& + b_{2,t-2}^{CF}\varphi_{1,t-2}\mathbf{e}_{\pi}'\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}_{t-2}^{4}\widehat{\mathbf{z}}_{t-2} \\
& + b_{3,t-2}^{CF}(\mathbf{e}_{q}'\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}_{t-2}^{2}\widehat{\mathbf{z}}_{t-2} + \mathbf{e}_{g^y}'\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}_{t-2}^{3}\widehat{\mathbf{z}}_{t-2}), \qquad (72)
\end{aligned}
$$

where

$$
\mathbf{K}_t \equiv (\mathbf{I} - \Omega_t \mathbf{A}_t)^{-1}. \qquad (73)
$$

Hence, the vector of cross-equation restrictions is given by

$$\begin{aligned}
\mathbf{e}'_\pi \mathbf{A}^2_{t-2} &= \widetilde{\rho}^{CF}_{2,t-2}\mathbf{e}'_\pi \mathbf{A}_{t-2} + (1-\tau)\widetilde{\rho}^{CF}_{2,t-2}\mathbf{e}'_\pi \mathbf{I} + (1-\phi)\widetilde{\lambda}^{CF}_{t-2}\mathbf{e}'_{mcdom}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2} \\
&\quad + \phi\,\widetilde{\lambda}^{CF}_{t-2}\mathbf{e}'_{mcim}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2} + b^{CF}_{0,t-2}\mathbf{e}'_\pi \mathbf{K}_{t-2}\mathbf{A}_{t-2} \\
&\quad + b^{CF}_{1,t-2}\mathbf{e}'_\pi \mathbf{K}_{t-2}\mathbf{A}^2_{t-2} + b^{CF}_{2,t-2}\mathbf{e}'_\pi \mathbf{K}_{t-2}\mathbf{A}^3_{t-2} \\
&\quad + b^{CF}_{2,t-2}\varphi_{1,t-2}\mathbf{e}'_\pi \mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^4_{t-2} \\
&\quad + b^{CF}_{3,t-2}(\mathbf{e}'_q \mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^2_{t-2} + \mathbf{e}'_{g^y}\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^3_{t-2}) \\
&\equiv \mathbf{g}^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}). \qquad (74)
\end{aligned}$$

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani. 2007. "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through." *Journal of International Economics*, 72(2): 481–511.

Altissimo, F., M. Ehrmann, and F. Smets. 2006. "Inflation Persistence and Price-Setting Behaviour in the Euro Area: A Summary of the IPN Evidence." National Bank of Belgium Working Paper No. 95, October.

Angeloni, I., L. Aucremanne, M. Ehrmann, J. Galí, A. Levin, and F. Smets. 2006. "New Evidence on Inflation Persistence and Price Stickiness in the Euro Area: Implications for Macro Modeling." *Journal of the European Economic Association*, 4(2/3): 562–574.

Ball, L.M., and D.H. Romer. 1990. "Real Rigidities and the Non-Neutrality of Money." *The Review of Economic Studies*, 57(2): 183–203.

Ball, L.M, S.G. Cecchetti, and R.J. Gordon. 1990. "Inflation and Uncertainty at Short and Long Horizons." *Brookings Papers on Economic Activity*, 1990(1): 215–254.

Barnes, M. L., F. Gumbau-Brisa, D. Lie, and G. P. Olivei. 2011. "Estimation of Forward-Looking Relationships in Closed Form: An Application to the New Keynesian Phillips Curve." Federal Reserve Bank of Boston Working Paper No. 11-3.

Barsky, R. B. 1987. "The Fisher Hypothesis and the Forecastablility and Persistence of Inflation." *Journal of Monetary Economics*, 19(1): 3–24.

Beechey, M., N. Bharucha, A. Cagliarini, D. Gruen, and C. Thompson. 2000. "A Small Model of the Australian Macroeconomy." Research Discussion Paper 2000-05, Reserve Bank of Australia.

Bernanke, B.S., and I. Mihov. 1998. "Measuring Monetary Policy." *The Quarterly Journal of Economics*, 113(3): 869–902.

Beveridge, S., and C.R. Nelson. 1981. "A New Approach to Decomposition of Economic Time Seires into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'." *Journal of Monetary Economics*, 7(2): 151–174.

Calvo, G. A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12(3): 383–398.

Chari, V.V., P. Kehoe, and E. McGrattan. 2008. "New Keynesian Models: Not Yet Useful for Policy Analysis." *American Economic Journal: Macroeconomics*, 1(1): 242–266.

Christiano, L.J., M.S. Eichenbaum, and C. Evans. 1999. "Monetary Policy Shocks: What Have We Learned and to What End?" *Handbook of Macroeconomics*, 1A. J.B Taylor and M. Woodword (eds.), 65-148. Amsterdam: Elsevier Science.

Christiano, L.J., M.S. Eichenbaum, and C. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1): 1–45.

Cogley, T., and A.M. Sbordone. 2008. "Trend inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve." *The American Economic Review*, 98(5): 2101–2126.

Cogley, T., and T.J. Sargent. 2005. "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S." *Review of Economic Dynamics*, 8(2): 262–302.

Cogley, T., G.E. Primiceri, and T.J. Sargent. 2010. "Inflation-Gap Persistence in the US." *American Economic Journal: Macroeconomics*, 2(1): 43–69.

de Brouwer, G., and N.R. Ericsson. 1998. "Modeling inflation in Australia." *Journal of Business and Economic Statistics*, 16(4): 433–449.

Fuhrer, J. 2009. "Inflation Persistence." Federal Reseve Bank of Boston Working Paper No. 9-14.

Fuhrer, J., and G. Moore. 1995. "Inflation persistence." *The Quarterly Journal of Economics*, 110(1): 127–159.

Galí, J. 1992. "How Well Does the IS-LM Model Fit Postwar US data?" *The Quarterly Journal of Economics*, 107(2): 709–738.

Galí, J. 2008. *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework.* Princeton: Princeton University Press.

Galí, J., and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics*, 44(2): 195–222.

Galí, J., and T. Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies*, 72(3): 707–734.

Galí, J., M. Gertler, and J. David López-Salido. 2005. "Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve." *Journal of Monetary Economics*, 52(6): 1107–1118.

Gruen, D., A. Pagan, and C. Thompson. 1999. "The Phillips Curve in Australia." *Journal of Monetary Economics*, 44(2): 223–258.

Jääskelä, J.P., and K.P. Nimark. 2011. "A Medium-Scale New Keynesian Open Economy Model of Australia." *Economic Record*, 87(276): 11–36.

Justiniano, A., and B. Preston. 2010. "Monetary Policy and Uncertainty in an Empirical Small Open Economy Model." *Journal of Applied Econometrics*, 25(1): 93–128.

Kimball, M. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model." *Journal of Money, Credit, and Banking*, 27(4): 1241–1277.

Kreps, D. 1998. "Anticipated Utility and Dynamic Choice." *Frontiers of Research in Economic Theory: The Nancy L. Schwartz Memorial Lectures, 1983–1997.* D.P Jacobs, E. Kalai and M.Kamien (eds.), 242-274. Cambridge, MA: Cambridge University Press.

Kuttner, K., and T. Robinson. 2010. "Understanding the Flattening Phillips Curve." *North American Journal of Economics and Finance*, 21(2): 110–125.

Lindé, J. 2005. "Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach." *Journal of Monetary Economics*, 52(6): 1135–1149.

Mankiw, N.G. 1985. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly." *The Quarterly Journal of Economics*, 529–537.

Mankiw, N.G., and R. Reis. 2002. "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics*, 117(4): 1295–1328.

Milani, F. 2007. "Expectations, Learning and Macroeconomic Persistence." *Journal of Monetary Economics*, 54(7): 2065–2082.

Monacelli, T. 2005. "Monetary Policy in a Low Pass-Through Environment." *Journal of Money, Credit, and Banking*, 37(6): 1047–1066.

Nakamura, E., and J. Steinsson. 2008. "Five Facts About Prices: A Reevaluation of Menu Cost Models." *The Quarterly Journal of Economics*, 123(4): 1415–1464.

Nimark, K. P. 2009. "A Structural Model of Australia as a Small Open Economy." *Australian Economic Review*, 42(1): 24–41.

Norman, D., and A. Richards. 2010. "Modelling Inflation in Australia." Research Discussion Paper 2010-03, Reserve Bank of Australia.

Pivetta, F., and R. Reis. 2007. "The Persistence of Inflation in the United States." *Journal of Economic Dynamics and Control*, 31(4): 1326–1358.

Rotemberg, J.J., and M. Woodford. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." *NBER Macroeconomics Annual 1997*, 12. B.S Bernanke and J.J Rotemberg (eds.), 297-346. Cambridge, MA: MIT Press.

Rudd, J., and K. Whelan. 2005. "New Tests of the New-Keynesian Phillips Curve." *Journal of Monetary Economics*, 52(6): 1167–1181.

Sbordone, A.M. 2005. "Do Expected Future Marginal Costs Drive Inflation Dynamics?" *Journal of Monetary Economics*, 52(6): 1183–1197.

Sims, C.A. 1992. "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy." *European Economic Review*, 36(5): 975–1000.

Stevens, G.R. 1999. "Six Years of Inflation Targeting." Reserve Bank of Australia *Bulletin*, May: 46–61.

Taylor, J.B. 1980. "Aggregate Dynamics and Staggered Contracts." *The Journal of Political Economy*, 88(1): 1–23.

Yun, T. 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics*, 37(2): 345–370.

Zbaracki, M.J., M. Ritson, D. Levy, S. Dutta, and M. Bergen. 2004. "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets." *Review of Economics and Statistics*, 86(2): 514–533.