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# A Response to Cogley and Sbordone's Comment on "Closed-Form Estimates of the New Keynesian Phillips Curve with Time-Varying Trend Inflation"*

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**Abstract**

In their 2010 comment (which we refer to as CS10), Cogley and Sbordone argue that: (*i*) our estimates are not entirely closed form, and hence are arbitrary; (*ii*) we cannot guarantee that our estimates are valid, while their estimates (Cogley and Sbordone 2008, henceforth CS08) always are; and (*iii*) the estimates in CS08, in terms of goodness of fit, are just as good as other, much different estimates in our paper. We show in this reply that the exact closed-form estimates are virtually the same as the "quasi" closed-form estimates. Our estimates are consistent with the implicit assumptions underlying the first-stage VAR used to form expectations, while the estimates in CS08 are not. As a result, the estimates in CS08 point towards model misspecification. We also rebut the goodness of fit comparisons in CS10, and provide a more credible exercise that illustrates that our estimates outperform CS08's estimates.

JEL Classification: E12, E31, E52

Keywords: closed form, model-consistent expectations, New Keynesian Phillips curve, forward-looking Euler equation, time-varying trend inflation.

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# 1 In a nutshell...

In their 2010 comment (which we refer to as CS10), Cogley and Sbordone argue that: ($i$) our estimates are not entirely closed form, and hence are arbitrary; ($ii$) we cannot guarantee that our estimates are valid, while their estimates (Cogley and Sbordone 2008, henceforth CS08) always are; and ($iii$) the estimates in CS08, in terms of goodness of fit, are just as good as other, much different estimates in our paper.

In this reply we show that:

- The exact closed-form (ECF) estimates are virtually the same as the estimates based on the "quasi" closed form.
- Our estimates (including the ECF estimates) generally satisfy the necessary and sufficient conditions for the validity of the two-stage estimation framework. These conditions guarantee determinacy and the proper specification of the first-stage VAR expectations.
- These necessary and sufficient conditions are often not satisfied in CS08. This indicates a violation in their own critical assumption that expectations can be estimated from a finite-order VAR with i.i.d. shocks.
- The goodness of fit exercise in CS08 and CS10 is misleading, as it almost reduces to fitting expected inflation with expected inflation itself. We provide more credible goodness of fit comparisons, which show that our estimates outperform CS08 estimates.

With the exception of the goodness of fit exercise, all of these issues are addressed in the current version of our paper. However, Cogley and Sbordone's comment deserves a direct point-by-point response. We think there is room for debate regarding the extent to which model-consistent constraints should be placed at the estimation stage, but our empirical results are supported by a number of alternative specifications, while those in CS08 are not. In this respect, we do not see much room for argument. Imposing a modest amount of model discipline on expectations already leads to considerable departures from the estimates in CS08. Moreover, just by changing the vantage point of expectations from $t-1$ to slightly earlier in time, the same specification as in CS08 yields estimates that are very similar to ours. Overall, our results show that accounting for time variation in trend inflation does not resolve the controversy around the source of inflation inertia.

## 2 A refresher of the main issues

Expressing variables as log-deviations from trends, the difference equation (DE) form of the New Keynesian Phillips curve (NKPC) is given by[1]

$$\begin{aligned}\pi_t^n &= \lambda_t E_t \pi_{t+1}^n + \zeta_t mc_t \\ &\quad + \gamma_t \varphi_t \sum_{i=0}^{\infty} \varphi_t^i E_t \left\{ q_{t+i,t+1+i} + g_{t+1+i}^y + (\theta - 1)\pi_{t+1+i}^n \right\} + u_{\pi,t}, \end{aligned} \tag{1}$$

where $\pi_t^n$ is the portion of inflation that is not predetermined at time $t$, that is,

$$\pi_t^n \equiv \pi_t - \rho\tau(\pi_{t-1} - g_t^\pi) - \rho(1-\tau)(\pi_{t-2} - g_t^\pi - g_{t-1}^\pi),$$

and $g_t^\pi$ denotes the growth rate in the exogenous trend for inflation. In CS08, predetermined inflation can only depend on the first lag of inflation ($\tau = 1$), while in our paper we allow indexation to depend on two lags of inflation. In equation (1), $mc_t$ is real marginal costs, $g_t^y$ is the growth rate of real output, and $E_t q_{t,t+1}$ is the expected real discount factor. Compared with the more standard formulation of the NKPC with no time-varying trend inflation, the expression above includes, in addition to real marginal costs, higher-order terms for expected inflation and real discounted output growth. The parameter $\varphi_t$ is bounded so that the series in (1) is finite. The NKPC estimates in CS08 are then obtained from the cross-equation restrictions that result from the DE form (1) when expectations are formed through a first-stage VAR.

Our paper provides a general discussion of the efficiency gains from imposing model-consistent restrictions on expectations that a DE specification such as (1) does not exploit. Our empirical applications produce very different NKPC estimates from the ones reported in CS08. Among other specifications, we consider a quasi closed-form (CF) version of the NKPC given by

$$\begin{aligned}\pi_t^n &= \zeta_t \sum_{j=0}^{\infty} \lambda_t^j E_t mc_{t+i} \\ &\quad + \gamma_t \varphi_t \sum_{j=0}^{\infty} \lambda_t^j \sum_{i=0}^{\infty} \varphi_t^i E_t \left\{ q_{t+i+j,t+1+i+j} + g_{t+1+i+j}^y + (\theta - 1)\pi_{t+1+i+j}^n \right\} + u_{\pi,t}. \end{aligned} \tag{2}$$

This version of the NKPC imposes model discipline on expectations that is not exploited when estimating the DE form. Model discipline on expectations is of central relevance when evaluating forward-looking models. The "quasi" closed form (2) does not fully restrict the evolution of expectations: Higher-order terms for expected inflation still appear on the right-hand side of the equation. Nonetheless, imposing model consistency through (2) adds model information to a critical part of inflation's driving process, expected future marginal costs.

CS10 raises the issue that our estimates from the CF specification could change once we consider *all* of the model restrictions that the exact closed-form version of the NKPC places on the behavior of expected

[1] See CS08 for a derivation.

inflation. While it makes sense to impose model restrictions only where they can have meaningful implications, we fully address this issue in the current version of the paper. Specifically, we also consider estimates from the exact closed form (ECF) associated with (1) and show that the results are still consistent with our findings. We also address CS10's concerns as to whether the CF and ECF estimates we obtain are correct, in the sense of being compatible with a forward determinate solution of the NKPC model. These results are summarized in Sections 3 and 4 of this reply. We show in Section 4 that the same conditions under which the CF and ECF estimates are correct also need to hold for the DE estimates, contrary to CS10's assertion. When these conditions are violated, the first-stage VAR estimates used to generate expectations are typically misspecified. We document in Section 5 that the estimates in CS08 often contradict the implicit assumptions underlying the first-stage VAR.

Finally, CS10 acknowledges that estimating $\tau$ in (1) produces different estimates for $\rho$, but dismisses these findings by arguing that $\tau < 1$ does not produce noticeable improvements in goodness of fit. Section 6 addresses this and other goodness of fit issues. Section 7 concludes and provides results on how the estimates in CS08 change simply by taking expectations earlier in time.

# 3 Exact closed-form estimates

The current version of our paper provides the derivation (in Section 3 and Appendix C) of the exact closed form of the NKPC with time-varying coefficients. This is given by

$$\begin{aligned} \pi_t^n &= \zeta_t \sum_{j=0}^{\infty} \xi_{1,t}^j \sum_{i=0}^{\infty} \xi_{2,t}^i E_t \left\{ mc_{t+i+j} - \varphi_t mc_{t+1+i+j} \right\} \\ &\quad + \gamma_t \varphi_t \sum_{j=0}^{\infty} \xi_{1,t}^j \sum_{i=0}^{\infty} \xi_{2,t}^i E_t \left\{ q_{t+i+j,t+1+i+j} + g_{t+1+i+j}^y \right\} + u_{\pi,t}, \end{aligned} \tag{3}$$

where $\xi_{1,t} + \xi_{2,t} = \lambda_t + \varphi_t + \gamma_t \varphi_t (\theta - 1)$ and $\xi_{1,t}\xi_{2,t} = \lambda_t \varphi_t$. The parameters $\xi_{1,t}$ and $\xi_{2,t}$ need to be bounded in order to guarantee that the geometric sums are well defined. We return to this point later in Section 4, where we also discuss uniqueness of the exact closed form. Note that when $\gamma_t \to 0$ we have that $\xi_{1,t} \to \lambda_t$ and $\xi_{2,t} \to \varphi_t$, and as a result the exact closed form (3) and the "quasi" closed form (2) are the same.[2] As long as $\gamma_t$ is small, these two representations of the NKPC are very similar. The driving process in the closed form contains the expected real discount rate, which can be obtained from the first-stage empirical VAR as a linear combination of the nominal discount factor and (VAR-based) expected inflation. In the closed form, non-predetermined inflation should depend only on current and expected future real variables.

[2] To see this point, notice that

$$\sum_{j=0}^{\infty} \lambda_t^j \sum_{i=0}^{\infty} \varphi_t^i E_t \left\{ mc_{t+i+j} - \varphi_t mc_{t+1+i+j} \right\} = \sum_{j=0}^{\infty} \lambda_t^j E_t mc_{t+j}.$$

.

Table 1 below reports the exact closed-form estimates (ECF), as well as the original CF estimates (*CF_ uncon* in our paper).

Table 1: Exact Closed-Form (ECF) Estimates:
Comparison with CF Estimates

| | $\rho$ | $\alpha$ | $\theta$ | $\tau$ |
|---|---|---|---|---|
| *ECF* | 0.90 | 0.88 | 12.38 | 0.69 |
| | (0.80, 0.98) | (0.82, 0.93) | (10.77, 20.79) | (0.50, 0.90) |
| *CF_ uncon* | 0.89 | 0.88 | 12.28 | 0.69 |
| | (0.76, 0.99) | (0.79, 0.93) | (10.77, 20.19) | (0.49, 0.90) |

Note: Point estimates are medians; 90-percent trust regions in parentheses.

It is apparent that the estimates do not change meaningfully when considering the exact closed form. In other words, the additional model restrictions on expectations imposed by the exact closed form add little information to the estimation procedure. This result stems from a very small estimated $\gamma_t$ for both specifications, implying that the influence of higher-order terms is negligible. The irrelevance of higher-order terms in the estimated NKPC is also true for the CS08 estimates. Figure 1 compares the predicted value of inflation based on their DE estimates, and the predicted value based on their DE estimates that set $\gamma_t = 0$ and shut down time-variation in the NKPC coefficients by setting the exogenous trends to zero (these estimates are reported in Table C.3 in CS08). The two lines are almost indistinguishable.[3]

# 4 Validity of parameter estimates

The NKPC equation (1) by itself does not impose structural parameter restrictions that guarantee that the present discounted values in (2) and (3) are finite, but the estimation framework hinges on additional assumptions because it relies on a first-stage VAR. We discuss here the conditions for finiteness of the present discounted values, and show that these conditions also guarantee the validity of the first-stage estimates. Regardless of the specification being estimated in the second stage (DE, CF, or ECF), the structural parameter estimates should satisfy these conditions to rule out misspecification in the first stage.

In the context of our two-stage estimation procedure, the NKPC model comprises (*i*) the structural NKPC equation and (*ii*) a model to form the expectations that appear in the NKPC. We have already introduced the first element. The second element is given by an unconstrained VAR (written in first-order form) that is used to form expectations:

$$\mathbf{z}_t = \mathbf{A}_t \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{4}$$

[3] We discuss in Section 6 issues associated with how well these models perform against the data. Here, we just note that by any reasonable metric, the two nested models shown in Figure 1 appear to have the same implications for inflation dynamics. Moreover, we find no economic relevance for $\gamma$ in any of the specifications estimated in our paper and in this response.

where $\mathbf{z}$ is a vector of variables that includes inflation and its driving process, and $\boldsymbol{\varepsilon}$ is the vector containing the *i.i.d.* reduced-form errors. The VAR has time-varying coefficients that evolve as a random walk, and the eigenvalues of $\mathbf{A}$ are restricted to lie inside the unit circle. The forecasting rule for a variable $x$ in $\mathbf{z}$ is

$$E_t\left\{x_{t+j}\right\} = \mathbf{e}_x' \mathbf{A}_t^{t+j} \mathbf{z}_t, \tag{5}$$

where $\mathbf{e}_x$ is a (column) vector that selects $x$ from $\mathbf{z}$. With this forecasting rule, we have for instance:

$$\sum_{i=0}^{\infty} \xi_{2,t}^i E_t\left\{mc_{t+i}\right\} = \mathbf{e}_{mc}' \sum_{i=0}^{\infty} \left(\xi_{2,t}\mathbf{A}_t\right)^i \mathbf{z}_t.$$

This series is finite if and only if the largest eigenvalue of $\xi_{2,t}\mathbf{A}_t$ lies within the unit circle, that is $\left\|\xi_{2,t}\mathbf{A}_t\right\| < 1$. It is then apparent that the necessary and sufficient condition for the geometric sums in the ECF specification (3) to be well-defined is

$$\delta_t^{\max} \equiv \max\left\{\left\|\xi_{i,t}\mathbf{A}_t\right\|\right\}_{i=1,2} < 1. \tag{6}$$

For the CF specification, the condition is $\|\lambda_t \mathbf{A}_t\| < 1$, but here we focus on (6) because it can be shown that this condition is more stringent (see Appendix C in our paper).[4]

Figure 2 below shows the distribution of $\delta_t^{\max}$ for the ECF estimates. The 95th percentile of the distribution is always below unity with the exception of a few violations during the mid-1970s. In all, the figure illustrates that the necessary and sufficient conditions for the validity of the ECF estimates are generally satisfied. This is also the case for the conditions that apply to the CF estimates (the results are reported in the paper).

The conditions associated with the existence of the ECF specification (3) play a crucial role in the estimation framework. CS10 fails to recognize that these conditions guarantee the validity of the first-stage estimated forecasting rule. This forecasting rule is the same for all specifications considered, including the DE estimates in CS08. If second-stage estimates violate the conditions in (6), the forecasting rule used in the first stage is misspecified.

To illustrate this point, consider for example the case in which $\|\xi_1\mathbf{A}\| > 1 > \|\xi_2\mathbf{A}\|$. Now the closed form can be written as

$$\begin{aligned}
\pi_t^n \quad = \quad & \frac{1}{\xi_{1,t-1}}\pi_{t-1}^n - \frac{\zeta_{t-1}}{\xi_{1,t-1}}\sum_{i=0}^{\infty}\xi_{2,t-1}^i E_{t-1}\left\{mc_{t-1+i} - \varphi_{t-1}mc_{t+i}\right\} \\
& -\frac{\gamma_{t-1}\varphi_{t-1}}{\xi_{1,t-1}}\sum_{i=0}^{\infty}\xi_{2,t-1}^i E_{t-1}\left\{q_{t-1+i,t+i} + g_{t+i}^y\right\} \\
& -\frac{1}{\xi_{1,t-1}}u_{\pi,t-1} + \eta_{\pi,t},
\end{aligned} \tag{7}$$

[4] We do not mention here the condition $\left\|\varphi_{1,t}\mathbf{A}_t\right\| < 1$, which is necessary for the validity of all specifications in our paper and in CS08, as this condition is always satisfied in the data and is not an immediate point of contention.

where $\eta_\pi$ is an expectational error, that is,[5]

$$\eta_{\pi,t} \equiv \pi_t^n - E_{t-1}\left\{\pi_t^n\right\}.$$

The solution (7) includes the time $t-1$ structural shock for inflation, $u_{\pi,t-1}$. Unlike in the ECF solution (3), this lagged structural shock is not guaranteed to cancel out with any of the elements in the driving process.[6] Indeed, the ECF, which is the unique determinate solution, is the only solution that does not involve predictable error terms. Instead, the closed form (7) is consistent with a general equilibrium model that is not invertible, and hence with a reduced form that has either a moving average error term or equivalently an infinite number of lags (see Fernandez-Villaverde et al., 2007). When the data-generating process for inflation follows (7), estimating a reduced-form VAR as in (4) in the first stage yields misspecified estimates of $\mathbf{A}$. This same issue arises when $\|\xi_i \mathbf{A}\| > 1$ for $i = 1, 2$, a case that is discussed in Appendix C of our paper.

In sum, when the NKPC solution takes the form in (7), forecasts obtained from (5) are incorrect not just because of the presence of truncation bias in $\hat{\mathbf{A}}$, but also because they are omitting relevant information. Under (7), the correct forecasting rule is

$$\begin{aligned} E_{t-j}\left\{\pi_{t+1}\right\} &= \mathbf{e}_\pi' \mathbf{A}_{t-j}^{j+1} \mathbf{z}_{t-j} - \mathbf{e}_\pi' \mathbf{A}_{t-j}^{j} \mathbf{b}_{t-j} u_{\pi,t-j} \\ &\neq \mathbf{e}_\pi' \mathbf{A}_{t-j}^{j+1} \mathbf{z}_{t-j}, \end{aligned} \tag{8}$$

where the vector $\mathbf{b} \neq \mathbf{0}$ relates the reduced-form shocks (the vector $\boldsymbol{\varepsilon}$), to the structural shock $u_\pi$. This illustrates that regardless of the NKPC specification being estimated, it is critical that the condition (6) be satisfied. This condition is necessary and sufficient for the VAR in (4) and the associated forecasting rule (5) to be correctly specified without imposing additional model assumptions.

CS10 argues in favor of estimating the DE specification of the NKPC over a closed form, because it requires fewer assumptions on the underlying general equilibrium model. Our discussion to this point shows that this is not true, given the two-stage estimation method: The ECF estimates are consistent with the only closed-form solution that does not open the possibility for the first-stage VAR-based expectations to be misspecified. Indeed, as we show in the next section, it is Cogley and Sbordone who need to make extra assumptions on which general equilibrium model generates the data, since their estimates of the NKPC point to an indeterminate solution. In particular, Cogley and Sbordone would need to place restrictions through additional structural model equations in order to guarantee that the forecasting rule (5) is valid.[7] Another possibility would be to argue that the NKPC holds exactly in the data, which would restrict $u_\pi = 0$ always. This and other possible model restrictions require assumptions that are questionable at best.

[5] In DSGE models this expectational error would typically be a (possibly indeterminate) function of the time $t$ structural shocks, plus a sunspot shock. As a result, this expectational error can co-vary with other endogenous time $t$ variables, and cannot be set to zero arbitrarily (see Lubik and Schorfheide 2004).

[6] See Appendix D in our paper.

[7] Lubik and Schorfheide (2004) discuss the restrictions needed for determinacy in a DSGE model. For a model with sunspots that has a finite-order VAR representation, see Farmer (1997).

# 5 Violation of first-stage assumptions in CS08

The DE estimates in CS08 do not satisfy the condition (6). Indeed, in their comment Cogley and Sbordone acknowledge that their estimates are not consistent with a determinate solution for the NKPC model. But contrary to their statement that indeterminacy "poses no difficulty for estimation," it certainly does: under indeterminacy the VAR-based expectations (5) are, at best, highly questionable (see equation (8) in the previous section).

Here, we stress only that violations of determinacy in CS08 estimates are pervasive. It is not just during the 1970s — a period when, according to some research, indeterminacy likely played a role[8] — that the condition (6) for determinacy is violated. Figure 3 shows that with CS08 estimates, over most of the estimated sample there is no compelling evidence in favor of a determinate solution, even for the most recent 25 years (1978 to 2003). The median of the distribution of $\delta_t^{\max}$ hovers around one most of the time.

It is nonetheless possible to estimate the DE specification from CS08 in a manner that guarantees congruence between the second-stage estimates and the first-stage forecasting rule assumptions. In the exercise reported in Table 2 below, we explicitly impose the condition that $\delta_t^{\max} < 1$ (for all $t$) in the second-stage estimation. We also report estimates of the same exercise but for the DE specification that estimates $\tau$ ($DE\_uncon$).

Table 2: Difference-Equation Estimates:

Restricted to Determinate Solution ($\delta_t^{\max} < 1, \forall t$)

| | $\rho$ | $\alpha$ | $\theta$ | $\tau$ |
|---|---|---|---|---|
| $DE\_con$ | 0.71 | 0.590 | 7.96 | 1 |
| | (0.00, 0.93) | (0.343, 0.831) | (5.69, 10.92) | – |
| $DE\_uncon$ | 0.85 | 0.608 | 11.47 | 0.60 |
| | (0.57, 1.00) | (0.504, 0.756) | (10.09, 13.34) | (0.42, 0.81) |

Note: Point estimates are medians; 90-percent trust regions in parentheses.

The value of $\rho$ is now much higher, although it is less precisely estimated. The median estimated values are now similar to the estimates obtained from the closed form. In sum, these estimates illustrate that making explicit model assumptions already built into the forecasting rule produces results that contradict CS08's findings. These results, obtained using their same DE specification, show that inflation inertia cannot be fully accounted for by their purely forward-looking version of the NKPC.

[8] See Clarida, Galí, and Gertler (2000) and Coibion and Gorodnichenko (2008).

# 6 Goodness of fit

In their comment, Cogley and Sbordone acknowledge that introducing an inflation indexation mechanism that depends not just on the previous quarter's value of inflation but on a weighted average of inflation over the previous two quarters changes the estimated role of lagged inflation substantially from the estimate reported in CS08. The median estimate for the parameter $\rho$ governing the role of lagged inflation in the NKPC increases from a median value of zero with one-period indexation to 0.64 with two-period indexation.[9] But CS10 dismisses these findings by claiming that the fit of the model with one-period indexation is no worse than the fit with two-period indexation. However, the measures of fit reported in Figure 4 of Cogley and Sbordone's comment are misleading. The reason is that the bulk of the fit comes from inflation essentially being matched by inflation itself. The estimation procedure matches expected inflation to the NKPC model forecast, and their goodness of fit is the simple correlation between these two variables. In the purely forward-looking DE specification estimated by Cogley and Sbordone this goodness of fit measure is (ignoring notation for time variation, and with hats denoting estimates):

$$Corr\left\{\hat{E}_{t-1}\pi_t,\ \hat{E}_{t-1}^{NKPC}\pi_t\right\}$$

with

$$\begin{aligned}\hat{E}_{t-1}\pi_t &\equiv \mathbf{e}_\pi'\hat{\mathbf{A}}\ \mathbf{z}_{t-1},\\ \hat{E}_{t-1}^{NKPC}\pi_t &\equiv \lambda\mathbf{e}_\pi'\hat{\mathbf{A}}^2\mathbf{z}_{t-1}+\zeta\mathbf{e}_{mc}'\hat{\mathbf{A}}\mathbf{z}_{t-1}+\gamma\mathbf{H}(\hat{\mathbf{A}})\mathbf{z}_{t-1},\end{aligned}$$

where the matrix $\mathbf{H}(\hat{\mathbf{A}})$ accounts for the higher-order terms in the NKPC. Most of the goodness of fit from this exercise comes from the fact that $\hat{E}_{t-1}\pi_t$ is highly is highly correlated with $\mathbf{e}_\pi'\hat{\mathbf{A}}^2\mathbf{z}_{t-1}=\hat{E}_{t-1}\pi_{t+1}$. It is then very difficult to assess the model's performance by just plotting $\hat{E}_{t-1}\pi_t$ against the model expectation $\hat{E}_{t-1}^{NKPC}\pi_t$ obtained from a DE representation like (1).

To illustrate this point, consider the same plot as in Figure 4 of CS10 (and Figure 3 in CS08). In Figure 4 below we replicate the plot for $\hat{E}_{t-1}\pi_t$ and Cogley and Sbordone's baseline estimate of $\hat{E}_{t-1}^{NKPC}\pi_t$. Now we add to the figure a plot of $\hat{\lambda}_{t-1}\mathbf{e}_\pi'\hat{\mathbf{A}}^2\mathbf{z}_{t-1}$ by itself. We do so to isolate the importance of $\hat{\lambda}_{t-1}\hat{E}_{t-1}\pi_{t+1}$ in matching $\hat{E}_{t-1}\pi_t$. It is clear from the figure that this term alone is responsible for the estimated model's "goodness of fit." Indeed, the correlation of $\hat{E}_{t-1}\pi_t$ with $\hat{\lambda}_{t-1}\hat{E}_{t-1}\pi_{t+1}$ is actually higher than the correlation for the complete DE specification, $\hat{E}_{t-1}^{NKPC}\pi_t$. Still, based on this "goodness of fit" exercise, it is highly doubtful that one would conclude that ignoring the driving process(es) results in a superior structural model for inflation.

A more credible measure of the model's fit would compare $\hat{E}_{t-1}\pi_t$ with the closed-form representation of inflation, because this form explicitly solves for inflation as a function of its driving process. As a consequence,

[9] CS10's point that such an increase in $\rho$ only raises the combined weight on lagged inflation in the DE representation of the NKPC to 0.26 is misleading, as this represents approximately half of the maximum possible value.

the closed-form representation is better suited to assess the contribution of the NKPC's driving process in explaining the dynamics of inflation. This way of assessing the goodness of fit is not uncommon in the literature even when the NKPC is estimated using a DE specification — a prominent example is Galí and Gertler (1999).[10] For this exercise, we use the estimates of the NKPC reported by CS08 that shut down the higher-order terms and the time-variation in the NKPC coefficients by setting trend inflation to zero.[11] We do so because these estimates are fully consistent with a determinate solution and, as shown earlier in Figure 1, time variation in the NKPC coefficients and higher-order terms do not change in any meaningful way the behavior of the model.[12] When constructing the closed-form solution from these DE estimates (which CS08 reports in Table C3), the match between $\hat{E}_{t-1}\pi_t$ and the model is not especially tight. This is shown in the top panel of Figure 5. While there is some congruence between the model and the VAR-based expectations in the mid-'70s and early '80s, it is evident that for most of the sample the model performs poorly. This is especially the case for the the post-1990 sample, where the NKPC model's standard deviation is almost 9 times larger than the VAR's. During this period the difference between the two series often exceeds 5 percent in absolute value.

Although still not supportive of the model, the DE estimates with two lags of indexation that CS10 dismisses provide improvements along a number of important dimensions of fit (see lower panel of Figure 5). When allowing for two lags of indexation, the raw correlation increases from 0.57 to 0.79. We mention the correlation measure as this is not uncommon in the literature and is also the metric emphasized by CS10. However, the correlation is invariant to proportional scaling, which is not a desirable feature for assessing how well the NKPC fits the level of inflation (or inflation expectations), where the scale does matter for the quality of the fit. As a result, we consider the root mean squared error (RMSE), which is useful in comparing the fit of two competing models. The RMSE, when allowing for two lags of inflation, is smaller by 21 percent over the entire sample, and by 32 percent in the post-1984 sample. Moreover, instead of considering median estimates, it is possible to exploit the information in each of the ensembles. For example, Figure 6 looks at the evolution of the relative RMSEs of the two specifications, computed across ensembles at each point in time. It is evident from the picture that the specification with two lags of inflation rarely performs worse, and typically overwhelmingly better, than CS08's specification. Needless to say, this kind of exercise can also be criticised along similar lines as our own criticism of CS10's, since lagged inflation ($\mathbf{e}_\pi'\mathbf{z}_{t-1}$) is explaining a part of the VAR-based expected inflation ($E_{t-1}\pi_t = \mathbf{e}_\pi'\hat{\mathbf{A}}_{t-1}\mathbf{z}_{t-1}$). Nevertheless, performing this analysis in closed form is conceptually different, as it allows one to disentangle the role of the driving process from the effect of autonomous inflation dynamics.

[10] See also Rudd and Whelan (2007) for a discussion of goodness of fit in the context of a closed-form NKPC with fixed coefficients.

[11] The estimation procedure still uses the time-varying VAR to form expectations, and allows trend inflation to vary over time in the long-run relationship linking trend inflation and trend marginal costs.

[12] These higher-order terms and time variation in the coefficients are necessary for the NKPC to be fully consistent with the long-run restriction. This is quite important in another dimension, as Table 2 illustrates. There we show that keeping time variation and higher-order terms in the NKPC, but imposing determinacy at the estimation stage to guarantee validity of the first-stage estimates, does have consequences for the estimation results.

Two things emerge from this discussion of model fit. First, allowing for a richer structure of indexation yields an improvement in model fit, even using the DE specification. Second, Figure 5 shows that the model is lacking, contrary to the "goodness of fit" measure reported by CS08. One could attempt to dismiss these findings by arguing that time variation and higher-order terms are critical for the model's fit. Alas, for such an argument to work it would be necessary to resort to a different goodness of fit metric than the one used in CS08 and CS10, as we showed in Figure 1.

One final observation concerns the model fit of the CF estimates. The correlation between the VAR-based inflation expectations and the NKPC-based inflation expectations is much higher for the CF estimates (close to 0.95), and the RMSE is 80 percent lower than in CS08's specification. We do not want to over-emphasize these results for the same reasons that we criticized Cogley and Sbordone's goodness of fit analysis using the DE estimates, and because the results, very much like CS08's estimates, cast doubt on the importance of marginal costs (as proxied by the labor share) as a driving process for inflation.[13] Nonetheless, these findings provide additional information on the relevance, accuracy, and fairness of Cogley and Sbordone's use of such an exercise.

# 7 Conclusions

Our paper provides a rationale for estimating forward-looking relationships in a way that takes into account the constraints that the forward-looking model places on expectations. While Cogley and Sbordone (2010) acknowledge such contribution, they object to our empirical application of their NKPC model with time-varying trend inflation. We show that imposing model-consistent constraints on the evolution of expectations yields strikingly different estimates for the NKPC deep structural parameters from the ones reported in Cogley and Sbordone (2008). In this reply we illustrate that their criticism of our findings is misplaced. If anything, Cogley and Sbordone's comment highlights flaws and inconsistencies in their original paper. The claim in CS10 that our estimates are not believable because they are not derived from an exact closed-form solution is unfounded. We provide exact closed-form estimates and show that our findings do not change in any relevant way. Moreover, and contrary to their claim, our estimates are consistent with the assumptions needed to derive the closed form. In this respect, Cogley and Sbordone's comment is self-defeating when it recognizes that the estimates in CS08 indicate that the inflation process is likely to be sunspot-driven over most of the sample. The problem with this finding, which is not an issue for our estimates, is that the presence of indeterminacy invalidates the VAR used in the first stage to generate expectations. Such a VAR is typically not consistent with an indeterminate equilibrium, unless one makes stringent additional (and questionable) assumptions. The only model solution that does not present these issues is estimated in our paper. The results in CS08, instead, indicate that the reduced-form representation of their estimated model

[13] Results are available upon request.

is different from the VAR used to form expectations. This is a troubling inconsistency when estimating a structural forward-looking relationship.

In this reply we also highlight flaws in the comparison of model fit made by Cogley and Sbordone in their comment, and provide an alternative goodness of fit comparison. In particular, we show that the fit of their model is poor: there is little to suggest that real marginal costs are an important driver for inflation — at least when marginal costs are proxied by the labor share. Still, the only estimates from our paper that Cogley and Sbordone are willing to discuss provide a sizeable model fit improvement, in a mean-squared error sense, over their own estimates reported in CS08. These estimates question CS08's claim that "when drift in trend inflation is taken into account, a purely forward-looking version of the model fits the data well, and there is no need for backward-looking components." (CS08, p. 2101). In our paper, we consider a wide array of different specifications, and all of them, aside from the particular DE specification considered in CS08, overwhelmingly point to the presence of inflation persistence that is not captured by the model. As a result, using the language of the NKPC model we find an important role for indexation in explaining inflation dynamics.

To conclude, we touch briefly on two additional points. First, our paper devotes considerable discussion to a set of estimates that is never mentioned in Cogley and Sbordone's comment. These estimates explicitly require expectations to be consistent with the NKPC, but only for a few quarters out instead of the entire future. They are based on specifications that, very much like the NKPC estimated in CS08, are also difference equations themselves, yet they provide overwhelming support to our CF and ECF estimates (see Table 2 in our paper). Such difference-equation estimates pose an important challenge to CS10, as none of the objections raised against our CF estimates would apply to these specifications in any event.

Second, if CS08's estimates were robust, the point in time from which expectations are taken should not matter much.[14] However, as shown in Table 3 below, this is not the case. The table presents estimates of the NKPC parameters using the same DE specification as in CS08, but with expectations taken at different points in time in the second stage of the estimation procedure.

[14] More specifically, here we refer to the timing of expectations applied on both sides of (1) in order to obtain the cross-equation restrictions used in the second stage of the estimation (see CS08 for details, or Appendix A in our paper). The intuition behind this is that if

$$E_{t-j}\{\pi_t\} = E_{t-j}\{NKPC_t\}$$

holds for $j = 1$, then it should hold for $j = 2, 3, ...$, as a result of the Law of Iterated Expectations.

Table 3: *DE_ con* Estimates

Changing the Time Perspective of Expectations

| | $\rho$ | $\alpha$ | $\theta$ |
|---|---|---|---|
| $t-1$ expectations (CS08) | 0.000<br>(0.000, 0.165) | 0.582<br>(0.447, 0.674) | 9.757<br>(7.655, 12.458) |
| $t-2$ expectations | 0.154<br>(0.000, 0.684) | 0.575<br>(0.486, 0.659) | 11.860<br>(9.670, 15.989) |
| $t-3$ expectations | 0.535<br>(0.040, 0.941) | 0.606<br>(0.506, 0.719) | 11.807<br>(10.246, 14.090) |
| $t-4$ expectations | 0.654<br>(0.207, 0.940) | 0.631<br>(0.541, 0.718) | 11.886<br>(10.305, 14.326) |
| $t-5$ expectations | 0.762<br>(0.435, 0.956) | 0.664<br>(0.571, 0.741) | 11.712<br>(10.270, 13.787) |
| $t-6$ expectations | 0.755<br>(0.436, 0.948) | 0.664<br>(0.574, 0.737) | 11.724<br>(10.270, 13.878) |
| $t-7$ expectations | 0.772<br>(0.470, 0.952) | 0.672<br>(0.582, 0.742) | 11.707<br>(10.271, 13.852) |
| $t-8$ expectations | 0.786<br>(0.460, 0.968) | 0.670<br>(0.578, 0.742) | 11.681<br>(10.269, 13.859) |

Note: All estimates obtained from the DE specification in CS08 (Cogley and Sbordone 2008). The estimates in the first row replicate CS08, and the subsequent specifications differ only in the time perspective at which expectations are taken in deriving the cross-equation restrictions.

The table shows that taking expectations further back in time increases the weight given to lagged inflation, from zero in the CS08 specification to 0.7 when expectations are taken from the perspective of $t-4$. This lack of robustness does not arise for the estimates that we put forth in our paper as alternatives to CS08's.[15]

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# Figures

**Figure 1:** *DE_ con* Model Expectations,
With Time-Varying Trends and With Trends Set to Zero

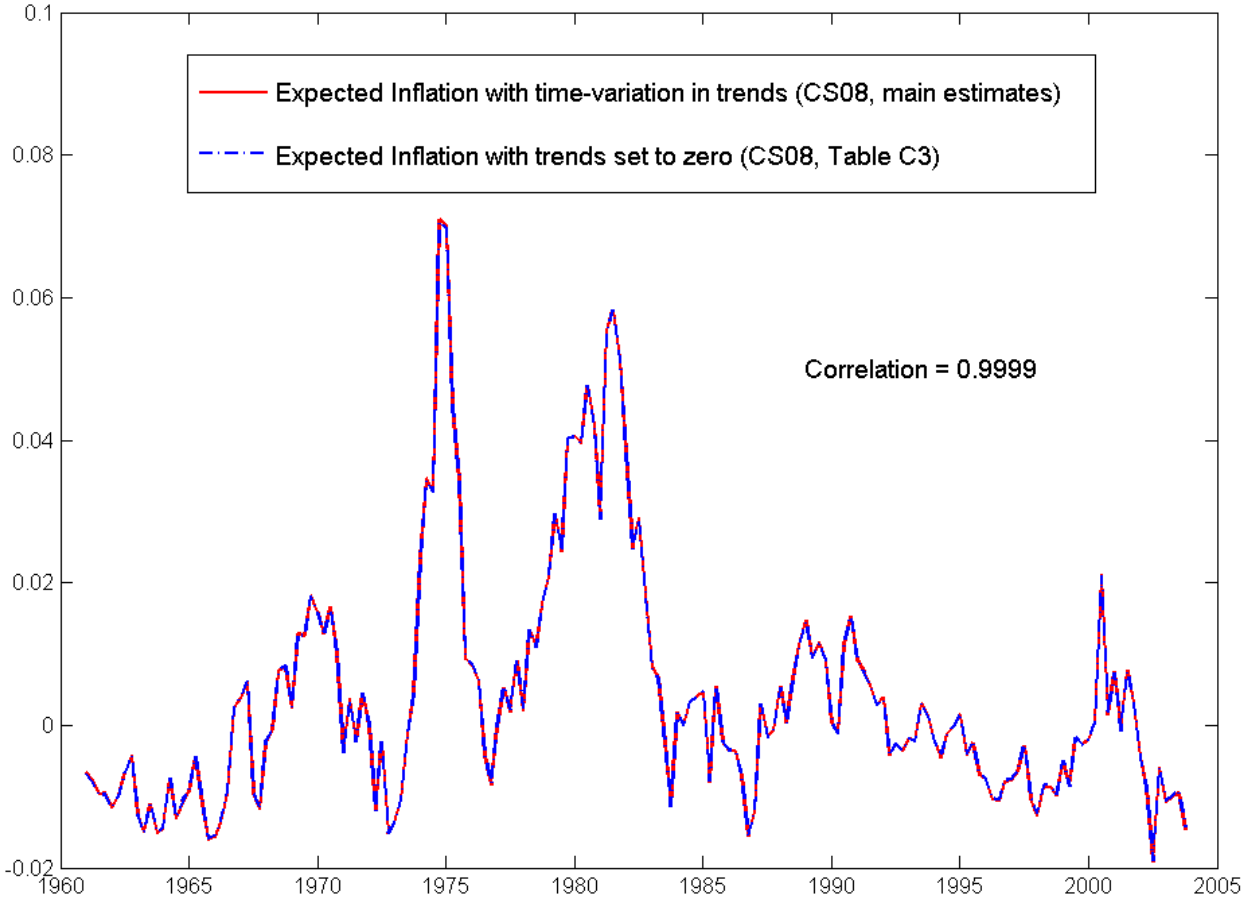


**Figure 2:** Exact Closed-Form Estimates

99th, 90th and 50th Percentiles of $\delta_t^{\max}$

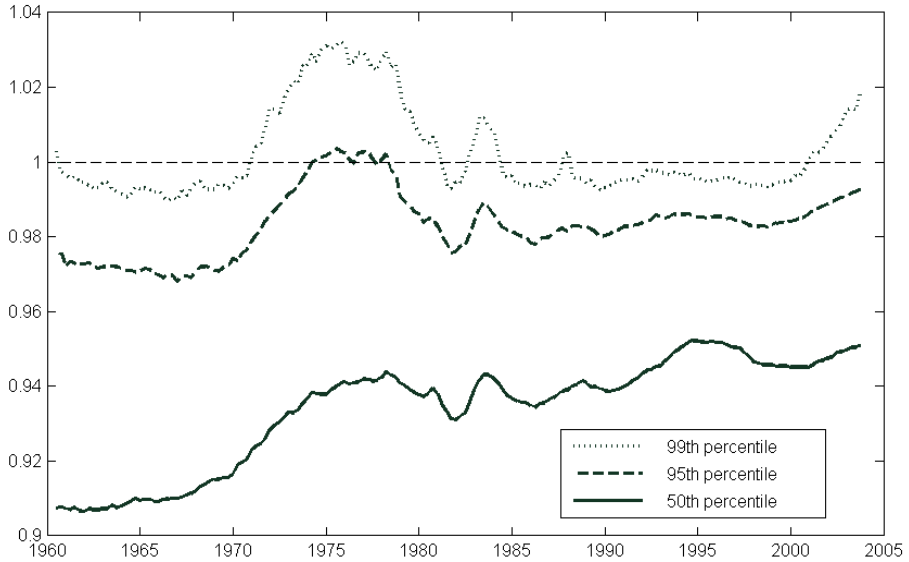



**Figure 3:** Difference-Equation Estimates (CS08)

99th, 90th and 50th Percentiles of $\delta_t^{\max}$

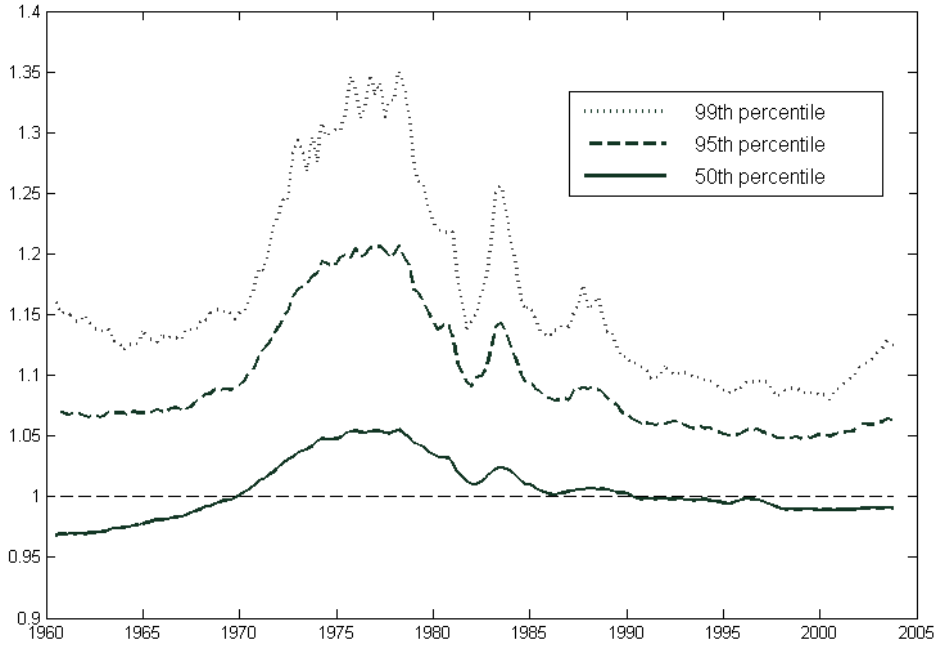


**Figure 4:** Cogley and Sbordone's "Goodness of Fit" Exercise, With and Without Driving Processes.

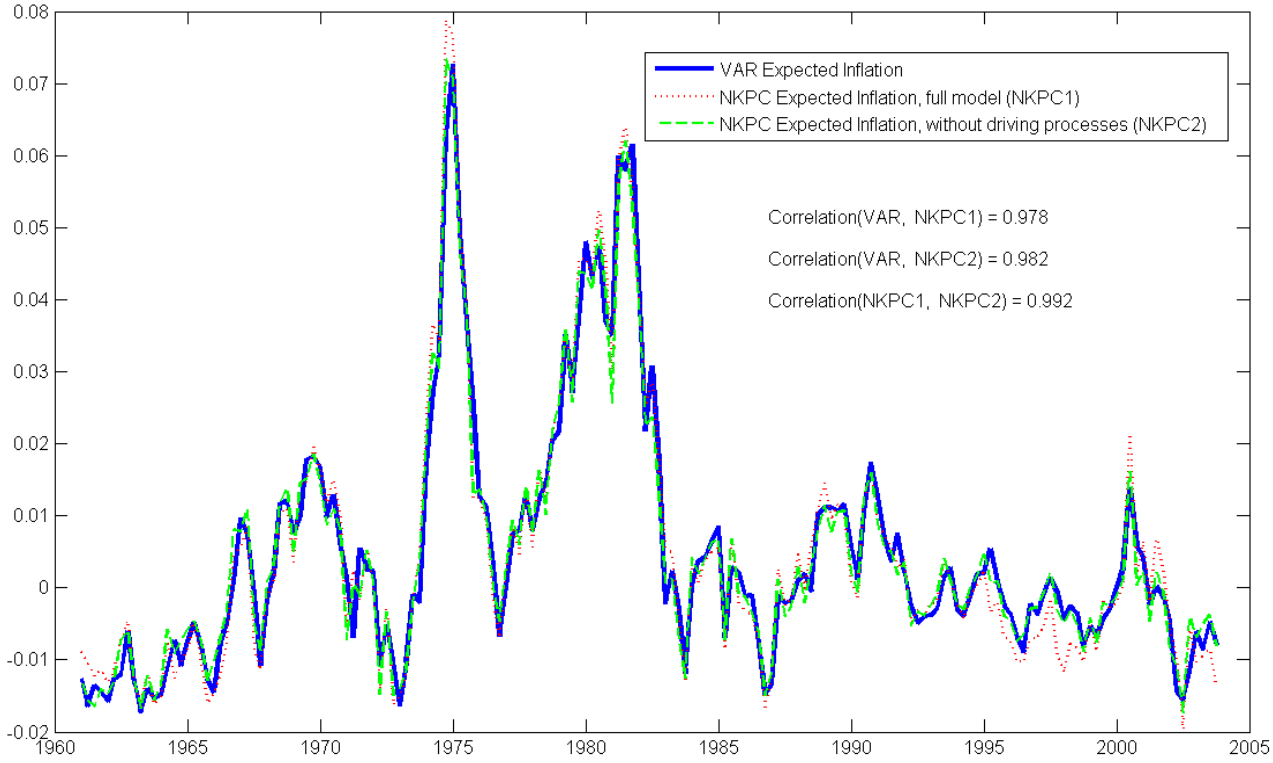



**Figure 5:** Model Fit from DE Estimates, Evaluated In Closed Form.

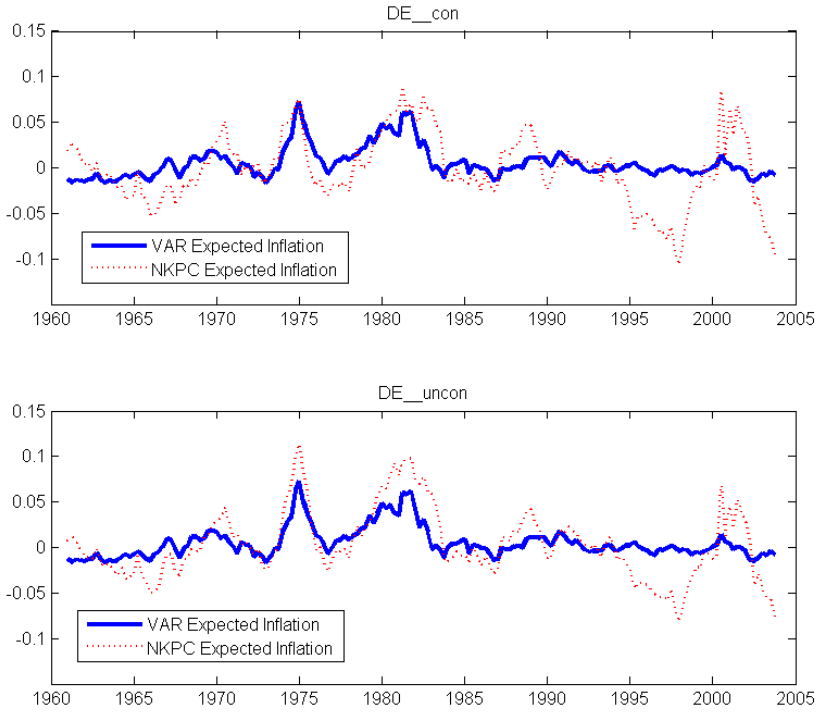
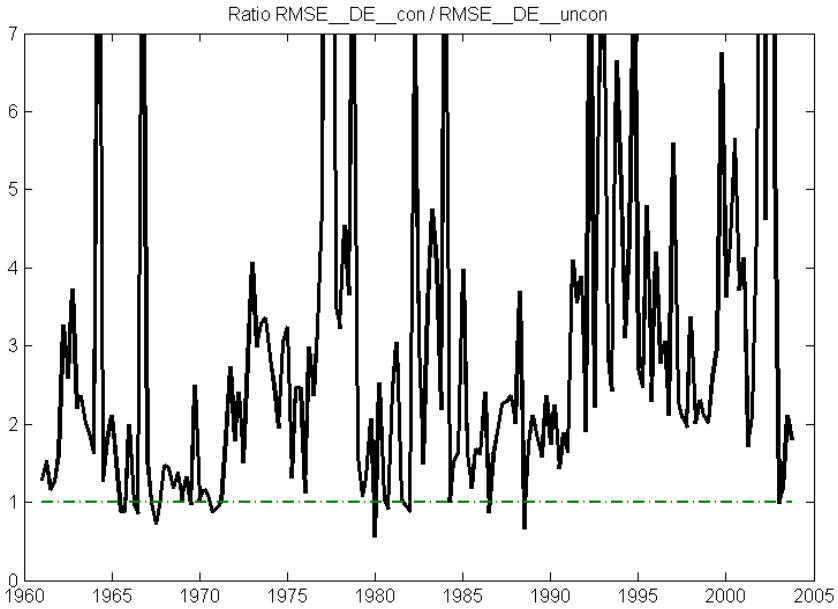


**Figure 6:** Comparison of Root Mean Squared Errors (RMSE)

RMSE from *DE_ con* Relative To RMSE from *DE_ uncon**




* Note: The vertical axis is truncated to facilitate identification of the ratio; only values below 7 are shown. The figure presents the RMSE from the *DE_ con* specification, divided by the RMSE from the *DE_ uncon* specification.