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Graham White

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Graham White

School of Economics and Political Science
The University of Sydney

Email: G.White@econ.usyd.edu.au

ABSTRACT

The aim of this paper is to shed light on the idea of demand-led growth and in particular debate of the last two decades on the appropriate direction of non-marginalist growth theory by exploring the relation between growth and autonomous demands in a fixed capital model proper. The paper developes a simplified two sector model producing a pure consumption good and a machine with variable efficiency. A "fixed-price" model is considered whereby relative prices and the real wage are held at their long-period equilibrium levels, so that disequilibrium in the model is limited to quantities. The dynamics of quantities are represented in terms of first order difference equation system in growth rates of demand, investment growth rates, utilization rates and the relative size of the two sectors. While the local stability of the steady state can be considered in terms of the characteristics of the relevant Jacobian for the model in question such analysis is largely inconclusive. Attention is instead focused on the results of dynamic simulation of the model. These latter results, show the dynamics to depend crucially on the assumptions made regarding the formation of producers' expectations about future growth in demand. Most importantly, expectations of future growth which make allowance for dispersion in past growth rates; as well as expectations which are partially dependent on expectations about autonomous demand growth may have a significant stabilizing effect on the growth of the economy.

Keywords:
JEL codes:

CONTENTS

I Introduction

The present paper is intended as a contribution to recent debate on the nature of a demand-constrained growth, specifically debate over the contours of a Keynesian view of long-run growth and its synthesis with a classical-Sraffian explanation of value and distribution.

Formal growth theory of the demand constrained type has its intellectual roots in the growth models of Harrod and Domar, though arguably the precedent for demand-constrained growth extends at least back to Marx. As is well known, the problem identified by both Harrod and Domar was that of obtaining the particular rate of growth of investment “warranted” or required to absorb saving generated by growing income, given the saving propensity and technology.

To the extent that neither Harrod nor Domar could identify any mechanism ensuring that the actual growth of investment would come into line with the required rate, their models left some intriguing unanswered questions. For instance, adopting the view advanced by both Pasinetti (1974) and Kalecki (1962) that long-run growth in the Harrodian system was ephemeral once the economy slipped off the warranted growth path, leads one inevitably to ask what forces other than investment based on the expectation of growing demand were driving demand in the long-run. Indeed, this question is at the heart of different versions of Kalecki’s model of cyclical growth (e.g. 1943, 1954, 1962). What is interesting in this line of thought is the implication that explanations of long-run growth in capitalist economies could not avoid references to exogenous components of demand.

An alternative path leading out of Harrod and Domar was the so-called Cambridge or neo-Keynesian perspective on long-run growth which provided a way around the uniqueness of Harrod’s warranted growth path, while still seeking to maintain the independence of investment in relation to saving and while eschewing a marginalist explanation of distribution. As is also well known, this approach centered on the argument that an independently determined rate of capital accumulation governed the distribution of income and ultimately the aggregate flow of saving out of income.

A more recent phase in the development of demand constrained growth theory has evolved out of research into the possibility of synthesising Sraffian pricing models and a Keynesian inspired explanation of output and employment. The early literature in this area dealt primarily with the openness of the Sraffian price system with respect to quantities; and, in particular, what this price system implied, if anything, about the relationship between demand and output capacity (Eatwell, 1983; Vianello, 1985; Ciccone, 1986, 1987; Kurz, 1986; Committeri, 1986; Dutt, 1986; White, 1989).

The subtext in the debate over this question was essentially the need to rethink the Keynesian principle of effective demand for a long-run context where capital accumulation was taking place; as well as the need to consider how the process by which capacity adapted to demand was related to the process by which relative prices gravitated around their long-period configuration. Subsequent work during the 1990's on these two matters (Kurz, 1990, Garegnani, 1992; Garegnani and Palumbo, 1998; Trezzini, 1995, 1998; Palumbo and Trezzini, 2003, Serrano, 1995; Cessarato, Serrano and Stirati, 2002, and Park, 1997, 2000,) highlighted the need to specify the nature of autonomous demands and their relation with both steady-state and non-steady-state growth paths, an integral part of that relation being the so-called "Sraffian supermultiplier". But this literature also brought into the open doubts about the Cambridge/neo-Keynesian approach as the appropriate framework within which to consider demand-constrained growth.[1]

For the most part however, this later work has taken place in the context of aggregative models, while explicit consideration of sectoral interdependencies has remained largely implicit and submerged in assumptions about relative prices and distribution. The present paper is an attempt to move further on the idea of demand-led growth and in particular the debate referred to above, by exploring the relation between growth and autonomous demands in a fixed capital model proper – viz., where used fixed capital is treated as both output and input. The dynamics are made more complex by this treatment of fixed capital, firstly, since depreciation allowances impact on net profit and hence on capitalists' consumption; and secondly, because both investment decisions and depreciation allowances are influenced by the age composition of the capital stock in each sector.

The paper considers a simplified two sector model producing a pure consumption good and a machine with variable efficiency and an autonomous component of demand for both commodities. Relative prices and the real wage are fixed at their long-period equilibrium levels - determined by technology and an exogenous rate of profit - so that disequilibrium in the model is limited to quantities. Thus there are no "cross-dual" dynamics, but only "dual" dynamics which are limited to the interaction of output and demand.

The dynamics of output and demand are governed in this model by the interaction of the income expenditure multiplier and what is in effect an accelerator mechanism. What drives investment, in other words, is the expectation of growing demand. This idea, combined with the multiplier concept is considered a useful starting point for two reasons: first, both ideas have an inherent plausibility and, in the present writer's view, an important place in an alternative, non-marginalist view of growth;[2] second, and by way of a challenge, these two notions when modeled together can, as is well known, give rise to unstable

[1] Given the rich vein of twentieth century thought (see for example some of the discussion in Setterfield, 2002) giving prominence to the idea that demand is the independent variable in the theory of output the continuing refusal of mainstream growth theory to seriously acknowledge this line of thought is intellectually inexcusable, not to mention theoretically indefensible.

[2] This is not to deny that the accelerator could be made to fit into a marginalist view of investment: by supposing that the implied desired capital to output ratio is determined in accordance with the postulate of a systematic relation between relative factor prices and the relative proportions of capital and labour used in production. Such an approach to determining the capital to output ratio is however rejected in the present paper.

behaviour in aggregate demand and output. As such, the challenge exists to provide reasonable accounts of how the interaction between multiplier and accelerator could be part of an explanation of growth, without giving rise to unstable outcomes. Of course, what is critical to the dynamic behaviour of the system encompassing these two components is the formation of expectations about future growth in demand. In this regard, the present paper suggests an important role for autonomous demand.

Section II developes the dynamics of quantities in terms of first order difference equations in growth rates of demand, investment growth rates, utilization rates and the relative size of the two sectors. Section III makes use of the structure of the model in attempting a clarification of the notion of exogenous versus endogenous growth. In effect, the model allows as it were for alternative "closures": a feature which may assist in clarifying different positions in recent literature on demand-constrained long-run growth.

Sections IV introduces the discussion of the model's dynamics with a consideration of both the local stability of the steady state growth path set by the growth of autonomous demand; and a dynamic computer simulation of the system's behaviour following a change to the rate of growth of autonomous demand. The simulation results in particular reveal an unstable system. Sections V and VI discuss two alternative approaches to the modeling of the expected growth forecast as a means of rendering the model more stable. Analysis of simulation results there suggests an improvement in stability is associated with both allowance for dispersion in past growth rates in the formation of the growth forecast and also when producers base their forecasts at least partially on expectations about growth in autonomous demands.

Section VII briefly takes up some of the wider issues raised by a modeling of expectations which includes forecasts of autonomous demand growth. Most notable of these issues is the rationale for such an approach, more particularly how producers might arrive at the weighting they would attach to autonomous demand in the formation of expectations about future demand growth. Section VIII provides some brief concluding notes.

II A two-sector fixed capital model

(i) Production

Consider an economy which produces each period a quantity of a consumption good as well as a quantity of new machines of a particular type. The production of each commodity requires a quantity of machines and a quantity of labour, which is assumed to be of a uniform type. Machines have a maximum (technical) life of two periods, and have a zero scrap value. It is assumed that disposal of two-year old machines is costless. Other than a quantity of one-period old machines in each sector at the end of each production period there is no joint production. Assume also that older machines are not transferable between sectors. The model ignores foreign trade as well as the government sector.[3]

It follows from these assumptions that production in this economy can be described in terms four processes producing four commodities: a consumption good, new machines, and two types of one-period old machines according to whether they are used in the production of the consumption good or new machines.

It is further assumed that machines have a variable efficiency. This is reflected in the requirement of greater quantities of labour per unit of output with the use of older machines as compared with new machines. More precisely, denoting the unit labour requirement of l^0_{it} associated with the use of new (zero-years old) machines in the production of a unit of output in sector i in period t, then the corresponding labour requirement with (one-year) old machines is given by

$$l^1_{it} = l^0_{it}.(1+\alpha) \qquad \text{(II.1)}$$

[3] Though demand arising from foreign trade and /or government intervention could well provide part of demand which is referred to here as "autonomous".

where α is positive and less than unity. The assumption of variable efficiency warrants a distinction between the components of total output corresponding to machines of different ages, if only because variable efficiency may entail different desired rates of utilization of different aged machines. To allow for this possibility, output in sector i during period t can be represented as follows:

$$Y_{it} = Y_{it}^{0} + Y_{it}^{1} \quad i = 1,2 \qquad \text{(II.2)}$$

where Y_{it}^{0} *and* Y_{it}^{1} refer to outputs of commodity i on new and old machines respectively. Actual utilization rates of new and one-year old-year old machines in sector i in period t are then given by

$$u_{it}^{j} = \frac{Y_{it}^{j}}{\beta_i M_{it}^{j}}, \quad j = 0,1, \; i = 1,2 \qquad \text{(II.3)}$$

M^{j}_{it} is the number of j-year old machines used in production in sector i in period t and β_i is the output capacity of a machine in sector i, i.e. in terms of units of commodity i. For simplicity, it is assumed that this output capacity is the same for new (j =0) and used (j = 1) machines, so that variation in efficiency over the life of machines amounts to more labour being required by older machines to produce the same output as new machines. u^{j}_{it} refers to theactual utilization rate in period t of j-years-old machines.

Differences in utilization rates on machines of different ages are treated in as simple a manner as possible by assuming a linear relation such that

$$u_{it}^{1} = \frac{u_{it}^{0}}{(1+\phi)} \qquad \text{(II.4)}\ [4]$$

It is also assumed that there is a desired utilization rate in each sector, specifically a desired rate in relation to newly installed plant, denoted as u^{n0}_{it}. In view of (II.4) above, this effectively also implies a desired rate of utilization on older plant.

Regarding the relationship between output and demand, this paper discusses two possibilities, with differing implications regarding the system's dynamics. The first, used in this section and in sections III-VI, assumes that output during period t is taken to be governed by the demand which materialises during period t. In other words, subject to the constraint provided by capacity, producers respond in the same period as that in which the demand is expressed. In this manner the utilization rate in each sector fluctuates in line with demand. For simplicity, inventories of finished goods are ignored. The second approach involves an explicit recognition of the lag between demand and production and thus the need for producers to forecast future demand in the planning of output. This approach is taken up in Section VII.

(ii) Investment

Over the longer-run, capacity in each sector is assumed to adjust to persistent variation in demand. Specifically, the decision about the size of capacity in each sector at the end of period t involves an estimate of demand through period t+1, and on that basis an estimate of the capacity required assuming a desired utilization rate of u^{n0}_{it+1} on newly installed capacity; and hence an estimate of the extent to which capacity at the end of period t is deficient or excessive. Hence, with demand D^{e}_{it+1} expected in sector i in period t+1, the firm would choose a capacity comprising one-year old machines in t+1, which were new in period t, and new machines to be installed for use in t+1, such that

[4] It is important to note the assumption of a constant ratio of utilization rates on new and old machines is not strictly speaking consistent with cost-minimization and the assumption of declining efficiency of machines. I am indebted to an anonymous referee for pointing out that declining labour efficiency would imply that older machines would only be used if, in order to meet demand, it was necessary to produce an output in excess of the normal utilization on new machines. Dealing with this implication however would considerably complicate the model, even if the desired utilization of older machines is taken to be a simple linear function: increasing in demand and decreasing in the growth rate of new capacity. Equation (II.4) is therefore used for the sake of simplicity, even though it implicitly assumes that output would always exceed the output potential of newly installed capacity.

$$D^e_{it+1} = \left(u^n_i . M^0_{it+1} + \frac{u^n_i . M^1_{it+1}}{(1+\phi)} \right) . \beta_i \tag{II.5}$$

Since $M^1_{it+1} = M^0_{it}$ and noting that investment demand I_{it} during period t leads to the installation of an amount of new capacity M^0_{it+1} for use in t+1, then expression (II.5) can be written as

$$D^e_{it+1} = \left(u^n_i . I_{it} + \frac{u^n_i . M^0_{it}}{(1+\phi)} \right) . \beta_i \tag{II.6}$$

Demand expected in t+1 is assumed to be based an extrapolation of demand observed in period t-1, so that

$$D^e_{it+1} = D_{it-1} . \left(1 + g^d_{it-1}\right)^2 \tag{II.7}$$

where D_{it-1} and g^d_{it-1} are demand for commodity i during t-1 and the rate of growth of demand for commodity i in t-1 respectively.

It follows from (II.6) and (II.7) that investment demand in sector i at time t can be expressed as

$$I_{it} = \frac{D_{it-1} . \left(1 + g^d_{it-1}\right)^2 . (1+\phi) - u^n_i . M^0_{it} . \beta_i}{u^n_i . \beta_i . (1+\phi)} \qquad i = 1,2 \tag{II.8}$$

Demand for new machines - commodity 2 - during period t is

$$D_{2t} = \sum_{i=1}^{2} \left(\frac{D_{it-1} . \left(1 + g^d_{it-1}\right)^2 . (1+\phi) - u^n_i . M^0_{it} . \beta_i}{u^n_i . \beta_i . (1+\phi)} \right) + I^{Aut}_t \tag{II.9}$$

where I^{Aut}_t represents autonomous investment demand for commodity 2 expressed during period t. Precisely what makes up I^{Aut}_t is not taken up for discussion here though it is necessary to note here that this demand is assumed to be non-capacity creating demand for new machines.[5] This assumption allows one to identify newly installed capacity with investment demand "induced" by the expectation of growing demand, viz., I_{it}.

Expression (II.9) can be rearranged given that, as noted above, $M^1_{it+1} = M^0_{it}$ and $M^0_{it} = I_{it-1}$; and with $D_{2t-1} = I_{1t-1} + I_{2t-1} + I^{Aut}_{t-1}$, then

$$D_{2t} = I^{Aut}_t + \frac{I^{Aut}_{t-1}}{(1+\phi)} + D_{2t-1} . \left(\frac{D_{12t-1} . \left(1 + g^d_{1t-1}\right)^2}{u^n_1 . \beta_1} + \frac{\left(1 + g^d_{2t-1}\right)^2}{u^n_2 . \beta_2} - \frac{1}{(1+\phi)} \right) \tag{II.10}$$

where $D_{12t-1}=D_{1t-1}/D_{2t-1}$. In turn, by dividing through by D_{2t-1}, equation (II.10) can be transformed into an expression for the growth rate of demand for commodity 2:

$$g^d_{2t} = \frac{D_{12t-1} . u^n_2 . \beta_2 . (1+\phi)\left(1 + g^d_{1t-1}\right)^2 + u^n_1 . \beta_1 . \left\{ \left(1 + g^d_{2t-1}\right)^2 (1+\phi) + F\left(ID^{Aut}\right) \right\}}{u^n_1 . u^n_2 . \beta_1 . \beta_2 . (1+\phi)} \tag{II.11}$$

where

$$F\left(ID^{Aut}\right) = u^n_2 . \beta_2 . \left[ID^{Aut}_{2t-1} \left(a + a\phi + \phi + 2\right) - 2 - \phi \right]$$

and where ID^{Aut}_{2t} is the ratio of autonomous demand for commodity 2 to total demand for commodity 2 at t and a is the exogenously given growth rate of autonomous demand I^{Aut}.

(iii) Consumption

It is assumed that consumption demand depends on income and is expressed with a lag of one period. In other words, consumption demand of workers in period t is based on wage income earned in period t-1. Likewise, consumption by capitalists in period t is based on profit flows generated in period t-1.

Demand for commodity 1 – consumption demand – during period t can be written as

$$D_{1t} = (1 - s_w) Y^w_{t-1} + (1 - s_c) . P^c_{t-1} + D^{Aut}_{1t} \tag{II.12}$$

with s_w, and s_c the saving propensities of workers and capitalists respectively, Y^w_{t-1} the income of workers and P^c_{t-1} the profit flow to capitalists. D^{Aut}_{1t} represents autonomous demand for commodity 1. As

[5] The most reasonable interpretation would be that such demand arises from the government sector and/or from export demand. It is however possible to conceive of such demand in the case of investment which arises from the appearance of new technology in turn requiring the early truncation of fixed capital. However, as has been noted by Caminati (1986), such a stimulus to demand cannot be taken for granted as a general effect of technical progress.

is well known with $s_w > 0$ some part of total profit will accrue to workers. In this case, income of workers in period t denoted as Y_t^w can be written as

$$Y_t^w = \frac{w}{1-\pi.s_w}\left(L_{1t} + L_{1t}\right). \qquad \text{(II.13)}^{6}$$

Here w is the real wage in terms of commodity 1 while the parenthetical term represents the total labour requirement for production in the economy in period t.

A first complexity which arises with the treatment of fixed capital as a joint product together with the assumption of variable efficiency of machines relates to the representation of the total labour requirement of equation (II.13). In effect the labour required in each industry in order to produce a given output will depend in part on the age-composition of the stock of physical capital. In other words, and bearing in mind equations (II.1) and (II.4), the total labour requirement for each sector can be written as

$$L_{it} = \left(M_{it}^0 .l_{it}^0 .u_{it}^0 + \frac{M_{it}^1 .l_{it}^0 \left(1+\alpha\right).u_{it}^0}{\left(1+\phi\right)} \right), \qquad i = 1,2 \qquad \text{(II.14)}$$

A second complication introduced by the joint production treatment of fixed capital relates to capitalists' consumption. The profit available to capitalists for consumption expenditure will be influenced by the size of depreciation allowances and these are dependent on relative prices and the rate of profit. More precisely, depreciation for sector i in period t, denoted λ_{it}, can be expressed as

$$\lambda_{it} = M_{it}^0\left(p_{21t} - p_{it}^{m11}\right) + M_{it}^1 .p_{it}^{m1} \qquad i = 1,2 \qquad \text{(II.15)}$$

where p^{m1}_{it} refers to the price of one-year old machines used in sector i in period t relative to the price of commodity 1. Taking commodity 1 as numeraire, profit available for consumption expenditure by capitalists in period t, denoted Π_t, is given by the sum of profits associated with the use of capacity at different ages, less depreciation allowances for each sector, viz.,

$$\Pi_t = \sum_{i=1}^{2}\sum_{j=0}^{1}\left\{\left(p_{i1} - l_{it}^j .w\right).Y_{it}^j - \lambda_{it}\right\} \qquad \text{(II.16)}$$

where p_{i1} is the price of commodity i in terms of the numeraire.[7] Taking c_p as the propensity of capitalists to consume, equations (II.13)-(II.16), together with equations (II.1)-(II.4) allow one to rewrite equation (II.12), representing the demand for commodity 1, as

$$D_t^1 = \frac{\left(1-s_w\right).w}{\left(1-\pi.s_w\right).\left(1+\phi\right)}.\sum_{i=1}^{2} l_i^0 .u_{it-1}^0 .\beta_i .\left\{M_{it-1}^0 .\left(1+\phi\right) + M_{it-1}^1 .\left(1+\alpha\right)\right\} + \left(1-s_c\right).\left[A^0 + A^1 - \pi.s_w .Y_{t-1}^w\right] + D_{1t}^{Aut}$$

with (II.17)

$$A^0 = \sum_{i=1}^{2} M_{it-1}^0 \left\{p_{i1} .u_{it-1}^0 .\beta_i - l_i^0 .u_{it-1}^0 .w.\beta_i - p_{21} + p_i^{m1}\right\}$$

$$A^1 = \sum_{i=1}^{2} M_{it-1}^1 \left\{\frac{p_{i1} .u_{it-1}^0 .\beta_i}{\left(1+\phi\right)} - \frac{l_i^0 .u_{it-1}^0 .w.\beta_i .\left(1+\alpha\right)}{\left(1+\phi\right)} - p_i^{m1}\right\}$$

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6 If we assume that the profit flow P^w_t accruing to workers through their saving is instantaneous then $P^w_t = \pi s_w Y^w_t$, and since $Y_t^w = W_t + P_t^w$ then $Y_t^w = \frac{w}{1-\pi s_w}\left(L_{1t} + L_{1t}\right)$. The more realistic assumption would of course be that profit flow to workers in period *t* is derived from saving out of income in period *t-1*. To work out workers' income in *t* thereby requires a knowledge of the growth in workers' income between *t* and *t-1* and *inter alia* how this growth differs between the two sectors. The simplifying assumption embodied in expression (II.13) means however that the profit income available in period *t* is based on saving undertaken in period *t*. The only possible "realistic" interpretation of this simplifying assumption might be along the lines that the income estimate on which workers based their consumption in period *t* includes the expectation of income (profit/interest) to be received in *t+1* from saving in period *t*.

7 The term p_{i1} in the expressions for A^0 and A^1 is of course equal to 1 for commodity 1 since the latter is numeraire. Additionally, since prices are assumed to be a long-period equilibrium levels (see next sub-section), the time-subscripts have been omitted.

In order to determine the growth rate of demand for commodity 1 it is necessary to define the growth rate of newly installed capacity in sector i between t and t-1 as

$$g_{it}^{m} = \frac{M_{it}^{0}}{M_{it-1}^{0}} - 1 = \frac{I_{it-1}}{I_{it-2}} - 1 \qquad \text{(II.18)}$$

as well as a number of ratios of capacity to demand. More precisely, given equations (II.2), (II.3) and (II.4), one can express the ratios of new capacity in either sector to demand for commodity 1 as:

$$\frac{M_{2t-1}^{0}}{D_{1t-1}} = \frac{1-\left(1/z_1\right)}{M_{12t-1}^{0} u_{it-1}^{0}.\beta_i.} \qquad \text{(II.19)}$$

$$\frac{M_{1t-1}^{0}}{D_{1t-1}} = \frac{(1+\phi)\left(1+g_{1t-1}^{m}\right)}{u_{it-1}^{0}.\beta_i.z_1}$$

where

$$z_i = \left(\phi.g_{it-1}^{m} + g_{it-1}^{m} + \phi + 2\right) \qquad i = 1,2$$

and M^{0}_{12t-1} refers to the ratio of newly installed capacities in the two sectors in period t-1. From equation (II.18), one can write for the ratio of older capacity in either sector to demand for commodity 1

$$\frac{M_{it-1}^{1}}{D_{1t-1}} = \frac{M_{it-1}^{0}}{D_{1t-1}} \bigg/ \left(1+g_{it-1}^{m}\right) \qquad \text{(II.20)}$$

Dividing through equation (II.17) by D^{1}_{t-1}, making use of (II.18)-(II.20) and bearing in mind $M_{it+1}^{1} = M_{it}^{0}$, one can arrive at an expression (albeit, somewhat complicated) for the rate of growth of demand for commodity 1 between t and t-1. Given technology, the rate of profit and the growth rate of autonomous demand, this growth rate is a function of growth rates, utilization rates, ratios of newly installed capacity, ratios of demand and the ratio of autonomous to total demand for commodity 1 all for period t-1. This relation is written here simply as

$$g_{1t}^{d} = g_1\left(g_{it-1}^{d}, g_{it-1}^{m}, u_{it-1}^{0}, D_{12t-1}, M_{12t-1}^{0}, CD_{t-1}^{Aut}\right) \qquad i = 1,2 \qquad \text{(II.21)}$$

where CD^{Aut}_{1t-1} refers to ratio of autonomous demand for commodity 1 to total demand for commodity 1 at t-1.

The growth rate of induced investment for each sector – i.e. the growth rate of new capacity – can be derived on the basis of equations (II.8) and (II.18). Thus

$$g_{it}^{m} = \frac{D_{it-2}.g_i^m A - \left(1+g_{it-2}^{d}\right)\left(M_{it-1}^{0} - M_{it-2}^{0}\right)u_i^{n1}.\beta}{D_{it-2}\left(1+g_{it-3}^{d}\right)^2 - \left(1+g_{it-2}^{d}\right)M_{it-2}^{0}u_i^{n1}.\beta}, \qquad i = 1,2 \qquad \text{(II.22)}$$

where

$$g_i^m A = g_{it-2}^{d}\left(1+\left(1+g_{it-2}^{d}\right)\left(2+g_{it-2}^{d}\right)\right) - g_{it-3}^{d}\left(2+g_{it-3}^{d}\right)$$

As noted above, u^{n0}_i represents the normal or desired utilization rate on newly installed plant. Given expression (II.4), this rate implies a normal or desired rate on older plant, denoted in equation (II.26) as u^{n1}_i and equal to $u^{n0}_i/(1+\phi)$. Expressions (II.18) and (II.19) allow one to eliminate the D_{it-1}'s and M^{0}_{it-1}'s in equation (II.22) and express g^{m}_{it} as a function of growth rates of demand for i from t-2 to t-3, the growth rate of capacity for t-1 and capacity utilization in t-1, given technology, normal utilization rates and the growth rate of autonomous demand. Thus

$$g_{it}^{m} = g^{m}\left(g_{it-j}^{d}, g_{it-1}^{m}, u_{it-1}^{0}\right) \quad i = 1,2;\ j = 2,3 \qquad \text{(II.23)}$$[8]

(iv) Prices

At this point it is necessary to explicitly deal with the price system, the determination of which is presupposed in the determination of the growth rate of demand for commodity 1. As noted in the Introduction, relative prices and the real wage are taken as given at their long-period equilibrium levels, consistent with a uniform rate of profit. With fixed capital treated as a joint product and considering the capital stocks and outputs of period t, evaluated at long-period equilibrium prices, relative prices are the solutions to the following price system:

[8] Note, the most recent demand level relevant to investment decisions determining g^{m}_{it} are those in *t-2*. Where growth rates of demand are constant across time at g^* then $g^{m}_{it} = g^*$, as one would expect.

$$M^0_{it}.p_{21t}.(1+\pi_t)+w_t.l^0_i.Y^0_{it} = Y^0_{it}.p_{i1}+M^0_{it}.p^{m1}_{it}$$
$$M^1_{it}.p^{m1}_{it}.(1+\pi_t)+w_t.l^1_i.Y^1_{it} = Y^1_{it}.p_{i1} \qquad i = 1,2 \qquad \text{(II.24)}$$ [9]

Taking account of equations (II.1) - (II.4), and assuming that long-period prices correspond with the normal (desired) utilization of productive capacity, price equations (II.24) can then be rewritten as

$$p_{21}.(1+\pi) = u^n_i.\beta_i.\left(p_{i1}-w.l^0_i\right)+p^{m1}_i$$
$$p^{m1}_i.(1+\pi) = u^n_i.\beta_i.\left(p_{i1}-w.l^0_i.(1+\alpha)\right)\Big/(1+\phi) \qquad i = 1,2 \qquad \text{(II.25)}$$

where u^n refers to the desired / normal utilization rate on newly installed capacity, which, as noted above, with ϕ also given implies a "desired" utilization rate on older machines. The price system (II.25) determines three relative prices and the real wage rate for an exogenously determined rate of profit.[10]

Completing the model requires modeling the behavior of utilization rates, the ratio of investments of the two sectors (M^0_{12}) and the ratios of autonomous demand to total demand (ID^{Aut} and CD^{Aut}).

Considering utilization rates first, equations (II.2), (II.3) and (II.4) imply that for sector i

$$Y_{it} - M^0_{it-1}.u^0_{it}.\beta_i = \frac{M^0_{it-1}.u^0_{it}.\beta_i}{1+\phi} \qquad i = 1,2$$

and therefore that

$$u^0_{it} = \frac{Y_{it}.(1+\phi)}{\beta_i.\left(M^0_{it}.(1+\phi)+M^0_{it-1}\right)} = \frac{D_{it}.(1+\phi)}{\beta_i.\left(I^0_{it-1}.(1+\phi)+M^0_{it-1}\right)} \qquad i = 1,2\ldots \qquad \text{(II.26)}$$

In view equation (II.8) for induced investment,

$$u^0_{it} = \frac{D_{it}.u^{n0}_i}{D_{it-2}.\left(1+g^d_{it-2}\right)^2} = \frac{\left(1+g^d_{it}\right).\left(1+g^d_{it-1}\right).u^{n0}_i}{\left(1+g^d_{it-2}\right)^2} \qquad i = 1,2 \qquad \text{(II.27)}$$

Note that (II.27) implies that equality of growth rates of demand for commodity i between t and t-2 means that actual utilization in period t is equal to the normal rate, assuming Ω = 1. Substituting expressions (II.21) and (II.11) respectively for the period t rate of growth of demand for commodities 1 and 2 in the corresponding version of equation (II.27) allows one to express utilization on newly installed plant in each sector as a function of variables in t-1 and t-2.

Regarding the ratio of induced investment of sector 1 relative to sector 2 at time t, M^0_{12t}, this can be expressed as

$$M^0_{12t} = \frac{\left(1+g^m_{1t}\right)M^0_{12t-1}}{1+g^m_{2t}} \qquad \text{(II.28)}$$

Substituting the corresponding version of expression (II.23) for g^m_{1t} and g^m_{2t} respectively will yield an expression for M^0_{12t} as a function of growth rates of both demands in t-1 and t-2, growth rates of capacity in t-1 and utilization rates in t-1.

Finally, concerning the ratio of autonomous demand to total demand for each sector, it is assumed for simplicity that both autonomous demands grow at a uniform rate, so that the ratio of the two autonomous demands is constant. The relation between the ratios of autonomous to total demand for the two sectors can be written as

$$CD^{Aut}_t = \frac{ID^{Aut}_t.\mu}{D_{12t}} \qquad \text{(II.29)}$$

where μ is the ratio of the two autonomous demands and D_{12t} is the ratio of total demands for the two sectors at time t. Equations (II.19) and (II.20) allow for a relation between D_{12t} and the ratio of the new capacities in the two sectors:

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[9] The value of the used machine is equal to the discounted profit per unit of output on the machine, where the discount rate is the rate of profit. It is also assumed here that equilibrium is maintained in the market for used machines to the extent that they exist.

[10] The obvious deficiency with such an approach is that it ignores the dependence of the desired rate of capacity utilization on relative prices and thus on the rate of profit. The more satisfying approach – not pursued here in the interest of simplicity – is one where normal utilization rates (meaning those which are implicit in the rate of profit used as a guide for investment decisions), are determined simultaneously with long-period equilibrium prices (*cf.*, White, 1996 and more recently Franke, 2000).

$$D_{12t} = \frac{\left(g^m_{2t}+1\right).M^0_{12t}.u^0_{1t}.\beta_1.z_1}{\left(g^m_{1t}+1\right).u^0_{2t}.\beta_2.z_2} \tag{II.30}$$

Where growth rates are uniform and utilization rates normal in each sector, this expression becomes

$$D_{12t} = \frac{M^0_{12t}.u^{n0}_1.\beta_1}{u^{n0}_2.\beta_2}$$

In effect, equations (II.29) and (II.30) allow us to eliminate CD^{Aut}_{t-1} from the expressions for g^d_{1t} and u^0_{1t} by substituting (in equations (II.21) and (II.27)) with a term in ID^{Aut}_{t-1}, μ, M^0_{12t-1}, and utilization and capacity growth rates for t-1. There remains to express the time path of ID^{Aut}:

$$ID^{Aut}_t = \frac{ID^{Aut}_{t-1}.(1+a)}{(1+g^d_{2t})} \tag{II.31}$$

Substituting from equation (II.30) to eliminate D_{12t-1} in expression (II.11) and in turn substituting for g^d_{2t} in equation (II.31) provides for a relation between ID^{Aut}_t and growth and utilization rates in t-1.

III Equilibrium: endogenous or exogenous growth?

The model outlined above essentially provides a system of equations describing the time path of eight variables: two growth rates of demand (g^d_{1t}, g^d_{2t}), two growth rates of new capacity (g^m_{1t}, g^m_{2t}), two utilization rates on newly installed machines (u^0_{1t}, u^0_{2t}), the ratio of induced investments in the two sectors (M^0_{12t}) and the ratio of autonomous investment demand to total investment demand (ID^{Aut}_t). Defining the following four additional variables

$$g^L_{it} = g^d_{it-1} \quad i = 1,2 \qquad \text{and} \qquad g^S_{it} = g^L_{it-1} \quad i = 1,2 \tag{III.1}$$

one can substitute g^L_{it-1} for g^d_{it-2} and g^s_{it-1} for g^d_{it-3} in expressions (II.23) and (II.27) above. This then allows the model to be written as the first-order difference equation system

$$x_t = f(x_{t-1}) \tag{III.2}$$

where x is the vector

$$x = \left(g^d_i, g^m_i, M^0_{12}, u^0_i, ID^{Aut}, g^L_i, g^S_i\right) \qquad i = 1, 2.$$

In other words, the model can be thought of in terms of 12 variables whose values in period t are functions of the same 12 variables at t-1. An equilibrium (fixed point) of the recursive system (III.2) is characterised by $g^d_{it} = g^d_{it-1} = g^m_{it} = g^m_{it-1} = g*$ and $u_{it} = u^n_i$ for i = 1,2

The existence of autonomous demand introduces a considerable complication in thinking about equilibria for system (III.2). In fact one of two distinct equilibria seem possible and these two possibilities correspond to two cases arising in recent literature on a long-run Keynesian approach to growth. The first possibility is that the rate of growth of autonomous demand is sufficiently low relative to "induced" growth of the economy that the ratio of autonomous demand to total demand declines over time ultimately reaching zero. This appears to be the approach of Park (2000). In this case, the equilibrium growth rate is endogenous. Equations (II.11) and (II.21) (after taking account of equation (II.30)) effectively provide two expressions in g* and the equilibrium relative size of the two sectors given by $D_{12}*$.

The alternative approach, which seems to underlie recent arguments by Trezzini (1995, 1998), Palumbo and Trezzini (2003), Serrano (1995) and Cesaratto, Serrano and Stirati (2003) and which is adopted here, does not place the same restriction on the rate of growth of autonomous demand. Here the argument is that the equilibrium rate of growth would be determined by the exogenous rate of growth of autonomous demand.[11] We then have what could be appropriately termed "exogenous growth". However, the system (III.2) in this case determines endogenously the ratio of autonomous demand to total demand for each sector, along with the relative size of the two sectors.

[11] It should be noted however that in Trezzini, 1995, 1998 and Palumbo and Trezzini, 2003 a key contention is the difficulty for the economic system in adjusting to changes in the growth rate of autonomous demand in such a way as to restore normal (desired) capacity utilization. On this basis, these authors question the usefulness of steady state growth paths characterized by the "full-adjustment" of capacity to demand.

In other words, setting all growth rates equal to a, the growth rate of autonomous demands, equations (II.11) and (II.25) would determine ID^{Aut} and D_{12} and equation (II.30) would determine CD^{Aut} for a given μ.[12]

IV Disequilibrium and stability

As mentioned in the Introduction, in both this and following sections the dynamic behaviour of system (III.2) is investigated. Two methods are available for this purpose. The first involves an analysis of the local stability of the steady state growth path for the "exogenous growth" case, i.e. where the steady state growth rate is set by the growth rate of autonomous demand. The second method for analyzing the dynamics of (III.2) involves computer simulation of the system's behaviour following an exogenous shock[13]

The first method presents complexities which unfortunately render the results rather inconclusive. These results are therefore dealt with in an appendix (Appendix A). We focus our discussion here instead on the alternative method of analyzing stability, viz., dynamic simulation of the model (see Appendix B for details).

Specifically, the simulation exercise examines the behaviour of the model over time, starting from a steady state and imposing on the model a shock in the form of a change in the rate of growth of the autonomous components of demand and thus a change to the steady state rate of growth. The values assigned to technology (including normal utilization rates) and the rate of profit and therefore (via equations (II.25)) to the real wage and relative prices for all simulations are given in Table 1 in Appendix A. Simulation results for system (III.2) are depicted in Figure 1 and clearly point to an unstable system, with explosive growth rates of demand and investment in the two sectors within the first ten periods: in the case of demand in the investment sector for example, the growth rate rises from a base of 4% per period to over 400%.

V Adjusting expectations I: growth rate dispersion

In view of these rather negative simulation results for the model (III.2) the question arises as to changes which could be made in order to render the system stable. One obvious way of modifying the model is to consider alternative formulations of the expected growth rate used in the investment decision.[14] In this section the first of two alternatives is considered. Instead of supposing that the expected growth rate in each sector is some fixed proportion of the most recent realized growth rate, we follow the lead taken

[12] Either of these two types of equilibria are fixed points of the system (III.2). In the first "endogenous growth" case, substitution of g^* for growth rates, $u^0_t = u^{n0}$ for actual utilization rates, and D_{12}^* and $ID^{Aut} = 0$ will yield g^* as the solution for all growth rates and u^{n0}_i for u^0_{it}. Similar substitutions of a for all growth rates, D_{12}^* and ID^{Aut*} will yield a as the solution for growth rates and u^n_i for u^0_{it} in the "exogenous growth" case.

It should be added that the "exogenous growth" case does allow for both an exogenous growth rate and an exogenous rate of profit and thus represents one possible of a Sraffian-based explanation of prices and distribution on the one hand and a Keynesian long-run view about output, to the extent that the latter is to be associated with exogenous sources of growth.

[13] The reader is reminded that "disequilibrium" refers here to disequilibria of quantities: in particular, divergence between growth rates of demand and capacity, and between these growth rates and the growth of autonomous demand; divergence between actual and expected growth rates of demand; and, divergence between actual and normal utilization rates. Throughout the analysis, relative prices and the real wage are assumed to remain at their long-period equilibrium levels, determined by technology and an exogenous rate of profit.

[14] We leave aside one possibility in this regard, viz., using an alternative form of equation (II.7) such as

$$D^e_{it+1} = D_{it-1}.\left(1+\Omega.g^d_{it-1}\right)^2 \text{, with } 0 < \Omega < 1 .$$

One conceptual difficulty with values of Ω less than 1 is that, in view of II.8, II.26 and II.27, a constant growth rate of demand would imply $u^0_{it} > u^{n0}_i$, i.e. overutilization of capacity. This seems difficult to reconcile with the steady state since such overutilization would presumably eventually result in either a revision of the normal utilization rate itself or a revision of the parameter Ω.

by Franke and Weghorst (1988) in relation to a similar model and suppose that producers use an average of the previous two realized growth rates, discounted by a factor which depends on the dispersion between those rates. More formally, expression (II.8) for investment is rewritten as:

$$I_{it} = \frac{D_{it-1}.\left(1+g_{it}^{de}\right)^2.\left(1+\phi\right)-u_i^n.M_{it}^0.\beta_i}{u_i^n.\beta_i.\left(1+\phi\right)}$$

with g^{de}_{it} as the expectation, held at the end of period t, about the rate of growth of demand for commodity i during period t+1. As noted earlier, the most recent growth rate of demand for commodity i observed at the time at which the investment demand is formulated is that during period t-1. As such g^{de}_{it} is determined as follows:

$$g_{it}^{de} = \frac{\left(g_{it-1}^d + g_{it-2}^d\right)}{2}.X_{it} \quad \text{where} \quad X_{it} = \frac{1}{1+\sigma.\left(g_{it-1}^d - g_{it-2}^d\right)^2} \qquad \text{(V.1)}$$

The main changes to the model associated with (V.1) are detailed in Appendix C.

The results of dynamic computer simulation of the revised model are depicted in Figure 2. Their most significant feature is that a high enough value of σ[15], which determines the size of the discount for dispersion in previous growth rates in the calculation of the expected growth rate, yields persistent fluctuations in growth rates of demand without any apparent tendency for the amplitude of the cycle to increase. More precisely, for values of σ = 500 and 1000, growth rates of demand explode after only a few periods (as in the simulations shown in Figure 1); but increasing σ to 2000, yields a reasonably stable set of fluctuations following some initial volatility. These results appear to support those of Franke and Weghorst regarding the stabilizing effect of allowing for growth rate dispersion in the formulation of the expected growth rate of demand.[16] To this extent at least, they also demonstrate that a joint product treatment of fixed capital does not alter the fact that the dynamic behaviour of the system depends critically on the hypothesis chosen about expectations formation.

VI Adjusting expectations II: autonomous demand as a stabilizing force

At this point we consider a second alternative formulation of the expected growth rate and one which includes the formulation of the previous section as a particular case. An alternative way of thinking about expectations in the context of an economy where there is persistent growth in autonomous components of demand would presumably allow for the development of expectations about growth in those components of demand. In particular, it may be assumed that for investment at the end of period t producers forecast growth in demand for the period t+1 based on two calculations: one based on the demand growth in their own sector observed at the conclusion of periods t-1 and t-2; and one based on the growth rates of autonomous demand. Hence, it is proposed here that the estimate of future growth in demand is a very simple weighted average of these two calculations, so that, modifying expression (V.1),

$$g_{it}^{de} = \varepsilon.g_t^{eAut} + \left(1-\varepsilon\right).\frac{\left(g_{it-1}^d + g_{it-2}^d\right)}{2}.X_{it} \qquad \text{(VI.1)}$$

where g_t^{eAut} refers to the expectation held at the end of period t about autonomous demand growth between t and t+1 while X_{it} is as in

[15] For example, with two growth rates for two adjacent periods being 0.04 and 0.05, and thus an average of 0.045, for *X* in expression (V.1) to be approximately 0.9 and thus the discount for dispersion in growth rates to be 10% (i.e. so that the expected growth rate equals 0.045x(1-0.1)) = 0.405, σ would have to be in the order of 10^3.

[16] Franke and Weghorst examine by means of computer simulation a one-sector model which allows for both fixed and circulating capital. Their treatment of fixed capital is along traditional lines, viz., exclusively as an input. They also allow for autonomous demand, in the form of a constant amount of autonomous consumption.

expression VI.1. An outline of the changes to the model (III.2) implied by (VI.1) is provided in Appendix D.

It is worth considering the intuition behind expression (VI.I). The second term on the right hand side of expression (VI.I) could be thought to reflect, albeit in a very simplified way, two considerations in the minds of producers: first, an indication of the exogenous growth forces acting on the "economy as a whole"; and second, a recognition that growth in their own sector is not completely independent of growth in the economy as a whole. Thus the weight ε reflects the views of producers as to the importance of autonomous demand for the two commodities in determining the growth of the economy as a whole as well as their views about the relation between the latter and the growth of their own industry.[17] It should be noted here that although in the present model it is assumed that there is autonomous demand for each of the two newly produced commodities (and that these demands are growing at a uniform rate), the more interesting case relates to sectors where autonomous demand is not a component of demand for the commodity in question. In these sectors, the effect of autonomous demands on sectoral growth is indirect and arguably through growth in the “rest of the economy” at least seen from the producers’ point of view. It is really this indirect influence, namely, via the view that autonomous demand growth drives the growth of the economy and that the latter will provide some guide to the growth of individual sectors, that is of interest in the present discussion.

Two further points assumptions are made about the forecast g_t^{eAut}: first, the growth rate of autonomous demand is the same for both sectors. Second, the forecast (uniform) growth rate of autonomous demand is based on a very simple adaptive rule, viz., the forecast growth rate is equal to the most recently observed growth rate.

As might be expected, this modification to the basic model of Section II generates somewhat more encouraging results at least with respect the simulation of the model; also relative to the modification introduced in the previous section.

The suspicion that the change in behaviour of the principal minors compared with the models of the previous two sections may signal some improvement in the stability aspects of the model is confirmed by the results of dynamic simulation, which are depicted in Figure 3. These results raise a number of interesting points. Figure 3 (i) depicts the growth rates of demand in the two sectors for two cases, ε = 0.4 and ε = 0.6895. The simulations results for these two cases indicates that larger values of ε have stabilizing effects on the cycle in growth rates.

However, the picture is more interesting than this interpretation would suggest. Figure 3 (ii) illustrates the case with ε = 0.745. At this value [18] the simulation produces a long-run convergence to the constant rate of growth of autonomous demand. In one sense, this result is not surprising: one would expect that if a high enough weighting is given to autonomous demand growth in the formation of expectations about future growth, the growth rate of investment, capacity and demand will converge on that rate of growth of autonomous demand. Additionally, numerical calculation of the eigenvalues of the relevant Jacobian for this case reveals them to be all less than unity in modulus.[19] On the other hand, a similar calculation for the case ε = 0.6895 reveals eigenvalues not all less than 1 , which implies that the equilibria are not asymptotically stable; yet the simulation results point to the system’s trajectory being nonetheless bounded. An economic interpretation of this would be that, although the system does not converge to the constant rate of

[17] Some reflection on this point may bring to mind what is now commonplace, viz., the existence of numerous economic commentators pronouncing on the likely path of those types of expenditure we might refer to as autonomous e.g. those associated with the public sector as well as export demand dependent on growth of the world economy, at least for the case of a small open economy.

There are of course wider considerations here, specifically ones which go to the question of how producers arrive at a value for ε. Discussion of this is left to the next section following a review of stability results for the model incorporating expression (VI.1).

[18] Arguably an unrealistically high value in that it implies that producers attribute almost 75% of the growth rate in their sector in any period as determined by the growth rate of autonomous demand (*cf.*, the next section).

[19] That is, even though the principal minors of **[I-K$^+$]** and not all positive, hence reflecting the non-necessity of these stability conditions for the eigenvalues to be < 1.

growth of autonomous demand for some $\varepsilon < 0.745$, a sufficient enough weighing of autonomous demand growth in forecasts of growth by producers (i.e. ε sufficiently > 0.4) would render stability at least in terms of a non-explosive fluctuations in growth rates of demand and investment.

Significant also is the fact that the simulations for the model using (VI.1) to determine the expected growth rate, were all run for the value of $\sigma = 1000$: a value of σ clearly associated with instability (in the sense of fluctuations of increasing amplitude) in the simulations for section VI.

VII Some further considerations concerning expectations

It is important at this point to reflect on an issue raised by the analysis and results of the previous section. This concerns the justification for the function (VI.1) determining the expected growth in demand in each sector.

What informs the use of VI.I above is the view that producers accept that autonomous components of demand can have a persistent influence on output; specifically, that a persistent change in the rate of growth of autonomous demands can lead to a persistent change in the growth rate of the economy. Implicit in this supposition is the notion that producers do not believe that in the long-run the rate of growth is constrained by the available supply of inputs, including labour.[20]

Importantly however it is also assumed that producers are uncertain about the nature of the relation between growth in individual sectors and the growth of autonomous demand. More precisely, there is uncertainty about the nature of three sets of dynamics:

(i) the dynamics of the growth rate(s) of autonomous demand(s);

(ii) the relation between the growth rate(s) of autonomous demand(s) and the growth rate of the economy;

(iii) the relation between the growth rate of the economy and the growth rate of demand in the individual producer's own sector.

[20] To the extent therefore that producers hold beliefs about the aggregate economy, it is therefore assumed that these beliefs are not necessarily in accordance with conventional wisdom about long-run steady states.

Moreover, part of the uncertainty at least with respect to (ii) above (and arguably (iii)) is, as Caminati (1998) has emphasised, uncertainty about how the nature of the economic system's dynamics is affected by the way producers form their expectations about future growth in demand in their own sector. Put alternatively, producers are unable to assess how "the actual function mapping the output realizations [.... , in our analysis] into current [values, …] changes with every change in the expectation function used to produce the forecast[s, ……] (p. 12)". In other words, with respect to the model with (VI.I), producers are unable to assess how the relation between growth in demand in their own sector and the rate of growth of autonomous demands is affected by producers' beliefs about the significance of autonomous demands in the economy's growth.

Having taken this view, the more difficult question concerns how producers arrive at a value for ε. It is in relation to this question that one confronts some of the issues which arise in relation to learning and stability in multiplier-accelerator models (cf. Caminati, ibid.). In particular, in the comments which follow, a tentative attempt is made to consider the issue of the weighting given to autonomous demand in forming expectations about future growth in the light of these issues.

It is useful to approach the matter of determining ε by considering what, if anything, the simulation results for the model with equation (VI.I) suggest. Consider that even for the case of long-run convergence of growth rates to that of the rate of growth of autonomous demand – cases with arguably, "high" values of ε (i.e. greater than 0.745) – in general, the growth rates of demand and growth rates of autonomous demand will be unequal at any point in time. From the perspective of producers who face the uncertainty referred to above, arguably this fact would work against assigning a "high" (e.g. sufficient for long-run convergence) value given to ε. Even noting that growth rates fluctuate around the constant growth rate of autonomous demand, even where there is no long-run convergence, this realization may require a period of observation of growth rates which is too long to be of use for investment decisions by producers. Moreover, even if producers believe in such a long-run gravitation of demand growth rates, the uncertainty about the precise dynamics of interaction between growth rates in their own sector and autonomous demands may arguably render this belief of little use for investment decisions.

One implication of these first set of considerations would seem to be that there may be little to recommend an assumption that ε is sufficiently high for long-run convergence of growth rates, or even high enough for non-explosive fluctuations in growth rates. In this case, the rationale for a positive value of ε would come to depend on the development over time of conventional beliefs about autonomous components of demand driving the growth of the economy.
The simulation results reported above also indicate (Figure 4) that forecast errors [21] are reduced over the long-run in the case where ε is sufficiently high enough to ensure long-run convergence of growth rates; while for cases where there is no long-run convergence, there is no long-run tendency for prediction errors to decline in magnitude. Thus cases where there is some stability - in the sense of irregular fluctuations showing no tendency to explode - without long-run convergence of growth rates exhibit systematic forecast errors. On the face of it, this might suggest an alternative basis for forecasting should be used.
However, as noted above, the problem is that producers do not have sample information which would allow inferences to be drawn about the long-run behaviour of prediction errors as ε changes. The lack of information here is no so much about past growth rates of demand; rather it is about the values of ε used by producers in general – information necessary to uncover the link between "the function mapping the output realizations into current [values and] the expectation function used to produce forecasts."
But the difficulty lies in uncovering information about the form of function VI.I. One could argue that producers can infer information about the expected growth rates of demand from past growth rates of the capital stock, knowledge of actual capital stocks and normal utilization rates. One might also argue that observations about past growth rates of demand including the growth rate of autonomous demand, together with inferences about expected growth rates would suggest a function which gives weight to both past sectoral growth rates and the growth rate of autonomous demand. The main difficulty, in terms of the present model, is proceeding from this information to values of ε. This would require inferences to be drawn about the value of σ, i.e. about the significance given by producers to dispersion of past growth rates in the determining expected growth rates of demand.[22]

VIII Concluding remarks

The preceding discussion represents one attempt to sketch a Keynesian approach to long-run growth and one consistent with a revived classical theory of value and distribution. A simplified fixed capital model has been constructed as a means of considering the relation between long-run growth and the existence of autonomous demand; for the most part, on the supposition that growth is dominated by the behaviour of the autonomous components of demand.
The analysis above underscores the intuitively plausible notion that the dynamic behaviour of the economy is crucially linked to the way in which producers form their expectations about future growth of demand and that this feature of multiplier-accelerator dynamics carries over to the case where fixed capital is treated as a joint product. The results above also suggest that expectations formed without reference to growth in autonomous demand can lead to unstable growth outcomes. Alternatively put, reference to autonomous demand growth in the formation of expectations can act as a stabilizing force, even where expectations about autonomous demand are formed in a very simplistic way.
Two possible avenues of further research are, first, consideration of the changes which would allow stability/convergence at lower values of ε compared with the exercises in section VI; and, second, to allow for an endogenous ε. In relation to the first avenue it is possible to show that additional stability can be provided by for example adding a lag

[21] Equal to $\left(g_{it-1}^{de} - g_{it}^{d}\right)^2 / 2$

[22] In other words, a given set of past growth rates and a given expected growth rate could be consistent with different values of ε, depending on the value of σ. An alternative way of taking the present analysis further, also considered in Caminati's analysis, would involve making ε a function of forecast errors, so that its value moves in response to the effect on prediction errors of past variations in ε (*ibid*., pp. 13-17). This is however properly the subject of a separate paper.

between production and demand (the model discussed above assumes that output in each period match demand during the period; and thus, in effect, a continuous short-term equilibrium between demand and output).[23] Regarding the second avenue, it would be useful to consider an endogenous determination of ε, specifically, one in which there is some attempt by producers to take into account prediction errors, though without unwarranted assumptions on the ability of producers to uncover the true dynamics of demand.

[23] Simulation results for such a case for the present model were discussed in an earlier, larger version of the present paper. Information about these results is obtainable from the author.

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Appendix A:

This Appendix provides a summary of an analysis of the eigenvalues of the Jacobian for system (III.2) and the reformulated system of equations with (V.1) and (VI.1) respectively determining the expected growth rate of demand in each sector. In essence, local stability of the steady state equilibrium requires that the moduli of the eigenvalues of the Jacobian matrix for the difference equation system (III.2), evaluated at equilibrium, are less than unity in absolute value. Given the dimension of the Jacobian and the fact that some of its components will necessarily be unambiguously negative under reasonably plausible economic conditions,[24] the only way to proceed is in terms of a numerical study of the principal minors of the matrix [**I** - $\mathbf{J}^+$]; where **I** is the identity matrix and the elements of $\mathbf{J}^+$ are the absolute values of the corresponding elements of **J**. In other words, the elements of [**I**-$\mathbf{J}^+$], as well as the equilibrium values of M^0_{12} and ID^{Aut} , can be expressed as polynomials in key parameters of the model, i.e. s_c, s_w, a, μ, ϕ, α.[25] In turn, variations in the sign of the leading principal minors of [**I**-$\mathbf{J}^+$] can be linked with these same parameters. A set of sufficient conditions for the above requirement on the eigenvalues of **J** is that all leading principal minors of [**I** - $\mathbf{J}^+$] are positive (Gandolfo 1980, pp.136-39).

For parameter values given in Table 1 below, for initial version of (III.2) as well with allowance for dispersion of growth rates in the calculation of expected growth (equation (V.1)), there exists no range of values of s_w and s_c for which all principal minors of [**I** - $\mathbf{J}^+$] are positive. In particular, in both cases, three principal minors are negative independently of the magnitude of the saving propensities, while four are zero.

Where expected growth rates of demand are determined according to equation (VI.1), apart from four equal to zero, all principal minors are positive for a range of s_w and ε values, although this range is arguably quite restrictive (i.e. $0.35 < \varepsilon < 0.425$).

In all three of the above cases, there exists a range of values of s_w and s_c for which the following two necessary conditions for local stability are satisfied: (i) Abs Trace **J** < order of **J**; (ii) Abs $|\mathbf{J}| < 1$.

[24] Namely, positive g^*, u^n_i's, M^0_{12} and ID^{Aut} and CD^{Aut} as well as both A^D and CD^{Aut} <1.

[25] Setting all growth rates equal to the exogenous rate of growth of autonomous demand, taking normal utilization in each sector as exogenously given, and making use of equations (II.29) and (II.30), equations (II.11) and (II.21) provide two expressions in M^0_{12} and ID^{Aut}. The initial values for M^0_{12} and ID^{Aut} in Table 2 are calculated on this basis.

Table 1

l^0_1	= 0.1	p^m_{11}	= 1.44322	w	= 0.26180
l^0_2	= 2	p^m_{12}	= 1.42869	α	= 0.1
β_1	= 2	p_{21}	= 2.9795	μ	= 0.2
$\beta 2$	= 0.8	π	= 0.04		

Appendix B: Simulation details

Simulations were undertaken using the MODEL-SOLVE procedure available in the program TSP. The models used for the purposes of simulation are based on equation (III.2) and the analogous equations for subsequent versions (cf. Appendices C, and D below). The models are completely recursive: in particular, apart from their dependence on parameter values, the current values of endogenous variables depend only on the solved values of variables for previous periods (i.e. the simulation thus proceeds by substituting solved values of endogenous variables for current values of those same variables). For the simplest version of the recursion (III.2), once the values of parameters and endogenous variables in previous periods are determined, the values of the endogenous variables for the current period are solved in the following order: g^d_1, g^d_2, g^m_1, g^m_2, u^0_1,u^0_2, ID^{Aut}, M^0_{12}.

Note (III.2) has two other variables for each sector, these representing growth rates of demand in t-2 and t-3. In terms of the simulation this means that for the initial period, values for the growth rate of demand in the previous three periods need to be known. These values are detailed in Table 2 below.

Table 2

g^d_{it-j} j=1…4	0.04	ID^{Aut}_{t-1}	0.159
g^m_{it-1}	0.04	M^0_{12t-1}	0.206
u^0_{it-1}	0.85		

As noted above, simulations of the model involve starting the model in a steady state (corresponding to a rate of growth of autonomous demand of 4%) and then subjecting it to a shock in the form of a permanent increase in the rate of growth of autonomous demand (from 4 to 5 %). The parameter values used for the simulations were the same as those in Table 1 above as for the analysis of the eigenvalues of the Jacobian matrix. Apart from this, calibration was only required for the coefficient σ (see footnote 15 above).

Appendix C: Adjusting expectations I

The determination of g^{de}_{it} associated with expression (V.1) effectively adds two variables to the recursion as well as requiring changes to expressions for g^{d}_{2t}, g^{m}_{1t}, g^{m}_{2t}, u^{0}_{1t}, u^{0}_{2t}, ID^{Aut}_{t} and M^{0}_{12t}.: growth rates of gross investment (the g^{m}_{it}'s) will now depend on the corresponding growth rates of demand in t-4, since investment demand at the end of period t-2 (required in the determination of the g^{m}_{it}'s) will now depend inter alia on demand in period t-4. In order then to represent the model in terms of a first-order difference equation system requires the definition of two new variables, g^{w}_{it}, along with two new equations, $g^{w}_{it} = g^{S}_{it-1}$ (i = 1,2). In view of expression (V.I) we substitute for expressions (II.11), (II.22)-(II.23) and (II.27) respectively the following equations

$$g^{d}_{2t} = \frac{\left(1+g^{m}_{1t-1}\right)\left(M^{0}_{12t-1}Q_{1t}-4\right)u^{0}_{2t-1}u^{n0}_{1}z_{2}+Q_{2t}}{\left(1+g^{m}_{1t-1}\right)^{2}\left(u^{0}_{2t-1}u^{n0}_{1}z_{2}\right)^{2}u^{n0}_{2}\beta_{2}} + \frac{1+\phi+u^{n0}_{2}\beta_{2}\left[ID^{Aut}_{t-1}\left(2+a+\phi+a\phi\right)-\left(2+\phi\right)\right]}{u^{n0}_{2}\beta_{2}\left(1+\phi\right)}$$

where

$$Q_{it} = \left(1+g^{m}_{jt-1}\right)u^{0}_{it-1}u^{n0}_{j}\left(2+(g^{d}_{it-1}+g^{d}_{it-2})X_{1t}\right)^{2}z_{1} \qquad \text{i = 1,2, j = 1,2, i} \neq \text{j;}$$

$$g^{m}_{it} = \frac{\left(1+g^{d}_{it-2}\right)\left\{4\left(1+g^{d}_{it-1}\right)\left(1+g^{m}_{it-1}\right)u^{n0}_{i}-u^{0}_{it-1}\left(2+\left(g^{d}_{it-2}+g^{d}_{it-3}\right)X_{it-1}\right)^{2}z_{i}\right\}}{4\left(1+g^{d}_{it-1}\right)\left(1+g^{d}_{it-2}\right)u^{n0}_{i}-u^{0}_{it-1}\left(2+\left(g^{d}_{it-3}+g^{d}_{it-4}\right)X_{it-2}\right)^{2}z_{i}}-1$$

and

$$u^{0}_{it} = \frac{4\left(1+g^{d}_{it}\right)\left(1+g^{d}_{it-1}\right)u^{n0}_{i}}{\left(2+\left(g^{d}_{it-2}+g^{d}_{it-3}\right)X_{it-1}\right)^{2}}$$

Given expressions for g^{d}_{1t} and g^{d}_{2t}, utilization rates at t can be expressed in terms of variables at t-1 and t-2. One can then proceed as outlined in section II in order to arrive at the new expressions for ID_{it}^{Aut} and M^{0}_{12t}.

Appendix D: Adjusting expectations II

Similarly as for the changes introduced in section V, expression (VI.1) adds two new variables to the model of section II (expression (III.2)). This can be handled in exactly the same manner as outlined in Appendix A. We simply list here the changes to the expressions for g^{d}_{2t}, g^{m}_{it} and u^{0}_{it}. In place of expressions (II.11), (II.22)-(II.23) and (II.27) we have respectively:

$$g^{d}_{2t} = \frac{1}{4}\left(\frac{4\left(1+a\varepsilon\right)^{2}-4u^{n0}_{2}\beta_{2}}{u^{n0}_{2}\beta_{2}}+g^{d}_{2A}+g^{d}_{2B}-\frac{4}{1+\phi}+ID^{Aut}_{t-1}\left(1+a+\frac{1}{1+\phi}\right)\right)$$

where

$$g^{d}_{2A} = \frac{\left(g^{d}_{2t-1}+g^{d}_{2t-2}\right)X_{2t}\left(\varepsilon-1\right)\left\{\left(g^{d}_{2t-1}+g^{d}_{2t-2}\right)X_{2t}\left(\varepsilon-1\right)-4\left(1+a\varepsilon\right)\right\}}{u^{n0}_{2}\beta_{2}},$$

$$g^{d}_{2B} = \frac{\left(1+g^{m}_{2t-1}\right)M^{0}_{12t-1}u^{0}_{1t-1}z_{1}\left(\left(g^{d}_{1t-1}+g^{d}_{1t-2}\right)X_{it}\left(\varepsilon-1\right)-2\left(1+a\varepsilon\right)\right)^{2}}{\left(1+g^{m}_{1t-1}\right)u^{0}_{2t-1}u^{n0}_{1}z_{2}\beta_{2}};$$

$$g^{m}_{it} = \frac{\left(1+g^{d}_{it-2}\right)\left\{4\left(1+g^{d}_{it-1}\right)\left(1+g^{m}_{it-1}\right)u^{n0}_{i}-u^{0}_{it-1}z_{i}\left(2+2a\varepsilon-\left(g^{d}_{it-2}+g^{d}_{it-3}\right)X_{it-1}\left(\varepsilon-1\right)\right)^{2}\right\}}{4\left(1+g^{d}_{it-1}\right)\left(1+g^{d}_{it-2}\right)u^{n0}_{i}-u^{0}_{it-1}z_{i}\left(2+2a\varepsilon-\left(g^{d}_{it-3}+g^{d}_{it-4}\right)X_{it-2}\right)^{2}z_{i}}-1;$$

and

$$u^{0}_{it} = \frac{4\left(1+g^{d}_{it}\right)\left(1+g^{d}_{it-1}\right)u^{n0}_{i}}{\left[\left(g^{d}_{it-2}+g^{d}_{it-3}\right)\left(\varepsilon-1\right)X_{it-1}-2\left(1+a\varepsilon\right)\right]^{2}}.$$