

# WORKING PAPERS IN ECONOMICS

STRATEGIC INVESTMENT, COMPETITION  
AND THE INDEPENDENCE RESULT\*

by

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No. 137

March 1990

DEPARTMENT OF ECONOMICS



The University of Sydney  
Australia 2006

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Abstract

The provision of capital in oligopoly is considered within the context of a two-stage capital then price or output game in which firms have non-zero conjectural variations in both stages. Capital is defined as any variable which affects demand or marginal cost. We demonstrate that there exist certain types of capital for which the *strategic effect*, which distinguishes two-stage models, can be signed independent of the level of competition in either stage. In addition we examine the role played by inconsistent second stage conjectures and distinct levels of competition across stages in determining the sign of the strategic effect.

\*This paper is drawn from my Ph.D. thesis at Queen's University, Kingston, Canada. Much of the work for this paper was done at Trent University, Peterborough, Canada and University of Sydney, Australia. I am indebted to by supervisor Richard Arnott for his patience, encouragement and insight.

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## INTRODUCTION

The purpose of our analysis is to begin the process of showing whether certain types of capital are over or under provided as compared to the welfare maximizing level independent of the level of collusion and independent of whether capital is chosen strategically or non-strategically. The type of capital that we are interested in can be defined as any variable which has durable affects on demand or marginal cost and thus can involve (i) R and D (ii) product differentiation (iii) advertising (iv) multiperiod learning by doing (v) and multiproduct firms. Our research is motivated by the multitude of results in the literature which have been obtained for particular types of capital and particular types of competition. Our analysis attempts to present existing works in a more general framework which thus serves to unify the literature and point out more general results. The focus of our present analysis is to compare the strategic and non-strategic equilibrium thereby leaving the actual welfare analysis to a sequel.

In strategic investment models capital investment is either irreversible or if not irreversible at least costlier to change than price or output. As a result firms can credibly commit to a level of capital before choosing price or output. Strategic investment models are thus modelled as two-stage capital then price or output games in which firms: take the level of capital as given when choosing output and; 'see through' to the next stage thereby taking into account the *strategic effect* when choosing capital.

In non-strategic models changes in capital and output or price are equally costly and thus firms are unable to credibly commit to the former before choosing the latter. Consequently the non-strategic provision of capital is modelled as a one-stage simultaneous capital and price or output game in which firms: cannot credibly take the level of capital as fixed when choosing output and thus; cannot credibly take into account the strategic effect when choosing capital.

The above discussion makes it clear that any comparison of the strategic and non-strategic equilibrium will centre around the strategic effect.<sup>1</sup> Despite the importance of the strategic effect and the wide use of strategic investment models<sup>2</sup> only a few authors have examined the strategic effect in detail. Furthermore those that have examined the strategic effect have done so under special circumstances. We now briefly review previous results regarding the signing of the strategic effect.

Fudenberg and Tirole (1984) and Bulow *et al.* (1985) and others have examined strategic investment in the context of a zero conjectural variation model in which capital investment by firm *i* does not affect the marginal profitability of expanding firm *j*'s

<sup>1</sup>Ziss (1989a) proves that a positive (negative) strategic effect will be both necessary and sufficient for the strategic levels of capital to exceed (be less than) their non-strategic counterparts provided a strong set of stability conditions hold. Consequently we shall use the terms positive (negative) strategic effect and overinvestment (underinvestment) interchangeably.

<sup>2</sup>For example two-stage models have been used to examine R and D by Brander and Spencer (1983), Dixon (1985, 1986), Lee (1986), Okuno-Fujiwara and Suzumura (1987) (hereafter OFS (1987) among others, product differentiation by Hotelling (1929), Neven (1985), Gal-Or (1987), Shaked and Sutton (1982) among others, learning by doing by Spence (1981) and Fudenberg and Tirole (1983b), advertising or product quality by Economides (1989).

price or output. Since second stage price or output conjectures are zero in these models firms strategically invest so as to deter rival output or raise rival price. Since conjectures are zero in the investment stage, and since investment does not effect the rival's marginal profitability of expanding price or output, then the slope of the rival's stationary reaction function, and the effect of firm *i*'s capital on the marginal profitability of expanding firm *i*'s price or output, will determine the sign of the strategic effect.

Eaton and Grossman (1984) and Yarrow (1985) extend the above models to consider non-zero second stage conjectures. As a result the aim of strategic investment is no longer to just deter rival output or raise rival price as in the zero conjectural variation case but rather to (i) reduce industry output or raise aggregate prices as well as to (ii) increase output or reduce price of the investing firm relative to rival firms. The degree of collusion in the price or output stage determines the relative importance of these two goals whereas, the size of the reaction function slope determines the relative extent to which these two effects occur in response to strategic changes in capital. As a result it is the difference between the actual and conjectured slope as opposed to the slope *per se*, along with the effect of firm *i*'s capital on the marginal profitability of expanding firm *i*'s price or output that is important in determining the sign of the strategic effect.<sup>3</sup>

<sup>3</sup>For example an increase in the degree of collusion in the second stage results in less output being produced at higher mark-ups over marginal cost. Consequently the strategic investor becomes more concerned with increasing output or reducing price relative to his rivals than with reducing industry output or raising industry prices. Furthermore, provided capital has no direct effect on rivals, a decrease in the second stage reaction function slope results in greater relative as opposed to aggregate

Our contribution lies in extending the above analysis even further to allow for non-zero conjectures in the capital stage and for the capital of firm  $i$  to affect the marginal profitability of expanding price or output for firm  $j$ . The latter feature is present in models of advertising, product differentiation and spillover R and D models among others yet was previously ignored. In our model the strategic effect is thus determined by (i) the degree of collusion in the price or output stage which determines the relative importance of strategically induced changes in aggregate as opposed to relative second stage magnitudes (ii) the degree of collusion in the investment stage and the type of capital which jointly determine the extent to changes in aggregate as opposed to relative second stage variables occur in response to strategic investment.<sup>4</sup>

The particular purpose of this paper then is to analyse under what circumstances the strategic effect can be proved to be positive or negative independent of the level of collusion. In light of the oft-heard criticism that 'in oligopoly models anything can happen' this would seem to be a daunting task. Surprisingly enough however we can prove just such an independence result provided the type of collusion is restricted in a reasonable manner and provided capital is of a certain type.

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movements in second stage variables in response to strategic changes in capital.

<sup>4</sup> For example the strategic effect will tend to be positive if firms price well above marginal cost and strategic investment serves to reduce the relative price or increase the relative output and thus profits of the investing firm. Changes in relative outputs or prices will occur when firms act competitively in the investment stage and thus offset each others strategic increases in capital, or if capital has a dissimilar effect on investing and rival firm marginal profitability of expanding price or output.

The type of capital for which independence holds has the following features. The relative effect of capital on investing versus rival firm marginal profitability of expanding price or output is sufficiently large so as to guarantee increased market share or reduced relative price of the investing firm yet; sufficiently small so as to guarantee aggregate output to fall and aggregate prices to rise in response to a given strategic change in capital. If the above two features are present then the strategic effect can be signed independent of the level of collusion in either the first or second stage. We point out that the independence result is a feature of the horizontal product differentiation models contained in Hotelling (1929) and Neven (1985), the informative advertising model contained in Grossman and Shapiro (1984) and the advertising or product quality model contained in Economides (1989).

Aside from the independence result described above we use our model to examine the role of inconsistent conjectures as well as the role of distinct levels of collusion across stages in determining the sign of the strategic effect. The three main features of our model within which the above results are obtained are now described and justified.

The first main feature of our analysis is that it is static in nature and thus firms are assumed to be able to accumulate capital instantaneously. If capital accumulation takes time then it may not be the case that the static Nash equilibrium emerges as the steady state equilibrium.<sup>5</sup> We make allowances for static

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<sup>5</sup>In fact it has been shown by Fudenberg and Tirole (1983a) that both the static Nash and joint profit maximum points emerge as possible symmetric steady state subgame perfect equilibria in the capital choice game.

equilibria which are non-Nash by allowing for non-zero conjectures in the capital choice stage.

The second feature of our analysis is that we consider market which consists of  $n$  symmetric firms and thus ignore questions regarding entry deterrence and Stackelberg leadership. Furthermore we assume the profit function of each firm to be continuously differentiable.

A final main feature of our model is that we assume that firms have non-zero conjectures in both the first and second stages. Although it has been argued that the conjectural variations approach suffers from methodological and conceptual problems because it attempts to model dynamic oligopoly interaction in an essentially static model we nonetheless justify their use on the grounds that dynamic models yield an embarrassingly rich set of results<sup>6</sup> or can only be modelled with great difficulty.<sup>7</sup> Thus rather than attempt the difficult task of modelling the capital then price (output) game as a dynamic process we opt for the simpler static framework which employs conjectural variations in order to capture, albeit in an *ad hoc* manner, some of the dynamic elements of oligopolistic interaction. In this way we are able to highlight the essential effects of commitment without having to tie ourselves to a particular model

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<sup>6</sup>For example, see Riordan (1985) for a model of negative dynamic conjectural variations and Cubbin (1983) for a model of positive dynamic conjectural variations. See also Kalai and Stanford (1985). Furthermore estimates by Applebaum (1982) and Iwata (1974) have found conjectures to take on a variety of levels ranging from price taking to joint profit maximizing levels.

<sup>7</sup>Fudenberg and Tirole (1983b) have made this argument for continuous time 'learning by doing' models.

of oligopolistic interdependence. In addition our approach allows us to examine the effect of collusion on commitment models in a very simple manner.<sup>8</sup>

Among the types of conjectures we consider are calculus-based<sup>9</sup> conjectures that are both partially<sup>10</sup> and exogenously<sup>11</sup> consistent in the second stage of the game. The reason that we adopt for this particular form of consistent conjecture is because of its tractability and because it has relatively few existence and uniqueness problems as compared to other notions of consistency.<sup>12</sup> Theoretical support for the notion of consistency is provided by Shaffer (1984) who argues that under uncertainty consistent conjectures avoid chaos whereas a theoretical indictment of consistency is contained in Makowski (1987). Empirical support for consistent conjectures is contained in Kolstad and Wolak (1986) in a non-experimental setting whereas Holt (1985) rejects the consistent conjectures hypothesis in an

<sup>8</sup>Eaton and Grossman (1984, 1986), Dixit (1988), Brander and Spencer (1985), Geroski (1984) and Dixon (1986) have also used conjectural variation models as a means of capturing varying levels of competition.

<sup>9</sup>Assuming that conjectures are calculus-based implies that each firm conjectures that the rival will react in continuous fashion to changes in its choice variable and thus that conjectures can be thought of as derivatives as opposed to more general mappings.

<sup>10</sup>Conjectures are partially consistent if the conjectural variation of firm  $i$  is equal to the first partial derivative (i.e. the slope) of the reaction function of the rival firm with respect to the choice variable of firm  $i$ .

<sup>11</sup>Conjectures are exogenously (endogenously) consistent if firms do not (do) take into account the effect of the first stage variable on the consistent level of the second stage conjectures.

<sup>12</sup>See Bresnahan (1981), Perry (1982), Boyer and Moreaux (1983a, 1983b), Robson (1983) and Kamien and Schwartz (1983) for a discussion of the existence and uniqueness of consistent conjectures.

experimental setting.<sup>13</sup>

The paper is organized as follows. In Section I we outline the model and analytically derive the strategic effect. In Section II we present our results and in Section III we mention avenues for future research.

## SECTION I THE MODEL

We shall carry out our analysis in the context of the following model. We consider a market consisting of  $n$  symmetric firms who have two choice variables : capital and output or price. For example, the capital variable can be interpreted as (i) marginal cost-reducing  $R$  and  $D$  or physical capital (ii) advertising or (iii) product differentiation. The static profit function of firm  $i$  can be expressed as follows

$$(1) \pi_i = \pi^i(S, K)$$

where  $K = (k_1, \dots, k_n)$  and  $S = (s_1, \dots, s_n)$  represent the vector of  $n$  first stage capital and second stage output (price) variables of the  $n$  firms in the industry respectively,  $\pi_i$  represents the profit variable and  $\pi^i$  represents the profit function. Firm  $i$ 's first stage variable is the strategic capital investment variable (denoted  $k_i$ ) whereas its second stage variable (denoted  $s_i$  for second stage) is either price or quantity.<sup>1</sup> In order to solve the strategic investment model we use the ubiquitous method of

<sup>13</sup> Furthermore consistent conjectures have been used in the optimal trade policy literature by Eaton and Grossman (1986) in the  $R$  and  $D$  literature by Dixon (1986) and Eaton and Grossman (1984), by Geroski (1984) to examine entry and in product differentiation models by Eaton and Kierzkowski (1984).

backward induction.

## 1.1 THE SECOND STAGE

Since we have assumed symmetry we shall analyse the behaviour of firm  $i$ , the representative firm. We shall denote firm  $j$  as any firm which is not firm  $i$  and firm  $h$  as any firm which is neither firm  $i$  nor firm  $j$ . In addition since we are using a conjectural variation model neither the first nor the second stage first order condition is represented by a partial derivative of the objective function and thus we shall use  $D_{s_i}^i$  and  $D_{k_i}^i$  to denote the first order condition associated with firm  $i$ 's choice of second stage variable and capital respectively. Given the aforementioned notational conventions firm  $i$ 's first order condition associated with its second stage choice of price or output is as follows

$$(2) D_{s_i}^i = \pi_{s_i}^i + (n-1)\lambda\pi_{s_j}^i = 0$$

$$\text{where } \pi_{s_i}^i = \frac{\partial \pi^i}{\partial s_i}, \quad \pi_{s_j}^i = \frac{\partial \pi^i}{\partial s_j} \quad \text{and } \lambda = \left. \frac{ds_j}{ds_i} \right|_{\text{con}. \text{ var.}}$$

$\lambda$  represents the firm  $i$ 's conjecture regarding the  $s_j$  response of firm  $j$  to a change in  $s_i$  taking into account the response of the remaining  $n-2$  firms and is thus characterized by a total derivative.  $\lambda$  also represents firm  $i$ 's best guess regarding the slope of firm  $j$ 's reaction function and is furthermore a measure of tacit collusion in the market.<sup>14</sup> In

<sup>14</sup>The actual (conjectured) slope of firm  $j$ 's reaction function can be derived from the following conceptual experiment. Suppose

particular an increase in  $\lambda$  will, in most cases imply an increase in collusion.<sup>15</sup>

Since we use the notation  $D_{s_i}^1$  to designate the first order condition associated with firm  $i$ 's choice of  $s_i$ , then the higher order derivative of  $D_{s_i}^1$  with respect to  $s_i$  shall be designated as  $D_{s_i, s_i}^1$  and so on for the remaining higher order derivatives. Furthermore since these higher order derivatives are cumbersome we shall replace them with short-hand notation developed in OFS (1987). Using our new shorthand notation we now impose the following conditions to insure that the second stage is unique and stable.

$$(3a) \quad \alpha - \beta < 0, \quad \alpha + (r-1)\beta < 0 \quad r = 1, \dots, n \quad \text{where}$$

$$(3b) \quad \alpha \equiv D_{s_i, s_i}^1 < 0 \quad (\text{second order condition})$$

agents are initially in equilibrium and now firm  $i$  contemplates a small deviation away from equilibrium. The deviation away from equilibrium implies that firm  $i$  will no longer be satisfying its first order condition and thus the aforementioned change in  $s_i$  can be thought of as exogenous. The slope of firm  $j$ 's reaction function (firm  $i$ 's conjecture regarding the slope of firm  $j$ 's reaction function) then measures how firm  $j$  responds (how firm  $i$  conjectures that firm  $j$  responds) to firm  $i$ 's deviation both directly and indirectly via the induced changes in the second stage choice variables of the remaining  $n-2$  firms. Reaction function shifts have similar interpretations.

<sup>15</sup>In particular if  $\lambda = 1$  and firms' pay-off functions are symmetric then each firm acts as if it were maximizing the sum of the pay-offs of all firms. If  $\lambda = 0$  then firm  $i$  takes the choice variable of the other firms as given and thus the non-cooperative Nash equilibrium ensues. In addition as  $\lambda$  increases firm  $i$  expects a larger  $s_j$  response of rivals to an increase in  $s_i$ . If output is the firm's second stage choice variable then firm  $i$  will be more reluctant to increase output the larger the expected output response of rivals. If price is the second stage variable then firm  $i$  will be more willing to increase price if it expected rivals to follow suit to a greater extent. In both cases an increase in  $\lambda$  results in higher prices and less output which in turn implies a more collusive outcome.

$$(3c) \quad \beta \equiv D_{s_i, s_j}^1 > 0^{16}$$

## 1.2 COMPARATIVE STATICS

Before carrying out our comparative static exercises we introduce the following notation regarding the second stage solutions.

$$(4a) \quad S = S(K) = [s^1(K), \dots, s^n(K)] \quad \text{where} \quad K = (k_1, \dots, k_n)$$

$$(4b) \quad s_i^1 \equiv \frac{\partial s^1(K)}{\partial k_1} = \left. \frac{ds_i}{dk_1} \right|_{\text{stat. comp.}} = s_j^j$$

$$(4c) \quad s_i^j \equiv \frac{\partial s^j(K)}{\partial k_1} = \left. \frac{ds_j}{dk_1} \right|_{\text{stat. comp.}} = s_j^i = s_j^h$$

The notation above can be explained as follows.  $S(K)$  represents the vector of second stage solutions as functions of the first stage vector of capital.  $s^1(K)$  or more concisely  $s^1$  represents the solution to the variable  $s_i$ . The partial derivatives of the  $s^1$  functions thus yield comparative static results which are derived in the appendix and given below.

<sup>16</sup>In most cases  $\beta > 0$  when price is the firm's choice variable since an increase in the price of firm  $j$  increases firm  $i$ 's demand, thereby increasing firm  $i$ 's marginal pay-off of increasing price and thus inducing firm  $i$  to raise price. Conversely, in most cases  $\beta < 0$  when output is the firm  $i$ 's choice variable because an increase in firm  $j$ 's output level will serve to drive down price and thus reduce firm  $i$ 's marginal pay-off of increasing output thereby inducing firm  $i$  to reduce output. Thus in most cases outputs are strategic substitutes whereas prices are strategic complements, to use the terminology of Bulow et al. (1985). More explicit expressions for  $\alpha$  and  $\beta$ , along with other shorthand notation developed later on in the paper, are given in the Appendix.

$$(5a) \quad s_i^l = s_j^l = \frac{\varepsilon(n-1)\beta - \sigma(\alpha + (n-2)\beta)}{\Delta}$$

$$(5b) \quad s_i^j = s_j^i = s_j^h = \frac{\sigma\beta - \varepsilon\alpha}{\Delta} \quad \text{where}$$

$$(5c) \quad \Delta = (\alpha - \beta)(\alpha + (n-1)\beta) > 0 \quad \text{by stability, and where}$$

$$(5d) \quad \sigma = D_{\varepsilon_i k_i}^i \quad \text{and} \quad \varepsilon = D_{\varepsilon_j k_i}^j \quad 17$$

The comparative statics reported above can now be used to derive the following result, which we proof in the Appendix, and which turns out to be useful in subsequent analysis

$$(6a) \quad s_i^j = g s_i^i - \frac{\varepsilon}{\alpha + (n-2)\beta} \quad \text{where}$$

$$(6b) \quad \left. \frac{ds_j}{ds_i} \right|_{\text{slope RF}_j} = g = \frac{-\beta}{\alpha + (n-2)\beta} \quad \text{and where}$$

<sup>17</sup>  $\varepsilon$  and  $\sigma$  can take on a variety of different signs depending on the type of capital and on whether price or output is the second stage variable. For example R and D models with zero spillover effects in which an increase in R and D by firm i serves to reduce the marginal cost of firm i and leave the marginal cost of rivals unaltered will increase firm i's marginal profitability of expanding output ( $\sigma > 0$ ) but reduce its marginal profitability of expanding price ( $\sigma < 0$ ). Furthermore since there are no spillover effects the marginal profitability of expanding either price or output for rival firms remains unaltered ( $\varepsilon = 0$ ).

An increase in informative advertising of firm i may make consumers more aware of the existence of close substitutes for each product and thus may tend to increase the demand elasticities of all firms in the industry. More elastic demand will tend to decrease the marginal profitability of expanding price for both firm i ( $\sigma < 0$ ) and for firm i's rivals ( $\varepsilon < 0$ ) whereas it would increase the marginal profitability of expanding output for firm i ( $\sigma > 0$ ) and for firm i's rivals ( $\varepsilon > 0$ ). In contrast had we considered persuasive advertising which induces brand loyalty thereby reducing as opposed to increasing demand elasticities the above signs of  $\sigma$  and  $\varepsilon$  would have been reversed.

$$(6c) \quad \left. \frac{ds_j}{dk_i} \right|_{\text{shift of RF}_j \text{ in response to } dk_i} = \frac{-\varepsilon}{\alpha + (n-2)\beta}$$

(6a) thus proves that the effect of  $k_i$  on  $s_j$  can be broken up into a movement along firm j's reaction function resulting from the endogenous  $s_i$  response of firm i to the increase in  $k_i$  ( $gs_i^i$ ) and a shift of firm j's reaction function resulting from the direct effect of  $k_i$  on  $s_j$ . Furthermore the stability conditions have the following implications for the slope of agent j's reaction function

$$(7a) \quad \text{sgn slope RF}_j = \text{sgn } \beta = \text{sgn } -\beta/\alpha \quad \text{and}$$

$$(7b) \quad -1/(n-1) < g < 1$$

The restrictions on the slope of the actual second stage reaction function given by (7b) lead us to place similar restrictions on firm i's conjecture regarding the slope of firm j's reaction function so as to make the latter plausible and so as to restrict the possible set of equilibria. The restrictions that we place on  $\lambda$  will nonetheless allow for Bertrand and Cournot competition ( $\lambda = 0$ ), perfect competition when quantity is the second stage variable and goods are homogeneous ( $\lambda = -1/(n-1)$ ), joint profit maximizing behaviour when either price or quantity is the second stage variable ( $\lambda = 1$ ). The restrictions on the agents' conjectures are given as follows

$$(8) \quad -1/(n-1) \leq \lambda \leq 1$$

### 1.3 THE FIRST STAGE

We now substitute the second stage solutions given by (4a) back into the profit function given by (1) to obtain

$$(9) \quad \pi_i = \hat{\pi}^i(K) = \pi^i[S(K), K]$$

We now maximize profits with respect to the first stage capital variable. Doing so we obtain the following first order conditions in symmetric form

$$(10a) \quad D_{k_i}^1 \equiv \pi_{s_i}^1 \bar{s}_i^1 + (n-1)\pi_{s_j}^1 \bar{s}_j^1 + \pi_{k_i}^1 + (n-1)\rho\pi_{k_j}^1 = 0, \text{ where}$$

$$(10b) \quad \bar{s}_i^1 \equiv s_i^1 + \rho(n-1)s_j^1 \quad \text{and}$$

$$(10c) \quad \bar{s}_i^j \equiv s_i^j + \rho s_j^j + (n-2)\rho s_h^j \quad h \neq j, i \text{ and}$$

$$(10d) \quad \rho \equiv \frac{dk_j}{dk_i} \Bigg|_{\substack{\text{conj.} \\ \text{var.}}}$$

We shall refer to  $\bar{s}_i^1$  and  $\bar{s}_j^1$  as the total responses of firm i and firm i's typical rival (firm j) to a change in the capital of firm i. The total responses include the direct effect of firm i's capital on the second stage variables and also the indirect effects which occur as a result of firm i conjecturing that its rivals will change their capital in response to a change in its capital.

The first stage conjectural variation term given by  $\rho$

measures firm i's conjecture regarding the  $k_j$  response of firm j to an increase in firm i's capital taking into account that firm j also responds to the induced change in the capital levels of the remaining n-2 firms. We now place the following restrictions on  $\rho$ .<sup>18</sup>

$$(11) \quad -1/(n-1) \leq \rho \leq 1$$

We now evaluate the first order condition of the first stage taking into account optimizing behaviour in the second stage. From (2) we obtain that optimal behaviour in the second stage is characterized by  $\pi_{s_j}^1 = -\lambda(n-1)\pi_{s_j}^1$ , which we thus substitute into (10a) to obtain

$$(10a)' \quad D_{k_i}^1 \equiv (n-1)\pi_{s_j}^1 \left[ \bar{s}_j^1 - \lambda \bar{s}_i^1 \right] + (n-1)\rho\pi_{k_j}^1 + \pi_{k_i}^1 = 0$$

which leads us to the following definition of the strategic effect.

DEFINITION 1 (strategic effect, positive/negative strategic effect) The strategic effect is defined as being equal to

$$(11a) \quad (n-1)\pi_{s_j}^1 \left[ \bar{s}_j^1 - \lambda \bar{s}_i^1 \right]$$

and shall be considered positive/negative whenever (11a) is greater than (less than) zero.

## II THE STRATEGIC EFFECT, COMPETITION AND THE TYPE OF CAPITAL

<sup>18</sup>Our justification for restrictions on  $\rho$  are similar to those given for the restrictions on  $\lambda$ .

We now decompose the strategic effect into two components. The decomposition that we have selected involves factoring  $\bar{s}_1^i$  from the strategic effect in (11a) and then substituting the symmetry assumption  $\pi_{s_j}^i = \pi_{s_i}^j$  in order to arrive at the following.

$$(11b) \text{ strategic effect} = (n-1)\pi_{s_i}^j \bar{s}_1^i \left[ \frac{\bar{s}_1^j}{\bar{s}_1^i} - \lambda \right]$$

We now use (11b) to formally define the top dog effect, fat cat effect, lean and hungry look and puppy dog ploy. In order to do so we first define investment as being either tough or soft.

DEFINITION 2 : Investment by firm i makes firm i tough (soft) if the total second stage response of firm i to an increase in its own capital serves to reduce (increase) the profits of rival firms.

Necessary and sufficient conditions for soft and tough are as follows

$$(12a) \text{ Tough} : \pi_{s_i}^j \bar{s}_1^i < 0$$

$$(12b) \text{ Soft} : \pi_{s_i}^j \bar{s}_1^i > 0$$

Investment makes the firm tough (soft) by inducing it to either expand (reduce) output or cut (increase) price thereby adversely affecting the profits of rival firms.

DEFINITION 3A (top dog effect) : The top dog effect occurs when

the strategic effect is positive and investment makes the firm tough.

DEFINITION 3B (fat cat effect) : The fat cat effect occurs when the strategic effect is positive and investment makes the firm soft.

DEFINITION 3C (lean and hungry look) : The lean and hungry look occurs when the strategic effect is negative and investment makes the firm soft.

DEFINITION 3D (puppy dog ploy) : The puppy dog ploy occurs when the strategic effect is negative and investment makes the firm tough.

We now make the following additional definitions concerning the conjectural variation terms and then follow with a proposition which uses all the definitions in this section and which outlines when the abovementioned four effects occur.

DEFINITION 4 : The second stage conjectures of firm i are low (high) if they are less than (greater than) the ratio (denoted  $\lambda^*$ ) of firm j's total response to an increase in the capital of firm i ( $\bar{s}_1^j$ ) to the total response of firm i to an increase in the capital of firm i ( $\bar{s}_1^i$ ).

In mathematical terms we obtain that necessary and sufficient conditions for high and low conjectures are as follows

(13a) High conjectures :  $\lambda > \lambda^*$

(13b) Low conjectures :  $\lambda < \lambda^*$  where

$$(13c) \lambda^* = \frac{\bar{s}_i^j}{\bar{s}_i^i} = \frac{s_i^j + \rho [s_j^j + (n-2)s_h^j]}{s_i^i + \rho(n-1)s_j^i}$$

PROPOSITION 1 : Necessary and sufficient conditions for the top dog effect, the fat cat effect, the lean and hungry look and the puppy dog ploy to occur are as follows

(14a) Top dog effect occurs if and only if

$$\text{investment is tough : } \pi_{s_i^i}^j \bar{s}_i^i < 0$$

$$\text{conjectures are high : } \lambda^* - \lambda < 0$$

(14b) Fat cat effect occurs if and only if

$$\text{investment is soft : } \pi_{s_i^i}^j \bar{s}_i^i > 0$$

$$\text{conjectures are low : } \lambda^* - \lambda > 0$$

(14c) Lean and hungry look occurs if and only if

$$\text{investment is soft : } \pi_{s_i^i}^j \bar{s}_i^i > 0$$

$$\text{conjectures are high : } \lambda^* - \lambda < 0$$

(14d) Puppy dog ploy occurs if and only if

$$\text{investment is tough : } \pi_{s_i^i}^j \bar{s}_i^i < 0$$

$$\text{conjectures are high : } \lambda^* - \lambda > 0$$

$$(14e) \text{ where } \lambda^* = \frac{\bar{s}_i^j}{\bar{s}_i^i}$$

Proof : Follows directly from Definitions 2, 3, and 4. Q.E.D.

We now illustrate Proposition 1 in figure 1. We label  $E_0$  as the second stage equilibrium which corresponds to the non-strategic equilibrium. In order to illustrate the strategic effect diagrammatically we now increase the capital of firm i by an arbitrarily small amount :  $\Delta k_i$  and allow the capital of firm j to change by the amount conjectured by firm i :  $\rho \Delta k_i$ . The changes in firm i's capital and the conjectured change in firm j's capital cause the second stage reaction functions to shift and result in a new intersection of the reaction functions at one of the four points marked  $E_1$ .<sup>19</sup> The line connecting  $E_0$  and  $E_1$  has a slope equal to the ratio of firm j to firm i's second stage response to a change in firm i's capital and to the associated conjectured change in firm j's capital and is thus equal to  $\lambda^*$  by definition.  $\lambda$  is illustrated by the slope of firm i's isoprofit contour at  $E_0$  along its reaction function.<sup>20</sup> We now use figure 1 to explain the top dog effect.

The top dog effect involves toughness, high conjectures and a  
<sup>19</sup> $E_1$  does not represent the strategic equilibrium since we have considered an arbitrarily small increase in firm i's capital and not the equilibrium changes in capital which arise when firm's choose capital strategically.  $E_1$  is simply used as a reference point to indicate whether or not there exist profitable and credible deviations away from the non-strategic equilibrium.  
<sup>20</sup>This result can be shown by totally differentiating an isoprofit contour, setting all the capital terms equal to zero, using symmetry and then substituting in the first order conditions of the second stage.

positive strategic effect and is illustrated by the movement from  $E_0$  to  $E_1^{TD}$ . Toughness is illustrated by the fact that firm  $i$ 's price falls or output rises as a result of increasing capital as we move from  $E_0$  to  $E_1^{TD}$  and thus the profits of firm  $j$  will fall.  $\lambda$  is illustrated as being high by the fact that the slope of the isoprofit contour, given by  $\lambda$ , is greater than (i.e. less negative) than the slope of the line connecting  $E_0$  and  $E_1^{TD}$ , given by  $\lambda^*$ . The positive strategic effect is illustrated by the fact that the isoprofit contour going through  $E_1^{TD}$  is farther from (closer to) the horizontal axis than the isoprofit contour going through  $E_0$  when price (quantity) is the second stage variable. Similar arguments can be used to explain how the fat cat effect, lean and hungry look and puppy dog ploy will involve movements from  $E_0$  to  $E_1^{FC}$ ,  $E_1^{LH}$  and  $E_1^{PD}$  respectively.

## II.1 THE INDEPENDENCE RESULT

In each of the results presented below<sup>21</sup> we shall presume the stability conditions as well as the restrictions on the first and second stage conjectures given by (11) and (8) respectively, to hold. Furthermore we shall make the following standard assumptions regarding the effect of rival price and output on profits

$$(15a) \pi_{s_i}^j > 0 \text{ when } s_i = \text{price of firm } i$$

$$(15b) \pi_{s_i}^j < 0 \text{ when } s_i = \text{output of firm } i^{22}$$

<sup>21</sup>Proofs for the results are given in the Appendix.

<sup>22</sup>(15a) states that an increase in the price of firm  $i$  increases the demand for firm  $j$ 's product, since the goods are substitutes,

Given the strategic effect as expressed in (11b) the above assumptions make it clear that if the strategic effect is proved positive for output it will have been proved negative for price. It should also be noted that the signs of  $\beta$ ,  $\epsilon$  and  $\sigma$  could also change as we switch from output to price.<sup>23</sup>

PROPOSITION 2 : (THE INDEPENDENCE RESULT) (i) If output is the second stage variable and, provided the zero strategic effect as outlined in proposition 5 does not occur, then the strategic effect will be positive (negative) independent of the level of collusion in either the first or second stage if and only if (16) holds (holds with the inequalities reversed). (ii) Furthermore if price is the second stage variable then the above independence result holds except with the words positive and negative interchanged.

$$(16a) \sigma + (n-1)\epsilon \leq 0 \text{ and}$$

$$(16b) \epsilon - \sigma \leq 0.$$

In order to make the intuition behind this result clear we state two results regarding necessary conditions for independence followed by four special cases.

COROLLARY 2A : If output is the second stage variable then the strategic effect will be positive (negative) independent of the

thereby increasing firm  $j$ 's profits *ceteris paribus*. Conversely (15b) states that an increase in the output of firm  $i$  will require a reduction in firm  $i$ 's price which will reduce the price of firm  $j$ 's product, since the goods are substitutes, thereby reducing the profits of firm  $j$  *ceteris paribus*.

<sup>23</sup>See footnote 16 for a discussion of the sign of  $\beta$  for price and output and footnote 17 for a discussion of the signs of  $\epsilon$  and  $\sigma$  under price or output competition for various types of capital.

level of collusion in either the first or second stage only if the firm anticipates that an increase in its capital will deter (encourage) rival output and only if  $\epsilon < 0$  ( $\epsilon > 0$ ).

COROLLARY 2B : If price is the second stage variable then the strategic effect will be positive (negative) independent of the level of collusion in either the first or second stage only if the firm anticipates that an increase in its capital will raise (lower) the rival's price and only if  $\epsilon > 0$  ( $\epsilon < 0$ ).

#### II.1A SPECIAL CASES OF THE INDEPENDENCE RESULT

In the four special cases that we consider in this section we hold the level of competition in one stage constant at either the most competitive or least competitive level (i.e. conjectures are set equal to  $-1/(n-1)$  or  $1$  respectively) and we prove the strategic effect to be positive for a given type of capital independent of the level of competition in the other stage.

COROLLARY 2C : (i) If output is the second stage variable and firms act as joint-profit maximizers in the output stage (i.e.  $\lambda = 1$ ) then the strategic effect will be positive (negative) independent of the level of collusion in the investment stage if and only if  $\epsilon - \sigma < 0$  ( $\epsilon - \sigma > 0$ ) (ii) Furthermore if price is the second stage variable then the above partial independence result holds except with the words positive and negative interchanged.

COROLLARY 2D : (i) If output is the second stage variable and  $\lambda = -1/(n-1)$  then the strategic effect will be positive (negative) independent of the level of collusion in the investment stage if and only if  $\sigma + (n-1)\epsilon < 0$  ( $\sigma + (n-1)\epsilon > 0$ ) (ii) Furthermore if price is the second stage variable then the above partial independence result holds except with the words positive and negative interchanged.

COROLLARY 2E : (i) If output is the second stage variable and firms act as joint-profit maximizers in the investment (i.e.  $\rho = 1$ ) then the strategic effect will be positive (negative) independent of the level of collusion in the output stage if and only if  $\sigma + (n-1)\epsilon < 0$  ( $\sigma + (n-1)\epsilon > 0$ ) (ii) Furthermore if price is the second stage variable then the above partial independence result holds except with the words positive and negative interchanged.

COROLLARY 2F : (i) If output is the second stage variable and  $\rho = -1/(n-1)$  then the strategic effect will be positive (negative) independent of the level of collusion in the output stage if and only if  $\epsilon - \sigma < 0$  ( $\epsilon - \sigma > 0$ ) (ii) Furthermore if price is the second stage variable then the above partial independence result holds except with the words positive and negative interchanged.

#### II.1B DISCUSSION OF THE INDEPENDENCE RESULT

We now point out the intuition behind Proposition 2 and

Corollaries 2a, 2b, 2e and 2f by using figure 2 to consider the special cases examined in Corollaries 2c and 2d. We argue the output case in detail and provide a sketch of the intuition behind the price case in footnotes.

If firms act as joint profit maximizers in the output stage then they produce relatively little output at a large mark-up over marginal costs. In order to increase its profits firm  $i$  must increase its output relative to its rivals.<sup>24</sup> Using changes in output alone firm  $i$  cannot bring about increases in its relative output because it expects one for one output responses from rivals which would leave relative output. As a result firm  $i$  seeks to induce increases in relative output via strategic capital investment. As stated in Corollary 2c the condition  $\epsilon - \sigma < 0$  is both necessary and sufficient for the later to occur and thus for the strategic effect to be positive independent of the level of competition in the investment stage.

If  $\lambda = -1/(n-1)$  then firms produce large amounts of output at close to marginal cost and thus seek to increase profits by reducing aggregate output.<sup>25</sup> As stated in Corollary 2d the condition  $\sigma + (n-1)\epsilon < 0$  is both necessary and sufficient for the

<sup>24</sup>At the joint profit maximum firm  $i$  operates at point A along its output isoprofit contour illustrated in figure 2a. In order to increase profits firm  $i$  wishes to move to the right of the tangent line aa. Since line aa has slope equal to  $\lambda = 1$  then points to the right of line aa are associated with higher relative output for firm  $i$ . Furthermore if price is the second stage variable then increasing market share is analogous to reducing relative price.

<sup>25</sup>If  $\lambda = -1/(n-1)$  then each firm operates at point B along its output isoprofit contour illustrated in figure 2a. In order to increase profits firm  $i$  wishes to move to the left of the tangent line bb. Since line bb has slope equal to  $\lambda = -1/(n-1)$  then points to the left of line bb are associated with lower industry output. Furthermore when price is the second stage variable then raising aggregate prices is analogous to reducing industry output.

later to occur and thus for the strategic effect to be positive independent of the level of competition in the investment stage.

If  $-1/(n-1) \leq \lambda \leq 1$  then the strategic effect will be positive provided a strategic increase in capital serves to both reduce industry output yet expand relative output of the investing firm.<sup>26</sup> As stated in the independence result the later effect will occur provided the conditions  $\sigma + (n-1)\epsilon \leq 0$  and  $\epsilon - \sigma \leq 0$  both hold.

Furthermore since falling market output and increasing market share of the investing firm imply that rival output must fall we thus obtain that the latter condition is necessary but not sufficient for independence. In fact rival output deterrence is only necessary and sufficient when competition in the output stage is Nash (i.e.  $\lambda = 0$ ). We now explain the intuition behind Corollaries 2e and 2f.

If firms act as joint profit maximizers in the investment stage then each firm believes that changes in its capital will be matched each rival one for one. As a result strategic increases in capital will have no effect on market share and will either cause industry output to rise or fall depending on whether  $\sigma + (n-1)\epsilon$  is greater or less than zero. In the former case profits fall and thus the strategic effect is negative whereas the reverse is true in the latter case.

When  $\rho = -1/(n-1)$  then rival capital will completely offset increases in firm  $i$ 's capital thereby leaving industry output

<sup>26</sup>If  $-1/(n-1) < \lambda < 1$  then firm  $i$  operates at some point like C in figure 2a. The lines cc and dd which are drawn through point C have slopes of 1 and  $-1/(n-1)$  respectively. The area to the right of line cc and to the left of line dd represents an area of higher profits, lower industry output and increased relative output of firm  $i$ .

constant and either causing an increase or a decrease in firm  $i$ 's relative output depending on whether  $\epsilon - \sigma$  is less than or greater than zero. In the former case profits rise and thus the strategic effect is positive whereas the reverse is true in the latter case.

### II.1c APPLICATIONS OF THE INDEPENDENCE RESULT

In Ziss (1989b) we determine that conditions guaranteeing independence are inherent in the: address models of horizontal product differentiation contained in Hotelling (1929) and Neven (1985) as well as the quality and/or advertising models contained in Economides (1989) and Grossman and Shapiro (1984). In both sets of models changes in capital serve to raise the sum of industry prices and yet reduce the relative price of the investing firm.<sup>27</sup>

In non-spillover R and D models the two conditions in (16) which guarantee independence cannot both hold since  $\epsilon = 0$ . As a result the sign of the strategic effect will depend critically on the degree of collusion. In particular, since increases in R and D serve to both expand industry output or lower aggregate prices and, increase the relative output or reduce the relative price of the investing firm, then the strategic effect effect will be positive provided firms price well above marginal cost as in the Cournot case considered by Brander and Spencer (1983) but, will be negative when firms price close to or at marginal cost as in the

<sup>27</sup>The aforementioned price changes are brought about by firms moving strategically towards the boundaries of the market in the former set of models and by reductions in product quality or advertising in the latter set of models.

Bertrand case considered in Lee (1986) or Dixon (1985).<sup>28</sup>

### II.2 PARTIAL INDEPENDENCE RESULTS

We now prove two sets of partial independence results which highlight the role played by divergences in the degree of competition across stages and by consistent conjectures in the second stage respectively. In the first (second) set of results we prove the strategic effect to be positive for a given level of capital independent of the actual level of competition but dependent only on whether the degree of competition in the second stage is greater than, less than or equal to competition in the other stage (the consistent level of competition in the second stage). In contrast to the independence result the partial independence results below involve only sufficient as opposed to both necessary and sufficient conditions for independence.

PROPOSITION 3 : (i) If output is the second stage variable, the zero strategic effect, as outlined in Proposition 5 does not hold, then (17a), (17b) and (17c) (with the inequalities reversed) represent three sets of sufficient conditions for which the strategic effect is positive (negative) (ii) If price is the second stage variable then the above partial independence result holds except with the words positive and negative interchanged.

<sup>28</sup>Furthermore the strategic effect will tend to be positive when the second stage is played competitively and negative when the second stage is played collusively. In the output case the strategic effect also becomes negative should the degree of R and D spillover, as modelled in d'Aspremont and Jacquemin (1988), be sufficiently high.

either (i)  $\epsilon - \sigma \leq 0, \epsilon \geq 0$  ( $\beta \leq 0$ )

(17a)  $\lambda > \rho, \sigma\beta - \epsilon\alpha \leq 0$  and

or (ii)  $\sigma + (n-1)\epsilon \geq 0, \epsilon \leq 0$

either (i)  $\epsilon - \sigma \geq 0, \epsilon \leq 0$

(17b)  $\lambda < \rho, \sigma\beta - \epsilon\alpha \leq 0$  and

or (ii)  $\sigma + (n-1)\epsilon \leq 0, \epsilon \geq 0, (\beta \geq 0)$

(17c)  $\lambda = \rho, \sigma\beta - \epsilon\alpha < 0$

where the term in brackets indicating the slope of  $\beta$  is not required *per se* but is implied by the other conditions.

Proposition 3, Corollaries 2c - 2f and all but case 6 of Proposition 5 are illustrated in figure 3 and together cover the whole range of competition in the two stages. The R and D and output competition model considered in Brander and Spencer (1983) is a special case of (17a) and (17c) and thus we show that a positive strategic effect occurs in these sorts of models for  $\lambda \geq \rho$  and not just for  $\lambda = \rho = 0$  as shown in Brander and Spencer (1983). Similarly we extend the R and D and price competition negative strategic effect result in Lee (1986) to include  $\lambda \leq \rho$  and not just  $\lambda = \rho = 0$ .

PROPOSITION 4 : (i) If output is the second stage variable, the zero strategic effect, as outlined in Proposition 5 does not hold, then (18a), (18b) and (18c) (with the inequalities reversed) represent three sets of sufficient conditions for which the strategic effect is positive (negative). (ii) If price is the

second stage variable then the above partial independence result holds except with the words positive and negative interchanged.<sup>29</sup>

(18a)  $\lambda > \rho, \rho \leq 0, \epsilon \leq 0$  and  $\sigma + (n-1)\epsilon \geq 0$

(18b)  $\lambda < \rho, \rho \geq 0, \epsilon \leq 0$  and  $\epsilon - \sigma \geq 0$

(18c)  $\lambda = \rho, \rho = 0, \epsilon < 0$

### III.3 ZERO STRATEGIC EFFECT

PROPOSITION 5 : Sufficient sets of conditions for which the strategic effect is zero are as follows

(19a) Case 1 :  $\lambda = \rho = -1/(n-1)$

(19b) Case 2 :  $\lambda = \rho = 1$

(19c) Case 3 :  $\lambda = -1/(n-1)$  and  $\sigma + (n-1)\epsilon = 0$

(19d) Case 4 :  $\lambda = 1$  and  $\epsilon - \sigma = 0$

(19e) Case 5 :  $\lambda = \rho$  and  $\sigma\beta - \epsilon\alpha = 0$

(19f) Case 6 :  $\lambda = \rho, \rho = \epsilon = 0$

(19g) Case 7 :  $\epsilon = \sigma = 0$

When  $\lambda = -1/(n-1)$  then firms are pricing close to marginal cost and thus invest strategically so as to increase aggregate prices or reduce aggregate output. In Cases 1 and 3 the strategic effect is zero because aggregate second stage variables remain constant. Aggregate second stage variables are left unchanged in

<sup>29</sup>Proposition 4, Corollaries 2c - 2f and all but Case 5 of Proposition 5 are illustrated in figure 4. Furthermore the extension of Economides (1989) contained in Ziss (1985) to allow quality to affect marginal production costs is a special case of both Propositions 3 and 4.

Case 1 because an increase firm i's capital is met by offsetting reductions in capital by rival firms and in Case 3 because an increase in the capital of firm i affects the marginal second stage profits of firm i and firm i's rivals in exactly offsetting fashion (e.g. advertising or quality as in Economides (1989)).

Similarly if  $\lambda = 1$  then firms act as joint profit maximizers in the second stage and thus use strategic investment so as to increase relative output. The conditions  $\rho = 1$  and  $\epsilon = \sigma$  guarantee that relative output stays constant in Cases 2 and 4 respectively, and thus that the strategic effect is zero.

In Case 5  $\sigma\beta - \epsilon\alpha = 0$  implies that a change in the capital of firm i has no effect on the second stage variable of firm j. As a result the ratio of rival to investing firm second stage responses in response to a strategic increase in firm i's capital is given by  $\rho$ , the conjectured response of rival to investing firm capital. The strategic effect is zero because  $\rho = \lambda$  where  $\lambda$  is equal to the ratio of rival to investing firm second stage responses which leaves profits constant.

In Case 6  $\lambda = g$  implies that firm i's second stage isoprofit contour is tangent to the reaction function of the rival firm. Thus firm i seeks to strategically invest so as to induce a shift of the rival reaction function. If  $\rho = \epsilon = 0$  then the strategic effect is zero because a change in the capital of firm i leaves firm j's reaction function stationary (as in the non-spillover R and D models of Brander and Spencer (1983) and Lee (1986)). As a result a strategic change in capital moves the equilibrium down the stationary reaction function of the rival and thus along

the same isoprofit contour of the investing firm.<sup>30</sup>

### III FUTURE RESEARCH

The next step is to derive particular expressions for  $\alpha$ ,  $\beta$ ,  $\sigma$  and  $\epsilon$  and thus determine whether the independence result or any of the partial independence results hold for a particular model. Second we could then determine the effect of collusion on the provision of capital by considering how both the strategic and non-strategic effects respond. For example we can use our analysis to examine the minimum/maximum product differentiation issue which arises in horizontal product differentiation models. The third step is to conduct a welfare analysis and to examine whether commitment improves or worsens welfare.

One interesting extension of our analysis is to examine strategic investment in the context of a principal-agent model in which each principal strategically commits to an incentive scheme for his agent before the rivaling agents engage in price or output competition. Examples of these sorts of principal-agent models are the owner-manager model contained in Sklivas (1987) and Fershtmann and Judd (1988), the government-domestic exporting firm model in Eaton and Grossman (1986), and the manufacturer-retailer model in Bonanno and Vickers (1988).

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<sup>30</sup>Case 7 is the trivial case in which the strategic effect is zero because capital has no effect on the second stage equilibrium.

## APPENDIX A : SHORTHAND NOTATION

Following Brander and Spencer (1985) we obtain the following second order condition for firm i when the conjectural variation term is non-zero

$$(A1a) \alpha \equiv D_{s_1 s_1}^i = \pi_{s_1 s_1}^i + (n-1)\lambda \pi_{s_1 s_j}^i + (n-1)\lambda \left[ \pi_{s_j s_1}^i + (n-1)\lambda \pi_{s_j s_j}^i \right] < 0$$

Substituting  $\pi_{s_1 s_j}^i = \pi_{s_j s_1}^i$  into (A1a) we obtain

$$(A1b) \alpha \equiv D_{s_1 s_1}^i = \pi_{s_1 s_1}^i + 2(n-1)\lambda \pi_{s_1 s_j}^i + [(n-1)\lambda]^2 \pi_{s_j s_j}^i < 0 \text{ (second order condition)}$$

$$(A2) \beta \equiv D_{s_1 s_1}^i = \pi_{s_1 s_j}^i + (n-1)\lambda \pi_{s_j s_j}^i = D_{s_j s_1}^j > 0$$

= effect of  $s_j$  on firm i's perceived marginal profitability of expanding  $s_1$

$$(A3) \sigma \equiv D_{s_1 k_1}^i = \pi_{s_1 k_1}^i + (n-1)\lambda \pi_{s_j k_1}^i = D_{s_j k_1}^j < 0$$

= effect of  $k_1$  on firm i's perceived marginal profitability of expanding  $s$

$$(A4) \epsilon \equiv D_{s_j k_1}^j = \pi_{s_j k_1}^j + (n-1)\lambda \pi_{s_1 k_1}^j = D_{s_1 k_1}^i < 0$$

= effect of  $k_1$  on firm j's perceived marginal profitability of expanding  $s_j$

## APPENDIX B : THE SLOPE AND SHIFT OF THE SECOND STAGE REACTION FUNCTION OF FIRM J

Following Kamien and Schwartz (1983) we totally differentiate the

first order condition of firm j with respect to all the second stage variables and  $k_1$  to obtain

$$(B1) \alpha ds_j + \beta ds_1 + (n-2)\beta ds_h + \epsilon dk_1 = 0$$

Setting  $ds_j = ds_h$  and  $dk_1 = 0$  and rearranging yields the slope of firm j's reaction function given by (6b). Similarly setting  $ds_j = ds_h$  and  $ds_1 = 0$  and re-arranging yields the shift of firm j's reaction function in response to a change in  $k_1$  given by (6c).

## APPENDIX C : COMPARATIVE STATICS

Following OFS (1987) we totally differentiate the first order conditions of firm i keeping all but firm i's capital constant. Having differentiated we then divide through by  $dk_1$  and use the notation given in the text to arrive at the following

$$(C1) \alpha s_1^i + (n-1)\beta s_1^j + \sigma = 0$$

We now totally differentiate the first order conditions of firm j keeping all but firm i's capital constant. Doing so we obtain the following

$$(C2) \alpha s_j^i + (n-2)\beta s_j^h + \beta s_j^j + \epsilon = 0$$

We now use the symmetry assumptions  $s_1^i = s_j^j$  and  $s_1^j = s_j^i = s_j^h$  and then use (C1) and (C2) to arrive at (5a) and (5b).

N.B. IN ALL SUBSEQUENT PROOFS WE SHALL ONLY PROVE THE CASE INVOLVING OUTPUT AND A POSITIVE STRATEGIC EFFECT. REMAINING CASES CAN BE PROVED IN SIMILAR FASHION.

## APPENDIX D : PROOF OF PROPOSITION 2 (SUFFICIENCY)

Substituting (5a) and (5b) into both the definition of  $\bar{s}_1^i$  and  $\lambda^*$  given by (10b) and (13c) respectively we come up with two possible sets of expressions given by

$$(D1a) \bar{s}_1^1 = \frac{\begin{matrix} (+) & (+) \\ [\sigma + (n-1)\epsilon] [\beta - \alpha] - (n-1) [1 - \rho] [\sigma\beta - \epsilon\alpha] \end{matrix}}{\begin{matrix} \Delta \\ (+) \end{matrix}}$$

$$(D1b) \lambda^* = \frac{-1}{n-1} + \frac{\begin{matrix} (+) & (+) \\ [\sigma + (n-1)\epsilon] [\beta - \alpha] [\rho + \frac{1}{n-1}] \end{matrix}}{\begin{matrix} \bar{s}_1^1 \Delta \\ (+) \end{matrix}}$$

OR

$$(D2a) \bar{s}_1^1 = \frac{\begin{matrix} (-) & (+) \\ [\epsilon - \sigma] [\alpha + (n-1)\beta] + [1 + \rho(n-1)] [\sigma\beta - \epsilon\alpha] \end{matrix}}{\begin{matrix} \Delta \\ (+) \end{matrix}}$$

$$(D2b) \lambda^* = 1 + \frac{\begin{matrix} (-) & (-) \\ [\epsilon - \sigma] [\alpha + (n-1)\beta] [\rho - 1] \end{matrix}}{\begin{matrix} \bar{s}_1^1 \Delta \\ (+) \end{matrix}}$$

(D1a) and (D1b) were obtained by adding and subtracting  $\beta\sigma + (n-1)\alpha\epsilon$  and  $\rho[\beta\sigma + (n-1)\alpha\epsilon]$  in the derivation of  $\bar{s}_1^1$  and  $\bar{s}_1^2$  respectively. (D2a) and (D2b) were obtained by adding and subtracting  $\alpha\epsilon + (n-1)\sigma\beta$  and  $\rho[\alpha\epsilon + (n-1)\sigma\beta]$  in the derivation of  $\bar{s}_1^1$  and  $\bar{s}_1^2$  respectively.

From (D1a) and (D2a) we can see that  $\bar{s}_1^1$  can either be positive or negative. We consider both cases. Suppose  $\bar{s}_1^1 > 0$  then since we have assumed output to be the second stage variable for the purpose of this proof we conclude that capital is tough.  $\bar{s}_1^1 > 0$ ,  $\sigma + (n-1)\epsilon \leq 0$  and (D1b) imply that  $\lambda^* \leq -1/(n-1)$  and thus that conjectures are always high provided either  $\lambda \leq -1/(n-1)$  or  $\sigma + (n-1)\epsilon \leq 0$  holds strictly. Thus tough investment implies high conjectures which together imply that the top dog effect occurs.

Now suppose  $\bar{s}_1^1 < 0$  and thus that investment is soft.  $\bar{s}_1^1 < 0$ ,  $\epsilon - \sigma \leq 0$  and (D2b) imply that  $\lambda^* \geq 1$  and thus that conjectures

are always low provided either  $\epsilon - \sigma \leq 0$  or  $\lambda \leq 1$  holds strictly. Thus soft investment has implied low conjectures which together both imply that the fat cat effect occurs. Thus we have shown that either the top dog effect or the fat cat effect occurs and thus that overinvestment always occurs. Q.E.D.

(NECESSITY) (BY CONTRADICTION)

suppose the independence result for overinvestment implies  $\sigma + (n-1)\epsilon > 0$ . Suppose  $\bar{s}_1^1 > 0$  (tough investment).  $\bar{s}_1^1 > 0$ ,  $\sigma + (n-1)\epsilon > 0$  and (D1b) imply that  $\lambda^* \geq -1/(n-1)$ . But  $-1/(n-1) \leq \lambda \leq \lambda^*$  implies that conjectures are low and thus that the lean and hungry look and underinvestment occurs which is a contradiction. Similarly we can use (D2b) to show that assuming  $\epsilon - \sigma > 0$  will also result in a contradiction.

#### APPENDIX E : PROOF OF COROLLARY 2A

##### Part 1 : output deterrence (by contradiction)

Assume that the independence result for overinvestment implies  $\bar{s}_1^1 \geq 0$ . Now suppose  $\bar{s}_1^1 > 0$  (investment is tough) then  $\lambda^* = \bar{s}_1^2/\bar{s}_1^1 \geq 0$  and thus  $\lambda < 0$  implies that conjectures are low. Low conjectures and tough investment imply the puppy dog ploy and underinvestment which is a contradiction. Q.E.D.

##### Part 2 : $\epsilon < 0$

Overinvestment and Proposition 2 imply  $\epsilon + (n-1)\sigma \leq 0$  and  $\epsilon - \sigma \leq 0$  with at least one of the two inequalities holding strictly. The above implies  $\epsilon < 0$ . Q.E.D.

N.B. ALL SUBSEQUENT PROOFS ARE SIMILAR TO PROOF OF PROPOSITION 2 AND INVOLVE THE FOLLOWING PROCEDURE.

- Determine whether the hypothesis implies  $\bar{s}_1^1 < 0$  or  $> 0$ .
- Show that (i)  $\bar{s}_1^1 < 0$  implies low conjectures and the fat cat

effect whereas (ii)  $\bar{s}_1^1 > 0$  implies high conjectures and the top dog effect.

- c) If it is not possible to sign  $\bar{s}_1^1$  then consider both parts of b).
- d) For Proposition 5 involving a zero strategic effect show that either that the comparative statics are zero or  $\lambda = \lambda^*$  (i.e. conjectures are neither high nor low).

#### APPENDIX F : PROOF OF COROLLARIES 2C-2F INCLUSIVE

##### Corollary 2c : $\lambda = 1$

Follows from (D2a) and (D2b). Both the fat cat and top dog effects occur. Q.E.D.

##### Corollary 2d : $\lambda = -1/(n-1)$

Follows from (D1a) and (D1b). Both fat cat and top dog effects occur. Q.E.D.

##### Corollary 2e : $\rho = 1$

Follows from (D1a) and (D2b). Fat cat effect occurs. Q.E.D.

##### Corollary 2f : $\rho = -1/(n-1)$

Follows from (D1b) and (D2a). Top dog effect occurs. Q.E.D.

#### APPENDIX G : PROOF OF PROPOSITION 3

Substituting (6a) into the definition of  $\lambda^*$  given by (13c) we obtain

$$(G1) \lambda^* = \rho + \frac{s_1^1 [1 - \rho] [1 + (n-1)\rho]}{\bar{s}_1^1}$$

Substituting (5a) and (5b) into the definition of  $\bar{s}_1^1$  (13c) we come up with the following two expressions

$$(G2a) \bar{s}_1^1 = \frac{\begin{matrix} (-) \\ -[\sigma + (n-1)\varepsilon] \left[ \alpha + \beta [n-2 - (n-1)\rho] \right] \\ \\ (-) \quad (+) \\ + (n-1)\varepsilon \left[ \alpha + (n-1)\beta \right] (1 - \rho) \end{matrix}}{\Delta}$$

OR

$$(G2b) \bar{s}_1^1 = \frac{\begin{matrix} (-) \quad (+) \quad (+) \\ [\varepsilon - \sigma] \left[ \alpha + \beta [n-2 - (n-1)\rho] \right] + \varepsilon [1 + \rho(n-1)] (\beta - \alpha) \end{matrix}}{\Delta}$$

where (G2a) and (G2b) were obtained from (13c) by adding and subtracting  $(n-1)\varepsilon \left[ \alpha + \beta [n-2 - (n-1)\rho] \right]$  and  $\varepsilon \left[ \alpha + \beta [n-2 - (n-1)\rho] \right]$  respectively.  $\alpha + \beta [n-2 - (n-1)\rho] < 0$  because of stability and because  $n-2 - (n-1)\rho$  is less than  $n-1$  in absolute value given the restrictions placed on  $\rho$ .

Case 1 :  $\sigma\beta - \varepsilon\alpha \leq 0$ ,  $\sigma + (n-1)\varepsilon \geq 0$ ,  $\varepsilon \leq 0$  and  $\lambda \geq \rho$

Follows from (G1), (G2a) and (5b). Top dog effect occurs.

Case 2 :  $\sigma\beta - \varepsilon\alpha \leq 0$ ,  $\varepsilon - \sigma \leq 0$ ,  $\varepsilon \geq 0$  and  $\lambda \geq \rho$

Follows from (G1), (G2b) and (5b). Top dog effect occurs.

Case 3 :  $\sigma\beta - \varepsilon\alpha \leq 0$ ,  $\varepsilon - \sigma \geq 0$ ,  $\varepsilon \leq 0$  and  $\lambda \leq \rho$

Follows from (G1), (G2b) and (5b). Fat cat effect occurs.

Case 4 :  $\sigma\beta - \varepsilon\alpha \leq 0$ ,  $\sigma + (n-1)\varepsilon \leq 0$ ,  $\varepsilon \geq 0$  and  $\lambda \leq \rho$

Follows from (G1), (G2a) and (5b). Fat cat effect occurs.

Case 5 :  $\sigma\beta - \epsilon\alpha < 0$  and  $\lambda = \rho$

Follows from (G1), (D1a), (D2a), and (5b). Both fat cat and top dog effect occur. Q.E.D.

APPENDIX H : PROOF OF PROPOSITION 4

Substituting (6a) into the definition of  $\lambda^*$  given by (13c) we obtain the following

$$(H1) \lambda = g + \frac{\rho s_1^1 [1 + g(n-1)] [1 - g] - \epsilon \left[ \frac{1 + (n-2)\rho - (n-1)\rho g}{\alpha + (n-2)\beta} \right]}{\bar{s}_1^1}$$

where  $1 + (n-2)\rho - (n-1)\rho g$  can be shown to be positive as a result of  $g$  and  $\rho$  having been restricted to take on levels between 1 and  $-1/(n-1)$ . We now re-write (5a) in one of two ways to arrive at the following

$$(H2a) s_1^1 = \frac{-[\sigma + (n-1)\epsilon][\alpha + \beta(n-2)] + (n-1)\epsilon[\alpha + (n-1)\beta]}{\Delta}$$

OR

$$(H2b) s_1^1 = \frac{[\epsilon - \sigma][\alpha + \beta(n-2)] + \epsilon(\beta - \alpha)}{\Delta}$$

where (H2a) and (H2b) were obtained from (5a) by adding and subtracting  $(n-1)\epsilon[\alpha + \beta(n-2)]$  and  $\epsilon[\alpha + \beta(n-2)]$  respectively.

Case 1 :  $\rho \geq 0, \lambda \leq g, \epsilon - \sigma \geq 0$  and  $\epsilon \leq 0$

Follows from (G2b), (H2b) and (H1). Fat cat effect occurs.

Case 2 :  $\rho \leq 0, \lambda \geq g, \sigma + (n-1)\epsilon \geq 0$  and  $\epsilon \leq 0$

Follows from (G2a), (H2a) and (H1). Top dog effect occurs.

Case 3 :  $\rho = 0, \lambda = g$  and  $\epsilon \leq 0$

Follows from (10b) (i.e.  $\bar{s}_1^1 = s_1^1$ ), (I2a) or (I2b) and (I1). Either the top dog or fat cat effect occurs. Q.E.D.

APPENDIX I : PROOF OF PROPOSITION 5

Case 1 :  $\lambda = \rho = -1/(n-1)$

Follows from (D1b) and (11b).  $\lambda = \lambda^* = -1/(n-1)$ .

Case 2 :  $\lambda = \rho = 1$

Follows from (D2b) and (11b).  $\lambda = \lambda^* = 1$ .

Case 3 :  $\lambda = -1/(n-1)$  and  $\sigma + (n-1)\epsilon = 0$

Follows from (D1b) and (11b).  $\lambda = \lambda^* = -1/(n-1)$ .

Case 4 :  $\lambda = 1$  and  $\epsilon - \sigma = 0$

Follows from (D2b) and (11b).  $\lambda = \lambda^* = 1$ .

Case 5 :  $\lambda = \rho$  and  $\sigma\beta - \epsilon\alpha = 0$

Follows from (G1), (5b) and (11b).  $\lambda = \lambda^* = \rho$ .

Case 6 :  $\rho = \epsilon = 0$  and  $\lambda = g$

Follows from (H1) and (11b).  $\lambda = \lambda^* = g$ .

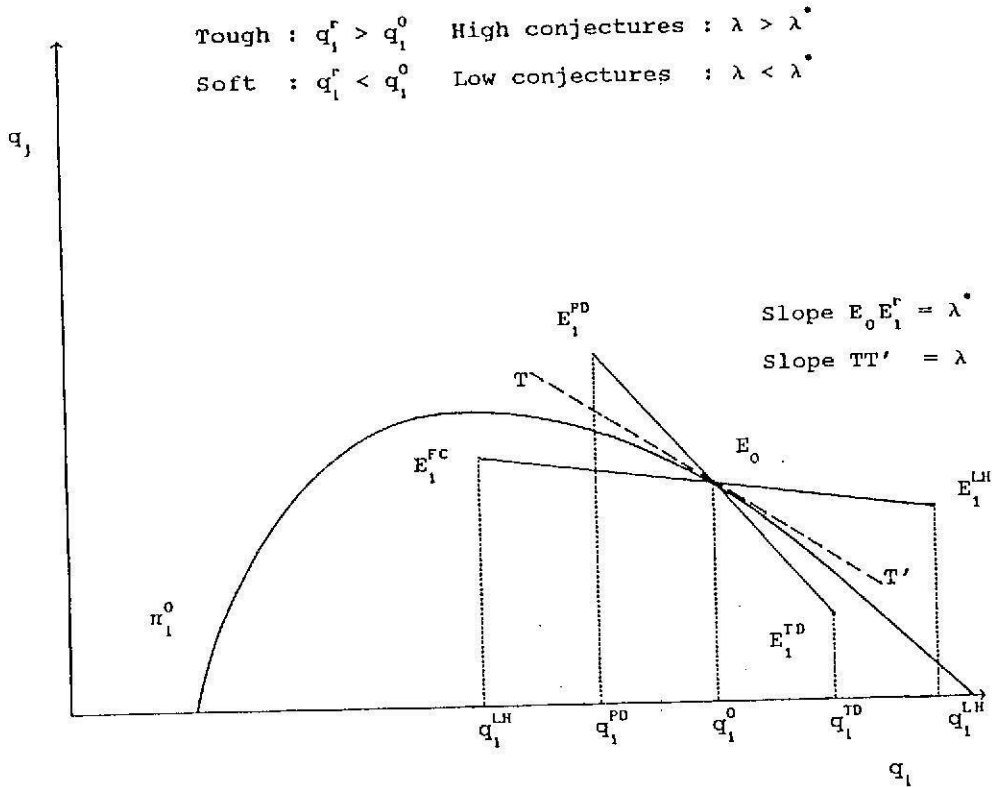
Case 7 :  $\epsilon = \sigma = 0$

$\epsilon = \sigma = 0$  implies that all the comparative statics are zero which in turn implies a zero strategic effect. Q.E.D.

FIGURE 1A : ILLUSTRATION OF PROPOSITION 1 (OUTPUT)

	investment	conjectures	strategic effect
TD = top dog effect	tough	high	positive
FC = fat cat effect	soft	low	positive
PD = puppy dog ploy	tough	low	negative
LH = lean and hungry look	soft	high	negative

Tough :  $q_1^r > q_1^0$  High conjectures :  $\lambda > \lambda^*$   
 Soft :  $q_1^r < q_1^0$  Low conjectures :  $\lambda < \lambda^*$

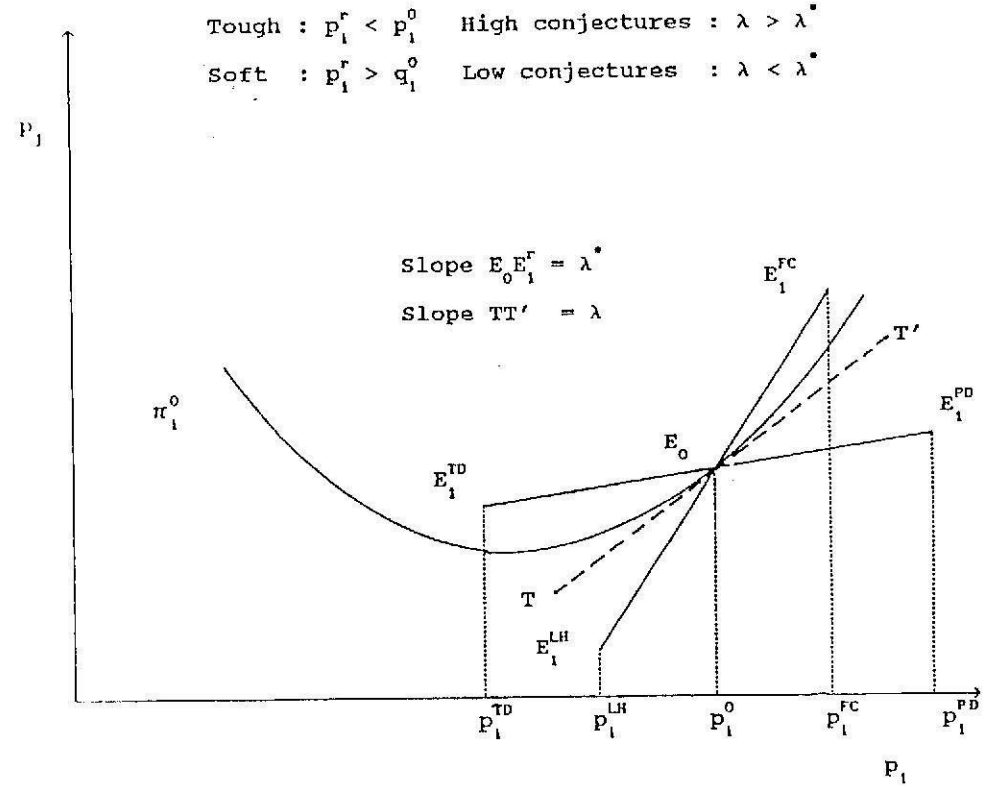


$E_0$  : initial (non-strategic) equilibrium  
 $E_1^r$  : equilibrium following an increase in  $k_1$  and the conjectured  $k_j$  response ( $r = TD, FC, PD, LH$ )

FIGURE 1B : ILLUSTRATION OF PROPOSITION 1 (PRICE)

	investment	conjectures	strategic effect
TD = top dog effect	tough	high	positive
FC = fat cat effect	soft	low	positive
PD = puppy dog ploy	tough	low	negative
LH = lean and hungry look	soft	high	negative

Tough :  $p_1^r < p_1^0$  High conjectures :  $\lambda > \lambda^*$   
 Soft :  $p_1^r > p_1^0$  Low conjectures :  $\lambda < \lambda^*$



$E_0$  : initial (non-strategic) equilibrium  
 $E_1^r$  : equilibrium following an increase in  $k_1$  and the conjectured  $k_j$  response ( $r = TD, FC, PD, LH$ )

FIGURE 2A : ILLUSTRATION OF PROPOSITION 2 (OUTPUT)

Point	$\lambda = \text{slope of tangent}$	Area of increased profit at margin (sufficient)	Occurs for all $\rho$ if
A	1	right of aa which is also the area where $q_i - q_j$ rises <sup>1</sup>	$\epsilon - \sigma < 0$
B	$-1/(n-1)$	left of bb which is also the area where $\sum q_i$ falls <sup>2</sup>	$\sigma + (n-1) < 0$
C	btwn 1 and $-1/(n-1)$	between cc and dd which is also the area where both $q_i - q_j$ rises and $\sum q_i$ falls	$\epsilon - \sigma < 0$ and $\sigma + (n-1) < 0$

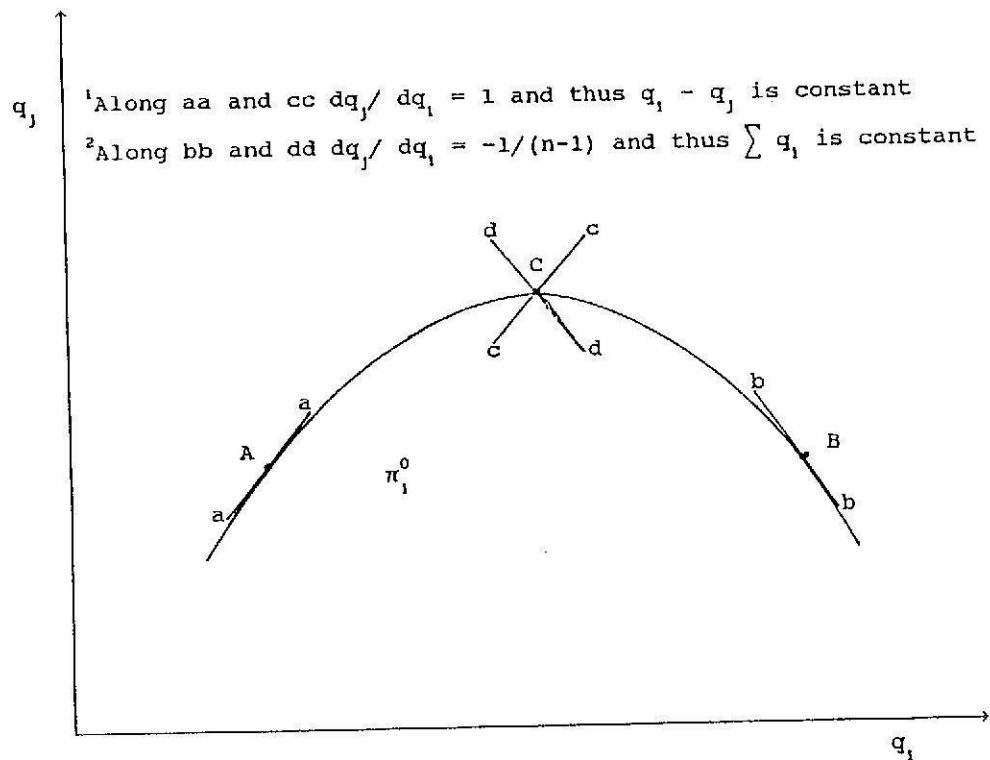


FIGURE 2B : ILLUSTRATION OF PROPOSITION 2 (PRICE)

Point	$\lambda = \text{slope of tangent}$	Area of increased profit at margin (sufficient)	Occurs for all $\rho$ if
A	1	left of aa which is also the area where $p_i - p_j$ falls <sup>1</sup>	$\epsilon - \sigma < 0$
B	$-1/(n-1)$	right of bb which is also the area where $\sum p_i$ rises <sup>2</sup>	$\sigma + (n-1) < 0$
C	btwn 1 and $-1/(n-1)$	between cc and dd which is also the area where both $p_i - p_j$ falls and $\sum p_i$ rises	$\epsilon - \sigma < 0$ and $\sigma + (n-1) < 0$

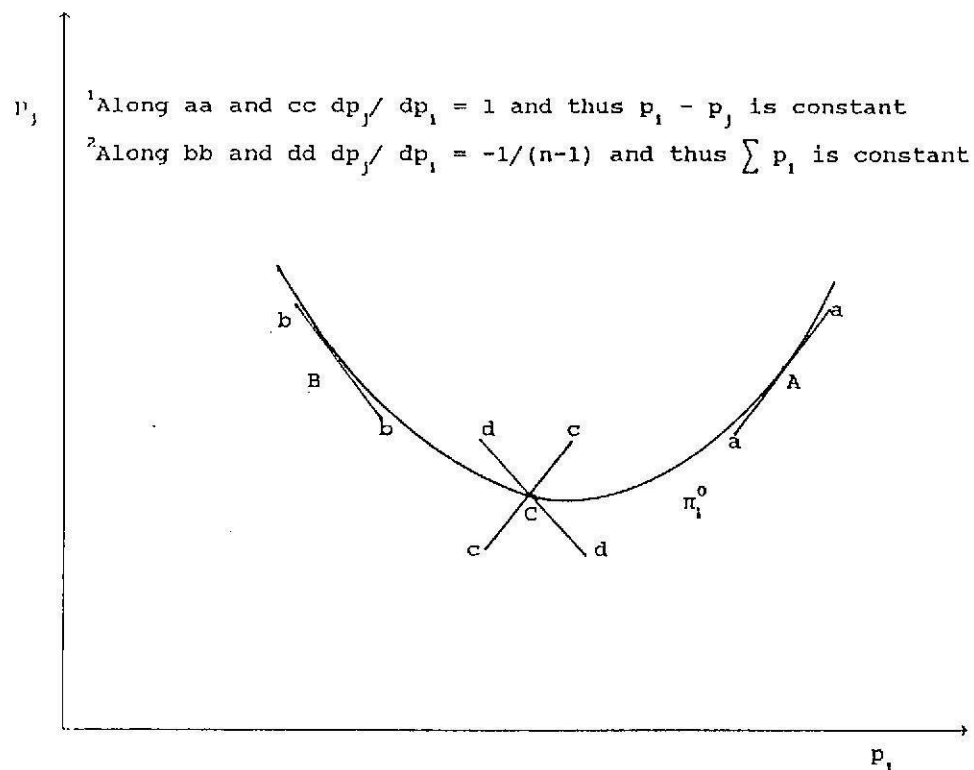


FIGURE 3 : ILLUSTRATION OF PROPOSITION 3, COROLLARIES 2C - 2F AND ALL BUT CASE 6 OF PROPOSITION 5

$\lambda$	$\frac{-1}{n-1}$		1
$\rho$			zero <sup>3</sup>
1		$\sigma + (n-1)\epsilon \approx 0$ either $\epsilon - \sigma \geq 0, \epsilon \leq 0^5$ $\sigma\beta - \epsilon\alpha \leq 0$ and or $\sigma + (n-1)\epsilon \leq 0,$ $\epsilon \geq 0^5 (\beta > 0)$	$\epsilon > 0$ $\epsilon < 0^2$
		$\sigma\beta - \epsilon\alpha < 0^4$ either $\epsilon - \sigma \leq 0, \epsilon \geq 0^5$ $(\beta \geq 0)$ $\sigma\beta - \epsilon\alpha \leq 0$ and or $\sigma + (n-1)\epsilon \geq 0, \epsilon \leq 0^5$	
$\frac{-1}{n-1}$	zero <sup>3</sup>	$\epsilon - \sigma \leq 0$	

- <sup>1</sup>If  $\sigma + (n-1)\epsilon = 0$  when  $\lambda = -1/(n-1)$  then the strategic effect is zero.
- <sup>2</sup>If  $\epsilon - \sigma = 0$  when  $\lambda = 1$  then the strategic effect is zero.
- <sup>3</sup>Indicates zero strategic effect.
- <sup>4</sup>Indicates that strategic effect is positive for  $\lambda = \rho$  provided  $\sigma\beta - \epsilon\alpha < 0$ . If  $\sigma\beta - \epsilon\alpha = 0$  when  $\lambda = \rho$  then the strategic effect is zero.
- <sup>5</sup>At least one of the 'either, or' inequalities must hold strictly or else  $\epsilon = \sigma = 0$  which would imply a zero strategic effect.

FIGURE 4 : ILLUSTRATION OF PROPOSITION 4, COROLLARIES 2C - 2F AND ALL BUT CASE 5 OF PROPOSITION 5

$\lambda$	$\frac{-1}{n-1}$	g	1
$\rho$			zero <sup>3</sup>
1		$\sigma + (n-1)\epsilon \leq 0$	$\epsilon > 0$ $\epsilon < 0^2$
		$\epsilon - \sigma \geq 0$ $\epsilon \leq 0^5$	
0		$\epsilon < 0^4$	$\epsilon > 0$ $\epsilon < 0^2$
		No Result	
$\frac{-1}{n-1}$	zero <sup>3</sup>	$\epsilon - \sigma \leq 0$	

- <sup>1</sup>If  $\sigma + (n-1)\epsilon = 0$  when  $\lambda = -1/(n-1)$  then the strategic effect is zero.
- <sup>2</sup>If  $\epsilon - \sigma = 0$  when  $\lambda = 1$  then the strategic effect is zero.
- <sup>3</sup>Indicates zero strategic effect.
- <sup>4</sup>Indicates that strategic effect is positive for  $\lambda = g$  and  $\rho = 0$  provided  $\epsilon < 0$ . If  $\epsilon = 0$  when  $\lambda = g$  and  $\rho = 0$  then the strategic effect is zero.
- <sup>5</sup>At least one of the inequalities must hold strictly or else  $\epsilon = \sigma = 0$  which would imply a zero strategic effect.

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