

WORKING PAPERS IN ECONOMICS

GENERAL EQUILIBRIUM COMPUTATION
APPLIED TO PUBLIC SECTOR ISSUES*

by John Piggott

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- * This paper has been prepared for inclusion in a volume of readings on aspects of public sector economics, edited by Paul Hare and to be published by Basil Blackwell, Oxford. The computations were done Jon Fuller. A version of the paper containing a FORTRAN program of the model used together with some sample data and results is available. I am grateful to Paul Hare and to other contributors for comments on earlier drafts. Financial support from the Reserve Bank of Australia and the Australian Research Grants Scheme is gratefully acknowledged.

I. INTRODUCTION

Over the last 30 years or so, economists have become increasingly aware of the limitations of the partial equilibrium model as a framework for analysing the economic effects of many public sector policies. While early writers on the subject, such as Ricardo, Wicksell and Walras, developed models of the public sector that were of a general equilibrium nature to consider the impacts of public sector policies, much of the analysis of the first half of the twentieth century was undertaken using partial equilibrium techniques.¹ In the 1950s, however, contributions in tax analysis by writers such as Rolph (1952) and Musgrave (1959), and in public expenditure analysis by Samuelson (1954), sought to cast the analysis of the incidence and efficiency effects of such policies within a general equilibrium framework once more. In Carl Shoup's (1969) happy phrase, economists had become aware that "a state of mind, if not the actual engine"² of general equilibrium analysis was required if public finance analysis was not to be misleading.

The rapidly increasing use of the general equilibrium model as a theoretical framework for public sector policy analysis posed a challenge to those economists concerned with the quantitative effects of alternative policies in actual economies. While a rigorous theoretical general equilibrium framework had been provided by Walras in his classic treatise "The Elements of Pure Economics", and had been formalised by Arrow and Debreu in the early 1950s, no corresponding quantitative techniques were available to make the general equilibrium model operational.

This essay focuses on a research field which has been developed to meet this challenge. Known variously as applied general-equilibrium analysis (AGE), computable general-equilibrium analysis (CGE), and sometimes numerical general equilibrium analysis, it involves the construction of multi-dimensional numerically specified general-equilibrium models and their application to public sector issues. As a broad research program, AGE can probably already be counted a success. Models have been constructed covering a number of countries, including the U.K. and U.S., and government agencies are already beginning to use such models to aid in policy formulation on a regular basis. In this sense, AGE modelling can be seen as the microeconomic analogue of the macroeconometric models which are now used as a matter of course by policymakers. In drawing this analogy, however, it should be emphasised that AGE models have no forecasting role, but rather provide "numerical insight" into the impacts of policy alternatives.

While standing as a research field in its own right, AGE analysis owes much for its approach to the work of Arnold Harberger, who was the first economist to undertake a numerical general equilibrium analysis of tax policy. By adapting earlier general equilibrium model formulations which had been applied to international trade, Harberger (1962, 1966) developed a 2 factor (capital and labour) 2 industry (incorporated and unincorporated), general equilibrium model that could be parameterised and solved in terms of factor and expenditure shares, and elasticities of substitution (in production) and of demand (in consumption). This model, which is nowadays described routinely in advanced public economics texts, can be given a geometric representation (see section II). In many ways, it

provides the intuitive basis for all subsequent numerical general-equilibrium analyses of public sector policies.³ Because it is accessible elsewhere, however, I will be referring to the Harberger model only briefly, in section II, for comparative and pedagogic purposes.

This contribution has three related aims. The first is to provide insight into the nature of the general equilibrium interactions flowing from public sector policy. Secondly, I hope to give the reader an appreciation of what is involved in building and applying an applied general equilibrium model of a national economy and public sector. Thirdly, I wish to illustrate the use of these models in policy evaluation. Results from models of the U.K. and U.S. economies are discussed to try to demonstrate the ways in which these models contribute to the policy formulation process. Each of the next three major sections of the paper are devoted, respectively, to these three goals. A concluding section summarises the strengths and limitations of the AGE approach, and indicates possible future research directions.

In writing this paper I have inevitably drawn on previous expositions of the field. The excellent surveys by Shoven (1983) and Shoven and Whalley (1984) are especially acknowledged. These articles, together with the monographs on the U.K. and U.S. models [Figgott and Whalley (1985a) and Ballard et al. (1985)], offer additional detail for the reader seeking further information and enlightenment. Further papers

are conveniently collected in two conference volumes, edited by Scarf and Shoven (1984) and Piggott and Whalley (1985b).

The research covered represents the central core of AGE analysis of taxation policy. It is not, however, entirely comprehensive. AGE models with alternative structures and numerical specification procedures have been built and are used for public sector policy evaluations, and models similar to those examined in this essay have been applied to policy issues other than taxation - in particular, trade policy. Reference will be made to these other projects where appropriate, and papers reporting these alternative approaches are included in the conference volumes mentioned above.

II. GENERAL EQUILIBRIUM AND TAXATION - A SIMPLE DIAGRAMMATIC ANALYSIS.⁴

The central idea underlying general equilibrium analysis of tax policy is that in order to evaluate the effects of changing a major tax, important economy-wide effects must be taken into account. Taxes distort the allocation of resources in the marketplace by causing resources to be used where productivity is lower than elsewhere and commodities to be offered to consumers at tax distorted prices adversely affecting consumer choice.

The basic analytics of general equilibrium tax models can be illustrated with the aid of diagrams depicting a two-factor, two-product, perfectly competitive economy, with fixed aggregate factor supplies, and consumers, who will be initially assumed to have identical, homothetic preference functions. In the absence of

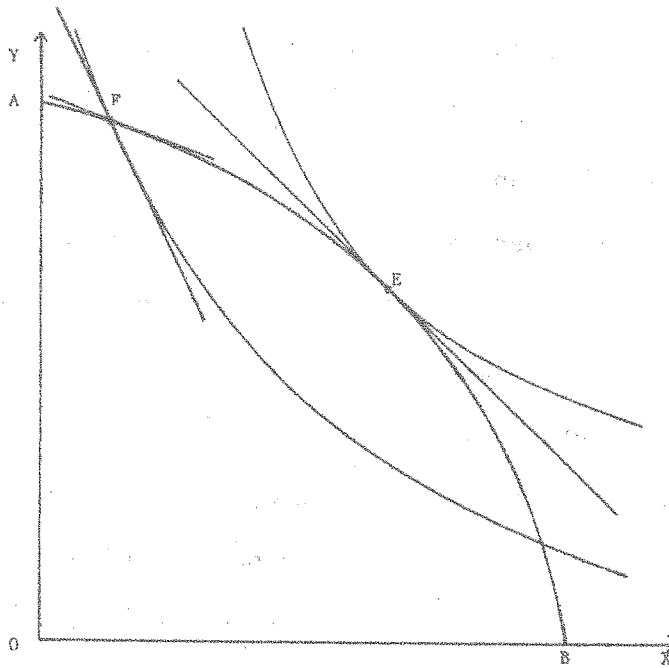
externalities and government interventions, such an economy will, in equilibrium, satisfy all the marginal conditions required for a Pareto optimal allocation. With distorting taxes this will no longer be true.

Figure 1 depicts the simple case of a consumption tax on product X in a two-commodity economy. I consider a single consumer who selects from alternative combinations of the two goods X and Y represented by the economy's production possibilities frontier. The revenues from the tax are returned in lump sum form to the single consumer. Productive efficiency is not affected since the economy remains on the production possibility frontier AB, but the product mix is altered by the tax. The tax produces a distortion between the marginal rate of transformation (MRT) (the net of tax price ratio facing producers) and the marginal rate of substitution (MRS) (the gross of tax price ratio facing consumers). In an equilibrium in the presence of the tax (point F), consumers adjust their purchases so that their marginal rate of substitution equals the gross of tax commodity price ratio. The consumer price ratio exceeds the ratio of prices received by producers since these are net of taxes. The distorted equilibrium at point F corresponds to a lower indifference curve than that associated with the undistorted equilibrium at point E. Because $MRS \neq MRT$ the allocation of resources corresponding to point F cannot be Pareto optimal.

A consumer tax which induces a move along the production possibility frontier from E to F may also have other effects beyond those represented in the diagram. One issue frequently analyzed is the incidence of such a tax. In the two-sector framework, this usually

FIGURE 1

SIMPLE ANALYSIS OF A DISTORTING
CONSUMER TAX ON GOOD X



involves functional incidence analysis: the impact on the income return to the two factors, capital and labour. The functional incidence of the tax in Figure 1 will depend upon the relative factor intensities of the two industries and cannot be analyzed simply by the same diagram. A proposition from the literature on the Harberger model (see Mieszkowski [1969]) is that the factor which is relatively intensively used in the taxed industry will bear some of the burden of the tax; whether it will fully bear the tax burden, however, depends on the parameters of demand and production functions.

The two sector framework represented in Figure 1 is not confined to analyzing tax distortions of choices by consumers among commodities, as one ordinarily thinks of them. Any household decision distorted through the tax system can be fitted into this framework.

Distortions of household choices between labour and leisure, and between present and future consumption can be analyzed analogously.

A similar framework can also be used to analyze tax distortions of production decisions. Figure 2 contains an Edgeworth box diagram showing the effects of a tax distortion on the production side of the economy induced by a tax on the use of one factor in one industry. We consider a tax on the use of capital in industry X. Because of the tax, the marginal rate of substitution between factors will differ between industries. In Figure 2, A is a point on the efficiency locus where the marginal rates of substitution between factors are equalized, while B is a point where, because of the tax on capital use in X, the marginal rates of substitution differ across industries. In this case the economy will not operate on its production possibility frontier in the presence of the tax, and

Pareto optimality is not attained. Either labour or capital can bear the burden of the tax, depending on the factor intensities and substitution possibilities in the two industries.

The indifference curve depicted in Figure 1 can only represent a single consumer, or a group of consumers with identical, homothetic preferences. In general, differences in preferences and endowments will provide the potential for gains from trade between consumers. Figure 3 depicts an economy with two groups of consumers, A and B, with an allocation of X and Y consistent with a single point on the production possibility frontier of Figure 1. The endowment point is denoted by E, a Pareto efficient equilibrium by H, and a distorted equilibrium by G. Here, the distortion is introduced by tax-distorted relative prices facing consumers - for example, a progressive income tax, which taxes the rich at a rate different from the poor.

So far, the analysis of this section has been purely qualitative. This diagrammatic discussion concludes with a brief discussion of the Harberger approach to quantifying the general equilibrium economic effects of public sector policy.

The Harberger model is based on standard neo-classical assumptions. He assumes fixed aggregate factor supplies; perfect factor mobility between industries; two factors and two products; perfect competition in factor and product markets; a closed economy (no foreign trade); linear homogeneous production functions; and a one-distortion economy.

FIGURE 2

SIMPLE ANALYSIS OF A FACTOR TAX DISTORTION

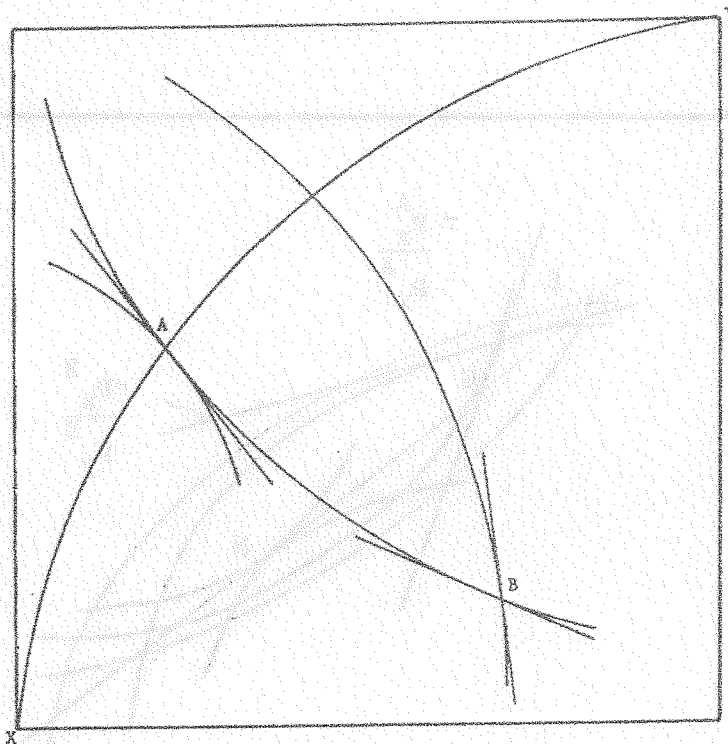
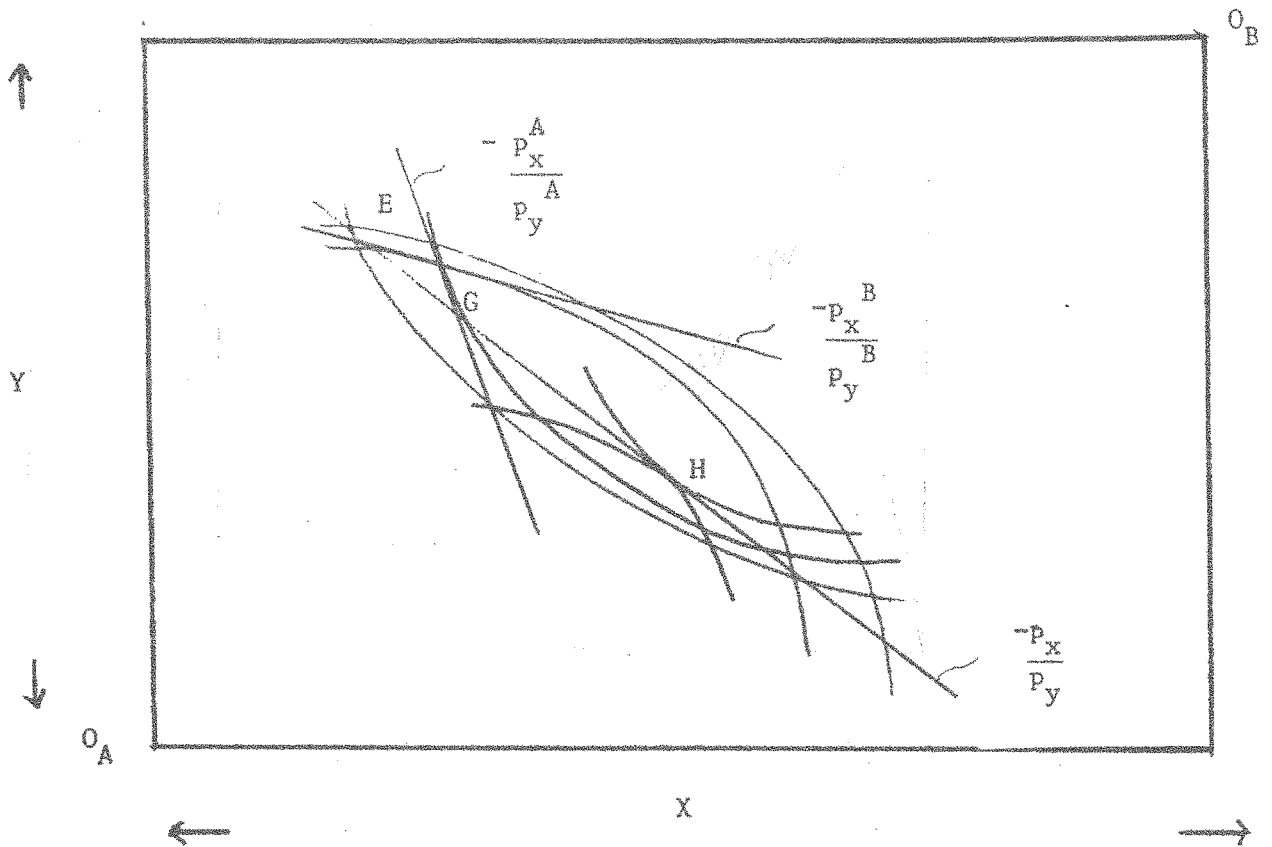


FIGURE 3

A SIMPLE ANALYSIS OF DIFFERENTIAL CONSUMPTION TAXES

This model, with particular values of elasticities of substitution in production and demand, is able to generate estimates of the incidence and efficiency effects of particular taxes. The model is represented by a reduced form of three equations, and changes in tax rates are evaluated through the model. Strictly speaking, only infinitesimally small changes in taxes and their comparative static impacts upon other variables can be considered, and for discrete changes the analysis provides only approximate results. Perhaps the most famous numerical finding from this model is that the US corporate tax is borne by all capital owners, whether or not their capital is used in incorporated enterprises.

Harberger also develops a procedure for estimating the size of the welfare cost of a distortionary factor tax. In the case of capital taxation distortions, he considers the economy to be represented by two sectors, "heavily taxed" and "lightly taxed". These are labelled sectors X and Y in Figure 4. Each sector uses capital in production, and marginal revenue product schedules are assumed to be linear (a local approximation). The economy has a fixed capital endowment. In the absence of any taxes, market forces will ensure that capital is allocated between the two sectors such that the rate of return r in each is equalised, and the capital endowment is fully employed. If, instead, a tax on capital income in sector X operates, the gross rate of return r_g in that sector must be such that the net rate of return r_n is equalized across the sectors, and capital is again fully employed. The difference between r_g and r_n is the tax on each unit of capital utilized in sector X.

In Figure 4, the tax and no tax situations are characterized by the capital allocations K_{x1} , K_{y1} , and K_{x0} , K_{y0} respectively. The area ABEF represents the loss in output in sector X when K_x decreases from K_{x0} to K_{x1} as the tax is imposed. GHIJ represents the increase in output of sector Y. Full employment guarantees that $K_{x0} - K_{x1} = K_{y1} - K_{y0}$. The area FECD (= ABEF-GHIJ) represents the efficiency cost of the tax, L, and is given by

$$L = 1/2 T \cdot \Delta K_x$$

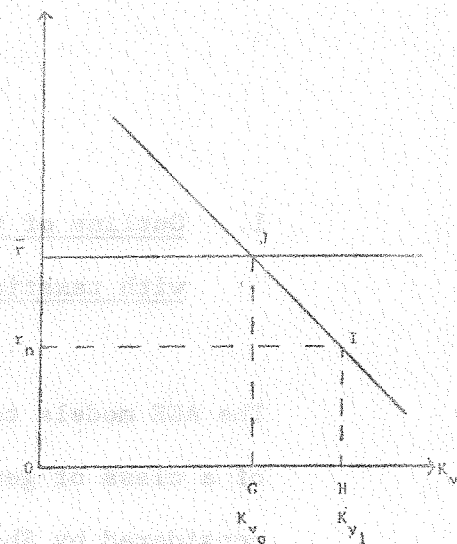
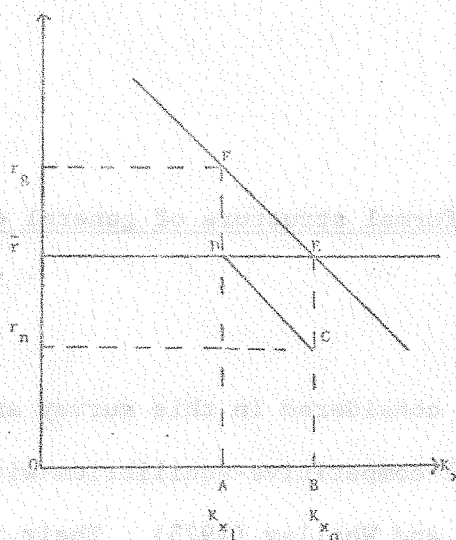
where T represents the tax distortion $r_g - r_n$, and K_x is the change in capital use in sector X from the removal of the tax. Harberger finds a solution for K_x by solving a system of three reduced form equations describing the local behaviour of the two sector general equilibrium model. This form of calculation has been generalized by Harberger (1964) into an extension of the famous welfare loss formula due to Hotelling (1938). In spite of the simplicity of the procedure, a number of difficulties are immediately apparent, not the least of which is the reliance on local approximations when the large changes often associated with tax distortions are being analyzed. In addition, it is unclear how the Harberger procedure can be applied where several distortions simultaneously operate and change together. The ACE approach, which we are about to discuss in detail, helps to overcome these difficulties.

FIGURE 4

HARBERGER'S TWO-SECTOR ANALYSIS OF EFFICIENCY IMPACTS
OF DISTORTIONARY CAPITAL TAXATION

Sector x
(Heavily taxed)

Sector y
(Lightly taxed)



III. BUILDING ON AGE MODEL

In this section, I begin by offering an overview of the basic structure of AGE models. I then outline the procedures used for building and solving them. Finally I work through the construction of a simple AGE model in detail.

1. Outline of the formal structure of general equilibrium models with taxation

The AGE models to be considered in this survey are specific examples of a class of general-competitive-equilibrium-with-taxation models considered by Shoven and Whalley (1973). Their formal structure is most easily presented within an activity analysis framework. In this class of models, producers are assumed to maximise profits and market demand functions are assumed to exist which are non-negative, continuous and homogeneous of degree zero in prices and tax revenue. The economy produces N commodities with a technology comprising M productive activities which can be operated at non-negative intensity, and comprises Q consumers. The symbol a_{ij} refers to the per unit use of the i^{th} good in the j^{th} activity. A producer tax system may then be represented by a matrix of tax coefficients, of which the typical element t_{ij} is the ad valorem tax rate applying to the use of the i^{th} commodity in the j^{th} activity. The consumer tax system also comprises a matrix of tax coefficients, of which the

typical element e_{iq} is the ad valorem tax rate associated with the purchase of the i^{th} good by the q^{th} consumer.

The quantities of each commodity demanded by all the consumers in the economy are represented by the market demand functions $\xi(\pi, R)$, where π is the vector of prices faced by consumers before payment of consumer taxes, and R denotes total tax revenues. R enters the demand functions because it is part of consumers and government income. The market demand functions must also satisfy Walras's Law, which in this context may be written:

$$\sum_{i=1}^N \pi_i \xi_i(\pi, R) + \sum_{q=1}^Q \sum_{i=1}^N e_{iq} \pi_i \xi_i(\pi, R) = \sum_{i=1}^N \pi_i w_i + R \quad (1)$$

where w_i is the initial endowment of the i^{th} commodity.

For this class of models, Shoven and Whalley have shown that an equilibrium will exist.⁵ It may be interpreted as the result of competitive processes where rival producers try to undercut each other so long as they do not make losses on any productive activity, and market prices adjust to eliminate any discrepancy between market demands and supplies.

A competitive equilibrium for this model is defined as a non-negative vector $P^* = (\pi^*, R^*)$ and a vector of non negative activity levels x^* such that

(a) demand equals supply for all commodities:

$$\xi_i(\pi^*, R^*) = w_i + \sum_{j=1}^M a_{ij} x_j^* \quad (i = 1, \dots, N) \quad (2)$$

(b) no activity makes positive profits after payment of taxes with those in use just breaking even:

$$\sum_{i=1}^N \pi_i^* a_{ij} - \sum_{i=1}^N \pi_i^* a_{ij} t_{ij} \leq 0 \quad (j = 1, \dots, M) \quad (3)$$

with strict equality if $x_j > 0$.

At an equilibrium, Shoven and Whalley (1973) show that the revenue disbursed to consumers or retained by the government equals that generated on the production side of the economy plus that collected on the demand side:

$$R^* = \sum_{j=1}^M \sum_{i=1}^N p_i^* t_{ij} a_{ij} x_j^* + E^* \quad (4)$$

where E^* is the value of consumer tax revenues.

The models discussed in this paper are of a less general form than that outlined above, in that a partition is drawn between factors of production and produced commodities. Factors of production are not

producibile, and their initial endowments in the economy must therefore be fixed in some sense. On the other hand, there are no initial endowments of produced commodities.

This is the case for three reasons. First, much of the traditional literature in public finance is cast in terms of such a partition. Second, national accounts and related data are organised around a partition between factors and produced goods, so that empirical implementation of the model follows far more naturally if this same partition is embodied in the model structure. Third, such a partition, when combined with constant returns to scale production functions, allows the search for equilibrium prices to be confined to factor prices, thus dramatically simplifying solution.

It should also be noted that in AGE models continuous "value added" production functions are used, instead of the activity analysis approach, which requires an explicit articulation of a number of specific productive processes. As well, demand functions are derivable by utility maximization from well defined utility functions. This implies that the assumptions on demand functions mentioned above must necessarily hold. The activity analysis approach is presented in detail in Piggott and Whalley (1985a) and summaries are also provided in Shoven (1983) and Shoven and Whalley (1984).

2. The structure of a partitioned AGE model.

We can imagine the structure and operation of partitioned AGE models as n-dimensional extensions of the two sector model already presented

geometrically. In one sense, all AGE amounts to is the numerical specification of the isoquants, indifference curves, endowments, taxes, and equilibrium conditions which are represented in figures 1-3.

Figure 5 represents the structure of a more detailed AGE model of the partitioned type. It allows us to make explicit the requirements which must be met for its implementation. Assuming for a moment a timeless fixed factor closed-economy model, these requirements are as follows:

(i) A list and description of agents, factors, produced goods and policies to be considered in the model must be provided.

(ii) The endowments of consumers must be numerically specified in units of capital and labour, together with their tax revenue receipt entitlements.

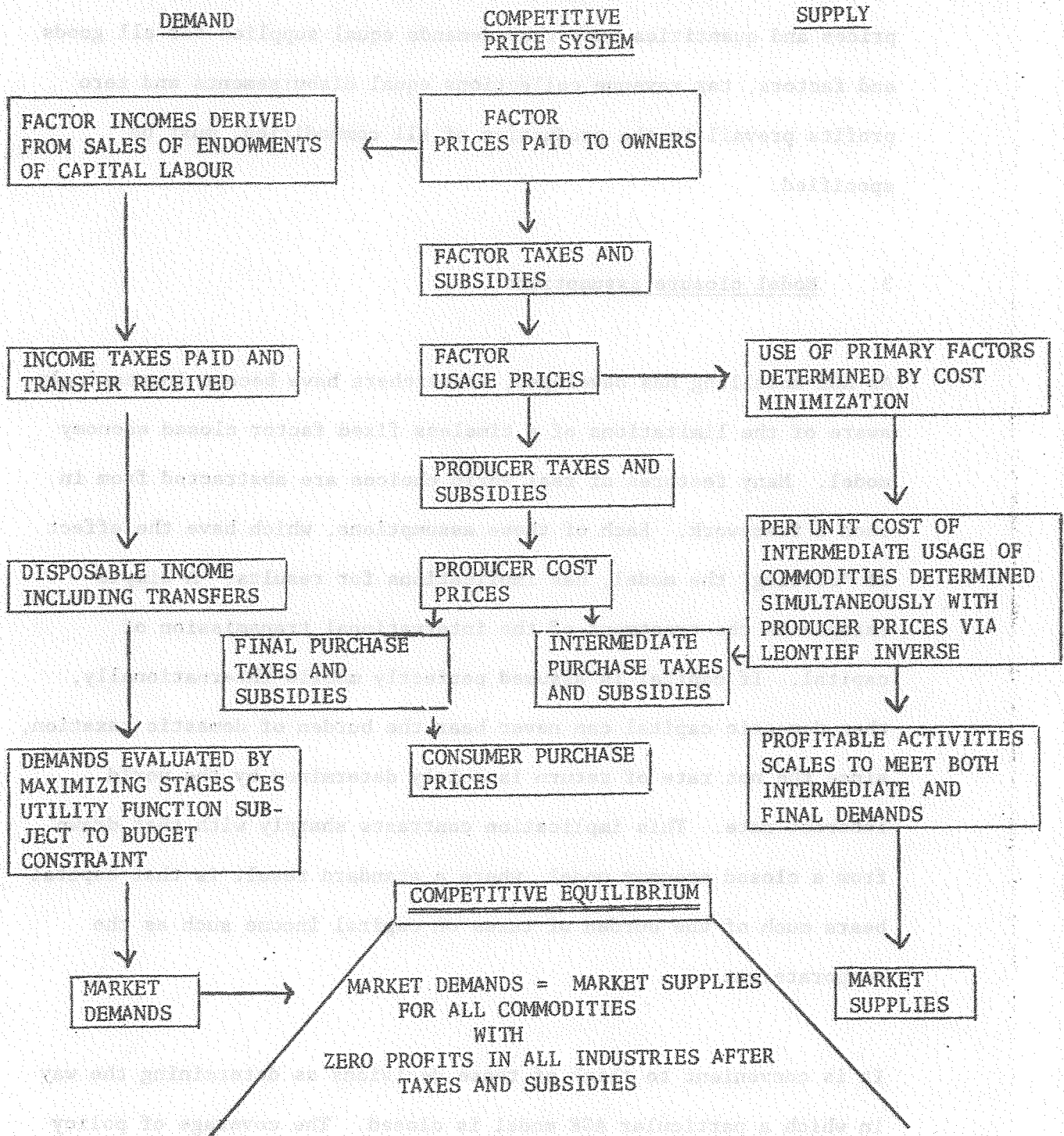
(iii) Some form of functional representation must be chosen to describe the behaviour of the various agents (consumers and producers, for example).

(iv) Parameter values must be provided for the functional representations. Items (iii) and (iv) provide a specification of preferences and the production technology.

(v) A complete description of policy parameters must be provided in model admissible form (e.g., ad valorem tax and subsidy rates).

FIGURE 5

STRUCTURE OF A TYPICAL APPLIED GENERAL EQUILIBRIUM
TAX MODEL



(vi) An equilibrium concept, given by a set of (equilibrium) prices and quantities such that demands equal supplies for all goods and factors, tax revenue collections equal disbursements and zero profits prevail in the production of all commodities, must be specified.

3. Model closure assumptions

As AGE modelling has developed, researchers have become increasingly aware of the limitations of a timeless fixed factor closed economy model. Many features of real world choices are abstracted from in such a framework. Each of these assumptions, which have the effect of "closing" the model, has implications for results. A simple example is the treatment of the international transmission of capital. If capital is assumed perfectly mobile internationally, then domestic capital can never bear the burden of domestic taxation, since the net rate of return is solely determined by the world interest rate. This implication contrasts sharply with that drawn from a closed economy model, where a standard result is that capital bears much of the burden of taxes on capital income such as the corporate tax.

It is convenient to think of these decisions as determining the way in which a particular AGE model is closed. The coverage of policy instruments, the treatment of the foreign sector, the operation of the public sector, the modelling of leisure and household production, and the treatment of uncertainty all require closure decisions of one kind or another. In intertemporal variants of AGE models, the

assumptions made about expectations represent a form of closure. The choice of closure assumptions will reflect the prior views of the model builder as well as the purposes for which the model has been constructed. They can have profound implications for model outcomes, and it is therefore very important for the model builder to consider his options carefully when deciding between alternative strategies for closing the model. Equally, it is important for policymakers to be aware of the closure assumptions made when interpreting the results of model simulations for policy assessment.

4. The Choice of Functional Forms for the Behavioural Equations in the Model

Before discussing the estimation of parameter values for the model equations, the considerations involved in the choice of particular functional forms for demand and production functions should be explained. The functional form used in the model must be consistent with the basic model assumptions, and the maximising responses of agents must be simple enough to make repeated solution in the sequences of calculations involved in equilibrium computations feasible. Tractable functional forms, therefore, must obviously be used to describe behaviour patterns of both producers and consumers. Household utility maximization problems must be readily soluble, as must industry cost minimization problems.

Inevitably, in practice the well-known family of convenient functional forms provides the candidate specifications for general equilibrium policy models of the type used here. Demand and cost functions derived from Cobb-Douglas, Stone-Geary, and CES (either

single stage or nested) utility and production functions tend to be used. More complex variants (such as Generalized Leontief functions) may also be considered although such functions raise more difficult estimation problems and substantially increase execution times required for equilibrium calculations. The choice of CES functions in many studies reflects the trade-off between complexity (and hopefully added realism) and tractability.

5. Alternative Approaches to Parameter Selection

Utility and production functions yield household demand and industry cost functions which depend upon all prices. In a reduced form of the model represented by interdependent excess demand functions, conventional estimation procedures are difficult to implement. For the parameters of any particular structural equation to be identified a large number of excluded exogenous variables or other identifying restrictions are required. This makes identification of all equations in the model by conventional methods (such as zero or other parameter value restrictions) impractical. For most AGE models, therefore, comprehensive econometric estimation is abandoned.

Instead, numerical specification proceeds by means of "calibration", and I will confine attention to this approach here.

The fundamental assumption made in 'calibrating' the model is that the economy is in equilibrium in a particular year. By modifying the National Accounts and other blocks of data for that year, a data set is generated in which all equilibrium conditions inherent in the model are satisfied. This is termed a 'benchmark equilibrium' data set. The requirement that the set of parameter values used in the

model be capable of replicating this 'observed equilibrium' as an equilibrium solution to the model is then imposed as a restriction in the process of parameter selection.

Parameter values are determined in a non-stochastic manner by solving the equations which represent the equilibrium conditions of the model. I use the data on prices and quantities which characterize the benchmark equilibrium. Whether the observed equilibrium alone is sufficient to uniquely determine the parameter values depends upon the functional forms used. For example, Cobb-Douglas functions imply constant shares. The benchmark equilibrium data set, which contains equilibrium share observations, can therefore be generated by only one set of Cobb-Douglas functions. In using CES production and demand functions extraneous estimates of elasticities of substitution (which are unit free) are incorporated into the procedure, serving together with the equilibrium replication requirement as identifying restrictions on the model. CES functions are used in the example constructed in section 8.

6. The Construction of a Benchmark Equilibrium Data Set

The assumption that an economy is in equilibrium implies that the model equilibrium conditions must be satisfied in any data used to determine parameter values. In a benchmark equilibrium data set all equilibrium conditions are satisfied. Demands equal supplies for all goods and factors, and non-positive profits are made in all industries.

In order for these equilibrium conditions to be satisfied by the data, various adjustments are necessary. The blocks of data used are available separately in National Accounts and related sources but not arranged on any synchronized basis. These data sets are modified so as to become mutually consistent. Table 1 presents an example of a simplified benchmark data set for a 2 factor, 2 commodity, 2 consumer model in which the equilibrium conditions are satisfied; demands equal supplies and zero profit conditions are satisfied by industry. This data set is used in sections 8 and 9.

In higher dimensional models, the data requirements for benchmark equilibrium data sets are extensive and a substantial amount of work is involved in constructing such a set. Data are needed on the use of productive inputs by industry, including the use of both domestic and imported commodities. On the demand side information is required on the expenditure patterns of different consumer groups, and on the composition of their incomes. To incorporate the public sector into the model, data are required on tax revenues, tax payments by agent, subsidy receipts, and transfers; and all as they relate to each producer, commodity, and consumer. Data on government expenditures on goods and services and foreign trade are also required.

The sources for these data are generally the publications or unpublished records of government agencies and the methods of manipulation and adjustment are frequently complex. Adjustments are needed for a number of reasons, such as unsuitable stock flow distinctions appearing in most published accounts, incomplete detail on taxes and subsidies, differences of definition between source

TABLE 1
A SIMPLE BENCHMARK EQUILIBRIUM DATA SET

		A. PRODUCTION				
COMMODITY	1	2	3	4	5	
	VALUE OF CAPITAL SERVICE	TAX ON CAPITAL USE	VALUE OF LABOUR SERVICE	TAX ON LABOUR USE	COST OF PRODUCTION [(1) + (2) + (3) + (4)]	
1	20	20	60	30	130	
2	20	10	25	5	60	
	40	30	85	35	190	
		B. CONSUMPTION				
COMMODITY	6	7	8	9	10	
	COST OF PRODUCTION	OUTPUT TAX	AGGREGATE CONSUMPTION OUTLAYS [(6) + (7)]	CONSUMER 1'S PURCHASES	CONSUMER 2'S PURCHASES	
1	130	65	195	135	60	
2	60	0	60	20	40	
	190	65	255	155	100	
		C. SOURCE OF INCOME				
COMMODITY	11	12	13	TOTAL [(11) + (12)]		
	CONSUMER 1	CONSUMER 2				
	30	10	40			
	60	25	85			
	65	65	130			
	155	100	255			

materials and concepts appearing in the model, and classification incompatibilities between the model and basic data sources.

The whole tax-subsidy system for any modern market-type economy can be fitted into this conceptual framework. Factor taxes can be identified which distort the use of factors by industry and commodities for intermediate purposes by industries. The income tax system can be modelled along with the transfers by government to persons. Subsidies can be incorporated, although modelling these usually presents major data problems as well as conceptual difficulties in defining what any particular subsidy actually is. Within the income tax the treatment of housing, progressivity in the tax rates, the absence of taxation of leisure, the tax treatment of saving, and other characteristics of the tax can be represented.

Use of these models to analyse the tax system in this way inevitably involves judgement as to the appropriate way to treat particular taxes. There is frequently active debate among public finance specialists as to the true character of a particular tax. The property tax, for instance, has been long debated as either an excise or a factor tax; the corporate tax has been alternately treated as a straight forward partial-factor tax, a tax on particular financing instruments available to firms, or as a lump sum tax. The social security tax is not regarded by some as a tax, but rather as an insurance premium. None of these debates is resolved by the particular model treatment adopted; they are merely truncated. A further point which should be emphasized is that other distortions in the economy are usually not simultaneously modelled in these exercises. Minimum wage controls, monopolies, and distortions from

the expenditure side of government policy, including the welfare system and unemployment insurance, are all excluded.

7. Units Conventions and the Use of the Benchmark Equilibrium Data Set

The benchmark equilibrium data set obtained by adjusting diverse data sets into mutually consistent form provides observations on equilibrium transactions in value terms. To obtain information separately on equilibrium prices and quantities, a units convention must be adopted to separate observations on price-quantity combinations into component parts.

As factors of production are treated in the model as perfectly mobile between alternative uses, the allocation of factors by industry in equilibrium will equalize the returns received net of taxes and gross of subsidies in all industries. It is therefore convenient to adopt a definition of physical units for all factors as that amount of a factor that can, in equilibrium, earn a reward of one currency unit (e.g., £1,000,000 or \$1,000,000) net of taxes, and after receipt of subsidies, in any of its alternative uses. Units for commodities are similarly defined as those amounts which, in equilibrium, sell for one currency unit net of all consumer taxes and subsidies.

The assumption that marginal revenue products of factors are equalized in all uses in equilibrium permits this convention to translate factor payments data by industry into observations on physical quantities for use in the determination of parameters for the model. In this way observed equilibrium transactions (products

of prices and quantities) are separated out into price and quantity observations. An observed equilibrium is characterized by an equilibrium price vector of unity, and ownership of a unit of labour or capital services yields a net income of one currency unit.

Ad valorem tax rates must be calculated to convert net-of-tax prices to gross-of-tax prices. In this survey, I will largely confine my attention to taxes whose rates are expressed as a percentage of the net-of-tax price, and for which average and marginal rates are equal. Tax rates can be calculated as the ratio of the value of revenue collected to the value of transactions taxed, not including the tax revenue.

In section 8, I illustrate these procedures by working through in detail the parameterisation of an AGE model to the benchmark equilibrium data set specified in Table 2. Section 9 illustrates solution procedures using this model. CES functions are used throughout.

8. Determining parameter values from the benchmark equilibrium data set - a simple 2 factor 2 sector 2 consumer example

The calibration procedure outlined in previous sections and illustrated here required the benchmark equilibrium data set to be used to determine parameter values for production and demand functions. The parameter values must be chosen so that the model, when solved with the benchmark policy structure, will reproduce the benchmark equilibrium data set. To achieve this, I make use of some

implications of profit maximizing behaviour by firms, utility maximisation by households, and competitive market structures.

Consider the production side of the model first. To keep things simple, I will assume that each industry produces a single output according to a (CES) production function, with two inputs, capital and labour. Constant returns to scale are assumed. The production function for a typical industry can then be written.

$$Q = \gamma \left(\alpha K^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) L^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} \quad (5)$$

where Q , K , and L represent output, capital and labour respectively, γ is a "scaling" parameter which permits us to determine the "size" of a unit of Q relative to units of input, α is the "distribution" parameter reflecting the relative importance of the inputs, and σ is the elasticity of substitution between capital and labour. Given the units convention outlined above, the benchmark equilibrium data set gives the values of Q , K and L directly. The value of σ is exogenously given on the basis of econometric estimates.

However, the values of α and γ are still unknown. To establish these, we recall that to cost minimise, a firm operating in competitive factor and product markets must employ inputs up to the point where the value of the marginal product of each is equal to its hire price, that is, where $p \cdot mp_K = r(1 + t_K)$ and $p \cdot mp_L = w(1 + t_L)$, p is the producer price of a unit of output, and $r(1 + t_K)$, $w(1 + t_L)$ are the prices paid by the firm for units of capital and labour.

Expressing these relationships in terms of relative factor prices yields

$$\frac{mp_K}{mp_L} = \frac{r(1+t_K)}{w(1+t_L)} \quad (6)$$

This yields

$$\frac{\partial Q}{\partial K} = \alpha K^{\frac{-1}{\sigma}} \gamma (\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \quad (7)$$

$$\frac{\partial Q}{\partial L} = (1-\alpha)L^{\frac{-1}{\sigma}} \gamma (\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \quad (8)$$

When (7) is divided by (8), the expression for the ratio of marginal products simplifies to

$$\frac{mp_K}{mp_L} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{K}{L} \right)^{\frac{1}{\sigma}} \quad (9)$$

Substituting (9) into (6) yields

$$\left(\frac{\alpha}{1-\alpha} \right) = \frac{r(1+t_K)}{w(1+t_L)} \left(\frac{K}{L} \right)^{\frac{1}{\sigma}} \quad (10)$$

in which, assuming a value for σ is determined, α is the only unknown. Representing the right hand side of (10) by the symbol A , we have

$$\alpha = A/(1 + A) \quad (11)$$

The value for γ follows from our units convention that the seller price of all units be equal to one. γ must be chosen such that the right hand side of (5) is equal to Q , that is,

$$\gamma = \frac{Q}{\left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}} \quad (12)$$

The solutions to (11) and (12), together with previously determined variables, permits a complete specification of (5).

On the demand side, an analogous procedure may be carried out using CES utility functions. Writing a CES utility function with two goods, X and Y , gives

$$U = \left(\beta X^{\frac{s-1}{s}} + (1-\beta) Y^{\frac{s-1}{s}} \right)^{\frac{s}{s-1}} \quad (13)$$

(Notice that since the size of "units" of utility have no significance, I drop the scaling parameter from this formulation.

I retain the exponentiation of the expression by the term $(s/s-1)$ to

preserve homogeneity of degree 1 in X and Y, since that property will be useful to us later. It has no immediate significance, since it is merely transforms U monotonically.)

Utility maximisation implies that

$$\text{mrs}_{yx} \left(= \frac{\partial U/\partial x}{\partial U/\partial y} \right) = \frac{p_x(1+t_x)}{p_y(1+t_y)} \quad (14)$$

The expression for mrs_{yx} can be obtained by differentiating (12) with respect to X and Y, and dividing the resulting expressions (a procedure analogous to that outlined in equations (7) - (10) for production functions). This yields

$$\left(\frac{\beta}{1-\beta} \right) = \frac{p_x(1+t_x)}{p_y(1+t_y)} \left(\frac{X}{Y} \right)^{\frac{1}{s}} \quad (15)$$

from which the value of β may be derived. [Values of X, Y, p_x , p_y , t_x and t_y are known from the benchmark equilibrium data set, and the value of s has been exogenously determined by reference to econometric evidence.

Representing the right hand side of (15) by B, and rearranging, we have

$$\beta = B/(1+B) \quad (16)$$

Table 2 presents the parameter values consistent with the benchmark equilibrium data set reported in Table 1, assuming all elasticities

of substitution, in both production and demand, are set at 0.5.

9. Model solution

The last subsection showed how an AGE model was built. Once built, however, the model still has to be solved in order to provide a basis for interpreting the outcomes of policy simulations. Solution can be thought of as comprising two different kinds of operations. The first involves the calculations of the quantity implications of any given set of prices for the model. That is, given some set of prices, excess demands must be calculated. The second involves the search for an equilibrium - that is, for a set of prices and revenues associated with zero excess demands. In practice, model solutions frequently involve iterating between these two operations. A price vector is chosen, the associated excess demands are calculated, and the values of those excess demands are used to guide the choice of the next candidate price vector, and so on, until an equilibrium price vector is found.

a) The calculation of excess demands

Recall the following features of our stylized AGE model. First, endowments of factor services are given. Second, constant returns to scale prevail in all industries. These two features combine to render tractable the calculation of excess demands in the model. The constant returns to scale assumption permits us to reduce the calculation of excess demands to cover factor services only. Since

TABLE 2

Specification of a simple AGE model calibrated from the benchmark equilibrium data set given in Table 1.

PRODUCTION.

	σ	α	γ
Sector 1	0.5	0.13	2.73
Sector 2	0.5	0.44	2.67

DEMAND

	s	β
CONSUMER 1	0.5	.97
CONSUMER 2	0.5	.60

the scale of production will not alter the marginal cost of production with factor prices held constant. I can force equality between the demand and supply of produced commodities for any set of factor prices by "scaling up" the level of production of any industry. Once all activity levels have been determined in this way, I can calculate the implied "derived" demands for capital and labour. With endowments subtracted, I am left with a vector of excess demands.

To see how this works for a particular functional form, consider again the production function (5) and the utility function (13). I will deal first with production. For cost minimisation, the marginal rate of technical substitution of capital for labour (the ratio of marginal products) must be set equal to the factor price ratio faced by the industry as specified in (10). This is my starting point in deriving an expression for the cost minimising levels of capital and labour required to produce output Q . Rearranging (10) gives

$$L = K \left(\frac{1-\alpha}{\alpha} \frac{r(1+t_K)}{w(1+t_L)} \right)^\sigma \quad (17)$$

Rewriting (5) as

$$\left(\frac{Q}{\gamma} \right)^\sigma = \alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}}$$

and substituting in (17) for L , I have

$$\left(\frac{Q}{\gamma} \right)^\sigma = K^{\frac{\sigma-1}{\sigma}} \left(\alpha + (1-\alpha)^\sigma \alpha^{(1-\sigma)} \left[\frac{r(1+t_K)}{w(1+t_L)} \right]^{(\sigma-1)} \right) \quad (18)$$

The cost minimizing value for K is thus given by

$$K = \frac{Q}{\gamma} (\alpha + (1-\alpha)^\sigma \alpha^{1-\sigma} \left[\frac{r(1+t_K)}{w(1+t_L)} \right]^{\sigma-1} \frac{\sigma}{\sigma-1})$$

which, after some tedious manipulation, can be written

$$K = \frac{Q}{\gamma} \frac{\alpha^\sigma}{[r(1+t_K)]^\sigma (\alpha^\sigma [r(1+t_K)]^{(1-\sigma)} + (1-\alpha)^\sigma [w(1+t_L)]^{(1-\sigma)})^{\sigma/(\sigma-1)}} \quad (19)$$

The cost minimizing level of capital required per output-unit is then

$$K^* = \frac{1}{\gamma} \frac{\alpha^\sigma}{[r(1+t_K)]^\sigma D} \quad (20)$$

where $D = (\alpha^\sigma [r(1+t_K)]^{(1-\sigma)} + (1-\alpha)^\sigma [w(1+t_L)]^{(1-\sigma)})^{\frac{\sigma}{\sigma-1}}$

Similarly, the cost-minimizing level of labour required per output-unit is given by

$$L^* = \frac{1}{\gamma} \frac{(1-\alpha)^\sigma}{[w(1+t_L)]^\sigma D} \quad (21)$$

It is then a simple matter to compute unit cost, which, since I have assumed constant returns to scale and competitive output markets, will be constant and equal to producer price:

$$P_X = r(1+t_K) K^* + w(1+t_L) L^* \quad (22)$$

I must now determine commodity demand. The demand function for X and Y can be derived from the utility function (13) by solving the appropriate Lagrangean. The demand for X by a typical consumer is given by

$$X = \frac{\beta^s}{p_x^c \left(\beta^s p_x^c (1-s) + (1-\beta)^s p_y^c (1-s) \right)} \quad (23)$$

and similarly, the demand for Y by a typical consumer may be written

$$Y = \frac{(1-\beta)^s}{p_y^c \left(\beta^s p_x^c (1-s) + (1-\beta)^s p_y^c (1-s) \right)} \quad (24)$$

where p_x^c , p_y^c are the constant prices of X and Y, equal to $p_x(1+t_x)$ and $p_y(1+t_y)$ respectively.

At any set of factor prices, incomes are determined by the value of endowments, taxes and transfers. Market demands are calculated by summing individual demands.

As I have already mentioned, the assumption of constant returns to scale and its implication of constant marginal cost for given factor prices can be exploited to simplify the solution of the model. Once market demands have been determined, I can simply scale production activities to satisfy those demands, thus ensuring equilibrium in all

produced commodities markets. Excess demands, and the search for equilibrium prices, can be confined to factor price space. When activities are scaled to satisfy commodity demands, there will be an implicit derived demand for capital and labour given by

$$K^d = K^* \cdot Q^* \quad (25)$$

$$L^d = L^* \cdot Q^* \quad (26)$$

where Q^* is the market demand for a typical good. Summed over all production activities, the aggregate derived demands K^D and L^D may be compared with aggregate factor supplies K^S and L^S . Excess demands

are given by $\xi^K = K^D - K^S$ (27)

$$\xi^L = L^D - L^S \quad (28)$$

The solution of the model will ensure that these excess demands are zero. In addition, a third condition, that revenue collections equal revenue disbursements, must prevail:

$$R^C = R^D \quad (29)$$

The zero profit condition must of course continue to hold.

Search procedures for a set of prices and a level of revenue for which ξ^K and ξ^L are zero, and which satisfies (29), will now be briefly considered.

b) Solution techniques

As mentioned in the introductory part of this survey, one major reason for the delay in the development of AGE models was the problem of solution. While it has been known since the work of Arrow and Debreu in the 1950s that even under fairly weak assumptions, at least one equilibrium existed, no computational procedure guaranteeing that the solution would be found was developed until the late 60s. The development of these solution techniques by Herbert Scarf and others was the intellectual breakthrough that provided the impetus for AGE model construction and application.

As it turned out, however, these fixed point algorithms, as they are known, are not usually required for solving the applied general equilibrium models which have been developed to date. Newton type procedures, which make linear approximations of various kinds, have been found to be satisfactory in most cases. Fixed point algorithms are discussed in a number of survey papers⁶ and will not be pursued here. To give some idea of how Newton type procedures may be applied, however, let me briefly sketch one such routine. More comprehensive examination of "naive" computational approaches has been undertaken: a useful selection appears in Manne (1985)

The basis of this solution method is the development of a Jacobian matrix of excess demands, of which the element $\frac{\partial \xi_1}{\partial p_j}$ is typical. These elements of the matrix are calculated from the benchmark equilibrium by successively varying each price by a small amount, and noting the resulting pattern of excess demands. This matrix, once developed,

provides guidance as to the likely impacts of relative price movements on the pattern of excess demands, and can in this way inform successive guesses in an iterative procedure. The magnitude of price movements is reduced as the values of excess demands decline with successive guesses. In the event that the Jacobian generated from the benchmark does not lead to a solution, a new Jacobian can be calculated, where the derivatives are evaluated at the set of prices and quantities representing the iteration at which it was decided that convergence using the old Jacobian would not occur.

10. Analysis of a simple tax replacement

The essential feature of the results generated from AGE tax models is that they are based upon a comparison between alternative economic equilibria. In most cases, the initial equilibrium (usually called the benchmark) is compared with an alternative computed equilibrium (usually called the replacement) which is consistent with a set of tax rates which is different from the benchmark set. Large scale AGE models can generate so many numbers that comparing equilibria becomes in large part a problem in digesting large sets of data. It may therefore be helpful to undertake a comparative static exercise for the simple 2 sector model developed throughout this section, as specified in Tables 1 and 2. The policy change considered is the abolition of the tax on capital use in sector 1.

An important point to note in interpreting equilibrium comparisons is that AGE models determine only relative prices. Comparisons between

variables that are dependent on the price level, or, to put it another way, on the normalisation rule, should be treated with great caution. In the results analysed here, the normalisation rule chosen is that the sum of the market prices of capital and labour equal unity in both equilibria. It is neither better nor worse than any other rule. It does, however, imply that the consumer price index falls from 100 in the benchmark to about 80 in the replacement, and results which depend on the normalisation rule, such as consumer incomes, should be interpreted with this in mind.

In the comparative static analysis, I will focus on measures of deadweight loss (or excess burden, or welfare cost), on price and quantity change, and on who gains and loses. Gains and losses of economic welfare, either by individual consumers or in aggregate, are usually measured by compensating and equivalent variations. These concepts, which derive from Hicks's work on the theory of demand, permit a dollar value to be placed on the welfare effects of a change. The compensating variation (CV) is the (algebraic) sum of money which must be taken from consumers affected by an external change to compensate (that is, to leave as well off as before) them for that change. It is envisaged that this compensation would take place after the change had been effected, and would therefore be at post-change prices. The equivalent variation (EV) is the (algebraic) sum of money which must be given to induce a welfare change equivalent to that which consumers would have experienced had a certain external change been effected. Here it is envisaged that the external change in question was not carried out, so the equivalent sum is valued at pre-change prices. The major difference between the two concepts is the level of utility at which the cost difference due to the price change

is measured, the CV being concerned with the original utility level and the EV with the final utility level.

By definition, the equivalent variation for the 1th any individual associated with moving from social state S_0 to social state S_1 is given by

$$EV_I = C_I(P_0; U_I^1[P_1, Y_1]) - C_I(P_0; U_I^1[P_0, Y_0]) \quad (30)$$

and the compensating variation by

$$CV_I = C_I(P_1; U_I^0[P_1, Y_1]) - C_I(P_1; U_I^0[P_0, Y_0]) \quad (31)$$

where C_I is the cost function of the I th consumer; its value is the minimum money cost, at prices p , of attaining the appropriate target level of utility.

Both EVs and CVs are widely used in applied welfare economics as measures of the economic gains and losses to individuals (or homogeneous groups of individuals). It should be emphasised, however, that in general, only the EV provides the basis for a satisfactory money metric. If we require of such a measure that it be capable of ranking all relevant price/quantity situations according to the preferences of the individual or the homogeneous group, then the CV is not admissible. This is essentially because the CV uses as its basic reference point a base level of satisfaction. This is consistent with a large number of alternative price vectors.

However, McKenzie (1983, pp 34 ff) has shown that a necessary and sufficient condition for the CV to provide the basis of a satisfactory money metric in the above sense is that the utility function be homothetic. In the results reported below, therefore, I am secure in my interpretation of the CV. For non-homothetic utility functions, this would not be so.

If the sum of EVs or (in our special case) CVs is used as a measure of welfare change, the underlying social welfare function is

$$W = \sum_{I=1}^N C_I (p; p, y) \quad (32)$$

where N gives the number of households or household groups, and is assumed to be constant between social states.

In general, the evaluation of these welfare measures involves the explicit solution of the cost (or expenditure) functions. Here, however, I make use of the homogeneity property of the particular specification of the utility function (13). For if the utility functions are homogeneous of degree 1 in income, then the EV may be calculated by

$$EV_I = \frac{U_I^1 - U_I^0}{U_I^0} \cdot y_I^0 \quad (33)$$

and the CV by

$$CV_I = \frac{U_I^1 - U_I^0}{U_I^1} \cdot y_I^1 \quad (34)$$

The sum of EVs or CVs still embody (32) as the underlying social welfare function.

The results of the tax abolition are reported in Tables 3 and 4. Price and quantity data are documented in Table 3. As a result of the tax abolition, revenues have fallen from \$130 to \$89.7. While this at first sight suggests that general-equilibrium interactions have led to a revenue loss considerably greater than the \$20 raised by the tax in the benchmark, in terms of consumer price purchasing power the revenue loss is \$18 in benchmark prices.⁴ Capital income has increased dramatically and labour income has fallen. Good 1 has increased in demand, while demand for good 2 has fallen. Sector 1 has become more capital intensive, while sector 2 has become more labour intensive.

Turning to Table 4, the EV from the tax abolition for consumer 1 is \$12.1, while the EV for consumer 2 is - \$8.1. Applying the social welfare function specified in (32), I get a sum of EVs measure of \$4 as the index of our gain in social welfare. The sum of CVs gives a welfare improvement index of \$3.2. Income changes for the two consumers are lower in absolute value than the welfare changes because the first consumer gains not only from the increase in the value of his endowments but from the change in relative consumer prices. Consumer 2, by contrast, loses from both the effects.

Finally, distributional effects may be noted, although in a 2 consumer model these are very artificial. As a result of this tax

TABLE 3

EQUILIBRIUM PRICES AND QUANTITIES BEFORE AND AFTER ABOLITION OF TAX ON
USE OF CAPITAL IN SECTOR 1.

Benchmark (B) consumer price index = 100
Replacement (R) = 80

<u>INCOME</u>	<u>CONSUMER 1</u>		<u>CONSUMER 2</u>	<u>TOTAL</u>
		B	65.0	65.0
Revenue receipts	R	44.9	44.9	89.8
Capital income	B	30.0	10.0	40.0
[Capital price = 1.18 in replacement]	R	35.4	11.8	47.2
Labour income	B	60.0	25.0	85.0
[Labour price = .82 in replacement]	R	49.2	20.2	69.4
Total income	B	155.0	100.0	255.0
	R	129.5	76.9	206.4
<u>CONSUMPTION</u>				
Quantity of good 1	B	90.0	40.0	130.0
[Price of good 1 = 1.12 in replacement.	R	99.0	39.0	138.0
Quantity of good 2	B	20.0	40.0	60.0
[Price of good 2 = 0.99] in replacement	R	19.1	33.8	52.9
<u>PRODUCTION QUANTITIES</u>				
	<u>GOOD 1</u>		<u>GOOD 2</u>	
Capital	B	20.0	20.0	40.0
	R	23.8	16.2	40.0
Labour	B	60.0	25.0	85.0
	R	60.7	24.2	84.9

TABLE 4

CHANGE IN INCOME AND WELFARE FROM TAX CHANGE SPECIFIED
IN TABLE 3 (NORMALISATION ADJUSTED TO EQUATE CONSUMER
PRICE INDEX BEFORE AND AFTER CHANGE)

- a) Change in income and welfare from abolition of tax on capital use in sector 1.

	EQUIVALENT VARIATION	COMPENSATING VARIATION	INCOME CHANGES
CONSUMER 1	12.1	11.7	+7.0
CONSUMER 2	- 8.1	- 8.5	-3.4
TOTAL	+ 4.0	+ 3.2	+ 3.6

- b) Change in welfare from marginal increase in tax rate on capital use in sector 1.

(1)	(2)	(3)
SUM OF EQUIVALENT VARIATIONS	CHANGE IN REVENUE	(1) as % of (2)
.21	.45	46.7

change, consumer 1, the richer, gets still richer, while consumer 2 is worse off.

In Table 4, I may also read off the welfare consequences of marginally increasing the tax rate on capital use in the production of good 1. This requires a separate simulation, in which the tax rate is marginally increased. The very substantial marginal welfare cost of raising an additional dollar of revenue, which is the interpretation to be placed on the percentage figure reported in column 3 of part b) of Table 4, is a result which has been substantiated, at least in terms of order of magnitude, by a number of writers. The intuition behind this finding is revealed by writing down the Harberger formula for the excess burden of an excise tax:

$$L = \frac{1}{2} \epsilon t^2 PQ \quad (35)$$

where P is the initial price, Q the initial quantity, and the tax rate $t = \Delta P/P$. The value of the loss thus increases with the square of the effective tax rate, while revenues, given by tPQ , increase linearly with the tax rate.

The average welfare changes reported in part a) of Table 4 may also be calculated by the Harberger formula given in section II of the paper. In benchmark prices, the Harberger formula gives

$$L = 1/2 T. \Delta K_x \quad (36)$$

$$= 1/2. 1. 3.8 = 1.9$$

(from table 3)

This compares with an estimated gain of \$4.0 using the full AGE model. One important reason for this discrepancy is that the partial equilibrium Harberger formula (36) takes no account of a further distortion in the model - the output tax on commodity 1 - which compounds with the tax on capital used in sector 1. The relationship between the Harberger and AGE results is thus consistent with the analysis of marginal welfare cost given above. Other sources of the discrepancy include the linearisation assumptions required by the Harberger approach. In addition, of course, the Harberger analysis does not permit any distributional analysis.

It should be noted that often, in considering the replacement of one system of taxes with an alternative system, the relevant policy constraint is that the replacement set of taxes should generate the same real government revenue as the original set. This is consistent with Musgrave's (1959) concept of "differential incidence". When economic behaviour is itself a function of tax rates, the rates required for matching the yields cannot be easily determined. In fact, a full general-equilibrium analysis is required to determine such rates correctly. The computational algorithms used can easily be extended to calculate not only an equilibrium for a new tax system but also a scalar that determines the level of tax rates. The user has some choice as to whether this scalar is additive or multiplicative to the rates in the tax system under examination and whether the adjustments of the equal-yield rate (determined by the scalar) apply to all agents and taxed activities or just to a subset of them. This technique is described in Shoven and Whalley (1977).

This is about as far as an investigation of a simple 2 dimensional AGE model will take me. In the next section, the ideas developed here will be applied to large scale models of the UK and US. While their basic structure is the same as that of the simple model I have used so far, a number of refinements and extensions have been incorporated into these more ambitious modelling attempts. I will be examining some of these extensions and reporting the results of the models for policy formulation.

IV. TAX APPLICATIONS

In this section I examine the findings of two AGE models, one calibrated to UK data and one applied to the US economy. In order to understand the significance of the results, it is necessary to describe certain extensions to the model outlined in section III, which have been incorporated in these more sophisticated policy models.

1. Extensions to the basic model

ENDOGENOUS FACTOR SUPPLIES

As developed in earlier sections, AGE models were characterised as requiring exogenously determined fixed aggregate supplies of factor services. At first sight, this requirement may appear to preclude the AGE analysis of the effects of taxation on labour supply and saving, both of which have received much attention in the public finance literature. In some sense, it must always be true that aggregate factor supplies are fixed in general-equilibrium models,

otherwise endowments could not be defined. At the same time, however, it has been possible to introduce mechanisms which allow households to choose how much labour to supply to the market, and how much investment to finance through saving for future consumption. Both models extend the basic structure to permit analysis of these issues.

a) Labour supply

Labour supply to the market is rendered endogenous by assuming that in addition to the labour which a consumer supplies to the market, there is a further quantity of labour which he reserves for his own use, and his labour endowment is then defined as the sum of these two quantities. In the models discussed here, that use is the direct consumption of leisure. Consumer preferences are at the same time modified so that at benchmark prices, households choose to consume directly as leisure exactly the same labour endowment increment. The utility function thus has as arguments not only produced commodities, but leisure as well, and the supply of labour to the market will vary with changes in the net-of-tax return to labour which can be commanded. In this sense, labour supply to the market is endogenous. Moreover, the pure distortion between labour and leisure introduced by income or commodity taxation is captured. There is still a fixed endowment of labour in the economy, of course - the variable supply of labour relates to its supply to market uses.

In order to calibrate this extended model so that it is consistent with the econometric literature on labour supply elasticities, the utility function (13) is modified to a "nested" form. Consumers are

thought of as making a series of consumption choices - first, choosing between goods and leisure, and secondly, choosing among goods. Thus, in general, (13) may be rewritten

$$U = U (L, C) \quad (38)$$

where L is leisure, and C is a composite of market goods, chosen to maximize the function

$$C = C (X, Y) \quad (39)$$

These functions may both have a CES or related form, with consistent price indices being constructed for "composite" goods. Each "level" of this nested structure has its own elasticity of substitution and this allows the calibration procedure to take account of differences between labour supply elasticities and commodity demand elasticities.

b) Saving

To incorporate endogenous saving behaviour utility functions must be further modified. The basic idea is to treat savings in any period as reflecting a decision to acquire a stream of consumption in the future. A decision by any household to save implies a decision to purchase an annuity with an associated anticipated income stream based upon the flow of capital services which will be yielded period by period from the stock of the asset which is acquired.

Expectations are formed on the future price of capital services which determine expected income in future periods. Savings decisions are modelled using myopic expectations and current period rental prices

on capital are thus assumed to dictate the expected real rate of return on capital in future periods. If the economy is on a balanced growth path, myopic expectations are equivalent to perfect foresight since the rental price of capital will not change as the economy moves along its balanced growth path. Under an assumption of balanced growth it is therefore possible to consider the behaviour of the economy in terms of an equivalent single period equilibrium problem. If the economy grows at the rate of growth of the labour force, the effect of savings distortions can be captured in terms of an equivalent one period general equilibrium model and evaluated through a comparison between balanced growth paths. Because labour endowments are assumed to be growing at some exogenously determined and constant rate, the associated balanced-growth rate of saving will be consistent with a unique capital-labour ratio. The higher the rate of saving, the more capital intensive the economy.

As the perceptive reader will already have discerned, however, figure 6 omits an important element of the welfare effects of a shift away from a discriminatory tax on savings. In order to move from the benchmark growth path to the replacement growth path in figure 6, the additional capital investment required to increase the capital labour ratio from K^0/L to K^1/L must be made available by foregoing current consumption. The actual path of the economy, following such a tax reform, would look like the dashed line in figure 6, where T^* denotes the timing of the introduction of the reform. This impact is captured in the US model by solving a "sequence" of equilibria. Each element in the sequence is characterised by a temporarily fixed endowment of capital determined from the initial capital endowment plus saving to that time. Thus, through the transition period

between balanced growth paths, capital is growing at a faster rate than labour, and this is reflected in the model simulations. Results are derived by comparing these sequences of equilibria, discounted where appropriate to present value terms.

The utility function (13) may now be written as

$$V = V (U_P (L_P, C_P), U_F (C_F))$$

where the subscripts P and F denote "present" and "future" respectively. This is incorporated in the CES case by adding a further level to the nesting structure. For both models, the pattern of the nesting structure is given by Figure 7.

If welfare changes are calculated in discounted present value terms, therefore, it is not necessarily the case that such a reform will be measured as welfare-improving, because of the early consumption loss which facilitates the capital deepening required to reach the new balanced growth path.

Once again, resource constraints are fixed in the economy. Here I am fixing first period capital and labour, and the rate of increase of the aggregate endowment labour in each time period. The amount of capital available in all periods following the initial one will depend on consumers' choices between current consumption and saving.

FIGURE 6

CONSUMPTION PROFILES UNDER A SIMPLE INCOME
AND CONSUMPTION TAX

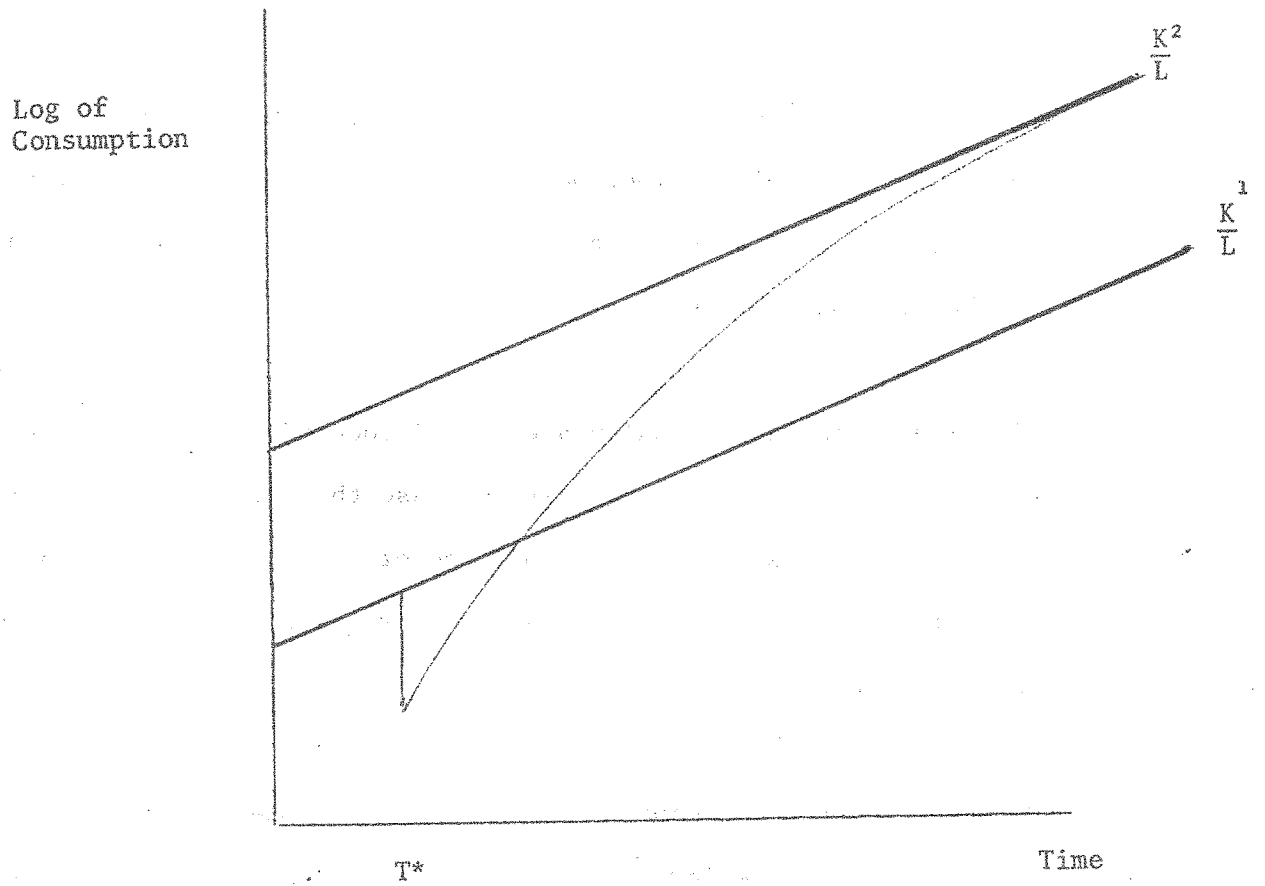
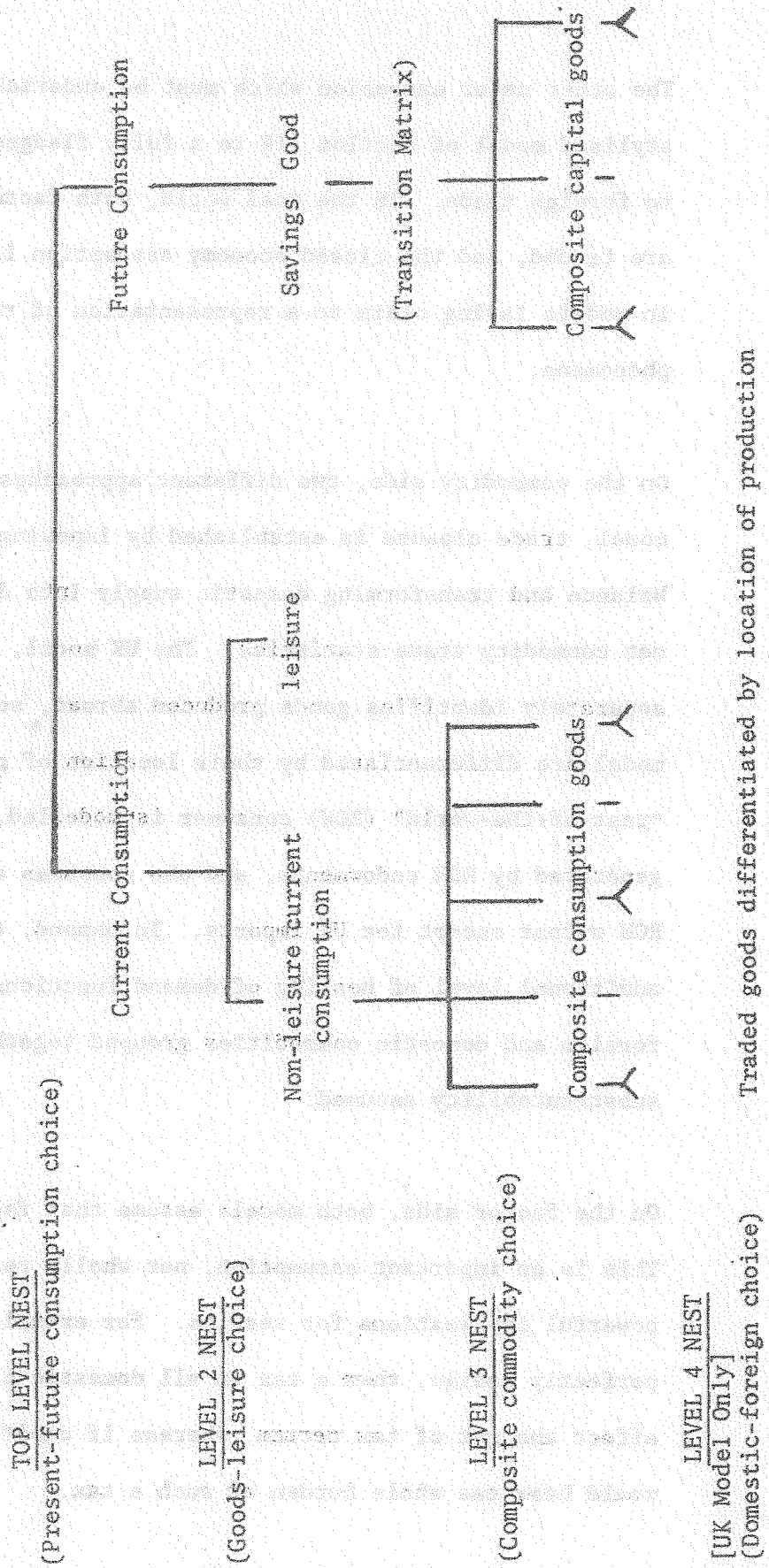


FIGURE 7

NESTING STRUCTURE OF CES UTILITY FUNCTIONS FOR
A DYNAMIC VARIANT OF A TYPICAL AGE TAX MODEL



FOREIGN TRADE

The other major extension which must be undertaken to transform the stylised model of section III to a fully fledged policy model relates to foreign trade. In the real world, both factors and produced goods are traded, and the closed economy assumption is a severe limitation in models laying claim to a representation of real world economic phenomena.

On the commodity side, two different approaches are used. In the US model, trade closure is established by imposing commodity trade balance and transforming domestic supply into domestic demand through net commodity trade statistics. The UK model, by contrast, separately identifies goods produced abroad, so that goods in the model are differentiated by their location of production. A separate "rest-of-the-world" (ROW) consumer is modelled, whose income is generated by ROW endowments, and who consumes all UK exports, and all ROW output except for UK imports. In demand, this requires an additional level of nesting of demand functions, with "corresponding" foreign and domestic commodities grouped together with a high substitutability assumed.

On the factor side, both models assume that factors are immobile. This is an important assumption, not wholly realistic, which has powerful implications for results. For example, if capital were perfectly mobile, then a tax on all domestic capital use would not affect the net of tax return, whereas if capital were immobile, it would bear the whole burden of such a tax.

A certain amount of sensitivity analysis has been carried out on this question in the U.S. model. Coulder et al (1983) report that outcomes are very sensitive to the assumptions employed. It would be fair to say that the issue of how most appropriately to close a tax model with respect to foreign trade is not yet resolved, and remains an important area for further research.

THE PUBLIC SECTOR

Both models close the public sector by assuming utility maximizing behaviour on a stylized "government" consumer. Government revenue is derived from tax revenues and capital holdings. There is thus no direct articulation of private demands for publicly provided goods. A preliminary attempt to incorporate the endogenous determination of public good supply with the UK model was reported in Figgott and Whalley (1985a, ch. 9), and has been taken further in subsequent work.

The basic structure of both models is captured in Figure 5. Table 5 gives a summary of differences between the two model specifications.

2. Results from the UK and US models

A large number of model simulations have been carried out with both the UK and US models. The questions asked are of two broad types. First, what is the economic impact of the overall tax system (or of some part of the tax system) relative to some hypothetical "neutral" alternative? Second, what is the economic effect of some specific

TABLE 5

Comparison of UK and US model structures

<u>UK</u>	<u>US</u>
100 household consuming agents	12 household consuming agents
33 produced commodities	19 produced commodities
Capital and labour as inputs	Capital and labour as inputs
Multi-stage CES demand functions	Multi-stage extended CES LES demand functions
CES production functions with intermediate production	CES production functions with intermediate production
Factor supply endogeneity incorporated via labour leisure choice and balanced growth comparisons associated with alternative tax regimes	Factor supply endogeneity incorporated via labour leisure choice and sequenced equilibria capturing transition between balanced growth paths associated with alternative tax regimes
International trade in commodities modelled by explicit articulation of "rest-of-world" trading partner. Factors of production assumed immobile internationally.	International trade in commodities closed. Factors of production assumed immobile internationally.

tax reform proposal (for example, a shift from income to consumption taxation)?

The UK model - a summary of findings

The first results to be discussed are generated by a model simulation in which all taxes and subsidies are abolished, and replaced by yield preserving, lump sum taxes levied at a rate proportional to household consumption. The "fixed factor" variant of the model is used here; neither labour nor capital supply to the market are variable.

The overall view presented of the UK tax/subsidy system is that the annual welfare costs of its distorting effects are significant, and marked redistribution occurs. This contrasts with an often asserted view that no significant redistribution occurs through the tax/subsidy system (save for direct transfers), and that welfare costs of distortions are small. This latter view leads to a policy prescription of attempts at further and increased redistributive taxation since costs are small. The model findings suggest the opposite policy orientation of more concern with efficiency and less with redistribution in tax reform.

The aggregate estimate of the static annual welfare loss from non-savings/non-leisure distortions produced from tax/subsidy policies in the UK of 6% to 9% of NNP at factor cost per year. This is an annual recurring loss which is larger than might be expected from an extrapolation of some of the currently available estimates of distortionary costs in the public finance literature.

The distributional impact of the tax/subsidy system is found to be strong in terms of welfare gain or loss as a fraction of household disposable income. The abolition of all taxes and subsidies with replacement by a yield preserving general sales tax makes the top 10% of households some 26% better off and the bottom 10% some 20% worse off. The impact on the Gini coefficient of size distribution of incomes is much smaller, however, changing by around 30 points out of 1000. These findings are consistent, since the Gini coefficient is least sensitive to movements in the tails of the distribution, and most sensitive to changes in the area of the mode. The redistribution appears to arise not from relative factor price movements so much as from the direct effects of the personal income tax and the income and welfare effects of consumer tax and subsidy abolitions.

The tax system substantially protects the housing industry. Removal of tax/subsidy distortions shows a sharp reduction in the provision of housing services. By contrast, the tax system penalizes the manufacturing industries. This occurs both because of the personal income tax break on owner-occupation and the subsidy to local authority housing. The housing industry is also emphasised when the impact of individual components of the subsidy system are investigated. One striking feature is that the abolition of local authority housing subsidies, replaced by a yield preserving subsidy, gives a gain to the top income group which is one half of the total gain they would receive by the complete abolition of the progressive income tax.

More generally, the results from simulations in which parts of the tax-subsidy systems are removed suggest that major welfare gains are achieved through the removal of distortions in the indirect tax system. Changes in the ratio of direct and indirect taxes which move in favour of direct taxes tend to be both redistributively and allocatively desirable.

When the variable factor supply variant of the model is used, neither labour supply nor savings distortions appear to increase substantially the welfare costs of the tax-subsidy system estimated with the fixed factor variant. The key elasticity values here are the savings elasticity, which is set at 0.4, and the uncompensated labour supply elasticity, which is set at zero. When the variable factor supply variant is used, it is found that moving from the existing income tax to a 'pure' income tax appears to be a welfare losing proposition; distortions in housing are removed but distortions of savings worsened. A move to a pure consumption tax is a significant gaining proposition.

The extended model was also applied to an analysis of tax inflation interactions. Inflation non-neutralities potentially produce significant welfare impacts. In particular, there were significant effects from the interlinkage between housing distortions and inflation since inflationary capital gains on houses are tax free.

The US model - a summary of findings

Perhaps because of its more complete modelling of intertemporal effects, the US model has been used extensively to analyse tax

reforms which might be expected to have an impact on savings and growth. These reforms include both proposals for integrating the personal and corporate income taxes, and transforming the income tax to a consumption tax, thus eliminating the "double taxation" of savings.

Total integration of the personal and corporate income taxes is shown to yield a present value of gains ranging from \$311 billion to \$695 billion in 1973 dollars, or about 0.5 to 1.0 % of the present value of consumption, depending on the yield-preserving tax. The corresponding results from partial integration plans were found to be generally lower, although they exceeded \$100 billion in every case. Simulations with the model suggest that the potential gains under integration from removal of intertemporal distortions would be significantly reduced if marginal income tax rates are raised, particularly if the higher-income groups, who are also larger savers, face larger rate increases.

In examining the economic effects of moving from income to consumption taxation, it must be realised that the US already has a partial consumption tax, since roughly half of saving is not subject to tax: 30 percent through retirement plans and life insurance, where the tax is deferred until withdrawal (as with a consumption tax), and 20 percent in the form of new housing construction. Housing must be purchased with after tax dollars (i.e., the saving/investment is not deductible), but the return on it, imputed or otherwise, is very lightly taxed. Thus, it is not taxed twice, as with an income tax; its treatment is more neatly analogous to a consumption tax.

Model simulations indicate that sheltering more savings from the current U.S. income tax could improve economic efficiency even if marginal tax rate increases are necessary in order to maintain government revenue. The present value of welfare gains for a policy of complete savings deduction with marginal rate adjustments (a consumption tax) is around \$500 billion to \$600 billion in 1973 dollars, more than 1% of the present value of consumption. Gains of this magnitude are very significant. The gains are smaller under the consumption tax when revenues are replaced with distortionary taxes, but larger when lump-sum scaling is used.

A combined policy of tax integration and savings deductions offers an even greater welfare improvement, with the present value figure lying between \$975 billion and \$1.3 trillion, or about 2% of the present value of consumption.

Among the most interesting simulations that have been carried out with the US model, however, relate not to specific proposals for tax reform, but to the total and marginal efficiency costs of the overall tax system. The estimate here for the hypothetical experiment removing the entire tax system and replacing it with a set of lump-sum levies showed that the present value of welfare would increase by \$3.3 trillion, which is roughly 6.7 percent of national income plus leisure, or 10 percent of national income. (This is consistent with the UK finding.)

The results of marginal increases in taxes, however, are even more dramatic. If all marginal rates are increased by 1%, for example, the marginal welfare cost is estimated at between 52 and 76 cents per

dollar of revenue raised. This kind of calculation, which cannot be appropriately carried out using partial equilibrium techniques, has important implications both for the assessment of the efficiency effects of marginal tax reform and for cost benefit analysis, which has traditionally ignored the marginal welfare cost component of the tax revenue cost of public projects.

V. Concluding remarks

This survey has aimed to introduce AGE analysis in a way which will give some sense of the contribution it has made to economic investigation of taxation policy, and provide an appreciation of the tasks involved in constructing and applying such models. In conclusion, I offer a summary of the benefits from AGE modelling, briefly list the strengths and weaknesses of the approach, and indicate new research directions.

AGE analysis has provided policymakers with a sophisticated tool for the analysis of microeconomic policy issues. These models are now providing more refined calculations of efficiency and distributional effects than were previously available. While policy decisions continue to be made, and while it is believed that such decisions are not wholly determined by competing groups lobbying for their own ends, these models will have a role to play in providing fresh insights on policy options not available from any other source. An example of such numerical insight is the marginal welfare cost calculation carried out with the US model, and summarised in section IV. The size of the marginal excess burden of a dollar of revenue was not fully appreciated prior to these calculations. At another

level, AGE models can force policymakers to think consistently about a policy problem, since to explain model results, the intuitions stemming from the general equilibrium model structure must be understood.

The empirical implementation of this approach permits a higher dimensional version of traditional two sector general equilibrium models. This yields a number of advantages over earlier attempts to deal with incidence and efficiency questions associated with tax policy.

Firstly, considerable disaggregation of commodities and consuming groups is possible. This allows incidence analysis to focus on the size distribution of personal income, rather than solely on factor incomes. It also permits a more general representation of tax distortions, which treats alternative agents, factors and commodities differently. Taxes and subsidies can be introduced on all transactions, so that multiple tax instruments can be represented and changed simultaneously. Extended models can cope with the endogenous variation of factor supplies, through introducing labour, leisure and intertemporal consumption choices. The two most exhaustively analysed distortions in the public finance literature can therefore be treated simultaneously.

Secondly, the extension of this approach to include equal yield alternatives makes possible differential incidence calculations. These are easier to interpret than the incidence experiments associated with the Harberger approach, or the incidence assumptions used in much of the partial equilibrium literature.

Thirdly, there is much more flexibility in the tax changes that may be considered. Taxes may be changed by discrete amounts, and a number of tax instruments simultaneously altered. This allows complex tax reform proposals to be assessed. It also opens the way to an analysis of the economic effects of the overall tax system which escapes some of the methodological difficulties confronting statistical tax burden calculations which have been stressed in the literature.

Like any other approach to the analysis of economic issues, however, AGE modelling has disadvantages as well. Firstly, these models are not macroeconomic, and they cannot address economic problems associated with the business cycle, such as unemployment or inflation. Nor can they address issues of interaction between macro variables and micro policy instruments, such as tax-inflation interactions, in a manner which takes account of unanticipated macroeconomic fluctuations. Secondly, many policies can only be represented in an AGE model in very crude terms, so that the general equilibrium impact of the actual policy may be quite different from the computed impact of the policy as modelled. Thirdly, data problems, particularly elasticity estimates, constrain the accuracy and further development of AGE analysis. Finally, not all policy instruments can be represented in any one AGE model; regulatory policy for example, may be omitted from a tax model. Those that are included may well, in reality, interact with those that are omitted. So policy assessment, while aided by AGE models, must still recognise the incomplete nature of the policy representation in the model.

New research directions include the incorporation of regulation policy [Imam (1985)], public goods [Piggott and Whalley (1987)], and price rigidities [Nguyen (1985)]. Research by Heady and Mitra, and others, has focused on the AGE computation of optimal public policies, thus addressing the important theoretical literature on optimal taxation. Heady and Mitra (1986) present a useful overview of this work. Issues such as the life cycle, human capital formation, and the impact of overlapping generations have yet to be addressed within this framework. Perhaps the greatest strength of AGE analysis is that significant questions keep on presenting themselves as research proceeds, thus stimulating and guiding a continuing research programme.

ENDNOTES

1. Musgrave (1959, pp. 385-404) provides a historical review of the work of these writers with special reference to taxation.
2. See Shoup (1969), p. 15.
3. Through the remainder of the 60s and into the 70s, papers by a number of authors appeared which modified and extended the Harberger model in various ways. For example, McLure (1969) has extended the model to encompass interregional incidence, and has introduced immobile factors (1971a). Thirsk (1972) has extended the analysis to the case of three goods, and Mieszkowski (1972) has considered the case of three factors. Anderson and Ballentine (1976) have extended the analysis to incorporate the case of monopoly. Finally, Vandendorpe and Friedlaender (1976) extend the Harberger formulation to encompass an initial situation with a number of distortions, although only one instrument may be altered at any one time.
4. This section draws on the introductory section of Piggott and Whalley (1985a).
5. Although the existence of equilibrium is assured, there is no guarantee that such an equilibrium will be unique. In an economy with distortions, no sufficient conditions for uniqueness have thus far been formulated. The possibility of multiple equilibria is clearly of concern if the results of AGE models are to be used for policy assessment, since two

alternative solutions may have quite different implications for efficiency and distribution, even though the specified tax reform is the same in both cases. Kehoe and Whalley (1985) have demonstrated for the US model referred to in section IV that the solution is unique. By and large, therefore, researchers operate on the presumption that uniqueness prevails.

6. The two best expositions are by Scarf (1981 and 1984).

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