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A STRATEGY FOR WINNING AT ROULETTE

by

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A STRATEGY FOR WINNING AT ROULETTE

1 *Introduction*

This paper examines the problems involved in exploring and exploiting the bias of a roulette wheel. Records are made of the winning numbers over a large number of spins of a roulette wheel. The analysis is in terms of the maximum of the frequencies of success of each of the 37 roulette numbers. Answers are sought to two questions.

> Question 1: What value of the maximum frequency is sufficient for us to assume that the wheel is biased?
>
> Question 2: If the wheel is assumed biased, what observed frequency of a number is sufficient to justify betting on that number in order to make money?

The answer to Question 1 depends, of course, on the number of spins observed and the level of significance, α. We find that frequency X, which will be exceeded with probability α by the maximum of the 37 frequencies from a perfectly unbiased wheel. A wheel will thus be assumed to be biased if its maximum frequency exceeds X.

Given the size of each bet, the size of the player's bank, the opportunity cost of the player's time (i.e., the minimum acceptable long-term average winning rate per hour) and the number of spins per hour, a critical probability q will be determined. This q will be such that the expected gain from betting on a number which has true probability of success q, exactly equals the expected loss from betting on a number with true success rate 1 in 37.

For a biased wheel with numbers with true probabilities of success $p_0, p_1, \ldots, p_{36}$, there is a frequency Q' such that the expected hourly advantage (taking into account the opportunity cost of the player's time) of a roulette number with Q' successes in n trials is exactly zero. It is then profitable to bet on numbers with frequency Q' or greater. However Q' depends upon the 37 values of the p_i. In this paper we find an upper bound, Q*, of the Q' over all sets of p_i for which at least one of the p_i is q or greater. The decision rule is to bet on those numbers with frequency Q* or greater in n trials.

By an appropriate choice of n and α, the same critical frequency can be used to answer Questions 1 and 2 simultaneously.

Some 33,000 observations have been collected from three roulette wheels. The theory has been used to select favourable numbers from these data. We calculate the results of betting on these numbers over that part of the data not used in the samples.

2 *Description of Roulette*

The roulette wheel is a mechanical device for, supposedly, randomly choosing one of the 37 numbers 0, 1, 2, ..., 36. At the rim of the wheel are 37 numbered compartments coloured red (18 compartments), black (18 compartments) and green (the number 0). In clockwise order, the numbers are

0 32 15 19 4 21 2 25 17 34 6 27 13 36 11 30 8 23
10 5 24 16 33 1 20 14 31 9 22 18 29 7 28 12 35 3 26.

The wheel is housed inside a large bowl. While the wheel is slowly rotating, a small ball is thrown into the bowl in the opposite direction. The ball eventually lands in one of the 37 compartments, thus determining the winning number.

The betting table features the numbers 1 to 36, and 0 arranged in the following pattern:-

0	3	6	9	12	15	18	21	24	27	30	33	36
	2	5	8	11	14	17	20	23	26	29	32	35
	1	4	7	10	13	16	19	22	25	28	31	34

A player makes a bet by placing a chip, which represents a certain amount of money, on a number (including 0) or group of numbers on the betting table. Any one bet can cover numbers which are adjacent on the betting table, or all the numbers in the same row or column or an adjacent group of rows or columns. It is also possible to bet on various groups of 18 numbers excluding 0: evens, odds, lows, highs, reds or blacks. It is possible for any one chip to cover selections of

1, 2, 3, 4, 6, 12, 18 or 24 numbers.

If the winning number is covered by a chip, the owner of the chip receives the chip back plus, respectively, a further

35, 17, 11, 8, 5, 2, 1, or ½ chips.

Otherwise the chip is lost.

If it is assumed that every number has an equal chance of winning, it can be readily calculated that the owner of the wheel has an expected return of 1/37th (2.70%) of all stakes bet. In some American casinos there are wheels which also have a 38th compartment, the double zero, 00. On such wheels the expected return is 2/38th (5.26%) of all stakes bet.

3 *Suspicion of Possibility of Bias*

In many popular books about gambling, for example Wykes [2], there is a story about an English engineer named William Jaggers who won a lot of money at Monte Carlo in 1891. Jaggers and several assistants recorded the winning numbers at various roulette wheels. After a month's recording Jaggers noted that the results were not likely to have been obtained from unbiased wheels, so he bet on the numbers occurring most frequently until he was banned from the casino. Many accounts state that because of this episode all roulette wheels at casinos are now carefully tested and examined every day.

I have strong reasons to disbelieve that roulette wheels are "carefully tested and examined every day." A person who had been employed in casinos in Australia and England for six years told me that testing involves the use of a spirit level every six months or so. Another croupier told me that at one casino the management once decided to check whether a wheel was biased. The "analysis" was based upon 200 spins. When I was recording numbers, I noticed a piece of paper of size one square centimetre in one of the compartments which was not removed for three weeks. I have even seen a roulette wheel used as an ashtray.

The author is aware of only one published account, Wilson [1], of the recording of a large number of spins. This book includes the results of 80,000 consecutive spins of a 38-numbered wheel. If this wheel was perfectly unbiased then the maximum frequency was 4 standard deviations above the mean.

An associate of the author had collected records of 19,000 spins from one wheel - Wheel 3 in Tables 3 and 4 of this paper. A preliminary inspection of these results lent further support to the view that roulette wheels are biased and that it might be possible to bet on favourable numbers and make a nice profit.

It is the author's belief that the management and the employees at casinos are unaware of any bias of roulette wheels. A biased wheel would mean that the house would lose money to smart, patient gamblers, such as Jaggers and Wilson. Furthermore, in conversation with numerous croupiers who have been employed in casinos throughout England, U.S.A., Australia and parts of Europe, the author has never heard any evidence of rigged wheels as are commonly seen in movies such as "The Sting".

If there was a bias which significantly favoured one or more numbers, the only potentially profitable bet would be the backing of individual numbers with odds of 35 to 1 or better. It would be unlikely that several favourable numbers could be covered with one chip, without also including numbers which were not favourable. Besides a bet of \$A on a group of m numbers is equivalent to simultaneous bets of \$(A/m) on each of the m individual numbers.

4 *Criteria for Determining Whether a Wheel is Biased*

Suppose that a wheel is completely unbiased. Provided that the number of spins, n, is sufficiently large, the frequency of success of any roulette number would follow a normal distribution with mean

$$\frac{n}{37} \tag{1}$$

and standard deviation

$$\sqrt{n \cdot \frac{1}{37} \cdot \frac{36}{37}} = \frac{6}{37}\sqrt{n} \tag{2}$$

by the normal approximation to the binomial distribution. Throughout this paper the values of n used will be sufficiently large so that the normal approximation may be used rather than the Poisson distribution.

The most commonly used test statistic to test whether the observed frequencies X_0, X_1 ...., X_{36} follow the above distribution is the sum of the squares of the standardised variates

$$\sum_{i=0}^{36} \left(\frac{X_i - \frac{n}{37}}{\frac{6}{37}\sqrt{n}}\right)^2 . \tag{3}$$

The value (3) is then compared with the χ^2 distribution with 36 degrees of freedom. One degree of freedom has been lost because of the dependence

$$\sum_{i=0}^{36} X_i = n . \tag{4}$$

However, it is more suitable for Question 1 when done in conjunction with Question 2, to use a test statistic based upon the maximum frequency, even though this test will not be as efficient as the χ^2 test. This is because there are some biases which would appear in the χ^2 test that would not be exploitable. For instance, suppose that numbers on one side of the wheel are slightly more likely to win. This in fact was the bias that the author observed on a toy roulette wheel. The χ^2 test is likely to confirm this, yet it may be possible that the bias is not sufficient to be exploitable. Another bias that would not interest us is where one number has considerably less chance of success than 1 in 37, but all other numbers have equal probability.

If a number, i, has been predetermined the critical frequency would be expression (1) plus U(α) times expression (2)

$$\frac{n}{37} + U(\alpha).\frac{6}{37}\sqrt{n}$$

because

$$\alpha = \text{Prob}\ (X_i > \frac{n}{37} + U(\alpha).\frac{6}{37}\sqrt{n}).$$

Here U(α) is the standard normal variate for which

$$\alpha = \text{Prob (random standard normal variate} > U(\alpha)\ ),$$

e.g. $U(0.05) = 1.645$.

If the wheel is unbiased and if we ignore the dependence (4), then $X_0, X_1, \ldots\ldots, X_{36}$ will be 37 independent values from the normal distribution with mean (1) and standard deviation (2). We wish to find a frequency Y which is exceeded by the maximum of the X_i with probability α.

Let β be the probability that Y is exceeded by a random normal variate with mean (1) and standard deviation (2). But

$$\begin{aligned}\alpha &= 1 - \text{Prob }(Y > X_0, Y > X_1, \ldots., Y > X_{36}) \\ &= 1 - (1-\beta)^{37} \\ &= 1 - (1 - 37\beta + (\tfrac{37}{2})\beta^2 - \ldots.) \\ &= 37\beta - (\tfrac{37}{2})\beta^2 + \ldots\end{aligned}$$

If we ignore the second and higher order terms, β is approximately $\alpha/37$.

Thus Y is

$$\frac{n}{37} + U(\frac{\alpha}{37}).\frac{6}{37}\sqrt{n} \ . \qquad (5)$$

This corresponds to an average number of spins per success of

n divided by expression (5). (6)

The critical frequency, (5), and success rate, (6), are displayed in Table 1 for various values of α and n.

5 *Criteria for Determining When it is Profitable to Bet on a Number*

Let A be the size of each bet. At the casino at which I recorded, the minimum bet was $2. Let B be the size of the player's bank. Let C be the opportunity cost of the player's time, i.e., the minimum acceptable average profit rate per hour. Let D be the average number of spins per hour. This is taken to be 100 throughout this paper.

5.1 Let us consider a roulette number for which the true probability of success is p. We shall find the mean and standard deviation of the number of successes of the number in n trials.

On any spin the probability of winning is p and the advantage is 35 units. The probability of losing is 1-p and the advantage is -1 unit. The expected value of the advantage from one spin is

$$35.p + -1.(1-p)$$

$$= 36p - 1. \tag{7}$$

The variance of the advantage from one spin is

$$(35 - (36p-1))^2.p + (-1 - (36p-1))^2.(1-p)$$

$$= (36 - 36p)^2.p + (-36p)^2.(1-p)$$

$$= 36^2\{(1-p)^2 p + p^2(1-p)\}$$

$$= 36^2.p(1-p)\{1 - p + p\}$$

$$= 36^2.p(1-p) \tag{8}$$

The expected value of the advantage from n spins is n times expression (7):

$$n.(36p-1) \quad . \tag{9}$$

The variance of the advantage from n spins is n times expression (8):

$$36^2.np(1-p)$$

so that the standard deviation is

$$36\sqrt{np(1-p)} \quad . \tag{10}$$

5.2 In one hour there are D spins so that if the unit bet is A, the expected advantage per hour is

$$AD.(36p-1) \quad .$$

When the opportunity cost of the player's time is taken into account, the expected advantage per hour is

$$AD.(36p-1) - C \quad . \tag{11}$$

If it is desired that expression (11) be positive, then

$$AD.(36p-1) - C > 0$$

$$\therefore\ 36p-1 > \frac{C}{AD}$$

$$\therefore\ p > \frac{1}{36} + \frac{C}{36AD} \quad .$$

Thus for $A = 2$, $C = 10$, $D = 100$, p must be greater than 1 in $34\frac{2}{7}$, if betting is to be profitable.

5.3 If p is the true probability of success of the roulette number on which we are betting units of \$A, we would like to calculate the probability of losing the bank, \$B.

Let $z(n)$ be the probability that the player would lose a bank of \$36An. Denote $z(1)$ by z.

To lose a bank of \$36A(n+1), one must first lose a bank of \$36An, and subsequently lose a further bank of \$36A. Thus

$$z(n+1) = z(n) \,.\, z \,.$$

This difference equation has solution

$$z(n) = z^n \,.$$

Suppose the bank is \$36A and that the present winnings of the player are \$36An. The probability of subsequently losing the bank is $z(n+1) = z^{n+1}$.

Consider the first 36 spins. The probability of i successes is

$$\binom{36}{i} p^i (1-p)^{36-1} \quad ,$$

in which case the winnings are \$36A(i-1). Thus

$$z = \sum_{i=0}^{36} \begin{array}{l} \text{(probability of i successes) times} \\ \text{(probability of losing a bank of \$36A subsequent} \\ \quad \text{to winning \$36A(i-1) in the first 36 spins)} \end{array}$$

$$= \sum_{i=0}^{36} \binom{36}{i} p^i (1-p)^{36-i} \,.\, z^i$$

$$= \sum_{i=0}^{36} \binom{36}{i} (pz)^i (1-p)^{36-i}$$

$$= (pz + (1-p))^{36} .$$

Solution of the equation

$$z = (pz + 1 - p)^{36} \tag{12}$$

yields

$z = 0.6784565544$ when $p = 1$ in 30
$z = 0.7289411474$ when $p = 1$ in 31
$z = 0.7807173352$ when $p = 1$ in 32
$z = 0.8337467098$ when $p = 1$ in 33
$z = 0.8879929300$ when $p = 1$ in 34
$z = 0.9434215736$ when $p = 1$ in 35.

Thus the probability of losing the bank, \$B, is

$$z^{\left(\frac{B}{36A}\right)} \tag{13}$$

5.4 Given values of A, C, D, we would like to bet on those numbers with true probability p for which expected hourly advantage is C or greater, i.e.,

$$AD.(36p-1) \geq C \ .$$

However we are only estimating the true probabilities and we may be liable to identify as favourable a number whose true probability is really 1 in 37. Rather, we shall content ourselves with trying to identify roulette numbers with true probability q or greater, where the expected advantage of a number with probability q equals the expected disadvantage obtained from choosing a number with probability 1 in 37:

$$AD.(36q-1) - C = C - AD.\left(\frac{36}{37} - 1\right)$$

$$\therefore \quad q = \frac{C}{18.AD} + \frac{38}{36.37} \quad . \tag{14}$$

5.5 In order to proceed further it is necessary to assume a specific distribution of the true probabilities of the numbers on a biased wheel: $p_0, p_1, \ldots, p_{36}$. Each frequency, X_i, follows a normal distribution with mean

$$np_i \tag{15}$$

and standard deviation

$$\sqrt{np_i(1-p_i)} \quad . \tag{16}$$

The probability that X_i exactly equals the integer Q is the probability that the normal distribution with mean (15) and standard deviation (16) lies between Q-½ and Q+½. This probability is closely approximated by the value of the probability density function of this normal distribution at Q, multiplied by the length of the interval, which is one:

$$\frac{1}{\sqrt{2\pi . np_i(1-p_i)}} . \exp\left(\frac{-(Q-np_i)^2}{2np_i(1-p_i)}\right) . \qquad (17)$$

The hourly advantage of betting on number i is expression (11):

$$AD.(36p_i-1) - C .$$

Consider a roulette number which has frequency of success exactly Q in n trials, where Q is considerably above the mean n/37. The expected hourly advantage of such a number is the weighted average of the expected hourly advantages of all 37 numbers, where the weight of any number i is the probability that the frequency i is exactly Q. Thus the expected hourly advantage of a number with frequency of success Q in n trials is

$$\frac{\sum_{i=0}^{36} \frac{AD.(36p_i-1)-C}{\sqrt{2\pi . np_i(1-p_i)}} . \exp\left(\frac{-(Q-np_i)^2}{2np_i(1-p_i)}\right)}{\sum_{i=0}^{36} \frac{1}{\sqrt{2\pi . np_i(1-p_i)}} . \exp\left(\frac{-(Q-np_i)^2}{2np_i(1-p_i)}\right)} . \qquad (18)$$

Let Q′ be the value of Q for which expression (18) equals zero. It is profitable to bet on all numbers whose frequency in n trials is Q′ or greater. However, since the denominator of expression (18) is positive, expression (18) will be zero at the same time as its numerator:

$$\frac{AD.(36p_i-1)-C}{\sqrt{2\pi . np_i(1-p_i)}} . \exp\left(\frac{-(Q-np_i)^2}{2np_i(1-p_i)}\right) . \qquad (19)$$

5.6 Given A, C, D, we wish to find any numbers with probability q or greater, where q is given by expression (14). This means, for a given value of n, we must find Q′ such that expression (19) equals zero. Expression (19) is a function of Q and the p_i. It is an increasing function of Q. We shall find the set of p_i for which Q′ is a maximum. This set of p_i must be that for which expression (19) is minimum as a function of Q.

Since we wish to isolate numbers with true probability q or greater, we can assume that one of the p_i is greater than or equal to q. Expression (19) will be less if any such probabilities equal q rather than exceed q. Also expression (19) will be less if there is only one q rather than several.

Expression (19) will be minimised if all the other p_i are equal. These p_i would each have the value

$$\frac{1}{36}(1-q)$$

which is less than 1 in 37. Instead we shall assume that each of the p_i has value 1 in 37. Besides making the calculations easier, expression (19) will again be lower, so that Q' will be slightly higher. We shall use the symbol Q^* to denote this value Q'.

Thus the set of p_i for which expression (19) is minimum and consequently Q^* maximum, is q and 36 values of 1 in 37.

5.7 For such a set of p_i we shall now find Q^*. The expected advantage of the number with probability q equals the expected disadvantage of any of the numbers with probability 1 in 37, so

$$\frac{AD.(36p_i-1) - C}{\sqrt{2\pi n}}$$

can be cancelled out of expression (19).

A summand of (19) when p_i = 1 in 37, is

$$-\frac{37}{6} \cdot \exp\left(\frac{-(37Q-n)^2}{72n}\right), \tag{20}$$

and when $p_i = q$, is

$$\frac{1}{\sqrt{q(1-q)}} \cdot \exp\left(\frac{-(Q-nq)^2}{2nq(1-q)}\right). \tag{21}$$

Thus when expression (19) equals zero, expression (21) plus 36 times expression (20) equals zero:

$$\frac{1}{\sqrt{q(1-q)}} \cdot \exp\left(\frac{-(Q-nq)^2}{2nq(1-q)}\right) - 6.37.\exp\left(\frac{-(37Q-n)^2}{72n}\right) = 0 .$$

Taking logarithms:

$$-\ln(\ 6.37.\sqrt{q(1-q)}\ ) - \frac{(Q-nq)^2}{2nq(1-q)} = -\frac{(37Q-n)^2}{72n} \quad .$$

Rearrangement gives a quadratic equation in Q:

$$\frac{Q^2}{n}\left(-\frac{1}{2q(1-q)} + \frac{1369}{72}\right) + Q\left(\frac{1}{1-q} - \frac{74}{72}\right) - \frac{nq}{2(1-q)} + \frac{n}{72}$$

$$- \ln(6.37.\sqrt{q(1-q)}\ ) = 0 \quad .$$

5.8 For various values of q and n, the solution Q* of the above quadratic equation is tabulated in Table 2.

Note when n = 5,000, that Q* increases as q decreases. This is surprising as we would have thought that the critical frequency Q* would decrease when q gets closer to 1 in 37.

This apparent paradox is solved by graphically inspecting the normal distributions. Q* is the intersection of the normal distribution with mean nq, and 36 times the normal distribution with mean n/37. In Fig. 1 it can be seen that when q = 1 in 34 the intersection is at the extreme right-hand tail of the distribution, so there is only a very slight chance of a number with probability 1 in 34 having frequency 178 or greater. On the other hand when q = 1 in 30, the intersection is at the centre of the distribution so that there is approximately an even chance that such a number has frequency 166 or greater.

(See Figure 1, page 13)

To take the above phenomenon into consideration we calculate, for values of n and q, the probability, h, that such a roulette number has frequency Q* or greater, and tabulate these alongside Q*.

5.9 Given values of A, B, C, D, e.g. A = \$2, B = \$2,000, C = \$10, D = 100, the procedure is as follows:

(i) Calculate q by formula (14). Here q = 1 in 31.94. We round this to obtain q = 1 in 32, and consequently C = \$9.80.

(ii) We calculate the probability of losing the bank when betting on a number with true probability q. Since q = 1 in 32, by equation (12), z = 0.7807173352. By formula (13) the probability of losing

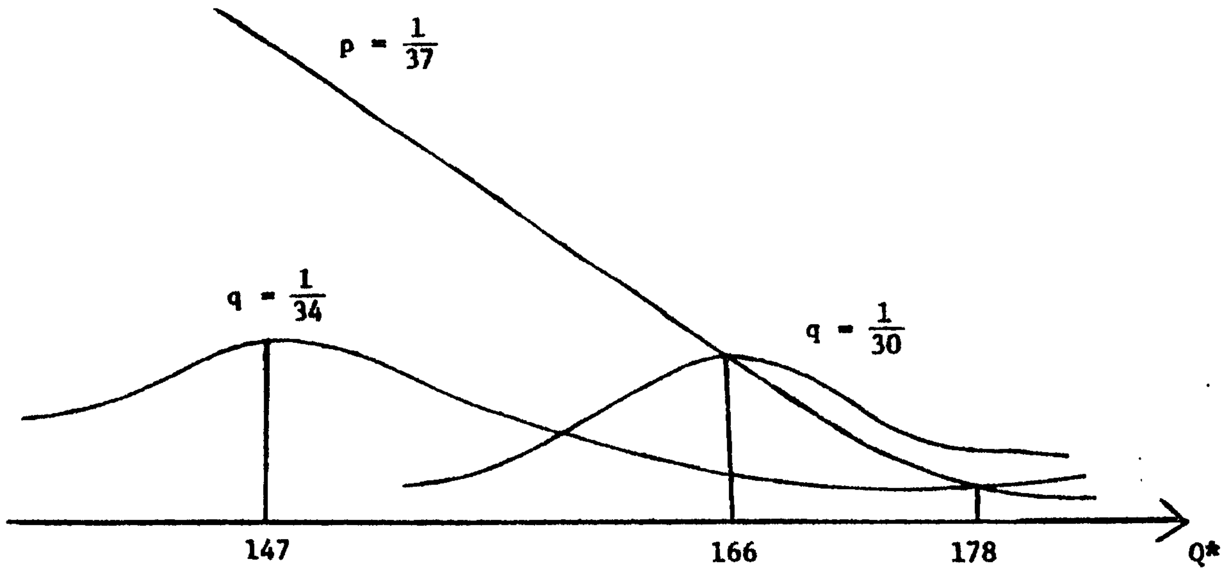

FIGURE 1

the bank is

$$(0.7807173352)^{(\frac{2000}{36.2})} = 0.0010 \ .$$

(iii) Corresponding to various values of n, we use Table 2 to find h, the probability that a number with true probability q will be identified in n trials. For the example values: $n = 1{,}000$, $h = 0.001$; $n = 2{,}000$, $h = 0.017$; $n = 3{,}000$, $h = 0.055$; $n = 5{,}000$, $h = 0.169$; $n = 8{,}000$, $h = 0.356$; $n = 10{,}000$, $h = 0.464$.

The cost of obtaining n observations is

$$\frac{C^*n}{D} \ ,$$

where C* is the hourly cost of collecting observations. Note that C* may be less than C if it is possible to hire people to do this duty for less than $C per hour. The benefit is an hourly rate of

$$h.(AD.(36q-1) - C) \ .$$

Probably the best way of comparing a sum of money and an hourly rate is by multiplying the hourly rate by H, the expected number of total hours that the player will be able to bet on this number. The value of n must be chosen for which

$$-\frac{C^*n}{D} + h.(AD.(36q-1) - C).H$$

is maximised (and greater than zero). With the numerical values above we assume that C* and H are such that the maximum occurs when $n = 5{,}000$.

(iv) With the chosen value of n, we again use Table 2 to obtain the integer nearest to Q*. In our case $Q^* = 168.02$, so we shall select roulette numbers which have frequency 168 or greater after 5,000 spins.

(v) We use Table 1 to find the probability that a frequency of Q* or greater could have been obtained on an unbiased wheel. This probability also approximately equals the probability that Q* or greater could have been obtained from any of the numbers with probability 1 in 37 on an unbiased wheel. When $n = 5{,}000$, $Q^* = 168$, this probability is about 7½%.

6 *Empirical Results*

The author decided to record the winning numbers from a roulette wheel. Two wheels at adjacent tables at a casino were selected because they could be recorded simultaneously. Furthermore each wheel had a high turnover averaging over 100 spins per hour for the off-peak nights Sunday, Monday, Tuesday, Wednesday and Thursday.

Recording was not an especially pleasant job. The casino does provide scorecards, and it is common to see several players at each table recording numbers in order to apply a favourite system to determine which bets to make. The management and employees regarded me with curiosity, but never objected to the recording. Sometimes various people were paid to record the numbers, but most of these people would give up after only 2 or 3 hours.

Wheels 1 and 2 are the wheels from which the author collected values. Wheel 3 is at another casino, and the scores were collected by an associate. Table 3 contains the results of the first 5,000 observations from each wheel. The frequency of a number is converted to standard deviations when it is more than one standard deviation above or below the mean. Table 4 contains the results of all observations from each wheel.

There was at least one number on each wheel with frequency 168 or greater, where 168 is the critical frequency corresponding to $A = \$2$, $C = \$9.80$, $D = 100$. They were Number 27 on Wheel 1, Number 5 on Wheel 2, and Numbers 34 and 9 on Wheel 3. According to the decision rules each wheel could be assumed biased at the 7½% level of significance.

The χ^2 test can also be performed as a useful comparison. The sums of squares, (3), are 54.565 for Wheel 1, 66.688 for Wheel 2, and 61.836 for Wheel 3. These correspond to normal variates of 2.02, 3.12 and 2.69 respectively. This is because of the approximation of

$$\sqrt{2\chi^2} - \sqrt{2n-1}$$

by the standard normal distribution. Thus the χ^2 test also confirms that all three wheels are biased.

Let us calculate the results of betting on numbers selected from the first 5,000 spins of each wheel, over the remaining observed spins.

We shall compare five ways of selecting roulette numbers: (i) numbers with frequency greater than or equal to 168; (ii) numbers with frequency more than two standard deviations above the mean, 159 plus; (iii) numbers with frequency more than one standard deviation above the mean, 147 plus; (iv) numbers with frequency more than two standard deviations below the mean, 112 or less; (v) numbers with frequency more than one standard deviation below the mean, 123 or less.

Beyond the first 5,000 spins, I have data for 1,290 more spins of Wheel 1, 3,140 more of Wheel 2, and 14,023 more from Wheel 3. At 100 spins per hour, betting on these 18,453 spins would take 184.5 hours. The number of roulette numbers in each category is (i) 1, 1, 2; (ii) 2, 1, 2; (iii) 6, 10, 8; (iv) 1, 2, 2; (v) 11, 9, 8, from each wheel respectively. Thus the total number of $2 bets in each category is (i) 32,476; (ii) 33,766; (iii) 151,324; (iv) 35,616; and (v) 154,634. The total number of wins at $72 each is (i) 913; (ii) 943; (iii) 4,196 and (v) 3,971. The cash results are (i) win $784; (ii) win $364; (iii) lose $536; (iv) lose $4,416; and (v) lose $23,356. Cash results per hour are (i) win $4.25; (ii) win $1.97; (iii) lose $2.91; (iv) lose $23.93; and (v) lose $126.59. Cash results per 100 $2 bets are (i) win $2.41; (ii) win $1.08; (iii) lose $0.35; (iv) lose $12.40; and (v) lose $15.10. These last figures can be compared with an expected loss of $5.41 per hour playing a number with probability 1 in 37.

These results certainly indicate that a player at roulette can achieve quite different results from the expected loss of 2.70%. The results also indicate that it is quite possible for a player to win money at roulette provided he/she has a lot of patience and a moderate capital. With $2 bets the hourly win rate was $4.25, so with $10 bets it would have been $21.25. However, it must be remembered that there are huge fluctuations in one's fortunes (even with the most favourable numbers there were often runs of over 100 spins without success), one's assets are tied up in cash, large amounts of cash have to be carried, there is no return for the first 50 hours, the working hours are very late, the atmosphere is very smoky, and in many places the casinos are illegal, although some of these casinos give patrons free drinks.

If a player using this system is winning heavily there is some chance that the casino may ban that person from playing. One should try to prevent this happening in the following way. Because the nightly fluctuations in one's fortunes are huge it is quite likely that, on average, one would win four times out of seven and lose three times out of seven. Thus, in conversation at the casino one should exaggerate one's losses and grumble about one's bad luck. Also the size of one's winnings is likely to be a very small proportion of the total betting turnover - I estimated the weekly turnover to be approximately two million dollars at the casino where I recorded.

One of the most interesting features of Table 4 is the sequence of 13 adjacent numbers from Number 25 to Number 5. Twelve of these numbers deviate from the mean by more than one standard deviation. Furthermore the "good numbers" are together and so are the "bad numbers". In Wilson's 80,000 numbers, [1], there are also sequences of "good numbers" and "bad numbers".

For a while the results for Wheel 2 were also recorded according to the direction of spin. 1,666 anti-clockwise spins and 1,386 clockwise spins revealed no interesting patterns.

TABLE 1

Critical maximum frequencies and success rates for determining whether a wheel is biased

α	20%		10%		5%		1%	
$U(\frac{\alpha}{37})$	2.55		2.78		3.01		3.46	
n	freq.	rate	freq.	rate	freq.	rate	freq.	rate
1,000	40.10	24.94	41.28	24.22	42.46	23.55	44.77	22.34
2,000	72.55	27.57	74.21	26.95	75.88	26.36	79.15	25.27
3,000	103.73	28.92	105.77	28.36	107.82	27.83	111.81	26.83
5,000	164.37	30.42	167.01	29.94	169.65	29.47	174.81	28.60
8,000	253.20	31.60	256.54	31.18	259.87	30.78	266.40	30.03
10,000	311.62	32.09	315.35	31.71	319.08	31.34	326.38	30.64

TABLE 2

Critical frequency, Q, for which a number with probability q is 36 times more likely to appear than a number with probability 1 in 37 in n trials. Probability, h, of the number with probability q having frequency Q* or greater in n trials*

q		1 in 30	1 in 31	1 in 32	1 in 33	1 in 34
n = 5,000	Q*	166.27	166.58	168.02	171.28	177.92
	h	0.512	0.337	0.169	0.052	0.005
n = 10,000	Q*	316.77	314.78	314.13	315.60	320.97
	h	0.821	0.670	0.464	0.233	0.056

n		1,000	2,000	3,000	5,000	8,000	10,000
q = 1 in 32	Q*	47.69	78.93	108.97	168.02	255.80	314.13
	h	0.001	0.017	0.055	0.169	0.356	0.464

TABLE 3

Results after 5,000 spins

Number	Wheel 1		Wheel 2		Wheel 3	
0	137		157	+1.90	135	
32	155	+1.73	123	-1.06	150	+1.30
15	123	-1.06	144		128	
19	123	-1.06	151	+1.38	140	
4	139		125		136	
21	143		124		143	
2	146		127		122	-1.15
25	134		129		125	
17	113	-1.93	135		130	
34	144		153	+1.56	169	+2.95
6	146		126		135	
27	171	+3.13	119	-1.41	117	-1.58
13	126		118	-1.49	139	
36	132		119	-1.41	145	
11	143		148	+1.12	140	
30	148	+1.12	128		116	-1.67
8	121	-1.23	142		103	-2.80
23	152	+1.47	155	+1.73	136	
10	136		147	+1.03	124	
5	122	-1.15	179	+3.83	127	
24	145		138		118	-1.49
16	161	+2.26	132		134	
33	145		116	-1.67	117	-1.58
1	123	-1.06	151	+1.38	143	
20	147	+1.03	141		147	+1.03
14	134		135		112	-2.02
31	123	-1.06	108	-2.37	149	+1.21
9	139		131		168	+2.87
22	141		113	-1.93	139	
18	101	-2.98	122	-1.15	146	
29	126		133		155	+1.73
7	132		150	+1.30	135	
28	119	-1.41	128		153	+1.56
12	123	-1.06	137		125	
35	118	-1.49	112	-2.02	117	-1.58
3	131		158	+1.99	129	
26	138		146		153	+1.56

TABLE 4

The complete results

Number	Wheel 1		Wheel 2		Wheel 3	
0	178		250	+2.05	488	-1.17
32	187	+1.32	207		497	
15	153	-1.32	229		509	
19	150	-1.56	226		498	
4	179		209		516	
21	186	+1.24	211		548	+1.51
2	183	+1.01	222		484	-1.35
25	174		201	-1.30	503	
17	149	-1.63	200	-1.37	508	
34	187	+1.32	252	+2.19	562	+2.14
6	179		201	-1.30	533	
27	204	+2.64	197	-1.57	474	-1.79
13	157	-1.01	187	-2.26	535	
36	172		200	-1.37	525	
11	169		238	+1.23	509	
30	175		219		480	-1.53
8	167		237	+1.16	496	
23	183	+1.01	263	+2.94	527	
10	165		239	+1.30	521	
5	164		262	+2.87	515	
24	178		233		478	-1.62
16	191	+1.63	225		500	
33	185	+1.17	221		474	-1.79
1	166		236	+1.09	536	
20	176		234		548	+1.51
14	159		230		458	-2.51
31	159		187	-2.26	519	
9	187	+1.32	232		572	+2.59
22	170		187	-2.26	542	+1.25
18	129	-3.19	187	-2.26	524	
29	158		217		533	
7	173		229		483	-1.39
28	158		201	-1.30	587	+3.26
12	148	-1.71	229		492	
35	146	-1.87	194	-1.78	452	-2.78
3	174		237	+1.16	536	
26	172		211		561	+2.10
Total	6,290 spins		8,140 spins		19,023 spins	

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