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A TEST
OF THE BLACK AND SCHOLES MODEL
OF OPTION VALUATION IN AUSTRALIA

by

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ABSTRACT

A model to explain option prices has been developed by Black and Scholes. The approach adopted here is to solve the Black and Scholes equation in reverse to obtain implied variance rates of return on the underlying stocks. These are then converted to a standard deviation basis and compared to various measures of historical standard deviation available at the time of trade and with the standard deviations which actually resulted after that date. It is shown that the model produces biased estimates and that part of this bias might be explained by incorrect pricing possibly due to a lack of experience with exchange-traded options in Australia. It is also shown that the bias has decreased substantially, indicating that a learning process may have occurred in the market.

A TEST OF THE BLACK AND SCHOLES MODEL
OF OPTION VALUATION IN AUSTRALIA

The pricing of options has attracted considerable comment in both the academic and commercial literature in the field of finance.¹ An article published by Black and Scholes [6] in 1973 was an important advance as it proposed a general equilibrium model for the pricing of call options which has been tested on U.S. data by the same authors [7] and by Latané and Rendleman [13]. To date, however, there appears to be no study which has tested the Black and Scholes approach using Australian data.² This paper seeks to remedy the deficiency.

1 Institutional Features of the Australian Options Market

Conventional stock options have long been a part of the securities industry in Australia but, as in the U.S., trading was limited because the market was not fully organised or continuous.³ In February 1976 this situation changed with the establishment of the Australian Options Market with a structure similar to that of the Chicago Board Options Exchange. Call options standardised in terms of exercise prices, maturity dates and parcel size were introduced while a system of registered traders was adopted to try to ensure orderly and continuous trading opportunities. The shares on which options could initially be traded were BHP, CSR and Western Mining Corporation (WMC). Options on the stock of Woodside-Burmah Oil (WBO)⁴ became available two weeks later and options on Bougainville Copper first entered the market in November 1976.

Exercise prices are set at standard intervals of 10¢ for share prices less than \$1, 25¢ between \$1 and \$2, 50¢ between \$2 and \$6 and \$1 for higher prices. The range of exercise prices is also set so that an investor should always be able to choose an option with an exercise price which is above, below or approximately equal to the current stock price. Only four days in any one year are maturity dates; these occur towards the end of March, June, September and December. As the longest maturity period is nine months, options maturing on only three of these dates are available at any one time. Immediately following a maturity date, options of three, six and nine months duration are available and over the next three months these progress towards zero, three and six months respectively. New nine month options then become available.

Commissions are charged according to the total premium paid in the case of an option transaction or the total amount paid in the case of the transfer of stock through exercising. An official booklet issued by the Market warns "transactions costs may be significant". [1, p6] Stamp duty is also payable.⁵ For a small investor buying, say, one option contract at 50¢ per share and selling at 60¢, the commission charges reduce the profit from 10¢ to 6¢ per option. An investor buying and selling twenty such options at a time would find the profit reduced to approximately 7½¢ per option.

A large number of complex strategies are possible using options but as it is not the purpose of this paper to detail them, only a brief indication of some of the simpler possibilities will be given here.⁶ Because an option writer in effect agrees to forego any capital gain above the exercise price he receives in return the option price which can then be viewed as a form of insurance against moderate price falls. Thus for investors with a long position in the stock, option writing is a means of hedging against market risk. The option buyer in such a transaction is advantaged in that he knows precisely the amount he may lose and by investing in riskless debt can hedge against realised losses. Such a policy is of course open also to the investor in the stock itself but the option buyer is able to obtain the same net position with a lower outlay.⁷

The Australian Options Market is therefore a very recent addition to the securities industry in Australia. Like its more illustrious Chicago forbear it promises considerable risks, profits and losses for the speculator and offers a range of useful investment strategies which take time, effort and experience to learn and exploit. It is therefore not surprising that this paper presents evidence which suggests that learning may have lagged somewhat behind trading activity.

2 *The Black and Scholes Valuation Model*

In their initial paper [6], Black and Scholes assume a perfect market in both the stock and the option. These assumptions are listed in detail below.

- (i) There exists a riskless short-term interest rate (r) at which investors can borrow and lend and which is constant through time.

- (ii) The stock price follows a continuous random walk with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any given time period is log-normal. The variance rate of the return on the stock (v^2) is constant.
- (iii) There are no transactions costs or taxes.
- (iv) Short selling is allowed with no restrictions on the use of the proceeds from such sales.

In addition, the problem is further simplified by assuming the following:

- (v) The stock pays no dividends or other distributions.
- (vi) The option may be exercised only on the maturity date (that is, a 'European' option).

The price (w) of an option under these assumptions is therefore a function only of the stock price (x), time (t) and variables that may be considered to be known constants.

$$w = w(x, t)$$

If $\frac{1}{\partial w / \partial x}$ options are sold short for every share held long, the equity in the portfolio is independent of stock price movements and must therefore yield the riskless return of assumption (i) as no arbitrage opportunities will exist in a perfect market.⁸ However, for the investment to remain riskless, rebalancing of the portfolio must be undertaken for every change in x and t .

These observations can be shown to lead to a partial differential equation in w :

$$\frac{\partial w}{\partial t} = rw - rx \frac{\partial w}{\partial x} - \frac{1}{2} v^2 x^2 \frac{\partial^2 w}{\partial x^2}$$

The boundary conditions are:

$$\begin{aligned} w(x, t^*) &= x - c && \text{for } x \geq c \\ w(x, t^*) &= 0 && \text{for } x < c \end{aligned}$$

where t^* is the maturity date and c is the exercise price. Solving this equation gives the well known result,

$$w(x, t) = x.N(d_1) - ce^{-rt}.N(d_2)$$

$$\text{where } d_1 = \frac{\ln(x/c) + (r + \frac{1}{2}v^2)t}{v\sqrt{t}}$$

$$\text{and } d_2 = d_1 - v\sqrt{t}$$

and $N(\cdot)$ is the cumulative normal density function

and t is now the period to maturity.

The option price in this formula is positively related to x , v^2 , r and t and negatively related to c (see [14] for proofs of these propositions). In general, the option value is sensitive to changes in x and v^2 and highly insensitive to changes in r (see Black [5]).

The equation is very attractive as a means of evaluating options. It is grounded in solid economic theory and is a general equilibrium solution rather than the result of some curve-fitting approach.⁹ The variables t , c and x have observable and reliable data available while proxies for r exist. Black and Scholes [7] suggest that v^2 might reasonably be estimated from historical series. Primarily because v^2 is assumed to be a known constant, the option value is independent of investors' expectations and is entirely free of such non-quantifiables as investors' attitudes toward risk.

3 Empirical Tests of the Model

The model has been tested by Black and Scholes [7] and by Latané and Rendleman [13]. Using data on conventional six-month call options, Black and Scholes simulated various buying and selling strategies at market and model prices. Each day a realised excess dollar return defined as $\Delta w - \frac{\partial w}{\partial x} \Delta x - [w - \frac{\partial w}{\partial x} x]r\Delta t$ was calculated and it was shown that such profit opportunities as there were would vanish once allowance was made for transactions costs. The tests also showed that the model performs better if actual variance rates of return are used rather than estimates based on historical prices.

Latané and Rendleman [13] used data on exchange-traded options from the Chicago market and derived implied standard deviations (ISD's) of stock price returns by solving the Black and Scholes equation in reverse

with a given option price. A weighted average of these implied standard deviations (WISD) was then calculated and was the central variable used to test the model.¹⁰ Various tests using WISD were undertaken but in the present context the most important were those which compared WISD and actual standard deviations. Alternative estimates of *ex post* and *ex ante* standard deviations were calculated for each underlying stock both before and within the sample period and correlations between these estimates and the mean of the WISD's of each stock were then calculated. It was found that WISD's out-performed the historical series as predictors of future standard deviations. The evidence on over- and under-pricing was conflicting: WISD's were too high compared to standard deviations computed using monthly data and too low compared to those based on weekly data. Nonetheless the authors felt that 'the preponderance of evidence would be toward options being over-priced [by the market, relative to the model], [13, p.377].

A number of reformulations of the model using less restrictive assumptions have been published and Appendix II lists some of these contributions and indicates which assumptions have been successfully relaxed. Rigorous empirical testing however has been limited in the literature to the basic Black and Scholes model. The present paper also seeks to test only the basic model and defers testing of the more sophisticated versions to future research.

4 *The Sample of Option Prices*

As noted above, the valuation formula is highly sensitive to stock price and it is therefore important to be as certain as possible that the value used for the stock price was actually the stock price ruling at the time the option was traded. Failure to do so has the potential to cause substantial errors. If data were collected at regular intervals (such as the daily closing time of the stock market) and if trading in both the stock and option markets took place continuously each day, then a perfect matching of stock and option prices would be ensured. In a high-volume market, such as the Chicago Board Options Exchange, it may not be unreasonable to assume that the closing prices of stock and option are in fact correct matches. It would, however, be dangerous to make such an assumption in relation to the Australian Options Market which, possibly because it is new and probably not yet widely understood, attracts very few

investors compared with other sectors of the capital market. The fact that trading in the underlying stocks is virtually continuous is a necessary but not a sufficient condition for using closing prices.¹¹

Attention has therefore been limited to options traded on those days in the sample period (February 1976 to March 1977 inclusive) when recorded stock prices varied no more than one cent during the day's trading and did not at any time during the day vary more than one cent from the closing price of the previous day.¹² A larger sample could have been obtained if the variation requirement were expressed in terms of a percentage equal to, say, a one cent variation in the lowest-priced stock, but this approach would be dubious because the price of an option is related much more to the absolute rather than to the relative difference between the exercise price and the stock price.¹³

The greater reliability obtained from using this strict selection criterion is of course not without cost. While the total size of the sample is still acceptable by econometric standards, the distribution between the underlying stocks is heavily biased towards WMC, Woodside-Burmah and Bougainville, while BHP did not qualify even once. There is also the possibility that the sample may be biased towards particular sub-periods and indeed this is probably true of CSR but seems not to have been a problem with the other stocks. In any case, if the market did actually behave according to the Black and Scholes model, the basis for selection of the sample would be irrelevant provided that the data were sound. A table showing the number of option prices and trading days included in the sample is given below

TABLE 1

*Number of Option Prices and Trading
Days Included in the Sample*

Company	No. of Option Prices	No. of Trading Days
Woodside-Burmah Oil	271	40
WMC	303	51
CSR	42	12
Bougainville Copper	64	23

5 The Data

Exercise prices, terms to maturity and stock prices were collected from the Sydney Stock Exchange Daily Quote Sheets. For every option in the sample, weekly variance rates of return on adjusted stock prices were calculated for five time periods viz. the previous 26 weeks, the previous 52 weeks and the periods from the date of trade to the first, second and third maturity dates. So that the historical variance rate of return always reflected the latest information available at the time the option was traded, a different weekly series was constructed for each day of the week.

The remaining variable to be considered is the riskless rate of interest which is not strictly observable but for which proxies may be found. The most suitable rate to use is that paid on a riskless instrument whose maturity corresponds to that of the option. However, any data source which could fulfil this requirement exactly would in effect need to consist of rates for all maturities from 1 day to 9 months (even though on any given day only three of these rates would be needed). Not only would such a data source be unlikely to exist but also the formula is often insensitive to the interest rate and, fortunately, this is especially true of the shorter periods to maturity when estimating the riskless rate is particularly hazardous.

The eventual compromise reached was to collect rates for each week as follows:

- (a) The gross redemption yields on the two non-rebatable Commonwealth Government Securities with maturity dates on either side of the required date. (*Source: Bain and Co., Sydney.*)
- (b) The 182-day Treasury Note yield.
(*Source: Reserve Bank of Australia, Monthly Statistical Bulletin, Sydney.*)
- (c) The 91-day Treasury Note yield.
(*Source: Reserve Bank of Australia, Monthly Statistical Bulletin, Sydney.*)

- (d) The mean of the buy and sell yield quotes for 60-day Treasury Notes. (*Source:* Trans City Discounts Ltd., Sydney.)
- (e) The mean of the buy and sell yield quotes for 30-day Treasury Notes. (*Source:* Trans City Discounts Ltd., Sydney.)

Except for those weeks in which the official Treasury Note yields changed, rates were assumed constant throughout the week in which they were recorded. In addition, a constant call money rate was assumed to hold throughout the sample period.¹⁴ By linear interpolation the required annual rates were obtained and then converted to weekly compounding rates.

6 Method of Testing the Model

Using a numerical search procedure Latané and Rendleman [13] solved the Black and Scholes equation in reverse to derive the variance rate of return implied by an actual option price on the assumption that investors are indeed behaving according to the model. These implied variances were then converted to implied standard deviations (ISD's). In principle, all ISD's should be equal for a given underlying stock at a given time regardless of exercise price, term to maturity or riskless rate of interest.¹⁵ In practice, however, this is most unlikely to be true. Firstly, the model itself may be imperfect. Secondly, option and stock prices are discrete variables and, especially in options that are way out-of-the money, this will distort the ISD actually found. Finally, a close estimation of the standard deviation is much more important for the investor in some cases than in others. At this stage of the analysis the first problem is assumed away and there is no obvious method of dealing with the second. Latané and Rendleman deal with the third problem by forming a weighted average implied standard deviation (WISD) each week for each underlying stock using as weights the partial derivative of the pricing formula with respect to the ISD. Thus higher weights are attached to ISD's resulting from trades in which a careful assessment of the stock price volatility was more likely to have been made. In this paper, a modified version of the WISD is calculated for each day in the sample but different tests are applied.

One of the difficulties inherent in this method is that it is possible that no solution will exist in some cases.

Recalling that

$$d_1 = \frac{\ln(x/c) + (r + \frac{1}{2}v^2)t}{v\sqrt{t}}$$

It follows that

$$\lim_{v \rightarrow 0} d_1 = \lim_{v \rightarrow 0} \left[\frac{\ln(x/c) + rt}{v\sqrt{t}} \right] + \frac{1}{2} \lim_{v \rightarrow 0} v\sqrt{t}$$

The first of these terms approaches infinity and the second approaches zero. Thus $d_1 \rightarrow \infty$ as $v \rightarrow 0$. Because $d_2 = d_1 - v\sqrt{t}$, then as $v \rightarrow 0$, $d_2 \rightarrow \infty$ though at a slightly slower rate than d_1 . Since $N(y) \rightarrow 1$ as $y \rightarrow \infty$, an option on even a very low volatility stock should always be worth at least $(x - ce^{-rt})$ according to the valuation formula. Any option price less than this value is inconsistent with the formula and an ISD does not exist. Of the 680 option prices studied, solutions at closing prices could be found in 95.3% of cases and a further 1.5% could be solved for the ISD at the other stock price recorded during the day's trading. However, so that the study as a whole would be based on closing prices and in the absence of any better alternative the 4.7% which were inconsistent had to be ignored.

There must obviously be some degree of error involved in adopting this course but there are reasons for believing that the error is probably negligible. Not only is the proportion of the sample small but also the extent of the inconsistency is low. If the option prices recorded in the inconsistent cases had been higher by an average of only 3.5% all of them would have been consistent and the problem would not exist. However, values of WISD would change very little. The extra ISD's which would then be recorded would be very low but so also would the weights to be attached to them. Galai and Masulis [11] give the partial derivative of the Black and Scholes equation with respect to the variance rate as

$$\frac{\partial w}{\partial v^2} = ce^{-rt} \cdot Z(d_2) \frac{\sqrt{t}}{2v}$$

where $Z(d_2)$ is the standard normal density at d_2 .

$$\text{Thus } \frac{\partial w}{\partial v} = ce^{-rt} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-d_2^2/2} \cdot \sqrt{t}$$

The limiting value of this expression as v approaches zero depends on

$$\lim_{v \rightarrow 0} e^{-d_2^2/2}$$

It was shown previously that $d_2 \rightarrow \infty$ as $v \rightarrow 0$.

$$\text{Thus } \lim_{v \rightarrow 0} e^{-d_2^2/2} = 0$$

Thus both the extra ISD's and the weights attached to them would probably be very small and consequently WISD would be approximately unaffected.

The weighting system used by Latané and Rendleman [13] is not employed here as it is shown in Appendix III that it is inappropriate. Instead, the weighting system used is

$$\text{WISD} = \frac{\sum_{i=1}^n S_i q_i}{\sum_{i=1}^n q_i}$$

where S_i is the ISD of option i and q_i is the weight (defined above) and n is the number of options analysed.

This system embodies the simple meaning normally associated with the term 'weighted average' and it is also shown in Appendix III that this will give consistently higher values for WISD than the system used by Latané and Rendleman.

7 Empirical Results

Using closing prices, implied standard deviations were calculated for all option prices consistent with the model. As there is no analytic solution for v in the Black and Scholes equation an iterative procedure was used and a solution deemed to have been found when the ISD would produce an error of less than \$0.0001 in the option price. Approximately 90% of option prices in the sample occurred on days on which two stock prices were recorded and ISD's were calculated for this group using the other stock price as well. A check of the potential error due to incorrect matching of prices was made by calculating both weighted and unweighted means of the absolute value of the percentage deviation of the ISD based on the alternative stock price from the ISD based on the closing price. The results are shown in the table below.

TABLE 2

*Mean Absolute Percentage Deviation in ISD's
Resulting from Alternative Stock Prices*

Company	Unweighted Mean (%)	Weighted Mean (%)
Woodside-Burmah Oil	4.4	3.6
WMC	4.2	3.4
CSR	2.8	2.3
Bougainville Copper	9.7	8.0

The lower deviation when weights are considered is the expected result; when the ISD is highly sensitive to stock price (given an option price), the option price is relatively insensitive to the estimate of standard deviation. Hence a lower weight is attached. If the criterion for inclusion in the sample were less strict the degree of potential variation would be higher and it would also be expected that the number of option prices inconsistent with the model would rise. Of course, it is possible that very little of this potential would be realised but at this stage the results would appear to justify the more cautious assumption.

For purposes of testing for differences between WISD's and measures of past and future standard deviations, series (1) was formed using only those WISD's resulting from ISD's from at least two of the possible three maturity dates. This marginally reduced the sample to 111 WISD's covering 628 option prices. Series (2) to (6) are the standard deviations of stock price returns for those stocks and on those dates matching series (1) using data from the following time periods:

Series (2): the 26 weeks prior to the date of trade.

Series (3): the 52 weeks prior to the date of trade.

Series (4): from the date of trade to the first (shortest) maturity date where this exceeds six weeks.

Series (5): from the date of trade to the second maturity date.

Series (6): from the date of trade to the third (longest) maturity date.

Thus series (1) comprises the estimates of volatility implied by the model, while (2) and (3) provide alternative estimates based on what was the recent history of share price movements at the time the option was traded. Series (4), (5) and (6) provide estimates of the volatility which actually occurred over the relevant future periods. Series of error terms were then formed for the following reasons:

Series (2) minus series (1) and series (3) minus series (1) :
to test for the extent to which WISD's are in line with historical experience.

Series (4), (5) and (6) minus series (1) :
to test the ability of WISD's to predict future standard deviations.

Series (4), (5) and (6) minus series (2) :
to be a standard of comparison for the error series above.

Series (4), (5) and (6) minus series (3) :
to be a second standard of comparison.

Tables 3 to 6 show the mean errors (correct to four significant figures) with t-statistics in brackets and mean error as a percentage of the variable being "predicted".¹⁶

TABLE 3

*Mean Errors in "Prediction" of Weekly Standard
Deviation Rate of Return on Stock Price*

Company : Woodside-Burmah Oil

Error Series	Mean Error and t-statistic	Mean Error as a Percentage of Mean Value of "Predicted" Variable
(2) - (1)	-.04719 (-20.68)*	-80.9
(3) - (1)	-.04355 (-17.17)*	-70.3
(4) - (1)	-.05695 (-12.24)*	-112.7
(4) - (2)	-.008140 (-2.28)∅	-16.1
(4) - (3)	-.01179 (-4.18)*	-23.3
(5) - (1)	-.05080 (-16.99)*	-92.9
(5) - (2)	-.003607 (-2.18)∅	-6.6
(5) - (3)	-.007248 (-5.29)*	-13.3
(6) - (1)	-.05411 (-22.02)*	-105.5
(6) - (2)	-.006919 (-5.86)*	-13.5
(6) - (3)	-.01056 (-9.02)*	-20.6

Notes: *, ≠, ∅

Significantly different from zero at .1%, 1% and 5% levels respectively.

Sample sizes

Series involving (4) : 16 WISD's; 120 prices.
All other series : 40 WISD's; 266 prices.

TABLE 4

Mean Errors in "Prediction" of Weekly Standard
Deviation Rate of Return on Stock Price

Company : Western Mining Corporation

Error Series	Mean Error and t-statistic	Mean Error as a Percentage of Mean Value of "Predicted" Variable
(2) - (1)	-.02385 (-8.51)*	-54.9
(3) - (1)	-.02387 (-9.98)*	-55.0
(4) - (1)	-.02715 (-8.08)*	-63.0
(4) - (2)	.0006891 (0.24)	1.6
(4) - (3)	.0001517 (0.05)	0.4
(5) - (1)	-.02699 (-12.93)*	-67.0
(5) - (2)	-.003137 (-1.54)	-7.8
(5) - (3)	-.003122 (-2.13)ϕ	-7.8
(6) - (1)	-.02594 (-12.67)*	-62.8
(6) - (2)	-.002094 (-1.25)	-5.1
(6) - (3)	-.002079 (-1.88)ϕ	-5.0

Notes: *, ≠, ϕ

Significantly different from zero at .1%
1% and 5% levels respectively.

Sample sizes

Series involving (4) : 23 WISD's; 141 prices.
All other series : 48 WISD's; 284 prices.

TABLE 5

*Mean Errors in "Prediction" of Weekly Standard
Deviation Rate of Return on Stock Price*

Company : C.S.R.

Error Series	Mean Error and t-statistic	Mean Error as a Percentage of Mean Value of "Predicted" Variable
(2) - (1)	-.01664 (-3.65) [‡]	-52.5
(3) - (1)	-.01511 (-3.51) [‡]	-45.5
(4) - (1)	-.01891	-62.5
(4) - (2)	.003725	12.3
(4) - (3)	.001570	5.2
(5) - (1)	-.01477 (-2.97) [♠]	-44.0
(5) - (2)	.001874 (0.53)	5.6
(5) - (3)	.0003371 (0.10)	1.0
(6) - (1)	-.01232 (-3.64) [‡]	-34.2
(6) - (2)	.004326 (-1.60)	12.0
(6) - (3)	.002789 (1.23)	7.7

Notes: *, †, ♠

Significantly different from zero at .1%, 1% and 5% levels respectively.

Sample sizes

Series involving (4) : 2 WISD's; 6 prices.
All other series : 7 WISD's; 30 prices.

TABLE 6

*Mean Errors in "Prediction" of Weekly Standard
Deviation Rate of Return on Stock Price*

Company : Bougainville Copper

Error Series	Mean Error and t-statistic	Mean Error as a Percentage of Mean Value of "Predicted" Variable
(2) - (1)	-.003285 (-0.97)	-7.2
(3) - (1)	-.003344 (-1.13)	-7.4
(4) - (1)	-.0005825 (-0.08)	-1.1
(4) - (2)	.008930 (1.90)	16.5
(4) - (3)	.009848 (1.39)	18.2
(5) - (1)	-.01225 (-3.84)*	-33.6
(5) - (2)	-.003964 (-5.15)*	-24.6
(5) - (3)	-.008905 (-7.39)*	-24.5
(6) - (1)	-.01008 (-3.47)†	-26.1
(6) - (2)	-.006794 (-3.86)*	-17.6
(6) - (3)	-.006735 (-5.49)*	-17.5

Notes: *, †, ∅

Significantly different from zero at .1%, 1% and 5% levels respectively.

Sample sizes

Series involving (4) : 4 WISD's; 11 prices.
All other series : 16 WISD's; 48 prices.

Clearly, some results are not highly reliable due to the low number of WISD's covered but these have been included mostly for the sake of completeness. Any error series involving series (4) must also be regarded with some suspicion as the periods are quite short even though, in general, the results for such series are quite in line with the other findings. However, many could be considered quite reliable especially those relating to WMC and Woodside-Burmah and a number of inferences may be drawn with reasonable confidence.

The results show a systematic bias in the application of the model to the Australian Options Market. A possible exception to this conclusion is Bougainville Copper, where the sample is drawn almost exclusively from 1977 trades. In comparing WISD's with the two measures of historical standard deviation both Woodside-Burmah and, to a lesser extent, WMC show a highly significant tendency to over-estimate the standard deviation. Options on CSR stock also show the same tendency though possibly not as severely. Thus options tended to be over-priced in relation to what the model would predict given the two historical measures of volatility.

Turning to the ability of WISD's to predict future standard deviations it is clear that in general the historical series were a much more reliable guide than WISD's. Again, however, Bougainville Copper must be regarded as a possible exception to this conclusion. The mean percentage errors are much higher for WISD's than for the historical series and naturally this is reflected in the highly significant t-statistics associated with WISD prediction errors. It should also be noted that in some cases the historical series also predicted with an error which was significantly different from zero, although in nearly all cases the percentage errors were within much more reasonable bounds than the errors associated with WISD's. These results suggest that during the period there has been a shift downward in some of the volatilities and that although the shift may not be that large in percentage terms the variance of the distribution of volatilities is low enough to produce statistically significant errors.

These conclusions are supported by the evidence of Table 7 which gives the F-values which result from pair-wise comparisons of the means of the various error series.

TABLE 7

*F-Values for Differences in
Means of the Error Series*

Error Series Being Compared	Company			
	W.B.O.	W.M.C.	C.S.R.	Bgv. Cop.
(4)-(1) & (4)-(2)	73.79*	40.90*	...	1.72
(4)-(1) & (4)-(3)	73.46*	37.74*	...	1.48
(4)-(2) & (4)-(3)	0.68	0.02	...	0.02
(5)-(1) & (5)-(2)	195.87*	68.12*	8.67 ϕ	0.87
(5)-(1) & (5)-(3)	179.90*	89.42*	7.18 ϕ	1.02
(5)-(2) & (5)-(3)	2.95	0.00	0.11	0.00
(6)-(1) & (6)-(2)	307.31*	83.16*	17.24 \neq	1.00
(6)-(1) & (6)-(3)	262.67*	107.31*	16.06 \neq	1.20
(6)-(2) & (6)-(3)	4.91 ϕ	0.00	0.22	0.00

Notes: *, \neq , ϕ

Significantly different means at .1%, 1% and 5% levels respectively.

Table 7 clearly shows that the performance of WISD in prediction is significantly different to the performance of the historical series, with the exception of Bougainville Copper. The table also shows that, as expected, the series (2) and (3) are very similar except in the case of Woodside-Burmah where a shift in volatility may have caused the two estimates to diverge. Hence, with this exception, there is no significant difference in predictive ability.

As Latané and Rendleman comment, "Market efficiency does not imply that *ex ante* expectations necessarily equal *ex post* realisations. However, efficiency in the options market does suggest that standard deviations which are used to price options bear some semblance to the actual variability of returns." [13, p. 375]. Thus imperfect forecasting is not in itself evidence of market inefficiency, but if it is consistently and grossly inferior to a naive hypothesis based on historical data, it might possibly be counted as such.

The most interesting features of the results lie in a comparison between apparent pricing behaviour in 1976 and 1977. Tables 8 - 11 disaggregate the results given in Tables 3 - 6 (respectively) into the 1976 and 1977 components. The final column of each table gives the F-value to test for differences in mean errors in the two sub-periods.

TABLE 8

*Mean Errors in "Prediction" of Weekly Standard Deviation
Rate of Return of Stock Price in 1976 and 1977*

Company : Woodside-Burmah Oil

Error Series	1976		1977		F-Value
	Mean Error	Mean error as % of mean value of "Predicted" Variable	Mean Error	Mean error as % of mean value of "Predicted" Variable	
(2)-(1)	-.05054	-82.8	-.03382	-71.6	11.09 [≠]
(3)-(1)	-.04775	-74.8	-.02676	-49.3	15.39*
(4)-(1)	-.06240	-121.5	-.03335	-70.8	10.23 [≠]
(4)-(2)	-.008675	-16.9	-.005823	-12.4	0.10
(4)-(3)	-.01129	-22.0	-.01394	-29.6	0.14
(5)-(1)	-.05557	-99.2	-.03173	-64.3	13.89*
(5)-(2)	-.005031	-9.0	-.002090	4.2	3.23
(5)-(3)	-.007818	-14.0	-.004966	-10.1	0.71
(6)-(1)	-.05737	-106.0	-.04108	-102.7	8.63 [≠]
(6)-(2)	-.006833	-12.6	-.007263	-18.2	0.02
(6)-(3)	-.009621	-17.8	-.01432	-35.8	2.76

Notes: *, ≠, ∅

Significantly different means at .1%, 1% and 5% levels respectively.

Sample sizes

1976 - Series involving (4) : 13 WISD's; 101 prices.
All other series : 32 WISD's; 225 prices.
1977 - Series involving (4) : 3 WISD's; 19 prices.
All other series : 8 WISD's; 41 prices.

TABLE 10

*Mean Errors in "Prediction" of Weekly Standard Deviation
Rate of Return on Stock Price in 1976 and 1977*

Company : C.S.R.

Error Series	1976		1977	
	Mean Error	Mean error as % of mean value of "Predicted" Variable	Mean Error	Mean error as % of mean value of "Predicted" Variable
(2)-(1)	-.01998	-63.1	-.008305	-26.2
(3)-(1)	-.01915	-58.9	-.005005	-14.3
(5)-(1)	-.01674	-47.9	-.009840	-32.6
(5)-(2)	-.003238	9.3	-.001535	-5.1
(5)-(3)	.002406	6.9	-.004835	-16.0
(6)-(1)	-.01475	-40.0	-.006240	-18.5
(6)-(2)	.005230	14.2	.002065	6.1
(6)-(3)	.004398	11.9	-.001235	-3.7

Notes: Sample sizes: 1976 : 5 WISD's; 24 prices.
1977 : 2 WISD's; 6 prices.

There are no observations for series (4) in 1977.

TABLE 11

*Mean Errors in "Prediction" of Weekly Standard Deviation
Rate of Return on Stock Price in 1976 and 1977*

Company : Bougainville Copper

Error Series	1976		1977		F-Value
	Mean Error	Mean error as % of mean value of "Predicted" Variable	Mean Error	Mean error as % of mean value of "Predicted" Variable	
(2)-(1)	-.01765	-43.1	.00003077	0.1	5.79 ^φ
(3)-(1)	-.01136	-24.0	-.001494	-3.3	1.92
(5)-(1)	-.02035	-53.1	-.01038	-28.8	1.66
(5)-(2)	-.002700	-7.1	-.01041	-28.9	3.78
(5)-(3)	-.008993	-23.5	-.008885	-24.7	0.00
(6)-(1)	-.02231	-61.4	-.007257	-18.6	5.73 ^φ
(6)-(2)	-.004653	-12.8	-.007288	-18.6	0.35
(6)-(3)	-.01095	-30.2	-.005763	-14.7	3.35

Notes: *, †, φ Significantly different means at .1%, 1%, 5% levels respectively.

Sample sizes 1976 : 3 WISD's; 10 prices.
1977 : 13 WISD's; 38 prices.

There are no observations for series (4) in 1976.

The relationship between WISD and the historical series (2) and (3) is noticeably closer in 1977 than in the previous year, although the error terms are still large in many cases and with one exception remain firmly on the side of over-estimation. The difference in means is statistically significant for both error series in the cases of Woodside-Burmah and WMC and for one of the two series in the case of Bougainville Copper. The movement in the other error term for Bougainville is in the same direction though it is not statistically significant, while for CSR the low number of observations in 1977 precludes a meaningful F-test but the little evidence which is available is consistent with the evidence for the other stocks.

In terms of the ability of WISD's to predict future standard deviations a similar change has occurred. Percentage errors are still large and WISD's are still consistently over-predicting, but in all cases the mean errors are smaller and most of these differences between the two sub-periods are statistically significant. As expected, the historical series appear to predict with about equal reliability, and only in the case of WMC are the mean prediction errors significantly different in the two years.

Taken together, these results support two main conclusions. Firstly, the model consistently produces estimates of standard deviations which in almost all cases are too high in relation to the estimates of historical and future standard deviations which were used. This indicates that the model tends to underprice options or, equivalently, that the market tends towards over-pricing. Secondly, market participants may have behaved in significantly different patterns in the two sub-periods.

8 Sources of Model Bias and Market Efficiency

One possible source of bias is the criterion used to select the sample of option prices. It is conceivable that the trading days which fall within the sample were unrepresentative of the trading days in the period as a whole, as the relevant stock prices on the selected days were more stable than those recorded at other times. However, if this momentary stability were to have any effect on market participants it could reasonably be expected to result in lower estimates of stock price volatility and would therefore be more likely to result in a bias in the opposite direction to that actually found.

The assumptions made to derive the model provide other possible sources of bias. Whereas the model assumes a non-dividend-paying stock, all stocks except Woodside-Burmah do in fact pay dividends. However, Merton [14] has shown that an option on a dividend-paying stock will never be worth more than an equivalent option on a stock which does not pay dividends. Hence this source of bias should be leading the model towards over- rather than under-pricing. Similarly, the model assumes zero taxes and transactions costs whereas in fact both exist. Scholes [19] has shown that the existence of income taxes should lead to over-estimation of prices at least in the case where there are no capital gains taxes, while Noti [17] has shown that ignoring the taxation of interest receipts will have a similar effect. The question of transactions costs seems not to have been explicitly considered in the theoretical literature but a simplified case of commission charges on exercising is easily incorporated. Assuming that these charges are some proportion, b , of the exercise price, an option at maturity will be worth zero to the holder if $x - (1+b)c < 0$. The boundary conditions for the partial differential equation therefore become

$$\begin{aligned} w(x, t^*) &= x - (1 + b)c & \text{for } x \geq (1 + b)c \\ w(x, t^*) &= 0 & \text{for } x < (1 + b)c \end{aligned}$$

This is equivalent to a higher exercise price and hence should result in lower option prices. Consequently these sources of bias also are unhelpful in explaining the bias evident in the empirical results.

The assumption that options are of the European type, whereas in the Australian market they are in fact of the American type, has some chance of explaining the bias. Merton [14] has shown that ignoring the right to exercise before maturity should cause the model to under-estimate option prices. However, Merton was also able to prove in the same article that whether an option is of the European or American type is irrelevant if a stock pays no dividends and hence this argument cannot be advanced to explain the apparent tendency of the market to over-price Woodside-Burmah options. For this case, resort would have to be made to the possibility of a 'jump' process to describe stock price dynamics rather than the diffusion-type process assumed in the model.¹⁷ This may not be entirely unreasonable given the nature of Woodside-Burmah's business.

The other possible explanation of the bias is, of course, simply that the market was wrong, particularly in the earlier stages of operation. Because an option is quite a sophisticated financial instrument it could be inferred that a process of market learning may have occurred and that as time passed option buyers became more cautious in forming their estimates of future stock price movement. Thus ISD's declined to a range closer to, but still higher than, the range suggested by historical standard deviations with consequent improvements in forecasting ability.

If options have indeed been over-priced by the market one would expect that option buyers would be experiencing difficulties and indeed such independent evidence as there is points in this direction. The losses have been large enough to attract the attention of at least one journalist who wrote, "It is easy to lose money on options as many investors in the relatively new Sydney market will testify from bitter experience. Those who lost out were mainly option buyers ... they did not have the experience of falling markets to give them the necessary degree of caution." [8, p.14]. However, it is not yet possible to say whether the recent apparent changes in behaviour will be permanent.

9 Conclusions and Suggestions for Future Research

The basic Black and Scholes model of option evaluation produced implied estimates of stock price volatility which were significantly upward-biased. This in turn suggests that the model is under-pricing options and/or that the market is over-pricing them. Latané and Rendleman [13] reached a similar conclusion for the Chicago Board Options Exchange and would have had more confidence in their conclusion had an inappropriate weighting system not been used.¹⁸

The more interesting question is to ask to what extent this bias was due to the model's assumptions diverging systematically from the facts of the case and to what extent it may have been due to sub-optimal behaviour on the part of option buyers. The market itself appears to have at least partly answered this question. In 1977 the behaviour of the market is consistent with a trend towards greater buyer caution in options pricing as evidenced by an apparently greater willingness to consider the historical behaviour of stock prices.

The findings of this paper also suggest a number of areas for future research. Included here would be the development of a formal model of the market-learning process, further testing of model bias and testing for the sensitivity of the results to alternative weighting systems used to derive WISD.

APPENDIX I

*Transaction Costs on the
Australian Options Market*

Commission is payable on both the buying and selling of options as shown in the schedule:

On the first \$5,000 of premium : 2.5%

On the next \$10,000 of premium : 2.0%

On the next \$35,000 of premium : 1.5%

On that amount exceeding \$50,000: 1.0%

All transactions are subject to a minimum
Commission charge of \$20.

Clearing charges are also payable. Stamp duty is payable at the rate of 3¢ for every \$100.

Normal commission charges and other transactions costs of share transfers are payable by both sides on exercise.

APPENDIX II

Relaxing the Assumptions of the Black and Scholes Model

Some of the advances that have been made in the theory of options pricing are given in this appendix. The numbering of the subsections follows the numbering of the list of assumptions given in the main body of the paper.

- (i) A stochastic rate of interest rather than a constant rate - see Merton [14].
- (ii) Allowance for a 'jump' process of stock price dynamics rather than a diffusion type process - see Cox and Ross [9] and Merton [16].
- (iii) Adjustment of the model to allow for taxes - see Noti [17] and Scholes [19].
- (iv) Thorpe (as reported in Smith [21]) has suggested that the assumption of no restrictions on the use of short sale proceeds is not necessary.
- (v) Allowance for continuous dividend payments has been discussed in Merton [14] and in a slightly different context in Merton [15] and it has been found that no closed form solution is available for the resulting equation in the more realistic cases. Schwartz [20] has developed a numerical technique and Black [5] has suggested approximate adjustments which might be made.
- (vi) Merton [14] has shown that it will never pay to exercise an American option prematurely if the stock pays no dividends. Thus provided assumption (v) is retained, (vi) may be discarded. If dividends are paid at discrete time intervals it may pay to exercise before maturity but only just prior to the ex-dividend date.

APPENDIX III

Weighting System Used for WISD

The weighting system used in [13] is, with a change of notation,

$$\text{WISD} = \frac{\sqrt{\sum_{i=1}^n S_i^2 q_i^2}}{\sum_{i=1}^n q_i}$$

where S_i is the ISD of option i

and q_i is the weight

and n is the number of options analysed.

This weighting system can be demonstrated to be inappropriate.¹⁹ If the model proved to be a perfect description of reality all ISD's on a given underlying stock at a given point in time would be equal. It is logical to require in these circumstances that WISD should also be equal to this single value. The weighting system above does not give this result except in the trivial case of $n=1$. In using this system the authors of [13] appear to have implicitly assumed that each ISD is based upon a different set of data; this is clearly inconsistent with the efficient markets assumptions underlying the Black and Scholes model. In an efficient market every price reflects all known information.

While there is probably not a unique best method some possible weighting systems consistent with these arguments are:²⁰

$$\frac{\sum_{i=1}^n S_i q_i}{\sum_{i=1}^n q_i}$$

(1)

$$\sqrt{\frac{\sum_{i=1}^n S_i^2 q_i^2}{\sum_{i=1}^n q_i^2}} \quad (2)$$

$$\sqrt{\frac{\sum_{i=1}^n S_i^2 q_i}{\sum_{i=1}^n q_i}} \quad (3)$$

While (1) is the weighted mean of the ISD's, (3) is the square root of the weighted mean of the implied variances. In this comparison the former appears superior as the weights are calculated in terms of the standard deviation and are intended to be weights on the implied standard deviations while (3) applies the weights to the variances. In choosing between (1) and (2), two factors deserve mention. Firstly, (2) gives greater weight to larger ISD's. This can best be seen by considering the three simple cases of $n=2$, $s_1=k$, $s_2=1.5k$ (where k is any positive constant) and (a) $q_1=3$, $q_2=1$, (b) $q_1=2$, $q_2=2$ and (c) $q_1=1$, $q_2=3$. Formula (1) gives WISD's of (a) $1.125k$, (b) $1.25k$, and (c) $1.375k$, while (2) gives WISD's of approximately (a) $1.061k$, (b) $1.275k$ and (c) $1.458k$. As there is no reason to believe that reliability is related to the size of the estimate of the ISD, (1) is to be preferred to (2). Secondly, there is no reason to prefer a more complex weighting system if a simpler formulation fulfils the requirements at least as well. Again, therefore, a preference is held for the use of (1) relative to (3) as the former embodies the simple meaning normally given to the term "weighted average".

It is easily shown that the use of the weightingsystem defined by equation (1) will always produce a higher value for WISD than the system used in [13] for $n>1$. Both systems have the same denominator so any difference between them will be due to differences in the numerator. The numerator in [13] is

$$\sqrt{\frac{\sum_{i=1}^n S_i^2 q_i^2}{\sum_{i=1}^n q_i^2}} = \sqrt{\left[\frac{\sum_{i=1}^n S_i q_i}{\sum_{i=1}^n q_i} \right]^2 - J}$$

where J is the sum of the cross-product terms all of which are positive.

Thus the numerator is less than

$$\sqrt{\sum_{i=1}^n S_i q_i}^2$$

$$= \sum_{i=1}^n S_i q_i, \text{ the numerator of (1).}$$

The authors of [13] reported a tendency in the market towards over-pricing. The above proof suggests that this tendency was under-stated.

FOOTNOTES

- 1 See the review article by Smith [21] for a bibliography of the academic literature. Many new papers have however become available since that article was published. For examples in the commercial literature see [4], [5] and [12]. Even magazines and newspapers have recently commented on options - see [8], [10] and [22].
- 2 Although Noti [17] has illustrated the effects of taxable interest income on options pricing using Australian data.
- 3 A 'conventional' stock option differs from an 'exchange-traded' option in that it is essentially a private contract between writer and buyer who negotiate every aspect of the agreement. Trading in secondary markets is therefore very limited. 'Conventional' options are also usually exercisable only at maturity and are normally dividend-protected. Further details may be found in [2] and [12].
- 4 The name was changed to Woodside Petroleum Ltd. in May 1977.
- 5 See Appendix I for a table showing transactions costs.
- 6 For further details see for example [2] and [12, pp. 70-76].
- 7 'Spreading' of options (for example, simultaneously buying and writing options with different exercise prices) is another useful strategy - See [5] and [12, pp. 78-91]. Options may also be used to limit tax liability - See [12, pp. 92-141].
- 8 See Ross [18] for a rigorous treatment of this approach to asset pricing.
- 9 Attempts to evaluate options and warrants by statistical estimation have a lengthy history. Gastineau [12] presents a summary of statistical models developed by Shelton and Kassouf. Bird and Henfrey [4] also present work in this tradition.
- 10 This approach is discussed in more detail in Section 6 and in Appendix III.

- 11 The necessary condition is assumed to hold throughout the period studied.
- 12 Even this strict requirement is imperfect. Variation is still present and the method will also be invalid to the extent that trading in the underlying stocks is discontinuous.
- 13 Referring back to the valuation formula, x and c are constants at any given time and will usually be of about the same order of magnitude. $N(d_1)$ and $N(d_2)$ will both be positive proper fractions while the term e^{-rt} converts c to a present value based on the riskless rate, r , and will also be a positive proper fraction. The formula could therefore be expressed as

$$f_1 \cdot x - f_2 \cdot c$$

where f_1 and f_2 are positive proper fractions.

- 14 This is the most ambitious assumption in theory but in the present context is of virtually no significance.
- 15 If the model is interpreted very strictly, so that the interest rate and the variance rate of return are fixed constants for all time, then even the limitation "at a given time" could be discarded. Arguments in this paragraph mostly follow those of [13].
- 16 The term "prediction" is being used here to cover also the closeness of the relationship between WISD and the historical series (that is (2)-(1) and (3)-(1)).
- 17 A 'jump' process allows for a positive probability of a sudden movement in stock price - See [9] and [16].
- 18 See Appendix III for a proof of this proposition.
- 19 I wish to thank Geoff Lewis for assistance in clarifying a number of the arguments in this Appendix.
- 20 It is possible that some definition of weighted median may also be suitable and possibly even superior to the formulae listed here. However, there are serious practical problems in choosing an appropriate definition of a weighted median when n is small.

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