

# WORKING PAPERS

RELATIVE PRICE EFFECTS AND THE DEMAND

FOR IMPORTS

by

Ulrich R. Kohli

No. 41

July 1980

# IN ECONOMICS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF SYDNEY

RELATIVE PRICE EFFECTS AND THE DEMAND

FOR IMPORTS

by

Ulrich R. Kohli

No. 41

July 1980

National Library of Australia Card Number and ISBN  
0 86837 018 5

## RELATIVE PRICE EFFECTS AND THE DEMAND FOR IMPORTS\*

### 1 Introduction

Estimation of import demand functions has received a great deal of attention in the empirical literature of international trade. In most studies, the quantity of imports is taken as a function of the price of imports relative to domestic prices, and of some activity variable.<sup>1</sup> Such a representation can easily be derived from production theory or utility theory, depending on whether imports are viewed as intermediate goods or as end-products. It should be noted, however, that this treatment of imports differs substantially from the viewpoint inherent in the Heckscher-Ohlin-Samuelson (HOS) model of international trade theory in which (i) foreign goods are perfect substitutes for domestic goods, (ii) commodities are necessarily end-products, and (iii) they can all be divided between importables and exportables.

We have examined elsewhere the trade theory that is implicit in much of the empirical work on the demand for imports.<sup>2</sup> In this paper we want to discuss some of the issues that arise in empirical applications and that are related to the choice of explanatory variables. Which data should be used to measure domestic prices, domestic activity? Should the same domestic price series be used to deflate import prices and domestic activity? The theory provides little guidance on this subject. Relatively little attention is devoted to these questions in empirical work; description of variables being used is often relegated to a footnote, while availability plays a major role in the selection process. Amongst the variables that have been used in the estimation of aggregated import demand functions one finds: (i) domestic prices - wholesale price, GNP deflator, domestic factor rental prices; and,

(ii) activity - GNP, national income, gross output, domestic factor utilization.

The issue of data selection is important when assessing estimated price and quantity effects. These are only meaningful insofar one does not dissociate them from the framework within which they were defined. For example, an own price elasticity of demand indicates, ceteris paribus, how the quantity demanded is affected by variations in the corresponding price. It merely indicates a partial effect; as to the total effect, the result might be quite different. In order to assess meaningfully a partial elasticity, it is essential to bear in mind what the "other things being held constant" actually are. Although the reader might feel that this qualification is obvious, it tends to be overlooked when estimated elasticities are used to assess the effects of changes in exogenous variables in a partial equilibrium framework (e.g. to evaluate the effect of a change in import tariffs on the quantity of imports), or when elasticity estimates drawn from different sources are compared with each other.<sup>3</sup>

In this paper we examine how various definitions of import price and quantity elasticities derived from the same structural model can meaningfully be compared with each other. We find that large differences between the various elasticities are possible, especially if the country's foreign trade sector is large. One cannot rule out the possibility of the own partial price elasticity of imports being positive in certain cases, even if the underlying model is well behaved. We also examine whether some specifications are more relevant than others for economic analysis.

Throughout this paper imports are treated as intermediate goods being inputs to the domestic technology. This treatment is in accordance with the fact that most internationally traded goods are intermediate goods; moreover finished imports are generally still subject to domestic handling, transportation and retail changes before reaching final demand.

The model we use is a relatively simple one, and it allows us to distinguish between only a limited number of price and activity variables. We ignore the effects of taxation, the possibility of market disequilibrium, or the existence of a nontraded good sector. The model is nevertheless sufficiently flexible to draw a distinction between several of the most important price and quantity variables being used in practice. Our approach is entirely deterministic, that is, we do not discuss any of the statistical problems that might be linked to some of the specifications. For simplicity we assume that the home country is a small open economy, i.e. it faces infinitely elastic foreign import supply and export demand functions. This permits omission of foreign demand and supply considerations from our model; this assumption is invoked, at least implicitly, in most of the empirical work in the area.

The remainder of this paper is organized as follows. In the next section we examine the case where the technology exhibits constant returns to scale; this enables us to show in a simple context how the various elasticities we define are related to each other. We turn to the more complex case of variable returns to scale in section 3. Some concluding comments are given in the last section.

## 2 Constant Returns to Scale

We assume that imports are inputs to the technology. They are used in conjunction with a composite domestic primary factor. In this section we assume that the technology exhibits constant returns to scale. Import decisions are assumed to be taken by profit maximizing firms operating under perfect competition in all markets.

Let the technology be given by the following aggregate production

function:

$$q_Y = f(q_M, q_D) \quad (1)$$

where the  $q$ 's indicate quantities, and the labels  $Y$ ,  $M$  and  $D$  stand for domestic gross output, imports and the domestic composite factor respectively. We assume that  $f(\cdot)$  is defined for all nonnegative input quantities, monotonically increasing, concave, at least twice continuously differentiable, and that it exhibits decreasing marginal returns. In addition, since constant returns to scale are assumed,  $f(\cdot)$  is linearly homogeneous.

It is convenient at times to represent the technology by a revenue function defined as follows:

$$R(q_M, q_D, p_Y) = f(q_M, q_D) p_Y \quad (2)$$

where  $p_Y$  is the price of  $Y$ . Alternatively,  $R(\cdot)$  can be viewed as the solution of the following (trivial) revenue maximization problem:

$$R(q_M, q_D, p_Y) = \max_{q_Y} \{p_Y q_Y : q_Y \leq f(q_M, q_D); q_Y \geq 0\} \quad (3)$$

for  $p_Y > 0$ , and  $q_M, q_D \geq 0$ . Since optimization is assumed, it is also possible to represent the technology by a cost function defined as:

$$C(p_M, p_D, q_Y) = \min_{q_M, q_D} \{p_M q_M + p_D q_D : f(q_M, q_D) \geq q_Y; q_M, q_D \geq 0\} \quad (4)$$

for  $p_M, p_D > 0$ ,  $q_Y \geq 0$ , and where  $p_M$  and  $p_D$  are the price of imports and the rental price of the domestic composite factor respectively. Under the assumptions made on  $f(\cdot)$ ,  $C(\cdot)$  is nondecreasing, linearly homogeneous and concave in prices; it is monotonically increasing in the quantity of gross output, and, under constant returns to scale, it is also linearly homogeneous in this variable:<sup>4</sup>

$$C(p_M, p_D, q_Y) = c(p_M, p_D) q_Y \quad (5)$$

where  $c(\cdot)$  is the unit cost function of gross output.

The demand for imports and for the domestic factor can be obtained by differentiation of  $C(\cdot)$ , a result implied by Shephard's (1953) lemma:

$$q_M = \partial C(\cdot) / \partial p_M = [\partial c(\cdot) / \partial p_M] q_Y = c_M(p_M, p_D) q_Y \quad (6)$$

$$q_D = \partial C(\cdot) / \partial p_D = [\partial c(\cdot) / \partial p_D] q_Y = c_D(p_M, p_D) q_Y \quad (7)$$

where the subscripts attached to functions indicate partial derivatives with respect to the corresponding argument. Under linear homogeneity and competitive behaviour we also have:

$$p_Y = \partial C(\cdot) / \partial q_Y = c(p_M, p_D) \quad (8)$$

Differentiating (6)-(8) logarithmically and using matrix notation, we get:

$$\begin{bmatrix} d \ln q_M \\ d \ln q_D \\ d \ln p_Y \end{bmatrix} = \begin{bmatrix} C_{MM} p_M / q_M & C_{MD} p_D / q_M & C_{MY} q_Y / q_M \\ C_{DM} p_M / q_D & C_{DD} p_D / q_D & C_{DY} q_Y / q_D \\ C_{YM} p_M / p_Y & C_{YD} p_D / p_Y & C_{YY} q_Y / p_Y \end{bmatrix} \begin{bmatrix} d \ln p_M \\ d \ln p_D \\ d \ln q_Y \end{bmatrix} \quad (9)$$

where  $C_{MD} = \partial^2 C(\cdot) / (\partial p_M \partial p_D)$  and so on.

From the linear homogeneity of  $C(\cdot)$  with respect to gross output it follows that:

$$C_{MY} q_Y / q_M = 1, \quad C_{DY} q_Y / q_D = 1, \quad C_{YY} q_Y / p_Y = 0 \quad (10)$$

Linear homogeneity of  $C(\cdot)$  with respect to prices implies that:

$$C_{MM} p_M / q_M + C_{MD} p_D / q_M = 0, \quad C_{DM} p_M / q_D + C_{DD} p_D / q_D = 0 \quad (11)$$

From (6)-(8) it follows that:

$$C_{YM}p_M/p_Y = \alpha, \quad C_{YD}p_D/p_Y = 1-\alpha \quad (12)$$

where  $\alpha = p_M q_M / (p_Y q_Y)$ ,  $0 < \alpha < 1$ . Symmetry of the Hessian, finally, implies that:

$$C_{DM}p_M/q_D = (C_{MD}p_D/q_M)\alpha/(1-\alpha) . \quad (13)$$

Let us denote  $C_{MM}p_M/q_M$  by  $(1-\alpha)^2 \epsilon_{MM}$ ,  $\epsilon_{MM} < 0$ .<sup>5</sup> Making use of (10)-(13), (9) can be rewritten as:

$$\begin{bmatrix} d \ln q_M \\ d \ln q_D \\ d \ln p_Y \end{bmatrix} = \begin{bmatrix} (1-\alpha)^2 \epsilon_{MM} & -(1-\alpha)^2 \epsilon_{MM} & 1 \\ -\alpha(1-\alpha) \epsilon_{MM} & \alpha(1-\alpha) \epsilon_{MM} & 1 \\ \alpha & 1-\alpha & 0 \end{bmatrix} \begin{bmatrix} d \ln p_M \\ d \ln p_D \\ d \ln q_Y \end{bmatrix} . \quad (14)$$

Thus, once that all prior information has been taken into account, the number of unknown parameters reduces to two ( $\alpha$  and  $\epsilon_{MM}$ ).

Had we used (2) instead of (4) to represent the technology, the inverse input demand functions could have been derived as follows:

$$p_M = \partial R(\cdot) / \partial q_M = f_M(q_M, q_D) p_Y \quad (15)$$

$$p_D = \partial R(\cdot) / \partial q_D = f_D(q_M, q_D) p_Y \quad (16)$$

and obviously:

$$q_Y = \partial R(\cdot) / \partial p_Y = f(q_M, q_D) . \quad (17)$$

Logarithmic differentiation of (15)-(17) yields:

$$\begin{bmatrix} d \ln p_M \\ d \ln p_D \\ d \ln q_Y \end{bmatrix} = \begin{bmatrix} R_{MM}q_M/p_M & R_{MD}q_D/p_M & R_{MY}p_Y/p_M \\ R_{DM}q_M/p_D & R_{DD}q_D/p_D & R_{DY}p_Y/p_D \\ R_{YM}q_M/q_Y & R_{YD}q_D/q_Y & R_{YY}p_Y/q_Y \end{bmatrix} \begin{bmatrix} d \ln q_M \\ d \ln q_D \\ d \ln p_Y \end{bmatrix} \quad (18)$$

where  $R_{MD} = \partial^2 R(\cdot) / (\partial q_M \partial q_D)$  and so on. In view of (14), (18) is equal to:

$$\begin{bmatrix} d \ln p_M \\ d \ln p_D \\ d \ln q_Y \end{bmatrix} = \begin{bmatrix} \epsilon_{MM}^{-1} & -\epsilon_{MM}^{-1} & 1 \\ -\epsilon_{MM}^{-1} \alpha / (1-\alpha) & \epsilon_{MM}^{-1} \alpha / (1-\alpha) & 1 \\ \alpha & 1-\alpha & 0 \end{bmatrix} \begin{bmatrix} d \ln q_M \\ d \ln q_D \\ d \ln p_Y \end{bmatrix}. \quad (19)$$

Comparing (14) with (19), one clearly sees how the direct own price elasticity of the demand for imports is related to the corresponding inverse elasticity; we have:

$$\partial \ln p_M(q_M, q_D, p_Y) / \partial \ln q_M = (1-\alpha)^2 [\partial \ln q_M(p_M, p_D, q_Y) / \partial \ln p_M]^{-1}. \quad (20)$$

(6) and (15) are two examples of how the demand for imports can be modelled, and (20) indicates how the corresponding price elasticities are related to each other. In what follows, we ignore inverse formulations such as (15), and we concentrate instead on direct formulations.<sup>6</sup> Even then there is a large number of possible specifications. We examine the following ones which are particularly relevant to empirical work:

$$\begin{aligned} q_M &= q_M(p_M, p_D, q_Y) & \text{I} \\ q_M &= q_M(p_M, p_Y, q_D) & \text{II} \\ q_M &= q_M(p_M, p_D, q_D) & \text{III} \\ q_M &= q_M(p_M, p_Y, q_Y) & \text{IV} \\ q_M &= q_M(p_M, p_Y, q_Z) & \text{V} \\ q_M &= q_M(p_M, p_Y, q_G) & \text{VI} \\ q_M &= q_M(p_M, p_G, q_G) & \text{VII} \end{aligned}$$

where  $q_Z = (p_Y q_Y - p_M q_M) / p_Y$  is real national income (alternatively real domestic value added),  $q_G = q_Y - q_M$  is real national product (conventional

measure), and  $p_G = (p_Y q_Y - p_M q_M)/q_G$  is the national product deflator. It should be emphasized that the functionals  $q_M(\cdot)$  will in general vary with the specification; for notational simplicity the demand for imports is denoted by  $q_M(\cdot)$  in all cases, but to avoid any confusions we will always indicate the relevant arguments.

Formulation (I) has been employed by Burgess (1974b) in estimating the U.S. demand for imports. Formulation (II) is similar to the one used by Kohli (1978) to estimate Canadian import demand functions. Specification (IV) has been applied to a number of countries by Adams et al. (1969), and by Meyer-zu-Schlochtern and Yajima (1970) for instance. Formulation (VI) is probably the one that is the most widely used in empirical work, with the wholesale price index for  $p_Y$ : see Ball and Marwah (1962) or Houthakker and Magee (1969) for example. Formulations (V) and (VII) are similar to (VI), except that the same price variable (the price of output and the GNP deflator respectively) is used to measure domestic prices and to deflate national product. The latter approach is used by Miller and Fratiani (1974).

For every one of the seven specifications, one can define an own partial price elasticity of import demand  $\partial \ln q_M(\cdot)/\partial \ln p_M$ , and corresponding cross price and activity elasticities. We now want to examine how these elasticities vary with the specifications.

Specification (I) has already been examined. From (14) we can write:

$$d \ln q_M = (1-\alpha)^2 \epsilon_{MM} d \ln p_M - (1-\alpha)^2 \epsilon_{MM} d \ln p_D + d \ln q_Y \quad (21)$$

Using (14) or (19) as a starting point, it is easy, although somewhat tedious, to obtain the following differentials:

$$d \ln q_M = \epsilon_{MM} d \ln p_M - \epsilon_{MM} d \ln p_Y + d \ln q_D \quad (22)$$

$$d \ln q_M = (1-\alpha) \epsilon_{MM} d \ln p_M - (1-\alpha) \epsilon_{MM} d \ln p_D + d \ln q_D \quad (23)$$

$$d \ln q_M = (1-\alpha)\epsilon_{MM} d \ln p_M - (1-\alpha)\epsilon_{MM} d \ln p_Y + d \ln q_Y \quad (24)$$

$$d \ln q_M = [\epsilon_{MM} + \alpha/(1-\alpha)] d \ln p_M - [\epsilon_{MM} + \alpha/(1-\alpha)] d \ln p_Y + d \ln q_Z \quad (25)$$

$$d \ln q_M = (1-\alpha)/(1-\delta)\epsilon_{MM} d \ln p_M - (1-\alpha)/(1-\delta)\epsilon_{MM} d \ln p_Y + d \ln q_G \quad (26)$$

$$d \ln q_M = (1-\alpha)^2 \epsilon_{MM} / [(1-\delta) - (\delta-\alpha)(1-\alpha)\epsilon_{MM}] (d \ln p_M - d \ln p_G) + d \ln q_G \quad (27)$$

where  $\delta = q_M/q_Y = \alpha(p_Y/p_M) > 0$ ; <sup>7</sup> the quantity  $(\alpha-\delta)$  thus is a measure of the extent to which the current relative price of imports differs from its base period value.

The various partial price and quantity elasticities can be read directly from (21)-(27). Several comments are in order. First, it appears that all demand functions are homogeneous of degree zero in prices; it is therefore legitimate to include the two price variables as a ratio. Secondly, under constant returns to scale, all activity elasticities are unity. More important though is the result that the own partial price elasticity of the demand for imports  $\partial \ln q_M(\cdot) / \partial \ln p_M$  varies widely from specification to specification. The variations are particularly large for  $\alpha$  large (i.e. a country with a large foreign sector, such as most small open economies), and for  $\delta$  different from  $\alpha$ ; as an illustration, if  $\epsilon_{MM} = -1$ ,  $\alpha = .4$  and  $\delta = .3$ , the own price elasticities would vary from  $-.33$  to  $-1$ . The meaning of  $\epsilon_{MM}$  also becomes clear: it is equal to the own price elasticity of the demand for imports for the second specification.

One can easily see that if  $\delta = \alpha$ , formulation (VI) becomes equivalent to formulation (II), and (VII) becomes identical with (III). This is because in that case  $q_G = q_D$ , and  $p_G = p_D$ . Real national product in this model is simply an index for the quantity of the domestic composite factor, and the national product deflator is an index for the rental price of D. Because of the functional forms that are being used to construct  $q_G$  and  $p_G$  (direct Laspeyres and direct Paasche respectively), however, these are relatively poor indexes for  $q_D$  and  $p_D$  if  $p_M$  varies substantially from  $p_Y$ , i.e. if  $\delta$  very different from  $\alpha$ . <sup>8</sup> As a

result, the price elasticities corresponding to (VI) and (VII) might differ substantially from those of (II) and (III); actually, one cannot rule out the possibility (for  $\delta$  sufficiently larger than  $\alpha$ ) of the own price elasticity of imports being positive should real national product (conventional measure) be used as the activity variable, even if the underlying technology is well behaved, i.e. even if cost function (4) is concave.

It is not possible in general to rank according to size the price elasticities corresponding to the different specifications. Ranking depends on the values of  $\epsilon_{MM}$ ,  $\alpha$  and  $\delta$ . Since  $0 < \alpha < 1$ , however, we can assert that:

$$\epsilon_{MM}^{II} < \epsilon_{MM}^{III} = \epsilon_{MM}^{IV} < \epsilon_{MM}^I < 0, \quad \epsilon_{MM}^{II} < \epsilon_{MM}^V \quad (28)$$

where the upperscript refers to the specification. Thus one cannot rule out the possibility that  $\epsilon_{MM}^V > 0$ . Regarding  $\epsilon_{MM}^{VI}$  and  $\epsilon_{MM}^{VIII}$ , they can take any value between  $-\infty$  and  $+\infty$ .

One question that comes to mind is which one of the seven formulations we have considered is the most meaningful. In a deterministic framework, it makes no difference which specification is retained since any six can be obtained from the seventh one. All seven specifications describe equally well the same technology. One might argue, however, that some formulations are more useful than others in the sense that the elasticities yielded are readily usable for economic analysis in a partial equilibrium framework, without further manipulations. Estimated import price elasticities are often used to assess the effect on the quantity of imports of an exogenous change in their price. For a small open economy - a price taker in the markets of its imports and exportables - with fixed domestic factor endowment and operating at full employment, it would seem best to treat the price of output and domestic factor utilization as exogenous next to the price of imports. In that sense (II) is our preferred specification.<sup>9</sup> The own price elasticities of imports derived from the other specifications would

only indicate the partial effect of a change in import prices since  $p_D$ ,  $q_Y$  or  $q_Z$  could hardly be expected to remain constant in such an instance. Correct assessment of the total effect would require some calculations and knowledge of  $\alpha$ . The same holds if real national product is used as the activity variable if  $\delta \neq \alpha$  as may well be expected.

It is also apparent from our analysis that no matter which specification is used, the import demand equation that is obtained is part of a system of three equations:<sup>10</sup> two direct and one inverse demand or supply equations such as equations (6)-(8). It is most efficient then to make use of all information available, and to estimate the entire system rather than the import demand equation in isolation as it is usually done.<sup>11</sup>

To conclude this section, it might be useful to say a word about technological change. Assume that the aggregate production function is given by:

$$q_Y = f[q_M, q_D(\tilde{q}_D, t)] \quad (1')$$

where  $q_D$  is now the augmented quantity of the domestic composite factor,  $\tilde{q}_D$  is the measured quantity, and  $t$  is time. It is convenient to assume that technological change is disembodied and occurs at constant exponential rate. In that case:

$$q_D = \tilde{q}_D e^{\mu t}, \quad p_D = \tilde{p}_D e^{-\mu t}, \quad \mu \geq 0 \quad (29)$$

where  $\mu$  is the rate of technological change, and  $p_D$  ( $\tilde{p}_D$ ) is the implicit (measured) rental price of D. (29) can be substituted into (I)-(III) to eliminate the unobservable variables  $q_D$  and/or  $p_D$ ; time then appears as an additional explanatory variable.

### 3 Variable Returns to Scale

We now turn to the somewhat more complex case where  $f(\cdot)$  can no longer be taken as linearly homogeneous;  $f(\cdot)$  may not even be homothetic. Variable returns to scale might occur for a number of reasons, the most obvious being perhaps the omission of an input or an output.<sup>12</sup> The case of variable returns to scale is somewhat more complex than the case examined in the previous section because it can generally no longer be assumed that optimization takes place in all markets, and it is no longer a matter of indifference whether one chooses to describe the technology by a cost function such as (4), a revenue function such as (2), or yet another representation. If the aggregate technology exhibits variable returns to scale, and if the economy faces less than infinitely elastic input supply and output demand functions, it is generally not true that the marginal product of each input equals its price in equilibrium. One must therefore ask oneself for which input(s) does this equality break down. The answer to this question determines the form by which the technology can most adequately be represented.

We have argued in the previous section that for a small open economy operating at full employment and with fixed factor endowment it is appropriate to take  $p_M$ ,  $p_Y$  and  $q_D$  as exogenous; the producer's problem is to maximize variable profits given the price vector  $(p_M, p_Y)$  and fixed national factor endowment  $q_D$ . The technology can thus be represented by the following variable profit function:

$$\pi(p_M, p_Y, q_D) = \max_{q_Y, q_M} \{ p_Y q_Y - p_M q_M : q_Y \leq f(q_M, q_D); q_Y, q_M \geq 0 \} \quad (30)$$

for  $p_M, p_Y > 0$  and  $q_D \geq 0$ . Given our assumptions on  $f(\cdot)$ ,  $\pi(\cdot)$  is monotonically increasing in  $q_D$ , nondecreasing in  $p_Y$ , nonincreasing in  $p_M$ , and homogeneous of degree one and convex in prices.<sup>13</sup>

The profit maximizing demand for imports and supply of output can be obtained by differentiation, a result implied by Hotelling's (1932) lemma:

$$q_M = - \partial \pi(\cdot) / \partial p_M = - \pi_M(p_M, p_Y, q_D) \quad (31)$$

$$q_Y = \partial \pi(\cdot) / \partial p_Y = \pi_Y(p_M, p_Y, q_D) \quad (32)$$

In addition, if the composite factor is mobile between firms:

$$p_D = \pi(p_M, p_Y, q_D) / q_D \quad (33)$$

i.e. the domestic factor rental price is determined residually by use of the national account identity  $p_Y q_Y - p_M q_M = p_D q_D$ . Thus the totality of variable profits (nominal national income) is allocated to the domestic composite factor.

The modus operandi would have been quite different had we used (4) instead of (30) to represent the technology. Under variable returns to scale and for output set at an arbitrary level, the marginal cost of output is generally not equal to its price (as determined by the accounting identity), and hence input prices are not equal to the corresponding marginal products. The first-order conditions for a solution to (4) only imply that the ratio of the marginal products of the two factors is equal to the ratio of their prices. The same would be if (3) were used to describe the technology assuming that factors are mobile between firms.

We now proceed as in the previous section, but using the variable profit function as our starting point. Logarithmic differentiation of (31)-(33) yields:

$$\begin{bmatrix} d \ln q_M \\ d \ln q_Y \\ d \ln p_D \end{bmatrix} = \begin{bmatrix} -\pi_{MM} p_M / q_M & -\pi_{MY} p_Y / q_M & -\pi_{MD} q_D / q_M \\ \pi_{YM} p_M / q_Y & \pi_{YY} p_Y / q_Y & \pi_{YD} q_D / q_Y \\ -\alpha / (1-\alpha) & 1 / (1-\alpha) & \pi_D / q_D - 1 \end{bmatrix} \begin{bmatrix} d \ln p_M \\ d \ln p_Y \\ d \ln q_D \end{bmatrix} \quad (34)$$

where  $\pi_{MY} = \partial^2 \pi(\cdot) / (\partial p_M \partial p_Y)$  and so forth;  $\alpha$  is defined as before.

Linear homogeneity in prices implies:

$$\pi_{MM}p_M/q_M + \pi_{MY}p_Y/q_M = 0, \quad \pi_{YM}p_M/q_Y + \pi_{YY}p_Y/q_Y = 0 \quad (35)$$

and

$$\pi_{DM}p_M/p_D + \pi_{DY}p_Y/p_D = \pi_D/p_D. \quad (36)$$

In addition it follows from the symmetry of the Hessian that:

$$\pi_{YM}p_M/q_Y = \alpha \pi_{MY}p_Y/q_M. \quad (37)$$

Let  $\epsilon_{MM} = -\pi_{MM}p_M/q_M < 0$ ,  $\eta_M = -\pi_{MD}q_D/q_M > 0$  and  $\eta_Y = \pi_{YD}q_D/q_Y > 0$ ;  $\epsilon_{MM}$  is again the own price elasticity of the demand for imports holding  $p_Y$  and  $q_D$  constant;  $\eta_M$  and  $\eta_Y$  are the elasticities of the demand for imports and of the supply of output with respect to domestic factor endowment for constant prices.

Making use of (35)-(37), (34) can be rewritten as:

$$\begin{bmatrix} d \ln q_M \\ d \ln q_Y \\ d \ln p_D \end{bmatrix} = \begin{bmatrix} \epsilon_{MM} & -\epsilon_{MM} & \eta_M \\ \alpha \epsilon_{MM} & -\alpha \epsilon_{MM} & \eta_Y \\ -\alpha/(1-\alpha) & 1/(1-\alpha) & \mu \end{bmatrix} \begin{bmatrix} d \ln p_M \\ d \ln p_Y \\ d \ln q_D \end{bmatrix} \quad (38)$$

where  $\mu = -[\alpha/(1-\alpha)]\eta_M + 1/(1-\alpha)\eta_Y - 1$ . It can be seen from (38) that all price and quantity effects can be captured by four parameters ( $\alpha$ ,  $\epsilon_{MM}$ ,  $\eta_M$  and  $\eta_Y$ ).

The import demand price and quantity elasticities corresponding to the various specifications can now be derived. For formulation (II), the elasticities can be read directly from (38); we have indeed:

$$d \ln q_M = \epsilon_{MM} d \ln p_M - \epsilon_{MM} d \ln p_Y + \eta_M d \ln q_D. \quad (39)$$

The elasticities corresponding to the other specifications can be obtained

from (38) by substitution; they are reported in table 1.

Several comments can be made. First, all import demand functions emerge yet again as homogeneous of degree zero in prices; cross elasticities are therefore not reported, being equal to minus the corresponding own price elasticities. One can also see that each formulation generally yields a different set of elasticities, some of them being rather complex expressions. It appears that for most formulations one cannot rule out that the own price elasticity of the demand for imports is actually positive: this is a definite possibility if formulations (I), (IV), (V), (VI) or (VII) are used, even if  $\epsilon_{MM}$  is negative as required by the convexity of  $\pi(\cdot)$ . It does not seem that this has ever been recognized in empirical work. A positive estimate thus does not necessarily imply that the estimating equation ought to be rejected.

It is not possible in general to rank the various price and quantity elasticities as most of them depend not only on  $\epsilon_{MM}$  and  $\eta_M$ , but also on  $\alpha$ ,  $\eta_Y$  and  $\delta$ . If  $\eta_M = \eta_Y = 1$ , however ( $\mu = 0$  then), the production function is linearly homogeneous, the elasticities simplify to their values in (21)-(27), and (28) is again valid. Of interest also is the case where  $f(\cdot)$  is homogeneous of degree  $n$ . We then have the following:<sup>14</sup>

$$\eta_M = 1 - (n-1)\epsilon_{MM}, \quad \eta_Y = n - (n-1)\alpha\epsilon_{MM}, \quad \mu = (n-1)/(1-\alpha). \quad (40)$$

As an illustration, we may consider the Cobb-Douglas production function:

$$q_Y = q_M^\alpha q_D^\beta \quad \text{with } n = \alpha + \beta, \quad 0 < \alpha < 1, \quad \beta > 0. \quad (41)$$

The variable profit function is then equal to:

$$\pi(p_M, p_Y, q_D) = \gamma p_M^{-\alpha/(1-\alpha)} p_Y^{1/(1-\alpha)} q_D^{\beta/(1-\alpha)}, \quad \gamma > 0, \quad (42)$$

from which one can easily see that:<sup>15</sup>

$$\epsilon_{MM} = -1/(1-\alpha), \quad \eta_M = \eta_Y = \beta/(1-\alpha). \quad (43)$$

The various import demand price and quantity elasticities corresponding to the Cobb-Douglas functional form are set out in table 2. There are still substantial differences between the various formulations, especially for  $\alpha$  large. All price elasticities, with the possible exception of  $\epsilon_{MM}^{VI}$ , tend to underestimate  $\epsilon_{MM}$  in absolute value; as we have argued earlier,  $\epsilon_{MM}$  is probably the most useful measure of the price elasticity of the demand for imports. For example, if  $\alpha = \delta = .3$  and  $\beta = .5$ , the price elasticities range from  $-.63$  to  $-1.43$  (the value of  $\epsilon_{MM}$ ), and the activity elasticities range from  $.71$  to  $1.25$ .

#### 4 Concluding Comments

What one would like to do at this stage is to take the various price and quantity elasticities reported in the literature to which we referred earlier, and translate them into the same framework for comparison purposes. This is unfortunately not possible since the necessary information is generally not available. Given the published elasticities, one would require the values of  $\alpha$ ,  $\eta_Y$  and possibly  $\delta$  to be able to construct estimates of  $\epsilon_{MM}$  and  $\eta_M$ . An average value for  $\alpha$  could be constructed without much difficulty in most cases. Knowledge of the data normalization used, however, is of crucial importance for the calculation of  $\delta$ . This information is often not at hand. The major difficulty though stems from the fact that nearly all estimated import demand functions are single equation estimates; hence no information on  $\eta_Y$  is available.<sup>16</sup> This means that in most instances, and unless one has a rather intimate knowledge of the actual study and of its data base, the estimates obtained are of limited usefulness for economic analysis.<sup>17</sup> In our judgement specification (II) is probably the most helpful since the elasticities it yields are usable for a variety of purposes without any further manipulations.

Specification (VI), which has been applied in a large number of studies, might be our second choice, since it is equivalent to (II) if  $\alpha = \delta$ . The validity of this assumption, however, is questionable. Many countries experience large changes in their terms of trade over time; in recent years, especially, most oil importing countries have been subject to sharp increases in the relative price of their imports. As a result  $\delta$  is likely to deviate considerably from  $\alpha$ , and real national product becomes a poor index of economic activity.

Among the elasticities that we have defined, there is none that is superior to all the other ones. From a strictly deterministic point of view, they all represent the same model, and hence are equivalent, even if some are more readily usable than others. In applying these elasticities, it is essential to keep in mind that they only indicate ceteris paribus price and quantity effects, and it is crucial to be aware of what are the other things that are being held constant. Without this precaution comparisons of elasticities drawn from different sources are meaningless. If one wishes to reconcile different estimates, one does find, as even our simple model reveals, that this is a complex exercise calling for more information than might actually be available.

TABLE 1: Import Demand Price and Quantity Elasticities for Alternative Specifications.

Specification	$\partial \ln q_M(\cdot) / \partial \ln p_M$	$\partial \ln q_M(\cdot) / \partial \ln q$
I $(p_M, p_D, q_Y)$	$\epsilon_{MM} \frac{(1-\alpha)(\eta_Y - \alpha\eta_M)}{\alpha(1-\alpha)\mu\epsilon_{MM} + \eta_Y}$	$\frac{(1-\alpha)\mu\epsilon_{MM} + \eta_M}{\alpha(1-\alpha)\mu\epsilon_{MM} + \eta_Y}$
II $(p_M, p_Y, q_D)$	$\epsilon_{MM}$	$\eta_M$
III $(p_M, p_D, q_D)$	$\epsilon_{MM}(1-\alpha)$	$\eta_M + (1-\alpha)\mu\epsilon_{MM}$
IV $(p_M, p_Y, q_Y)$	$\epsilon_{MM} \left(1 - \alpha \frac{\eta_M}{\eta_Y}\right)$	$\frac{\eta_M}{\eta_Y}$
V $(p_M, p_Y, q_Z)$	$\epsilon_{MM} + \frac{\alpha\eta_M}{(1-\alpha)(1+\mu)}$	$\frac{\eta_M}{1+\mu}$
VI $(p_M, p_Y, q_G)$	$\epsilon_{MM} \frac{\eta_Y - \alpha\eta_M}{\eta_Y - \delta\eta_M}$	$\frac{\eta_M(1-\delta)}{\eta_Y - \delta\eta_M}$
VII $(p_M, p_G, q_G)$	$\frac{\epsilon_{MM}(1-\alpha)(\eta_Y - \alpha\eta_M)}{(\eta_Y - \delta\eta_M) - (\delta-\alpha)(\eta_Y - \alpha\eta_M)\epsilon_{MM}}$	$\frac{(1-\delta)\eta_M - (\delta-\alpha)(\eta_Y - \alpha\eta_M)\epsilon_{MM}}{(\eta_Y - \delta\eta_M) - (\delta-\alpha)(\eta_Y - \alpha\eta_M)\epsilon_{MM}}$

where  $q = q_Y, q_G, q_Z$  or  $q_D$  as the case may be

$q_M$  = quantity of imports

$q_D$  = quantity of domestic composite factor

$q_Y$  = quantity of gross output

$q_Z$  = real national income (real national value added)

$q_G$  = real national product (conventional measure)

$p_M$  = price of imports

$p_D$  = rental price of domestic composite factor

$p_Y$  = price of gross output

$p_G$  = national product deflator

TABLE 2: Import Demand Price and Quantity Elasticities for Alternative Specifications, Cobb-Douglas Functional Form.

$$q_Y = q_M^\alpha q_D^\beta, \quad 0 < \alpha < 1, \beta > 0$$

Specification	$\partial \ln q_M(\cdot) / \partial \ln p_M$	$\partial \ln q_M(\cdot) / \partial \ln q$
I $(p_M, p_D, q_Y)$	$-\beta / (\alpha + \beta)$	$1 / (\alpha + \beta)$
II $(p_M, p_Y, q_D)$	$-1 / (1 - \alpha)$	$\beta / (1 - \alpha)$
III $(p_M, p_D, q_D)$	$-1$	$\beta(\alpha + \beta) / (1 - \alpha)$
IV $(p_M, p_Y, q_Y)$	$-1$	$1$
V $(p_M, p_Y, q_Z)$	$-1$	$1$
VI $(p_M, p_Y, q_G)$	$-1 / (1 - \delta)$	$1$
VII $(p_M, p_G, q_G)$	$-1$	$1$

where  $q = q_Y, q_D, q_Z$  or  $q_G$  as the case may be.

## FOOTNOTES

\* I wish to thank P.S. Andersen, K.W. Clements, W.P. Hogan and A.J. Phipps for a number of helpful comments on an earlier draft of this paper, but I remain alone responsible for any errors or omissions.

1. See Houthakker and Magee (1969) for instance. For an account of the methodology, see Leamer and Stern (1970).
2. See Kohli (1979).
3. There is no mention of this problem in Stern et al. (1976) for instance.
4. See Diewert (1974) for a proof.
5. The meaning of  $\epsilon_{MM}$  will become clear below;  $q_M > 0$  is assumed.
6. See Burgess (1974a) for the estimation of an inverse import demand function such as (15).
7. In obtaining (25)-(27), we have used the following results:
 
$$\begin{aligned} d\ln q_Z &= d\ln p_D + d\ln q_D - d\ln p_Y \\ d\ln q_G &= 1/(1-\delta)d\ln q_Y - \delta/(1-\delta)d\ln q_M \\ d\ln p_G &= 1/(1-\alpha)d\ln p_Y - \alpha/(1-\alpha)d\ln p_M + (\delta-\alpha)/(1-\alpha)(d\ln q_M - d\ln q_G) . \end{aligned}$$
8. We have shown elsewhere (Kohli, 1979) that if  $p_M/p_Y = 1$  initially, a finite change in this ratio, for given factor endowment, necessarily decreases real national product. If the terms of trade improve sufficiently, real national product can actually become negative!
9. In the case of a small open economy with fixed factor endowment and operating under full employment, formulation (II) would also be preferable from an econometric point of view.
10. The framework used here can easily be extended to account for more than three inputs or outputs; see Burgess (1974b) and Kohli (1978) for instance.
11. See Burgess (1974b) and Kohli (1978) for instance.
12. The absence of constant returns to scale could also be due to a misspecification of the model following inappropriate aggregation of inputs and/or outputs. Burgess (1974b) rejects the two input one output representation in the case of the United States.
13. See Diewert (1974) for a proof.
14. From (30) we have:

$$\begin{aligned} p_M/p_Y &= f_M(q_M, q_D) . \\ \therefore dq_M &= - (f_{MD}/f_{MM})dq_D + (1/f_{MM})d(p_M/p_Y) . \end{aligned}$$

It follows that:

$$\epsilon_{MM} = (f_M/f_{MM})(1/q_M), \quad \eta_M = - (f_{MD}/f_{MM})(q_D/q_M) .$$

We also have:

$$d \ln q_Y = \alpha d \ln q_M + [f_D/f(\cdot)] q_D d \ln q_D$$

$$\therefore \eta_Y = \alpha \eta_M + [f_D/f(\cdot)] q_D .$$

Homogeneity of degree  $n$  implies:

$$(n-1)f_M = f_{MM}q_M + f_{MD}q_D$$

$$\therefore \eta_M = 1 - (n-1)\epsilon_{MM} .$$

Homogeneity of degree  $n$  also implies:

$$nf(\cdot) = f_Mq_M + f_Dq_D$$

$$\therefore \eta_Y = \alpha \eta_M + [n - f_Mq_M/f(\cdot)] = n - (n-1)\alpha \epsilon_{MM} .$$

15. Note that  $\eta_M = \eta_Y$  here because of the unit elasticity of substitution of the Cobb-Douglas functional form. From the previous footnote:

$$\eta_Y - \eta_M = (n-1)[1 + \epsilon_{MM}(1-\alpha)] = q_D(n-1)[f_D f_M - f(\cdot) f_{MD}] / [f(\cdot) f_{MM} q_M] .$$

Hence  $\eta_Y = \eta_M$  iff  $n = 1$  or the elasticity of substitution between imports and the domestic factor is unity.

16. One exception is provided by Burgess (1974b) who estimates the full system of demand and supply functions. In addition, Burgess assumes constant returns to scale.
17. Note that this problem does not arise if the import demand function is used in simulations in conjunction with relationships that endogenize the other price and activity variables (e.g. in macroeconomic models).

## REFERENCES

- Adams, F.G., H. Eguchi and F. Meyer-zu-Schlochtern (1969) An Econometric Analysis of International Trade (Paris: OECD)
- Ball, R.J. and K. Marwah (1962) "The U.S. Demand for Imports, 1948-1958", Review of Economics and Statistics 44, 395-401.
- Burgess, D.F. (1974a) "Production Theory and the Derived Demand for Imports", Journal of International Economics 4, 103-117.
- Burgess, D.F. (1974b) "A Cost Minimization Approach to Import Demand Equations", Review of Economics and Statistics 56, 225-234.
- Diewert, W.E. (1974) "Applications of Duality Theory", in M.D. Intriligator and D.A. Kendrick (eds) Frontiers of Quantitative Economics, vol. 2 (Amsterdam: North-Holland).
- Hotelling, H. (1932) "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions", Journal of Political Economy 40, 577-616.
- Houthakker, H.S. and S.D. Magee (1969) "Income and Price Elasticities in World Trade", Review of Economics and Statistics 51, 111-125.
- Kohli, U.R. (1978) "A Gross National Product Function and the Derived Demand for Imports and Supply of Exports", Canadian Journal of Economics 11, 167-182.
- Kohli, U.R. (1979) "Imports as an Input to the Technology", University of Sydney, mimeo.
- Leamer, E.E. and R.M. Stern (1970) Quantitative International Economics (Boston: Allen and Bacon).
- Meyer-zu-Schlochtern, F. and A. Yajima (1970) "OECD Trade Model 1970 Version", OECD Economic Outlook, Occasional Studies (Paris: OECD).
- Miller, J.C. and M. Fratiani (1974) "The Lagged Adjustment of U.S. Trade to Prices and Income", Journal of Economics and Business 26, 191-198.
- Shephard, R.W. (1953) Cost and Production Functions (Princeton: Princeton University Press).
- Stern, R.M., J.H. Francis and B. Schumacher (1976) Price Elasticities in International Trade (London: Macmillan).

WORKING PAPERS IN ECONOMICS

- \* 1 I.G. Sharpe A Framework for Analysis of the Money Supply Process in Australia
- \* 2 I.G. Sharpe & R.G. Walker Asset Revaluation and Stock Market Prices
- \* 3 N.V. Lam Incidence and Stabilization Impact of Tin Export Taxation in West Malaysia
- \* 4 V.B. Hall & M.L. King Inflationary Expectations in New Zealand: A Preliminary Study
- \* 5 A.J. Phipps Strike Activity and Inflation in Australia
- 6 N.V. Lam Incidence of the Rice Export Premium in Thailand
- \* 7 I.G. Sharpe Secondary Reserve Requirements, the Monetary Base and the Money Supply in Australia
- 8 P. Saunders Labour Demand Functions and the Quasi-Fixity Hypothesis: Some Empirical Results for U.K. Manufacturing Industries, 1963-1974
- 9 W.P. Hogan Economic Strategies for Recovery
- 10 T.P. Truong Asset Revaluation and Share Prices: A Study using the M.S.A.E. Regression Technique
- 11 S. Kim Instability of Primary Exports, Income Stabilization Policies and Welfare
- \*12 I.G. Sharpe & P.A. Volker Institutional Change, Specification Error, Inflation and the Stability of the Demand for Money Function in the United States, 1901-1974
- \*13 I.G. Sharpe & P.A. Volker The Impact of Institutional Changes on the Australian Short-Run Money Demand Function
- \*14 W.P. Hogan The Connections Between Foreign Trade and Economic Development: An Empirical Study
- \*15 F. Gill The Case of the Black Exodus from the Southern U.S., 1910-1970: Some Lessons for Theory and Applied Theory
- \*16 A.J. Phipps The Impact of Wage Indexation on Wage Inflation and Strike Activity in Australia
- \*17 V.B. Hall Pricing Behaviour in Australia: A Data Evaluation Study
- \*18 I.G. Sharpe Australian Money Supply Analysis: Direct Controls and the Relationship Between the Monetary Base, Secondary Reserve and the Money Supply
- \*19 L. Haddad Economic Systems: Towards a New Classification
- 20 G. Lewis A Strategy for Winning at Roulette

- 21 R.L. Brown A Test of the Black and Scholes Model of Option Valuation in Australia
- \* 22 V.B. Hall Pricing Behaviour in Australian Manufacturing Industry: Hypothesis Testing 1955-1956 to 1967-1968
- \*23 I.G. Sharpe & P.A. Volker The Selection of Monetary Policy Instruments: Evidence from Reduced Form Estimates of the Demand and Supply of Money in Australia
- \*24 V.B. Hall Excess Demand and Expectations Influences on Price Changes in Australian Manufacturing Industry
- \*25 I.G. Sharpe & P.A. Volker The Tradeoff Between Improved Monetary Control and Market Interest Rate Variability in Australia: An Application of Optimal Control Techniques
- \*26 Evan Jones with the assistance of Mary MacDonald An Examination of Earnings Differentials in Australian Manufacturing Industry
- 27 W.P. Hogan Questions on Structural Adjustment Policies
- 28 P. Saunders Price and Cost Expectations in Australian Manufacturing Firms
- 29 W.P. Hogan, I.G. Sharpe & P.A. Volker Regulation, Risk and the Pricing of Australian Bank Shares 1957-1976
- 30 W.P. Hogan Quicksands of Policy-Making
- 31 C. Emerson Taxing Natural Resources Projects
- 32 R.W. Bailey, V.B. Hall & P.C.B. Phillips A Small Model of Output, Employment, Capital Formation and Inflation, applied to the New Zealand Economy
- 33 W.P. Hogan Eurofinancing: Currencies, Loans and Bonds
- 34 S. Adee The Growth of Foreign Investment and Economic Nationalism in Post Independence Ghana: 1957-1975
- 35 W.P. Hogan The 40 Per Cent Investment Allowance
- 36 W.P. Hogan Controlling Eurofinance Markets
- 37 R.T. Ross Disaggregate Labour Supply Functions for Married Women: Preliminary Estimates for New Zealand
- 38 U.R. Kohli Beginning- and End-of-Period Specifications of Asset Market Equilibrium in Balance-of-Payments

- 39 G. Mills Government Incentive Contracts with Private  
Companies: Some Lessons from the Channel  
Tunnel
- 40 C.G.F. Simkin Closer Economic Relations Between  
Australia and New Zealand
- 41 U.R. Kohli Relative Price Effects and the Demand  
for Imports

Papers marked with an asterisk are out of stock. Copies of the others are available upon request from:

Department of Economics  
The University of Sydney  
N.S.W. 2006

Working Papers in Economics that have  
been accepted for publication elsewhere

- 2 I.G. Sharpe &  
R. G. Walker Journal of Accounting Research,  
Fall, 1975, 293-310.
- 3 N. V. Lam Journal of the Developing Economies  
Vol. 17, No. 1, March 1979.
- 4 V. B. Hall &  
M. L. King New Zealand Economic Papers, 1976,  
118-51.
- 5 A. J. Phipps Economic Record, September 1977,  
297-319.
- 6 N. V. Lam Journal of Development Studies  
Vol. 14, No. 1, October 1977.
- 7 I.G. Sharpe Australian Journal of Management,  
April 1976, 85-106.
- 9 W. P. Hogan Economic Papers, No. 55, The Economic  
Society of Australia and New Zealand,  
29-57.
- 13 I. G. Sharpe &  
P. A. Volker Kredit und Kapital, Vol. 12, No. 1,  
1979
- 14 W. P. Hogan Some Calculations in Stability and  
Inflation, A.R. Bergstrom et.al. (eds.),  
John Wiley and Sons, 1978.
- 21 R. L. Brown Australian Journal of Management,  
Vol. 3, No. 1, April 1978, 17-36.
- 23 I.G. Sharpe &  
P. A. Volker The Australian Monetary System in the  
1970s, M. Porter (ed.), Supplement to  
the Economic Record, 1978.
- 24 V. B. Hall Economic Record, Vol. 56, No. 152,
- 27 W. P. Hogan "Dividing Structural Adjustment  
Policies, Malayan Economic Review,  
Vol. 24, No. 1., pp. 69-81.
- 30 W. P. Hogan Australian Economic Papers, Vol. 18,  
No. 33, December 1979.