

WORKING PAPERS IN ECONOMICS

Saving Behavior in a Pure
Life-Cycle Model with
Income Uncertainty

by

I.J. Irvine and S. Wang*

No. 220

June 1995

DEPARTMENT OF ECONOMICS



The University of Sydney
Australia 2006

**Saving Behavior in a Pure
Life-Cycle Model with
Income Uncertainty**

by

I.J. Irvine and S. Wang*

No. 220

June 1995

Abstract

Several models of economic behavior currently compete for an explanation of individual wealth accumulation. However, most focus upon a very limited set of saving motives. In this paper we build a general model of economic behavior, set in a stochastic environment, which incorporates multiple motives and which yields closed form solutions for wealth accumulation and consumption functions. Depending upon the degree of prudence assumed, total private wealth holdings can be attributed to retirement, income uncertainty, lifetime uncertainty and intertemporal substitution motives. The paper then addresses the claims of a number of recent contributions in the area. An exact measure of the equivalent *precautionary premium* is developed and estimated.

*Irvine is an Associate Professor, Department of Economics, Concordia University, Montreal, P.Q., Canada, H4X 1T9. Wang is an Assistant Professor at the HK University of Science & Technology, Hong Kong. The authors wish to thank participants at the Concordia Economics workshop for their helpful comments on the paper.

National Library of Australia Card Number and ISBN 0 86758 887 X.

CONTENTS

	Page
1. Introduction	1
2. The Model	4
2.1 The Setup	4
2.2 The Working Period	6
3. Parameterization	10
4. Results	11
5. Conclusions	17
References	25
Addendum	27

1. Introduction

Since the 1950s, Franco Modigliani and Richard Brumberg's life cycle model (1954) has been viewed as providing perhaps the most significant reason for private wealth accumulation. In the life cycle model individuals save for a retirement period which may be characterized by an uncertain date of death.

But two developments in the 1980s made economists review their beliefs on this proposition. One was the examination of the role of income uncertainty, the other was the recognition that intergenerational transfers of wealth could be important. The income uncertainty models of Jonathan Skinner (1988), Stephen Zeldes (1989) and Ricardo Caballero (1991) all imply that income uncertainty could account for a substantial proportion of private wealth. An exception to this finding is the work of Luigi Guiso, Tullio Jappelli and Daniel Terlizzese (1992). Zeldes obtained his results using numerical methods, Skinner by approximating the Euler equation resulting from a constant relative risk aversion (CRRA) utility function, and Caballero by choosing a specific utility function and income generating process which result in closed form solutions. These are all representative agent simulation models. In contrast, Guiso *et al* use survey data to test the implications of the theory of prudent behavior, and find that income uncertainty may account for as little as two percent of household wealth.

While slow wealth decumulation patterns of the elderly has long been recognized (e.g. Anthony Shorrocks, 1975, or Paul Menchik and Martin David, 1983), it was only with Lawrence Kotlikoff and Lawrence Summers' paper (1981) that the extent to which such transfers could be important was recognized.

The purpose of the present paper is to provide a framework within which several wealth accumulation motives can be analyzed. From a methodological standpoint, the reasons for saving and wealth accumulation can be assessed more realistically in this way. When taken together, models which focus upon a single, or limited set of, motives imply that the wealth stock can be explained several times over: for example, Modigliani (1988) believes that about 70% of private wealth is life cycle wealth, Kotlikoff and Summers (1981) believe that 80% is attributable to intergenerational transfers, and Skinner (1988) and Caballero (1991) believe that income uncertainty might account for as much as 60% of such wealth.

A significant obstacle to life cycle model building in the presence of income uncertainty is that closed form solutions for consumption and saving equations can only be derived for very particular functional forms. Guiso, Jappelli and Terlizzese (1991) and Caballero (1991) obtain such solutions by choosing a constant absolute risk aversion (CARA) utility function and a random walk specification for income. Their models have no retirement period (during which income is zero or non stochastic), and their solutions require that the rate of time preference equal the rate of interest. Irvine and Wang (1994) relaxed the assumption that the rate of interest equal the rate of time preference, and their results indicate great sensitivity to this seemingly small generalization.

Not only do the CARA class of models discussed above require the same recursive stochas-

tic income process until the time of death, but so too do the numerically optimized models of Skinner (1988) and Zeldes (1989). Introducing a retirement motive means that the nature of the income generating process changes at the point of retirement, and the standard Bellman techniques cannot be applied to the whole horizon. We use a solution algorithm which yields closed form solutions for the consumption, saving and wealth accumulation equations, despite this discontinuity.

While the model we develop is also a member of the CARA class and involves some technical detail, it is a more realistic representation of behavior than the other members of its class which have addressed the question of why people save. The assumption of a stochastic income process to the time of death, without an explicit retirement phase, might be thought justifiable on the grounds that, even though the incomes of the retired are not stochastic, their needs are unpredictable. Hence the assumption of a stochastic income to the time of death might be assumed to proxy this. (For example, health expenditures are highly unpredictable). However, there is a major difference between the income uncertainty experienced during the working life cycle and the expenditure uncertainty during retirement: In the latter case well developed insurance markets exist, while in the former case the presence of moral hazard means that full insurance is not available. Consequently, achieving a target wealth level at the point of retirement means that needs-related uncertainties can be eliminated. Furthermore, the numerical results we report indicate that the explicit treatment of a retirement period characterized in this way has a significant impact on the roles played by the various savings motives.

The second point to note about the retirement phase is that if individuals are risk averse in the presence of an uncertain date of death, they may behave in a manner which is consistent with the possibility of a long retirement phase. This means that retirement can be more important in accumulation and decumulation patterns (relative to the working phase) than if individuals behave in accordance with certainty equivalence - intending to accumulate sufficient wealth only for the expected value of their lifetime.

We model behavior by generalizing what has become a fairly widely recognized framework - the maximization of expected utility over the lifecycle, where the instantaneous utility function is of the exponential type and the income stream is a random walk. Both the tractability characteristics and the peculiarities of this model are well known. For example, Philippe Weil (1990, 1993) has developed functions based on the approach of Kreps and Porteus (1978), which do not constrain the intertemporal elasticity of substitution to equal the coefficient of relative risk aversion, nor suffer from the problem of a finite marginal utility of consumption when consumption tends towards zero. While the latter problem may permit negative consumption as part of an optimal program, such an event occurs with an extremely small probability in the model we develop.

The paper proceeds as follows. In the next section the basic model is developed and equations are presented for the consumption, saving and wealth accumulation equations. The derivations are relegated to the appendix. We also develop an exact monetary measure of the cost of income uncertainty. This is similar to Miles Kimball's (1990) equivalent precautionary

premium. This provides a money metric of the utility cost of income uncertainty. In section 3 the model is parameterized and simulated subject to a variety of assumptions. This enables us to evaluate the relative importance of the different savings motives. In the final section we offer conclusions, and examine some of the well recognized characteristics of the CARA model.

2. The Model

2.1. The Setup

Consider an overlapping generations (OG) economy in which each agent can live for a maximum of $T+N$ periods. Individuals are identical at birth, with the same preferences and endowment (initial nonhuman wealth and future income process). These individuals may die at the end of any particular period with probability $1-p$. Such an occurrence is termed an **accidental death**. Individuals who survive to period $T+N$ die of a **natural death** at the end of that period. Following the development of Caballero (1990, 1991) and Irvine-Wang (1994), the population size is normalized at 1 for each period. Accordingly, the number of individuals dying accidentally in any period is $1-p$, and the number of individuals having a natural death in any period is $\frac{1-p}{1-p^{T+N}}p^{T+N}$, which is equal to the number of individuals who survive to the natural life span $T+N$. The number of births in each period is assumed to be $\frac{1-p}{1-p^{T+N}}$, which equals the sum of deaths from accidental and natural causes, so that the population size is maintained at 1.

Each individual has the same tastes and these are defined by the exponential utility function which has constant absolute risk aversion. Accordingly a representative individual faces the following lifetime utility maximization problem:

$$\left\{ \begin{array}{l} V(A_0) = \max_{E_0} \sum_{t=1}^{T+N} -\frac{1}{\theta} e^{-\theta c_t} \left(\frac{p}{1+\delta} \right)^{t-1} \\ \text{s.t. } A_t = (1+r)A_{t-1} + y_t - c_t, \quad \forall t \\ y_{t+1} = y_t + w_t, \quad t = 1, \dots, T-1 \\ y_{T+1} = 0, \quad \tau = T, \dots, T+N \\ A_{T+N} = 0, \text{ given } A_0, \end{array} \right. \quad (2.1)$$

where E_t is the expectations operator conditional on information available at time t , θ is the coefficient of absolute risk aversion, c_t is consumption, y_t is income, A_t is nonhuman wealth, r is the interest rate, δ is the rate of time preference, p is the per period survival probability, and $\{w_t\}$ is i.i.d. and $w_t|F_t \sim N(0, \sigma^2)$, $\forall t$.

This is not a recursive problem and consequently the standard Bellman approach cannot be applied directly. To get around this difficulty, we break the optimization into two periods, each of which has the same recursive structure and within which the Bellman approach can therefore be applied.

- There are two periods: working period $t = 1, \dots, T$; and retirement period $t = T+1, \dots, T+N$.
- In each period, there are T and N years respectively.

In the working period, the consumer problem is

$$\left\{ \begin{array}{l} U(A_0, A_T) = \max_{E_0} \sum_{t=1}^T -\frac{1}{\theta} e^{-\theta c_t} \left(\frac{p}{1+\delta} \right)^{t-1} \\ \text{s.t. } A_t = (1+r)A_{t-1} + y_t - c_t, \\ y_{t+1} = y_t + w_t, \\ \text{given } A_0 \text{ and } A_T. \end{array} \right. \quad (2.2)$$

This is a recursive problem, and therefore can be solved using the Bellman equation.

In the retirement period, the consumer problem is

$$\left\{ \begin{array}{l} U(A_T) = \max_{E_0} \sum_{t=T+1}^{T+N} -\frac{1}{\theta} e^{-\theta c_t} \left(\frac{p}{1+\delta} \right)^{t-1} \\ \text{s.t. } A_t = (1+r)A_{t-1} - c_t, \\ A_{T+N} = 0, \\ \text{given } A_T. \end{array} \right. \quad (2.3)$$

Since this is also a recursive problem, it can be solved using the Bellman equation. The overall problem for lifetime optimization is

$$\left\{ \begin{array}{l} V(A_0) = \max_{A_T} U(A_0, A_T) + U(A_T) \\ \text{given } A_0. \end{array} \right. \quad (2.4)$$

The initial wealth A_0 is determined by the intergenerational equilibrium condition that the wealth stock of those dying in any period is passed on to those who are born in that period. We assume furthermore that such wealth is distributed evenly among the new born.¹

To facilitate the development of the results we use the following notation.

$$\alpha \equiv \frac{1}{1+r}, \quad \beta \equiv p \frac{1+r}{1+\delta}, \quad \gamma \equiv \frac{1}{\theta} \ln \beta, \quad \Gamma \equiv \frac{1}{2} \theta \sigma^2, \quad \Gamma^* \equiv \Gamma + \gamma.$$

Also denote

$$t_p \equiv \frac{1-p}{1-p^T} \sum_{t=1}^T t p^{t-1} = \frac{1}{1-p} - \frac{T p^T}{1-p^T},$$

which is the average age of the working individuals.

Since the derivations and proofs are all quite long they are presented in the appendix. However, as implied by equations (2.1) – (2.4) the solution strategy is simply to optimize in each of the two periods conditional upon a fixed wealth target A_T and then choose an optimal A_T as the final step in the optimization.

¹The assumption that the inheritance is received at the beginning of the economic life can be relaxed without affecting the closed form nature of the solutions. The assumption that bequests are equal is necessary, since this is a representative agent model. However, since we are interested primarily in the aggregate stock of wealth rather than its distribution within a given age cohort, this restriction is not too serious.

2.2. The Working Period

The utility maximization problem (2.2) for the working period gives the consumption function:

$$c_t = y_t + \frac{r}{1 - \alpha^{T+1-t}} (A_{t-1} - \alpha^{T+1-t} A_T) + \left[\frac{(T+1-t)\alpha^{T+1-t}}{1 - \alpha^{T+1-t}} - \frac{1}{r} \right] \Gamma^*, \quad (2.5)$$

Let $s_t \equiv y_t - c_t$ be saving. Then, by (2.5) and the budget equation, we have

$$\begin{cases} s_t = \left[\frac{1}{r} - \frac{(T+1-t)\alpha^{T+1-t}}{1 - \alpha^{T+1-t}} \right] \Gamma^* + \frac{r}{1 - \alpha^{T+1-t}} (\alpha^{T+1-t} A_T - A_{t-1}) \\ A_t = \frac{1}{\alpha} A_{t-1} + s_t \\ \text{given } A_0 \text{ and } A_T. \end{cases} \quad (2.6)$$

Recursively using (2.6) gives the individual wealth profile:

$$A_t = \frac{1 - \alpha^{T-t}}{1 - \alpha^T} A_0 + \frac{\alpha^{T-t} - \alpha^T}{1 - \alpha^T} A_T + \frac{1}{r} \left(t - \frac{\alpha^{T-t} - \alpha^T}{1 - \alpha^T} T \right) \Gamma^*, \quad (2.7)$$

for $t = 1, 2, \dots, T$. Finally, the maximum expected utility for the working period can be shown to be

$$U(A_0, A_T) = -\frac{1 - \alpha^T}{\theta(1 - \alpha)} e^{-\theta \left[y_0 + \frac{r}{1 - \alpha^T} (A_0 - \alpha^T A_T) + \gamma - t_\alpha \Gamma^* \right]}. \quad (2.8)$$

2.3. The Retirement Period

The utility maximization problem (2.3) for the retirement period gives the consumption function:

$$c_t = \frac{r}{1 - \alpha^{T+N+1-t}} A_{t-1} + \left[\frac{(T+N+1-t)\alpha^{T+N+1-t}}{1 - \alpha^{T+N+1-t}} - \frac{1}{r} \right] \gamma. \quad (2.9)$$

Let $s_t \equiv -c_t$ be the saving. Then, by (2.9) and the budget equation, we have

$$\begin{cases} s_t = \left[\frac{1}{r} - \frac{(T+N+1-t)\alpha^{T+N+1-t}}{1 - \alpha^{T+N+1-t}} \right] \gamma - \frac{r}{1 - \alpha^{T+N+1-t}} A_{t-1} \\ A_t = \frac{1}{\alpha} A_{t-1} + s_t \\ \text{given } A_T. \end{cases} \quad (2.10)$$

Recursively using (2.10) gives the individual wealth profile:

$$A_t = \frac{1 - \alpha^{T+N-t}}{1 - \alpha^N} A_T + \frac{1}{r} \left(t - T + \frac{\alpha^N - \alpha^{T+N-t}}{1 - \alpha^N} N \right) \gamma, \quad (2.11)$$

for $t = T, \dots, T+N$. The maximum expected utility for this period is

$$U(A_T) = -\frac{\alpha^T \beta^T (1 - \alpha^N)}{\theta(1 - \alpha)} e^{-\theta \left[\frac{r}{1 - \alpha^N} A_T + \left(\frac{N\alpha^N}{1 - \alpha^N} - \frac{1}{r} \right) \gamma \right]}. \quad (2.12)$$

2.4. The Lifetime Problem

The optimal A_T from the lifetime maximization problem (2.4) is obtained from the solutions (2.8) and (2.12):

$$\hat{A}_T = \frac{1 - \alpha^N}{1 - \alpha^{T+N}} A_0 + \frac{(1 - \alpha^T)(1 - \alpha^N)}{r(1 - \alpha^{T+N})} \left[y_0 - t_\alpha \Gamma + \left(\frac{T}{1 - \alpha^T} - \frac{N\alpha^N}{1 - \alpha^N} \right) \gamma \right]. \quad (2.13)$$

Substituting this in turn into (2.8) and (2.12) yields the maximum lifetime utility

$$V = -\frac{1 - \alpha^{T+N}}{(1 - \alpha)\theta} e^{-\theta \left[\frac{1 - \alpha^T}{1 - \alpha^{T+N}} (y_0 - t_\alpha \Gamma) + \frac{1 - \alpha^N}{1 - \alpha^{T+N}} W + \left(\frac{T + N}{1 - \alpha^{T+N}} - \frac{1}{r} - T - N \right) \gamma \right]}. \quad (2.14)$$

With optimal A_T^* , by (2.7) and (2.11), the individual wealth profile is

$$\begin{aligned} \hat{A}_t &= \frac{1 - \alpha^{T+N-t}}{1 - \alpha^{T+N}} A_0 + \frac{(1 - \alpha^N)(\alpha^{T-t} - \alpha^T)}{r(1 - \alpha^{T+N})} \left[y_0 - t_\alpha \Gamma + \left(\frac{T}{1 - \alpha^T} - \frac{N\alpha^N}{1 - \alpha^N} \right) \gamma \right] \\ &\quad + \left(t - \frac{\alpha^{T-t} - \alpha^T}{1 - \alpha^T} T \right) \frac{\Gamma^*}{r}, \end{aligned} \quad (2.15)$$

for $t = 1, \dots, T$, and

$$\begin{aligned} \hat{A}_t &= \frac{1 - \alpha^{T+N-t}}{1 - \alpha^{T+N}} A_0 + \frac{(1 - \alpha^T)(1 - \alpha^{T+N-t})}{r(1 - \alpha^{T+N})} \left[y_0 - t_\alpha \Gamma + \left(\frac{T}{1 - \alpha^T} - \frac{N\alpha^N}{1 - \alpha^N} \right) \gamma \right] \\ &\quad + \left(t - T + \frac{\alpha^N - \alpha^{T+N-t}}{1 - \alpha^N} N \right) \frac{\gamma}{r}, \end{aligned} \quad (2.16)$$

for $t = T, \dots, T+N$.

2.5. Aggregate Wealth

The aggregate wealth in this economy, W , is the sum of individual wealth holdings:

$$W \equiv \frac{1 - p}{1 - p^{T+N}} \sum_{t=1}^{T+N} p^t A_t. \quad (2.17)$$

Following our earlier assumption on A_0 , the equilibrium condition on the transfer of wealth is

$$\frac{1-p}{1-p^{T+N}}A_0 = (1-p)W. \quad (2.18)$$

The right side is the expected total wealth left from the newly dead, and the left side is the total amount of initial wealth endowment with each new-born getting A_0 . Substituting (2.18) into (2.15) and (2.16) gives A_t , $t = 1, \dots, T+N$, depending on W ; then substituting these A_t in turn into (2.17) determines a unique aggregate wealth:

$$W^* = \frac{1}{r} \left(\frac{\alpha^T - p^T}{\alpha^{T+N} - p^{T+N}} - \frac{1-p^T}{1-p^{T+N}} \right) \left[y_0 - t_\alpha \Gamma + \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma \right] + \frac{1}{r} \left(\frac{1}{p^{T+N} - \alpha^{T+N}} + \frac{1}{1-p^{T+N}} \right) \left[(1-p^T)(t_p - t_\alpha)\Gamma^* + \left(\frac{p-\alpha}{1-\alpha} \frac{1-p^N}{1-p} + \frac{\alpha^N - p^N}{1-\alpha^N} N \right) p^T \gamma \right]. \quad (2.19)$$

In view of this model's purpose, several comments are in order. First, consumption in the working period is stochastic, yet saving is non stochastic. This is because consumption adjusts fully to the income shocks in each period. A corollary of this is that each individual of the same age holds the same amount of wealth regardless of past income innovations. This explains why aggregate wealth W in (2.19) is nonstochastic, in the sense of being independent of the actual realizations of y_t .

Second, the aggregate stock of wealth depends upon all of the parameters which are contained in (2.19). Consequently, the effect of particular savings motives upon the economy's aggregate wealth stock can be examined by setting these parameters to specific limiting values. For example, to ascertain the effect of income uncertainty the parameter σ^2 can be permitted to tend to zero; or to ascertain the role of intertemporal substitution the rate of time preference δ can be set equal to the rate of interest r .

Third, even though considerable stocks of wealth can be passed between generations, there is no bequest motive. Consequently, inheritances are received because of an uncertain date of death, yet the *amount* of such inheritances will depend upon the strength of all savings motives. For example, an individual who is strongly risk averse to her uncertain income stream will leave a larger (expected) bequest than one who is less risk averse or who has a smaller income variance. This illustrates that the aggregate wealth stock is not simply the addition of several components, each corresponding to one particular saving motive. At the same time, there is no reason why a non zero value for A_{T+N} could not be specified in the model as it stands. Such a terminal condition would obviously lead to an increase in the wealth stock.

Fourth, the implications of the assumptions regarding insurance markets in this model should be clarified. To this point we have assumed that insurance is available neither for income nor lifetime uncertainty. The absence of income insurance is realistic and standard in these models, due to the moral hazard problems which would be associated with its intro-

duction. While we have not introduced an insurance market for lifetime uncertainty, Yaari (1965) has shown that the conditions governing the evolution of the equations of motion for savings and consumption are the same in the no-lifetime uncertainty case as in the case where lifetime is uncertain but with an annuity market where the rate of interest is 'fair'. Thus, if the terminal conditions on the problem were the same in each of these cases, the actual path of wealth accumulation would be the same and the inclusion of a market for life insurance would be equivalent to the path under a certain lifetime. However, Kotlikoff (1988) has argued that 'fair' annuities are difficult to supply in practice because of agent heterogeneity and the self selection which this implies.

Fifth, it is interesting to examine the type of individual wealth profiles which emerge. Several such profiles are presented in figures 1 and 2 for a variety of specifications on the model parameters, which we discuss more fully in the next section of the paper. Generally, wealth reaches a peak at the point of retirement, but not in all cases. It is shown in the appendix that the growth rate gap in the individual wealth profile at retirement is

$$\left. \frac{\partial A_t^*}{\partial t} \right|_{T-} - \left. \frac{\partial A_t^*}{\partial t} \right|_{T+} = \left[\left(\frac{1}{\ln \alpha} + \frac{1}{1-\alpha} + T \right) \Gamma - y_0 \right] \frac{\ln \alpha}{r}. \quad (2.20)$$

Finally, following the work of Miles Kimball (1990), this model lends itself to a computation of compensating and equivalent precautionary premia. These are simply the changes in mean income y_0 which compensate for, or are equivalent to, a change in the variance of the income process σ^2 . We show in the appendix that the total cost of σ^2 measured in terms of goods y_0 , evaluated at the utility level where σ^2 is not zero, is

$$\text{cost}(\sigma) = \frac{(1-\alpha^T)t_\alpha - (1-p^T)t_r}{p^T - \alpha^T} \Gamma. \quad (2.21)$$

3. Parameterization

The basic set of values we have chosen for the parameters is given in the first column of Table 1. The real interest rate r is set at 4% and the rate of time preference δ is set at 2%. The difference between these two values determines the degree to which the preference for a growing consumption stream - which we also term 'intertemporal substitution' - generates saving and wealth. We normalize the income stream such that $Y_0 = 100$ in the certain lifetime case, and set the coefficient of absolute risk aversion θ at 3%. The value of average consumption C , used in computing the coefficient of relative risk aversion, which is consistent with the normalization on Y_0 and the values of the model's parameters, is obtained numerically through the equations of the model. This gives rise to a coefficient of relative risk aversion θC in the range 3.3 to 4.2. Since average consumption equals average income minus average savings, average consumption can either be greater or less than average income, depending upon the initial stock of assets inherited, A_0 . The coefficient of variation on the income process σ/Y is set at 0.07. This is consistent with the values chosen by other authors. For example, Caballero suggests a value of 0.10 based upon the findings of MaCurdy (1982), although Guiso *et al* suggest a value as low as 0.02.

The expected length of the economic life of an agent is set equal to 50 years, with a working life of 40 and a retirement period of 10 years in the certain lifetime case. When the lifetime is uncertain there is an infinite number of combinations of p and T which will give the same expected lifetime of 50 years. We choose $T + N = 57$ and $p = 0.99523$ as one such combination, simply because it is approximately what Caballero chooses and therefore it provides us with a comparison point.

This set of parameter values generates a wealth to consumption ratio W/C of 5.38, which is a typical value for a developed economy. In this sense the parameter set can be considered reasonable.

4. Results

We can now address the question posed at the outset: why do people save and which saving motives are most important? Several conceptual experiments can be designed to answer this question. But before examining the simulation results we reiterate that the impact of any particular saving motive will depend upon what other motives are in the model. Accordingly, when we examine the effect of removing income uncertainty, the result is conditional upon the general behavior assumed for the individual - as summarized by the parameter values specified. With this in mind, consider the first set of results which are in Table 1.

The basic set of parameter values for the complete model is given in the top half of the first column. The lower half of the table yields the values for the variables of interest: aggregate consumption of the population C , the coefficient of relative risk aversion θC , aggregate wealth W , the ratio of aggregate wealth to aggregate consumption W/C , the expected bequest A_0 , the maximal asset value over the life cycle A_{\max} , the expected value of lifetime utility U , and the cost of σ^2 - defined as the number of units of Y_0 which would be equivalent to abolishing income uncertainty.

Two constraints govern the comparisons. One is that the expected income stream be the same in each simulation, the other is that the expected lifetime be the same. Columns (2), (3) and (4) define the marginal effect on wealth accumulation of eliminating one motive from the agent's optimization. The final column contains the results for the case of no uncertainty and where the interest rate equals the rate of time preference. It can be considered analogous to the simplest type of lifecycle model developed by Modigliani and Brumberg (1954) or Atkinson (1971).

If we consider the reduction in aggregate wealth in the economy, following a restriction placed on the model reflecting a particular saving motive being omitted, then columns (2), (3) and (4) paint a very clear picture. The most influential saving motive is clearly lifetime uncertainty, followed by the intertemporal substitution motive, and lastly income uncertainty. If there were no lifetime uncertainty the aggregate wealth stock would be reduced by 37.8% [(751.54 - 467.14)/751.54]. If individuals have a preference for a constant consumption stream over the lifecycle ($r = \delta$), saving would fall and the wealth stock would be reduced by 21%. The elimination of income uncertainty would reduce aggregate wealth by just 3.1%. Finally, a pure lifecycle model would have a wealth stock equal to 43.6% of the more complete representation of the economy given in column (1).²

These results have sharp implications for the two debates we outlined earlier. First, the result that the income uncertainty motive, when interpreted in this way, explains so little of the aggregate wealth stock surprised us greatly. After all, Caballero, Skinner and Zeldes each built simulation models quite similar in spirit to this one (although less general) and found that income uncertainty might account for half of all wealth accumulation. This raises

²Note that we cannot examine the effect of omitting the retirement period without violating the constraint that the expected horizon in the model be the same in all simulations.

two questions: (i) is the degree of income uncertainty which we specify too low? (ii) Could income uncertainty be interacting with some of the other motives in such a way as to be really responsible for a greater fraction of total wealth than appears from Table 1?

To examine the first possibility, we doubled the value of the σ/Y to 14% and reestimated the aggregate wealth stock for each of the cases considered in Table 1. In this case we found that the marginal effect of income uncertainty on the wealth stock was 11.3%, still far below the numbers alluded to above, but consistent with the findings of *Guiso et al.* (We have also investigated the effect of the intertemporal substitution effect using other, equal value, combinations of r and δ . While choices of r and δ certainly influence the aggregate wealth level, choosing different equal values has no perceptible effect on the role played by income uncertainty.)

The second possibility, that income uncertainty may be interacting with other motives is examined in Table 2. Here, in columns (6), (7) and (8) motives are eliminated pairwise. The first and last columns are the same as in Table 1. For example column (6) defines the joint influence of income uncertainty and a preference for a rising consumption stream (intertemporal substitution). What emerges is that the joint effect of the motives on the wealth stock is not very different from the sum of their marginal effects as given in Table 1, though the motives interact with one another differently. That is, the joint effect of income uncertainty and lifetime uncertainty exceeds the sum of their marginal effects, while the joint effect of lifetime uncertainty and intertemporal substitution is less than the sum of their marginal effects.

Despite the seemingly small effects of income uncertainty on wealth accumulation, this finding says nothing about the utility cost of income uncertainty. Indeed, the cost of income uncertainty, in terms of Kimball's precautionary premium, is high — if σ/Y were reduced from 7% to zero, an individual would be willing to sacrifice 9.0% of expected annual income, every year, while attaining the same level of expected utility. Furthermore, if, from a value of 14%, σ/Y is reduced to zero the precautionary premium is 35.3% of expected annual income. Consequently, even though income uncertainty may have a small effect on wealth accumulation it still has a very great effect on individual welfare. These outcomes imply that, given that individuals save for a variety of other motives, utility optimization dictates only a moderate saving response to an increase in uncertainty, despite the utility loss.

The reason why our results are so different from those of Caballero, Skinner and Zeldes is relatively straightforward. By introducing a retirement period into the model, when income is zero, we recognize two aspects of real world behavior which are omitted by income uncertainty models where the stochastic income stream continues until the time of death. Not only is the period of income uncertainty reduced from an expected value of 50 years to 40, but those last years in the lifecycle have a much greater variance than earlier years. For example, with the random walk income process, the variance of income in year 46 is 46 times the variance in year 1, or twice the variance of income in year 23. Consequently, assuming that income is stochastic in the retirement period, when in fact it is not, (incorrectly) adds additional years

of income uncertainty whose variance is relatively high.

The second issue upon which these results can shed light is the Kotlikoff /Summers - Modigliani debate. Our results indicate that a simple lifecycle model with retirement as the sole reason for saving can account for just under half (43.6%) of the wealth stock associated with a more complete characterization of the economy, while lifetime uncertainty can account for about one third of the wealth stock. This 43.6% number lies between Modigliani's estimate of three quarters and Kotlikoff and Summers' estimate of less than one quarter. We have not incorporated a bequest motive, and accordingly bequests arise because agents die at an unknown time. In this context, the average bequest (A_0) in Table 1 amounts to as much as one quarter of the aggregate wealth stock.

Kotlikoff and Summers' work focuses primarily upon *how much* of the wealth stock can be attributed to a pure life cycle model and how much to non life cycle influences, rather than upon simply bequests induced by lifetime uncertainty. Accordingly, in comparing our results with those of Kotlikoff and Summers, it would not be unreasonable to compare our wealth accumulation figures, *net of pure life cycle effects*, with theirs. Our value would thus be 56.4% ($100.0 - 43.6$), while theirs is about 80%. A further way of making comparisons would be to take the typical bequest (A_0) and cumulate it at the interest rate to some point in time. But, as has been pointed out by Kessler and Masson (1989), the effect of bequests upon wealth accumulation depends on how saving behavior is influenced by inheritances. In our model, inherited wealth is treated exactly like wealth from savings.

In concluding this discussion, we should report that we have also examined the behavior of the wealth stock by calculating wealth elasticities with respect to various parameters. Specifically we have calculated the percentage change in the wealth stock resulting from a percentage change in the interest rate, the variance of the income process and the length of the retirement period. These elasticities corresponded to what the preceding discussion led us to expect: the elasticity with respect to extending the retirement period was large, with respect to the income variance very small, and with respect to the interest rate moderate.

Several wealth accumulation profiles for a typical individual are portrayed in Figures 1 and 2, each corresponding to a particular column, as labeled, in Tables 1 and 2. For the set of values in those tables each profile peaks at the point of retirement. However, this is not a necessary outcome of the model: when we set σ^2 or r high enough, peaks can occur well before retirement. The rate of growth in the profiles is very different, even for profiles which peak at the same age. Likewise, the rate at which wealth starts to decumulate, as shown in equation (2.20), varies.

TABLE 1
CONDITIONAL EFFECT OF SAVING MOTIVES
ON WEALTH ACCUMULATION

	Complete model	Zero income uncertainty	Zero lifetime uncertainty	No intertemporal substitution	Retirement as sole motive
	1	2	3	4	5
r	0.04			0.03*	0.03*
δ	0.02			0.03*	0.03*
p	0.99523		1.0*		1.0*
T	40				
N	17		10*		10*
θ	0.03				
σ/Y	0.07	0.0*			0.0*
C	139.67	138.73	118.69	127.43	109.84
θC	4.19	4.16	3.56	3.82	3.30
W	751.54	728.21	467.14	593.98	327.89
W/C	5.38	5.25	3.94	4.66	2.99
A_0	179.29	173.72	0.0	141.70	0.0
A_{max}	1303.37	1443.22	805.96	1075.95	766.32
$cost(\sigma)$	12.57	—	11.38	13.75	—
Γ	0.883	0.0	0.735	0.883	0.0
γ	0.488	0.488	0.647	-0.159	0.0
U	-61.88	-43.35	-89.10	-64.06	-59.66

* denotes a change in value relative to col (1)

TABLE 2
JOINT CONDITIONAL EFFECTS OF SAVING MOTIVES
ON WEALTH ACCUMULATION

	Complete model	No income uncertainty, no intertemporal substitution	No income uncertainty, no lifetime uncertainty	No intertemporal substitution, no lifetime uncertainty	Retirement as sole motive
	1	6	7	8	5
r	0.04	0.03*		0.03*	0.03*
δ	0.02	0.03*		0.03*	0.03*
p	0.99523		1.0*	1.0*	1.0*
T	40				
N	17		10*	10*	10*
θ	0.03				
σ/Y	0.07	0.0*	0.0*		0.0*
C	139.67	127.03	117.12	110.90	109.84
θC	4.19	3.81	3.51	3.33	3.30
W	751.54	580.68	427.80	363.22	327.89
W/C	5.38	4.57	3.65	3.28	2.99
A_0	179.29	138.53	0.0	0.0	0.0
A_{max}	1303.87	1239.20	890.97	672.54	766.32
$cost(\sigma)$	12.57	—	—	12.24	—
Γ	0.883	0.0	0.0	0.735	0.0
γ	0.488	-0.159	0.647	0.0	0.0
U	-61.88	-44.16	-65.06	-52.97	-59.66

* denotes a change in value relative to col (1)

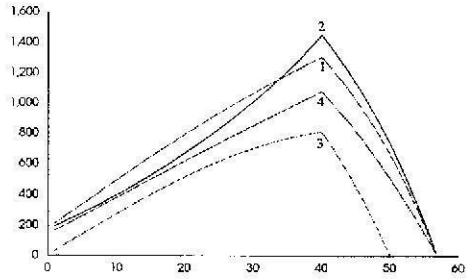


Figure 1. Wealth Profiles with Single Parameter Restrictions

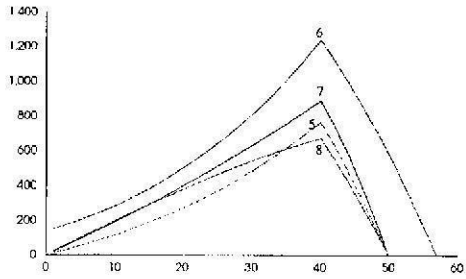


Figure 2. Wealth Profiles with Double Parameter Restrictions

5. Conclusion

In this paper we have assessed the contribution of different saving motives to the explanation of the stock of wealth, within the confines of a specific overlapping generations model. Rather than reiterate our conclusions, some comments on the nature of the framework we have used are appropriate: First, the use of instantaneous utility of the exponential type is restrictive, and the particular characteristics of this function are well known. However, to this point we are unaware of any models which are as general as the one we have developed here, in the sense of incorporating a retirement period and both lifetime and income uncertainty. This model supports a number of results, particularly on the role of income uncertainty, which could not otherwise be addressed.

A fairly common criticism of the exponential utility framework is that it may lead to the choice of negative consumption values as part of the optimal consumption program. For example, Deaton (1992, p180) and Weil (1993) have made this point. However, the probability of this occurrence is very small with the set of parameter values we have chosen. Some Monte Carlo experiments indicate that it will occur less than 1% of the time. The reason for this low occurrence is that, given the nature of the income generating process, a negative value for *income* will likely take several periods before materializing. But by this point, a significant addition to the inherited wealth stock A_0 will have taken place. Consequently, this stock will considerably reduce the possibility of negative consumption. Further, if an explicit bequest target were introduced, wealth accumulation would be greater and the probability of negative consumption values correspondingly even less.

We have not addressed the issue of binding liquidity constraints, primarily because the intergenerational nature of the model means that inheritances are received and agents are, therefore, generally not constrained. Thus even if we were to set the rate of time preference high relative to the interest rate as, for example, Christopher Carroll (1992) does, this would not necessarily lead to the consumer being liquidity constrained. At the same time we note that consumption, as defined in equation (2.5), responds strongly to changes in current income values, and thus corresponds to the concerns that reasonable consumption processes respond “excessively” to current income changes.

Appendix

A.1. The Working Period — Section 2.2

The Bellman equation for (2.2) is

$$U(A_{t-1}, A_T) = \max_{c_t} \left\{ -\frac{1}{\theta} e^{-\theta c_t} + \frac{P}{1+\delta} E_t U(A_t, A_T) \right\}, \quad \text{subject to the same conditions.}$$

The first-order condition and Envelope Theorem imply the Euler equation:

$$e^{-\theta c_t} = p \frac{1+r}{1+\delta} E_t e^{-\theta c_{t+1}}.$$

One can easily verify that the solution is

$$c_{t+1} = \Gamma^* + c_t + w_t,$$

Then

$$c_T - c_t = (T-t)\Gamma^* + y_T - y_t. \quad (\text{A.1})$$

By the budget constraint, (A.1) implies

$$c_t = (1+r)A_{T-1} - A_T + y_t - (T-t)\Gamma^*. \quad (\text{A.2})$$

The budget constraint and (A.2) then imply

$$(1+r)A_{t-1} - A_t = c_t - y_t = (1+r)A_{T-1} - A_T - (T-t)\Gamma^*.$$

Then,

$$A_{t-1} - \alpha A_t = A_{T-1} - \alpha A_T - \alpha(T-t)\Gamma^*. \quad (\text{A.3})$$

Let $S \equiv$ one step forward. Then (A.3) becomes

$$(1-\alpha S)A_{t-1} = A_{T-1} - \alpha A_T - \alpha(T-t)\Gamma^*,$$

implying

$$A_{t-1} = \frac{1}{1-\alpha S} [A_{T-1} - \alpha A_T - \alpha(T-t)\Gamma^*] = \sum_{i=0}^{\infty} \alpha^i S^i [A_{T-1} - \alpha A_T - \alpha(T-t)\Gamma^*].$$

Then

$$A_{t-1} = \frac{A_{T-1} - \alpha A_T}{1-\alpha} - \Gamma^*(T-t) \frac{\alpha}{1-\alpha} + \Gamma^* \left(\frac{\alpha}{1-\alpha} \right)^2,$$

which is a special solution of (A.3).³ Since the solution for the homogeneous equation $A_{t-1} - \alpha A_t = 0$ is $C \left(\frac{1}{\alpha} \right)^t$, where C is some constant, the general solution of (A.3) is

$$A_{t-1} = C \left(\frac{1}{\alpha} \right)^t + \frac{A_{T-1} - \alpha A_T}{1-\alpha} - \Gamma^*(T-t) \frac{\alpha}{1-\alpha} + \Gamma^* \left(\frac{\alpha}{1-\alpha} \right)^2. \quad (\text{A.4})$$

³We can actually verify this directly.

By initial condition $A_{t-1} = A_{T-1}$ when $t = T$, (A.4) implies

$$A_{T-1} = C \left(\frac{1}{\alpha} \right)^T + \frac{A_{T-1} - \alpha A_T}{1-\alpha} + \Gamma^* \left(\frac{\alpha}{1-\alpha} \right)^2. \quad (\text{A.5})$$

Eliminating C using (A.4) and (A.5) then gives the solution of (A.3):

$$A_{t-1} = \frac{1-\alpha^{T+1-t}}{1-\alpha} A_{T-1} - \frac{\alpha(1-\alpha^{T-t})}{1-\alpha} A_T + [\alpha - (T-t)(1-\alpha) - \alpha^{T+1-t}] \frac{\alpha \Gamma^*}{(1-\alpha)^2}. \quad (\text{A.6})$$

Substituting (A.6) into (A.2) then gives the optimal consumption:

$$c_t = y_t + \frac{1-\alpha}{\alpha(1-\alpha^{T+1-t})} A_{t-1} - \frac{(1-\alpha)\alpha^{T-t}}{1-\alpha^{T+1-t}} A_T + \left[\frac{(T+1-t)\alpha^{T+1-t}}{1-\alpha^{T+1-t}} - \frac{\alpha}{1-\alpha} \right] \Gamma^*. \quad (\text{A.7})$$

(2.6) implies

$$A_t = \frac{1-\alpha^{T-t}}{1-\alpha^{T+1-t}} A_{t-1} + \frac{(1-\alpha)\alpha^{T-t}}{1-\alpha^{T+1-t}} A_T + \left[\frac{\alpha}{1-\alpha} - \frac{(T+1-t)\alpha^{T+1-t}}{1-\alpha^{T+1-t}} \right] \Gamma^*. \quad (\text{A.8})$$

We can then use (A.8) recursively to find the closed-form solution of A_t :

$$A_t = \frac{1-\alpha^{T-t}}{1-\alpha^T} A_0 + A_T \sum_{i=1}^t \frac{1-\alpha^{T-t}}{1-\alpha^{T-i}} \frac{\alpha^{T-i}(1-\alpha)}{1-\alpha^{T+1-i}} + \Gamma^* \sum_{i=1}^t \frac{1-\alpha^{T-t}}{1-\alpha^{T-i}} \left[\frac{\alpha}{1-\alpha} - \frac{(T+1-i)\alpha^{T+1-i}}{1-\alpha^{T+1-i}} \right], \quad (\text{A.9})$$

for $t = 1, 2, \dots, T$. Let us now simplify the formulas. We have

$$\frac{1}{1-\alpha^{T-i}} - \frac{1}{1-\alpha^{T+1-i}} = \frac{\alpha^{T-i}(1-\alpha)}{(1-\alpha^{T-i})(1-\alpha^{T+1-i})}.$$

Then,

$$\sum_{i=1}^t \frac{(1-\alpha^{T-t})\alpha^{T-i}(1-\alpha)}{(1-\alpha^{T-i})(1-\alpha^{T+1-i})} = \frac{\alpha^{T-t} - \alpha^T}{1-\alpha^T}.$$

We also have

$$\frac{T+1-i}{1-\alpha^{T+1-i}} - \frac{T-i}{1-\alpha^{T-i}} = \frac{1-\alpha}{\alpha} \frac{1}{1-\alpha^{T-i}} \left[\frac{\alpha}{1-\alpha} - \frac{(T+1-i)\alpha^{T+1-i}}{1-\alpha^{T+1-i}} \right].$$

Then,

$$\sum_{i=1}^t \frac{1-\alpha^{T-t}}{1-\alpha^{T-i}} \left(\frac{\alpha}{1-\alpha} - \frac{(T+1-i)\alpha^{T+1-i}}{1-\alpha^{T+1-i}} \right) = \frac{\alpha}{1-\alpha} \left(T \frac{1-\alpha^{T-t}}{1-\alpha^T} - T + t \right).$$

(A.9) can then be simplified to (2.7). Substituting (2.7) into (2.6) then gives the saving function (2.8).

The Euler equation is

$$e^{-\theta c_{t-1}} = \beta E_{t-1} e^{-\theta c_t},$$

Then,

$$E_{t-2} e^{-\theta c_{t-1}} = \beta E_{t-2} e^{-\theta c_t}.$$

Thus,

$$e^{-\theta c_{t-2}} = \beta E_{t-2} e^{-\theta c_{t-1}} = \beta^2 E_{t-2} e^{-\theta c_t}.$$

Recursively,

$$e^{-\theta c_t} = \beta^{t-1} E_1 e^{-\theta c_t}. \quad (\text{A.10})$$

Using (A.10), we then have

$$U(A_0, A_T) = E_0 E_1 \sum_{t=1}^T -\frac{1}{\theta} e^{-\theta c_t} \left(\frac{p}{1+\delta} \right)^{t-1} = -\frac{1}{\theta} E_0 \sum_{t=1}^T \left(\frac{p}{1+\delta} \right)^{t-1} E_1 e^{-\theta c_t}.$$

Thus,

$$U(A_0, A_T) = -\frac{1-\alpha^T}{\theta(1-\alpha)} \left(E_0 e^{-\theta c_1} \right). \quad (\text{A.11})$$

By (2.7),

$$c_1 = w_0 + a,$$

where

$$a \equiv y_0 + \frac{1-\alpha}{\alpha(1-\alpha^T)} A_0 - \frac{(1-\alpha)\alpha^{T-1}}{1-\alpha^T} A_T + \left(\frac{T\alpha^T}{1-\alpha^T} - \frac{\alpha}{1-\alpha} \right) \Gamma^*.$$

We then have

$$E_0 e^{-\theta c_1} = e^{-\theta a} E_0 e^{-\theta w_0} = e^{-\theta a} e^{\frac{1}{2}\theta^2 \sigma^2}.$$

Substituting this into (A.11) then gives

$$U(A_0, A_T) = -\frac{1-\alpha^T}{\theta(1-\alpha)} e^{\frac{1}{2}\theta^2 \sigma^2 - \theta a} = -\frac{1-\alpha^T}{\theta(1-\alpha)} e^{\theta(\Gamma^* - a)},$$

which is (2.9).

A.2. The Retirement Period — Section 2.3

The derivation process is similar to the one for the working period.

A.3. The Lifetime Problem — Section 2.4

The first order condition for (2.4) is

$$U(A_0, A_T^*) \frac{\theta(1-\alpha)\alpha^{T-1}}{1-\alpha^T} + U(A_T^*) \left[-\frac{(1-\alpha)\theta}{\alpha(1-\alpha^N)} \right] = 0,$$

implying

$$\frac{1-\alpha}{\alpha} \left(\frac{\alpha^T}{1-\alpha^T} + \frac{1}{1-\alpha^N} \right) A_T^* = \left(\frac{T\alpha^T}{1-\alpha^T} - \frac{\alpha}{1-\alpha} \right) \Gamma^* + \left(T + \frac{\alpha}{1-\alpha} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma + y_0 + \frac{1-\alpha}{\alpha(1-\alpha^T)} A_0 - \Gamma.$$

This equation immediately gives (2.15). By (2.9) and (2.14), we have

$$V = -\frac{1-\alpha^T}{\theta(1-\alpha)} e^{-\theta \left[y_0 + \frac{1-\alpha}{\alpha(1-\alpha^T)} A_0 - \frac{(1-\alpha)\alpha^{T-1}}{1-\alpha^T} A_T^* + (1-t_a)\gamma - t_a\Gamma \right]} - \frac{\alpha^T \beta^T (1-\alpha^N)}{(1-\alpha)\theta} e^{-\frac{(1-\alpha)\theta}{\alpha(1-\alpha^N)} A_T^* + \left(\frac{\alpha}{1-\alpha} - \frac{N\alpha^N}{1-\alpha^N} \right) \theta \gamma}.$$

Substituting (2.15) into this and using the fact $A_0 = (1-p^{T+N})W$ gives

$$V = -\frac{1-\alpha^T}{(1-\alpha)\theta} e^{-\theta \left[\frac{1-\alpha^T}{1-\alpha^T+N} (y_0 - t_a\Gamma) + \frac{(1-\alpha)(1-\alpha^{T+N})}{\alpha(1-\alpha^T+N)} W + \frac{1-\alpha^T}{1-\alpha^T+N} \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma + \left(\frac{N\alpha^N}{1-\alpha^N} - \frac{\alpha}{1-\alpha} - T \right) \gamma \right]} - \frac{\alpha^T \beta^T (1-\alpha^N)}{(1-\alpha)\theta} e^{-\theta \left[\frac{1-\alpha^T}{1-\alpha^T+N} (y_0 - t_a\Gamma) + \frac{(1-\alpha)(1-\alpha^{T+N})}{\alpha(1-\alpha^T+N)} W + \frac{1-\alpha^T}{1-\alpha^T+N} \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma + \left(\frac{N\alpha^N}{1-\alpha^N} - \frac{\alpha}{1-\alpha} - T \right) \gamma \right]}.$$

Since $\theta T \gamma = T \ln \beta = \ln(\beta^T)$, we then have

$$V = -\frac{\beta^T (1-\alpha^{T+N})}{(1-\alpha)\theta} e^{-\theta \left[\frac{1-\alpha^T}{1-\alpha^T+N} (y_0 - t_a\Gamma) + \frac{(1-\alpha)(1-\alpha^{T+N})}{\alpha(1-\alpha^T+N)} W + \frac{1-\alpha^T}{1-\alpha^T+N} \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma + \left(\frac{N\alpha^N}{1-\alpha^N} - \frac{\alpha}{1-\alpha} - T \right) \gamma \right]},$$

which can be simplified a little bit further to (2.16).

A.4. Aggregate Wealth — Section 2.5

(2.19) and (2.20) imply

$$\frac{1}{1-p} A_0 = \sum_{t=1}^{T-1} p^{t-1} A_t^* + p^{T-1} A_T^* + \sum_{t=T+1}^{T+N} p^{t-1} A_t^*. \quad (\text{A.12})$$

Substituting (2.15), (2.17) and (2.18) into (A.12) gives

$$\begin{aligned}
& \left(\frac{1}{1-p} - \sum_{t=1}^{T-1} p^{t-1} \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} - p^{T-1} \frac{1-\alpha^N}{1-\alpha^{T+N}} - \sum_{t=T+1}^{T+N} p^{t-1} \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} \right) A_0 \\
&= \frac{\alpha \Gamma^*}{1-\alpha} \sum_{t=1}^{T-1} p^{t-1} \left(t - \frac{\alpha^{T-t} - \alpha^T}{1-\alpha^T} T \right) \\
&+ \frac{\alpha(1-\alpha^N)}{(1-\alpha)(1-\alpha^{T+N})} \left[y_0 + \left(\frac{T\alpha^T}{1-\alpha^T} - \frac{1}{1-\alpha} \right) \Gamma + \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma \right] \sum_{t=1}^{T-1} p^{t-1} (\alpha^{T-t} - \alpha^T) \\
&+ p^{T-1} \frac{\alpha(1-\alpha^T)(1-\alpha^N)}{(1-\alpha)(1-\alpha^{T+N})} \left[y_0 + \left(\frac{T\alpha^T}{1-\alpha^T} - \frac{1}{1-\alpha} \right) \Gamma + \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma \right] \\
&+ \frac{\alpha \gamma}{1-\alpha} \sum_{t=T+1}^{T+N} p^{t-1} \left(t - T + \frac{\alpha^N - \alpha^{T+N-t}}{1-\alpha^N} N \right) \\
&+ \frac{\alpha(1-\alpha^T)}{(1-\alpha)(1-\alpha^{T+N})} \left[y_0 + \left(\frac{T\alpha^T}{1-\alpha^T} - \frac{1}{1-\alpha} \right) \Gamma + \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma \right] \sum_{t=T+1}^{T+N} p^{t-1} (1-\alpha^{T+N-t}).
\end{aligned}$$

Since

$$\sum_{t=1}^{T+N} p^{t-1} (1-\alpha^{T+N-t}) = \frac{(1-p)\alpha^{T+N} - (1-\alpha)p^{T+N} + p - \alpha}{(1-p)(p-\alpha)},$$

(A.12) becomes

$$\begin{aligned}
& \frac{(1-\alpha)^2(p^{T+N} - \alpha^{T+N})}{\alpha(1-p)(p-\alpha)(1-\alpha^{T+N})} A_0 = \\
& \Gamma^* \sum_{t=1}^{T-1} p^{t-1} \left(t - \frac{\alpha^{T-t} - \alpha^T}{1-\alpha^T} T \right) + \gamma \sum_{t=T+1}^{T+N} p^{t-1} \left(t - T + \frac{\alpha^N - \alpha^{T+N-t}}{1-\alpha^N} N \right) \\
& + \left[y_0 + \left(\frac{T\alpha^T}{1-\alpha^T} - \frac{1}{1-\alpha} \right) \Gamma + \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma \right] \cdot \\
& \cdot \left[\frac{1-\alpha^N}{1-\alpha^{T+N}} \sum_{t=1}^{T-1} p^{t-1} (\alpha^{T-t} - \alpha^T) + \frac{1-\alpha^T}{1-\alpha^{T+N}} \sum_{t=T}^{T+N} p^{t-1} (1-\alpha^{T+N-t}) \right].
\end{aligned}$$

This equation gives an explicit solution of A_0 . Using (2.20), the aggregate wealth is then

$$\begin{aligned}
W &= \frac{\alpha(1-p)(p-\alpha)(1-\alpha^{T+N})}{(1-\alpha)^2(p^{T+N} - \alpha^{T+N})(1-p^{T+N})} \left\{ \left[y_0 + \left(\frac{T\alpha^T}{1-\alpha^T} - \frac{1}{1-\alpha} \right) \Gamma + \left(\frac{T}{1-\alpha^T} - \frac{N\alpha^N}{1-\alpha^N} \right) \gamma \right] \cdot \right. \\
& \cdot \left. \left[\frac{1-\alpha^N}{1-\alpha^{T+N}} \sum_{t=1}^{T-1} p^{t-1} (\alpha^{T-t} - \alpha^T) + \frac{1-\alpha^T}{1-\alpha^{T+N}} \sum_{t=T}^{T+N} p^{t-1} (1-\alpha^{T+N-t}) \right] \right\}.
\end{aligned}$$

$$+ \Gamma^* \sum_{t=1}^T p^{t-1} \left(t - \frac{\alpha^{T-t} - \alpha^T}{1-\alpha^T} T \right) + \gamma \sum_{t=T+1}^{T+N} p^{t-1} \left(t - T + \frac{\alpha^N - \alpha^{T+N-t}}{1-\alpha^N} N \right) \Big\}.$$

Further simplification reduces this formula to (2.21).

A.5. Growth Pattern of Individual Wealth — Equation (2.20)

For $t \leq T$, by (2.17), we have

$$\frac{\partial A_t^*}{\partial t} = \frac{\alpha^{T-t} \ln \alpha}{1-\alpha^T} A_0^* - \frac{\alpha^{T-t} \ln \alpha}{1-\alpha^T} A_T^* + \frac{\alpha}{1-\alpha} \left(1 + \frac{\alpha^{T-t} \ln \alpha}{1-\alpha^T} T \right) \Gamma^*, \quad (\text{A.13})$$

and thus,

$$\left. \frac{\partial A_t^*}{\partial t} \right|_{T-} = \frac{\ln \alpha}{1-\alpha^T} A_0^* - \frac{\ln \alpha}{1-\alpha^T} A_T^* + \frac{\alpha}{1-\alpha} \left(1 + \frac{\ln \alpha}{1-\alpha^T} T \right) \Gamma^*. \quad (\text{A.14})$$

For $t \geq T$, by (2.18), we have

$$\frac{\partial A_t^*}{\partial t} = \frac{\alpha^{T+N-t} \ln \alpha}{1-\alpha^N} A_T^* + \frac{\alpha}{1-\alpha} \left(1 + \frac{\alpha^{T+N-t} \ln \alpha}{1-\alpha^N} N \right) \gamma, \quad (\text{A.15})$$

and thus,

$$\left. \frac{\partial A_t^*}{\partial t} \right|_{T+} = \frac{\alpha^N \ln \alpha}{1-\alpha^N} A_T^* + \frac{\alpha}{1-\alpha} \left(1 + \frac{\alpha^N \ln \alpha}{1-\alpha^N} N \right) \gamma. \quad (\text{A.16})$$

(A.14) and (A.16) can then be substituted into the definition of Δ in (2.22) to give (2.22).

A.6. Cost of Income Variability — Equation (2.21)

The term in V^* that relates y_0 to σ is

$$y_0 - t_\alpha \Gamma + \frac{1-p^{T+N}}{1-\alpha^T} r W^*. \quad (\text{A.17})$$

After substituting W^* in (2.19) into (A.17), we find that only this term relates y_0 to σ :

$$\begin{aligned}
\xi &= y_0 - t_\alpha \Gamma + \frac{1-p^{T+N}}{1-\alpha^T} \left(\frac{\alpha^T - p^T}{\alpha^{T+N} - p^{T+N}} - \frac{1-p^T}{1-p^{T+N}} \right) (y_0 - t_\alpha \Gamma) \\
&+ \frac{1-p^{T+N}}{1-\alpha^T} \left(\frac{1}{p^{T+N} - \alpha^{T+N}} + \frac{1}{1-p^{T+N}} \right) (1-p^T)(t_p - t_\alpha) \Gamma. \quad (\text{A.18})
\end{aligned}$$

(A.18) can then be simplified to

$$\xi = \frac{(p^T - \alpha^T)(1 - \alpha^{T+N})}{(p^{T+N} - \alpha^{T+N})(1 - \alpha^T)} \left[y_0 - t_\alpha \Gamma + \frac{(1-p^T)(t_p - t_\alpha)}{p^T - \alpha^T} \Gamma \right]. \quad (\text{A.19})$$

This is the only term in V^* that relates y_0 to σ . With $\sigma = 0$, suppose the value of ξ is ξ_0 . With any given σ , suppose y_0 is increased by Δy_0 to make $\xi = \xi_0$. By (A.19), Δy_0 satisfies the equation:

$$\xi_0 = \frac{(p^T - \alpha^T)(1 - \alpha^{T+N})}{(p^{T+N} - \alpha^{T+N})(1 - \alpha^T)} \left[y_0 + \Delta y_0 - t_\alpha \Gamma + \frac{(1 - p^T)(t_p - t_\alpha)}{p^T - \alpha^T} \Gamma \right].$$

We can easily solve for Δy_0 :

$$\Delta y_0 = \left[t_\alpha - \frac{(1 - p^T)(t_p - t_\alpha)}{p^T - \alpha^T} \right] \Gamma, \quad (\text{A.20})$$

which is the cost of σ , measured in terms of goods y_0 . (A.20) can be simplified to (2.21).

References

- [1] Atkinson, Anthony (1971). "The Distribution of Wealth and the Individual Life Cycle", *Oxford Economic Papers*, vol 23, 239–254.
- [2] Caballero, Ricardo (1991). "Earnings Uncertainty and Aggregate Wealth Accumulation", *American Economic Review*, vol 81, 859–872.
- [3] ——— (1991). "Consumption Puzzles and Precautionary Saving", *Journal of Monetary Economics*, vol 25, 113–136.
- [4] Carroll, Christopher (1992). "The Buffer-Stock Theory of Saving: Some Macroeconomics Evidence", *Brookings Papers on Economic Activity*, no.2, 61–135.
- [5] Deaton, Angus (1992). *Understanding Consumption*. Oxford: Oxford University Press.
- [6] Guiso, Luigi, Tullio Jappelli and Daniele Terlizzese (1991). "Earnings Uncertainty and precautionary saving". *Journal of Monetary Economics*, vol 30, 307–337.
- [7] Irvine, Ian and Susheng Wang (1994). "Earning Uncertainty and Wealth Accumulation: Comment", *American Economic Review*, forthcoming.
- [8] Kessler, D. and A. Masson (1989). "Bequests and Wealth Accumulation: Are some Pieces of the Puzzle missing?" *Journal of Economic Perspectives*, vol 3, 141–152.
- [9] Kimball, Miles (1990). "Precautionary Saving in the Small and in the Large", *Econometrica*, 1990, vol 58, 53–73.
- [10] Kotlikoff, Laurence (1988). "Intergenerational Transfers and Saving", *Journal of Economic Perspectives*, vol 2, 41–58.
- [11] Kotlikoff, Laurence and Summers, Lawrence (1981). "The Role of Intergenerational Transfers in Aggregate Capital Accumulation", *Journal of Political Economy*, vol 89, 706–732.
- [12] Kreps, David and Porteus, E (1978). "Temporal Resolution of Uncertainty and Dynamic Choice Theory", *Econometrica*, vol 97, 91–100.
- [13] MaCurdy, Thomas (1992). "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis", *Journal of Econometrics*, vol 18, 83–114.
- [14] Menchik, Paul and Martin David (1983). "Income Distribution, Lifetime Savings and Bequests". *American Economic Review*, vol 73, 672–690.
- [15] Modigliani, Franco, "Measuring the Contribution of Intergenerational Transfers to Total Wealth: Conceptual Issues and Empirical Findings", in Kessler and Masson, editors, *Modeling the Accumulation and Distribution of Wealth*, Clarendon Press, Oxford, 1988.

- [16] Modigliani, Franco (1988). "The Role of Intergenerational Transfers and Life Cycle Saving in the Accumulation of Wealth". *Journal of Economic Perspectives*, vol. 2, 15-40.
- [17] Shorrocks, Anthony (1975). "The Age-Wealth Relationship". *Review of Economic and Statistics*, vol 57.
- [18] Skinner, Jonathan (1988). "Risky Income, Life Cycle Consumption, and Precautionary Savings", *Journal of Monetary Economics*, vol 22, 237-255.
- [19] Weil, Philippe (1993). "Precautionary Savings and the Permanent Income Hypothesis", *Review of Economic Studies*, vol 60, 367-383.
- [20] ——— (1990). "Nonexpected Utility in Macroeconomics", *Quarterly Journal of Economics*, vol 105, 29-42.
- [21] Yaari, Menahem (1965). "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *Review of Economic Studies*, vol 32, 137-150.
- [22] Zeldes, Stephen (1989). "Optimal Consumption with Stochastic Income", *Quarterly Journal of Economics*, vol 104, 275-298.

Working Papers in Economics

- 176 J.B. Towe Aspects of the Japanese Equity Investment in Australia; June 1992
- 177 P.D. Groenewegen Alfred Marshall and the Labour Commission 1891-1894; July 1992
- 178 D.J. Wright Television Advertising Regulation and Programme Quality; August 1992
- 179 S. Ziss Moral Hazard with Cost and Revenue Signals; December 1992
- 180 C. Rose The Distributional Approach to Exchange Rate Target Zones; December 1992
- 181 W.P. Hogan Markets for Illicit Drugs; January 1993
- 182 E. Jones The Macroeconomic Fetish in Anglo-American Economies; January 1993
- 183 F. Gill Statistics in the Social Sciences A Mixed Blessing? March 1993
- 184 Y. Varoufakis & S. Hargreaves-Heap The Simultaneous Evolution of Social Roles and of Cooperation; April 1993
- 185 C. Karfakis & D.M. Moschos The Information Content of the Yield Curve in Australia; April 1993
- 186 C. Karfakis & A. Parikh Uncovered Interest Parity Hypothesis for Major Currencies; May 1993
- 187 C. Karfakis & A.J. Phipps Do Movements in the Forward Discount on the Australian Dollar Predict Movements in Domestic Interest Rates? Evidence from a Time Series Analysis of Covered Interest Parity in Australia in the late 1980s; May 1993
- 188 J.B. Towe Citation Analysis of Publications on the Australian Tariff Debate, 1946-1991; August 1993
- 189 C. Karfakis & S.J. Kim Exchange Rates, Interest Rates and Current Account News: Some Evidence from Australia; September 1993
- 190 A.J. Phipps & J.R. Sheen Unionisation, Industrial Relations and Labour Productivity Growth in Australia: A Pooled Time-Series/Cross-Section Analysis of TFP Growth; September 1993
- 191 W.P. Hogan Market Value Accounting in the Financial Sector; November 1993
- 192 Y. Varoufakis & W. Kafouros The Transferability of Property Rights and the Scope of Industrial Relations' Legislation: Some Lessons from the NSW Road Transport Industry; November 1993
- 193 P.D. Groenewegen Jacob Viner and the History of Economic Thought; January 1994
- 194 D. Dutta & A. Hussain A Model of Share-Cropping with Interlinked Markets in a Dual Agrarian Economy; March 1994
- 195 P.E. Korsvold Hedging Efficiency of Forward and Option Currency Contracts; March 1994
- 196 J. Yates Housing and Taxation: An Overview; March 1994
- 197 P.D. Groenewegen Keynes and Marshall: Methodology, Society and Politics; March 1994
- 198 D.J. Wright Strategic Trade Policy and Signalling with Unobservable Costs; April 1994
- 199 J. Yates Private Finance for Social Housing in Australia; April 1994

- 200 L. Hatclad The Disjunction Between Decision-Making and Information Flows: The Case of the Former Planned Economies; April 1994
- 201 P.D. Groenewegen & S. King Women as Producers of Economic Articles: A Statistical Assessment of the Nature and the Extent of Female Participation in Five British and North American Journals 1900-39; June 1994
- 202 P.D. Groenewegen The French Connection: Some Case Studies of French Influences on British Economics in the Eighteenth Century; June 1994
- 203 F. Gill *Inequality and the Wheel of Fortune: Systemic Causes of Economic Deprivation*; July 1994
- 204 M. Smith The Monetary Thought of Thomas Tooke; July 1994
- 205 A. Aspromourgos Keynes on the Australian Wages System; July 1994
- 206 W. Kafourios & Y. Varoufakis Bargaining and Strikes: From an Equilibrium to an Evolutionary Framework; July 1994
- 207 A. Oswald & I. Walker Rethinking Labor Supply: Contract Theory and Unions; July 1994
- 208 J.B. Towe & D.J. Wright The Research Output of Australian Econometrics and Economics Department: 1988-93; July 1994
- 209 F. Gill & C. Rose Discontinuous Payoff Functions under Incomplete Information; August 1994
- 210 S-J Kim Inflation News in Australia: Its Effects on Exchange Rates and Interest Rates; October 1994
- 211 Y. Varoufakis Moral Rhetoric in the Face of Strategic Weakness: Modern Clues for an Ancient Puzzle; November 1994
- 212 E. Jones Bureaucratic Politics and Economic Policy: The Evolution of Trade Policy in the 1970s and 1980s; December 1994
- 213 R. Hataiseree & A.J. Phipps The Relationship Between Money and Income in Thailand: Some Evidence for the 1980s Using a Cointegration Approach; January 1995
- 214 G. White Classical Competition, Keynesian Adjustment and Composite Dynamics: A Critical Perspective; February 1995
- 215 C. Karfakis & A.J. Phipps Treasury Note and Bank Bill Rates, the Risk Premium and Australian Monetary Policy; February 1995
- 216 P.D. Groenewegen The Post 1945 Internationalization of Economics The Australian Experience; April 1995
- 217 S-J Kim Modeling Changes in Daily \$A Exchange Rates: An Application of GARCH; June 1995
- 218 R. Hataiseree & A. Phipps The Degree of Capital Mobility in Thailand: Some Estimates Using a Cointegration Approach; June 1995
- 219 I.J. Irvine & W.A. Sims Measuring Consumer Surplus with Unknown Hicksian Demands; June 1995
- 220 I.J. Irvine & S. Wang Saving Behavior in a Pure Life-Cycle Model with Income Uncertainty; June 1995

Copies are available upon request from:

Department of Economics
The University of Sydney
N.S.W. 2006, Australia

Working Papers in Economics Published Elsewhere

- 2 I.G. Sharpe & R.G. Walker *Journal of Accounting Research*, 13(2), Autumn 1975
- 3 N.V. Lam *Journal of the Developing Economies*, 17(1), March 1979
- 4 V.B. Hall & M.L. King *New Zealand Economic Papers*, 10, 1976
- 5 A.J. Phipps *Economic Record*, 53(143), September 1977
- 6 N.V. Lam *Journal of Development Studies*, 14(1), October 1977
- 7 I.G. Sharpe *Australian Journal of Management*, April 1976
- 9 W.P. Hogan *Economic Papers*, 55, The Economic Society of Australia and New Zealand, October 1977
- 12 I.G. Sharpe & P.A. Volker *Economics Letters*, 2, 1979
- 13 I.G. Sharpe & P.A. Volker *Kredit und Kapital*, 12(1), 1979
- 14 W.P. Hogan *Some Calculations in Stability and Inflation*, A.R. Bergström et al (eds.), J. Wiley & Sons, 1978
- 15 F. Gill *Australian Economic Papers*, 19(35), December 1980
- 18 I.G. Sharpe *Journal of Banking and Finance*, 3(1), April 1978
- 21 R.L. Brown *Australian Journal of Management*, 3(1), April 1978
- 23 I.G. Sharpe & P.A. Volker *The Australian Monetary System in the 1970s*, M. Porter (ed.), Supplement to Economic Board 1978
- 24 V.B. Hall *Economic Record*, 56(152), March 1980
- 25 I.G. Sharpe & P.A. Volker *Australian Journal of Management*, October 1979
- 27 W.P. Hogan *Malayan Economic Review*, 24(1), April 1979
- 28 P. Saunders *Australian Economic Papers*, 19(34), June 1980
- 29 W.P. Hogan I.G. Sharpe & P.A. Volker *Economics Letters*, 6 (1980), 7 (1981)
- 30 W.P. Hogan *Australian Economic Papers*, 18(33), December 1979
- 32 R.W. Bailey, V.B. Hall & P.C.B. Phillips *Keynesian Theory, Planning Models, and Quantitative Economics*, G. Gandolfo and F. Marzano (eds.), 1987
- 38 U.R. Kohli *Australian Economic Papers*, 21(39), December 1982
- 39 G. Mills *Journal of the Operational Research Society* (33) 1982
- 41 U.R. Kohli *Canadian Journal of Economics*, 15(2), May 1982
- 42 W.J. Merrilees *Applied Economics*, 15, February 1983
- 43 P. Saunders *Australian Economic Papers*, 20(37), December 1981
- 45 W.J. Merrilees *Canadian Journal of Economics*, 15(3), August 1982
- 46 W.J. Merrilees *Journal of Industrial Economics*, 31, March 1983
- 49 U.R. Kohli *Review of Economic Studies*, 50(160), January 1983
- 50 P. Saunders *Economic Record*, 57(159), December 1981
- 53 J. Yates *AFSI, Commissioned Studies and Selected Papers*, AGPS, IV 1982
- 54 J. Yates *Economic Record*, 58(161), June 1982
- 55 G. Mills *Seventh Australian Transport Research Forum-Papers*, Hobart 1982
- 56 V.B. Hall & P. Saunders *Economic Record*, 60(168), March 1984

- 57 P. Saunders *Economic Record*, 59(166), September 1983
- 58 F. Gill *Économie Appliquée*, 37(3-4), 1984
- 59 G. Mills & W. Coleman *Journal of Transport Economics and Policy*, 16(3), September 1982
- 60 J. Yates *Economic Papers*, Special Edition, April 1983
- 61 S.S. Joson *Australian Economic Papers*, 24(44), June 1985
- 62 R.T. Ross *Australian Quarterly*, 56(3), Spring 1984
- 63 W.J. Merrilees *Economic Record*, 59(166), September 1983
- 65 A.J. Phipps *Australian Economic Papers*, 22(41), December 1983
- 67 V.B. Hall *Economics Letters*, 12, 1983
- 69 V.B. Hall *Energy Economics*, 8(2), April 1986
- 70 F. Gill *Australian Quarterly*, 59(2), Winter 1987
- 71 W.J. Merrilees *Australian Economic Papers*, 23(43), December 1984
- 73 C.G.F. Simkin *Singapore Economic Review*, 29(1), April 1984
- 74 J. Yates *Australian Quarterly*, 56(2), Winter 1984
- 77 V.B. Hall *Economics Letters*, 20, 1986
- 78 S.S. Joson *Journal of Policy Modeling*, 8(2), Summer 1986
- 79 R.T. Ross *Economic Record*, 62(178), September 1986
- 81 R.T. Ross *Australian Bulletin of Labour*, 11(4), September 1985
- 82 P.D. Groenewegen *History of Political Economy*, 20(4), Winter 1988 and *Scottish Journal of Political Economy*, 37(1) 1990
- 84 F.M.A. Gross, W.P. Hogan & I.G. Sharpe *Australian Economic Papers*, 27(50), June 1988
- 85 F. Gill *Australian Bulletin of Labour*, 16(4), December 1990
- 94 W.P. Hogan *Company and Securities Law Journal*, 6(1), February 1988
- 95 J. Yates *Urban Studies*, 26, 1989
- 96 B.W. Ross *The Economic and Social Review*, 20(3), April 1989
- 97 F. Gill *Australia's Greatest Asset: Human Resources in the Nineteenth and Twentieth Centuries*, D. Pope (ed.), Federation Press, 1988
- 98 A.J. Phipps *Australian Economic Papers*, 31(58), June 1992
- 99 R.T. Ross *Australian Bulletin of Labour*, 15(1), December 1988
- 100 L. Haddad *Hetsa Bulletin*, (11), Winter 1989
- 101 J. Piggott *Public Sector Economics - A Reader*, P. Hare (ed.), Basil Blackwell, 1988
- 102 J. Carlson & D. Findlay *Journal of Macroeconomics*, 13(1), Winter 1991
- 102 J. Carlson & D. Findlay *Journal of Economics and Business*, 44(1), February 1992
- 104 P.D. Groenewegen *Decentralization, Local Government and Markets: Towards a Post-Welfare Agenda*, R.J. Bennet (ed.) Oxford University Press, 1990
- 107 B.W. Ross *Prometheus*, 6(2), December 1988
- 108 S.S. Joson *Rivista di diritto valutario e di economia internazionale*, 35(2), June 1988
- 112 P. Groenewegen *NeoClassical Economic Theory 1870 to 1930*, K. Hennings and W. Samuels (eds.), Boston Kluwer-Nijhoff, 1990
- 113 V.B. Hall, T.P. Truong & V.A. Nguyen *Economic Record*, 59(166), September 1983
- 114 V.B. Hall, T.P. Truong & V.A. Nguyen *Australian Economic Review*, (87) 1989(3)
- 115 F. Gill *Australian Journal of Social Issues*, 25(2), May 1990
- 116 G. Kingston *Economics Letters*, 15 (1989)
- 117 V.B. Hall & D.R. Mills *Pacific and Asian Journal of Energy*, 2(2), December 1988
- 118 W.P. Hogan *Abacus*, 25(2), September 1989
- 120 P. Groenewegen *Flattening the Tax Rate Scale. Alternative Scenarios & Methodologies*, (eds) J.G. Head and R. Krever. 1990
- 122 W.P. Hogan & I.G. Sharpe *Economic Analysis and Policy*, 19(1), March 1989
- 123 G. Mills *Journal of Transport Economics and Policy*, 23, May 1989
- 126 F. Gill *The Australian Quarterly*, 61(4), 1989
- 128 S. Lahiri & J. Sheen *The Economic Journal*, 100(400), 1990
- 130 J. Sheen *Journal of Economic Dynamics and Control*, 16, 1992
- 135 Y. Varoufakis *Économie Appliquée*, 45(1), 1992
- 136 L. Ermini *The Economic Record*, 69(204), March 1993
- 138 D. Wright *Journal of International Economics*, 35, (1/2) 1993
- 139 D. Wright *Australian Economic Papers*, 32, 1993
- 141 P. Groenewegen *Australian Economic Papers*, 31, 1992
- 143 C. Karfakis *Applied Economics*, 23, 1991
- 144 C. Karfakis & D. Moschos *Journal of Money, Credit and Banking*, 22,(3), 1990
- 147 J. Yates *Housing Studies*, 7, (2), April 1992
- 158 W.P. Hogan *Economic Papers*, 10(1), March 1991
- 159 P. Groenewegen *Local Government and Market Decentralisation: Experiences in Industrialised, Developing and Former Eastern Block Countries*, R. J. Bennett (ed.) UN University Press, 1994
- 160 C. Karfakis *Applied Financial Economics*, 1(3), September 1991
- 162 Y. Varoufakis *Erkenntnis*, 38, 1993
- 163 Y. Varoufakis *Science and Society*, 56(4), 1993
- 173 C. Rose *The Rand Journal of Economics*, 24(4), Winter 1993
- 177 P. Groenewegen *European Journal of the History of Economic Thought*, 1(2) Spring 1994
- 178 D. J. Wright *Economic Record*, 70(211), December 1994
- 187 C. Karfakis & A. J. Phipps *Australian Economic Papers*, 33(62), June 1994
- 189 C. Karfakis & S-J Kim *Journal of International Money and Finance*, 14(4) August 1995
- 190 A.J. Phipps & J.R. Sheen *Labour Economics and Productivity*, 6(1), March 1994
- 193 P. Groenewegen *Contributions to Political Economy*, 13, 1994
- 199 J. Yates *Housing Policy Debate*, 5(2), 1994
- 202 P. Groenewegen *Dix-Huitième Siècle* (26), 1994
- 203 F. Gill *Australian Economic Papers*, 33(62), June 1994
- 208 J.B. Towe & D.J. Wright *Economic Record*, 71(212), March 1995