Measuring Consumer Surplus with Unknown Hicksian Demands

by

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Abstract

The objective of this paper is to introduce the Slutsky demand curve as a tool in welfare analysis. It is shown that the compensating or equivalent variation can in most cases be measured to within a fraction of a percent of their true value without any numerical integration techniques. Two well-known examples in the literature are explored. A theoretical measure of the accuracy of the Slutsky based measure, relative to the Marshallian measure, is developed. The approach is locally path independent and can be used to measure the money value of a ration. Finally, its application to models of labor supply is explored and an error in the literature is corrected.

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1. Introduction

The measurement of deadweight loss has been a change in focus since the papers of Willig (1976) and Hausman (1981). At the present time, the major emphasis is on cases where the Hicksian demands cannot readily be obtained through integration. As a consequence, numerical methods which are applicable to either single or multi equation systems dominate. Since Vartia’s paper (1983), Hayes and Porter-Hudak (1987), Porter-Hudak and Hayes (1986, 1991), Hausman and Newey (1992), and Breslow and Smith (1995a, 1995b), among others, have proposed numerical methods for calculating the compensating variation, CV, and equivalent variation, EV, and their associated standard errors.

In this paper we propose a simple one-step procedure for estimating the CV or EV associated with one or more price changes where the Hicksian demands are unknown. This method is applicable without any numerical integration and requires only an understanding of a basic graduate course in consumer theory. The paper proceeds as follows: In section 2 we illustrate the ideas generally and then apply them in sections 3 and 4 to two well known examples in this literature - Hausman’s (1981) gasoline example, and McKenzie and Pearce’s (1976) two good indirect aditioal example. We find that our method approximates the CV and EV to within a fraction of a percent. In developing these results, we demonstrate an equivalence between the method we propose and what has become known in the literature as the McKenzie and Pearce (MP) method for the Hausman linear demand example. In section 5 we develop the theoretical properties of our method. We show that it is locally path independent and develop a ‘Willig type’ result on relative errors which supports the results from the numerical examples. In section 6 we examine the welfare cost of a ration, and the measurement of the deadweight loss of wage taxes.

2. A Slutsky compensation based algorithm

A standard exercise in introductory graduate microeconomics is to examine the relationship between Hicksian and Slutsky compensated demand curves. Figure 1 is typically used in such exercises – see, for example, Friedman (1962) or Silberberg (1990). We shall use the term Slutsky compensation to denote the income compensation required so that a consumer can afford the bundle of goods purchased prior to a price change. The amount of a good purchased at a given set of prices with such compensation we will denote by \( x^S = x^S(p, x^m) \) where \( x^m \) is the specific good or vector of goods which the consumer chooses at the initial price vector \( p^S \). In contrast, Hicksian compensation is the amount required to reach a particular level of indifference. The associated demand is \( x^H = x^H(p, x^H) \) where \( x^H(x^H) \) is the initial level of utility attainable. The Marshallian demand will be denoted by \( x^M = x^M(p, y) \) where \( y^* \) is the initial income and \( x \) is assumed to be a normal good. The standard relationship between these demands is that the Slutsky demand is above, but tangent to the Hicksian demand at the point where they intersect the Marshallian demand at point \( f \) in Figure 1. Slutsky compensation is the basis of revealed preference and index number theory (e.g. Diewert, 1976), but to date has not been used as a basis for measuring \( CV \) or \( EV \).

When the Hicksian demand is known, the area to the left of the Hicksian demand represents the monetary value to the consumer of a price change. When the integration is with respect to the original level of utility, \( u^S \), the resulting measure is the \( CV \) and when it is with respect to the new (post price change) utility level it is the \( EV \). When the Hicksian demand is unknown an approximation is required. The proposal presented in this paper is to use the Slutsky demand to approximate these measures. In addition to being straightforward to derive from an estimated Marshallian demand, we show that it provides an accurate approximation to the \( CV \) and/or \( EV \), even for very large price changes or where the share of a good whose price is charged forms a significant component of the consumer’s budget. In the case of a single good the proposal is simply to measure the area \( p^S dp \) in Figure 1 and use it as an approximation of the unknown area \( p^S dp \). Estimation of the \( CV \) is only slightly more complicated in the mixed good case, but essentially the same tools can be used.

To examine the relationship between the Slutsky demand-based measure of the \( CV \) and other methods, it is convenient to define the \( CV \) mathematically. Letting \( e \) denote the expenditure function, the \( CV \) is given by

\[
CV = e(p^S, u^S) - e(p, u^S)
\]  

(2.1)

where \( p \) is a vector of prices at the initial \( (p^S) \) and final \( (p) \) equilibria. Expanding \( e(p, u^S) \) around the initial price and utility combination by means of a Taylor series, and considering only one price change for the moment, we obtain

1. The demand in Figure 1 come from a Cobb-Douglas utility function \( u = x_1 x_2 \) with a budget constraint \( x_1 + 2x_2 = 4 \). The proximity of \( x^S \) to \( x^H \) is thus not simply constructed to support our approach. All figures are developed with Maple V. 66 using exact functions, and are presented to scale. We have omitted part of the horizontal axis in some figures so that the various demands can be distinguished.

2. If the outer Marshallian demand would be the steepest of the three demands and the Slutsky demand curve would be everywhere beneath the Hicksian curve, except at the point of tangency. The relationship between these three demand functions is investigated further in Appendix B.
\( e(p', y') = e(p^0, y^0) + \frac{\partial e(p^0, y^0)}{\partial p} \Delta + 0.5 \frac{\partial^2 e(p^0, y^0)}{\partial p^2} \Delta^2 + R \)  \hspace{1cm} (2.2)

where \( R \) is the remainder term in the series, and \( \Delta \) is the price change \((p' - p^0)\).

If the quadratic terms alone form a good approximation, then the \( CV \) may be approximated by

\[ CV \approx -v^m(p^0, y^0) \Delta - 0.5 \frac{\partial v^m(p, y^0)}{\partial p} \Delta^2 \]  \hspace{1cm} (2.3)

since the derivative of the expenditure function is the Hicksian demand. McKenzie and Pearce (1976) - henceforth MP - suggested that \( \frac{\partial v^m(p, y^0)}{\partial p} \) can be evaluated by the Slutsky equation for small changes in \( p \). That is:

\[ \frac{\partial v^m(p, y^0)}{\partial p} = \frac{\partial x^m(p, y^0)}{\partial p} + x^0 \frac{\partial x^m(p, y^0)}{\partial y} \]  \hspace{1cm} (2.4)

and this gradient is therefore obtainable from a knowledge of the parameters in the Marshallian demand function. Geometrically, the MP method amounts to taking the tangent \((Z)\) to the Hicksian demand at the initial equilibrium and approximating the area \( p^f x p \) by \( p^f x p \). Clearly, if the Hicksian demand is reasonably close to being linear (i.e. if the remainder in the Taylor series expansion is small), this will yield a good estimate of the \( CV \).

A variation of the MP approach is to break down the price change into a series of smaller changes and to compensate the consumer at each stage. Such a procedure will reduce the error for large deviations from the original equilibrium. This is essentially the approach adopted in Breslaw and Smith (1994a, 1994b). They also present estimates for the variances of \( CV \) and \( EV \).

We now examine the accuracy of the Slutsky compensation method of measuring the \( CV \). While this method need be used only in cases where the Hicksian demands are unobtainable, we test it by using three well-known examples for which exact measures are available. We use Hausman's gasoline example (1981) and labour supply example (1980, 1981) for the case of a single price change, and MP's example for multiple price changes.

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1In the interest of brevity we omit the development and estimation of the \( EV \), despite its desirable characteristics [Kay, 1986].
3. Hausman's gasoline example

Hausman's frequently used example is of a linear demand function for gasoline of the form

\[ x^H = \alpha p + \delta y + \gamma \]  \hspace{1cm} (3.1)

where \( p \) is the price of gasoline, \( y \) is income and \( \gamma \) a constant. The corresponding indirect utility function is

\[ u(p, y) = e^{\delta y} \left[ y + \frac{\gamma}{\alpha} (\alpha p + \delta y + \gamma) \right] \]  \hspace{1cm} (3.2)

The values Hausman chose are \( y = 729 \), initial price \( p = 0.75 \) and new price \( p' = 1.5 \). The estimated parameter values are \( \alpha = -4.22 \), \( \delta = 0.063 \), \( \gamma = 4.95 \). The utility values for each price are \( u_0 = -1,377.154 \) and \( u_1 = -1,410.019 \). The Marshallian, Slutsky and Hicksian demands are shown in figure 2. The Hicksian demand is

\[ x^H = u' \delta \delta p - \alpha \delta \]  \hspace{1cm} (3.3)

and the Slutsky demand is obtained by substituting the initial consumption bundle for \( y \) in 3.1

\[ x^S = (\alpha + \delta x^H) p + \delta x^H + \gamma \]  \hspace{1cm} (3.4)

where \( x \) is a composite of all other goods consumed at the normalized price of unity.

For the linear Marshallian demand curve, the Hicks compensated demand is concave rather than convex. The exact value of the compensating variation (the area left of \( x^H \) between \( p = 0.75 \) and \( p' = 1.5 \)) is -37.167, with the net of tax revenue deadweight loss component being 5.172. The residual is accounted for by the tax revenue generated of 31.995. The Marshallian approximation to the CV is -35.994 - an error of 3.17%. Finally, it should be noted that when he discusses deadweight loss, Hausman always refers to a triangular area. However, as emphasized in Zabala (1982),

the CV corresponding to this tax induced price increase, when considered net of the tax revenue raised, is the trapezoid \( abfk \) in figure 2. This accounts for the difference in deadweight loss measures reported here (5.176) and by Hausman (2.88).

Integrating behind the Slutsky demand, 3.4, between \( p' \) and \( p \) yields:

\[ \int_p^{p'} x^S dp = \left[ \frac{1}{2} (\alpha + \delta x^0) p^2 + \delta x^0 p + \gamma p \right] \bigg|_{p' = 1.5}^{p = 0.75} \]  \hspace{1cm} (3.5)

The value of this integral is -37.2242, representing an error of one seventh of one percent.\(^4\)

There is an interesting equivalence between our Slutsky based measure and what we term the MP measure in the case of the linear Marshallian demand curve: in this case they are identical. Graphically this simply means that the tangent to \( x^H \) at \( f \) intersects the line \( p = 1.5 \) at the same point as the Slutsky compensated demand function. But in this instance the Slutsky demand curve is linear in \( p \) (see equation 3.4) and thus is itself the tangent to \( x^H \) at \( f \) since by definition \( \partial x^H / \partial p = \partial x^f / \partial p \) at \( f \).

\(^4\)In contrast the CV is a triangular area in each case and can be obtained using the Hicksian demand through the new equilibrium at point \( x^H (p', y) \).

\(^5\)The Slutsky measure of deadweight loss is 5.176, an error of about 1.1%. This compares with an error of over 25% for the equivalent Marshallian measure.
4. Multiple price changes.

The welfare effects of multiple price changes can also be estimated using Slutsky demands. To illustrate this, consider the example from MP. Their example differs from Hausman’s substantially. In Hausman’s case the price change considered was very large -100%, but the share of the budget attributable to the good whose price change was arguably small - 5.6% at base prices. In the MP example the price changes are smaller - between 10% and 15%, but the budget shares are much larger: at base prices good 1 accounts for 66.7% of expenditure and good 2 accounts for 33.3%.

Consider the indirect utility function

\[ v(p, y) = y \left( \frac{1}{p_1} + \frac{1}{p_2} \right) \]  

(4.1)

This is a restricted form of the indirect Addilog\(^7\), and the demands are given by

\[ x^N = \frac{y}{p_1 + p_2 / p_1}; \quad x^N = \frac{wp_2^2}{(p_1 + p_2)^2}; \quad x^E = p_0 x^N + p_1 x^N \frac{p_0}{p_1} \]  

(4.2)

The expenditure function is obtained by inverting 4.1 and is of the form

\[ e = \frac{yp_1 p_2}{p_1 + p_2} \]  

(4.3)

MP set \( y = 220, p_1^0 = 1.0, \) and \( p_2^0 = 2.0 \). They change the price of good 1 to \( p_1 = 1.1 \) and then lower the price of good 2 to \( p_2 = 1.692338 \). The prices are changed so that final utility equals initial utility: \( u = v^0 = 330 \).

The value of the CV attributable to the increase in \( p_1 \) is -14.1933. The demands in market 1 are illustrated in figure 2. A Marshallian approximation to the CV in market 1, obtained by integrating behind the Marshallian demand curve, is -13.75145.

The Slutsky estimate of the CV is obtained by integrating \( x^E \). That is,

\[ \int x^E dp_1 = \int \frac{x_1 + x_2}{1 + c_1 p_1} dp_1 = \frac{c_1}{p_1} \left[ \frac{1}{1 + c_1 p_1} dp_1 + c_2 \right] \int \frac{1}{p_0 (1 + c_0^0 p_1)} dp_1 \]

where \( c_1 = x_1^0, \quad c_2 = p_0 x_2^0, \quad c_3 = p_2^0 \).

\(^7\) The direct utility function corresponding to this is \( u(x_1, x_2) = x_1 + x_2 + 2x_1 x_2 \). This is a CES form, with the distribution parameters set equal to unity and an elasticity of substitution equal to 2. It thus exhibits a relatively high degree of substitutability.
$$= \frac{c_2}{x_1} \int x_2 \frac{1}{1 + c_2 \theta} \, dp_1 + \frac{c_2}{x_1} \int x_2 \frac{1}{p_1 - c_2} \, dp_1 = \frac{c_2 \ln(1 + c_2 \theta) + c_2 \ln(p_1) - c_2 \theta}{c_2}$$

Evaluating this integral yields a value of -14.2032, an error of 0.068 of one percent. The ratio of the Marshallian error to the Slutsky error is 45:1.

Consider now the measurement of the CV in market 2 after the price change in market 1. As a result of a change in p_1, the demands for good 2 shift. The Hicksian demands are shown in figure 4. The CV associated with this price change is given as the area behind the new Hicksian demand x^H_2(p_1, \theta, p_2, p_1, p_1 = 1.1), i.e. x^H_2 \theta. The CV associated with the joint price change in p_1 and p_2 is invariant with respect to the order in which these integrals are evaluated when using the Hicksian demands. The Hicksian measure of the CV of this price fall in market 2 is 1.1395.

The Slutsky estimate of the CV is obtained by again integrating behind the Slutsky demand curve. Care must be exercised at this point in defining the appropriate Slutsky demand for the integration. To obtain the Slutsky demand which is tangent to the Hicksian, x^S_2(p_1, \theta, p_1 = 1.1) at p_2 = 2.0, define x as the vector of Hicksian demands at the intermediate price vector \tilde{p}_1 (p_1 = 1.1, p_1 = 2.0). Then this is the bundle upon which the Slutsky demand in market 2 must be conditioned, because at \tilde{p}_1, x^S_2(\tilde{p}_1, \theta, \tilde{p}_1) = x^S_2(\theta, \tilde{p}_1, \theta, \tilde{p}_1), where x yields utility level \theta^S. The two demands are not only equal at this point, but are tangent. In figures 3 and 4, x_1 and x_2 can be read from the Hicksian demand curves at (p_1 = 1.1, p_2 = 2.0). In practice the Hicksian demand curves are unknown, but the vector x can readily be inferred by using MP's suggestion that the change in quantity consumed can be approximated by the Slutsky equation. Thus in market 1, x_1 can be obtained from the approximation

$$\Delta x^H_1 = (\partial x^H_1 / \partial p_1 + x_1 \partial x^H_1 / \partial \theta) \Delta p_1$$

and in market 2, x_2 can be obtained similarly:

$$\Delta x^H_2 = (\partial x^H_2 / \partial p_1 + x_2 \partial x^H_2 / \partial \theta) \Delta p_1$$

The Slutsky demand curve x^S_2(\theta, \tilde{p}_1, \theta, \tilde{p}_1) is given in figure 4. The estimate of the CV obtained in this way is 14.1925 - an error of less than one thousandth of one percent, without any knowledge of the underlying utility function.

Finally, the estimate of the CV in market 2 using the Marshallian demand curve is 13.7545. It is worth noting that, in the case analyzed, the Marshallian demands yield the correct value of the total welfare impact of the multiple price change (i.e. zero); this despite large errors in the measurement of the welfare impact of each individual price change. As is shown in Appendix A, this result is not general, and occurs in this case because the underlying utility is homogeneous and because the price changes analyzed were designed to hold utility constant. In reality, these circumstances will rarely, if ever, arise and, even if they do, this will generally not be known. As a result, it is best to adopt that procedure which yields the most accurate estimate of the welfare impact of each individual price change i.e. the Slutsky based measure.
5. Theoretical Properties

As is clear from the examples provided, there is strong evidence that the Slutsky demand provides an excellent method of approximating the welfare effects of price changes. In this section, the theoretical relationship between the errors from measuring CV with the Slutsky and Marshallian demands are developed in a manner which does not depend on the functional forms of the demands. In addition, the theoretical path independence of the Slutsky approach is investigated.

5.1. A Willig type result

Robert Willig (1976) derived a measure of the error associated with the use of a Marshallian, rather than a Hicksian demand to estimate the CV or EV. Willig's result can be illustrated, following Bowdy and Bruce (1984, 218), with the use of figure 5. A fall in price from $p^2$ to $p^1$ yields a welfare gain, as measured by the CV, equal to the area $p^1\Delta d$. The Marshallian approximation to this is $p^2\Delta d\Delta p$, yielding an error $ad$. Geometrically this can be approximated by $\frac{1}{2}d\Delta p$ and an expression for this is easily derived: $dc$ is the income effect associated with an income change of $CV (= p^2\Delta d)$. That is $dc = \frac{\partial C}{\partial m} d\Delta m$ where $\Delta m = CV$. Accordingly the Marshallian error is given by

$$ad = \frac{1}{2} \frac{d}{m}(CV) \Delta p$$

(5.1)

where $\eta$ is the income elasticity of demand.

The error associated with using the Slutsky demand is given by $adf$. As before we can easily derive an expression for the distance $df$. The movement from $c$ to $f$ is attributable to the Slutsky compensation - in this case negative. The Slutsky compensation is defined by $a^2\Delta p (= p^2\Delta e)$ in the case of a single good. It therefore follows that the Slutsky income effect, $df$, is due simply to an increase in income defined by the area $ade$. Accordingly the error associated with using the Slutsky demand is

$$adf = \frac{1}{2} \frac{d}{m}(adf) \Delta p$$

(5.2)

The area $ade$ can be interpreted as a gain from the elimination of a deadweight loss if the price fall resulted from eliminating a tax equal to $\Delta p$. Referring
to the deadweight loss, $DWL$, we therefore have the result that the ratio of the Slutsky error to the Marshallian error is approximately $DWL/CV$.

A corresponding result can be derived for a price increase or for the EV. The magnitude of the ratio, for a given initial consumption, depends upon the price elasticity of the Hicksian demand. Specifically, the smaller its price elasticity the greater is the error from using the Marshallian demand. In the limit, if the good in question is perfectly complementary with the aggregate of other goods, the Slutsky and Hicksian demands coincide and $DWL$, as measured by the Hicksian demand tends to zero.

While these results are remarkably simple, they essentially illustrate that there is no need to use Marshallian demands to estimate CV or EV when the Hicksian demand cannot be obtained. If the Marshallian demand can be integrated, so can the Slutsky demand, and it yields an estimate of the welfare change which is an order of magnitude smaller than that which comes from the Marshallian demand.

5.2. Path Independence

An important property of Hicksian demands, which is of special concern when determining the welfare effects of multiple price changes, is path independence. This characteristic of demand systems requires symmetric cross partial price derivatives: $\frac{\partial x^i}{\partial p_j} = \frac{\partial x^j}{\partial p_i}$, Slutsky demands possess this property locally. That is, at the conditioning price vector $p = p^0$, $\frac{\partial x^i}{\partial p_j} = \frac{\partial x^j}{\partial p_i}|_{p = p^0}$ because the functions $x^i$ and $x^j$ are tangent. While path independence thus holds only locally, it should be noted that Marshallian demands generally do not have this property even locally.\textsuperscript{14}

\textsuperscript{12}Making triangular assumptions is equivalent to using the linear terms of a Taylor series expansion.

\textsuperscript{13}This suggests the existence of an interesting anomaly. While it was shown in section 3 that the imposition of a tax resulted in a deadweight loss, as measured with CV, which was not a triangle, but rather a trapezoid, the removal of the same tax leads to a reduction in deadweight loss, again measured with CV, which is a triangle.

\textsuperscript{14}We have experimented with some two homogeneous functions for moderate price changes and found that the order of price changes has an imperceptible effect on the measurement. However, as mentioned earlier, Marshallian demands derived from homogeneous utility functions, such as in the McKenzie-Pearce paper, do exhibit global path independence.
6. Extensions

6.1. Quantity constraints

The application of this approach to measuring the welfare cost of a ration where the Hicksian demand function is unknown, but the parameters of the Marshallian demand function are known, is straightforward\footnote{Neary and Roberts (1980) have developed the theory underlying behavior under binding rations in terms of cost functions and virtual prices. Our focus is upon cases where the cost function and Hicksian demands are not known.}. It simply requires that a set of virtual prices be computed which support the constrained equilibrium. The welfare value of removing or imposing the ration, either in whole or in part, can then be evaluated at whatever set of prices is desired. For purposes of illustration, consider again Hausman's gasoline example. We need no new calculations, merely a reinterpretation of the example.

Suppose a ration is imposed which is designed to reduce consumption by 20% from its current level of 33.335, which is demanded at \( p = 0.75 \). At an income level of $720, it turns out that the gasoline price which supports such an equilibrium is exactly \( p = 1.5 \). The cost of the ration can therefore be interpreted as the deadweight loss measured in section 3. We have already seen that the Slutsky compensation method we propose gives a very accurate measure of this welfare cost.

6.2. Labor Supply

The use of Slutsky demands in measuring the cost of taxes in the labor market provides a stringent test of the technique's usefulness, due to the large 'income' effects which accompany a change in the value of the endowment as a result of an income tax.

The most widely cited case is again from Hausman (1980). It is a model which is discussed extensively even in recent reviews (e.g. McCurdy, Green and Paarsch, 1992), because it affords a means of testing the reasonableness of imposing Slutsky conditions on the labor supply function. The linear Marshallian labor supply is given by

\[ L^H = n\omega + \delta y + s \]  \hspace{1cm} (6.1) 

where \( \omega \) is the wage rate, \( y \) is nonlabor income and \( s \) a constant dependent upon demographics. This can be integrated to yield a utility function whose indirect
form is

$$u = e^{-w(y + \frac{\alpha}{\delta}w + \frac{s}{\delta} - \frac{\alpha}{\delta^2}y)}$$ \hspace{1cm} (6.2)$$

where \( e \) is the exponential operator. The Hickson labor supply function \( L^H \) can be obtained from 6.2 and is

$$L^H = \frac{\alpha}{\delta} + \delta e^{-w} u$$ \hspace{1cm} (6.3)$$

where \( u \) can be either the initial (\( u^0 \)) or final (\( u' \)) level of utility attained. Finally, the Slutsky labor supply, \( L^S \), is

$$L^S = w(\alpha - \delta L^H) + \delta v^0 + s$$ \hspace{1cm} (6.4)$$

where \( c \) is a composite of all goods consumed, except leisure, at the normalized price of unity. For the set of values given in Haushan (1980 and 1981)\(^{10}\) these demands are illustrated in figure 6. Again the Hickson labor supply has a positive second derivative. The true value of the CV is \(-$1,246.95\) and the CV computed as the integral behind the Slutsky demand is \(-$1,238.3\). The error is thus less than one percent. In contrast, the welfare measure using the Marshallian supply function is \(-$1,315.4\) - an error of almost 6%.

It is of interest to note that Haushan’s estimate of the CV incorporates a serious error. His estimate is \(-$2,066\), which, as he points out, imputes an error of 44% to the Marshallian approximation (1981, p.672). However, this is clearly at odds with the well known relationship which implies, in the case of a single price reduction, that the absolute value of the Marshallian measure will be bounded from below by the absolute value of the CV and from above by the absolute value of the EV. The EV value is \(-$1,382.93\). It follows that, as a measure of the true welfare loss, the Marshallian measure is not as faulty as Haushan proposes and his injunction against the use of Willig’s approach is not fully justified.

Notwithstanding this, as a measure of the pure deadweight loss component, the Marshallian measure deviates considerably from the CV or EV based measures, due to the greater than unit elastic value associated with his labor supply function (elasticity = 1.15). The deadweight loss using CV is $101.80. The Marshallian approximation is $170.55 and the Slutsky is $93.45.

In conclusion, this example provides an illustration of the use of the Slutsky labor supply function. In general, the Hickson labor supply cannot be obtained from estimated econometric equations, but the Slutsky demand can. It is also

\(^{10}\) \( y \) is set at \$32.26, \( u^0 \) is \$4.15, \( w^0 \) is \$3.32. The estimated parameter values are \( \alpha = 495.1, \ \delta = -0.225, \ s = 765.4\). These values yield utility of \( u^0 = -27.387.2, \ w = -29.210.4\).
7. Conclusion

The recent literature on welfare measurement has focussed upon cases where exact theoretical measures are unavailable - i.e. where the cost function or Hicksian demands cannot be generated from estimated Marshallian demands. Our objective in this paper has been to show that a simple and intuitive measure of welfare changes associated with price movements is readily available without any numerical integration techniques. An understanding of the relationships between Hicksian, Marshallian and Slutsky demand functions as described, for example, by Milton Friedman (1952) is all that is required. Tests of the procedure indicate that it is accurate to within a fraction of a percent, even in cases where price changes or budget shares are large. On a theoretical level, following Willig’s methodology, it is straightforward to demonstrate that the error associated with the Slutsky demand is an order of magnitude smaller than that associated with the Marshallian demand and is easy to obtain. Since the Slutsky demand based measure is also locally path independent, we conclude first that there is no reason ever to have recourse to Marshallian based measures of welfare, and second that the numerical methods recently proposed in the literature need not be used unless an error of a fraction of one percent is deemed unreasonably large.
A. Measuring the welfare effects of multiple price changes with Marshallian demands

It was noted in Section 4 that the overall welfare impact of a multiple price change was measured correctly using Marshallian demands, when the example from MP was used. This result is not general and, as shown below, is a result of the particular utility function chosen and the type of price change investigated. In particular, it is shown that if the Marshallian demand is based on a homothetic utility function and if the multiple price changes are designed to hold utility constant, the resulting change in welfare always equals the correct value, despite the fact that the welfare impacts of individual price changes, as measured by the Marshallian approximation to CV, can be exceedingly inaccurate!

Assume that \( U = U(x_1, x_2) \) is homothetic. A well known result is that the indirect utility function is then separable in \( p \) and \( y \):

\[
U = v(p, y) = \psi(p)y
\]

and by Roy’s identity the Marshallian demand will always take the form,

\[
x_i^M = \frac{\partial \psi}{\partial p_i} y = f(p)y
\]

and the Marshallian approximation to any CV is thus,

\[
CV_i^M = \int_{p_i^0}^{p_i^2} x_i^M dp_i = \int_{p_i^0}^{p_i^2} \frac{\partial \psi}{\partial p_i} y = \left[ y \ln \frac{y}{\psi(p)} \right]_{p_i^0}^{p_i^2} = \left[ y \ln \frac{y}{\psi(p, y)} \right]_{p_i^0}^{p_i^2}
\]

Now following MP, assume that \( p_1 \) increases from \( p_1^0 \) to \( p_1^2 \) and \( p_2 \) falls from \( p_2^0 \) to \( p_2^1 \) such that \( U \) remains constant:

\[
v(p_1^1, p_2^1, y) = v(p_1^2, p_2^2, y)
\]

The overall welfare change resulting from these two price changes is the sum of two \( CV^x \): one related to the increase in \( p_1 \) (\( CV_1 \)) and the other to the decline in \( p_2 \) (\( CV_2 \)). These are approximated as,

\[
CV^M = CV_1^M + CV_2^M = \left[ y \ln \frac{y}{\psi(p_1, y)} \right]_{p_1^0}^{p_1^2} + \left[ y \ln \frac{y}{\psi(p, y)} \right]_{p_2^0}^{p_2^1} = y \left[ \ln \frac{y}{\psi(p_1, y)} \right]_{p_1^0}^{p_1^2} - \ln \frac{y}{\psi(p_1, p_2^1, y)}
\]

and this equals zero from A.4.

B. The Slutsky Demand

The Slutsky demand can be derived by solving the following problem:

\[
\text{Maximize } U(x) \text{ subject to } px = x
\]

where, \( x \) is a vector of commodities, \( p \) are the corresponding prices, \( x^0 \) is the bundle of goods on which the demand is conditioned and \( U \) is a strictly quasi-concave utility function. The resulting Slutsky demand takes the general form,

\[
x_1 = x_1^S(p, x^0)
\]

It is also clear that the following identity holds:

\[
x_i^0(p, x^0) = x_i^M(p, \sum p_k x_k^0) = x_i^2(p, v(p, \sum p_k x_k^0))
\]

where \( v \) is the indirect utility function, \( x_i^0 \) is the Marshallian demand and \( x_i^M \) is the Hicksonian demand. Differentiating (B.3) with respect to \( p_i \), and applying Roy’s Identity ,

\[
\frac{\partial x_i^M}{\partial p_i} = \frac{\partial x_i^H}{\partial p_i} + (x_i^0 - x_i^M) \frac{\partial x_i^M}{\partial p_i}
\]

where \( x_i^H \) is the demand for \( x_i \) when \( m = \sum p_k x_k^0 \) and \( \frac{\partial x_i^M}{\partial p_i} \) is the slope of the Hicksonian demand, \( x_i^M \), conditioned on \( U(x^0) \), at any \( p \).

Two results follow from this, when comparing the Hicksonian demand conditioned on \( U(x^0) \) and the Slutsky demand conditioned on \( x^0 \):

i) When prices and income are such that the quantity actually demanded \( (x_i^M) \) is equal to the bundle on which the Slutsky is conditioned \( (x_i^0) \), then the Hicksonian and the Slutsky demands have the same slope. That is they are tangent as is shown in figure 1.

ii) When prices and income are such that \( x_i^M > x_i^0 \), then the Hicksonian is steeper than the Slutsky demand at \( x_i^0 \), assuming that \( x_i \) is a normal good. Conversely, when prices and income are such that \( x_i^M < x_i^0 \), then the Hicksonian is flatter than the Slutsky demand at \( x_i^M \), assuming that \( x_i \) is a normal good.

This demonstrates that any particular Hicksonian demand is an envelope of Slutsky demands, since this result can be demonstrated for any arbitrary bundle, \( x \), that corresponds to a point on the Hicksonian demand conditioned on \( U(x^0) \).

If (B.3) is differentiated with respect to \( p_i \), the resulting equation demonstrates the local path independence property discussed in section 5.2.
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