WORKING PAPERS IN ECONOMICS

MULTINATIONAL PRODUCERS IN AN ARROW-DEBREU TYPE GENERAL EQUILIBRIUM MODEL

by

Michael C. Blad and Ernestine M. Gross

No. 89 March 1987

DEPARTMENT OF ECONOMICS

The University of Sydney
Australia 2006

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National Library of Australia Card Number and ISBN

0 949269 68 9

1. INTRODUCTION

In this paper we extend the Arrow-Debreu general equilibrium model of resource allocation with production by introducing a new agent, a multinational producer into a model of an economy which is partitioned into a finite number of separate economies, called national economies. This approach is inspired by the enormous amount of literature on so-called multinational corporations which has accumulated during the past 30 years.* A substantial amount of this work has been carried out by members of the industrial organisations school (Brash 1966, Caves 1971, 1974, 1982, Dunning 1974, 1978, Hogan 1967, Hymer 1960, Kindleberger 1969, Vernon 1966).

The methodology of this school differs from the axiomatic approach of the Arrow-Debreu-McKenzie school in some important respects. Without even attempting to outline the methodological differences and the respective merits of each of these two approaches to economic analysis, a maintained difference is that whereas general equilibrium theory achieves determinate positive results and welfare theorems based on assumptions about behaviour and environment of economic agents, "the field of economics that studies actual product markets - industrial organisations - by contrast takes an empirically oriented approach to its subject matter, eschewing general equilibrium and depending on made-to-measure varieties of oligopoly theory". (Caves 1974, p. 1)

With respect to multinational corporations no doubt very valuable information has been gained on actual behaviour of firms from the work of the industrial organisations school. Moreover, several hypotheses have emerged from the empirically oriented studies about the causes of so-called multinational corporations. But, there seems to be no unanimously agreed

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* A useful review of the literature is found in Caves (1982).

upon definition on what a multinational corporation is (Dunning 1978, p. 3). Most authors use the term for real world firms who produce goods and services in more than one country. While most hypotheses about the causes of so-called multinational corporations appear individually plausible, upon closer examination the majority of these contain aspects which are relevant for firms, multinational or not[†]. Hence the distinction between multinational and non-multinational corporations is either blurred or at best a matter of degree rather than of kind. The implications of so-called multinational corporations for the welfare of consumers remains an unsettled issue, although much of the empirical research has been stimulated by welfare questions.[⊖]

The empirically founded literature has reached a stage where it seems useful to exploit the merits of the axiomatic approach in order to shed light on the conditions under which it is conceivable to have agents with at least some of the features ascribed to so-called multinational corporations. Our paper presents a first step towards this end.

After introducing the theoretical framework of our model, we describe in the remaining part of section 2 an Arrow-Debreu-type 'world' economy which is partitioned into a given finite number of 'national economies' and a given finite set of 'multinational producers', the latter being a subset of all producers in the 'world' economy. Each national economy consists of a subset of the given finite number of all consumers and non-multinational producers who interact with each other in the form of exchange and production of 'nationally available' commodities. The 'nationally available commodity space' is a sub-space of the 'world' commodity space.

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† This point is discussed in a separate forthcoming paper by E. Gross.

⊖ See for example Caves (1982), Dunning (1978), Shapiro (1983)

Our notion of a nation differs from that found in Debreu (1959). It attempts to interpret the contention that firms have a 'natural national identity' due to cultural, political and linguistic differences between groups of people (Hymer, 1960, Caves, 1974). This hypothesis is taken as the sine qua non by some members of the industrial organisations school for treating so-called multinational corporations as a phenomenon to be explained (Hymer, 1960, Caves, 1974).

A multinational producer in our model is an agent entirely characterised by his production technology, thus remaining within the conceptual framework of the Arrow-Debreu model. However, our description of the multinational producer's technology aims to take into account at least some important features of enterprises which, according to the empirically oriented literature, are typical of so-called multinational corporations. These features are: (1) production may take place in more than one country. (2) So-called multinational corporations "have to adapt products sold in one country to the tastes of consumers and legal requirements prevailing in another country".* We interpret this to mean that a multinational producer has some knowledge of production possibilities which are indigenous to the 'national economies'. (3) So-called multinational corporations "must have an advantage over non-multinational corporations" (Hymer 1960), which was seen to lie in oligopolistic market structures and/or technological know/how (Hymer, 1960) and/or 'intangible capital' (Caves, 1974, 1982). We focus on the 'technological advantage' of such enterprises. In the conceptual framework of our model, producer j' can be said to have a 'technological advantage' over producer j in several ways. (i) j' may be 'technically more efficient' than j in the sense that the production set of the latter, Y_j is a subset of the production set of the former, i.e.

* Caves 1971, 1974. Chamber of Commerce of the United States, Report on Multinational Enterprise Survey (1960, 1970).

$Y_j \subset Y_{j'}$, strict inclusion. Using this definition, multinational producers could be differentiated from 'national producers' by assuming that the former have a technically more efficient production set than the latter. But this is not satisfactory for our purposes since multinational producers would be no more than technically more efficient producers. (ii) j' can be said to have a 'technological advantage' over j if j' can make available a commodity which cannot be made available by j. Applying this definition, multinational producers can be differentiated from non-multinational producers by assuming that each multinational producer, by combining commodities from at least two national economies, can make available at least one commodity in at least one national economy which cannot be made available by any 'national producer'. We adopt the second interpretation of 'technological advantage' because it allows us to relate it to the multinational production aspect of the agent.

These very general statements about important features of multinational corporations and our interpretation thereof are made precise in section 2.2.3.

While the essential difference between a 'national' and a 'multinational producer' is described by their respective technologies, the applied theoretical framework is capable of dealing with two further aspects of so-called multinational corporations; the first relating to the ownership of multinational producers, the second to market behaviour. These aspects are discussed in section 3.

Section 4 contains definitions of 'feasible' and 'attainable states', modified to take in account the special structure of our model. These definitions will be used in subsequent sections.

In section 5 the notion of a free disposal equilibrium is defined for a competitive 'world economy', consisting of a finite number of 'national economies' and a finite set of 'multinational producers' with no trade between national economies. Existence of a free disposal equilibrium is proved (Theorem 1). For the specific case of our model, it is necessary to assume that each MNP has at least one shareholder in each of those national economies where production is possible. Without this assumption Walras' Law cannot be obtained (Proposition 1, sec. 5.3).

In section 6 the natural concept of a Walras' Equilibrium is defined and Theorem 2 contains the existence result for the economy described in section 5.

The structure of our 'world economy' suggests two definitions of Pareto efficiency, 'global' and 'national' Pareto efficiency (sec. 7.2). The first and second fundamental welfare theorems are proved to hold for our 'world economy' with respect to 'global' Pareto efficiency (Theorems 3 and 4). It is shown that 'global' Pareto efficiency does not imply 'national' Pareto efficiency (Remark 1, sec. 7.5).

The distinction between 'global' and 'national' Pareto efficiency is useful when appraising the debate on the apparent conflict between the operations of so-called multinational corporations and national governments (Vernon, 1977, Kindleberger, 1984), which has been a recurring theme in the literature, in particular in the field of developmental economics (Hymer, 1971, Todero, 1985, Lecraw, 1985). Similarly, the contention that so-called multinational corporations are a vehicle for developed countries to 'exploit' less developed countries (Penrose, 1968) reemerges from time to time. As is evidenced in a recent symposium in honour of Hymer's contribution to the theory of direct foreign investment (American Economic Review, May 1985) these issues seem to elicit debate rather than theoretical results. Given the space constraint of this paper it is not possible to discuss these issues in relation to the properties of our model. However, the case used to demonstrate the result in section 7.5 may be interpreted as one where consumers in one national economy are 'unintentionally exploited' by the consumers in another national economy via the profit maximising production decision of price taking (competitive) MNPs. The exploitation is unintentional in the sense that there is no motivational postulate in our model which accounts for this result but it is the consequence of a specific assumption about the structure of consumption and production technologies in our 'world economy' model. (see section 7.6)

Finally, in section 8 we briefly review those aspects of the empirically oriented literature on so-called multinational corporations which, by their nature cannot be dealt with in the Arrow-Debreu model or which were left for future research.

2. THE MODEL

2.1 Theoretical framework

The theoretical framework of our model is the Arrow-Debreu model of a private ownership economy with m consumers, n producers, and L commodities. Such an economy is described by

$$E = (I, F, \mathbb{R}^L, (X_i, \precsim_i, \omega_i)_{i\in I}, (Y_f)_{f\in F}, (\theta_{if})_{\substack{i\in I\\ f\in F}})$$

where $I = \{1, \ldots, m\}$ is a finite set of consumers, $F = \{1, \ldots, n\}$ is a finite set of producers and $\mathbb{R}^L$, $L \in \mathbb{N}$ is the L-dimensional Euclidean space, interpreted as the commodity space. The possible choices for agents among all commodities are represented by consumption sets, $X_i \subseteq \mathbb{R}^L$ and production sets $Y_f \subseteq \mathbb{R}^L$ which, for every i and every f, are subsets of the commodity space. Each consumer is assumed to have a preference $\precsim_i$ over consumptions possible for him. Further, each consumer is assumed to have initial endowments of commodities, $\omega_i \in \mathbb{R}^L \setminus \{0\}$ and to own shares θ_{if} to profits made by producers. Here θ_{if} denotes the ith consumer's share of ownership rights to the fth producer's profits. It is assumed that only non-negative shares are available, $\theta_{if} \geq 0 \ \forall_{i,f}$, and that firms are completely owned by consumers, $\sum_{i\in I} \theta_{if} = 1, \forall f$.

The economy E may be interpreted as a world economy to which all agents belong and within which they interact through exchange and production of commodities; there is one world market.

2.2 Market segmentation: 'national economies' and 'multinational producers'

We consider the case where the world economy is partitioned into a finite number of 'national economies' and a finite set of 'multinational producers' (MNPs). From the point of view of our model the number K of national economies is given, as is the set of MNPs, Z. Let n^Z denote the number of elements in Z.

2.2.1 Partitioning of agents

Consumers: let I^k denote the set of consumers which belong to the kth national economy. We assume that I^k is a non-empty subset of I for k = 1, ..., K. Let m^k denote the number of elements in I^k. Furthermore, a consumer in the world economy belongs to exactly one national economy, i.e. $I^k \cap I^{k'} = \emptyset$ all $k' \neq k$, and all consumers belong to a national economy, $I = \bigcup_{k=1}^{K} I^k$.

Producers: we assume there are two types of producers, 'national' (NPs) and 'multinational' (MNPs). Let J denote the set of all NPs while Z denotes the set of all MNPs. We assume that J and Z are two non-empty subsets of F. Each producer in the world economy is either a NP or a MNP. Hence $J \cap Z = \emptyset$ and $F = J \cup Z$. Let J^k denote the set of NPs which belong to the kth national economy and let n^k denote the number of elements in J^k. We assume J^k is a non-empty subset of J for k = 1, ..., K. Each NP belongs to exactly one national economy, i.e. $J^k \cap J^{k'} = \emptyset$ all $k' \neq k$, and all NPs belong to a national economy, $J = \bigcup_{k=1}^{K} J^k$.

2.2.2 Commodities

Let $\ell \in \mathbb{N}$ denote the number of 'nationally' available commodities and $\alpha \in \mathbb{N}$ the number of 'MNP' commodities. We assume $K \leqq \ell$ and $L = \ell + \alpha$ with $\mathbb{R}^L = \mathbb{R}^\ell \times \mathbb{R}^\alpha$. $\mathbb{R}^\ell$ and $\mathbb{R}^\alpha$ are interpreted as the 'nationally' available commodity space and the 'MNP' commodity space respectively. Let $\ell_k \in \mathbb{N}$ and $\alpha_k \in \mathbb{N} \cup \{0\}$ denote the number of 'national' and 'MNP' commodities which are available in the kth national economy. We assume $\ell_k > 0$ all k, $\alpha_k \geqq 0$ all k, $\alpha_k > 0$ at least one k with $\mathbb{R}^\ell = \mathbb{R}^{\ell_1} \times \ldots \times \mathbb{R}^{\ell_k}$ and $\mathbb{R}^\alpha = \mathbb{R}^{\alpha_1} \times \ldots \times \mathbb{R}^{\alpha_k}$. We define the 'nationally traded' commodity space by $\mathbb{R}^{L_k} := \text{proj}_{(\mathbb{R}^{\ell_k} \times \mathbb{R}^{\alpha_k})} \mathbb{R}^L$.

2.2.3 Description of agents

<u>Consumers:</u> the set of possible consumptions of the ith consumer in the kth national economy, X_i^k is assumed to be restricted to be a subset of the nationally traded commodity space, i.e. $\forall_k, \forall_{i \in I^k} \quad X_i^k \subseteq \mathbb{R}^{L_k}$. Each consumer is assumed to have preferences over consumptions which are possible for him, $\succsim_i^k$ is defined on X_i^k, and each consumer is endowed with a vector of nationally available commodities, $\omega_i^k \in \mathbb{R}^{\ell_k} \setminus \{0\}$.

<u>Producers:</u> the jth NP in the kth economy is characterised by a production possibility set, Y_j^k which is a subset of the nationally available commodity space, $\mathbb{R}^{\ell_k}$, i.e. $\forall k, \forall j \in J^k \; Y_j^k \subseteq \mathbb{R}^{\ell_k}$.

All NPs belonging to the kth national economy are assumed to be owned by consumers in this economy. Let θ^k_{ij} denote the share of profits of the jth producer in the kth national economy which is owned by the ith consumer in the kth national economy. We assume

$$\theta^k_{ij} \geq 0 \text{ all } j \varepsilon J^k \text{ and } i \varepsilon I^k \text{ and } \sum_{i\varepsilon I^k} \theta^k_{ij} = 1 \quad \forall j \varepsilon J^k, \forall k.$$

As was stressed in section 1 we wish to define a MNP as a producer who is entirely characterised by his production technology. However, this technology should have the following features: (1) production may take place in more than one national economy, (2) the MNP has some knowledge of production possibilities which is indigenous to national economies, (3) MNPs have a 'technological advantage' over NPs in the sense that some commodities can only be made available by the former. We have interpreted these very general notions of the characteristics of MNP agents to be defined in the following way. Assume that each member of the set of MNPs has a production set of the form,

$$\forall_{z \varepsilon Z} \; Y_z = \prod_{k=1}^{K} Y^k_z \times Y^M_z \subseteq \mathbb{R}^\ell \times \mathbb{R}^\ell_- \times \mathbb{R}^\alpha_+$$

Y^k_z may be $\{0\}$ for some k but we assume

$$\exists\, k, k', k' \neq k, \; \forall \; \gamma \varepsilon \{k,k'\} \; \exists \; j \varepsilon J^\gamma, \quad Y^\gamma_z = Y^\gamma_j \,, \; Y^\gamma_j \neq \{0\}$$

and $\forall \; y_z \; \varepsilon \; Y_z, \; y_z = ((y^k_z)^K_{k=1}, \; y^M_z)$:

$$\forall_{k} \; \exists_{k' \neq k} \; \text{proj}\,(\mathbb{R}_{-}^{\ell k} \times \mathbb{R}_{-}^{\ell k'} \times \mathbb{R}_{+}^{\alpha k} \times \mathbb{R}_{+}^{\alpha k'})(y_z^M) = (y_z^{Mk}, y_z^{Mk'}, y_z^{\alpha k}, y_z^{\alpha k'})$$

fulfills the following condition

$$(y_z^{\alpha k} \neq 0 \vee y_z^{\alpha k'} \neq 0) \Rightarrow (y_z^{Mk} \neq 0 \wedge y_z^{Mk'} \neq 0)$$

The production set Y_z is composed of sets of national production possibilities, Y_z^k, $k = 1, \ldots, K$, and a set of multinational production possibilities, Y_z^M. Y_z^k is a national production possibility set for the MNP in the sense that it is a subset of the nationally available commodity space in the kth economy, $\mathbb{R}^{\ell k}$. Each Y_z^k is therefore akin to the production set of an arbitrary NP^k. In order to ensure that the MNP has 'some knowledge of production technologies indigenous to national economies' we have assumed that Y_z^k is identical to the (non-zero) production set of at least one NP in each of at least two national economies. The Y_z^M component of the MNP's production technology is 'multinational' in the sense that any MNP commodity, α_h, can only be produced from (input) commodities obtained from at least two national economies. The vector y_z^{Mk} denotes the zth MNP's vector of input commodities from the kth national economy, $y_z^{\alpha k}$ denotes the vector of outputs of MNP commodities of the zth MNP made available in the kth economy. Examples of MNP productions are:

(1) $y_z = ((y_z^k)_{k=1}^K = 0, \;\; y_z^M = 0)$

(2) $y_z = ((y_z^k)_{k=1}^K \neq 0, \;\; y_z^M = 0)$

(3) $y_z = ((y_z^k)_{k=1}^K = 0, \;\; y_z^M \neq 0)$

(4) $y_z = ((y_z^k)_{k=1}^K \neq 0, \;\; y_z^M \neq 0)$

Case one is simply that of no production. In the second case the MNP's production is equivalent in its nature to that of a set of NPs, one in each of $k = 1, \ldots, K$ national economies, each choosing a production $y_j^k \in Y_j^K$ (of which some might be zero). Case three applies when the MNP is exclusively producing MNP commodities while case four describes the case where the MNP produces nationally available as well as MNP commodities.

We consider this break-down of the MNP's production into national and multinational productions a highly desirable feature of our model since it potentially sheds light on the discussion in the literature on the relative 'behaviour and performance' of multinational and non-multinational corporations in terms of welfare implications for consumers in one or all national economies.[†] Quite clearly, if solving the MNP's decision problem leads to a y_z of type 2 above in each national economy, then the MNP is indistinguishable from NPs in each of these economies. An important question is under which conditions in the world economy does one obtain this solution and under which conditions does one obtain any one of the alternatives. In the concluding section of this paper we indicate how our general model may be applied to study these conditions.

The above description of the MNP technology also lends itself to study the locational choice of direct foreign investment. Again in section 8 we give an example of how Dunning's (1981) 'locational advantage of a country' can be described in our model.

Each MNP is assumed to be owned by consumers. Let $\theta_{iz}^k \geq 0$ denote the ownership share of the ith consumer in the kth national economy of profits of the zth MNP. We assume

$$\sum_{k=1}^{K} \sum_{i \in I^k} \theta_{iz}^k = 1, \; \forall z \in Z.$$

† see for example Dunning (1978)

2.2.4 National economies

National economy, E^k, $k = 1, \ldots, K$, is now described by

$$E^k = (I^k, J^k, \mathbb{R}^{L_k}, (X_i^k, \succsim_i^k, \omega_i^k)_{i \in I^k}, (Y_j^k)_{j \in J^k}, (\theta_{if}^k)_{\substack{i \in I^k \\ f \in J^k \cup Z}})$$

We stress that if it were assumed that the set of MNPs is empty then each national economy would be completely segmented from all others. Hence our model contains that of autarky of national economies as a special case.

Further, note that our definition of a national economy captures a sociological as well as a spacial aspect of the notion of a 'nation'. The partitioning of the set of agents I and J into a finite given number of subsets may be interpreted as the case where cultural, political and linguistic factors distinguish one group of individuals from another and which in turn defines membership of an individual to a national group. The spacial aspect of the notion of a national economy is taken into account by the definition of a commodity. A commodity is a good, defined with respect to a 'location' of availability. In fact one may choose the devision of space over which economic life takes place in such a way that the notion of a 'nation' coincides with that of a 'location' (Debreu, 1959). Our definition of a national economy contains the Debreu definition of a nation as a special case, namely that where all commodities available in one national economy, i.e. ℓ_k, are goods available at location k. Clearly, our definition of a national economy is broader in the sense that we may have more than one location in one national economy and alternatively, we may have more than one national economy in one location.†

However having assumed Z to be non-empty, there are at least two national economies, E^k and $E^{k'}$, $k \neq k'$, in our world economy E which are linked to each other via an MNP. This link occurs because any MNP's production technology 'intersects' the production technology of at least one NP in each of at least two national economies. Each MNP is also 'attached' to at least one national economy in either one or both of the following ways. The MNP commodity space, $\mathbb{R}^{\alpha}$ 'intersects' the space of nationally traded commodities, $\mathbb{R}^{L_k}$, in at least one national economy, E^k. Finally each MNP is also linked to at least one set of national consumers through the ownership distribution $(\theta^k_{iz})_{i \in I^k,\ k=1, \ldots K}$. Although it sometimes might be natural to assume that the zth MNP, which is owned by consumers in the kth economy, either produces locally available or MNP commodities in this economy, it is not a necessary feature of our model.

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† The relative merit of our definition over the basic Debreu concept depends on the purpose of the model. If sociological aspects of economic life are judged to be unimportant then there is no apparent gain from adopting our definition. However, the empirically oriented literature on so-called multinational corporations takes the proposition of a firm having a 'natural national identity due to sociological factors' as the <u>sine qua non</u> for treating multinational corporations as a phenomenon to be explained. (Caves 1974)

3. BEHAVIOURAL DESCRIPTION OF AGENTS

We assume that the world economy E is competitive. In each national economy E^k, $k = 1, \ldots, K$, there is a national price system, $p^k \in \mathbb{R}_+^{L_k}$, which all agents who interact in this economy through exchange or production of commodities take as given.# There is no trade between national economies. A world price system p for E is an array of national price systems.

Producers: each producer, NP^k and MNP, is assumed to choose a production which is technologically possible for him and which maximises profits.

The jth producer's decision problem for $j \in J^k$, $k = 1, \ldots, K$, denoted by NPP, is

$$\text{NPP:} \quad \text{given } Y_j^k \subseteq \mathbb{R}^{\ell_k},\ p^k \in \mathbb{R}_+^{L_k}$$

$$\text{choose } \bar{y}_j^k \in Y_j^k \text{ s.t.}$$

$$p^k \bar{y}_j^k \geq p^k y_j^k \text{ all } y_j^k \in Y_j^k$$

The zth MNP's decision problem for each $z \in Z$, denoted by MNPP, is

$$\text{MNPP:} \quad \text{given } Y_z = \prod_{k=1}^{K} Y_z^k \times Y_z^M \subseteq \mathbb{R}^{\ell} \times \mathbb{R}_-^{\ell} \times \mathbb{R}_+^{\alpha}$$

$$\text{choose } \bar{y}_z = ((\bar{y}_z^k)_{k=1}^K, \bar{y}_z^M) = ((\bar{y}_z^k)_{k=1}^K, (\bar{y}_z^{Mk})_{k=1}^K,$$

$$(\bar{y}_z^{\alpha k})_{k=1}^K) \in Y_z \quad \text{such that}$$

#There is no agreement in the industrial organisations literature on the market behaviour of so-called MNCs. (Caves, 1974: MNCs may increase or decrease price competitiveness; Vernon, 1977: MNCs may earn short run monopoly profits on some products but are price competitive in the long run)

(1) $p^k \tilde{y}_z^k \geqq p^k y_z^k$ all $y_z^k \in Y_z^k$ all k and condition (2) stated below.

The zth MNP maximises profits in each national economy and hence maximises total profits from its national production technology, $\prod_{k=1}^{K} Y_z^k$.

Since we have assumed that there is no trade between national economies the MNP's profit maximisation decision on the multinational part of its technology, Y_z^M, is constrained in the following way. The MNP can acquire inputs y_z^{Mk} in the kth national economy only from profits on its national production, $p^k y_z^k$, and from revenue of output of MNP commodities made available in the kth national economy, $p^k y_z^{\alpha k}$. Hence, for each k, the MNP can only choose y_z^{Mk} in the 'budget constrained' subset of Y_z^M, defined by

$$\gamma_z^k(p) = \tilde{\gamma}_z^k(p^k) = \{\mathrm{proj}_{(\mathbb{R}^{\ell k + \alpha k})}(y_z^M), y_z^M \in Y_z^M \mid p^k(-y_z^{Mk} - y_z^{\alpha k}) \leqq \max p^k Y_z^k\}$$

Even though the kth budget correspondence, γ_z^k, only depends upon p^k, we have written it as depending upon all price vectors $(p^1, p^2, \ldots, p^K)$ in order to be able to define the total budget correspondence γ_z below.

However, note that the MNP's multinational production technology, Y_z^M, may be such that inputs y_z^{Mk} can be combined with inputs $y_z^{Mk'}$, $k \neq k'$, without being technologically able to make available any MNP commodities in the kth national economy, i.e. $\mathrm{proj}_{(\mathbb{R}^{\alpha k})}(Y_z^M) = \{0\}$. The total 'budget constrained' production set of the zth MNP is defined by $\gamma_z((p^k)_{k=1}^K) = \sum_{k=1}^{K} \gamma_z^k(p)$.

The second part of the MNP's profit maximisation problem is hence

(2) choose $\bar{y}_z^M \varepsilon \gamma_z(.)$ s.t. $p\bar{y}_z^M \geq py_z^M$ all $y_z^M \varepsilon \gamma_z(.)$.

A few comments on the MNPP are in order. We have assumed that each MNP maximises total global profits subject to a budget constraint on the multinational part of its technology. While this assumption of global profit maximisation is appealing on the grounds of adhering to the standard assumption of the firm's behaviour in the Debreu model, it raises serious questions about the ownership aspect of such agents. We have also assumed that any MNP may be owned by consumers in more than one national economy and that the ownership distribution is such that globally each MNP is entirely owned by consumers, i.e. $\sum_{k=1}^{K} \sum_{i \varepsilon I^k} \theta_{iz}^k = 1, \forall z \varepsilon Z$. Since the national economies $E', \ldots, E^K$ are segmented from each other except for each MNP's production, profits cannot be transferred from one national economy to another in order to distribute global profits according to the ownership distribution $(\theta_{iz}^k)_{i \varepsilon I^k, \; k=1, \ldots, K}$. In the national economy, E^k, the zth MNP's total profit for distribution is equal to $p^k(\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k})$. All of this profit is to be distributed within the national economy in proportion $(\theta_{iz}^k)_{i \varepsilon I^k}$. The zth MNP profit is nationally distributed using the shares $(\delta_z^k \theta_{iz}^k)_{i \varepsilon I^k}$ where

$$\delta_z^k = \begin{cases} 1 / \sum_{i \varepsilon I^k} \theta_{iz}^k & \text{if } \sum_{i \varepsilon I^k} \theta_{iz}^k > 0 \\ 0 & \text{if } \sum_{i \varepsilon I^k} \theta_{iz}^k = 0 \end{cases}$$

Clearly the zth MNP may make positive profits in a national economy where there is only one shareholder with, say, .01 of the total share ownership and zero profits in another national economy where the share ownership of the consumers in this economy sums to .99. Hence one consumer would obtain the entire profit earned by this MNP while all other owners would obtain zero profits. This raises serious doubt as to whether is is reasonable to assume that MNPs maximise total profits irrespective of the effect this has on the wealth of their shareholders worldwide.

A different approach to total profit maximisation would be to assume that an MNP maxmises profits in the economy to which its shareholders belong, subject to a zero profit condition in other national economies. However, this approach poses problems too. In the case where an MNP is assumed to be owned by consumers in more than one national economy it is not clear at all what the objective of the MNP should be; letting the MNP maximise total shareholder wealth implies that the problem of transferrring profits between two national economies re-emerges. Alternatively, if it is assumed that the MNP maximises profits subject to wealth maximisation of shareholders in each national economy, then there may be conflict of interest between groups of shareholders. For example, would it be reasonable to assume that shareholders in a specific national economy agree to the above objective of the firm if they collectively hold a majority of shares of the MNP?

Since the primary purpose of our model is to introduce MNPs into a Debreu-type model we retain the assumption of total profit maximisation. But we stress that profit maximisation behaviour by MNPs need not entail shareholder wealth maximisation in the sense of the Debreu model nor is profit maximisation a goal which may be taken to be unanimously agreed upon by shareholders in a partially segmented world economy with MNPs and no trade. The issue of shareholder unanimity in this model will be the subject in a forthcoming paper, following the approach suggested by Dreze (1984).

Next we turn to the consumers. The ith consumer's decision problem for $i \varepsilon I^k$, $k = 1, \ldots, K$, denoted by CP is

CP: given $X_i^k \subseteq \mathbb{R}^{L_k}$, $p^k \varepsilon \mathbb{R}_+^{L_k}$, $\bar{y}_z \varepsilon Y_z \; \forall z$ and $\bar{y}_j \varepsilon Y_j^k \; \forall j \varepsilon J^k$

choose $\bar{x}_i^k \varepsilon \mathbb{R}^{L_k}$ such that

(1) $\bar{x}_i^k \varepsilon X_i^k$

(2) $\bar{x}_i^k$ max $\succsim_i^k$ on the set

$$\Gamma_i^k (p , w_i^k) = \{x_i^k \varepsilon X_i^k \mid p^k x_i^k \leq w_i^k\}$$

where w_i^k denotes the ith consumer's wealth defined by

$$w_i^k = p^k \omega_i^k + \sum_{j \varepsilon J^k} \theta_{ij}^k \, p^k \bar{y}_j^k + \sum_{z \varepsilon Z} \delta_z^k \, \theta_{iz}^k \quad p^k (\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k})$$

4. ATTAINABLE STATES

In an abstract economy with consumers and producers we say that a set of consumptions (x_i) and productions (y_j) is attainable if for each consumer i (producer j) the consumption x_i (production y_j) is possible, i.e. $x_i \varepsilon X_i$ $(y_j \varepsilon Y_j)$ and the state $((x_i), (y_j))$ is feasible, i.e. $\sum_i x_i - \sum_j y_j = \sum_i \omega_i$. *
We shall now apply this general notion to the economies considered in the present context.

4.1 Definition of the set of attainable states for the economy $E = (E^1, \ldots, E^K, Z, (Y_z))$

The set of consumptions and productions possible in the kth national economy (k=1,...K) is $\tilde{D}^k = (\prod_{i \varepsilon I^k} X_i^k) \times (\prod_{j \varepsilon J^k} Y_j^k) \times (\prod_{z \varepsilon Z} Y_z^k) \subseteq \mathbb{R}^{\tilde{\beta}^k}$

where $\mathbb{R}^{\tilde{\beta}^k} = \mathbb{R}^{(\ell k + \alpha k)m^k} \times (\mathbb{R}^{\ell k})^{n^k} \times (\mathbb{R}^{\ell k})^{n^Z}$

The set of consumptions and productions possible in the world economy E is given by

$\tilde{D} = \tilde{D}^1 \times \ldots \times \tilde{D}^K \times (\prod_{z \varepsilon Z} Y_z^M) \subseteq \mathbb{R}^\beta$

where $\mathbb{R}^\beta = \mathbb{R}^{\tilde{\beta}^1} \times \ldots \times \mathbb{R}^{\tilde{\beta}^K} \times (\mathbb{R}^L)^{n^Z}$

Let M^k denote the set of feasible states in the kth national economy, E^k, (k=1, ..., K), then

$$M^k = \{((x_i^k), (y_j^k), (y_z^k), (y_z^{Mk}, y_z^{\alpha k})) \varepsilon \mathbb{R}^{\beta k} \times (\mathbb{R}^{\beta k})^{n^Z} \mid \sum_{i \varepsilon I^k} x_i^k - \sum_{j \varepsilon J^k} y_j^k - \sum_{z \varepsilon Z} (y_z^k + y_z^{Mk} + y_z^{\alpha k}) = \sum_{i \varepsilon I^k} \omega_i^k\}$$

where $\mathbb{R}^{\beta k} := \mathrm{proj}_{(\mathbb{R}^{\ell k} \times \mathbb{R}^{\alpha k})} \mathbb{R}^\beta$

The set of feasible states for the economy E is $M = M^1 \times \ldots \times M^K$.

---

*Note for the purpose of section 5 below feasibility is defined as net demand being at most equal to total resources, i.e. $\sum_i x_i - \sum_j y_j \leq \sum_i \omega_i$.

Finally, the set of states in the economy E, which are possible and feasible, is the set of attainable states $D = \tilde{D} \cap M$.

4.2 Definition of the set of attainable states in the national economy E^k, $k = 1, \ldots, K$

A state $((x_i^k),(y_j^k),(y_z^k, y_z^{Mk}, y_z^{\alpha k}))$ is attainable in the kth national economy, E^k, if and only if there exists an attainable state $((x_i^k),(y_j^k),(y_z^k, y_z^{Mk}, y_z^{\alpha k}))_{k=1}^K$ for the economy E, which has the given state as its kth coordinate. Hence the set of attainable states in E^k is $D^k := \mathrm{proj}_{(\mathbb{R}^{Lk})}(D)$ where $\mathbb{R}^{Lk}$ is situated in its canonical spot in the product space $\mathbb{R}^{L1(\cdot)} \times \ldots \times \mathbb{R}^{LK(\cdot)}$.

Note that $D = \mathrm{proj}_{(\mathbb{R}^{L1})}(D) \times \ldots \times \mathrm{proj}_{(\mathbb{R}^{LK})}(D)$.

5. FREE DISPOSAL EQUILIBRIUM

5.1 Definition of a free disposal equilibrium (F.D.E.)

A set of consumption vectors $((\bar{x}_i^k)_{i \epsilon I^k})_{k=1}^K$, productions $((\bar{y}_j^k)_{j \epsilon J^k})_{k=1}^K$, $(\bar{y}_z)_{z \epsilon Z}$ and prices $\bar{p} = (\bar{p}^1, \ldots, \bar{p}^K)$ is called a free disposal equilibrium for the economy $E = (E', \ldots, E^K, Z, (Y_z))$ if the following conditions are fulfilled:

(1) $\forall_k \; \forall_{i \epsilon I^k} \; \bar{x}_i^k$ is a solution to CP

(2) $\forall_k \; \forall_{j \epsilon J^k} \; \bar{y}_j^k$ is a solution to NPP

(3) $\forall_z \; \bar{y}_z$ is a solution to MNPP

(4) $\forall_k \; \bar{\mu}^k = \sum_{i \epsilon I^k} \bar{x}_i^k - \sum_{j \epsilon J^k} \bar{y}_j^k - \sum_{z \epsilon Z} (\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) - \sum_{i \epsilon I^k} \omega_i^k \leq 0$

with $\bar{p}^k \geq 0$ all k and $(\bar{p}^1, \ldots, \bar{p}^K) \cdot (\bar{\mu}^1, \ldots, \bar{\mu}^K) = 0$

5.2 Theorem 1

The economy $E = (E^1, \ldots, E^K, Z, (Y_z))$ has a free disposal equilibrium if for k = 1, ..., K:

$\forall i \epsilon I^k$ A1: X_i^k is non-empty, compact and convex

A2: There is no satiation consumption in agent i's attainable consumption set, $\hat{X}_i^k$, defined by

$$\hat{X}_i^k = \{x_i^k \varepsilon X_i^k \Big|_{i'\varepsilon I^k}^{\forall} \ \ \overset{\exists}{x_{i'}^k \varepsilon X_{i'}^k} \ \ \overset{\forall}{f\varepsilon J^k \cup Z} \ \ \overset{\exists}{y_f^k \varepsilon Y_f^k}$$

$$\overset{\forall}{z\varepsilon Z} \ \ \overset{\exists}{(y_z^{Mk}, y_z^{\alpha k}) \varepsilon \ \gamma_z(p)} \ : \ \dot{\mu}^k \leqq 0\}$$

A3: The preference relation $\succsim_i^k$ is closed

A4: The preference relation $\succsim_i^k$ is convex

A5: $\exists \ \overset{\circ}{x}{}_i^k \varepsilon X_i^k$ such that $\text{proj}_{(\mathbb{R}^{\ell k})}(\overset{\circ}{x}{}_i^k) << \omega_i^k$ and $\text{proj}_{(\mathbb{R}^{\ell k})}(\overset{\circ}{x}{}_i^k) \varepsilon X_i^k$

$\forall \ j\varepsilon J^k$ A6: Y_j^k is non-empty, compact, convex and contains 0

A7a: Y_z^k is non-empty, compact, convex and contains 0

A7b: Y_z^M is non-empty, compact, strictly convex and contains 0.

Let $\mathcal{D}^k$ be the set of MNP commodities made available in the kth national economy, E^k. (Note: $\mathcal{D}^k$ may be empty for some k but since $\alpha > 0$ there exists at least one national economy where $\mathcal{D}^k$ is non-empty.)

A8a: if $h\varepsilon\mathcal{D}^k$, $i\varepsilon I^k$, $x_i^k \varepsilon X_i^k$, then $\exists \ \lambda^k > 0$ such that $(x_i^k + \lambda^k \beta^h) \succ_i^k x_i^k \ \ \forall i\varepsilon I^k$, where β^h is the positive unit vector of the $(\ell_k + h)$th axis in $\mathbb{R}^{\ell k + \alpha k}$

$\mathcal{D} = \bigcup_{k=1}^{K} \mathcal{D}^k$ denotes the set of MNP commodities in the economy E which are 'always desired'.†

A8b: $\forall k \ \ \forall z\varepsilon Z : \text{proj}_{(\mathbb{R}_+^{\alpha k})}(Y_z^M) \neq \{0\} \Rightarrow \text{proj}_{(\mathbb{R}_-^{\ell k})}(Y_z^M) \neq \{0\}$.

(If it is possible for the zth MNP to make available MNP commodities in the kth national economy then it is technologically given that he uses inputs from the kth national economy for the production of MNP commodities.)

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† Our definition of always desired commodities constitutes an application of the Arrow-Debreu (1954) concept to our model.

Let $Y^M = \sum_{z \in Z} Y_z^M$ and for $k = 1, \ldots, K$ define

$$H^k := \{h \varepsilon (\sum_{j=1}^{k-1} L_j + 1, \ldots, \sum_{j=1}^{k-1} L_j + \ell_k \mid \forall\, y^{Mk} \varepsilon \operatorname{proj}_{(\mathbb{R}_-^{\ell_k})}(Y^M \setminus \{0\}) : y_h^{Mk} < 0\}$$

and let

$$H = \bigcup_{k=1}^{K} H^k$$

The set H contains all nationally available commodities in the economy E which, given the multinational production technology, Y^M, are input commodities for multinational production. Since we have assumed $\alpha > 0$ there are at least two national economies $\tilde{k}$, k, $\tilde{k} \neq k$ in E for which $H^{\tilde{k}}$, H^k are non-empty. (Note, in conjunction with assumption A9 below, H^k is non-empty for all $k = 1, \ldots, K$, while $\mathcal{D}^k$ is non-empty for at least one k).

Let $y^{kM} := \operatorname{proj}_{(\mathbb{R}^{L_k})}(\sum_{z \in Z} y_z^M)$.

A8c: if $y^M \varepsilon Y^M$, $y^M = (y^{kM})_{k=1}^{K}$ and $h \varepsilon H^k$ $\forall k$

then

$$\exists\ y^{M'} \varepsilon\, Y^M,\ \exists\, k' \neq k :$$

$$y_h^{kM'} < y_h^{kM}$$

$\exists\, h_o \varepsilon \mathcal{D}^{k'}$: $\quad y_{h_o}^{k'M'} > y_{h_o}^{k'M}$

$\exists\, h_o' \varepsilon H^{k'}$: $\quad y_{h_o'}^{k'M'} \leqq y_{h_o'}^{k'M}$

$\tilde{h} \varepsilon H \cup \mathcal{D} \setminus \{h, h_o', h_o\}$: $y_{\tilde{h}}^{M'} = y_{\tilde{h}}^{M}$

(Each commodity in H is internationally jointly productive. For any $h \varepsilon H$ one can find $h \varepsilon H^k$, $h_o' \varepsilon H^{k'}$ such that by using strictly more input of $h \varepsilon H^k$ and at least as much input of $h_o' \varepsilon H^{k'}$ the output of $h_o \varepsilon \mathcal{D}^{k'}$ (an always desired commodity) can always be increased.)

A9: $\neg(\exists\, k = 1, \ldots, K$, such that $proj_{(\mathbb{R}^{L_k})}(\sum_{z \varepsilon Z} Y_z^M) = \{0\})$, i.e. we assume that none of the national economies E^k, $k = 1, \ldots, K$ is in a state of autarky due to the absence of an MNP link - see section 2.2.4.*

A10: $\forall_{k=1 \ldots K}\ \forall_{z \varepsilon Z}$ if $proj_{(\mathbb{R}^{L_k})}(Y_z) \neq \{0\}$ then $\exists\, i \varepsilon I^k$ such that $\theta_{iz}^k > 0$, i.e. we assume that each MNP has shares owned by consumers in each of the national economies where the "intersection" of the nationally traded commodity space with his production set is non-zero.

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*The case of autarky of a national economy is not interesting. A9 is merely a convenient assumption to exclude the case of autarky of one or several national economies.

Proof

We follow the Debreu (1982) method of proving existence of a free disposal equilibrium by transforming the economy E into a social system, A, then proving existence of an equilibrium for A and then demonstrating that an equilibrium for A is an equilibrium for the economy E. However, there are some modifications to this approach which stem from the specific structure of the economy E. We have to describe the social system such that this structure is preserved. Specifically, since no trade is allowed between the national economies it seems as if we might need K market agents, each being characterised by a choice set P^k, a subset of $\mathbb{R}_+^{Lk}$, and a utility function t^k, to be defined below. Moreover, we cannot ignore the fact outlined above that the MNP's choice of a multinational production, y_z^M, given the choices of all other agents in A, including the choices of the K market agents, is restricted by a budget constraint.

Our social system A is denoted in the following way:

$$A = (((X_i^k)_{i\varepsilon I^k}, (Y_j^k)_{j\varepsilon J^k}, P^k)_{k=1}^K, (Y_z)_{z\varepsilon Z}, ((u_i^k)_{i\varepsilon I^k}, (v_j^k)_{j\varepsilon J^k}, t^k)_{k=1}^K,$$

$$(v_z)_{z\varepsilon Z}, ((\Gamma_i^k(.))_{i\varepsilon I^k}, (Y_j^k)_{j\varepsilon J^k}, P^k)_{k=1}^K, ((Y_z^k)_{k=1}^K, \gamma_z(.))_{z\varepsilon Z})_N$$

Here

X_i^k, Y_j^k, Y_z, $\Gamma_i^k(.)$, $\gamma_z(.)$ are defined as in sections 2 and 3.

The closedness of preference relations (A3) implies that there exist continuous utility functions u_i^k for each $i\varepsilon I^k$, $k = 1, \ldots, K$, representing $\succsim_i^k$ in the sense that $x_i^k \succsim_i^k x_i'^k$ is equivalent to $u_i^k(x_i^k) \geq u_i^k(x_i'^k)$ (Debreu, 1954)

v_j^k denotes the utility function of the jth producer in the kth national economy, defined by

$$v_j^k(((x_i^k)_{i\varepsilon I^k})_{k=1}^K, ((y_j^k)_{j\varepsilon J^k})_{k=1}^K, (y_z^k, y_z^{MK}, y_z^{\alpha k})_{z\varepsilon Z}, p^1, \ldots, p^K) = p^k y_j^k$$

v_z is the utility function of the zth MNP, defined by

$$v_z(((x_i^k)_{i\varepsilon I^k})_{k=1}^K, ((y_j^k)_{j\varepsilon J^k})_{k=1}^K, (y_z^k, y_z^{Mk}, y_z^k)_{z\varepsilon Z}, p^1, \ldots, p^K) = v_z^1 + v_z^2, \text{ where } v_z^1 = \sum_{k=1}^K p^k y_z^k \text{ and } v_z^2 = \sum_{k=1}^K p^k(y_z^{Mk}, y_z^{\alpha k})$$

t^k is the utility function of the kth market agent defined by

$$t^k(((x_i^k)_{i\varepsilon I^k})_{k=1}^K, ((y_j^k)_{j\varepsilon J^k})_{k=1}^K, (y_z^k, y_z^{Mk}, y_z^{\alpha k})_{z\varepsilon Z}, p^1, \ldots, p^K) = p^k(\sum_{i\varepsilon I^k} x_i^k - \sum_{j\varepsilon J^k} y_j^k - \sum_{z\varepsilon Z}(y_z^k + y_z^{Mk} + y_z^{\alpha k}) - \sum_{i\varepsilon I^k} \omega_i^k)$$

$N = m + n + K$

In order to apply the Debreu (1982) theorem of existence of equilibrium in a social system to the above system A, the following conditions must be fulfilled:

(1) $\forall_k \forall_{i\varepsilon I^k} \forall_{j\varepsilon J^k} \forall_{z\varepsilon Z}$ X_i^k, Y_j^k, Y_z, P^k are non-empty compact, convex subsets of $\mathbb{R}^L$,

(2) The utility functions u_i^k, v_j^k, v_z, t^k are real valued, continuous and quasi-concave for all agents in A.

(3) The point to set maps Γ_i^k and γ_z, which determine the restricted choice sets of the ith consumer in the kth economy and the zth MNP, given the choices of all other agents, are non-empty, continuous and convex valued.

Two aspects of our social system immediately call for special attention:

(a) How to define the price sets P^k, $k = 1, \ldots, K$, in order to fulfil condition 1?

Suppose we use the price simplex defined for each national market over all nationally traded commodities, i.e.
$P^k = \{p^k \varepsilon \mathbb{R}_+^{L_k} | \sum_h p_h^k = 1, \text{ all } h \varepsilon \{ \sum_{j=1}^{k-1} L_j + 1, \ldots, \sum_{j=1}^{k-1} L_j + L_k \}\}$

Quite clearly, the equilibrium condition 3 in the definition of a F.D.E. is not homogeneous of degree zero with respect to p^k. We recall that the profit maximising problem for the multinational production part of each MNP is: choose $\bar{y}_z^M \varepsilon \gamma_z(.)$ such that $p\bar{y}_z^M \geq py_z^M$ all $y_z^M \varepsilon \gamma_z(.)$, $py_z^M = \sum_{k=1}^{K} p^k(y_z^{Mk}, y_z^{\alpha k})$.

Note that $\forall_k \; \tilde{\gamma}_z^k(p^k)$ is homogeneous of degree zero in p^k, i.e.

$$\forall_k \; \tilde{\gamma}_z^k(p^k) = \tilde{\gamma}_z^k(\lambda \, p^k), \; \lambda > 0.$$

Moreover, for $(\lambda^1,\ldots,\lambda^K)\epsilon\mathbb{R}^K_{++}$ we then have

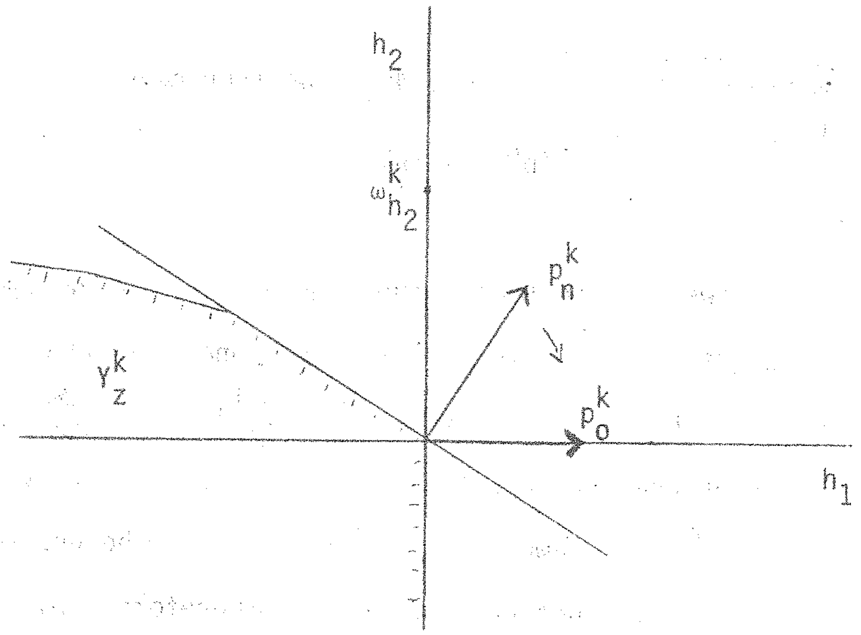
$$\sum_{k=1}^{K} \tilde{\gamma}_z^k(\lambda^k p^k) = \sum_{k=1}^{K} \tilde{\gamma}_z^k(p^k) = \gamma_z(p) \ .$$

However, the profit function py_z^M is not homogeneous of degree zero in the different national economies, since we have that $(\hat{p}^1, \ldots, p^K)\cdot((\bar{y}_z^{M1}, \bar{y}_z^{\alpha 1}), \ldots, (\bar{y}_z^{MK}, \bar{y}_z^{\alpha K}))$ is in general not equal to $(\lambda^1 p^1, p^2, \ldots, p^K)\cdot((\bar{y}'^{M1}_z, \bar{y}'^{\alpha 1}_z), \ldots, (\bar{y}'^{MK}_z, \bar{y}'^{\alpha K}_z))$ where the y'-vectors are the new profit maximising production vectors, and therefore typically different from the y-vectors due to changes in 'relative prices' between national economies.

(b) The budget correspondence γ_z^k may not be continuous for $p^k \geqq 0$. The last problem can be illustrated as follows. Let $\alpha^k = 0$. The kth budget correspondence for the zth MNP, γ_z^k, is defined by the set $\{\text{proj}_{(\mathbb{R}^{\ell k})}(y_z^M),\ y_z^M \in Y_z^M \mid$

$$p^k(-y_z^{Mk}) \leqq \max p^k Y_z^k\} \ .$$

Consider the price sequence p_n^k which approaches p_o^k and the profit sequence $\max p_n^k Y_z^k = \{0\}$, all n, as shown in Figure 1 below. Let p_o^k be such that the price of commodity h_2 is zero. As long as $p_n^k \neq p_o^k$ the MNP's acquisition of inputs for the multinational production is zero, i.e. $y_z^{Mk} = \{0\}$ all n. But at p_o^k the MNP may acquire the entire endowment of this commodity, $\omega_{h_2}^k$.



(Figure 1)

However, for later reference, we note that if the price system $p = (p^1, \ldots, p^K)$ is multiplied by $\lambda > 0$, i.e. $\lambda p = (\lambda p^1, \ldots, \lambda p^K)$ then all equilibrium conditions are unchanged.

We therefore define a new social system

$$\tilde{A} = ((X_i^k)_{i \varepsilon I^k}, (Y_j^k)_{j \varepsilon J^k})_{k=1}^K, (Y_z)_{z \varepsilon Z}, ((u_i^k)_{i \varepsilon I^k},$$

$$(v_j^k)_{j \varepsilon J^k})_{k=1}^K, (v_z)_{z \varepsilon Z}, ((r_i^k(.))_{i \varepsilon I^k}, (\gamma_j^k)_{j \varepsilon J^k})_{k=1}^K,$$

$$((\gamma_z^k)_{k=1}^K, \gamma_z(.))_{z \varepsilon Z}, P, t)_{\tilde{N}}$$

where

$$P = \{(p^1, \ldots, p^K) \varepsilon \mathbb{R}_+^{L1} \times \ldots \times \mathbb{R}_+^{LK} \mid \sum_{k=1}^{K} \sum_{h=\tilde{h}}^{\tilde{\tilde{h}}} p_h^k = 1,$$
$$p_h^k \geqq e \quad \forall\, h \varepsilon H^k,\ k=1,\ldots K\}$$

where $\tilde{h} = \sum_{j=1}^{k-1} L_j + 1, \quad \tilde{\tilde{h}} = \sum_{j=1}^{k-1} L_j + L_k$

$e \in \mathbb{R}_{++}$

As will be shown later, due to assumption A8 the equilibrium price of each commodity belonging to the set H is strictly positive. Moreover, assumption A9 implies that there is at least one commodity of type H in each of the k = 1, ..., K national economies. Consequently, $\forall\, k \quad 0 \leqq \bar{p}^k \neq 0$. By choosing $e > 0$ such that $\forall k\ \forall h \varepsilon H\ e \leqq \bar{p}_h^k$ we can therefore, without loss of generality, confine the price set to the closed set P, as we have done above.

$$t = \sum_{k=1}^{K} t^k[((x_i^k)_{i\varepsilon I^k})_{k=1}^K, ((y_j^k)_{j\varepsilon J^k})_{k=1}^K, (y_z^k, y_z^{Mk}, y_z^{\alpha k})_{z\varepsilon Z}, (p^k)_k^K$$

$$= \sum_{k=1}^{K} p^k\left(\sum_{i\varepsilon I^k} x_i^k - \sum_{j\varepsilon J^k} y_j^k - \sum_{z\varepsilon Z} (y_z^k + y_z^{Mk} + y_z^{\alpha k}) - \sum_{i\varepsilon I^k} \omega_i^k\right).$$

$\tilde{N} = m + n + 1$

All other elements of $\tilde{A}$ are defined as for A.

We now show that (i) an equilibrium exists for $\tilde{A}$ and (ii) this equilibrium is also an equilibrium for A.

(i) By assumptions A1, A6, and A7 condition 1 is fulfilled for all consumers and all producers' choice sets in $\tilde{A}$.

P is non-empty, convex and compact.

$\forall_k \ \forall_{i\,\varepsilon\, I^k} \ u_i^k$ is continuous by A3 and quasi-concave by A4 (Debreu, 1959)

The continuity and quasi-concavity requirement is also fulfilled for v_j^k, $\forall_k$ $\forall_{j \in J^k}$, v_z, $\forall_{z \in Z}$, and t.

We now prove that (a) $\forall_{z \in Z}$ $\gamma_z(.)$ is non-empty and continuous (Lemma 1) and (b) that $\Gamma_i^k(.)$ for $i \in I^k$, $k = 1, \ldots, K$ is non-empty and continuous. The proofs will rely on the following results:

<u>(Lemma 2, Debreu 1982)</u>

If Y is a non-empty, compact subset of $\mathbb{R}^\ell$ and M is a map from $\mathbb{R}^\ell$ to $\mathbb{R}$, defined by $M(p) = \max pY = \max_{y \in Y} py$, then M is continuous.

<u>(Lemma 3, Debreu 1982)</u>

If $X \subset \mathbb{R}^\ell$ is non-empty, compact and convex and w^o is a real number with $w^o > \min p^o X$, then the correspondence $\Gamma(p,w) = \{x \in X \,/\, px \leq w\}$ is continuous at (p^o, w^o).

<u>(Debreu, 1959, 1.8(4))</u>

If X, Y are metric spaces, ϕ is a continuous correspondence from X to Y with compact values and f is a continuous map from $Gr\phi$ to $\mathbb{R}$ then the map m from X to $\mathbb{R}$, defined by $m(x) = \max_{y \in \phi(x)} f(x,y)$ is continuous and ψ: $X \to Y$ defined by $\psi(x) = \{y \in \phi(x) \,/\, f(x,y) = m(x)\}$ is u.h.c. correspondence with compact values.

(a) $\forall_{z \varepsilon Z}$ $\gamma_z(.)$ has non-empty values:

$(0 \varepsilon Y_z^k \ \forall k \text{ and } 0 \varepsilon Y_z^M) \Rightarrow 0 \varepsilon \gamma_z(p), \ \forall \ p \varepsilon P$

Lemma 1

If $\forall \ z \varepsilon Z \ \ Y_z^k$ is non-empty, compact and contains 0 for $k = 1, \ldots, K$, Y_z^M is strictly convex and contains 0 $\forall \ z \varepsilon Z$ and A8,

Proof

The proof proceeds in several steps:

(1) $\forall k$ $\tilde{\gamma}_z^k$ is u.h.c. and has compact values.

Choose $(y_z^{Mk}, y_z^{\alpha k}) \varepsilon \ \text{proj}_{(\mathbb{R}^{L_k})}(Y_z^M)$ and let $\tilde{y}_z^k = (y_z^{Mk} + y_z^{\alpha k})$.

The domain of $\tilde{\gamma}_z^k$ is $P^k = \{p^k \ \varepsilon \ \mathbb{R}_+^{L_k} \mid \exists \ p \varepsilon P: \text{proj}_{(\mathbb{R}_+^{L_k})}(p) = p^k\}$.

Observing that $P^k = \text{proj}_{(\mathbb{R}_+^{L_k})}(P)$, P is compact, and proj is a continuous map, it follows that P^k is compact.

The graph of $\tilde{\gamma}_z^k$, Gr $\tilde{\gamma}_z^k$ is the set

$\{(p^k, \tilde{y}_z^k) \varepsilon \ P^k \times \text{proj}_{(\mathbb{R}^{L_k})}(Y_z^M) \mid \tilde{y}_z^k \ \varepsilon \ \tilde{\gamma}_z^k(p^k)\}$.

Let $(p_n^k, \tilde{y}_{zn}^k)$ be a sequence in Gr $\tilde{\gamma}_z^k$ which converges to $(p_o^k, \tilde{y}_{zo}^k)$, i.e. $p_n^k \to p_o^k$ and $\tilde{y}_{zn}^k \to \tilde{y}_{zo}^k$.

We now show that $(p_o^k, \tilde{y}_{zo}^k) \ \varepsilon$ Gr $\tilde{\gamma}_z^k$.

Since Y_z^k is compact max $p_n^k Y_z^k$ is continuous in p^k (Lemma 2, Debreu, 1982). Hence max $p_n^k Y_z^k \to$ max $p_o^k Y_z^k$.

$(p_n^k \to p_o^k \text{ and } \tilde{y}_{zn}^k \to \tilde{y}_{zo}^k) \Rightarrow p_n^k \tilde{y}_{zn}^k \to p_o^k \tilde{y}_{zo}^k$ as $n \to \infty$.

As $p_n^k(-\tilde{y}_{zn}^k) \leqq \max p_n^k Y_z^k$ (because $\tilde{y}_{zn}^k \varepsilon \tilde{\gamma}_z^k(p_n^k)$)

$\lim_{n\to\infty} p_n^k(-\tilde{y}_{zn}^k) \leqq \lim_{n\to\infty} \max p_n^k Y_z^k$.

Hence $p_o^k(-\tilde{y}_{zo}^k) \leqq \max p_o^k Y_z^k$, i.e. $(p_o^k, \tilde{y}_{zo}^k) \ \varepsilon$ Gr $\tilde{\gamma}_z^k$.

So Gr $\tilde{\gamma}_z^k$ is closed, γ_z^k is u.h.c. and since Y_z^M is compact, has compact values.

(2) $\forall\, z\varepsilon Z \quad \gamma_z$ is u.h.c. as we saw above that $\forall k$
$\tilde{\gamma}_z^k$ is u.h.c. and has compact values.

(3) $\forall k \quad \gamma_z^k$ is l.h.c.

We first prove that $\tilde{\gamma}_z^k$ is l.h.c.

The proof consists of two parts.

Consider the sequence $p_n^k \to p_o^k$, and $\tilde{y}_{zo}^k \varepsilon\ \tilde{\gamma}_z^k(p_o^k)$, i.e.
$\exists\, \tilde{\tilde{y}}_{zo} \varepsilon\ Y_z^M$: $\text{proj}_{(\mathbb{R}^{L_k})}(\tilde{\tilde{y}}_{zo}) = \tilde{y}_{zo}^k$ with $p_o^k(-\tilde{y}_{zo}^k) \leqq \max p_o^k Y_z^k$

(i) We construct a sequence $(\tilde{y}_{zn}^k)$ such that

(1) $\tilde{y}_{zn}^k \varepsilon\ \tilde{\gamma}_z^k(p_n^k) \quad \forall\, n$

(2) $\tilde{y}_{zn}^k \to \tilde{y}_{zo}^k$

Suppose first that $\tilde{\tilde{y}}_{zo}^k$ is an interior point in Y_z^M.

Suppose that all prices are strictly positive.

Define $\tilde{y}_{zn}^k$ coordinate wise as follows:

$$\tilde{y}_{znh}^k := \frac{p_{oh}^k}{p_{nh}^k} \cdot \tilde{y}_{zo}^k + \frac{\max(-p_h^k + p_o^k)Y_z^k}{L_k \cdot p_{nh}^k}$$

$$h = \sum_{j=1}^{k-1} L_j + 1, \ \ldots, \ \sum_{j=1}^{k-1} L_j + L_k$$

As $n \to \infty$ we first note that

$$\tilde{y}_{znh}^k \to (1.\tilde{y}_{zoh}^k + \frac{0}{L_k \cdot p_{oh}^k}) = \tilde{y}_{zoh}^k \quad \forall\, h$$

So $\tilde{y}_{zn}^k \to \tilde{y}_{zo}^k$

Secondly,

$$p_n^k(-\tilde{y}_{zn}^k) = -(\sum_{h=\nu}^{\nu+L_k} p_{nh}^k . y_{zoh}^k + \sum_{h=\nu}^{\nu+L_k} p_{nh}^k . \frac{\max(-p_n^k + p_o^k)Y_z^k}{L_k \cdot p_{nh}^k})$$

$$\text{where} \quad \nu = \sum_{j=1}^{k-1} L_j + 1$$

$$= -(p_o^k . \tilde{y}_{zo}^k - \max p_n^k Y_z^k + \max p_o^k Y_z^k)$$

and as we know that $p_o^k(-\tilde{y}_{zo}^k) \leqq \max p_o^k Y_z^k$, we have
$p_n^k(-\tilde{y}_{zn}^k) \leqq \max p_n^k Y_z^k$

Whenever $p^k_{nh} = 0$ we set $\tilde{y}^k_{znh} := \tilde{y}^k_{zoh}$.

Note that given assumptions A8 and A9, $p^k \nleq 0 \quad \forall k$.

(ii) Suppose now that $\tilde{y}^k_{zo}$ corresponds to a boundary point $\tilde{\tilde{y}}_{zo} \varepsilon Y^M_z$, then two cases arise:

(a) the constructed sequence $\tilde{y}^k_{zn}$ corresponds to a sequence $\tilde{\tilde{y}}_{zn}$ with $\tilde{\tilde{y}}_{zn} \varepsilon Y^M_z$ except for a finite number. Here the proof in (i) above works without modification.

(b) If for an infinite number of n there does not exist $\tilde{\tilde{y}}_{zn} \varepsilon Y^M_z$, then the following modification is made to the construction of the $\tilde{y}^k_{zn}$ sequence:

From the proof above we have a sequence of hyperplanes, Hyp^k_n, where

$Hyp^k_n = \{y^k_z \varepsilon \mathbb{R}^{Lk} \mid p^k_n y^k_z = p^k_n \tilde{y}^k_{zn}\}$

Let $Y^M_z \quad = proj_{(\mathbb{R}^{Lk})}(Y^M_z)$.

Since $0 \varepsilon Y^M_z$ and $0 \varepsilon Y^k_z \ \forall k \ \forall z \varepsilon Z$, $0 \varepsilon \tilde{Y}^k_z(p^k_n) \ \forall \ n, \forall \ p^k_n \varepsilon P^k$.

Consider the sequence $(\tilde{y}^k_{zn})$ constructed above and suppose for all n $\tilde{y}^k_{zn} \notin Y^{M_k}_z$.

We now substitute each point $\tilde{y}^k_{zn}$ with another point $\tilde{y}'^k_{zn} \varepsilon Y^{M_k}_z$ such that $\tilde{y}'^k_{zn} \to \tilde{y}^k_{zo}$ and $\tilde{y}'^k_{zn} \varepsilon Hyp^k_n$ in the following way.

For each n we know that Hyp^k_n is a non-empty convex subset of $\mathbb{R}^{Lk}$, containing 0 and $\tilde{y}^k_{zn}$.

As $Y^{M_k}_z$ is convex and contains $\tilde{y}^k_{zo}$ and 0 it contains the set $V = \{y^k_z \varepsilon \mathbb{R}^{Lk} \mid \tilde{y}^k_{zo} \leq y^k_z \leq 0\}$.

For sufficiently large n, $Hyp_n^k \cap V \neq 0$.
As $\tilde{y}_{zn}^k \to \tilde{y}_{zo}^k$ and $p_n^k \to p_o^k$ for $n \to \infty$ it is possible for each n sufficiently large to choose a point $\tilde{y}_{zn}^{\prime k} \in Hyp_n^k \cap V$ such that $\tilde{y}_{zn}^{\prime k} \to \tilde{y}_{zo}^k$.
So by construction $\tilde{y}_{zn}^{\prime k} \in \tilde{\gamma}_z^k(p_n^k)$ for all n sufficiently large.

(4) $\forall z \in Z \quad \gamma_z((p^k)_{k=1}^K) = \sum_{k=1}^K \gamma_z^k(p)$ is l.h.c.
since γ_z^k is l.h.c. $\forall k$, as $\tilde{\gamma}_z^k$ is l.h.c. $\forall k$.

Hence γ_z is continuous. ∎

(b) $\forall i \in I^k$, $k = 1, \ldots, K$, $\Gamma_i^k(p, w_i^k)$ is non-empty and continuous in p:
Given the definition of w_i^k and P, $\Gamma_i^k(p, w_i^k)$ can be written as

$$\tilde{\Gamma}_i^k(p) = \{x_i^k \in X_i^k \mid p^k x_i^k \leqq p^k \omega_i^k + \sum_{j \in J^k} \theta_{ij}^k p^k \bar{y}_j^k$$
$$+ \sum_{z \in Z} \delta_z^k \theta_{iz}^k (p^k \bar{y}_z^k + p^k (proj_{(\mathbb{R}^{L^k})}(\bar{y}_z^M))$$

$\forall j \varepsilon J^k$ $\quad 0 \varepsilon Y_j^k$ implies that $p^k \bar{y}_j^k = \max p^k Y_j^k \geqq 0$

$\forall z \varepsilon Z$ $\quad 0 \varepsilon Y_z^k$ implies that $p^k \bar{y}_z^k = \max p^k Y_z^k \geqq 0$

As by definition $(\bar{y}_z^{Mk}, \bar{y}_z^{\alpha k})_{k=1}^K \varepsilon \gamma_z(p)$, we have

$p^k(\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) \geqq 0$, $k = 1, \ldots, K$.

$\theta_{ij}^k \geqq 0$ $\forall$ $i \varepsilon I^k$, $\forall j \varepsilon J^k$, $k = 1, \ldots, K$

$\delta_z^k \theta_{iz}^k \geqq 0$, $\forall i \varepsilon I^k$, $k = 1, \ldots, K$, $\forall z \varepsilon Z$

To finish proving that $\tilde{\Gamma}_i^k(\cdot)$ is non-empty we show that $\min p^k X_i^k < p^k \omega_i^k$ where it is given that $X_i^k \subseteq \mathbb{R}^{Lk}$ and $\omega_i^k \varepsilon \mathbb{R}^{\ell k} \backslash \{0\}$, $p^k \varepsilon \mathbb{R}_+^{Lk}$.

Let $\mathbb{R}^{Lk} = \mathbb{R}^{\ell k} \times \mathbb{R}^{\alpha k}$, $\alpha_k \geqq 0$.

A8 and A9 imply that $p^k \geqq 0$.

A5 implies that for $p^k \geqq 0$: $p^k \overset{\circ}{x}_i^k < p^k \omega_i^k$.

Next we show that $\tilde{\Gamma}_i^k(\cdot)$ is continuous in p. The proof is divided into two steps:

(1) $\quad w_i^k(p)$ is continuous in p:

$\forall j \varepsilon J^k$ $\quad Y_j^k$ is compact $\Rightarrow$ $p^k \bar{y}_j^k$ is continuous in p^k

and

$\forall z \varepsilon Z$ $\quad Y_z^k$ is compact $\Rightarrow$ $p^k \bar{y}_z^k$ is continuous in p^k

(Lemma 2, Debreu 1982)

Finally consider the map $p \to p^k \cdot \text{proj}_{(\mathbb{R}^{Lk})}(\bar{y}_z^M)$.

Uniqueness (strict convexity of Y_z^M) of the solution to the maximisation problem defined by

$$m(p) = \max_{y_z^M \varepsilon \gamma_z(p)} p y_z^M$$ combined with the second part of

1.8(4), Debreu, 1959, above (which we know is applicable) imply that $m(\cdot)$ and therefore $\text{proj}_{(\mathbb{R}^{L_k})}(\bar{y}_z^M)$ varies continuously with p. Therefore the map is continuous.

(2) To prove that $\Gamma_i^k(p, w_i^k)$ is a continuous correspondence with respect to p we first note that $w_i^k = w_i^k(p)$ varies continuously with p.

Define $g(p) := (p, w_i^k)$. The function g is then continuous in p.

Now, $\Gamma_i^k(p, w_i^k) = \Gamma_i^k(g(p))$.

Using Debreu (1959), 1.8(2), then gives that if the correspondence $\tilde{\Gamma}_i^k(y)$ is continuous with respect to y, then the composite correspondence $\Gamma_i^k(p, w_i^k)$ is continuous with respect to p. To obtain $\tilde{\Gamma}_i^k(p)$ continuous with respect to p, note that $\min p^k X_i^k < p^k \omega_i^k$ for each $p^k \varepsilon P^k$, therefore the continuity of $\Gamma_i^k(p)$ follows from Lemma 3, Debreu, 1982, A1 and Lemma 1 above.

As $\gamma_z(\cdot)$ and $\Gamma_i^k(\cdot)$ are convex valued, all conditions for existence of an equilibrium in the social system $\tilde{A}$ are fulfilled. Hence there is $((\bar{x}_i^k)_{i\varepsilon I^k}, (\bar{y}_j^k)_{j\varepsilon J^k}, (\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k})_{z\varepsilon Z}, \bar{p}^k)_{k=1}^K$ such that for $k = 1, \ldots, K$, $\forall i\varepsilon I^k$ $\bar{x}_i^k$ maximises u_i^k on $\Gamma_i^k(\cdot)$ and $\forall j\varepsilon J^k$ $\bar{y}_j^k$ maximises v_j^k on Y_j^k relative to $\bar{p}^k$, $\forall z\varepsilon Z$ $(\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k})$ maximises v_z^1 on $\prod_{k=1}^K Y_z^k$ and v_z^2 on $\gamma_z^k(\cdot)$ relative to $\bar{p} = (\bar{p}^1, \ldots, \bar{p}^K)$ and $\bar{p} = (\bar{p}^1, \ldots, \bar{p}^K)$ maximises the utility function t on P.

(ii) An equilibrium in $\tilde{A}$ is also an equilibrium in A:

In order to get from A to $\tilde{A}$ we made the following substitutions.

The price sets $P^k \subset \mathbb{R}_+^{L_k}$, $k = 1, \ldots, K$ in A were replaced by the set $P \subset \mathbb{R}_+^{L_1} \times \ldots \times \mathbb{R}_+^{L_K}$ in $\tilde{A}$ and the utility functions $t^k = p^k(., ., .,)$, $k = 1, \ldots, K$ in A were replaced by $t = \sum_{k=1}^{K} p^k(., ., .,)$. $N = m + n + K$ was replaced by $\tilde{N} = m + n + 1$.

As $P = P^1 \times \ldots \times P^K$ and $p = (p^1, \ldots, p^K)$ it is a straightforward application of Debreu (1959, 3.4(1)) to get:

$(\bar{p}^1, \ldots, \bar{p}^K) \cdot (\bar{\mu}^1, \ldots, \bar{\mu}^K) = \max\ (\bar{p}^1, \ldots, \bar{p}^K) \cdot D$

if and only if $\forall k = 1, \ldots, K$, $\bar{p}^k\bar{\mu}^k = \max\ \bar{p}^k D^k$,

where $D = D^1 \times \quad \times D^k$, and D^k is the set of attainable states in E^k, $k = 1, \ldots, K$.

An equilibrium in A is a F.D.E. for E:

It remains to be shown that an equilibrium for A, $((\bar{x}_i^k)_{i\varepsilon I^k}, (\bar{y}_j^k)_{j\varepsilon J^k}, (\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k})_{z\varepsilon Z}, \bar{p}^k)_{k=1}^K$ satisfies the conditions 1 to 4 in the definition of a F.D.E. for E.

Given the above equilibrium allocation for the social system A we have

$\forall i\varepsilon I^k$, $k = 1, \ldots, K$

$$p^k\bar{x}_i^k \leq \bar{p}^k\omega_i^k + \sum_{j\varepsilon J^k} \theta_{ij}^k \bar{p}^k\bar{y}_j^k + \sum_{z\varepsilon Z} (\delta_z^k\theta_{iz}^k)\bar{p}^k(\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k})$$

Summing over $i\varepsilon I^k$, rearranging terms and noting that

$\sum_{i\varepsilon I^k} \theta_{ij}^k = 1$, $\forall j\varepsilon J^k$ and $\sum_{i\varepsilon I^k} \delta_z^k\theta_{iz}^k = 1$, $\forall z\varepsilon Z$, yields

$$\bar{p}^k[\sum_{i\varepsilon I^k} \bar{x}_i^k - \sum_{i\varepsilon I^k} \omega_i^k - \sum_{j\varepsilon J^k} \bar{y}_j^k - \sum_{z\varepsilon Z} (\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}^{\alpha}k)] \leq 0$$

The term in the square bracket is identical to the definition of $\bar{\mu}^k$. Hence $\bar{p}^k\bar{\mu}^k \leqq 0 \quad \forall k$.

We have to show that $(\bar{p}^1, \ldots, \bar{p}^K)\cdot(\bar{\mu}^1, \ldots, \bar{\mu}^K) = 0$, i.e. $\forall_k \; \bar{p}^k\bar{\mu}^k = 0$.

Fix $k\varepsilon\{1,2, \ldots, K\}$. Let N^k denote the set of all commodities traded in the kth national economy. (The number of elements in N^k is $\ell_k + \alpha_k$).

We know that $\quad \sum_{h\varepsilon H^k} \bar{p}_h^k\bar{\mu}_h^k \; + \sum_{h\varepsilon N^k\setminus H^k} \bar{p}_h^k\bar{\mu}_h^k \leqq 0. \qquad (1)$

Claim 1: $\quad \bar{\mu}_h^k \geqq 0 \quad \forall \; h\varepsilon H^k \cup \mathcal{D}^k$

Proof:

(i) Suppose $\bar{\mu}_h^k < 0$ for some $h\varepsilon\mathcal{D}^k$.

Thus $\bar{p}_h^k = 0$.

By A8a $\exists\lambda^k > 0$ such that $(\bar{x}_i^k + \lambda^k\beta^h) \succ_i^k \bar{x}_i^k \quad \forall \; i\varepsilon I^k$, where β^h is the positive unit vector of the $(\ell_k + h)$th axis in $\mathbb{R}^{\ell k+\alpha k}$.

$\bar{p}_h^k = 0 \implies \bar{p}^k\cdot(\lambda^k\beta^h) = \lambda^k\bar{p}_h^k = 0$ and $\bar{p}^k(\bar{x}_i^k + \lambda^k\beta^h) = \bar{p}^k\bar{x}_i^k$.

Hence $(\bar{x}_i^k + \lambda^k\beta^h)\varepsilon\Gamma_i^k(p)$.

By A3 $(\bar{x}_i^k + \lambda^k\beta^h) \succ_i^k \bar{x}_i^k \implies u_i^k(\bar{x}_i^k + \lambda^k\beta^h) > u_i^k(\bar{x}_i^k)$.

But $\bar{x}_i^k$ maximises u_i^k on $\Gamma_i^k(p)$.

Thus, $\bar{\mu}_h^k < 0$ cannot correspond to an equilibrium.

We conclude that $\bar{\mu}_h^k \geqq 0$ and $\bar{p}_h^k > 0 \quad \forall \; h\varepsilon\mathcal{D}^k$

(ii) Suppose for some $h\varepsilon H^k$ $\bar{\mu}_h^k < 0$.

Thus $\bar{p}_h^k = e > 0$.

We now show that $\bar{\mu}_h^k < 0$ cannot be an equilibrium excess demand as this would lead to a contradiction.

Let $\bar{y}^{kM} = \text{proj}_{(\mathbb{R}^{L_k})}(\sum_{z\varepsilon Z} \bar{y}_z^M)$ be the total equilibrium multinational production in $\mathbb{R}^{L_k}$, corresponding to $\bar{\mu}^k$, and let $\bar{y}_h^{kM}$ denote the hth coordinate in $\mathbb{R}^{L_k}$. Assumption A8c implies that for the given $h\varepsilon H^k$ $\exists\, y^M \varepsilon Y^M$, $\exists\, k' \neq k$ such that

$$y_h^{kM} < \bar{y}_h^{kM}$$

$$\exists\, h_o \varepsilon \mathcal{D}^{k'}: \qquad y_{h_o}^{k'M} > \bar{y}_{h_o}^{k'M}$$

$$\exists\, h_o' \varepsilon H^{k'}: \qquad y_{h_o'}^{k'M} = \begin{cases} \bar{y}_{h_o'}^{k'M} & \text{if } \bar{y}_{h_o'}^{k'M} \neq 0 \\ < 0 & \text{if } \bar{y}_{h_o'}^{k'M} = 0 \end{cases}$$

$$\tilde{h}\varepsilon H \cup \mathcal{D} \setminus \{h, h_o', h_o\}: \qquad y_{\tilde{h}}^M = \bar{y}_{\tilde{h}}^M$$

The contradiction is now obtained by showing that y^M implies a gain in profit:

$$\bar{p}_h = e > 0 \implies e y_h^{kM} < e \bar{y}_h^{kM} < 0$$

$$\text{From (i), } \bar{p}_{h_o} > 0 \implies \bar{p}_{h_o} y_{h_o}^{k'M} > \bar{p}_{h_o} \bar{y}_{h_o}^{k'M}$$

$$\text{Let} \quad \delta = \bar{p}_{h_o} y_{h_o}^{k'M} - \bar{p}_{h_o} \bar{y}_{h_o}^{k'M} > 0$$

$$\bar{p}_{h_o'}^{k'} y_{h_o'}^{k'M} = \bar{p}_{h_o'}^{k'} \bar{y}_{h_o'}^{k'M} \quad \text{or} \quad \bar{p}_{h_o'}^{k} y_{h_o'}^{k'M} < 0$$

$$\bar{p}_{\tilde{h}} y_{\tilde{h}}^M = \bar{p}_{\tilde{h}} \bar{y}_{\tilde{h}}^M$$

For given prices and due to convexity of Y^M one can choose y^M such that $e(-y_h^{kM} + \bar{y}_h^{kM}) < \delta/2$ and $\bar{p}_{h_o'}^{k'}(-y_{h_o'}^{k'M}) < \delta/2$.

Therefore profits with the vector $y^{kM} = \text{proj}_{(\mathbb{R}^{L_k})}(\sum_{z\varepsilon Z} y_z^M)$ are larger than those from the vector $\bar{y}^{kM} = \text{proj}_{(\mathbb{R}^{L_k})}(\sum_{z\varepsilon Z} \bar{y}_z^M)$ which correspond to the "equilibrium" excess demand vector $\bar{\mu}_h^k < 0$, $h\varepsilon H^k$. Thus $\bar{\mu}_h^k < 0$ cannot be an equilibrium excess demand.

We conclude that $\bar{\mu}_h^k \geqq 0$ and $\bar{p}_h^k \geqq e$ $\forall h\varepsilon H^k$.

Claim 2: $\bar{\mu}^k_{h_o} \leqq 0 \quad \forall \; h_o \epsilon N^k \setminus H^k$

Proof:

Suppose $\bar{\mu}^k_{h_o} > 0$ for $h_o \epsilon N^k \setminus H^k$.

$\bar{p}$ maximises the market agent's utility function on P. Hence $p^k \bar{\mu}^k \leqq 0$

for each $p \epsilon P$.

Choose a price vector $p \epsilon P$ such that

$p_h = e \quad \forall \; h \epsilon H$

$p_h = 0 \quad \forall \; h \notin H, \; h \neq h_o$

$p^k_{h_o} = 1 - \sum_{h \neq h_o} p_h \quad = \quad (1 - n^H e) > 0$

where n^H is the number of elements in H

$0 < e \leqq 1/(2n^H)$

The value of excess demand, $p^k \bar{\mu}^k$, is then strictly positive

which is a contradiction of (1) above.

Claim (1) and (2) $\Rightarrow$ $\bar{\mu}^k_h = 0 \quad \forall h \epsilon D^k \cup H^k$

$\bar{\mu}^k_h \leqq 0 \quad \forall h \epsilon N^k \setminus H^k \cup D^k$

Hence $\bar{\mu}^k \leqq 0$, and therefore $\forall k \;\; \bar{\mu}^k \leqq 0$.

Given assumption A2 it follows directly from Debreu (1982) that

$\bar{p}^k \bar{\mu}^k = 0 \quad \forall k$.

5.3 Proposition 1: suppose assumptions A1 to A9 are fulfilled and assume $\exists k_\circ \;\; \exists z' \epsilon Z$: $p^{k_\circ}(\bar{y}_{z'}^{k_\circ} + \bar{y}_{z'}^{Mk_\circ} + \bar{y}_{z'}^{\alpha k_\circ}) > 0$. If assumption A10 is not fulfilled then Walras Law does not hold.

Proof

Given an equilibrium allocation for the social system A we know that $\forall i \epsilon I^k$, $k = 1, \ldots, K$,

$$\bar{p}^k \bar{x}_i^k \leqq \bar{p}^k \omega_i^k + \sum_{j \epsilon J^k} \theta_{ij}^k \bar{p}^k \bar{y}_j^k + \sum_{z \epsilon Z} (\delta_z^k \theta_{iz}^k) \bar{p}^k (\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) \qquad (1)$$

Now suppose assumption A10 is not fulfilled, i.e.

$$k = 1, \ldots, K \quad \exists z \epsilon Z \quad (\neg(\mathrm{proj}_{(\mathbb{R}^{L^k})}(Y_z) \neq 0) \vee \forall i \epsilon I^k \; \theta_i^k = 0)$$

By convexity of preferences and non-satiation, $\bar{p}^k \bar{x}_i^k = \bar{p}^k w_i^k \;\; \forall i \epsilon I^k$, i.e. equality in (1) is obtained.

Summing over $i \epsilon I^k$, noting that $\sum_{i \epsilon I^k} \theta_{ij}^k = 1 \;\; \forall j \epsilon J^k$, we obtain

$$\bar{p}^k [\sum_{i \epsilon I^k} \bar{x}_i^k - \sum_{i \epsilon I^k} \omega_i^k - \sum_{j \epsilon J^k} \bar{y}_j^k - \sum_{z \epsilon Z} \sum_{i \epsilon I^k} (\delta_z^k \theta_{iz}^k)(\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k})] = 0$$

$$\leqq \bar{p}^k [\sum_{z \epsilon Z} (\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) - \sum_{z \epsilon Z} \sum_{i \epsilon I^k} (\delta_z^k \theta_z^k)(\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k})] \qquad (2)$$

with strict inequality in (2) for $k = k_\circ$.

With our assumptions the value of aggregate consumption is equal to the value of aggregate wealth available for consumption. However, the value of aggregate consumption is less than the value of aggregate supply in the economy if $\bar{p}^k \cdot \sum_{z \epsilon Z} \sum_{i \epsilon I^k} (\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) > 0$. The 'price mechanism' does no longer work in equating the value of demand to the value of supply. The reason for this is that the specification of the 'price mechanism' as the maximisation of the value of excess demand is insufficient to ensure Walras Law. In the terminology of an abstract economy, the market agent in our social system A would have to be given more information. His choice problem, MP, for example would have to be

MP: choose $\bar{p} \varepsilon P$ such that

(i) $\sum_{k=1}^{K} \bar{p}^k(\mu^k) \geqq \sum_{k=1}^{K} p^k(\mu^k)$ all $p \varepsilon P$.

(ii) $\forall_k$: $\mu_h^k < 0 \Rightarrow \bar{p}_h^k = 0$ all h.

If a solution to MP exists then $\bar{p}^k \bar{\mu}^k = 0$ since $\bar{p}^k \bar{\mu}^k \leqq 0$ and $\bar{\mu}^k \leqq 0$, k= 1, ..., K.

For the specific case of our economy E with assumptions A1 to A9 a solution to MP does not exist since the condition $\mu_h^k < 0 \Rightarrow \bar{p}_h^k = 0$ violates assumption A8 ($p_h > 0 \quad \forall h \varepsilon H$)

6. WALRAS EQUILIBRIUM

6.1 Definition of a Walras Equilibrium (W.E.)

A set of consumptions $((\bar{x}_i^k)_{i \varepsilon I^k})_{k=1}^K$, productions $((\bar{y}_j^k)_{j \varepsilon J^k})_{k=1}^K$, $(\bar{y}_z)_{z \varepsilon Z}$ and prices $\bar{p} = (\bar{p}^1, \ldots, \bar{p}^K)$ is called a W.E. for the economy $E = (E', \ldots, E^K, Z, (Y_z)_{z \varepsilon Z})$ if the following conditions are fulfilled:

(1) $\forall_k \forall_{i \varepsilon I^k}$ $\bar{x}_i^k$ is a solution to CP

(2) $\forall_k \forall_{j \varepsilon J^k}$ $\bar{y}_j^k$ is a solution to NPP

(3) $\forall_{z \varepsilon Z}$ $\bar{y}_z$ is a solution to MNPP

(4) $\forall_k \quad \sum_{i \varepsilon I^k} \bar{x}_i^k - \sum_{j \varepsilon J^k} \bar{y}_j^k - \sum_{z \varepsilon Z} (\bar{y}_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) - \sum_{i \varepsilon I^k} \omega_i^k = 0$

In the following we wish to relax the assumption that the production set of each producer in the economy E is compact and convex. However, following Debreu (1959), we assume that the total production set of the economy (as defined below) is closed. Moreover, we assume that non-increasing returns to scale prevail in the aggregate, that is, the total production set is convex.

The total production set of the economy $E = (E', \ldots, E^K, Z, (Y_z)_{z \varepsilon Z})$ may be thought of in several ways.

1. The total production set of the economy E may be defined in the global commodity space $\mathbb{R}^L$ as the sum of all national

production sets and the total multinational production set.

i.e. $Y = \sum_{k=1}^{K} \left(\sum_{f \in J^k \cup Z} Y_f^k \right) + \sum_{z \in Z} Y_z^M$.

2. Alternatively, one may define K + 1 total production sets, comprising K total national production sets, $Y^k = \sum_{f \in J^k \cup Z} Y_f^k$, $k = 1, \ldots, K$, and one total multinational production set, $Y^M = \sum_{z \in Z} Y_z^M$. Note that Y^k is a subset of the corresponding national commodity space, $\mathbb{R}^{\ell k}$, and Y^M is a subset of $\mathbb{R}^L$.

3. Finally, one may define K total production sets, one for each national economy, E^k, taking into account the subset of the total multinational production set which 'intersects' the kth economy's nationally traded commodity space, i.e. $Y^{kM} = Y^k + Y^{M_k}$ where $Y^k = \sum_{f \in J^k \cup Z} Y_f^k$ $Y^{M_k} = \text{proj}_{(\mathbb{R}^{L_k})} (Y^M)$, $Y^M = \sum_{z \in Z} Y_z^M$.

The third approach is particularly useful for our purpose, given our assumption of no trade between national economies. We therefore adopt this definition in the following.

6.2 Theorem 2

The economy $E = (E^1, \ldots, E^K, Z, (Y_z)_{z \in Z})$ has a W.E. if for $k = 1, \ldots, K$, assumptions A2 to A5 and A8 to A10 of Theorem 1 hold and assumptions A1, A6 and A7 are replaced by A1', A6' and A7' as follows:

A1': $\forall i \epsilon I^k$	X_i^k is closed, convex and has a lower bound for $\leqq$
A6'a: $\forall f \epsilon J^k \cup Z$	Y_f^k contains zero
A6'b: $\forall z \epsilon Z$	Y_z^M is strictly convex, closed and contains zero
A7'a: $\forall$ k	$Y^{kM} \cap (-Y^{kM}) = \{0\}$
A7'b:	Y^{kM} is convex and closed
A7'c:	$Y^{kM} \supset \mathbb{R}_-^{Lk}$ $(Y^k \supset \mathbb{R}_-^{\ell k})$

Proof

We apply Debreu's (1982) method of proving existence of a W.E. for the economy E by constructing an economy $\hat{E} = (\hat{E}^1, \ldots, \hat{E}^K, Z, (\hat{Y}_z = \prod_{k=1}^{K} \hat{Y}_z^k \times \hat{Y}_z^M))$ for which the assumptions of Theorem 1 are fulfilled, hence a F.D.E. exists for $\hat{E}$. The existence of a F.D.E. for $\hat{E}$ then yields a W.E. for E.

There are several steps in this proof.

(1) We construct an economy

$\bar{\dot{E}} = ((\bar{\dot{E}}^1, ((\bar{\dot{Y}}_z^1), (\bar{Y}_z^{M1}))_{z\epsilon Z}), \ldots, (\bar{\dot{E}}^K, ((\bar{\dot{Y}}_z^K), (\bar{Y}_z^{MK}))_{z\epsilon Z}))$

which is identically defined to the economy E except that for k = 1, ..., K the production set Y_j^k, $\forall j \epsilon J^k$, is replaced by $\bar{\dot{Y}}_j^k$, the closure (–) of the convex hull (·) of Y_j^k. This substitution is also made for the national parts of each MNP's production technology, so $\forall z \epsilon Z$, Y_z^k is replaced by $\bar{\dot{Y}}_z^k$.

By A6'b Y_z^M is convex. So is $Y_z^{M_k} = \text{proj}_{(\mathbb{R}^{Lk})}(Y_z^M)$, $\forall z \epsilon Z$, $\forall k$. We substitute $\bar{Y}_z^{Mk}$ for Y_z^{Mk} $\forall z \epsilon Z$, $\forall k$. Thus, all production sets which form part of the total production set in each national economy $\bar{\dot{E}}^k$ are closed and convex.

Next we show that the allocation $((\bar{x}_i^k), (\bar{y}_j^k), (\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k}))_{k=1}^K$ is an equilibrium allocation for the economy E iff it is an equilibrium allocation for the economy $\bar{\dot{E}}$.

(i) The attainable production and consumption sets in $\bar{\dot{E}}$ are bounded:

Consider an attainable state in the economy $\bar{\dot{E}}$, i.e.

$((x_i^k)_{i\varepsilon I^k}\ ,\ (y_j^k)_{j\varepsilon J^k}\ ,\ (y_z^k)_{z\varepsilon Z}\ ,\ (y_z^{Mk},\ y_z^{\alpha k})_{z\varepsilon Z}\ )_{k=1}^K$

Attainability implies,

$$\forall_k \quad \sum_{i\varepsilon I^k} x_i^k - \sum_{i\varepsilon I^k} \omega_i^k \leqq \sum_{j\varepsilon J^k \cup Z} y_f^k + \sum_{z\varepsilon Z} (y_z^{Mk} + y_z^{\alpha k})$$

Since X_i^k is bounded below by a vector x_i^k, $\forall i\varepsilon I^k$, $\forall k$, total production in each national economy, including total MNP inputs and outputs is bounded below, i.e.

$$\sum_{f\varepsilon J^k \cup Z} y_f^k + \sum_{z\varepsilon Z} (y_z^{Mk} + y_z^{\alpha k}) \geqq \sum_{i\varepsilon I^k} x_i^k - \sum_{i\varepsilon I^k} \omega_i^k \quad \forall k.$$

By assumptions A7'a, A7'b, and A7'c, it follows from Smale (1982, Lemma 2) - applying it K times - that the attainable production sets in $\bar{\dot{E}}^k$, $\hat{\dot{Y}}^k$ $\forall$ $f\varepsilon J^k \cup Z$ is bounded and $\hat{Y}_z^M{}^k$ is bounded $\forall$ $z\varepsilon Z$, $\forall k$.

Since $\sum_{i\varepsilon I^k} x_i^k$ is bounded above and each x_i^k is bounded below, each attainable consumption x_i^k $i\varepsilon I^k$ is bounded. Hence $\hat{X}_i^k$ is bounded.

(ii) A consumption x_i^k is attainable in E^k, k = 1, ... K, iff it is attainable in $\bar{\dot{E}}^k$, k = 1, ... K:

The total production set Y^{kM} in $\bar{\dot{E}}^k$ is identical to the corresponding total production set in E^k and endowments ω_i^k , $\forall i\varepsilon I^k$, k = 1, ..., K are identical in both national economies. (Debreu, 1982)

(iii) (a) if $\bar{y}_f^k$ maximises $\bar{p}^k y_f^k$ on Y_f^k, it also maximises $\bar{p}^k y_f^k$ on $\bar{\dot{Y}}_f^k$ for $\forall f \varepsilon J^k \cup Z$, k = 1, ..., K (Debreu, 1959, (5.7))

(b) If $\bar{y}_z^M$ maximises $\bar{p}y_z^M$ on the set $\gamma_z(p)$ where $\gamma_z(p) = \sum_{k=1}^K \tilde{\gamma}_z^k(p^k)$

and

$$\forall_k\ \tilde{\gamma}_z^k(p^k) = \{\text{proj}_{(\mathbb{R}^{\ell k} + \alpha k)}(y_z^M),\ y_z^M \varepsilon Y_z^M \mid \bar{p}^k(-y_z^{Mk} - y^{\alpha k}) \leqq \max \bar{p}^k Y_z^k\}$$

then it also maximises $\bar{p}y_z^M$ on $\bar{\dot{Y}}_z(p) = \sum_{k=1}^{K} \bar{\dot{Y}}_z^k(p^k)$,

where

$$\forall_k \; \bar{\dot{Y}}_z^k(p^k) = \{\text{proj}_{(\mathbb{R}^{\ell k} + \alpha_k)}(y_z^M),\; y_z^M \varepsilon Y_Z^M \mid \bar{p}^k(-y_z^{Mk} - y_z^{\alpha k}) \leqq \max \bar{p}^k \bar{\dot{Y}}_z^k \}$$

From (a) above, if max $\bar{p}^k Y_z^k$ exists, max $\bar{p}^k \bar{\dot{Y}}_z^k$ also exists and max $\bar{p}^k Y_z^k$ is equal to max $\bar{p}^k \bar{\dot{Y}}_z^k$. Since Y_z^M is strictly convex we have the result.

Thus the budget set of each $i \varepsilon I^k$ is unaffected by the substitution of $\bar{\dot{Y}}_f^k$ for Y_f^k, $k = 1, \ldots, K$.

(2) Compactification of $\bar{\dot{E}}$ to obtain $\hat{E}$.

Since all attainable consumption and production sets in $\bar{\dot{E}}$ are bounded we can choose for each economy $\bar{\dot{E}}^k$ a compact, convex set with centre zero, $C^k \subset \mathbb{R}^{Lk}$ such that C^k contains in its interior all attainable consumption sets, $\hat{X}_i^k$ and production sets, $\hat{\bar{\dot{Y}}}_f^k$, $\hat{\bar{Y}}_z^{Mk}$. Let $C = C^1 \times \ldots \times C^K \subset \mathbb{R}^L$. The set C is also compact and convex ((7) of 1.6, Debreu, 1959), with centre zero. Finally let C' be a strictly convex subset of $\mathbb{R}^L$ such that $C' \supset C$.

Define for k = 1, ..., K

$\forall i \varepsilon I^k \quad \hat{X}_i^k = X_i^k \cap C^k$

$\forall f \varepsilon J^k \cup Z \quad \hat{Y}_f^k = Y_f^k \cap C^k$

$\forall z \varepsilon Z \quad \hat{Y}_z^M = Y_z^M \cap C'$

In order to construct an economy

$\hat{E} = (\hat{E}^1, \ldots, \hat{E}^K, Z, (\hat{Y}_z = \prod_{k=1}^{K} \hat{Y}_z^k \times \hat{Y}_z^M))$ from the economy

$\bar{\dot{E}} = ((\bar{\dot{E}}^1, ((\bar{\dot{Y}}_z^1), (\bar{Y}_z^{M1}))_{z\varepsilon Z}), \ldots, (\bar{\dot{E}}^K, ((\bar{\dot{Y}}_z^K), (\bar{Y}_z^{MK}))_{z\varepsilon Z}))$

$\hat{X}_i^k$ is replaced by $\widehat{X}_i^k$ $\forall i \epsilon I^k$, $\hat{\bar{Y}}_f^k$ by $\widehat{Y}_f^k$ $\forall f \epsilon J^k \cup Z$, and

$\forall$ $z \epsilon Z$ $\widehat{Y}_z^M$ is substituted for $\tilde{Y}_z^M$, where $\tilde{Y}_z^M = \hat{Y}_z^{M1} \times \ldots \times \hat{Y}_z^{MK}$. Note that C' contains $\tilde{Y}_z^M$ in its interior. By the definition of $\widehat{X}_i^k$, $\widehat{Y}_f^k$ and $\widehat{Y}_z^M$, the state $((x_i^k),(y_j^k),(y_z^k, y_z^{Mk}, y_z^{\alpha k}))_{k=1}^K$ is attainable in $\bar{\dot{E}}$ if it is attainable in $\widehat{E}$.

(3) The economy $\widehat{E}$ has a F.D.E.: $\widehat{X}_i^k$ is clearly compact and convex for each $i \epsilon I^k$, k = 1, ..., K. To satisfy A1 of Theorem 1 it remains to be shown that $\widehat{X}_i^k$ is non-empty. According to A5 of Theorem 2 the state $\Sigma_{i \epsilon I^k} \overset{\circ}{x}_i^k << \Sigma_{i \epsilon I^k} \omega_i^k$, in which the ith consumer in the kth national economy consumes $\overset{\circ}{x}_i^k$ $\forall i \epsilon I^k$, $y_f^k = 0$ $\forall$ $f \epsilon J^k \cup Z$ and $(y_z^{Mk} = 0,\ y_z^{\alpha k} = 0)$ $\forall z \epsilon Z$ is attainable in E^k, k = 1, ..., K, hence in $\widehat{E}^k$. Hence $\overset{\circ}{x}_i^k \epsilon \widehat{X}_i^k$ $\forall i \epsilon I^k$ $\forall k$ and $0 \epsilon \widehat{Y}_f^k$ $\forall f \epsilon J^k \cup Z$ and $0 \epsilon \widehat{Y}_z^M$ $\forall z \epsilon Z$. Thus $\widehat{X}_i^k$ is non-empty.

Assumption A2 of Theorem 1 is fulfilled, given the definition of $\widehat{X}_i^k$ and assumption A4 of Theorem 2 as the proof in Debreu, 1982, Theorem 5, applies directly.

Assumptions A3 and A4 in Theorem 1 are also fulfilled for the economy $\widehat{E}$ since these assumptions are unaffected by the construction of the economy $\widehat{E}$, and they are assumed to be fulfilled for E.

The production sets $\widehat{Y}_j^k$ and $\widehat{Y}_z^k$ in $\widehat{E}$ are compact and convex for $\forall j \epsilon J^k$, $\forall z \epsilon Z$, k = 1, ..., K. Since each of these production sets contains zero they are non-empty and hence satisfy A6 and A7a of Theorem 1. $\widehat{Y}_z^M$ is compact, contains zero, and by A6'b of Theorem 2, is strictly convex.**

** Given C' is a strictly convex subst of $\mathbb{R}^L$ and $Y_z^M \subset \mathbb{R}^L$ is strictly convex, then $\widehat{Y}_z^M = Y_z^M \cap C'$ is strictly convex.

Assumption A8 of Theorem 1 is also fulfilled for the economy $\hat{E}$: by A8a of Theorem 2, if $h \varepsilon \mathcal{D}^k$, $i \varepsilon I^k$, $x_i^k \varepsilon X_i^k$, then $\exists \lambda^k > 0$ such that $(x_i^k + \lambda^k \beta^h) \succ_i^k x_i^k$, $\forall i \varepsilon I^k$, where β^h is the positive unit vector of the $(\ell_k + h)$th axis in $\mathbb{R}^{\ell k + \alpha k}$. Let $x_i^k \varepsilon \hat{X}_i^k$ and $(x_i^k + \lambda^k \beta^h) \varepsilon X_i^k$. Since X_i^k is convex and $\hat{X}_i^k$ is in the interior of C^k we can find a consumption $(x_i^k + \gamma \lambda^k \beta^h) \varepsilon \hat{X}_i^k$ for $\gamma \varepsilon]0,1[$. By A4 $(x_i^k + \gamma \lambda^k \beta^h) \succ_i^k x_i^k$ all $i \varepsilon I^k$, $k = 1, \ldots, K$.

Using a similar argument assumption A2 of Theorem 1 can be shown to be fulfilled for each E^k, $k = 1, \ldots, K$.

Thus all assumptions of Theorem 1 are satisfied and a F.D.E. exists for the economy $\hat{E}$.

(4) A F.D.E. for $\hat{E}$ yields a W.E. for the economy E .

(i) A F.D.E. for $\hat{E}$ provides an attainable state for the economy E :

let $((\bar{x}_i^k), (y_j^k), (y_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k}), \bar{p}^k)_{k=1}^K$ denote a F.D.E. allocation for the economy $\hat{E}$ and let

$$\mu^k = \sum_{i \varepsilon I^k} \bar{x}_i^k - \sum_{j \varepsilon J^k} y_j^k - \sum_{z \varepsilon Z} (y_z^k + \bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) - \sum_{i \varepsilon I^k} \omega_i^k$$

be the associated excess demand for the national economy $\hat{E}^k$, $k = 1, \ldots, K$.

We know that $\forall k \ \mu^k \leqq 0$ and $\mu_h^k = 0 \ \forall h \varepsilon \mathcal{D}^k$.

Since $Y^k \supset \mathbb{R}_-^{\ell k}$ it follows that

$$\forall_k \sum_{i \varepsilon I^k} \bar{x}_i^k - \sum_{f \varepsilon J^k \cup Z} \bar{y}_f^k - \sum_{z \varepsilon Z} (\bar{y}_z^{Mk} + \bar{y}_z^{\alpha k}) - \sum_{i \varepsilon I^k} \omega_i^k = 0$$

where

$$\forall f \varepsilon J^k \cup Z \quad \bar{y}_f^k \varepsilon Y_f^k \text{ such that } \sum_{f \varepsilon J^k \cup Z} \bar{y}_f^k = \sum_{f \varepsilon J^k \cup Z} y_f^k + \mu^k.$$

(The proof in Theorem 5, Debreu, 1982, applies directly.

(ii) $\forall i\ I^k$ $\bar{x}_i^k$ is a solution to CP, k=1, ..., K

(iii) $\forall j\ J^k$ $\bar{y}_j^k$ is a solution to NPP, k = 1, ..., K

(iv) $\forall z\ Z$ $\bar{y}_z^k$ is a solution to MNPP.

The proof of (ii) to (iv) follows directly from Debreu (1982).

7. Pareto efficiency

The structure of the world economy described in the previous sections suggests two applications of the Pareto efficiency criterion. First, the criterion is applied to the world economy as a whole (Global Pareto Efficiency) and secondly, the criterion is applied to each national economy separately (National Pareto Efficiency).

7.1 Definition of Pareto Efficiency

a) Global Pareto Efficient (G.P.E.)

Let D be the set of attainable states - as defined in section 4.1 - for the economy $E = (E^1, \ldots, E^K, Z, (Y_z))$.

The state

$d = ((x_i^k), (y_j^k), (y_z^k, y_z^{Mk}, y_z^{\alpha k}))_{k=1}^K \in D$ is G.P.E. if

$\nexists\ d' = ((x_i'^k), (y_j'^k), (y_z'^k, y_z'^{Mk}, y_z'^{\alpha k}))_{k=1}^K \in D$

such that $x_i'^k \succsim_i^k x_i^k$ all $i \in \bigcup_{k=1}^K I^k$ and $x_i'^k \succ_i^k x_i^k$ for some $i \in \bigcup_{k=1}^K I^k$.

b) National Pareto Efficient (N.P.E.)

Let $D^k := \mathrm{proj}_{(\mathbb{R}^\tau)}(D)$, where $\tau = m^k L_k + n^k \ell_k + n^Z(\ell_k + 2L_k)$

The state $d^k = ((x_i^k), (y_j^k), (y_z^k, y_z^{Mk}, y_z^{\alpha k})) \in D^k$ is N.P.E. if $\nexists\ d'^k = ((x_i'^k), (y_j'^k), (y_z'^k, y_z'^{Mk}, y_z'^{\alpha k})) \in D^k$ such that $x_i'^k \succsim_i^k x_i^k$ all $i \in I^k$ and $x_i'^k \succ_i^k x_i^k$ for some $i \in I^k$.

For the purpose of the following theorem we need the notion of an equilibrium relative to a price system

7.2 Definition of an equilibrium relative to a price system

A state $((\bar{x}_i^k), (\bar{y}_j^k), (\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k}))_{k=1}^K$ of the economy E is called an equilibrium relative to the price system $p = (p^1, \ldots, p^K) \varepsilon \mathbb{R}^{L_1} x \ldots x \mathbb{R}^{L_K}$ if

(a) $\bar{x}_i^k$ is a greatest element of the set $\{x_i^k \varepsilon X_i^k \mid p^k x_i^k \leq p^k \bar{x}^k\}$ for $\succsim_i^k \ \forall i \varepsilon I^k, k = 1, \ldots, K$

(b) (i) $\bar{y}_f^k$ maximises $p^k y_f^k$ on Y_f^k, $\forall f \varepsilon J^k \cup Z$, $k = 1, \ldots, K$

(ii) $\bar{y}_z^M = ((\bar{y}_z^{Mk}), (\bar{y}_z^{\alpha k}))$ maximises $p y_z^M$ on $\gamma_z(p)$, $\forall z \varepsilon Z$

(c) $\sum_{i \varepsilon I^k} \bar{x}_i^k - \sum_{f \varepsilon J^k \cup Z} \bar{y}_f^k - \sum_{z \varepsilon Z} (\bar{y}_z^{Mk} + \bar{y}_z{}^k) = \sum_{i \varepsilon I^k} \omega_i^k \quad \forall k.$

Clearly, if $((\bar{x}_i^k), (\bar{y}_j^k), (\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k}), \bar{p}^k)_{k=1}^K$ is an equilbrium of the economy E then the state $((\bar{x}_i^k), (\bar{y}_j^k), (\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k}))_{k=1}^K$ is an equilibrium relative to the price system $\bar{p} = (\bar{p}^1, \ldots, \bar{p}^K)$.***

*** The converse result may always be obtained by a suitable definition of the distribution of initial endowments. See Debreu, 1959, section 6.2

7.3 Theorem 3: an equilibrium relative to a price system for the economy E is G.P.E. if

(i) X_i^k is convex, $\forall i \epsilon I^k$, $k = 1, \ldots, K$

(ii) $\succsim_i^k$ is convex, $\forall i \epsilon I^k$, $k = 1, \ldots, K$

(iii) no $\bar{x}_i^k$ is a satiation consumption

(iv) Y_z^M is strictly convex, $\forall z \epsilon Z$

Proof

The idea of the proof is to show that given the price system $p = (p^1, \ldots, p^K)$ and total resources in each national economy, ω^k, $k = 1, \ldots, K$, for each consumer $i \epsilon I^k$, $k = 1, \ldots, K$, the equilibrium consumption $\bar{x}_i^k$ minimises expenditure on the set of possible consumptions which are at least as preferred as the equilibrium consumption and that the equilibrium consumption is at least as preferred as any greatest element of the set of possible consumptions which are at most as expensive as the equilibrium consumption. Hence the equilibrium consumption $((\bar{x}_i^1), \ldots, (\bar{x}_i^K))$ is Pareto efficient as there does not exist another admissible consumption $((x_i^1), \ldots, (x_i^K))$ such that $x_i^k \succsim_i^k \bar{x}_i^k$ $\forall i \epsilon I^k$ with $\succ_i$ for at least one $i \epsilon \bigcup_{k=1}^{K} I^k$. Formally, let

$$G = \sum_{k=1}^{K} \left(\sum_{i \epsilon I^k} X_i^{k\bar{x}} - \sum_{f \epsilon J^k} Y_f^k \right) - \sum_{z \epsilon Z} Y_z^{M\bar{y}} \subset \mathbb{R}^L$$

where

$$X_i^{k\bar{x}} = \{ x_i^k \epsilon X_i^k \mid x^k \succsim_i^k \bar{x}_i^k \} \quad \forall\, i \epsilon I^k,\ k = 1, \ldots, K$$

$$Y_z^{M\bar{y}} = \{ y_z^M \epsilon Y_z^M \mid \mathrm{proj}_{(\mathbb{R}^\ell_-)}(y_z^M) \leqq \mathrm{proj}_{(\mathbb{R}^\ell_-)}(\bar{y}_z^M) \}$$

The set $X_i^{k\bar{x}}$ is the set of all possible consumptions which the ith consumer in the kth national economy prefers at least as much as the equilibrium consumption $\bar{x}_i^k$. The set $Y_z^{M\bar{y}}$ consists of all MNP productions which are possible for the zth MNP and which use at least as much of inputs as the equilibrium production, $\bar{y}_z^M$.

Consider the function from $\mathbb{R}^L$ to $\mathbb{R}$, defined by $a \to p \cdot a$, where $a = (a^1, \ldots, a^K) \in \mathbb{R}^{L1} \times \ldots \times \mathbb{R}^{Lk}$, $p = (p^1, \ldots, p^K) \in \mathbb{R}^{L1} \times \ldots \times \mathbb{R}^{LK}$. Note that $x_i^k \in \mathbb{R}^L$ has zero coordinates everywhere except for the coordinates corresponding to $\mathbb{R}^{Lk}$.

As $\bar{x}_i^k$ is an equilibrium consumption we know that $p^k x_i^k \leqq p^k \bar{x}_i^k \Rightarrow \bar{x}_i^k \succsim_i^k x_i^k$. By assumptions (i) and (iii) the conditions of (2) of 4.9, Debreu, 1959, are fulfilled. Hence we have $x_i^k \succsim_i^k \bar{x}_i^k \Rightarrow p^k x_i^k \geqq p\bar{x}_i^k$, $\forall\ i \in I^k$, $k = 1, \ldots, K$. Therefore $\bar{x}_i^k$ minimises $p \cdot a$ on $X_i^{k\bar{x}}$ $\forall\ i \in I^k$, $\forall k$.

Condition (b)(i) of 7.2 above implies that $\bar{y}_f^k$ minimises $p \cdot a$ on $-Y_f^k$, $\forall f \in J^k \cup Z$.

From (b)(ii) of 7.2 above, $\bar{y}_z^M$ minimises $p \cdot a$ on $-Y_z^{M\bar{y}}$ $\forall z \in Z$. Note that $\forall z \in Z$ $\min p(-Y_z^{M\bar{y}}) = \max p\gamma_z(p)$, given that $\bar{y}_z^k$ minimises $p^k(-Y_z^k)$ $\forall k$, moreover, strict convexity of Y_z^M implies that $\bar{y}_z^M$ is unique, $\forall z \in Z$.

From (c) of 7.2 above, $\sum_{i\in I^k} \bar{x}_i^k - \sum_{f\in J^k \cup Z} \bar{y}_f^k = \sum_{i\in I^k} \omega_i^k + \sum_{z\in Z} (\bar{y}_z^{Mk} + \bar{y}_z^{\alpha k})$, $\forall k$

Summing over k,

$$\sum_k \left(\sum_{i\in I^k} \bar{x}_i^k - \sum_{f\in J^k \cup Z} \bar{y}_f^k \right) = \sum_k \sum_{i\in I^k} \omega_i^k + \sum_{z\in Z} \bar{y}_z^M$$

By (1) of 3.4 of Debreu (1959),

$$\sum_k \left(\sum_{i\in I^k} \bar{x}_i^k - \sum_{f\in J^k \cup Z} \bar{y}_f^k \right) - \sum_{z\in Z} \bar{y}_z^M \text{ minimises } p\cdot a \text{ on } G.$$

The rest of the proof follows directly from 6.3 of Debreu, 1959.

Since assumptions (i) to (iv) above are fulfilled for the economy E of Theorem 2, an equilibrium for this economy is G.P.E.

7.4 <u>Theorem 4</u>: a G.P.E. allocation $((\bar{x}_i^k), (\bar{y}_j^k), (\bar{y}_z^k, \bar{y}_z^{Mk}, \bar{y}_z^{\alpha k}))_{k=1}^{K}$ is an equilibrium for the economy $E = (E^1, \ldots, E^K, Z, (Y_z))$ relative to a price system $p = (p^1, \ldots, p^K)$, $p^k \neq 0$ $\forall k$, if

1. $\forall_k \ \forall_{i\in I^k}$ $\quad \bar{x}_i^k$ minimises $p^k x_i^k$ on $\{x_i^k \in X_i^k \mid x_i^k \succsim_i^k \bar{x}_i^k\}$
2. $\forall_{f\in J^k \cup Z}$ $\quad \bar{y}_f^k$ maximises $p^k y_f^k$ on Y_f^k
3. $\forall_{z\in Z}$ $\quad \bar{y}_z^M = ((\bar{y}_z^{Mk}), (\bar{y}_z^{\alpha k}))$ maximises $p y_z^M$ on $Y_z(p)$
4. for some $i \in \bigcup_{k=1}^{K} I^k$ $\quad \bar{x}_i^k$ is not a satiation consumption
5. $\forall_k$ $\quad Y^k = \sum_{f\in J^k \cup Z} Y_f^k$ is convex
6. $\forall_{z\in Z}$ $\quad Y_z^M$ is strictly convex.

Proof

Let $\overset{\circ}{X}{}^{k\bar{x}}_{i,i'} = \{x^k_i \varepsilon X^k_i \mid x^k_i \succ^k_i \bar{x}^k_i\}$, $i' \varepsilon \bigcup_{k=1}^{K} I^k$

$$\overset{\circ}{G} = \overset{\circ}{X}{}^{k\bar{x}}_{i,i'} + \sum_k \Big(\sum_{i \varepsilon I^k} X^{k\bar{x}}_i - \sum_{f \varepsilon J^k \cup Z} Y^k_f \Big) - \sum_{z \varepsilon Z} Y^{M\bar{y}}_z$$

where

$X^{k\bar{x}}_i$ and $Y^{M\bar{y}}_z$ are defined as for Theorem 3.

The proof follows from 6.4 of Debreu (1959), noting that $\sum_{z \varepsilon Z} Y^{M\bar{y}}_z$ is convex and that min py_z on $-Y^{M\bar{y}}_z$ is equal to max py_z on $\gamma_z(p)$.

7.5 Remark 1 : global Pareto efficiency does not imply national Pareto efficiency in general.

Consider a G.P.E. allocation, $(((\bar{x}^1_i), (\bar{y}^1_f), (\bar{y}^{M1}_z, \bar{y}^{\alpha 1}_z)), \ldots, ((\bar{x}^K_i), (\bar{y}^K_f), (\bar{y}^{MK}_z, \bar{y}^{\alpha K}_z))) \varepsilon D$; then

$$\forall_k \quad \sum_{i \varepsilon I^k} \bar{x}^k_i - \sum_{f \varepsilon J^k \cup Z} \bar{y}^k_f = \sum_{i \varepsilon I^k} \omega^k_i + \sum_{z \varepsilon Z} (\bar{y}^{Mk}_z + \bar{y}^{\alpha k}_z).$$

Now consider the case where the zth MNP's profit maximising production in the kth national economy, E^k, is such that $\bar{y}^{Mk}_z < 0$ and $\bar{y}^{\alpha k}_z = 0$. This is the case where the MNP uses inputs, $\bar{y}^{Mk}_z$ from the kth national economy but does not make available MNP commodities in this economy. The inputs $\bar{y}^{Mk}_z$ are acquired from the profit, max $p^k Y^k_z$ which the MNP makes in the kth national economy by using the "national part" of his production technology. By assumption (iii) of Theorem 3, $\bar{x}^k_i$ is a non-satiation consumption for each consumer $i \varepsilon I^k$. The resources $\bar{y}^{Mk}_z$ could be split among the consumers in the kth national economy by giving each agent the amount $(1/n^k)(-\bar{y}^{Mk}_z)$. As $0 \varepsilon Y^M_z \; \forall z \varepsilon Z$ one can find an attainable state in the economy $E = (E^1, \ldots, E^K, Z, (Y_z))$, $(((x_i^1), (y_f^1)\ (y_z^{M1}, y_z^{\alpha 1})), \ldots, ((x_i^K), (y_f^K), (y_z^{MK}, y_z^{\alpha K}))) \in D$ such that $x_i^k \succ_i^k \bar{x}_i^k$ all $i \in I^k$ with $x_i^k = \bar{x}_i^k + (1/n^k)(-\bar{y}_z^{Mk})$ and $y_z^{Mk} = 0$.

7.6 Note, the case described to obtain the result of the above Remark may be called 'unintended exploitation'. From the perspective of the consumers in the kth national economy resources which could be consumed in the absence of the MNP's multinational production are no longer available for that purpose. In this sense the consumers in the kth national economy are exploited. Exploitation is 'unintended' in the sense that price taking behaviour and profit maximisation are usually considered desirable properties of firm behaviour.

At times it is argued that so-called multinational corporations bestow a benefit to 'host countries' in the form of 'new' technologies.[1] Again from the perspective of the kth national economy MNP commodities are 'new' in the sense that they cannot be produced with national production technologies. However, in the case considered no such benefit occurs ($\alpha_k = 0$). Moreover, given our assumption that $\forall k\ \ \forall z \in Z\ \exists\ j \in J^k$ such that $Y_z^k = Y_j^k$ the national part of the MNP's production technology is identical to that of at least one national producer. Thus, again there is no 'technological benefit' for the kth national economy.

It should also be noted that unintended exploitation does not occur if the assumption of Y_j^k is convex is replaced by the assumption of Y_j^k is a convex cone with vertex zero $\forall j \in J^k\ \forall k$. In this case $\max p^k Y_j^k = 0\ \forall\ j \in J^k\ \forall k$ and therefore $\max p^k Y_z^k = 0$ $\forall z \in Z\ \forall k$. In this case $\bar{y}_z^{Mk} = 0$.

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[1] see for example Kindleberger (1984)

8. Concluding Comments

The purpose of the foregoing work is to take a first step towards integrating some of the observations and claims found in the empirically oriented literature about the nature of so-called multinational corporations into the Arrow-Debreu general equilibrium model. It is appropriate now to briefly review those aspects of the empirically oriented literature which, by their nature, cannot be dealt with in the Arrow-Debreu model or which were left for future research and to outline further research which is in progress but cannot be reported here.

Many studies of so-called multinational corporations are concerned with the question of how a firm obtains a technological advantage which enables it to become a multinational corporation. Vernon's (1966) product cycle theory is perhaps the best known and most thorough example of this type of work. Empirical work which examines the 'research and development' activities of so-called multinational corporations also belongs to this body of literature. Quite clearly we did not ask the question of how some firms acquire production technologies which enable them, so to speak, to operate in more than one socio-economic environment — the question of how any firm obtains technological knowledge, represented by the production set, is not asked in the Arrow-Debreu model. The starting point is to take production technologies as given. Parenthetically it should be mentioned that if one wishes to study how technological knowledge is obtained or developed by firms then this question should be asked for all firms otherwise confusion ensues as to the definition of a multinational versus a non-multinational corporation.

There is no agreement in the empirically oriented literature as to the market behaviour of so-called multinational corporations. Until very recently the prevailing view was that such firms belong to oligopolistic markets. This view became associated with Hymer (1960). Oligopolistic market behaviour in the sense of non-price-taking behaviour was assumed by Helpman (1984, 1985) for firms in the 'corporate sector' of a two country-two sector trade model. In his model a 'corporation' is a producer of a 'differentiated product' and a 'multinational corporation' is a corporation with the added assumption that the differentiated product is 'internationally mobile'. However, as is evident in the writings of Caves (1974) and others, so-called multinational corporations are judged to either increase or decrease the 'degree of competitiveness' of an industry depending on whether they decrease or increase the level of concentration in this industry. We have chosen to model a competitive economy.

Recently, Hymer's (1960) thesis was severely criticized on the grounds of having been "misleading in its emphasis upon market power" (Teece, 1985)[1]. Without wishing to enter the current debate on Hymer's contribution to the theory of direct foreign investment[2] it is worth noting that Hymer followed the then prevailing 'structure → conduct → performance' paradigm of the industrial organisation school (see for example Pickering, 1974)[3]. Moreover, a careful reading of Hymer (1960)

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[1]See also Dunning and Rugman (1985)

[2]See American Economic Review, May 1985.

[3]So-called multinational corporations were observed to operate in 'highly concentrated industries'.

shows that Hymer considered 'market power' as only one of three possible sources of an advantage enjoyed by so-called multinational corporations which he postulated to be a prerequisite for their existence. The other two sources were reduction of risk through diversification in the spirit of the Markowitz (1952, 1959) portfolio theory of diversification and the possession of a technological advantage ("more efficient production function", p.41). Moreover, Hymer cautioned his readers about the limitations of his analysis when he wrote, "It is important to stress that these are merely isolated partial-equilibrium considerations. By themselves, they provide no guide for policy; in a broader theory which may someday be developed, they may be useful." (p.211) It is idle but tempting to speculate on the course of Hymer's research for his PhD dissertation had he been directed to Debreu (1959) rather than the contemporary, Bains, who quite clearly influenced Hymer's work. In subsequent work by Horst (1972) and others, surveyed by Hufbauer (1975), the notion of 'more efficient production function' is expressed by making an assumption about the numerical value of the scalar λ in a Cobb-Douglas production function of the form $X = \lambda L^{\alpha} K^{\beta}$. This notion is akin to what we have called 'technical efficiency' (see sec. 1).

With few exceptions, there is agreement in the literature that so-called multinational corporations aim to maximise global profits. We have adopted global profit maximisation as behavioural assumption. Nevertheless, as was discussed in section 3 above, this assumption turned out to pose some questions for our model which will be approached in a forthcoming paper, following the work of Dreze (1984). Essentially the problem is that global profit maximisation in our model does not entail shareholder wealth maximisation in the sense of the Debreu model nor is profit maximisation a goal which may be taken to be unanimously agreed upon by shareholders in our world economy model with MNPs and no trade.

Given the assumptions of our world economy model, described in section 5 above, assumption A10 was proved to be necessary to obtain Walras Law (Proposition 5.3). The problem of loosing Walras Law stems essentially from the case where a MNP makes strictly positive profits in, say the kth national economy, and these profits cannot be transferred to shareholders belonging to another national economy. Alternatively put, given the structure of our world economy model Walras Law obtains if all profits are distributed in the national economy where they are made. Assumption A10 ensures this. One may conceive of several alternative ways of 'disposing' of MNP profits. For example one could modify the model by introducing an agent (government) who taxes all MNP profits which cannot be distributed to a national shareholder and then redistributes (wealth) to all national consumers. Another possibility would be to introduce (limited) migration, conditional on employment in MNP production, into the model. The latter approach raises obvious questions about the definition of National Pareto Efficiency.

Finally, a few words should be said about our concept of a MNP in relation to international trade theory and location theory. Firstly, in contrast to Helpman's (1985) model where direct foreign investment is always a unilateral phenomenon between any pair of countries, our model accommodates unilateral direct foreign investment as a special case. This is a desirable feature of our model because direct foreign investment was observed to be typically multilateral.

Secondly, in his "eclectic approach" to the theory of international production, Dunning (1981) reminds of the importance of studying the locational choice of international production. A country which attracts direct foreign investment is said to have a "locational advantage" due to

"possession of resources, material and labour costs, markets, government policy, economies of product specialisation and concentration, the need to be near customers, after sales servicing, etc." (1981, p. 49)
If the 'locational choice of a MNP is interpreted as non- zero production in a national economy where production is possible, then our model can be applied to describe 'locational advantage' of a national economy due to the 'possession of resources, material and labour costs, and markets'. For example, consider three national economies, k = 1, 2, 3. Let $\ell_1 = \ell_2 > 0$, $\alpha_1 = \alpha_2 = 0$, $\alpha_3 > 0$. For simplicity we treat each national economy as one location and assume that ℓ_1, ℓ_2 are ℓ identical goods. Furthermore, assume $\exists\, z \in Z$ such that $Y_z^1 \neq \{0\}$, $Y_z^2 \neq \{0\}$ and 'identical' in terms of input-output vectors of the ℓ goods, $proj_{(\mathbb{R}^{\ell_1})}(Y_z^M) \neq \{0\}$, $proj_{(\mathbb{R}^{\ell_2})}(Y_z^M) \neq \{0\}$, $proj_{(\mathbb{R}^{L_3})}(Y_z^M) \neq \{0\}$, with 'identical' input vectors y_z^{M1}, y_z^{M2}, in the national economies E^1 and E^2. Then national economy E^1 has a locational advantage due to 'markets' (including material and labour costs) over national economy E^2 if the solution to the MNP's profit maximisation problem is given by the set of vectors
$$((\bar{y}_z^1 \neq \{0\}, \bar{y}_z^2 = \{0\}, \{0\}), (\bar{y}_z^{M1} < 0, \bar{y}_z^{M2} = \{0\}, \bar{y}_z^{M3} < 0), (0, 0, \bar{y}_z^{\alpha 3} > 0)).$$
This is the case when $\max p^1 Y_z^1 > \max p^2 Y_z^2 = 0$ and $\max p Y_z \geqq 0$.

In a forthcoming paper we show that our model also lends itself to describe 'enclave economies'.

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