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NONJOINT TECHNOLOGIES

by

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1 Introduction

The assumption that production is nonjoint in input quantities (separate production functions) plays a crucial role in many areas of economic theory. We need only think of international trade theory, public finance, macroeconomics, growth theory or traded-nontraded good models of international finance. In fact, it is safe to say that more often than not production is assumed nonjoint in input quantities in multiple-output models.

Little work has been done to test the empirical validity of the assumption of nonjointness in input quantities. Samuelson (1966) studies the characterization of nonjoint technologies in terms of the transformation function. As pointed out by Hall (1973), however, Samuelson's results do not seem very useful for econometric work. Hall studies the characterization of nonjointness in input quantities in terms of the joint cost function; this leads to a number of restrictions which can easily be imposed or tested for, and which were used in empirical work by Burgess (1976) and by Kohli (1979). Lau (1972), finally, distinguishes between two cases of nonjointness (nonjointness in input quantities and nonjointness in output quantities) and characterizes them in terms of the profit function.

In this paper we distinguish between four cases of nonjointness and we characterize them in terms of the joint cost function and the variable (or normalized restricted) profit function. The new forms of nonjointness that we define may be useful to describe technologies at the level of the firm, and maybe even more so, at the aggregate level; some may be useful concepts in other areas of economics as well, such as consumer theory. By distinguishing between the various forms of nonjointness one obtains a number of testable restrictions on the corresponding joint cost or variable profit functions. A number of comparative statics results are also obtained.

The remainder of this paper is organized as follows. Two alternative ways to describe the technology are discussed in section 2. In section 3 we define the four forms of nonjointness, and some of our main results regarding the characterization of the technology are obtained. Further results are obtained in section 4 by working in the price space. In section 5 we examine the special case where the number of inputs equals the number of outputs. Fixed coefficient technologies are examined in section 6, and section 7 treats of the general case of an unequal number of commodities and factors. A number of comparative statics results are derived in section 8, and some concluding comments are offered in the final section.

2 Alternative Representations of the Technology

Assume a large number of profit maximizing firms that operate under perfect competition in all markets. These firms choose their vectors of input and output quantities subject to the technology and input and output prices. Assume that the country's endowments of factors are fixed, and that output prices are exogenously determined. It is well known that if factors are mobile between firms the competitive equilibrium can be viewed as the solution of maximizing gross national product subject to the technology, national factor endowments and output prices. This programming problem yields the competitive factor rental prices as dual variables.[1]

Assume that there are I outputs (commodities) and J primary factors. We denote the vector of output quantities by $y = (y_1,\ldots,y_I)'$ and the vector of factor endowments by $x = (x_1,\ldots,x_J)'$. The vector of commodity prices is $p = (p_1,\ldots,p_I)'$ and the vector of factor (rental) prices is $w = (w_1,\ldots,w_J)'$. Let T be the production possibility set, i.e. the set of feasible input and output combinations (y,x). We assume that T is a closed, nonempty, convex set, that the components of y are bounded from above for given x, and that it permits free disposal of inputs. In addition, we assume that the technology exhibits constant returns to scale, i.e. T is a cone. The economy's variable profit (or gross national product) function can be defined as follows:

$$\pi(p,x) \equiv \max_y \{p'y : (y,x)\varepsilon T, \; y \geq 0\} \quad \text{for } p \gg 0,\ x \geq 0 \ .$$

It can be shown that given the assumptions made on T, $\pi(\cdot)$ is nonnegative and well defined for all $p >> 0$, $x \geq 0$; $\pi(\cdot)$ is continuous, nondecreasing, linearly homogeneous and convex in the components of p, and it is continuous, nondecreasing, linearly homogeneous and concave in the components of x. In addition, it can be shown that there is a one to one relationship between variable profit functions and production possibility sets, so that the former one may be chosen to describe the technology.[2]

Alternatively, the technology can be represented by the joint cost function defined as follows:

$$C(y,w) \equiv \min_x \ \{w'x : (y,x)\varepsilon T,\ x \geq 0\} \quad \text{for } w >> 0,\ y \geq 0 .$$

One can show that $C(\cdot)$ is nonnegative and well defined for all $w >> 0$ and $y \geq 0$; it is continuous and nondecreasing in all its arguments, and it is linearly homogeneous and convex in the components of y, and linearly homogeneous and concave in the components of w.[3]

An important property of the variable profit function is that the subdifferential of $\pi(\cdot)$ at (p^*,x^*), if it exists, coincides with the solution set of the GNP maximization problem and the set of dual variables:

$$y^* \ \varepsilon\ \nabla_p\ \pi(p^*,x^*)$$

$$w^* \ \varepsilon\ \nabla_x\ \pi(p^*,x^*)$$

where $\nabla_p\ \pi(\cdot)$ and $\nabla_x\ \pi(\cdot)$ denote the subdifferentials of $\pi(\cdot)$ with respect to the components of p and x respectively.[4]

If the variable profit function is twice continuously differentiable, we can define the Hessian of $\pi(\cdot)$ as:

$$H(\pi) = \begin{bmatrix} \pi_{pp} & \pi_{px} \\ \pi_{xp} & \pi_{xx} \end{bmatrix}$$

where $\pi_{px} = \partial^2\pi(\cdot)/\partial p\partial x'$ is the sub-Hessian of $\pi(\cdot)$ with respect to the components of p and x, and similarly for π_{pp}, π_{xp} and π_{xx}. The curvature properties of $\pi(\cdot)$ imply that π_{pp} is positive semi-definite and that π_{xx} is negative semi-definitive. Linear homogeneity in the components of p and of x implies that π_{pp} and π_{xx} are singular. The components of π_{px} indicates the effect of a change in factor endowment on the supply of commodities (at unchanged commodity prices), π_{xp} indicates the effect of a change in commodity prices on factor prices. Symmetry of the Hessian matrix yields Samuelson's (1953-4) reciprocity conditions.

Similar results can be obtained if one works with the joint cost function. The subdifferential of $C(\cdot)$ at (y^*,w^*), if it exists, coincides with the solution set of the cost minimization problem and the set of dual variables:

$$p^* \in \nabla_y C(y^*,w^*)$$

$$x^* \in \nabla_w C(y^*,w^*)$$

where $\nabla_y C(\cdot)$ and $\nabla_w C(\cdot)$ are the subdifferentials of $C(\cdot)$ with respect to components of y and w respectively.[5]

If the joint cost function is twice continuously differentiable, then we can define the Hessian of $C(\cdot)$ as:

$$H(C) = \begin{bmatrix} C_{yy} & C_{yw} \\ C_{wy} & C_{ww} \end{bmatrix}$$

where $C_{yw} = \partial^2 C(\cdot)/\partial y \partial w'$ and so on. Convexity of $C(\cdot)$ with respect to the components of y indicates that C_{yy} is positive semi-definite, and concavity with respect to the components of w implies that C_{ww} is negative semi-definite. Because of the linear homogeneity of $C(\cdot)$, C_{yy} and C_{ww} are singular. The elements of C_{wy} indicate the effect on factor demand of a change in the output of commodities: C_{wy} is thus the matrix of marginal input-output coefficients $a_{ij} = \partial x_j/\partial y_i$; these coefficients will in general not be constant, but depend on relative factor prices instead. The elements of C_{yw} indicate the effect of a change in factor prices on the price of commodities. Symmetry of the Hessian implies that $\partial x_j/\partial y_i = \partial p_i/\partial w_j$.

If both $\pi(\cdot)$ and $C(\cdot)$ are continuously differentiable, then it follows that:

$$(dy', dw')' = H(\pi)\ (dp', dx')'$$
$$(dp', dx')' = H(C)\ (dy', dw')'$$

hence

$$H(C) = [H(\pi)]^{-1},$$

i.e. if the Hessians of $\pi(\cdot)$ and of $C(\cdot)$ both exist, the two Hessians are the inverse of one another.

The representation of the technology given so far is valid for a large variety of technologies, including ones which allow for intermediate products and joint production. The variable profit function and the joint cost function are actually particularly well suited to handle cases of joint production. In this paper, however, it is the cases of nonjoint production which are of interest to us. We examine in the following sections the additional properties of $\pi(\cdot)$ and $C(\cdot)$ when production is nonjoint.

3 Four Forms of Nonjointness

The first two cases of nonjointness we consider are cases of nonjointness in quantities.

Definition 1. The technology is said to be nonjoint in input quantities if there exists individual quasi-concave, nonnegative, nondecreasing production functions $f^i(\cdot)$ such that:

$$y_i \leq f^i\ (x_{1i}, \ldots, x_{Ji}) \qquad i\varepsilon I$$

$$\sum_j x_{ji} \leq x_j \qquad j\varepsilon J$$

$$(y,x)\varepsilon T\ .$$

Definition 2. The technology is said to be nonjoint in output quantities if there exists individual quasi-convex, nonnegative, nondecreasing factor requirements functions $g^j(\cdot)$ such that:

$$x_j \geq g^j\ (y_{1j},\ldots,\ y_{Ij}) \qquad j\varepsilon J$$

$$\sum_j y_{ij} \geq y_i \qquad i\varepsilon I$$

$$(y,x)\varepsilon T\ .$$

These two definitions are consistent with the definitions given by Diewert and by Lau.[6] We also consider the following case of nonjointness, defined by Kohli (1979).

Definition 3. The technology is said to be nonjoint in output prices if there exists individual quasi-convex, nonnegative, nondecreasing factor requirements functions $h^j(\cdot)$ such that:

$$x_j \geq h^j\ (y_1,\ldots\ y_I) \qquad j\varepsilon J$$

$$(y,x)\varepsilon T\ .$$

Finally, we may define the following form of nonjointness:

Definition 4. The technology is said to be nonjoint in input prices if there exists individual quasi-concave, nonnegative, nondecreasing production functions $k^i(\cdot)$ such that:

$$y_i \leq k^i\ (x_1,\ldots,x_J) \qquad i\varepsilon I$$

$$(y,x)\varepsilon T\ .$$

The first case of nonjointness is best known to economists. Nonjointness in input quantities implies that each commodity is produced by a separate production function, with the supply of factors allocated among the different industries. Nonjointness in input quantities is frequently assumed in the literature. Actually, this assumption is probably more often invoked than not (the qualification "in input quantities" is usually omitted), and it plays an important role in many multi-sector models of international trade theory, public finance, international finance, macroeconomics, and growth theory. Figure 1a illustrates the case of nonjointness in input quantities in a 2 by 2 situation. The two unit value isoquants are drawn for the given commodity prices. Relative factor prices are then given by (minus) the slope of the tangent to the two curves, and the two tangency points indicate factor intensities in the respective industries. Given factor endowments x, the allocation of factors between the two industries can be determined. Production remains diversified as long as the factor endowment point lies within the cone embraced by the two factor intensity lines (Woodland, 1977).

The case of nonjointness in output quantities is familiar as well. It arises when an activity uses one input only, which is "cracked" into a number of commodities.[7] One might think of the oil industry, where crude oil is used to produce gasoline and kerosene, or the sheep industry where sheep are used to produce wool and meat as examples of technologies that are nonjoint in output quantities. The world technology in the neoclassical model of international trade can serve as another example of nonjointness in output quantities. Total supply of each commodity is the sum of the output of each production unit (e.g. each country). Figure 2a illustrates an occurrence of nonjointness in output quantities in the 2 by 2 case. The two unit cost production possibility frontiers are drawn for given factor prices. The slope of the tangent to the two curves indicates relative commodity prices. The two tangency points determine the corresponding relative output mixes, and the output of each production unit can be determined once that global output is given at y.

The case of nonjointness in output prices is less familiar to economists, yet it probably arises frequently in reality. It describes a situation where a line of commodities must go through a number of production stations. Each station is manned by a single factor with a transformation function $x_i = h^i(y)$. Since each commodity must pass through each station, total supply of each commodity is equal to the smallest quantity treated by any of the individual stations. Figure 3a illustrates a situation of nonjointness in output prices in the 2 by 2 case. The two production possibility frontiers are drawn for a given endowment of the two factors. For the given commodity prices, the supply of commodities is shown to be at y, which is at the intersection of the two production possibility frontiers.

The case of nonjointness in input prices seems more difficult to visualize, at least in the context of production theory. We mainly consider it for sake of completeness, and because it yields some interesting restrictions on the form of the joint cost function: since these restrictions are rather straightforward ones, it is useful to be aware of their meaning insofar as the technology is concerned. In addition, the concept of nonjointness in input prices is potentially a very useful one in household theory in the context of characteristic analysis.[8] We could thus consider our I outputs as being characteristics "produced" by the functions $k^i(\cdot)$ defined over all our J inputs. There is no reason then why any given quantity x_j should not enter all "production processes" at the same time. Figure 4a illustrates a situation of nonjointness in input prices in the 2 by 2 case. The isoquants corresponding to the two production functions are drawn for a given commodity output. For the given factor prices, factor input is shown to be x, which is at the intersection of the two isoquants.

The relevance of any of these four cases of nonjointness is essentially an empirical matter. None of the four cases examined above is likely to hold exactly at any level of aggregation, but it may be that one or the other form of nonjointness is a good approximation to reality, and thus may yield useful restrictions on the variable profit or the joint cost functions, or interesting comparative statics results. This point is further investigated in this and the following sections.

We may first observe that the various cases of nonjointness have interesting implications on the economy's production possibility set T and on the functions introduced in definitions 1 - 4. It is useful to define the input requirements set V(y) as:

$$V(y) \equiv \{x : (y,x) \varepsilon T\} .$$

V(y) is a subset of T; it is defined by the intersection of a convex set and a hyperplane and hence it is itself a convex set. In addition, it can easily be seen that if $x \varepsilon V(y)$, then $\lambda x \varepsilon V(\lambda y) (\lambda \varepsilon R^+)$. We may also define the producible output set P(x) as:

$$P(x) \equiv \{y : (y,x) \varepsilon T\} .$$

It is clear that $P(x)$ is also a convex set, and that for $y \in P(x)$, $\lambda y \in P(\lambda x)$.

Consider now the case of nonjointness in input quantities. We can define the i^{th} commodity's input requirements set $V_f^i(y_i)$ as follows:

$$V_f^i(y_i) \equiv \{x : f^i(x) \geq y_i, \; y_i \geq 0\} \qquad i \varepsilon I \; .$$

The quasi-concavity of $f^i(\cdot)$ ensures that $V_f^i(y^i)$ is convex. It is quite clear that under nonjointness in input quantities:

$$V(y) = \sum_{i \varepsilon I} V_f^i(y_i) \; .$$

In addition, it can easily be shown that if $x \varepsilon V_f^i(y_i)$, then $\lambda x \varepsilon V_f^i(\lambda y_i)$.[9] Hence, the individual production functions $f^i(\cdot)$ must be linearly homogeneous and concave.

Similarly, if production is nonjoint in output quantities, we define the j^{th} factor's producible output set as:

$$P_g^j(x_j) \equiv \{y : x_j \geq g^j(y)\} \qquad j \varepsilon J$$

and

$$P(x) = \sum_{j \varepsilon J} P_g^j(x_j) \; .$$

It can again be shown that $P_g^j(x_j)$ is convex and that if $y \varepsilon P_g^j(x_j)$, then $\lambda y \varepsilon P_g^j(\lambda x_j)$; hence the factor requirements functions $g^j(\cdot)$ must be linearly homogeneous and convex.

If production is nonjoint in output prices, the j^{th} factor's producible output set is defined as:

$$P_h^j(x_j) \equiv \{y : x_j \geq h^j(y)\} \qquad j \varepsilon J$$

and it is clear that:

$$P(x) = \bigcap_{j \varepsilon J} P_h^j \; .$$

In addition, it can easily be seen that the $P_h^j(\cdot)$'s are convex, and that if $y \in P_h^j(x_j)$, then $\lambda y \in P_h^j(\lambda x_j)$.[10] Hence the factor requirements functions $h^j(\cdot)$ must be linearly homogeneous and convex.

If production is nonjoint in input prices, finally, the i^{th} commodity's input requirements set is defined as:

$$V_k^i(y_i) \equiv \{x : y_i \leq k^i(x)\} \qquad i \epsilon I .$$

It follows that:

$$V(y) = \bigcap_{i \epsilon I} V_k^i(y_i) .$$

The linear homogeneity and the concavity of $k^i(\cdot)$ can easily be demonstrated.

We can now demonstrate the main results of this section.

Theorem 1 (Hall, 1973): A necessary and sufficient condition for nonjointness in input quantities is that the joint cost function can be written as:

$$C(y,w) = \Sigma\, y_i\, \phi^i(w)$$

where the $\phi^i(\cdot)$'s are nonnegative, nondecreasing, linearly homogeneous and concave.

Proof: The proof follows from repeated application of McFadden's (1978) rule 5 (p.51) and from our earlier result that the individual production functions $f^i(\cdot)$ are nondecreasing, linearly homogeneous and concave (Diewert, 1974a).

QED

The functions $\phi^i(\cdot)$ can be interpreted as unit cost functions. The cost minimizing demand for inputs can be obtained directly by differentiation, a result known as Shephard's lemma (Diewert, 1974a). The following results also follow from theorem 1.

Corollary 1.1: Nonjointness in input quantities implies that the sub-Hessian C_{yy} exists and is zero.

Corollary 1.2: Nonjointness in input quantities implies that the marginal cost of each output is independent of the output mix.

Given relative factor prices, the optimum input mix can be determined independently of the level of output. The competitive price of each commodity therefore only depends on factor prices. Factor requirements can be determined as soon as the level of output is indicated.

Theorem 2: A necessary and sufficient condition for nonjointness in output quantities is that the variable profit function can be written as:

$$\pi(p,x) = \Sigma\, x_j\, \gamma^j(p)$$

where the $\gamma^j(\cdot)$'s are nonnegative, nondecreasing, linearly homogeneous and convex.

Proof: The proof follows from repeated application of McFadden's (1978) rule 4 (p.99) and from the properties of revenue functions that correspond to nonnegative, nondecreasing, linearly homogeneous and convex factor requirements functions (Diewert, 1974b).

QED

Corollary 2.1: Nonjointness in output quantities implies that the sub-Hessian π_{xx} exists and is zero.

Corollary 2.2: Nonjointness in output quantities implies that the marginal revenue of each factor is independent of factor endowments.

Given output prices, the output mix of each activity and the revenue of each factor can be determined. Total output is obtained as soon as factor endowments are known.

Theorem 3. A necessary and sufficient condition for nonjointness in output prices is that the joint cost function can be written as:

$$C(y,w) = \Sigma\, w_j\, h^j(y)$$

where the $h^j(\cdot)$'s are nonnegative, nondecreasing, linearly homogeneous and convex.

Proof: The proof follows immediately from definition 3 and from our earlier results on the properties of the factor requirements functions $h^j(\cdot)$.

QED

Corollary 3.1: Nonjointness in output prices implies that the sub-Hessian C_{ww} exists and is zero.

It is quite clear from definition 3 that if the technology is nonjoint in output prices, factor requirements depend on the output mix only. They are independent of factor prices.

Theorem 4: A necessary and sufficient condition for nonjointness in input prices is that the variable profit function can be written as:

$$\pi(p,x) = \Sigma\, p_i\, k^i(x)$$

where the $k^i(\cdot)$'s are nonnegative, nondecreasing, linearly homogeneous and concave.

Proof: The proof follows immediately from definition 4 and from our earlier results on the properties of the production functions $k^i(\cdot)$.

QED

Corollary 4.1: Nonjointness in input prices implies that the sub-Hessian π_{pp} exists and is zero.

It is obvious from definition 4 that if the technology is nonjoint in input prices, the output mix depends on factor endowments only; it is independent of commodity prices.

Theorems 1 - 2 can also be interpreted at the light of figures 1b - 2b which are drawn for the 2 by 2 case and assuming neoclassical conditions. Figure 1b illustrates the case of nonjointness in input quantities. The isovalue curves corresponding to the given commodity prices are drawn for each industry. Equilibrium factor prices are shown at w, which is at the intersection of the two curves. The factor intensity of each industry is given by the gradient of the two curves at w (Woodland, 1977). Factor utilization in the two industries adds up to total factor endowments x.

Figure 2b shows the two isorevenue curves corresponding to the two factors and to the given factor prices for the case of nonjointness in output quantities. The output mix produced by each factor is given by the gradients of the two curves at p, and the supply of each production unit adds up to total supply y.

4 Nonjoint Technologies: A Dual Approach

In this section, we use a dual representation of the technology. This provides further insight into the concept of nonjointness.

We now define the price possibility set W as the set of all feasible factor price and commodity price combinations:

$$W \equiv \{(p,w): w'x \geq p'y, \ (y,x)\varepsilon T\}$$

It can be shown that the variable profit function is equivalent to:[11]

$$\pi(p,x) = \min_{w} \{w'x: \ (p,w)\varepsilon W, \ w >> 0\} \text{ for } p >> 0, \ x \geq 0 \ .$$

Similarly, the joint cost function is equivalent to:

$$C(y,w) = \max_{p} \{p'y = (p,w)\varepsilon W, \ p >> 0\} \text{ for } w >> 0, \ y \geq 0 \ .$$

We can now demonstrate the following theorems which apply to the two cases of nonjointness in prices.

Theorem 5: The technology is nonjoint in output prices if and only if there exists individual nonnegative, nondecreasing, linearly homogeneous, convex factor revenue functions $\Theta^j(\cdot)$ such that:

$$w_j = \Theta^j(p_{1j},\ldots,p_{Ij}) \qquad j\varepsilon J$$

$$p_i = \sum_j p_{ij} \qquad i\varepsilon I$$

$$(p,w)\varepsilon W \ .$$

Proof: Following the same steps as the ones leading to the demonstration of theorem 1, it can easily be shown that the above conditions are necessary and sufficient for the joint cost function to be written as:

$$C(y,w) = \Sigma\, w_j\, h^j(y)\ .$$

Application of theorem 3 then completes the proof.

QED

This theorem indicates that if production is nonjoint in output prices, the price of each commodity can be written as the sum of the value added at each phase of production, i.e. at each production station.

<u>Theorem 6</u>: The technology is nonjoint in input prices if and only if there exists individual nonnegative, nondecreasing, linearly homogeneous, concave cost functions $\kappa^i(\cdot)$ such that:

$$p_i = \kappa^i(w_{1i}, \ldots w_{Ji}) \qquad i\varepsilon I$$

$$w_j = \sum_i w_{ji} \qquad j\varepsilon J$$

$$(p,w)\varepsilon W\ .$$

<u>Proof</u>: Following the same steps as the one leading to the demonstration of theorem 2, it can easily be shown that the above conditions are necessary and sufficient for the variable profit function to be written as:

$$\pi(p,x) = \Sigma\, p_i\, k^i(x)\ .$$

Application of theorem 4 completes the proof.

QED

Nonjointness in input prices means that a given input is involved in several activities. The price of each input can then be written as the sum of the value added in the various activities.

Theorems 5 and 6 should also shed some light on the terminology that we are using in this paper. In view of definitions 1 and 2, the duality that exists between nonjointness in quantities and nonjointness in prices is complete.

The case of nonjointness in prices, and the meaning of theorems 5 and 6 can be shown graphically with the help of figures 3 and 4 for the 2 by 2 case, and assuming again that neoclassical conditions prevail. Assuming nonjointness in output prices, the two unit-cost isorevenue lines are drawn in figure 3b for given factor endowment (x_1, x_2). The slope of the tangent to the two curves indicates the relative supply of the two commodities, and the rays through the tangency points show how commodity prices are allocated between the two factors. The decomposition of the commodity prices is also shown in figure 3a.

Figure 4b represents a case of nonjointness in input prices. The two unit-value isocost curves are drawn for the given level of output (y_1, y_2). The slope of the tangent to the two curves gives relative factor requirements, and the rays through the tangency points indicate how factor prices are imputed to the cost of the two commodities. Factor revenues drawn from the different activities are also shown in figure 4a.

5 Same Number of Factors and Commodities

In this section, we assume that the number of factors equals the number of commodities. The case I = J has often been favoured. As noted by Samuelson (1953-4), it is an interesting special case which leads to a number of remarkable results, but it should be viewed with some caution, since many of these results do not generalize. The main result in this section is theorem 7 (and its counterpart, theorem 8), which is the source of some well-known theorems of international trade theory.

Theorem 7: If the number of factors equals the number of commodities, if all commodities are produced and if the joint cost function is once continuously differentiable, nonjointness in input quantities implies that the variable profit function can be written as:

$$\pi(p,x) = \sum_j x_j \, w^j(p) \; .$$

Similarly, if the number of factors equals the number of commodities, if all factors are fully employed and if the variable profit function is once continuously differentiable, nonjointness in output quantities implies that the joint cost function can be written as:

$$C(y,w) = \Sigma\, y_i\, p^i(w) \ .$$

Proof: By theorem 1, under nonjointness in input quantities the joint cost function can be written as:

$$p_i = \phi^i(w) \qquad\qquad i\varepsilon I$$

where the equalities follow from the assumption that all commodities are produced. Each one of the $\phi^i(\cdot)$'s is once continuously differentiable, and we can safely assume that the Jacobian of this system is nonsingular (otherwise some of the individual production functions would be identical up to a factor of proportionality, and some commodities could be aggregated violating the assumption I = J): hence, from the implicit function theorem, the factor prices can be solved as functions of the commodity prices:

$$w_j = w^j(p) \qquad\qquad j\varepsilon J \ .$$

The second part of the theorem can be proved in the same way.

QED

Corollary 7.1: If the number of factors equals the number of commodities, and if all commodities are produced, nonjointness in input quantities implies that the sub-Hessian π_{xx} exists and is zero. If the number of factors and commodities is the same, and if all factors are fully employed, nonjointness in output quantities implies that the sub-Hessian C_{yy} exists and is zero.

The first part of theorem 7 contains as a result the well-known factor price equalization theorem (Samuelson, 1953-4). A word of caution might be in order: it does not follow from theorems 1, 2 and 7 that nonjointness in input quantities implies nonjointness in output quantities and vice-versa when I = J. If the technology is nonjoint in input quantities, for instance, and if the other assumptions underlying theorem 7 are fulfilled, the $w^j(\cdot)$'s will in general not satisfy the properties of the $\gamma^j(\cdot)$'s as set in theorem 2. In particular, it is well-known that (for $I = J > 1$) the $w^j(\cdot)$'s will not be nondecreasing in all the components of p (Stolper and Samuelson, 1941).

It is rather remarkable that the famous factor price equalization result only holds in the special case described in the assumptions to theorem 7 when production is nonjoint in input quantities, but always holds if production is nonjoint in output quantities.

Corollary 7.1 follows directly from theorem 7, but it is instructive to prove it using an alternative way. Consider the case of nonjointness in input quantities, and assume that $H(C)$ and $H(\pi)$ both exist. As we know, $H(\pi) = [H(C)]^{-1}$, and $C_{yy} = 0$. Hence:

$$H(\pi) = \begin{bmatrix} 0 & C_{yw} \\ C_{wy} & C_{ww} \end{bmatrix}^{-1} = \begin{bmatrix} -C_{wy}^{-1} C_{ww} C_{yw}^{-1} & C_{wy}^{-1} \\ C_{yw}^{-1} & 0 \end{bmatrix}$$

since C_{yw} (and its transpose, C_{wy}) is assumed square and nonsingular. This shows clearly that π_{xx} is zero.[12]

Theorem 7 is useful for empirical applications if one chooses to use a variable profit function (joint cost function) to represent the technology, and if one wants to impose or test for nonjointness in input (output) quantities. A similar theorem holds for the case of nonjointness in prices.

<u>Theorem 8</u>: If the number of factors equals the number of commodities, if all factors are fully active, and if the joint cost function is once continuously differentiable, nonjointness in output prices implies that the variable profit function can be written as:

$$\pi(p,x) = \Sigma\, p_i\, y^i(x)\ .$$

Similarly, if the number of factors is equal to the number of commodities, if all commodities are produced to capacity, and if the variable profit function is once continuously differentiable, nonjointness in input prices implies that the joint cost function can be written as:

$$C(y,w) = \Sigma\, w_j\, x^j(y)\ .$$

<u>Proof</u>: The proof is analogous to the proof of theorem 7.

QED

Corollary 8.1: If the number of factors equals the number of commodities, and if all factors are fully active, nonjointness in output prices implies that the sub-Hessian π_{pp} exists and is zero. Similarly, if the number of factors equals the number of commodities, and if all commodities are produced to capacity, nonjointness in input prices implies that the sub-Hessian C_{ww} exists and is zero.

6 Nonjointness in Inputs and Outputs

In this section, we consider cases where the technology may be nonjoint in more than one way at a time. Our main results are contained in two theorems.

Theorem 9: If the technology exhibits constant input-output coefficients, nonjointness in input quantities implies nonjointness in output prices, and vice-versa; the joint cost function can then be written as:

$$C(y,w) = \Sigma\Sigma\, a_{ij}\, y_i\, w_j \qquad\qquad a_{ij} \geq 0 \ .$$

Proof: Assume that the technology is nonjoint in input quantities and that the individual production functions are of the fixed coefficient type. By solving the first-order conditions of the cost minimizing problem, one can show that the joint cost function can be written as $C(y,w) = \Sigma\Sigma\, a_{ij}\, y_i\, w_j$ where the a_{ij}'s are the actual input-output coefficients and are positive. It follows from theorem 3 that the technology is nonjoint in output prices. Similarly, assume that production is nonjoint in output prices and that the factor requirement functions are of the fixed coefficient type. Solving the first-order conditions of the cost minimization problem shows again that the joint cost function can be written as above. It follows, by theorem 1, that production is nonjoint in input quantities as well.

QED

What is usually called a Leontief technology is thus nonjoint in both input quantities and in output prices.[13] In such a system the marginal cost of output depends on input prices only, and factor requirements depend exclusively on the output mix. It also follows that the sub-Hessians C_{yy} and C_{ww} are zero.

Theorem 10: If the technology exhibits constant input-output coefficients, nonjointness in output quantities implies nonjointness in input prices, and vice-versa; the variable profit function can then be written as:

$$\pi(p,x) = \Sigma\Sigma\, b_{ij}\, p_i\, x_j \qquad\qquad b_{ij} > 0 .$$

Proof: The proof follows along the same lines as the proof of theorem 9.

QED

Imagine a technology that is nonjoint in output quantities, and where each factor produces the various commodities in fixed proportions. The neoclassical model of international trade might qualify as an example of such a technology in the short run if all factors are immobile between firms. Both the domestic and the foreign production possibility frontiers have then the shape of an inverted L. Theorem 10 indicates that such a technology is also nonjoint in input prices; this result gives additional credit to the concept of nonjointness in input prices. It follows that in such a model world output depends on factor endowments only, and factor revenues solely depend on commodity prices.[14] It can easily be seen also, that for such a technology the sub-Hessians π_{pp} and π_{xx} are both zero.

We conclude this section by commenting on the case where the number of factors equals the number of commodities. Assume that the technology is nonjoint in input quantities and in output prices (Leontief technology), and that $H(C)$ and $H(\pi)$ both exist. We know from earlier results that $H(\pi) = [H(C)]^{-1}$ and that $C(y,w) = \Sigma\Sigma\, a_{ij}\, y_i\, w_j$. It follows that $\pi(p,x) = \Sigma\Sigma\, \alpha_{ij}\, p_i\, x_j$; where $[\alpha_{ij}] = [a_{ij}]^{-1}$. It is not true, however, that this implies that the technology is also nonjoint in output quantities or in input prices as the matrix $[\alpha_{ij}]$ will in general not be nonnegative. The only case where the technology may be nonjoint according to all four definitions is where $[a_{ij}]$ is a diagonal matrix. It can be shown in the same way that when $I = J$, and if all Hessians exist, nonjointness in output quantities and in input prices implies that $C(y,w) = \Sigma\Sigma\, \beta_{ij}\, y_i\, w_j$, where $[\beta_{ij}] = [b_{ij}]^{-1}$. It does not follow, however, that the technology is nonjoint in input quantities and in output prices as well.

7 Unequal Number of Goods and Factors

It is well known (e.g. Samuelson, 1953-4) that under nonjointness in input quantities the passage from factor prices to commodity prices is straightforward. Indeed, corollary 1.2 above indicates that p is a function of w only. Factor requirements, however, depend on both factor prices and on the level of output (C_{ww} and C_{wy} are nonzero in general).

The passage from commodity prices and factor endowment to factor prices and output quantities is somewhat more delicate when $I \neq J$, yet it is of crucial importance, for instance in international trade theory where p and x are usually given. Assume first that $I < J$, i.e. more factors than commodities. It can easily be seen that the supply of commodities and factor revenues will generally depend on both commodities prices as well as on factor endowments. It is instructive to look at the Hessian of C and at its inverse if both matrice exist:

$$H(\pi) = \begin{bmatrix} \pi_{pp} & \pi_{px} \\ \pi_{xp} & \pi_{xx} \end{bmatrix} = \begin{bmatrix} 0 & C_{yw} \\ C_{wy} & C_{ww} \end{bmatrix}^{-1} .$$

π_{xx} is of dimension $J \times J$, and hence it cannot be zero, for otherwise $H(\pi)$ would be singular (as $J > I$) and $H(C)$ would not exist. The result of theorem 7 clearly does not hold in this case: the factor price equalization theorem breaks down.

Consider next the case of $I > J$, i.e. more commodities than factors. It is well known that in such a situation, for arbitrary commodity prices, some of the commodities may not be produced. Let n be the number of commodities produced. Three possibilities may be considered. Firstly, it may be that $n < J$, the number of commodities produced is less than the number of factors (if the factor endowment point lies outside all diversification cones): this case is similar, for all practical purposes, to the case $I < J$ examined above. Secondly, it is possible that $n = J$ (this is probably the most likely case), a situation similar to the one examined in section 5. Finally, it may be that commodity prices and factor endowment are such that $n > J$. In that case, the supply of commodities is undeterminate. Although the value of $\pi(\cdot)$ is defined for given p and x, the variable profit function is not differentiable with respect to the component of p, and the Hessian $H(\pi)$ does not exist. This is not surprising since $H(C)$ is singular as C_{yy}, which is of dimension $n \times n$, $n > J$, is zero.

The other cases of nonjointness can be examined in a similar way. Generally speaking the passage from commodity prices and factor endowments to commodity supply and factor revenue is trivial if the technology is nonjoint in output quantities, or nonjoint in input prices, or both. Corollaries 2.1 and 4.1 provide some useful results in this respect. The passage from commodity output and factor prices to factor requirements and commodity prices is less straightforward. Under nonjointness in output quantities, it is important to distinguish the cases where the number of employed factors is less than, equal to, or larger than the number of commodities; in the latter case factor requirements is indeterminate. If production is nonjoint in input prices, one must distinguish the cases where the number of commodities produced at full capacity is less than, equal to, or larger than the number of factors; in the latter case commodity prices are indeterminate.

If production is nonjoint in output prices, finally, the passage from factor prices and commodity output to commodity prices and factor requirements is straightforward; corollary 3.1 provides some useful results in this respect. The reverse exercise is more delicate, and here one must distinguish the cases where the number of fully employed factors is less than, equal to, or larger than the number of commodities produced; in the latter case factor prices remain indeterminate.

8 Comparative Statics Results

A number of comparative statics results can readily be obtained in view of the properties of π and C, and particularly in view of the positive semi-definiteness of π_{pp} and C_{yy} and of the negative semi-definiteness of π_{xx} and C_{ww}. In this section, however, we limit our attention to the popular 2 x 2 case (I = J = 2), and we show how some interesting results may be derived graphically. Throughout this section, we assume monotonically increasing and continuously differentiable production, factor requirements, cost and revenue functions, and we assume interior solutions.

Woodland (1977) has shown how figures 1a and 1b can be used to give a simple proof of some fundamental theorems of international trade theory. It is apparent from figure 1a that an increase in the endowment of one factor (e.g. an increase in x_1 leading to a rightward shift of x) leads to an absolute increase in the supply of the commodity intensive in this factor (commodity 1 here) and to an absolute decrease in the supply of the other commodity (the Samuelson-Rybczynski theorem). Figure 1b can be used to demonstrate the Stolper-Samuelson theorem: an increase in a commodity price (say p_1, leading to a downward shift in the corresponding isocost curve) leads to an absolute increase in the price of the factor used intensively in the production of that commodity (factor 1 here) and to an absolute reduction in the price of the other factor.

It is clear that in the case of nonjointness in output quantities, similar theorems hold. We first need the following definition.

Definition 5: Let $x_j = g^j\ (y_{1j},\ldots,y_{Ij})$ a factor requirements function. When $I = 2$, the commodity intensity of factor j at a point (y_{1j}^*, y_{2j}^*) is defined as the ratio y_{2j}^*/y_{1j}^*. When $J = 2$ as well, factor 1(2) is said relatively commodity 1 intensive if $y_{21}^*/y_{11}^* < (>)\ y_{22}^*/y_{12}^*$.

Theorem 11: In the 2 by 2 case, and if production is nonjoint in output quantities, an increase in the output of one commodity leads, for given factor prices, to an absolute increase in the requirements of the factor relatively intensive in that commodity and to an absolute reduction in the requirement of the other factor.

Proof: The proof follows immediately from figure 2a. An increase in y_1, for instance, displaces y to the right. Since commodity prices do not change (theorem 7), commodity intensities remain the same, and hence the demand for x_1 (which is relatively intensive in commodity 1) increases while the demand for x_2 falls. The other possible cases can be examined in the same way.

QED

Theorem 12: In the 2 by 2 case, and if production is nonjoint in output quantities, an increase in the price of one factor leads, at unchanged output, to an absolute increase in the price of the commodity produced relatively intensively by that factor, and to an absolute reduction in the price of the other commodity.

Proof: The proof follows from figure 2b. An increase in w_1 for instance shifts the corresponding iso-revenue curve outwards, thus leading to an increase in the price of commodity 1 (which is produced intensively by factor 1 here) and to a reduction in p_2. All other cases can be treated in the same way.

QED

If factor endowments and commodity prices are viewed as the exogenous variables (rather than the reverse), it becomes more difficult to obtain comparative statics results when production is nonjoint in output quantities. Yet, from theorem 2 and the properties of the revenue functions $\gamma^j(\cdot)$, $j = 1,2$, we can assert that an increase in a commodity price leads to an increase in both factor prices, that an increase in the endowment of a factor leads to an increase in both factor prices, and that an increase in the endowment of a factor leads to an increase in the supply of both commodities; as indicated by corollary 2.2, the change in factor endowments has no effect on factor prices.

Some interesting results for changes in factor endowments or commodity prices can be obtained if production is nonjoint in output prices. A rather remarkable result is that adapted versions of both the Stolper-Samuelson and the Samuelson-Rybczynski theorem hold in this case.

<u>Theorem 13</u>: In the 2 by 2 case, and if production is nonjoint in output prices, an increase in the price of a commodity leads, at unchanged factor endowments, to an absolute increase in the revenue of the factor that produces that commodity relatively intensively and to an absolute decrease in the revenue of the other factor.

<u>Proof</u>: We give a graphical proof based on figure 3b. An increase in p_1 for instance shifts p to the right, and since y remains unchanged (this is implied by theorem 8), p_{11} and p_{21} both increase, while p_{12} and p_{22} both fall. From the monotonicity of the revenue functions $\theta^j(\cdot)$, $j = 1, 2$, it follows that w_1 must increase and that w_2 must fall. All other cases can be treated in the same way.

<u>Theorem 14</u>: In the 2 by 2 case, and if production is nonjoint in output prices, an increase in the endowment of one factor leads, at unchanged commodity prices, to an absolute increase in the supply of the commodity that is produced relatively intensively by that factor and to an absolute

reduction in the supply of the other commodity.

Proof: Consider figure 3a. An increase in x_1 for instance shifts the corresponding production possibility frontier outwards, leading to an increase in the supply of the first commodity (which is produced relatively intensively by factor 1) and to a decrease in the supply of commodity 2. All other cases can be treated in an analogous manner.

QED

If the technology is nonjoint in input quantities and in output prices, the Stolper-Samuelson and the Samuelson-Rybczynski theorems hold on two counts. It can easily be verified in the 2 by 2 case that if the technology is of the Leontief type, and if commodity i is relatively factor j intensive, then factor j is relatively commodity i intensive: hence the Stolper-Samuelson theorem and theorem 13 on one hand, and the Samuelson-Rybczynski theorem and theorem 14 on the other hand give consistent predictions.

It can easily be demonstrated that Jones's (1965) magnification effect also holds if the technology is nonjoint in output prices. Assume that factor 1 is relatively commodity 1 intensive, and that $\hat{x}_2 > \hat{x}_1$ (the hats indicate proportional changes); it is then true that:

$$\hat{y}_2 > \hat{x}_2 > \hat{x}_1 > \hat{y}_1 .$$

Similarly, if $\hat{p}_2 > \hat{p}_1$:

$$\hat{w}_2 > \hat{p}_2 > \hat{p}_1 > \hat{w}_1 .$$

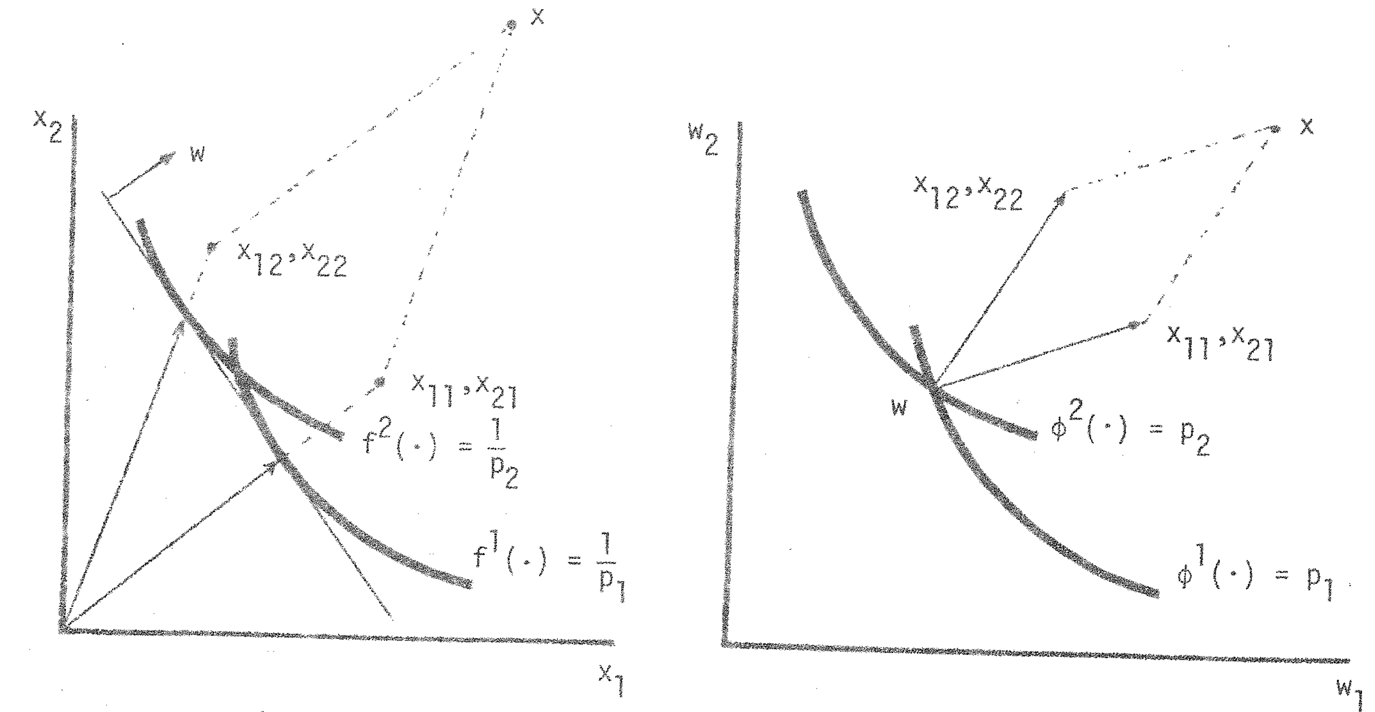
The counterpart of theorems 13 and 14 can easily be demonstrated for the case of nonjointness in input prices (for exogenous changes in commodity output or factor prices). The graphical proof is analogous to the proofs of theorems 11 - 14.

9 Concluding Comments

We have examined in this paper some general technologies which can adequately be described by either a variable profit function or a joint cost function. We have defined four cases of nonjointness, and we have obtained a number of results which are useful to characterize the description of the corresponding technologies. A number of comparative statics results have been obtained as well.

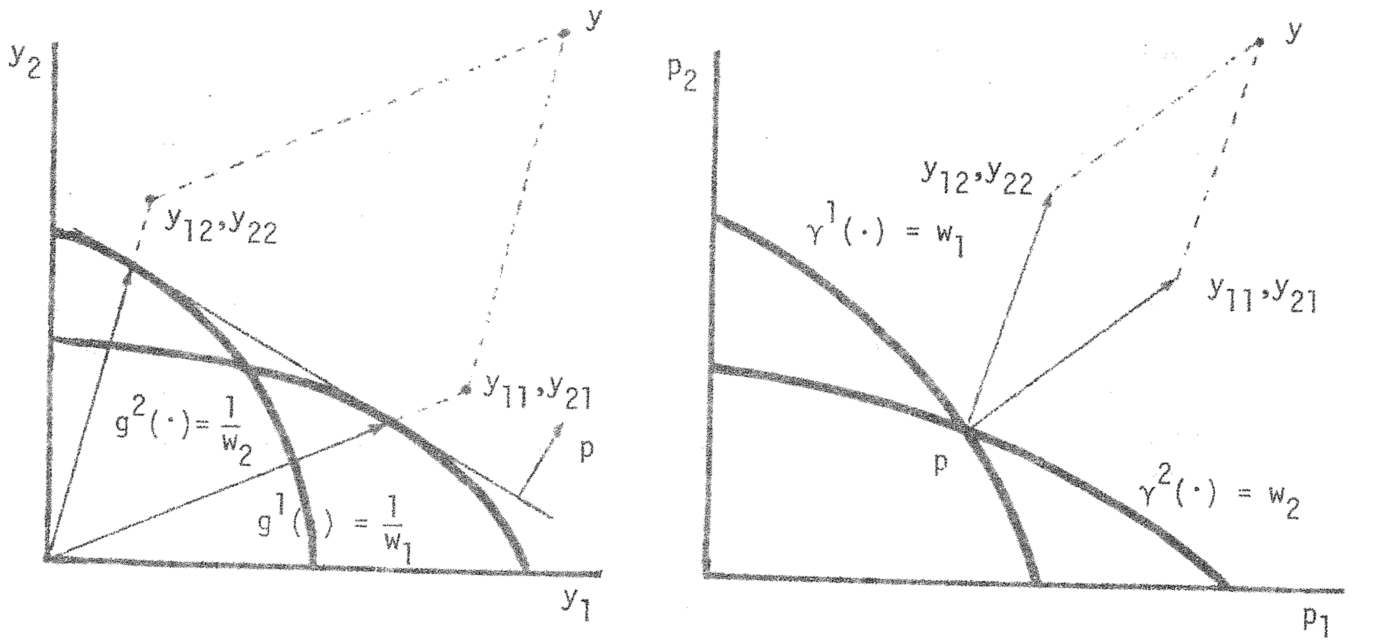
Although the case of nonjointness in input quantities is by far the most common in the literature, the other cases of nonjointness that we have defined are also of interest, both at the level of the firm and at the aggregate level. One form of nonjointness at the firm level does not exclude another type of nonjointness at the aggregate level. Consider the neoclassical model of international trade, for instance, where, for fixed factor endowments, each country has its own production possibility frontiers; although nonjointness in input quantities is assumed at the level of the firm, the world technology is nonjoint in output quantities; if, in addition, factors are immobile between firms, the world technology is nonjoint in input price as well.

We have shown that the various forms of nonjointness lead to a number of restrictions on the corresponding variable profit or joint cost functions. These restrictions are empirically testable, hence their validity can be empirically evaluated. In spite of the popularity of the assumption of nonjointness in input quantities in economic theory, there has been little work done to test its empirical validity.
Recent exceptions include Burgess (1976), who tests and rejects (at the 95% confidence level) this hypothesis in the case of the United States; Kohli (1979), using a somewhat different disaggregation of output, finds, however, that the assumption that the U.S. technology is nonjoint in input quantities cannot be rejected, although the assumptions of nonjointness in output prices, as well as of nonjointness in input quantities and in output prices, are not supported by the data. To our knowledge, there has been no empirical investigation of the other forms of nonjointness. The relevance of nonjointness in production can only be determined by further empirical work in this area.



a) b)

Figure 1
Nonjointness in input quantities



a) b)

Figure 2
Nonjointness in output quantities

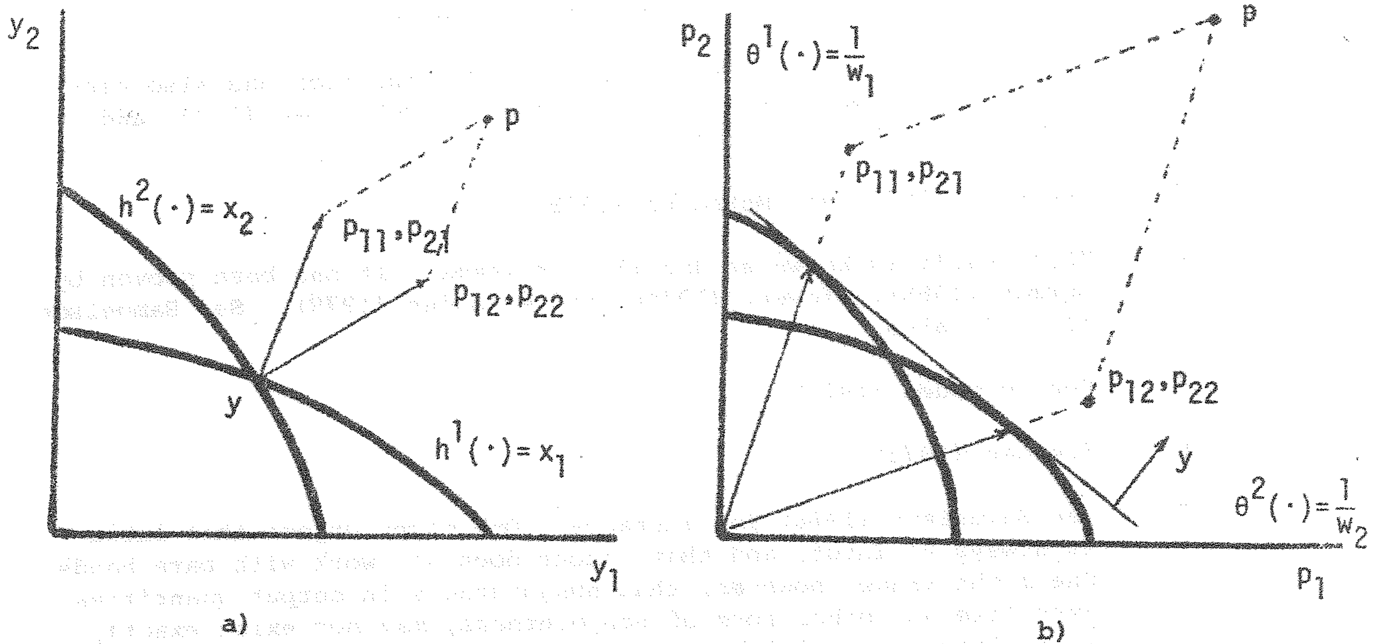


Figure 3

Nonjointness in output prices

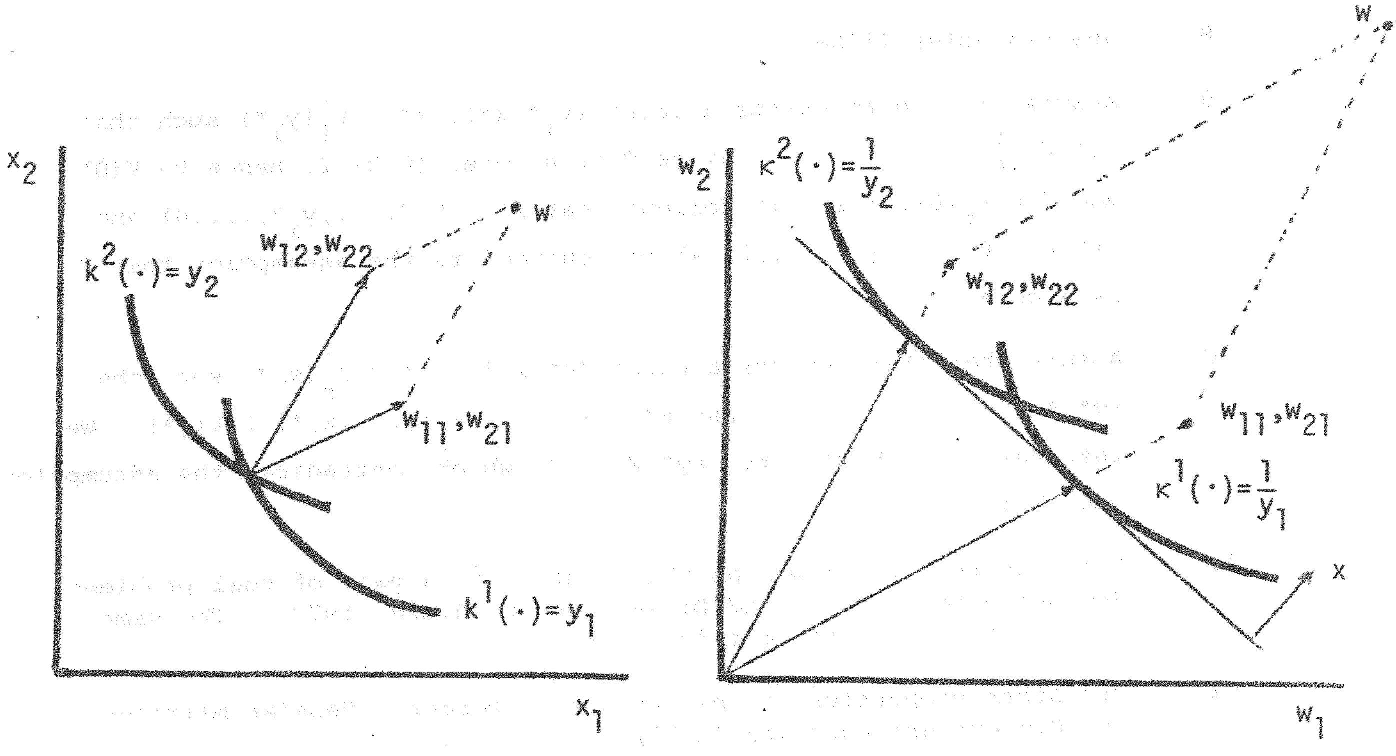


Figure 4

Nonjointness in input prices

FOOTNOTES

* I wish to thank, without implicating, R.T. Ross for a number of useful comments on an earlier version of this paper.

1 See Diewert and Woodland (1977) for instance.

2 See Diewert (1974a). The variable profit function has also been discussed by Samuelson (1953-4), Gorman (1968), Lau (1976) and McFadden (1978) among others.

3 See Hall (1973) and McFadden (1978).

4 This result is known as Hotelling's lemma. It has been proven by Gorman (1968), Diewert (1974a) and McFadden (1978). See Samuelson (1953-4) also.

5 See McFadden (1978).

6 See Lau (1972).

7 See Samuelson (1966) for instance. One might object that labour is always an input, and that labour does not work with bare hands. One might argue, however, that nonjointness in output quantities, just like any other form of nonjointness, may not exist exactly in reality, but still be a good approximation. In addition, any form of nonjointness considered in this paper may well hold at the aggregate level even if it does not hold at the firm level. Finally, the single factor considered here may well be a composite factor: if the technologies of the firms are separable between inputs and outputs, for instance, the supply side of the technology can be considered as nonjoint in output quantities.

8 See Lancaster (1966).

9 Assume that there exists a point (y_i^*, x^*), $x^* \in V_f^i(y_i^*)$ such that $\lambda x^* \notin V_f^i(\lambda y_i^*)$, $\lambda \geq 0$. Since T is a cone, $(0,0) \in T$, hence $0 \in V(0)$ and $0 \in V_f^i(0)$, $i \in I$. It follows that $x^* \in V(0,\ldots,y_i^*,\ldots,0)$ and $\lambda x^* \notin V(0,\ldots,\lambda y_i^*,\ldots,0)$ which contradicts the assumption that T is a cone.

10 Assume that there exists a point (y^*, x_j^*), $y^* \in P_h^j(x_j^*)$ such that $\lambda y^* \notin P_h^j(\lambda x_j^*)$, $\lambda \geq 0$. Let $x^* = x_1^*,\ldots,x_j^*,\ldots,x_J^*) \in V(y^*)$. We then have $y^* \in P(x^*)$, but $\lambda y^* \notin P(x^*)$ which contradicts the assumptions that T is a cone.

11 $\pi(\cdot)$ can thus be viewed as the solution to a pair of dual problems. See Samuelson (1958), and Diewert and Woodland (1977). The same applies to the joint cost function.

12 For other properties of the inverse of bordered Hessian matrices, see Diewert and Woodland (1977).

13 Thus the Leontief technology is another (familiar) example of a technology that is nonjoint in output prices.

14 In this example the two factors are composite factors obtained by aggregating domestic and rest-of-the-world resources.

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