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Trading Floors as Separating Devices

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ABSTRACT

Traders with specific characteristics operating in a pairwise exchange market may prefer to meet other traders with similar (or complementary) characteristics, while other categories of traders may not have such preferences. The existence of the second type imposes a negative externality on the first. Under conditions which are not very restrictive, establishing a trading floor designated for the former type induces the two types of traders to separate themselves - with one type trading on the floor and the other trading on the 'street'. Separation may require assessing a fee for entry. The consequences for efficiency are mixed.

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CONTENTS

1 Introduction

It is by now well recognised that the walrasian construct of the auctioneer hides a serious omission in the study of the market process, that of *market-making* and intermediation. While market microstructure has been studied extensively in the finance literature, its contours and consequences in markets for goods and services has received relatively less attention until recently. The only survey of microstructure in non-financial markets that I am aware of dates as recently as 1996 (Spulber 1996).

The literature identifies several significant reasons why market-makers exist. The most prominent of these is to provide liquidity and immediacy in a market where potential agents may arrive at disparate times. Intermediaries also match traders who may be searching for compatible trading partners, and may avert market failure stemming from information problems. One may of course add to these the traditional reasons for the existence of retail trade, such as economies of scale arising from nonconvexities in storage costs.

Starting with Demsetz (1968), the literature has devoted some attention to rationale and consequences of the actions of market-makers within an exchange. Significantly less interest has been evoked by the fact that exchanges exist at all. One may intuitively dismiss the question as less than interesting because the exchange itself is so obviously a coordinating device, offering a *focal point in space and time for traders to assemble. If this is* so, however, then it remains to be explained why trades are also executed *outside the ambit of the exchange as well*, i.e. on the "street".

In this paper, I consider a market for a homogeneous good, where traders differ in the sizes of trades they wish to execute. Time is valuable, so that agents who wish to execute large trades would prefer to meet other similar sized agents and conclude trading swiftly. However, they do not know each other's characteristics until after they meet. Agents spend time searching for each other. Thus if a large trader finds that she has been paired with a small one, then she has no reason to decline a small trade, since this can only reduce the remaining time she expects to spend in the market.

The existence of small traders speed up trade, since it increases the total mass of agents in the market. Indeed, if the presence of small traders had no impact on the arrival rate of large traders, then it would be welcomed by the large agents. Since there is a probability that two small traders in sequence may arrive before the first large trader does, to that extent the presence of small traders would reduce the expected waiting time.

However, the existence of small traders also imposes a negative externality on the big agents. This is because the big agent who meets a small agent first can trade only part of her endowment, and thus herself becomes a small agent. For the remaining big agents, this reduces the rate at which large partners arrive, and makes it more likely that they will need to make several trades. If this negative effect outweighs the positive gain from speeding up trade, then the large agents will gain by separating and trading exclusively among themselves.

This separating function of formal markets has not been much remarked upon in the literature. In markets where agents are heterogeneous and must search for partners, certain types of agents may prefer not to be matched with certain other types, whereas this dispreference is not reciprocated. An example is a gambling resort populated by both vacationers and professional

card-sharps. Resorts often solve this problem by placing betting limits on tables. Low limits make tables unattractive to the professional, thus shielding the vacationers from unanticipated ruin.

Bose and Pingle (1995) show that, in markets populated by weak and strong bargainers, separation may be effected by a store which trades at fixed bid and ask prices, profiting from the difference. Weak bargainers go to the store to avoid meeting strong bargainers one-on-one, while the latter trade bilaterally without going to the store. In Bose (1996), patient and impatient traders flow through a matching and bargaining market where patient agents obtain an advantage if they are matched with impatient ones. A trading floor which charges an entry fee can be constructed such that in equilibrium all impatient agents trade on the floor (having paid the entry fee), while all patient agents trade outside.

Market segmentation by size or other characteristics is a fairly widespread phenomenon. A major example is the separation of wholesale and retail trade, which occur in simultaneous but essentially non-overlapping circles. Price "clubs" and bulk retailers screen customers by making it known that they will only sell in large consignments and often by charging a membership fee. Finally, the necessity of membership in a stock exchange restricts casual buyers and sellers, who instead conduct their trades on an over-the-counter market, or through a dealer.

Two characteristics of this separation are significant, and are shared by the model in this paper. First, the market-maker who implements the mechanism to separate the types does not have any information not possessed by the other agents, nor does he have a technological advantage in executing trades. Secondly, separation is a consequence of a specific price being charged by the market-maker, in particular the types cannot be separated if the price charged is too low. Thus the profit from this kind of intermediation is not subject to competitive erosion.

* * *

In what follows, we consider two variations of a simple model inspired by Diamond (1982). In this paper, we ignore production and concentrate on the exchange aspect of the model. Agents arrive in the market with some amount of a homogeneous good to trade. The need for exchange is driven by a taboo that an individual agent cannot consume the unit he has himself produced. Without unnecessary complicating the model, this captures the fact that in modern economies one typically consumes very little of one's own output, and the need for exchange is paramount.

Some of the entering agents have a large endowment of the good (two units), while others have a small endowment (one unit). In section 2 we assume that agents arrive continuously at some exogenous rate, with the result that at any given time there is a mass of agents in the market searching for trading partners. The rate at which agents meet each other is determined by the size of this mass. For a given agent, the probabilities of meeting large or small agents are determined by the relative frequencies of the two types. Some large agents inevitably meet small agents and trade one of their two units. They then remain in the market as small agents, and thus inflate the steady-state proportion of small agents in the market. If this effect is strong enough the large agents would benefit from organizing a separate floor, which charges a price for entry. It is shown that there is a range for price (bounded in both directions) such that large traders will prefer to trade on the floor, while small traders will opt to remain outside.

In section 3 I consider the case where traders arrive one at a time at some exogenously given rate. Each arriving trader is either large or small with some given probability. When a trader arrives, there is either another trader already waiting, in which case they trade, or the market is empty, in which case the trader waits for a partner to arrive. The extent of the externality imposed by small traders is much stronger in this case.

In what follows, "market" refers to the context where both large and small traders trade together, "floor" refers to a context where only large traders trade in isolation from small traders, and "street" refers to a context where only small traders trade among themselves.

2 Simultaneous Entry

Agents (also called 'traders') arrive in the market at an exogenous rate, endowed with either one unit or two units of a homogeneous good. Depending on his/her endowment, s/he is designated a "big" or a "small" agent. An entering agent is big with probability π, and small with probability $1-\pi$.

Apart from the good, each agent also has an endowment of money, which may be thought of as a proxy for other commodities. Money payments will occasionally be made to acquire the right to trade in a specific environment, or to coax an unwilling partner to conduct a trade.

In the marketplace, the trader encounters other agents at a rate $\lambda(.)$, determined by the number of agents who are currently on the market. He exchanges his endowment against that of others, as described below, and consumes once his entire endowment has been exchanged. If he consumes x units, he gets an instantaneous utility of $x+p$, where p is any net addition to his money holding that may occur in course of the trade. Agents discounts the future at an instantaneous rate r. Since all agents possess the same homogeneous good, we assume that the nominal price in exchanges is unity. Effecting the exchange is in itself costless.

When two agents of the same endowment type meet each other, they can immediately exchange their entire endowment. However, when a large agent meets a small agent, she can at most trade one of the two units in her endowment, and must then wait for another trading opportunity. In the process she is transformed into a small agent. Since waiting is costly, a large agent would rather meet another large agent than a small agent. However, if she does meet a small agent, she would under some circumstances prefer to exchange part of her endowment with him, for then she is assured of completing trade the next time she meets another agent.

The equilibrium concept used is that of a steady-state, in which the numbers of each type of agent in the market remains constant. I first establish the expected utility of each type of trader entering the steady-state market. Then I presume that the large and small traders trade separately (on the 'floor' and in the 'street', respectively), and find the expected utility of an entering trader of each type. It is found that, under a broad range of parameter values, the large traders gain from this separation, while the small traders lose.

2.1 The Unseparated Market

At any given time, there are a number of agents searching for partners in the market. For each agent, trading partners arrive according to a Poisson process with an arrival rate dependent on the market population. For a population M of agents, we denote this arrival rate by $\lambda(M)$. $\lambda(.)$ must be bounded above by unity, and we assume that it is weakly increasing and weakly concave in M.

The meeting rate function $\lambda(M)$ plays an important role in what follows, so it is useful to point out the implications of its shape. In general, all the conclusions of this paper are strengthened if $\lambda(.)$ is 'flatter', they are weakened if it increases rapidly with M. Suppose first that λ is a constant independent of M, i.e., the rate at which a given trader meets partners is independent of the size of the market. Then the number of traders being matched at each instant is λM, which is linearly increasing in the size of the market. This may be identified with constant returns to scale, with market size as input and completed trades as output. At the other extreme, suppose $\lambda(.)$ itself is linearly increasing in M, so an individual trader meets partners at a rate λM, where λ is some constant. Here the individual benefits if the trading community is large. The number of traders being matched each instant is then λM^2, which is a case of increasing returns. Clearly this rate cannot be sustained for all M, for the number of matched traders must be bounded above by the size of the market. Thus the restrictions placed on $\lambda(.)$ are more than reasonable.

Let m be the number of traders entering the market at each instant. Of these πm are large and $(1-\pi)m$ are small. Let M_0 be the number of agents who are in the market at any given time in steady-state, with fractions θ and $1-\theta$ being large and small, respectively. For notational convenience we shall write λ_0 for $\lambda(M_0)$.

On arriving at the marketplace, then, the small agent's expected utility is:

$$W_0^S = \int_0^\infty \lambda_0 e^{-\lambda_0 t} e^{-rt} dt = \frac{\lambda_0}{r+\lambda_0} \tag{1}$$

where t is the time elapsed after his arrival, and $\lambda_0 e^{-\lambda t}$ is the density function for the arrival time of the first trading partner.

For the large trader, the expected payoff depends on whether the first trader she meets is large or small. The former case has probability θ and trade is concluded with the arrival of the first trader. The latter case has probability $1-\theta$, and trade is concluded with arrival of the second trader. Noting that the density function for the time of the second arrival is $\lambda_0{}^2 te^{-\lambda_0 t}$, and that a large agent gets two units of consumption upon completing trade, we get the expected payoff W_0^L where:

$$\begin{aligned} W_0^L &= 2\left[\theta\int_0^\infty \lambda_0 e^{-\lambda_0 t}e^{-rt}dt \quad + \quad (1-\theta)\int_0^\infty \lambda_0^2 te^{-\lambda_0 t}e^{-rt}dt\right] \\ &= 2\left[\frac{\lambda_0}{\lambda_0+r}\theta \quad + \quad \frac{\lambda_0^2}{(\lambda_0+r)^2}(1-\theta)\right] \end{aligned} \tag{2}$$

Of the θM_0 large agents who are on the floor, a fraction λ_0 trade at each instant and thus cease to be large. Simultaneously, πm large agents enter the market. Since this is a steady state, we have

$$\theta M_0\lambda_0 = \pi m \tag{3}$$

Of the agents undertaking trade, a fraction $(1-\theta)$ become small agents and remain in the market. In addition, $(1-\pi)m$ new small agents enter the market. At the same time, of the $(1-\theta)M_0$ extant small agents, a fraction λ_0 conclude trade and exit the market. Again invoking the steady-state we have

$$(1-\theta)\theta M_0\lambda_0 + (1-\pi)m = (1-\theta)M_0\lambda_0 \tag{4}$$

From the two above equation, we eliminate $M_0\lambda_0$ to get

$$\pi = \frac{\theta}{1-\theta(1-\theta)} \tag{5}$$

which is greater than θ for $\theta < 1$. In other words,

Observation 1 : *In steady-state, if all matched pairs of traders conduct trades, then the fraction of large traders in the market is smaller than the fraction of large traders among the entering agents.*

At each time, some of the large agents trade with small agents and become small. So the number of small agents that appear in the market at each point of time is larger than the number that enter, whereas the only new large agents are the entering ones. This raises the proportion of small agents above the entering proportion. Thus the existence of small traders imposes a negative externality on the large traders, by reducing the probability that any given meeting will end in a *completed* trade. We will next show that, for some values of π, there is an equilibrium where large traders refuse to trade with their small counterparts and thus avoid this externality (section 2.2), while for other values of π this externality is strong enough to make it worthwhile for the large traders to trade on a separate floor (section 2.3).

2.2 Voluntary Separation

In the preceding section we assumed that a large trader will trade with a small partner, because this increases the probability that she will complete trading on the next encounter. Suppose, however, that in general large traders refuse to trade with small partners. Can a deviant large trader nevertheless gain by such trading, or is there an equilibrium in which large traders may voluntarily restrict themselves to trading only with other large traders?

Consider a large trader with two units of the good who has just met a small trader. Suppose she expects that other large traders whom she might meet in the future will only trade with large partners. If she conducts trade with the small trader, then she will become small, else she will remain large.

In the former case, she will complete trade only by meeting another small partner. In case she declines the present trade, she will complete trade when she meets the next large partner.[1]

We can therefore treat the opportunity for a large trader to make a small trade as a choice between remaining a large trader and converting to a small trader. Clearly, the choice which the agent will make hinges on the expected delay in meeting a partner of one or the other type. If large traders arrive more frequently, then it is more profitable to remain large, and not trade with a small partner, whereas if small traders arrive more frequently then it is better to conduct the trade and become small. If there is no trading between types, then the frequency of arrival of a given type of trader is monotonic in the rate at which traders of that type arrive in the market. In particular if $\pi > \frac{1}{2}$ then large traders are more numerous, and large partners will therefore on average arrive faster than small partners.

Therefore if $\pi \geq \frac{1}{2}$ the following strategy profile is a candidate for equilibrium: a small trader trades with any matched partner who is willing to trade, and a large trader trades only if matched with a large partner. The large trader declines to trade with a small partner because she would remain in the market with one unit of the good, and if she thereafter meets a large partner the latter will refuse to trade with her. Since large partners arrive more frequently, it is optimal to decline the small trade and remain in the market with two units of the good rather than one.

This equilibrium, however, is not always robust if we allow the small traders to offer additional incentives to induce large agents to trade with them. Consider a small trader who has just met a large trader. If he can

[1] We are implicitly using the single deviation property here: if she does not find it profitable to conduct trade with the current small partner, then she will not find it profitable to trade with the next small partner she meets. This is suficiently obvious not to require a formal proof.

induce the latter to trade, then he leaves the market immediately, and gets unit utility from consumption. Otherwise, he stays in the market until the next small agent arrives. The large partner will stay in the market whether she trades or not, the difference on her part being that between waiting for a small as opposed to a large partner. If this difference is not too large, then the small trader will be able to compensate her for the expected loss in utility and still retain a benefit. As a consequence, the value of π for which such voluntary separation is an equilibrium is significantly in excess of $\frac{1}{2}$.

To see this, suppose that the strategy profile described above is indeed an equilibrium. Let the steady-state population of the large traders in this equilibrium be M_L, and that of the small traders be M_S. Let the corresponding arrival rates be λ_L and λ_S respectively. It is easy to verify that since $\pi \geq \frac{1}{2}$, $M_L \geq M_S$ and $\lambda_L \geq \lambda_S$.

Now suppose a small trader meets a large trader. In the given equilibrium they will not trade, so the small trader will wait for the next small partner. Discounted to the present instant, this has expected utility $\frac{\lambda_s}{\lambda_s+r}$. However, if he were able to induce the large partner to trade, he would get unit instantaneous utility. Thus he is willing to pay a premium of up to $1-\frac{\lambda_s}{\lambda_s+r}$.

The large trader, on the other hand, would wait for the next large partner, which has expected utility $\frac{2\lambda_L}{\lambda_L+r}$. If he were to trade with the small partner, he would then have to wait for the next small arrival, and get expected utility $\frac{2\lambda_s}{\lambda_s+r}$. Thus his loss from trading with the current small partner is $2[\frac{\lambda_L}{\lambda_L+r}-\frac{\lambda_s}{\lambda_s+r}]$.

It follows that the small trader would be able to successfully bribe the large partner into trading if:

$$1-\frac{\lambda_s}{\lambda_s+r} \geq 2[\frac{\lambda_L}{\lambda_L+r}-\frac{\lambda_s}{\lambda_s+r}] \tag{6}$$

which reduces to

$$\lambda_L \leq 2\lambda_s + r \tag{7}$$

if $r > 0$.

Thus for voluntary separation to be an equilibrium, we need $\lambda_L > 2\lambda_s + r$. Clearly this requires $M_L > M_s$, i.e. $\pi > \frac{1}{2}$. Let $\bar{\pi}$ be the value of π such that (7) holds with equality.

Proposition 2.1 *If $\pi > \bar{\pi}$, then there is an equilibrium in which the large agents trade only with other large agents, and refuse to trade with the small traders.*

Numerical example: Suppose that λ is linear in M, i.e. $\lambda(M) = \lambda M$ for some positive constant λ. Then $\bar{\pi} > 0.8$. If $\lambda(.)$ is concave, the proportion of large agents entering the floor must be even larger to satisfy (7).

2.3 A Trading Floor

Now suppose that a floor were established where only large agents are invited to trade. Small agents continue to trade on the street, which is the remnant of the undistinguished market we have already considered.

We have not assumed that the endowment type of an agent is publicly observable before he is matched with a trading partner, so entry into the floor can be neither enforced nor restricted. We are interested in finding out whether there are conditions under which agents will voluntarily separate themselves in this way. For the moment suppose that they do so. We want to find the steady-state in each sub-market, and the payoff to each type of agent.

First consider the floor. Let M_f be the steady-state population of agents, and correspondingly let $\lambda_f = \lambda(M_f)$ be the rate at which an agent meets a trading partner. Since all agents are large, each meeting results in both

agents leaving the floor. Thus the rate at which agents leave the floor is $M_f\lambda_f$. Large agents enter at the rate πm. Hence

$$M_f\lambda_f = \pi m \tag{8}$$

From (3) and (8) above, it follows that

$$\frac{M_f}{\theta M_0} = \frac{\lambda_0}{\lambda_f} \tag{9}$$

which is greater than unity if the meeting rate is strictly increasing in the steady-state population. In other words,

Observation 2 *In steady-state, the population (of large agents) on the floor is larger than the population of large agents in the combined market.*

This is of course a direct consequence of the externality referred to in the earlier observation 1.

The expected payoff of the large agent on the floor is

$$W_f^L = 2\frac{\lambda_f}{\lambda_f + r} \tag{10}$$

derived in the same way as in (1).

For the large traders to prefer to trade on the floor rather than in the combined market described in the earlier section, we need that this payoff must exceed their expected payoff in the market, i.e.,

$$\Delta_{f0}^L \equiv W_f^L - W_0^L \geq 0 \tag{11}$$

Using (2) and (10) this reduces to the requirement that

$$\frac{\lambda_f}{\lambda_0} \geq \frac{\lambda_0 + \theta r}{(2-\theta)\lambda_0 + r} \tag{12}$$

For $\theta < 1$ the right-hand-side of the above expression is necessarily less than unity. Intuitively, the LHS is closer to unity when the function $\lambda(.)$ is more concave, i.e., when an increase in the density of agents on the market contributes less to an increase in the meeting-rate. This would be the situation in which the large agents would prefer to trade separately—since the existence of small agents on the street does not appreciably speed up trade, and instead impose the negative externality referred to earlier.

For a numerical illustration, suppose that $\lambda_0 = 4r$. (This yields an expected payoff of 0.8 for the small trader entering the market. If the expected time of waiting is one "period", then it corresponds to a discount factor of 0.8). Then (12) would be satisfied for a value of θ smaller than one-third, which corresponds to a value for π of less than 3/7. Thus the prospect of a separate floor would be attractive to large traders even when they constitute a minority in the market.

Next consider the small traders that have been excluded from the floor, and hence trade on the street. Let M_R be the steady-state population on the street. Repeating the arguments that precede (8), we obtain

$$M_R\lambda_R = (1-\pi)m \tag{13}$$

The expected payoff of the small trader on the street is

$$W_R^S = \frac{\lambda_R}{\lambda_R + r} \tag{14}$$

Under what conditions is this separation of traders—by size between floor and street an equilibrium? It will turn out that, depending on the matching technology and the relative sizes of the two groups, such separation may occur voluntarily. In cases where the traders do not voluntarily separate, we can find a price (or subsidy) scheme which will induce them to separate. As intuition would suggest, the separation is not viable when the proportion of large agents in the total entering population is sufficiently small.

Suppose that all large agents are trading on the floor, and all small agents were trading on the street. If a large trader were to deviate and trade on the street, she would only meet small traders, and hence would need to make two trades to complete exchanging her endowment. Letting λ_R be the arrival rate of agents on the street, her expected payoff is then

$$W_R^L = 2\left[\frac{\lambda_R}{\lambda_R + r}\right]^2 \tag{15}$$

Thus her gain from trading on the floor as opposed to trading on the street is $W_f^L - W_R^L$, which reduces to

$$\Delta_{fR}^L = 2\frac{r\lambda_f(\lambda_R + r) + r\lambda_R(\lambda_f - \lambda_R)}{(\lambda_f + r)(\lambda_R + r)^2} \tag{16}$$

This is positive if the numerator is positive. One way to write this condition is

$$r\frac{\lambda_f}{\lambda_R} > (\lambda_R - 2\lambda_f). \tag{17}$$

The LHS is positive, so the condition will certainly be satisfied if the RHS is less than or equal to zero, i.e., if $2\lambda_f \geq \lambda_R$. To see what this implies for the relative sizes of the large and small populations, take the extreme case where $\lambda(.)$ is linear in trading population, i.e. $\lambda(M) = \lambda M$. Then using (8) and (13) we get

$$\frac{2\lambda_f}{\lambda_R} = 2\sqrt{\frac{\pi}{1-\pi}}$$

which is ≥ 1 if $\pi \geq \frac{1}{5}$. If $\lambda(.)$ is strictly concave (as in natural to assume), smaller values of π will do. Thus large traders would prefer to trade on the floor rather than defecting to the street as long as large traders constitute at least a fifth of the population, and perhaps less.

Define π^* to be the value of π such that $\Delta_{fR}^L \geq 0$ if and only if $\pi > \pi^*$. It is easy to show that, given a strictly increasing function λ and the discount rate r, Δ_{fR}^L is completely determined by π, and that the monotonicity properties required by the above definition do obtain. We also know that for concave λ, $\pi^* \leq \frac{1}{5}$.

Turning to the possibility of deviation on the part of the small traders, we note that these agents will want to deviate only if the steady-state population of traders on the floor (and hence the meeting-rate) is greater than that on the street. This is because each small agent needs only one meeting to exchange his endowment—the size of the trading partner does not matter. Since the steady-state population is increasing in the arrival-rate of traders, the small trader will want to deviate only if $\pi > \frac{1}{2}$.

If he does deviate, his payoff on the floor is

$$W_f^S = \frac{\lambda_f}{\lambda_f + r} \tag{18}$$

and his gain from the deviation is

$$\Delta_{fR}^S = \frac{r(\lambda_f - \lambda_R)}{(\lambda_f + r)(\lambda_R + r)} \tag{19}$$

Comparing to this with the gain of the large trader, we find

$$\Delta_{fR}^L - \Delta_{fR}^S = \frac{r\lambda_R(\lambda_f - \lambda_R) + r\lambda_f(\lambda_R + r) + r\lambda_R(\lambda_f + r)}{(\lambda_f + r)(\lambda_R + r)^2} \tag{20}$$

which is definitely positive if $\lambda_f > \lambda_R$. But from (19), this latter condition is fulfilled whenever *the small traders gain by entering the floor. In other* words, if the small traders can gain by trading on the floor, then the large traders always have even more to gain by doing so. This is intuitively obvious, since small traders benefit only from a higher meeting-rate, while large traders also benefit because they avoid the need for multiple meetings.

It follows from (20) that if the small traders have incentive to deviate, then we can find a price for entry into the floor which will discourage the small traders but not the large traders. The conclusions of this section are collected in the following propositions.

Proposition 2.2 *If condition (12) is fulfilled, then large traders will prefer to trade on a separate floor, rather than in the combined market.*

Proposition 2.3 *If $\pi \geq \pi^*$ (as defined following equation (17)) then separation of the large and small traders between the floor and the street is viable in the following sense:*
(a) If $\pi \in [\pi^*, \frac{1}{2}]$ then neither type of trader has an incentive to deviate from his assigned trading place.
(b) If $\pi > \frac{1}{2}$, then we can set a price $p \in [\Delta_{fR}^S, \Delta_{fR}^L]$ for entry into the floor, such that the large traders will prefer to pay this entry price and trade and the floor, while small traders will prefer to not pay the price and will trade on the street.

The last part is evident, since Δ_{fR}^i is the relative gain of a type-i trader from trading on the floor rather than on the street. By (20), the interval in proposition 2.3 (b) is non-empty.

2.4 Does the floor enhance efficiency?

Under appropriate conditions, then, the creation of the floor increases the utility of the large traders, but at the cost of the well-being of the small traders. Thus a Pareto comparison between the two alternative structures is not directly possible. Some other, less direct criterion must be called upon as a basis for efficiency calculations. [2]

The source of inefficiency *lies in the fact that individuals have to wait to* conduct exchanges. Thus some measure of the aggregate waiting time would

[2] The most satisfactory basis for comparison would be one using a Rawlsian "veil of ignorance". Suppose that, prior to entry, each agent only knows that he will get a large endowment with probability π, and a small endowment with probability $1 - \pi$. For each market structure, the agent can then determine his expected utility, as calculated before his endowment is known. The structure that yields the higher expected utility is the more efficient one. Unfortunately, the algebra associated with this calculation turns out to be intractable.

constitute a proxy for inefficiency. However, the question is complicated by the existence of the two types of traders. A large trader who waits for a given time suffers a greater loss than does a small trader who waits the same length of time.

Recall that a large trader does not consume while she is waiting, even though she may have succeeded in exchanging one of her units. Thus the number of unconsumed units that are extant on the market at any given time is precisely a weighted sum of the number of traders that are waiting, with weights corresponding to the endowment sizes of the traders. In the steady-state, this number will be larger if each trader, on average, waits a longer time. Thus the outstanding number of unconsumed units in the steady-state provides a measure of inefficiency that is generated by the market structure.[3]

Intuitively, the comparative efficiency of the separated market relative to the combined market depends on two factors. The combined market slows down the large traders because it reduces the mass of large traders outstanding, and thus the likelihood of trades being completed in one shot. On the other hand, it increases the mass of traders with which a given agent can trade (even though large traders will need two of these trades before they can get out of the market). If the second factor outweighs the first, then the combined market will be more efficient. In turn, this requires that, in the combined market, the speed at which an individual trader completes trades must be appropriately higher. In other words, the function $\lambda(M)$ must increase sufficiently steeply in M, i.e., the matching technology must exhibit increasing returns.

[3] This has a flavor of overlapping generations models of social security. If a certain stock is carried over unconsumed through time, then the utility from consuming this stock is delayed infinitely. If the economy could switch to a regime in which a smaller stock is carried over, then agents could consume the difference and therefore gain a one-time increase in welfare. Other things being equal, the switch to the smaller stock is a pareto improvement.

To see this, Suppose that the technology merely exhibits constant returns. Then the rate at which matches are made, $M\lambda(M)$, is linear in M, i.e. $\lambda(.)$ is a constant independent of M. Thus each agent's likelihood of meeting partners does not depend on the number of agents in the market; the total number of matches increases with market size merely because there are more agents to be matched. But then the payoff of the small agents is insensitive to the structure of the market. The large agents also meet partners at the same rate irrespective of whether they trade on the combined or the separated market. However, in the latter they do not run the risk of delay caused by meetings with small agents. Thus large agents are better off in the separated market, which is consequently more efficient.

If $\lambda(M)$ is increasing in M, on the other hand, then there is loss in matching efficiency when the market is separated, and this must be balanced against the delay which large agents face in the combined market. Roughly speaking, this loss is greater when the proportion of large agents is greater. Hence which structure is more efficient will depend upon the relationship between the function $\lambda(.)$ and the proportion π of large agents. The calculations which follow use θ instead of π for ease of algebra. The relation between the two is given by equation (5).

We first calculate the outstanding number of unconsumed units in steady-state in the combined market. Recall that large agents do not consume anything until they complete trading. The number of unconsumed units is therefore equal to the number of small traders plus twice the number of large traders that are looking to trade at any given time. Of the M_0 agents that are on the market, a fraction θ are large, and $(1-\theta)$ are small. But some of the latter entered the market as large agents, and must be counted as such for the present purpose. It is easily shown that these constitute a fraction

θ of the currently small agents.[4] Thus the total number of agents who are waiting with two units of the good is $(\theta + \theta(1-\theta))M_0$, while $(1-\theta)^2 M_0$ are waiting with one unit. Hence the total number of unconsumed units is $[2\theta(2-\theta) + (1-\theta)^2]M_C$. Using (3) this can be written as:

$$X_0 = \frac{1+2\theta-\theta^2}{1-\theta+\theta^2}\frac{m}{\lambda_0} \tag{21}$$

For the separated market, we use equations (3), (8), (4) and (13) to obtain the outstanding number of units as:

$$X_{(J+R)} = \frac{2\theta}{1-\theta+\theta^2}\frac{m}{\lambda_J} + \frac{(1-\theta)^2}{1-\theta+\theta^2}\frac{m}{\lambda_R} \tag{22}$$

The separated market is more efficient if and only if $X_0 > X_{(J+R)}$, which reduces to the condition:

$$\frac{1+2\theta-\theta^2}{\lambda_0} > \frac{2\theta}{\lambda_J} + \frac{(1-\theta)^2}{\lambda_R} \tag{23}$$

As discussed above, if the matching technology does no better than constant returns to scale, then this condition is always fulfilled. This can be seen by putting $\lambda_0 = \lambda_J = \lambda_R$, in which case (23) is strictly satisfied for all $\theta \in]0,1[$. In general, for any value of θ within that range, the condition will be negated if λ_0 is sufficiently larger than λ_J and λ_R, which is another way of saying that the matching technology displays strong enough increasing returns to scale.

A numerical example

Suppose that $\lambda(M) = \lambda.M^{\frac{1}{2}}$ for some constant λ. Then in a population of M traders there will be $\lambda.M^{\frac{3}{2}}$ traders meeting partners each instant, i.e., there is increasing returns to the size of the market. Using this together with (3), (5) and (21) we get:

$$X_0 = [\frac{m}{\lambda(1-\theta+\theta^2)}]^{\frac{2}{3}}(1+2\theta-\theta^2).$$

Similarly, using (8), (3), (13), (4), (5) and (22), we get:

$$X_{(J+r)} = [\frac{m}{\lambda(1-\theta+\theta^2)}]^{\frac{2}{3}}[2\theta^{\frac{2}{3}} + (1-\theta)^{\frac{4}{3}}].$$

So the condition that the separated market is more efficient reduces to

$$1+2\theta-\theta^2 - 2\theta^{\frac{2}{3}} - (1-\theta)^{\frac{4}{3}} > 0.$$

Machine computation shows that this requires a value for θ in excess of 0.313255, which implies that π must be greater than 0.399115. Thus with this particular increasing returns matching technology, the separated market becomes more efficient (in the sense specified) than the combined market once the proportion of large agents rises to about 40%. It is interesting to highlight the following conclusion.

Observation 3 *: There are non-empty ranges of parameter values for which the separated market is an equilibrium outcome, even though it is less efficient than the combined market.*

3 Sequential Entry

We next turn to a market where traders arrive sequentially according to a Poisson process with arrival rate λ. Each arriving trader is large with probability π and small with probability $1-\pi$. This is equivalent to considering two independent Poisson processes with rates $\lambda\pi$ and $\lambda(1-\pi)$ respectively. As in the previous section, we first consider the case where both types of agents trade on the same market.

[4] Let α be the fraction of the small agents who were originally large. The flow into this set consists of the large agents who trade with small agents, i.e. $\lambda\theta M_0$ times $1-\theta$. The flow out of the group consists of all those agents who conclude a trade, i.e. $\lambda\alpha(1-\theta)M_0$. In steady-state the inflow must equal the outflow.

An arriving agent may find the marketplace empty, in which case s/he must wait for the next arrival. Otherwise s/he will find an agent already waiting, in which case the two will trade. If the two agents have similar endowments, then both will leave, and the market will be empty until the next arrival. If they have unequal endowments (large/small), then the large agent will be reduced to a small one, and will wait for the next arrival. Under no circumstances will there be more than one agent waiting to trade.

At any given time, the market is thus in one of three states – it is either empty, or it has a small agent waiting to trade, or it has a large agent waiting to trade. Refer to these as states s_0, s_1, and s_2 respectively (by the number of units available for trade). The expected payoff of a given agent who arrives on the market will depend upon the probabilities of each of these states prevailing, as well as on λ and π. As in the last section, our propositions will hinge on a comparison of these expected payoffs with those that could be obtained on a separated market.

Denote the probabilities of the three states p_0, p_1, p_2 which sum to unity. There is a change of state each time a new agent arrives on the market. Now consider the state s_0. This may have occurred in two ways: *either* the previous state was s_1 (probability p_1) and the last arrival was a small trader (probability $1-\pi$), or the previous state was s_2 (probability p_2) and the last arrival was a large trader (probability π). Thus we have:

$$p_0 = (1-\pi)p_1 + \pi p_2 \tag{24}$$

Similarly, state s_1 can occur in three ways: state s_0 followed by the arrival of a small trader, state s_1 followed by the arrival of a large trader, or state s_2 followed by the arrival of a small trader. Finally, state s_2 can occur only if the previous state was s_0 and the last arrival was a large trader.

These yield two further conditions:

$$p_1 = (1-\pi)p_0 + \pi p_1 + (1-\pi)p_2 \tag{25}$$

$$p_2 = \pi p_0 \tag{26}$$

Only two of the three above equations are independent, but since the three states are exhaustive we can solve for the probabilities, which turn out to be:

$$p_0 = \frac{1}{2(1+\pi)}, \qquad p_1 = \frac{1}{2}, \qquad p_2 = \frac{\pi}{2(1+\pi)} \tag{27}$$

It is interesting to note that the probability of state s_1—that a small trader will be in the market– is independent of the density of small traders in the population. The intuition for this is as follows. Suppose that at some given time, the state of the market is *not* s_1, and a small trader arrives. Then the state must change to s_1. Now if a large trader arrives he will find a small trader to trade with, and hence will remain on the market with one unit of the good to exchange, i.e., the state will remain s_1. A change *out of* this state requires that another small trader must arrive on the market.

Thus each time a small trader arrives on the market, the state switches between s_1 and not-s_1 and stays so until the next arrival. Hence on average *the state must be s_1 half the time*, regardless of the relative density of small traders (as long as this density is positive).

If the arrival rate of large traders is sufficiently high, then once again there is an equilibrium in which large traders only trade with other large traders. The analysis of section 2.2 carries over unaltered, and proposition 2.1 applies to the present case. The only difference is that the arrival rates of large and small traders, λ_L and λ_s are now given by $\lambda\pi$ and $\lambda(1-\pi)$ respectively, so condition (7) reduces to:

$$\lambda\pi \leq \lambda(1-\pi) + r \tag{28}$$

So the voluntary separation equilibrium becomes viable for smaller values of π, though $\bar{\pi}$ is still greater than $\frac{1}{2}$.

3.1 Separation

Consider a large trader arriving in the market. With probability p_2 he finds another large trader waiting in the market, and concludes trade instantaneously. With probability p_1 he finds a small trader and trades half his endowment, then waits for the next arrival to conclude trade. With probability p_0, the market is empty. If the next trader to arrive is large (probability π) he concludes trade, if the next arrival is small, he has to wait until the second trader arrives to conclude. Using (27) and calculations familiar from the previous section, we obtain the large traders expected payoff as:

$$V_0^L = 2\left[\frac{\pi}{2(1+\pi)} + \left(\frac{1}{2} + \frac{\pi}{2(1+\pi)}\right)\left(\frac{\lambda}{\lambda+r}\right) + \frac{1-\pi}{2(1+\pi)}\left(\frac{\lambda}{\lambda+r}\right)^2\right] \qquad (29)$$

Once again we want to show that, under suitable conditions, it would be in the interest of the large traders to establish a floor exclusively for their own transactions. Suppose there were such a floor and all the large traders traded on the floor, while all small traders traded on the street. The rate of arrival of traders on the floor is $\lambda\pi$, and each trader needs only one encounter to conclude trade. A newly arrived trader either finds a partner already waiting, or waits for the next arrival—each contingency occuring with equal probability. Thus the expected payoff of the large trader is:

$$V_F^L = 2\left[\frac{1}{2} + \frac{1}{2}\frac{\lambda\pi}{\lambda\pi+r}\right] \qquad (30)$$

The large trader's gain from establishing a floor is $V_F^L - V_0^L$. It turns

out that:

$$V_F^L - V_0^L \geq 0 \quad \Leftrightarrow \quad \frac{\lambda\pi r + \lambda r + r^2}{\lambda^2} \geq 1 - 2\pi \qquad (31)$$

Since the LHS is strictly positive, the condition must be satisfied for all values of π larger than some $\hat{\pi} < \frac{1}{2}$. Thus once again the large traders would prefer to be seggregated on a floor even when they are in a minority.[5]

If the large traders are indeed in a minority, then the small traders will be content to trade on the street, since the arrival rate there is thicker, and this is all that matters for them. However, if $\pi > \frac{1}{2}$, then we must ensure that a price can be found which deters the small traders but not the large ones. So we must once again calculate the gain that a small trader makes by trading on the floor rather than on the street, and compare it with the corresponding gain for the large trader. If the latter is larger than the former, then a price situated between the two would deter the small traders from entering the floor, but would not discourage the large traders.

On either the floor or in the street, an arriving traders finds a ready partner with probability half, and has to wait for the next arrival with probability half. For a small trader, a single encounter concludes trade on either forum. For a large trader a single encounter is *sufficient on the floor*, but two encounters are necessary on the street since all traders on the street are small. Together with the different arrival rates, this yields the differential gains for the two types (we use Γ corresponding to the Δ of the last section):

$$\Gamma_{FR}^L = 1 + \frac{\lambda\pi}{\lambda\pi+r} - \frac{\lambda(1-\pi)}{\lambda(1-\pi)+r} - \left[\frac{\lambda(1-\pi)}{\lambda(1-\pi)+r}\right]^2 \qquad (32)$$

$$\Gamma_{FR}^S = \frac{1}{2}\left[\frac{\lambda\pi}{\lambda\pi+r} - \frac{\lambda(1-\pi)}{\lambda(1-\pi)+r}\right] \qquad (33)$$

[5] It can be shown that they would prefer to separate for all positive values of π if the discount rate r is large enough, specifically if $r > \lambda[\frac{1}{2}\sqrt{5} - 1]$.

From the two equations above we get

$$\Gamma_{FR}^{L} - \Gamma_{FR}^{S} = \{1 - [\frac{\lambda(1-\pi)}{\lambda(1-\pi)+r}]^2\} + \frac{1}{2}[\frac{\lambda\pi}{\lambda\pi+r} - \frac{\lambda(1-\pi)}{\lambda(1-\pi)+r}] \quad (34)$$

The term in braces on the RHS is always positive, so a sufficient condition for the whole expression on the RHS to be positive is that the term in square brackets be non-negative, which is true if $\pi \geq 1-\pi$. In other words, if the density of large traders is at least as great as that of small traders, then a price can be found which separates the two types. But this is the only case where the small traders would want to infiltrate the floor anyway. Thus separation can be sustained whenever it is in the interest of the large traders to do so.

Proposition 3.1 *If condition (31) is fulfilled, then large traders will prefer to trade on a separate floor, rather than in the combined market.*

Proposition 3.2 *If $\pi \geq \hat{\pi}$ (as defined following equation (31)) then separation of the large and small traders between the floor and the street is viable in the following sense:*
(a) If $\pi \in [\hat{\pi}, \frac{1}{2}]$ then neither type of trader has an incentive to deviate from his assigned trading place.
(b) If $\pi > \frac{1}{2}$, then we can set a price $p \in [\Gamma_{FR}^{S}, \Gamma_{FR}^{L}]$ for entry into the floor, such that the large traders will prefer to pay this entry price and trade and the floor, while small traders will prefer to not pay the price and will trade on the street.

3.2 Efficiency

In keeping with the approach used in the earlier section, the appropriate efficiency comparison involves comparing the expected or average number of unconsumed units that are carried in the market at a time. This number is

determined easily for the separated market. Whether on the floor or on the street, the market is empty half the time, and has one outstanding agent the other half of the the time. Thus on average the number of units carried on the street is $\frac{1}{2}.1 + \frac{1}{2}.0 = \frac{1}{2}$, and on the floor this number is 1. Thus for the separated market we have the average number of units carried as:

$$Y_{F+R} = \frac{3}{2} \quad (35)$$

In the combined market, the number of units carried at any given time is clearly 2 when the state is s_2, and 0 if the state is s_0. When the state is s_1, the number depends on the original endowment of the agent who is in the market. This can be determined by considering the preceding state and the endowment of the last arriving agent. Specifically, the number of unconsumed units is 1 if the previous state was s_0 and the last agent to arrive was small, which has the probability $p_0(1-\pi)$. For any other sequence of events leading to state s_1, the agent on the market was originally large, and two units are unconsumed.

Using the information above, we get the probability that two unconsumed units are being carried to be $\frac{3\pi}{2(1+\pi)}$, the probability that one unit is being carried to be $\frac{1-\pi}{2(1+\pi)}$, and that of an empty market as $\frac{1}{2(1+\pi)}$. This yields the expected number of unconsumed units to be

$$Y_0 = \frac{1+5\pi}{2(1+\pi)} \quad (36)$$

Comparing the two expressions above, it follows that $Y_{F+R} > Y_C$ for all values of $\pi < 1$. Thus the combined market is always more efficient than the separated market, even though the latter is an equilibrium for a wide range of parameter values.

Proposition 3.3 *If agents arrive sequentially, and the proportion of small agents is strictly positive, then the separated market is strictly less efficient than the combined market.*

4 Conclusion

When markets comprise heterogeneous types of agents, agents of some types may face costs resulting from chance meetings with 'unwelcome' partners. In the example considered here, the type of an agent is determined by the history of transactions she has previously engaged in, and the mixing of heterogeneous types results in a negative externality for large agents. While the present example is cast in terms of the sizes of endowments, it seems likely that similar externalities may be generated by other bases for heterogeneity, such as degrees of patience (e.g. Bose 1996) and levels of information. A possible response to this situation is that traders of the offended type seggregate themselves in a notional trading establishment, and keep the unwelcome types out by making it costly to enter.[6] Such multiplicity of trading environments for essentially the same commodity is widely observed in real life.

This paper attempts to make this point in the simplest possible manner, and the model used is less than suitable to investigate extensions which immediately suggest themselves. In the context of the present example, in particular, there are two obvious questions which may be of interest. The first relates to the realistic case where the number of endowment types is larger than two. Would we still expect the market to separate into only two segments, or more? One conjecture is that, with discrete endowment types and a suitable distribution of agents between those types, we should

[6] This is a concept familiar from the economic theory of social situations such as clubs, Veblenesque class, and old-boy netweorks.

observe as many segments as there are types. The heuristics are as follows. Suppose there were three types, with endowments of one, two and three units respectively. Suppose that the three unit traders did separate into a floor of their own. Then the remainder of the population parallels the case examined above, and should separate into two trading environments. But then the choice for the three-unit traders is to remain separated or to join one of the other types. But again by arguments similar to those used here, they would not want to join any *one* other type. Hence under suitable conditions three-way separation should be an equilibrium. However, the argument does not carry over to a case with a continuum of types.

The second question is more immediately interesting: is there scope for brokers or middlemen to arise, who trade on the floor on behalf of a number of small traders at a time? It is certainly true that such intermediation is ubiquitous in financial markets, and perhaps in other markets as well. However, this kind of contracting is not best analysed within the present model, for it raises a wider and perhaps a different set of issues.

References

Bose, Gautam and Mark Pingle (1995), Stores, *Economic Theory*, 6, 251 262

Bose, Gautam (1996): Bargaining Economies with Patient and Impatient Agents: Equilibria and Intermediation. *Games and Economic Behavior* 14, 149- 172.

Demsetz, Harold (1968): The Cost of Transacting. *Quarterly Journal of Economics*, pp.33-53.

Diamond, P. (1982), Aggregate Demand Management in Search Equilibrium, *Journal of Political Economy*, 90, 881-894

Spulber, D. (1996): Market Microstructure and Intermediation, *Journal of Economic Perspectives* 10 Summer, p 135-52.

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