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STOCHASTIC STABILITY IN A RATIONAL EXPECTATIONS MODEL OF A SMALL OPEN ECONOMY

by

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1. Introduction

Recently some interesting applications have been formulated within the model of macroeconomic activity proposed by Dornbusch (1976). The solution to that model often has a saddlepoint property. The associated instability is "handled" by the device, suggested by Sargent and Wallace (1973), of nominating an endogenous variable which "jumps" by exactly the amount required to keep the model on the stable manifold. This auxiliary assumption has the advantage of allowing the introduction of "forward looking" variables into macroeconomic models and of suggesting an explanation for the sharp changes which have occurred in asset prices in general and exchange rates in particular. However, there is no mechanism in the model itself to motivate the auxiliary assumption. There are, for example, no arbitrage profits to be made if the variables in Dornbusch's model follow the unstable trajectory.[1]

These observations lead us to enquire into the possibility of other ways of ensuring stability while maintaining the usefulness and plausibility of the perfect foresight-sticky wage/price macroeconomic framework. Our starting point is to note that many models are specified in an entirely deterministic form, presumably for ease of manipulation, certainly not for realism. Now it can be shown that the addition of stochastic shocks can make an otherwise unstable deterministic model stable, in a stochastic sense. Thus it may be possible to add to the realism of macromodels while simultaneously maintaining their stability. This is the direction taken in this paper.

In fact, in this particular application, attention is directed to the stabilising function of stochastic processes by excluding the possibility of any variable "jumping", so that the deterministic model would be strictly unstable.

The paper is set out as follows. In section 2 the deterministic model is set out. Section 3 introduces the concept of stochastic stability. In section 4 the design of a series of stochastic simulation experiments is explored with alternative sets of parameters of the deterministic model representing, broadly, Keynesian and monetarist positions, and with parameters of stochastic parts chosen to ensure stability and to explore the consequences of varying the "signal-to-noise" ratios of structural equations. Section 5 summarises the results of the simulations in terms of regressions, time series properties and spectral decompositions.

2. The Deterministic Model

The model is straightforward: we have no contribution to make in terms of deterministic modelling. The model is very similar to the one employed by Dornbusch (1976) and is represented in four equations.

$$m = ky - \lambda r + p \qquad k,\lambda > 0 \tag{1}$$

$$y = -\gamma(r - Dp) + \delta(e - p) \qquad \gamma,\delta > 0 \tag{2}$$

$$Dp = \phi y \qquad \phi > 0 \tag{3}$$

$$De = r \tag{4}$$

m nominal money balances

p domestic price levels

y real income

r domestic interest rate

e nominal exchange rate (domestic currency price of foreign currency)

D differential operator, d/dt

All variables except r are in logs. The equations are the LM curve (1), the IS curve (2), Phillips curve (3), the interest parity condition (4). The world price level and the world interest rate are assumed constant and omitted. Otherwise the term in the IS curve would read $\delta(e-p + p_w)$ and the interest parity condition would have r_w subtracted from the right-hand side. Variables y, r, Dp and De are endogenous in the short-run.

Defining real money balances ℓ = m-p and the real exchange rate c = e-p the system has the vector first order state-space representation, used by Buiter and Miller (1981)

$$(5) \quad \begin{bmatrix} D\ell \\ Dc \end{bmatrix} = A \begin{bmatrix} \ell \\ c \end{bmatrix}$$

$$\text{with} \quad A = 1/\Delta \begin{bmatrix} \phi\gamma & \phi\lambda\delta \\ 1 & \delta(\phi\lambda-k) \end{bmatrix}$$

and $\Delta = \gamma(\phi\lambda-k) - \lambda$.

With the parameters having signs as indicated and with $\Delta < 0$ the matrix A has two eigenvalues, one with positive real part, the other with negative real part. Denoting the roots by $\lambda 1$, $-\lambda 2$ with $\lambda 1, \lambda 2 > 0$ the equilibrium trajectory for c, say is given by $c^* = B_1 e^{\lambda_1 t} + B_2 e^{-\lambda_2 t}$ (unstable) eigenvector dominating for large t. A "jump" in c* to the stable trajectory is equivalent to setting B_1 to zero.

Our alternative approach to maintaining stability is introduced in the next section.

3. Stochastic Stability

In this section we extend the modelling so as to describe ℓ and c as stochastic processes. For this purpose we start with the deterministic model (5) and add continuous martingales to ℓ and c. As the deterministic part has the Markov property; that is, all the information relevant to determining the change in ℓ and c is embodied in the current values of ℓ and c, it is convenient and elegant to retain this property in modelling the stochastic part. With the additional assumption that the covariance matrix is continuous with respect to ℓ,c and t, ℓ and c can be taken to be diffusion processes. Thus we write

$$(6) \qquad \begin{bmatrix} d\ell \\ dc \end{bmatrix} = A \begin{bmatrix} \ell \\ c \end{bmatrix} dt + \Sigma db$$

where b is a vector Brownian motion containing independent standard motions and Σ is a matrix of functions of ℓ and c specifying the instantaneous covariance structure. Even though the matrix A in (5) has an eigenvalue with a positive real part, indicating instability of the 'deterministic' part of the system, the stochastic process given by (6) can be stable in that there exists an invariant joint distribution for ℓ, c and the ergodic process ℓ,c converges to a stationary stochastic process with the invariant distribution as its limiting stationary joint distribution (Kushner, 1967). Examples in one dimension provide an illustration and an introduction to the technicalities involved.

Consider the one dimensional system in x given by $dx = \alpha x\, dt + \sqrt{\beta} x\, db$ for b a one dimensional standard Brownian motion. The deterministic part is unstable for $\alpha > 0$. However this equation can be explicitly solved for x in terms of t and b yielding $x(t) = x(0) \exp(\alpha t - \beta/2t + \sqrt{\beta} b(t))$ so that x(t) tends to zero with probability one provided $\beta/2 > \alpha$ (Kushner 1967, pp.55-56). The point x=0 is an absorbing state for this process for if $x(t) = 0$ then $x(t+s) = 0$ for all $s > 0$.

In order to avoid an absorbing state at zero we alter the equation to $dx = \alpha x\, dt + (\gamma + \beta x^2)^{1/2} db$. A positive γ insures that there is no absorbing state and the variance rate is approximately proportional to x^2 as in the above example. This example is explicitly studied by Rozanov (1969, p.274) where it is shown that for $\beta/2>0\alpha>0$ and $\gamma<\beta$, $x(t)$ converges to a stationary stochastic process with invariant distribution given by $p(x) = A(x^2+\gamma/\beta)^{\alpha/\beta-1}$. Note that x has infinite variance as $y=\mathrm{sign}(x)|x|^{\theta}$, has finite variance only for $\theta<1/2 - \alpha/\beta$. Hence, as sign preserving power transforms of x are finite variance stationary stochastic processes, their dynamic structures may be analyzed by the application of spectral and time series analysis, which is taken up in later sections.

We turn now to the stabilization of the two dimensional system (6). Firstly we define Σ to be a diagonal 2×2 matrix with diagonals $(\gamma_1+\beta_1\ell^2)$, $(\gamma_2+\beta_2 c^2)$, heteroskedastic disturbances which have been studied in the discrete time case by Engle (1982).

Note that without loss of generality one may suppose the equilibrium values of l and c to be zero as they are log variables. The proposed diffusion specification makes variance rates proportional to the absolute deviation from equilibrium, independent of direction. This is fairly reasonable for one would expect forces of changes to be stronger the further the system deviates from equilibrium and variance rates essentially measure the uncertainty in the results of such forces.

For parameter restrictions consistent with the stability of the two dimensional diffusion processes (6) we employ the criteria developed by Bhattacharya (1978), which is explained fully in the appendix.

The stabilization strategy we adopted was to set γ_1, γ_2 at .05, a relatively small value, to avoid zero becoming an absorbing state, and to choose β_1, β_2 such that the model is stochastically stable. In operational terms this means that a variable betabar, defined in the appendix is less than unity. An intuitive grasp of the role and interpretation of betabar can be obtained by referring, once again, to the one dimensional case $dx = \alpha x\, dt + \sqrt{\beta} x\, db$.

In this case betabar equals $1/2 + \alpha/\beta$ so that as betabar rises towards unity, the signal to noise ratio is increasing. Returning to the two dimensional case, it is simple to show, by calculating betabar at $\ell = c$ that, for betabar < 1, the trace of A in equation (5) must be negative. Hence our stabilising procedure cannot stabilise a deterministic system in which both roots are unstable. Furthermore, in our system with just a single unstable root another restriction is necessary for stability. With rows of A represented by a, b and c, d we need $a+d+|b+c| < 0$. Thus our choices of parameters have been made subject to this constraint. Within this limitation we have chosen the parameters to reflect Keynesian and monetarist beliefs. The specific choices are explained in the next section.

4. The Simulations

The simulations were designed to explore the significance of three dimensions of the modelling: the traditional Keynesian-monetarist distinction, the relative importance of the deterministic and stochastic contributions, and the relative stochastic contributions of ℓ and c. This makes eight possible simulations. In fact we conducted only six. The Keynesian variant proved difficult to stabilise, requiring large values for β_1 and β_2 (the slope coefficients in the heteroskedastic error processes). Thus there are no Keynesian simulations with low variance stochastic parts.

To generate an alternative Keynesian model would have required us to increase the variances even more, giving rise to a model which would be stable but dominated by white noise. The various models are summarised in table 1.

Table 1

Parameter Values for Simulations

	Monetarist				Keynesian	
	equal high	equal low	unequal high	unequal low	equal high	unequal high
Parameters	MEQH	MEQL	MUQH	MUQL	KEQ	KUQ
k	2.0				0.75	
λ	0.5				2.5	
γ	2.5				2.5	
δ	0.25				0.1	
p	3.0				0.5	
γ_1	0.05	0.05	0.025	0.025	0.05	0.025
β_1	2.0	0.5	1.0	0.25	2.0	1.0
γ_2	0.05	0.05	0.05	0.05	0.05	0.05
β_2	2.0	0.5	2.0	0.5	2.0	2.0
eigenvalue 1	0.1	0.1	0.1	0.1	0.04	0.04
eigenvalue 2	-4.31	-4.31	-4.31	-4.31	-1.08	-1.08
betabar	0.57	0.72	0.56	0.72	0.97	0.93

The word "high" means β_2=2.0 and "low" means β_2=0.5. "equal" means $\beta_1=\beta_2$ and "unequal" means $\beta_1 = 1/2\ \beta_2$. Thus the first four simulations - all monetarist - explore the consequences of varying the variances in the diffusions for ℓ and c, both in absolute and in relative terms. The two Keynesian simulations are both "high variance", the "low variance" alternatives being unstable.

Turning to the choice of the parameters of equations 1 to 4, we applied the following principles. The income elasticity of money demand, k, was put at 2.0 in the monetarist case and 0.75 in the Keynesian case. The interest semi-elasticity, λ, is set at 0.5 and 2.5 in the monetarist and Keynesian cases, giving a relatively steep LM curve for the monetarist model as (at least a) textbook exposition might lead us to expect is the case. The slope of the IS curve in each case is 2.5 while the elasticity of domestic demand to changes in the real exchange rate, δ, is 0.25 in the monetarist case and 0.1 in the Keynesian case, arguably reflecting the additional enthusiasm with which monetarists tend to view the efficacy of changes in relative prices. Finally the slope of the Phillips curve is taken at 3.0 for the monetarist variant and 0.5 in the Keynesian variant, again a rather natural interpretation of the differing views.

The remaining information in the table is related to the stability of the model. Note that, in the Keynesian case, both the stable eigenvalue (-1.08) and the unstable one (0.04) are rather small. Also note that betabar is close to its upper bound of unity in both cases (0.97 and 0.93) imposed by the requirements of stochastic stability. The monetarist values for betabar are lower, the corresponding values (MEQH and MUQH) being 0.57 and 0.56.

In simulating the diffusion of equation (6) we followed the procedure developed by Yavin (1985, p.69). A unit interval of time was partitioned into 1000 intervals of length Δ=.001 and the discrete time approximation of Yavin (equations 3.48 and 3.49) were used to construct $x(t+\Delta)$ from $x(t)$.[3] In this way observations on simulated values of lt, ct at multiples of unit time were generated. In order to induce finite variance, a sign preserving power transformation was performed defining $y_1 = \mathrm{sign}(\ell)\ |\ell|^{\theta}$, $y_2 = \mathrm{sign}(c)|c|^{\theta}$, with $\theta=0.1$.

The choice of θ equal to 0.1 may be motivated by noting that, in the one dimensional example used above, a finite variance variable will be generated provided $\theta < 1/2 - \alpha/\beta$, or, equivalently θ less than (1-betabar). We have used this relationship as a guide in the two dimensional case. Thus $\theta=0.1$ emerges as a value which leaves a wide range for betabar, may not "flatten" the original variables too much and which can be used for all simulations, a requirement reflecting our view that a common power transformation is desirable if we are to make comparisons across simulations. This said, it remains the case that our choice of a value for θ has been only partially justified.

In the next section we analyse, using regression, time series and spectral analysis, the simulated series of on ℓ, c and y1, y2 for the simulations summarised in table 1.

5. <u>Results</u>

The previous section listed six models for simulation. Three had equal variances in the heteroskedastic error terms and the other three set the variance in the $d\ell$ equation equal to one half the variance in the dc equation. In fact, the equal and unequal variance results were almost identical, so the "unequal variance" simulations are not reported. We are left with three models: monetarist high variance, (MEQH), monetarist low variance (MEQL) and Keynesian (KEQ) for each of which 100 observations are simulated. Initially the results of the spectral analysis are discussed. Figures 1 and 2 display the spectral densities for real balances and the real exchange rate.[2] Figure 1 shows that the monetarist high variance and the Keynesian models have quite similar spectra, with the monetarist high variance model having two significant peaks and the Keynesian model possibly possessing three, while the monetarist low variance model has most of the spectral density concentrated at low frequencies, with no pronounced cycles. This pattern is reproduced in figure 2 for the real exchange rate.

In all models the cyclical patterns for real balances and the real exchange rate are very similar. All models have a long cycle, between 39 and 56 years, while the monetarist high variance model and Keynesian models also possess a medium term cycle of about 7 years and, additionally, the Keynesian model plausibly has a short cycle of 4-5 years.[3]

It is important at this point to spell out the nature of the comparison between MEQH, MEQL and KEQ. Two comparisons are obvious: MEQH with MEQL, and MEQH with KEQ. MEQH and MEQL are based on identical deterministic models and differ only in the values of β_1 and β_2. Thus the differences exposed by the figures are solely due to the stochastic influence. MEQH and KEQ are, obviously, based on different parameter sets, as the upper half of table 1 shows. Their stochastic parts are, however, identical, as the lower half of the table shows, so that comparison between MEQH and KEQ reveals the effects of the deterministic differences which frequently form the basis of monetarist-Keynesian debates. Note now, and this is important, that MEQL and KEQ can form the basis of a useful comparison. These two models differ in their deterministic parts and their stochastic parts. However they have similar values, by construction, for betabar (.72 and .97). On our interpretation betabar is related to the signal to noise ratio for stochastic processes. Thus the MEQL and KEQ simulation have similar relative contributions from the stochastic parts.

An important feature of the results, revealed in figures 1 and 2 is that, by increasing the heteroskedastic error component in the monetarist model (i.e. going from monetarist low variance to monetarist high variance), it is quite possible to so change the frequency characteristics of the

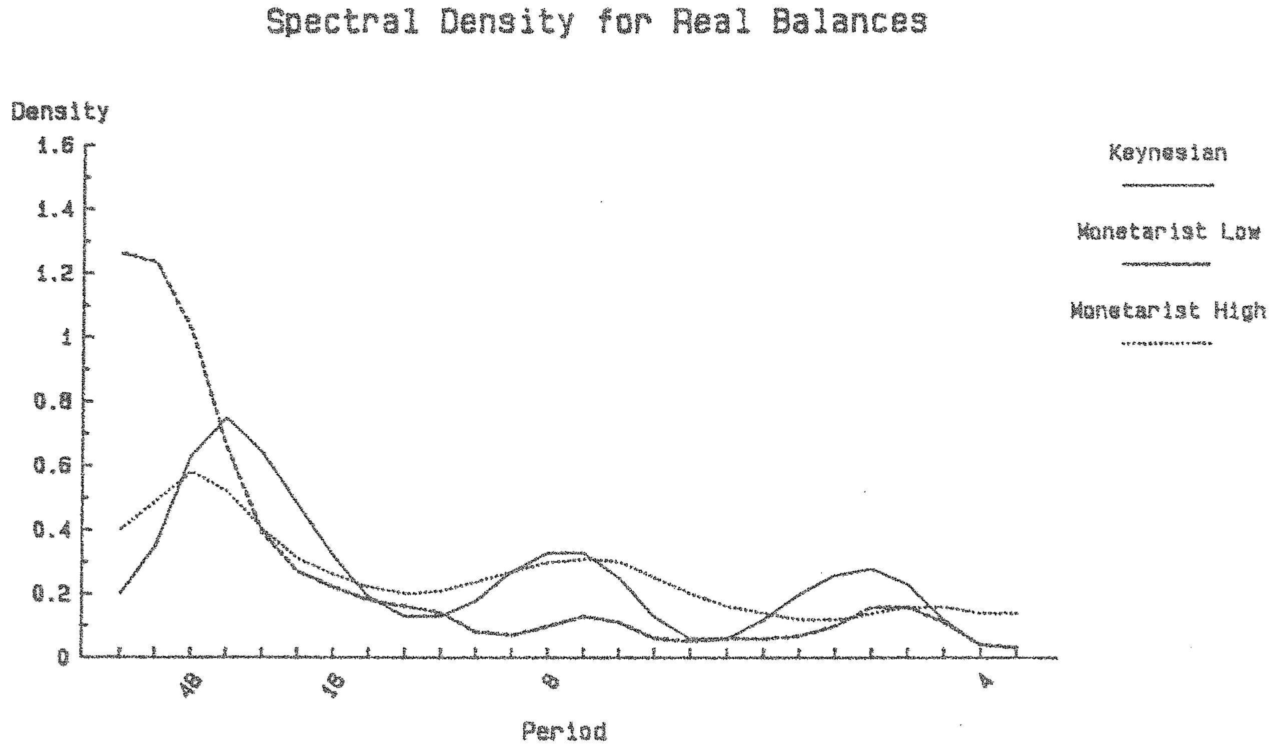

Figure 1

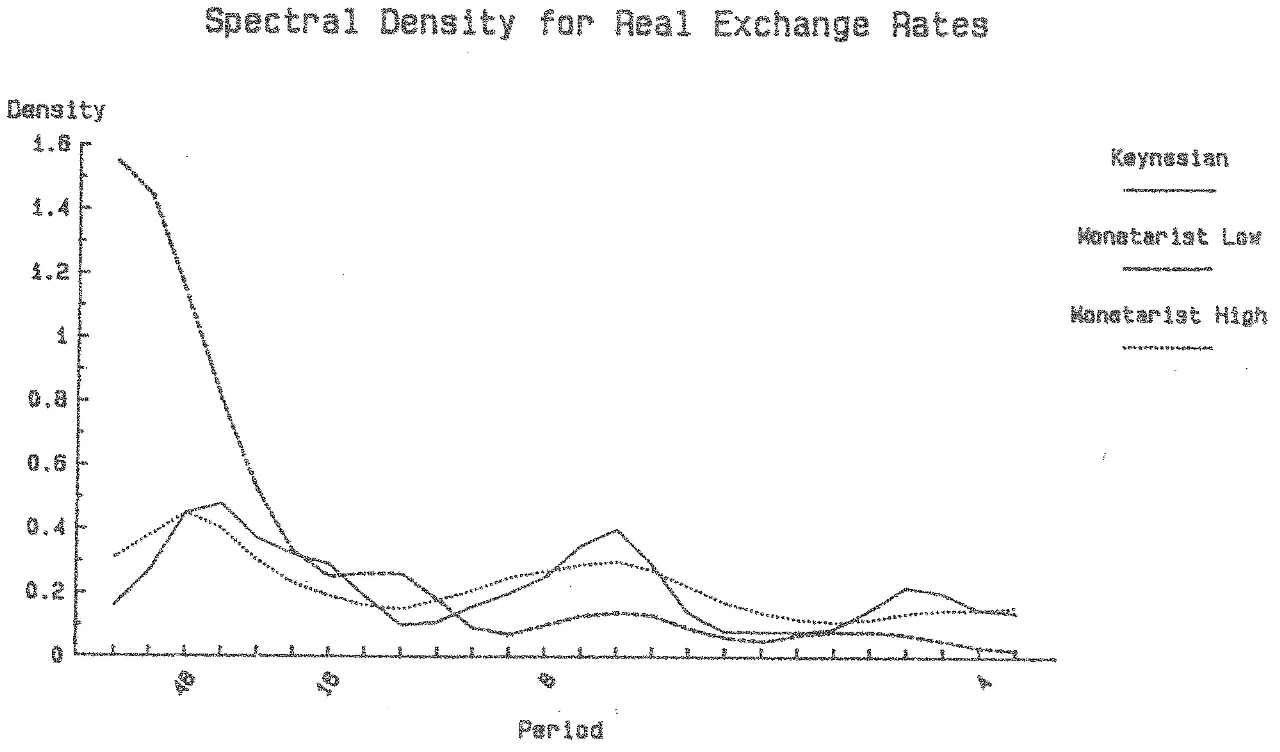

Figure 2

monetarist model that they resemble the frequency pattern of the Keynesian model. Or, equivalently, as the monetarist models have identical deterministic parts and different stochastic parts, their dissimilar spectra are due, in this case, to differences in stochastic parameters.

We now turn to the evidence from the cross spectral analysis in figures 3 and 4. Figure 3 shows the spectral densities for real balances, the real exchange rate and their coherence for the monetarist low variance model, and figure 4 shows the same data for the Keynesian model.[4]

Figure 3 shows that at the lower frequencies which dominate the monetarist low variance model, real balances and the real exchange rate have high coherence. In figure 4 this relationship between dominant frequencies and high coherence is repeated for the Keynesian model, though the figures appear to be quite different. In the Keynesian model low, medium and high frequency cycles are all important and, at those frequencies the coherence is relatively high, though not as high as for either monetarist model. The variables are out of phase, by 180° in the monetarist low variance model, by between 160° and 175° in the monetarist high variance model and the first two cycles of the Keynesian model, and by between 130° and 150° for its third cycle.

The results for the time series analysis are summarised in table 2. The monetarist high variance and the Keynesian models yield very similar results: real balances is ARMA (2,0) and the exchange rate is ARMA(1,0) with similar coefficient estimates. Real balances, according to the time series model, follow a damped cycle of eight years, picking up the second cycle revealed by the spectral analysis. The real exchange rate is very "noisy" so that the time series analysis fails to pick up a second cycle, though, given the high coherence noted above, one would expect it to be present.

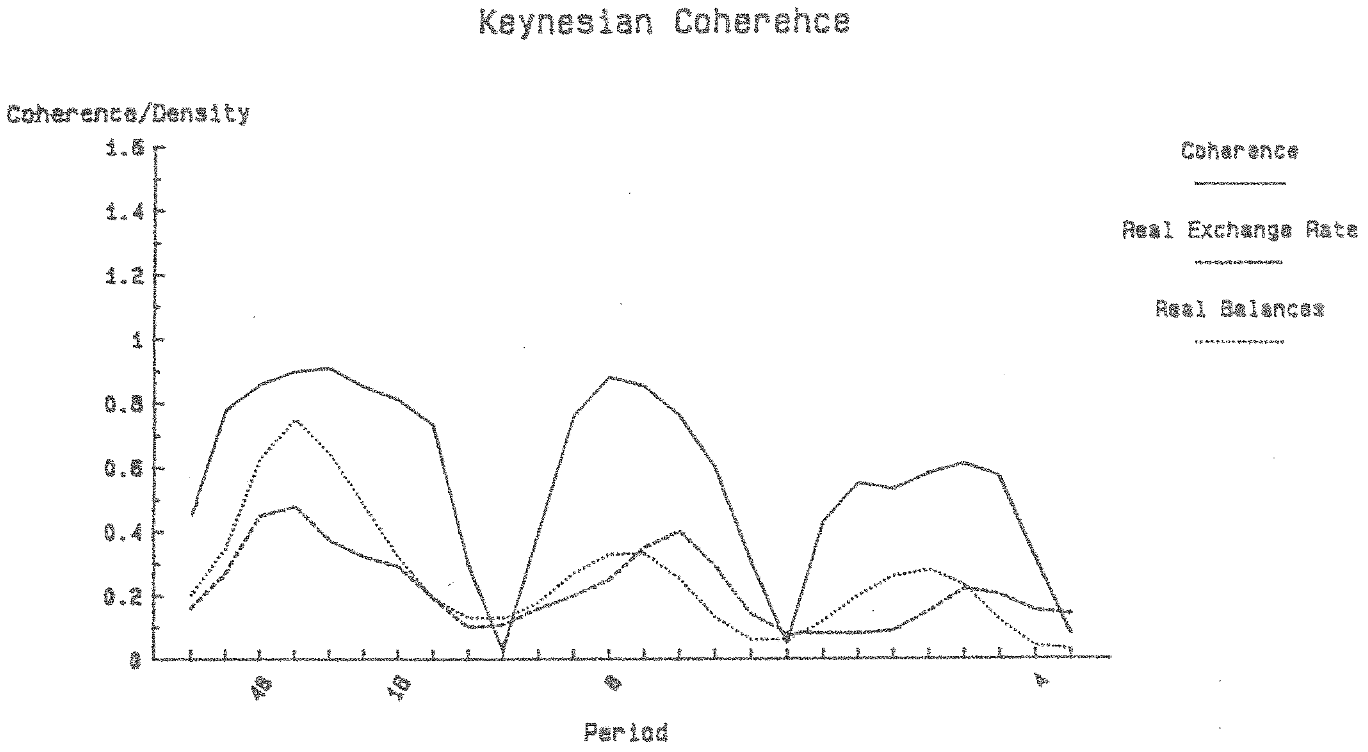

Figure 3

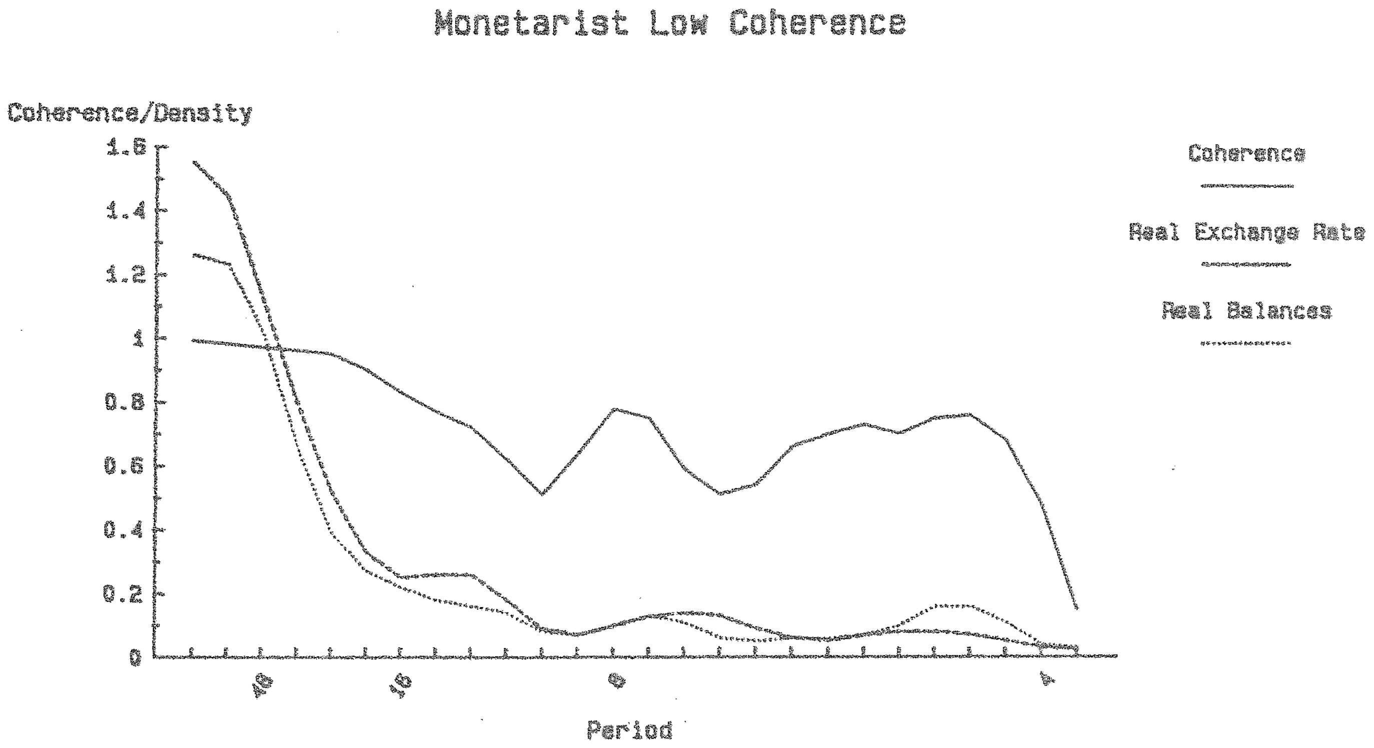

Figure 4

Table 2: Time Series Analysis

Variable		ARMA model	Coefficients on		
			AR1	AR2	MA1
MEQL	1	(1,1)	.8840	-	.507
			(12.62)		(3.91)
	2	(1,0)	.8023	-	-
			(13.30)		
MEQH	1	(2,0)	.6628	-.2125	-
			(6.68)	(-2.14)	
	2	(1,0)	.3526	-	-
			(3.73)		
KEQ	1	(2,0)	.6481	-.1341	-
			(6.44)	(-2.33)	
		(1,0)	.3466	-	-
			(3.66)		

In the monetarist low variance model real balances are ARMA (1,1), with a great deal of persistence, past disturbances receiving a relative weight of 12/13.[5] The real exchange rate is ARMA (1,0), the closeness of the AR coefficient to unity suggesting that changes in the exchange rate are approximately white noise, as some proponents of the efficient markets hypothesis would assert.

Some results from regression analysis are shown in table 3. The statistics have been derived from OLS equations in which real balances and the real exchange rate are regressed on five variables: real balances lagged one and two periods, the real exchange rate lagged one and two periods and a constant. The results reveal that all series are difficult to "explain", with the ratio of standard error of regression to standard deviation of original series close to unity. The R^2 reveal that the monetarist low variance model has less unexplained variation than either of the other models, a result which sits squarely with the absence of a pronounced cyclical pattern, the large AR coefficients in the time series representation and the relatively small heteroskedastic shocks used to stabilise the deterministic, and hence highly predictable, model.

Table 3

Real balances	std. dev.	std. error	R^2
MEQL	.14	.09	.54
MEQH	.84	.69	.38
KEQ	.53	.40	.47
Real Exchange Rate			
MEQL	2.52	1.53	.66
MEQH	17.88	16.72	.18
KEQ	5.53	5.24	.16

Summarising these results it is clear that the monetarist low variance model is the "odd one out". It has longer cycles, inertia in real balances, approximately white noise exchange rate changes and higher predictability. If the monetarist high variance model had not been simulated we might be tempted to record a view that the differences between the Keynesian and monetarist low variance models were evidence of the importance of the parameter differences emphasised in macroeconomic debate. However the differences between monetarist low and high variance spectra demonstrate that this is not the important source of the differences.

Although our original concern was with stability it is clear that the results have an additional important feature. The addition of stochastic shocks has give rise to cycles, as originally demonstrated by Adelman and Adelman (1959). Also, our spectral results for the Keynesian model bear a resemblence to the spectra shown in Sargent (1979, pp.216-218) for aggregate U.S. time series data.

The monetarist low and high variance models have identical deterministic parts but very different time series representations. Thus it is difficult to resist the conclusion that the major difference between the monetarist low variance and the other two models lies with the stochastic modelling. Not, of course, that the monetarist high variance and the Keynesian models have identical properties. For example, the Keynesian models appear much more likely to have a short cycle with a period of approximately four years. Nevertheless the differences between those two models are small when compared with the differences between either of them and the monetarist low variance model.

By combining monetarist deterministic parameters with relatively small shocks we have been able to generate a model which is consistent with a monetarist view of the world. The ease with which this model is stabilised also fits in neatly. By contrast, the Keynesian vision emphasises the difficulties inherent in stabilising the economy, in explaining it, and the importance of short cycles. These are precisely the results obtained. The Keynesian deterministic model requires large shocks for stability and, as a consequence of those shocks, the regressions have relatively low explanatory power, and cycles are suggested by both time series and spectral analyses.

Finally we must allude to a possible econometric problem suggested by these results, without claiming to have carried out the detailed work necessary to clinch the point. Two of the models produce simulated data which are similar in their time series properties. We therefore raise the question: if real world data exhibit characteristics similar to the data generated by our models, can econometric estimation discriminate between monetarist and Keynesian parameters?[6]

6. Conclusion

The observation that deterministically unstable models may be stochastically stable led us to investigate the perfect foresight, sticky prices model due to Dornbusch (1976). By adding heteroskedastic diffusions to the deterministic components of the motion in real balances and real exchange rate, we were able to stabilise the model in a stochastic sense. Choosing, broadly, Keynesian and monetarist values for the deterministic parameters and attaining stability by stochastic specification we analysed the simulated output of three models, Keynesian, and monetarist low and high variance. Our results indicate that the monetarist low variance and Keynesian models consistently represent monetarist and Keynesian views of the world. The former has long lags in adjustment, white noise behaviour of real exchange rates and greater predictability. The latter is more difficult to stabilise and possesses some short cycles of approximately 4-5 years. Interestingly these differences are blurred when the stochastic specification of the monetarist model is changed to a monetarist high variance case. The spectral decomposition, time series and regression results on the simulated output of this case are quite similar to the

Keynesian case. This suggests that stochastic specification is important in model description with the differences between the monetarist properties of the monetarist low variance case and the Keynesian properties being due more to differences in stochastic specification than to the choice of deterministic parameters.

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Footnotes

1. Of course the unstable trajectory implies that some price is tending to zero as t tends to infinity. If forward markets existed for many periods into the future there would be a case for neglecting the unstable solution. This would also be the case if we were to require the agents in the model to be infinite horizon discounted utility maximiser (Brock, 1975). Both rationales are, we feel, misplaced in a short-run macroeconomic model.

2. In section 3 above we pointed out that the distribution function generating ℓ and c has an infinite variance. As the spectral density describes the frequencies which make up the variance, the application of spectral methods in this case presents a problem. Our approach was to calculate spectra for the variables of interest and for their tenth roots. For all three models the original real balances and their respective tenth roots possessed similar qualitative and quantitative cyclical patterns. This conclusion applies also to exchange rates for the two monetarist models. In the Keynesian model the original variable possesses an additional higher frequency cycle. In view of these close associations we present only the spectra for the original series.

3. These statistics are obtained by averaging the density over five frequencies around the peak frequencies.

4. The coherence for the monetarist high variance model is not shown as it is close to unity over all frequencies.

5. The ARMA (1,1) model for real balances, given by $\ell_t = \alpha\ell_{t-1} + \varepsilon_t + \beta\varepsilon_{t-1}$, can be rewritten $\ell_t = \varepsilon_t + (\alpha+\beta)/(1-\alpha)\left[(1-\alpha L)^{-1}\varepsilon_{t-1}\right]$ and, using α, β from the table, $(\alpha+\beta)/(1-\alpha) \approx 12$.

6. The continuous time econometrics of heteroskedastic diffusion disturbances is as yet quite undeveloped. Some progress has been made by Engle (1982) in studying discrete time conditional heteroskedastic models.

Appendix

Stabilisation of the diffusion process by parameterisation of the covariance structure

We apply conditions (3.25) and (3.26) of Bhattacharya (1978, p.550) with $z=0$. In order to apply these conditions we use definitions (3.9) of Bhattacharya to define $A(x)$, $B(x)$, $C(x)$ $\bar{\beta}(r)$, $\underline{\alpha}(r)$, and $\bar{I}(r)$. We drop the subscript z as we are taking $z=0$. First with $x=(l,c)$ let $S(x)$ be the 2×2 diagonal matrix with diagonal $((\gamma_1+\beta_1\ell^2),(\gamma_2+\beta_2c^2))$ and let $C(x)=2x^TAx$ where A is as in equation (5). Now define $A(x) = x^TS(x)x/x^Tx$ $B(x) = \text{trace } S(x)$ and $\bar{\beta}(r) = \sup\{(B(x) - A(x) + C(x))/A(x) \mid \|x\|=r\}$. Note that in our case $(B(x) + C(x))/A(x)$ is homogeneous of degree zero in x when $\gamma1=\gamma2=0$. Non zero values of $\gamma1$, $\gamma2$ are dominated by x for $\|x\|$ sufficiently large and $\bar{\beta}(r)$ tends to a constant β as $r\to\infty$. Hence for $r0$ sufficiently large, $\bar{I}(r)$ is $\ell n(r/r0)^{\beta}$ and $\bar{\beta}<1$ is sufficient for (3.25). For (3.26) observe that $A(x)$ is homogeneous of degree 2 in x when $\gamma1=\gamma2=0$ and so the degree 2 homogeneity holds arbitrarily closely as $\|x\|\to\infty$. Therefore $\underline{\alpha}(u)$ is approximately cu^2 and $e^{I(u)}/\underline{\alpha}(u)$ is $(u/u0)^{\rho-2}$. $\bar{\beta}<1$ is therefore also sufficient for (3.26).

Note $\beta<1$ just if $\sup(B(x)+C(x))/2A(x) < 1$ for r sufficiently large. We define betabar to equal $\sup\{(B(x)+C(x))/2A(x) \mid \|x\|=r\}$ for large r and calculate it using a one dimensional search procedure. The value of betabar was calculated at $r=10$ and $r=100$ and observed to be the same before it was accepted as the limiting value.

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