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THE DISTRIBUTIONAL APPROACH TO EXCHANGE RATE TARGET ZONES

by

Colin Rose

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CONTENTS

The Distributional Approach to Exchange Rate Target Zones*

Upon the foundation stone laid by Krugman (1991), there has grown a prolific literature examining the effect of target zones on exchange rate behaviour. Without exception, this literature uses the Krugman model as a base, and then expands or modifies it in some manner. This has given birth to models with discrete intervention (Flood and Garber), with imperfectly credible bands combined with realignments (Bertola and Caballero), with constant drift (various), with mean reverting fundamentals (Delgado and Dumas), and with stochastic devaluation risk (Bertola and Svensson), amongst many others. By contrast, this paper presents an alternative perspective on target zones, as outlined recently in Rose (1992). Whereas Krugman models assume that fundamentals follow a stochastic process, we assume that the exchange rate itself is stochastic in a discrete time, continuous state space model. This has five central implications, as summarised below.

The approach is virtuous in its simplicity. In particular, the problem reduces to modelling a stochastic variable with a doubly bounded distribution. As a result, there is no need for stochastic calculus, smooth pasting conditions, nor second-order differential equations. Despite these differences, it is appealing to make comparisons with Krugman models, for it is shown that under a free-float, the exchange rate can follow the same path in both models. Intuitive analytical solutions are easily derived. Moreover, the techniques introduced in this paper have general application to the modelling of bounded stochastic processes.

Secondly, it allows the exchange rate to be viewed in intertemporal space. A target zone is then shown to have two manifestations: (a) it defines a *feasible zone* within which the exchange rate must fluctuate. This *feasible zone* acts as a window through which we view exchange rate activity. Moreover (b), the zone implies that the expected future exchange rate is a non-linear S-shaped function of the current rate. Only a segment of this S-curve will lie within the feasible zone. In the presence of a drift process, the shape of this (internal) segment may not resemble an 'S' at all.

*It is with considerable pleasure that I thank Jeff Sheen for many enlightening discussions, the clarity and elegance of which, have greatly benefited this paper. I am grateful for comments by Lars Svensson.

Thirdly, the paper generates new results. It is shown that under a perfectly credible band, the expected exchange rate will converge (without oscillation) to an intertemporal equilibrium, even when the stochastic process contains a constant drift term. The actual exchange rate is shown to converge stochastically under a rational expectations hypothesis that has expository value. These results extend naturally to the interest differential, and it is shown that a target zone regime converges to the same interest differential as a perfectly fixed exchange rate regime: that is, to a zero differential.

Fourthly, the model appears to fit the facts well, for it can explain empirical phenomena which Krugman models have had problems with. In particular, Krugman models predict that in the long run, the exchange rate should be more frequently near the limits of the band than near the centre of the band. Empirically, the opposite is true; our model predicts this empirical result, and it does so with a perfectly credible band, and without mean-reverting interventions. We also proffer an explanation as to why non linear models of estimation have failed to outperform linear ones.

Finally, by adopting a distributional approach to target zones, rather than using the fundamental asset-pricing approach of Krugman models, the basic model presented here does not need to make any assumption about the existence or absence of market rationality. By contrast, it has been suggested that Krugman models require excessive rationality on the part of investors by imposing saddle-path solutions. Indeed, Krugman and Miller argue that it is this assumption of market rationality that "is the most serious objection to the target zone model" (1992b, p.14).

The paper is structured over 8 sections. *Section I* provides the basic model for a perfectly credible band. *Section II* discusses the S-shaped curve and the feasible zone. *Section III* considers issues of stability (the marriage effect), convergence, and equilibrium. *Section IV* links the model with empirical evidence, and argues that it is consistent, though somewhat paradoxical, for non-linearity to be theoretically important and simultaneously empirically insignificant. *Section V* shows that we expect the interest differential to tend to zero under a perfectly credible target zone regime. *Section VI* illustrates how the distributional approach can be used when the intervention strategy is explicitly known. *Section VII* extends the analysis to intra-marginal intervention. *Section VIII* provides concluding comments and appendices follow.

I. The Model

We proceed by comparing a free float regime with a target zone regime.

Under a Free Float Regime

It is assumed that, under a free float, the exchange rate follows a random walk:

① $s_{t+1} = s_t + k + \varepsilon_{t+1}$ where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ (Gaussian White Noise)

where s denotes the spot exchange rate (domestic price of foreign currency, and measured in natural logarithms), k denotes a constant drift term that may be zero, and ε denotes Gaussian white noise. The above equation implies

$$s_{t+1} \sim N(s_t + k,\ \sigma_t^2)$$

or more generally, for any time period $t + \Delta$, the transition density is...

② $s_{t+\Delta} \sim N(s_t + k\Delta,\ \sigma_\varepsilon^2 \Delta)$ with pdf $\phi(s)$ and distribution function $\Phi(s)$

This process[2] is central to the paper and has three motivations.

The first is empirical, for it is now widely accepted that it is difficult to outperform a random walk in forecasting nominal exchange rates under a free float. Mussa (1979) finds that the natural logarithm of the spot exchange rate follows approximately a random walk. Meese and Rogoff (1983) "find that a random walk model would have predicted major-country exchange rates during the recent floating-rate period as well as any of our candidate models [in terms of out-of-sample forecasting]" (p.3). Similarly, Fratianni, Hur and Kang (1987) state: "If there is a central message to the paper, this involves the reiteration of the robustness of the random walk as a way to explain the behaviour of the spot exchange rate" (p.511). More recently, see Liu and Maddala (1992) amongst many others. This is not to say that exchange rates *should* follow random walks, but merely that it is empirically difficult to distinguish them from random walks. In essence, a random walk is still the benchmark to beat.

The second motivation is analytical: by assuming a random walk, the essential features that distinguish a target zone from a free-float can be elegantly derived.

The third motivation is that in Krugman models, the free float exchange rate follows a random walk (continuous time) – see Appendix B. In essence

[2] Equation ② may also be derived from a continuous-time perspective. In particular, if s follows the absolute Brownian motion $ds = k\,dt + \sigma dz$, where z is a Wiener process, then equation ② still holds, where Δ is now interpreted as a continuous variable.

then, the *free-float exchange rate in this paper* **can** *follow the same* stochastic path as it does in Krugman models. We like to think of this as 'pasting to the literature'. Nevertheless, we stress the word 'can' to emphasise an important distinction between the present paper and Krugman models. As King, Wallace and Weber (1992) have recently shown, exchange rates are able to display randomness unrelated to fundamentals. More generally, tests for excess volatility in financial markets suggest that asset prices may be affected by more than fundamentals – an empirical result consistent with the theoretical literature on fads, bandwagon effects and rational stochastic speculative bubbles. Since Krugman models only capture fundamental sources of randomness, they may not capture the full stochastic nature of exchange rates.

Under a Target Zone Regime

We now establish a perfectly credible target zone which places lower and upper bounds on the exchange rate, so that $s \in (\underline{s}, \bar{s})$. Then, given s_t, the transition density of $s_{t+\Delta}$, conditional on $\underline{s} < s < \bar{s}$ is a doubly truncated normal distribution with pdf:

$$\left(\phi(s) \,\middle|\, \underline{s} < s < \bar{s}\right) = \begin{cases} \phi(s)/\psi & \text{For } \underline{s} < s < \bar{s} \\ 0 & \text{Otherwise} \end{cases} \qquad \text{where } \psi = \Phi(\bar{s}) - \Phi(\underline{s})$$

From an economic standpoint, this statistical technique can be justified from two quite different perspectives. The first perspective argues that the bounds of an announced target zone are always known. Contrariwise, the specific intervention strategy that is used to keep the exchange rate within its bounds is typically unknown, and is almost certainly time variant. In particular, assume that a perfectly credible band is announced but that the intervention strategy that sustains it is not known, so that the announcement of the band is the only difference between the free-float information set Ω_t^{FF} and the target-zone information set Ω_t^{TZ}. If economic agents use all available information under each regime, then in their target-zone calculations, they must replace the free-float pdf $\phi(s)$, with the conditional distribution of s, conditional on $\underline{s} < s < \bar{s}$ as above.
If, contrary to this assumption, there are other differences between Ω_t^{FF} and Ω_t^{TZ}, this information must be modelled explicitly. This approach, where the intervention strategy *is* assumed known, is the second perspective. It is discussed in *Section VI* where we illustrate the explicit intervention strategy that yields the doubly-truncated transition density above.

Exactly half-way between the bounds, there exists a centre value s_0. In this sense, every zone can be thought of as being symmetric around some s_0.

We wish to derive the expected future exchange rate as a function of the spot rate, under each regime. For notational convenience, we define:

③ $\quad {}_t s_{t+\Delta}^{FF} = E\left[s_{t+\Delta} \middle| \Omega_t^{FF}\right]$ — the **free float** expectation of $s_{t+\Delta}$ (conditional on the **free float** information set at time t)

④ $\quad {}_t s_{t+\Delta}^{TZ} = E\left[s_{t+\Delta} \middle| \Omega_t^{TZ}\right]$ — the **target zone** expectation of $s_{t+\Delta}$ (conditional on the **target zone** information set at time t, so that agents know that $s_{t+\Delta}$ lies within the bounds of the target zone.)
where:
$(\underline{s} < s_{t+\Delta} < \bar{s}) \in \Omega_t^{TZ}$

The **free-float** solution (③) is given immediately by equation ② as:

③* $\quad {}_t s_{t+\Delta}^{FF} = s_t + k\Delta$

The **target-zone** solution (④) is given by simply calculating the expectation of a random variable with doubly truncated normal distribution which yields (see Appendix A):

④* $\quad \boxed{{}_t s_{t+\Delta}^{TZ} = s_t + k\Delta - \frac{\delta}{\psi}\sigma^2\Delta}$

where: $\delta = \phi(\bar{s}) - \phi(\underline{s})$

$$\psi = \Phi(\bar{s}) - \Phi(\underline{s}) = Prob[\underline{s} < s_{t+\Delta} < \bar{s}] \text{ under a free float} \qquad (0 < \psi < 1)$$

Thus, the target zone expectation (${}_t s_{t+\Delta}^{TZ}$) is calculated using the pdf and cdf of the *unbounded* free float variable $s_{t+\Delta}$. Since s_t is always known, as are the bounds, ${}_t s_{t+\Delta}^{TZ}$ is fully specified, given σ and k. The expectation of next *period's exchange rate is obtained by simply setting $\Delta = 1$, and yields*

$${}_t s_{t+1}^{TZ} = s_t + k - \frac{\delta}{\psi}\sigma^2$$

which forms the basis for much of our analysis. This result is also intuitive... Under a free float (③*), the expected exchange rate shifts each period by the drift term k. Under a target zone, there are 2 sources of change: a) the drift term k as before, and b) the expected appreciation or depreciation ($\frac{\delta}{\psi}\sigma^2 \gtrless 0$) created by the presence of the target zone itself. This term creates non-linearities[3], and is discussed in greater detail in Proposition 2, below.

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[3]Not only is the exchange rate non-linear under a target zone regime, but it is also heteroskedastic. This can be shown by calculating the variance of a doubly truncated random variable. This is illustrated in Rose (1992) – also, see Svensson (1991) who demonstrates heteroskedasticity within the framework of a Krugman model.

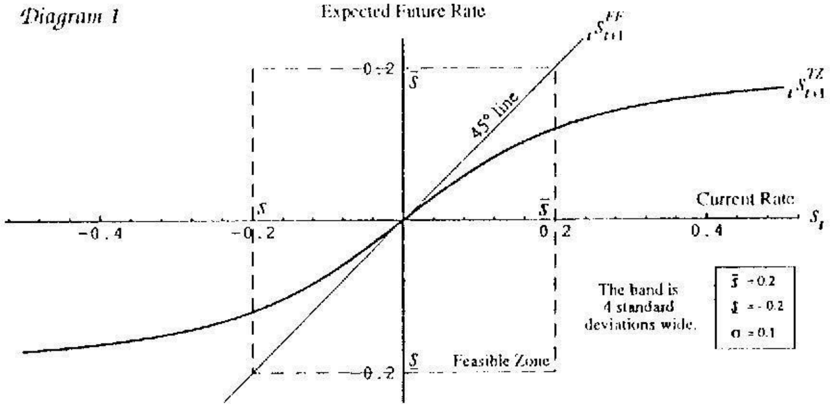
II. *The S-shaped Curve and the Feasible Zone*

The target-zone solution (④*) defines a non-linear Δ-order expectational difference equation ${}_{t}s^{TZ}_{t+\Delta} = f(s_t \mid k, \sigma, \Delta, \underline{s}, \bar{s})$, where f is a function defined by ④*. Plotting ${}_{t}s^{TZ}_{t+\Delta}$ against s_t yields the facinorous S-shaped curve, but now in the form of a phase diagram. This is illustrated[4] in *Diagram 1*, with $\Delta = 1$, and with the drift term $k = 0$ (non-zero drift is discussed later). By contrast, the free-float solution (③*) is just the 45° line passing through the origin (for $k = 0$).

Since $s_t \in (\underline{s}, \bar{s})$, *AND* since ${}_{t}s^{TZ}_{t+\Delta} \in (\underline{s}, \bar{s})$, the bounds of the target zone define a square-shaped feasible zone within which both s_t and ${}_{t}s^{TZ}_{t+\Delta}$ must be located. The presence of an exchange rate target zone thus has two manifestations: namely, a S-curve, and a feasible zone. The intersection set of the feasible zone and the S-curve defines the **relevant segment** of the S-curve, i.e. that segment of the S-curve satisfying $\left(s_t, {}_{t}s^{TZ}_{t+\Delta}\right) \in (\underline{s}, \bar{s})$.

As one would expect with a doubly truncated normal distribution[5], and as apparent from *Diagram 1*, ${}_{t}s^{TZ}_{t+1}$ approaches its bounds asymptotically. Thus, a portion of the S-curve must lie outside the feasible zone, and we see that the tails of the S are 'irrelevant', so to speak, in this zero-drift case.

Diagram 1



Zero Drift

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[4] *Diagram 1* is a computer-generated plot drawn by *Mathematica*. The *Mathematica* package/notebook that generated the basic target-zone diagrams in this paper, is available in Rose (1992).

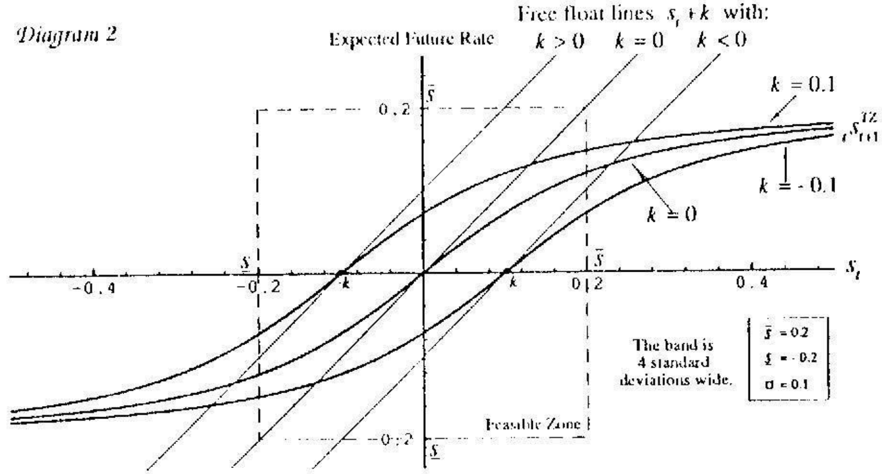
[5] Recall from ④ that ${}_{t}s^{TZ}_{t+1}$ is the expectation of s_{t+1} conditional on s_{t+1} lying within its bounds. Hence, ${}_{t}s^{TZ}_{t+1}$ must always lie strictly within the bounds. By contrast, in Krugman models, the S-curve smooth-pastes to its bounds within some feasible zone. This is discussed further in *Section VI*.

Proposition 1a When $k \neq 0$, the shape of the S-curve does not change; it simply shifts $k\Delta$ units to the left (that is, when $k > 0$, it shifts to the left, and when $k < 0$, it shifts to the right).

Proof ④* can be re-expressed as ${}_{t}s^{TZ}_{t+\Delta} = f^*(\mu \mid \sigma^2\Delta, \underline{s}, \bar{s})$, where $\mu = s_t + k\Delta$, since $s_{t+\Delta} \sim N(s_t + k\Delta, \sigma_\varepsilon^2\Delta)$, and where f^* is assumed to be an invertible function, as S-shaped curves are. Suppose we now constrain our attention to any *specific* value of ${}_{t}s^{TZ}_{t+\Delta}$, which we denote $\hat{s}^{TZ}_{t+\Delta}$. Then, $\hat{s}^{TZ}_{t+\Delta}$ relates uniquely to some specific value of $\mu = s_t + k\Delta$, namely $\hat{\mu}$. This, in turn, defines a unique value of s_t as a function of k, namely $\hat{s}_t = \hat{\mu} - k\Delta$. *Q.E.D.*

Proposition 1a is verified by plotting the S-curve ${}_{t}s^{TZ}_{t+1}$ ($\Delta = 1$) for various values of k. This is done in *Diagram 2*, which plots the S-curve for $k > 0$, $k = 0$, and $k < 0$.

Diagram 2



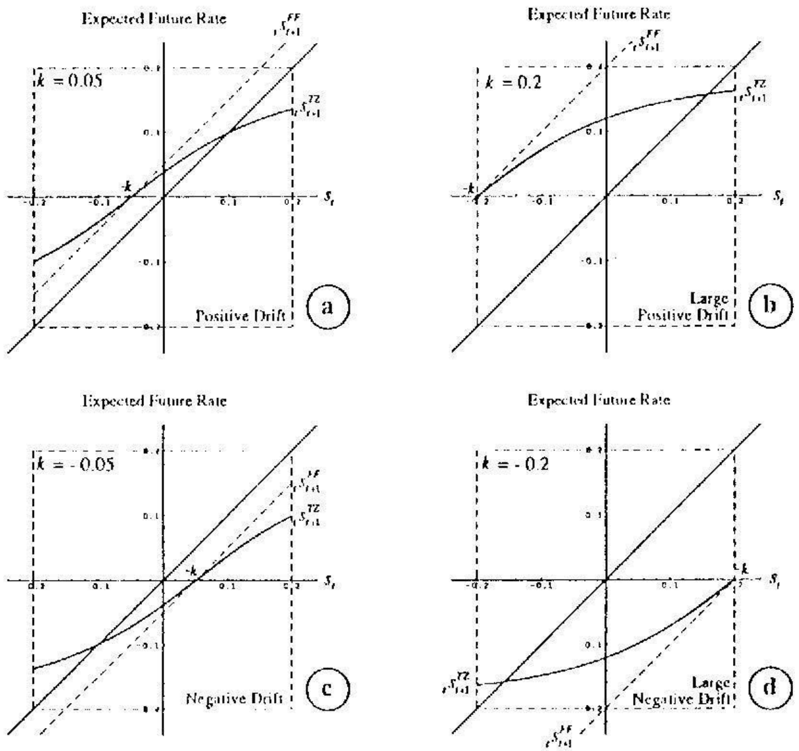
The S-curves *are* horizontally k units away from each other, even though the gap between them 'seems' widest in the middle, and narrowest in the tails. This is a perceptual distortion.

Proposition 1b As k varies, the shape of the **relevant segment** of the S-curve ***does*** change, and may not resemble an 'S'-shape at all.

Proof Follows from 1a: see discussion immediately below.

It is useful to think of the feasible zone as a fixed window through which we view S-curves. Imagine that you are looking through the fixed window at the S-curve in *Diagram 1* where $k = 0$ and $\Delta = 1$. To derive the S-curve for all $k \neq 0$, simply apply Proposition 1a.... that is, put your hand through the window, take hold of the S-curve and pull it k units to the left (by 1a). A different portion of the S-shaped curve is now seen in the window; instead of looking at the middle of the curve, you will now be viewing more of the tails. Thus, although the S-curve itself does not change shape, the relevant segment does change, and may not resemble an S at all! *Diagram 3* (a-d) gives the relevant segment of the S-curve for various k values with $\Delta = 1$. Note that when $|k|$ is large (3b and 3d), the relevant segment will be either concave (3b) *OR* convex (3d), and hence quite unlike an S.

Diagram 3



III. The Marriage Effect and Convergence

Proposition 2 $\left|{}_t s^{TZ}_{t+\Delta} - s_0\right| < \left|{}_t s^{FF}_{t+\Delta} - s_0\right|$ for all $s_t + k\Delta \neq s_0$

$\left(\text{If } s_t + k\Delta = s_0, \text{ then } {}_t s^{TZ}_{t+\Delta} - {}_t s^{FF}_{t+\Delta} = s_0\right)$

The exchange rate is more 'stable' under a target zone regime, than under a free-float.

Proposition 2 is a formal statement of what we term the *marriage effect*, akin perhaps to Krugman's *honeymoon effect*, and refers to the stability created when a government **commits** itself to an exchange rate target zone. All it says is that the deviation of all future expected exchange rates from s_0 (the band's centre) will always be smaller under a target zone regime than under a free float. Although this is self-evident from *Diagram 1*, the proof below formalises the result, and in doing so, provides some intuition about δ.

Proof Since the normal distribution is symmetric around its mean, $\phi(\bar{s})$ will equal $\phi(\underline{s})$ iff the mean of the distribution is the midpoint between $\bar{s}$ and $\underline{s}$. The mean of $s_{t+\Delta}$ is $s_t + k\Delta$, and s_0 is the midpoint. So if $s_t + k\Delta > s_0$, the mean lies closer to $\bar{s}$ than $\underline{s}$, so that $\phi(\bar{s}) > \phi(\underline{s})$, in which case $\delta > 0$. Thus, the sign of δ is given by:

⑤a $\delta \gtreqless 0$ if and only if $s_t + k\Delta \gtreqless s_0$

Since both ψ and σ are strictly positive, ④* implies that:

⑤b ${}_t s^{TZ}_{t+\Delta} \lesseqgtr s_t + k\Delta$ if and only if $\delta \gtreqless 0$

Thus ⑤a and ⑤b yield:

⑤c ${}_t s^{TZ}_{t+\Delta} \lesseqgtr s_t + k\Delta$ if and only if $s_t + k\Delta \gtreqless s_0$

By substituting ${}_t s^{FF}_{t+\Delta}$ for $s_t + k\Delta$ in ⑤c, this may be expressed as:

⑤d ${}_t s^{TZ}_{t+\Delta} - s_0 \lesseqgtr {}_t s^{FF}_{t+\Delta} - s_0$ if and only if ${}_t s^{FF}_{t+\Delta} - s_0 \gtreqless 0$ *Q.E.D.*

As an aside, note that an S-curve is made out of a convex section and a concave section. It is convex for $\mu < s_0$, and concave for $\mu > s_0$, where $\mu = s_t + k\Delta$. When $k = 0$, this simplifies to convex for $s_t < s_0$, and concave for $s_t > s_0$ (as per *Diagram 1*).

Convergence

In this section, we demonstrate exchange rate convergence properties under a target zone regime. We do so over two parts: Part 1 illustrates convergence of the **actual** exchange rate under a rational expectations hypothesis that has expository value. In Part 2, this restrictive assumption is dropped and convergence of the **expected** exchange rate is established.

Part 1 *Convergence of the Actual Exchange Rate*

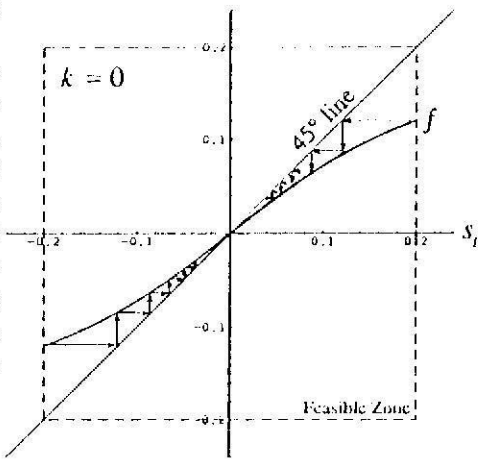
Under a rational expectations hypothesis,

$$s_{t+1} = {}_ts^{TZ}_{t+1} + \xi_{t+1} \qquad \text{where } \xi_{t+1} \text{ is an uncorrelated process}$$

which has both a deterministic component (${}_ts^{TZ}_{t+1}$) and a stochastic component (ξ_{t+1}). In this section, we demonstrate the convergence of the deterministic component. To do so, we ignore[6] the stochastic component so that $s_{t+1} = {}_ts^{TZ}_{t+1}$. With $\Delta = 1$, ④* defines a non-linear expectational first-order difference equation of the form ${}_ts^{TZ}_{t+1} = f(s_t \mid k, \sigma, \underline{s}, \bar{s})$. If $s_{t+1} = {}_ts^{TZ}_{t+1}$, ④* may be re-written as $s_{t+1} = f(s_t \mid \cdot\ )$. Re-writing at $t+1$ and substituting in yields $s_{t+2} = f(f(s_t)) = f^2(s_t)$, and more generally $s_{t+\Delta} = f^{\Delta}(s_t)$. Then the entire time path is immediately given by a single S-curve, now interpreted as a simple phase line, as illustrated in *Diagram 4*, where $k = 0$, with the arrows indicating the time path. Moreover, an S-curve has all the desirable properties we could hope for:

a) its slope (f') is always positive so that the resulting time path is non-oscillatory, and b) $f' \leq 1$, so[7] that the time path is convergent. By definition, the resulting intertemporal equilibrium is achieved when $s_{t+1} = s_t$. Graphically, this occurs at the intersection between the phase line f and the 45° line passing through the origin.

Diagram 4



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[6] Alternatively, this exercise can be interpreted as a perfect foresight assumption.

[7] If f denotes an S-curve, then its slope (f') must reach a maximum at the point of inflexion where the S-curve changes in shape from convex (${}_ts^{TZ}_{t+1} > {}_ts^{FF}_{t+1}$) to concave (${}_ts^{TZ}_{t+1} < {}_ts^{FF}_{t+1}$). At this point ${}_ts^{TZ}_{t+1} = {}_ts^{FF}_{t+1}$: ie where the S curve cuts the free-float line, which has a slope of 1.

Thus, when $k = 0$, equilibrium s^* occurs at the centre of the zone s_0. If $k > 0$, then $s^* > s_0$ (to see why, apply Proposition 1a and pull the S-curve k units to the left: it now cuts the 45° equilibrium line to the right of s_0). Similarly, if $k < 0$, then $s^* < s_0$. We have thus proved Proposition 3a:

Proposition 3a

Under a rational expectations hypothesis, the exchange rate converges to an equilibrium at s^* where:

$$s^* \gtreqless s_0 \qquad \text{if and only if} \qquad k \gtreqless 0$$

By ④*, at any such equilibrium, if ${}_ts^{TZ}_{t+1} = s_t$, then $k = \sigma^2\delta/\psi$, so that in equilibrium, the drift in the exchange rate is exactly offset by the expected depreciation due to the bounds of the target zone.

We have shown that under a rational expectations hypothesis, the actual exchange rate will converge deterministically to a well-defined equilibrium, if the uncorrelated stochastic component ξ_{t+1} is ignored. More generally, when ξ_{t+1} is included, the deterministic path to equilibrium will be replaced by a stochastic path. If k is zero or small, this will yield a hump-shaped ***equilibrium*** density function -- see Appendix D. When the rational expectations hypothesis is dropped, the essence of these results still holds true for the expected exchange rate, as now discussed.

Part 2 *Convergence of the Expected Exchange Rate*

The target zone solution ④* expresses the expected future exchange rate ${}_ts^{TZ}_{t+\Delta}$ as a function of s_t, not only for next period ($\Delta = 1$), but for *all* time periods into the future. For each and every Δ, there corresponds a unique S-curve. The issue of convergence can then be handled within the model, and without additional assumptions, by seeing what happens to the S-curve as Δ increases. If, as argued above, the exchange rate converges as time progresses, then this convergence should be reflected in the shape of the S-curve as Δ increases. The derivation follows....

Stated simply, a target zone is a stochastic process with a lower and upper bound. The *tightness* of this zone is measured by how wide it is in standard deviation units; that is by $w = \frac{\bar{s} - \underline{s}}{\sigma\sqrt{\Delta}}$. Then, as $w \to \infty$, the target zone approaches a free-float. Contrariwise, as $w \to 0$, the target zone approaches a fixed exchange rate regime. Intuitively then, [illegible] the size of the feasible zone will pull the S-curve toward the 45° free-float line for given σ. To obtain the fixed exchange rate solution, let $w \to 0$ by decreasing the size of the band whilst holding σ constant. As $(\bar{s} - \underline{s})$ decreases, the feasible zone shrinks in upon its centre until, in the limit, it is just a point at s_0. A different perspective can be obtained by again letting $w \to 0$, but this time by increasing σ, whilst holding the size of the band constant. In the limit, as $w \to 0$, this must yield the same solution s_0 for every s_t in the feasible zone, even though $(\bar{s} - \underline{s})$ may be physically large (but infinitesimal relative to σ). By doing so, we see that the target zone representation of a fixed exchange rate regime is not just the point s_0, but rather the horizontal line passing through s_0 (i.e. the horizontal axis in *Diagram 1*). This result is argued more formally in Appendix C; namely, that an increase in σ, given the bounds, pulls the S-curve towards the horizontal line through s_0. Re-stating this, as $w \to 0$, the S-curve approaches the horizontal line through s_0. Proposition 3b then follows:

Proposition 3b $\quad {}_t s_{t+\Delta}^{TZ} \to s_0$ as $\Delta \to \infty$ IF $k = 0$

In the absence of drift, the target zone expectation of $s_{t+\Delta}$ converges upon the centre of the band s_0, as Δ increases.

Proof $\quad$ As $\Delta \to \infty$, $w \to 0$, since $w = \dfrac{\bar{s} - \underline{s}}{\sigma\sqrt{\Delta}}$.

Note the similarity between Propositions 3a (actual outcomes) and 3b (expected outcomes). Proposition 3b is illustrated by *Diagram 5a* which plots the S-curve for $\Delta = 1, 2, 4$, and 25, with $k = 0$. *Diagram 6a* provides the time plot of ${}_t s_{t+\Delta}^{TZ}$ as Δ increases from 1 through 20, for six different starting values of s_t.

Proposition 3b was restricted to $k = 0$. Recall that $\mu = s_t + k\Delta$. Hence, when $k \neq 0$, there are two effects associated with an increase in Δ. Firstly, the S-curve will get pulled towards the horizontal axis, as discussed above. Secondly, by Proposition 1a, the S-curve gets shifted $k\Delta$ units to the left. Consequently, the dynamics when $k \neq 0$ are complex. *Diagram 5b* ($k > 0$) and *Diagram 5c* ($k < 0$) provide the S-curves for $\Delta = 1, 2, 4$, and 25. *Diagram 6b* ($k > 0$) and *Diagram 6c* ($k < 0$) provide the time plot of ${}_t s_{t+\Delta}^{TZ}$ as Δ increases from 1 through 20. It is clear that if $k > 0$, the expected exchange rate converges to a value greater than s_0, and if $k < 0$, the expected exchange rate converges to a value smaller than s_0, which yields a more general form for Proposition 3b, and completes the analogy with Proposition 3a.

Diagram 5
The S-curve as Δ changes

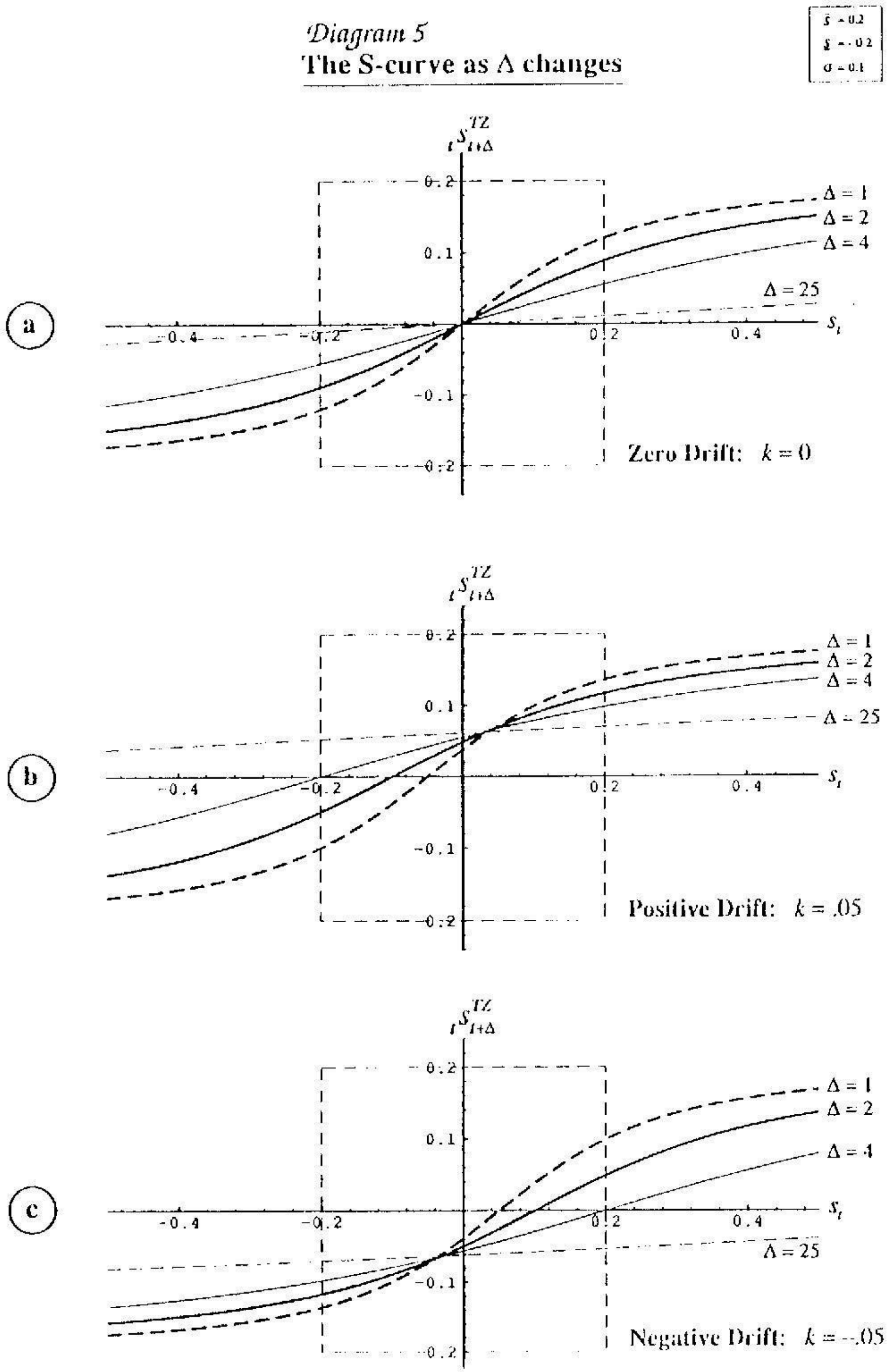
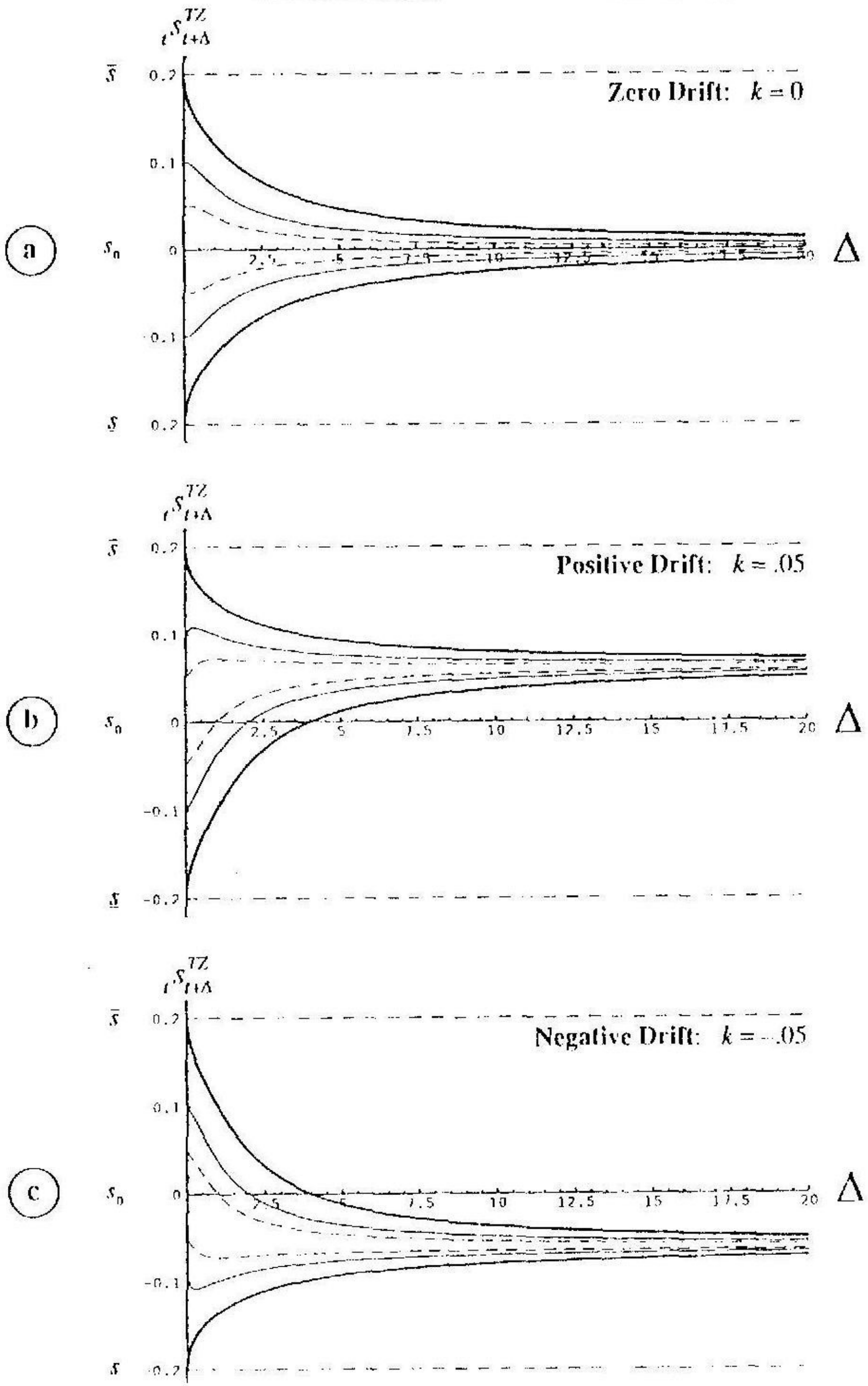


Diagram 6

Time Plots: Plotting ${}_t s^{TZ}_{t+\Delta}$ against Δ



IV. Linking Theory with Empirical Results

— and a paradox on the importance of non-linearity —

Two results stand out from the available empirical literature.

Empirical Result Exchange rates are more frequently found at the centre of their bands than at the limits of their bands, and hence have a hump-shaped distribution (see Bertola and Caballero (1992)).

This is predicted by the present model. Assume $k = 0$ for the moment. Then, it was illustrated that if we assume rational expectations and ignore the uncorrelated stochastic component, the exchange rate will converge deterministically to the centre of the band. More generally, when the stochastic component is included, the exchange rate will converge stochastically to the centre of the band. Finally, dropping rational expectations altogether, by proposition 3b, we will always expect the exchange rate to converge upon the centre of its band, for $k = 0$. Indeed, *unless the drift term is very large*[8] *relative to the band, we will still expect* the exchange rate to converge to a value near the centre of the band.

By comparison, first-generation Krugman models do not fare quite so well... indeed, as Svensson (1991) has shown, standard Krugman models predict that in the long run, the exchange rate should be more frequently near the limits of the band than near the centre of the band! This dichotomy between evidence and theory is reinforced by the work of Smith and Spencer (p.223) who find that Krugman models predict greater kurtosis in exchange rate levels than are observed in reality.

Second-generation Krugman models have attempted to address this issue by introducing more complex intervention policies. For instance, Delgado and Dumas (1992) and Lindberg and Söderlind (1992) propose that in *addition to marginal interventions at the edge of the band, there may be* intra-marginal mean-reverting (leaning-against-the-wind) interventions as well. If so, this *will* yield a hump-shaped distribution (see Lindberg and Söderlind), but this result is achieved somewhat tautologically – with even a small amount of leaning, it is the leaning that does the work, and not the band, as is apparent from Svensson (1993). That is, it is the managed float that generates these results – not the target zone[9] itself. By contrast, we show that the very existence of a perfectly credible band is sufficient to generate the hump-shaped relationship.

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[8] *In any case, one would not expect large drift terms under the exchange rate mechanism of the EMS, although this is an empirical matter itself.*

[9] A second attempt has been made to explain the hump-shaped distribution using the Krugman model. Specifically, Bertola and Caballero (1992) show that *if bands are*

Empirical Result S-curves are of course non-linear. Empirically, however, non-linear estimation would appear to have no consistent out-of-sample advantage over linear estimation: see for instance Meese and Rose (1990), or Flood, Rose and Mathieson (1991).

We proceed in two stages: (a) What are the implications of the model presented here for the importance of non-linearity? and (b) Does this have relevance for Krugman models (and hence for the empirical work to date)?

(a) This paper suggests that it is *because* non-linearity is theoretically important, that it may be empirically insignificant. The theoretical target zone model presented here is non-linear[10]. The non-linearity of the relevant segment of the S-curve is greatest near the edges of the band, and smallest near the centre, assuming k is not large. This non-linearity in expectations is of critical importance because it forces the exchange rate away from the edges, and towards the centre, as discussed above. Thus, under fairly general conditions, we expect to find the exchange rate mostly near its centre where non-linearities are relatively unimportant. We thus have a paradox of sorts: most of the time, non-linearity will be empirically *unimportant* because the exchange rate will be near the centre of the band; why then is the exchange rate near the centre? Because non-linearity is theoretically important (at least near the edges of the band).

(b) The above argument should also apply more generally to Krugman models. In Krugman models, the S-curve represents a non-linear relationship between the actual exchange rate and fundamentals. If, as empirically observed, the exchange rate is most of the time at the centre of its band, then once again, most of the time, non-linearities will be relatively unimportant. The above is a possible explanation of why non-linearity has been empirically insignificant; we do not mean to imply that non-linearity *should* be empirically insignificant in a target zone model. Certainly, there would appear to be a need for conditional empirical tests of non-linearity – conditional on the exchange rates position in the band.

imperfectly credible, if realignments in such bands are large, and if the probability of *realignment is high*, then it is possible to show that the asymptotic distribution of the exchange rate *will* have more weight in the centre than near the boundaries. However, this approach cannot explain why a perfectly credible band will exhibit a hump-shaped distribution. As is apparent from Svensson (1992b), over the period March 1979 - April 1992, expected rates of devaluation have generally fallen. Indeed, some currencies no longer exhibit symptoms of imperfect credibility (the Belgian franc and the Netherlands guilder relative to the Deutsche mark). Yet, these currencies still exhibit hump-shaped distributions. The relevance of this second attempt in explaining the hump shaped distribution should thus be treated with some care.

[10]Note that the non-linearity is limited insofar as the end tails of the S-curve are not relevant, so to speak, since they lie outside the feasible zone: see *Section II*.

V. *The Convergent Interest Differential*

Svensson (1992a) argues that a relatively narrow target zone is likely to exhibit small risk premia. In the case of a perfectly credible band, as per this paper, the risk premium will be insignificant and may be disregarded (see Svensson 1992a). In the absence of risk premia, uncovered interest parity (UIP) implies that the interest differential is equal to the expected rate of depreciation. Proposition 4 then follows:

Proposition 4 Assuming UIP, the interest differential under a perfectly credible target zone is expected to tend to zero with time.

Proof If i_t, i_t^* denote the nominal yield over $(t, t+1)$ on domestic and foreign bonds respectively, then:

$i_t - i_t^* = {}_t s_{t+1} - s_t$ *UIP*

Using ③*, the free-float solution is simply:

$i_t - i_t^* = k$ *Free-Float Solution*

By UIP, the target-zone solution is:

⑥a $i_t - i_t^* = {}_t s_{t+1}^{TZ} - s_t$ *Target-Zone Solution*

Re-writing ⑥a at time $t+\Delta$ yields:

⑥b $i_{t+\Delta} - i_{t+\Delta}^* = {}_{t+\Delta} s_{t+\Delta+1}^{TZ} - s_{t+\Delta}$

By taking the expectations operator through ⑥b, conditional on the target zone information set at time t, and applying the law[11] of iterated expectations, this may be expressed as:

⑥c ${}_t i_{t+\Delta}^{TZ} - {}_t i_{t+\Delta}^{TZ*} = {}_t s_{t+\Delta+1}^{TZ} - {}_t s_{t+\Delta}^{TZ}$

If, as argued above, ${}_t s_{t+\Delta}^{TZ}$ converges as $\Delta \to \infty$, then ${}_t s_{t+\Delta+1}^{TZ} - {}_t s_{t+\Delta}^{TZ} \to 0$, as $\Delta \to \infty$. Thus, the right-hand-side of ⑥c approaches zero as $\Delta \to \infty$. It follows that *under a target zone regime*, it is expected that the interest differential will tend to zero with time. Thus, the target zone regime converges (over time) to the same interest differential as a purely fixed exchange rate regime. *By comparison, the free-float interest differential converges* instantaneously to the drift term (which for example, may reflect inflation differentials).

[11]For any variable x, and an information set $\Omega_t \subset \Omega_{t+i}$, the law of iterated expectations states that $E\{E[x|\Omega_{t+i}] | \Omega_t\} = E[x | \Omega_t]$ $i = (1, 2, 3 \ldots)$.

VI. *Explicit Models of Intervention*

Under a free-float, we assumed that the natural logarithm of the exchange rate follows a random walk, with or without drift. Then next period's exchange rate has distribution $s_{t+1} \sim N(s_t + k,\ \sigma_\varepsilon^2)$; more generally $s_{t+\Delta} \sim N(s_t + k\Delta,\ \sigma_\varepsilon^2\Delta)$, as per equation ②. Suppose we now move to a perfectly credible target zone by announcing lower and upper bounds $(\underline{s}, \bar{s})$ on this distribution. If the announcement of these bounds is the only difference between the free-float information set Ω_t^{FF} and the target zone information set Ω_t^{TZ}, then in our calculations, we must replace the unconditional pdf $\phi(s)$, with the conditional pdf $\left(\phi(s) \middle| \underline{s} < s_{t+\Delta} < \bar{s}\right)$, and this yields precisely the doubly truncated distribution which has been used thus far in this paper. If, however, there are other differences between Ω_t^{FF} and Ω_t^{TZ}, then this specific information must be modelled, as now discussed.

If we (perhaps naively) assume that the underlying stochastic nature of the exchange rate does not change when bounds are added, then at each point in time, the original distribution can be divided into two parts; namely, that part lying *inside* the bounds, and that part lying *outside* the bounds. A defining characteristic of the basic target zone model is that the exchange rate fluctuates freely provided it lies within its band, with intervention only occurring when the exchange rate reaches its bounds. That is, it is assumed that intervention only takes place at the boundaries. In terms of this discussion, this means that the inside pdf remains exactly the same, whilst intervention converts the outside pdf into some internal form, the nature of which will depend on the intervention policy of the authority. In this section, we briefly mention several possible intervention policies, and attach to each of these the relevant *post-intervention* distribution.

Intervention Policy 1 The Doubly Censored Approach

Should the exchange rate reach one of its bounds, the authority intervenes by an amount just sufficient to keep the exchange rate at that bound. This is equivalent to the Cox and Miller (1965, p.61) definition of a reflecting barrier, and is consistent with Krugman-style infinitesimal intervention. Thus, the inside pdf does not change, whilst each outside pdf gets compressed into a point at each bound. This is just the specification of a doubly censored normal distribution. If $\tilde{s}$ denotes this *post-intervention* exchange rate, whilst s denotes the *pre-intervention* exchange rate, then:

$$\tilde{s} = \begin{cases} \bar{s} & \text{if } s \geq \bar{s} \\ s & \text{if } \underline{s} < s < \bar{s} \\ \underline{s} & \text{if } s \leq \underline{s} \end{cases}$$

Intervention Policy 2 The Recto-Normal Approach

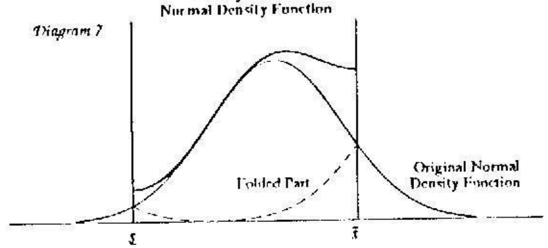
In stark contrast to the doubly censored approach which collapsed each outside pdf into a point at each band, the authority could intervene in such a way as to distribute the outside pdf with uniform density over the inside section. The resulting distribution has been termed the ***Recto-Normal Distribution*** by the author.

The censored approach and the recto-normal approach represent extreme policy positions. However, a convex combination of them yields an entire continuum of possible policy responses.

Intervention Policy 3 The Doubly Folded Approach

Under this scenario, the authority takes the outside pdf and by means of intervention, folds it about the bound, as per *Diagram 7*. This is reminiscent of a light wave reflecting off a mirror; indeed, this approach is equivalent to specifying that the bands are reflecting barriers in the standard sense. The resulting *post-intervention* distribution is obtained by simply adding the folded outside parts to the inside pdf. The resulting distribution has been termed the *Generalised Doubly Folded*[12] *Normal Distribution* by the author.

Diagram 7



This distribution captures rather elegantly[13] the idea that a) in reality, intervention forces the exchange rate into the inside of the zone, rather than just keeping the exchange rate at its bounds; b) given that intervention will typically force the exchange rate back into the band, the size of this intervention is *not* a discrete constant, but is distributed over a continuous state-space; c) that the probability density of intervention (the folded part) decreases with the size of intervention; and d) the closer the current exchange rate is to a bound, the greater the probability that

[12] After Leone, Nelson and Nottingham (1961) who introduce the folded normal distribution as a normal distribution which has a single fold about zero (on the lower tail). This has been used in industrial practice in situations where measurements are recorded in absolute value (hence the fold at zero).

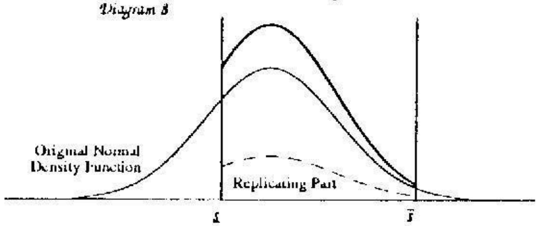
[13] Unfortunately, this distribution is only intuitively appealing when the mean of the distribution lies between the bounds; as a consequence, it should only be used when $k = 0$.

intervention will force the exchange rate towards the centre of the zone (to see this, draw the distribution of the exchange rate when the mean of the distribution (s_t) is near a bound).

Intervention Policy 4 The Doubly Truncated Approach

Here, the inside pdf $\phi(s)$ (defined on $(\underline{s}, \bar{s})$) is considered to be ideal by the authority – see *Diagram 8*. Thus, in converting the outside pdf into an internal form, the authority aims to replicate the internal pdf. This replicating (intervention) pdf will have density $\alpha\phi(s)$ defined on $(\underline{s}, \bar{s})$ where α is a scaling factor that[14] ensures that the area under the replicating pdf equates to the area of the outside pdf. The resulting *post-intervention* distribution is obtained by simply adding the replicating pdf to the inside pdf. The result is just the doubly truncated normal distribution used in *Section I*, but now derived by means of an explicit intervention policy.

Diagram 8



Like the doubly folded approach, this distribution captures the idea that intervention forces the exchange rate back *into* the band, and that the size of such intervention is not a constant, but is itself stochastic with its own density function. This approach has already been discussed in some detail (Appendix A). Statistical properties and algebraic solutions for the other intervention[15, 16] scenarios are provided in Rose (1993).

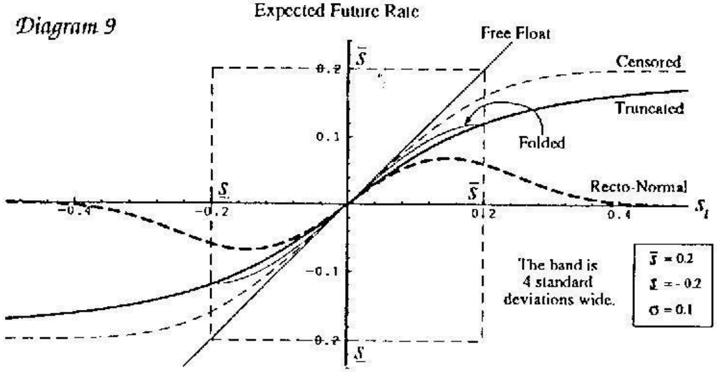
[14] The area of the inside pdf is $\psi = \int_{\underline{s}}^{\bar{s}} \phi(s)ds$. Hence, the area of the outside pdf is $1 - \psi$. Then, the area under the replicating pdf (defined on $(\underline{s}, \bar{s})$) must also be $1 - \psi$. Let its density be $\alpha\phi(s)$. Then α must satisfy $1 - \psi = \int_{\underline{s}}^{\bar{s}} \alpha\phi(s)ds = \alpha\psi$. Hence, $\alpha = \frac{1 - \psi}{\psi}$.

[15] In fundamental asset-pricing models, s is a known function of fundamentals. Thus, a desired exchange rate is achieved by intervening with fundamentals (there is no intervention in the forex market). By contrast, implicit in *Section VI*, a desired exchange rate is achieved by means of direct intervention in the forex market (selling reserves etc).

[16] In Krugman models, the S-curve smooth-pastes to the boundary. Why doesn't this happen in *Diagram 9* below? The answer is easy: Krugman-models yield a S-curve between s_t and fundamentals. Here, the S-curve relation is between the expected future rate and s_t. It is possible to plot this relationship whilst using the Krugman framework. Doing so yields similar results to those shown here (non-linear, S-shaped, no smooth pasting) – see Rose and Svensson (1991, p.6 & Fig.2).

Diagram 9 plots and contrasts the various intervention scenarios discussed above.

Diagram 9



Zero Drift

It is interesting to note that standard Krugman models implicitly assume an intervention policy equivalent to the doubly censored approach. In such models, the S-curve smooth-pastes to the bounds of the target zone, and this effect can be seen in *Diagram 8*. Also note that the centre of the band acts as an asymptote under recto-normal intervention. This is intuitive, for as the mean of the distribution moves away from the centre of the band, the ratio of the outside pdf to the inside pdf steadily increases, until in the limit the entire pdf lies outside the band. By recto-normal intervention, in this limit, the entire pdf then gets distributed uniformly over the inside of the band, yielding an expected value equal to the centre of the band.

Explicit models of intervention require that a) the pdf of the underlying variable does not change following the introduction of a target zone, b) that intervention is triggered at some predictable point, and here we adopted the standard assumption of a trigger at the boundary, and c) that the actual intervention policy is known, i.e. that it is an element of Ω_t^{TZ}. Whereas the bounds of a target zone *are* always known, the particular intervention strategy of the authority is typically an unknown, and is almost certainly not constant. As this section has shown, if the intervention strategy is not publicly available information, there is no unique or obvious way to model intervention. Rather, there exists a whole package of intervention strategies: the censored and recto-normal approaches are hard to justify, but a convex combination of these seems intuitively attractive, as does the doubly folded approach and the doubly truncated approach.

VII. *Extensions*

Empirical evidence for some EMS currencies has caused the target zone literature to expand in two directions: the analysis has extended from purely marginal interventions[17] to intra-marginal interventions (a managed float), and from perfectly credible bands to imperfectly credible bands.

Leaning-against-the-wind is an example of intra-marginal intervention that has attracted some attention (Delgado and Dumas implicitly (1992), Lindberg and Söderlind (1992), and Svensson (1993). Under this scenario, the total sum of intervention is leaning-against-the-wind inside the band (the managed-float), plus a further intervention policy at the edge of the band (the target-zone). There are, of course, an infinite number of possibilities for the latter, but we shall confine our attention to those discussed thus far (*Section VI*). We now have three regimes to contrast: a free-float, a managed-float, and a managed-float *with* a target-zone. It is remarkably simple to add leaning-against-the-wind....

Ignoring drift, the free-float process is given by:

⑦ $s_{t+1} = s_t + \varepsilon_{t+1}$ where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and thus $s_{t+1} \sim N(s_t, \sigma_\varepsilon^2)$

With leaning-against-the-wind, the managed-float process is:

⑧ $s_{t+1} = s_t - \lambda(s_t - s^*) + \varepsilon_{t+1}$ and thus $s_{t+1} \sim N\big((1-\lambda)s_t + \lambda s^*, \sigma_\varepsilon^2\big)$

where $\lambda \in [0, 1]$ measures the degree of leaning, and s^* denotes the desired exchange rate. For simplicity, we set $s^* = s_0 = 0$.

Once again, we wish to derive the expected future exchange rate as a function of the current rate, under each regime.

The **free-float** solution is given immediately by ⑦ as:

$${}_t s_{t+1}^{FF} = s_t$$

The **managed-float** solution is given immediately by ⑧ as:

$${}_t s_{t+1}^{MF} = (1-\lambda)s_t + \lambda s^*$$

The **managed-float *with* target-zone** solution will, of course, depend on the type of target-zone that is implemented. That is, it will depend on what type of intervention takes place at the edge of the band: whether it is doubly-folded, doubly-truncated, et hoc genus omne. In fact, the functional

[17] That is, intervention that is triggered at the margin – as per *Section VI*.

form of the solution has already been derived! To see this, note that the free-float ⑦ and the managed-float ⑧ both share the identical distributional form[18] (normal), and differ only in the parameter μ in this single-period setting. Hence, the managed-float *with* target-zone must have exactly the same functional form as the free-float *with* target-zone which has already been described in this paper. For example, if intervention is doubly-truncated with a managed-float, then the post-intervention transition pdf must once again be doubly-truncated normal. Recall that Appendix A (equation ⑥) calculates the expected value of a random variable with this distribution. Substituting $(1-\lambda)s_t + \lambda s^*$ for μ in ⑥ yields the expected future exchange rate under a managed-float *with* target-zone as:

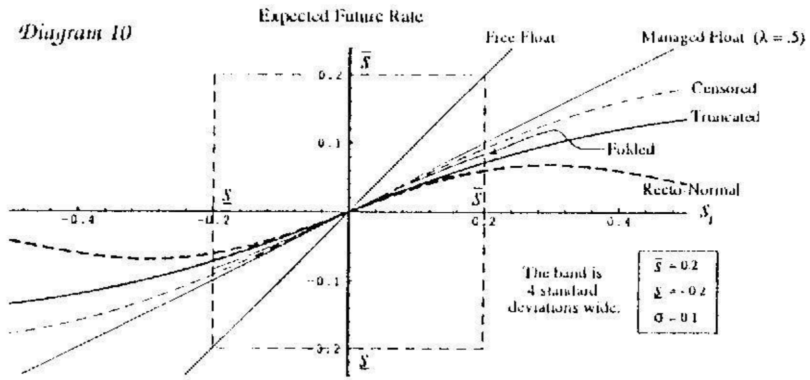
$${}_t s_{t+1}^{MFTZ} = (1-\lambda)s_t + \lambda s^* - \frac{\delta}{\psi}\sigma^2$$

where: $\delta = \phi(\bar{s}) - \phi(\underline{s})$ where $\phi(s)$ now denotes the managed-float pdf of s

$\psi = \Phi(\bar{s}) - \Phi(\underline{s})$ where $\Phi(s)$ now denotes the managed-float cdf of s

More generally, the expected future rate can be plotted as a function of s_t under the three regimes: free-float, managed-float, and managed-float *with* target-zone (various). *Diagram 10* illustrates for $\lambda = 0.5$ and $s^* = s_0 = 0$.

Diagram 10



[18] In continuous time analysis, the free-float process and the managed-float process will still share the same distributional form...
For a free-float, let s follow the absolute Brownian motion $ds = \sigma dz$ where z is a Wiener process. Then, given $s(t)$, the transition pdf is $s(t+\Delta) \sim N(s(t), \sigma^2\Delta)$.
For a managed-float with leaning-against-the-wind, let s follow the Ornstein-Uhlenbeck process $ds = -\lambda s\,dt + \sigma dz$ (for $s^* = s_0 = 0$). Then, given $s(t)$, the transition pdf is:

$$s(t+\Delta) \sim N\left(s(t)\exp(-\lambda\Delta),\ \frac{\sigma^2\left[1-\exp(-2\lambda\Delta)\right]}{2\lambda}\right)$$

This is based on Karlin and Taylor (1981; p.172 and p.217/8).

Imperfectly credible bands are beyond the scope of this paper. Nevertheless, we briefly note that the solution for an imperfectly credible band will have form:

$$ {}_{t}\hat{s}_{t+\Delta} = p \cdot {}_{t}s^{TZ}_{t+\Delta} + (1-p) \cdot {}_{t}s^{Crash}_{t+\Delta} $$

where ${}_{t}\hat{s}_{t+\Delta}$ denotes the expected future exchange rate under an imperfectly credible band, $p \in [0, 1]$ denotes the probability the band still exists at time $t+\Delta$, ${}_{t}s^{TZ}_{t+\Delta}$ denotes the expected future rate conditional on the band surviving, and ${}_{t}s^{Crash}_{t+\Delta}$ denotes the expected future rate conditional on the band collapsing. The latter must of course be modelled in its own right. We merely wish to stress that the perfectly credible result is needed to obtain the imperfectly credible solution.

VIII. Coda

This paper has demonstrated the distributional approach to exchange rate target zones, by assuming that the exchange rate itself is stochastic in a discrete-time continuous state space model. A different perspective on exchange rate target zones was obtained by viewing the exchange rate in intertemporal space. The notion of a feasible zone was introduced, and this zone was used as a window through which S-curves were viewed. This window also defined the relevant segment of the S-curve. The distributional approach proved to be tractable. In particular, the exchange rate was shown to be more 'stable' under a target zone than under a free-float (the so-called marriage effect). Moreover, the expected exchange rate was shown to converge, whilst the stochastic convergence of the actual exchange rate was demonstrated under the assumption of rational expectations. These results extended to the interest differential, and it was shown that a target zone regime converges to the same interest differential as a perfectly fixed exchange rate regime.

The model was also able to explain empirical phenomena which Krugman models have had difficulty with. Specifically, we were able to explain why exchange rates are more frequently found at the centre of their bands than at the limits of their bands, and we were able to do so without introducing intra-marginal interventions. We also provided a possible explanation as to why non-linear models of estimation have failed to yield better results than linear models. Finally, the distributional approach was shown to extend naturally as a means of modelling intervention explicitly, for both a free-float *with* target-zone and a managed-float *with* target-zone.

The interested reader may consult Appendix B which discusses the qualitative similarity between the distributional approach and the standard Krugman model.

Appendix A

The expectation of a random variable with doubly truncated normal distribution.

Let $Z \sim N(0,1)$ with pdf $h(z)$ and distribution function $H(z)$
Let $S \sim N(\mu,\sigma^2)$ with pdf $\phi(s)$ and distribution function $\Phi(s)$

If $Z = \frac{S-\mu}{\sigma}$ one can show that:

❶a $\sigma\phi(s) = h(z)$ ❶b $\Phi(s) = H(z)$

❷ by direct integration $\int_{\underline{z}}^{\infty} z\,h(z)dz = h(\underline{z})$, and similarly $\int_{\bar{z}}^{\infty} z\,h(z)dz = h(\bar{z})$.

❸ It follows immediately that $\int_{\underline{z}}^{\bar{z}} z\,h(z)dz = h(\underline{z}) - h(\bar{z})$.

If we now doubly truncate Z, such that $\underline{z} < Z < \bar{z}$, then the pdf of this doubly truncated variable is $g(z)$:

❹ $g(z) = \frac{h(z)}{H(\bar{z}) - H(\underline{z})}$ for $\underline{z} < Z < \bar{z}$, and zero otherwise.

To see why, recall that the area under a pdf must equal 1. Having doubly truncated the parent distribution $h(z)$, the part that remains constitutes only $[H(\bar{z}) - H(\underline{z})]$ of the unit area of the original distribution.

The expectation of any doubly truncated *standard* normal variable Z is then given by:

❺ $$E\left[Z \mid \underline{z} < Z < \bar{z}\right] = \int_{\underline{z}}^{\bar{z}} z\,g(z)dz = \frac{1}{H(\bar{z}) - H(\underline{z})} \int_{\underline{z}}^{\bar{z}} z\,h(z)dz = -\frac{h(\bar{z}) - h(\underline{z})}{H(\bar{z}) - H(\underline{z})}$$ (by ❸, ❹).

The expectation of any doubly truncated normal variable S, where $S = \mu + \sigma Z$, is then given by:

$$E\left[S \mid \underline{s} < S < \bar{s}\right] = \mu + \sigma E\left[Z \mid \underline{z} < Z < \bar{z}\right]$$ where $\underline{z} = \frac{\underline{s}-\mu}{\sigma}$, $\bar{z} = \frac{\bar{s}-\mu}{\sigma}$

$$= \mu - \sigma \frac{h(\bar{z}) - h(\underline{z})}{H(\bar{z}) - H(\underline{z})}$$ {to get this, apply ❺

❻ $$= \mu - \sigma^2 \frac{\phi(\bar{s}) - \phi(\underline{s})}{\Phi(\bar{s}) - \Phi(\underline{s})}$$ {to get this, apply ❶a and ❶b

From ④, ${}_{t}s^{TZ}_{t+\Delta} = E[s_{t+\Delta} \mid \underline{s} < s_{t+\Delta} < \bar{s}]$ where $s_{t+\Delta} \sim N(s_t + k\,\Delta, \sigma_\varepsilon^2\Delta)$. Now apply ❻ by replacing μ with $s_t + k\,\Delta$, and by replacing σ_ε^2 with $\sigma_\varepsilon^2\Delta$ so that:

④* $${}_{t}s^{TZ}_{t+\Delta} = s_t + k\,\Delta - \frac{\delta}{\psi}\sigma^2\Delta$$ where:

$$\delta = \phi(\bar{s}) - \phi(\underline{s})$$

$$\psi = \Phi(\bar{s}) - \Phi(\underline{s}) = Prob\left[\underline{s} < s_{t+\Delta} < \bar{s}\right] \quad (0 < \psi < 1)$$

Appendix B

This appendix converts the target solution ④ into a form comparable to the solution of Krugman models.*

The essence of a Krugman-style model is the starting equation:

❼ $$s = v + \gamma E(ds)/dt \quad \text{where} \quad dv = \eta dt + \sigma dz$$

where v denotes economic fundamentals and follows a continuous-time random walk with η and σ constant, and where z denotes a standard Wiener process. Under a free-float, $E(ds)/dt = 0$ in a bubbles-free world, so that the exchange rate in a Krugman-style model follows the same continuous-time random walk as v (that is, $ds = dv = \eta dt + \sigma dz$). In this paper, the free-float path of the exchange rate is given by equation ① and we see that the exchange rate can[19] follow precisely the same random walk, but now in discrete time. In other words, the free-float exchange rate can follow the same stochastic process in both models. Both models then impose the same target zone bounds $(\underline{s}, \bar{s})$. It should then be intuitively appealing that both derive qualitatively similar solutions. The essence of the similarity is that ④* can be expressed as ${}_t s^{TZ}_{t+1}$ = Free Float $\frac{\delta}{\psi}\sigma^2$ where $\frac{\delta}{\psi}\sigma^2$ = expected depreciation due to target zone regime, whilst any solution satisfying equation ❼ can be written s = free float rate − expected depreciation due to target zone regime. This appendix shows this qualitative similarity formally...

The solution of the present model was given by equation ④*. Setting $\Delta = 1$ for simplicity and using ③* allows the solution to be expressed as:

$${}_t s^{TZ}_{t+1} = \text{Free Float} - \frac{\delta}{\psi}\sigma^2 \qquad \text{where } \delta \text{ and } \psi \text{ may be expressed as:}$$

$$\delta = \frac{1}{\sigma\sqrt{2\pi}}\left[e^{-\bar{z}^2/2} - e^{-\underline{z}^2/2}\right] \qquad \text{where} \quad z = \frac{s-\mu}{\sigma}$$

$$\psi = \int_{\underline{z}}^{\bar{z}} h(z)dz = \frac{A^*}{\sqrt{2\pi}} \qquad \text{where } A^* = \int_{\underline{z}}^{\bar{z}} e^{-z^2/2}dz > 0$$

It follows that:

$${}_t s^{TZ}_{t+1} = \text{Free Float} - \frac{\sigma}{A^*}\left[e^{-\bar{z}^2/2} - e^{-\underline{z}^2/2}\right] \qquad (A^* > 0)$$

By comparison, with symmetric bounds, the Krugman model has solution (see equation 8 of Krugman 1991):

$$s = \text{Free Float} - A\left[e^{\rho v} - e^{-\rho v}\right] \qquad \rho = \frac{1}{\sigma}\left(\frac{2}{\gamma}\right)^{1/2} \quad \text{and} \quad A > 0$$

Note however that whereas A is a constant in the Krugman model, A^* is a variable.

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[19] Once again, we use the word *can* to emphasise a distinction between the present paper and Krugman-style models. As King, Wallace and Weber (1992) have shown, exchange rates can display randomness unrelated to fundamentals. Since Krugman models only capture fundamental sources of randomness, they may not capture the full stochastic nature of exchange rates. By contrast, this paper treats the stochastic nature of the exchange rate as an empirical phenomenon, and thus avoids such problems.

Appendix C

To Prove: an increase in σ causes the S-curve to get pulled towards the fixed-exchange rate solution (the horizontal axis), as discussed on pages 11 and 12.

Using ④* and the quotient rule, it follows that:

❼ $$\frac{\partial\, {}_t s^{TZ}_{t+\Delta}}{\partial \sigma} = -\delta\left(\frac{\Delta\sigma}{\psi}\right)J \qquad \text{where } J = \left[2 + \frac{\partial \delta}{\partial \sigma}\frac{\sigma}{\delta} - \frac{\partial \psi}{\partial \sigma}\frac{\sigma}{\psi}\right]$$

The gist of the proof is then quite simple. Suppose J is always positive. Then, the sign of $\frac{\partial\, {}_t s^{TZ}_{t+1}}{\partial \sigma}$ is given by $-\delta$, since Δ, σ and ψ are all strictly positive. Now, the sign of δ has already been derived in ⑥$_\sigma$ of Proposition 2. Using this information, it follows immediately that:

$$\frac{\partial\, {}_t s^{TZ}_{t+\Delta}}{\partial \sigma} \lesseqgtr 0 \quad \text{if and only if} \quad s_t \gtreqless s_0 - k\Delta$$

Thus, if $s_t > s_0 - k\Delta$ (the concave part of the S-curve), an increase in σ causes ${}_t s^{TZ}_{t+\Delta}$ to fall, so that the S-curve approaches the horizontal axis. Alternatively, if $s_t < s_0 - k\Delta$ (the convex part), an increase in σ causes ${}_t s^{TZ}_{t+\Delta}$ to rise, so that the S-curve once again approaches the horizontal axis. It is easy to see this from *Diagram 1* where $k = 0$.

The above proof is complete if J is always positive. J is positive, but the derivation of this result is less than elegant, and requires numerical analysis at the final stage......

Recall that $\phi(x)$ denotes the pdf of $s_{t+\Delta} \sim N(s_t + k\Delta, \sigma_\varepsilon^2\Delta)$.

Then, $\phi(x) = \frac{1}{\sigma\sqrt{2\pi\Delta}}\exp\left[-\frac{(x-\mu)^2}{2\Delta\sigma^2}\right]$ and $\frac{\partial\phi(x)}{\partial\sigma} = \frac{1}{\sigma}\left[\frac{(x-\mu)^2}{\Delta\sigma^2}\phi(x) - \phi(x)\right]$. Thus...

❺ $$\frac{\partial\delta}{\partial\sigma} = \frac{\partial\phi(\bar{s})}{\partial\sigma} - \frac{\partial\phi(\underline{s})}{\partial\sigma} = \frac{1}{\sigma}\left[\xi - \delta\right] \qquad \text{where } \xi = \frac{(\bar{s}-\mu)^2\phi(\bar{s}) - (\underline{s}-\mu)^2\phi(\underline{s})}{\Delta\sigma^2}$$

❻ $$\frac{\partial\psi}{\partial\sigma} = \int_{\underline{s}}^{\bar{s}}\frac{\partial\phi(x)}{\partial\sigma}dx = \frac{1}{\sigma}\left[\frac{1}{\Delta\sigma^2}\int_{\underline{s}}^{\bar{s}}(x-\mu)^2\phi(x)\,dx - \int_{\underline{s}}^{\bar{s}}\phi(x)\,dx\right]$$

$$= \frac{1}{\sigma}\left[\frac{E\left[(x-\mu)^2 \,\middle|\, \underline{s} < x < \bar{s}\right]\cdot\psi}{\Delta\sigma^2} - \psi\right]$$

$$= -\frac{\psi}{\sigma}\left[1 - \frac{\text{Conditional Variance}}{\text{Actual Variance}}\right]$$

$$= \frac{\psi}{\sigma}\lambda \qquad (0 < \lambda < 1)$$

Moreover, it is possible to show that $\lambda = \Delta\sigma^2\frac{\delta^2}{\psi^2} + \frac{1}{\psi}\left[(\bar{s}-\mu)\,\phi(\bar{s}) - (\underline{s}-\mu)\,\phi(\underline{s})\right]$

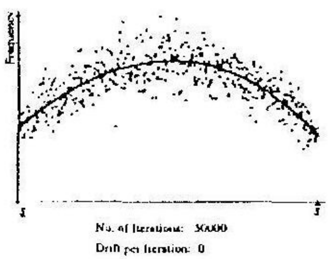
Substituting ❺ and ❻ into ❼ yields $J = \left[1 + \frac{\xi}{\delta} + \lambda\right]$ where ξ and λ are defined above. Numerical analysis can now be easily conducted and indicates that $\inf(J) = 0$.

Appendix D

Simulating the Equilibrium Distribution of the Exchange Rate when the Transition Density is Doubly Truncated Normal

In *Sections III* and *IV*, we argued that if the drift parameter is not large relative to the band, then the exchange rate will converge to a value near the centre of the band. That is, the exchange rate will exhibit a hump-shaped *equilibrium* density function. To verify this, we simulate the *equilibrium* density by letting the exchange rate iterate for 75000 time periods, given that its *transition* density each period is doubly truncated normal.

Zero Drift



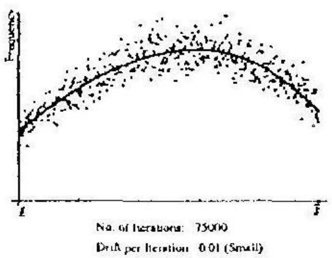
$\bar{s} = 0.25$

$\underline{s} = -0.25$

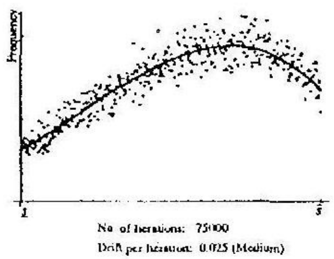
$\sigma = 0.15$

The smooth line is a fitted 3rd order polynomial regression.

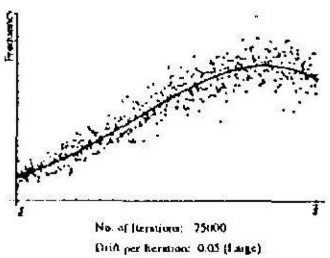
With Small Drift



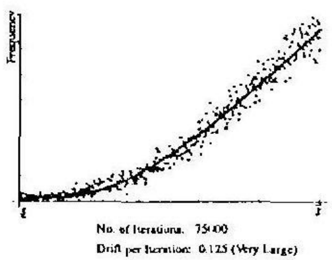
With Medium Drift



With Large Drift



With Very Large Drift



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