

WORKING PAPERS IN ECONOMICS

MULTIPLE REPUTATIONS IN
FINITELY REPEATED GAMES

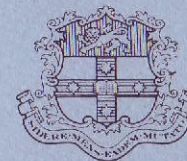
by

Shaun Hargreaves-Heap
&
Yanis Varoufakis

No. 129

October 1989

DEPARTMENT OF ECONOMICS



The University of Sydney
Australia 2006

MULTIPLE REPUTATIONS IN
FINITELY REPEATED GAMES

by

Shaun Hargreaves-Heap
&
Yanis Varoufakis

No. 129 October 1989

National Library of Australia Card Number and ISBN 0 949269 92 1

I. Introduction

A reputation is an end in itself for many people. But for others, a reputation is only valued instrumentally as a means to some other end. This paper is concerned with the possibly less noble, instrumental sort, and it builds on the Kreps and Wilson (1982) analysis of such reputations in repeated games.

The Kreps and Wilson approach is not uncontroversial and we sidestep the controversy here.¹ We use it because we believe it offers many interesting insights on reputation formation, and we note that recent experimental evidence is also favourable with respect to their predictions on reputation (see Camerer and Weigelt (1988)).

In doing this, we are following Fudenberg and Kreps (1987). They add breadth to the original Kreps and Wilson by considering a case where a single defender faces multiple opponents simultaneously. This yields a concept of multiple reputations where there is a reputation for each opponent. In this paper, we add another dimension to the concept of multiple reputations by allowing each player in the finitely repeated 2 person game to have more than one reputation. A reputation is contingent on the arena of the contest and two players may be engaged in contests with each other in more than one arena. This development produces further insights on gaming when reputation is at stake which depend on the assumed links between these multiple reputations. In particular, it offers a formal account of such well known pieces of strategic advice like 'attack is the best form of defence', 'puppy dog submission' and Machiavelli's paeon on imitating both the 'fox and the lion'.

Section II reviews the original Kreps and Wilson model, where each player has a single reputation, because this is a building block in the subsequent analysis (see Kreps and Wilson (1982) and Fudenberg and Kreps (1987) for a full analysis of these basic contests). Section III considers contests involving multiple reputations, in our sense, under four different informational assumptions concerning the relationship

between reputations in different arenas. Finally, Section IV offers some illustrations where these models of multiple reputations have an economic significance.

II. Conflict and Reputation

1. The One-shot Game:

The backbone of the analysis is a simple contest which is fully characterised by table 1. The challenger must choose between launching an 'attack' and 'holding back'; and the defender deliberates, when faced by an attack, between an aggressive response and capitulation.

The outcome depends on each players' pay-offs and his/her beliefs about the type of opponent s/he faces. There are two types of player, 'weak' and 'strong', where the pay-offs to a 'weak' player are given by the vectors in columns 2 and 4 and 'strong' players' pay-offs are captured by the vectors in columns 1 and 3; and the probability pair [p,q] captures the beliefs of each player about the type of opponent they face.

Reputation matters even in a one shot version of this game. Thus, of instance, a weak challenger will attack when the reputation of the defender for strength is such that $p < [b/(1+b)]$, and a weak defender will acquiesce. Hence, a weak defender has reason to value a reputation for belligerence (p) since it can prevent an attack by a weak challenger. When such games are repeated, the sequential equilibrium concept throws light on how such reputations can be built.

		Challenger (A)		Defender (B)	
		'strong'	'weak'	'strong'	'weak'
Challenger (A)	holds back	0	0	a	a
	attacks and defender(B) fights	c	-1	0	-1
	attacks and defender acquiesces	b	b	-1	0
	probabilities (reputations)	q	1-q	p	1-p

table 1

2. Finite horizon, continuous repetition and one-sided uncertainty:

We begin by assuming the challenger is weak (i.e. set $q=0$). The countdown of the game begins at $t=1$ and ends at $t=0$. At any point of time in this interval, the challenger can launch an attack which will be met either by a fight or acquiescence. In the latter case, since only weak defenders acquiesce, the game continues in this vein with b and 0 pay-offs per period until the game ends at $t=0$. In the case of the conflict, the challenger receives -1 as does a weak defender while a strong one gets 0 . The conflict lasts for a vanishing short period and may be followed by further challenges.²

The strong defender will always fight an attack, but the weak defender has to decide whether reputation building through fighting dominates the acquiescence strategy. The appropriate decision rule specifies the probability with which the weak defender acquiesces as a sequential equilibrium. We offer a sketch here of that equilibrium.

The basic idea is that some initial doubt about the type of defender can radically alter the game as contrasted with the case of perfect information. When the defender is known to be weak, then attack will definitely occur in the last moment of the game and the backward induction process takes hold to produce the perfect equilibrium of attack and acquiescence throughout the interval. However, if there is a possibility that the reputation of the defender in the last instance is such that an attack at that moment is not guaranteed (i.e. $p \geq b/(1+b)$), then an alternative sequential equilibrium becomes possible. In particular, it is possible for a weak defender to start from a smaller initial p_0 (prior belief of the challenger concerning the defender $=p_0$) and build a reputation, by fighting attacks in earlier instances of the contest, which makes attack in the last instance uncertain. And once attack in the last instance is uncertain, the backward induction process which is responsible for the perfect equilibrium does not take hold. Instead, this process of reputation building helps define the alternative sequential equilibrium.

Reputation building of this sort is only possible when there is some non zero probability of the challenger attacking, otherwise the defender never gets the opportunity to demonstrate strength through fighting. So, consider the challenger at time t : s/he fears not only a

strong defender but also a weak one who is playing tough for reputation purposes. The expected return from attacking in these circumstances is given by the left hand side of equation (1). When this equals 0, the return from not attacking, the challenger will be indifferent between the two strategies and will randomise.

$$bt(1-p_t)\pi_t - [1-(1-p_t)\pi_t] = 0 \quad \dots (1)$$

where π_t is the probability that a weak defender gives in following an attack in time t .

The defender knows this and will set π_t as in (2) to make the challenger randomise.

$$\pi_t = 1/(1-p_t) (1+bt) \quad \dots (2)$$

A display of toughness under this rule will improve the reputation of a defender. In the challenger's eyes, the probability of acquiescence during the interval $(t, t+\delta t)$ is $(1-p_t)\pi_t \delta t$ up to terms that are $O(\delta t)$. By applying Bayes' rule, this means the reputation of the defender will evolve as in (3).³

$$\dot{p}_t = p_t (1-p_t)\pi_t \quad \dots (3)$$

Substitution of (2) into (3), followed by integration under the assumption that p must reach a value in the last instance which makes attack uncertain, gives the time path for p in (4) (see figure 1).⁴

$$p = (1+bt)^{-1/b} \quad \dots (4)$$

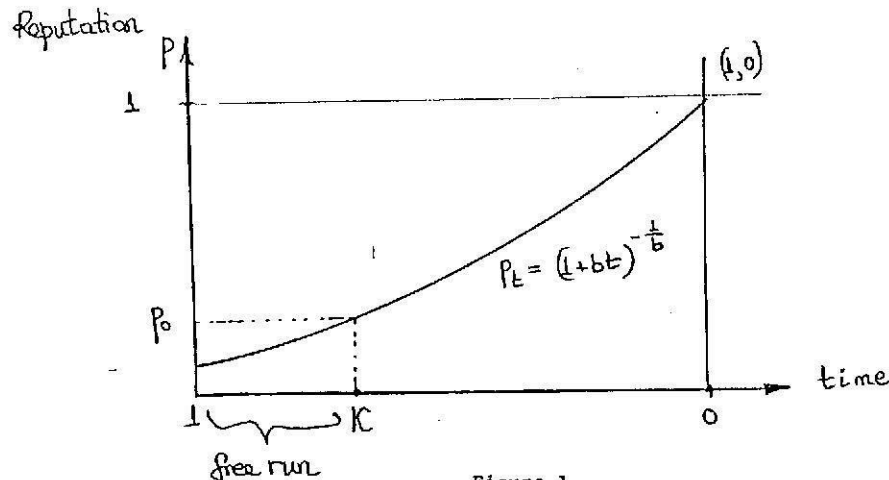


Figure 1

This defines the time path for p under this process of reputation building. It is a risky business for the defender and it may not reach this conclusion because there is always a chance that a weak defender will acquiesce (since $\pi_t \neq 0$ when reputation building is happening), at which point p would drop to 0 and there would be attack followed by acquiescence in the remainder of the contest. Hence the defender will not embark on reputation building until it becomes necessary. Accordingly, when the initial p exceeds the value given by this path, there is no need for the defender to risk his/her reputation. The defender will set $\pi=0$ since s/he does not have to encourage the challenger yet for reputation building purposes, and consequently the challenger will hold back. This gives the defender a free run period in the game given by $1-k$ in the figure, where k is defined in (5).

$$k = [1/b][p_0^{-b} - 1] \quad \dots (5)$$

To summarise, the sequential equilibrium is characterised by a period where the challenger holds back and this is followed by attack with some non-zero probability, which will be met by a fight with a probability less than 1. When a fight occurs, reputation is enhanced and the attack with some non zero probability which is met by a fight with probability less than 1 recurs. But, when there is acquiescence,

reputation drops to zero and attack and acquiescence occurs throughout the remainder of the game.

3. Two-sided uncertainty

Reputation also matters when there is doubt over the type of a challenger (i.e. $q \neq 0$). In the one shot version of the game, a weak defender expects returns in 6 from a strategy of fighting.

$$(1-q)a - [1-(1-q)a] \quad \dots (6)$$

where $1-a$ is the probability of an attack from a weak challenger.

If a weak defender could credibly commit him/herself to fight when attacked, then this commitment would make sense only when (6) is greater than 0 (i.e. when (7) holds).

$$a > 1/(1-q)(1+a) \quad \dots (7)$$

Note that in the single shot version of the game $a = \Pr(p > b/(1+b))$. Hence, reputation matters for the challenger because the larger is q the less likely is 7, and so the less likely is it that a weak defender will commit themselves to fighting an attack when this option is available.

Of course, it is precisely this option which is available to the defender in the continuous version of the game in section 2. The defender can credibly commit themselves to fighting, at least with some probability $(1-\pi_t)$, because it helps to build a reputation. Hence, continuous repetition of the game over the period (1,0) when there is two sided uncertainty now gives the challenger an opportunity to build a reputation to good effect as well. By attacking when weak, the challenger can take the sting out of defender's tail by undermining his/her reputation building strategy of fighting when there is an attack. Formally, the game has been transformed into one of chicken/attrition and the side which chickens out first loses their reputation, and along with it the strategic advantage which comes from having one. Specifically, the challenger attempts to undermine the resolve of the defender by setting a as in (8), as this makes the expected returns to the defender from fighting when weak equal to zero.

$$(1-q_t)a_t = 1-(1-q_t)a_t \quad \dots (8)$$

In effect, the weak challenger is randomising with a probability given by (9). Application of Bayes' rule gives the evolution for the challenger's reputation in (10), which will be traced provided the reputation is not compromised under the randomisation strategy by the occurrence of a non-attack.

$$a = 1/(1-q_t)(1+a_t) \quad \dots (9)$$

$$\dot{q}_t = q_t(1-q_t)a_t \quad \text{or} \quad q_t = (1+a_t)^{-1/a} \quad \dots (10)$$

The game proceeds as follows. At $t=1$, $[p_0, q_0]$ denotes each player's beliefs about their opponents (the reputation of each player). When

$$p_0 < q_0 [(1+b)/(1+a)]^{a/b} \quad \dots (11)$$

the defender is forced to randomise first with a probability which is sufficient to turn (11) into an equality, in the event of a randomisation producing the fight option. Otherwise, the game ends immediately with the challenger always attacking. When the inequality in (11) is reversed, the roles are reversed. Either this part of the game ends immediately because the challenger does not attack, or the equality is achieved. Once this has been achieved from either initial condition in 11, the future contest is conducted with symmetric randomisations given by (2) and (9), while (4) and (10) give the evolution of each reputation.

III. Multiple Reputations

1. One-shot and Repeated Games with Reputational Isolation:

The asymmetry of the basic game, which makes one player a challenger and the other a defender, can be avoided by allowing the roles to be reversed in another arena. Player A becomes the defender and B becomes the challenger in the second arena. The structure of the game in the second arena is identical to the first and is encapsulated by table 2. The full contest is set out in extensive form in figure 2.

Challenger (B)	Challenger (B)		Defender (A)	
	'strong'	'weak'	'strong'	'weak'
holds back	0	0	f	f
attacks while A fights	e	-1	0	-1
attacks while A acquiesces	d	d	-1	0
probabilities/reputations	r	1-r	s	1-s

Continuous repetition of this contest in the unit interval (1,0) allows each player to use the initial uncertainty to build reputations. In the simple case where the two information sets [p,q] and [r,s] are independent, the contest is fully separable into two games each fully characterised by the analysis of section II.3.

To illustrate this application, we shall consider the one-sided version where both challengers are known to be weak (q=r=0). The initial defensive reputations are given by [p₀,s₀]. Both arenas will experience peace for time 1-k and 1-k' respectively, where k' is given by (12) in analogous fashion to 5.

$$k' = (1/d)[s_0^{-d} - 1] \dots (12)$$

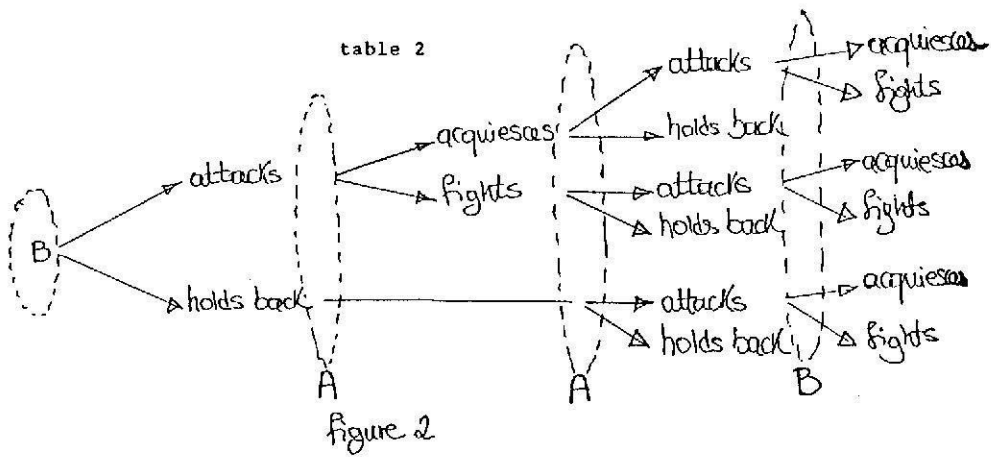
After t=k, A will attack B with a non-zero probability and B will reciprocate in the second arena after t=k'. The development of the conflict in arena 1 follows that already sketched in II.2. And the conflict in arena 2 follows a similar pattern, so long as B's attacks are not successful because they are fought by A whose reputation will be evolving along the path given in 13. Hence, who attacks first depends on initial reputations and the relative sizes of the pay-offs b and d, and the subsequent development of the contest depends on the respective randomisations once the free run periods in each game are exhausted.

$$s = [1+d.t]^{-\frac{1}{d}} \dots (13)$$

Although, we shall argue in section IV that there are some interesting examples of contests which fit this structure, nothing new has been added to the literature on reputation formation by this analysis. The idea of multiple reputations here is no different than the addition of several independent single reputations. It is in the next three sections that the idea of multiple reputations is given new content.

2. Single Reputation revisited with coincident information sets:

We consider the opposite informational assumption in this section by allowing the two reputations for strength to collapse into one (i.e. p=r and q=s). Each player is thought to be either weak or strong in both



When the contest between the two players occurs simultaneously in the two arenas and it is played once and there is informational isolation between the reputations, we note the following cases can materialise:

- (i) when p=q=r=s=0
Both players will attack in the arena where they can take the initiative as challengers and acquiesce as defenders.
- (ii) when p=q=r=s=1
There is 'total war', with attacks followed by fights in both arena.
- (iii) when p > b/(1+b), s > d/(1+d), q=r=0
There is no conflict as both A and B recognise each other's reputations for defence are high enough to deter attack.
- (iv) when p < b/(1+b), s < d/(1+d), and q=r=0
A(B) attacks in arena 1(2) but B(A) refrains from attacking in arena 2(1).

arenas, and a failure to attack or fight in either arena is interpreted as a reflection of overall frailty. In these circumstances, each player has a restricted set of possible actions because of the informational constraints placed on each reputation.

The simplest way to model this contest is by assuming there are simultaneous moves in both arenas. In effect, the two arenas collapse into one and there is a game of two-sided uncertainty where each player's action set is a joint move combining either 'attack and fight' or 'hold back and acquiesce'. During the period of chicken in this contest, B and A will randomise with probabilities of acquiescence given respectively by: $2/((1-p_t) ((f+b)t + 2))$; $2/((1-q_t) ((a+d)t + 2))$. While the battle is raging, the reputation of each player evolves along the paths of (14).

$$\dot{p}_t = 2p_t/((f+b)t + 2) \quad \dot{q}_t = 2q_t/((a+d)t + 2) \dots (14)$$

Let us suppose A chickens out first. A is known to be a weak defender in arena 2 and a weak attacker in arena 1. This means that in arena 2, A will acquiesce as B attacks. However, there remains the possibility of A launching intermittent attacks on B in arena 1 because the game in that arena has become one of one-sided uncertainty, as in section II.2. As far as B is concerned, the interesting point to note is that the chicken phase of the game, where there are attacks in both arenas, will have raised the defensive reputation which B takes into the one-sided uncertainty part of the game in arena 1 once A 'chickens out'. For B, this demonstrates neatly the adage that 'attack is the best form of defence'. Thus, we infer proposition 1.

Proposition 1:

When two players each have single reputations for strength which are tested in attack and defence, and play each other simultaneously in two arenas a finite number of times, where the roles of challenger and defender are reversed between arenas, the play of the contest corresponds in part to the adage that 'attack is the best form of defence'.

3. Interdependent Reputations

Binmore (1987) has recently observed that 'naive Bayesian rationality apparently endows its fortunate adherents with the capacity

to pluck their beliefs from the air' (p. 211). It is unfortunate that priors appear in this manner out of thin air. However, we believe that the analysis of this section and the next can go some way towards removing this anomaly by linking the reputation in one game to the reputation which has been built in another game. Naturally, this will still leave the criticism to worry some original set of beliefs which provide the anchors for others via linkage, but at least these will now be smaller in number. Furthermore, as the examples in part IV of the paper will illustrate, we believe there are interesting economic examples of reputational linkages.

We first set $r=0$. Thus, B is known to be weak as an attacker and only has a reputation as a strong defender in this version of the contest. This produces a chicken game in arena 1 and a game of one-sided uncertainty in arena 2. Conflict occurs immediately at $t=1$ in the chicken game and at this stage B will not be planning a counter attack in arena 2 until period k' (as defined in 12) has been reached. Consequently, A will be expecting a return of $(1-k')f$ from the arena 2 part of the contest.

The twist to the analysis now comes from making A's reputation as a strong defender (s) depend positively on his/her reputation for strength in attack (q), as in (15).

$$s = g(q) \text{ where } g' > 0 \text{ for all } q > 0 \dots (15)$$

As the s reputation is not tested in arena 2 until k' , it will only be affected initially by developments in arena 1 where q is affected. Both p and q are revised during the game of chicken in arena 1, and a rise in q in this part of the contest will fuel s via (15). This improvement in s raises the free run period in arena 2 via (12). Let x be the increase in the pay-offs to A in arena 2 from the continuation of the chicken game in arena 1 for an infinitesimally short period. The expected payoffs at t in arena 1 for A must now be slightly altered to reflect this gain which will be lost if B acquiesces there. (16) replaces (1) in the analysis.

$$(1-p_t) * p_t (bt-x) - (1-[1-p_t] * p_t) \dots (16)$$

And B's equilibrium strategy is, as before, to set x so that this is equal to zero.

$$r_c = 1/(1-p_c) (bt-x+1) \dots (17)$$

$$\text{where } x = [d/dt] [(1-k')f] - [\partial(1-k')f/\partial k'] [\partial k'/\partial s] [\partial s/\partial q] [\partial q/\partial t] \\ - f s_o^{-(d+1)} [\partial s/\partial q] [q_c/(1+at)]$$

Hence, the greater the responsiveness of A's defensive reputation to his/her offensive reputation ($\partial s/\partial q$), the greater the probability that B will acquiesce in arena 1. The linkage of the reputations makes B more apprehensive about the chicken part of the contest because although it would be nice to win it, the cost of sustained conflict there is compounded by the gains A is reaping in the other arena as the free run period is extended. Thus, B is more willing to yield a defensive reputation in arena 1 because this enhances his or her offensive possibilities in arena 2. In short, this type of linkage between reputations helps make sense of the puppy dog strategy where players 'roll over' (concede a defensive reputation) to enhance their success elsewhere. Hence we infer proposition 2.

Proposition 2:

When two players play each other simultaneously in two separate arenas where the roles are reversed and one player has a reputation for both defence and attack while the other player only has a reputation for defence, then the player with only the defensive reputation is more likely to concede this reputation when there is a positive linkage between the reputations of his/her opponent than when there is reputational isolation. Hence, the play of the game with asymmetric reputations and reputational linkage yields behaviour which corresponds in part to the puppy dog strategy.

4. The Fox and the Lion

So far the roles assigned to players in the second arena have balanced those in the first. In this section, there is a sole defender who is concerned to deter attacks in two separate arenas. Table 3 gives the pay-offs for each player in the two arenas. The challenger sees each arena as distinct because it requires special skills in each to overcome a weak defender. To bring out this distinction, we shall refer to the

first arena as the plains and the second arena as the forest, and we will give the game a hunting interpretation.

To attack in the plains, you release wolves; whereas you lay snares to attack in the forest. --- Wolves are useless in forests because they cannot climb trees, and likewise snares on the plains because they cannot be concealed there. --- A defender can be a fox or a lion. If it is a fox in the plains then it will be caught by the release of the wolves (producing a pay-off vector given by column two); but if it is a lion, the wolves are dinner (producing pay-offs given by the third column). Conversely, if the defender is a lion in the forests then the snares will trap the defender (yielding pay-offs given by the second column; but if the defender is a fox in the forests it can tread lightly enough to remove the bait without triggering the trap (yielding the pay-offs in the third column).

		Arena 1 ('plains')			Arena 2 ('forest')		
		A	B		A	B	
			fox	lion		lion	fox
			weak	strong		weak	strong
A	holds back	0	a	a	0	h	h
	attacks	-1	-1	0	-1	-1	0
			B acquiesces			j	
		b	0	-1		0	-1
probabilities/reputations			1-p	p		1-r	r

Table 3

Since the defender can be either a fox or a lion but not both, it is assumed $p=1-r$. In a single shot version of this game, there is a simple taxonomy for the outcomes which depends on the information set of the challenger.

(i) when $p=0$:

The challenger holds back in the plains, but sets snares in the forests.

(ii) when $p=1$:

The challenger releases wolves in the plains, but holds back in the forests.

(iii) when $p < b/(1+b)$

The challenger attacks in the plains.

(iv) when $p > 1/(1+j)$

The challenger attacks in the forest.

Now, we consider a repeated version of this game. In all cases we assume that moves in each arena are made simultaneously at any moment in time. Two different situations can be distinguished: (X) when $b/(1+b) > 1/(1+j)$; and (Y) when $b/(1+b) < 1/(1+j)$.

In situation (X), it is known that one of the conditions (i)-(iv) above must hold in the last instance since p must lie in the $(0,1)$ interval. When it is known that one of the conditions (i)-(iv) will be satisfied in the last play then, there will be an attack somewhere and it is in the interest of the defender that it occurs in the arena where s/he is weak. Thus the defender will have interest in declaring his/her identity in the last play, and will have no reason to develop a reputation in the previous play for being something other than what s/he is. Hence, in the previous play of the game the returns to the defender are the same as in the one shot version and the defender will also find it in his/her interest to declare his/her identity. Thus the defender has no need to carry a reputation from the play before that, and so on: the problem unravels back to the present courtesy of the usual backward induction process. The defender will want to declare his/her identity immediately; and if there is no direct mechanism of communication, the challenger will attack in one or other arenas (or both, depending on the value of p) as this will afford the defender the opportunity to declare his/her identity through action in response to this attack.

In situation (Y), things are rather different. It is now possible for p to lie in the region given by (v), in the last play of the game and thus none of the conditions (i)-(iv) might obtain in that play, with the result that no attack occurs in either arena.

(v) when $b/(1+b) < p < 1/(1+j)$. Hence, in situation (Y), an attack in the last play is not guaranteed and an alternative sequential equilibrium exists for the game (strictly speaking a weak inequality is sufficient to

make attack in one arena or the other less than certain). Starting from any initial p , condition (v) can obtain in the last play of the game with appropriate reputation building behaviour in the course of the game, provided the interval in (v) actually exists (i.e. $j \cdot b < 1$). The appropriate reputation building behaviour, as in II.2 an 3, defines this alternative sequential equilibrium. It is given below in a) and b).

a) When p lies in the interval $[1/(1+j), b/(1+b)]$ then, trivially, the defender does not have to build a reputation in either arena to deter attacks in the final instance and consequently there are no attacks throughout the game.

b) When p lies outside the interval $[1/(1+j), b/(1+b)]$ then either the reputation for being a fox or the reputation for being a lion is inadequate at its current level to deter attacks in the final play, but both reputations cannot be inadequate. Assuming $p < b/(1+b)$, so that it is the lion reputation in the plains which is inadequate, then the equilibrium strategies are given by the analysis of the game of one sided uncertainty in II.2 for the plains arena, and by no attack in the forest. In other words, defining k as before $(= (1/b)\{p_0^{-b} - 1\})$, there will be no attacks in either arena until $t=k$, and thereafter there is a positive probability of attack and a positive probability of acquiescence in response to the attack in the plains. This is always the case provided the defender maintains a reputation for being a lion when $t < k$. This reputation grows with fights but it falls to zero if there is an acquiescence. Thus, the moment an acquiescence occurs, there are attacks followed by acquiescence for the remainder of the game in the plains. Meanwhile there are no attacks in the forest. Alternatively, if $p > 1/(1+j)$, then the equilibrium strategies are given by the analysis of one sided uncertainty in II.2 for the forest and by no attack in the plains.

It is worth making two observations about the equilibria defined in a) and b) for situation (Y). Firstly, the defender gains through the doubt over his/her identity as the expected returns exceed those which are obtained under situation (X) where attack and acquiescence in one arena occurs throughout the contest. The source of the gain is the free run period that is generated in the arena where reputation building occurs. This is the basis for our third proposition.

Proposition 3

When one player has two reputations which are arena specific and where there is reputational linkage such that the player cannot be strong in both arenas, then the player can benefit from the doubt as to his/her true identity. This benefit corresponds to the famous remark from Machiavelli (1950) on the value of imitating both the fox and the lion:

'the prince ... must imitate the fox and the lion, for the lion cannot protect himself from traps and the fox cannot defend himself from wolves. One must, therefore, be a fox to recognise traps and a lion to frighten wolves. Those who wish to be lions do not understand this.' (p. 60)

Secondly, while the defender benefits from this capacity to imitate both the lion and the fox, the defender's greatest gain arises when the attacker is particularly badly informed. To illustrate this observation, suppose there is no difference in the returns to the defender in the two arenas that arises from differences in the matrix of pay-offs, so that $a=h$, and there is likewise no difference for the attacker ($b=j$). This means the free run period is of the same duration and yields the same returns whichever arena the attack and reputation building occurs in. In these circumstances, the defender will prefer the attacker to attack in the arena where s/he is strong since reputation building here involves no losses whereas attacks in an arena of weakness do generate losses. Thus, the greatest gain will arise when the attacker is badly informed, in the sense that his/her probability assessment actually leads him/her to test the defender's reputation in the arena where the defender is strong rather than weak. So, not only is a bit of doubt a good thing from the defender's point of view, but ignorance, in the sense above, is even better.

IV. Discussion

In this section, we give some illustrations of games where multiple reputations of the sort discussed in section III arise.

1. The fox and the lion and the oligopoly/union game:

We offer two illustrations of the fox and lion multiple reputations at work. Firstly, consider a firm playing an oligopoly game

of the chain store variety, in arena 1, with a single firm for a finite period (or a finite number of firms) when the firm is unionised. The firm is playing a game with the union which has a similar structure of pay-offs, in arena 2, and will be repeated for the same number of periods.

To motivate this assumption, it is possible to think of a product protected by patents and the like and this defines the same length of time for both games since the two firms and the union are in conflict over the distribution of the rents associated with this product during the patent period. The doubt which is common to both games concerns the cost structure of the dominant firm. A reputation for low costs equates with 'strength' in the oligopoly game played with the other firm because it means the dominant firm can cut prices very low to ward-off the competition. However, the union believes that a low cost firm will acquiesce in response to a high wage demand, whereas a high cost firm will have to fight a high wage demand to preserve its minimal profit levels. Thus, low cost equates with 'weakness' in the union game when the union is considering a challenge in the form of a high wage demand.

The game is not exactly the same as the fox and lion presented in III.4 as there are two attackers, but this has no effect on the analysis provided it is assumed that the two attackers share the same information set and start the contest with the same prior probability assessments concerning low and high costs. — It can easily be complicated to allow for differences in these information sets. — Under these assumptions, it is interesting to observe that the model from III.4 predicts that initially the dominant firm will gain from being able to 'imitate being both the fox and the lion', but that this will give way to a period where there is fighting or striking in one arena or the other, but not both. Thus the model predicts that should the firm acquiesce, then the rents associated with the produce are either shared with other firms or the union, but not both. In short, to put this slightly differently, unions obtain higher wages on average in less competitive industries as compared with more competitive industries and dominant firms enjoy higher profits in the early stages of the life cycle of its products. These are both familiar findings.

2. the fox and lion and the union/firm game:

Consider the standard bilateral monopoly model of wage bargaining where the wage employment bargain lies on the Leontief contract curve, and allow for a deterioration in union power that is expected to last for a finite period of time. The conventional generalised Nash solution would point to an increase in the firm's share of the producer surplus through a gradual decline in employment and, possibly a fall in wages.⁵ This solution rests on the assumption of fixed threat points which are known to both parties, and it predicts that the adjustment will be achieved without a strike. We relax this informational assumption and cast the problem in a way that fits the informational structure of the fox and lion game, with the result that strikes are a possibility.

We suppose that there are two actions which a firm can take to obtain more surplus as its power increases: it can cut wages or it can rationalise production by lowering employment; and these define the two arenas of the contest, the wage and employment arenas. The firm is uncertain about the response of the union to either measure because the militance of the 'rank and file' within the union is not known.

A union needs to develop an appropriate response to each of these actions and in doing so it must have an eye on carrying the membership in these actions. Carrying the membership is a function both of the level of the militancy of the 'rank and file' and the perceived 'reasonableness' of the action because this determines public support which in turn feeds back to influence the membership during a defensive action. Striking is perceived as reasonable for a wage cut but not for a modest rationalisation, while the reasonable response to rationalisation is a go slow. These perceptions of reasonableness define the action of 'fighting' in each arena because an action by the employer cannot be defeated unless the union's response is publicly perceived as 'reasonable'. 'Reasonableness' and militancy combine in the following ways to affect the efficacy of fighting and acquiescing in each arena. In the employment arena, a go-slow undertaken by a workforce with low militancy produces the pay-offs of a strong defender because the action is consonant with public perceptions and the level of militancy, while a go-slow undertaken by a workforce of high militancy produces the pay-offs of a weak union because the membership becomes disenchanted with the low level engagement, they prefer inaction to this kind of tokenism. In the

wage arena, the reverse is the case, the fighting action of striking here will only generate the strong union's pay-offs when the level of militancy is high, when the level of militancy is low the union membership again becomes alienated from the action to produce the weak union's pay-offs.

Unfortunately, the union leadership cannot choose the level of militancy to suit the measure undertaken by the firm. The membership is either 'highly' or 'lowly' militant, and the game corresponds to the fox and lion reputational game in III.4.

By construction, this version of the union/firm bargaining game leads to the abandonment of the Leontief contract curve and it predicts wage rigidity when the union has a reputation such that the firm decides to attack first in the rationalisation arena. Perhaps the most interesting implication, is the advantage that the union derives from this uncertainty over the level of militancy; it gains a free-run period in one of the arenas before the employer takes offensive action and this would not have happened if the employer knew the level of militancy. This is an interesting result for at least two reasons. Firstly, it offers an explanation of why there might be lags in the adjustment by firms to a change in their bargaining power (i.e. the frequent observation of 'wage stickiness' and/or labour hoarding in the literature). Secondly, it makes postal ballots on industrial action suddenly look a significant change in the industrial relation's landscape because they are a potential device for removing employer doubt over a level of worker militancy.

3. Single Reputations, reputation linkage and entry versus acquisition.

It is not difficult to think of oligopoly settings where firms face each other in two separate markets and where the roles of defender and challenger are reversed. Nor would it be surprising to associate a single reputation with each firm in these circumstances, thus producing the model in II.2. Indeed, the asymmetry in the basic chain-store model, where one firm is granted the exclusive role of incumbent while the other is the entrant, might be thought somewhat atypical. Doubtless there are often dominant and subordinate firms like this in particular markets, but it is rare for this asymmetry to be reproduced in all markets where the

two firms might meet. What is perhaps a little more difficult, is to imagine a situation where the information and reputations are such that the puppy dog strategy of II.3 becomes appropriate. However, one case where this might arise is the following.

Suppose arena 1 is the standard chain-store example where one firm is considering entering another's market, and we now suppose that the second arena is the stock market where the incumbent/defender firm from the first arena is considering whether to mount a takeover bid for another firm which directly competes with the challenger from the first arena in some other market. The challenger from the first arena has the choice in the second arena of fighting this takeover plan or acquiescing. In other words, the incumbent firm from the first arena takes the battle to challenger's own market, as above, only the nature of the challenge made in the second arena is to mount a takeover bid for some other firm operating in that market. In these circumstances, it is plausible that reputations for fighting takeover battles are related positively to reputations for fighting competitive moves in produce markets, but not in such a way as to reduce one to the other. Thus, A's reputation for strength in challenges in product markets can be related to its strength in equity markets, as in 15. Equally, it is possible that the incumbent (B) in arena 1 could have lost his/her reputation for being strong in takeover battles, and yet still have a reputation for strength in product markets, arena 1.

4. Reputation Isolation and the Battle between Central Banks

Consider the situation where everyone agreed that relative interest rates between the US and Germany must change, with US rates rising relative to those in Germany. But there is disagreement over how it should be achieved. Germany believes that the overall state of the world economy points to the need for a tightening of monetary conditions and so the adjustment should be made by the US raising its interest rates. While the US believes that the state of the world economy points to the need for monetary easing and so the adjustment should occur through a fall in the German rates.

In these circumstances, the US can encourage Germany to lower interest rates by buying DMs with gold and purchasing German treasury bills, as this will put pressure on the exchange rate in an upward and

interest rates in a downward direction. This action is the equivalent of 'attacking' and Germany can either 'fight' by selling DMs, accepting gold and issuing bills, or 'acquiesce' by lowering interest rates. Fighting is costly for Germany because of the interest costs of issuing bills and because it forestalls the adjustment of relative interest rates. But, acquiescing while producing the adjustment in relative interest rates, also comes with a cost because of the belief that monetary conditions in the world as a whole should be tightened. And so, it is perfectly possible that Germany could either rank fighting higher than acquiescing (i.e. be 'strong') because of tightening monetary conditions was perceived as more important than a change in relative interest rates, or rank them vice versa (i.e. be 'weak'). From the US point of view, it is plain that a German acquiescence is preferred to a fight, and a fight is worse than holding back because there are transactions costs associated with mounting an offensive on the DM.

This, then, describes arena 1, the market for DM, where the US is an attacker and there is doubt about how Germany rates the need for nominal adjustment as compared with real adjustment and this translates into Germany being either strong or weak. There is a second arena, the market for dollars where Germany can attack by selling dollars and the US can respond by buying dollars with gold or acquiescing in a rise to US interest rates. The analysis of this arena follows that of the first, where the US could be 'strong' or 'weak' depending on how getting 'nominal' things right, with the correct general level of interest rates, is valued relative to getting 'real' things right with an adjustment of relative interest rates. It should be noted that although the two arenas appear very similar, they are not the same because central bank intervention in exchange markets uses a third currency, gold in this instance.

Of course, the designation of the US and Germany is hypothetical, but this illustration of the multiple reputations from II.1 might well have some relevance for the events in the world economy which preceded the October 1987 crash. To draw out just one implication of this model, there is the interesting prediction of delays in the adjustment of relative interest rate, even though all are agreed that they should occur.

V. Conclusion

This paper has extended the concept of multiple reputations by allowing players to compete in several arenas and by giving players arena specific reputations. This not only enables a formal account of attributes of gaming which are part of strategic folk-lore, like the imitation of the fox and the lion, but we also believe that it has a ready applicability to a range of economic settings. As such, we hope it has reinforced the general proposition that the key to understanding a variety of behaviours in the economy is to be found in the informational setting of these activities.

Notes

1. The Kreps and Wilson concept of sequential equilibrium and its implications for reputation building in finite horizon games has had a mixed reception, ranging from unqualified acceptance (see Backus and Driffil (1985) and Sobel (1985), through attempts at refinement (Grossman and Perry (1986) and Cho and Kreps (1987)), to the disputatious (Fudenberg and Maskin (1987) and Sugden (1988).
2. One of the advantages of this analysis of reputation creation is that it generates conflict as part of the optimal plans of instrumentally rational agents. Linking conflict with rationality is quite a feat and it relies on both the finite horizon and uncertainty. Selten (1978) shows that in a complete information environment with finite horizons conflict is never optimal. Similarly, Gul and Sonnenschein (1988) demonstrate that one sided uncertainty in continuous time leads to no conflict outcomes when horizons are infinite.
3. In A's mind the probability of a fight when there is an attack is $1 - (1-p_t) \pi_t \delta t + O(\delta t)$.
Bayes' rule implies that:
 $P(B \text{ is strong at } t-\delta t | B \text{ fighting during } (t, t-\delta t))$
 $= [P(\text{fight} | \text{strong})P(\text{strong at } t)] / P(\text{fight during } (t, t-\delta t))]$
 $\rightarrow P_{t-\delta t} = p_t / [1 - (1-p_t) \pi_t \delta t + O(\delta t)]$
Subtracting p_t from both sides and dividing by δt yields:
 $\dot{p}_t = p_t (1-p_t) \pi_t$ as $\delta t \rightarrow 0$
4. A reputation $p=b/(1+b)$ in the last instance of the game together with the weak defender's π rule implies a concluding value of $p=1$.
5. See MacDonald and Solow (1981, 1986).

References

- Backus, D. and Driffill, J. (1985) 'Inflation and Reputation', *American Economic Review*, 75, 530-38.
- Binmore, K. (1987) 'Modeling Rational Players: Part I', *Economics and Philosophy*, 3, 179-214.
- Camerer, C. and Weigelt, K. (1988) 'Experimental Tests of a Sequential Equilibrium Reputation Model'. *Econometrica*, 56, 1-36.
- Cho, I. and Kreps, D. (1987) 'Signalling Games and Stable Equilibria', *Quarterly Journal of Economics*, 102, 179-221.
- Fudenberg, D. and Kreps, D. (1987) 'Reputation in the Simultaneous Play of Multiple Opponents'. *Review of Economic Studies*, LIV, 541-68.
- Fudenberg, D. and Maskin, E. (1986), 'The Folk Theorem in Repeated Games with Discounting or with Incomplete Information', *Econometrica*, 54, 533-44.
- Grossman, S. and Perry, M. (1986) 'Perfect Sequential Equilibrium', *Journal of Economic Theory*, 39, 97-119.
- Gul, F. and Sonnenschein, H. (1988) 'On Delay in Bargaining with One-Sided Uncertainty', *Econometrica*, 56, 601-11.
- Kreps, D. and Wilson, R. (1982) 'Reputation and Imperfect Information', *Journal of Economic Theory*, 27, 253-79.
- McDonald, I. and Solow, R. (1981) 'Wage Bargaining and Employment', *American Economic Review*, 71, 896-908.
- MacDonald, I. and Solow, R. (1986) 'Wages and Employment in a Segmented Labour Market', *Quarterly Journal of Economics*, 100, 1115-1142.
- Machiavelli, N. (1950) *The Prince and the Discourse*, New York: Modern Library.
- Selten, R. (1978) 'The Chain Store Paradox', *Theory and Decision*, 9, 127-59.
- Sobel, R. (1975) 'Re-Examination of the Perfectness Concept for Equilibrium in Extensive Games', *International Journal of Game Theory*, 4, 22-25.
- Sugden, R. (1988) 'Backward Induction and the Finitely Repeated Prisoners' Dilemma', mimeo University of East Anglia.
83. C.G.F. Simkin *Popper's Methodology and Economic Theory*; July 1985
84. E.M.A. Gross *Market Information and Potential*
W.P. Hogan *Insolvency of Australian Financial*
I.G. Sharpe *Institutions July 1985*
85. F. Gill *Over-Award Payments; Result of a Survey conducted*
in 1982; December 1985
86. S.K. Kim *Short Run Policy Analysis of Employment, Food*
Price and Rural-Urban Migration for a Labour-
Abundant Developing Economy; January 1986
87. E. Kiernan & *Stochastic Stability in a Rational*
D.B. Madan *Expectations Model of a Small Open*
Economy; March 1986
88. E. Gross *A Note on the Testability of Fama's Efficient*
Capital Market Hypothesis; February 1986
89. M.C. Blad & *Multinational Producers in an Arrow-Debreu*
E. Gross *Type General Equilibrium Model; March 1987*
90. P. Saunders *Explaining International Difference in Public*
Expenditure: An Empirical Study; August 1986
91. W.P. Hogan *International Debt and Foreign Exchange Markets;*
January 1987
92. M.C. Blad & *Union-Firm Bargaining as a Repeated*
N. Oulton *Prisoner's Dilemma; January 1987*
93. R.T. Ross *The Heckman Procedure for Estimating Static*
Disaggregate Labour Supply Function: An
International Comparison of Estimates for
Married Women; March 1987
94. W.P. Hogan *Assessing Insider Trading; June 1987*
95. J. Yates *Housing Policy Reform: A Constructive Critique;*
June 1987
96. B.W. Ross *The Leisure Factor in Entrepreneurial Success*
during the 'Robber Baron' Era; July 1987
97. F. Gill *Determination of Wage Relativities under the*
Federal Tribunal: 1953-1974; August 1987
98. A.J. Phipps *Union Objectives, Wage Bargaining and the Phillips*
Curve; October 1987
99. R.T. Ross *The Labour Market Position of Aboriginal People in*
New South Wales; November 1987
100. L. Haddad *List Revisited: Dynamic Consideration of Trade and*
Protection; November 1987
101. J. Piggott *General Equilibrium Computation Applied to Public*
Sector Issues; December 1987
102. J.A. Carlson & *Relative Prices, Wage Indexation and*
D.W. Findlay *Unemployment; December 1987*
103. M. Waterson *A Model of Product Differentiation and*
Profitability; December 1987
104. P.D. Groenewegen *Taxation and Decentralisation: A Reconsideration*
of the Costs and Benefits of a Decentralised
Tax System; March 1988
105. L. Ermini *Some New Evidence on the Timing of Consumption*
Decisions and on Their Generating Process;
March 1988
106. G. Mills *Spatially-Differentiated Trucking Markets:*
Equilibria under Price Regulation without
Entry Restrictions; April 1988

107. B.W. Ross Strategic Commitment, Unknowledge and the Nature of Entrepreneurial Activity; April 1988
108. S.S. Joson Offsets and Development of Defence Support Industries in Small Economies; April 1988
109. B.W. Ross The Conglomerate and the Focussed Agglomerate: Modern Forms of the Leader-Commanded Firm; July 1988
110. L. Ermini Inertial Behavior on Schedule and Hierarchical Decomposition; July 1988
111. L. Ermini The Limits of Systems Control Theory in Economic Policy-Making; July-October 1988
112. P.D. Groenewegen Neo-Classical Value and Distribution Theory: The English Speaking Pioneers; September 1988
113. V.B. Hall, T.P. Truong & V.A. Nyuyen An Australian Fuel Substitution Tax Model: ORANI-LFT; October 1988
114. V.B. Hall, T.P. Truong & V.A. Nyuyen Responses to World Oil and Coal Shocks, in an Australian Short-Run Fuel Substitution Tax Model; October 1988
115. F. Gill Social Justice and the Low-Paid Worker; October 1988
116. G. Kingston Theoretical Foundations of Constant-Proportion Portfolio Insurance; October 1988
117. V.B. Hall & D.R. Mills Is Medium Temperature Solar Thermal Process Steam Viable for Australia? Some Preliminary Results; November 1988
118. W.P. Hogan Insider Information and Market Adjustment: November 1988
119. L. Ermini Reinterpreting a Recent Temporally Aggregated Consumption-Cap Model; December 1988
120. P.D. Groenewegen Progressive Personal Income Tax - A Historical Perspective; December 1988
121. M.C. Blad & N. Oulton Rubinstein's Solution of the Bargaining Some Problem: Generalisations and Extensions; December 1988
122. W.P. Hogan and I.G. Sharpe Prudential Regulation of Bank Ownership and Control; January 1989
123. G. Mills The Reform of Australian Aviation; June 1989
124. L. Ermini Transitory Consumption and Measurement Errors in the Permanent Income Hypothesis; June 1989
125. E. Kiernan Is Austerity Necessary?; July 1989
126. F. Gill Labour Market Flexibility - To What End?; August 1989
127. E. Kiernan Financial Reform: A Perspective; September 1989
128. S. Lahiri & J. Sheen On Optimal Dumping; September 1989
129. S. Hargreaves-Heap & Y. Varoufakis Multiple Reputations in Finitely Repeated Games; October 1989

Copies are available upon request from:

Department of Economics,
The University of Sydney,
N.S.W. 2006, Australia.

Working Papers in Economics Published or Accepted for Publication Elsewhere

1. I.G. Sharpe & R.G. Walker
1. N.V. Lam
4. V.B. Hall & M.L. King
5. A.J. Phipps
6. N.V. Lam
7. I.G. Sharpe
9. W.P. Hogan
12. I.G. Sharpe
P.A. Volker
13. I.G. Sharpe & P.A. Volker
14. W.P. Hogan
15. F. Gill
18. I.G. Sharpe
21. R.L. Brown
23. I.G. Sharpe & P.A. Volker
24. V.B. Hall
25. I.G. Sharpe & P.A. Volker
27. W.P. Hogan
28. P. Saunders
29. W.P. Hogan,
I.G. Sharpe & P.A. Volker
29. W.P. Hogan,
I.G. Sharpe & P.A. Volker
30. W.P. Hogan
32. R.W. Bailey,
V.B. Hall & P.C.B. Phillips
38. U.R. Kohli
39. G. Mills
41. U.R. Kohli
42. W.J. Merrilees
43. P. Saunders
45. W.J. Merrilees
46. W.J. Merrilees
49. U.R. Kohli
50. P. Saunders
53. J. Yates
54. J. Yates
55. G. Mills
- Journal of Accounting Research, 13(2), Autumn 1985
- Journal of the Developing Economies, 17(1), March 1979
- New Zealand Economic Papers, 10, 1976
- Economic Record, 53(143), September 1977
- Journal of Development Studies, 14(1), October 1977
- Australian Journal of Management, April 1976
- Economic Papers, 55, The Economic Society of Australia and New Zealand
- Economic Letters, 2, 1979
- Kredit and Kapital, 12(1), 1979
- Some Calculations in Stability and Inflation, A.R. Bergstrom et.al. (eds.), John Wiley and Sons, 1978
- Australian Economic Papers, 19(35), December 1980
- Journal of Banking and Finance, 3(1), April 1978
- Australian Journal of Management, 3(1), April 1978
- The Australian Monetary System in the 1970s, M. Porter (ed.), Supplement to the Economic Board 1978
- Economic Record, 56(152), March 1980
- Australian Journal of Management, October 1979
- Malayan Economic Review, 24(1), April 1979
- Australian Economic Papers, 19(34), June 1980
- Economic Letters, 6 (1980)
- Economic Letters, 7 (1981)
- Australian Economic Papers, 18(33), December 1979
- Keynesian Theory, Planning Models, and Quantitative Economics, G. Gandolfo and F. Marzano (eds.), 2, 703-767, 1987
- Australian Economic Papers, 21(39), December 1982
- Journal of the Operational Research Society 33, 1982
- Canadian Journal of Economics, 15(2), No. 2, May 1982
- Applied Economics, 15, February 1983
- Australian Economic Papers, 20(37), December 1981
- Canadian Journal of Economics, 15(3), August 1982
- Journal of Industrial Economics, 31, March 1983
- Review of Economic Studies, 50(160), January 1983
- Economic Record, 57(159), December 1981
- AFSI, Commissions Studies and Selected Papers, AGPS, IV 1982
- Economic Record, 58(161), June 1982
- Seventh Australian Transport Research Forum-Papers, Hobart, 1982

56. V.B. Hall & P. Saunders Economic Record, 60(168), March 1984
57. P. Saunders Economic Record, 59(166), September 1983
58. F. Gill Economie Appliquee, 37(3-4), 1984, pp. 523-541
59. G. Mills & W. Coleman Journal of Transport Economics and Policy, 16(3), September 1982
60. J. Yates Economic Papers, Special Edition, April 1983
61. S.S. Joson Australian Economic Papers, 24(44), June 1985
62. R.T. Ross Australian Quarterly, Vol. 56(3), Spring 1984
63. W.J. Merrilees Economic Record, 59(166), September 1983
65. A.J. Phipps Australian Economic Papers, 22(41), December 1983
67. V.B. Hall Economics Letters, 12, 1983
69. V.B. Hall Energy Economics, 8(2), April 1986
70. F. Gill Australian Quarterly, Vol. 59, No. 2, Winter 1987
71. W.J. Merrilees Australian Economic Papers, 23(43), December 1984
74. J. Yates Australian Quarterly, 56(2), Winter 1984
77. V.B. Hall Economics Letters, 20, 1986
78. S.S. Joson Journal of Policy Modeling, 8(2), Summer 1986
79. R.T. Ross Economic Record, Vol. 62, No. 178, September 1986
81. R.T. Ross Australian Bulletin of Labour, 11(4), September 1985
82. P.D. Groenewegen History of Political Economy, 20(4), Winter 1988
84. E.M.A. Gross, W.P. Hogan, I.G. Sharpe Australian Economic Papers, 27(50), June 1988
94. W.P. Hogan Company and Securities Law Journal, 6(1), February 1988
97. F. Gill Australia's Greatest Asset: Human Resources in the Nineteenth and Twentieth Centuries, D. Pope (ed.), Federation Press, 1988
99. R.T. Ross Australian Bulletin of Labour, 15(1), December 1988
100. L. Haddad Hetsa Bulletin, (11), Winter 1989
101. J. Figgott Public Sector Economics - A Reader, P. Hare (ed.), Basil Blackwell, 1988
107. B.W. Ross Prometheus, 6(2), December 1988
108. S.S. Joson Rivista Di Diritto Valutario e Di Economia Internazionale, 35(2), June 1988
116. G. Kingston Economics Letters, 15 (1989)