Cellularity of Twisted Semigroup Algebras of Regular Semigroups

Stewart Wilcox

A thesis submitted in fulfillment of
the requirements for the degree of
Master of Science

Pure Mathematics
University of Sydney

December 2005

Acknowledgements

Firstly I would like to thank my supervisor, David Easdown; despite having so many students, he manages to spend so much time and effort on each. Thanks for reading my drafts with meticulous care and commenting not only on typographical and mathematical errors, but also on points of style and consistency. Thanks also for supervisor meetings which were friendly and informal and full of immensely useful discussion.

I would also like to thank James East for many helpful conversations, especially for suggesting the partition algebra as an application. Thanks to Shona Yu for numerous valuable discussions and advice, and in particular for suggesting the extension of cellularity to infinite indexing sets.

Statement

This thesis contains no material which has been accepted for the award of any other degree or diploma. All work in this thesis, except where duly attributed to another person, is believed to be original.

CONTENTS

CHAPTER 1

Introduction

There has been much interest in algebras which have a basis consisting of diagrams, which are multiplied in some natural diagrammatic way. Examples of these so-called *diagram algebras* include the partition, Brauer and Temperley-Lieb algebras. These three examples all have the property that the product of two diagram basis elements is always a scalar multiple of another basis element. Motivated by this observation, we find that these algebras are examples of *twisted semigroup algebras*. Such algebras are an obvious extension of *twisted group algebras*, which arise naturally in various contexts; examples include the complex numbers and the quaternions, considered as algebras over the real numbers.

The concept of a *cellular algebra* was introduced in a famous paper of Graham and Lehrer [**6**]; an algebra is called cellular if it has a basis of a certain form, in which case the general theory of cellular algebras allows us to easily derive information about the semisimplicity of the algebra and about its representation theory, even in the non-semisimple case. Many diagram algebras (including the above three examples) are known to be cellular. The aim of this thesis is to deduce the cellularity of these examples (and others) by proving a general result about the cellularity of twisted semigroup algebras. This will extend a recent result of East [**4**].

In Chapters 2 and 3 we discuss semigroup theory and twisted semigroup algebras, and realise the above three examples as twisted semigroup algebras. Chapters 4 to 7 detail and extend slightly the theory of cellular algebras. In Chapter 8 we state and prove the main theorem, which shows that certain twisted semigroup algebras are cellular. Under the assumptions of the main theorem, we explore the *cell representations* of twisted semigroup algebras in Chapter 9. Finally in Chapter 10, we apply the theorem to various examples, including the three diagram algebras mentioned above.