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FACTORS AFFECTING THE BEHAVIOUR OF COMPOSITE STEEL AND CONCRETE COLUMNS

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SUMMARY

It is now well known that the concrete encasement of columns in a steel frame building can significantly increase their load carrying capacity. Most of this information has come from tests on pin-ended composite columns containing a single steel I-section, either a rolled steel joist or a universal beam or column section. However, built-up composite columns containing two or more rolled steel sections, mainly channels or angles which are laced and battened together, were in use long before the encased single steel I-section. In recent years, the use of built-up composite columns has declined, most likely because of the added cost of the battens or lacing which have traditionally been provided in the same manner as for bare steel built-up columns and because research has not kept pace with their application in practice. With the increasing use of composite construction in general, especially in high rise office and apartment buildings, built-up composite columns could provide a suitable alternative to the encased steel I-section with the added advantage of increased flexibility in the arrangement of the steel components which can be spaced to give any desired stiffness to resist the actions of the applied loads and moments. Their use is both a matter of economics and structural performance which are related to the requirements for battens or lacing.

Accordingly, the initial stage of this investigation was an experimental examination of the behaviour of eccentrically and concentrically loaded pin-ended built-up composite columns in which no battens or lacing were used to connect the steel components (two channel sections) ,the concrete alone being relied upon to transmit any shear. A study was made of the effect on the load carrying capacity of varying the eccentricity, slenderness and axis of bending. The case of bending in double curvature, which represents the maximum shear condition for a pin-ended column, was also examined. A similar study was made of the behaviour of square concrete-filled steel tubes which were treated as another form

of composite columns. Analytical methods for determining the theoretical load-deflection relationships, load-strain relationships and load carrying capacities under short term static loading have been developed as extensions of the original theory(13) for encased rolled steel joists. The validity of these methods was checked by comparison with the test results.

The second stage of the investigation was concerned with the influence of the time-dependent properties, creep and shrinkage, on the behaviour of built-up composite columns, again containing no battens or lacing. The effect of creep on the deformations of columns under sustained concentric and eccentric load was examined. The influence of shrinkage stresses and cracks, present in the column prior to loading to failure, on the load capacity and the deformations under short term static loading was observed. Theoretical procedures were developed for predicting the time-dependent deformation of such columns and the test results were used to verify the theory.

Results have indicated that the absence of battens or lacing is unlikely to have any detrimental effect on the performance of built-up columns under short term static loading. The analyses have been shown to provide theoretical results which accurately predict the actual behaviour of these columns. Creep under sustained load has been found to increase the initial deformations by several orders of magnitude which may or may not, depending on the loading conditions, result in a reduction in the load capacity of the columns. Shrinkage cracks in the column prior to loading have been shown to significantly reduce the short term load carrying capacity of the pin-ended member. Relatively simple theoretical models for creep and shrinkage have been found to give a satisfactory representation of the actual time-dependent deformations of the columns.

PREFACE

All of the work described in this thesis was carried out by the writer at the School of Civil Engineering, University of Sydney during the period March 1967 to March 1974. This study of built-up composite columns forms part of an overall programme of research into the behaviour of composite steel and concrete structures. The research is under the direction of Professor J.W. Roderick who has summarized the extent of the programme in a recent paper (12). The column aspect of this work was first looked at by Roderick and Rogers (13) who developed an inelastic instability analysis which accurately predicted the behaviour of encased rolled steel joists bent about their minor axes in single curvature. This study was extended by Loke(14) who made both a theoretical and experimental investigation into the behaviour of eccentrically loaded pin-ended columns made from small scale rolled steel joists encased in concrete and subjected to short term static loading applied about either or simultaneously about both principal axes of the crossection. The writer has further extended this work to the cases of square concrete-filled tubes and composite columns containing more than one steel component and in particular, two steel channel sections between which there was no other connection except that provided by the concrete alone. The behaviour under both short term static load and constant sustained load has been examined. The effects of shrinkage cracking on column performance have also been investigated.

The writer submits that the new work described in this thesis is substantially an original contribution to knowledge in the field of composite steel-concrete structures. The theoretical analytical methods for built-up columns and concrete-filled tubes under short term static loading have been developed as extensions of Roderick's(13) original analysis and Loke's(14) modifications. The basic principles involving equilibrium and compatibility are similar in all these analyses. However, some further important modifications have been made by the writer to make the method applicable to columns having any shape or form of crossection containing steel sections of any number and type and to account for residual stresses, the tensile strength of concrete, stress-strain relationships for the materials of any shape and differing end moments such that the column can be bent in either single or double curvature.

The theoretical considerations with regard to sustained loading and restrained shrinkage are claimed to be original. Any information used and derived from other sources is indicated by specific reference in the text.

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CHAPTER 1

INTRODUCTION

1.1 DEVELOPMENTS IN COMPOSITE CONSTRUCTION

In the building industry, the term 'Composite Construction' has become synonymous with the use of structural steel sections encased in or connected to concrete as opposed to the use of steel reinforcing bars embedded in concrete which is known appropriately by the generic term 'Reinforced Concrete'. Composite construction has its origins in steel frame construction. During the first decade of this century, rolled structural steel sections were readily available and steel frame buildings were becoming commonplace, especially in the United States with some structures as high as 30 storeys. At this time, the need for fire protection became apparent as it was found that the load carrying capacity of steel was significantly reduced at high temperatures. This aspect was of considerable importance considering the increasing size and occupancy of buildings. The general practice was therefore to encase the structural steelwork in concrete as a means of fireproofing. However, the use of structural concrete was in its infancy so it was not surprising that the concrete encasement was not considered to contribute to the strength and stiffness of the steel as designers were not yet convinced of the reliability of concrete which, compared to today's standards, was of low quality and strength. There were some isolated cases where City Building Regulations permitted the concrete to be considered as assisting in carrying the load(3) but this practice was not widespread.

The use of reinforced concrete in structures expanded so rapidly that by the 1930's, it was becoming a serious and efficient competitor to structural steelwork, not so much in Great Britain and the United States where perhaps consultants were not prepared to take risks with a client's capital by using a new material, but more so in Europe where contractors usually included for design in their tenders. Accordingly, designs were more competitive and designers were willing to attempt new forms of construction to obtain economy. Because of the continuity in reinforced concrete structures, they had to be analysed as statically indeterminate frames and designer were therefore forced, rather fortunately, to take some account of rigid frame action. This consideration was limited to behaviour in the elastic range as only elastic methods of analysis and design were available. Steel construction was placed at somewhat of a disadvantage as no account was taken of rigid frame action, the orthodox 'simple design' method dating back to the turn of the century still being used in which the beams were assumed to be simply supported with their reactions close to the face of the columns. A further disadvantage was that the concrete encasement was an additional dead load which was assumed to be carried by the steel frame, its only use being for fire protection. It seemed incredible that the philosophy of using the steel frame to carry construction and other dead loads and the steel, composite with the concrete, to carry live and additional dead loads was not apparently appreciated although the concept had certainly been established(2)(3).

A fillip was given to the steel industry with the advent and acceptance of welding as a means of connection. This made it possible to construct a fully rigid frame in steel and this was reflected in the design provisions of codes such as BS 449-1948 in which alternatives to the simple design method were permitted, namely semi-rigid design and fully-rigid design. At last some recognition had also been given to the structural action of the concrete, albeit minor, in which the concrete was considered to provide lateral stiffness to steel columns thereby reducing the slenderness for buckling, under axial load, about the minor axis. This resulted in higher allowable axial stresses for the steel. However, these were limited to a maximum of 1½ times that for the bare steel section. The concrete was considered not to carry any load, an entirely inconsistent approach for how can concrete provide stiffness without also adding to the load carrying capacity.

By the 1950's, there was a growing realization that as the actual behaviour of structures is different from the elastic behaviour assumed in analysis, then design methods should account for this difference. This led to a new philosophy of design in which safety factors were based on the load to cause collapse of the structure rather than in a more arbitrary manner by limiting elastic stresses, calculated for working loads, to some 'safe' value. This development, especially in the field of steel structures, owes much to the work of Sir John Baker(60) and his colleagues who, over a period of many years, had developed an extensive theory for the post-elastic behaviour of steel structures culminating in what now is known as the 'plastic design' method. It was found that the method led to economies in steel, especially for simple frames. At that time, an ultimate load approach was also being contemplated for reinforced concrete structures. Some building regulations had already permitted individual members to be designed on an ultimate load basis. But what of composite construction?

Compared to the developments in steel and reinforced concrete construction, very little progress had been made. In BS 449-1959, some recognition was given to the fact that the concrete in a composite column really did carry load in addition to providing stiffness to buckling about the minor axis. However, the paradox still remained that no such stiffness was considered to be provided about the major axis. The allowable axial load for the composite column was limited to twice that for the bare steel section. This was particularly restrictive for columns containing light steel sections(13). For the range of possible steel sections that could be used, the rationale of this requirement is not apparent and it is difficult to conceive of a similar requirement ever being entertained for reinforced concrete.

For steel beams bent about their major axis, the allowable moment capacity was effectively increased, if encased in concrete, by permitting higher steel stresses. This resulted from the concrete being considered to provide additional stiffness about the minor axis hence reducing the slenderness of the beam and the possibility of lateral torsional buckling. The strength of the concrete in resisting applied moments was neglected. Both the strength and the stiffness of the concrete were apparently neglected when considering the moment capacity of columns.

During the 1960's, there have been considerable developments in steel and concrete construction. Advances in concrete technology have resulted in the use of higher strength concrete with improved quality control. New concrete construction techniques have resulted in increased rates of construction e.g. slip-forming. The use of high yield stress steels, both for structural sections and reinforcing bars, has led to savings in steel. Steel sections such as tubes and thin cold-formed sections of special shapes have been used to advantage. The problem of corrosion for steel has been reduced with the advent of the so-called 'weathering steels'. The performance of connections between steel members has been improved with the use of high-tensile torqued bolts. With improved welding techniques, large steel sections have been fabricated from plate. In some situations, the speed of erection has been increased with the use of prefabricated structural elements. The use of computers has enabled more refined analyses of structural behaviour to be made resulting in better estimations of the actual forces in the structure and associated improvements in design. Ultimate strength design methods have been recognized and their use has led to some savings in materials.

Unfortunately, the developments in the use of steel remained parallel to those in concrete with little interrelation. Perhaps the notable exception was in the use of beams composed of structural steel sections, with or without concrete encasement, which act in conjunction with an in-situ reinforced concrete slab, the two elements being connected by some means to form a composite section as a whole. Although this type of construction had been used earlier, it was not until the advent of the stud shear connector with its high rate of attachment that the composite floor/deck system could compete economically with other forms of construction. During this period, it was significant that composite construction was at last recognized, with the publication of CP 117:Pt.I:1965 and CP 117:Pt.II:1967, as a particular form of construction with its own unique characteristics requiring a separate code of practice rather than being an appendage to steel or reinforced concrete codes. The first two parts of the new code have dealt with beams in building and bridges. It is expected that later parts will deal with complex and special beams in building and columns in buildings and bridges.

The latest developments are most exciting as composite construction is now emerging as a serious competitor to structural steel and reinforced concrete construction in the field of high rise office and apartment buildings. The Skidmore, Owings and Merril team headed by Fazlur Khan(11), after considerable experience with the two conventional building types, realized that it was possible to use both concrete and steel in a way that exploited their advantages and eliminated their disadvantages. The "tube" concept was used in which closely spaced concrete perimeter columns and spandrels acted together as a rigid framed tube to resist lateral forces. Three notable composite buildings of this type have been erected, the 24 storey CDC Building in Houston, the 36 storey Gateway Tower in Chicago and the 50 storey One Shell Square Tower in New Orleans(10). The use of composite construction is said to have cut structural costs by 15% and steel requirements by 50%(11). The perimeter columns were actually slender steel members forming part of an overall steel frame which were encased in concrete as the construction progressed upwards. The steel columns were designed to support nothing more than the temporary construction loads of up to 8 or 10 floors. Concrete filled steel decking, which provided both early access to tradesmen and a stabilizing diaphragm to the exposed steel frame, followed closely behind the steel erection. Internal beam fireproofing and mechanical work were then commenced. Prefabricated reinforcing cages for the columns and spandrel beams were then positioned. Forms were erected and the concrete for the beam and columns cast to form the rigid frame external tube which was designed to carry additional dead, live and lateral loads as a composite frame. The fulfilment of the early design philosophy mentioned early was at last being achieved.

Future trends in composite construction will depend not only on improvements in structural efficiency but also on the economy that can be achieved. The latter will be influenced by market forces existing at the time such as availability of materials, financial arrangements, the state of the construction industry and the relative roles of the developer, architect, consultant and contractor in the development of the project. The success of composite construction may well depend on integration within the construction industry which has traditionally been divided into steelworkers and concrete crews. In this regard, the successful completion of the three high rise composite buildings mentioned above is encouraging. High rise steel structures have had the advantage of a high percentage of rentable area and a fast rate of construction with reduced labour costs, reduced interest repayments on loans and a quicker return on the money initially borrowed. These benefits have been offset by the high material costs and the extra material and effort needed to stiffen and stabilize the steel frame to resist wind forces. Reinforced concrete structures have been in rather the reverse position with the main advantages being low material costs and the inherent stiffness of the structures. These features have generally been balanced by high labour costs associated with the slow rate of construction and the considerable amount of formwork and materials handling needed. This balancing of material costs against labour costs is perhaps the reason why the steel and reinforced concrete construction industries have remained in steady competition for so long. To ensure economy, the innovation of any new system should, where possible, lead to savings in both labour and material costs. It is believed that this can be achieved with composite construction in several ways namely, the use of composite floor/deck systems, columns and beams containing steel sections other than the single rolled steel I-section, concrete filled tubes and integrated steel and concrete construction techniques. These points are discussed in turn below.

(i) Rather than use concrete-filled steel decks for the floors in which the steel deck is only used as reinforcement, the steel deck can be made composite with the concrete slab. This composite deck then can also be made composite with the steel supporting beams by connecting the steel deck to the beams with stud shear connectors. Existing systems for doing this are available. The bond between the steel deck and the concrete has generally been found to be insufficient to enable the full moment capacity, assuming full composite action, to be achieved. There have been recent developments in which a mechanical type of connector is rolled into the steel deck during the forming process. These have been found to significantly improve the composite action. The advantages of the composite floor system are that efficient use is made of the steel, both as a temporary working platform and as combined formwork and reinforcement to the concrete slab. The floor is effectively a composite T-beam in the transverse direction to the span of the decking. No removal of formwork is required and scaffolding and propping are unnecessary, except for long spans where some temporary props may be needed to limit the initial deflections of the steel deck before composite action is attained with the development of concrete strength.

(ii) In general, composite construction has been limited to the single steel I-section either fully encased or attached to the concrete with shear connectors. Where the structural elements are subjected to bending moments, more efficient use of the steel could be made by using more than one steel section. These steel sections could then be spaced and positioned towards the extremities of the crossection to provide a better resistance to the applied moments. Use of this concept has been made by Civil and Civic Pty.Ltd. with their patented Progressive Strength beam and slab floor system in which standard open web steel trusses, consisting of angle sections and round steel rods with main trusses at large spacings and smaller secondary trusses at close spacings, are designed to support the formwork, plastic concrete and construction loads without propping. As the concrete develops strength, the role of the steel trusses changes progressively from a concrete support system to its final role as concrete reinforcement. Some of the advantages are speed of construction as stripping of the forms is not dependent to any great extent on the development of concrete strength, reduced labour costs by the use of standard trusses and standard lightweight polypropylene coffer forms supported on the bottom chords of the secondary trusses and more usable column free space as long spans can be achieved without propping. It is considered that similar effective use could also be made of built-up steel sections in composite columns which are subjected to bending. As for the beams, their use will be a question of economics and efficiency.

(iii) In the limit, perhaps the most efficient means of using the steel is to have it right at the extremities of the crossection as in concrete filled tubes. However, this then raises the question of fireproofing. Some designers, in their desire to have a steel exterior yet still have protection against fire, have resorted to such schemes as water-filled tubes which are connected into a water reticulation system. The use has even been suggested of ablative coatings such as those developed by the aero-space industry for heat protection of space vehicles on re-entry into the earth's atmosphere. At the present time, the high cost of such coatings makes them economically prohibitive. There are situations where an external fireproofing cover to the steel may not be necessary as in multistorey car parking stations where recent tests have indicated that the risk of a conflagration starting from a local car fire and spreading to all the parked cars is low. The use of concrete-filled tubes could result in a small crossectional size for the columns with more available car parking space.

(iv) It may well be that the success of composite construction in the future could depend not only on an efficient integration of the materials but also on the integration of the construction methods and techniques that have been developed for economical erection of either steel or reinforced concrete structures. For example, it is envisaged that slipforming techniques could be used in which the forms are supported on the steel frame above rather than on the concrete cast below. This could result in high speed of erection as almost continuous casting of the concrete could be achieved as the movement of the forms would be in no way restricted by the development of strength in the concrete cast below. Some interesting innovations in the construction of composite buildings have already been developed(10).

However, full advantage of the composite form will not result until more is known about the behaviour of connections between beams and columns and slabs and columns and full use can be made of the concrete in transmitting moment and shear between these elements. Roderick(12) has already made the point that when the moment-rotation characteristics of truly composite connections can be reliably forecast, it should be possible to develop plastic design methods for the rigid framed composite steel and concrete structure which could lead to significant savings.

Recent trends are towards the use of limit state design methods where the design of a structure is not only required to provide an adequate safety factor against collapse, be it by a mechanism with the formation of plastic hinges or by elastic or inelastic instability, but also to limit deflections so as to prevent damage to fittings and finishes, to limit cracking of concrete eg. for prevention of reinforcement corrosion in a marine environment, to prevent excessive vibrations and to limit accelerations to a comfortable human level. To be able to account for these factors, an understanding is needed of the actual and theoretical behaviour of structures over the full range of loading, rather than just at collapse or at working loads. With regard to the limiting of cracking and deflections, a knowledge of the influence of the time-dependent properties, creep and shrinkage, of concrete on the behaviour of structural elements containing concrete is becoming of increasing importance.

In reviewing the development of composite construction, the writer has discovered a number of problems related to the behaviour, performance and construction of composite structures, some of which have been revealed above. Considering the wide range of problems, it has not been possible to cover all of these in this thesis. Perhaps one of the more critical areas of concern is the stability of columns. With reference to tall buildings, this will be of particular importance, not so much for the columns in the lower floors which will tend to be stocky because of the high axial loads, but more so for the columns in the upper floors which will be more slender with a higher ratio of moment to load. There are a considerable number of factors which can influence the behaviour of columns. A few of these have already been examined. The writer has considered other contributing factors for which research has been lacking.

1.2 COMPOSITE COLUMNS - RESEARCH AND PRACTICE

The composite column, as it is known today, generally consists of a single rolled steel I-section encased in concrete. However, columns of this form are only a relatively recent development in the history of composite columns, the use of which dates back to the start of this century. At that time, typical compression members in bare steel (Fig.1.1) consisted of two or more steel sections, usually angles, channels or rolled steel joists, which were built-up into a single member with the use of battens or lacing (Fig.1.2). When the need was realised for protecting the steel against fire in buildings, the simplest solution was to encase the built-up steel sections in concrete, this type of column being referred to hereafter in this thesis as a built-up composite column. These columns, sometimes containing cast-iron cores instead of steel, were used fairly extensively for the first 30 or so years(3)(4). Perhaps one of the most interesting applications was in a block of 950 flats erected in Leeds in 1939(9) which consisted of a series of steel frames, five to eight storeys high, prefabricated on the ground, erected on prepared base plates and then connected together by secondary beams. Precast concrete slabs were used for the floors and the columns were encased in concrete. The columns were formed from two channels placed back to back. Each frame consisted of three columns with the beams continuous for two spans, each beam passing between the channels, supported on the column batten plates and connected by welding. After encasement with concrete, the concrete in-fill of the columns was accepted as adding 28% to the load carrying capacity.

By this time, research into the behaviour of built-up composite columns had virtually come to a standstill. The investigations that had been carried out were largely experimental and confined to tests on single isolated columns subjected to short term static loading. In every case the loading was concentric. In some cases, the end conditions were not defined and often the properties of the materials were not given much attention, sometimes not mentioned at all. Only a limited range of column lengths were tested and no real attempt was made to investigate the reduction in axial load capacity with increasing slenderness in what might be considered the slender column range. As expected, the results indicated that composite columns were considerably stronger than similar unencased steel columns. For the stocky columns tested, it was generally found that the maximum load was equal to the sum of the yield strength of the steel sections and the compressive strength of the concrete crossection. This was a result of the fact that for the materials used in the tests, the strain to cause yielding of the steel was lower than the strain corresponding to the maximum stress for the concrete. The concrete strength was found to be further enhanced by providing spiral reinforcement in the concrete encasement. The theoretical aspects of the behaviour of composite columns were practically ignored although by this time, rational buckling theories had been developed for pin-ended bare steel members. Consequently the results of the investigations did not yield sufficient information for any significant changes to be made in design codes and building specifications.

Despite the lack of research, built-up composite columns have still been used in practice. A recent example was the State Office Block in Sydney, a building of some 34 storeys built in 1962. A reinforced concrete core, carrying most of the lateral load, was connected by steel floor beams to perimeter columns consisting of four small steel channels which were battened and laced and then encased in concrete. A typical column section is shown in Fig. 1.3. The channels were arranged so that beams could frame between the channels. Although not covered by existing code requirements, full composite action between the steel and concrete of the column was assumed in the design which was based on the results of ultimate load tests made at the University of Sydney on similar columns. This is a typical case of where practice is ahead of research. This may not be the case in the future as development of computer techniques will facilitate the simulation of the actual behaviour of structures.

The position with regard to composite columns containing a single rolled steel I-section is much better. Since Faber's(15) tests in 1955, which could be considered as marking the beginning of the study of the composite column on a planned basis, this type of column has received considerable attention, one of the most recent investigations being carried out at this University by Loke(14) who has critically reviewed most of the previous research. The pin-ended column bent about the minor, major and simultaneously about both axes was examined in some detail. Some allowance for composite action in designing column members of this type is permitted by design codes such as BS 449:Pt.1:1971 and ACI 318-71. In the latter, the full contribution of the concrete to the load and moment capacity of the section is permitted to be utilized as for a reinforced concrete section. However, the isolated pin-ended composite column rarely exists in practice. The behaviour of the continuous column under various conditions of loading and end restraint still requires extensive study before the full advantage of the composite form can be realized.

1.3 OBJECT AND SCOPE OF THIS INVESTIGATION

The aim of this investigation has been to examine several important factors that influence the behaviour, performance, efficiency and economy of composite columns. One aspect considered has been the relative position of the steel within the crossection. There is no doubt that where members are subjected to bending moment, more efficient use can be made of the steel by positioning it towards the extremities of the crossection to provide greater resistance to the applied moment. Two possibilities have been examined; built-up composite columns containing two channel sections spaced apart and encased in concrete with an adequate cover for fire protection and composite columns with the steel on the outside viz. square steel tubes filled with concrete (Fig. 1.4).

While the efficiency of composite columns might not be questioned, there is the problem of economy. Previous practice has been to batten or lace the steel components together, as in Fig. 1.2, prior to encasement. The fabrication time and associated high labour costs in providing battens or lacing has probably been one of the main reasons why built-up composite columns have not been in favour in recent times. If the battens or lacing could be reduced, or in the limit, eliminated altogether without affecting the performance of built-up composite columns, their use might then become an economically viable proposition. To this end, concentric and eccentric load tests have been performed on pin-ended built-up composite columns in which the channels were left unconnected. No secondary reinforcement was used in the concrete encasement. The purpose of this has been to determine whether the concrete alone was sufficient to transmit the shear between the two steel components. This has been examined under maximum shear conditions for a pin-ended member by loading a column with equal but opposite end eccentricities such that it was bent in double curvature. The effect of absence of battens has been examined positively by comparing the experimental behaviour of similar columns, either with or without battens, loaded under identical conditions.

Compared to concentrically loaded concrete-filled tubes, the behaviour of which has been the subject of a number of previous investigations, there is relatively little information on their behaviour under eccentric load, especially for tubes of practical sizes. Under the latter loading condition, the question is whether the bond between the concrete and the tube walls is sufficient to enable the ultimate load, assuming full composite action, to be attained. Eccentric load tests have therefore been conducted on square concrete-filled tubes, including some bent about axes inclined to a face of the tube.

One of the main objectives of this work has been to study the stability of composite columns of different section to those containing a single steel core. This has been done experimentally with a number of tests on pin-ended columns, as mentioned above, in which the influence of a number of variables viz. relative position of the steel in the crossection, slenderness, eccentricity of loading, axis of bending and mode of bending, on column stability has been examined. Column stability has also been studied from a theoretical viewpoint and analytical methods have been developed to predict the behaviour, under short term loading, of composite columns with any form of crossectional shape. The analysis has been developed as an extension of the theoretical method presented by Roderick and Rogers(13). The validity of the analytical methods has been verified by comparing theoretical results from the analysis with experimental results from the tests.

Perhaps one of the more important aspects of the present investigation has been a study of the influence of material properties on column stability. It could fairly be said that many of the significant advances in the analysis and design of structures have come as a result of a better understanding of the nature of the materials used in the structures. To accurately predict behaviour, column analyses require the determination of realistic load-moment-curvature relationships for the column crossection. To do this, the stress-strain characteristics for the materials must be known, not just from control specimens alone but for the actual materials in the column itself under its own unique set of loading conditions. An attempt has been made to account for this aspect in the analysis.

The effects of the time-dependent nature of concrete on column behaviour have been examined. Two aspects have been treated; the effect of sustained loading in which deflections increase with time due to concrete creep with the possibility of creep instability at lower values of load than the short term loading capacity; the influence of shrinkage cracks, present in the column before loading, on column stability for short term loading, the cracks resulting from the restraint provided by the steel core to the concrete shrinkage. Sustained loading and restrained shrinkage tests have been carried out on built-up composite columns which were identical in section to those tested to failure under short term loading. These creep and shrinkage tests are of particular importance and significance as they form the only known investigation into the long term behaviour of composite columns. Theories have been developed to account for the effects of creep and shrinkage on column behaviour. The correlation between the test and theoretical results has been used to verify the validity of the assumptions used in the analyses.

It is realized that the behaviour of pin-ended columns, loaded eccentrically or concentrically through roller bearings in a testing machine, does not truly represent the behaviour of a column continuous through a multistorey building frame. However, an understanding of the former is necessary if methods of analysis, based on sound assumptions, are to be established from which rational design procedures may eventually be determined.

This investigation is conveniently divided into two parts, one dealing with the short term loading behaviour and the other with the long term behaviour. In Chapter 2, a review of the existing literature pertaining to the subject is reviewed. The material properties, column theory, built-up column tests and concrete-filled tube tests for short term loading are described in Chapters 3, 4, 5 and 6 respectively. The material properties, column theory, restrained shrinkage tests and sustained load tests associated with the time dependent nature of the concrete (creep and shrinkage) are presented in Chapters 7, 8, 9 and 10 respectively. The design of built-up composite columns is discussed in Chapter 11 in which the results of the writer's tests are compared with design methods in existing codes of practice. The experimental and theoretical results are discussed at the end of each individual test chapter and the conclusions reached from the present investigation are presented in Chapter 12.

CHAPTER 2

REVIEW OF PREVIOUS RESEARCH

2.1 INTRODUCTION

Research into the behaviour of composite columns has been in progress for the last 70 years since the initial tests by Emperger(1)(2) in 1905. Between 1905 and 1935, most interest was in the composite column containing built-up steel and cast-iron sections, braced or battened together, and cast-iron cores, both solid and hollow tubes. These latter type of columns were generally known as "von Emperger Columns"(5) owing to the work done by Emperger in promoting their use in structures both in Europe and in the United States. As far as can be ascertained, no further investigations were reported during the period 1935 to 1955.

Since 1955, the emphasis has been almost exclusively on the study of composite columns containing the single rolled steel joist, the initial tests being by Faber(15) and followed by many others. This is probably because the built-up sections involved shapes that were generally complex requiring a considerable amount of fabrication, especially in the lacing or batten detail, the cost of which was unlikely to be economical. The single rolled-steel joist had the advantage of ease and speed of erection in a practical structure which led to the almost complete abandonment of work on the built-up section. Research on the encased rolled steel section has been reviewed in detail by Loke(14) and as it is generally outside the scope of the present investigation, it need not be repeated here.

Since 1935, the only tests known to the writer on encased built-up sections have been by Bondale(8). In fact, these tests form the only known data on the behaviour of such columns subjected to eccentric loading as all previous tests were for concentric loading. Research into the behaviour of encased built-up sections, as defined in Chapter 1, is reviewed in Section 2.2.

When the present investigation by the writer was commenced in 1967, the behaviour of concentrically loaded concrete filled tubes, including the increase in strength due to concrete confinement, had been established from several investigations such as those by Swain & Holmes(16), Klopell and Goder(17), Salani and Sims(18) and Gardner and Jacobson(19). However, there was a lack of information on the behaviour of eccentrically loaded concrete filled tubes. It was therefore decided to include some eccentric load tests on square tubes filled with concrete in the test program on encased built-up sections. By using square tubes, the effect of confinement was minimized. The inelastic instability analysis developed by Roderick(20) for predicting the behaviour of encased rolled steel joists bent about the minor axis was modified, as discussed in Chapter 4, to predict the behaviour of the concrete filled tubes. The results of the tests and the theoretical predictions are given in Chapter 6. However, since 1967, the results of several investigations into the behaviour of concrete filled tubes, both concentrically and eccentrically loaded, have been published. These have been by Furlong(21)(22), Knowles and Park(23)(24) and Neogi, Sen and Chapman(25). Only the latter investigators have proposed a theoretical analysis capable of predicting the behaviour of eccentrically loaded members, the others having proposed empirical methods of determining beam-column strength. These and other earlier investigations are reviewed in Section 2.3.

The bare steel built-up sections of a composite column to be used in a practical structure must be capable of withstanding the construction and other loads before concrete encasement. As described in Chapter 5, two tests on bare steel sections were carried out by the writer to enable their load-deflection behaviour and load carrying capacity to be determined experimentally so that a comparison could be made with their behaviour when encased. However, as the study of bare steel sections is generally outside the scope of the present investigation, a review of the fairly extensive research into the behaviour of these members has not been included. It is sufficient here to state that built-up bare steel columns have been investigated closely and can be designed for a particular load capacity using current analytical techniques.

A summary and some general conclusions drawn from the investigations covered in this review are presented at the end of the chapter in Section 2.4.

2.2 BUILT-UP COMPOSITE COLUMNS

In Section 2.2.1, an account is given of concentric load tests on composite columns where the main purpose appeared to be the examination of the effect of concrete encasement in improving the strength above that of the bare metal sections. Although columns of various lengths were tested, no apparent attempt was made to determine the influence of column length on the load-carrying capacity.

In Section 2.2.2, some concentric load tests are described in which not only the improvement in strength by concrete encasement was investigated but in which account was also taken of the effect of column length. A range of lengths was tested.

In Section 2.2.3, a description is given of concentric load tests on stocky columns in which both the increase in strength due to concrete encasement and the non-linear behaviour of the concrete were observed. An attempt was made to determine the actual stresses in the concrete.

In Section 2.2.4, Bondale's (8) eccentric load tests, where the improvement in both axial and flexural stiffness was demonstrated, are discussed.

2.2.1 Concentric Load Tests - Column Strength

(a) Emperger(1)(2) and Mensch(4)(5)

As already stated, research on composite columns was initiated by the well-known pioneer in concrete practice, F. von Emperger, in 1905. It was then that he proposed the "law of superposition" or "law of addition" whereby the strength of a concentrically loaded composite column was equal to the sum of the strengths of the steel (or cast-iron) column alone and the concrete column alone. This was contrary to what he called the "equivalence coefficient" or "elasticity relationship" being used at that time when the theory of elasticity for columns was generally accepted whereby the ratio of the steel stress to the concrete stress was equal to the ratio of the corresponding moduli of elasticity i.e. the modular ratio. His first concentric load tests in 1905 to demonstrate this "law of addition" were on two steel channel sections, battened together, and encased in plain concrete which, by coincidence, is the same type of crossection used for the present investigation as shown in Fig. 1.4. In each case it was found that the column strength was less than the sum of the strengths of the structural steel and concrete, probably because of the inability to control numerous imperfections that can affect concentric load behaviour. His tests on cast-iron columns embedded in plain concrete gave still more unsatisfactory results. It was found that the plain concrete failed first before the cast-iron had attained its full strength. Mensch(4)(5) found from other tests on similar cast-iron columns that the ultimate strain of cast-iron at the point of failure probably exceeds 0.01 in. per in. which is well in excess of the strain at maximum stress for the concrete of approximately 0.002 in. per in. This indicated that the concrete had reached its maximum stress well before the cast iron had reached anywhere near its ultimate strength. The instability or failure of these composite columns was therefore similar to that demonstrated in Fig. 4.4 and discussed later in Section 4.1.3 where the maximum load is less than the sum of the strengths of the two materials.

The combination of a cast-iron core surrounded by spirally reinforced concrete was the next stage of Emperger's investigation. The spiral provided lateral restraint to the concrete hence increasing its stress capacity and, even more important, its strain capacity such that it was compatible with the strain capacity of the cast-iron. Results of these tests have been reported by Mensch(4) (5) and showed that the strength of such columns was at least equal to and even greater than the combined strength of the spirally reinforced concrete and the cast-iron core alone. This result was verified by Mensch(4) in a series of concentric load tests on solid steel and cast iron cylindrical columns embedded in spirally reinforced concrete at the Armour Institute of Technology in 1918 and the University of Illinois in 1930. A summary of the results is given in Table 2.1. It can be seen that the concrete had a larger stiffening effect on the slender 2 inch dia. cores than on the stockier 4 inch and 5 inch dia. cores. Unfortunately, the columns as tested had ends in which the metal cores were not always even with the concrete. No details were given of the end loading conditions and hence the effect of any end restraint cannot be determined. The concrete properties for each column, if any, were not recorded and it was assumed that the concrete strength was identical to that obtained from the column test without any metal core. All of these factors may account for the variations shown in the results.

In most of his investigations over a large number of years, Emperger tested fairly stocky composite columns containing a variety of metal sections, some of which are shown in Fig. 2.1. Unfortunately, precise information of the results is not readily available. However, he was the first to demonstrate the considerable advantages of encasing the bare metal members in concrete. When reviewing the results of his tests at the First IABSE Congress at Paris in 1932(2), he concluded that the short column strength, P_{sh}, of composite steel-concrete columns could be given by

$$P_{sh} = f_{sy} A_s + f''_c A_c \qquad \text{———————(2.1)}$$

provided that for increasing strain, the steel reached its yield strength, f_{sy}, before or at the point when the concrete reached its maximum strength, f''_c. Equation (2.1) is in agreement with equation (4.7) given later in Section 4.1.3 for the identical case, illustrated in Fig. 4.1 when the strain, ε_{sy}, for steel yield is less than the strain, ε_o, corresponding to the maximum stress for the concrete.

(b) Stang, Whittemore and Parsons(7)(26)

The steel cores used in these tests were approximately half-scale, except for the length, of the tower columns used for the George Washington Bridge across the Hudson River at New York City. The bare steel sections were tested in 1935 by Stang and Whittemore(26). An investigation in 1936 by Stang, Whittemore and Parsons(7) determined the strength and stiffness under load of four identical steel columns encased in reinforced concrete. These massive composite members of 3ft. 1½in. square cross-section were 24ft. long and built-up out of angles and plates as shown at (a) in Fig. 2.2. The crossectional area was 159 sq.ins. for the steel and 1245 sq.ins. for the concrete, the steel percentage being 11.3. The steel sections were stiffened near the ends and at 3ft. 1½in. centres along their length by diaphragms made from 4in. x 4in. x 9/16 in. angles. The concrete encasement contained ½ inch dia. binders at 6 in. centres along the column length. The columns were tested with flat ends and were subjected to 25 cycles of repeated concentric loading before loading to failure. The maximum and minimum loads for each cycle were 88% and 67% of the average test ultimate strength of the members. The columns were completely unloaded at the end of the 20th and 25th cycles. For two columns, the load was applied to the steel and concrete simultaneously and for the other two columns, the load was applied to the steel alone which projected approximately ½ inch from the ends of the concrete. There was no significant difference in the column behaviour for the two types of load application and it was suggested that for the members loaded on the steel alone, the concrete came into contact with the platens of the testing machine after the first cycle or two of loading. In fact, it was found that on removal of some of the crushed concrete, after final failure had occurred, some of the angles near the ends of the members had buckled locally, probably during the early load cycles.

For the final loading to failure of the four composite columns, the stress-axial strain relationships obtained are shown in Fig. 2.3 together with the relationships for the two corresponding bare steel columns. The quasi-stress referred to for the composite columns is the column load divided by the crossectional area of the steel sections alone. The lateral deflections recorded were extremely small, of the order of 0.1 inch in 24ft. at a point where the steel had just yielded, and only became significant at axial strains in excess of 0.026 in./in. which is well beyond the yield strain for the steel. At this point, the load was removed. The column stress-axial strain curves are similar in shape to a typical stress-strain curve for mild steel with a well defined yield point, a yield plateau and a strain hardening region. For all four composite columns, strains in excess of 0.026 in. per in. were recorded which indicated that some strain hardening in the steel had occurred. The effectiveness of the binders was also demonstrated by the large strains that the concrete had attained without a reduction in the load carried by the concrete. The small fluctuations in the stress-strain curves, after initial yield had taken place, occurred whenever some local concrete crushing took place on the face of the columns resulting in a small decrease in stress.

From tensile tests on coupons taken from the plates and angles of every column, the average yield stress was 34 kips/sq.in. This yield stress was verified by the load tests on the bare steel sections as shown in Fig. 2.3. Knowing the steel yield stress, the steel area, the concrete area and the average maximum load for the composite columns, the average maximum concrete strength, f''_c, in the columns can be calculated to be 2.13 kips/sq.in. from equation (2.1). The average cylinder strength, f''_c, for the concrete used in all the columns, was determined from tests on 6 in. by 12 in. cylinders stored under water for 2 months. This was found to be 2.96 kips/sq.in. However, for the concrete in the columns, the forms were stripped 48 hours after casting and the concrete cured for 2 months under normal atmospheric conditions before testing. The ratio of concrete strength to cylinder strength is 0.72 which is not an unlikely figure, as discussed later in Section 3.2.3, considering the difference in size and curing conditions and the fact that it would be more difficult to compact the concrete in the columns than in the much smaller cylinders.

The results of these tests again indicate, as proposed by Emperger, that the strength of short composite columns containing mild steel and concrete of practical strengths is equal to the sum of the strengths of the structural steel sections and the concrete as given by equation (2.1).

(c) Saliger(27)

Concentric load tests were carried out by Saliger in 1931 on five stocky composite columns containing two steel channel sections battened together and encased in concrete which was spirally reinforced. The four different types of crossection tested are shown in Fig. 2.4. Type 4 was the only crossection that had the concrete reinforced with rectangular binding rather than spiral reinforcement. The length of all the columns was 9 ft. 10½ in. The length to diameter ratio varied from 7.5 to 10.7. The columns were tested with flat ends and were loaded on the steel, the steel sections protruding 0.4 inch from the concrete at the ends. To prevent local end failures, a steel collar of the same diameter as columns, 9½ in. deep and 0.4 in. thick was attached to both ends of the column when the concrete was cast.

The yield stress of the steel was determined from coupons cut from the flanges and the webs of the various sizes of channels used. The concrete strength was determined from compression tests on large prisms, 8 in. x 8 in. x 31½ in, having a similar size of crossection and subjected to similar curing conditions as the columns. Axial shortening strains were measured on both the steel and the concrete. Strain measurements at the last load increment before failure indicated that the average difference between the steel strain and the concrete strain was only 4.7%, probably within the order of accuracy of the strain measuring equipment. Apart from the strain differences being small, they were also random; in some cases the steel strain was higher and in other cases, the concrete strain was higher. It could therefore be assumed that the strains in both the steel and the concrete were identical indicating that the steel-concrete bond was maintained throughout the tests.

A summary of the test results is given in Table 2.2. As was found previously by Emperger and Mensch, the column strength in all cases was equal to or greater than the sum of the strengths of the concrete and the structural steel, the additional strength being provided by the confinement of the concrete by the spiral reinforcement. The average increase strength by the use of spiral reinforcement was 10.7%.

It is interesting to note that for the Type 4 column which only had rectangular binders, there was little or no additional strength provided by these binders because of their shape. The influence of binders of this type on concrete behaviour under load will be discussed in Section 3.2.4 where it is shown that although rectangular binders do not have much influence on concrete strength, they can considerably improve the ductility of the concrete. The Type 3 column was identical to the Type 2 column except that the size of the spiral reinforcement used was larger which resulted in a further improvement of strength, as would be expected by the increase in effective confinement of the concrete, of 3.2%. The influence of binder size on concrete behaviour under load will also be discussed in Section 3.2.4.

These tests have again shown that for composite columns containing structural sections of mild steel, the short column strength can be determined by the use of equation (2.1). Additional strength can be obtained by the use of spiral reinforcement in the concrete encasement.

2.2.2 Concentric Load Tests - Effect of Length

(a) Talbot and Lord(28)

A report was made in 1912 by Talbot and Lord of an investigation into the value of concrete as reinforcement for structural steel columns. Concentric load tests were made on ten plain steel members built up out of eight steel angles as shown at (b) in Fig. 2.5, the angles being battened together at 16 in. centres. According to Buel in subsequent discussion of a paper by Burr(3), this type of column was designed by J.H.Gray and had been used in many important buildings in the United States. The crossectional area of the steel was 13 sq.ins. and the radius of gyration, k, was 3.9 sq.ins. Five column lengths, 2ft. 4 in., 4ft. 8 in., 10ft. 0in., 15ft. 4 in. and 19ft. 4 in., were used to examine the effect of length on the load carrying capacity. Two columns of each length, 1, were tested. The range of slenderness ratio, $\frac{l}{k}$, was therefore 7.2 to 59.5. The columns were tested with flat ends. The maximum load carried by each length of column is shown in column(2) of Table 2.3 and have been plotted in Fig. 2.6(a) and Fig. 2.7.

Tests were made on twelve similar steel members with a core of concrete but no outer fireproofing concrete steel, the external dimensions of the concrete being shown at(b) in Fig. 2.5 as the CC Retype outline. The four column lengths tested were the same as for the bare steel columns. This was done so that a direct comparison could be made. Two columns of each length were tested with a concrete core having a mix with proportions 1:2:4. To examine the effect of variation of concrete strength, an additonal four columns of 10ft. length were tested, two with a concrete core having mix proportions of 1:1:2, the other two with a 1:3:6 concrete core. The columns were again tested with flat ends. The average maximum load carried by each column is shown in column(5) of Table 2.3. The results for the columns with the 1:2:4 concrete core are also shown in Fig. 2.6(b) and Fig. 2.7.

Some interesting results of this investigation are reproduced in Fig. 2.6 where for particular values of axial strain, ε, the corresponding stress, taken as the column load divided by the steel area, has been plotted for each length of column. The axial strains were measured by means of extensometers attached to the exposed flanges of the steel angles. It is rather surprising to see that for any particular value of axial strain before failure, the axial stress decreased with increasing length even though the columns were nominally of the same crossection and all were concentrically loaded. This effect became more predominant as the particular value of strain was increased. Perhaps the columns were not exactly axially loaded or that significant initial curvature was present in the columns. If this was the case, some inelastic behaviour associated with lateral deformation and hence bending of the longer columns could have occurred. Unfortunately no details of any measurements of lateral deformation or of any strain gradient across the section were available. At failure, the effect of length on load carrying capacity was clearly demonstrated, the average stress and hence the load at failure decreasing significantly with increasing length. This is as expected irrespective of whether the tangent modulus or initial curvature approach is used to explain the effect of length on the axial load carrying capacity of columns.

By comparing the two sets of curves in Fig. 2.6, it can be seen that for a particular value of axial strain, the slopes were very nearly the same for both the composite and bare steel members which indicated that the influence of increasing length on the strength of the columns was almost identical for the two types of columns. Within the limits tested, the authors concluded from these results that the stress taken by the concrete could be considered to be independent of the length or slenderness ratio and that the stress in the steel of the composite columns could be considered to be identical to that taken by the bare steel member of the same length. For the ultimate condition, the same result was noted in a simpler manner by Andrews(29) who plotted the average maximum load for each length of the bare steel members and the composite members containing the 1:2:4 concrete. The results are reproduced in Fig. 2.7. Both curves were almost parallel which indicated that the increase in strength due to the concrete encasement was approximately the same for all lengths. This increase in strength was approximately equal to the concrete strength as obtained from ancillary tests on plain concrete prisms as can be seen by comparing columns(4) and (6) of Table 2.3.

This above behaviour indicated that at the ultimate load, the concrete contributed little or nothing to the flexural stiffness when the columns failed by buckling, the steel carrying the same stress at failure whether encased in concrete or not. The writer considers this is probably due to the following facts. The steel configuration was such that the bare steel columns were relatively stiff, the steel area being concentrated at the outer edges of the columns. The steel percentage was relatively high at 11 per cent and the slenderness ratios tested were relatively low. Therefore, the strains attained by the bare steel columns at buckling failure were fairly high, being of the order of 0.0015 in./in. At this strain level, the concrete would be at or near its maximum strength. Hence the value of tangent modulus for the concrete would be approaching zero and the contribution of the concrete to the total flexural stiffness would therefore be negligible. However, for very slender steel sections where the stresses and hence the strains at buckling are low, the effect of concrete encasement can be quite marked such that the stresses in the steel at failure for a composite column are well in excess of those for the bare steel sections. This effect is shown in Table 2.1 where for tests by Mensch(4) on slender 2 in. dia. steel rods, 6ft. in length, the load at failure for the bare steel rods was 63.9 Kips yet when encased in concrete to 7½ in.dia., the load considered to be carried by the steel at failure was 176.6 Kips, an increase in steel stress of 275%!

The effect of concrete strength on the load carrying capacity of the composite columns is shown in column (7) of Table 2.3 where the increase in strength over the bare steel column strength was 25% for the weak 1:3:6 concrete, 33% for the stronger 1:2:4 concrete and 54% for the 1:1:2 concrete, this being the strongest concrete used even though its strength was only 2120 lb/sq.in.

Some additional columns were tested with a fireproofing shell, the outline of which is shown at (b) in Fig. 2.5. This shell was either plain concrete or reinforced with 0.75 or 1.0 per cent spiral reinforcement. The columns with spiral reinforcement attained higher loads than those without. Again, as previously found by Emperger and Mensch, it was demonstrated that the restraint to the concrete provided by the spiral considerably improved the stress and strain capacity of the concrete and hence the strength of the composite columns.

(b) Mensch(30)

In order to verify the results of Emperger's tests, under American conditions, on hollow cast-iron cylinders encased in spiral reinforced concrete as shown at (a) in Fig. 2.1, Mensch had some tests conducted at the Bureau of Standards at Pittsburgh in 1916 on similar columns. Twelve composite columns were made consisting of a centre cast-iron core of pipe, 6 in. in external diameter and 3/4 in. thick with concrete encasement to a diameter of 12 in., this concrete being reinforced with 0.6% each of spiral and longitudinal reinforcement. The average concrete strength from cylinder tests was 4100 lb./sq.in. The column lengths were 6ft., 8ft., 10ft., 12ft. and 14ft., two columns of each length being tested except for the 10ft. lengths where four columns were tested. A concentric load was applied to the flat ends of the columns. The stress on the cast iron cores at failure, ranging from 30,000 lb./sq.in. for the long columns to 51,000 lb./sq.in. for the short column, was found to be considerably less than for Emperger's tests where the stress ranged from 50,000 lb./sq.in. to 100,000 lb./sq.in. at failure. This was ascribed to the common grade of cast-iron used whereas Emperger used a high grade cast-iron and to the poor quality of the hollow cast-iron pipes which showed great uneveness in thickness. No details were given of any measurements of strain or lateral deformation that might have been taken.

The maximum loads for each length of column have been plotted in Fig. 2.8. As expected, the effect of increasing length was to decrease the load carrying capacity. The straight line of best fit to this data, as determined by Tucker(5), is given by

$$\frac{P}{P_{sh}} = 1 - 0.0179 \frac{l}{D} \qquad \text{——————(2.2)}$$

where P is the load carrying capacity corresponding to the length to diameter ratio; $\frac{l}{D}$, and P_{sh} is the short column strength for zero length. For Talbot and Lord's data, as shown in Figs. 2.6 and 2.7, from tests on a different type of composite column, the following relation was obtained by Tucker(5) where

$$\frac{P}{P_{sh}} = 1 - 0.0126 \frac{l}{D} \qquad \text{——————(2.3)}$$

Both equations (2.2) and (2.3) are empirical relationships based on test results and their linear nature cannot be considered to represent the effect of length over a full range of slenderness.

2.2.3 Concentric Load Tests - Concrete Stiffness

(a) Burr(3)

In 1912, Burr reported some tests on concentrically loaded columns made in 1908 in order to ascertain the relative strength of plain steel columns consisting of an outer frame of braced steel compression members compared with composite columns with a similar outer frame filled with concrete. Two types of steel section were used, one consisting of four steel angles connected with single diagonal lacing and the other consisting of four steel channels battened together at intervals. The crossections of these two types are shown at (a) in Fig. 2.5. Two bare steel and two composite columns of each type were tested. All columns were 7ft. in length and tested between hemispherical end bearings to ensure pin-ended conditions. Specimens of the steel taken from the angles and channels of these columns showed an average yield stress of 40.8 kips/sq.in. Evidently no concrete specimens were tested as the concrete strength was not reported. However, a standard 1:2:4 mix was used and from other investigations at the same time using similar mixes (see Table 2.3), the likely strength would be of the order of 1500 lb./sq.in. This was verified by the results for the stresses attained by the concrete in the composite columns as shown in column (9) of Table 2.4. A compressometer reading to 1/10,000 inch over a gauge length of 70 inches was used to measure the axial shortening strains of the columns. The lateral deflections due to any flexure were measured by screw micrometers.

The results for the load tests on the bare steel columns are shown in Figs. 2.9 and 2.10 and in columns (7) and (8) of Table 2.4. The lateral deflections at the centre of the columns were less than 0.1 inch in a length of 7ft, except for column No.5. This indicated that the columns were fairly straight initially and the concentric load was applied accurately. The steel stress at failure was less than the yield stress for the material as can be seen from column (8) of Table 2.4. This was due to the modes of failure whereby local buckling of the channels or angles occurred near the ends between the bracing or battens, except for column No.5 which failed by general flexure, probably because of some initial end eccentricity or initial curvature. The stress attained by the angles with diagonal bracing was higher than that attained by the channels with battens which indicated that the diagonal lacing was more effective in bracing the longitudinal compression members than was the battening as is now well-known.

The results for the load tests on the composite columns are also shown in Figs. 2.9 and 2.10 with additional computed results being shown in Table 2.4. It can be seen that for low strains, the slopes of the composite curves are steeper than the bare steel curves which indicated the initial increase in axial stiffness provided by the concrete. However, for increasing strains, the slope for the composite columns decreased at a greater rate than for the bare steel columns which demonstrated the reduction in stiffness for the concrete with increasing stress. It should be noted that the curves shown for the bare steel columns do not truly reflect the behaviour of the steel as a material as the measured strains at higher values included deformations produced by local buckling of the angles and channels and the maximum stress attained by the columns was less than the yield stress for the material. Burr recognised that although a fair and close value for the modulus of elasticity, E_s, for the mild structural steel in the columns could be taken as 29×10^6 lb./sq.in., the modulus of elasticity, E_c, for the concrete was not nearly so well defined. Therefore he used the following equations to determine the concrete elastic modulus, E_c, the concrete stress, σ_c, and the steel stress, σ_s, for the composite columns.

$$P = A_s E_s \varepsilon + A_c E_c \varepsilon \qquad (2.4)$$

$$E_c = \frac{\sigma_c}{\varepsilon} \qquad (2.5)$$

$$E_s = \frac{\sigma_s}{\varepsilon} \qquad (2.6)$$

The assumptions were that the modulus of elasticity of the steel was constant at 29×10^6 lb./sq.in. and that the axial shortening strain, ε, was identical for both the steel and the concrete. The values of P and ε were given by the test curves shown in Figs. 2.9 and 2.10 and the values of E_c were determined from equation (2.4). It should be noted that the value of E_c determined in this manner is the secant modulus which is the slope of the line connecting the particular concrete stress under consideration to the point of zero stress on the concrete stress-strain curve. If the ascending portion of the stress strain curve is assumed to be the parabolic curve used by Hognestad(31) and shown in Fig. 3.4, the secant modulus to maximum stress is half the initial modulus at zero stress. This has been added to Burr's results by the writer in column (10) of Table 4. The steel stress, concrete stress and concrete secant modulus calculated by Burr using equations (2.6),(2.5) and (2.4) are shown in columns (8), (9) and (10) of Table 2.4 respectively. As the curves for the composite columns in Figs. 2.9 and 2.10 were approximately linear up to a load of 120 Kips, the value of the concrete secant modulus up to this load can be taken as the initial elastic concrete modulus.

The importance of Burr's tests was to demonstrate the manner in which the steel and concrete shared the load in a concrete composite column. Although he showed that the concrete modulus was not constant but decreased with increasing stress, no comments were made on this phenomenon and it is not clear whether he realized the significance of this finding. Another important fact to come out of these tests was that the concrete was sufficient to brace the longitudinal compression members such that the yield stress of the steel was attained before local buckling of these members occurred. It was also found that the use of concrete filling led to a 50 per cent increase in ultimate strength for both types of steel columns.

(b) Warren(32)

In 1921, Warren reported some concentric load tests on a plain mild steel column consisting of four steel angles tied together with diagonal bracing and on two similar composite columns. In one composite column only the core was filled with concrete and for the other, an additional fire-proofing cover of 1½ inches was provided. Details of the column sections are given at (c) in Fig.2.5.

The three columns were all 10ft. 7in. long. Test pieces taken from the steel angles gave the yield stress at 39.25 kips/sq.in. and the elastic modulus, E_s, at 29×10^3kips/sq.in. However, the concentric load test on the bare steel column indicated that the yield stress was 43.0 kips/sq.in., the maximum load for the column being 363 kips. The axial shortening strains were measured at a few load increments for all three columns. The concrete strength from cylinder or prism tests was apparently not measured. The results of the tests and some computed results, as discussed below, are given in Table 2.5.

Warren stated in his text that "Prof.Talbot has brought out an interesting point in connection with the stress and deformation relation in regard to reinforced concrete columns. It is well known that the relation between deformation of a column of concrete and the stress producing it may be represented, with considerable accuracy, by a parabola". The parabolic function used is that shown later in Fig. 3.4 and was the same as that proposed earlier by Ritter(33) in 1899 and used extensively by Hognestad(31) since 1951 where

$$\sigma_c = f''_c[2(\frac{\varepsilon_c}{\varepsilon_o}) - (\frac{\varepsilon_c}{\varepsilon_o})^2] \qquad \text{———}(2.7)$$

and f''_c is the maximum concrete stress and ε_o is the strain corresponding to this stress. For this parabola

$$f''_c = \frac{E_c\varepsilon_o}{2}$$

By substitution, then

$$\sigma_c = E_c\varepsilon_c[1 - \frac{\varepsilon_c}{2\varepsilon_o}] \qquad \text{———}(2.8)$$

Using the assumptions that the steel is elastic and that the strains in the steel and concrete are identical then the stress in the steel, σ_s, is given by

$$\sigma_s = E_s \varepsilon_c \tag{2.9}$$

From equations (2.8) and (2.9) then

$$\frac{\sigma_s}{\sigma_c} = \frac{E_s}{E_c} \cdot \frac{1}{\left(1 - \frac{\varepsilon_c}{2\varepsilon_o}\right)} \tag{2.10}$$

The load, P, carried by the composite column can be written as

$$P = \sigma_s A_s + \sigma_c A_c \tag{2.11}$$

Warren first assumed that at failure, the steel was still elastic and the concrete had reached its maximum stress at which crushing took place. At that time, it was not known that the concrete stress-strain curve had a descending portion because of the test methods used in its determination. This is discussed later in more detail in Section 3.2.1 and Appendix A. For this maximum concrete stress condition,

$$\frac{\sigma_s}{\sigma_c} = 2\,\frac{E_s}{E_c} \tag{2.12}$$

as determined from equation (2.10). The stresses in the steel and concrete of the composite columns at failure calculated by Warren using equations (2.11) and (2.12) are shown in columns (9) and (10) of Table 2.5 respectively. Using the same approach as used previously by Burr, the initial value for the concrete elastic modulus, E_c, was calculated using equation (2.4) for a relatively low load value on the composite column at which the axial shortening strain was still essentially proportional to the applied load.

Warren then assumed that the stress in the steel for the composite columns was capable of reaching the yield stress, as was found for the bare steel column. The values of the steel stress and concrete stress at maximum load obtained by using this assumption are shown in columns (7) and (8) of Table 2.5. It can be seen that there is a discrepancy between the values for the two different assumptions. To be able to determine the maximum load for concentrically loaded short columns, the full stress-strain relationships for both the steel and the concrete must be known. A means of calculating this maximum load for a combination of differing stress-strain curves for concrete and steel is described later in Section 4.1.3. From this, it appears to the writer that the values given in columns (7) and (8) of Table 5 are the most likely. Unfortunately, from the scant details given by Warren, it was not too clear which was the larger; the strain at steel yield or the strain at maximum stress for the concrete. At least Warren was aware of the inelastic nature of the concrete and attempted to account for it. He also clearly demonstrated that composite columns were considerably stronger than the bare steel section and, for his tests, exhibited an increase in ultimate strength of approximately 100 per cent.

(c) Memmler, Bierett and Gruning(34)

In 1934, a comprehensive series of concentric load tests were completed by Memmler, Bierett and Gruning on columns made up from two steel channels either placed back to back (Type II) or toe to toe (Type V). A total of thirty-three columns were tested in which the channels were spaced apart at different distances and battened together. For most, the core between the channels was filled with concrete. Details of the columns are given in Fig. 2.11. On the average, the concentric load test for each Type and Mark of column was repeated twice. The columns were all 10ft. 8 in. in length and were tested between end rocker bearings to ensure pin-ended conditions. The yield stress for the steel was determined for each channel section used in every column. The concrete strength was determined from tests on 8 in. x 8 in. x 32 in. prisms taken from the concrete used in the manufacture of each column. The axial shortening strains were measured on both the concrete core and the steel channels and it was found the strains were sensibly the same. The lateral deflections were measured at the centre and quarter points along the length of the column. Except for the slender column Type II Mk.1, the lateral deflections at failure were evidently small. For example, the maximum lateral deflection for column Type V Mk.2, just prior to failure, was reported to be 0.013 inches which is a small deflection in a length of 10ft. 8 in. This indicated the care that was taken to ensure that the columns were concentrically loaded.

A summary of the test results are shown in Table 2.6, the average load for the repeated tests on each Type and Mark of column being recorded. For the bare steel columns, it can be seen from columns (9) and (10) of Table 2.6 that the stress in the steel of the column sections had reached the yield stress at maximum load. It could be safely assumed that this was also the case for the stiffer composite columns as was indicated by the axial strain measurements. Using this assumption, the load and hence the stress in the concrete at maximum load were computed and are shown in columns (11) and (12) respectively. The ratio of this maximum concrete stress to the prism strength is shown in column (14). Excluding the Type II Mk 1 column which was the most slender composite column tested and which was reported to have failed in general flexure due to either initial end eccentricity or initial curvature, the average maximum concrete stress to prism strength ratio was 0.71. This is not altogether an unreasonable value as the concrete in the columns would be more difficult to compact and was also subjected to different curing conditions to the prisms.

These "size" effects are discussed later in more detail in Section 3.2.3 where other investigators are shown to have found a ratio of 0.85 to be a reasonable value for their tests. An important result to come out of these tests was that the relative column strengths were not affected by increasing the spacing between the longitudinal compression members, in this case, the channels.

Memmler, Bierett and Gruning calculated the stresses in the concrete and steel for increasing load from equations (2.4)(2.5) and (2.6) using the same assumptions as used previously by Burr(3) where the elastic modulus had a constant value (29.7×10^6 lb./sq.in.) up to the yield stress and the axial shortening strain was identical for both the steel and the concrete. The variation of steel stress and concrete stress with increasing axial strain for the Type II Mk.3 composite column is shown in Fig. 2.12. The stress-strain curve for the concrete appears a little flattened on top probably because of the assumption that the steel was elastic right up to its yield stress when probably residual stresses were present in the steel sections. The results indicated that the maximum load was attained after the steel had yielded and when the concrete had reached its maximum stress. For the materials used in these tests, this occurred close to the same value of axial strain. The column strength for these columns can therefore be given by equation (2.1) as proposed by Emperger.

A summary of the load sharing between the steel and the concrete for all the composite columns, as calculated above, is shown in Fig. 2.13. It can be seen that for the lower load values, the proportion of the load carried by the steel remained constant which indicated that for low stress values, the concrete was essentially elastic. However, for high load values, the proportion of load taken by the steel increased which demonstrated the inelastic nature of the concrete. The proportion taken by the steel for any load value was also increased by decreasing the area of the concrete.

2.2.4 Eccentric Load Tests

Since 1935, the investigation by Bondale(8) is apparently the only known research into the behaviour of built-up composite columns. In fact, theoretical research on these members was virtually non-existent until 1966 when Bondale presented his column analysis which was applicable to bending about either principal axis. A general theory of instability for initially straight pin-ended columns was proposed. This theory was an extension of von Karman's(35) basic concept of inelastic column action incorporating Westergaard and Osgood's(36) modification of assuming the deflected shape to be part of a half cosine wave which further simplified the analysis. In fact, this method had been applied much earlier, in 1957, to reinforced concrete columns by Ernst, Hromadik and Riveland(37) and further extended by Broms and Viest(38) in 1961 using an identical approach to that proposed later by Bondale. Because of its application to composite columns, a brief description of the method is given below.

Consider an initially straight pin-ended column of length, L, subjected to a load, P, applied at equal end eccentricity, e, as shown at (a) in Fig. 2.14. Assuming that the effects of residual stresses, shear, local buckling and lateral-torsional buckling on the deflections and critical load are negligible, then the deflected shape, assuming it to be part of a half cosine wave, can be expressed as

$$y = y_o \cos\left(\frac{\pi z}{l}\right)$$

with origin at O.
The curvature, ρ, is therefore given by

$$\rho = -\frac{d^2y}{dz^2} = \frac{\pi^2}{l^2} \cos\left(\frac{\pi z}{l}\right) \qquad \text{———(2.13)}$$

At the ends of the member then

$$e = y_o \cos\left(\frac{\pi L}{2l}\right) \qquad \text{———(2.14)}$$

At the centre of the member

$$y_o = e + \delta_o = y_r + V_r \qquad \text{———(2.15)}$$

where δ_o is the central deflection.

With reference to the strain distribution and crossection shown at (b) in Fig. 2.14, consideration of the equilibrium of axial force and moments about the centroidal axis for the rth section along the length of the member leads to the following equations:

$$P = \int_A \sigma dA = F_1(\rho_r, \varepsilon_{ar}) \qquad \text{———(2.16)}$$

$$M_r = Py_r = \int_A \sigma y' dA = F_2(\rho_r, \varepsilon_{ar}) \qquad \text{———(2.17)}$$

where σ is the stress acting on the elemental area dA.

Considering only the centre section of the member, then the load and moment at this section are given by

$$P = F_1(\rho_o, \varepsilon_{ao}) \qquad \text{———(2.18)}$$

$$M_o = Py_o = F_2(\rho_o, \varepsilon_{ao}) \qquad \text{———(2.19)}$$

and from equation (2.13), the curvature, ρ_o, can be expressed as

$$\rho_o = \frac{\pi^2 y_o}{l^2} \qquad \text{———(2.20)}$$

Since equations (2.18) and (2.19) for P and M_o are both functions of the strain distribution as defined by the centroidal axis strain, ε_{ao}, and the curvature, ρ_o, it is possible to obtain a set of increasing values of M_o

corresponding to a set of increasing values of ρ_o for a given value of P. That is

$$M_o = F_3(\rho_o) \qquad \text{——————(2.21)}$$

for constant P.

An eccentrically loaded column of given length, L, and subjected to a given load, P, becomes unstable with increasing eccentricity, e, when

$$\frac{d_e}{dy_o} = 0 \qquad \text{——————(2.22)}$$

this condition being known as the critical eccentricity criterion. Suitable use of equation (2.22) with equations (2.14)(2.19) and (2.20) leads to the following relationship:

$$\frac{1}{\frac{\pi L}{2l}\tan\left(\frac{\pi L}{2l}\right)} = \frac{\frac{M_o}{2\rho_o}}{\frac{dM_o}{d\rho_o}} - \frac{1}{2} \qquad \text{——————(2.23)}$$

The stability problem now reduces to the simultaneous solution of equations (2.14)(2.18)(2.19)(2.20)(2.21) and (2.23) resulting ultimately in sets of values for the ultimate load P corresponding to a set of values of L and e. This process is described briefly below.

(i) For a given value of buckling load, P, a set of y_o corresponding to a set of M_o values and hence a set of ρ_o values can be obtained from equations (2.19) and (2.21).

(ii) A related set of l values corresponding to the set of y_o and ρ_o values can be found from equation (2.20).

(iii) From the simultaneous use of equations (2.21) and (2.23), a set of L values corresponding to the set of y_o values can be computed.

(iv) Knowing y_o and L, the corresponding e values are obtained from equation (2.14).

(v) A curve of e against L for the given load P chosen to be the buckling load can now be drawn.

(vi) By repeating steps (i) to (v), a family of curves of e against L for a range of P values from zero to a maximum can be determined.

(vii) The buckling load, P, for a column of given length, L, and eccentricity, e, can be interpolated from within the family of curves.

It should be noted that in this analysis, equilibrium is satisfied only at the centre section of the column. Provided that the deflections are small and the flexural stiffness is constant and equal to the stiffness at the centre of the column, equilibrium is automatically satisfied at every other section along the length of the column as the part-cosine curve is the theoretical correct shape. In actual fact, as the centre section is subjected to the largest moment, its flexural stiffness will be smaller than for any other section along the length of the column as concrete decreases in stiffness with increase in stress. As the part-cosine wave assumption implies that the flexural stiffness as calculated for the centre section is constant along the length of the column, the computed deflections tend to be larger than the actual deflections and the computed ultimate loads tend to be lower than the actual ultimate loads. This was confirmed by the results for every eccentrically loaded column tested by Bondale. The actual loads were always greater than the computed values as shown in the last column of Table 2.7.

The above analysis is involved and is laborious, even by computer standards. While the $\frac{M_o}{\rho_o}$ term of equation (2.23) can be determined fairly accurately by interpolation of known calculated points on the curve defining the moment-curvature relation for constant load, the differential term $dM_o/d\rho_o$, which is the slope of the $M_o - \rho_o$ curve, is difficult to calculate with reasonable accuracy. It should be noted that for a column of given length and eccentricity, the buckling load cannot be computed directly using this analysis.

Sixteen small scale encased columns were tested so that correlation with the above theoretical analysis could be made. Eight were of a 4 in. x $1\frac{3}{4}$ in. @ 5 lb. steel joist encased in concrete with a 1 inch cover all round, four being subject to loading applied about the minor axis, the other four to loading about the major axis. A similar number of columns subjected to the same loading conditions were made up from four 1 in. x 1 in. x 3/16 in. latticed angles with a cover of $\frac{3}{4}$ in. all round as shown at (b) in Fig. 2.2. The four lengths tested for bending about either axis were 5ft. 0 in., 6ft. 8 in., 8ft. 4 in. and 10ft. 0 in. and the corresponding eccentricities were 3 in., 2 in., 1 in. and 0 in. such that the eccentricity decreased as the length increased. A summary of results is shown in Table 2.7. For the theoretical solutions, the stress-strain relationship for both the structural steel and the reinforcing steel was assumed to be linear elastic - plastic with no strain hardening and the relationship for the concrete was assumed to be that proposed by Hognestad(31) as shown in Fig. 3.4. The yield point for the steel in all columns was taken as 20 tons/sq.in. (44.8 kips/sq.in.) and the concrete cylinder strength for all the concrete used as 2 tons/sq.in. (4.48 kips/sq.in.).

As expected, the theoretical maximum loads for the eccentrically loaded columns were less than the experimental test loads. This was probably due to the part-cosine wave assumption as discussed previously. There was up to 24% discrepancy between the theoretical and test loads, the average being 11%, indicating a rather poor correlation.

The theoretical solutions for the concentrically loaded columns were obtained using the assumption that the buckling load is given by the tangent-modulus load. The tangent modulus loads were in excess of the test loads, especially for the members bent about the major axis. For these latter members, Bondale attributed this behaviour to premature failure on account of observed initial deformation in the steel sections, although no measurements were given, and also possibly because of the tendency of these columns to deflect laterally but again no supporting experimental evidence of this was provided. The tangent modulus load was calculated assuming the steel was linear elastic, no account being taken of any residual stress effects. The effect of residual stress on the tangent-modulus load is well known and forms the basis of the design procedure in the current AISC Specification(39) for concentrically loaded columns. Residual stress effects would have reduced the tangent modulus for the steel and hence reduced the tangent modulus load for the composite columns. Better agreement with the test results would have resulted. If significant initial curvature was present, the tangent modulus approach would still overestimate the load capacity.

Bondale attempted to examine the effects of axis of bending, eccentricity and length on the load carrying capacity. Unfortunately, both the same length and eccentricity were matched for loading about either principal axis such that the effect of varying either the length or eccentricity alone could not be determined. The properties of the materials were not available for each individual column and as discussed in the following Chapter 3, variations in material properties have considerable influence on column behaviour. Deflection data over the full range of loading from zero to maximum load was not reported for each individual column. Consequently, these tests are of limited use in checking other analytical solutions. However, several important conclusions came out of this investigation as listed below.

(1) The structural behaviour of these small scale encased built-up columns bent about the major axis by an eccentric load was essentially flexural and showed no tendency to lateral torsional buckling. The same conclusion was reached in earlier studies as reviewed by Loke(14) although these were confined to encased single rolled steel joists.

(2) Even though the 3/16 in. dia. lattice used to connect the four angles was relatively light compared to that which might be used for a bare steel column, no premature failures resulted that could be attributed to this, indicating that once the steel sections were encased in concrete, the concrete provided sufficient bracing to maintain integrity up to failure.

(3) The encased lattice angle sections compared favourably with the encased joists in regard to load carrying capacity even though the quantity of steel used in the built-up sections was smaller and the concrete quantities were approximately the same for both types of composite members. This was attributed to the better placement of the steel in the built-up sections, particularly for the columns bent about the minor axis.

(4) From (3) above, it could be said that built-up sections have the advantage of flexibility in the placement of the steel compared to the single rolled steel joist which is limited by its form. With suitable placement of the steel to resist the actions on the column, the writer believes that the use of composite built-up sections could lead to a saving in steel weight and hence structural costs providing the lateral bracing, traditionally required for connecting the steel sections, can be kept to a minimum.

2.3 CONCRETE FILLED TUBULAR COLUMNS

2.3.1 Concentric Loading

The bulk of the test results have been for concentric loading, mainly for circular steel tubes. A few tests have been reported for square steel tubes. The behaviour of such columns depends on the method of load application to the ends of the columns. Three possible methods can be used

(i) Loading of Steel Only

Because of the Poisson's ratio effect, the steel tube will separate, under load, from the unloaded concrete when the bond between the two is broken. For a short column, the maximum load is attained when either the steel yields or local buckling of the tube walls, as for thin-walled tubes, occurs. As the concrete is well separated from the steel tube, it can not offer any restraint to this local buckling. This was verified by Gardner and Jacobson(19) in tests on short 3 in. dia. tubes, 6 in. long with a wall thickness of 0.067 in., where it was shown that loading the steel alone for short concrete filled tubes did not increase the maximum load above that obtained by loading similar hollow tubes. In this case, failure was by local buckling. For slender columns, the steel tube will commence overall buckling at the same load as for a hollow tube. However, once the steel has deflected sufficiently to make contact with the unloaded concrete core, the writer feels that this core will act as a restraint to any further deformation. To cause complete buckling of the tube and core, further load would have to be applied. No details of any tests were available to confirm this assumption.

(ii) Loading of the Concrete Alone

The steel provides a lateral confining stress to the concrete which is then in a state of triaxial stress and can therefore sustain longitudinal stresses well in excess of the unconfined strength. This behaviour has been utilized in columns by Lohr(40). The action is similar to that of spirally reinforced concrete columns where it has been found that at ultimate load, the spiral steel is more effective than an equivalent volume of longitudinal steel. For concrete filled tubes, considerable lateral pressure will be exerted on the concrete by the steel as the load and hence the axial strain in the concrete is increased. This confining pressure ensures that bond by friction between the concrete and steel will exist. Therefore, some axial strain and hence stress will be induced in the steel which will reduce the yield stress in the circumferential direction thereby reducing the maximum possible lateral confining pressure on the concrete and hence the maximum longitudinal stress capacity of the concrete. Even though this loss in concrete axial load capacity is compensated somewhat by the additional axial load induced in the steel by bond, Gardner and Jacobson(19) found that for short columns, loading the concrete alone did not increase the maximum load above that obtained by loading both the steel and concrete simultaneously.

(iii) Loading of Both Steel and Concrete

The concrete and steel are subjected to the same end shortening and both materials would develop lateral tangential strains which are dependent on their respective Poisson's ratios. Initially, the steel has a higher Poisson's ratio than the concrete and hence a higher lateral strain which could lead to a breaking of the bond between the steel tube and the concrete core. This was verified by Furlong(22) who found that short pieces of the steel tube, cut from the columns after testing, could be removed from the concrete without any trace of concrete adhering to the steel. As the load is increased, the Poisson's ratio for the concrete increases until it reaches the steel value and with further loading, it would eventually exceed the steel value. At this stage, when the volume of the concrete exceeds the enclosed volume within the steel tube, the concrete may be expected to exert lateral pressure on the steel. The critical axial stress at which unconfined concrete ceases to decrease in volume and commences to expand was found by Hognestad et al(41) to be between 71 and 96 percent of the unconfined concrete strength. Knowles and Park(23) obtained a value of 95.4 percent of the strength with a coefficient of variation of 4.5 percent. It could therefore be assumed that when concrete is stressed to a value equal to its maximum unconfined strength, the steel could be expected to exert a confining pressure on the concrete and the concrete would then be capable of carrying axial stresses in excess of its maximum unconfined strength due to the triaxial stress condition.

For slender columns, buckling could occur well before the concrete has reached this stress and hence no increase in concrete strength due to confinement can be expected. This result was confirmed in tests by Knowles and Park(23)(24) as shown in columns (8) and (12) of Table 2.8. Assuming no increase in concrete strength due to triaxial effects, the theoretical column strengths for their circular tubes with lengths greater than 44 inches, calculated using the tangent modulus theory, were in agreement with the actual column strengths. For the column lengths less than 44 inches, the actual column strengths became progressively greater than the theoretical values as the length decreased. Similar results were obtained by Gardner and Jacobson(19) who found that for length to diameter ratios greater than 15, there was no apparent increase in strength due to triaxial effects. It is interesting to note from Table 2.8 that there is little evidence of any increase in concrete strength due to confinement for the square tubes, even for quite short lengths. For these tubes, the only effective confinement can occur in the corners of the tube. This difference in behaviour between the circular and square tubes is directly analogous to the well-known difference in behaviour between concrete columns reinforced with spiral and similar columns reinforced with square ties. The spiral can considerably enhance the concrete strength but it has generally been found that even closely spaced square ties do not significantly increase the concrete strength although the improvement in ductility is quite marked. This aspect is discussed in more detail in the later Section 3.2.4.

The majority of tests were on columns where the load was applied to both the concrete and the steel and therefore, the following discussion refers only to tests with this type of loading. The load was generally applied to the columns through stiff steel end fittings of various types. These stiff fittings would ensure that the end shortening and hence the axial strains for both the concrete and the steel were identical. Furlong(22) found that there was virtually no difference in the axial load or eccentric load behaviour of initially unbonded or bonded tubes. For the unbonded tubes, the bond was prohibited by greasing the inside of the tubes. The result is not surprising for the axially loaded tubes as the axial stiffness does not depend on bond where stiff end fittings are used. For the eccentrically loaded tubes, the stiff end fittings would have ensured that the end shortening and end rotation were identical for both the concrete and the steel. As the concrete was contained by the steel, the deformed shape of both must also have been identical. As a result, the two materials were forced to interact rather than act independently, irrespective of whether the tubes were bonded or not.

Some theoretical aspects of tubular column behaviour and comparisons with test results are given below.

The axial load, P, carried by a concrete filled tube can be given by

$$P = \sigma_s A_s + \sigma_c A_c$$

where σ_s and σ_c are the axial stress in the steel and concrete respectively. Assuming that the axial strain, ε, in both the concrete and the steel are identical, then the maximum load for a short column, which is assumed not to buckle, occurs when

$$\frac{dP}{d\varepsilon} = 0$$

i.e. $$A_s \frac{d\sigma_s}{d\varepsilon} + A_c \frac{d\sigma_c}{d\varepsilon} = 0$$

If E_{tc} and E_{ts} are the tangent moduli at the same strain for the concrete and steel respectively, then the above condition can be expressed as

$$E_{tc} = -\frac{A_s}{A_c} E_{ts} \qquad \text{———————(2.24)}$$

For the case where the steel reaches its yield stress, f_{sy}, before the concrete reaches its maximum stress, f_c'', then E_{ts} is zero. For equation (2.24) to be satisfied, then E_{tc} has to be zero which is when the concrete has reached its maximum stress f_c''. Therefore, the maximum short column load, P_{sh}, for this case is given by

$$P_{sh} = f_{sy} A_s + f_c'' A_c \qquad \text{———————(2.25)}$$

Further theoretical aspects of short column behaviour are discussed in detail later in Section 4.1.3. With further increase in axial strain beyond the value at maximum load, the concrete could be expected to press against the steel tube. If a failure theory for the steel, such as the maximum shear stress criterion, is assumed, then the steel is incapable of exerting any confining pressure on the concrete as it has already yielded in the longitudinal direction and would have no strength in the circumferential direction as assumed from the failure criterion. Therefore the maximum stress in the concrete, f_c'', as used in equation (2.25) would be the unconfined concrete strength. For Furlong's tests(21) on short concrete filled tubes with a yield stress between 42 to 48 kips/sq.in. for the steel, which is therefore likely to yield before the concrete reaches its maximum strength, equation (2.25) gave a good estimate of the actual column strengths. Furlong proposed a lower bound estimate of this strength by using a concrete stress in equation (2.25) corresponding to the strain at which the steel first yields rather than the maximum concrete strength, f''_c.

For the case, as when using high strength steels, where the concrete reaches its maximum stress before the steel yields, the maximum load for short columns is attained when the condition of equation (2.24) is satisfied. For square tubes, where the steel tube is incapable of exerting any significant lateral pressure on the concrete, the stress-strain curve for the concrete is likely to be similar to that shown in Fig. 3.3 obtained under uniaxial compression and therefore exhibits a descending portion. As E_{ts} for the steel has a positive value before yield, E_{tc} must be negative for equation (2.24) to be satisfied. Hence, the concrete is strained past its maximum stress and will therefore have a value less than the maximum. Similarly, the steel will be below the yield stress and equation (2.25) would over-estimate the short column strength for this case. Unfortunately no test results could be found in the literature for tubes of high-strength steel filled with concrete for the above hypothesis to be tested. However, some test results for circular tubes were available but the conditions are somewhat different. For circular tubes, a triaxial stress condition can be produced in the concrete and the stress-strain curve for the concrete will not fall once the maximum unconfined concrete strength is reached for it is at this stage that the steel tube will commence to exert a confining pressure on the concrete as discussed previously. Hence, the longitudinal stress in the concrete will rise with increasing longitudinal strain provided that the steel continues to exert an increasing lateral pressure on the concrete. This is possible providing the steel remains elastic. Assuming the maximum shear stress failure theory for the steel as used for steel tubes by Gardner and Jacobson(19) where

$$\sigma_{sl} + \sigma_{st} = f_{sy}$$

then any tangential tensile stress, σ_{st}, induced in the steel tube will lower the maximum stress capacity, σ_{sl}, in the longitudinal direction. Gardner and Jacobson(19) conducted a series of tests on short concrete filled circular tubes with several different wall thicknesses and with steel yield stresses (0.2 percent offset) ranging from 53 kips/sq.in. to 92 kips/sq.in. which were therefore likely to yield at strains in excess of the strain corresponding to the maximum unconfined concrete strength, this strain being of the order of 0.002. It was found that the experimental loads were significantly greater than those given by equation (2.25) which is the sum of the individual strengths of the concrete and steel. This indicated that the increased load capacity of the concrete by virtue of increasing its longitudinal stress capacity with triaxial compression more than compensated for the reduction in load capacity of the steel due to the presence of the tangential tensile stress in the steel which decreased its longitudinal stress capacity. Gardner and Jacobson(19) also found that local buckling, which occurred for the thin-walled hollow tubes, was prevented by the presence of the concrete core providing the load was applied to both the steel and concrete and not the steel alone as discussed previously. This is because any separation that occurs between the concrete core and steel tube is only small when the load is applied simultaneously to the concrete and steel. The concrete core is therefore able to force the local buckling associated with both inward and outward deformation of the tube walls into a post-yield mode of purely outward deformation of the tube walls. The tube walls are thereby stabilized within the elastic range for the steel ensuring the potential development of the longitudinal yield stress.

For the more slender columns, where overall buckling can occur, several tangent modulus approaches have been used to determine the axial load capacity. These are the Combined Tangent Modulus approach where the axial strain and curvature for both the steel and concrete are assumed identical, this condition corresponding to perfect bond between the steel and concrete such that the column buckles as a fully composite column; the Individual Tangent Modulus approach where the buckling load is calculated as the sum of the tangent modulus buckling loads of the steel column and the concrete column taken independently, and the Maximum Tangent Modulus approach where no bond is assumed to exist between the steel and the concrete and for increasing load on the column, the material that has reached its independent tangent modulus load first is assumed to be restrained by the other material until it reaches its independent tangent modulus buckling load. These three approaches are discussed separately below.

(a) Combined Tangent Modulus

This theory assumes that the axial strain and curvature for both the concrete and the steel are identical. Salani and Sims(18) used the following expression to determine the axial load capacity, P_o

$$P_o = \frac{\pi^2 E_t I_g}{l^2} \qquad \text{———— (2.26)}$$

where I_g is the moment of inertia for the gross area, A_g. Now

$$A_g = A_s + A_c$$

and because of the symmetry of circular and square tubes, the principal axes of the concrete area, A_c, and the steel area, A_s, coincide, hence

$$I_g = I_s + I_c$$

where I_s and I_c are the moment of inertia of the steel and concrete areas respectively. The value of the effective tangent modulus, E_t, was determined by Salani and Sims(18) from an axial load test on a short concrete filled tube for which the stress on the gross area was plotted against the axial strain the slope of the curve giving the tangent modulus. The use of this value of E_t is erroneous. The value of E_t required for equation (2.26) is the effective tangent modulus in flexure for the gross area. This can be shown to be dependent on the configuration of the steel and concrete and not just only on their respective areas. Within the elastic range for both the steel and the concrete, the effective modulus for axial load conditions, E_{ta}, is given by

$$E_{ta} = \left(\frac{A_s}{A_g}\right) E_s + \left(1 - \frac{A_s}{A_g}\right) E_c \qquad \text{———— (2.27)}$$

where E_s and E_c are the elastic moduli for the steel and concrete respectively. For circular or square tubes, the effective modulus in flexure is given by

$$E_{tb} = \left(2\frac{A_s}{A_g} - \frac{A_s^2}{A_g^2}\right) E_s + \left(1 - \frac{A_s}{A_g}\right)^2 E_c \qquad \text{———— (2.28)}$$

Equations (2.27) and (2.28) have been derived in this form by Furlong(21). For low stress values within the elastic range, the difference in the above two equations demonsrates the error if equation (2.27) is used in equation (2.26) to determine the tangent modulus buckling load instead of using equation (2.28). This is the possible source of the discrepancy between the theoretical and experimental maximum found by Salani and Sims(18) from their tests. They also reported the only known tests where the concrete core did not improve the load carrying capacity of the steel tube. Although this phenomenon occurred only for the very small diameter tubes, no explanation for this behaviour was put forward.

Furlong(22) also used equation (2.26) to determine the axial load capacity, P_o, for slender columns. The columns were assumed to be elastic for values of $P_o \leq P_{sh}/2$ where P_{sh} was the short column load such as might be given by equation (2.25). The correct value of the effective tangent modulus, as given by equation (2.28), was used. Substitution of this value of E_{tb} into equation (2.26) yielded the elastic buckling or Euler load, P_E where

$$P_o = P_E = \frac{\pi^2 E_{tb} I_t}{l^2} \qquad \text{——— (2.29)}$$

$$\text{for } P_o \leq \frac{P_{sh}}{2}$$

The value of the effective tangent modulus, E_{ti}, in the inelastic range, where $\frac{P_{sh}}{2} \leq P_o \leq P_{sh}$ was obtained from an empirical expression where

$$E_{ti} = 4 \frac{P_o}{P_{sh}}\left(1 - \frac{P_o}{P_{sh}}\right) E_{tb} \qquad \text{——— (2.30)}$$

Use of this value of the tangent modulus in equation (2.26) and then substituting the value of P_E given by equation (2.29) then

$$\frac{P_o}{P_{sh}} = 1 - \frac{1}{4} \frac{P_{sh}}{P_E} \qquad \text{——— (2.31)}$$

$$\text{for } \frac{P_{sh}}{2} < P_o \leq P_{sh}$$

Equation (2.31) is the Johnson(42) parabola and is identical in form to the CRC(43) column strength curve as adopted by the AISC Specification(39) for the axial load capacity of steel columns which are assumed to contain residual stresses of half the yield stress of the steel. It is therefore unlikely that equation (2.31) would accurately represent the axial load capacity of concrete filled tubes and although it is simple in form, it depends on the convenient choice of the empirical expression of equation (2.30) for the effective tangent modulus of the concrete filled tubes. The tangent modulus is the first derivative of the stress strain function and hence equation (2.30) defines a stress-strain function for the composite material. The writer has shown later in this section that this stress-strain function is exponential in form. Unfortunately, the concrete stress-strain curve is unlikely to be of this form. Its shape can be shown to be influenced by many factors as discussed in detail in Chapter 3. When applied to the test results of Klöpell and Goder(17) and Salani and Sims(18) for columns of varying slenderness, Furlong(22) found equations (2.29) and (2.31) gave conservative results, averaging about 10% below the experimental loads with the maximum variation being 20%. It can be seen that these equations may have the advantage of simplicity but do not accurately predict the axial load capacity probably because the actual material properties are not adequately represented by the empirical expression of equation (2.30).

Both Gardner and Jacobson(19) and Neogi, Sen and Chapman(25) used the combined tangent modulus approach in the following form where the maximum load, P_o, is that value of the load which simultaneously satisfies the equations

$$P = \frac{\pi^2(E_{ts}I_s + E_{tc}I_c)}{l^2} \qquad \text{———(2.32)}$$

and $$P = \sigma_s A_s + \sigma_c A_c \qquad \text{———(2.33)}$$

where for a particular value of strain, E_{ts} and σ_s are the tangent modulus and stress for the steel and E_{tc}

and σ_c are the corresponding values for the concrete. For their tests on slender columns of circular crossection where augmentation of the maximum load, due to triaxial stress conditions in the concrete, was unlikely, Gardner and Jacobson(19) found that the average difference between the theoretical tangent load and the experimental maximum loads was 6% which is better agreement than was found by Furlong(22). The stress-strain curves for the steel and concrete were obtained from tests on the materials.

For concrete filled square tubes, where the concrete core is unlikely to be in a state of triaxial compression, the concrete could have a stress-strain curve with a descending portion similar to that shown in Fig.3.3. It would therefore be possible for the simultaneous solution of equations (2.32) and (2.33) to occur for a negative value of the concrete tangent modulus, E_{tc}. This would be likely for steel tubes manufactured from high yield stress steel. Unfortunately the square tubes tested by Furlong(21) and Knowles and Park(23) contained relatively low yield stress steel so it was not possible to test the validity of using the negative tangent modulus for the concrete.

(b) Individual Tangent Modulus

Furlong's(22) results, showing no difference between bonded and unbonded axially loaded specimens, were interpreted by Knowles and Park(23)(24) to mean that the concrete filled tube could be considered to be two independent columns, one of concrete and the other of steel. Using this assumption then either the concrete or the steel will reach the strain corresponding to its individual tangent modulus load before the other as the load is applied. Assuming the concrete reaches its buckling load first, as was the case for the materials and crossection used in Knowles and Park's tests, then on further application of load and hence end shortening of the column, the concrete does not carry any further load but maintains its buckling load, P_{tc}. The maximum load,

P_o, is reached when the steel reaches its tangent modulus buckling load, P_{ts}. The maximum load is therefore given by the sum of the individual tangent modulus loads for the concrete and steel alone where

$$P_o = P_{tc} + P_{ts} \qquad \text{——— (2.34)}$$

P_{tc} is obtained from the simultaneous solution of the expressions in equation (2.35)

$$\left.\begin{aligned} P &= \frac{\pi^2 E_{tc} I_c}{l^2} \\ P &= \sigma_c A_c \end{aligned}\right] \qquad \text{——— (2.35)}$$

and P_{ts} from the simultaneous solution of the expressions in equation (2.36)

$$\left.\begin{aligned} P &= \frac{\pi^2 E_{ts} I_s}{l^2} \\ P &= \sigma_s A_s \end{aligned}\right] \qquad \text{——— (2.36)}$$

The summation of the independent buckling loads assumes that once the concrete has reached its tangent modulus load, it cannot sustain any further load and hence axial strain. However, as the applied load is increased beyong this stage, the writer believes that the stiff end fittings usually used in tests will ensure that further end shortening of both the concrete and steel will take place. As the concrete is now assumed to have a constant load and therefore axial strain, any further end shortening of the concrete must be accomodated by lateral deformation and curvature of the concrete core. Although no bond is assumed between the concrete core and steel tube, any separation between the two due to differential expansion will be small. Therefore any end shortening due to curvature of the concrete will also be small, the steel tube restraining the concrete core against any further lateral deformation once the two come in contact. That the separation between the core and steel tube is extremely small is evidenced by the fact that local buckling of thin-walled tubes is prevented by the concrete core as discussed earlier in this Chapter. Hence, with further loading, the writer feels that the axial strain in the concrete also continues to increase and the load carried by the concrete will be dependent on this strain rather than remain constant at the tangent modulus buckling load. For their tests on long, slender columns, where a triaxial stress condition in the concrete core was unlikely, Knowles and Park(23) found that the average difference between the theoretical loads, calculated by the summation of the individual tangent modulus loads, and the experimental loads was 5%. The accuracy of the method is therefore similar to that obtained by the combined tangent modulus method as used by Gardner and Jacobson(19). However, the writer feels that the individual tangent modulus method does not correctly estimate the proportion of the total load carried by the steel and concrete individually. Knowles and Park(23) did not investigate this aspect and strain results were not available from the reports of their tests to enable a check to be made.

(c) Maximum Tangent Modulus

For this approach, when either the steel or concrete reaches its individual tangent modulus load, the material that has not reached its individual tangent modulus load acts as a restraint on the material that has. It is assumed that no bond exists between the concrete and steel. On further loading, the axial strains in both materials increase until the tangent modulus load for the restraint material is reached. Assuming that the concrete reaches its individual tangent modulus load first, then the axial load capacity, P_o, is given by

$$P_o = P_{ts} + \sigma_c A_c \qquad (2.37)$$

where σ_c is the concrete stress corresponding to the strain at which the tangent modulus load for the steel is attained. Knowles and Park(23) found that this approach over-estimated the axial load capacity for the longer tubes as can be seen by comparing columns (4) and (15) of Table 2.8. For the longer tubes, it also gives higher loads than if the concrete were assumed bonded to the steel tube as for the combined tangent modulus approach. This can be seen by comparing columns (11) and (15) of Table 2.8. Such behaviour could not be considered as being possible and this is demonstrated by the actual maximum experimental loads that were attained for the long columns.

The three tangent modulus approaches are compared in Table 2.8 for test data given by Knowles and Park(24). The results for the individual tangent modulus approach were calculated by Knowles and Park and the results for the combined and maximum tangent modulus approaches were calculated by the writer. The stress-strain curve for the concrete is that proposed by Hognestad(31) and shown in Fig. 3.4. Knowles and Park did not use an explicit stress-strain curve for the steel but obtained the tangent modulus buckling load, P_{ts}, from the following equations

$$\frac{P_{ts}}{P_s} = 1 - \frac{1}{4}\frac{P_s}{P_E} \quad \text{for} \quad \frac{P_s}{2} < P_{ts} \le P_s \qquad (2.38)$$

$$P_{ts} = P_E \quad \text{for} \quad P_{ts} \le \frac{P_s}{2} \qquad (2.39)$$

These equations define the parabolic column strength curve as proposed by the CRC(43) and adopted for the AISC Specification(39). The stress-strain curve for the steel has been derived by the writer in the following manner.

Now $P_s = A_s f_{sy}$ ——— (2.40)

$$P_{ts} = A_s \sigma_s = \frac{\pi^2 E_{ts} I_s}{l^2} \qquad (2.41)$$

$$P_E = \frac{\pi^2 E_s I_s}{l^2} \qquad (2.42)$$

$$E_{ts} = \frac{d\sigma_s}{d\varepsilon} \qquad (2.43)$$

From equations (2.39)(2.40)(2.41) and (2.42) then

$$\sigma_s = E_s \varepsilon \quad \text{for} \quad \sigma_s \leq \frac{f_{sy}}{2} \qquad (2.44)$$

Rearranging equation (2.38) then

$$\frac{P_{ts}}{P_E} = 4 \frac{P_{ts}}{P_s} (1 - \frac{P_{ts}}{P_s}) \qquad (2.45)$$

Substitution of equations (2.40) to (2.43) into equation (2.45) yields

$$E_{ts} = 4E_s \frac{\sigma_s}{f_{sy}} (1 - \frac{\sigma_s}{f_{sy}}) = \frac{d\sigma_s}{d\varepsilon} \qquad (2.46)$$

This can be seen to be identical in form to the empirical equation (2.30). Solution of the differential expression of equation (2.46) gives

$$\sigma_s = (\frac{e^k}{1+e^k}) f_{sy} \quad \text{for} \quad \frac{f_{sy}}{2} < \sigma_s \leq f_{sy} \qquad (2.47)$$

where $k = \frac{4E_s \varepsilon}{f_{sy}} - 2$

Equations (2.44) and (2.47) therefore define the stress-strain curve for the steel and is that used, although not in this form, in the AISC Specification(39) for determining the axial load capacity by the tangent modulus method for steel columns. It is interesting to note that the yield stress is only attained for strain values approaching infinity. This stress-strain curve is therefore only an approximation to the real stress-strain curve for a steel section with residual stresses of one half the yield stress. The real stress-strain curve will depend not only on the magnitude of the residual stresses but also on their distribution throughout the steel section. Using the assumed stress-strain curves for the concrete and the steel, it can be seen from columns (7), (11) and (15) of Table 2.8 that the axial load capacity, P_o, calculated using the three different tangent modulus approaches do not differ very much except that for the longer columns, the maximum tangent modulus approach gives high values as discussed earlier in this Section. The major difference between the approaches is in the load sharing between the concrete and the steel, especially for the longer columns. By comparing columns (5) and (6) with columns (9) and (10) of Table 2.8 it can be seen that the combined tangent modulus approach gives higher concrete loads but lower steel loads than the individual tangent modulus approach. This results from the strain corresponding to the combined tangent modulus load being greater than the strain corresponding to the tangent modulus buckling load for the concrete alone but less than the strain corresponding to the tangent modulus buckling load for the steel alone.

Although the internal load sharing varies, it would appear from these results that the axial load capacity, P_o, is predicted with about the same accuracy using either the individual or the combined tangent modulus approach. The short column loads shown in column (16) of Table 2.8 were calculated using the criterion of equation (2.24). As the theoretical stress-strain curve used was for unconfined concrete and hence exhibited a distinct drooping characteristic, the theoretical short column loads for the circular tubes are therefore less than the experimental loads for the short 10 inch and 20 inch column lengths where triaxial compression of the concrete core is possible. For the square tubes, where triaxial effects are not significant, there is good agreement between the theoretical short column loads and the experimental loads for the short 10 inch and 20 inch column lengths.

2.3.2 Eccentric Loading

Tests on concrete filled tubes subjected to both axial force and bending moment have been performed by Furlong(21)(22), Knowles and Park(23)(24) and Neogi, Sen and Chapman(25).

In Furlong's tests, the moment was applied to the ends of the columns by a load, provided by an hydraulic ram, applied at a constant eccentricity to an outrigger attached to each end of the column. A further concentric load was applied by a testing machine acting through spherical bearings. The concentric axial load was adjusted for each change in the eccentric load so that the total axial load remained constant while the moments were increased up to a maximum value. In all, 39 circular or square tubes with diameters or side lengths varying from 4 in. to 6 in. and wall thicknesses varying from 0.061 in. to 0.189 in. were tested in this manner for a range of axial loads. Fourteen similar columns were tested with purely concentric loading. No details were given for the length of the columns. Knowles and Park(24) reported that Furlong's 14 concentrically loaded columns were of 36 in. effective length and it has been assumed that the eccentrically loaded columns were of the same length. The results of the eccentric load tests are plotted in Fig. 2.15 in the form of an interaction diagram as presented by Furlong. For a particular value of applied load, P_u, the corresponding maximum moment, M_u, was the actual moment at midheight of the column and was calculated as the sum of the product of the column deflection times the total axial force plus the eccentric force times its constant moment arm. The axial load capacity, P_o, for zero moment was obtained from concentric load tests on similar columns. The ultimate moment capacity, M_o, for zero applied load was calculated as for ultimate strength reinforced concrete theory using the assumptions of a limiting strain of 0.003 for the concrete, a simplified stress block for the concrete such as that used by Hognestad et.al.(31), zero tensile strength for the concrete and all the steel being at the yield stress in either tension or compression. From flexure tests on concrete filled tubes, simply supported at each end and subjected to two transverse point loads, Knowles and Park(23) found that the ratio of the experimental to the theoretical M_o values, as calculated using the above simplifying assumptions, only varied between 0.93 and 1.15 which indicated reasonable agreement. These results also indicate that in flexure, the steel tube and concrete core act compositely and not as independent members, otherwise the ultimate moment capacity would have been that for the steel tube alone, the concrete core having no contribution to the moment capacity once it had cracked in tension. It can be seen from Fig. 2.15 that the simple straight line interaction formula

$$\frac{P_u}{P_o} + \frac{M_u}{M_o} = 1 \qquad \text{———— (2.48)}$$

which has been used to represent the beam column behaviour of bare steel I-sections(43), is very conservative when applied to concrete filled tubes. Furlong(21) suggested that the simple elliptical interaction function

$$\left(\frac{P_u}{P_o}\right)^2 + \left(\frac{M_u}{M_o}\right)^2 = 1 \qquad \text{———— (2.49)}$$

would provide a better estimate of the beam-column strength but this can still be seen to be very conservative in the majority of cases yet unconservative in a few cases. The difficulty in the use of interaction formulae of the type given by equations (2.48) and (2.49) is that they require the maximum moment, M_u, to be known which in turn requires the deflected shape of the column to be known. To overcome this problem, Furlong(22) later proposed the use of a moment magnification factor identical in form to that used for bare steel sections as in the AISC Specification(39) where

$$M_u = \frac{M_e}{1 - \frac{P_u}{P_{cr}}} \qquad \text{(2.50)}$$

and M_e is the known moment produced by the eccentric force times its moment arm and P_{cr} is the elastic critical buckling load in the plane of the applied moment. The use of moment magnification factors is discussed later in Chapter 11.

Knowles and Park(23) conducted similar eccentric load tests except that a single load was applied to the ends of the column through knife edges centred in a groove in a loading plate which could be offset to provide the desired initial end eccentricity, in this case being 0.3 in. and 1.0 inches. As with Furlong's tests, the same eccentricity was used at both ends such that the columns were bent symmetrically in single curvature. The load was increased on the columns until a maximum load, P_u, was reached at which point the central deflections were noted so that the maximum moment, M_u, at the centre of the columns could be calculated. Flexure tests and concentric load tests were also carried out to determine the ultimate moment capacity, M_o, and the axial load capacity, P_o, respectively. Details of the tests are given in Table 2.9. The test results are shown in Fig. 2.16 in the form of an interaction diagram in an identical manner as for Furlong's tests in Fig. 2.15. Knowles and Park(23) also proposed the use of the straightline interaction formula given by equation (2.48). It can be seen from Fig. 2.16 that several of the experimental results were below this straight line. This is in direct contrast to Furlong's results in Fig. 2.15 where all the results were above and generally well away from the straight line relationship. No reasonable explanation was put forward by Knowles and Park to account for the experimental points being so far below the straight line relationship except that they noted that for these points, tensile strains were developed in the columns. However, Knowles and Park reported that the loading assembly was slightly unstable and the full experimental load may not have been reached in all cases. In fact, the square tube with an effective length of 56 in. and an initial eccentricity of 1.0 in. was observed to have jumped out the testing apparatus at the end of its loading and it was thought that this column certainly did not reach its maximum load. It is interesting to note that the 3.0 in. square tube of 32 in. length is the only column, loaded at an eccentricity of 1.0 in., which gave an experimental point above the straight line interaction relationship. From column (7) of Table 2.9, this can be seen to be the only column tested with a 1.0 in. eccentricity that had a significantly larger central deflection than a similar column tested with the smaller 0.3 in. eccentricity. It seems surprising that for the circular tubes, the columns loaded with 1.0 in. eccentricity all gave smaller central deflections at maximum load than for the columns of similar length loaded with the smaller 0.3 in. eccentricity. The reverse is generally found from the results of tests on eccentrically loaded columns, where the central deflection at maximum load is found to increase with increasing eccentricity, rather than decrease as was the case for Knowles and Park's tests. This indicates that instability of the testing assembly or some other unusual but unnoticed behaviour must have occurred during the tests at 1.0 in. eccentricity before the full load and hence moment capacity was reached. It should also be noted that the deflection at maximum load is a difficult quantity to define accurately owing to the instability of the column at this point. It is therefore felt that little importance should be attached to the experimental results for the tubes tested with 1.0 in. eccentricity. If these results are ignored, the straight line interaction equation (2.48) can again be considered to give a conservative estimate of the load-moment capacity of concrete filled tubes. It is again stressed that the moment, M_u, used in this equation is the actual maximum moment at the centre of the column which includes the moment produced by the deflection of the column.

As can be seen from the above discussion, Knowles and Park(23) and Furlong(21)(22) only considered the condition at maximum load and moment and attempted to estimate the beam-column capacity using simple interaction formulae such as those given by equations (2.48) and (2.49). Neogi, Sen and Chapman(25) were concerned with the behaviour over the full range of loading. Two methods for determining the load-deflection behaviour of eccentrically loaded concrete-filled tubes were considered and compared with the results from tests on eighteen columns, all of circular crossection, with a variety of wall thicknesses and lengths.

The first theoretical method used the simplifying assumption that the deflected shape could be approximated by part of a half-cosine wave. This was also used by Bondale(8) for composite columns and as discussed earlier in Section 2.2.4. Neogi's approach involves a simpler computation procedure than that of Bondale and can also be used to determine the falling branch of the load-deflection curve. A brief description for comparison is therefore given below.

With reference to Fig.2.14 together with the boundary condition $y = e$ at $z = {}^{L}/2$, the curvature,

ρ_o, at the centre of the column for $z = 0$ can be calculated from equation (2.13). Neogi has shown this to be

$$\rho_o = \frac{4}{L^2}\left[\cos^{-1}\left(\frac{e}{e + \delta_o}\right)\right]^2 (e + \delta_o) \qquad \text{———— (2.51)}$$

From equation (2.51), the central curvature, ρ_o, is related directly to the central deflection, δ_o. The process for calculating the load-deflection curve is as follows.

(i) Select a value of the central deflection, δ_o.

(ii) From equation (2.51) calculate ρ_o.

(iii) Select a trial value of centroidal axis strain, ε_{ao}

(iv) Using the values of ρ_o and ε_{ao}, calculate the load, P, and the moment at the centre of the column, M_o, from equations (2.18) and (2.19)

(v) For equilibrium

$$M_o = P(e + \delta_o) \qquad \text{———— (2.52)}$$

If equation (2.52) is not satisfied to a pre-assigned tolerance, improve the value of ε_{ao} using a Newton-Raphson technique of successive iterations. When equation (2.52) is satisfied, a value of the load, P, corresponding to the selected central deflection, δ_o, is obtained.

(vi) By successively incrementing the value of δ_o and repeating steps (ii) to (v) for each value of δ_o, the full load deflection curve is established. It should be noted that equilibrium is satisfied only at the centre of the column. To overcome this objection, Neogi et al(147) used a second more exact method which accounted for equilibrium at every section along the length of the column. This markedly increased the amount and difficulty of computation as can be seen below.

The half length of the column, shown in Fig. 2.14, is divided into n sections, each of length h. At the rth crossection, the moment, M_r, is given by

$$M_r = P(y_o - v_r) \qquad \text{———}(2.53)$$

and the curvature, ρ_r, at this section can be expressed in finite difference form as

$$\rho_r = \frac{1}{h^2}(v_{r-1} - 2v_r + v_{r+1}) \qquad \text{———}(2.54)$$

Since $v_o = 0$ and by symmetry $v_1 = v_{-1}$, then from equation (2.54)

$$v_1 = \frac{1}{2} h^2 \rho_o \qquad \text{———}(2.55)$$

The computation procedure is as follows.

(i) Choose an initial value of the central curvature, ρ_o.

(ii) Select a trial value of central centroidal axis strain, ε_{ao}.

(iii) From equations (2.18) and (2.19) calculate the value of load, P, and moment at the centre of the column, M_o, corresponding to ρ_o and ε_{ao}.

(iv) Using P and M_o, calculate the value of y_o from equation (2.53) for $r = 0$.

(v) Use ρ_o to calculate the offset distance, v_1, from equation (2.55).

(vi) Use the values of P, v_1 and y_o to calculate the moment, M_1, at section 1(r=1).

(vii) Using P and M_1, solve the simultaneous equations (2.16) and (2.17) to obtain the corresponding values of ρ_1 and ε_{a1} at section 1.

(viii) Calculate v_2 from equation (2.54) using the values of ρ_1, v_o and v_1.

(ix) Repeat steps (vi) to (viii) using the value v_2 then v_3 etc. for all the sections along the length of the column until the offset distance v_n and hence y_n is obtained.

(x) For compatibility

$$y_n = e \qquad (2.56)$$

If equation (2.56) is not satisfied to a pre-assigned tolerance, modify the value of ε_{ao} and repeat steps (ii) to (x) until it is. The corresponding values of P and y_o and hence δ_o represent a point on the load deflection curve.

(xi) By successively incrementing the value of ρ_o and repeating steps (ii) to (x), the complete load-deflection curve is established.

It should be noted that while it is relatively simple, as in step (iii), to obtain the load, P, and moment, M_r, for a crossection with a given strain distribution as defined by the curvature, ρ_r, and centroidal axis strain, ε_{ar}, the reverse procedure of determining ρ_r and ε_{ar} for a given value of P and M_r, as in step (vii), is much more difficult. Neogi et al.(25) gave no indication of their technique for doing this but it is likely that a procedure, such as that by Gurfinkel and Robinson(44), was used. A similar procedure developed by the writer is described in a later Section 4.2.4(d).

The results for the eighteen columns tested by Neogi, together with eight column tests conducted at Tokyo University by Kato and Kanatani(45), are shown in Table 2.10. The predicted load-central deflection and load-strain curves using the two above theoretical methods are compared with the experimental results of one of the tests, M5, in Figs. 2.17 and 2.18 respectively. The strain plotted was the longitudinal strain at the centre of the column on the concave face. From these figures and columns (6) and (7) of Table 2.10, it can be seen that in all cases, the theoretical approach using the cosine wave assumption gave slightly lower maximum loads than the theoretical approach using the exact deflected shape of the column. This difference is as expected as discussed in Section 2.2.4. The cosine wave assumption appeared to be reasonable for predicting the load-deflection and load-strain curves except for the descending portion of the load-strain curve. For the columns with $\frac{l}{d}$ ratios greater than 15, reasonable agreement was obtained between the theoretical and experimental loads, the former being calculated using stress-strain curves obtained from uniaxial tests on concrete and steel specimens. It could be concluded that no concrete confinement took place and that triaxial effects were unimportant for these more slender columns. Similar behaviour was also observed for slender columns subjected to concentric loading as discussed in detail in Section 2.3.1. Except for column C8 which failed at the quarter point, perhaps due to bad filling, the four shorter columns, C5 to C8, which were all tested with a very small eccentricity, exhibited some augmentation in maximum load due to triaxial effects. This increase in load was up to 30% above the theoretical maximum loads calculated using the stress-strain curve for unconfined concrete and was similar to the increase in maximum load for short columns subjected to concentric load as shown in Table 2.8 for the tubes of circular crossection. However, as indicated by the Tokyo University test results, this triaxial effect was insignificant for practical values of eccentricity as clearly demonstrated by the results for the short columns, BC7 to BC20, with $\frac{l}{d}$ ratios of only 8.8. It could therefore be inferred that for columns with practical values of slenderness and end eccentricity, triaxial effects would be relatively insignificant. It is interesting to note that for Neogi's tests, curvatures of the order of 0.01 in^{-1} and strains in excess of 0.03 in./in. were attained without any apparent local failures. As a check, a tube was cut open at the centre section and the concrete was found to have retained cohesion.

Some results of tests for determining the fire resistance of concrete filled tubes have been reported by Boué(6). These tests were conducted at the Federal Materials Testing Institute, Berlin - Dahlem, under the direction of Dr.-Ing. Seekamp. Seamless tubes of 36 kips/sq. in. yield stress steel, an overall diameter of 4.8 in., a wall thickness of 0.2 in., a length of 15 ft. 9 in. and a slenderness ratio of 116 were subject to a design axial load of 19.8 kips applied to the steel alone, this representing an axial steel stress of 8.7 kips/sq.in. The fire tests were in accordance with German Standard DIN 4102. The times to reach a steel temperature of 350°C and column collapse were 7 minutes and 10 minutes respectively for the hollow tubes, 10 minutes and 14 minutes for the concrete filled tubes and 64 minutes and 67 minutes for concrete filled tubes with 1.2 in. of lime rendering over the steel. These tests indicated that the concrete filling alone did not improve the fire resistance significantly above that of the bare steel tubes yet the provision of a cover of only a small thickness was sufficient to markedly improve the fire resistance. A further interesting aspect was demonstrated during these tests. Towards the end of the fire testing, the walls of two of the concrete filled tubes burst open and steam poured out. Further experiments immediately followed to determine how the build-up of dangerous steam pressure could be avoided. It was found that this could be achieved by providing drill holes, 1.86 sq.in. in area, through the tube walls at no more than 16 ft. spacing, a minimum of two holes being required. The rendered cover was found not to impede the escape of steam. A similar behaviour was reported by Furlong(22) in tests carried out at Braunschweig in Germany by Prof. Kordina who warned that entrapped moisture can cause an explosion of the steel tube during a fire. German Building Regulations now require the placement of holes in the steel tube approx. $\frac{3}{4}$ in. in diameter (0.44 sq.in.) at every 2ft. or so throughout the height of the column.

2.4 SUMMARY

2.4.1 Built-up Composite Columns

1. It has been shown from early tests that the column strengths of axially loaded built-up steel sections are considerably enhanced by concrete encasement. In most tests, the material properties were such that the strain at steel yield, f_{sy}, was less than the strain at the maximum stress, f''_c, of the concrete and the load capacity for stocky columns could therefore be estimated by the use of equation (2.1). Where spirals were used as reinforcement, the concrete strength was further enhanced and strengths in excess of that given by equation (2.1) were attained.

2. As for reinforced concrete and bare steel columns, the axial load capacity was found to decrease with increasing slenderness. However, no theoretical work was developed to investigate the effect of slenderness although empirical relationships, derived from curve fitting to experimental data, were developed.

3. The inelastic nature of concrete was observed from tests and simple parabolic stress-strain curves for the concrete have been used to predict the sharing of axial load between the concrete and steel.

4. A simplified inelastic analysis, assuming a part cosine wave for the deflected shape and satisfying equilibrium only at the centre of the column, has been developed to determine the load capacity for eccentrically loaded columns. Although not entirely accurate, encouraging results were obtained. Unfortunately, the method cannot be used directly to obtain the load capacity for a column of given length and end eccentricity.

5. The effect of concrete encasement increased the torsional stiffness of the bare steel sections such that the

behaviour was essentially flexural in the plane of the applied moments with little or no tendency for lateral torsional buckling.

6. The concrete was shown to provide considerable lateral restraint to the individual steel components of the built-up sections. Buckling of individual elements, which is a problem for the bare steel members, was not evidenced until after crushing of the concrete when it is no longer capable of offering restraint.

7. For similar steel area, built-up sections encased in concrete were capable of carrying higher loads at the same eccentricity than solid I-sections. This can be explained by the fact that built-up sections can be better placed to advantage within the concrete encasement to resist the applied moments.

8. Long term effects, such as creep and shrinkage, have not been considered. Residual stresses in the steel have not been accounted for. The influence of material properties, especially for the concrete, have only been taken into account in a very simplified manner. Only a very small number of eccentric load tests have been conducted. The effectiveness of the concrete encasement in providing the shear transfer between the longitudinal steel elements instead of using battens or diagonal tie plates has not been examined.

2.4.2 Concrete-Filled Tubes

1. Concentric load tests were numerous. Triaxial effects in the concrete have been investigated. Providing high yield stress steel is used the axial load capacity of short concrete-filled circular tubes can be considerably enhanced by these triaxial effects. Triaxial effects have been shown to be insignificant for square tubes.

2. For the more slender columns, buckling under concentric load took place before any augmentation in concrete strength, due to confinement, could occur. The axial load capacity, for a range of slenderness ratios, has been predicted with reasonable accuracy using tangent modulus methods incorporating typical stress-strain curves for the concrete and the steel, the latter including the effects of residual stresses. For the upper range of slenderness ratios, the tangent modulus method has sometimes been found to over-estimate the actual load capacity. This was generally attributed to initial curvature, accidental end eccentricities or other imperfections. The tangent modulus method assumes no lateral deformation until the buckling load is reached and therefore does not account for initial curvature or geometrical imperfections which can be significant for the longer columns. Neogi et al(25) accounted for initial curvature and initial end eccentricity in their analysis by computing an equivalent end eccentricity that produced the same lateral deformation and strain distribution as measured at the centre of the column under test. In Sections 4.2 and 4.3 an analysis developed by the writer is described which accounts for the inelastic nature of the materials, residual stresses, initial curvature and end eccentricity.

3. Empirical equations have been used with little success to estimate the beam-column strength of eccentrically loaded columns. An analytical method, using the full load-moment-curvature relationships incorporating realistic material properties, has been shown to give reasonable agreement with the results from experimental tests. The writer has developed a similar analysis of this type which is described in detail in Sections 4.2 and 4.3. The relative number of eccentric load tests compared to concentric load tests was small. No biaxial bending tests have apparently been conducted on square or rectangular concrete filled tubes.

CHAPTER 3

MATERIAL PROPERTIES UNDER SHORT TERM LOADING

3.1 INTRODUCTION

The engineering theory of bending, based on Navier's theory using both Bernoulli's hypothesis of plane sections remaining plane and Hooke's Law of stress being proportional to strain, was well known at the turn of the century. This was applied to both steel members and reinforced concrete members, the "standard" elastic theory of cracked sections for reinforced concrete being attributed to Coignet and Tédesco(46). These theories became so widely accepted that they were often used beyond their range of validity owing to the inability of Hooke's Law to represent the real stress-strain characteristics of the materials. It was a better understanding of the nature and behaviour of the materials that led to the development of the inelastic or ultimate theories for the behaviour of reinforced concrete and composite structures and the plastic methods of analysis for steel structures. As a starting point, any study of the behaviour of structural members therefore requires a knowledge of the strain behaviour of the constituent materials under stress i.e. their stress-strain characteristics. As an example, the influence of three different concrete stress-strain curves, shown as Types I, II and III, on the load-deflection curves for an eccentrically loaded composite column is shown in Fig. 3.1. Type I was initially used by Roderick and Rogers(13) as a simple representation of the concrete stress-strain curve during the early development of their analysis of the behaviour of encased I-section. This was then modified to the Type II curve to obtain a better agreement with the experimental results for a range of columns with varying eccentricities and lengths. The writer has further modified this curve to the Type III curve to give an even more realistic representation of the stress-strain behaviour of the concrete. From Fig. 3.1, the influence of the concrete stress-strain curves is clearly demonstrated where the shapes of the load-central deflection curves reflect the Type of stress-strain curve used. Full details of the Types I, II, and III curves are shown later in Fig. 3.23.

There are many factors which differentiate one concrete from the next such as cement type, cement content, grading and mechanical properties of the aggregates, aggregate content, water content, mixing of the concrete, compaction techniques and curing conditions. Similarly for the steel there are such factors as grade of ore, conversion processes, carbon content, addition of alloying elements, rate of cooling and the amount of work-hardening that will distinguish one steel from the next. However, once the particular materials have been used to make a structural member, there are other external factors which will influence their behaviour and hence the behaviour of the member itself. It is these factors which are discussed in this chapter.

3.1.1 Concrete

Although the inelastic nature of concrete was observed in the early 1900's, its significance was not generally realized. For short term loading tests to failure carried out in 1908 on concentrically loaded composite columns consisting of latticed angles or battened channels encased in concrete, Burr(3) reported that with increase in loading, the stresses in the steel and concrete did not remain in constant proportion to one another and from these results, calculated the change in concrete modulus with load. McMillan(47) in 1921 reviewed load test data on reinforced concrete columns and also found that the steel stresses were considerably higher than those predicted using elastic theory. Lyse, Slater, Staehle and Richart(48)(49)(50)(51)(52), in their comprehensive study of reinforced concrete columns for the American Concrete Institute in the 1930's, also recognized the inelastic nature of the concrete. The elastic theory for concentrically loaded columns was therefore abandoned and an ultimate strength theory, based on the compressive strength of concrete and the yield stress of steel, was developed, this forming the basis of the design formula still used in Australian Standard AS CA2 - 1963(53).

The compressive strength of concrete is usually determined from concentrically loaded prism tests where final collapse takes place shortly after the maximum stress is reached, this being related to the energy stored in the testing machine and therefore to the stiffness of the testing machine. This aspect is discussed in Section 3.2.1 and Appendix A. By using suitably stiff testing machines or similar devices, stress-strain relationships showing the descending portion of the curve can be obtained as shown in Figs. 3.2 and 3.3. It was not until the publication of Hognestad's work in 1951(31)(54) on the study of combined bending and axial force in reinforced concrete members that use was made of the descending portion of the stress-strain relationship in their analysis and design. Hognestad's assumed stress-strain curve is reproduced in Fig. 3.4 and has formed the basis of concrete behaviour used by many investigators(8)(44)(55)(56) in the study of structural members having a concrete component. Ultimate strength theories for reinforced concrete in flexure, as used in most design codes, now rely on the fact that strains in excess of the strain at maximum stress can be attained with safety.

3.1.2 Steel

Referring to the diagrammatic stress-strain curve in Fig. 3.24, the normal elastic extension terminates at the upper yield stress, f_{uy}, then deformation proceeds at a decreased yield stress, f_{sy}. The deformation at this stage is not homogenous, as observed by Piobert(57) in 1842, Luders(58) in 1860 and Hartmann(59) in 1986 who observed bands or "stretcher strains" in the specimen. The specimen consists of regions, generally known as Luders' bands, where the strain has already reached the strain hardening strain, ε_{sh}, and other regions which are still elastic. Deformation proceeds by the growth of these Luders' bands along the specimen until its entire length has been strained by the amount, ε_{sh}. Deformation then proceeds, with increased stresses, in an essentially homogenous manner until failure occurs. This statement represents the general behaviour of a mild steel specimen under axial load, and much of this behaviour, through the observations of Luders and others, was certainly known in the 19th century. However, no reference was apparently made to the presence of an upper yield point, possibly due to the crude methods of testing. Referring to information from the text by Baker et al.(60), Wicksteed(61) in 1886 produced stress-strain curves that showed some semblance of a drop in stress and Kennedy(62), with improved testing methods, confirmed the pronounced stress drop in a discussion of Wicksteed's paper. Finally, in 1913, Robertson and Cook(63) developed a test apparatus that eliminated the small eccentricities that had previously masked the upper yield stress in axial tension tests and produced stress-strain curves similar to Fig. 3.24.

The elastic theories(64) being developed at that time were not capable of representing the plastic behaviour of steel members and although Ewing(65) in 1899 formulated the simple plastic-moment capacity for a rectangular beam section, no experiments were carried out to confirm the result. Bleich(66) also lists several attempts after this to ulitize the plastic properties of steel in redundant structures but the studies were of little consequence as they lacked completeness and supporting experimental evidence. It was probably the work of Professor H. Maier-Leibnitz(67) that stimulated other investigators to examine the behaviour of steel members loaded into the plastic range. He loaded encastred and continuous beams through the elastic range and into the plastic range and showed that loads well in excess of the load at first yield could be sustained. This work led to the extensive programme of research into the plastic behaviour of structures initiated by Sir John Baker and Professor J.W.Roderick in 1936. This investigation was initially carried out at the University of Bristol and after the Second World War at the University of Cambridge(60). Account was taken of the upper yield stress. The influence of this work on other investigators such as Professor L.S. Beedle of Lehigh University in the United States led to the establishment of other research teams doing analogous work.

The effect of the strain hardening range on structural behaviour was considered by Horne(68) in 1949 who demonstrated the effect of strain hardening on the equalization of moments at hinge locations in continuous beams. Much later, Lay and Smith(69) showed that strain hardening was necessary for moment redistribution. The effects of strain hardening have also been accounted for in lateral torsional buckling of steel members by White(70) and others.

Some of the earliest work on residual stress effects was by Baker and Horne(71), Osgood(72) and Beedle et al.(73)(74) in the early 1950's. Since then, the amount of literature pertaining to residual stress effects has been growing at an increasing rate. For instance, investigations into the effect of residual stresses on steel column behaviour at Lehigh University led to the development of the Column Research Council(43) basic column strength curve for axially loaded members based on the tangent modulus load for a straight column containing residual stresses up to half the yield stress. This curve was adopted for use in the AISC Design Specification(39).

Therefore, it can fairly be claimed that an appreciation of the nature and the behaviour of materials has led to the significant advances in understanding structural behaviour.

3.2 STRESS-STRAIN CHARACTERISTICS OF CONCRETE FOR SHORT TERM LOADING

The departure from linearity of the stress-strain relationship for concrete has been attributed entirely to creep by Glanville(75) but is also affected by the onset and development of cracking, some of which may also be time dependent. This time-dependent nature of concrete is evident even for loading over short time durations as demonstrated by Rusch(76), his results for loading at a constant strain rate for durations of up to one day being shown in Fig. 3.2.

Therefore, "short term" will be defined by the writer as those durations of loading to failure that have negligible effect on the ultimate load capacity and where any creep strains occurring during that time are small compared to the creep strain that can be attained for an infinite duration of loading. For the writer's short term column tests described in Chapters 5 and 6, loading to failure took an average of four hours, this being considered as short term loading.

3.2.1 The Effect of Testing Conditions

The major influence on the stress-strain behaviour of concrete in concentric compression is the stiffness of the load applying device, be it a testing machine or some structural arrangement. As would be expected, the stiffness of the testing machine does not affect the stress-strain curve for stresses up to and including the maximum stress, this being confirmed by Sigvaldason(77). However, it limits the strain to which the descending portion of the curve can attain before instability occurs to be followed immediately by sudden failure. This phenomenon was discussed by Whitney(78) in 1943. The present writer has given a simple analysis in Appendix A in which the testing machine stiffness, k_m, is defined and its effect on the termination of the concrete stress-strain is demonstrated in subsection (a) of this Appendix. This analysis is general and also includes the effect of stiffness, k_b, provided to the concrete by other structural members and the effect of the stiffness, k_c, provided by any longitudinal steel within the concrete. With reference to this Appendix, a stiff testing machine is therefore defined as one with a large value of k_m and a soft testing machine as one with a low value of k_m.

Very stiff testing machines were used by Ramaley and McHenry(79) at the U.S. Bureau of Reclamation and by Barnard(80) at the University of Cambridge and the full stress-strain curves obtained are shown in Fig. 3.3 and 3.5 respectively, loading being at a constant strain rate. The important result of these tests was that there was no sudden failure of the concrete and that the specimens were able to sustain very large strains without ceasing to carry stress, even though cracked or spalled regions existed in the specimens at this stage. Tests, carried out by Hognestad, Hanson and McHenry,(41) using a soft (or less stiff) testing machine, resulted in the stress-strain curves shown at (b) of Fig. 3.6. The curves terminated, with sudden failure associated with the release of strain energy from the testing machine, when the slopes of their descending portions became tangential to the testing machine curve of slope, E_t, as defined in Appendix A. This affected the higher strength concretes owing to the steeper slope of their descending portions of the curves. To have obtained the full stress-strain curves for these concretes, the testing machine stiffness and thereby the slope of the testing machine curve would have had to have been increased to a slope greater than the maximum slope of the descending portions of the stress-strain curves. In some structural systems, this amount of stiffness may not be present and the full stress-strain curve for the concrete then can not be relied upon to be available. The writer therefore believes that a limiting strain at which concrete can no longer carry stress (sudden failure) is not an inherent property of the concrete but is rather a consequence of the testing conditions. This also applies to concrete in flexure as discussed in Section 3.2.6. The theoretical stress-strain curves derived by the writer in the following Section 3.2.7 therefore have an extended descending portion. However, for the writer's inelastic column analysis described in Chapter 4, the stiffness of the testing system is accounted for by assuming a limiting strain of 0.005, this being an average value found from the writer's tests.

The general assumption is that concrete is "brittle" in tension and therefore has a straight or linear stress-strain relation and a limited ultimate strain at which sudden failure or cracking occurs. Recent tests by Hughes and Chapman(81) and Evans and Marathe(82) indicated that under certain test conditions, it was possible to produce very large tensile strains without failure as indicated by their results, in the form of stress-strain curves, in Figs. 3.7 and 3.8. These curves are similar in shape to those obtained in compression as in Fig. 3.3. This tensile strain behaviour was also obtained by Holford(83) from strains measured alongside large openings in slabs under tension. Evans and Marathe(82) pointed out that a large proportion of the total strain was due to microcracks. To propagate these microcracks which causes tensile failure, a supply of energy greater than the amount used up in the formation of cracks is required. This will depend on the loading conditions and structural arrangement of the concrete. Test stiffness will therefore influence the tensile behaviour in much the same manner as for compression, the descending portion of the stress-strain curve being terminated when its slope reaches the test instability modulus, E_t, as defined in Appendix A. As the descending portions of the tensile stress-strain curves are associated with considerable microcracking and require an extremely stiff testing system to attain, the writer has assumed, for his analysis, that once the maximum tensile stress is reached, the concrete cracks and is incapable of carrying any further stress with increasing strain. The effect of neglecting the descending portion has been examined analytically by the writer and has been found to have an insignificant influence on the load-moment-curvature relationship for a composite section. In fact, ignoring the tensile strength of the concrete altogether has been found to have little influence also, except for small curvatures and strains, but has been included in the writer's analysis in Chapter 4 for completeness.

3.2.2 The Effect of Strain Rate

The effect of the rate of strain on the shape of the stress-strain curve is best illustrated in Fig. 3.9 for results obtained by Rasch(84) and reported by Rusch(76). These results indicated that the faster the strain rate, the higher the maximum stress and the lower the strain corresponding to this stress. This behaviour was confirmed by Clark, Gerstle and Tulin(85). Strain rate was noted to have a large influence on the descending portions of the curves where the faster the strain rate, the more rapid the decrease in stress with increase in strain as in Fig. 3.9. Again, strains well in excess of the strain at maximum stress were attained. Because of the time-dependent nature of the shape of the curves, any stress-strain curve used in theoretical studies of members containing concrete must take account of the effect of strain rate. The writer has attempted to do this as discussed in Section 3.2.7.

3.2.3 The Effect of Size

There is some evidence that larger specimens show less strength than small specimens of similar concrete. The results of the USBR tests reported by Blanks and

McNamara(86) are shown in Fig. 3.10 for compression tests on cylinders of varying sizes. This type of behaviour has been explained on Griffith crack theory by Endersbee(87). However, it is difficult to separate the size effect for different masses from other effects such as differing casting conditions, differing compacting conditions, differing curing conditions such as temperature due to heat of hydration and different bond crack restraint. Assuming all of these effects are part of the so-called "size" effect, it has generally been found that the strength of concrete in columns is less than the cylinder strength as obtained from tests on standard cylinders. This may also be a function of the strain rate as cylinders are tested at a much faster strain rate than the concrete in a column test to failure (see Section 3.2.2). Richart and Brown(88) found that from tests on numerous vertically-cast concentrically loaded reinforced concrete columns, the average value of concrete strength, f''_c, in the columns was given by

$$f''_c = 0.85\ f'_c \qquad \text{———— (3.1)}$$

where f'_c was the strength of 6 in. by 12 in. cylinders. For tests on eccentrically loaded reinforced concrete members reported by Hognestad(31), the concrete strength given by equation (3.1) was found to be a reasonable estimate. On the basis of Hognestad's work, Roderick and Rogers(13) used the same value for the concrete strength of encased rolled steel joists and their test results indicated that this was a reasonable estimate. The concrete strength, f''_c, given by equation (3.1) is also adopted by the writer for the theoretical considerations in Chapter 4 as a result of an investigation described later in Section 3.2.7.

As far as can be ascertained, the effect of size on the shape of the stress-strain curve has not been investigated.

3.2.4 The Effect of Reinforcements

Although the effect of spiral reinforcement on the behaviour of concrete under load is well known, the provision of transverse reinforcement such as binders, stirrups or ties in rectangular prismatic concrete members has also been found to influence the stress-strain characteristics of the concrete. From compression and flexural tests on prismatic members, Shah and Vijay Rangan(89) and Soliman(90) both found that by increasing the quantity of transverse reinforcement, hereafter referred to as binders, the slope of the descending portions of the stress-strain curves were reduced and hence the possibility of instability due to test stiffness, as discussed in Appendix A, was also reduced. Some increase in the maximum stress for the concrete was also obtained. If the area under the stress-strain curve is taken as a measure of the ductility, energy absorbed or "toughness", the provision of binders can therefore be said to improve the ductility. Three variables were considered by Shah and Vijay Rangan(89) and Soliman(90): The binder spacing, the size or diameter of the bars forming the binders and the volume or quantity of binders. Their results are shown in Figs. 3.11, 3.12, 3.13 and 3.14. For the flexural tests, the stress-strain curves were calculated using a similar technique to that used by Hognestad et.al.(41). From these figures, it can be seen that the binder spacing has the most significant influence on the shape of the stress-strain curves, especially on the descending portions. This increase in ductility of columns by the use of closely spaced binders is of particular importance in the area of seismic design and it has been suggested by Park and Sampson (91) that two different stress-strain curves be used in the determination of the moment-rotation capacity of columns; a less ductile curve for the unconfined concrete outside the binders and a more ductile curve for the confined concrete within the binders, similar to the shapes marked (a) and (b) on Fig. 3.12 respectively.

For the writer's column tests, as described in Chapters 5, 6, 9 and 10, the concrete encasement was intentionally unreinforced to represent perhaps the most adverse condition, especially in the unloading behaviour of the columns. Varghese(92) found that the provision of longitudinal reinforcement with binders in concentrically loaded encased rolled steel joists led to a more ductile behaviour after the maximum load was reached but did not have any noticeable effect on the ultimate load capacity when compared to members tested without any binders. For eccentric loads on similar members, Loke(14) also found that the absence of binders did not appear to have affected their load carrying capacity. A similar behaviour was found by Hudson(93) in tests on reinforced concrete columns containing longitudinal steel with or without binders. The concrete encasement apparently provided sufficient restraint against reinforcement buckling up to the point of compressive failure of the concrete and the absence of binders seemingly had no effect on the maximum load capacity of the columns. However, for the continuous composite column in the building frame and certainly for columns subjected to seismic loads where not only the load carrying capacity is of importance but where a good moment-rotation capacity is also required, the provision of binders would be desirable.

3.2.5 The Effect of Strain Reversal

This effect is best illustrated by the results of tests by Karsan and Jirsa(94) on concentrically loaded prisms as shown in Fig. 3.15 where the strains for a given cycle were increased until the stress began to decrease. The specimen was then unloaded to zero stress then reloaded and the cycle repeated. It was found that the envelope curve for a specimen coincided with the stress-strain curve for a similar specimen under monotonic loading to failure. From other tests, the same envelope curve was obtained regardless of the strain accumulated prior to a particular cycle or the stress level and range used for each cycle. Tests by Sinha, Gerstle and Tulin(95) led to a similar conclusion where the stress-strain relationship of concrete under compressive load was considered to possess an envelope curve which was unique and identical with the stress-strain curve obtained under constantly increasing strain.

The intersections of the loading and unloading curves were termed common points and were considered by Sinha et al.(95) to be the points above which stresses would produce additional strains which could eventually lead to failure while stresses below these points would result in a looping of the stress-strain curves. The locus of these common points could therefore be considered as a stability limit. Karsan and Jirsa(94) found that by unloading and then reloading until the initial unloading curve was reached again, a new common point was obtained which was lower than that from the previous loading and unloading cycle. However, with more cycles, it was found that the common points stabilized and this was considered to be the stability limit as for any further cycles up to this limit, no additional strain was accumulated. This behaviour is shown in Fig.3.16.

The important conclusion from the investigations into the behaviour of concrete under cyclic loading is that the loading and unloading curves starting from a point within the envelope were not unique and were dependent on the peak stress and strain of the previous cycle. Therefore it is not possible to obtain a single expression to represent the behaviour of concrete subjected to cycles of loading and unloading.

As the load is increased on a column where the load is applied at a constant end eccentricity, the moment at a section, for example the central section at midheight, increases at a greater rate than the load owing to the deflection of the column. This can result in strain reversal on the convex side of the column accompanied by a shift of the neutral axis towards the concave or compression face of the column. This behaviour was observed in the tests conducted by the writer and discussed later in Chapters 5 and 6. Fortunately, this strain reversal took place at relatively low strain and hence stress levels in the concrete. For strain reversal occurring at a stress level of 40% of the maximum concrete stress as shown in Fig. 3.17, it can be seen that the unloading stress-strain curve is similar to and could be approximated by the loading curve. For the analytical treatment of eccentrically loaded columns described in Chapter 4 the writer has therefore assumed that where strain reversal occurs, the relationship between stress and strain is identical to that for monotonically increasing strains.

3.2.6 The Effect of Strain Gradient

There has been some doubt that, for any particular fibre in a prismatic concrete member, the stress-strain relationship obtained under conditions of uniaxial compression (zero strain gradient) was similar to that obtained under flexural conditions where the prism is subjected to a strain gradient. Apart from Sturman, Shah and Winter(96[2]), other investigators(41)(85)(96[1])(97) now generally agree that there is practically no difference in the stress-strain relationships, at least up to the point of maximum stress. Results by Hognestad et.al.(41), Clark et.al.(85) and Karsan and Jirsa(96[1]) are shown in Figs. 3.6, 3.18 and 3.19 respectively. However, in trying to establish the descending portions of the curves, the writer believes that each of the three investigations were affected by their testing conditions, similar to the effects described in Appendix A. As the equilibrium conditions differ from those of the concentrically loaded specimens discussed in Appendix A, this aspect is therefore considered in Appendix B with reference to these three investigations.

Hognestad et al.(41) used a testing arrangement whereby one face of the concrete prism was kept constant at zero strain while the strain at the other face was increased this being Case I of Appendix B. Hence the strain rate differed for each fibre across the section and the strain rate effects discussed in Section 3.2.2 could have influenced the results. Although the force producing the applied moment was stated as being capable of being reduced without a sudden release of energy and accompanying instability, the major thrust on the section was stated to have increased until a maximum was reached at failure. This condition is given by equations(B.11) and (B.14) of Appendix B. No unloading occurred which indicated a relatively soft testing machine applying the major thrust, the machine probably being similar to the one used in the concentrically loaded prism tests, the results of which are shown in Fig.3.6 (b) where strains much in excess of the strain at maximum load were not possible.

Clark et al.(85) examined the effect of strain gradient alone by using a testing arrangement whereby the strain gradient was kept constant during loading to failure, this being Case II of Appendix B. Each fibre could then be subjected to the same strain rate throughout the test, hence removing any objections to the tests by Hognestad et al.(41). However, the writer believes that a relatively soft testing machine must have been used to apply the main axial thrust. This is shown by the fact that the maximum strain attained by the concentrically loaded prisms was only slightly greater than the strain at maximum stress (see Appendix A). Therefore failure occurred when the maximum load was reached, this condition being given by equations (B.19) and (B.20) of Appendix B. If a stiff testing machine had been used the writer feels that stress-strain curves, such as those shown in Figs. 3.2, 3.3 and 3.5 with extended descending portions could have been obtained.

Karsan and Jirsa(96[1]) used a testing arrangement similar to Clark et al. (85) but where the strain at one face was maintained at zero while the strain on the opposite face was increased this being the same test condition as Hognestad et.al(41) and treated as Case I in Appendix B. Hence, the strain rate differed for each fibre across the section and a constant strain rate was not maintained. The difference in the descending portions of the stress-strain curves of Fig. 3.19 could have been due to this strain rate effect as discussed in Section 3.2.2, this fact being recognized by the authors also. A stiff testing machine was used so that large strains were developed, in excess of three times the strain at maximum stress. A factor of six was obtained by Soliman(90) for a similar test on a plain concrete specimen (no binders) as shown in Fig. 3.12.

In the light of the above discussion, the treatment given in Appendix B and the few results by Hognestad et al. (41) for the lower strength concretes where the descending portions of the stress-strain curves were not terminated owing to testing machine stiffness, the writer considers it reasonable to assume that the full stress-strain curves for concrete are unaffected by the presence of a strain gradient. If this were not the case, the prediction of the behaviour of concrete structural members, which will contain varying strain gradients throughout their lengths, would be a formidable task indeed!

3.2.7 Evaluation of the Stress-Strain Relationship

From the discussion in the preceding sections of this Chapter, the writer has concluded that the main factors influencing the stress-strain relationship are the testing conditions, the strain rate (time dependent properties) and the so-called "size" effects as discussed in Sections 3.2.1, 3.2.2 and 3.2.3 respectively. For accuracy, this relationship would best be evaluated from the actual concrete in the structural member under consideration for its own unique size and loading conditions.

Several methods have been used to determine the stress-strain curve for concrete prismatic members subjected to flexure. Referring to Appendix B and using the technique of keeping the strain, ε_L, at one face zero and varying the strain at the other face, equations (B.10) and (B.15), rearranged and repeated below,

$$\sigma_H = \frac{\varepsilon_H}{bd}\frac{dP_c}{d\varepsilon_H} + \frac{P_c}{bd} \qquad (3.2)$$

$$\sigma_H = \frac{2\varepsilon_H}{bd^2}\frac{dM_c}{d\varepsilon_H} + \frac{P_c}{bd} + \frac{4M_c}{bd^2} \qquad (3.3)$$

were used by Hognestad et al.(41) to determine the shape of the stress-strain curve. Equations (3.2) and (3.3.) give the concrete stress, σ_H, in terms of the corresponding strain, ε_H, for the known load, P_c, and known moment, M_c, measured at this strain. By calculating in small increments, the differentials can be approximated by the finite differences $\frac{\Delta P_c}{\Delta\varepsilon_H}$ and $\frac{\Delta M_c}{\Delta\varepsilon_H}$. The stress-strain relationship can be computed independently by the above equations and hence the accuracy of the test data can be checked. This method has also been used by Soliman and Yu(90). A variation was used by Smith(98) who combined equations (3.2) and (3.3) to obtain

$$\sigma_H = \frac{P_c d + 4M_c - 2P_c\frac{dM_c}{dP_c}}{bd^2 - 2bd\frac{dM_c}{dP_c}} \qquad (3.4)$$

where ε_H was eliminated such that any numerical error in the differential terms did not include any error from the strain measurements.

For tests on concrete prisms by Clark et al.(85), the technique of keeping the strain gradient constant and increasing the strains at a constant strain rate was used. The stress-strain curve was determined by using a segmental method to arrive at a series of overlapping parabolas. Referring to Appendix C and expressing the stress as a parabola between the two limits of strain, ε_H and ε_L, then from equation (C.4)

$$\sigma_c = a_1\varepsilon_c + a_2\varepsilon_c^2 \qquad \text{———————(3.5)}$$

From equations (C.5) and (C.6) then

$$\frac{(\varepsilon_H^2 - \varepsilon_L^2)}{2} a_1 + \frac{(\varepsilon_H^3 - \varepsilon_L^3)}{3} a_2 = \frac{\phi P_c}{b} \qquad \text{———————(3.6)}$$

$$\frac{(\varepsilon_H^3 - \varepsilon_L^3)}{3} a_1 \quad \frac{(\varepsilon_H^4 - \varepsilon_L^4)}{4} a_2 = (\frac{\varepsilon_H + \varepsilon_L}{2}) \frac{\phi P_c}{b} + \frac{\phi^2 M_c}{b} \qquad \text{———————(3.7)}$$

The solution of equations (3.6) and (3.7) gave Clark et.al.(85) the values of a_1 and a_2 and hence the parabolic segment of the curve between the values of ε_H and ε_L. For a number of loading stages, further parabolic segments were computed and the full stress-strain curve consisted of a series of overlapping segments as shown in Fig. 3.20 which were averaged to give a continuous curve as in Fig. 3.18. This method is satisfactory for small gradients and hence small differences between the maximum and minimum strains for a particular load. If the strain gradient is large, the writer believes that the use of a simple parabola to approximate a large portion of the stress-strain curve will be inadequate resulting in large differences between the overlapping segments.

Much of the knowledge of the compressive stress-strain behaviour of concrete is based on prism tests such as those described above where attempts have been made to simulate some of the conditions that exist in the compression zones of concrete structural members. For an eccentric load test to failure on a long composite column, the concrete will be subjected to a variety of strain gradients, the strain rates for each fibre will not only be different but vary with time and the size effects described in Section 3.2.3 will be present. This makes the task of determining the stress-strain curve, especially the descending portion, for the actual concrete in the column, from the results of cylinder or prism tests, a difficult one. In fact, with varying strain rates for each fibre, there is no unique stress-strain curve that can be used for every fibre of the concrete. The best that can be achieved is to determine a single stress-strain curve that represents the "average" behaviour of the concrete. It therefore should be evaluated from strain data obtained from the actual concrete in the member under its own unique loading conditions. For singly reinforced concrete beams, a finite difference method for doing this was developed by Prentis(99) but necessitated graphical differentiation of some of the parameters. A method of least squares, where a polynomial curve is used to fit the experimental data was developed by Smith(100). This method was suitable for digital computation and was shown to be more accurate than the Prentis' method.

The writer has applied this method of least squares to data obtained from his tests on the eccentrically loaded composite columns, numbers CC1 and CC2, described in detail Chapter 5, to enable the general shape of the curve to be established. For the central section of the columns, the strains across the section, the load, the central deflection and hence the moment were measured for approximately twenty loading stages up to the maximum load and into the unloading range for the columns.

As the geometrical properties of the steel channels and the stress-strain relationship for the steel were known within experimental accuracy, the load and moment taken by the steel channels were computed from the strain data and subtracted from the total load and moment to give the load and moment taken by the concrete alone, this being done for each loading stage. Using this concrete data, the method of least squares, described in Appendix C, was used to compute the stress-strain curve. Appendix C is essentially the analysis developed by Smith(100) except that it has been modified by the writer to incorporate a weighting procedure to account for the relative importance of the experimental data. The strain distributions for the two columns were such that low strain values were included for every loading stage whereas high strain values were attained only for the few loading stages approaching and beyond the maximum load when the deflections and hence the curvatures increased at a more rapid rate. The higher strain values are important in determining the descending portion of the curve and the experimental data containing these strains was weighted to ensure that it had more influence in determining the shape of the curve in this region than the data at lower strain levels. Therefore the weighting, W_r, as used in equation (C.7) of Appendix C, was made proportional to the strain range covered by the strain distribution at each loading stage. The results of the least squares analysis for columns CC1 and CC2 are shown in Figs. 3.21 and 3.22 respectively. The writer found that four terms of the polynomial given by equation (C.4) of Appendix C were sufficient to define the curve with reasonable accuracy. The differences in the shapes of the curves obtained from concentrically loaded cylinders and from the column test data were as expected owing to the difference in strain rates, two minutes to failure and four hours to failure respectively, and the size effects. The ratio of the concrete strength, f_c'', to the cylinder strength, f_c', was 0.89 for column CC1 and 0.78 for column CC2. The average value of 0.835 compares well with the value of 0.85 obtained by other investigators, as discussed in Section 3.2.3.

Once the general shape of the curve was established by using the method of least squares, the writer decided to approximate the shape by the Type III theoretical curve given in Fig. 3.23 rather than having to apply the least squares analysis to every column tested. A similar curve has been used by Warner(101). All that is required to establish the numerical values for this theoretical curve are the concrete strength, f_c'', and the elastic modulus, E_c, where E_c is the chord modulus to 40% of the concrete strength. This information can be obtained directly from cylinder tests. The cylinder strength, f_c', can be obtained from tests on 12 in. by 6 in. cylinders in accordance with ASTM Standard C39 and the concrete strength, f_c'', taken as 0.85 of this value. From Figs. 3.21 and 3.22, it can be seen that the initial portions of the stress-strain curves obtained from cylinders and from the column tests are almost identical, hence E_c can be determined from tests on standard cylinders in accordance with ASTM Standard C469, which gives the chord modulus to 40% of the cylinder strength.

A comparison between the theoretical curve and the curve obtained from the least squares analysis is shown in Fig. 3.21 for column CC1 and in Fig. 3.22 for column CC2. Although there was some discrepancy for the results for column CC1, good agreement was obtained for column CC2. This theoretical curve, shown as Type III in Fig. 3.23, was used by the writer to determine the theoretical load-deflection behaviour of composite columns and then to compare this with the actual behaviour under load. The justification in the use of this theoretical curve was the good agreement of the experimental and theoretical load - deflection behaviour as discussed later in Chapters 5 and 6.

3.3 STRESS-STRAIN CHARACTERISTICS OF STEEL

The extended elastic range of steel made it more amenable than concrete to the application of elastic theories and it was not until the nature of concrete was understood that the development of suitable inelastic theories led to its accepted use in a great variety of structures. Much the same could be said about the development of plastic analysis for steel structures. The increasing use of steel in a multitude of applications was accompanied by a similar increase in the quantity of corresponding literature on the behaviour of steel as a material. It is not proposed to discuss that literature here but to consider briefly a few aspects that influence stress-strain characteristic of steel and hence the behaviour of structural members containing steel; in particular, mild steel as the sections used in the writer's composite columns contained steel conforming to Australian Standard A149(102) with a minimum yield stress of 36 kips/sq.inch.

3.3.1 The Yield Point Phenomenon

Although the sharp yield behaviour of mild steel, as shown by the full line in Fig. 3.24, was known at the turn of the century, its mechanism was not explained until the advent of the dislocation theories. Low and Gensamer(103), in 1944, demonstrated that almost complete removal of carbon and nitrogen from low carbon steel removed the yield point, a typical curve being shown in Fig. 3.24 as a dashed line. Only a 0.001 per cent addition of either of these elements caused its reappearance. Cottrell and Bilby(104) proposed that these "impurity" atoms of carbon and nitrogen were not scattered randomly but formed a cloud or atmosphere around the dislocations which locked them. For plastic flow, the dislocations had to break away from this locking. The concept of dislocation locking is now generally accepted but Cottrell and Bilby's static unlocking theory for yield had difficulty in explaining the effects of temperature, strain rate and grain size on the upper and lower yield points. Yielding, in terms of dislocation dynamics, was examined by Johnston(105) (106) and Hahn(107) initially and followed by others. Account was taken of the balancing of the strain rates of the crystals and the testing machine and they reported that the most influential factors determining the sharp yield and subsequent yield drop were the initial mobile dislocation density and the dislocation velocity. An account of these factors and others and their role in the mechanism of yielding is found in text by Hall(108) which summarizes, with numerous references, the current state of research into yield point phenomena in terms of dislocation theories. It is sufficient to state that under normal conditions of use, mild steel will exhibit an upper and lower yield point.

3.3.2 The Effect of Strain Rate

The influence of the rate of straining on the value of the yield stress was appreciated by many early investigators(60) and these studies were summarized by Cook(109). The conclusions reached were that both the upper and lower yield stresses decreased as the strain rate decreased and that the rate had to be exceedingly slow before the lower yield stress could be considered to approach a constant minimum value. Hence, the concept of zero strain rate as defining the lower limit of the yield stress. This above phenomena is explained on dynamic dislocation theory as in Hall's text(108). Results obtained by Steidel and Makerov(110) are shown in Fig. 3.25. This effect of strain rate on the yield stress has been considered by investigators such as Roderick and Phillips(111) in predicting the behaviour of beams loaded into the plastic range.

The yield plateau, drawn to a small scale as in Fig. 3.24, appears as a straight horizontal line but examined at a larger scale as in Fig. 3.26, it reflects the discontinuous nature of yielding. If the straining is completely stopped after the upper yield stress has been reached, the stress drops to the static level as shown in Fig. 3.26. If the stress is then raised to the dynamic yield stress, straining continues at a relatively constant rate. This behaviour has been examined by Forscher(112). The effect of strain rate on the ratio of the dynamic yield stress to the static yield stress has been studied by Beedle and Tall(113) and their findings are reproduced in Fig. 3.27 which includes results by Gozum and Huber(114) who found that at a very low strain rate of a microstrain per second, the ratio was approximately 1.05. Hence, if a steel specimen is tested such that the strains are always increasing, it is apparently the dynamic yield stress that is measured.

Most specifications for the supply of steel specify a maximum strain rate for the determination of the yield stress. Australian Standard A 147-1971 "General Requirements for the Supply of Steel for Structural Purposes" requires that the loading rate should not exceed 0.6 tons/sq. inch/second when nearing the yield point. This is equivalent to a strain rate of 45 microstrain per second. Such tests are suitable for quality control but may not give a true indication of the value of the yield stress at very low rates of strain.

3.3.3 Effect of Testing Conditions

Again, reference is made to Appendix A. If the testing machine stiffness, k_m, is low for a tension or compression test on a steel specimen, the sharp yield drop will not be recorded as illustrated in Fig. 3.28. After the upper yield stress, f_{uy}, is reached, the stress strain curve rapidly follows the testing machine curve with slope, E_t, until an equilibrium position for the material is reached. For very soft machines, this position may be on the strain hardening portion of the curve and no value of the lower yield stress, f_{sy}, is recorded. These effects have been studied by Welter(115).

In tests on simply supported, rectangular, steel beams under symmetric loading, Robertson and Cook (63) confirmed that the extreme fibres reached the upper yield stress. With further straining, the stress in these fibres decreased to the lower yield stress as the stress in the inner fibres increased elastically. For this behaviour, the stresses corresponding to the increased strains must equilibrate the applied moment.

The moment at first yield, M_Y, is given by

$$M_Y = f_{uy} Z \qquad (3.8)$$

where Z is the usual elastic section modulus and the full plastic moment, M_P, ignoring strain hardening is given by

$$M_P = f_{sy} S \qquad (3.9)$$

where S is the usual plastic section modulus. From equations (3.8) and (3.9) then

$$\frac{M_P}{M_Y} = \frac{f_{sy} S}{f_{uy} Z} \qquad (3.10)$$

For M_P to be greater than M_Y, then

$$\frac{f_{sy}}{f_{uy}} > \frac{S}{Z} \qquad (3.11)$$

For this condition, the moments increase beyond the value at first yield, the maximum moment being the full plastic moment. However, if the ratio of the lower to upper yield stress is lower than the value of the shape factor, S/Z, the maximum moment is reached when the extreme fibres just reach the upper yield stress. This behaviour is possible only in normalized specimens. In structural sections, residual stresses are present and variations in the properties across the section are possible. Hence yielding will not necessarily progress unfiormly from the extreme fibres towards the centre with increasing strains. This was confirmed by Roderick(116) in tests on rolled steel joists where the plastic zones differed from those predicted using the simple plastic theory. However the load-deflection curves were similar to those obtained from his previous tests(111) on annealed specimens taking into account the upper yield stress as shown in Fig. 3.29. Good agreement was obtained between the observed and calculated full plastic moments using equation (3.9). In some cases, strain hardening led to higher observed values. Roderick and Heyman(117) also showed that by utilizing the true stress-strain relationship, the deflections of the members throughout the plastic range could be predicted accurately.

3.3.4 The Effect of Strain Reversal

As discussed in Section 3.2.5 for the concrete, strain reversal in the steel sections also occurred on the convex side of the columns for the short term tests described later in Chapters 5 and 6. As this occurred while the steel was still in the elastic range, it can be assumed that the unloading curve and the loading curve are identical. If unloading had occurred when the steel was in the plastic range, the unloading curve, when plotted on a small scale, would have appeared to be a straight line and parallel to the initial linear-elastic loading curve. If, however, as in Fig. 3.30 by Dalby(118), the strains are plotted to a sufficiently large scale, what appears to be a straight line is really a loop and neither in loading or unloading is the material truly linear elastic. However, it is reasonable to assume that the modulus of elasticity for mild steel is the same for unloading as loading.

3.3.5 The Effect of Loading Sense

It is generally assumed that the yield phenomenon for mild steel is identical in compression and tension. Morrison(119) confirmed this with the results of a careful investigation in 1939. Good agreement between observed and theoretical behaviour for numerous tests on steel members has been found using this assumption. It forms the basis for the simple plastic theory for steel structures.

However, in comparing the stress-strain behaviour of steel in both compression and tension, it must be ensured that the conditions of stress are identical for both. Because of the shortness of compression test specimens required to prevent instability, radial deformations at the ends are of the specimen are restrained by friction with the testing machine head and a triaxial state of stress exists. Likharev(120) found that by using thick walled cylindrical specimens and suitable end platens, no change occurred in the state of stress or in the distribution of stress components along the axis of the specimen for strains up to 0.1 in./in. Likharev's test results are shown in Fig. 3.31. The elastic modulus in tension and compression are identical and the yield stresses approximately the same. However, the strain hardening curves are different, the steel in compression offering a higher resistance to plastic deformation than in tension. However, provided that large plastic strains do not occur in a member under load, the stress-strain curves for tension and compression have been assumed by the writer to be identical.

3.3.6 The Effect of Residual Stress

Residual stresses can be defined as the system of stresses that remain in a body even though external forces, including body forces, are not acting on it. In rolled structural sections, residual stresses arise for several reasons. Surface stresses can be induced by surface rolling, short peening, machining or grinding. If a member is cold straightened by bending, as is sometimes done before the section leaves the mill or at the fabrication shop, residual stresses will result on unloading as plastic deformation has to occur for the straightening process to succeed. This behaviour is demonstrated in Fig. 3.32. However, just as these residual stresses are established by non-uniform plastic flow, they can be relieved by uniform plastic flow as shown in Fig. 3.33.

However, the main cause of residual stresses in hot rolled structural sections is thermal effects produced by the non-uniform volume changes resulting from transient temperature gradients on cooling. Compressive stresses develop in the portions that cool most rapidly with balancing tensile stresses in the remaining protions of the crossection. For a channel section, it would be expected that the tips of the flanges and the centre of the thinner web would be in compression whilst the area around the junctions of the flange and web would be in tension. This is confirmed by the results, shown in Fig. 3.34, of measurements taken by O'Connor(121) on locally produced channel sections. The smaller the channel size, the lower the values of residual stress that were measured.

The 3 in. by 1½ in. channel sections, used for the writer's column tests described in Chapters 5, 9 and 10, were assumed to have residual stress values similar to those for the 4 in. by 2 in. channels measured by O'Connor. A likely distribution of the residual stresses is that shown at (a) in Fig. 3.35. The writer has approximated this by the theoretical linear distribution shown at (b) in Fig. 3.35 where σ_{r1} σ_{r2} and σ_{r3} are the peak values of the stresses at the flange tips, web and flange junction and the centre of the web respectively. For doubly symmetric I-sections, axial force equilibrium is sufficient to define any symmetric stress distribution. However, both axial force and moment equilibrium are required to define the stress distribution for the monosymmetric channel. For the linear distribution, the stresses σ_{r1}, σ_{r2} and σ_{r3} must be in a definite ratio to one another. Assuming the flange thickness, T, and the web thickness, t, are small compared to the flange width, B, and web depth, D, it can be shown that

$$\frac{\sigma_{r2}}{\sigma_{r1}} = -2 \qquad \text{(3.12)}$$

and

$$\frac{\sigma_{r3}}{\sigma_{r1}} = \frac{2BT}{Dt} + 2 \qquad \text{(3.13)}$$

For 3 in. by 1½ in. by 4.5 lb./ft. channels

$$\frac{\sigma_{r3}}{\sigma_{r1}} = 3.34 \qquad \text{(3.14)}$$

and for 4 in. by 2 in. by 7 lb./ft. channels

$$\frac{\sigma_{r3}}{\sigma_{r1}} = 3.25 \qquad \text{(3.15)}$$

Using equations (3.12) and (3.15) and an average value of 1.3 kips/sq.in. for σ_{r1} from measured values by O'Connor(121) for the 4 in. by 2 in. channels, it can be seen from Fig. 3.36 that the theoretical linear distribution gives reasonable agreement with O'Connor's measured results.

As discussed briefly in Section 3.1.2, the effect of residual stresses on member behaviour has been known for some time and has been incorporated in design specifications. If a steel stub column section containing residual stresses is loaded in compression, the sharp yield phenomenon is masked as some elements in the section commence to yield before others. Any variation in the yield stress across the section would also have a similar effect. The shape of the average stress-strain curve reflects the distribution of the yield stresses as discussed by Beedle and Tall(122). The slope of this curve can be used to determine the tangent modulus buckling load(43) for a long column. Therefore, two stub column specimens were cut from each 3 in. by 1½ in. channel length used in the manufacture of the writer's composite columns subsequently tested. The specimens were tested in accordance with Technical Memorandum No.2 of the Column Research Council(43). The results, in the form of an average stress-strain curve, are shown in Fig. 3.37 for Column CC1. The results are typical of those obtained for the rest of the columns tested. First yield commenced at the point where the sum of the applied average stress and the maximum compressive residual stress was equal to the yield stress, this point depicted by the deviation of the stress-strain curve from the initial elastic line. Hence, a value of the maximum compressive residual stress can be determined, this being the difference between the yield stress and the average stress at first yield for the section. Loading was continued until the whole crossection had yielded and some strain-hardening was attained. The average maximum compressive residual stress, σ_{r3}, from all the stub column tests was found to be 25% of the yield stress.

The average stress-strain curve in tension, although more difficult to obtain experimentally for structural sections, would have a similar shape to Fig. 3.37, the point of first yield depending on the magnitude of the maximum tensile residual stress.

3.3.7 Evaluation of the Stress-Strain Relationship

In addition to the stub column tests described in Section 3.3.6, four tension test specimens, two from the web and two from the flanges, were cut from each channel length used for the writer's columns and tested in accordance with Australian Standard A 23-1960(123). To minimize the strain rate effects discussed in Section 3.3.2, the strain rate nearing yield was kept to a low value, of the order of 1 microstrain per second, to simulate the low rate of strain for the steel during the column tests. In most cases, tests were also carried out on Hounsfield Tensometer specimens as a check on the yield stress and to determine its variation across the crossection. Details of these are given in each test Chapter. The tension test results for Column CC1 are shown in Fig. 3.38, together with the stub column results, these being typical of the results for the rest of the columns. The differences between the curves in tension and compression are similar to those found by Likharev(120) and shown in Fig. 3.31. The elastic modulus, E_s, for both tension and compression varied little from an average value of 30 x 10^3 kips/sq.in. From all tests, the strain hardening modulus, E_{sh}, had an average value of $E_s/30$ in compression and $E_s/50$ in tension. The ratio of the strain, ε_{sh}, at commencement of strain hardening to the strain, ε_{sy}, at the commencement of yield had an average value of 6 in compression and 8 in tension. For ASTM A36 steel, which is equivalent to the AS A149(102) steel used by the writer, Adams and Galambos(124) state that E_{sh} varies approximately from $E_s/35$ to $E_s/45$ and that a normally accepted value of $\varepsilon_{sh}/\varepsilon_{sy}$ is 12. The writer's values of E_{sh} are in agreement with these values although the writer's ε_{sh} values are somewhat lower.

In light of the previous discussion in Sections 3.3.1 to 3.3.6 and the actual test results above, the relationship between strain and average stress, shown in Fig. 3.39, was adopted by the writer. Good agreement, in the range where residual stresses are effective, was obtained between this theoretical curve and the results for the stub column tests as shown in Fig. 3.37. The theoretical curve of Fig. 3.39 is used in later theoretical considerations described in Section 4.2.4(a) of Chapter 4, where although residual stresses are not accounted for directly, their effect is simulated in an approximate manner by the use of this average curve.

In the theoretical considerations in Section 4.2.4(b) of Chapter 4, residual stresses are accounted for directly. The residual stress distribution is assumed to be linear as at (b) in Fig. 3.35. Using equations (3.12) and (3.14) and a value for the maximum compressive residual stress, σ_{r3}, of 0.25 f_{sy} as obtained from the average of all the writer's stub column tests, then

$$\left.\begin{aligned} \sigma_{r3} &= 0.25\ f_{sy} \\ \sigma_{r2} &= -0.15\ f_{sy} \\ \sigma_{r1} &= 0.075\ f_{sy} \end{aligned}\right| \qquad \text{———} (3.16)$$

tensile stresses taken as negative. These peak stresses define the writer's adopted residual stress distribution as shown at (b) in Fig. 3.35. If the steel crossection is divided into a large number of small elements as is done in Section 4.2.4(b) of Chapter 4, then each element can be assumed to have a uniform residual stress over its area, the magnitude and sense depending on the location of the element's centroid in relation to the residual stress distribution. Each element can then be assumed to have an elastic-plastic stress-strain curve as shown in Fig. 3.40, the magnitude of the yield stress depending on the sense of the residual stress, σ_r, in relation to the sense of the applied stress. The upper yield stress, apart from being difficult to quantify accurately, has been neglected, its effect being assumed to be negligible.

CHAPTER 4

THEORY FOR COMPOSITE COLUMNS - SHORT TERM BEHAVIOUR

In the three sections of this chapter, the theoretical aspects of the behaviour of composite columns under short term loading have been examined, each section dealing with a particular category of column. The three categories, in order of discussion, are

(i) the concentrically loaded short column,

(ii) the eccentrically loaded short column and

(iii) the eccentrically loaded long column.

The concentrically loaded long column is treated by the writer as a particular case of the eccentrically loaded long column with zero end eccentricity.

4.1 CONCENTRICALLY LOADED SHORT COLUMNS

4.1.1 Introduction

The first question that has to be answered is: what constitutes a short column? From a study of a number of concentrically loaded composite columns of rectangular crossection, Brettle(126) found that the ultimate strength was not significantly affected by slenderness for ratios of length to minimum dimension of crossection less than about 15. The writer has therefore assumed this value of 15 to be a limit to the validity of the theory derived in Section 4.1.3. As discussed in detail in Chapter 2 where full reference is made to the relevant investigations, concentric load tests on short columns form the bulk of previous research into the behaviour of built-up composite columns where the main consideration was the determination of column strength. In Section 4.1.3, the writer has demonstrated that the theoretical determination of short column behaviour, as for long column behaviour, is a problem involving instability as well as strength and is dependent on the loading conditions and the stress-strain characteristics of the materials.

4.1.2 Elastic Behaviour

For a composite column containing an area of steel, A_s, and an area of concrete, A_c, and subjected to a load, P, such that the axial strain in both the steel and concrete is ε_a, then

$$P = \varepsilon_a E_s A_s + \varepsilon_a E_c A_c = P_s + P_c \qquad (4.1)$$

where E_s and E_c are the elastic moduli for the steel and concrete respectively and P_s and P_c are the axial load carried by the steel and concrete respectively. Rearranging, then

$$P = \varepsilon_a E_c A_T \qquad (4.2)$$

where $A_T = mA_s + A_c \qquad (4.3)$

and $m = \frac{E_s}{E_c}$

A_T is defined in the normal manner as the transformed area where the crossectional area of the steel can be considered to have been transformed to an equivalent area of steel by use of the modular ratio, m.

4.1.3 Inelastic Behaviour

Provided that the end fittings or loading platens used in a test are sufficiently stiff and the concrete and steel are loaded simultaneously, as was the case for the writer's column tests described in Chapter 5, the strain distribution should be constant over the section and the strains in the concrete and steel identical. This would normally be the case at sections away from the ends. Therefore, the assumptions used in the following analysis of the behaviour of a pin-ended composite column concentrically loaded in a testing machine are

(i) plane sections remain plane,

(ii) the strains in the concrete and steel are identical,

i.e. $\varepsilon_s = \varepsilon_c$ ——————(4.4)

(iii) the concrete stress is a function of strain only,

i.e. $\sigma_c = F(\varepsilon_c)$ ——————(4.5)

and (iv) the effect of testing machine stiffness is as described in Appendix A.

The behaviour is best illustrated by diagrams and four possible variations in the behaviour are summarized in Figs. 4.1, 4.2, 4.3 and 4.4 where the stress-strain curves for the concrete and steel and the load-axial strain behaviour of the composite column are shown. Column instability is defined by the writer as that point reached on the load-axial strain relationship for which an equilibrium position of the column, subject to a particular set of test conditions, can not be maintained for any further increase in strain. For a dead load test, this will occur when the maximum load is reached. For a set of test conditions where stiffness can be provided by other parts of the structure connected to the column or by the testing machine or device applying the load, instability will occur in the unloading range for the column. The unloading behaviour of concentrically loaded members has been treated in Appendix A.

(a) Behaviour Illustrated in Fig. 4.1

For a composite column of length L_c containing an area of steel, A_s, and an area of concrete, A_c, the maximum short column load, P_{sh}, is reached when the concrete reaches its maximum stress, the steel having previously yielded. This maximum load and its corresponding strain, ε_{ps}, are given by

$$\varepsilon_{ps} = \varepsilon_o \qquad (4.6)$$

and $$P_{sh} = f_{sy} A_s + f_c'' A_c \qquad (4.7)$$

Instability of the column occurs when the slope of the descending portion of the stress-strain curve for the concrete reaches a critical value of the test instability modulus given by equation (A.17) of Appendix A. The steel yield strain is normally less than the concrete strain at its maximum stress for normal structural materials used in practice. This was the case for the materials used in the writers column tests described in Chapters 5 and 6. Therefore, equation (4.7) defines the maximum short column or squash load for those columns.

(b) Behaviour Illustrated in Fig. 4.2

The maximum short column load is reached when the steel yields, the concrete having previously passed its maximum stress. The values of P_{sh} and ε_{ps} for this condition are given by

$$\varepsilon_{ps} = \varepsilon_{sy} \qquad (4.8)$$

and $$P_{sh} = f_{sy} A_s + \sigma_c A_c$$

$$= f_{sy} A_s + F(\varepsilon_{sy}) A_c \qquad (4.9)$$

Again, instability of the column is defined by equation (A.17) of Appendix A.

(c) Behaviour Illustrated in Fig. 4.3

The maximum short column load is reached before the steel yields, the concrete having previously passed its maximum stress. Using equations (4.4) and (4.5), the load P is given by

$$P = \sigma_s A_s + \sigma_c A_c$$

$$= E_s A_s \varepsilon_c + F(\varepsilon_c) A_c \qquad \text{———— (4.10)}$$

Differentiating with respect to ε_c and equating to zero gives the maximum load, P_{sh}, when

$$\frac{dP}{d\varepsilon_c} = E_s A_s + F'(\varepsilon_c) A_c = 0$$

That is, when $F'(\varepsilon_c) = \dfrac{d\sigma_c}{d\varepsilon_c} = -\dfrac{E_s A_s}{A_c}$

$$= -k_c \frac{L_c}{A_c} \qquad \text{———— (4.11)}$$

where $k_c = \dfrac{E_s A_s}{L_c}$

and is as defined in Appendix A. The value of ε_{ps} is obtained from the solution of equation (4.11) and the value of P_{sh} can then be determined from

$$P_{sh} = E_s A_s \varepsilon_{ps} + F(\varepsilon_{ps}) A_c \qquad \text{———— (4.12)}$$

Instability of the column takes place after the steel has yielded and is therefore defined by equation (A.17) of Appendix A.

(d) Behaviour Illustrated in Fig. 4.4

The behaviour is identical to that given in Fig. 4.3 except that instability of the column occurs before the steel has yielded and is therefore defined by equation (A.20) of Appendix A.

(e) Discussion

The descending portion of the column load-axial strain curve can be used structurally providing that redistribution of load can be achieved within the structural system. The limit to the axial strain capacity, ε_t, of the concrete of the composite column will therefore depend on the test stiffness as defined in Appendix A and illustrated in Figs. 4.1, 4.2, 4.3 and 4.4. For axial strains in excess of the value of ε_t, which defines the point of instability, a lower bound to the load carrying capacity is the load, P_{sy}, carried by the steel alone and given by

$$P_{sy} = f_{sy} A_s \qquad (4.13)$$

For a dead load test on a pin-ended composite column, instability occurs at a strain, ε_{ps} corresponding to the maximum load, P_{sh} (see equations (A.18) and (A.21) of Appendix A).

It should be noted that for short composite columns containing high strength steel such that $\varepsilon_{sy} > \varepsilon_o$ and where there is strain compatibility between the concrete and the steel as is generally the case for such columns, the maximum or ultimate load capacity, P_{sh}, is less than the sum of the individual maximum load capacities of the steel $(A_s f_{sy})$ and concrete $(A_c f_c'')$ as these are not attained for the same value of strain in both materials.

4.2 ECCENTRICALLY LOADED SHORT COLUMNS

4.2.1 Introduction

Again, the question is: what constitutes a short column? Brettle(126) used a simple ultimate strength analysis, based on similar assumptions as are now normally used to determine the ultimate strength of reinforced concrete sections, to determine the load-moment capacity of single steel I-sections encased in concrete, slenderness effects not being considered. Brettle compared this analysis with the results of tests by several investigators on composite columns of rectangular crossection containing a single steel I-section encased in concrete, with length to minimum dimension of crossection ratios ($\frac{l}{b}$) ranging from 2.4 to 15.7. The columns were loaded at a constant end eccentricity, the eccentricity values ranging from 0.4 in. to 8 inches. The eccentricity at each end of the columns was kept equal such that the columns were bent in single curvature. For the low $\frac{l}{b}$ ratio value of 2.4, the experimental maximum load capacities were always equal to or slightly greater than the theoretically predicted maximum loads. However, for all $\frac{l}{b}$ values greater than 2.4, the experimental maximum loads were always less than the theoretical loads for all values of end eccentricity. The difference between the experimental and theoretical loads ranged from 6% for a low $\frac{l}{b}$ value of 4.4 to 38% for a higher $\frac{l}{b}$ value of 11.9. Brettle's results indicate that for eccentrically loaded columns, the slenderness can affect the load carrying capacity for even relatively short columns. Therefore the theoretical treatment in this section on eccentrically loaded short columns will be concerned with the analytical methods for determining the load-moment-curvature relationship for the crossection alone of a built-up composite column. Slenderness effects are dealt with later in Section 4.3.

The strain distribution across a given crossection can be defined by the curvature, the centroidal axis strain (or the strain at any other known point in the crossection) and the angle between the neutral axis and a principal axis of the crossection. An analytical method for determining the load-moment-curvature relationship and suitable for programming in a digital computer has been developed by Roderick and Rogers(13) for encased rolled steel joists bent about their minor principal axis. The stress-strain relationships used for both the concrete and steel were those shown as Types I and II in Figs. 3.23 and 3.39 respectively. Loke(14) extended this method to the case of bending about the major axis and for the more complex condition of simultaneous bending about both principal axes for the encased rolled steel joist. Again, the simplified stress-strain relationships of Type I and Type II were used to represent the material properties. The writer has further extended the original analysis by Roderick(13) to take account of

(i) composite columns containing steel sections other than single I-sections and bent about either the major or minor principal axis,

(ii) simultaneous bending about both principal axes using a similar technique to Loke(14),

(iii) the non-linear stress-strain characteristics of the steel and concrete by using multipart stress-strain curves whereby the actual stress-strain relationships can be approximated by any desired number of straight line segments, the curves used being designated Type III as shown in Figs. 3.23 and 3.39,

(iv) crushing of the concrete by using a limiting value for the concrete strain, the concrete being assumed to be incapable of carrying stress for strains greater than this limiting value. A value of 0.005 in./in. was used which was obtained from observations of crushing in the actual column tests for the particular stiffness of the test system used, this aspect being discussed earlier in Section 3.2.1 and Appendices A and B; and

(v) the tensile strength of the concrete taken as one tenth of the compressive strength, this value being the order of the tensile strengths as obtained from split cylinder tests.

The details of this analysis are presented in Section 4.2.4(a). It should be noted that residual stresses present in the steel sections before loading are accounted for only in an average manner whereby every fibre of the steel is assumed to have a stress-strain characteristic given by the Type III curve in Fig. 3.39 which in fact gives the relationship between strain and average stress over the whole crossection.

Residual stresses, resulting from a number of causes, can be present in both the concrete and the steel of a composite section as explained below.

(i) Non-uniform cooling of the hot rolled structural steel sections such that compressive stresses develop in the portions of the steel crossection that cool most rapidly with balancing tensile stresses in the remaining portions. This aspect has previously been discussed in Section 3.3.6. The assumed pattern of this type of residual stress for the channel sections, used by the writer in his tests, is shown at (b) in Fig. 3.35, the peak values being given by equation (3.16).

(ii) Shrinkage of the concrete resulting in compressive stresses being developed in the steel with balancing tensile stresses in the concrete, the ability of concrete to creep in tension helping to reduce the magnitude of these stresses. For the writer's tests, the channel sections were placed symmetrically within the concrete encasement such that the centroids of the individual steel and concrete areas were coincident. The writer has assumed that the shrinkage strains are uniform across the concrete section and the forces developed in the concrete and steel are therefore axial with no resulting warping of the section. Hence, the stresses developed can be considered to be uniform across the concrete and steel areas. The writer's analysis for determining the magnitude of these stresses for known shrinkage and tensile creep properties of the concrete is described later in Section 8.2.2 of Chapter 8.

(iii) Construction loads and moments applied to the bare steel sections. The situation envisaged is typical of multi-storey construction, that of commencing erection of the bare steel frame and following with the concrete encasement at some later date by which time the bare steel column sections already have load and moment applied to them by construction loads and the weight of the structure being erected above. This mode of fabrication induces initial stresses in the bare steel sections that are conveniently quantified in terms of residual stresses, preceding and adding to the subsequent loading of the column as a composite column. Knowing the crossectional properties of the bare steel sections, these residual stresses can be determined from the loads and moments calculated using normal frame analysis techniques.

An analysis developed by the writer for determining the load-moment-curvature relationship for composite sections containing residual stresses, such as those listed above, is described in Section 4.2.4(b). The steel and concrete areas in the crossection are broken into many small discrete rectangular areas in a similar manner to that used by Basu(127), Warner(128), Brettle(126) and Virdi and Dowling(129). The concrete stress-strain relationship used for comparison with experimental results is that given in Fig. 3.23 and the steel stress-strain relationship is the elastic-plastic-strain hardening characteristic shown in Fig. 3.40 as the residual stresses in the steel are taken into account for each individual fibre rather than in the average manner shown in Fig. 3.39.

4.2.2 Elastic Behaviour

It is realized that a treatment of the elastic behaviour of a section bent about any required axis can be found in any standard engineering mechanics text. However, a brief summary of the development of the relevant expressions relating the load, moment and curvature are presented below so that reference can be readily made to these elastic expressions when considering the more complex case of inelastic biaxial bending later in this Chapter.

Consider the symmetrical composite section shown in Fig. 4.5 which is subjected to a gradient of strain, defined by its curvature, ρ_x, in the Ox direction and centroidal axis strain, ε_a, such that the stresses developed are still within the elastic range for the concrete and steel. Plane sections are assumed to remain plane so that the strain distribution is linear over the crossection. This has been verified by many investigators, reference here being made to the well known work of Hognestad(31)(41). As shear forces are generally small in pin-ended columns, the bond between the steel and concrete is assumed to be sufficient to maintain full composite action between the materials. In Fig. 4.5 a general case of biaxial bending is taken where the angle between the neutral axis and the major principal axis, OY, is β measured anticlockwise from the latter axis. The curvature in the Ox plane perpendicular to the neutral axis, is defined as ρ_x, the curvature in the mutually perpendicular plane, Oy, being zero. Let I_{cY} and I_{sY} be the second moments of area of the concrete area, A_c, and the steel area, A_s, about the major principal axis OY respectively, I_{cX} and I_{sX} being the corresponding values about the minor principal axis OX. As the principal axes of the steel area and the concrete area coincide with the principal axes for the whole crossection, then their second moments of area about the Ox and Oy axes, where Oy is parallel to the neutral axis, are given by

$$\left.\begin{aligned} I_{sx} &= I_{sX} \cos^2\beta + I_{sY} \sin^2\beta \\ I_{cx} &= I_{cX} \cos^2\beta + I_{cY} \sin^2\beta \\ I_{sy} &= I_{sY} \cos^2\beta + I_{sX} \sin^2\beta \\ I_{cy} &= I_{cY} \cos^2\beta + I_{cX} \sin^2\beta \end{aligned}\right] \qquad \text{———(4.14)}$$

For the rotated set of axes Ox and Oy,

$$\left.\begin{aligned} x &= X \cos \beta + Y \sin \beta \\ y &= Y \cos \beta - X \sin \beta \end{aligned}\right] \qquad \text{———(4.15)}$$

The moment, M_y, about the Oy axis is given by

$$M_y = \rho_x E_s I_{sy} + \rho_x E_c I_{cy} = M_s + M_c$$

$$= \rho_x E_c I_{Ty} \qquad \text{———(4.16)}$$

where $I_{Ty} = m I_{sy} + I_{cy}$ ———(4.17)

and $m = \dfrac{E_s}{E_c}$

and M_s and M_c are the moments carried by the steel and concrete respectively. The moment, M_x, about the Ox axis is given by

$$M_x = \rho_x E_s I_{sxy} + \rho_x E_c I_{cxy}$$

$$= \rho_x E_c I_{Txy} \qquad \text{———(4.18)}$$

where $I_{Txy} = m I_{sxy} + I_{cxy}$ ———(4.19)

The product moments of area can be shown to be

$$I_{sxy} = \tfrac{1}{2}(I_{sX} - I_{sY})\sin 2\beta$$

and

$$I_{cxy} = \tfrac{1}{2}(I_{cX} - I_{cY})\sin 2\beta \qquad \text{——— (4.20)}$$

Substituting equation (4.20) into equation (4.19) yields

$$I_{Txy} = \tfrac{1}{2}[mI_{sX} + I_{cX} - (mI_{sY} + I_{cY})]\sin 2\beta$$

$$= \tfrac{1}{2}[I_{TX} - I_{TY}]\sin 2\beta \qquad \text{——— (4.21)}$$

where $I_{TX} = mI_{sX} + I_{cX}$

and $I_{TY} = mI_{sY} + I_{cY}$ ——— (4.22)

With the use of equations (4.22) and (4.14), equation (4.17) can be rearranged to give

$$I_{Ty} = I_{TY}\cos^2\beta + I_{TX}\sin^2\beta \qquad \text{——— (4.23)}$$

where I_{TY} and I_{TX} are the second moments of area of the whole crossection, transformed to concrete, about the major and minor principal axes respectively. From equations (4.18) and (4.21), M_x will have a value of zero only when $\beta = 0$ or $\frac{\pi}{2}$, or when $I_{TX} = I_{TY}$, these conditions occurring when the plane of curvature, Ox, coincides with either the major or minor principal axes or when sections are used where the second moments of area about both the major and minor principal axes are equal as for circular or square concrete filled tubes. In general, $M_x \neq 0$ and the plane of curvature and the plane of the applied moment, M, where

$$M = \sqrt{M_Y^2 + M_x^2} \qquad \text{——— (4.24)}$$

do not coincide, the angle $(\alpha-\beta)$ between these two planes being given by

$$\tan(\alpha-\beta) = \frac{M_x}{M_y} \qquad \text{———————(4.25)}$$

where the angle, α, defines the direction of the plane of the applied moment. Substitution of equations (4.16) (4.18) (4.21) and (4.23) into equation (4.25) yields

$$\tan(\alpha-\beta) = \frac{(I_{TX} - I_{TY})\sin^2\beta}{2(I_{TY}\cos^2\beta + I_{TX}\sin^2\beta)} \qquad \text{————(4.26)}$$

The total moment, M, can be considered to be equivalent to a force, P, applied at an eccentric point defined by its co-ordinates, e_x and e_y, with respect to the Ox and Oy axes. These can be converted to the co-ordinates, e_X and e_Y, with respect to the OX and OY axes by the use of equation (4.15). The values of e_x and e_y can be determined from the following

$$M_x = Pe_y$$

$$M_y = Pe_x$$

Substitution of these values into equation (4.24) gives

$$M = \sqrt{(Pe_x)^2 + (Pe_y)^2} = Pe \qquad \text{———————(4.27)}$$

where e is the eccentricity measured from an axis passing through the centroid. The direction of this axis is defined by the angle, α, as shown in Fig. 4.5. The value of the force, P, is given by equation (4.1).

4.2.3 Biaxial Bending Considerations

For a composite section with a load, P, being applied at a point of constant eccentricity, e, defined by its co-ordinates, e_X and e_Y, as in Fig. 4.5, the plane of the applied moment is defined by the angle, α, measured from the OX axis where

$$\tan \alpha = \frac{e_Y}{e_X} \qquad (4.28)$$

The plane of curvature and therefore the direction of the neutral axis, as defined by the angle, β, can be calculated from equation (4.26) using the value of α determined from equation (4.28). Provided that the section remains wholly elastic, the value of β is constant for all values of load, P.

Warner(128) computed values of $(\alpha-\beta)$ corresponding to a range of loads on a typical reinforced concrete column section, the angle β being held constant. A typical non-linear stress-strain relationship for the concrete, similar to that shown in Fig. 3.4, and an elastic-plastic stress-strain relationship for the steel, similar to that in Fig. 3.40, were used in the analysis. It was found that the value of $(\alpha-\beta)$ did not remain constant but varied as the load increased, this being a result of the non-elastic nature of the materials, especially the concrete.

The biaxial bending problem is further complicated when the length of the column is considered. Early investigations by Horne(60) into the behaviour of single steel I-section columns, subject to biaxial loading, showed that not only did these columns deflect in a different plane to the applied end moments but also twisted as the load increased. It was shown that the solution of the differential equations of equilibrium from a direct stability analysis was exceedingly complex. A recent review of the biaxial bending problem for steel columns by Chen and Santathadaporn(130) indicates the complexity of the problem and large number of investigations that have been concerned with the problem.

It can be well imagined that a lateral torsional buckling analysis of a built-up composite column, containing two or more steel components and two materials having totally different properties, would be an even more complex problem, if not impossible. To make the problem more amenable to analysis, it was therefore decided to make the same simplifying assumptions as used by Loke(14) so that a theoretical estimate of the load-deformation behaviour could be made for the biaxially loaded columns tested by the writer as described later in Chapters 5 and 6.

Procter(131) has noted that lateral-torsional buckling failures have not been observed in tests on composite columns or in actual composite structures. Procter also demonstrated that the torsional rigidity of universal beams and columns, with concrete encasement to normal values encountered in practice, could be up to one thousand times greater than the torsional rigidity of the bare steel section. For biaxially loaded encased columns of similar section with a length to minimum crossection dimension ($\frac{l}{b}$) ratio of 12, Loke(14) found that the behaviour of these members was essentially flexural without any evidence of twisting. Sherman and Lukas(132) conducted a series of tests on torsionally stiff hollow rectangular steel tubes for a range of lengths where the load was applied eccentrically about the major axis. Even when failure occurred by lateral buckling about the minor axis such that the centre section of the column was in a state of biaxial bending, the maximum recorded twist at this section for all tests was only 0.46 degrees.

The first assumption therefore is that for torsionally stiff columns such as composite columns of normal proportions, the twist can be neglected together with any warping of the crossection due to any torsions or shears at the section as these are generally small for members of solid section.

The results of the biaxial loading tests by Loke(14) on concrete encased rolled steel joists demonstrated that not only was the behaviour of these members essentially flexural without any evidence of twisting but that the deformations also occurred in the plane of the applied moment exerted by the eccentric load. In these tests, the eccentric load was applied vertically through case hardened steel rockers which ensured that the columns were pin-ended in the plane of the applied moments but effectively fixed ended in the lateral direction. Therefore, the end conditions were such that the columns were restrained at the ends to deform in the plane of the applied end moment.

The second assumption is therefore that the curvature is in the plane of the applied moment. It is realised that while this assumption is valid for the particular columns under consideration, it may not be satisfactory in other cases such as for columns of narrow rectangular section where the second moments of area about the two principal axes differ considerably and for very slender columns where the effects of end restraint are reduced. For these cases, techniques such as those proposed by Virdi and Dowling(129) and Warner(133) could be useful.

It should be noted that the above assumption affects the theoretical considerations for only five of the writer's twenty-five tests, the remaining twenty tests being loaded eccentrically or concentrically to bend about either the major or minor principal axis where the plane of curvature automatically coincides with the plane of the applied moment provided lateral buckling for the columns bent about the major axis does not occur. This is unlikely for composite columns of normal proportions.

4.2.4 Inelastic Behaviour

This section has been divided into four subsections (a), (b), (c) and (d). In subsection (a), the analysis for the determination of the axial load and the moment about the centroidal axis from a given strain distribution, defined by its curvature and centroidal axis strain, is described. The method used is essentially that developed by Roderick and Rogers(13) and extended by Loke(14) for the case of biaxial bending. The writer has further extended the method to account more accurately for the non-linear stress-strain characteristics of the materials, especially the concrete. For a given crossection, the stress distribution across the section can be readily computed from the strain distribution and the stress-strain relationships for the two materials. Knowing the stress distribution, the load and moment can then be calculated from the geometrical properties of the section. Therefore, the load and moment can be calculated directly for a given strain distribution. No account is taken of residual stresses in the materials except that residual stresses in the steel resulting from the rolling process are included in an average manner by the use of an average steel stress-strain curve of the type shown in Fig. 3.39.

In subsection (b), a procedure developed by the writer is presented for determining the axial load and moment for a given strain distribution. This differs from that given in (a) in that residual stresses from all sources are accounted for directly using a technique in which the crossection is divided into small elemental areas.

In subsection (c), the procedure for determining the moment from given values of load and curvature is described. The moment cannot be calculated directly and an iterative process is required. However, this technique is useful when determining the deflected shape of a column for a particular value of applied load. Values of moment can be computed for a range of corresponding values of curvature thus establishing the relationship between moment and curvature for a constant value of load. This can ultimately be used to compute the deflected shape of the column by integration of the curvatures at a number of discrete sections along the length of the column. This technique was used by Roderick and Rogers(13).

In subsection (d), the procedure developed by the writer is described for determining the strain distribution for given values of the load and moment. The strain distribution cannot be calculated directly and a more involved iterative procedure than that used in subsection (c) is needed. However, this technique is also useful in the determination of the deflected shape of a column by integration methods as the curvature at discrete sections along the length of the column can be calculated from the load and moment values at these sections. In this method the curvature at only the discrete sections are computed whereas in subsection (c) the process is such that a list of moment-curvature values is established where the moment values are not necessarily exactly those at the discrete sections. The curvatures corresponding to the moment values at the discrete sections are found by interpolation in the moment-curvature list. It could be said that the method of (c) includes interpolation errors, which can be reduced by using small increments of curvature when establishing the moment-curvature list but thereby increasing the computational time whereas the method of (d) could include further errors associated with the extra iteration required and a more lengthy computational time if the length of the column is divided into a large number of discrete sections. This time can be reduced by decreasing the number of sections along the column length. It has been found that deflections computed using 20 discrete sections over the length are within 5% of the deflections computed using 200 discrete sections for all load values up to the maximum load.

(a) Determination of the Resultant Axial Load and Moment at a Composite Section for a given Strain Distribution across the Section - No Residual Stresses.

Consider a section of a composite column, as shown in Fig. 4.6, subjected to a gradient of strain defined by its curvature, ρ_x, and centroidal axis strain, ε_a. The assumptions are

(i) Plane sections remain plane such that the strain distribution is linear.

(ii) The moment plane and curvature plane coincide as discussed in Section 4.2.3.

(iii) The bond between the steel and concrete is sufficient to maintain full composite action.

(iv) The relationship between stress and strain for the concrete and steel can be expressed in the form of the Type III curves in Figs. 3.23 and 3.39 respectively. These have been simplified in Fig. 4.6 for demonstration purposes only. It will be seen that the analytical procedure is identical for stress-strain curves approximated by any number of straight line segments. The slopes of the straight line segments are expressed in terms of the initial elastic moduli, E_c and E_s, by the use of the corresponding coefficients ν_{cc}, ν_{ct}, ν_{sc} and ν_{st}.

The strain distribution selected in Fig.4.6 covers the full range of behaviour of the materials including cracking and crushing of the concrete and yielding of the steel in both tension and compression. It can be seen that the actual stress distributions for the steel and concrete, as shown by the full lines, can be obtained by the removal of the triangular and trapezoidal stress blocks, as shown bounded by the dashed lines, from an assumed fully elastic stress distribution. Therefore, the load, P, and moment, M, about the centroidal axis resulting from the actual stress distribution can be obtained by the removal of the loads and moments produced by the actions of the triangular and trapezoidal stress blocks on their corresponding areas of concrete and steel, as shown in Figs. 4.7 and 4.8 respectively, from the load $(P_c + P_s)$ and moment $(M_c + M_s)$ for a fully elastic section as shown in Fig.4.5. From equations (4.1) and (4.16) for a wholly elastic section where the concrete is able to sustain tension, the load, P_c, and moment, M_c, carried by the concrete and the load, P_s, and moment, M_s, carried by the steel are given by

$$\left.\begin{aligned} P_c &= \varepsilon_a E_c A_c \\ P_s &= \varepsilon_a E_s A_s \\ M_c &= \rho_x E_c I_{cy} \\ M_s &= \rho_x E_s I_{sy} \end{aligned}\right] \qquad \text{———(4.29)}$$

where I_{cy} and I_{sy} are the second moments of area of the total concrete area, A_c, and the total steel area, A_s, about the centroidal axis parallel to the neutral axis. I_{cy} and I_{sy} can be calculated from the second moments of area about the two principal axes by the use of equation (4.14). With reference to Figs. 4.5, 4.7 and 4.8, the resultant axial force, P, is given by

$$P = P_c + P_s - \sum_{i=1}^{3} P_{cci} + \sum_{i=1}^{2} P_{cti} - \sum_{i=1}^{2} P_{sci} + \sum_{i=1}^{2} P_{sti}$$

———(4.30)

The element forces given in equation (4.30) can further be expressed in terms of the strain distribution, the geometrical properties and the material properties of the section as follows

$$
\left.
\begin{aligned}
P = {} & \varepsilon_a E_c A_c + \varepsilon_a E_s A_s \\
& - E_c \rho_x \sum_{i=1}^{j-1} (\nu_{cci} - \nu_{cci+1}) [(\bar{x}_{cci} - x_{cci}) A_{cci}] \\
& - E_c \rho_x \nu_{ccj} [(\bar{x}_{ccj} - x_{ccj}) A_{ccj}] \\
& - f_{cu} [A_{ccj}] \\
& + E_c \rho_x \sum_{i=1}^{k-1} (\nu_{cti} - \nu_{cti+1}) [(\bar{x}_{cti} - x_{cti}) A_{cti}] \\
& + E_c \rho_x \nu_{ctk} [(\bar{x}_{ctk} - x_{ctk}) A_{ctk}] \\
& + f_{ct} [A_{ctk}] \\
& - E_s \rho_x \sum_{i=1}^{n-1} (\nu_{sci} - \nu_{sci+1}) [(\bar{x}_{sci} - x_{sci}) A_{sci}] \\
& + E_s \rho_x \sum_{i=1}^{p-1} (\nu_{sti} - \nu_{sti+1}) [(\bar{x}_{sti} - x_{sti}) A_{sti}]
\end{aligned}
\right] \quad \text{——— (4.31)}
$$

The geometrical terms of this equation are similar in form to those used by Roderick(13) and Loke(14), but the terms representing the material properties differ owing to the use of different stress-strain relationships. In equation (4.31) and the following equation (4.33), the values of x_{cci}, $\bar{x}_{cci}$, x_{sci} and $\bar{x}_{sci}$ are taken as positive when measured to the right of the centroidal axis and the values of x_{cti}, $\bar{x}_{cti}$, x_{sti} and $\bar{x}_{sti}$ are taken as positive when measured to the left of the centroidal axis. The absolute values of the concrete crushing strength, f_{cu}, and concrete tensile strength, f_{ct}, have been used. For the particular case shown in Fig. 4.6, j=3, k=2, n=3 and p=3. A normal value for ν_{cc1}, ν_{ct1}, ν_{sc1} and ν_{st1} would be 1.0.

With reference to Figs. 4.5, 4.7 and 4.8, the resultant moment, M, about the centroidal axis is given by

$$M = M_c + M_s - \sum_{i=1}^{3}(M_{cci} + P_{cci}\bar{x}_{cci}) - \sum_{i=1}^{2}(M_{cti} + P_{cti}\bar{x}_{cti})$$

$$- \sum_{i=1}^{2}(M_{sci} + P_{sci}\bar{x}_{sci}) - \sum_{i=1}^{2}(M_{sti} + P_{sti}\bar{x}_{sti}) \qquad \text{———(4.32)}$$

The elemental moments given in equation (4.32) can further be expressed in terms of the strain distribution, the geometrical properties and the material properties of the section as follows

$$M = E_c\rho_x I_{cy} + E_s\rho_x I_{sy}$$

$$-E_c\rho_x \sum_{i=1}^{j-1}(\nu_{cci} - \nu_{cci+1})[I_{cci} + (\bar{x}_{cci} - x_{cci})A_{cci}\bar{x}_{cci}]$$

$$-E_c\rho_x\nu_{ccj}[I_{ccj} + (\bar{x}_{ccj} - x_{ccj})A_{ccj}\bar{x}_{ccj}]$$

$$-f_{cu}[A_{ccj}\bar{x}_{ccj}]$$

$$- \ldots\ldots\ldots\ldots$$

$$
\left. \begin{aligned}
&-E_c \rho_x \sum_{i=1}^{k-1} (\nu_{cti} - \nu_{cti+1})[I_{cti} + (\bar{x}_{cti} - x_{cti}) A_{cti} \bar{x}_{cti}] \\
&-E_c \rho_x \nu_{ctk} [I_{ctk} + (\bar{x}_{ctk} - x_{ctk}) A_{ctk} \bar{x}_{ctk}] \\
&-f_{ct} [A_{ctk} \bar{x}_{ctk}] \\
&-E_s \rho_x \sum_{i=1}^{n-1} (\nu_{sci} - \nu_{sci+1})[I_{sci} + (\bar{x}_{sci} - x_{sci}) A_{sci} \bar{x}_{sci}] \\
&-E_s \rho_x \sum_{i=1}^{p-1} (\nu_{sti} - \nu_{sti+1})[I_{sti} + (\bar{x}_{sti} - x_{sti}) A_{sti} \bar{x}_{sti}]
\end{aligned} \right] \qquad (4.33)
$$

I_{cci}, I_{cti}, I_{sci} and I_{sti} are the second moments of area of the corresponding areas A_{cci}, A_{cti}, A_{sci} and A_{sti} about their own centroidal axes parallel to the neutral axis.

It can be seen that both equations (4.31) and (4.33) are of a form suitable for computation using a digital computer. The geometrical terms, shown inside the square brackets, can be conveniently separated from the terms involving the material properties and the strain distribution. The areas A_{cci}, A_{cti}, A_{sci} and A_{sti} can be located in terms of their corresponding edge distances x_{cci}, x_{cti}, x_{sci} and x_{sti} measured from the neutral axis where

$$\left.\begin{aligned} x_{cci} &= \frac{\varepsilon_{cci} - \varepsilon_a}{\rho_x} \\ x_{cti} &= \frac{\varepsilon_{cti} - \varepsilon_a}{\rho_x} \\ x_{sci} &= \frac{\varepsilon_{sci} - \varepsilon_a}{\rho_x} \\ x_{sti} &= \frac{\varepsilon_{sti} - \varepsilon_a}{\rho_x} \end{aligned}\right] \qquad \text{———} (4.34)$$

and using the absolute values of ε_{cci}, ε_{cti}, ε_{sci} and ε_{sti}. Values can now be calculated for these areas and their corresponding centroid distrances $\bar{x}_{cci}$, $\bar{x}_{cti}$, $\bar{x}_{sci}$ and $\bar{x}_{sti}$ from which the geometrical terms in the square brackets for equations (4.31) and (4.33) can be calculated. Where a large number of differing strain distributions are to be considered, this process could be a tedious one. A simpler numerical technique is to determine values for the geometrical terms for a number of discrete values of x_{cc}, x_{ct}, x_{sc} and x_{st} across the whole width of the section and store these in an array. The value of the geometrical terms for particular values of x_{cc}, x_{ct}, x_{sc} and x_{st} calculated from equation (4.34) can then be found by interpolation within the array. This procedure is described below and is similar to that used by Loke(14).

With reference to Fig. 4.9, the concrete width is divided into nc strips of equal thickness T_{nc} and the steel width is divided into ns strips of equal thickness T_{ns}. These strips are numbered 1 to nc and 1 to ns starting initially from the extreme right hand edge of the concrete and steel respectively. The writer has used nc = ns = 100. For the case of major or minor axis bending where the strips are parallel to the web or the flanges of the channels, this ensured that the web or flange thickness was divided into at least 5 strips. For the concrete in compression, consider an area, A_{cc}, containing q strips where q can have any integer value from 1 to nc. The edge distance, x_{cc}, measured from the centroidal axis, that locates this area is given by

$$x_{cc} = (\frac{nc}{2} - q)T_{nc} \qquad (4.35)$$

Any of the strips, numbered i, in this area can be approximated to a rectangle of thickness, T_{nc}, and a depth equal to its midlength L_{ci}. This midlength is at a distance $\bar{x}_{ci}$ from the centroidal axis where

$$\bar{x}_{ci} = (\frac{nc}{2} - i + \tfrac{1}{2})T_{nc} \qquad (4.36)$$

Using equations (4.35) and (4.36), the geometrical terms in the square brackets for the concrete in compression can therefore be expressed as

$$\left.\begin{aligned}
[A_{cc}] &= T_{nc} \sum_{i=1}^{q} L_{ci} \\
[A_{cc}(\bar{x}_{cc} - x_{cc})] &= T_{nc}^{2} \sum_{i=1}^{q} (q-i+\tfrac{1}{2})L_{ci} \\
[A_{cc}\bar{x}_{cc}] &= T_{nc}^{2} \sum_{i=1}^{q} (\frac{nc}{2} -i+\tfrac{1}{2})L_{ci} \\
[I_{cc}+(\bar{x}_{cc} - x_{cc})A_{cc}\bar{x}_{cc}] &= \\
&= T_{nc}^{3} \sum_{i=1}^{q} \frac{L_{ci}}{12} + (q-1+\tfrac{1}{2})(\frac{nc}{2} -i+\tfrac{1}{2})L_{ci}
\end{aligned}\right] \qquad (4.37)$$

Equation (4.37) can therefore be used to establish values for the geometrical terms corresponding to values of x_{cc} by incrementing values of q from 1 to nc in steps of 1, the values of x_{cc} being given by equation (4.35). The only unknown quantity in equation (4.37) is the midlength L_{ci} for the ith strip which can be calculated in the following manner. The outline of both the concrete section and the steel section in Fig. 4.9 consist of a series of straight line segments. The straight lines can be defined initially in terms of their end co-ordinates (X,Y) with respect to the principal axes system YOX and then transformed to (x,y) co-ordinates with respect to the required yOx system by use of equation (4.15). The magnitude of L_{ci} for the ith strip is obtained by a simple but tedious search technique in which tests are made in a systematic manner to determine the possible intersections of the mean line $x = \bar{x}_{ci}$, the value of $\bar{x}_{ci}$ being given by equation (4.36), with the various straight lines forming the outline of the section. The length, L_{ci}, is then easily determined from the differences between the y co-ordinates of the intersection points.

By taking a value of $x_{ct} = x_{cc}$, it can be seen that for the symmetrical section shown in Fig. 4.9, $A_{ct} = A_{cc}$, $\bar{x}_{ct} = \bar{x}_{cc}$ and $I_{ct} = I_{cc}$ and hence the geometrical terms in the square brackets of equations (4.31) and (4.33) for concrete in tension can also be given by the expressions of equation (4.37) for concrete in compression where the numbering of the strips is considered to commence from the extreme left hand edge of the section rather than the extreme right hand edge. If the section is unsymmetrical, the full procedure above has to be repeated for a range of discrete values of x_{ct}.

The identical procedure to that above can be used to determine values of the geometrical terms for the steel in compression or tension. The relevant equations for the steel compression terms are

$$
\left.\begin{aligned}
x_{sc} &= (\tfrac{ns}{2} - q)T_{ns} \\
\bar{x}_{si} &= (\tfrac{ns}{2} - i + \tfrac{1}{2})T_{ns} \\
[A_{sc}(\bar{x}_{sc} - x_{sc})] &= T^2_{ns} \sum_{i=1}^{q} (q-i+\tfrac{1}{2})L_{si} \\
[I_{sc}+(\bar{x}_{sc} - x_{sc})A_{sc}\bar{x}_{sc}] &= \\
&= T^3_{ns} \sum_{i=1}^{q} (q-i+\tfrac{1}{2})(\tfrac{ns}{2} - i+\tfrac{1}{2})L_{si} + \frac{L_{si}}{12}
\end{aligned}\right] \quad \text{——— (4.38)}
$$

where the terms are defined in Fig. 4.9(b).

Provided that the shape of the crossection can be reasonably approximated by a series of straight line segments defined by the co-ordinates at their ends, the above processes can be used to determine the relationship of load and moment with curvature for any required shape of crossection for a composite member where the stresses are outside the elastic range for the materials.

(b) Determination of the Resultant Axial Load and Moment at a Composite Section for a given Strain Distribution across the Section - Inclusion of Residual Stresses.

Consider a section of a composite column shown in Fig. 4.10 for which residual strains and hence stresses are present in the steel and concrete before any loading of the section as a composite section takes place. An axial load, P, and moment, M, about the centroidal axis, Oy, is applied to the composite section resulting in additional strains with a linear distribution defined by its curvature, ρ_x, and centroidal axis strain, ε_a. The problem is to determine the values of P and M for this strain distribution.

The following method is applicable to any composite section containing any pattern of steel where the steel and concrete areas can be subdivided into any required number of rectangular segments whose sides are parallel to a chosen orthogonal axes system Y'O'X'. It is convenient to initially choose the origin, O', at the bottom left hand corner as all the co-ordinates for the steel and concrete segments are then positive. The writer has found that this reduces data preparation errors. The number of segments used depends not only on the geometry of the section but also on the size of the segment over which the residual strains can be considered to be linear in both directions i.e. represented by a plane surface.

A possible source of residual strains is shrinkage. For the symmetrical section shown in Fig. 4.10, shrinkage will produce only axial forces in the steel and concrete. The writer has therefore assumed that the resulting strains in the concrete and steel will be uniform. For the steel section, further sources of residual strain are those produced by construction loading prior to encasement and those associated with the rolling process. For applied loads and moments the usual assumption of plane sections remaining plane has been taken. However, as discussed earlier in Section 3.3.6, the rolling and cooling process can produce residual strains and stresses with a non-linear variation across the section as shown at (a) in Fig. 3.35. The writer has assumed a linear variation, as shown at (b) in the same Figure, with discontinuities at the junction of the web and flange and at the centre of the web. Hence the choice of eight segments for the steel and yet only one segment for the concrete for the section in Fig. 4.10.

For a general crossection, the concrete is divided into nc rectangular segments and the steel into ns rectangular segments. For the particular case in Fig. 4.10, nc = 1 and ns = 8. The concrete segments are numbered from 1 to nc and the steel elements from nc + 1 to nc + ns. It should be noted that the concrete segments are taken as gross areas which may contain areas of steel. The concrete displaced by the steel is taken account of when considering the steel segments. The position and size of the nth rectangular segment can be completely defined by its bottom left hand co-ordinates, X'(n) and Y'(n), and dimensions WX'(n) and WY'(n) in the X' and Y' directions respectively. The nth segment is then sub-divided into smaller elements by taking NX'(n) strips in the X' direction and NY'(n) strips in the Y' direction. For the ith strip in the X' direction and the jth strip in the Y' direction, the n,i,jth element has an area A(n,i,j) where

$$A(n,i,j) = \frac{WX'(n)}{NX'(n)} \cdot \frac{WY'(n)}{NY'(n)}$$

The gross area, A_g, is given by

$$A_g = \sum_{n=1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)$$

only the concrete segments being summed as these include the steel areas. The steel area, A_s, is given by

$$A_s = \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)$$

and the concrete area, A_c, is then

$$A_c = A_g - A_s$$

The co-ordinates of the centroid of the n,i,jth element are given by

$$X'(n,i,j) = X'(n) + \frac{2i-1}{2} \frac{WX'(n)}{NX'(n)}$$

$$Y'(n,i,j) = Y'(n) + \frac{2j-1}{2} \frac{WY'(n)}{NX'(n)}$$

The co-ordinates of the centroid, $\bar{X}'$ and $\bar{Y}'$, are evaluated as follows

$$\left.\begin{aligned}
(E_cA_c+E_sA_s)\bar{X}' &= E_c \sum_{n=1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)X'(n,i,j) \\
&\quad +(E_s-E_c) \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)X'(n,i,j) \\
(E_cA_c+E_sA_s)\bar{Y}' &= E_c \sum_{n=1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)Y'(n,i,j) \\
&\quad +(E_s-E_c) \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)Y'(n,i,j)
\end{aligned}\right]$$

————(4.39)

where E_c and E_s are the tangent moduli for the steel and concrete respectively at zero strain for the materials.

As the residual strains are assumed to have a linear variation over a segment, three values of residual strain, $\varepsilon_{r1}(n)$, $\varepsilon_{r2}(n)$ and $\varepsilon_{r3}(n)$ at three corners of the segment, as shown in Fig. 4.11, are sufficient to define the plane of the residual strain. The values of $\varepsilon_{r1}(n)$, $\varepsilon_{r2}(n)$ and $\varepsilon_{r3}(n)$ for all segments are data that has to be established for the particular set of conditions being examined. The residual strain, $\varepsilon_r(n,i,j)$, on the n,i,jth element is given by

$$\varepsilon_r(n,i,j) = \varepsilon_{r1}(n) + \left[\frac{(2i-1)}{2} \frac{\varepsilon_{r2}(n) - \varepsilon_{r1}(n)}{NX'(n)}\right]$$

$$+ \left[\frac{2j-1}{2} \frac{\varepsilon_{r3}(n) - \varepsilon_{r1}(n)}{NY'(n)}\right]$$

The co-ordinates of all elements are transformed to the new axes system yOx where the origin, O, is at the centroid as defined by equation (4.39). The Oy axis is the centroidal axis about which the moment is to be calculated, the curvature, ρ_x, being in the Ox plane. For the YOX axes system, the co-ordinates of the n,i,jth element are

$$X(n,i,j) = X'(n,i,j) - \bar{X}'$$

$$Y(n,i,j) = Y'(n,i,j) - \bar{Y}'$$

For a rotation, β, of these axes to the yOx position, the x co-ordinates of the n,i,jth element are calculated using equation (4.15) from which

$$x(n,i,j) = (X'(n,i,j)-\bar{X}')\cos\beta + (Y'(n,i,j) - \bar{Y}')\sin\beta$$

The concrete stress-strain relationship is that given in Fig. 3.23. The writer has found it more convenient to use the actual function for the stress-strain curve rather than the approximation of straight line segments to the curve. The method of evaluating the function has been described in Section 3.2.7 with reference to Appendix C where it has been shown that the stress-strain function can be conveniently expressed in polynomial form where

$$\sigma_c = F_c[\varepsilon_c]$$

and $$F_c[\varepsilon_c] = a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 + a_4\varepsilon^4 \quad \text{for } \varepsilon > \varepsilon_{ct}$$

$$= 0 \quad \text{for } \varepsilon < \varepsilon_{ct}$$

ε_{ct} being the tensile strain at which the concrete cracks. The constants a_1, a_2, a_3 and a_4 are determined using a method of least squares as presented in Appendix C.

As the residual stresses are treated independently, the stress-strain relationship for the steel in both tension and compression can be considered as linear elastic-plastic where

$$\sigma_s = F_s[\varepsilon]$$

and

$$
\begin{aligned}
F_s[\varepsilon] &= E_s\varepsilon && \text{for } -\varepsilon_{sy} < \varepsilon < \varepsilon_{sy} \\
&= f_{sy} && \text{for } \varepsilon > \varepsilon_{sy} \\
&= -f_{sy} && \text{for } \varepsilon < -\varepsilon_{sy}
\end{aligned}
$$

ε_{sy} being the yield strain.

The strain at the centroid of the n,i,jth element due to the applied load, P, and moment, M, about the centroidal axis is given by

$$\varepsilon(n,i,j) = \varepsilon_a + \rho_x\, x(n,i,j) \qquad \text{———— (4.40)}$$

Compressive stresses and strains are taken as positive and tensile strains and stresses as negative.

Noting that a correction has to be applied for the amount of concrete displaced by the steel, the axial load, P, is given by

$$
\begin{aligned}
P = & \sum_{n=1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)F_c[\varepsilon(n,i,j)+\varepsilon_r(n,i,j)] \\
& + \sum_{n=nc+1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)F_s[\varepsilon(n,i,j)+\varepsilon_r(n,i,j)] \\
& - \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)F_c[\varepsilon(n,i,j)+\varepsilon_r(n,i,j)] \\
& - \sum_{n=1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)F_c[\varepsilon_r(n,i,j)] \\
& - \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)F_s[\varepsilon_r(n,i,j)] \\
& + \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} A(n,i,j)F_c[\varepsilon_r(n,i,j)]
\end{aligned}
\qquad (4.41)
$$

Similarly, the moment, M, about the centroidal axis can be expressed as

$$
\begin{aligned}
M = & \sum_{n=1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} x(n,i,j)A(n,i,j)F_c[\varepsilon(n,i,j)+\varepsilon_r(n,i,j)] \\
& + \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} x(n,i,j)A(n,i,j)F_s[\varepsilon(n,i,j)+\varepsilon_r(n,i,j)] \\
& - \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} x(n,i,j)A(n,i,j)F_c[\varepsilon(n,i,j)+\varepsilon_r(n,i,j)] \\
& - \sum_{n=1}^{nc} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} x(n,i,j)A(n,i,j)F_c[\varepsilon_r(n,i,j)] \\
& - \ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

$$- \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} x(n,i,j)A(n,i,j)F_s[\varepsilon_r(n,i,j)]$$

$$+ \sum_{n=nc+1}^{nc+ns} \sum_{i=1}^{NX'(n)} \sum_{j=1}^{NY'(n)} x(n,i,j)A(n,i,j)F_c[\varepsilon_r(n,i,j)] \Bigg] \qquad (4.42)$$

By using a suitable range of values of ε_a and ρ_x in equation (4.40), the load-moment-curvature relationship for a composite section containing residual strains can be calculated from equations (4.41) and (4.42). It can be seen that these equations are of a form suitable for solution using a digital computer. In other methods (126) (127) (128) (129) where the crossection has been divided into elements, the same size grid was generally used for both the concrete and the steel. As the elemental forces acting on the steel elements are considerably larger than for similar sized concrete elements at the same strain, the writer feels that for accuracy, the steel should be divided into elements of considerably smaller size than the concrete. In the above method, this presents no problem as the number of elements is defined for each individual segment by NX'(n) and NY'(n) which can be given any required values. For the sections of the composite columns and concrete filled tubes tested by the writer, it was found that concrete elements of the order of 0.2 in. square and steel elements 0.02 in. square gave a sufficiently accurate answer. Elements two times and ten times these sizes gave loads and moments differing from those obtained for the original sizes by approximately 1% and 9% respectively.

(c) Determination of the Resultant Moment at a Composite Section for a given Load and Curvature.

Consider a section of a composite column, as shown in Fig. 4.6, subjected to a load, P, such that the resulting curvature is ρ_x and the stresses are outside the elastic range for the materials. The first estimate of the centroidal axis strain, $\varepsilon_a(1,1)$, is obtained from equation (4.1) by assuming the section wholly elastic where

$$\varepsilon_a(1,1) = \frac{P}{E_s A_s + E_c A_c}$$

The values of $\varepsilon_a(1,1)$ and ρ_x are substituted into equation (4.31) or (4.41) to obtain the value of axial load $P(1,1)$ corresponding to the strain distribution defined by $\varepsilon_a(1,1)$ and ρ_x. As the section is inelastic, $P(1,1)$ will not be equal to P. A second estimate of the centroidal axis strain $\varepsilon_a(1,2)$ is found using linear interpolation where

$$\varepsilon_a(i,2) = \varepsilon_a(i,1) \frac{P}{P(i,1)} \qquad \text{———(4.43)}$$

Using the values of $\varepsilon_a(1,2)$ and ρ_x in equation (4.31) or (4.41), the value of the axial load, $P(1,2)$, is calculated. If $P(1,2)$ has not converged to P, a better estimate, $\varepsilon_a(1,3)$, of the centroidal axis strain is found by using Newton-Raphson linear interpolation where

$$\varepsilon_a(i,m) = \varepsilon_a(i,m-1) + \frac{\varepsilon_a(i,m-1) - \varepsilon_a(i,m-2)}{P(i,m-1) - P(i,m-2)} (P - P(i,m-1))$$

———(4.44)

Using the values of $\varepsilon_a(1,3)$ and ρ_x in equation (4.31), the axial load, $P(1,3)$, is calculated. If $P(1,3)$ has not converged to P, the above process is repeated until $P(1,m)$ converges to P, this generally occurring for $m=4$ with a limit of accuracy given by

$$0.999 < \frac{P(i,m)}{P} < 1.001 \qquad \text{———} (4.45)$$

The resultant moment, M, is then calculated from equation (4.33) or (4.42) using ρ_x and the value of $\varepsilon_a(1,m)$ at load convergence. The above procedure canbe repeated to obtain values of the moment, M, corresponding to a range of curvature values, ρ_x, thus establishing the moment curvature relationship for a constant value of axial load.

(d) Determination of the Resultant Strain Distribution at a Composite Section for a given Load and Moment.

Consider a section of a composite column, as shown in Fig. 4.6, subjected to an axial load, P, and moment, M, such that the maximum stresses are outside the elastic range for the materials. First estimates of the centroidal axis strain, $\varepsilon_a(1,1)$, and the curvature, $\rho_x(1,1)$ are obtained from equations (4.1) and (4.16) respectively by assuming the section to be wholly elastic where

$$\varepsilon_a(1,1) = \frac{P}{E_c A_c + E_s A_s}$$

$$\rho_x(1,1) = \frac{M}{E_c I_{cy} + E_s I_{sy}}$$

The values of $\varepsilon_a(1,1)$ and $\rho_x(1,1)$ are used in equation (4.31) or (4.41) to obtain the value of axial load $P(1,1)$ corresponding to the strain distribution defined by $\varepsilon_a(1,1)$ and $\rho_x(1,1)$. As the section is inelastic, $P(1,1)$ will not be equal to P. The curvature $\rho_x(1,1)$ is held constant and the procedure given in subsection (c) above, using equations (4.43) and (4.44), is repeated until the load $P(1,m)$ has converged to P for a value of centroidal axis strain, $\varepsilon_a(1,m)$, this generally occurring for $m=4$ using the same limit of convergence given by equation (4.45). The resultant moment, $M(1,1)$, is then calculated from equation

(4.33) or (4.42) using the values of $\rho_x(1,1)$ and $\varepsilon_a(1,m)$ at load convergence. $M(1,1)$ will most likely not be equal to M. The centroidal axis strain, $\varepsilon_a(1,m)$ is now held constant and a new estimate of the curvature, $\rho_x(1,2)$ is obtained by using linear interpolation where

$$\rho_x(i,2) = \rho_x(i,1) \frac{M}{M(i,1)}$$

The values of $\varepsilon_a(1,m)$ and $\rho_x(1,2)$ are used in equation (4.33) or (4.42) to obtain the value of moment, $M(1,2)$. If $M(1,2)$ has not converged to M, a better estimate, $\rho_x(1,3)$ of the curvature is found by using Newton-Raphson linear interpolation where

$$\rho_x(i,n) = \rho_x(i,n-1) + \frac{\rho_x(i,n-1) - \rho_x(i,n-2)}{M(i,n-1) - M(i,n-2)} (M - M(i,n-1))$$

Using the values of $\varepsilon_a(1,m)$ and $\rho_x(1,3)$ in equation (4.33) or (4.42), the moment, $M(1,3)$, is calculated. If $M(1,3)$ has not converged to M, the above process is repeated until $M(1,n)$ converges to M, this generally occurring for $n=4$ with a limit of convergence given by

$$0.999 < \frac{M(i,n)}{M} < 1.001$$

One cycle of load balancing and moment balancing is now completed. Using the values of $\varepsilon_a(1,m)$ and $\rho_x(1,n)$ in equation (4.31) or (4.41), the axial load, $P(2,1)$ is calculated which may not be equal to P. If this is the case then $\rho_x(2,1)$ is set equal to $\rho_x(1,n)$ and $\varepsilon_a(2,1)$ is set equal to $\varepsilon_a(1,m)$ and a second complete load and moment balancing cycle is carried out. This procedure is repeated for i cycles until both load and moment have converged for the same strain distribution defined by the centroidal axis strain $\varepsilon_a(i,n)$ and curvature $\rho_x(i,m)$, this generally occurring for $i=4$.

The above process could be carried out at any required number of sections along the length of a column to determine the curvatures resulting from the load and moment at the sections.

4.3 ECCENTRICALLY LOADED LONG COLUMNS

4.3.1 Introduction

In this section a finite difference method suitable for programming on a digital computer is presented for predicting the theoretical flexural behaviour of pin-ended built-up composite columns subjected to combined axial load and bending moment about any required axis using the assumptions for biaxial bending given in Section 4.2.3. An analysis was originally developed by Roderick and Rogers(13) for the case of a single rolled steel joist encased in concrete and bent about the minor axis. The end eccentricities were kept equal at both ends such that the column was bent in symmetrical single curvature. Loke(14) extended this method to be applicable to major axis and biaxial bending for the same members again bent in symmetrical single curvature. The writer has further extended the analysis for the case of composite columns containing any required form for the steel sections and bent about any required axis using any combination of end eccentricity to produce any desired form of deflected shape either in single or double curvature.

4.3.2 Elastic Behaviour

Consider the two columns shown in Fig. 4.13 subjected to two different loading conditions, the column at (a) being bent in single curvature and the column at (b) in double curvature. The column length, L, is divided into n equal elements, each of length $\frac{L}{n}$ and the sections at each end of the elements are numbered 1 to n+1 commencing at the origin at the bottom of the column. The distance, z_i, to the ith section is given by

$$z_i = (i-1)\frac{L}{n} \qquad \text{———(4.46)}$$

The load, P, is applied at eccentricities, e_1 and e_{n+1}, from the centroidal axis at the bottom and the top of the column respectively. The crossection is that shown in Fig. 4.6 where it can be seen that for an eccentricity, e, in the positive x direction, the deflection, u_i at the ith section would be in the negative x direction and vice versa. The convention adopted in Fig. 4.13 and the following analysis is that the end eccentricity has a positive value when it produces deflection in the positive x direction.

In addition to the assumptions given in Sections 4.2.2 and 4.2.3, the following assumptions are made:

(i) The deflections and therefore slopes are sufficiently small to express the curvature, ρ_x, in the form

$$\rho_x = \frac{d^2u}{dz^2}$$

where u is the deflection in the x direction and z is the distance along the column length.

(ii) As in the well known Perry(134) solution for struts, the imperfections are considered as an initial lack of straightness of the form

$$u_{oi} = c_{ox} \sin\frac{\pi z_i}{L} \qquad \text{———(4.47)}$$

where u_{oi} is the initial deflection at the ith section and c_{ox} is the initial deflection in the x direction at the mid-height of the column. The right hand side of equation (4.47) can be considered to be the first term of a Fourier series that could be used to approximate the exact initial deflected shape. It has been shown by Timoshenko(135) that as the applied load approaches the buckling load, this first term is the most significant. The writer has assumed that c_{ox} can be related to the initial imperfections, c_{oX} and c_{oY}, in the principal directions in a similar manner as for the second moments of area given by equation (4.14) such that

$$c_{ox} = c_{oX} \cos^2\beta + c_{oY} \sin^2\beta$$

This expression is represented graphically in Fig. 4.12. The values of c_{oX} and c_{oY} are assumed to be related to the steel section imperfections. As in the well-known Perry-Robertson strut formula, these can conveniently be expressed in terms of the geometrical properties of the steel section in the form

$$c_{oX} = \frac{\gamma L r_Y}{d_X}$$

$$c_{oY} = \frac{\gamma L r_X}{d_Y}$$

where γ = constant

L = Length of the column

r_Y and r_X = radius of gyration of the steel section about the OY and OX axes respectively

d_Y and d_X = minimum distance of the extreme fibre of the steel section measured from its centroid in the Y and X directions respectively as defined in Fig. 4.6

For the encased rolled steel joist, Roderick and Rogers (13) recommended a value of 0.00075 for γ which the writer has found to be satisfactory for both the built-up composite columns and concrete filled tubes.

The deflection, u_i, at the ith section is given by

$$u_i = e_1\left[\frac{\sin\mu(L-z_i)}{\sin\mu L} - \frac{(L-z_i)}{L}\right] + e_{n+1}\left[\frac{\sin\mu z_i}{\sin\mu L} - \frac{z_i}{L}\right]$$

$$+ \left[\frac{\mu^2L^2}{\pi^2-\mu^2L^2}\right]c_{ox} \sin\frac{\pi z_i}{L} \qquad (4.48)$$

where $\mu^2 = \dfrac{P}{E_cI_{cy} + E_sI_{sy}}$

A derivation of equation (4.48) in this form can be found in the text by Timoshenko(135).

The end slope, θ_1, of the column at section 1 at the origin is given by

$$\theta_1 = e_1\left(\frac{1}{L} - \frac{\mu}{\tan\mu L}\right) + e_{n+1}\left(\frac{\mu}{\sin\mu L} - \frac{1}{L}\right) + c_{ox}\left(\frac{\pi^3}{\pi^2L - \mu^2L^3}\right)$$

$$(4.49)$$

These equations can be used provided the materials are within their elastic range. Considering the material properties shown in Fig. 4.6 and assuming $\nu_{ccl} = \nu_{ctl}$ and $\nu_{scl} = \nu_{stl}$, which is generally accepted for concrete and steel, then the column can be considered elastic if the concrete strains are between ε_{ctl} and ε_{ccl} and the steel strains are between ε_{stl} and ε_{scl}. This will occur provided that the moment, M_i, at every section of the column is less than the smallest value of the four moments calculated as follows

$$\left.\begin{aligned}
M_{ccl} &= \frac{\nu_{ccl}E_c\varepsilon_{ccl}I_{Ty}}{x_c} - \frac{PI_{Ty}}{A_T x_c} \\
M_{ctl} &= \frac{\nu_{ctl}E_c\varepsilon_{ctl}I_{Ty}}{x_c} + \frac{PI_{Ty}}{A_T x_c} \\
M_{scl} &= \frac{\nu_{scl}E_s\varepsilon_{scl}I_{Ty}}{mx_s} - \frac{PI_{Ty}}{A_T x_s} \\
M_{stl} &= \frac{\nu_{stl}E_s\varepsilon_{stl}I_{Ty}}{mx_s} + \frac{PI_{Ty}}{A_T x_s}
\end{aligned}\right] \quad \text{——— (4.50)}$$

where x_c and x_s are the distances from the centroidal axis to the extreme fibre of the concrete and steel respectively as defined in Fig. 4.6; $m = \frac{E_s}{E_c}$ and the strain values ε_{ccl}, ε_{ctl}, ε_{scl} and ε_{stl} are taken as positive values.

The moment, M_i, at the ith section is given by

$$M_i = P(e_1 + (e_{n+1} - e_1)\frac{z_i}{L} + u_i + u_{oi}) \quad \text{——— (4.51)}$$

4.3.3 Inelastic Behaviour

Consider a column of length, L, subjected to a load, P, at end eccentricities, e_1 and e_{n+1}, as shown in Fig. 4.13, where the stresses at sections of the column may be outside the elastic range for the materials. The eccentricities are such that the deflections are in the Ox plane as defined in Fig. 4.6 where a typical crossection is shown. The procedure for determining the deflected shape is as follows.

(i) An initial set of deflections, u_i, for i=1 to n+1 and the end slope, θ_1, are calculated from equations

(4.48) and (4.49) respectively, assuming the column to be elastic.

(ii) The initial imperfections, u_{oi}, for i=1 to n+1 are calculated from equation (4.47).

(iii) The values of u_{oi} and u_i are substituted into equation (4.51) to obtain values for the moment, M_i, at every section of the column.

(iv) The elastic and inelastic sections of the column are determined by comparing the values of M_i with the smallest value of moment given by equation (4.50).

(v) The values of curvature, ρ_{xi}, at the elastic sections are calculated using the simple elastic equation (4.16). Values of ρ_{xi} at the inelastic sections are calculated using either equation (4.33) or (4.42) and the procedure described in either (c) or (d) of Section 4.2.4.

(vi) The slopes, θ_i, are calculated by integrating the curvatures, ρ_{xi}, using Simpson's trapezoidal rule where

$$\theta_i = \theta_1 + \frac{L}{2n}(\rho_{x1} + 2\rho_{x2} + 2\rho_{x3} + \ldots + 2\rho_{xi-1} + \rho_{xi})$$

(vii) The deflections, u_i, are calculated by integrating the slopes, θ_i, again using Simpson's trapezoidal rule where

$$u_i = \frac{L}{2n}(\theta_1 + 2\theta_2 + 2\theta_3 + \ldots + 2\theta_{i-1} + \theta_i)$$

as $u_1 = 0$. A linear correction, as proposed by Newmark(136), is applied to these deflections to satisfy the boundary condition $u_{n+1} = 0$. The same technique has been used in an analysis by Basu and Hill(137). The corrected deflections, u_i', are given by

$$u_i' = u_{i+1} - \frac{i.u_{n+1}}{n} \quad \text{for } i=1 \text{ to } n.$$

The end slope is also corrected to a value θ_1' where

$$\theta_1' = \theta_1 - \frac{u_{n+1}}{L}$$

(viii) The values of u_i' and θ_1' are set equal to u_i and θ_1 which are now used as starting values for step (iii). Steps (iii) to (vii) are repeated until the defelcted shape at the end of a cycle, defined by u_i' and θ_1', coincides with the deflected shape at the start of the cycle, defined by u_i and θ_1, within the desired limit of accuracy. The writer has used the following limit

$$0.999 < \frac{u_i}{u_i'} < 1.001 \quad \text{for } i=1 \text{ to } n+1$$

where a value of $n=200$ has been used.

The load-deflection relationship for the ith section is established by increasing the load in increments, dP, of predetermined value and calculating the deflections, u_i, at each increased value of load using steps (i) to (viii) above. The maximum load for the column is overstepped when the iteration procedure for determining the deflection ceases to be convergent indicating that the column has become unstable. The load increment is reduced in magnitude and the maximum load is approached commencing from the last value of load for which convergence was obtained. The load increment is successively reduced whenever the iteration procedure for the deflections ceases to be convergent. The maximum load is considered to have been attained when the last reduced load increment, dP, is less than some small predetermined fraction of the last value of load, P_{max}, for which convergence was obtained. The writer has used the following limit

$$dP < \frac{P_{max}}{5000}$$

Computation is terminated at this point.

CHAPTER 5

SHORT TERM TESTS ON BUILT-UP COMPOSITE COLUMNS

5.1 INTRODUCTION

In this chapter an account is given of concentric and eccentric load tests on

(i) Nine small-scale pin-ended columns containing two steel channel sections encased in concrete as shown in Fig. 5.1 and

(ii) two pin-ended steel columns of the same steel crossection but without any concrete encasement.

A description of the column specimens is given in Section 5.2 with details of the materials and their properties being described in Section 5.3. The experimental apparatus and testing procedures are reported in Section 5.4 and the actual individual column tests are described in detail in Section 5.5. The test results are compared with the theoretical values in Section 5.6 and are further discussed in Section 5.7.

It will be recalled from Chapter 2 that only a limited number of tests have been reported elsewhere, these being confined almost exclusively to concentric loading with the only known tests since 1935 being by Bondale(8). The writer's test programme, outlined in Table 5.1, was therefore designed to provide experimental data which would contribute to a better understanding of the structural behaviour of eccentrically loaded built-up composite columns. These tests also provided the source of reliable experimental data by which the validity of the theoretical procedures, as developed in Chapter 4 for predicting the behaviour of composite columns, could be substantiated.

The behaviour of a bare steel column of built-up section under short term loading is dependent not only on such factors as the strength of the steel, the shape and disposition of the steel components forming the crossection, the axis of loading, eccentricity of loading and the slenderness ratio but also on the quantity, nature and type of the battens or lacing used to connect the individual components. Because of the apparent complexity of the analytical treatment of these 'frameworks' and as the cost of battens or lacing can add considerably to the total cost of the column, built-up steel columns are not often used in practice. When encased in concrete, their behaviour would be further influenced by the quantity, strength and stiffness of the concrete encasement and the type and percentage of secondary reinforcement used in this encasement. The previous investigations reviewed in Chapter 2 showed that not only did the concrete add considerable stiffness and strength to the bare steel columns but that local buckling failures of the individual steel components, a feature of tests on bare steel columns, were prevented. The writer decided to take an even more basic condition and to omit the battens and lacing altogether and to rely on the concrete alone to transmit the shear between the steel components. No secondary reinforcement was used in the concrete encasement as this represented perhaps the most adverse condition and allowed the effect of the absence of battens or lacing to be examined without being influenced by the presence of binders, stirrups or ties. The effect of these types of reinforcement has already been discussed in Section 3.2.4.

In the limited time available, it was not possible to examine all the factors, mentioned above, that might influence the behaviour of built-up composite columns. Within a limited number of tests, the writer felt that the following factors were the most significant.

(i) Eccentricity of loading

(ii) Axis of loading

(iii) Slenderness

(iv) Absence of battens or lacing.

In the test programme outlined in Table 5.1, the columns are designated by numbers in ascending order, the prefix CC referring to columns containing steel channel sections. In all cases the steel crossection consisted of two 3in. x 1½in. x 4.60 lb/ft steel channels placed toe to toe with a separation as shown in Fig. 5.1. For the composite columns, the steel sections were encased in unreinforced concrete to an overall size of 8in. x 7in. giving 2 inches of clear cover to the steel all round. The eccentricities were kept relatively small so that it was not necessary to provide haunches at the ends of the column. As the crossection was rectangular, only relatively simple formwork was required which resulted in ease of fabrication of the columns. The formwork previously used by Loke(14) for the manufacture of encased rolled steel joists was adapted for use in the production of the encased built-up sections. The relatively small scale of the section was chosen so that the ratios of the test column lengths to the least dimension of the composite crossection were of the order of those that might be used in practice.

For each column, the applied load was increased in increments, strain and deflection measurements being taken at each increment until the maximum load was reached. For the majority of the tests, the stiffness of the testing machine was sufficient for the unloading behaviour (increasing deflection with decreasing load) to be measured. The steps in the development of the test programme are summarised below.

The first stage consisted of testing six columns all with a length of 7 feet and with no battens or lacing between the channels. The initial test was on column CC1 which was loaded at an eccentricity, e, of 1.5 inches to the minor principal axis as defined at (a) in Fig. 5.1. This was the only column bent about the minor axis as in this mode of bending, the absence of battens would not be expected to influence the behaviour. The aim of the test was to establish the basic behaviour of a composite column where shear transfer between the steel components was not a significant factor and to check the test equipment and procedures for the more important column tests to follow.

Columns CC2, CC3 and CC4 were loaded at eccentricities of 0.8, 1.5 and 0.0 inches respectively to the major principal axis as shown at (b) in Fig. 5.1. Column CC5 was bent simultaneously about both principal axes by loading at an eccentricity of 0.8 inches to a diagonal axes 49° to the major principal axes as shown at (c) in Fig. 5.1. For bending about these axes, the absence of battens might be expected to be a significant factor with only the concrete being available to transmit the shear between the two steel channels but as the columns were bent in symmetrical single curvature, the magnitude of the shear forces developed with bending was only small.

To examine the ability of the concrete alone to transmit the shear between individual steel components, column CC11 was tested with equal but opposite end eccentricities of 2 inches to the major axis such that the column was bent in double curvature. This represented the maximum shear condition to which the column could be subjected. The eccentricity of 2 inches was the maximum that could be applied to the end of the column using the same end fittings as for the previous tests without having to resort to providing haunches at the ends of the column.

The second stage of the test programme consisted of testing two bare steel columns, one with battens along its length and the other without, and then testing two similar steel columns encased in concrete and loaded at the same eccentricity, a value of 0.8 inches being used. A column length of 10 feet was selected, this being the maximum that could be accomodated in the testing machine. For the battened columns, the size, spacing and attachment of the battens were in accordance with the requirements of Australian Standard AS CA1 - 1968 which are similar to those of British Standard BS 449:Part 1:1970. The four columns initially comprising this second stage were columns CC13, CC13R, CC14 and CC15 with details as given in Table 5.1. The aim of these tests was to determine experimentally the increase in load carrying capacity provided by the concrete encasement and to determine directly whether the presence of battens had any influence on the behaviour of the columns.

An additional 10ft. column CC16 was tested at an eccentricity of 1.5 inches to the major axis. This enabled the effect of slenderness to be examined by comparing the results of tests on columns both 7ft. and 10ft. in length for at least two different values of end eccentricity.

5.2 MANUFACTURE OF THE COLUMN SPECIMENS

The writer was fortunate that the same formwork, end plates and rocker bearings that had been designed and used by Loke(14) in the testing of encased rolled steel joists could also be used for the built-up composite column tests. Only minor modifications were needed to the end plates so that they could be attached to the double channel section rather than the single I-section. The procedures used in the preparation of a typical column specimen are as follows.

The two lengths of 3in. x 1½in. channel required for each column specimen were cut from a single length of channel, together with the corresponding material test specimens, as shown in Fig. 5.2. The ends of the column lengths were machined plane and finished by grinding.

The initial deformed shape of the column lengths were measured with an optical level and a millimetre scale to within $\frac{1}{10}$th of a millimetre. To do this, the column was supported horizontally on a table to minimize any self weight deflections. Levels were taken at the ends, the centre and the quarter points on the three outer faces of each channel length as defined in Fig. 5.2. Using the plane through the ends of each outer face as reference, a deformation in the direction from the outer face towards the centroid of the channel section was taken as positive for the results tabulated in Table 5.2.

The average values of the crossectional dimensions of the two channels forming the steel core of the column, as defined by the section shown in Fig. 5.2, are tabulated in Table 5.3 for all eleven columns. The values were obtained from micrometer measurements taken on the web and both flanges of each column length at the same points at which levels were taken. The crossectional areas were computed from the measured dimensions. The validity of this method was checked by finding the volume and hence the area, using a water immersion weighing method, of two 1in. long specimens with ground ends cut from two of the column lengths. The result was within ½% of the area computed from the dimensions. For comparison, the values of the dimensions given in Australian Standard A1-1965 "Dimensions of Hot-rolled Steel Shapes and Sections for Structural Purposes" for a standard 3in. x 1½in. x 4.60 lb/ft channel section are given at the foot of Table 5.3.

Each channel length was wire brushed to remove rust and mill scale and then cleaned with a solvent. At the section at the centre of the length of the column, the surface of the channels was ground to a smooth finish at the positions indicated in Fig. 5.1 to which 20mm, 120 ohm electrical resistance strain gauges were cemented and water-proofed. For column CC11 tested under maximum shear conditions, strain gauges and rosettes were attached at the positions indicated in Fig. 5.3.

An 8in. x 7in. x $\frac{3}{4}$in. ground steel end plate was attached to the bottom end of the two channels forming the steel core of the column by four $\frac{3}{16}$in. allen screws into tapped holes in the corners of the channels at the web and flange junctions. The two channels were orientated so that their initial deformations were in the direction of the deflection to be produced by the applied eccentric load. Where the initial deformations of the individual channels were in opposite directions to each other, the channels were orientated so that the one with the larger initial deformation was bent in the direction of the deflection to be produced by the applied eccentric load.

For columns CC14 and CC15 only, battens were welded to the channels in the positions shown in Fig.5.4. These battens were in accordance with the requirements of Australian Standard AS CA1-1968 "SAA Steel Structures Code". For column CC13 which was tested as a bare steel unbattened column and then as the composite column CC13R, battens were welded to the ends of the channels but no intermediate battens were attached. The use of end battens enabled the same end plates attached by four small allen screws to be used rather than having to weld the channels to the end plates. It also represented the minimum attachment between the channels that would be expected for an actual column in practice.

The two channels were set up vertically inside steel formwork with the attached end plate at the bottom. The formwork, shown in Plate I, consisted of two straight steel 7in. x 3in. x 14 lb/ft. channels 10ft. in length and spaced at 8 inches between the webs which formed the 7in. faces of the composite column. The two 8in. faces of the composite column were formed by flat steel plates bolted to the flanges of the formwork channels. The bottom of the formwork was bolted onto a horizontal ground steel plate embedded in a concrete pedestal. The formwork was rigidly held at an intermediate height by stays attached to an existing reinforced concrete column forming part of the laboratory. The two channels forming the core of the composite column were maintained at the correct 1 inch spacing and held securely in position within the formwork by two steel crosspieces as shown at (a) in Plate II. These crosspieces were attached to the ends of the channels using allen screws into the same tapped holes in the channels as used for attaching the steel end plates. The required 2in. cover all round was obtained by adjusting the threaded rods with knurled ends as indicated at (b) in Plate II.

The channels were then encased in concrete to within ½in. from the top in not greater than 3ft. lifts, the concrete being deposited through openings in two faces of the formwork as shown at (a) in Plate I. The concrete was vibrated internally using a pneumatic immersion-type vibrator to ensure good compaction. To prevent the likelihood of settlement shrinkage cracks, as observed by Loke(14), the top 12in. of the column was lightly revibrated internally about an hour after casting. The concrete was mixed in one batch in a Bennet pan-type mixer of 7 cu.ft. capacity. Thirteen 6in. x 12in. control cylinders were taken from the batch and compacted using the same vibrator as for compacting the column.

Each column was cured for 7 days in the formwork as at (b) in Plate I, the open end being covered by wet sand. The forms were stripped after 7 days and the column stored in the normal atmospheric conditions of the laboratory until tested, this being from 18 to 43 days after casting, the curing period for each column being listed in Table 5.4. The column curing conditions were simulated for the control cylinders by removing them from the moulds after 1 day, storing them for 6 days in wet hessian and thereafter uncovered next to the column.

Three weeks after casting, the surface of the concrete was prepared at the positions indicated in Fig. 5.1 for the attachment of electrical resistance strain gauges with a length of 20mm and a resistance of 120 ohms. The surface was ground to a smooth finish just exposing the aggregate and the strain-gauges then cemented in position. For column CC11 tested under maximum shear conditions, the positions of the strain gauges and rosettes are shown in Fig. 5.3. For the measurement of deflections, using dial gauges at the ends, quarter points and centre of the column in both principal directions, the surface of the concrete at these positions was inspected to determine whether it was smooth and flat. Where necessary, a thin layer of polyester resin was applied to ensure a plane finish. This was needed at midheight where both strains and deflection were to be measured at coincident points as shown in Fig. 5.1. For column CC11 which was bent in double curvature, additional positions were prepared to enable the deformations in the plane of bending to be measured at the eighth points.

A few days before testing, the top of each column was capped with a stiff cement mortar premixed with a small amount of aluminium powder to prevent shrinkage of the mortar. The capping technique to ensure uniform contact between the end plate and the concrete was developed by Loke(14) for capping his encased rolled steel joists. The method is described below to give the complete picture of all the stages in the preparation of a column specimen. The column was supported horizontally with the 8in. side uppermost so that the 1in. space between the channels was vertical. An 8in. x 7in. x $\frac{3}{4}$in. steel end plate, identical to that attached to the bottom end of the channels before casting, was attached to the top end in the same manner using four $\frac{3}{16}$in. dia. allen screws. A spirit level was used to check that the sides of the top end plate were parallel to the sides of the bottom end plate. A steel collar, as shown at (a) in Plate III and with internal dimensions identical to the external dimensions of the column crossection, was fitted and clamped, as shown at (b) in Plate III, to the top end of the column to cover the ½in. gap between the concrete and the end plate. The uppermost strap of the collar was removed and the mortar deposited into the gap and compacted with a steel rod making sure that the mortar completely filled the central core between the two channels. The strap was replaced and the column rotated through 180 degrees. The uppermost strap was removed to check that the mortar had been compacted to the full depth. Where necessary, additional mortar was added and compacted and the top strap replaced. The top straps were left in place until just before testing to cure the mortar.

Just prior to the column being set up in the testing machine, case hardened steel rockers were attached to the end plates by allen screws into tapped holes in the end plates. These tapped holes were positioned so that the rockers could be attached at a range of eccentricities to the required axis of bending. The arrangement for the end rockers is illustrated in Plate IV, that shown at (a) for bending about the major axis and that at (b) for biaxial bending about a diagonal axis of the crossection. The centre of rotation of both rockers coincided with the junction between the steel end plate and the concrete thus making the effective length the same as the actual length of the column.

At this stage a column specimen was ready for placing in the testing machine.

5.3 MATERIALS

5.3.1 Steel

The channels used were supplied in lengths of 30ft. and conformed to the requirements in Australian Standard A149-1965 "Mild Steel for General Structural Purposes". The column lengths and steel control specimens for each column were cut from a single length as indicated in Fig. 5.2. The following control tests were conducted.

(a) Stub Column Tests

The average compressive yield stress for each column was determined from two squash tests on short lengths designated * SA and * SB in Fig. 5.2. The test procedure given by the Column Research Council(43) was used as a guide for these tests which were carried out on an Avery 250,000 lb capacity universal testing machine as shown at (a) in Plate V. The ends of the stub columns were machined and ground plane to be perpendicular to the longitudinal axis of specimens. Four electrical resistance strain gauges were attached to the flanges and the web of the specimens which were then whitewashed so that the progress of yielding could be examined visually by observing the flaking of the mill scale. Two dial gauges set diametrically opposite, as shown at (a) in Plate V, were used to measure the axial shortening of the stub column during testing. With increasing load, the pattern of flaking indicated that yielding was initiated in the centre of the web, this occurring at a load about three quarters that to produce yielding over the whole crossection. The fact that yielding initiated in the web indicated that the maximum compressive residual stress occurred in the web and was of the order of one quarter the yield stress. This supported the use of the theoretical residual stress distribution as proposed in Fig. 3.36. Typical stress-strain curves are shown in Figs. 3.37 and 3.38 which also indicate that first yield commenced at an average stress of about 75% of the yield stress. The average values of compressive yield stress for each column are listed in column (1) of Table 5.4.

(b) Coupon Tension Tests

The average tensile yield stress was determined from four coupon specimens tested in accordance with Australian Standard AS A23-1964 "Methods of Tensile Testing of Metals". The coupons had dimensions as indicated at (a) in Fig. 5.5 and were cut from the lengths designated * TA and * TB in Fig. 5.2 at the positions marked at (b) in Fig. 5.5. They were tested in a Mohr and Federhaff 88000 lb capacity universal testing machine in the manner shown at (b) in Plate V. An extensometer with a least count of 0.0005 in. was used to measure the strain over a 2 in. gauge length. A typical stress strain curve is shown in Fig. 3.38. The average values of lower yield stress and modulus of elasticity for each column are listed in columns (2) and (4) respectively of Table 5.4.

(c) Hounsfield Tension Tests

In some cases, tests were also carried out on Hounsfield Tensometer specimens with dimensions shown at (c) in Fig. 5.5. The specimens were cut from the lengths designated *HA and * HB in Fig. 5.2 at the seven positions indicated at (d) in Fig. 5.5. A typical plot obtained from the Hounsfield testing machine is shown at (e) in Fig. 5.5. A typical variation of lower yield stress with position of specimen is plotted at (f) in Fig. 5.5. This indicated that the yield stress was higher at the tips of the flanges and the centre of web where the material has a greater reduction in thickness during the rolling process. However the maximum difference from the mean value was only 5%. The average values of lower yield stress for each column are listed in column (3) of Table 5.4.

5.3.2 <u>Concrete</u>

The cement : sand : aggregate proportions by weight were 1 : 2½ : 2½ and the water/cement ratio was 0.74. The weight of cement used in a single batch was 139 lb which yielded 6½ cub.ft. of concrete, sufficient to cast one column and thirteen standard 12in. x 6in. control cylinders. Prior to mixing each batch, the moisture content of the sand and aggregate was measured and the amount of water added was adjusted accordingly. The average slump was 3½ inches.

The cement used in the manufacture of all columns was Union brand normal portland cement. A full chemical analysis is given in Table 5.5. This information is of use when considering the creep and shrinkage properties of the concrete which are discussed later in Chapter 7.

A river sand was used. The sand grading curve shown at (a) in Fig. 5.6 indicates the range of grading obtained over the test series.

The aggregate was a crushed basalt of $\frac{3}{8}$in. maximum size. The aggregate grading curve, showing the variation in grading over the test series, is displayed at (b) in Fig. 5.6. The combined aggregate-sand grading curve is shown at (c) in Fig. 5.6.

For each column, the standard 28 day cylinder compressive strength was obtained from three cylinders which were stored at 70°F and 100% relative humidity in a fog room for 28 days and then tested. The results are listed in column (5) of Table 5.4. The average compressive strength (4 cylinders), split tensile strength (3 cylinders) and modulus of elasticity (3 cylinders) for the concrete at the age of testing the corresponding column were determined in accordance with the following respective ASTM Standards : ASTM C39 "Compressive Strength of Moulded Concrete Cylinders"; ASTM C496 "Splitting Tensile Strength of Moulded Concrete Cylinders"; ASTM C469 "Static Young's Modulus of Elasticity and Poisson's Ratio in Compression of Cylindrical Concrete Specimens". The respective average values for each column are listed in columns (7), (8) and (9) of Table 5.4. The test arrangement for measuring the elastic modulus is shown in Plate VI. The same Mohr and Federhaff 200,000 lb. capacity testing machine was used to measure the compressive and split tensile strengths.

5.4 EXPERIMENTAL APPARATUS AND TEST PROCEDURE

For columns CC2, CC3, CC4 and CC5, the longitudinal compressive strains developed in the steel channels by the shrinkage of the concrete, prior to loading to failure in the testing machine, were measured by electrical resistance strain gauges affixed to the channels. Readings were taken immediately the columns were removed from the formwork and allowed to dry, this being at an age of 7 days. The axial shortening strains recorded during the drying period are plotted in Fig. 5.7. The corresponding tensile stress in the concrete was computed by simple statics from the crossectional areas and the elastic moduli of the concrete and steel sections. A knowledge of the magnitude of the residual stresses due to shrinkage was required so that values could be included in the analysis, used to predict the experimental behaviour of the columns, as described in Chapter 4. The average curve, shown as a broken line in Fig. 5.7, was also used to estimate the magnitude of the residual stresses, at the time of testing, in the columns for which shrinkage measurements were not taken.

The shrinkage strains of plain concrete specimens, cast from the same batch of concrete as the corresponding columns, were measured, as shown at (a) in Plate VII, by a dial gauge placed between brass targets, an Invar bar being used as reference. As displayed at (b) in Plate VII, these targets were cast into the specimens by the use of a plastic yoke, placed over the top of the formwork, which held the targets at the required gauge distance of 8 inches. The specimens had the same crossectional area as the column section and a length of 24 inches. On removal from the formwork after 7 days moist curing, their ends were sealed with a coating of Bondcrete, a waterproofing and sealing agent, followed by a layer of polythene film on top as can be seen at (a) in Plate VII. This was done to simulate the drying conditions of the long columns. The recorded shrinkage strains are plotted in Fig. 5.8. As these specimens and the columns were allowed to dry under normal atmospheric conditions existing in the laboratory, the temperature and humidity variations were measured using the recorder shown at (a) in Plate VII. The temperature tended to remain reasonably uniform over the early drying period but the humidity fluctuated over a wide range from 20% to 100%. For high values of humidity above 80% the shrinkage reversed and expansion occurred. Despite these fluctuations, the variation of shrinkage strain with time was approximately the same for all four specimens as demonstrated in Fig. 5.8.

The general arrangement for testing the columns is shown in Plate VIII for the columns bent about a principal axis, Plate IX for column CC5 bent in biaxial bending about a diagonal axis and Plate X for column CC11 bent in double curvature about the major principal axis with equal but opposite end eccentricities. All the tests were carried out in an Amsler 1,000,000 lb capacity compression testing machine. The frame shown to surround the testing machine was used as a reference frame to which dial gauges could be attached to measure the deflections. This frame was originally designed and used by Loke(14) for testing encased rolled steel joists. Because of the similar nature of the tests, the test procedures used by the writer and presented below for completeness were also similar to those used previously by Loke(14). The main difference was that the writer observed the behaviour well into the unloading range for the column taking care that measurements were taken only when the column was in stable equilibrium in this range.

A column was initially positioned in the testing machine by the following procedure. An end fitting with a hardened steel bearing plate, as displayed at (a) in Plate XI, was attached to the top loading platen of the testing machine so that the bearing plate was central on the platen. The spherical seat was wedged in position so that this platen was fixed. A similar end fitting, shown at (b) in Plate XI was attached to the bottom loading trolley which could be located centrally on the loading ram by a spigot. With the ram lowered, the trolley was wheeled away from the machine and column placed with its bottom end rocker approximately in the centre of the bearing plate in the position indicated at (b) in Plate XII. The trolley was then rolled back over the ram which was then raised until the top end rocker on the column came into contact with the centre of the top bearing plate in the position indicated at (a) in Plate XII. With a load of about 300 lb. applied, the trolley wheels were then wedged to keep the column in position without having to rely on the ram. The alignment screws shown in both Plates XI and XII were attached, these also serving to hold the column in position. The steel frame shown in Plates VIII, IX and X was then erected. Dial gauges for measuring deflections were attached to this frame and located on the prepared positions on the column as indicated in Fig. 5.1. The strain gauges were connected to a recording bridge as displayed in Plates VIII and IX.

The final positioning at the beginning of each test was carried out as follows. The machine ram was pumped up and the wedges removed from beneath the trolley. The load was then reduced to 300 lb. and the alignment screws at the top and bottom of the testing machine were adjusted so that the line of contact of the end rockers coincided with the plane of loading of the testing machine as defined by the centroids of the two screwed columns of the testing machine. The verticality of these two screwed columns was checked to be true with a theodolite. The member was then centred sideways between the two screwed columns by a small mechanical jack inserted between the side face of the member and the adjacent screwed column. The position in the plane of bending was checked again and where necessary, small re-adjustments were made with the alignment screws. The dial gauges were then adjusted to their final correct positions.

For the biaxially bent column CC5, the rotation at the central section was measured with three equally spaced dial gauges on one face of the column as shown at (a) in Plate XIII. For ease of reference, the columns were orientated in the testing machine so that the compression (concave) face was always facing north as indicated in Fig. 5.1. For column CC11 bent in symmetrical double curvature, the orientation was such that the compression face faced north for the lower half and south for the upper half of the column as exhibited at (b) in Plate XIII. Dial gauges were always located on the compression face of the column such that the column moved away from the gauges. This was done to prevent them being damaged in the event of a sudden failure with a rapid increase in deflections.

Readings of the dial gauges and strain gauges were recorded at the initial load of 300 lb which was then increased to 2000 lb at which point the alignment screws were released leaving the end rockers free to rotate as the column deflected. A set of readings were taken at this load, the deflections being recorded to the nearest 0.0001 in. and the strains to the nearest 1×10^{-6} in./in. The load was increased in increments of approximately one tenth of the estimated maximum, each increment taking about 2 minutes to apply. After reaching the required load, the testing machine was allowed to stabilize for about two minutes to ensure the load was constant. A complete set of readings were recorded at each load increment, each set taking about 3 mins. to complete. The column was continually examined for cracking or spalling of the concrete and their position and extent recorded. When nearing the maximum load, the increments were halved. The maximum load was reached after about 1½ hours of testing.

On reaching the maximum load, the deflections tended to increase rapidly. The load then commenced to fall. As the testing machine had considerable stiffness, the unloading behaviour was recorded, whenever possible, using the following procedure.

A particular value of central deflection was chosen. As the deflection of the column neared this value, the exhaust valve of the testing machine was opened and adjusted until the column deflection stopped on the target value. At this stage, the load indicator on the testing machine continued to fall. With the deflection held constant, the exhaust valve was gradually adjusted until the load indicator had stabilized on a constant value, this taking about 10 to 15 minutes. With the column in stable equilibrium, a set of readings of the gauges was recorded.

Another increased target value of deflection was chosen. The inlet valve of the hydraulic ram was temporarily opened to induce further shortening and hence deflection of the column. The load indicator registered a temporary rise during this procedure. Once the column deflections commenced to increase steadily, the inlet valve was closed and the outlet valve manipulated as before until the deflections were steady on the target value and the load had settled on a constant stable value. Readings were taken and the procedure repeated. The unloading behaviour of the columns was recorded in this manner, the test being stopped when considerable spalling of the concrete had extended over a fair length of the column. The load was released and the loading trolley wedged to hold the column in position in the testing machine. A photographic record of the test was then made.

5.5 DESCRIPTION OF COLUMN TESTS

5.5.1 Minor Axis Bending

Using the test procedure described in Section 5.4, column CC1 was loaded at an eccentricity of 1.5 inches to the minor principal axis as defined at (a) in Fig.5.1.

The resulting load-deflection relationship and strain results are shown in Figs. 5.9 and 5.10 respectively. As the load was increased, first cracking was observed near midheight on the convex face of the column at a load of 69 kips although the strain gauges at that position indicated that cracking occurred at a load of 40 kips, this being the last load increment that sensible strain readings were recorded on this face. From this and all the subsequent tests, it was found that if the strain gauge was placed directly over a crack position, the strain readings would increase suddenly to a large tensile value once the crack had formed often resulting in a tensile failure of the gauge. If the gauge was placed between crack positions, it was found that once the cracks formed, the strain readings ceased to increase in tension and then tended to decrease towards zero as the cracks widened and further cracks formed with increasing load. In general, it was found that the strain gauges indicated that cracking had commenced before the cracks were visible to the eye.

From the load-deflection relationship given in Fig. 5.9, it can be seen that for loads up to 50 kips, where the observed strains indicated that the materials could still be considered to be reasonably linear-elastic, the load-deflection relationship is not linear. This is expected as the theoretical elastic relationship between load and central deflection, as given by equation (4.48) in Chapter 4, is also non-linear. Above 50 kips, the rate of development of deflection increased as the sections of the column became progressively inelastic. Immediately before the maximum load of 117 kips was reached, the deflections increased rapidly. On attaining the maximum load, the load then began to decrease with further increase in deflection. The drooping characteristic of the load-deflection relationship was obtained from measurements at several equilibrium positions for the column using the technique described in Section 5.4. When the load had decreased to 106 kips, spalling of the concrete was observed to commence near midheight on the concave face of the column. With further decrease in load yet increase in deflection, the zone of concrete crushing extended further over the central region of the column as indicated at (a) in Plate XIV. The test was stopped when the load had decreased to a value of 45 kips at which point the central deflection was 1.5 inches. At this stage, the cracking on the convex face of the column had extended nearly over the full length of the column as shown at (b) in Plate XIV. The maximum recorded sideways deflection perpendicular to the plane of the applied end moments was only 0.008 inches. This confirmed the column did indeed deflect in the plane of the applied end moments as is normal for minor axis bending.

It can be seen from the strain results in Fig. 5.10 that the neutral axis moved towards the compression face of the column with increasing load. Therefore strain reversal occurred in some fibres of the column.

5.5.2 Major Axis Bending - 7ft. Columns

Columns CC2, CC3 and CC4 were loaded at eccentricities of 0.8 in., 1.5 in. and 0 in. respectively to the major principal axis as defined at (b) in Fig. 5.1. The resulting load-deflection relationships and strain results are shown in Figs. 5.11 and 5.13 respectively.

For column CC2, the rate of development of deflection increased as the load increased until just before the maximum load of 195.5 kips was reached at which point the deflection increased rapidly. Up to this stage, no cracking of the concrete was observed, the small eccentricity of 0.8 inches being such that the strains were primarily compressive over the crossection at midheight. This was indicated by the strain distribution near the maximum load as shown in the top left hand corner of Fig. 5.13 where it can be seen that only a small tensile strain was recorded on the convex face. On reaching the maximum load, the column became unstable and the load dropped off rapidly with increasing deflection and it was not until the load had decreased to 120 kips that an equilibrium position of the column could be maintained by suitable adjustments of the testing machine. At this point the central deflection was 0.5 inches, spalling of the concrete had commenced just below midheight of the column on the concave face as shown at (a) in Plate XIV and the concrete had cracked locally in the central region of the column as shown at (b) in Plate XIV. The strain reversal on the convex face is demonstrated in Fig. 5.13 where the strains were initially compresssive and changed to tension as the load increased.

For column CC3 loaded at a larger eccentricity of 1.5 inches, the deflections increased steadily with increasing load. Cracking was first observed on the convex face at 100 kips while the strain gauges on that face indicated that cracking had commenced at 90 kips. Once the maximum load of 159.0 kips was reached, the load decreased slowly with large increases in deflection, this unloading behaviour being gentle in nature and easily controlled. Spalling of the concrete was first observed when the load had fallen to 105 kips, this occurring in a region about one quarter the height from the bottom, possibly due to some local weakness in the concrete at that region. Deflections and strains were recorded until a load of 57 kips and a central deflection of 1.7 inches were reached. At this stage the crushed region had extended over about one fifth the length of the column as indicated at (a) in Plate XIV. The cracking of the concrete on the convex face of the column had extended nearly over the full length of the column, as shown at (b) in Plate XIV, and was similar to the pattern of cracking for column CC1 shown in the same Plate.

Column CC4 was concentrically loaded and showed little growth in deflection as the load increased. It was observed that the column commenced to bend in double curvature. The deflection profiles for several load values are plotted in Fig. 5.12. Initially, the prominent deflection was towards the south for the top half of the column, this being the intended direction of the deflections assuming the initial deformed shape before loading to be in that direction. However, as the load was increased, the deflection towards the north for the bottom half of the column became predominant, the column unwrapping and tending to go into single curvature. This mode of behaviour is not unexpected in concentrically loaded columns in which the magnitude and sign of the imperfection and possible errors in position of loading can exert a dominant effect. A maximum load of 270 kips was reached at which stage the maximum deflection was towards the north at the lower quarter point of the column and had a value of only 0.025 inches. The load then dropped slightly but suddenly to a value of 265 kips at which point crushing took place on the concave face (south face) in the lower half of the column as indicated at (b) in Plate XIV. The failure was sudden and explosive as expected for this "snap-through" mode of behaviour with the column unwrapping from double curvature into single curvature. A similar failure was recorded by Loke(14) for a concentric load test on an encased rolled steel joist. As the midheight deflection and hence curvature were only small approaching failure, the strain distribution at midheight was almost constant over the crossection as indicated in the bottom left hand corner of Fig. 5.13. Although the curvatures would have been considerably greater at the section where failure eventually occurred, strain gauges were not positioned at this section as it was not anticipated that failure would have occurred there.

For the above three columns, the maximum recorded sideways deflection was only 0.017 inches which indicated that the deflections were essentially in the plane of the applied end moments.

5.5.3 Diagonal Axis Bending

The 7ft. column CC5 was loaded at an eccentricity of 0.8 in. to the diagonal axis as indicated at (c) in Fig. 5.1. The load-deflection relationship and strain results are recorded in Figs. 5.14 and 5.15 respectively. The deflection increased steadily with increasing load. Cracking was first observed on the convex corner just below midheight at 112 kips while the strain gauges on this corner indicated that cracking commenced at 100 kips. On reaching the maximum load of 158.2 kips, the load decreased steadily with fairly large increases in deflection. Spalling of the concrete was first observed on the concave corner just below midheight when the load had fallen to 110 kips. The drooping characteristic of the load-deflection curve was recorded until the load had dropped to 70 kips at which point the central deflection was 1.08 inches. It can be seen from Fig. 5.15 that the experimental strain distributions are reasonably linear except for the strain, measured on the compression corner of the column, which was always low. This could have been due to the strain gauge being placed directly over a small shrinkage crack. Rapid drying occurs in the corners as two faces are available for moisture to be dried from the concrete. The corners are restrained from shrinking by the rest of the concrete in the column which does not dry and hence does not shrink as rapidly as the corner. This can result in cracking of the corners.

The extent of the final crushing and cracking of the concrete is shown in Plate XV. In Fig. 5.14, the deflections in the direction of the major axis (OY) and the minor axis (OX) have been plotted against each other. A linear relationship was obtained even into the unloading range of the column. From dial gauges mounted at midheight to measure the rotation of the column, as shown at (a) in Plate XIII, the maximum recorded rotation had an insignificant value of only 00°00' 48".

5.5.4 Major Axis Bending - Maximum Shear Condition

The 7ft. column CC11 was loaded at each end with an eccentricity of 2 in. but in opposite directions. The resulting load - deflection relationship for selected points is shown in Fig. 5.16, the deflection profiles in Fig. 5.17, the results of longitudinal strain measurements in Fig. 5.18 and the results of strain measurements with rosettes on the concrete at midheight in Fig. 5.19. With increasing load, the deflections increased steadily until the maximum load of 158.8 kips was reached. The deflection profiles in Fig. 5.17 indicated that the southwards deflections in the lower half of the column were larger than the northwards deflections of the upper half of the column. This was as expected as the initial imperfections of the steel channels were orientated towards the south. At the maximum load, spalling of the concrete commenced at the bottom end of the column on the northern concave face. As the crushing of the concrete continued, the load held steady for awhile and then dropped off suddenly with the region of concrete crushing extending well into the depth of the crossection at the bottom end. The extent of crushing is displayed in Plate XVI. No failure which could be attributed to shear was observed.

From the longitudinal strains measured on the compression faces of the column, as plotted in Fig. 5.18, it can be seen that the strains were larger for the bottom half of the column than for corresponding positions on the top half of the column as was expected from an observation of the deflection profiles in Fig. 5.17. The maximum strain adjacent to the bottom end of the column was 0.0052 in./in., this being the last value recorded before spalling of the concrete commenced. The maximum moment also occurred at this end position. The deflections were not sufficient for the maximum moment to occur within the length of the member.

At midheight, where the shear force is a maximum, two strain rosettes were placed where the major axis intersects the side faces of the column. The experimental stress condition at these gauges was calculated using the normal linear-elastic reduction method for strain rosettes based on the conversion of strains on the Mohr's circle of strain to stresses assuming the concrete to be isotropic and linear elastic. The value taken for the elastic modulus of the concrete was 3.38×10^6 lb./sq.in., as given in Table 5.4, and Poisson's ratio was taken as 0.15. The variation with applied load of shear stress and tensile stress on the vertical plane between the two channels and containing the major axis is shown in Fig. 5.19. The tensile stress recorded on this major axis plane may have been due to the initial curvatures of the individual channels being such that the channels curved away from each other. The channels would therefore tend to bow further away from each other as the load was applied, thus subjecting the concrete between them to a tensile force.

The variation of experimental principal tension with load at the two strain gauge rosette points is also shown in Fig. 5.19, a maximum value of 215 lb./sq. in being calculated.

5.5.5 Major Axis Bending - Battened and Unbattened Columns

Four columns were tested, all 10ft. in length and loaded at the same eccentricity of 0.8 inches to the major principal axis as defined at (b) in Fig. 5.1. The load-deflection relationships are shown in Fig. 5.20 and the results of strain measurements are given in Figs. 5.21, 5.22 and 5.23.

The bare steel column CC13 with battens at the ends only, as indicated in Fig. 5.4, was loaded until a maximum value of 21.3 kips was reached. At this point the deflection of the northern channel increased rapidly with the load falling slightly to 21 kips. The load was removed when this channel came into contact at midheight with the southern channel. The channels were initially at a one inch spacing before loading. On removal of the load, the channels returned to within 0.074 in. of their initial unloaded shape. The maximum recorded residual strain remaining in the channels after unloading was only 47×10^{-6} in./in. This indicated that failure was by elastic buckling. The strains recorded on the channels during loading, as shown in Fig. 5.21, demonstrated that the two channels behaved independently and it was only for loads below 5 kips that the strain distribution was linear across the whole crossection.

As the channels of column CC13 had not suffered any significant permanent damage during loading, the same channels were encased in concrete to become composite column CC13R. On loading this column, the deflection increased steadily with load. First cracking was observed near midheight on the convex face at a load of 140 kips while the strain gauges indicated that cracking had commenced at 130 kips. With a further slight increase in load the deflections increased rapidly. On reaching the maximum load of 146.6 kips the load dropped rapidly with further increase in deflection until a load of 126 kips was reached where it was again possible to maintain an equilibrium position of the column with the testing machine. When the load was further decreased to 115 kips, spalling of the concrete just below midheight of the column was first observed. The test was stopped when the load had been further reduced to 112 kips at which point the central deflection of the column was 1.06 inches. The extent of crushing and cracking of the concrete on the column is shown in Plate XVII. The results of measurements of strains at midheight of the column are shown in Fig. 5.23. The strain reversal on the convex side of the column is again in evidence.

The bare steel column CC14, fully battened in accordance with AS CA2-1968 as shown in Fig. 5.4, was loaded until a maximum load of 53.3 kips was reached. At this point yielding of the steel at midheight on the northern face commenced. The load then was decreased with increasing deflection until a load of 37.5 kips was reached at which point local buckling of the northern channel took place between two of the battens as shown in Plate XVIII. At this point the load commenced to drop rapidly and the test was stopped. The strains measured at midheight and recorded in Fig. 5.22 indicated that the strain distribution was essentially linear over the whole crossection. This, together with the fact that local buckling took place well after the maximum load had been reached, demonstrated that the battens were sufficient to transmit the shear between the channels and to ensure that the two channels acted conjointly.

Composite column CC15 contained the same battened steel section as column CC14. On loading, first cracking was observed at 140 kips on the convex face near midheight while the strain gauges indicated that cracking commenced at 130 kips. On reaching the maximum load of 150.5 kips, the load dropped rapidly with further deflection until a load of 132 kips was reached where it was possible to maintain an equilibrium position of the column with the testing machine. When the load was further decreased to 110 kips, spalling of the concrete was first observed a little above midheight on the convex face. The test was stopped when the load had been further reduced to 101.3 kips at which point the central deflection was 1.23 inches. The extent of crushing and cracking of the concrete is shown in Plate XIX. By comparing this Plate with Plate XVII for column CC13R, the close similarity between the failure patterns can be observed.

5.5.6 Major Axis Bending - 10ft. Columns

The two 10ft. composite columns CC13R and CC15 described in the previous section 5.5.5 also form part of this investigation. An additional composite column CC16 was loaded at an eccentricity of 1.5 in. to the major principal axis as defined at (b) in Fig. 5.1. The load-deflection relationship and strain results are both shown in Fig. 5.24. For increasing load, first cracking was observed at 90 kips while the strain gauges at midheight on the convex face of the column indicated that cracking commenced at 80 kips. On reaching the maximum load of 110.2 kips, the load decreased slowly with large increases in deflection, this unloading behaviour being gentle in nature and easily controlled. When the load was decreased to 96.5 kips, first spalling of the concrete, just below midheight on the convex face, was observed. The test was stopped when the load had fallen to 65 kips and the deflection increased to 1.36 inches. The extent of concrete crushing and cracking is shown in Plate XX. From (b) in this plate, it can be seen that cracking of the convex face extended over the whole length of the column, similar to the patterns for the 7ft. columns CC1 and CC3 (Plate XIV) which were also loaded at 1.5in. eccentricity.

5.6 COMPARISON OF THEORETICAL WITH EXPERIMENTAL RESULTS

5.6.1 Introduction

As the load-deflection relationships, the load-strain relationships and the maximum load carrying capacity are the most important characteristics of the isolated pin-ended member, they are used as the criteria for determining the validity of the analytical procedures developed in Chapter 4. The relevant comparisons are presented in Figs. 5.9 to 5.25 inclusive. In these figures the experimental points are shown as full circles and the curve of best fit through these points as a full line. The theoretical relationships are shown as broken lines. A comparison between the observed and theoretical maximum loads is given in Table 5.1.

The following information was required for the analysis to determine the theoretical behaviour of the composite columns.

(i) The load-moment-curvature relationship for a composite section was determined in accordance with Section 4.2.4 (b).

(ii) The analytical procedure used for determining the deflected shape and the maximum load for the columns is that described in Section 4.3.3.

(iii) The stress-strain relationship for the concrete was the Type III curve presented in Fig. 3.23.

(iv) The compressive strength, f''_c, of the concrete in the column was taken as $0.85f'_c$ where f'_c was the compressive strength of cylinders tested at the same age as the column tests. The values of f'_c for each column are given in column (7) of Table 5.4.

(v) The values of initial elastic modulus, E_c, of the concrete for each column are listed in column (9) of Table 5.4.

(vi) The stress-strain curve for the steel was the elastic-plastic-strain hardening relationship shown in Fig.3.40.

(vii) The yield stress, f_{sy}, of the steel for each column was taken as the value obtained from stub column tests as listed in column (1) of Table 5.4.

(viii) The modulus of elasticity, E_s, for the steel was taken as 30×10^6 lb./sq.in. The variation of the values shown in column (4) of Table 5.4 were most likely the result of errors in measurement.

(ix) The rolling residual stresses in the channels were assumed to have a distribution as shown at (b) in Fig. 3.35, the value of the maximum compressive residual stress being taken as $0.25\ f_{sy}$ as observed from the stub column tests.

(x) The tensile stresses in the concrete and the compressive stresses in the steel, developed as a result of concrete shrinkage prior to loading, were obtained from Fig. 5.7 for the particular ages at testing, as listed in column (6) of Table 5.4, for each column. For columns CC1, CC11, CC13R, CC15 and CC16, where shrinkage strains were not recorded, the average curve, shown as a broken line in Fig. 5.7, was used to estimate the shrinkage stresses.

(xi) The dimensions of the steel channels used in the analysis are listed in Table 5.3 for each column.

(xii) The initial deformed shape, prior to loading, for each column is as defined earlier in Section 4.3.2 and has the form used in the well-known Perry-Robertson strut formula.

For the battened bare steel column CC14 where the channels acted conjointly, the same analysis for the composite columns was used whereby the concrete elements were removed and only the steel elements and their material properties were specified.

For the unbattened bare steel column CC13 which failed by elastic buckling, a second order elastic analysis, developed by a colleague, Assoc. Prof. H.B.Harrison (183), was used to determine the elastic buckling load. The column was treated as a simple frame consisting of two long slender steel columns connected at each end by a stiff short beam (end batten) with the load being applied 0.8 in. from the centre of these beams.

5.6.2 Minor Axis Bending

An observation of the load-deflection relationship for column CC1 in Fig. 5.9 shows that good agreement was obtained between the observed and theoretical values, especially at low loads, bearing in mind that computations were not made beyond the maximum load from which the drooping characteristic of the curve might have been obtained. The theoretical relationship converged to a maximum load 7 percent below the observed. A likely explanation is that the actual strength of the concrete in the column may have been greater than 85% of the concrete cylinder strength, this value being adopted in the analysis.

The relationship between load and the longitudinal compressive strain on the concave face of the column at midheight is shown in Fig. 5.10. Up to the point where the theoretical relationship converged to a maximum load, excellent agreement was obtained. A few selected strain distributions are also shown in Fig. 5.10. The theoretical strain distribution agree remarkably well with the observed distributions, even up to a load of 109 kip which is just below the theoretical maximum load of 109.4 kips. This indicated that the theoretical load-moment-curvature relationships were an accurate representation of the real behaviour.

In this mode of minor axis bending, the absence of battens would be expected to have no affect on the column behaviour. In fact, similar results would have been expected from an equivalent single rolled steel joist encased in concrete. As this was the initial test of the series, the good agreement between the theoretical and test behaviour was encouraging. This augured well for the following tests on columns bent about the major axis where the absence of battens might be expected to influence the behaviour. This would then be shown up when comparing the test results with the theoretical results as the analysis assumes that the steel acts integrally and compositely with the concrete no matter what the orientation of loading on the column.

5.6.3 Major Axis Bending - 7ft. Columns

The load-deflection relationships for the three columns tested in this category are shown in Fig. 5.11. A close agreement was obtained between the experimental and theoretically determined curves. Again there were some slight discrepancies near the maximum loads where the columns are tending to become unstable prior to unloading as evidenced in the drooping characteristic of the curves.

The relationships between load and the longitudinal strains at midheight on the concave and convex faces of the column are shown in Fig. 5.13. Reasonable agreement was obtained for the eccentrically loaded columns CC2 and CC3 but the results for the concentrically loaded column CC4 showed some divergence. This is also evidenced in the strain distribution at midheight as shown in the bottom left hand corner of Fig. 5.13. It will be recalled from Section 5.5.2 that this column was initially bent in double curvature and that failure occurred in the lower quarter of the column (see Fig. 5.12) where the deflection and curvature were a maximum. The deflection and particularly the curvature were small at midheight. This accounts for the experimental strain distribution being almost constant over the midheight crossection with zero curvature. However, the theoretical distribution at midheight has a maximum curvature as the analysis assumes that the initial and hence the loaded deformed shape is of a symmetrical single curvature form.

The close agreement between the experimental and theoretical behaviour, demonstrated in Figs. 5.11 and 5.13, indicates that the steel section did indeed act compositely with the concrete. It could be said that there was no discrepancy between the experimental theoretical results that could be directly attributed to the absence of battens on the steel sections.

5.6.4 Diagonal Axis Bending

A similar agreement was obtained between the observed and theoretical load-deflection relationship, as shown in Fig. 5.14, for column CC5 bent about a diagonal axis as was obtained in Fig. 5.9 for column CC1 bent about the minor axis. Again excellent agreement was obtained at low loads with some divergence for loads approaching the maximum load. It will be discussed in a following Section

5.7.3 how variations in the shape of the concrete stress-strain curve can significantly affect the shape of the load-deflection relationship.

The relationships between load and the longitudinal strains on the concave and convex corners of the column are shown in Fig. 5.15. Good agreement was obtained up to the point where the theoretical relationship converged to a maximum load.

5.6.5 Major Axis Bending-Maximum Shear Condition

The positions of maximum deflection for column CC11 were close to the upper and lower quarter points as can be seen from the deflection profiles in Fig. 5.17. The theoretical and experimental load-deflection relationships for the centre and quarter points are shown in Fig. 5.16. Although reasonable agreement was obtained, the observed deflections at the quarter points were always larger than the theoretical values. Divergence commenced at low loads where the materials could be considered to be linear elastic. It can be observed from Table 5.4 that although the compressive strength of the concrete for column CC11 was the same as that for columns CC2 and CC3, the value of modulus of elasticity, measured from cylinders and used in the analysis, was higher than the corresponding values for columns CC2 and CC3. A possible source of error could be that the actual elastic modulus for the concrete in the column was lower than the value measured from cylinders.

The observed deflections at midheight for the column, although relatively small, were in good agreement with the analytically predicted values. For symmetric double curvature loading, the deflection at midheight is principally a function of the imperfections in the column. This close agreement obtained for double curvature bending indicated that the assumed initial deformed shape for the column, of the form used in the Perry-Robertson strut formula, was a realistic estimate of the imperfections.

The strain results in Fig. 5.18 again indicate reasonable agreement between observed and theoretical values. At the bottom end where failure finally occurred, the measured strains at gauge N1 increased at a greater rate with loading than predicted whilst better agreement was obtained at the corresponding position at the top of the column. This may have been due to some local weakness at the bottom end, perhaps incomplete compaction.

The stress condition at midheight on the vertical plane between the two channels and containing the major axis is shown in the upper two diagrams in Fig. 5.19. The values were calculated from strain rosette measurements on the surface as described in Section 5.5.4. The broken line shown in the upper diagram of Fig. 5.19 is the relationship between load and the average concrete shear stress on this plane across the full width of the crossection. This was calculated using the simple elastic expression given below. In this case, the steel channels were transformed to concrete by use of the modular ratio, $m = \frac{E_S}{E_C}$. The following assumptions were made.

(i) There is no slip between the concrete.

(ii) Plane sections remain plane as the member deforms.

(iii) The material in the composite section are linear-elastic.

The average shear stress, T_{av}, on the major axis plane is given by

$$T_{av} = \frac{VA_T \bar{Y}_T}{I_{TY} b}$$

where V = Shear force at the section

I_{TY} = second moment of area of the composite section, transformed to concrete, about its major axis

A_T = the area, transformed to concrete, of that part of the section lying above or below the plane on which the shear stress is to be calculated

$\bar{y}_T$ = the distance of the centroid of A_T from the centroidal axis, the major axis in this case

b = the width of the section at the plane on which the shear stress is to be determined.

The value of shear force at the midheight section was determined from the column analysis which gave the deflections, the bending moment and hence the rate of change of bending moment (shear force) at all sections along the length of the column. With reference to Fig. 4.13, the first order value of shear force, V, at any section along the length of the column, ignoring the column deflected shape, is given by

$$V = \frac{P(e_1 - e_{n+1})}{L}$$

For column CC11, the deflections were relatively small and the actual shear force at midheight was closely approximated by this first order value. This accounts for the theoretical dashed curve in the upper diagram of Fig. 5.19 being essentially linear and in good agreement with the average of the two measured load-stress relationships with divergence occurring only for loads close to the maximum value. Although no information was obtained on the likely distribution of shear stress on the major axis plane within the concrete section, the agreement between theoretical and measured values would indicate that the shear stress may be reasonably uniform on the plane between the channels.

5.6.6 Major Axis Bending - Battened and Unbattened Members

The close agreement obtained in Fig. 5.20 for the observed deflections and the deflections calculated from the theoretical elastic buckling analysis indicated that the bare steel column CC13 with no intermediate battens did indeed fail by elastic buckling. This was confirmed by the close agreement between the experimental and theoretical strain distributions in Fig. 5.21 for the individual channels which can be seen to have acted independently rather than conjointly.

For the fully battened bare steel column CC14, very good agreement was obtained between the observed and analytically predicted load-deflection relationships in Fig. 5.20. Close agreement was also obtained for the strain distributions in Fig. 5.22, the observed values being a little less than predicted. This agreement demonstrated that the analysis could be just as readily used for a column of one material as for a composite column.

The theoretical and experimental load-deflection relationships for the two composite columns are shown in Fig. 5.20. While the agreement was good for composite column CC15 with a fully battened steel core, the agreement was even better for column CC13R with an unbattened steel core. From the two observed load-deflection curves, it would appear that the battens may have had some slight stiffening effect for column CC15. There would be some slight increase in the column stiffness at the discrete points where the battens were placed but this would be insignificant when considering the large spacing of the battens in Fig. 5.4. It might be inferred that the shear taken by the battens reduced any shear deformation of the concrete but this would be small considering the solid crossection and the small shear values. It should be remembered that the steel section in column CC13R had already been tested as the bare steel column CC13 and that small residual deformations and stresses remained in the steel section on unloading. These represented additional imperfections for the composite column CC13R and could have resulted in the slightly increased deflections above those for column CC15 at the same load.

A comparison between the theoretically predicted and the observed strains is given in Fig. 5.23. Close agreement was obtained for the strain distributions but the theoretical strains calculated for the concave face at midheight tended to be larger than those measured by strain gauges. It can be seen that the strain gauge readings on this face were lower than the corresponding values obtained from the line of best fit through all the gauge values across the section. If the latter values had been plotted on the diagram on the right hand side of Fig. 5.23, a better agreement would have been obtained with the theoretical values. The low strain gauge readings were probably the result of surface shrinkage cracks as discussed in Section 5.5.3 for column CC5.

5.6.7 Major Axis Bending - 10ft. Columns

The measured and predicted load-deflection and load-strain relationships for the concave face are both shown in Fig. 5.24. A very close agreement was obtained with only some slight divergence as the load approached a maximum, the theoretical maximum load being slightly lower than the observed value. This again could have been due to the actual strength of the concrete in the column being slightly greater than 85% of the cylinder strength as assumed for the analysis.

5.6.8 Summary of the Comparisons

It is evident from Figs. 5.9 to 5.24 that, in the main, there was good agreement between the observed test results and those predicted by the analysis given in

Chapter 4 and using the information listed in Section 5.6.1. A comparison between the observed and theoretical maximum loads is given in column (8) of Table 5.1 in which the ratio of the observed to the theoretical maximum load is listed. It can be seen that excellent agreement was achieved between the two sets of maximum load, as listed in columns (6) and (7) of Table 5.1, with ratios ranging from 0.97 to 1.07, the mean value being 1.03. In all but two cases, the theoretically predicted values were slightly less than the observed values.

The observed and theoretical results for the composite columns are summarized in Fig. 5.25 which shows the relationship between the maximum load and the eccentricity of loading. From these curves, a study can be made of the influence of the principal variables, namely eccentricity of loading, axis of bending and column length or slenderness, on the maximum load carrying capacity. The maximum load has been expressed as a ratio of the short column or squash load, calculated using equation (4.7) and listed in column (9) of Table 5.1. As expected, the maximum load capacity decreases with increasing eccentricity for a constant column length and axis of bending. As shown in the upper diagram of Fig. 5.25 for major axis bending, the reduction in strength was both orderly and significant, being more pronounced for eccentricities up to 1.0in. and then falling off more gradually for larger eccentricities.

The influence of increasing the column length on the load capacity is also demonstrated, the capacity decreasing with increasing length for a constant eccentricity. For an increase in length from 7ft. to 10ft., the theoretical curves indicate a reduction in the load capacity from 13% down to 5% for eccentricities ranging from 0in. to 2.0in.

The effect of rotating the axis of bending is evident in the lower diagram of Fig. 5.25. As would be expected, the columns bent about the major axis and therefore having a greater initial stiffness sustained a higher maximum load than the columns bent about the minor axis and hence having the least initial stiffness. As expected, the columns bent about a diagonal axis and having an intermediate stiffness, exhibited a load carrying capacity between the values for major and minor axis bending.

It is evident from the summary of results in Fig. 5.25 that there was good agreement between the theoretical relationships and the experimental results for the particular values of eccentricity, column length and axis of bending examined. The relationships shown are for columns bent in symmetrical single curvature. The experimental value of maximum load for column CC11 has also been plotted in Fig. 5.25. This indicates, as expected, that a column bent in double curvature has a higher load capacity than a similar column bent in single curvature with the same value of end eccentricity. For pin-ended members, the symmetrical single curvature mode of bending gives the lower bound to the load carrying capacity of eccentrically loaded columns.

5.7 DISCUSSION

5.7.1 Battening or Lacing of the Steel Components

Perhaps the most important question to be answered when considering the behaviour up to collapse of composite columns containing built-up sections, is whether the individual steel components act compositely and conjointly with the concrete even when the components are not connected by battens or lacing.

The 7ft. columns CC2, CC3 and CC4 bent about the major axis and column CC5 bent about a diagonal axis in symmetrical single curvature, contained no battening or lacing between the two channel sections, the concrete alone serving as the connecting medium. No reinforcing rods and ties were used in the concrete encasement. No evidence could be found from an observation of the failure patterns, as shown in Plates XIV and XV, that the channels did not act integrally with the concrete. Close agreement was obtained between the observed and theoretical results as displayed in Figs. 5.11 to 5.15. This indicated that the channels deformed compositely with the concrete as the theory assumes that plane sections remain plane with no slip between the concrete and the steel. It is appreciated that in this mode of bending, the magnitude of the shear forces is small, the maximum shear force occurring at the ends of the member which are connected to stiff end fittings. However, even when a column of similar crossection was subjected to a more severe shear condition, as was the case for column CC11with a relatively short length of 7ft and bent in double curvature with equal but opposite end eccentricities of 2in., no evidence was observed that the behaviour was other than flexural. This can be observed from the failure pattern in Plate XVI which shows the failure at the bottom end where the moment was a maximum, the applied load on the column falling off when crushing of the concrete on the compression face commenced.

At the maximum load of 158.8 kips for this latter column (CC11), the maximum calculated principal tension was 220 lb./sq.in. This is a significant value when compared with the split cylinder tensile strength of 510 lb./sq.in. At what might be considered to be a working load of one half to one third of the maximum load, the corresponding values of principal tension are only 125 to 90 lb./sq.in. which could be sustained safely by the concrete. It is realised that principal tension due to shear is not necessarily a measure of the ultimate shear capacity of a section but it can indicate whether failure at the section due to shear is likely.

It can be appreciated that the ultimate shear behaviour of built-up composite members, which may contain a variety of different steel sections, is complex. As a result of considerable research and many tests, an understanding of the ultimate shear behaviour of reinforced concrete members is now just at the point where empirical design formula are being included in new design codes such as Australian Standard AS CA2-1973 "SAA Concrete Structure Code" where reference is made to the particular work on which the code requirements are based. Further research may be necessary for similar requirements to be established for built-up composite sections.

As indicated in the upper diagram of Fig. 5.19, the average measured shear stress in the concrete at midheight was 300 lb./sq.in. This is above the value of shear stress that AS CA2-1973 would permit to be carried by the concrete alone for a reinforced concrete member with a similar steel percentage. This would indicate that secondary reinforcement consisting of longitudinal bars with transverse ties should be included in the concrete encasement to satisfy the requirements of existing design codes. It is interesting to note from the writer's tests that crushing of the concrete did not take place, except for the concentrically loaded column which failed suddenly as might be expected, until well after the maximum load had been reached and the columns were unloading with increasing deflection. It could therefore be said that the absence of secondary reinforcement in the encasement did not adversely affect the behaviour of these pin-ended members and was not significant in regard to attaining the maximum load. Of course any additional longitudinal steel in the concrete would lead to a higher value of maximum load. It is appreciated that the above comments can not yet be applied to the continuous column forming part of the building frame. For this case, particular attention would have to be paid to the reinforcement of the connection between the columns and the beams framing into them.

It has been shown in Section 5.6.6 that the addition of battens to the steel sections had little or no influence on the behaviour of the built-up columns and that the observed behaviour of the columns was predicted accurately by an analysis which assumed full steel-concrete interaction. Therefore, the indications are that the absence of battens does not have any significant effect on the behaviour of built-up composite columns, at least for sections and loading conditions similar to those used by the writer.

Another question to be answered is whether adequate bond exists between the concrete and the steel for full composite action to be maintained at all levels of load for eccentrically loaded columns. It is difficult to make measurements on the concrete-steel interface that might give some indications. For the writer's tests, the only surface preparation done to the steel was degreasing with a solvent. The close agreement between all the observed and theoretical results would serve to indicate that good bond was attained as the theory assumes that perfect bond exists between the steel and the concrete. The use of end plates on the columns made it impossible to measure any slip at the ends as might be done in a beam test. However, evidence that good bond existed can be observed from the crack patterns, especially that for column CC16 shown in Plate XX. The cracks marked with an I were intermediate cracks that were observed to form between cracks that had opened up previously. If adequate bond were not available, these initial cracks would have opened wider without the formation of intermediate cracks. The close spacing of the cracks also indicates that good bond was achieved.

5.7.2 Torsional Displacements

It has been well established, by Timoshenko(135) and others(130) that columns constrained to bend about an axis inclined to the minor principal axis can fail in an instability mode involving torsional displacements with the deflections being outside the plane of the applied end moments. The significant results from the writer's tests on composite columns bent about an axis other than the minor axis, was that the behaviour of these members was essentially flexural without any evidence of twisting. This was a consequence of the considerable increase in torsional stiffness provided by the concrete encasement to the bare steel sections. In fact, for column CC5 bent about a diagonal axis, the maximum recorded twist was only 48 seconds of arc, even for deformations occurring beyond the maximum load and into the unloading range for the column. It was significant that this column deflected in one plane without any indication of torsional displacement for all values of applied load. Of even more importance was the fact that the plane of the deformations practically coincided with the plane of the applied end moments as demonstrated by the angles β on the diagrams in Fig. 5.14, the difference between the two planes being only 0.6 degrees. This behaviour justified the assumptions for the theoretical analysis given in Chapter 4 where for torsionally stiff columns such as composite columns of normal practical proportions, the twist was assumed to be zero and the column assumed to deform in the plane of the applied end moments.

As shown originally by Loke(14), these assumptions resulted in considerable simplification in the analysis when compared to the more exact but more complex methods of analysis which include lateral-torsional displacements. Such methods may be almost impossible to implement for composite columns containing two or more steel components and materials having totally different material properties which can be non-linear and time dependent, especially the concrete as already discussed in Chapter 3. The above simplifying assumptions may not be applicable in all cases and would have to be carefully reviewed, especially for slender columns of narrow crossection which could fail in a lateral-torsional buckling mode.

5.7.3 The Influence of Material Properties

In the writer's tests, the main variables being examined were the eccentricity of loading, axis of bending, column length and the absence or presence of battens. By using the same concrete mix and constituent materials and using steel in accordance with Australian Standard AS A149-1965 it was hoped that each column had similar material properties so that the above variables could be examined without being influenced by material properties. It can be seen from Table 5.4 that the compressive strength of the concrete varied from 3430 lb./sq.in. to 4450 lb./sq.in. and the initial concrete elastic modulus varied from 2.72×10^6 lb./sq.in. to 3.38×10^6 lb./sq.in. The steel was less variable with the compressive yield stress ranging from 40,300 lb.sq.in. to 45,300 lb./sq.in. The variation in strengths above necessitated the use of a non-dimensional maximum load in Fig. 5.25 when comparing the observed and theoretical relationships between maximum load and eccentricity.

In attempting to predict the behaviour of a composite column for which the basic information such as the dimensions of the concrete and steel sections and the strength and elastic moduli of the materials are known, the variables that might be expected to influence the results of the analysis are the stress-strain characteristics of the materials, the concrete tensile strength, the rolling residual stresses in the steel and the stresses developed in the steel and the concrete from shrinkage of the concrete prior to loading.

The effect of these variables on the theoretical load-deflection relationship for a typical column (column CC2) is demonstrated in Fig. 5.26. The concrete tensile strength was taken as one tenth the cylinder compressive strength, this being 453 lb./sq.in. compared to the split cylinder strength of 430 lb./sq.in. The rolling residual stresses were assumed to have a distribution as at

(b) in Fig. 3.35, the maximum compressive residual stress having a value of 10,100 lb./sq.in., this being 25% of the compressive yield stress. The tensile stress in the concrete and the compressive stress in the steel due to shrinkage were taken as 140 lb./sq.in. and 2750 lb./sq.in. respectively as determined from Fig. 5.7. The stress-strain curve for the steel was the elastic-plastic-strain hardening relationship as shown in Fig. 3.40. The Type I and Type II stress strain curves for the concrete, shown in Fig. 3.23, are the approximations to the concrete stress-strain relationship used by Roderick(13) using two and three straight lines respectively. The Type III stress-strain curve for the concrete is the relationship adopted by the writer as a result of an investigation into the stress-strain behaviour of the actual concrete in the column as discussed in detail in Chapter 3 with reference to Appendix C. It is realised that this theoretical stress-strain relationship may not necessarily represent the actual stress-strain behaviour of the concrete in each individual column. It can be seen in Fig. 3.22 that the Type III curve is in close agreement with the stress-strain relationship obtained from the least squares analysis for column CC2 yet the agreement for column CC1 in Fig. 3.21 is nowhere as close.

Curve 1 in Fig. 5.26 is the theoretical load-deflection relationship using the Type III concrete stress-strain curve and taking into account all the variables. The effect of ignoring the rolling residual stresses in the steel is demonstrated by comparing curve 2 with curve 1. It can be seen that the rolling residual stresses had little influence on the column behaviour. This is to be expected as the steel forms only 5% of the total crossectional area and the maximum residual stress was only 25% of the steel yield stress.

The effect of neglecting shrinkage stresses in the analysis is demonstrated with curve 3. By comparing with curve 1, it can be seen that the influence of these stresses on the analysis was small which is not unexpected as the magnitude of these stresses was small. With compressive residual stress in the steel sections, steel would commence to yield and hence reduce in stiffness for a lower value of applied load than for a column without these residual stresses. Similarly, the presence of a tensile residual stress in the concrete would lead to a slight increase in stiffness, the concrete becoming inelastic at a higher load than for a column without this residual stress. These two effects tend to compensate for each other and it is not surprising that close agreement was obtained between curve 3 and curve 1.

The effect of varying the shape of the concrete stress-strain curve is most significant. The Type I relationship was used in the analysis to obtain curve 4 and the Type II relationship to obtain curve 5 in Fig. 5.26. It can be seen that the shape of the stress-strain curves is also reflected in the load-deflection relationship. Curve 4 is reasonably linear up to the maximum load and the corresponding Type I stress-strain relationship is linear-elastic up to the maximum stress. Curve 5 has a distinct reduction in stiffness with increased deflections at about half the maximum load and the corresponding Type II stress-strain relationship has a reduced modulus of one third the initial elastic concrete modulus between one half and the full maximum stress. It could therefore be appreciated that any slight discrepancy between the observed and theoretical results shown in Figs. 5.9 to 5.24 could be due to the Type III stress-strain curve not exactly representing the stress-strain characteristics of the concrete in the actual test columns.

An interesting point to note from Fig.5.26 is that for loads above half the maximum load, the load-deflection relationships, as represented by curves 1, 4 and 5 and using the Type III, Type I and Type II stress-strain relationships respectively, are quite different yet the maximum load values are quite similar, being 201.2, 211.7 and 195 kips respectively. This represents a variation from the maximum to the minimum value of only 8.2% of the average value. This indicates that the load capacity is not influenced to any great extent by quite marked variations in the stress-strain curve. This is similar to reinforced concrete beam behaviour where the use of differing compressive stress blocks (differing stress-strain relationships for the concrete), such as from rectangular to parabolic stress blocks, does not significantly alter the calculated value of the ultimate moment capacity although the curvature at which the maximum value occurs could be altered significantly.

5.7.4 Column Stability

It should be noted from the writer's tests that no spalling of the concrete was in evidence at the maximum load. Crushing of the concrete was observed later when the columns were unloading. For column CC11, where a strain gauge was directly over the position of commencement of spalling, the last strain value recorded before crushing was 0.0052 in./in. which was well in excess of the concrete strain at maximum stress, this being of the order of 0.002 in./in. The unloading or drooping characteristic of the load-deflection curve, a typical example of which is shown in Fig. 5.9 for column CC1, was obtained as a result of the testing machine having considerable stiffness such that equilibrium positions of the column could be maintained in the unloading range. If the columns had been tested in a testing machine of zero stiffness, i.e. a dead load test, the column would have failed on reaching the maximum load, the deflections increasing rapidly under this constant maximum load with crushing of the concrete and yielding of the steel. The column would be in a dynamic state as an equilibrium position for the column could not be attained. This type of instability under dead load is usually associated with slender columns but similar behaviour occurs for short columns or, in the limit, a crossection subjected to an axial load and bending moment.

In Fig. 5.27, the relationship between load, moment and curvature for the writer's composite section, bent about the major axis, is shown. This was determined using the analysis described in Section 4.2.4(b) and the information listed in Section 5.6.1 for the column tests. The crossection and material properties for column CC2 were used. Fig. 5.27 has been made dimensionless by dividing the applied load by the short column load, the applied moment by the maximum moment for zero axial (ultimate moment capacity of the section as in a beam) and multiplying the curvature by the depth of the section. Contours of constant curvature have been plotted.

Some interesting points can be observed from Fig. 5.27. It can be seen that there is an envelope to the surface which defines the combination of maximum load and moment that the section can withstand. In design codes, such as AS CA2-1973 and ACI 318-71, similar curves to this envelope, hereafter referred to as the interaction envelope, are used for calculating the load and moment capacity of both short and long reinforced concrete columns. These codes determine the interaction envelope by calculating the load and moment for a range of curvatures holding the maximum compressive strain constant at what is termed a "maximum usable" value of 0.003 in./in. It is permitted to use a simple rectangular stress block for the concrete and a linear elastic-plastic stress strain relationship for the steel when considering load and moment equilibrium for a particular strain distribution.

The points on the curvature contours in Fig. 5.27, corresponding to a maximum compressive strain of 0.003 in./in., have been indicated by a circled cross. It can be seen that the locus of these points is very close to being coincident with the interaction envelope. This indicated that the same simplifying assumptions used for reinforced concrete could be useful when determining the load-moment interaction for a composite section. For very small curvatures when the section is close to being axially loaded, the value of 0.003 in./in. is too high for estimating the point on the interaction envelope. This is demonstrated in Fig. 5.27 for the non-dimensional curvature, ρd, of 0.0004. In this case, a strain value corresponding to the maximum stress on the concrete stress-strain curve, usually of the order of 0.002 in./in. for short term loading, would give a closer estimate.

As the concrete stress-strain curve had a drooping characteristic, it can be seen in Fig. 5.27 that it is possible to have negative applied moments, this occurring for high values of strain. It should be noted that strain reversal was not taken into account, the relationship between load and moment for each value of curvature being calculated for increasing strain.

In Fig. 5.28, the moment-curvature relationships for two cases of loading are shown, the values being taken from Fig. 5.27. In the upper diagram, the relationship for a section subjected to pure moment is displayed. This represents a vertical section through the surface, defined in Fig. 5.27, containing the moment axis. It can be seen that once the maximum moment (ultimate moment capacity as a beam) is reached, the moment remains reasonably constant for increasing values of curvature then falls off slowly for large values of curvature.

In the lower diagram of Fig. 5.28, the moment-curvature relationship for a section subjected to loading at a constant eccentricity is shown. The loading case used is that for column CC11, bent in double curvature. It will be remembered that this column failed at the end where the moment was a maximum. The maximum moment was therefore directly proportional to the applied load, with no amplification of the moment due to deflection of the column.

It can be seen that for this case, once the maximum moment (and load) is reached, there is a rapid drop in the moment for further increase in curvature indicating a more brittle nature for this type of loading when compared with that in the upper diagram of Fig. 5.28. In the actual test on column CC11, it was not possible to record this steep drooping characteristic as the testing machine had insufficient stiffness.

So far, the discussion has been confined to the behaviour of a composite section. The effect of length on the theoretical column behaviour is demonstrated in Fig. 5.29. The results were obtained by determining the load-deflection behaviour using the column analysis for columns ranging in length from 0 to 20ft and eccentricities from 0 to 8 inches. The geometrical and material properties for column CC2 were used. As for Fig. 5.27, the load and moment have been made dimensionless. The full lines in Fig. 5.29 are the loci of the points of maximum load and the midheight moment corresponding to that load for each combination of column length and eccentricity. The curve shown for zero length is the interaction envelope shown as a broken line in Fig. 5.27. Only the load-deflection relationships, replotted as load-midheight moment relations, for an eccentricity of one inch have been shown, these being the broken lines for lengths of 0, 7, 10, 15 and 20ft. in Fig. 5.29.

It can be seen from Fig. 5.29 that for lengths up to 10ft., the maximum applied load is attained when the load and midheight moment reach the interaction envelope indicating that the full load and moment capacity of the section has been attained. The effect of length on the midheight moment is clearly demonstrated by comparing the 7ft. and 10ft. broken line curves for an eccentricity of 1 inch with the zero length curve (straight line). The midheight moment has been clearly amplified by the additional moments due to the $P-\Delta$ effect, the amplification increasing with increasing column length. For column lengths greater than 10ft, values of 15ft. and 20ft. being shown, it can be seen that the maximum laod is reached within the interaction envelope. This indicates that for slender columns, the maximum load is reached before the full load and moment capacity of the section can be utilised. If the maximum load is defined as the point of instability, then this behaviour could be considered as overall column instability as opposed to crossection instability for the stockier columns where the full load and moment capacity of the section is attained. It is of interest to note that the maximum column length tested was 10ft. Hence, in all the writer's tests, the maximum load was reached when the section of the column, subjected to the largest load and moment which includes the P-Δ effect, reached its load-moment capacity as defined by the interaction envelope.

The curves shown in Fig. 5.29 are of little use for design purposes as only the axial load and moments at the ends of the column would be known initially from a conventional analysis of a structure and not the value of the maximum moment in the column which includes the P-Δ effect. The horizontal portions of the full line curves in this Figure, corresponding to the maximum load for a particular column length, are a result of using an initial deformed shape to represent the column imperfections. Therefore, in Fig. 5.30, the maximum loads have been replotted against the end moments rather than the maximum moments. The results are then in the form of interaction curves similar to those often used in the design of steel or reinforced concrete columns. It should be noted that in these curves, the maximum load has been made non-dimensional by dividing by the axial load capacity, P_o, of the column, which includes length effects, and not by the short column load, P_{sh}. It can be seen in Fig. 5.30 that the simple interaction equation

$$\frac{P}{P_o} + \frac{M}{M_o} = 1$$

shown as a dashed straight line, does not represent the behaviour very well, being grossly over-conservative for very short columns yet unconservative for the very slender columns. Further considerations regarding the design of composite columns are presented later in Chapter 11.

CHAPTER 6

TESTS ON CONCRETE FILLED STEEL TUBES - SHORT TERM BEHAVIOUR

6.1 INTRODUCTION

6.1.1 General

In this Chapter, tests on six 8in. x 8in. x $\frac{3}{8}$in. and two 6in. x 6in. x ¼in. concrete filled square steel tubes are described. It will be recalled from the review of previous research in Chapter 2 that the majority of tests on concrete filled tubes have been concentric load tests, mainly on circular tubes. When the writer's tests were commenced there was little information available on the behaviour of concrete filled tubes subject to eccentric loading. However, investigations by Furlong(21)(22), Knowles and Park(23)(24) and Neogi, Sen and Chapman(25) have been reported since then and have included eccentric load tests. Both small diameter circular tubes and small size square tubes were tested, the circular tubes being in the majority. No tests have apparently been reported on square or rectangular tubes subject to biaxial loading. Only Neogi, Sen and Chapman(25) presented an analysis by which the load-deformation relationships, load-strain relationships and maximum load values for eccentrically loaded concrete filled tubes could be estimated. This analysis has already been summarised and discussed in Chapter 2.

The writer's tests were therefore designed to examine the effect of the following variables,

(i) eccentricity of loading,

(ii) slenderness and

(iii) the inclination of the loading axis (biaxial bending), as defined in Fig. 6.1,

on the behaviour of concrete filled square steel tubes subject to short term loading to collapse. These tests were also aimed at providing an accurate source of experimental data which could be used to test the validity of the column analysis presented in Chapter 4. It will be recalled from Chapter 4 that the writer extended Roderick's(13) original analysis for encased rolled steel joists bent about the minor axis to account for the cases of composite columns consisting of steel and concrete of any shape and form and bent about any required axis in either double or single curvature. Full composite action between the steel and concrete is assumed.

The geometrical properties for each test column are given in Table 6.1 which includes the details of the eccentricity of loading and the inclination of the loading axis. The effect of eccentricity of loading was examined for three different end eccentricities (column(8) of Table 6.1) which gave five different eccentricity to depth ratios, $\frac{e}{D}$ (column(12) of Table 6.1) as two different size tubes were tested. So that the effect of varying the inclination of the loading axis could be examined independently of the other variables, three columns, SHC-1, SHC-3 and SHC-4, were tested with a constant length to depth, $\frac{L}{D}$, and eccentricity to depth, $\frac{e}{D}$, ratio. The three values of loading axis inclination used were 0, 30 and 45 degrees as indicated in Fig. 6.1. As the maximum length that could be accomodated in the testing machine was 10ft., two 6in. square steel tubes of that length were used so that column slenderness, $\frac{L}{D}$, could be examined over a reasonable range which is shown in column (11) of Table 6.1. The ¼in. wall thickness for the 6in. square tubes was chosen as this was the nearest size available that would give a depth to wall thickness ratio, $\frac{D}{TW}$, approximately equal to that for the 8in. tubes. This ensured that the two different sized tubes were of the same scale and could therefore be directly compared. Values of $\frac{D}{TW}$ for each tube are given in column (10) of Table 6.1. All the columns were tested with the end eccentricity being equal at both ends such that the columns were bent in single curvature.

The procedures used for manufacturing the columns, determining the material properties and testing the columns were similar to those for the built-up composite columns as already described in Chapter 5. Therefore, only a brief description of these procedures is given below, concentrating mainly on those aspects which differ from the tests on the built-up composite columns.

6.1.2 Manufacture of the Columns

The column length, together with a short length of tube for steel control specimens, were cut from the supplied tube length, usually 24ft. in length. Both ends of the column length were machined and ground square to the correct length as listed in column (7) of Table 6.1. Levels were taken along the length of the column for each face to determine the initial deformed shape. The maximum deformation at the centre of the length, measured from a plane containing the two ends of each face, is listed in column (6) of Table 6.1 for each column. It can be seen that the tubes were quite straight as supplied. Electrical resistance strain gauges were then attached at midheight in the positions indicated in Fig. 6.1. An end plate was attached to the bottom of each tube by allen screws. The column was then set up vertically for casting in the concrete laboratory as shown in Plate XXI.

The hollow tube was filled with concrete in one continuous lift, the concrete being compacted progressively with an immersion type vibrator. After about one hour, any excess water and slurry that had risen to the top was removed, some fresh concrete added to within ½ in. of the top and this top 6in. or so of the concrete was revibrated. A steel plate was then placed on top of the tube to prevent drying of the top concrete. After two weeks of curing in this manner, the ½ in. space left at the top was filled with mortar to which was added a small amount of aluminium powder to prevent shrinkage. While this mortar was still fluid, the top end plate was then screwed into position by four allen screws, the excess mortar squeezing out between the end plate and the end face of the steel tube. This was done to ensure that the load, when applied, would be transmitted to both the concrete and the steel at the ends.

Just prior to placing the column in the testing machine, loading rockers at the required eccentricity and loading axis inclination, identical to those shown in Plate IV for the built-up columns, were attached to the two end plates by allen screws.

6.1.3 Material Properties

(a) Concrete

The same cement, sand, aggregate and concrete mix was used as for the built-up composite columns. Details for these have been given in Section 5.3 with the aggregate grading curves being shown in Fig. 5.6. The same cylinder control tests were taken. The concrete properties obtained from these tests are shown in Table 6.2 for each tube column.

(b) Steel

Only Hounsfield control tests were used. It can be seen from Table 5.4 for the built-up composite columns that the values of yield stress obtained from Hounsfield tests were in reasonable agreement with values obtained from stub column and coupon tension tests. Sixteen Hounsfield specimens were cut from the tube in the positions indicated in Fig. 6.2. A typical variation of the yield stress with the position of the specimen is shown in the same Figure. It can be seen that yield stress has quite marked variations with the maximum values being obtained in the corners. This is probably the result of strain hardening at these points when the square tube was formed. The maximum variation from the average yield stress was 11 percent. The average yield stress values for each tube are listed in column (2) of Table 6.2.

Values of elastic modulus for steel, measured from tests on specimens cut from the steel channels used for the built-up composite columns, have already been shown in Table 5.4. The slight variations shown in these values are more likely a result of errors in measurement as any variation in the material properties. The average value of 30 x 10^6 lb./sq.in. was therefore adopted for the steel in the tubes.

6.1.4 Experimental Apparatus and Testing Procedures

These are identical to those used for the built-up composite columns as already described in Section 5.4. Typical test arrangements are shown in Plate XXII for the two cases of the loading axis being parallel to and inclined to a side face of the tube.

6.2 DESCRIPTION OF TESTS

6.2.1 Loading Axis Inclined at 0° to a Face (β = 0°)

Four columns SHC-1, SHC-2, SHC-7 and SHC-8 with dimensions as in Table 6.1 were tested with β = 0° as defined at (a) in Fig. 6.1. Steel tubes of 8in. x 8in. x $\frac{3}{8}$in. section were used for columns SHC-1 and SHC-2 and tubes of 6in. x 6in. x $\frac{1}{4}$in. section were used for columns SHC-7 and SHC-8. The resulting load-central deflection relationships are shown in Figs. 6.3 and 6.4 and strain results in the form of strain distributions at midheight of the columns are presented in Figs. 6.5 and 6.6.

The 7ft. column SHC-1 was loaded at an eccentricity of 1.5in. ($\frac{e}{D}$ = 0.187). As the load increased, the deflections increased at a fairly constant rate. Yielding of the steel was first observed on the northern concave face at midheight at a load of 390 kips. For loads above this value, large increases in deflection were produced by only small increases in load. On reaching the maximum load of 439.7 kips, the load then commenced to reduce slowly for very large increases in deflection. Using the technique described in Section 5.4, the testing machine was adjusted so that values of load and deflection could be obtained for equilibrium positions of the column in the unloading range. The test was stopped when the central deflection had reached 2.7 in., the corresponding value for the load being 365 kips. The extent of yielding of the steel tube in the central region of the column, for both the northern compression face and the southern tensile face, are shown in Plate XXIII. It is interesting to note in Fig. 6.5 that once the steel had commenced to yield on the compression face at midheight, the strains recorded by a gauge on the centre of this face increased rapidly with increasing load. The strain value for this face is shown to be well above the linear strain distribution drawn through the strain values recorded at points which had not yielded. This strain behaviour is discussed later in Section 6.4.1.

The 10ft. column SHC-2 was concentrically loaded ($\frac{e}{D} = 0$) and showed little growth in deflection as the load increased. This column commenced and continued to bend in single curvature. For increasing loads above 600 kips, the rate of deflection began to increase as indicated in Fig. 6.3. However, when the load reached its maximum value of 645 kips, yielding at both ends occurred as shown in Plate XXIV where flaking of the whitewash applied to the steel tube can be observed. This was a result of the end plates being of insufficient thickness such that they deformed under the action of the line load applied by the rockers. This caused a concentration of stress directly under the line of the rockers rather than a uniform stress over the whole crossection at the end. However, the load of 645 kips was very close to the maximum for column instability. This is demonstrated in Fig. 6.3 where it can be seen that the slope of the load-deflection curve was approaching zero at a load of 645 kips.

The 10ft. column SHC-7 was loaded at an eccentricity of 1.5in. ($\frac{e}{D}$ = 0.25). The deflections increased at a fairly constant rate with increasing load. Yielding of the steel at midheight on the concave face, as indicated by flaking of the whitewash, was observed at a load of 135 kips. For further loading above this value, the deflections increased rapidly as indicated by the load-deflection curve in Fig. 6.4. On reaching the maximum load of 152.8 kips, the load could be reduced slowly for large increases in the central deflection. The test was stopped when the load had been reduced to 93.5 kips at which point the central deflection was 3.6 inches. The extent of yielding at this stage is shown in Plate XXV, the yielding extending over three quarters of the length of the column. The strain distributions in Fig. 6.6 again demonstrated that once yielding had commenced on the concave face at midheight, the strains recorded by a gauge on the centre of this face increased rapidly with further loading, the strain values being well above the linear strain distribution drawn through strains measured at other points on the crossection which had not yielded.

The 10ft. column SHC-8 was loaded at an eccentricity of 2.5 in. ($\frac{e}{D}$ = 0.417). As shown in Fig. 6.4, the behaviour was similar to that for column SHC-7 except that the deflections were larger and the maximum load (115.3 kips) was lower as a result of the larger eccentricity used. Similar strain distributions were also obtained as demonstrated in Fig. 6.6, column SHC-8 having a larger curvature at midheight than column SHC-7 for the same value of applied load. The extent of yielding is shown in Plate XXVI where it can be seen that compressive yielding extended over the full length of the column on the concave face yet tensile yielding extended only over the central half of the column on the convex face. The large deflections sustained by this column are also in evidence in this Plate.

6.2.2 Loading Axis Inclined at 30° to a Face (β = 30°)

Two 8in. x 8in. x $\frac{3}{8}$in. concrete filled steel tubes were tested with β = 30° as defined at (b) in Fig. 6.1. Both were loaded at an eccentricity of 1.5in. ($\frac{e}{D}$ = 0.187), column SHC-3 being 7ft. in length and column SHC-5 being 10ft. in length to enable the effect of varying the slenderness to be examined for constant eccentricity and loading axis inclination. The load-deflection relationships are shown in Fig. 6.7. These relationships are similar in nature, the longer column SHC-5 having larger deflections and a lower maximum load as expected. As for the tubes tested with β = 0°, the deflections increased rapidly once yielding of the steel commenced, in this case on the concave corner at midheight. Once the maximum load had been reached, 490 kips for column SHC-3 and 458 kips for column SHC-5, the columns were able to continue to carry high loads for very large increases in the central deflection.

The deflections in the two principal directions, OX and OY, have been plotted against each other in Fig. 6.8. The deflection values for column SHC-3 show the relationship obtained up to the maximum load and the deflection values for column SHC-5 show the relationship obtained into the unloading range. In both cases, a linear relationship existed between the deflections in the two principal directions.

Strain distributions at midheight are shown in Fig. 6.9. Up to the load to cause first yielding of the steel, the strain distributions are linear. Once yielding commenced on the concave corner, the strain value recorded at this corner remained static or increased only slightly for further increases in load. This recorded value remained well below the linear strain distributions drawn through the strain values at other points in the crossection which had not yielded. This strain behaviour is in direct contrast to that obtained for the tubes with 0° loading axis inclination as shown in Figs. 6.5 and 6.6 and is discussed later in Section 6.4.1.

The extent of yielding and the residual deformation remaining in the columns after unloading are shown in Plates XXVII and XXVIII, the northern view showing the concave compression corner, the southern view showing the convex tensile corner and the eastern and western views showing the deformed shape to advantage.

6.2.3 Loading Axis Inclined at 45° to a Face (β = 45°)

Two 8in. x 8in. x $\frac{3}{8}$in. concrete filled tubes were tested with $\beta = 45°$ as defined at (c) in Fig. 6.1. Column SHC-4 was 7ft. in length and loaded at an eccentricity of 1.5in. ($\frac{e}{D} = 0.187$). Column SHC-6 was 10ft. in length and loaded at an eccentricity of 2.5 in. ($\frac{e}{D} = 0.313$). The load-central deflection relationships are shown in Fig. 6.10. The relationships are similar to those obtained for the tubes tested with $\beta = 30°$. The deflections were observed to increase rapidly once yielding of the concave edge at midheight had commenced. As expected, the longer column SHC-6, which was loaded at the larger eccentricity of 2.5in., had larger deflections and a lower maximum load than column SHC-4. Once this load had been reached, 486 kips for column SHC-4 and 365 kips for column SHC-6, the unloading characteristic of the columns was such that high values of load could be sustained for comparatively large increases in the value of the central deflection.

The relationship between the deflections in the two principal directions, OX and OY, are shown in Fig. 6.11. Deflection values up to the maximum load are shown for column SHC-4 and deflection values extending into the unloading range are shown for column SHC-6. A linear relationship between the deflections in the two principal directions is shown tc exist for both columns.

The strain distributions at mid-height are shown in Fig. 6.12. As for the columns tested with $\beta = 30°$ (Fig. 6.9), the recorded strain on the concave corner remained static or increased only slightly once yielding had commenced on this corner. The strain value remained well below the linear strain distributions drawn through the strain values at other points of the crossection which had not yielded.

The extent of yielding and the deformed shapes of the columns after unloading are shown in Plates XXVII and XXVIII.

6.3 COMPARISON OF THEORETICAL WITH EXPERIMENTAL RESULTS

6.3.1 Introduction

Comparisons of the observed and theoretical load-central deflection relationships, load-strain relationships and the load capacities are shown in Figs. 6.3 to 6.12 inclusive. It is considered that these characteristics are the most important for the isolated pin-ended member and are therefore suitable for determining the validity of the analytical procedures presented previously in Chapter 4. The experimental points are shown as full circles with the curve of best fit through these points being shown as a full line. The theoretical results computed from the analysis are shown as the dashed lines. The observed and theoretical maximum loads for each column are listed in columns (8) and (10) respectively of Table 6.3. These are compared in column (11) where the ratios of the observed to the corresponding theoretical maximum loads are listed.

The following points are made with regard to the analytical procedures used to determine the theoretical behaviour of the columns.

(i) The load-moment-curvature relationship for the crossection was determined in accordance with the procedure presented in Section 4.2.4(a). It will be recalled that residual stresses are not accounted for in this procedure. Assuming that the residual stresses in the steel tube are zero, the yield strain for every fibre of the steel tube would be given by the yield stress divided by the elastic modulus. Using the measured values of yield stress given in column (2) of Table 6.2 and an elastic modulus of 30×10^6 lb./sq.in., this assumed yield strain would be 1230×10^{-6} in./in. for the 6in. x 6in. x ¼in. steel tubes and range from 1400×10^{-6} to 1540×10^{-6}in./in. for the 8in. x 8in. x $\frac{3}{8}$in. steel tubes. It can be seen from the strain results for the columns, shown in Figs. 6.5, 6.6, 6.9 and 6.12, that yielding on the concave face for the columns tested with $\beta = 0°$ and on the concave edge for the tubes tested with $\beta = 30°$ or $45°$ was observed to commence at strain values of the order of those calculated above. It could therefore be inferred that the magnitude of the residual stresses was small. As the tubes were completely sealed during the period from casting to testing, it has been assumed that there was no shrinkage of the concrete and therefore no resultant shrinkage stresses were produced in the steel and concrete.

(ii) The Type III stress-strain relationship shown in Fig. 3.23 was used for the concrete.

(iii) The compressive strength, f''_c, of the concrete in the columns was taken as 85% of the cylinder compressive strengths listed in column (5) of Table 6.2.

(iv) The elastic modulus, E_c, for the concrete in the columns was taken as the values obtained from cylinder tests and listed in column (7) of Table 6.2.

(v) The Type III stress-strain relationship for the steel, shown in Fig. 3.39, was used to simulate, in an approximate manner, the effect of small residual stresses in the tubes. It has already been discussed in (i) above that the magnitude of these stresses was indeed small.

(vi) The yield stress, f_{sy}, for the steel in the tubes was taken as the value obtained from tests and listed in column (2) of Table 6.2.

(vii) A value of 30×10^6lb./sq.in. was used for the elastic modulus of the steel.

(viii) The geometrical properties used for each column were those listed in Table 6.1.

(ix) The initial deformed shape for the columns had the same form as used in the Perry-Robertson strut formula for steel columns, details of which have already been given in Section 4.3.2.

(x) The analytical procedures described in Section 4.3.3 were used to determine the deflected shape and maximum loads for the columns.

6.3.2 Load-Deflection Relationships

The relationships for the eight columns tested are shown in Figs. 6.3, 6.4, 6.7 and 6.10. In all cases, irrespective of the loading axis inclination, excellent agreement was obtained between the observed and theoretical load-central deflection relationships, bearing in mind that for the latter, no computations have been made beyond the maximum load from which the drooping characteristic might have been obtained. For loads up to a value where yielding was first observed, the observed deflections were predicted exactly by the analysis. This demonstrated that the geometrical and material properties, used to calculate the load-moment-curvature relationships, accurately represented the actual properties of the columns.

For loads above the value at which yielding was first observed, some of the observed load-deflection relationships deviated slightly from the theoretical relationships. These slight variations may well have been due to

(i) errors in determining the value of the yield stress of the steel in the tubes from the control specimens,

(ii) variation of the yield stress of the steel across the tube sections as demonstrated in Fig. 6.2, the mean value only being used in the analysis,

(iii) the actual strength of the concrete in the columns differing from the value of 85% of the cylinder strength assumed in the analysis and

(iv) the residual stresses having a distribution which was not adequately simulated by the average stress-strain curve for the steel as used in the analysis.

6.3.3 Strain Distributions

The comparisons between the measured and theoretical strain distributions at midheight, for a range of values up to the maximum value, are shown in Figs. 6.5, 6.6, 6.9 and 6.12. Excellent agreement was obtained, especially for loads up to the value where yielding was first observed. The close agreement between the experimental and theoretical curvatures is reflected in the close agreement between the experimental and theoretical load-deflection relationships as the theoretical deflections were calculated by integration of the theoretical curvatures along the length of the column.

As already mentioned earlier in Section 6.2, strains recorded at yielded portions of the tube did not retain a linear relationship with strains recorded at unyielded points on the crossection. However, the theoretical linear strain distributions can be seen, in Figs. 6.5, 6.6, 6.9 and 6.12, to remain in close agreement with the linear strain distributions drawn through the strain values measured at the unyielded points on the crossection and ignoring the values measured at the yielded points which are discussed later in Section 6.4.1.

6.3.4 Maximum Loads

The experimental and theoretical values are given in columns (8) and (10) respectively of Table 6.3. These two sets of values are compared in column (11) in which the ratios of the observed to the corresponding theoretical maximum loads are listed for each column. It can be seen that close agreement was obtained between the two sets of values of maximum load. The load ratios ranged from 0.94 to 1.02, the mean value being 0.99. Apart from column SHC-8 which was the most slender column and was loaded at the largest eccentricity to depth ratio of 0.417, the theoretical maximum loads are within 4% of the observed values.

6.3.5 Summary of Comparisons

It is evident from the preceding sections that the theoretical results, computed from the analysis presented in Chapter 4, were in close agreement with the experimental results obtained from tests. These results are summarised in Fig. 6.13 in which the variation of maximum load with eccentricity of loading, inclination of the loading axis and slenderness are shown. The maximum loads have been expressed as a ratio of their corresponding short column load. The short column loads were calculated from equation (4.7) and are listed in column (6) of Table 6.3. The use of a non-dimensional maximum load was so that the influence of the three main variables, eccentricity, loading axis inclination and slenderness, on the maximum load could be examined without the results being influenced by undesired variations in the material properties which are given in Table 6.2.

The theoretical results shown in Fig. 6.13 were computed for an 8in. square tube with a wall thickness of 0.363 in. such that the depth to wall thickness ratio was 22, this being the median value between the ratios for the 8in. x 8in. x $\frac{3}{8}$in. and 6in. x 6in. x ¼in. tubes as listed in column (12) of Table 6.1. This was done so that the observed results for both the 8in. and the 6in. square tubes could be compared simultaneously with the theoretical results. The material properties used in the analysis were the mean values of those listed in Table 6.2 for all the columns. This was done so that the theoretical relationships would be continuous and smooth and would enable the influence of each of the main variables to be examined without the results being affected by any fluctuations or discontinuities arising from variations in material properties.

From the upper diagram in Fig. 6.13, it can be seen that the maximum load is not greatly influenced by variations in the loading axis inclination, β, for the particular eccentricity to depth ratio used ($\frac{e}{D}$ = 0.187). For a slenderness, $\frac{L}{D}$, of 10.5, the theoretical maximum load was increased by only 5% by varying β from 0° to 45°. For the more slender columns, the effect can be seen to be even less, the maximum load for a slenderness, $\frac{L}{D}$, of 20 being increased by only 2% by varying β from 0° to 45°. The experimental results agree closely with the theoretical results although it would appear from the tests that the trend is for the maximum load to decrease with increase in β whereas the theoretical relationships show a reverse trend. However, Fig. 6.13 has been drawn to an enlarged load scale and in fact there is only 3% variation between the highest and the lowest of the three experimental load values. This is hardly enough variation to establish any significant trend in the results. The three experimental load values are also within 5% of the corresponding theoretical values. For the results shown in Fig. 6.13, it could therefore be assumed that the loading axis inclination has little, if not negligible effect on the maximum load capacity of square concrete filled tubes.

With this assumption in mind, the observed maximum loads for all eight columns tested were compared simultaneously in the lower diagram of Fig. 6.13. The theoretical relationships between maximum load, eccentricity and slenderness were computed for β = 0°. It can be seen that close agreement was obtained between the observed and the theoretical maximum loads for all the columns, irrespective of the loading axis inclination. This again would indicate that the loading axis inclination has an insignificant effect on the maximum load capacity of square concrete filled tubes.

As for the built-up composite columns, the maximum load decreased with increasing eccentricity for a constant slenderness. It can be seen in Fig. 6.13 that this reduction in strength was continuous and quite pronounced for eccentricity to depth ratios, $\frac{e}{D}$, up to 0.25 but became less significant for higher values of $\frac{e}{D}$. Similarly, the maximum load decreased with increasing slenderness for a constant eccentricity of loading.

6.4 DISCUSSION

6.4.1 Influence of the Concrete Core on Tube Behaviour

This can best be examined by comparing the theoretical load-moment interaction curves for a bare steel tube and a concrete filled tube as shown in Fig. 6.14 for tubes of the dimensions and material properties as tested by the writer. These curves are the load-moment envelopes to the surface which defines the relationship between load, moment and curvature, values of which have been plotted on a set of orthogonal axes. Such a surface, with the curvatures being represented as a series of contours, has already been shown in Fig. 5.27 for the writer's built-up composite column sections. Similar surfaces were computed for the tubular columns using the analysis described in Section 4.2.4(a). It will be recalled from Chapter 4 that the values of load, moment and curvature calculated from this analysis are also used in the determination of the theoretical deflected shape of the column. As already discussed in Section 5.7.4, the envelope to the surface defines the combination of maximum load and moment that the section (column of zero length) can sustain.

From the bare steel and composite envelopes in Fig. 6.14, it can be seen that the concrete core provides 30% of the axial load capacity yet only 7½% of the pure moment capacity of the composite tube section. Under this pure moment condition, such as in a beam, the addition of concrete therefore adds little to the moment capacity of the hollow steel tube, providing that local buckling of the tube walls does not control the bare steel moment capacity. The same conclusion was reached by Furlong(21)(22) who recommended, from a design point of view, that the pure moment capacity of concrete filled tubes of practical proportions could be approximated by the moment capacity of the bare steel tube. However, for increasing load, the concrete provides an increasing proportion of the moment capacity of the composite section which indicates that the contribution of the concrete is enhanced by axial load.

A comparison between the load-deflection relationships of a concrete filled tube and a hollow tube is given in Fig. 6.3 for an 8in. square tube loaded at 1.5in. eccentricity (Column SHC-1). The bare steel column curve was computed using the same analysis for the composite tube but with the concrete removed. For low load values, the deflections of the hollow tube were only 25% greater than those for the concrete filled tube. This is due to the steel tube providing about 75% of the flexural stiffness of the composite section, assuming the concrete to be uncracked and the materials within their elastic ranges. The high proportion of the stiffness provided by the steel is a result of it being placed at the extremities of the crossection where its higher relative stiffness (E_s:E_c) can be used to advantage under the action of applied moment. The addition of the concrete increased the maximum load from 325 kips to 440 kips, the concrete therefore contributing 26% of the load capacity of this eccentrically loaded concrete filled tube which compares with the value of 30% for the axially loaded short tube as obtained from Fig. 6.14.

Whether adequate bond exists between the concrete and the tube walls has always been a problem which has concerned investigators(21)(22)(23)(24)(25) examining the behaviour of concrete filled tubes and the present writer is no exception. The analysis used by the writer to predict the behaviour of these members, is based on the assumption that no slip occurred between the concrete and the steel with perfect bond existing between the two materials such that plane sections remained plane after bending. The close agreement between the experimental and theoretical results, as demonstrated in Figs. 6.3 to 6.13 inclusive, would suggest that this assumption was correct. It will be recalled from Figs. 6.5, 6.6, 6.9 and 6.12 that the strains measured on the concave faces or corners, after yielding had occurred at these regions, did not retain their linear relationship with strains measured at unyielded positions on the tube. The strains measured at yielded corners for tubes with $\beta = 30°$ or $45°$ remained below the linear relationships for strains at unyielded points yet strains measured in the centre of a yielded face for tubes with $\beta = 0°$ increased rapidly above the unyielded linear relationship as the load increased. This phenomenon may have been due to a breakdown in the steel-concrete bond at the yielded points on the tube resulting in small local buckling deformations at the centre of the unsupported width of the yielded face with subsequent increase in the measured strain. At the supported corners, the lack of bond could have resulted in no further strain being transmitted to the corners. It could be suggested that the strain results were in error, the readings being affected by yielding of the steel. However, strain values of the same magnitude were successfully measured on the built-up composite columns. The consistency of the two different trends for tubes tested with $\beta = 0°$ and tubes with $\beta = 30°$ or $45°$ would indicate that the strain measurements were in order. No evidence of any end slip was observed when the end plates were removed after unloading the columns although no instrumentation was used to accurately record whether there was or was not any slip. It could fairly be said the question of bond is one which has not been satisfactorily answered and should be the subject of further research. If there was any breakdown in bond, the close agreement between the experimental and theoretical results would suggest that it did not significantly lower the load carrying capacity of the tubes below that calculated assuming perfect bond, at least for the range of columns tested by the writer.

From the load-deflection relationships given in Figs. 6.3, 6.4, 6.7 and 6.10, it can be observed that the eccentrically loaded tubes were subjected to quite large deformations beyond the maximum load and into the unloading range. The tubes were capable of withstanding quite a high proportion of their maximum loads while in this grossly deformed state. The considerable amount of residual curvature remaining in the test columns on unloading is shown in Plates XXV, XXVI and XXVIII. This above behaviour demonstrated the degree of "tenacity" or "toughness" of the concrete filled tube. Tests by Gardner and Jacobson(19) on hollow steel tubes have shown that local buckling of the tube walls, in the elastic range for thin-walled tubes and in the plastic range for thick-walled tubes, results in failure of the tubes. No such local buckling, at least to the eye, was observed in the writer's tests as can be seen from the faces of the tubes in Plates XXV to XXVIII. For columns SHC-7 and SHC-8, yielding extended over the full length of the column on the compression face yet no local buckling on this face was detected visually. Neogi, Sen and Chapman's(25) test results confirm this finding where large values of curvature and strain, of the order of 0.01 $in.^{-1}$ and 30000×10^{-6} in./in. respectively, were obtained without any apparent local failures of the tube walls. The tenacity or toughness of the concrete filled tubes could therefore be attributed to the concrete which stabilizes the walls of the tube thus preventing local buckling failures.

It has already been discussed in the literature review of Chapter 2 that the strength of concrete in an axially loaded circular tube can be considerably increased by the confinement to the concrete provided by the steel tube which places the concrete in a state of triaxial compression. Knowles and Park(23) found that there was little evidence of any increase in concrete strength due to confinement for the square tubes, the only confinement possible being localized in the corners. From tests on eccentrically loaded tubes, Neogi, Sen and Chapman(25) demonstrated that for eccentricity to depth ratios $(\frac{e}{D})$ above 0.125, there was no significant increase in the concrete strength, even for circular tubes. The writer's results confirm the findings above. The close agreement between the theoretical and experiment maximum loads indicated that there was no enhancement of the concrete strength due to confinement as the concrete strength and the concrete stress-strain relationship used in the analysis were obtained from tests on unconfined concrete subjected to longitudinal stress only.

6.4.2 Torsional Displacements

For the two principal axes, OX and OY, which are taken as parallel to the tube sides as in Fig. 6.7, the corresponding transformed second moments of area, I_{TX} and I_{TY}, for square concrete filled tubes are equal in value, assuming the concrete to be uncracked. From equation (4.23), the transformed second moment of area, I_{Ty}, about an axis Oy inclined at β to the principal axis OY therefore has the same value,

$$\text{i.e.} \quad I_{Ty} = I_{TY} = I_{TX}$$

For these conditions, it can be seen from equation (4.26) that the plane of curvature and the plane of the applied end moments are coincidental for elastic materials. It is therefore not surprising that for low load values, where the materials might be considered to be elastic, the measured deflections in the two principal directions for the biaxially bent tubes had a linear relationship with the resultant deflection being in the plane of the applied end moments. This behaviour is shown in Figs. 6.8 and 6.11 for not only low load values but also into the unloading range of the columns.

For columns SHC-4 and SHC-6 tested with $\beta = 45°$, the section is doubly symmetric about the Ox and Oy axes and a moment in the Ox plane will only produce curvature in the same plane, irrespective of yielding of the steel, cracking of the concrete or the non-linear nature of the concrete stress-strain relationship. The same cannot be said for columns SHC-3 and SHC-5 tested with $\beta = 30°$, the section not being symmetric about the Ox and Oy axes. Therefore, any steel yielding, concrete cracking or stresses in the concrete not being proportional to strain would cause the plane of curvature to rotate out of the plane of bending. However, the considerable torsional rigidity of the concrete filled tubes was such that this twisting was prevented as demonstrated in Fig. 6.8 where even for loads beyond the maximum load and into the unloading range, the tubes continued to deflect in the plane of the applied end moments with no evidence of twisting. This justified the simplifying assumptions used in the analysis presented in Chapter 4 where it was assumed that the columns deflected in the plane of the applied end moment and that twisting could be neglected. Because of the high torsional rigidity of tubes which is further enhanced when filled with concrete, it is likely that these assumptions are applicable to concrete filled tubes of the normal size and slenderness that would be used in practice.

6.4.3 Stability

Much of the discussion already presented in Section 5.7.4 for the built-up composite columns also applies for the concrete filled tubes. The theoretical load-moment interaction curves for the tubes, as shown in Fig.6.15, were obtained by computing the load-deflection relationships for a series of columns with eccentricities ranging from 0 to 8inches and lengths of 7,10,15 and 20ft. The material properties were taken as the average of those in Table 6.2. The interaction curves for this range of lengths did not differ greatly from the interaction envelope (L = 0ft.) which defines the maximum load-moment capacity of the section. Even for the slender 20ft. columns, the maximum load is reached when the full load-moment capacity of the section has almost been attained. The same could not be said for the built-up composite columns, as demonstrated in Fig. 5.29 where for a slender 20ft. column of approximately the same overall crossection (8in. x 7in.) as the tubes, the maximum load was reached well within the interaction envelope. This difference in behaviour is a result of the steel providing the majority of the stiffness for the concrete filled tubes whereas the concrete provides the majority of the stiffness for the built-up composite columns. With increasing load, the stiffness of the tubes remains fairly constant until yielding of the steel occurs at which point the stiffness of the section decreases rapidly as yielding progresses. This is demonstrated in column (5) of Table 6.3 where the load at first yield is expressed as a ratio of the maximum load. On the average, yielding commenced at 87% of the maximum load. From a design point of view, the load at first yield could be used as a rough estimate of the maximum load for concrete filled tubes providing, of course, that this load was less than the elastic buckling load. For the built-up composite columns, the concrete provides the bulk of the stiffness and the behaviour is therefore governed more by the non-linear nature of the concrete stress-strain relationship rather than the steel properties.

The interaction curves of Fig. 6.15 have been replotted in Fig. 6.16 where the load capacity has been expressed as a ratio of the axial load capacity and the end moment has been used rather than the maximum moment at mid-height. The form of Fig. 6.16 is more suitable for design purposes as it is the end moments that would be computed from a conventional frame analysis rather than the maximum value in the column which includes the additional moment produced by the column deflection. The curves shown in Fig. 6.15 are similar in nature to curves for bare steel members, some of which have been produced in a text by Galambos(138). That the tubes are more similar in behaviour to steel rather than concrete members is there is little evidence of any "nose" to the interaction diagram which is typical for steel sections where the moment capacity is not enhanced by the application of axial load. It can be seen from the corresponding Fig. 5.29 for the built-up composite sections that there is a distinct "nose". This is typical of reinforced concrete sections where the moment capacity can be considerably increased by the application of axial load which reduces the cracking of the concrete in the section. Further design aspects with regard to concrete filled tubes are presented in Chapter 11.

An interesting comparison between the interaction envelopes for tubes with $\beta = 0°$ and $\beta = 30°$ is given in Fig. 6.14. The pure moment capacity, M_o, used is the value for the 0° axis tubes. Although the curves are similar, it can be seen that for load ratios, $\frac{P}{P_{sh}}$, up to 0.3, the 0° axis tubes had more moment capacity. For $\frac{P}{P_{sh}}$ values above 0.3, it was the 30° axis tubes that had more moment capacity. This latter behaviour is the reason for the 30° axis tubes having a slightly greater theoretical load capacity than the 0° axis tubes when tested at a constant eccentricity ($\frac{e}{D} = 0.187$) as demonstrated in the upper diagram of Fig. 6.13. The interaction envelopes for the 30° and 45° axis tubes are almost coincidental. This is also apparent in the upper diagram of Fig. 6.13 where the theoretical maximum loads for both these tubes were almost identical.

CHAPTER 7

MATERIAL PROPERTIES UNDER LONG TERM LOADING

7.1 INTRODUCTION

The time dependent nature of concrete was recognized in the early 1900's when tests by Hatt(139) in 1907 demonstrated that deflections of reinforced concrete beams increased with time under sustained load. Further early observations were reviewed by Davis and Davis(140) in 1931. Since then, the volume of published material on creep and time-dependent deformations has been growing at an increasing rate. According to Neville(141), there are over 1300 papers and reports dealing with this subject as evidenced by the annotated bibliography by the American Concrete Institute(142) containing 487 items and a non-annotated list by Lorman(143) containing 792 items. Many of these have been reviewed by Neville(141) in his text and need not be repeated here.

In this chapter, only the concrete will be considered. It is assumed that any creep of steel is negligible when compared to the concrete, especially at the temperatures of about 20°C that existed during the writer's tests. Composite columns which contain concrete will exhibit time-dependent deformations which will be demonstrated in the experimental Chapters 9 and 10 where the results of variations of strain and deflection with time for eccentrically and concentrically loaded composite columns are presented. Deflections of the order of 6 times those obtained under short term loading can be achieved. It would be expected that this behaviour would affect the maximum load carrying capacity of such members. To be able to assess this behaviour, it is necessary to have an understanding of the time-dependent characteristics of the concrete.

Both shrinkage and creep are associated with the transfer of moisture out of or into the cement gel. The physical mechanism by which this occurs is still not agreed as indicated by the final RILEM(144) report on the physical and chemical causes of creep and shrinkage of concrete which stated that "the models for hardened cement paste and the suggestions for explaining the creep process still diverge to a considerable extent". In spite of the lack of agreement on the mechanisms of creep and shrinkage, extensive tests over a long period have established a considerable bank of experimental data on which creep and shrinkage and the factors which influence them can be assessed. Factors affecting creep and shrinkage have been summarized by Campbell-Allen(145), Alexander(146) and Patten and Welch(147). Neville(141) devotes five full chapters of his text to these influences. They can be conveniently divided into two categories.

(a) Internal factors such as cement properties, aggregate size and type, mix proportions, aggregate and water contamination, admixtures, compaction and air-entrainment.

(b) External factors such as curing conditions, temperature and humidity conditions, shape and size of specimen, reinforcements, time and in the case of creep, additional factors such as the stress level, change in strength under load and age at loading.

For the writer's restrained shrinkage and sustained load tests described in Chapters 9 and 10, most of these factors were controlled. The same cement type, aggregate type (except for column CC7), mix proportions and compaction technique was used for each test. No admixtures were used. The columns and companion control specimens were cured at 100% relative humidity and a constant temperature of 70°F in a fog-room for at least 35 days such that by this time the strength and initial elastic modulus for short-term loading were reasonably constant. Control specimens for determining the creep and shrinkage strains had the same crossectional area as the columns (except for the tensile creep specimens) and had their ends sealed to simulate the radial drying characteristics of the longer columns. Drying and loading commenced at the same time and continued under conditions of constant relative humidity and temperature of 50% and 70°F respectively. The use of constant temperature and humidity conditions was to eliminate the fluctuations in the observed strains, such as those for shrinkage shown in Fig. 7.1, with varying temperature and humidity. This enabled the comparisons between the observed and theoretical behaviour to be made more easily without the results or trends being masked by uncontrolled variations in the atmospheric conditions.

Only those factors which directly influenced the behaviour of the concrete in the test columns under the conditions above are discussed below.

7.2 DEFINITION OF TERMS

Creep and shrinkage strains generally occur simultaneously and are inextricably related. However, it is common practice to consider the two additive such that the creep strain is the difference between the increase in strain of a stressed and drying specimen (creep + shrinkage) and the increase in strain of a similar unstressed specimen (shrinkage) as defined at (b) in Fig. 7.2. This definition is used for all subsequent Chapters. The creep may be further separated into drying creep and basic creep as at (d) in Fig. 7.2. The former is associated with moisture movement and the latter being the creep strains measured on a specimen in hygral equilibrium with its environment as at (c) in Fig. 7.2. If the age at loading is such that the elastic modulus does not alter significantly with time, the true elastic strain is approximated by the nominal elastic strain. For the writer's tests described in Chapters 9 and 10, the age of the concrete, when the load was applied, was sufficient for this approximation to be reasonably accurate.

7.3 CREEP UNDER CONSTANT STRESS

The effect of sustained loads on concrete is summarized in Fig. 7.3 from test results by Rusch(76). For a particular value of stress, the creep strain increases with time until it approaches a limiting value of strain where for low stresses, it continues to carry stress at the limiting strain and for high stresses, failure occurs. The value of sustained stress that just produces time-failure is known as the static fatigue limit or sustained load strength. Rusch's tests showed that this sustained load strength amounted to at least 75% and on the average to about 80% of the strength determined in a short-term test. Neville(141) quotes values ranging from 70% to 90%. From Fig. 7.3, it can be seen that the higher the stress level, the greater the creep strain at a particular time, indicating creep increases as the applied stress.

An investigation by Neville(148) on the influence of the type of cement on the creep of mortar specimens led to the development of a relationship between stress, strength and creep where for a particular mix, the creep strain is proportional to the stress and inversely proportional to the strength at the time of application of the stress. Neville's original data for mortars is shown in Fig. 7.4. In a following paper, Neville(149) verified the proportionality of creep and stress-strength ratio for concrete using test data from a number of other investigators. What is not certain is the upper limit of proportionality. Freudenthal and Roll(150) found the stress-strength ratio at the limit to be as low as 23% whilst Mamillan(151) found it to be as high as 75%. Above the limit of proportionality, the creep increases with stress at an increasing rate as illustrated in Fig. 7.5 from results by Gvozdev(152), where the stress-strength ratio at the limit of proportionality is about 0.4. However, if the stress and strength (maximum stress) are converted to their corresponding short term strains by the use of a stress-strain curve obtained from a short term test, in this case that shown in Fig. 3.23, and then Gvozdev's results replotted, the relationship shown in Fig. 7.6 is obtained. It should be noted that for the short term stress-strain curve of Fig. 3.23, the strain corresponding to the maximum stress (strength) is a function of both the concrete strength and the initial elastic modulus and which, for a small range of concrete strengths, should be reasonably constant. This assumption was verified from short term tests on cylinders by Rusch(153) as shown in Fig. 7.7 where for concrete strengths ranging between 3000 lb/sq.in. and 8000 lb/sq.in. the strain at maximum stress is remarkably constant. This is generally the case provided that the same constituent materials are used for each test specimen as was the case for the writer's tests described in Chapters 9 and 10 where the same concrete mix was used for all column specimens and the variation in strengths was small. From Fig. 7.6, it can be seen that the limit of proportionality has been extended considerably and that for stress-strength ratios up to 0.7 the creep strain, $\varepsilon_{cc}(t)$ at a particular time, t, is proportional to the applied short term strain, ε_c, assuming the short term strain at maximum stress to be constant as discussed above. The ratio of the creep strain at a particular time, t, to the short term or applied strain is defined as the creep coefficient, $\phi_c(t)$, where

$$\phi_c(t) = \frac{\varepsilon_{cc}(t)}{\varepsilon_c} \qquad \text{——————(7.1)}$$

and is the definition as used by Neville(141). Goyal and Jackson(154) found that for stress-strength ratios up to 0.75, the value of the creep coefficient was constant for a particular time, again indicating that the creep strain is proportional to the applied short term strain. Goyal and Jackson's results are shown in Fig. 7.8 which includes the variation of $\phi_c(t)$ with time. Therefore, the variation of creep strain with time for various stress levels and hence corresponding short term strain values can be reduced to a single relationship between the creep coefficient, $\phi_c(t)$, and time under load, t.

It is generally assumed that the shrinkage strains are unaffected by stress. Whilst this is convenient, there is some doubt of its validity as noted by Gluklich(155). L'Hermite(156) considers creep to be covariant with shrinkage such that creep does not add to shrinkage but combines with it "with a minimum of action".

The above discussion relates to concrete in compression as concrete is usually designed to make use of its high compressive strength and is rarely designed to carry tension, the usual assumption being that its tensile strength is zero. However, when shrinkage is present in an unloaded composite column, the steel acts as a restraint to the shrinkage by virtue of its bond to the concrete. Compressive stresses are developed in the steel and tensile stresses in the concrete which may lead to cracking. The ability of the concrete to creep in tension helps to relieve these stresses. Unfortunately, there has been little information published on the tensile creep behaviour of concrete. Initial tests suggested that creep in tension was higher than the creep in compression. This was confirmed by Illston(157) as illustrated in Fig. 7.10. From the same tests, Illston found that creep in tension was proportional to the stress-strength ratio up to a limit of 0.5, this being shown in Fig. 7.9. Gvozdev(152) found an even higher limit of proportionality.

From the results of tests by other authors, as discussed above, and from the results of his own tests described later in Chapters 9 and 10, the writer has concluded that it is reasonable to assume that for a particular value of time, t, the creep strain, $\varepsilon_{cc}(t)$, both in tension and compression, is proportional to the applied short term strain, ε_c. This assumption is used in the writer's analytical treatment of the long term behaviour of composite columns given in Chapter 8. The variation with time, t, of the creep coefficient, $\phi_c(t)$, which is the ratio of $\varepsilon_{cc}(t)$ to ε_c, can therefore be deduced from a single load test on a specimen with a constant applied stress and hence a constant value of short term strain, ε_c, together with a companion stress free specimen to obtain the shrinkage strains. For any particular value of time, t, the creep strain, $\varepsilon_{cc}(t)$, is obtained by deducting the shrinkage strain at that time from the shrinkage plus creep strain at that time obtained from the specimen under stress.

7.4 THE EFFECT OF SIZE AND SHAPE

It has been found by other investigators that there are considerable differences in both the creep behaviour and shrinkage behaviour for specimens of different size. This aspect has been looked at in detail by Hansen and Mattock(158). It was found that the major factor influencing both the shrinkage strains and the creep strains was the volume to surface-area ratio as shown in Figs. 7.11 and 7.12 respectively. The shape had a much smaller influence as shown by their results in Fig. 7.13 for the shrinkage of a cylinder and a T section having the same volume to surface-area ratio.

Hansen and Mattock(158) assumed that the shrinkage-time and creep-time curves could be represented by the following hyperbolic equations, which have also been used by Ross(159) and Lorman(160) and found to give satisfactory results.

$$\varepsilon_{cs}(t) = \frac{\varepsilon_{cs}(\infty)t}{N_s + t} \qquad \text{——(7.2)}$$

and

$$\varepsilon_{cc}(t) = \frac{\varepsilon_{cc}(\infty)t}{N_c + t} \qquad \text{——(7.3)}$$

where $\varepsilon_{cs}(t)$ and $\varepsilon_{cc}(t)$ are the shrinkage strain and creep strain at time, t, respectively; $\varepsilon_{cs}(\infty)$ and $\varepsilon_{cc}(\infty)$ are the final or limiting values of shrinkage and creep strain and N_s and N_c are constants which can be shown to be equal to the time at which $\varepsilon_{cs}(t)$ and $\varepsilon_{cc}(t)$ are equal to half their final values $\varepsilon_{cs}(\infty)$ and $\varepsilon_{cc}(\infty)$. The variation of N_s and N_c with the volume to surface-area ratio, $\frac{v}{s}$, are both shown in Fig. 7.14. It was found that the trend of the data could be represented equally well by the same equation where

$$N_c = N_s = 26\ e^{0.36\,(\frac{v}{s})} \qquad (7.4)$$

indicating that the shapes of the shrinkage-time and creep-time curves were identical and that the volume to surface-area ratio affected both the shrinkage and the creep in an identical manner.

Using this assumption, then for a particular time, t,

$$\underset{\substack{\text{4 in.dia.}\\ \text{cylinders}}}{\left[\frac{\varepsilon_{cc}(t)}{\varepsilon_{cs}(t)}\right]} = \underset{\substack{\text{8 in. by 7 in.}\\ \text{specimens}}}{\left[\frac{\varepsilon_{cc}(t)}{\varepsilon_{cs}(t)}\right]} \qquad (7.5)$$

Use of equation (7.5) was made by the writer for the restrained shrinkage tests described in Chapter 9 where some knowledge of the tensile creep behaviour was required. Because of the difficulty in performing a tension test on a specimen of the same size as the column crossection, this being 8in. by 7 in., the tensile creep strains were obtained by applying a constant tensile stress to smaller 4 inch dia. specimens. These creep strains were then converted to equivalent creep strains for an 8in. by 7in. specimen by using equation (7.5). However, it should be noted that this technique may not always be successful as demonstrated by the scatter of points for the N_c values in Fig. 7.14 which indicates that equation (7.4) and hence equation (7.5) are not entirely accurate.

7.5 THE EFFECT OF STORAGE CONDITIONS

The storage conditions can be separated into two distinct phases.

(a) The actual amount of water present in the concrete before loading. This depends on the relative humidity of the storage during this period, termed the curing conditions.

(b) The loss of water after the load is applied. This depends on the relative humidity of the storage after loading, termed the drying conditions.

It has been established that drying concrete attains a higher value of ultimate creep and creeps at a higher rate than concrete which either remains wet or dry.

The influence of the drying conditions is demonstrated in Fig. 7.15 from results obtained by Troxell, Raphael and Davis(161). It is clearly shown that the shrinkage decreases with increasing humidity. In fact, for 100% relative humidity, an expansion rather than shrinkage occurs. A similar behaviour is shown in Fig. 7.1 for shrinkage strains measured by the writer for one of his shrinkage specimens wherefor humidities above 80%, an expansion resulted. The influence of drying and therefore shrinkage during creep is also demonstrated in Fig. 7.15 where for increasing humidity, the creep decreases.

The influence of curing conditions on creep is shown in Fig. 7.16 from results obtained by Wierig(162) where a higher water content at the time of loading led to a higher creep. Dutron(163) and Hansen(164) have examined the effects of both curing conditions and drying conditions and their results have been discussed in detail by Neville(141).

The conclusions that can be drawn from this work are that creep is increased by

(a) curing the concrete at a high relative humidity to ensure that a large amount of evaporable water is present in the concrete before the application of load and

(b) drying the concrete at a low relative humidity to ensure a rapid moisture loss from the concrete to the surrounding atmosphere when the load is applied.

To accentuate the creep deformations in the writer's sustained load tests of Chapter 10, the columns were cured at a temperature of 70°F and a relative humidity of 100% in a fog room and then removed to a controlled environment with a temperature of 70°F and a low relative humidity of 50% where the columns were quickly placed under load before much drying could take place.

7.6 THE EFFECT OF AGGREGATE TYPE

Roper(165) has shown that for locally produced aggregates, the aggregate type could markedly affect the shrinkage and hence the creep. His results, for four petrological classes of aggregates, are summarized in Fig. 7.17 where

(a) The quartzose river gravel is shown as curve 5,

(b) the limestone as curve 1,

(c) the basic intrusive rocks such as dolerite, teschenite, essexite and picrite as curves 2,3,4 and 7 respectively, the picrite exhibiting high shrinkage as a result of the presence of the dimensionally unstable clay mineral montmorillonite, and

(d) the volcanic agglomerates or breccias as curves 6 and 8. The absorption of water by the breccias is high and they exhibit a high shrinkage on drying, hence the term "shrinking aggregates" used by Roper(166).

Therefore, to accentuate the cracking tendencies for the restrained shrinkage tests of Chapter 9 and the creep deformations for three of the sustained load tests of Chapter 10, the writer used breccia as the coarse aggregate in the manufacture of the concrete for these column specimens. A crushed basalt, similar to the dolerite shown as curve 2 in Fig. 7.17, was used in the manufacture of the concrete for the columns tested under short term loading. The use of breccia instead of crushed basalt resulted in:

(a) Higher shrinkage and creep similar to the behaviour shown in Fig. 7.17.

(b) Higher concrete strengths for the same mix proportions. This was caused by the breccia absorbing water from the mix thus reducing the water-cement ratio for the paste.

(c) Lower values of initial elastic modulus even though the strength was increased. This was due to the breccia aggregate itself having a lower elastic modulus than the basalt as discussed by Roper(167).

7.7 THE EFFECT OF AGE AT LOADING

For the same stress acting, the amount of creep at a particular time increases with decreasing age at loading. Results of several investigations were summarized by L'Hermite(156) and are shown in Fig. 7.18. An explanation for this phenomenon is that for earlier ages at loading, the concrete strength is lower and hence the stress-strength ratio is higher. Similarly, if drying is occurring at the same time, the amount of evaporable water present in the concrete will be greater for earlier ages. Both of these factors result in higher creep as discussed in Sections 7.3 and 7.5 respectively.

For ages greater than about 28 days the influence of age at loading is negligible as shown in Fig.7.19

from results by Hummel, Wesche and Brand(168). This is probably due to the fact that by this time, the concrete strength is reasonably constant or that the concrete is in reasonable hygral equilibrium with the surrounding atmosphere. A similar observation was made by Glanville(169) who found that after about a month of loading, the rate of creep was independent of the age at loading.

The variation of creep with both time under load and age at loading can be represented in the form of a creep surface as proposed by McHenry(170). A typical surface showing the variation of creep coefficient, $\phi_c(t)$, with time under load and age at loading is shown in Fig. 7.20. The information provided by this surface is required in the principle of superposition method, proposed by McHenry(170), for the calculation of creep under varying stress as treated in the following Section 7.8.

If it is assumed that the rate of creep is not affected appreciably by the age at loading, the creep surface of Fig. 7.20 can be represented in a two dimensional form as in Fig. 7.21 where the creep coefficient with time curve for the age at first loading can be used to determine the creep coefficient with time curves for any other later age at loading. The creep information in this form is used in the rate of creep method for calculating creep under varying stress as treated in Section 7.8.

The principle of superposition method requires the variation of creep with time for many different ages at loading which, experimentally, requires the loading of a number of specimens at different ages and the measurement of creep strain with time under load for each. For the rate of creep method, the creep with time curves for loading at later ages can be deduced from the creep with time curve obtained by loading a single specimen at the initial age at loading, thus reducing the amount of experimental work considerably. This method was used by the writer for the tests described later in Chapters 9 and 10.

7.8 CREEP UNDER VARIABLE STRESS

The above sections have dealt mainly with creep under constant stress. If this were the case in structures, estimation of the effects would present no problem as this is the condition for which most creep data has been obtained experimentally. However, in actual structures, the stress in the concrete varies with time and the problem is to predict the creep and its influence on structures from data obtained from tests under constant stress. Ross(171) has examined this problem. His first tests were designed to examine the simplest case of stress-reversal, that of a period of constant stress followed by complete removal of the stress. The results of one test are shown in Fig.7.22. It can be seen that on removal of the stress, there was an instantaneous elastic recovery followed by an additional gradual recovery called creep recovery. The residual deformation or irrecoverable creep is generally much larger than the creep recovery. Ishai(172) found that the irrecoverable creep increased with an increase in the period under load whilst the creep recovery tended to a constant value. In Ross's other tests, the stresses were not varied continuously but in either finite increments or decrements. The experimental results were compared with three methods generally used to predict creep behaviour with variable stress, viz. the effective modulus method, the rate of creep method and the principle of superposition method, these three methods being discussed below. It is assumed that the applied stress, σ_c, can be related to the applied short term strain, ε_c, by a function such that

$$\varepsilon_c = F(\sigma_c) \qquad \text{———(7.6)}$$

For low stresses, where the concrete can be assumed to be reasonably elastic, then

$$\varepsilon_c = \frac{\sigma_c}{E_c}$$

where E_c is the short term elastic modulus.

(i) Effective modulus method.

From a single curve of creep coefficient, $\phi_c(t)$, with time, t, since first loading such as that given by Fig. 7.8, then the creep strain $\varepsilon_{cc}(t)$ at a time t for a variable stress, σ_c, can be determined from equations (7.1) and (7.6) such that

$$\varepsilon_{cc}(t) = F(\sigma_c)\phi_c(t) \qquad (7.7)$$

The creep strain at a particular time, t, depends only on the stress, σ_c, acting at that time. This method disregards the previous stress history of the material and also predicts the complete recovery of strain on the removal of the stress, which does not occur in tests as can be seen from Fig. 7.22. This method therefore tends to underestimate the creep strain for decreasing stress.

(ii) Rate of Creep Method.

Using the same creep data as for the effective modulus method, then

$$\varepsilon_{cc}(t) = \int_0^t F(\sigma_c) \frac{d\phi_c(t)}{dt} dt \qquad (7.8)$$

This method attempts to account for the stress history by integrating all the elemental increments of creep but for removal of stress, no creep recovery results. Thus, this method tends to overestimate the creep strain for decreasing stress.

(iii) Principle of Superposition Method.

This method attempts to account for the influence of age at loading and allows for creep recovery. Each stress increment is considered as producing a resulting deformation component continuing for an infinite time and each component can be superimposed on preceding deformation components produced by previous stress increments. A stress decrement is taken as a stress increment of negative sign. The variation of creep coefficient, $\phi_c(t)$, with time, t, since first loading is required for various ages, K, at which further stress increments are applied. The creep coefficient can therefore be expressed as $\phi_c(t,K)$ being both a function of time since first loading and age at further loading. For a varying stress system consisting of an initial stress, $\sigma_{c,o}$, applied at age, K_o, followed by n stress increments such that the applied stress at age, K_i, is $\sigma_{c,i}$, then the creep strain, $\varepsilon_{cc}(t)$, at a time, t_i, since first loading such that all the stress increments have been applied is given by

$$\varepsilon_{cc}(t) = F(\sigma_{c,o})\phi_c(0,K_o) + \sum_{i=1}^{n}[F(\sigma_{c,i})-F(\sigma_{c,i-1})]\phi_c(t_i,K_i) \qquad (7.9)$$

If the stress, σ_c, varies continuously with time, t, integration is possible such that

$$\varepsilon_{cc}(t) = F(\sigma_{c,o})\phi_c(0,K_o) + \int_o^t \frac{dF(\sigma_{c,t})}{dt}\phi_c(t,K)dt \qquad (7.10)$$

Ross(171) has compared the three methods with experimental tests on concrete prisms subjected to a variety of stress increments and decrements. A typical result is shown in Fig. 7.23 for a specimen subjected to both increments and decrements of stress. As would be expected, the effective modulus method leads to the greatest error. The rate of creep method and superposition method show significantly better agreement and both have about the same degree of accuracy, the only difference being that the superposition method allows for some creep recovery and could therefore be considered as superior to the rate of creep method. The stress changes shown in Fig. 7.23 are quite severe whereas in the case of a concrete column under sustained load, the decrease in stress might be expected to be more gradual. Ross has also examined this case by considering the relaxation in stress of a concrete member held at a constant strain, the results of which are shown in Fig. 7.24. Although the superposition method still gives the best estimate, the effective modulus method was almost as good and differed little in accuracy when compared with either the rate of creep method or the superposition method.

7.9 THE PREDICTION OF CREEP AND SHRINKAGE

It is often inconvenient to perform long term tests to determine the shrinkage-time and creep-time relations and several equations have been suggested to approximate the relationships, mostly of the hyperbolic or exponential type. Ross(159) proposed a hyperbolic expression for the variation of creep strain, $\varepsilon_{cc}(t)$, with time, t, where

$$\varepsilon_{cc}(t) = \frac{t}{a + bt} \qquad \text{———(7.11)}$$

where a and b are constants that have to be determined experimentally. A plot of $t/\varepsilon_{cc}(t)$ against t is a reasonable straight line, the slope giving the value of b and the intercept on the $t/\varepsilon_{cc}(t)$ axis giving the value of a. The ultimate creep, $\varepsilon_{cc}(\infty)$ for large time is given by

$$\varepsilon_{cc}(\infty) = \frac{1}{b} \qquad \text{———(7.12)}$$

Hence, the variation of creep with time and the ultimate creep can be deduced from experimental data at early ages. Of course, the longer the time over which creep is actually measured, the better the prediction. A similar hyperbolic expression was proposed by Lorman(160) and exponential expressions have been suggested by Thomas(173) and McHenry(170).

For the cases where experimental data is not available for shrinkage and creep, the approach has been to use standard curves which are modified to allow for such factors as the humidity of the environment, age at loading, size of member, the mix proportions and time at which initial shrinkage is considered. This approach was first proposed by Jones, Hirsch and Stephenson(174) and by Wagner(175) and more recently, it has been recommended by the Comité European du Beton(176) in 1970 and by ACI Committee 209(177) in 1972.

Campbell Allen(178) and Roper(165) have shown that variations in shrinkage, and hence creep, resulting from changes in materials alone, especially aggregates, are at least as important as those arising from changes in mix design. The above recommendations do not cover this aspect. Hence, modified standard curves may be of use only when experimental data is not available. They are certainly no substitute for controlled environmental tests on specimens containing the same materials, mix proportions and cross-sectional dimensions as used in the structure to simulate actual behaviour as was the case for the writer's long-term tests of Chapters 9 and 10.

CHAPTER 8

COLUMN THEORY - LONG TERM BEHAVIOUR

8.1 INTRODUCTION

8.1.1 Sustained Loading

It has been shown in Chapter 7 that concrete under load continues to deform with time as a result of creep. Even with no applied load, concrete can deform with time as a result of shrinkage. A column containing concrete will therefore be subjected to similar deformations. The case examined is that of a pin-ended composite column subjected to a constant sustained load applied eccentrically at the ends. When the load is first applied, the column will take up its initial deflected shape. An analysis for determining this deflected shape for short term loading has been presented in Chapter 4. These deflections will increase with time due to creep. The extra deflection results in increased bending moments along the length of the column which in turn produce additional applied strain to the concrete and hence further creep. The deflections will continue to increase in this manner as long as the concrete continues to creep, providing that the column does not become unstable, this depending upon the magnitude of the applied load for a particular column. It is therefore necessary to determine the boundary between the values of sustained load which will lead to a shakedown of the column into a stable state and load values which will lead to instability or creep buckling resulting in a catastrophic collapse of the column.

Troxell, Raphael and Davis(161) observed that the creep deformations of plain concrete under sustained load continued to increase over the entire period of observation, which in some cases was as long as 28 years, and that shrinkage of unloaded cylinders also continued to increase over the same length of time but that the rates of change of deformation at later ages were very small. The rates of change of deformation of an eccentrically loaded column would therefore be very small at these later ages also. However, while there is a positive rate of creep, there is still the likelihood that instability of a loaded column could occur, this possibility diminishing as the rate of creep continues to decrease with further increase in age. The boundary between shakedown and collapse is therefore a function of the required life-span for the column. For an infinite life span, the load defining this boundary would be at its minimum value but it is not often that structures would be required to carry their loads forever without collapsing. The design life span for a composite column could well be of the order of 50 to 100 years.

The time-dependent behaviour of a column will depend on its own particular geometrical properties, short term (initial loading) and long term (creep and shrinkage) material properties and loading conditions. Therefore, there is no simple way of establishing the shake-down - collapse boundary and it can best be done by determining the relationship between load, deformation and time for the specified life-span of the column using a full range of the three parameters. Not only would the boundary be established by this relationship but also the time dependent behaviour of the column at all stages from initial loading. Based on the superposition, rate of creep and effective modulus methods for determining creep under variable stress as discussed in Chapter 7, analytical methods have been used to determine the load-deformation-time behaviour of reinforced concrete columns. Many of these studies have been referred to in recent work by Hellesland and Green(179)(180) and Goyal and Jackson(154) and need not be listed here. Many of these methods use mathematical functions to represent the creep and shrinkage characteristics of the concrete which may or may not represent the true nature of the material. It will be seen in the following Sections that the analyses presented by the writer are suitable for use with either mathematical functions or actual strain data recorded from creep and shrinkage tests.

The concrete in both reinforced and composite columns under sustained load will be subjected to similar varying stress conditions. Therefore similar methods of analysis are applicable to both types of columns. The difference in behaviour between the two types of columns is in that the composite column contains a steel section of considerable stiffness which is capable in itself of carrying considerable load whereas the steel reinforcing of the reinforced concrete column is only capable of carrying load when used in conjunction with the concrete. The writer's analyses, presented in the following sections, account for this difference in stiffness. As the concrete still provides additional stiffness to that provided by the steel even after creep and shrinkage have taken place, the following hypothesis is proposed. Provided the sustained eccentric load on a composite column is less than the load carrying capacity of the bare steel section loaded at the same eccentricity, creep instability of the composite column will not occur.

In Section 8.3.3, the writer's analysis for predicting the behaviour of composite columns subjected to constant sustained load using the effective modulus method is described. As discussed earlier in Section 7.8, the effective modulus method disregards the previous stress history prior to the particular time being considered. However, its use will be shown to lead to an analysis which requires only minor extensions to the analyses for short term loading which have been developed by the writer and have been described in detail in Chapter 4. A similar simplified approach for the sustained loading of reinforced concrete columns was used by Goyal and Jackson(154). Although there were some discrepancies, the general agreement between predicted and observed behaviour was good.

In Section 8.2.3, an alternative analysis developed by the writer using the rate of creep approach is described. This method accounts for previous stress history by the summation of elemental increments of creep over small increments of time. The change in strain during an increment of time is dependent on the strain determined at the end of the preceding increment such that any error in previous increments continues to affect all subsequent increments. As this analysis requires iterative procedures, errors tend to be cumulative. Therefore, although the rate of creep method provides a better model of the actual creep behaviour than the effective modulus method, the actual analytical procedures required are a little more complex and more susceptible to error.

8.1.2 Restrained Shrinkage

For composite columns, where drying has taken place before any load is applied, tensile stresses in the concrete and compressive stresses in the steel result from the concrete shrinkage being restrained by the steel, the stresses being transmitted by the bond between the concrete and the steel sections. This behaviour is referred to in the following Sections simply as 'restrained shrinkage'. The creep of concrete in tension reduces the magnitude of the stresses that would have developed by shrinkage alone. These stresses can be considered to be residual stresses in the column before load is applied. An analytical technique to account for the effect of residual stresses on the short term behaviour of composite columns has already been described in Section 4.2.4(b). The rate of creep and effective modulus methods of analysis for predicting the magnitude of these stresses are described in Sections 8.2.2 and 8.3.2 respectively. For the case of restrained shrinkage, the rate of creep analysis involves only a simple computational procedure whereas more complex procedures were required for the case of sustained load.

It should be noted that depending on the rate of shrinkage and creep and the percentage of steel in the crossection, restrained shrinkage could lead to cracking of the concrete. The effect of shrinkage cracks, present in the column before loading, on the subsequent load behaviour of composite columns has been examined experimentally by the writer and is discussed later in Chapter 9.

8.2 RATE OF CREEP METHOD OF ANALYSIS

8.2.1 General

From the definition of terms shown in Fig. 7.2 for concrete under sustained load, it can be seen that the nominal elastic strain component of the total strain is the strain resulting from the applied stress for short term loading. For low levels of applied stress, this strain could be considered elastic, the stress and strain being related by the short term elastic modulus, E_c, for the concrete. For higher levels of applied stress, the resulting strain for short term loading cannot be considered elastic as already shown in Fig. 3.2 showing typical stress-strain curves for concrete under short term loading. The writer has therefore preferred to use the term 'applied concrete strain' in the following analyses rather than the term 'nominal elastic strain'. The applied concrete strain is defined as that which is related to the applied stress using a stress-strain relationship for short term loading. The total strain for concrete under stress is therefore the sum of the applied concrete strain, the creep strain and the shrinkage strain. As already defined in Section 7.3, the ratio of the creep strain, $\varepsilon_{cc}(t)$, to the applied concrete strain, $\varepsilon_c(t)$, at some time, t, is termed the creep coefficient, $\phi_c(t)$.

The case considered in the following analyses is a typical one in which the column is moist cured for a period up to an age, K_d, followed by drying resulting in tensile stress in the concrete due to restrained shrinkage followed by a sustained load being applied at a later age, K_o, resulting in primarily compressive stress in the concrete. With reference to Chapter 7, the following assumptions are made.

(i) As the crossection dimensions of the composite columns are relatively small, the shrinkage strains are considered to be the same at all points of the crossection such that the shrinkage strain distribution is uniform. The shrinkage data can therefore be represented by a single curve, as in Fig. 8.1(a), showing the variation of shrinkage strain $\varepsilon_{cs}(K)$ with age, K, where age is measured from when the concrete was first manufactured i.e. when hydration commenced. The curve in Fig. 8.1(a) is a typical one showing a period of moist curing prior to the age, K_d, at which drying commenced. During this period, the shrinkage strain is taken as zero. In fact, some expansion may occur but Davis(181) has found this to be small, the expansion strain generally not exceeding 30×10^{-6} in./in.

(ii) The creep data can be represented by a single relationship between the creep coefficient, $\phi_c(t)$ and time, t, such as that shown in Fig. 7.8. This is a result of the assumption that the ratio of the creep strain, $\varepsilon_{cc}(t)$ to the applied concrete strain, $\varepsilon_c(t)$, is constant for a particular value of time, t, this ratio being termed the creep coefficient, $\phi_c(t)$. The basis of this assumption has already been discussed in Section 7.3 of Chapter 7.

(iii) For restrained shrinkage, the required tensile creep data is therefore represented by a single curve, as in Fig. 8.1(b) which shows the variation of tensile creep coefficient, $\phi_c(T)$ with drying time, T, this time being measured from age, K_d, at which drying and shrinkage commences for the concrete. For sustained loading, the compressive creep data is also represented by a single curve as in Fig. 8.1(c) showing the variation of compressive creep coefficient, $\phi_c(t)$, with loading time, t. This time is measured from age, K_o, at which the sustained load and hence compressive stress is first applied to the concrete.

(iv) The applied concrete strain is related to its corresponding stress by a stress-strain relationship for short term loading. The writer's theoretical curve for short term loading has already been shown in Fig. 3.23. This curve may be modified to account for the effect of ageing. However, if the age at which stress is first applied is such that most of the hydration process has been completed, the short term stress-strain curve at this age can be considered to apply for all later ages.

(v) Plane sections remain plane such that the total strain distribution over a column crossection is linear.

(vi) Perfect bond is maintained such that no slip occurs between the steel and concrete over the entire length of the column.

Consider a composite column of symmetrical crossection, as shown in Fig. 8.2, with an area of concrete, A_c, and an area of steel, A_s. A section of the column is subjected to a sustained constant axial load, P, and constant moment, M. After a time, t_k, of loading, the creep and shrinkage will have increased the total centroidal axis strain and curvature (centroidal axis strain and curvature for the steel) from their initial values at first loading to the increased values of $\varepsilon_{as}(t_k)$ and $\rho_s(t_k)$ respectively. Similarly, the initial applied concrete centroidal axis strain and applied concrete curvature at first loading will have been decreased to the new values, $\varepsilon_{ac}(t_k)$ and $\rho_c(t_k)$. The difference between the total strain and the applied concrete strain at any point a distance, x, from the centroidal axis is the creep strain, $\varepsilon_{cc}(t_k)$ and the shrinkage strain, $\varepsilon_{cs}(t_k)$, that have occurred up to time, t_k. During an additional small interval of time, Δt_k, similar small changes in centroidal axis strain and curvature will occur with the total strain being increased to that defined by $\varepsilon_{as}(t_k + \Delta t_k)$ and $\rho_s(t_k + \Delta t_k)$ and the applied concrete strain being decreased to that defined by $\varepsilon_{ac}(t_k + \Delta t_k)$ and $\rho_c(t_k + \Delta t_k)$ as indicated by the dashed lines in Fig. 8.2. The sum of the increment, $\Delta\varepsilon_s$, in the total (steel) strain and the decrement, $\Delta\varepsilon_c$, in the applied concrete strain at any point a distance, x, from the centroidal axis must be equal to the sum of the shrinkage strain, $\Delta\varepsilon_{cs}$, and the creep strain, $\Delta\varepsilon_{cc}$, that occurred during the same time interval, Δt_k. Hence

$$\Delta\varepsilon_s + \Delta\varepsilon_c = \Delta\varepsilon_{cs} + \Delta\varepsilon_{cc} \qquad (8.1)$$

Now

$$\Delta\varepsilon_s = \varepsilon_{as}(t_k+\Delta t_k) + \rho_s(t_k+\Delta t_k).x - [\varepsilon_{as}(t_k) + \rho_s(t_k).x] \qquad (8.2)$$

and

$$\Delta\varepsilon_c = \varepsilon_{ac}(t_k) + \rho_c(t_k).x - [\varepsilon_{ac}(t_k+\Delta t_k) + \rho_c(t_k+\Delta t_k).x] \qquad (8.3)$$

and

$$\Delta\varepsilon_{cs} = K_s(t_k)\Delta t_k \qquad (8.4)$$

where $K_s(t_k)$ is the shrinkage rate during the time interval, Δt_k which starts at time, t_k, as defined in Fig. 8.1(a). By choosing a sufficiently small value of Δt_k, the average applied concrete strain during the time interval can be assumed to be the median value of the strain at the start and the strain at the end of the time interval.

Therefore

$$\Delta\varepsilon_{cc} = \tfrac{1}{2}[\varepsilon_{ac}(t_k)+\rho_c(t_k).x+\varepsilon_{ac}(t_k+\Delta t_k)+\rho_c(t_k+\Delta t_k).x]K_c(t_k)\Delta t_k \tag{8.5}$$

where $K_c(t_k)$ is the creep coefficient rate during the time interval, Δt_k which starts at time, t_k, as defined in Fig. 8.1(c). Substitution of equations (8.2) to (8.5) into equation (8.1) and simplifying results in

$$\varepsilon_{as}(t_k+\Delta t_k)-\varepsilon_{as}(t_k)+\varepsilon_{ac}(t_k)-\varepsilon_{ac}(t_k+\Delta t_k)$$

$$= K_s(t_k)+\tfrac{1}{2}[\varepsilon_{ac}(t_k)+\varepsilon_{ac}(t_k+\Delta t_k)]K_c(t_k)\Delta t_k \tag{8.6}$$

and

$$\rho_s(t_k+\Delta t_k)-\rho_s(t_k)+\rho_c(t_k)-\rho_c(t_k+\Delta t_k)$$

$$= \tfrac{1}{2}[\rho_c(t_k)+\rho_c(t_k+\Delta t_k)]K_c(t_k)\Delta t_k \tag{8.7}$$

If the concrete and steel are assumed to be stressed within their elastic range, then for constant axial force, P,

$$E_cA_c\varepsilon_{ac}(t_k)+E_sA_s\varepsilon_{as}(t_k) = E_cA_c\varepsilon_{ac}(t_k+\Delta t_k)+E_sA_s\varepsilon_{as}(t_k+\Delta t_k) \tag{8.8}$$

where E_c is the short term elastic modulus for the concrete and E_s is the elastic modulus for the steel.
For constant moment, M,

$$E_cI_c\rho_c(t_k)+E_sI_s\rho_s(t_k) = E_cI_c\rho_c(t_k+\Delta t_k)+E_sI_s\rho_s(t_k+\Delta t_k) \tag{8.9}$$

where I_c and I_s are the second moments of area about the centroidal axis for the concrete and steel respectively. Substituting equation (8.8) into equation (8.6), simplifying and rearranging, then

$$\varepsilon_{ac}(t_k+\Delta t_k) = \varepsilon_{ac}(t_k) - \frac{K_s(t_k)E_{ca}\Delta t_k+\varepsilon_{ac}(t_k)K_c(t_k)E_{ca}\Delta t_k}{1 + \frac{1}{2}K_c(t_k)E_{ca}\Delta t_k} \qquad (8.10)$$

where

$$E_{ca} = \frac{E_sA_s}{E_sA_s+E_cA_c}$$

and

$$\varepsilon_{as}(t_k+\Delta t_k) = \varepsilon_{as}(t_k) + \frac{K_s(t_k)E_{sa}\Delta t_k+\varepsilon_{ac}(t_k)K_c(t_k)E_{sa}\Delta t_k}{1 + \frac{1}{2}Kc(t_k)E_{ca}\Delta t_k} \qquad (8.11)$$

where

$$E_{sa} = \frac{E_cA_c}{E_sA_s+E_cA_c}$$

Substituting equation (8.9) into equation (8.7), simplifying and rearranging, then

$$\rho_c(t_k+ t_k) = \rho_c(t_k) - \frac{\rho_c(t_k)K_c(t_k)E_{cm}\Delta t_k}{1 + \frac{1}{2}K_c(t_k)E_{cm}\Delta t_k} \qquad (8.12)$$

where

$$E_{cm} = \frac{E_s I_s}{E_s I_s + E_c I_c}$$

and

$$\rho_s(t_k+\Delta t_k) = \rho_s(t_k) + \frac{\rho_c(t_k)K_c(t_k)E_{sm}\Delta t_k}{1 + \frac{1}{2}K_c(t_k)E_{cm}\Delta t_k} \quad \text{———(8.13)}$$

where

$$E_{sm} = \frac{E_c I_c}{E_s I_s + E_c I_c}$$

Compressive strains have been taken as positive for the above expressions which are used in the following Section 8.2.2 for the restrained shrinkage analysis.

If the average applied concrete strain during the small time interval, Δt_k, is assumed to be the applied concrete strain at the start of the time interval, then equation (8.5) becomes

$$\Delta\varepsilon_{cc} = [\varepsilon_{ac}(t_k)+\rho_c(t_k).x]K_c(t_k)\Delta t_k \quad \text{———(8.14)}$$

This simplifying assumption is made use of later in the sustained loading analysis of Section 8.2.3 where its value will be made obvious. Using equation (8.14) in place of equation (8.5) in the above analysis results in

$$\varepsilon_{ac}(t_k+\Delta t_k) = \varepsilon_{ac}(t_k)-K_s(t_k)E_{ca}\Delta t_k-\varepsilon_{ac}(t_k)K_c(t_k)E_{ca}\Delta t_k$$

———(8.15)

$$\varepsilon_{as}(t_k+\Delta t_k) = \varepsilon_{as}(t_k)+K_s(t_k)E_{sa}\Delta t_k+\varepsilon_{ac}(t_k)K_c(t_k)E_{sa}\Delta t_k \qquad (8.16)$$

$$\rho_c(t_k+\Delta t_k) = \rho_c(t_k) - \rho_c(t_k)K_c(t_k)E_{cm}\Delta t_k \qquad (8.17)$$

$$\rho_s(t_k+\Delta t_k) = \rho_s(t_k) + \rho_c(t_k)K_c(t_k)E_{sm}\Delta t_k \qquad (8.18)$$

These expressions are used in the following Section 8.2.3 for the sustained load analysis.

8.2.2 Restrained Shrinkage

As shown in Fig. 8.1, this will occur from age K_d to K_o during which period drying and hence shrinkage is taking place but where there are no externally applied loads or moments to the column. As the column crossection shown in Fig. 8.2 is symmetrical, the shrinkage stresses developed are uniform across the section resulting in only axial forces in the steel and concrete. As the stresses developed in the concrete are tensile, the concrete can reasonably be assumed to be elastic with a modulus, E_c, up to a stress equal to the tensile strength, f_{ct}, at which point the concrete cracks and is no longer capable of carrying tensile stress. The steel can reasonably be assumed to be elastic, with an elastic modulus, E_s, up to the yield stress, f_{sy}. As already discussed in Section 7.8 of Chapter 7 for the rate of creep method, the creep strain occurring under a varying stress and hence varying applied concrete strain is calculated by integrating the elemental increments of creep over the time period under stress. To perform this integration, the time period from age, K_d, to age, K_o, is divided up into m small intervals of time where ΔT_j is the jth interval of time commencing at time, T_j. The origin for time, T_j, is at age, K_d, where drying and hence shrinkage commence. Therefore, T_1 is zero and corresponds to age, K_d, and

T_{m+1} is the time at which sustained loading commences at age, K_o. The intervals of time, ΔT_j, need not be equal. For accuracy of analysis, it is better to choose the size of each interval such that the change in slope of $K_s(T_j)$, as defined in Fig. 8.1(a), from the jth to the j+1th time interval is kept reasonably constant for all values of j from 1 to m.

By replacing the time, t_k, in equation (8.10) with time, T_j, the applied concrete centroidal axis strain, $\varepsilon_{ac}(T_{n+1})$ at age, K_o, is calculated by using successive applications of this equation for each of the m time intervals in turn such that

$$\varepsilon_{ac}(T_{m+1}) = \varepsilon_{ac}(T_1) - \sum_{j=1}^{m} \frac{K_s(T_j)E_{ca}\Delta T_j + \varepsilon_{ac}(T_j)K_c(T_j)E_{ca}\Delta T_j}{1 + \frac{1}{2}K_c(T_j)E_{ca}\Delta T_j} \qquad \text{——— (8.19)}$$

$\varepsilon_{ac}(T_1)$ is zero for this case and $\varepsilon_{ac}(T_{m+1})$ will be of negative sign indicating the strain is tensile, the slopes, $K_s(T_j)$ and $K_c(T_j)$ being taken as positive. The tensile stress, $\sigma_c(T_{m+1})$, in the concrete at age, K_o, is given by

$$\sigma_c(T_{m+1}) = \varepsilon_{ac}(T_{m+1})E_c \qquad \text{——— (8.20)}$$

From simple statics, the total compressive (steel) strain, $\varepsilon_{as}(T_{m+1})$ at age, K_o, is given by

$$\varepsilon_{as}(T_{m+1}) = -\varepsilon_{ac}(T_{m+1}) \frac{E_c A_c}{E_s A_s} \qquad \text{——— (8.21)}$$

The compressive stress, $\sigma_s(T_{m+1})$, in the steel at age K_o is therefore given by

$$\sigma_s(T_{m+1}) = \varepsilon_{as}(T_{m+1})E_s$$

If the magnitude of $\sigma_c(T_{m+1})$ is greater than f_{ct}, the concrete is assumed to have cracked and $\sigma_c(T_{m+1})$ is therefore zero. In an actual column, the cracks will be at finite spacings. The concrete stress at the crack will of course be zero but tensile stresses may still exist in the concrete between the cracks, the magnitude of the stresses being dependent on the crack spacing.

8.2.3 Sustained Loading

A constant sustained load, P, is applied at age, K_o, the time period from age, K_o, to age, K_f, being divided into p small intervals of time where Δt_k is the kth interval of time starting at time, t_k, as shown in Fig. 8.1(c). The age, K_f, can be any required age, the writer using a value where the rate of creep and shrinkage are sufficiently small such that the corresponding rate of increase in deflections for a stable column is also small and tending to zero. The origin for time is taken at the age, K_o, when the sustained load is first applied. Therefore t_1 is zero and t_{p+1} is the time corresponding to the required final age, K_f. As for the restrained shrinkage analysis above, the intervals of time, Δt_k, need not be equal. They are better chosen such that the change of slope of $K_c(t_k)$, as defined in Fig. 8.1(c), from the kth to the k+1 th time interval is kept reasonably constant for all values of k from 1 to p.

As for the analysis of the short term column behaviour, the column length, L, is divided into n equal elements, each of length $\frac{L}{n}$. The sections at each end of the element are numbered 1 to n+1 commencing at the bottom of the column. As shown in Fig. 4.13, the load, P, is applied at end eccentricities, e_1 and e_{n+1}, from the centroidal axis. The procedure for determining the variation of the deflected shape (values of $u_i(t)$) with time, t, for a constant sustained load of P is as follows, with reference to the analysis for short term loading, as in Chapter 4, where necessary.

(i) At time, t_1, just prior to the application of the load, P, the total strain and the applied concrete strain resulting from restrained shrinkage are given by equations (8.21) and (8.19) respectively, this strain condition being shown at (a) in Fig. 8.3. These strains are initially taken as residual strains present in the column before loading. At time, t_1, immediately after the load, P, is applied, the strain condition at the ith section is as shown at (b) in Fig. 8.3. The corresponding deflections, $u_i(t_1)$, moments, $M_i(t_1)$, centroidal axis strains, $\varepsilon_{asi}(t_1)$ and $\varepsilon_{aci}(t_1)$, and curvatures, $\rho_{si}(t_1)$ and $\rho_{ci}(T_1)$, at the ith section for all values of i from 1 to n+1 are calculated using the analytical procedures for short term loading already described in Sections 4.3.3 and 4.2.4(b).

(ii) k is set to 1

(iii) At time, t_{k+1}, at the end of the time interval, Δt_k, estimates for the values of $\varepsilon_{asi}(t_{k+1})$, $\varepsilon_{aci}(t_{k+1})$, $\rho_{si}(t_{k+1})$ and $\rho_{ci}(t_{k+1})$ are calculated from equations (8.15), (8.16), (8.17) and (8.18) respectively for i = 1 to n+1. These expressions assume that $M_i(t_{k+1}) = M_i(t_k)$ and that the stresses are within the elastic range for the materials.

(iv) Values for the deflections, $u_i(t_{k+1})$, are calculated by integrating the total curvatures, $\rho_{si}(t_{k+1})$, using the same integration procedure described in Section 4.3.3.

(v) Values for the moments, $M_i(t_{k+1})$, corresponding to these deflections, $u_i(t_{k+1})$ are calculated from equation (4.51) where

$$M_i(t_{k+1}) = P(e_1 + (e_{n+1} - e_1)\frac{z_i}{L} + u_i(t_{k+1}) + u_{oi})$$

(vi) New values of the curvatures, $\rho_{si}(t_{k+1})$ and

$\rho_{ci}(t_{k+1})$, and centroidal axis strains, $\varepsilon_{asi}(t_{k+1})$ and $\varepsilon_{aci}(t_{k+1})$, corresponding to the axial load, P, and moment, $M_i(t_{k+1})$, at each section are calculated using the iterative procedure already developed in Section 4.2.4(d) and the load-moment-curvature relationships described in Section 4.2.4(b) with modifications as follows.

In the iterative procedure of Section 4.2.4(d), both $\varepsilon_{asi}(t_{k+1})$ and $\varepsilon_{aci}(t_{k+1})$ must be subjected to identical changes until the load converges to P. Similarly, $\rho_{si}(t_{k+1})$ and $\rho_{ci}(t_{k+1})$ must be subjected to identical changes until the moment has converged to $M_i(t_{k+1})$. This is done so that the difference between $\varepsilon_{asi}(t_{k+1})$ and $\varepsilon_{aci}(t_{k+1})$ and the difference between $\rho_{si}(t_{k+1})$ and $\rho_{ci}(t_{k+1})$ remains constant. This is a result of the assumption, as used in equation (8.14), that the creep strain occurring during the time interval, Δt_k, is also constant and dependent only on the known value of the applied concrete strain at time, t_k, at the start of the time interval, Δt_k, and is not affected by any change in applied concrete strain that might take place during the time interval. The above can be expressed in mathematical terms by substracting equation (8.15) from equation (8.16) which results in

$$\varepsilon_{asi}(t_{k+1}) - \varepsilon_{aci}(t_{k+1}) = \varepsilon_{asi}(t_k) - \varepsilon_{aci}(t_k)$$
$$+K_s(t_k)\Delta t_k + \varepsilon_{aci}(t_k)K_c(t_k)\Delta t_k$$

and subtracting equation (8.17) from equation (8.18) which results in

$$\rho_{si}(t_{k+1}) - \rho_{ci}(t_{k+1}) = \rho_{si}(t_k) - \rho_{ci}(t_k) + \rho_{ci}(t_k)K_c(t_k)\Delta t_k$$

In section 4.2.4(b), equation (4.40), which defines the applied strain to both the concrete and the steel, is replaced by the two separate expressions below.

The applied concrete strain, $\varepsilon_{ci}(t_{k+1})$, at a distance, x, from the centroidal axis is given by

$$\varepsilon_{ci}(t_{k+1}) = \varepsilon_{aci}(t_{k+1}) + \rho_{ci}(t_{k+1}).x$$

and the applied steel (total) strain, $\varepsilon_{si}(t_{k+1})$, at a distance, x, from the centroidal axis is given by

$$\varepsilon_{si}(t_{k+1}) = \varepsilon_{asi}(t_{k+1}) + \rho_{si}(t_{k+1}).x$$

The stresses associated with these strains are calculated from the same short term stress-strain relationships for the materials as used for the short term loading analysis in Chapter 4.

(vii) Steps (iv) to (vi) are repeated in cycles until the deflected shape at the end of a cycle, defined by values of $u_i'(t_{k+1})$, coincides with the deflected shape at the start of a cycle, defined by values of $u_i(t_{k+1})$, within the desired limit of convergence. The values of $u_i'(t_{k+1})$ therefore define the deflected shape at time, t_{k+1}. The strain condition is shown at (c) in Fig. 8.3.

(viii) Steps (iii) to (vii) are repeated for $k = 2$ to p. The values of $u_i'(t_{p+1})$ define the final deflected shape at time t_{p+1} (age K_f). At this stage, the relationship between deflection and time for a constant load, P, has been obtained.

(ix) The complete load-deflection-time relationship is established by repeating steps (i) to (viii) for a range of values of the sustained load, P.

8.3 REDUCED MODULUS METHOD OF ANALYSIS

8.3.1 General

The effective modulus method ignores any previous strain history. The creep strain, $\varepsilon_{cc}(t_k)$, at a particular time, t_k, is assumed to be related only to the applied stress and hence the applied concrete strain, $\varepsilon_c(t_k)$, at that time. The same assumption, as for the rate of creep method, is used where the ratio of the creep strain to the applied concrete strain (creep coefficient $\phi_c(t_k)$) is constant for any particular time, t_k.

Now $$\frac{\varepsilon_{cc}(t_k)}{\varepsilon_c(t_k)} = \phi_c(t_k)$$

Therefore

$$\varepsilon_{cc}(t_k) + \varepsilon_c(t_k) = [1 + \phi_c(t_k)]\varepsilon_c(t_k) \qquad \text{———(8.22)}$$

If $\sigma_c(t_k)$ is taken as the concrete stress at time, t_k, and the concrete is assumed elastic, then by definition

$$\frac{\sigma_c(t_k)}{\varepsilon_c(t_k)} = E_c \qquad \text{———(8.23)}$$

where E_c is the elastic modulus for short term loading. From equations (8.22) and (8.23), the ratio of the applied stress to the sum of the applied and creep strains is given by

$$\frac{\sigma_c(t_k)}{\varepsilon_c(t_k) + {}_{cc}(t_k)} = \frac{E_c}{1 + \phi_c(t_k)} \qquad \text{———(8.24)}$$

This equation defines what is known as the reduced modulus.

Consider a section of a composite column, as shown in Fig. 8.2, subjected to an axial load, P, and moment, M. From Fig. 8.2, it can be seen that the total (steel) strain, $\varepsilon_s(t_k)$, at a distance, x, from the centroidal axis is equal to the sum of the applied concrete strain, $\varepsilon_c(t_k)$, the creep strain, $\varepsilon_{cc}(t_k)$ and the shrinkage strain, $\varepsilon_{cs}(t_k)$ at a time, t_k, where

$$\varepsilon_s(t_k) = \varepsilon_c(t_k) + \varepsilon_{cc}(t_k) + \varepsilon_{cs}(t_k)$$

Substituting equation (8.22) into this expression gives

$$\varepsilon_s(t_k) = [1 + \phi_c(t_k)]\varepsilon_c(t_k) + \varepsilon_{cs}(t_k)$$

Expressing this in terms of the centroidal axis strains and curvatures, then

$$\varepsilon_{as}(t_k) + \rho_s(t_k).x = [1 + \phi_c(t_k)]\varepsilon_{ac}(t_k) + \varepsilon_{cs}(t_k) + [1 + \phi_c(t_k)]\rho_c(t_k).x$$

From this expression it can be seen that

$$\varepsilon_{as}(t_k) = [1 + \phi_c(t_k)]\varepsilon_{ac}(t_k) + \varepsilon_{cs}(t_k) \quad \text{———} (8.25)$$

and

$$\rho_s(t_k) = [1 + \phi_c(t_k)]\rho_c(t_k) \quad \text{———} (8.26)$$

Equations (8.25) and (8.26) can also be expressed as

$$\varepsilon_{ac}(t_k) = \frac{\varepsilon_{as}(t_k) - \varepsilon_{cs}(t_k)}{1 + \phi_c(t_k)} \quad \text{———} (8.27)$$

and

$$\rho_c(t_k) = \frac{\rho_s(t_k)}{1 + \phi_c(t_k)} \qquad (8.28)$$

If the concrete and steel are assumed to be stressed within their elastic range, then the axial force, P, is given by

$$P = \varepsilon_{as}(t_k)E_sA_s + \varepsilon_{ac}(t_k)E_cA_c \qquad (8.29)$$

Substitution of equation (8.25) into this expression gives

$$P = [E_sA_s+E_sA_s\phi_c(t_k)+E_cA_c]\varepsilon_{ac}(t_k)+E_sA_s\varepsilon_{cs}(t_k) \qquad (8.30)$$

or

$$P = \frac{[E_sA_s+E_sA_s\phi_c(t_k)+E_cA_c]\varepsilon_{as}(t_k)-E_cA_c\varepsilon_{cs}(t_k)}{1 + \phi_c(t_k)} \qquad (8.31)$$

The moment, M, is given by

$$M = \rho_s(t_k)E_sI_s + \rho_c(t_k)E_cI_c$$

Substitution of equation (8.26) into this expression gives

$$M = [E_sI_s+E_sI_s\phi_c(t_k)+E_cI_c]\rho_c(t_k) \qquad (8.32)$$

or

$$M = \frac{E_sI_s+E_sI_s\phi_c(t_k)+E_cI_c}{1 + \phi_c(t_k)} \rho_s(t_k) \qquad (8.33)$$

8.3.2 Restrained Shrinkage

As shown in Fig. 8.1, restrained shrinkage will occur from age, K_d, to age, K_o, during which period drying and hence shrinkage will take place but there are no externally applied loads and moments. As the composite section is symmetrical, no curvatures are developed.

By replacing time, t_k, in equations (8.30) and (8.31) by the drying time, T_{m+1}, corresponding to an age, K_o, and setting P equal to zero then

$$\varepsilon_{ac}(T_{m+1}) = - \frac{E_s A_s \varepsilon_{cs}(T_{m+1})}{E_s A_s + E_s A_s \phi_c (T_{m+1}) + E_c A_c} \qquad (8.34)$$

and

$$\varepsilon_{as}(T_{m+1}) = \frac{E_c A_c \varepsilon_{cs}(T_{m+1})}{E_s A_s + E_s A_s \phi_c (T_{m+1}) + E_c A_c} \qquad (8.35)$$

where $\varepsilon_{ac}(T_{m+1})$ and $\varepsilon_{as}(T_{m+1})$ are the applied concrete strain and total (steel) strain at age, K_o, just prior to the application of the sustained load, P.

8.3.3 Sustained Loading

As for the analysis of the short term column behaviour, the column length, L, is divided into n equal elements each of length $\frac{L}{n}$. As shown in Fig. 4.13, the load, P, is applied at end eccentricities, e_1 and e_{n+1}, from the centroidal axis. A constant sustained load, P, is first applied at age, K_o. The procedure for determining the deflected shape at a required time, t_k, measured from when the load was first applied, is almost identical to that for determining the deflected shape under short term loading as already described in Section 4.3.3. A summary of this procedure, with relevant comments relating to the long term behaviour, is given below.

(i) Select the time, t_k, at which the deflected shape is required.

(ii) The applied concrete strain and the total (steel) strain resulting from restrained shrinkage prior to loading are calculated from equations (8.34) and (8.35) respectively. These strains are used as the residual strains, ε_r, in the procedure described in Section 4.2.4(b) for the determination of the load-moment-curvature relationship for the composite section.

(iii) An initial set of deflections, $u_i(t_k)$, and end slope, $\theta_1(t_k)$, corresponding to the load, P, are calculated from the elastic equations (4.48) and (4.49) respectively assuming the column to be elastic with a reduced elastic modulus for the concrete of

$$\frac{E_c}{1 + \phi_c(t_k)}$$

(iv) The initial imperfections, u_{oi}, for $i = 1$ to $n+1$ are calculated from equation (4.47)

(v) Values for the moments, $M_i(t_k)$, corresponding to the deflections, $u_i(t_k)$ and initial imperfections, u_{oi}, are calculated using equation (4.51) where

$$M_i(t_k) = P(e_1+(e_{n+1}-e_1)\ \frac{z_i}{L} + u_i(t_k)+u_{oi})$$

(vi) The values of the total curvature, $\rho_{si}(t_k)$, corresponding to $M_i(t_k)$ at the elastic sections of the column are calculated using equation (8.33). The values of total curvature, $\rho_{si}(t_k)$, corresponding to $M_i(t_k)$, at the inelastic sections of the column are calculated using the iterative procedure of either Section 4.2.4(c) or Section

4.2.4(d) and the load-moment-curvature relationships described in Section 4.2.4(b). In this latter Section, equation (4.40), which defines the applied strain to both the concrete and steel, is replaced by the following two expressions. The applied concrete strain, $\varepsilon_{ci}(t_k)$, at a distance, x, from the centroidal axis is given by

$$\varepsilon_{ci}(t_k) = \varepsilon_{aci}(t_k) + \rho_{ci}(t_k).x$$

and the total (steel) strain, $\varepsilon_{si}(t_k)$, is given by

$$\varepsilon_{si}(t_k) = \varepsilon_{asi}(t_k) + \rho_{si}(t_k).x$$

The stresses associated with these strains are calculated using the short term stress-strain relationships for the materials.

The total curvature, $\rho_{si}(t_k)$, and the applied concrete curvature, $\rho_{ci}(t_k)$, are directly related by the expressions given in equations (8.26) and (8.28). Similarly the applied concrete centroidal axis strain, $\varepsilon_{aci}(t_k)$ and the total (steel) centroidal axis strain, $\varepsilon_{asi}(t_k)$, are directly related by the expressions given in equations (8.25) and (8.27). In these expressions, the concrete shrinkage strain, $\varepsilon_{cs}(t_k)$, is the shrinkage that takes place after age, K_o, as shown in Fig. 5.1(a), as the shrinkage prior to this age has already been included in the residual strains calculated in Step (ii). From the above, it can be seen that any changes in $\varepsilon_{asi}(t_k)$, as done in the iterative procedure of Section 4.2.4(c) for the load to converge to P, must result in identical changes in $\varepsilon_{aci}(t_k)$. Similarly, any changes in $\rho_{si}(t_k)$, as done in the iterative procedure of Section 4.2.4(d) for the moment to converge to $M_i(t_k)$, must result in identical changes in $\rho_{ci}(t_k)$.

(vii) New values for the deflections, $u_i(t_k)$ are calculated by integrating the total curvatures $\rho_{si}(t_k)$

obtained in Step (vi). The same integration procedure described in Section 4.3.3 is used.

(viii) Steps (v) to (vii) are repeated in cycles until the deflections, $u_i'(t_k)$, at the end of a cycle have converged to the deflections, $u_i(t_k)$ at the start of the cycle within the desired limit of convergence. The values of $u_i(t_k)$ therefore define the deflected shape at time, t_k.

(ix) Steps (iii) to (viii) are repeated for a range of increasing values of sustained load, P, until a maximum load is reached, thus establishing the load-deflection relationship up to the maximum load for a particular time, t_k. The above procedure can be seen to be identical to that described in Section 4.3.3 except that for sustained loading, the applied concrete strain is not the same as the applied steel strain.

(x) The complete load-deflection-time relationship is established by repeating steps (i) to (ix) for a range of values of time, t_k.

CHAPTER 9

RESTRAINED SHRINKAGE TESTS

9.1 INTRODUCTION

In this chapter, tests are described on two built-up composite columns which, after moist curing, were subjected to a period of drying at a low relative humidity before loading to failure. A concrete of known high shrinkage was used in the manufacture of the columns. The aim of the tests was firstly to examine, during the drying period, the build-up of compressive stress in the steel sections and tensile stress in the concrete resulting from shrinkage and the restraint provided by the steel, and then to load the columns to failure to examine the effects of any shrinkage cracks on the short term behaviour of the columns.

The two columns were 7ft. in length and were of identical section to that for the built-up composite columns described in Chapter 5. No battens or lacing were used to connect the two channel sections. The columns, CC6 and CC12, were loaded at an eccentricity of 1.5in. to the minor and major principal axes respectively as defined at (a) and (b) in Fig. 5.1.

In addition, two model columns, CC6-M12 and CC6-M34, were cast in conjunction with column CC6. Both model columns were 30in. in length and 4in. in diameter. CC6-M12 contained two 1in. x 1in. x $\frac{3}{16}$in. steel angles and CC6-M34 contained two 1½in. x 1½in. x ¼ in. angles placed back to back in a cruciform section. These sections are shown to scale in Fig. 9.7. The two different steel sections were used so that the effect of two different degrees of restraint to the shrinkage could be examined.

9.2 MANUFACTURE OF THE SPECIMENS

Columns CC6 and CC12, having an overall size of 8in. x 7in., were manufactured in an identical manner to the built-up composite columns described in Chapter 5. The only difference in the concrete mix was that the $\frac{3}{8}$in. basalt aggregate was replaced by a $\frac{3}{8}$in. breccia aggregate known to produce a high shrinkage of the concrete (see Fig. 7.17). The grading of the aggregate was identical to that given in Fig. 5.6. The dimensions of the steel channels, the maximum initial deflection and the steel properties obtained from control tests are given in Table 9.1. The dimensions and steel properties for the angles used in the two model columns are given in the same Table.

The following control specimens were cast from the same batch of concrete used in each column.

(i) One plain concrete block, 24 in. in length, its crossection of 8in. x 7in. being identical to that of the columns. The formwork is shown in Plate XXIX. This block was not loaded and was used to determine the free shrinkage strains of the concrete for similar drying conditions to the concrete in the column.

(ii) Four 4in. diameter cylinders, 30in. in length with circular steel end plates cast on each end. Two cylinders were used to determine the free shrinkage of the concrete for this particular size. A typical cylinder can be seen in Plate XXX. The other two cylinders were used to determine the creep in tension of the concrete for two different levels of applied stress.

(iii) Twenty-four 8in. x 4in. standard control cylinders. These were used to determine the compressive strength, split cylinder tensile strength and elastic modulus at various ages over the duration of the test programme.

After curing two days, the columns and control cylinders were removed from the formwork and moulds and stored in a fog room at 70°F and 100% relative humidity. After seven days, the columns were removed temporarily from the fog room and brass plugs, ¼in. diameter and $\frac{3}{16}$in. long with a small steel ball embedded in the top, were inserted tightly into holes drilled in the concrete with a masonry drill at the positions which can be seen in Plate XXXVI. The four plugs spaced at 50cm on each face along the length of the column were used to measure the axial shortening of the column with shrinkage together with waterproofed strain gauges attached at midheight to the steel sections before casting. The brass plugs spaced at 10cm across each face at the centre and quarter points of the length were used to measure the lateral shrinkage strains. After attaching these plugs, the columns were returned to the fog room. Similar plugs were also attached at the same time to each face of the 8in. x 7in. x 24in. shrinkage blocks as shown in Plate XXX.

Columns CC6 and CC12 and their corresponding control specimens were removed from the fog room at an age of 36 days and 28 days respectively and then placed in a laboratory with a constant temperature of 70°F and a constant relative humidity of 50%. Measurements of the shrinkage and creep strains for the columns and the control specimens were commenced at this stage.

9.3 EQUIPMENT AND PROCEDURES FOR DETERMINING THE CONCRETE PROPERTIES

9.3.1 Concrete Properties under Short Term Loading

The compressive strength, split tensile strength and the elastic modulus were obtained from the normal tests on standard control cylinders using the same procedures as described in Chapter 5. These properties were measured at the age when drying commenced, the age when the columns were loaded to failure and at another intermediate age. The values of these properties are listed in Table 9.2, each value being the average from two or three cylinder tests. It can be seen from column (8) of this Table that the elastic modulus in compression was reasonably constant over the test period. Unfortunately it was omitted to measure the value at the commencement of drying for column CC6. For column CC12, this value was slightly higher than the values at later ages when a reverse trend is normally expected. This could have been a result of surface shrinkage cracking on the specimens. This cracking is discussed later in Section 9.4.2. It has been assumed that the elastic modulus in compression and also in tension was constant over the test period. For a constant applied stress, the corresponding applied concrete strain can therefore be considered as constant over this time period.

9.3.2 Measurement of the Free Shrinkage Strains

Two galvanised steel bands, with three steel balls embedded at equal intervals around the band to give gauge lengths 120° apart, were clamped at each end of the 4in.dia. by 30in. long free shrinkage specimens to give a spacing of 50cm. As shown in Plate XXXII, a Huggenberger Tensotast demountable extensometer, which can be read to a strain of 2×10^{-6} in./in. over a 50 cm gauge length, was used to measure the shrinkage at daily intervals.

The same extensometer was used to measure the shrinkage on the 8in. x 7in. shrinkage blocks. To simulate the drying of the longer columns, the ends of these blocks were sealed with a waterproofing agent followed by a sheet of thin plastic. An Invar bar with a low coefficient of expansion was used as a reference for the measurements. It can be seen from Plate XXXI that both a vertical and a horizontal reference bar were used. When measuring a vertical deformation, the vertical reference bar was used and the extensometer was held in the vertical position until the measurements were taken. A similar procedure was used for horizontal measurements. This technique was developed by

Holford(182) who had used this same extensometer for measuring shrinkage strains on both slabs and cylinders. He found that it eliminated a source of error, possibly due to inertia or gravitational effects on the mechanical extensometer.

The results of shrinkage measurements on the 4in.dia. and 8in. x 7in. free shrinkage specimens for both columns CC6 and CC12 are shown in Fig. 9.1 as the variation of shrinkage strain with time.

9.3.3 Measurement of Tensile Creep

The concrete creep data for columns CC6 and CC12 was obtained from measurements on two 4in.dia. cylinders 30in. in length, cast from the concrete used in each column and subjected to a sustained tensile stress of constant value. The experimental equipment used was developed by Holford(182) and is described here for completeness. The circular steel end plates cast onto the ends of the cylinders were fitted with four short but different lengths of $\frac{3}{16}$in.dia. screwed rod as shown in Plate XXXIII. There was sufficient bond between the rods and the concrete to develop the tensile strength of the concrete. An end fitting with a spherical seat to ensure a uniform tensile stress, as shown in Plate XXXIII, was then bolted onto the end plates. The two cylinders were then placed in the creep rig shown in Plate XXXIV. A tensile force was applied to the cylinders through the end fittings by a lever, mounted on knife edges, to which was attached a bucket. Lead shot was poured into the bucket until the desired tensile stress was attained, 100 lb./sq.in. in one and 150 lb./sq.in. in the other cylinder. The deformations of these cylinders, subjected to both shrinkage and creep, were measured with the extensometer between galvanised steel bands clamped to the ends of the cylinders as for the shrinkage measurements. Using the well established technique, the strains due to creep alone were obtained by subtracting the shrinkage strains for the unloaded cylinders from the creep and shrinkage strains for the two loaded cylinders. The tensile creep strains measured with time for column CC6 and column CC12 are shown in Fig. 9.2 and Fig. 9.3 respectively.

The tensile creep strain values, defined by the curve of best fit through the experimental points plotted in Figs. 9.2 and 9.3, were divided by the strain initially applied to the concrete by the load (applied concrete strain) and these values were then plotted in Figs. 9.4 and 9.5 respectively. By definition, the ratio of creep strain to the applied concrete strain is the creep coefficient, $\phi_c(T)$. Figs. 9.4 and 9.5 therefore show the variation of creep coefficient, $\phi_c(T)$, with time, T. It can be seen from either of these Figures that there is good agreement between the two curves obtained for two different levels of applied stress and hence applied strain. This verified the writer's assumption used in the analyses of Chapter 8 that the creep strain at any particular value of time is proportional to the applied concrete strain at that time.

When the two built-up composite columns, CC6 and CC12, were finally loaded to failure at an age of 75 days and 84 days respectively and no further creep data was therefore required, increments of load were added to the creep cylinders until tensile failure occurred as shown in Plate XXXV. This established the direct tensile strength values listed in column (7) of Table 9.2. It can be seen that the direct tensile strength values are of the order of 70% of the split cylinder tensile strengths at the same age and listed in column (6) of Table 9.2. For column CC12, one of the 4in.dia. shrinkage cylinders was also placed in the creep rig and loaded quickly to failure with increments of load.

The short term elastic modulus of the concrete in tension was determined from strain measurements taken at each load increment. The results are shown in Fig. 9.6 and also listed in column (9) of Table 9.2. It would appear from the results for column CC12 that after a period of sustained loading, the short term elastic modulus in tension decreases with an increase in the level of the sustained stress. The results for column CC6 show a slight reverse trend. On the basis of these few tests, no definite conclusions could be reached regarding the effect of sustained loading on the short term elastic modulus. It is interesting to note from Fig. 9.6 that the direct tensile strength of the concrete was not significantly affected by previous sustained loading. In fact, the cylinders with the highest level of sustained stress (150 lb./sq.in.) tended to have a slightly higher tensile strength. Further testing would be necessary to fully examine this behaviour.

9.4 COLUMN TESTS

9.4.1 Model Columns

The axial shortening strains for the 4in.dia. 30in. long model columns, CC6-M12 and CC6-M34, were measured with the Huggenberger extensometer in the same manner as for the 4in.dia. creep and shrinkage specimens. When the concrete had cracked within the gauge length, a sudden change in strain was recorded. Beyond this point, the strain measurements became erratic as a result of further cracking. The variation of axial shortening strain with time up to the age when the concrete first cracked is shown in Fig. 9.7. Cracking commenced after 21 days of drying for column CC6-M12 with 5.6% steel yet after only 13 days of drying for column CC6-M34 with 11% steel. The extra steel in column CC6-M34 provided more restraint to the shrinkage which resulted in the earlier age at cracking. The tensile stress, $\sigma_c(T)$ in the concrete at time, T, can be computed from the axial shortening strain, $\varepsilon_{as}(T)$, at that time by simple statics where

$$\sigma_c(T) = \frac{\varepsilon_{as}(T)\ E_s\ A_s}{A_c} \qquad \text{———— (9.1)}$$

and E_s, A_s and A_c are the steel elastic modulus, steel crossectionsl area and concrete crossectional area respectively. It is assumed that the steel is elastic and perfectly bonded to the concrete such that the axial shortening strain of both the steel and the concrete are identical. The tensile strength of the concrete in the model columns was computed from equation (9.1) using the axial shortening strain measured just prior to cracking of the concrete. The tensile strength was 410 lb./sq.in. for column CC6-M12 and 500 lb./sq.in. for column CC6-M34. These values are in reasonable agreement with the values of 390 lb./sq.in. and 460 lb./sq.in. for the direct tensile strengths measured on companion 4in.dia. cylinders in the manner described in Section 9.3.3. This comparison indicated that the direct tensile strength may be a reasonable estimate of the stress to cause cracking in columns in which the shrinkage is restrained by the steel.

The crack patterns on the two model columns can be seen in Plate XXX. For column CC6-M12 with 5.6% steel, a series of six cracks developed, each crack completely encircling the cylinder and being roughly equally spaced along the length of the cylinders. This indicated that the steel-concrete bond was good and that the steel angle sections had provided the desired restraint in the longitudinal direction. The maximum crack width was 0.1mm. Although these cracks were visible to the eye on close inspection, they were marked with a felt-tipped pen so that their positions would be clear in a photograph.

For column CC6-M34 with 11% steel, only two circumferential cracks formed together with one longitudinal crack which extended along the whole length of the cylinder. The steel was to within ½in. of the surface as shown by the crossection in Fig. 9.7. These larger angles provided restraint not only to the longitudinal shrinkage but to the lateral shrinkage as well resulting in a longitudinal crack in line with the toe of one of the angles where the effective area in tension was a minimum. The maximum crack width was 0.5 mm. As can be seen in Plate XXX, the cracks are plainly visible and a felt-tipped pen was not required to mark their positions.

9.4.2 Built-up Composite Columns-Restrained Shrinkage

The axial shortening strains on columns CC6 and CC12 were measured on the surface of the concrete with the extensometer over the brass plugs previously inserted in the concrete and on the steel sections with a strain bridge connected to the electrical resistance strain gauges attached to the steel sections. The columns were held in a vertical position as shown in Plate XXXVI. The variation of axial shortening strain with time is shown at (a) in Fig. 9.8 for column CC6 and at (a) in Fig. 9.9 for column CC12, the average values over the length of the column being plotted.

The axial shortening strain for column CC6 increased steadily but with a decreasing rate with time over the entire drying period of 39 days prior to loading the column to failure. At this latter stage, the axial shortening strain was 220×10^{-6} in./in. which corresponded to a tensile stress in the concrete of 330 lb./sq.in. (equation (9.1)). This value was below the lowest measured value of direct tensile strength of 390 lb./sq.in. No visible cracking which could have extended across the whole crossection, was observed. However, when the faces of the column were examined with a magnifying crack torch, an extensive pattern of surface shrinkage cracks was observed, the cracks being of the order of 0.01mm wide. This pattern was traced at the top of one of the faces with a felt tipped pen and can be seen in Plate XXXVII. As these cracks were in profusion and extended over all faces, it was not attempted to trace the complete pattern. These surface shrinkage cracks result from the shrinkage on the drying surfaces of the concrete being greater than the inner portions which restrain the surface shrinkage. The same pattern of surface shrinkage cracks was also observed on the free shrinkage specimens which contained no steel.

The drying period for column CC12 before loading to failure was 56 days. Visible cracking was observed for this column after only five days. With further drying, the axial shortening strain remained reasonably constant at a value of 110 x 10^{-6} in./in. as shown at (a) in Fig. 9.9. On cracking, further shrinkage only caused the cracks to open wider with no nett increase in the overall axial shortening. From equation (9.1), the tensile stress in the concrete at cracking corresponding to the axial shortening strain at that time was only 160 lb./sq.in. The split cylinder tensile strength and direct tensile strength were measured on a single 4in.dia. cylinder at the time of cracking (age of 33 days). The respective values were 567 and 437 lb./sq.in. It would appear that the column concrete cracked at a considerably lower stress than the direct tensile strength. However, the computed value of 160 lb./sq.in. for the cracking stress could be in error as it was based on the full crossectionsl area of the concrete being effective. With surface shrinkage cracks of any appreciable depth, the effective area in tension could be considerably reduced. Although it was not possible to measure their depth, the surface shrinkage cracks observed for column CC12 would probably have been deeper than those for column CC6 as both the shrinkage strains and the rate of shrinkage were higher for the concrete in column CC12 than in column CC6. This can be seen in Fig. 9.1.

The crack patterns in Fig. 9.10 for column CC12 show the cracks on all four faces which were plainly visible to the eye, the maximum crack width being 0.1mm. The surface shrinkage cracks that could be seen with the aid of a crack torch were not traced, their pattern being similar to that for column CC6 shown in Plate XXXVII. Similar surface shrinkage cracks were observed on the free shrinkage specimens. It can be seen in Fig. 9.10 that there were at least seven distinct cracks that completely encircled the column. It could be assumed that these cracks extended right through the crossection. It was interesting to note that longitudinal cracks formed in the centre of the north and south faces. This was due to the lateral shrinkage being restrained by the webs of the channels in the north and south faces compared to the east and west faces where the concrete was not restrained from shrinking, the two channels being free to move with the concrete because of the absence of battens between the channels.

9.4.3 Built-up Composite Columns - Loading to Failure

The procedures used were identical to those described in Chapter 5 for the short term testing of built-up composite columns.

Column CC6 was loaded at an eccentricity of 1.5in. to the minor axis. It will be recalled from the previous Section 9.4.2 that there were no visible cracks which extended through the crossection although small shrinkage cracks were present on the surface. The resulting load-deflection curve up to the maximum load is shown at (b) in Fig. 9.8. Cracking at midheight on the convex face was observed when the load had been increased to 20 kips. On reaching the maximum load of 147.0 kips, the load was decreased with increasing deflection. When the load had dropped to 135 kips crushing of the concrete commenced on the concave face slightly above midheight as shown at (a) in Plate XXXVII. The test was stopped when the load had dropped to 57 kips at which point the central deflection was 1.5 in. The extent of the concrete crushing on the concave face at this stage can be seen at (b) in Plate XXXVII, the spalled concrete being removed by hand to show the crushing zone to advantage. The pattern of cracking on the convex face is shown in Plate XXXVIII, a felt-tipped pen being used to highlight the position of the cracks.

Column CC12 was loaded at an eccentricity of 1.5in. to the major principal axis. For this column, quite distinct cracks were present in the column before loading, many of which encircled the column and were likely to extend clear through the crossection. The observed load-deflection relationship is shown at (b) in Fig. 9.9. With increasing load, the visible cracks on the compression face were observed to close while the cracks on the tension face remained open and widened. On reaching the maximum load of 154.8 kips, the deflections were increased further for decreasing load. Concrete crushing was first observed on the concave face below midheight when the load had been decreased to 121 kips. The column test was stopped when the central deflection had been increased to 1.2in. at which point the load had dropped to 81.3 kips. The extent of crushing and cracking on all four faces of the column are shown in Plate XXXIX. The tensile crack opposite the crushing zone had opened quite considerably and is plainly visible without being marked by a felt-tipped pen. The failure below mid-height coincided with the position of a large proportion of the shrinkage cracks which were observed before loading and are shown in Fig. 9.10.

9.5 DISCUSSION OF RESULTS

9.5.1 Prediction of Restrained Shrinkage Strains

The observed and theoretical variation of axial shortening strains with time are compared in Figs. 9.7, 9.8(a) and 9.9(a) for the model columns, column CC6 and column CC12 respectively. The rate of creep analysis presented in Section 8.2.2 was used to determine the theoretical values, in particular equations (8.19) and (8.21). The creep and shrinkage data required in this analysis are the variation of free shrinkage strain with time and the variation of creep coefficient with time. For the 4in.dia. model columns CC6-M12 and CC6-M34, the shrinkage data is given in Fig. 9.1 by the curve marked CC6 4in.dia. and the creep data by the average of the two curves given in Fig. 9.4.

For the built-up composite columns, CC6 and CC12, the shrinkage data is given in Fig. 9.1 by the curves for the corresponding 8in. x 7in. specimens. However, no creep data was obtained from tests on 8in. x 7in. specimens because of their size and the problems associated with applying a constant sustained tensile stress to such specimens. The effect of size and shape on shrinkage and creep has been discussed in Section 7.4. It will be recalled from this Section that Hansen and Mattock(158) found that the shrinkage-time and creep-time curves were similar in shape and that the volume to surface area ratio affected these shapes in an identical manner. It has therefore been assumed that

$$\left[\frac{\phi_c(T)}{\varepsilon_{cs}(T)}\right]_{\substack{\text{4in.dia.}\\ \text{cylinders}}} = \left[\frac{\phi_c(T)}{\varepsilon_{cs}(T)}\right]_{\substack{\text{8in.x 7in.}\\ \text{specimens}}} \qquad (9.2)$$

where $\phi_c(T)$ and $\varepsilon_{cs}(T)$ are the creep coefficient and shrinkage strain respectively at time, T. As values of $\phi_c(T)$ and $\varepsilon_{cs}(T)$ with time, T, are known for 4in.dia. cylinders and values of $\varepsilon_{cs}(T)$ with time, T, are known for 8in. x 7in. specimens from columns CC6 and CC12, the variation of $\phi_c(T)$ with time, T, for the 8in. x 7in. specimens can be computed from equation (9.2). It was in this manner that equivalent tensile creep data was established for the built-up columns CC6 and CC12.

It can be seen from the comparisons in Figs. 9.7, 9.8(a) and 9.9(a) that the analytical results were in close agreement with the observed results which indicated that the rate of creep method was sufficiently accurate in predicting restrained shrinkage behaviour. It should be noted that the analysis is only valid while the section remains uncracked. By specifying a tensile strength for the concrete, the analysis can also be used to determine the time at which cracking will occur.

Typical results that can be computed with the analysis are given in Fig. 9.11 where the variation of steel and concrete stress with time are shown for columns having steel percentages of 1%, 5%, 10% and 20%. The creep and shrinkage data used for the results in Fig. 9.11 was that for column CC6 with an 8in. x 7in. crossection. It can be seen that for high percentages of steel, the compressive stresses in the steel are low while the corresponding tensile stresses in the concrete are high with the likelihood that the concrete will crack. For low percentages of steel, the tensile stresses in the concrete are low and hence the possibility of cracking is reduced but this is at the expense of high compressive stresses being developed in the steel.

9.5.2 Effect of Shrinkage Cracks on the Short Term Loading Behaviour

Of particular interest are the load-deflection curves shown in Figs. 9.8(b) and 9.9(b). The observed load-deflection curves are compared with the theoretical curves computed using the analysis given in Chapter 4. From the short term tests in Chapter 5, this analysis is known to give an accurate prediction of the actual behaviour under load. For column CC6 which was effectively uncracked before loading, excellent agreement was again obtained between the observed and theoretical load-deflection relationships.

This was not the case for column CC12 which, prior to loading, was known to contain visible shrinkage cracks which encircled the column and were therefore likely to extend through the crossection. The comparison in Fig. 9.9(b) between the experimental deflections and the theoretical values, the latter computed assuming an initially uncracked section, demonstrates the effect of shrinkage cracks on the column behaviour. The portion OA of the experimental curve indicates the region where the cracks on the compression side were closing up. It can be seen that the stiffness of the column in this region was less than that for an initially uncracked column yet greater than that for a bare steel column provided the two steel sections are constrained to maintain their relative crossectional positions. This indicated that the concrete between the cracks was still providing additional stiffness to that of the bare steel section. Once point A had been reached on the experimental curve, it was evident from the change in stiffness that the cracks on the compression side had completely closed and the column acted as a normal uncracked member but with somewhat higher steel stresses and an increased column deflection. The shrinkage cracking acted as a large initial imperfection which increased the deflections and lowered the ultimate load. The theoretically predicted maximum load was 178.4 kips and the observed maximum load was 154.8 kips which indicated a significant reduction in the load carrying capacity of 14 percent.

The results of the short term loading test on column CC3 have been presented in Chapter 5. This column was identical in length and section to column CC12 and was loaded at exactly the same eccentricity of 1.5in. to the major principal axis. The experimental maximum loads for these two columns cannot be directly compared as the steel yield stress and concrete compressive strength were not the same for both columns as shown in Table 9.3. It has already been discussed in Section 7.6 how the use of breccia aggregate instead of basalt aggregate gives higher concrete strengths for the same concrete mix. Therefore, the observed maximum loads are expressed in Table 9.3 as a ratio of their respective short column loads, calculated from equation (4.7) which takes account of the steel and concrete strengths. The ratio for column CC3 is 0.495 and yet the ratio for column CC12 is only 0.407. On the basis of these ratios, the presence of shrinkage cracks reduced the load carrying capacity by 18 percent.

The test results for column CC1 have been presented in Chapter 5. This column was identical in length and section to the shrinkage column CC6 and was loaded at the same eccentricity of 1.5in. to the minor principal axis. The observed maximum loads for these two columns are also expressed in Table 9.3 as a ratio of their respective short column loads, the ratio for column CC1 is 0.394 and the ratio for column CC6 is 0.378 which indicates a reduction in load carrying capacity of only 4 percent. This slight reduction could have been the result of the surface shrinkage cracks present in column CC6 prior to loading but the percentage difference is small and probably within the order of accuracy of the determination of the material strengths, especially the concrete. By comparing Plates XIV and XXXVIII, it can be seen that the crack patterns on the tensile face, as a result of the eccentric loading, are almost identical for the comparison columns CC1 and CC6 with the cracks roughly equally spaced over the full length of both columns. This also indicated the close similarity in the behaviour of the two columns.

The effect of visible shrinkage cracking on the behaviour of column CC12 is further demonstrated in Fig. 9.12 where the observed and theoretical strain distributions at midheight are compared. The observed strain distributions have been based on the strains measured on the steel sections as the shrinkage cracks affected the strains on the concrete surface. The strain gauge on the compression (north) face was positioned directly over a shrinkage crack and for the first increment of load of 10 kips, it can be seen that a very large strain value was recorded as a result of this crack closing up under load. The strain gauge on the tensile (south) face was positioned adjacent to a crack and it therefore registered little strain as deformation on this face resulted in the cracks only opening wider. The comparison between the theoretical and observed strain distributions shows that the actual strains on the steel were well in excess of those predicted for an initially uncracked section. It has already been shown in Chapter 5 that there was good agreement between the observed and theoretical strain distributions for initially uncracked columns. The higher steel strains resulted from the steel sections at the crack positions having to resist most of the applied load and moment until the cracks had closed on the compression side and the concrete became fully effective.

It is appreciated that in the symmetrical single curvature mode of bending used, the magnitudes of the shear forces developed with bending are low. Therefore the effect of shrinkage cracks on the ability of the concrete alone to transmit the shear between the steel components was not examined in these tests. This problem should be the subject of a further investigation.

CHAPTER 10

BUILT-UP COMPOSITE COLUMNS - SUSTAINED LOAD TESTS

10.1 INTRODUCTION

10.1.1 General

Four columns, identical in section to the built-up columns subjected to short term loading as described in Chapter 5, were subjected to a constant sustained load. All four columns were bent about the major principal axis as defined at (b) in Fig. 5.1 as this was also the principal mode of bending for the short term tests described in Chapter 5. With the limited time available for this phase of the work, it was not intended to develop an extensive research programme to cover all aspects of the sustained load behaviour of composite columns. Rather the tests were designed to observe the load-deflection behaviour with time and to obtain reliable experimental data for a select range of loading conditions such that this data could be used to check the validity of the analyses presented in Chapter 8. If the analyses could be verified by these experimental results, it would then be considered reasonable to establish the entire load-deflection-time relationship with the analyses rather than with further tests which require measurements to be taken over a long period of time.

The first column placed under test was column CC7, 7ft. in length. This was subjected to a sustained load of 30% of the short term load carrying capacity. This sustained load of 48 kips was considered to represent a typical value of the working load for such a column. To perform the tests, two creep rigs had to be designed and manufactured, one for the column and the other for a plain concrete specimen from which the creep data for the analysis could be obtained. The writer was then fortunate that four creep rigs, used by Holford (182) to examine the behaviour under sustained load of concrete slabs with holes representing diaphragms from concrete box girders, became available. These rigs were then modified to take columns although the maximum column length was restricted to a value of 68 inches, this being shorter than the 84 in. lengths used for the short term loading tests. Three of these creep rigs were used to apply sustained loads of 96, 112 and 80 kips at eccentricities of 0, 0.8 and 2.0 in. to columns CC8, CC9 and CC10 respectively, the latter column being bent in double curvature. The values of sustained load selected were 40% of the short term load capacity, these again representing typical working load values for the columns. Much higher values of load that might produce creep instability were not possible as the creep rigs were limited to a load capacity of 112 kips. The remaining creep rig was used to apply load simultaneously to three plain concrete specimens cast with the three columns which were also loaded at the same time as the loading of the creep specimens.

10.1.2 Manufacture of the Test Specimens

The columns were manufactured in the same manner as described in Chapter 5 for the built-up columns subjected to short term loading. As for the restrained shrinkage tests of Chapter 9, a breccia aggregate was used to produce a concrete with a high shrinkage and creep so as to magnify and hence highlight the creep deformations of the columns. The following control specimens were cast with each column.

(a) Two plain concrete prisms, 24 in. in length having the same overall crossection as the columns of 8in. x 7in. The formwork used is shown in Plate XXIX. One concrete block was used to determine the free shrinkage strains and the other to obtain the creep strains under a constant applied stress in a creep rig.

(b) Ten standard 6in. x 12in. concrete cylinders, these being all that could be cast from one concrete batch together with the column and the creep and shrinkage blocks.

Two days after casting, the column and control specimens were removed from their forms and placed in a fog room. As for the restrained shrinkage tests of Chapter 9, the column and the creep and shrinkage blocks were removed temporarily from the fog room so that brass plugs could be inserted into the concrete. These were to be used to measure the creep and shrinkage strains on the blocks and the strains at midheight on the column with the Huggenberger extensometer. The columns and control specimens were then returned to the fog room.

10.2 EXPERIMENTAL EQUIPMENT AND PROCEDURES

Columns CC7, CC8, CC9 and CC10 were removed from the fog room after 74, 56, 49 and 42 days moist curing respectively at 100% relative humidity and a temperature of 70°F and transferred to a laboratory with a controlled environment of 70°F and 50% relative humidity. At this point drying of the concrete commenced. Columns CC8, CC9 and CC10 were removed from the fog room at the same time having previously been cast at weekly intervals. Column CC7 and its companion creep block were immediately placed under load. Columns CC8, CC9 and CC10 were allowed to dry for 9 days before loading. This was to enable ERS strain gauges to be attached to a dry surface of the concrete on the columns and the creep blocks.

Details of the creep rigs for column CC7 are shown in Fig. 10.1 and Plate XLI. The creep rigs for columns CC8, CC9 and CC10 were similar in form and are shown in Plates XLII and XLIII. With reference to Fig. 10.1 and the Plates, the procedure for loading a column is as follows:

(i) The adjustment plates and rockers shown in Fig. 10.2 were attached to the loading plates at the required end eccentricity, the bottom loading plate having previously been attached to the steel channels prior to casting and the top loading plate being attached after casting using the capping procedure described in Section 5.2.

(ii) The column was then lifted into position between the top and bottom loading beams of the creep rig with a portable crane.

(iii) The nuts above the top loading beam on the four 1 in. dia. screwed tension rods were tightened sufficiently to hold the column in position.

(iv) The column end rockers were centred on the bearing plates, attached to the top and bottom loading beams, by tapping the column into position with a leather hammer.

(v) The dial gauge frame was then attached to the loading plates at the ends of the column. The manner in which this is done is demonstrated in Plate XL. The dial gauge frame is pivoted on the bottom plate by a ball race and is held against a roller bearing at the top by a spring. This technique ensured that the dial gauge frame maintained a constant reference plane in relation to the column irrespective of any axial shortening or end rotation of the column under load. It also had the advantage that any movement of the creep rig as a whole did not affect the deflection measurements.

(vi) Dial gauges with a least count of 0.0001 in. were attached to the frame to measure the deflections at the centre and the quarter points. For column CC10 bent in double curvature, the deflections were also measured at the eighth points.

(vii) Four 40,000 lb capacity hydraulic jacks were inserted between the intermediate plate and the base plate at the bottom of the creep rig and connected to a common pressure line.

(viii) The load was applied to the column by jacking between the base plate and the intermediate plate which compressed the springs putting compressive load in the column and tension in the four screwed rods. Details of the springs are given in Table 10.1. The load was increased in six increments up to the desired sustained load, measurements of deflection and the strains at midheight being taken at each increment.

(ix) On reaching the selected sustained load (48, 96, 112 and 80 kips for columns CC7, CC8, CC9 and CC10 respectively), the nuts underneath the intermediate plate were screwed up finger tight against this plate. The hydraulic jacks were then removed and placed under the next creep rig.

(x) The plain concrete blocks for determining the creep of the concrete under a constant uniform stress were loaded in the same manner with a load of 64 kips. To do this, ground steel plates were attached to the ends of the blocks with dental plaster and checked with a spirit level to ensure that the ends were parallel to each other and perpendicular to the longitudinal axis of the specimens. A load/unit area of 1140 lb./sq.in. was applied to all of the creep blocks, this being approximately the mean value for the range of stresses developed in the columns. Similar concrete blocks for measuring the shrinkage strains were left unloaded near the creep rigs.

(xi) Measurements of column deflections, column strains, creep strains and shrinkage strains were taken at daily intervals for the first two weeks and then at less frequent intervals as the rate of creep and shrinkage diminished. The positions of the ERS strain gauges and the extensometer targets on the specimens can be seen clearly in Plates XLI, XLII and XLIII.

(xii) The load loss in the springs due to shortening of the specimens was measured by dial gauges between the intermediate plate and the bottom loading beam to obtain the change in length of the springs. To keep the losses to within 5%, it was found necessary to adjust the load for column CC7 with the hydraulic jacks after 161 days under load. As shown in Table 10.1, the spring rate for the column CC7 creep rigs was a lot higher than the spring rate for the columns CC8, CC9, CC10 creep rigs. The maximum loads for the springs in this Table were the loads for the springs to go solid. The safe loads were based on a shear stress in the wire of the spring of 100,000 lb./sq.in. The spring manufacturer recommended that the stress be limited to this value to prevent relaxation losses in the springs.

(xiii) For column CC7, the creep rigs were designed to have adequate clearance between the screwed tension rods and the side faces of the column to enable strain measurements to be taken on these faces with the extensometer. This was not the case for the other creep rigs which had previously been used to load slabs (182). It was therefore necessary to use ERS strain gauges and it was for this reason that columns CC8, CC9 and CC10 were allowed to dry for a period so that ERS gauges could be affixed. To check the stability of these gauges over a long time period, similar gauges were also attached to the creep blocks in the same positions where strains were measured with the extensometer.

10.3 MATERIAL PROPERTIES

10.3.1 Short Term Loading Properties

(a) Steel

Control specimens were cut from the channels and tested in the same manner as described in Section 5.3.1. The values of yield stress obtained from stub column, coupon tension and Hounsfield tension tests are listed in Table 10.2 together with the dimensions of the channel sections. The elastic modulus of the steel has been taken as 30×10^6 lb./sq.in.

(b) Concrete

Because of the limited number (ten) of cylinders that could be cast from each batch of concrete, the concrete properties were measured from five cylinders at two ages only; 4 days after the columns and creep blocks were loaded and approximately 950 days later. Two cylinders were used to measure the elastic modulus, two for the compressive strength and one for the split cylinder strength. As the stresses for the two modulus cylinders were only taken up to 50% of the compressive strength, one of these cylinders was tested to failure in compression and the other in tension. The elastic modulus at time of loading was measured from the incremental loading of the creep blocks. The results of these control tests are shown in Table 10.3.

Some interesting trends can be observed from this Table. The compressive strength, split tensile strength and modulus of elasticity all decreased with increasing time and drying. These results show a reverse trend to those attributed to Gilkey(197) which have so often been quoted in tests such as Troxell, Davis and Kelly(198) that it is generally accepted that concrete tested in the air dry condition has higher strengths than the same concrete tested wet and that strength increases with age whether cured moist or dry. More recent literature such as that by Butcher(199) and Grieb and Werner(200) would indicate that this is by no means always the case. Roper(201) has examined the influence of breccia aggregate on the concrete properties. He found that for concrete cylinders cured moist, the strength and modulus increased with time and that at 90 days, the values remained reasonably constant. However, after a further period of drying in air, the values for the concrete containing basalt or river gravel aggregate still remained reasonably constant whereas the values for concrete containing breccia aggregate decreased which verified the writers results. This reduction in strength is probably due to cracking between the aggregate and paste which was observed by Roper(202) in petrographic studies of concrete containing shrinking aggregate. As for the restrained shrinkage tests described in Chapter 9 in which breccia aggregate was also used, extensive surface cracking could be observed with a crack torch on the faces of the concrete.

10.3.2 Time-dependent Properties

(a) Shrinkage

The strains measured on the companion free shrinkage blocks for all four columns are shown in Fig.10.3. The ends of the blocks were sealed to simulate the drying conditions of the columns. It can be seen that quite large shrinkage strains of the order of 1000×10^{-6} in./in. were obtained after 1000 days of drying. At this stage the concrete was still shrinking at a steady but slow rate, most of the shrinkage taking place in the first 200 days.

(b) Creep

The combined creep and shrinkage strains were measured from the creep blocks loaded at a constant stress of 1140 lb./sq.in. The values measured with the ERS strain gauges were almost identical to those measured with the extensometer even after 900 days of loading. This demonstrated that the ERS gauges were still stable after a long period of time and that the results of strains measured on the columns with similar ERS gauges could be used with confidence. The creep strains were obtained by subtracting the shrinkage strains measured on the free shrinkage blocks from the combined creep and shrinkage strains. The variation of creep strain with time for all four columns is shown in Fig. 10.4. It can be seen that the creep strains were considerably higher for column CC7. This was a result of loading the column and the creep block immediately drying commenced whereas the other three columns and creep blocks were not loaded until after 9 days drying.

The creep coefficient, $\phi_c(t)$ at any particular time, t, was obtained by dividing the creep strain at that time by the applied concrete strain, the latter being the ratio of the applied stress to the short term elastic modulus at that time. Taking into account the measured spring losses in the creep rigs and using a linear interpolation for the short term elastic modulus between the ages at which it was measured, the variation of creep coefficient, $\phi_c(t)$ with time, t, of loading for each of the four columns is as shown in Fig. 10.5.

10.4 COMPARISON OF THEORETICAL WITH EXPERIMENTAL RESULTS

10.4.1 Concentric Loading

Column CC8 was subjected to a sustained axial load of 96 kips, this being 30% of the maximum load capacity under short term loading. At first loading, the central deflection was 0.0010 in. and after 987 days of sustained load, this had increased to only 0.0019 in. indicating that the column was indeed concentrically loaded. Therefore the variation of axial shortening strain with time was plotted in Fig. 10.6 rather than the variation of central deflection with time which was extremely small.

The observed axial strains were compared in Fig. 10.6 with theoretical values calculated using both the rate of creep analysis in Section 8.2 (equations (8.19) and (8.21)) and the effective modulus method in Section 8.3 (equation 8.31). The curvature was assumed to be zero and only axial strains were computed. The creep and shrinkage data used in the analyses were obtained from the curves in Fig. 10.5 and Fig. 10.3 respectively for column CC8. It can be seen that the rate of creep analysis gives a much better representation of the actual behaviour than the effective modulus method. It will be recalled from Chapter 8 that the rate of creep method attempts to account for previous stress history by summing the elemental increments of creep over small increments of time whereas the effective modulus method ignores any previous stress history, the creep at a particular time being related only to the applied stress, and hence the applied strain, and the creep coefficient at that time. The rate of creep method is therefore most likely to be a better model of the real creep behaviour than the effective modulus method.

It is interesting to note the changes in the load sharing with time between the concrete and steel. After nine days drying before the load was applied, a computed compressive stress of 2160 lb./sq.in. was developed in the steel and a tensile stress of 110 lb./sq.in. in the concrete. On application of the load at 9 days, the compressive stress in the steel was increased to 12720 lb./sq.in. and the concrete was placed in compression with a stress of 1170 lb./sq.in. After 906 days of drying with the sustained load applied during which time further creep and shrinkage took place, the compressive stress in the steel had been increased to 37800 lb./sq.in. (yield stress 44,210 lb./sq.in.) and the stress in the concrete had reduced to a value of 80 lb./sq.in. in tension. With further drying, it could be conceived that the concrete could crack in tension as it can be seen from Figs. 10.3, 10.5 and 10.6 that the shrinkage, creep and axial strain respectively were all still increasing, albeit slowly, even after 906 days of drying. The phenomenon of a concrete column, containing longitudinal steel and carrying a compressive axial load, in which the concrete is in tension has also been observed by Troxell, Raphael and Davis(161) from long term loading tests on stocky reinforced concrete columns for which there was no likelihood of creep instability.

10.4.2 Eccentric Loading - Single Curvature

The 7ft. column CC7 was subjected to a sustained constant load of 48 kips at an eccentricity of 1.6 in., this load being 29% of the theoretical load capacity for short term loading. The load was applied at the same time drying commenced. Column CC9, 5ft. 8in. in length, was subjected to a constant sustained load of 112 kips at 0.8 in. eccentricity, this being 42% of the theoretical short term load carrying capacity. The load was applied 9 days after drying commenced. The variation with time of the central deflection is shown in Fig. 10.7 for both these columns. Deflections of the order of six times and four times the initial deflections at first loading were obtained for columns CC7 and CC9 respectively, most of the creep deformation taking place in the first 200 days.

The deflections computed with the effective modulus method, shown as broken lines in Fig. 10.7, compare favourably with the experimental deflections, the maximum variation in the deflections at any value of time being 9% for column CC7 and 10% for column CC9. Goyal and Jackson(154) found similar close agreement using an effective modulus approach to predict the deflections of reinforced concrete columns under sustained eccentric loading. On the basis of the results shown in Fig. 10.6 for the concentrically loaded column CC8, the writer is of the opinion that the rate of creep analysis would give an even closer agreement. The rate of creep analysis described in Section 8.2.3 for eccentrically loaded columns has been used to obtain theoretical results but the writer has not been prepared to accept these results as there have been convergence problems associated with some cases of loading, the analysis involving a number of iterative procedures. Therefore, no conclusions can yet be reached concerning the accuracy of the rate of creep analysis for eccentrically loaded columns until it has been verified for a range of loading cases.

10.4.3 Eccentric Loading-Double Curvature

Column CC10 was subjected to a constant sustained load of 80 kips at equal but opposite end eccentricities of 2.0 in. This load represented 42% of the theoretical short term load capacity. The manner in which the column was loaded is clearly shown in Plate XLII. The load was applied 9 days after drying commenced. The variation with time of the upper quarter point deflection is shown in Fig. 10.7 and deflection profiles for a range of values of time under load are shown in Fig. 10.8. With reference to this latter Figure, it can be seen that the initial imperfection was such that the deflections on first loading were larger for the upper half of the column than the lower half. The larger moments and hence stresses in the upper half resulted in the creep deflections also being larger in this region and increasing with time at the expense of the lower half deflections which decreased slightly with time. It was not until after 48 days of loading that the deflections in the lower half began to increase slowly with time.

Although the theoretical deflections, computed with the effective modulus analysis and shown as broken lines in Fig. 10.8, did not coincide with the observed deflections, reasonable agreement was still obtained over most of the column length. This loading case of large equal but opposite end eccentricities for a short column represents the condition of maximum shear for a pin-ended column. Although no battens or lacing were used for shear transmission between the steel components, there were no indications that the concrete alone was not capable of doing this, even under sustained load with the possibility of creep in shear of the concrete. The test columns are still under load and measurements being taken. Until all the results of strains measured at midheight with strain rosettes, linear strain gauges and the extensometer have been fully processed, analysed and compared with a reliable theoretical model of concrete creep in shear, no definite conclusions can be reached regarding the influence of shear creep of the concrete on the column behaviour. This aspect is still under investigation by the writer. However, the indications are that shear effects are relatively small as evidenced by the agreement in Fig.10.8 between the observed and theoretical deflections, the latter being determined from an effective modulus analysis in which the shear deformations were not accounted for.

10.5 DISCUSSION

As reasonable agreement was obtained between the observed and theoretical deflections, it was considered that the effective modulus analysis could be used to extend the results and to determine the full load-deflection-time relationships for the eccentrically loaded columns. Load-deflection relationships were computed for each column with the effective modulus analysis for six different values of loading time. These are shown as the full lines in Figs. 10.9, 10.10 and 10.11 for columns CC7, CC9 and CC10 respectively.

For columns CC7 and CC9 bent in single curvature, it can be seen that the maximum load capacity decreases with time as indicated by the broken line marked "failure limit". The boundary between the loads that will cause collapse and the loads that will lead to shakedown with time is clearly defined in Fig. 10.9 and 10.10. The value of load at this boundary would be lower still if the time under load was increased beyond the values of 1389 and 897 days used in these Figures. These were the times at which creep and shrinkage data was last measured prior to computing the results. Measurements are still being recorded on the creep and shrinkage blocks and the columns which will enable the boundary value to be obtained for later ages.

The load factor against collapse for the test column CC7 was 3.42 for short term loading (t = 0 days) but this had reduced to 2.0 after 1389 days under load. For column CC9, the load factor was reduced from 2.35 to 1.64 after 897 days under load.

It can be seen from Fig. 10.11 that there was no reduction in load capacity with time for column CC10 bent in double curvature. Although the deflections increased with time, their magnitude was insufficient for the point of maximum moment to occur within the length of the member. The maximum occurred at the ends of the column and was therefore notamplified by creep deflections, the value of moment being given simply by the load times the end eccentricity. For the columns bent in single curvature, the maximum moment occurred at midheight of the column, its value increasing with time as the deflections increased. The load capacity for the double curvature column CC10 was therefore governed only by the strength of the end section for which the moment was proportional to the applied load. In the analysis, the compressive strength of the concrete in column CC10 was assumed constant at a value of 85% of 6000 lb./sq.in., this being the average of the cylinder strengths listed in column (5) of Table 10.3. If the concrete strength was assumed to have decreased with time under load as suggested by Rusch(76) and indicated in Fig. 7.3, the broken line indicating the failure limit would also have shown a decrease in load capacity with time.

It could fairly be said that the pin-ended column bent in symmetrical single curvature represents the case for which the maximum possible reduction in load capacity with time under sustained loading is obtained. For all other cases, including columns with end restraints, the creep deflections and the associated amplification of moments and reduction in load capacity with time will not be as severe.

CHAPTER 11

DESIGN OF COMPOSITE COLUMNS

11.1 INTRODUCTION

It could be said that the reinforced concrete column normally used in practice is relatively stocky compared with the bare steel column in normal use and that stability is therefore a more serious problem for the latter. The practical range of slenderness values for composite columns would most likely lay between that for steel columns and that for reinforced concrete columns. Slenderness effects are therefore likely to be important for composite columns. Ideally, a design procedure for columns should involve a rational second order structural analysis to determine the forces and moments in each member of the structure and then a means of proportioning the crossection to resist these forces and moments. The analysis should consider the effects of column slenderness, end restraints, column deflections and overall frame deflections on the forces and moments and the effects of these forces and moments on the member stiffnesses. Although such analyses have been developed for both steel and reinforced concrete columns (eg. those given by references(183) (184) and (185)), most design codes still allow empirical methods which are simple and generally conservative.

A common method used in steel codes is one in which an equivalent pin-ended member, having a length equal to the effective length (between points of contraflexure), is used to represent the restrained column continuous in the frame. The early background to this method can be found in the three reports of the Steel Structures Research Committee published by HMSO between 1931 and 1936 for the Department of Scientific and Industrial Research in Britain. Much of the work of this Committee was embodied in British Standard BS 449. In the

United States, the equivalent beam-column approach has been based on work by Winter(186) and associates and is discussed in the Column Research Council Design Criteria for Metal Compression Members(43). A similar method for reinforced concrete columns has been discussed by MacGregor, Breen and Pfrang(187).

It is only in recent times that design methods for composite columns have been included in design codes in which the concrete is permitted to provide both additional strength and stiffness to that of the bare steel column. Some present methods, based on the equivalent beam-column approach, are discussed below.

11.2 DESIGN METHODS

11.2.1 Working Stress Method

Both BS449 : Part : 1971(188) and AS CA1-1968 (189) are similar in nature and are based on working stresses in the steel alone. Composite columns are restricted to the cases of single rolled steel I-sections encased in concrete and two channels encased in concrete with the channels placed back to back and spaced not more than the depth of the channel with battening or lacing as required for a bare steel column. Therefore built-up composite columns are virtually excluded by these two codes.

The basic design requirement is that

$$\frac{f_c}{p_c} + \frac{f_{bc}}{p_{bc}} \not> 1$$

where f_c = calculated average axial compression stress at working loads

f_{bc} = calculated compressive stress due to bending from all sources

p_c = allowable compressive stress in axially loaded struts

p_{bc} = appropriate allowable compressive stress in bending

In determining the value of p_c for axial compression, the concrete is considered to provide additional stiffness about the minor axis but none about the major axis. No account is apparently taken of the concrete in evaluating f_{bc}. The value of p_{bc} is the permissible bending stress for the steel section alone. This latter requirement is particularly conservative, especially for bending about the major axis of the steel section where the concrete can provide considerable torsional rigidity thereby preventing lateral torsional buckling associated with this mode of bending. Another restrictive requirement is that the maximum working load is limited to twice the capacity of the bare steel section.

Roderick and Rogers(13) have already compared this design method with a range of encased rolled steel joists and encased universal sections and have shown that the method is very conservative and somewhat irrational. In all cases, higher load factors are required by the codes for composite columns than for bare steel sections. Higher load factors were also required for eccentrically loaded composite columns than those normally specified for the design of concentrically loaded members.

11.2.2 Ultimate Strength Method

An ultimate strength design method for composite compression members has been included in Section 10 of the ACI Building Code 318 - 71(190) where such members shall include "all concrete compression members reinforced longitudinally with structural steel shape, pipe or tubing, with or without longitudinal bars". The writer has assumed that this definition applies to his columns. The method is essentially the same as for the reinforced concrete columns. From the published Commentary(191) on this Code, the section on composite columns has been based on investigations(22)(192)

(193) into the behaviour of encased steel I-beams and concrete filled tubes. No reference was made to any work on composite columns containing built-up steel sections. So that reference can be made conveniently to the design method, it is briefly summarised below using the same notation as ACI 318-71.

(i) The method is only applicable for members with slenderness, kl_u/r, less than 100 where kl_u is the effective length of the column and r is the radius of gyration.

(ii) The radius of gyration, r, of the composite section shall not be greater than

$$r = \sqrt{\frac{\frac{1}{5}E_c I_g + E_s I_t}{\frac{1}{5}E_c A_g + E_s A_t}} \qquad \text{(11.1)}$$

where E_c = concrete elastic modulus

E_s = steel elastic modulus

A_g = gross area of the section

A_t = area of structural steel

I_g = second moment of area of the gross section about the centroidal axis

I_t = second moment of area of the structural steel section about the centroidal axis

(iii) Compression members shall be designed using the ultimate axial load, P_u, from a conventional frame analysis and a magnified ultimate moment, M_c, where

$$M_c = \delta M_2 \qquad \text{(11.2)}$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{\phi P_{cr}}} \geq 1.0 \qquad \text{(11.3)}$$

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} \qquad \text{(11.4)}$$

$$EI = \frac{\frac{E_c I_g}{5} + E_s I_t}{1 + \beta_d} \qquad \text{(11.5)}$$

and $$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \nless 0.4 \qquad \text{(11.6)}$$

for members braced against sidesway.

M_1 = value of the smaller end moment, positive if member is bent in single curvature and negative if bent in double curvature.

M_2 = value of the larger end moment, always positive.

ϕ = capacity reduction factor.

β_d = ratio of maximum design dead load moment to maximum design total load moment.

(iv) A member of adequate size is attained when the point defined by the ultimate axial load, P_u, and the ultimate magnified moment, M_c, is on or within the bounds of the interaction envelope for the section.

It can be seen that the above method assumes that the full load and moment capacity of the crossection is attained when the maximum load is reached. This is identical to the behaviour demonstrated in Fig. 5.29 for the 7ft. and 10ft. built-up composite columns where the load and moment capacity of the section, as defined by the interaction envelope, can be achieved. However, it can be seen that for the slender 15ft. and 20ft. columns that the load capacity for these columns is reached at a point well within the interaction envelope. The ACI method places a limit on the slenderness of the column of $k_u l/r \leq 100$ for which the ACI Commentary(191) states as "representing the upper range of actual tests on slender compressions members". In this case, the reference is to reinforced concrete columns. There is no reference to the fact that for slender columns, the maximum load is reached at a point within the interaction envelope. Using this limit of $kl_u/r \leq 100$ and typical values of E_c, I_g, A_g, E_s, I_t and A_t for the writer's built-up composite columns, this would place a limit of 12ft. on the length that could be designed by the ACI method. It can be seen from Fig. 5.29 that the interaction curve for a 12ft. column would lay within the interaction envelope having intermediate values between those for the 10ft. and 15ft. curves shown. From a design point of view, it would not differ greatly from the interaction envelope, but the writer feels that this slenderness limit could be a little high, say 20%. It should be noted however that this limit represents very slender columns that would be encountered in practice only on the odd occasion.

The form of the moment amplification factor given in equation (11.3) is identical to that used by the AISC Specification(39) and Australian Standard AS 1250-1972 "SAA Steel Structures Code" for steel columns. The $\dfrac{1}{1 - \dfrac{P_u}{\phi P_{cr}}}$ portion of equation (11.3) is the moment magnification factor for the moment at the centre of a column bent in symmetric single curvature and is Timoshenko's(135) approximation to the exact solution for an elastic column. For the case of a column with unequal end moments, the C_m factor is used, as stated in the ACI Commentary(191), to "give an equivalent uniform moment, $C_m M_2$, which would lead to the same maximum moment when magnified". In other words, the C_m factor converts the column with unequal end moments into an equivalent column bent in symmetric single curvature with equal end moments. The expression for C_m in equation (11.6) was first proposed by Austin(193) when considering the in-plane behaviour of metal beam-columns and has now been adopted by the ACI for the in-plane behaviour of concrete columns. Similar expressions to equation (11.6) had been proposed earlier by Masonnet(194) (195) and Horne(196) but were principally derived for the case of I-shaped columns loaded eccentrically to produce bending about the strong axis where the assumed failure was by combined bending and twisting, i.e. lateral torsional buckling.

The ϕ factor in equation (11.3) is to account for small adverse variations in material properties, dimensions and workmanship which individually may be within acceptable limits and limits of good practice yet may combine to result in a reduced capacity of the column.

The β_d term in equation (11.5) is to account for the reduction in stiffness of the section due to creep of the concrete under sustained load. The ACI commentary states that this factor "gives a correct trend when compared to both analyses and tests on columns under sustained loading". For the worst case of all the load being dead load, the use of the β_d factor has the effect of reducing the stiffness of the column by a maximum of one half which for small steel percentages is roughly equivalent to using a reduced modulus for the concrete of one half the initial elastic modulus for short term loading. As shown in the chapters on the effect of long term loading, the writer has measured reduced modulus values for the concrete of up to $\frac{1}{6}$ th the initial short term elastic modulus. For some concrete materials which exhibit high creep and shrinkage tendencies, the suggested β_d value may not be sufficient to account for the reduced concrete modulus under sustained loading.

The use of one fifth the concrete modulus in equation (11.5) is to approximate the reduction in stiffness due to cracking and non-linearity of the concrete stress-strain relationship. The ACI Commentary(191) has demonstrated that equation (11.5) is very conservative for low values of applied load but becomes more accurate for higher values of load for reinforced concrete sections. This is to be expected with the use of a constant value of $\frac{E_c}{5}$ to represent cracking and non-linearity of the concrete for all load values. It should be noted that ACI 318-71 allows a more accurate value of EI to be used in lieu of equation (11.5) such as from moment-curvature relationships based on the integration of acceptable non-linear stress-strain diagrams for concrete in flexure.

11.3 COMPARISON OF THE ACI ULTIMATE STRENGTH DESIGN METHOD WITH EXPERIMENTAL RESULTS

11.3.1 Built-up Composite Columns

The ACI design method described above has been compared with the results of the writer's tests in Figs. 11.1 and 11.2 for the 7ft. and 10ft. columns respectively. As the columns shown were bent in symmetric single curvature, the value of C_m in equation (11.3) is unity. The value of ϕ has been taken as unity as the columns were manufactured under laboratory conditions with accurate measurements taken of the dimensions and the material properties determined from carefully controlled tests on an adequate number of specimens. From equations (11.2) and (11.3), the moment, M_c, at the centre of the column can be expressed as a function of the end moment, M_2, where $M_2 = Pe$, P being the applied load and e the end eccentricity. Hence

$$M_c = \frac{Pe}{1 - \frac{P}{P_{cr}}} \qquad \text{———— (11.7)}$$

Equation (11.7) is shown as the broken line curves in Figs. 11.1 and 11.2. Although it could not be expected that this simple expression would give values of moment at midheight in agreement with the observed values, as determined from the observed central deflection, Δ, where $M_c = P(e + \Delta)$, it can be seen that the theoretical and observed relationships exhibit similar trends. In all cases, the ACI method gave moments at midheight which were larger than the observed values for all values of applied load. Although the load-moment relationships show some divergence, the theoretical maximum loads, as defined by the intersection of the dashed lines in Figs. 11.1 and 11.2 with the interaction envelope, are in fairly close agreement with the observed values. A comparison is given in Table 11.1. The maximum load value for CC11 was obtained from the intersection of the $e = 2$in. line with the interaction envelope in Fig. 5.27. For this column, the lower limit of unity for δ (equation (11.3)) applied, indicating that the maximum moment occurred at the end of the column, a fact that was observed from the actual test.

Apart from the concentrically loaded column CC4, the agreement between the observed and ACI maximum load values is to within 9%. The ACI values are always lower than the observed values which is on the conservative side from a design point of view. The reason for the ACI load being 20% less than the observed value for the concentrically loaded column CC4 is that the ACI Code specifies a minimum eccentricity of 5% of the crossection depth or 1 inch, whichever is the greater. The writer has used the value of 0.05 d in Fig. 11.1 as it was felt that the limit of 1 inch was much too severe for a column of only 8in. x 7in. crossection, this being smaller than the sizes of columns normally encountered in practice. Although the use of a minimum eccentricity criterion is necessary in design to account for such things as imperfect positioning of the axial load and the steel within the section, non-uniformity of materials and minor discrepancies between the assumptions made in the analysis and the actual behaviour, the test on column CC4 was carried out under laboratory conditions where the above discrepancies would be minimised. It is therefore not surprising that the ACI method gives a somewhat lower maximum load than was actually measured.

The unloading characteristic for the test columns can also be observed in Figs. 11.1 and 11.2. On reaching the maximum load, both the load and midheight moment commenced to fall. The close agreement between these points of maximum load and moment and the theoretical interaction envelope again indicated that, for the crossection and length of columns tested by the writer, the maximum load was reached when the section subjected to the largest moment had attained its full load-moment capacity as defined by the interaction envelope.

11.3.2 Concrete-Filled Square Tubes

The comparison of the ACI design method with the writer's results of tests on concrete-filled tubes are shown in Figs. 11.3 to 11.6 inclusive. It can be seen that equation (11.7) relating the applied load, P, to the moment, M, at midheight is in very close agreement with the experimental values for loads up to 50% of the maximum value. Timoshenko(135) has shown that for this range of load, equation (11.7) is accurate to within 2% for an elastic column. The close agreement is therefore not surprising as the materials could be considered elastic for low load values especially the steel which also provides 75% of the total flexural stiffness for tubes of the writer's crossection.

What was a little disturbing was that the maximum loads calculated by the ACI method, except for the concentrically loaded column SHC-2, were all in excess of the observed values as shown in Table 11.1, the maximum variation being 13%. The lower ACI load value for column SHC-2 was a result of using the minimum allowable eccentricity value of 0.05D rather than the actual test value of zero.

In the ACI method, the stiffness of the column up to the maximum load is given by equation (11.5) in which the concrete has a reduced modulus of $\frac{E_c}{5}$ but the steel modulus is left constant at its elastic value of E_s. In the actual column tests, the load to cause yielding of the steel always preceded the maximum load. This resulted in the observed load-moment relationships tending to flatten out once yielding of the steel commenced whereas the ACI load-moment relationships (equation (11.7)) continued to rise steadily, intersecting the interaction envelope at higher values of maximum load than those observed. The writer therefore suggests that a reduced stiffness for the steel section be used in equation (11.5) such that the ACI load-moment relationships would be below the observed relationships but would intersect the interaction envelope at values of load equal to or just less than the observed maximum values. In an ultimate design method, it is initially the agreement at the ultimate load that is important rather than at intermediate load values. Of course serviceability requirements at working loads also have to be satisfied. For the writer's concrete-filled tubes, the above criterion is met by using a reduced steel stiffness of $0.4E_sI_t$ in equation (11.5) for which the ACI maximum loads are equal to or below the observed values to within 9 percent. For tubes of other dimensions, a different reduced stiffness for the steel may be required to meet this condition. For the built-up composite columns where the concrete provides the majority of the initial stiffness, it has already been shown in Figs. 11.1 and 11.2 that the use of a reduced modulus for the concrete of $\frac{E_c}{5}$ was sufficient for the design loads to be equal to or less than the observed values.

It is demonstrated in Figs. 11.3 to 11.6 that the maximum load for the tubes, as obtained from the tests, was reached when the section subjected to maximum moment (midheight) had or almost attained the theoretical load-moment capacity of that section as defined by the interaction envelope. This confirmed the theoretical results in Fig.6.15 where it can be seen that the full load-moment capacity of the section (curve for L = 0ft.) could almost be attained for quite slender tubes.

11.4 DISCUSSION

The ACI Building Code(190) permits columns to be designed by either the simple equivalent beam-column method described in Section 11.2.2 or by a more rational method using an analysis which accounts for the influence of axial load and variable moment of inertia on member stiffness and the end moments resulting from end restraint and includes the effects of deflection on the moments and forces. For columns with a slenderness $(\frac{k\ l_u}{r})$ greater than 100, a rational analysis must be used. Apart from the effects of end restraints, the analysis presented in Chapter 4 by the writer for pin-ended columns meets all these requirements.

It can be seen by comparing the observed/ACI maximum load ratios in Table 11.1 with the observed/theoretical maximum load ratios in Table 5.1 for the built-up composite columns and Table 6.3 for the concrete filled tubes that a rational analysis gives more accurate results as would be expected. It is considered that the behaviour of a composite column continuous through a building frame, which could be subjected to a variety of different loading conditions, may not be represented adequately by an equivalent pin-ended member loaded concentrically or eccentrically at its ends as in the ACI method. A number of alternative approaches are possible but these would be best considered when more is known of the behaviour of the composite structure as an entity and the effects of end restraint for a variety of loading conditions has been considered. This aspect is now being studied at the University of Sydney. However, a study of the behaviour of pin-ended members is a convenient starting point in establishing analytical methods based on sound assumptions from which rational design procedures for composite columns in structures may eventually be determined.

Any rational analysis should include the effects of the duration of loads. For the ACI simple design method, these are taken into account in an empirical manner by the use of the β_d term in equation (11.5). The writer's analyses in Chapter 8 provide a more rational means by which the behaviour of pin-ended composite columns under sustained load can be predicted. The long term behaviour of restrained composite columns is an area in which further research is necessary.

CHAPTER 12

CONCLUSIONS

12.1 BEHAVIOUR UNDER SHORT TERM LOADING

Why built-up columns? This question was first posed at the outset of this investigation and this question is one of both efficiency and economy. Where the loading is mainly axial, the position of the steel within the crossection will have little effect on the load carrying capacity except for slender members which are likely to fail in a mode approaching elastic instability. However, where the loading is such that the column is subjected to a significant applied moment in addition to the axial load, there is no doubt that the use of built-up sections in composite columns can lead to a saving in steel as the steel sections, which may be of any desired shape, can be positioned towards the extremities of the crossection to make the most efficient use of the steel to resist the applied moment. The problem of economy is therefore one of labour costs and fabrication time as built-up steel sections used in composite columns have traditionally been laced or battened following the requirements for bare steel built-up columns.

Perhaps one of the most significant aspects of the investigation was therefore the short term loading to failure of pin-ended composite columns containing two channels which were in no way connected except by plain concrete. These tests have shown that the absence of battens or lacing between the steel components has no detrimental effect on the load carrying capacity of the columns. This has been demonstrated for a range of loading conditions including columns bent in single curvature about the major, minor and diagonal axes and in double curvature about the major axis with relatively large end eccentricities, this representing the maximum shear condition for the column. In this latter case, significant principal tension was developed in the concrete between the channels at the maximum load but this was still well below the tensile stress to cause cracking of the concrete. In all cases, the behaviour was essentially flexural with no evidence of any failure that could be attributed to shear. The correlation between the test behaviour of two similar columns, one with and one without battens, has positively confirmed that the absence of battens is not detrimental to the performance of built-up composite columns, even when the steel components, in this case two channels, are extremely slender in the unbattened state and fail by elastic instability at a considerably lower load than that for a similar steel column provided with battens.

While flexibility of arrangement of the steel sections in a built-up composite column might be an advantage from a practical point of view, it poses quite a problem when it comes to an investigation of their behaviour as an almost limitless number of different possible crossections can be conceived. Whether the same conclusions regarding the absence of battens are applicable to all these cases has yet to be determined. However, the results of tests by Hudson(93) on reinforced concrete columns containing longitudinal reinforcing rods, both with and without binders, offer some encouragement. The concrete encasement was found to provide sufficient restraint against buckling of the reinforcement such that the absence of binders had no effect on the maximum load capacity. However, the columns without binders were found to have a more "brittle" behaviour in the unloading range.

Of course some battens or lacing may be necessary for construction loads to be carried prior to encasement in concrete. The determination of the minimum amount of battens or lacing required under these conditions is an area in which some further study is required as existing design methods are based on codes which have established requirements that are sufficient for the full strength of the built-up steel sections to be developed.

It has long been established(130) that bare steel members constrained to bend about an axis other than the minor principal axis are prone to fail in a lateral torsional buckling mode involving torsional displacements. The results of the writers tests on built-up composite columns and preceding tests by Loke(14) on encased rolled steel joists, bent about axes other than the minor axis, are significant in that this torsional characteristic is absent. Not only were torsional displacements not in evidence but the columns deflected in the plane of the applied end moments with the behaviour of the columns at all stages being entirely flexural. This behaviour fully justified using the same assumptions as Loke(14) in the analysis given in Chapter 4, without which more exact analytical procedures would have been required to take into account lateral-torsional displacements which, for built-up composite columns containing any desired arrangement of the steel sections, would be exceedingly complex indeed. For concrete-filled tubes with their high torsional rigidity and built-up composite columns of roughly square crossection, torsional displacements are not likely to be significant. For built-up columns with their freedom of arrangement for the steel components, it is possible that slender columns of narrow rectangular section might be considered. The possibility of failure in a lateral-torsional buckling mode for this case is an aspect of composite column behaviour that has yet to be examined.

An important observation from the column tests is the ability of a composite column to continue to carry a substantial proportion of its maximum load for further deformation beyond that at the maximum load. The concrete-filled tubes, with a higher steel percentage and with the steel at the extremities of the crossection, were superior to the built-up composite columns in this regard. It is likely that the load and moment capacity of the steel governs the behaviour in the unloading range as the concrete becomes less and less effective as cracking and crushing of the concrete extends along the column with increasing deformation. This unloading characteristic is of importance when considering structures subjected to seismic loads where a good moment-rotation capacity for the columns is needed for energy absorption. This can be enhanced for the built-up column by the provision of binders in the concrete encasement which have been shown (Section 3.2.4) to improve the ductility of the concrete.

For concrete filled tubes, the tube walls can be considered as a continuous binder. Not only is this steel beneficial to the concrete but vice-versa as local buckling failures of the tube walls, in the elastic range for thin-walled tubes and the plastic range for thick walled tubes loaded as hollow tubes, are prevented by the concrete which stabilizes the tube walls. This has been demonstrated in the tests, especially by the cases where yielding extended over the full length of the tubes on the compression face yet there was no visible evidence of any local buckling of this face.

Varying the inclination, β, of the loading axis to a face of a square concrete filled tube from 0° to 45° has little effect on its load and moment capacity, being of the order of a 5% variation for tubes of the writer's cross-section. Tubes loaded at small eccentricities (high axial load) with $\beta = 45°$ have a slightly higher load and moment capacity than similar tubes loaded with $\beta = 0°$ while the reverse holds for large eccentricities (low axial load). For the purposes of design calculations, it is considered that these variations could be ignored. For the built-up composite columns where the stiffness of the section increases in magnitude as the axis of bending is rotated from the minor to the major principal axis, the load capacity for a constant eccentricity also increased with rotation of the axis, the maximum value being obtained for bending about the major axis as expected.

Built-up composite columns and concrete filled tubes are no different from other types of columns in the fact that increasing the slenderness results in a reduction in the load capacity. For the two types of columns tested, the load-slenderness-eccentricity relationships cannot be considered the same. This is to be expected as they are dependent on the material properties and the percentage of steel, its shape and relative position within the crossection. It is a simple matter to account for these variables with the analysis presented in Chapter 4. Rather than attempt to establish an empirical method to account for the effects of slenderness and eccentricity (applied moment) for all possible types of column crossection, it is suggested that a rational column analysis, such as that in Chapter 4, be used. As the use of computers increases, analyses such as this will play an increasingly important role in design procedures.

The difference in behaviour of the two types of composite columns tested is evident in the shape of their respective interaction envelopes (Fig. 5.30 for the built-up composite columns and Fig. 6.16 for the concrete-filled tubes) which define the load-moment capacity of the crossection: The interaction envelope for the built-up composite columns, where the concrete provides the major contribution to the strength, has a distinct "nose", the moment capacity being increased by the application of some axial load to the section in much the same manner as the moment capacity of a concrete beam is enhanced by prestressing; the interaction envelope for the concrete-filled tubes, where the steel provides the major contribution to the strength, shows little or no enhancement of the moment capacity with axial load, the behaviour being more like that expected for a bare steel section.

The effect of slenderness on the column behaviour also differs for the two types. Even for quite slender concrete filled tubes with a length to depth ratio of 20, the maximum load is reached when the section of the column subjected to the maximum load and moment has essentially attained the full load-moment capacity of the section as defined by the interaction envelope. This has been termed a "material failure" by MacGregor et.al.(187). For built-up columns of similar slenderness, the maximum load is reached before the section subjected to maximum load and moment has attained its full capacity i.e. at a point inside the interaction envelope. This has been termed a "stability failure" by MacGregor et.al.(187). It could be said that the built-up columns were more affected by slenderness than the tubes. For reinforced concrete columns, Broms and Viest(38) showed that an increase in the proportion of the load carried by the steel reinforcement led to a more stable column whereas columns with low percentages of steel tended to be most affected by length. The writer's results for the tubes and built-up columns confirm this behaviour.

A major objective of this investigation was the extension and modification of the original inelastic buckling analysis by Roderick and Rogers(13) for pin-ended encased rolled steel joists, bent about the minor axis in symmetrical single curvature under short term static loading, to the case of composite columns, under similar loading conditions, containing any shape, arrangement and form of steel sections and bent about any axis in either single or double curvature with any combination of end eccentricity. The modifications included taking residual stresses into account and refining the shapes of the assumed stress-strain relationships for the materials to give an even more accurate representation of the real nature of the materials. That this objective has been achieved is verified by the excellent agreement of the theoretical load-deflection and load-strain relationships and the maximum loads computed with the analysis with those obtained from a number of varied tests on both built-up composite columns and square concrete filled tubes. Within the range of columns tested and the principal variables examined viz. eccentricity, slenderness and axis of loading, it is considered that this phase of the investigation has been carried out successfully and is claimed to be an important contribution to the study of the behaviour of composite columns.

A logical extension of the analytical methods is the study of the effects of construction loads being applied to the steel prior to encasement followed by further loading with the steel composite with the concrete. The effect of the construction loads can be conveniently considered as a residual stress as far as the composite column is concerned. Some preliminary results from the analysis, which accounts for residual stress, have indicated that some preload on the steel, prior to final loading as a composite column, may even be beneficial when compared to a similar column loaded entirely as a composite column. For the same total applied load, the preload case leads to higher steel stresses yet lower concrete stresses than the latter. Depending on the relative contributions of the concrete and steel to the column stiffness, the increase in concrete stiffness resulting from lower concrete stresses can more than compensate for the reduction in steel stiffness resulting from higher steel stresses. The overall increase in stiffness is more significant for slender columns and can lead to an increase in maximum load above that for a column loaded entirely as a composite column. For short columns, the maximum load for a constant eccentricity is not significantly influenced by the order in which the load is applied to the steel and concrete. These theoretical aspects of column behaviour, which are pertinent to the construction sequence of composite columns, are still in need of some experimental verification.

As would be expected, variations in the shapes of the stress-strain curves for the materials result in similar variations in the shape of the load-deflection and moment-curvature relationships for the columns. However, variations in the shapes of the stress-strain curves, providing the same compressive strength for the concrete and yield stress for the steel are used, do not produce significant changes in the values of the maximum load computed for relatively short eccentrically loaded columns for which "material failure" governs. This is analagous to the case of under-reinforced concrete beams where the use of concrete stress-strain curves ranging from parabolic to rectangular in shape do not significantly alter the calculated value of the ultimate moment capacity. Use of this has been made in reinforced concrete design codes to simplify computations. However, for concentrically loaded or slender eccentrically loaded columns for which "stability failure" governs, their behaviour will be more sensitive to any changes in the shapes of the stress-strain relationships and hence the tangent moduli for the materials.

Up to the present time, much of the work on composite steel and concrete structures has been devoted to the behaviour of isolated beam and column elements. This has been necessary to lay the foundations for further studies into the behaviour of rigid frame composite steel and concrete structures. The present investigation falls into this category. However, composite construction cannot be used to its full advantage until means of providing a truly composite connection between beam and column elements have been developed which make full use of the concrete in transmitting moment and shear. As part of the overall research programme on composite construction at this University, Ansourian(203) has studied the behaviour of connections between composite T-beams and universal columns (steel I-sections) encased in concrete and connections between steel beams and rectangular concrete-filled tubes(204), this latter work being done while the author was on study leave at the University of Liège. Both cases of an internal and an external column in a structure have been examined. This work is soon to be presented in full as a thesis for the degree of Doctor of Philosophy. A logical extension of this work would be to examine the types of connections that could be used to connect beams with built-up composite columns. For an internal column at least, the connection could be quite simple with the steel components of the column being arranged to allow the beams to frame continuously through the columns and the concrete, perhaps with reinforcement, providing the rigidity. In fact, there may be no need for any complicated connection detail for the steel at all. For example, a simple joint using high tensile bolts for speed of erection could be used to connect a beam to an external column. The end of the beam could be considered as simply supported in the bare steel state during construction when the loads are small yet fully rigid after encasement with concrete when the live load and additional dead load have to be carried. The requirements for reinforcement in the area of the connections may need careful examination.

With a knowledge of the behaviour of beams, columns and connections, the next logical step is a study of the frame structure. Research is now in progress at this University on the experimental and theoretical behaviour of simple frames consisting of a single column, containing a single universal steel column encased in concrete, with two beams at each end which provide restraint about both the major and minor principal axes. The development of analytical methods capable of accurately predicting the behaviour of even simple composite frames such as these is still a complex problem as a variety of possible loading conditions and end restraints have to be considered. That such a task is now possible is a result of having first established a proper understanding of both the experimental and theoretical behaviour of the pin-ended composite column.

12.2 INFLUENCE OF TIME-DEPENDENT PROPERTIES OF CONCRETE ON COLUMN BEHAVIOUR

Apart from dead loads which are constant and sustained for the life of a member, a composite column in a structure is also subjected to live loads which may be mainly static and sustained over long periods or more fluctuating in nature but sustained over short periods. The loading history in the latter case may be quite complex. The time-dependent properties of the concrete viz. creep and shrinkage, are likely to affect the behaviour of such a column under load. With drying, concrete shrinkage will have induced deformations and associated stresses in the concrete and steel even before any load is applied.

It has been shown from tests that concrete shrinkage can result in cracking of the concrete in an unloaded composite column. The extent of cracking depends on the rate of shrinkage and the amount of restraint, which is a function of the percentage of steel in the crossection. Cracking can be prevented by using low percentages of steel but this is at the expense of having high steel stresses, perhaps even approaching the yield stress. Subsequent short term loading of a pin-ended composite column containing shrinkage cracks reveals a significant reduction in the load capacity compared to that for an initially uncracked column. It could be considered that shrinkage cracks act as a large initial imperfection which increases the central deflection and lowers the maximum load of the column.

If load is applied to the column before much drying and hence shrinkage of the concrete has taken place, cracking will be prevented but the creep deformations under load will be increased as the rate of creep is at its highest at early ages when drying has just commenced. This rate decreases with time as moisture is dried from the concrete. Tests on pin-ended columns subjected to a constant sustained load, which is applied immediately or soon after drying commences, have shown that deflections of the order of six times the initial deflections at first loading can be attained after three years of loading, most of the creep deformation taking place in the first 200 days. The interesting phenomenon is demonstrated of a built-up composite column loaded in compression with a constant sustained concentric load of one third its short term loading capacity in which the concrete stress changes from compression to tension with time. This complete transfer of load from the concrete to the steel is more likely to occur for composite columns which generally have higher percentages of steel than reinforced concrete columns.

Analytical procedures have been developed which enable the creep and shrinkage deformations of composite columns to be predicted to within 10 percent. The effective modulus method of analysis has the advantage in that it can be developed as a logical extension of analyses, as in Chapter 4, for predicting column behaviour under short term loading. While being more complex, the rate of creep analysis, which has yet to be verified, has the advantage that some account is taken of the previous stress history of the concrete and may therefore prove to be more accurate for the case of sustained loads which fluctuate with time, this being the more likely case for columns in an actual structure.

Following the correlation between the tests and the theory, the analysis has been used to determine the full load-deformation-time relationship for the test columns. For pin-ended columns bent in single curvature, the load capacity is significantly reduced with increasing time under load. This is considered to be the most severe case for sustained loading as the moment at midheight, being a maximum at this point, is continually amplified with time by the creep deflection. For columns bent in double curvature, there may be no reduction in load capacity with time providing the creep deflections are insufficient to increase the column moments, at sections within the length of the column, above the maximum value at the end section.

Analysis has indicated that the levels of sustained load applied to the test columns are well below the values which would lead to collapse within three years of loading. If the creep and shrinkage were to continue at their present rates, which are now extremely low, it could be conceived that eventual failure might occur within a finite time. However, long term creep and shrinkage tests(161) over 30 years have shown that the rate of creep and shrinkage continues to decrease with time. It is therefore unlikely that collapse will ever occur in any of the writer's tests. In one case (column CC7), the bare channels are themselves theoretically capable of carrying the sustained load assuming provision is made for shear transmission such as with battens or lacing. It has been demonstrated that the concrete encasement alone serves this function adequately for short term loading. It therefore follows that the influence of creep cannot possibly reduce the load capacity of the composite column below that for the bare channels providing that there is no reduction in the shear transfer capacity of the concrete with time under sustained load. This latter proviso is one for which there is yet no conclusive evidence. However, in one of the writer's creep tests, a built-up composite column without battens was bent in double curvature, which represents the maximum shear condition for a pin-ended column, with a value of sustained load well above the load capacity for the bare steel sections assuming adequate shear transmission. The results of this test have shown that adequate shear transfer capacity has been maintained with time. The deflections were also predicted with reasonable accuracy using an analysis in which no account was taken of shear deformations.

It is proposed to eventually unload the columns and then to reload them to failure in a testing machine. Under sustained loading, load is transferred from the concrete to the steel due to creep and shrinkage. On unloading, the elastic recovery of the steel will result in extensive cracking of the concrete. As for the columns with shrinkage cracks, it is likely that this extensive cracking will result in increased deflections and a marked decrease in load capacity on reloading compared to those for a similar uncracked column. It is envisaged that the investigation into this aspect of column behaviour could be extended to examine the cases of columns subjected to a constant sustained base load, representing an applied dead load, on which is superimposed a further sustained load, represent the live load, which is applied over a relatively short period, temporarily removed and then reapplied.

This cycle could be repeated many times to represent live load fluctuations. The writer has not attempted to include this particular problem in his study of the composite column.

Under the present design requirements, it is unlikely that creep will lead to failure of composite columns in an actual structure because of the large factors of safety that are still required for such members. This may not be the case when full use is made of the stiffness and load carrying capacity of the concrete resulting in reduced load factors and more slender columns. Other contributing factors are that in general, it is most unlikely that the full design load will be constantly applied for the whole life of the structure and that when the shrinkage and creep potential are at their highest at early ages during the construction stages, the structure is only lightly loaded. By the time the structure is completed and the full dead load and some live load is applied to the structure, most of the shrinkage strain will have been developed and the rate of creep for the concrete will have been reduced to a relatively low value. If the column forms part of a redundant structure, the restraint to the column may further reduce the creep deflections although at the expense of a redistribution of moments throughout the structure which may or may not be beneficial. A case in mind is that of two adjacent columns, rigidly connected by beams, which have differing rates of axial shortening due to creep under load. This will result in an increase in the negative moment at one end of the beam yet a decrease at the other end.

Notwithstanding the limitations of the eccentrically loaded pin-ended composite column in relation to the column continuous in the building frame, it is claimed that the present investigation into the effects of shrinkage and creep on the behaviour of composite columns has revealed a number of characteristics of the composite column which are basic to any examination of the more complex problem of the continuous column.