Appendix F: INFLUENCE COEFFICIENT MATRIX

Consider an arbitrary system as depicted in Fig. F.1. Let the vector $\{x\}$ represent a set of input applied simultaneously to the system and the vector $\{r\}$ represent the response of the system. The response $\{r\}$ can also be considered as the difference between the final state of the system and the initial state in which no input is applied. The relationship between the input and the response for a linear system is shown in Eq. (F.1).

$$[Q]\ \{x\} = \{r\} \tag{F.1}$$

The matrix $[Q]$ contains all the properties and characteristics of the system, i.e. it defines the system. For a complex system, the matrix $[Q]$ is unknown.

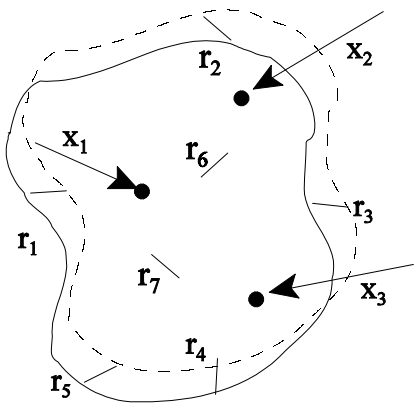

Figure F.1 An arbitrary system subjected to certain loads and its response.

Relating this to pure displacement shape control, displacement values $\{r\}$ are prescribed and the applied voltages $\{x\}$ are to be determined. The relationship between these two electro-mechanical state variables is represented by the analytically unknown $[Q]$. The concept is to construct an Influence Coefficient matrix $[C]$ to model the real system represented by the unknown $[Q]$.

When the system is linear, $[C]$ is constructed by recording the responses due to each independent actuation. For example when there is three actuation degrees of freedom $\{x_1, x_2, x_3\}$, then obtain 3 separate responses, $\{q_1\}$, $\{q_2\}$, $\{q_3\}$ from 3 different unit input at the corresponding

states as shown in Eq. (F.2)

$$[Q]\begin{Bmatrix}1\\0\\0\end{Bmatrix} = \{q_1\}\ ; \qquad [Q]\begin{Bmatrix}0\\1\\0\end{Bmatrix} = \{q_2\}\ ; \qquad [Q]\begin{Bmatrix}0\\0\\1\end{Bmatrix} = \{q_3\}\ ; \tag{F.2}$$

Then use the $\{q\}$ vectors to form columns of the $[C]$ matrix which is the influence coefficient matrix as shown in Eq. (F.3)

$$[C] = \left[\{q_1\} \quad \{q_2\} \quad \{q_3\}\right] \tag{F.3}$$

Note that when Eq. (F.2) is inserted into Eq. (F.3) then $[C]$ is equivalent to the discretized system matrix $[Q]$ as shown in Eq. (F.4).

$$[C] = \left[[Q]\begin{Bmatrix}1\\0\\0\end{Bmatrix} \quad [Q]\begin{Bmatrix}0\\1\\0\end{Bmatrix} \quad [Q]\begin{Bmatrix}0\\0\\1\end{Bmatrix}\right] = [Q]\left[\begin{Bmatrix}1\\0\\0\end{Bmatrix} \quad \begin{Bmatrix}0\\1\\0\end{Bmatrix} \quad \begin{Bmatrix}0\\0\\1\end{Bmatrix}\right] = [Q]\ [I] = [Q] \tag{F.4}$$

Algebraically, this proof seems trivial, but note that the Q-matrix is unknown, and the C-matrix is obtained by sampling the responses at the 3 states. So the main mechanism in the IC technique is the ability to obtain responses from the system due to given inputs by using $[Q]$ as a black box operator (e.g. the FE process) although the system $[Q]$ itself is not explicitly known. Several items to note:

1. The method presented above can be applied to a system with n states in general although it was illustrated as a 3-state system.
2. The $[C]$ and $[Q]$ matrices are in general non-square matrices. As shown in Fig. F.1 there are 3 degrees of actuation freedom and 7 degrees of response freedom. The latter depends on how many points of interest the analyst wishes to observe.
3. Following from the last point, it is obvious that in general, $[C]$ is non-invertible. Thus the solution Eq.(F.1) cannot be obtained by inverting $[C]$ or the system matrix $[Q]$ even if it is known.