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**Abstract**

The paper represents a preliminary attempt to shed light on the following question: in the context of demand-led growth, how does learning by agents about the economic system's structure and the determinants of long-run growth affect the long-run dynamics of the economy? Analysis is conducted in terms of an extension of the simplified two-sector model with autonomous demands in White (2008). The focus of the analysis is on the impact of learning about two mechanisms in particular: about how the growth of autonomous demand influences growth of the economy as a whole; and about how expectations about growth affect the dynamics of growth. The mechanics of learning are twofold: first, a simple gradient-descent rule, whereby key coefficients in the investment function relating producers expectations about growth to past growth in their own sector and in the economy are modified in a way which aims to minimize forecast errors; and, second, a more ambitious mechanism whereby producers attempt to uncover aspects of the true relation between past growth rates and expected growth rates. Analysis of the system's dynamics is primarily by means of computer simulation.

# DEMAND-LED GROWTH IN A MULTI-COMMODITY MODEL WITH LEARNING: SOME PRELIMINARY RESULTS

## I Introduction

The present paper is a tentative step towards exploring the significance of learning for the dynamics of growth, where the latter is ultimately driven by demand. The analysis is conducted in terms of two-sector model explored in White (2008) and represents a natural extension of that analysis, which considered the dynamics of demand-led growth but without allowing for agents to learn about those dynamics. The present paper attempts to take the analysis a step further by supposing that some or all producers can engage in one of two types of learning: one simplified and which does not attempt to uncover the dynamics of the system; and a more complex learning which seeks a deeper understanding of the system's dynamics.

As in White (2008) the focus of the discussion is on the dynamics of quantities, so that we follow the fairly restrictive assumptions used there with regard to the determination of relative prices and income distribution, viz., the distribution of income between wages and profits. Specifically, prices are assumed to be determined by competition and tend towards their long-period configuration, consistent with a uniform rate of return across sectors, consistent with a uniform rate of return across production processes. In other words, with constant returns to scale in the production of each commodity, relative prices and the real wage expressed in terms of one or any combination of the three commodities are fully determined by the technology of production – i.e. the relative proportions of labour, circulating and durable capital in the three production processes – and the equilibrium value of the rate of interest.

The analysis is developed in terms of a discrete-time, dynamic model, specifically a first-order difference equation system and within that framework the focus is on how the dynamics of growth reflects the dynamics of demand and on how agents' realizations as to the nature of these dynamics feed back on the system's growth. The complexity focussed upon, for the purposes of the proposed research, lies in the determination of investment and more specifically, in the determination of expectations on the part of producers about the growth of demand. As noted above, it is through this channel and thus via investment decisions that the effects of learning by producers are assumed to impact most on the process of growth.

Sections II – V develops the model without learning – following the approach of White (2008); with section V directed at a specific discussion of how expectations about demand

growth are formed. Sections VI and VII introduce the possibility of learning in the development of these expectations; firstly in terms of a simple modification of a key parameter on the basis of the evolution of forecast errors; and, secondly, by means of a more sophisticated approach which allows for some producers to model the dynamics of demand and the role of expectations in those dynamics. Section VIII provides some brief concluding comments.

## II A two-sector fixed capital model

*(i) Production*

Consider an economy which produces each period a quantity of a consumption good as well as a quantity of new machines of a particular type. The production of each commodity requires a quantity of machines and a quantity of labour, which is assumed to be of a uniform type. Machines have a maximum (technical) life of two periods, and have a zero scrap value and can be disposed of costlessly. Other than a quantity of one-period old machines in each sector at the end of each production period there is no joint production. The model ignores foreign trade as well as the government sector.[1]

Production in such an economy can be described in terms four processes producing four commodities: a consumption good, new machines, and two types of one-period old machines according to whether they are used in the production of the consumption good or new machines.

It is assumed that machines have a variable efficiency reflected in greater quantities of labour per unit of output being required with the use of older machines as compared with new machines. Denoting the unit labour requirement of $l^0_{it}$ associated with the use of new (zero-years old) machines in the production of a unit of output in sector $i$ in period $t$, the corresponding labour requirement with (one-year) old machines is given by

$$l^1_{it} = l^0_{it}.(1+\alpha) \qquad \text{…..(II.1)}$$

[1] Though demand arising from foreign trade and /or government intervention could well provide part of demand which is referred to here as “autonomous”.

where $\alpha$ is positive and less than unity. The assumption of variable efficiency warrants a distinction between the components of total output corresponding to machines of different ages, so that output in sector *i* during period *t* can be represented as follows:

$$Y_{it} = Y_{it}^{0} + Y_{it}^{1} \qquad i = 1,2 \qquad \text{…..(II.2)}$$

where $Y_{it}^{0}$ *and* $Y_{it}^{1}$ refer to outputs of commodity *i* on new and old machines respectively. Actual utilization rates of new and one-year old-year old machines in sector *i* in period *t* are then given by

$$u_{it}^{j} = \frac{Y_{it}^{j}}{\beta_i M_{it}^{j}}, \qquad j = 0,1, \;\; i = 1,2 \qquad \text{…..(II.3)}$$

$M^{j}_{it}$ is the number of *j*-year old machines used in production in sector *i* in period *t* and $\beta_i$ is the output capacity of a machine in sector *i*, i.e. in terms of units of commodity *i*. For simplicity, it is assumed that this output capacity is the same for new (*j* =0) and used (*j* = 1) machines, so that variation in efficiency over the life of machines amounts to more labour being required by older machines to produce the same output as new machines. $u^{j}_{it}$ refers to the actual utilization rate in period *t* of *j*-years-old machines.

Differences in utilization rates on machines of different ages are treated in as simple a manner as possible by assuming a linear relation such that

$$u_{it}^{1} = \frac{u_{it}^{0}}{(1+\phi)} \qquad \text{…..(II.4)}^{2}$$

It is also assumed that there is a desired utilization rate in each sector, specifically a desired rate in relation to newly installed plant, denoted as $u^{n0}_{it}$. In view of (II.4) above, this effectively also implies a desired rate of utilization on older plant.

Regarding the relationship between output and demand, this paper assumes a one-period lag between demand and production and thus the need for producers to forecast future demand in the planning of output.

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**2** See White 2008 (note 2) regarding the conceptual difficulties associated with such a simplistic approach to the modeling of utilization.

With time divided into discrete periods, together with the assumption that demand is expressed at each junction of two periods, planning production during period *t* requires producers to estimate the demand forthcoming at the conclusion of period *t*.

The assumption made here is that production planned for period *t* is based on demand realized at the end of period *t-1* as well as the expected growth rate of demand between periods *t* and *t-1*. Hence the relation between demand and output is

$$Y_{it} = D_{it-1}\left(1 + g^{de}_{it-1}\right) \qquad \text{……..(II.5)}$$

where $g^{de}_{it-1}$ is the expected growth rate of demand for sector *i* between periods *t+1* and *t*. The determination of $g^{de}_{it-1}$, is taken up in section V below. Since $D_{it-1} = D_{it} / (1+g^{d}_{it})$, where $g^{d}_{it}$ is the realized growth rate of demand in sector *i* between periods *t* and *t-1*, expression (II.5) implies that

$$Y_{it} = \frac{D_{it}\left(1 + g^{de}_{it-1}\right)}{1 + g^{d}_{it}} \qquad \text{……..(II.6)}$$

*(ii) Investment*

Over the longer-run, capacity in each sector is assumed to adjust to persistent variation in demand. Specifically, the decision about the size of capacity in each sector at the end of period *t* involves an estimate of demand forthcoming at the end of period *t+1*, and on that basis an estimate of the capacity required assuming a desired utilization rate of $u^{n0}_{it+1}$ on newly installed capacity; and hence an estimate of the extent to which capacity at the end of period *t* is deficient or excessive. Hence, with demand $D^{e}_{it+1}$ expected in sector *i* in period *t+1*, the firm would choose a capacity comprising one-year old machines in *t+1*, which were new in period *t*, and new machines to be installed for use in *t+1*, such that

$$D^{e}_{it+1} = \left(u^{n}_{i}.M^{0}_{it+1} + \frac{u^{n}_{i}.M^{1}_{it+1}}{(1+\phi)}\right).\beta_i \qquad \text{…..(II.7)}$$

Since $M^{1}_{it+1} = M^{0}_{it}$ and noting that investment demand $I_{it}$ during period *t* leads to the installation of an amount of new capacity $M^{0}_{it+1}$ for use in *t+1*, then expression (II.5) can be written as

$$D^{e}_{it+1} = \left( u^{n}_{i}.I_{it} + \frac{u^{n}_{i}.M^{0}_{it}}{(1+\phi)} \right).\beta_{i} \qquad \text{…..(II.8)}$$

Demand expected in *t+1* is assumed to be based an extrapolation of demand observed in period *t-1*, so that

$$D^{e}_{it+1} = D_{it-1}.\left(1+g^{de}_{it-1}\right)^{2} \qquad \text{…..(II.9)}^{3}$$

where $D_{it-1}$ and $g^{d}_{it-1}$ are demand for commodity *i* during *t-1* and the rate of growth of demand for commodity *i* in *t-1* respectively.

It follows from (II.6) and (II.7) that investment demand in sector *i* at time *t* can be expressed as

$$I_{it} = \frac{D_{it-1}.\left(1+g^{de}_{it-1}\right)^{2}.(1+\phi) - u^{n}_{i}.M^{0}_{it}.\beta_{i}}{u^{n}_{i}.\beta_{i}.(1+\phi)} \qquad i = 1,2 \qquad \text{…..(II.10)}$$

Demand for new machines - commodity 2 – at the end of period *t* is

$$D_{2t} = \sum_{i=1}^{2} \left( \frac{D_{it-1}.\left(1+g^{de}_{it-1}\right)^{2}.(1+\phi) - u^{n}_{i}.M^{0}_{it}.\beta_{i}}{u^{n}_{i}.\beta_{i}.(1+\phi)} \right) + I^{Aut}_{t} \qquad \text{…..(II.11),}$$

where $I^{Aut}_{t}$ represents autonomous investment demand for commodity 2 expressed during period *t*. Precisely what makes up $I^{Aut}_{t}$ is not taken up for discussion here though it is necessary to note here that this demand is assumed to be non-capacity creating demand for new machines. This assumption allows one to identify newly installed capacity with investment demand "induced" by the expectation of growing demand, viz., $I_{it}$.

Expression (II.9) can be rearranged given that, as noted above, $M^{1}_{it+1} = M^{0}_{it}$ and $M^{0}_{it} = I_{it-1}$; and with $D_{2t-1} = I_{1t-1} + I_{2t-1} + I^{Aut}_{t-1}$, then

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**3** The squaring of the expression in brackets arises here because producers in the investment sector could not estimate expected growth in demand on the basis of realised demand at the end of period t-1; since they cannot know this magnitude prior to making their own investment decisions. For simplicity, we assume this to be the case for investment decisions in both sectors.

$$D_{2t} = I_t^{Aut} + \frac{I_{t-1}^{Aut}}{(1+\phi)} + D_{2t-1}.\left( \frac{D_{12t-1}.\left(1+g_{1t-1}^{de}\right)^2}{u_1^n.\beta_1} + \frac{\left(1+g_{2t-1}^{de}\right)^2}{u_2^n.\beta_2} - \frac{1}{(1+\phi)} \right)$$

…..(II.12)

where $D_{12t-1}=D_{1t-1}/D_{2t-1}$. In turn, by dividing through by $D_{2t-1}$, equation (II.10) can be transformed into an expression for the growth rate of demand for commodity 2:

$$g_{2t}^d = \frac{D_{12t-1}.u_2^n.\beta_2.(1+\phi)\left(1+g_{1t-1}^{de}\right)^2 + u_1^n.\beta_1.\left\{\left(1+g_{2t-1}^{de}\right)^2(1+\phi) + F\left(ID^{Aut}\right)\right\}}{u_1^n.u_2^n.\beta_1.\beta_2.(1+\phi)}$$

…..(II.13)

where

$$F\left(ID^{Aut}\right) = u_2^n.\beta_2.\left[ID_{2t-1}^{Aut}(a+a\phi+\phi+2)-2-\phi\right]$$

and where $ID^{Aut}_{2t}$ is the ratio of autonomous demand for commodity 2 to total demand for commodity 2 at $t$ and $a$ is the exogenously given growth rate of autonomous demand $I^{Aut}$ for machines.

*(iii) Consumption*

It is assumed that consumption demand depends on income and is expressed with a lag of one period. In other words, consumption demand of workers in period $t$ is based on wage income earned in period $t$-$1$. Likewise, consumption by capitalists in period $t$ is based on profit flows generated in period $t$-$1$.

Demand for commodity 1 – consumption demand – at the end of period t can be written as

$$D_{1t} = (1-s_w)Y_{t-1}^w + (1-s_c).P_{t-1}^c + D_{1t}^{Aut} \qquad \text{…..(II.14)}$$

with $s_w$, and $s_c$ the saving propensities of workers and capitalists respectively, $Y^w_{t-1}$ the income of workers and $P^c_{t-1}$ the profit flow to capitalists. $D^{Aut}_{1t}$ represents autonomous demand for commodity 1. As is well known with $s_w > 0$ some part of total profit will accrue to workers. In this case, income of workers in period $t$ denoted as $Y_t^w$ can be written as

$$Y_t^w = \frac{w}{1-\pi.s_w}\left(L_{1t} + L_{1t}\right). \qquad \text{…..(II.15)}$$ [4]

Here $w$ is the real wage in terms of commodity 1 while the parenthetical term represents the total labour requirement for production in the economy in period $t$.

As to the latter magnitude, the assumption of variable efficiency of machines means that the labour required in each industry in order to produce a given output will depend in part on the age-composition of the stock of physical capital. In other words, and bearing in mind equations (II.1) and (II.4), the total labour requirement for each sector can be written as

$$L_{it} = \left( M_{it}^0.l_{it}^0.u_{it}^0 + \frac{M_{it}^1.l_{it}^0\left(1+\alpha\right).u_{it}^0}{\left(1+\phi\right)} \right), \qquad i = 1,2 \qquad \text{…..(II.16)}$$

A second complication introduced by the joint production treatment of fixed capital relates to capitalists' consumption. The profit available to capitalists for consumption expenditure will be influenced by the size of depreciation allowances and these are dependent on relative prices and the rate of profit. More precisely, depreciation for sector $i$ in period $t$, denoted $\lambda_{it}$, can be expressed as

$$\lambda_{it} = M_{it}^0\left(p_{21t} - p_{it}^{m11}\right) + M_{it}^1.p_{it}^{m1} \qquad i = 1,2 \qquad \text{…..(II.17)}$$

where $p^{m1}{}_{it}$ refers to the price of one-year old machines used in sector $i$ in period $t$ relative to the price of commodity 1. Taking commodity 1 as numeraire, profit available for consumption expenditure by capitalists in period $t$, denoted $\Pi_t$, is given by the sum of profits associated with the use of capacity at different ages, less depreciation allowances for each sector, viz.,

$$\Pi_t = \sum_{i=1}^{2}\sum_{j=0}^{1}\left\{\left(p_{i1} - l_{it}^j.w\right).Y_{it}^j - \lambda_{it}\right\} \qquad \text{…..(II.18)}$$

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[4] As noted in White 2008, the simplifying assumption implicit in expression (II.13) means that the profit income available in period $t$ is based on saving undertaken in period $t$ and that this also really assumes that the income estimate on which workers based their consumption in period $t$ includes the expectation of income (profit/interest) to be received in $t+1$from saving in period $t$.

where $p_{i1}$ is the price of commodity $i$ in terms of the numeraire.[5] Taking $c_p$ as the propensity of capitalists to consume, equations (II.13)-(II.16), together with equations (II.1)-(II.4) allow one to rewrite equation (II.12), representing the demand for commodity 1, as

$$D_t^1 = \frac{(1-s_w).w}{(1-\pi.s_w).(1+\phi)}.\sum_{i=1}^{2} l_i^0.u_{it-1}^0.\beta_i.\left\{M_{it-1}^0.(1+\phi)+M_{it-1}^1.(1+\alpha)\right\}+(1-s_c).\left[A^0+A^1-\pi.s_w.Y_{t-1}^w\right]+D_{1t}^{Aut}$$

with …..(II.19)

$$A^0 = \sum_{i=1}^{2} M_{it-1}^0 \left\{ p_{i1}.u_{it-1}^0.\beta_i - l_i^0.u_{it-1}^0.w.\beta_i - p_{21} + p_i^{m1} \right\}$$

$$A^1 = \sum_{i=1}^{2} M_{it-1}^1 \left\{ \frac{p_{i1}.u_{it-1}^0.\beta_i}{(1+\phi)} - \frac{l_i^0.u_{it-1}^0.w.\beta_i.(1+\alpha)}{(1+\phi)} - p_i^{m1} \right\}$$

In order to determine the growth rate of demand for commodity 1 it is necessary to define the growth rate of newly installed capacity in sector $i$ between $t$ and $t-1$ as

$$g_{it}^m = \frac{M_{it}^0}{M_{it-1}^0} - 1 = \frac{I_{it-1}}{I_{it-2}} - 1 \qquad \text{…..(II.20)}$$

as well as a number of ratios of capacity to demand. More precisely, given equations (II.2), (II.3) and (II.4), one can express the ratios of new capacity in either sector to demand for commodity 1 as:

$$\frac{M_{2t-1}^0}{D_{1t-1}} = \frac{(1+g_{1t-2}^{de})(1+g_{1t-1}^m)(1+\phi)}{(1+g_{1t-1}^{de})M_{12t-1}^0 u_{1t-1}^0.\beta_1.z_1}$$

$$\frac{M_{1t-1}^0}{D_{1t-1}} = \frac{(1+g_{1t-2}^{de})(1+g_{1t-1}^m)(1+\phi)}{(1+g_{1t-1}^{de})u_{it-1}^0.\beta_i.z_1} \qquad \text{…..(II.21)}$$

where

$$z_i = \left(\phi.g_{it-1}^m + g_{it-1}^m + \phi + 2\right) \qquad i = 1,2$$

[5] The term $p_{i1}$ in the expressions for $A^0$ and $A^1$ is of course equal to 1 for commodity 1 since the latter is numeraire. Additionally, since prices are assumed to be a long-period equilibrium levels (see next sub-section), the time-subscripts have been omitted.

and $M^0_{12t-1}$ refers to the ratio of newly installed capacities in the two sectors in period *t-1*. From equation (II.18), one can write for the ratio of older capacity in either sector to demand for commodity 1

$$\frac{M^1_{it-1}}{D_{1t-1}} = \frac{M^0_{it-1}}{D_{1t-1}} \Big/ \left(1 + g^m_{it-1}\right) \qquad \text{…..(II.22)}$$

Dividing through equation (II.17) by $D^1_{t-1}$, making use of (II.18)-(II.20) and bearing in mind $M^1_{it+1} = M^0_{it}$, one can derive an expression for the rate of growth of demand for commodity 1 between *t* and *t-1*. Given technology, the rate of profit and the growth rate of autonomous demand, this growth rate is a function of growth rates, utilization rates, ratios of newly installed capacity, ratios of demand and the ratio of autonomous to total demand for commodity 1 all for period *t-1*. This relation is written here simply as

$$g^d_{1t} = g_1\left(g^d_{it-1}, g^m_{it-1}, u^0_{it-1}, D_{12t-1}, M^0_{12t-1}, CD^{Aut}_{t-1}\right) \qquad i = 1,2 \qquad \text{…..(II.23)}$$

where $CD^{Aut}_{1t-1}$ refers to ratio of autonomous demand for commodity 1 to total demand for commodity 1 at *t-1*.

The growth rate of induced investment for each sector – i.e. the growth rate of new capacity – can be derived on the basis of equations (II.8) and (II.18). Thus

$$g^m_{it} = \frac{D_{it-2}.g^m_i A - \left(1 + g^d_{it-2}\right)\left(M^0_{it-1} - M^0_{it-2}\right)u^{n1}_i.\beta}{D_{it-2}\left(1 + g^d_{it-3}\right)^2 - \left(1 + g^d_{it-2}\right)M^0_{it-2}u^{n1}_i.\beta}, \qquad i = 1,2 \text{ …..(II.24)}$$

where

$$g^m_i A = g^d_{it-2}\left(1 + \left(1 + g^d_{it-2}\right)\left(2 + g^d_{it-2}\right)\right) - g^d_{it-3}\left(2 + g^d_{it-3}\right)$$

As noted above, $u^{n0}_i$ represents the normal or desired utilization rate on newly installed plant. Given expression (II.4), this rate implies a normal or desired rate on older plant, denoted in equation (II.26) as $u^{n1}_i$ and equal to $u^{n0}_i/(1+\phi)$. Expressions (II.18) and (II.19) allow one to eliminate the $D_{it-1}$'s and $M^0_{it-1}$'s in equation (II.22) and express $g^m_{it}$ as a function of growth rates of demand for *i* from *t-2* to *t-3*, the growth rate of capacity for *t-1* and capacity utilization in *t-1*, given technology, normal utilization rates and the growth rate of autonomous demand. Thus

$$g^m_{it} = g^m\left(g^d_{it-j}, g^m_{it-1}, u^0_{it-1}\right) \quad i = 1,2;\ j = 2,3 \qquad \text{…..(II.25)}$$

*(iv) Prices*

At this point it is necessary to explicitly deal with the price system, the determination of which is presupposed in the determination of the growth rate of demand for commodity 1. As indicated in the Introduction, the emphasis in the present paper is on the dynamics of quantities so that effectively, the model considered here is a "fixed-price" model. But the level at which prices are "fixed" is not arbitrary: instead it is assumed relative prices and the real wage are at their long-period equilibrium levels, consistent with a uniform rate of profit. With fixed capital treated as a joint product and considering the capital stocks and outputs of period *t*, evaluated at long-period equilibrium prices, relative prices are the solutions to the following price system:

$$\begin{aligned} &M_{it}^{0}.p_{21t}.(1+\pi_t)+w_t.l_i^0.Y_{it}^0 = Y_{it}^0.p_{i1}+M_{it}^0.p_{it}^{m1} \\ &M_{it}^{1}.p_{it}^{m1}.(1+\pi_t)+w_t.l_i^1.Y_{it}^1 = Y_{it.}^1 p_{i1} \end{aligned} \qquad i = 1,2 \;\ldots..(\text{II.26})$$ [6]

Taking account of equations (II.1) - (II.4), and assuming that long-period prices correspond with the normal (desired) utilization of productive capacity, price equations (II.24) can then be rewritten as

$$\begin{aligned} &p_{21}.(1+\pi) = u_i^n.\beta_i.\left(p_{i1}-w.l_i^0\right)+p_i^{m1} \\ &p_i^{m1}.(1+\pi) = u_i^n.\beta_i.\left(p_{i1}-w.l_i^0.\left(1+\alpha\right)\right)\Big/\left(1+\phi\right) \end{aligned} \qquad i = 1,2 \;\ldots..(\text{II.27})$$

where $u^n$ refers to the desired / normal utilization rate on newly installed capacity, which, as noted above, with $\phi$ also given implies a "desired" utilization rate on older machines. The price system (II.25) determines three relative prices and the real wage rate for an exogenously determined rate of profit.[7]

Completing the model requires modeling the behavior of utilization rates, the ratio of investments of the two sectors ($M^0_{12}$) and the ratios of autonomous demand to total demand ($ID^{Aut}$ and $CD^{Aut}$).

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[6] The value of the used machine is equal to the discounted profit per unit of output on the machine, where the discount rate is the rate of profit. It is also assumed here that equilibrium is maintained in the market for used machines to the extent that they exist.

[7] The approach here ignores the dependency of the desired rate of capacity utilization on relative prices and thus on the rate of profit.

Considering utilization rates first, equations (II.2), (II.3) and (II.4) imply that for sector *i*

$$Y_{it} - M^{0}_{it-1}.u^{0}_{it}.\beta_i = \frac{M^{0}_{it-1}.u^{0}_{it}.\beta_i}{1+\phi} \qquad i = 1,2$$

and therefore that

$$u^{0}_{it} = \frac{Y_{it}.(1+\phi)}{\beta_i.\left(M^{0}_{it}.(1+\phi) + M^{0}_{it-1}\right)} = \frac{D_{it-1}.\left(1+g^{de}_{it-1}\right).(1+\phi)}{\beta_i.\left(I^{0}_{it-1}.(1+\phi) + M^{0}_{it-1}\right)} \qquad i = 1,2 \quad \text{.....(II.28).}$$

In view equation (II.8) for induced investment,

$$u^{0}_{it} = \frac{\left(1+g^{d}_{it-1}\right)\left(1+g^{de}_{it-1}\right)u^{0}_{it-1}z_i}{\left(1+g^{de}_{it-2}\right)\left(1+g^{m}_{it-1}\right)\left(2+g^{m}_{it}+\phi+g^{m}_{it}\phi\right)} \qquad i = 1,2 \qquad \text{.....(II.29)}$$

Regarding the ratio of induced investment of sector 1 relative to sector 2 at time *t*, $M^{0}_{12t}$, this can be expressed as

$$M^{0}_{12t} = \frac{\left(1+g^{m}_{1t}\right).M^{0}_{12t-1}}{1+g^{m}_{2t}} \qquad \text{.....(II.30)}$$

Substituting the corresponding version of expression (II.23) for $g^{m}_{1t}$ and $g^{m}_{2t}$ respectively will yield an expression for $M^{0}_{12t}$ as a function of growth rates of both demands in *t-1* and *t-2*, growth rates of capacity in *t-1* and utilization rates in *t-1*.

Finally, concerning the ratio of autonomous demand to total demand for each sector, it is assumed for simplicity that both autonomous demands grow at a uniform rate, so that the ratio of the two autonomous demands is constant. The relation between the ratios of autonomous to total demand for the two sectors can be written as

$$CD^{Aut}_{t} = \frac{ID^{Aut}_{t}.\mu}{D_{12t}} \qquad \text{.....(II.31)}$$

where μ is the ratio of the two autonomous demands and $D_{12t}$ is the ratio of total demands for the two sectors at time *t*. Equations (II.19) and (II.20) allow for a relation between $D_{12t}$ and the ratio of the new capacities in the two sectors: using (II.19) to derive an analogous expression for $M^{0}_{2t-1}/D_{2t-1}$ and in turn solving forward for $M^{0}_{1t}/D_{1t}$ and $M^{0}_{2t}/D_{2t}$ allows one to write

$$D_{12t} = \frac{\left(1+g_{1t}^{d}\right).\left(1+g_{2t-2}^{de}\right).\left(1+g_{2t-1}^{m}\right).M_{12t-1}^{0}.u_{1t-1}^{0}.\beta_1.z_1}{\left(1+g_{2t}^{d}\right).\left(1+g_{1t-1}^{de}\right).\left(1+g_{1t-1}^{m}\right).u_{2t-1}^{0}.\beta_2.z_2} \qquad \text{…..(II.32)}$$

Where growth rates are uniform and utilization rates normal in each sector, this expression becomes

$$D_{12t} = \frac{M_{12t}^{0}.u_1^{n0}.\beta_1}{u_2^{n0}.\beta_2}$$

In effect, equations (II.29) and (II.30) allow us to eliminate $CD^{Aut}_{t-1}$ from the expressions for $g^{d}_{1t}$ and $u^{0}_{1t}$ by substituting (in equations (II.21) and (II.27)) with a term in $ID^{Aut}_{t-1}$, μ, $M^{0}_{12t-1}$, and utilization and capacity growth rates for *t-1*. There remains to express the time path of $ID^{Aut}$:

$$ID_t^{Aut} = \frac{ID_{t-1}^{Aut}.(1+a)}{(1+g_{2t}^{d})} \qquad \text{…...(II.33)}$$

Substituting from equation (II.30) to eliminate $D_{12t-1}$ in expression (II.11) and in turn substituting for $g^{d}_{2t}$ in equation (II.31) provides for a relation between $ID^{Aut}_{t}$ and growth and utilization rates in *t-1*.

**III Equilibrium: endogenous or exogenous growth?**

The model outlined above essentially provides a system of equations describing the time path of eight variables: two growth rates of demand ($g^{d}_{1t}$, $g^{d}_{2t}$), two growth rates of new capacity ($g^{m}_{1t}$, $g^{m}_{2t}$), two utilization rates on newly installed machines ($u^{0}_{1t}$, $u^{0}_{2t}$), the ratio of induced investments in the two sectors ($M^{0}_{12t}$) and the ratio of autonomous investment demand to total investment demand ($ID^{Aut}_{t}$). With suitable substitutions, the model outlined above can be written as the first-order difference equation system

$$x_t = f\left(x_{t-1}\right) \qquad \text{…..(III.2)}$$

where *x* is the vector

$$x = \left(g_i^{d}, g_i^{de}, g_i^{m}, M_{12}^{0}, u_i^{0}, ID^{Aut}\right) \qquad i = 1, 2.$$

In other words, the model can be thought of in terms of 12 variables whose values in period *t* are functions of the same 12 variables at *t-1*. An equilibrium (fixed point) of the

recursive system (III.2) is characterised by $g_{it}^{d} = g_{it-1}^{d} = g_{it}^{m} = g_{it-1}^{m} = g*$ and $u_{it} = u_{i}^{n}$ for $i$ = 1,2

**IV Disequilibrium and stability**

In both this and following sections the dynamic behaviour of system (III.2) is investigated. Two methods are available for this purpose. The first involves an analysis of the local stability of the steady state growth path for the "exogenous growth" case, i.e. where the steady state growth rate is set by the growth rate of autonomous demand. The second method for analyzing the dynamics of (III.2) involves computer simulation of the system's behaviour following an exogenous shock

The present paper focuses exclusively on the second of these two methods – computer simulations of system III.2. Each simulation exercise examines the behaviour of the model over time, starting from a steady state and imposing on the model a shock in the form of a change in the rate of growth of the autonomous components of demand and thus a change to the steady state rate of growth. The values assigned to technology (including normal utilization rates) and the rate of profit and therefore (via equations (II.25)) to the real wage and relative prices for all simulations are given in Tables 1 and 2 in the *Appendix*.

**V Expectations about growth – the basic model without learning**

In order to close the model for the purposes of simulation there remains to determine explicitly the expected growth rate of demand which underpins decisions about investment (II.8). For the "basic" model on which learning behavior is superimposed in subsequent sections it is assumed that for the purposes of investment at the end of period *t* producers forecast growth in demand for the period *t+1* based on two calculations: one based on the demand growth in their own sector observed at the conclusion of periods *t-1* and *t-2*; and one based on the growth rates of autonomous demand. With regard to the first calculation, it is further assumed following the lead taken by Franke and Weghorst (1988) that producers use an average of the previous two realized growth rates, discounted by a factor which depends on the dispersion between those rates.

Thus the estimate of future growth in demand is given by the weighted average

$$g_{it}^{de} = \varepsilon . g_{t}^{eAut} + (1-\varepsilon) . \frac{\left(g_{it-1}^{d} + g_{it-2}^{d}\right)}{2} . X_{it} \qquad \text{……..(V.1)}$$

where $g_{it}^{de}$ and $g_t^{eAut}$ refer to the expectation held at the end of period $t$ respectively about demand growth in sector $i$ between the end of periods $t+1$ and $t$; and about autonomous demand growth between $t$ and $t+1$. $X_{it}$ refers to the abovementioned dispersion factor and is given by

$$X_{it} = \frac{1}{1+\sigma.\left(g_{it-1}^{d} - g_{it-2}^{d}\right)^2} \qquad \text{……..(V.2)}$$

It has been demonstrated in White 2005 and 2008 that system (III.2) combined with (V.1) and (V.2) allows for steady state growth equal to the rate of growth of autonomous demand.[8]

Computer simulations of system III.2 discussed below are based on parameter values which entail a steady state equal to the rate of growth of autonomous demand (assumed uniform for the two sectors). In this case the system (III.2) in this case determines endogenously the ratio of autonomous demand to total demand for each sector, along with the relative size of the two sectors. In other words, setting all growth rates equal to $a$, the growth rate of autonomous demands, equations (II.11) and (II.25) would determine $ID^{Aut}$ and $D_{12}$ and equation (II.30) would determine $CD^{Aut}$ for a given.[9]

As to the intuition behind expression (V.1), we suppose that persistent growth in autonomous components of demand leads to the development of expectations about growth in those components of demand. Thus the right hand side of expression (V.1) could be seen as reflecting, albeit in a very simplified way, two considerations in the minds of producers: first, an indication of the exogenous growth forces acting on the "economy as a whole"; and second,

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**8** These analyses also indicate a second type of steady state - certainly more plausible where expectations are not based on expectations about autonomous demand growth along the lines of (V.1) and (V.2) – where the steady state rate of growth is in effect "endogenous": the steady state rate of growth of income exceeds the rate of growth of autonomous demands so that the ratio of autonomous demand to income converges on zero over time.

**9** Either of these two types of equilibria are fixed points of the system (III.2). In the first "endogenous growth" case, substitution of $g^*$ for growth rates, $u^0_t = u^{n0}$ for actual utilization rates, and $D_{12}^*$ and $ID^{Aut} = 0$ will yield $g^*$ as the solution for all growth rates and $u^{n0}_i$ for $u^0_{it}$. Similar substitutions of $a$ for all growth rates, $D_{12}^*$ and $ID^{Aut*}$ will yield $a$ as the solution for growth rates and $u^n_i$ for $u^0_{it}$ in the "exogenous growth" case.

It should be added that the "exogenous growth" case does allow for both an exogenous growth rate and an exogenous rate of profit and thus represents one possible of a Sraffian-based explanation of prices and distribution on the one hand and a Keynesian long-run view about output, to the extent that the latter is to be associated with exogenous sources of growth.

a recognition that growth in their own sector is not completely independent of growth in the economy as a whole.

Hence the value given to ε reflects the views of producers as to the importance of autonomous demand for the two commodities in determining the growth of the economy as a whole as well as their views about the relation between the latter and the growth of their own industry. In a more general model (where autonomous demand does not exist for some commodities) the effect of autonomous demands on sectoral growth may be indirect and arguably through growth in the "rest of the economy" at least seen from the producers' point of view. It is really this indirect influence, namely, via the view that autonomous demand growth drives the growth of the economy and that the latter will provide some guide to the growth of individual sectors, that is of interest in the present discussion.

Two further points assumptions are made about the forecast $g_t^{eAut}$: first, the growth rate of autonomous demand is the same for both sectors. Second, the forecast (uniform) growth rate of autonomous demand is based on a very simple adaptive rule, viz., the forecast growth rate is equal to the most recently observed growth rate.

Growth rates of demand for this basic model without learning are depicted in Figure 1. The critical parameter here is ε. Panels (a) and (b) indicate that for certain parameter values – including values of σ and ε – the system shows stability in the sense that there exists no tendency for fluctuations to increase in amplitude. For sufficiently high values of ε (> 0.745), the model III.2 – V.1 produces damped cycles in growth rates around the new equilibrium. Thus although the system does not converge to the constant rate of growth of autonomous demand for some ε < 0.745, a sufficient enough weighting of autonomous demand growth in forecasts of growth by producers (i.e. ε sufficiently > 0.2) would render stability at least in terms of a non-explosive fluctuations in growth rates of demand and investment.

**VI Learning I**

As noted above, what informs the use of expression VI.I is the view that producers accept that autonomous components of demand can have a persistent influence on output; specifically, that a persistent change in the rate of growth of autonomous demands can lead to a persistent change in the growth rate of the economy. Implicit in this supposition is the notion that producers do not believe that in the long-run the rate of growth is constrained by the available supply of inputs, including labour.

Importantly however it is also assumed that producers are uncertain about the nature of the relation between growth in individual sectors and the growth of autonomous demand. More precisely, we assume that there exists uncertainty about the nature of three sets of dynamics:

(i) the dynamics of the growth rate(s) of autonomous demand(s);

(ii) the relation between the growth rate(s) of autonomous demand(s) and the growth rate of the economy;

(iii) the relation between the growth rate of the economy and the growth rate of demand in the individual producer's own sector.

This uncertainty however points to one obvious limitation with the analysis up to this point, at least where producers are capable of learning: the parameter ε, representing the weighting of elements in the formation of producers' expectations about growth, is exogenous and independent of the economy's behavior over time; and most importantly, independent of errors in producers' growth forecasts.

One obvious change to the model would then involve making ε endogenous, as a proxy for making making endogenous the views of producers about the relative importance of growth in their own sector and growth in the economy as a whole (and not the significance of autonomous demands in the latter) in their forecasts of growth in demand for their own output.[10]

For this purpose we make use of a procedure suggested by Caminati (1998), viz., the application of a gradient-descent rule applied to the determination of ε for each of the two sectors of the basic model. More precisely it is assumed that e for each sector is modified in each period on the basis of forecast errors, specifically, how the forecast error itself responded to the change in this weighting the previous period.

The relevant forecast error here is

$$E^{d}_{it+1} = (1/2)\left(g^{d}_{it+1} - g^{de}_{it}\right)^{2} \qquad \text{(VI.1)}$$

so that the adjustment in the weight, $\varepsilon_{it}$, is given by

[10] Recall that the justification for having views about autonomous demand influencing expectations about growth is as an indicator of the growth rate of the economy as whole.

$$\varepsilon_{it} = \varepsilon_{it-1} - \eta . \frac{\partial E_{it}^{d}}{\partial \varepsilon_{it-1}} \qquad \text{(VI.2)}$$

Figures 2 depicts for ,number of different starting values of ε – 0.2, 0.4 and 0.6 - for each sector, the effect of having ε determined endogenously in accordance with expressions (VI.1) and (VI.2); allowing for a number of values of the coefficient η. What stands out immediately is firstly the divergence in ε values for the two sectors even though they were started at the same value for each sector; and secondly, how, the effect of this simple form of learning is to bring about a long-run change in the ε values for both sectors. A third interesting feature (at least tentatively) suggested by these results is that there appears to exist a gravitation around a long-run "equilibrium" set of ε values: around 0.4. Thus for low starting values – 0.2 – ε for both sectors is driven over time up, while starting values of 0.6 see ε values driven down over time. From VI.2, this behavior in the former case implies increases in ε have the effect at least for a range of ε values of reducing forecast errors; after which ε appears to stabilize; while in the latter case, the opposite holds, so that reductions in ε reduce forecast errors (relatively speaking). The further implication is of course that there is a set of ε values (at least for $\varepsilon_1$, $\varepsilon_2 <$ 0.6), which minimize forecast errors and the simple learning embodied in expressions VI.1 and VI.2 are able to lead the system to this stabilizing set of ε values. However, by way of qualification to this last point, a fourth feature suggested by figure 2 is that too high a rate of learning (reflected in the value of η) tends to make more volatile the long-run path of ε values. This is suggested by panel (c) in Figure 2, showing starting values of ε = 0.6, but values of η = 2 and 3 respectively.[11]

One final point in relation to these results is worth remarking on. The tendency for ε values to rise from low starting values is of some importance the regarding the question of how producers might arrive at a value of ε to begin with. As noted in White (2008), "even for the case of long-run convergence of growth rates to that of the rate of growth of autonomous demand – cases with arguably, 'high' values of ε [i.e. greater than 0.745] – in general, the growth rates of demand and growth rates of autonomous demand will be unequal at any point in time. Arguably this fact would work against producers assigning a 'high' (e.g. sufficient for

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**11** This last result appears to be in keeping with suggestions by the learning literature (*cf*. Caminati, **op.cit**. p. *****)

long-run convergence) value given to ε. Even if producers believe in a long-run gravitation of growth rates around the rate of growth of autonomous demand, uncertainty about the precise dynamics of interaction between growth rates in their own sector and autonomous demands may render this belief of little use for investment decisions. [Moreover], cases where there is some stability—in the sense of irregular fluctuations showing no tendency to explode—without long-run convergence of growth rates exhibit systematic forecast errors" (pp. 22-23).

The results depicted in Figure 2 suggest one possible way – via the application of a simple learning rule - in which values of ε evolve to a magnitude consistent with non-exploding fluctuations in growth rates, even though forecast errors are not completely eliminated.

**VII Learning II: agent's modeling of the dynamics of demand**

We turn now to discussion of what might be considered a more "sophisticated" approach to learning *vis a vis* that of the preceding section. It is now assumed that at least some producers in each sector recognize that the realised growth rates for individual sectors, depend, for a given set of growth rates of the economy, past sectoral growth rates, and past growth rates of autonomous demands, on producers' expectations about future growth rates. What is unknown is the nature of the function linking expected growth rates to actual realised growth rates - i.e. not only how past growth rates feed into expectations about future growth, but how changes in expected growth rates feedback on realized growth rates.[12] What is unknown in other words is the objective structure of the dynamics of the economy.

Thus, any agent wishing to make predictions about growth on the basis of some understanding of the dynamics of demand can be seen to be faced with two tasks in particular: first, working out how other producers in general would generate their expectations about growth on the basis of past growth rates; and, second, working out how these expectations feed into the dynamics of demand.

As to the first task, following the suggestion in Caminati (1989, p. 18), we assume that such an agent – referred to here as 'producer *i*' is able to infer something about the expectations of other producers about sectoral growth rates. If technical coefficients (including

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**12** Put alternatively, producers are unable to assess how "the actual function mapping the output realizations […. , in our analysis] into current [values, …] changes with every change in the expectation function used to produce the forecast[s, ……] (p. 12)".

especially, desired capital to output ratios) are known, existing capital stocks at the end of *t-1* for example are known and investment expenditures at the end of *t-1* are known, it is possible for producer *i* to form inferences about anticipated sectoral growth rates, the implicit assumption being that producer *i* assumes other producers decide on investment in similar way to himself/herself.

Modeling the second task on the other hand raises more difficult issues. In particular, it requires imparting to producer *i* the capability of or access to modeling of the interaction between investment and aggregate demand, including how autonomous elements of demand fit into that picture. In preceding down this track, the intention here is not to proceed down the "perfect-foresight path" ; viz., which, in the present model, would amount to assuming producer *i* has access to the system (III.2)-(V.1)-(VI.1)-(VI.2) above – in other words, access to (what amounts for the purposes of the present paper to) the "correct" model of the real world. Instead, a more reasonable hypothesis is that producer *i* may have access to a "model" of the present model; e.g. an aggregative one-sector simple-multiplier accelerator model at least for the purposes of explaining how expected growth rates of aggregate demand and growth rates of autonomous demand feed into realized growth rates. As such, that modeling task would involve problems well known to the forecaster, such as assigning values to aggregative versions of sectoral parameters e.g. an aggregate version of the sectoral β's (representing the output capacity of machines) .

With regard to the first of the above-mentioned tasks facing producer *i*, viz., an estimate of the expectations about future growth held by other producers at the end, we assume that estimate, made at the end of period *t*, is equivalent to the determination of $g_{it}^{de}$ in expression (V.1) except that the relevant value of ε is that for period *t-1*.[13] Hence producer *i*'s estimate, $g_t^{eij}$, formed at the end of period *t* of expected growth rates held by other producers in sector *j* about growth in demand between *t+1* and *t* is given by

[13] Since, in view of the if producer *i* supposes in accordance VI.1 and VI.2, that ε in each sector is determined endogenously, $\varepsilon_t$ could only be determined on the basis of demand in each sector at the end of *t* being known. But such demand cannot be known prior to producer *i* calculating his/her own investment demand. Effectively, expression (VII.1) implies that producer *i* adjusts the value of ε each period in a simple adaptive expectations manner.

$$g_t^{eij} = \varepsilon . g_t^{eAut} + (1-\varepsilon) . \frac{\left(g_{it-1}^d + g_{it-2}^d\right)}{2} . X_{it} \qquad \text{.....(VII.1)}$$

The first difficulty which arises however is that producer *i* seeks a simplified "workable" representation of expectations about growth held by other producers, but the ("real world") model of sections II – V above is multi-sectoral. Hence producer *i* is faced with the task of reducing different expectations about demand growth in each the two sectors into an "aggregate" expectation about growth. To this end it is assumed that producer *i* uses a weighted average of the $g_t^{eij}$ estimates for the two sectors where the weighting is determined by an historical average of the relative sizes of the two sectors, in turn measured by the most recent observation (end of period *t-1*) of the relative size of demands for the two sectors.

Thus producer *i*'s estimate of the "aggregate" growth expectation held by other producers at the end of period *t* (about growth in demand between the end of periods *t* and *t+1*) is given by

$$g_t^{ei} = \kappa . M_{12t}^0 . \frac{\beta_1}{\beta_2} . \frac{u_{1t}^0}{u_{2t}^0} . g_t^{ei1} + (1-\kappa) . M_{12t}^0 . \frac{\beta_1}{\beta_2} . \frac{u_{1t}^0}{u_{2t}^0} . g_t^{ei2} \qquad \text{.....(VII.2)}$$

where the term $M_{12t}^0 . \frac{\beta_1}{\beta_2} . \frac{u_{1t}^0}{u_{2t}^0}$ is taken as a proxy for the relative size of demands for the two commodities during period *t*. Having estimated other producers' collective views about future growth at the end of period t, producer *i* turns to the second of the above-mentioned tasks, viz., working out how this expectation feeds into the actual investment decisions and thus into actual growth, specifically, between the end of periods *t* and *t+1*. And on this basis, producer *i* will, it is assumed, make his/her own decisions about production and investment,

To this end and following the suggestion above, it is supposed that producer *i* constructs a simplified aggregate version of the model (III.2)-(V.1)-(VI.1)-(VI.2) which can serve as the basis of producer *i*'s prediction of future demand growth in the economy. The simplest form of such a model could be written as:

$$\begin{aligned} D_t &= C_t + I_t + D_t^{Aut} \\ C_t &= c . Y_{t-1} \end{aligned} \qquad \text{.....( VII.3)}$$

In view of expression II.6, the consumption function can be written as

$$C_t = c.\frac{D_{t-1}\left(1+g_{t-2}^{de}\right)}{1+g_{it-1}^{d}} \qquad \text{…..( VII.4)}$$

The modeling of investment follows broadly the approach adopted in the model (III.2)-(V.1)-(VI.1)-(VI.2) though we assume that producer *i*'s modeling of this is somewhat simpler, in particular by collapsing the analysis of two sectors down to an analysis of one sector. Hence, a one-sector version of expressions II.6 – II.9 yields the corresponding one-sector version of II.10,

$$I_t = \frac{1}{\beta}\left\{D_{t-1}.\left(\frac{\left(1+g_t^{de}\right)^2}{u^n} - \frac{\left(1+g_{t-1}^{de}\right)\left(1+g_t^m\right)}{u_t^0\left(2+g_t^m+\phi+g_t^m\phi\right)}\right)\right\} \qquad \text{…..( VII.5)}$$

where $g_t^m$ represents (as earlier, though for the one-sector case) the growth rate of the capital stock (machines) between periods *t* and *t-1*, which it is assumed producer *i* takes as equal to the rate of growth of demand for commodity 2, $g_{2t-1}^{d}$, which can be observed between the end of periods *t-1* and *t-2*.

Solving expression (VII.5) backward for $I_{t-1}$, and combining with (VII.3)-(VII.4) allow one to derive an expression for the growth rate of aggregate demand, $g_t^{mod}$ ,between the end of periods *t* and *t-1*; as a function of past growth rates in demand, the capital stock, expected demand as well as expected demand growth at the end of period t and the ratio of autonomous demand $D_t^{Aut}$ to total demand $D_t$. This function represents in effect producer *i*'s prediction of the growth of demand which would take place between the end of periods *t* and *t-1* on the basis of expectations about future growth; this latter being estimated in turn by producer *i* on the basis of expression (VII.2) above. Thus

$$g_t^{mod} = g^{mod}\left(g_{t-1}^{mod}, g_{t-k}^{ei}, g_t^m, u_t^{mod\,0}, D_{mod\,t}^{AD}\right) \qquad k = 0, 2 \text{ and } 3 \quad \text{…..( VII.6)}$$

where $u_t^{mod\,0}$ and $D_{mod\,t}^{AD}$ refer respectively to producer *i*'s modeled values of aggregate utilization during period *t* and the ratio of autonomous demand to total demand at the end of period *t*. With regard to $u_t^{mod\,0}$, producer *i*'s modeling of utilization for period *t* can be based on observables when the estimate is calculated and it is assumed that this estimate will be a weighted average of the utilization rates across the two sectors.

Modeling of $D_{mod\,t}^{AD}$ on the other hand is slightly more complex. Part of the complexity lies in the fact that at the end of period *t* when producer *i* has to model $g_t^{mod}$, $D_{mod\,t}^{AD}$ cannot be

observed, since $D_t$ and thus $I_t^{Aud}$ cannot be observed . There is also the realization by producer *i* that this ratio is subject to considerable fluctuation from period to period. In view of these two considerations, it is assumed that take an average of past $D_{mod}^{AD}$ 's to calculate $D_{mod\,t}^{AD}$. For this purpose it is assumed that producer *i* has access to national accounts information about the economy, sufficient to enable a calculation of $D_{mod\,t}^{AD}$ for any period *t* in the past, at least in nominal terms. More precisely, the ratio of autonomous demand measured in nominal terms as a proportion of nominal aggregate demand which would be implied by the model (III.2)-(V.1)-(VI.1)-(VI.2)

$$D_{mod\,t}^{AD} = \frac{ID_{t-1}^{Aut}.\left(p_{21}.u_2^{n0}.\beta_2 + M_{12t}^{0}.u_2^{n0}.\beta_1.\mu\right)}{\left(M_{12t}^{0}.u_1^{n0}.\beta_1 + p_{21}.u_2^{n0}.\beta_2\right)} \qquad \text{…..( VII.7)}$$

Hence, it is assumed that producer *i* calculates $g_t^{mod}$ as

$$g_t^{mod} = g^{mod}\left(g_{t-1}^{mod}, g_{t-k}^{ei}, g_t^{m}, u_t^{mod\,0}, D_{Histt-1}^{AD}\right) \qquad \text{…..( VII.8)}$$

where

$$D_{Hist}^{AD} = \sum_{i=1}^{n} \chi_i.D_{mod\,t-i}^{AD} \qquad \text{…..( VII.9)}$$

An analogous approach is adopted for the consumption propensity, '*c*' in expression (VII.3), so that

$$c = c_t^{Hist} = \sum_{i=1}^{n} \psi_i.c_{t-i}^{mod} \qquad \text{…..( VII.10)}$$

where

$$c_t^{mod} = \frac{\left(1+g_{1t}^{d}\right).\left(1+g_{1t-1}^{d}\right).M_{12t-2}^{0}.u_1^{n0}.\beta_1}{\left(1+g_{1t-2}^{de}\right).M_{12t-2}^{0}.u_1^{n0}.\beta_1 + \left(1+g_{1t-2}^{de}\right).p_{21}.u_2^{n0}.\beta_2} \qquad \text{…..( VII.11)}$$

Similarly to $D_{mod\,t}^{AD}$ of expression (VII.7) above, the RHS of expression above is what $c_t^{mod}$ what the model (III.2)-(V.1)-(VI.1)-(VI.2) would generate as the value of consumption demand as a share of aggregate demand and thus what producer *i* could glean each period from a set of national accounts.

Of course the task for producer *i* who is planning investment and output like any other producer is really to forecast growth in demand between the end of periods *t+1* and *t*, so that the modeling task for producer *i* therefore involves solving expression (VII.6) forward by one period and thus solving

$$g_{t+1}^{mod} = g^{mod}\left(g_t^{mod}, g_{t-k}^{ei}, g_{t+1}^{m}, u_{t+1}^{mod\,0}, D_{Histt}^{AD}\right) \qquad k = -1,\ 1 \text{ and } 2 \quad \text{.....( VII.12)}$$

As indicated by expression (VII.12), producer *i*'s forecast of growth in demand between the end of periods *t+1* and *t* will be based in part on producer *i*'s forecast of growth in demand between the end of periods *t* and *t-1*. Indeed, the arguments $g_{t+1}^{m}$ and $u_{t+1}^{mod\,0}$ on the RHS of expression (VII.12) will at the end of period *t* also be "forecasts".

With regard to the forecast $u_{t+1}^{mod\,0}$ an aggregative, one-sector version of (II.28) is

$$u_t^0 = \frac{Y_t.(1+\phi)}{\beta.\left(M_t^0.(1+\phi) + M_{t-1}^0\right)} = \frac{D_{t-1}.\left(1+g_{t-1}^{de}\right).(1+\phi)}{\beta.\left(I_{t-1}^0.(1+\phi) + M_{t-1}^0\right)} \qquad \text{.....(VII.13),}$$

so that, in view of expression (VII.5), and solving forward for period t+1, one can arrive at

$$u_{t+1}^{mod\,0} = u^0\left(g_{t-h}^{mod}, g_{t-k}^{ei}, u_{t-n}^{mod\,0}, g_{t-j}^{m}\right) \qquad h = 0,1;\ k = 0,\ 1,\ 2;\ n = 0,\ 1;\ j = 0,\ 1$$

.....(VII.14)

With regard to the forecast $g_{t+1}^{m}$ it is assumed for simplicity that producer *i* takes as an indicator of the likely growth of capacity between periods *t+1* and *t* his/her estimate of the expectation of demand growth between the end of these two periods, viz., $g_{t+1}^{ei}$. In turn, it is further assumed that producer *i* determines along the same lines as $g_t^{ei}$ in (VII.2) above, so that

$$g_{t+1}^{ei} = \left(\frac{\kappa.Q_t}{1+\kappa.Q_t}\right).g_{t+1}^{ei1} + \left(1 - \frac{\kappa.Q_t}{1+\kappa.Q_t}\right).g_{t+1}^{ei2} \qquad \text{.....(VII.15)}$$

where

$$Q_t = M_{12t}^0.\frac{\beta_1}{\beta_2}.\frac{u_{1t}^0}{u_{2t}^0}$$

and $g_{jt+1}^{ei}$ determined as

$$g_{t+1}^{eij} = \varepsilon_{t-1}.g_t^{eAut} + \left(1-\varepsilon_{t-1}\right).\frac{\left(g_t^{mod} + g_{jt-1}^{d}\right)}{2}.X_{jt+1} \qquad \text{.....(VII.16)}$$

with

$$X_{jt+1} = \frac{1}{1+\sigma.\left(g_t^{mod} - g_{jt-1}^{d}\right)^2} \qquad \text{.....(VII.17)}$$

In other words, it is supposed that producer *i* estimates $g_{t+1}^{ei}$ by assuming that other producers form their expectations along the lines of (V.1), (V.2) and (VI.1), (VI.2) (as for the estimation of $g_t^{ei}$), but the value of the growth rate $g_{jt}^{d}$ required by that calculation (for each of the two

sectors, $j$ = 1,2) is the rate modeled $g_t^{mod}$ by producer $i$ at the end of period t. The weighting of the two sectors in the calculation of $g_{t+1}^{ei}$ will be analogous to that in the calculation in VII.2 above, viz., on the basis of the relative size of the two sectors.

In summary, expressions (VII.1) – (VII.17) provide a means by which producer $i$ in any sector could model the growth rate of aggregate demand between periods $t$ and $t+1$. The final question concerns the weighting that producer $i$ places on this modeled growth rate in formulating his/her own investment decision at the end of period $t$ and his/her own decision about output to be produced in period $t+1$. It is assumed that producer $i$ in any sector $j$, calculates the expected growth rate of demand in his/her own sector between periods $t+1$ and $t$ as a weighted average of the modeled growth rate of the economy - $g_{t+1}^{mod}$ (expression (VII.12)) - and the growth rate, $g_{jt}^{ewl}$ which he/she would have calculated on the basis of past growth rates of demand, in the absence of any modeling (i.e. (V.1), (V.2), (VI.1) and (VI.2)); Thus producer $i$'s calculated growth rate for his/her own sector $j$ between periods $t+1$ and $t$ is therefore

$$g_{jt}^{dei} = \omega . g_{t+1}^{mod} + (1-\omega) . g_{jt}^{ewl} \qquad \text{.....(VII.18)}$$

Finally, we suppose that only some producers in any sector formulate expectations along the lines of producer $i$ [14] so that for each sector $j$ as a whole, the expectation of growth in demand held at the end of period $t$ will be given by

$$g_{jt}^{de} = \rho . g_{jt}^{dei} + (1-\rho) . g_{jt}^{ewl} \qquad \text{.....(VII.19)}$$

**True learning: some preliminary results**

We briefly discuss here the results of some preliminary simulations of the model with "true learning"; a positive value of the coefficient $\rho$ (equation (VII.19)), representing the magnitude of learning within the aggregate formation of expectations in each sector. For the purposes of these simulations it was assumed that $\eta$ = 0.5, so that the majority of producers

[14] In other words, some (probably most) producers formulate their expectations as in the simulation model without "true learning".

who were not engaged in "true learning" nonetheless applied a gradual gradient-descent rule to the determination of the coefficient ε.

Results for these simulations are depicted in Figure 3. Three different cases are examined by way of variations on the case with only gradient-descent learning for starting ε's = 0.6, η = 0.5 and ρ = 0 (Panel (a)) for a different value of the coefficient ρ from expression (VII.19), which represents the significance of modeling by producer's like producer *i* in the formation of growth expectations generally within each sector. The values of ω and κ are the same (ω = 0.5, κ = 1) across all four simulations. For each simulation case, Figure 3 shows growth rates of demand, values of ε for the two sectors, a comparison of actual demand growth and producer *i*'s modeled growth rate, $g_{t+1}^{mod}$, as well the evolution of the ratio of autonomous demand for machines as a proportion of total investment demand, $ID_t^{Aut}$.[15]

Two aspects of these results stand out fairly immediately. The first is that a relatively low ρ (= 0.1) appears to affect the long-term dynamics of demand only marginally (compare Panels (a) and (b)), while a relatively high value for ρ (= 0.5) destabilizes the system (panel (c)). Of particular interest in relation to the latter case (ρ = 0.5) is the evolution of the ε values for both sectors and in particular the significant fall in both ε values. Since these are both tied to the evolution of forecast errors by producers (equations (VI.1) and (VI.2)), excepting producer *i*, this case indicates a significant fall in forecast errors associated with a reduction in the value of ε and thus with a much lower weighting given to autonomous demand growth in forecasting total demand growth. The evolution of the $ID_t^{Aut}$ gives some idea as to the reason for this result: specifically the significant drop in that ratio close to zero for a significant part of the simulation. In turn this requires significant growth in total demand relative to autonomous demand. And the source of the latter can be found in the growth of demand for the two commodities compared with the modeled growth in demand by producer *i*; the latter being on average much higher.

A similar dynamic, though less stabilizing can be observed with the intermediate case ρ = 0.25 (panel (d)). Here again the long-term growth rate of demand in both sectors is higher than

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**15** Recall, as equation (II.31) indicates, this ratio can be linked to the corresponding ratio relating to demand for the consumption good.

the assumed uniform growth rate of autonomous demands, again, reflected in the convergence of the ratio $ID_t^{Aut}$ towards zero. Once again also the source for this result is to be found in the higher modeled growth rate by producer *i*, *vis a vis* the realized growth rates of demand. The difference between this case and the previous case where ρ = 0.5 is that the lower value of ρ means that this modeled growth rate has a lower weighting in producer *i*'s own estimates of future demand growth and in turn in the expectations about growth of producers as a whole.

This last case however does open up the interesting possibility that the type of learning discussed in the present section allows the long-run growth rate of demand to stabilize at a higher rate than the rate of growth of autonomous demands; so that the ratio of autonomous demands to total demand approaches zero in the longer run (as does producer *i*'s estimate of the ratio of autonomous demand to total demand). Of course, the qualification to this scenario, at least suggested by the results here is that persistent divergence between realized growth rates of demand and the future rate modeled by producer *i* $g_{t+1}^{mod}$ would lead such that producer to alter aspects of their model, including the weighting gives to such modeling in their own growth expectations, i.e. weights such as ρ. In turn this would suggest the application of something like gradient-descent rules (analogous to equations (VI.1) and (VI.2)) in relation to aspects/parameters of producer *i*'s own model.

## VIII Concluding remarks

The analysis above represents a tentative attempt to enquire into the significance of learning in a demand-led growth model. Though preliminary in nature, the results canvassed above are sufficient to indicate some impact of learning either in terms of a simple gradient-descent rule or a more sophisticated modeling procedure on the part of some producers on the dynamics of demand; and interestingly on the relation between growth rates of aggregate demand and growth in autonomous components of demand.

But a considerable amount of further research is required in order to ascertain in particular whether and under what conditions learning is likely to have a stabilizing or destabilizing impact on growth in a demand-constrained system.

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## Appendix: Simulation details – values for parameters and lagged endogenous variables

**Table 1**

| | | | | | |
|---|---|---|---|---|---|
| $l^0_1$ | = 0.1 | $p^m_{11}$ | = 1.44322 | $w$ | = 0.26180 |
| $l^0_2$ | = 2 | $p^m_{12}$ | = 1.42869 | $\alpha$ | = 0.1 |
| $\beta_1$ | = 2 | $p_{21}$ | = 2.9795 | $\mu$ | = 0.2 |
| $\beta 2$ | = 0.8 | $\pi$ | = 0.04 | | |

**Table 2**

| | | | |
|---|---|---|---|
| $g^d_{it-j}$ $j=1\ldots4$ | 0.04 | $ID^{Aut}_{t-1}$ | 0.159 |
| $g^m_{it-1}$ | 0.04 | $M^0_{12t-1}$ | 0.206 |
| $u^0_{it-1}$ | 0.85 | | |

As noted above, simulations of the model involve starting the model in a steady state (corresponding to a rate of growth of autonomous demand of 4%) and then subjecting it to a shock in the form of a permanent increase in the rate of growth of autonomous demand (from 4 to 5 %). Apart from this, calibration was only required for the coefficient $\sigma$ (see footnote 14 above).

**FIGURE 1 – Basic model without learning**

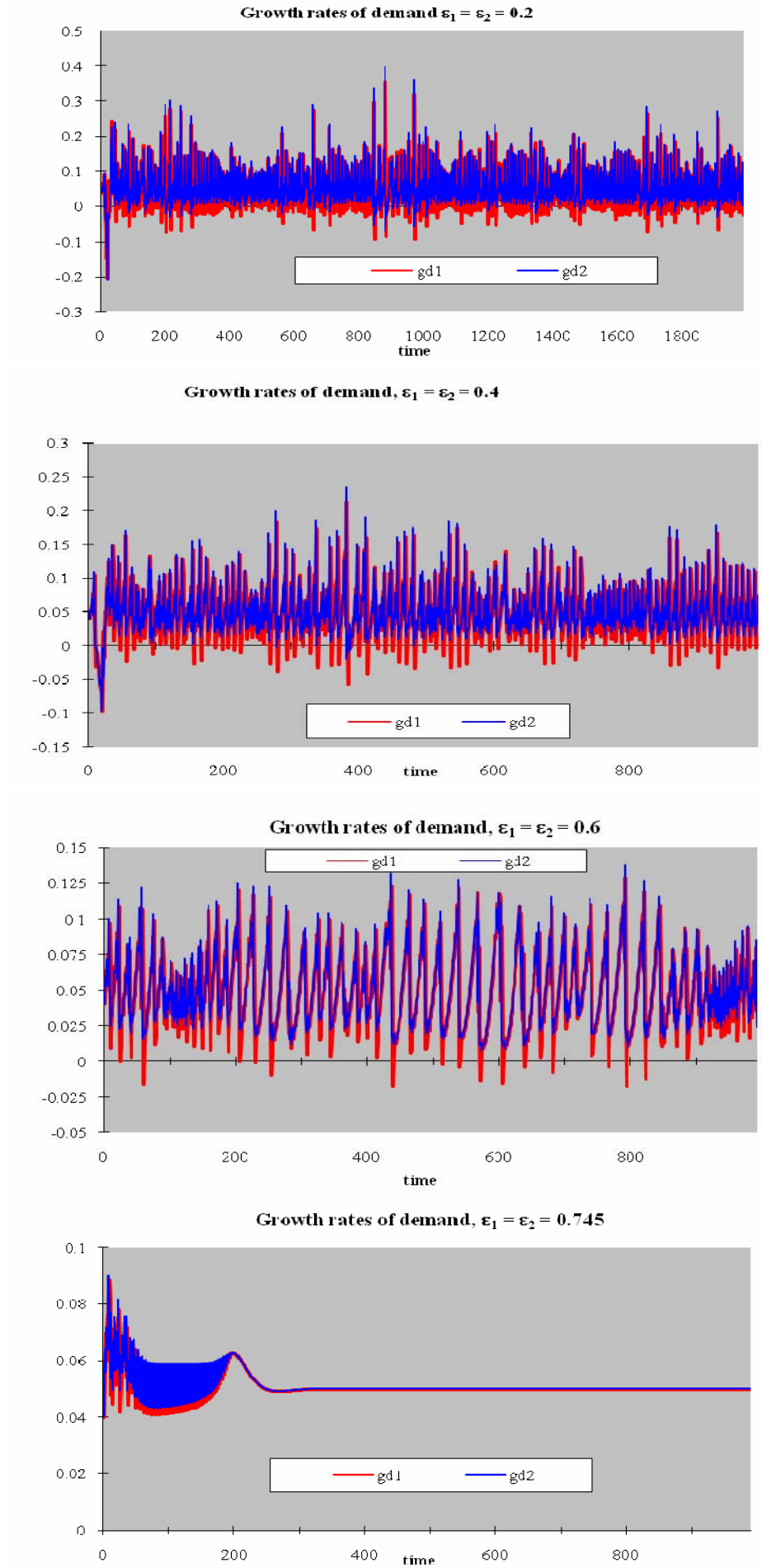


**FIGURE 2 (a): Simple learning (gradient-descent rule)**

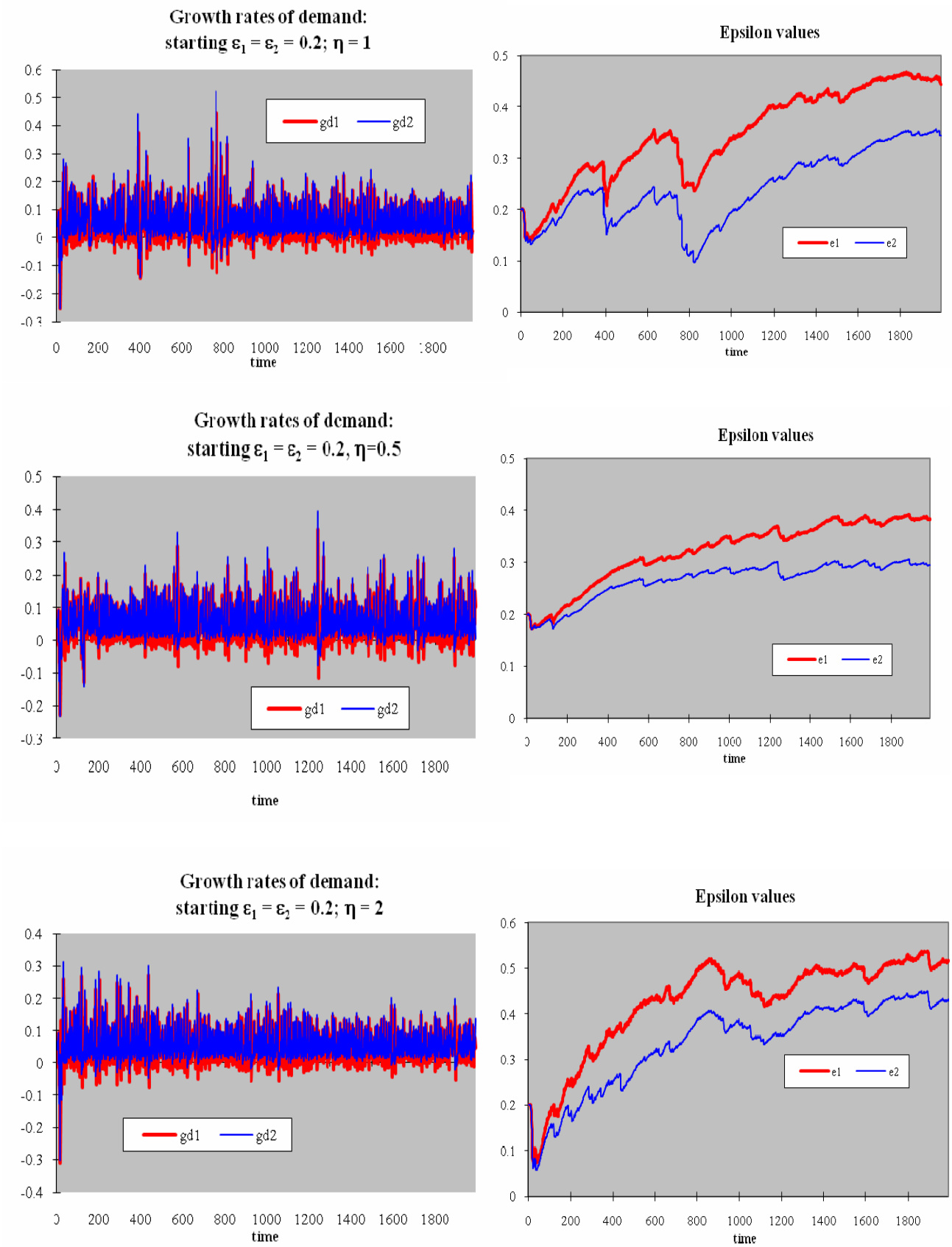


**FIGURE 2 (b): Simple learning (gradient-descent rule)**

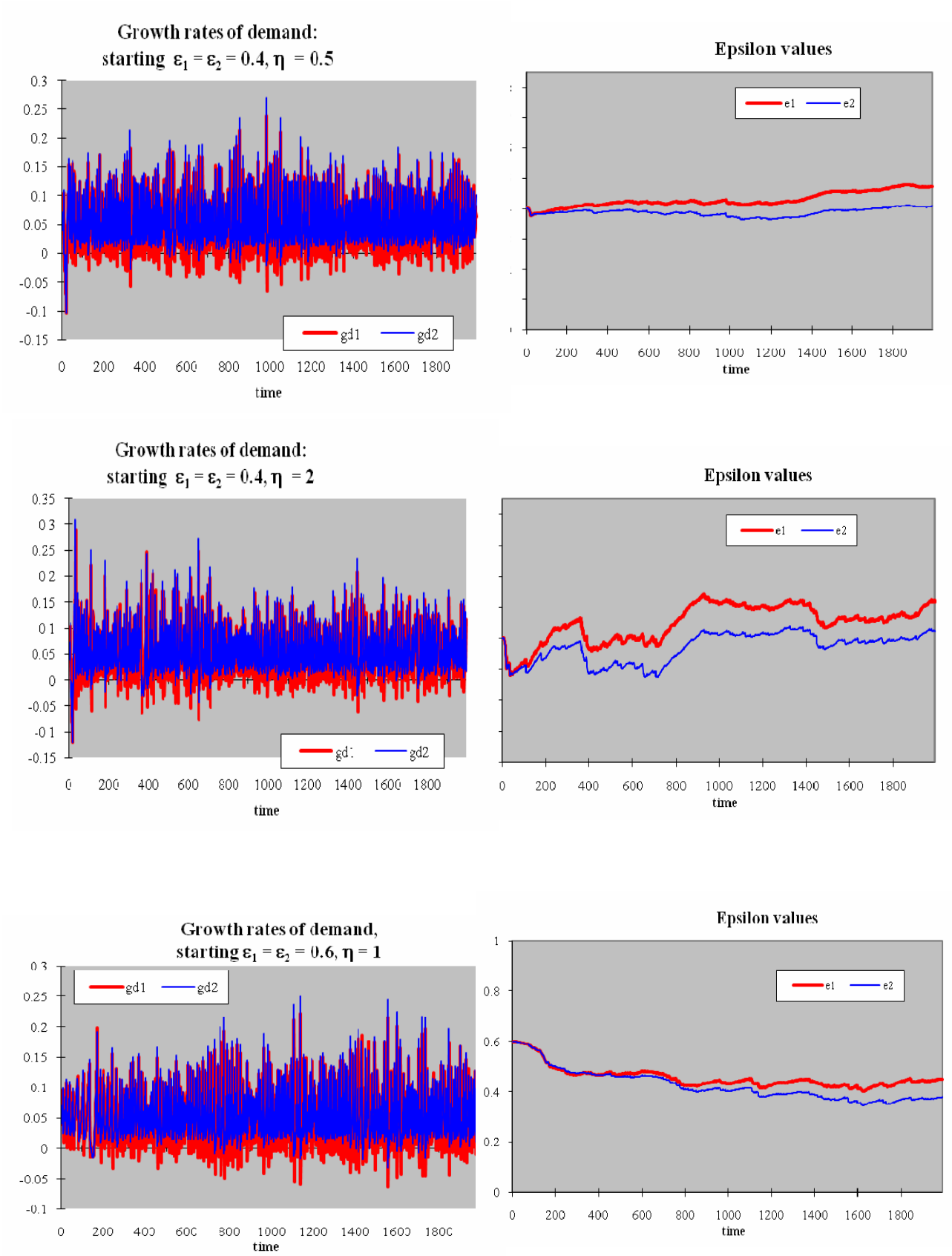
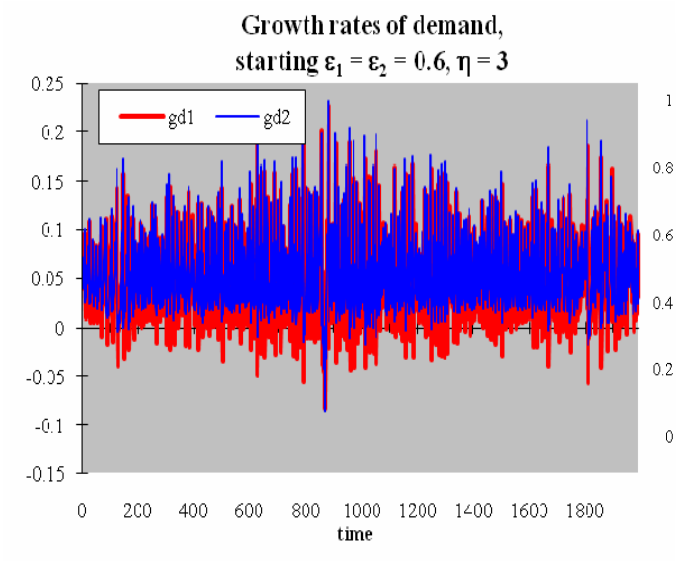
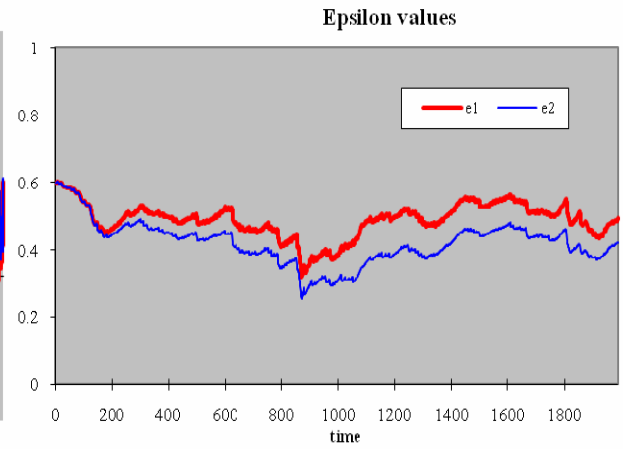
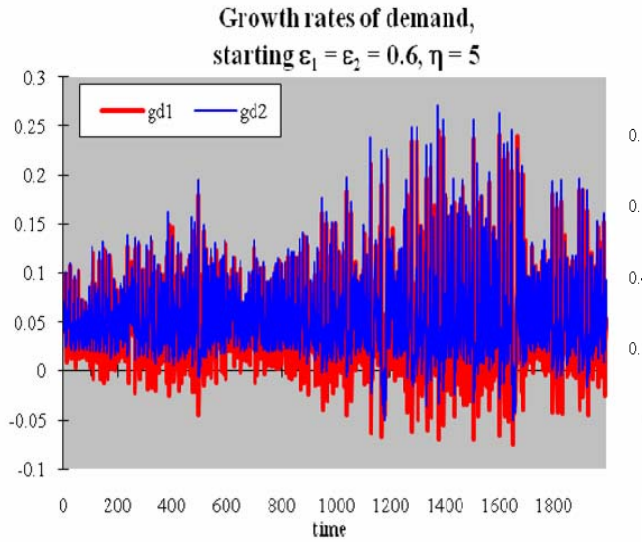


**FIGURE 2 (c): Simple learning (gradient-descent rule)**



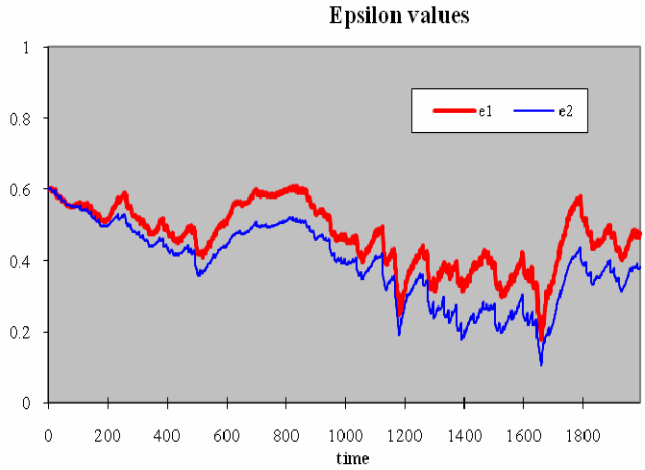
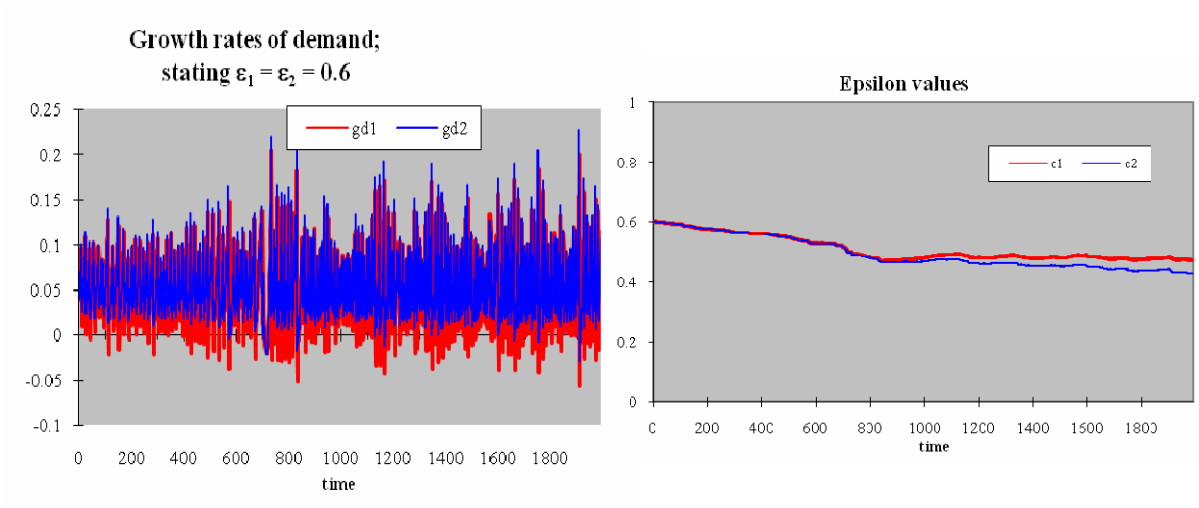
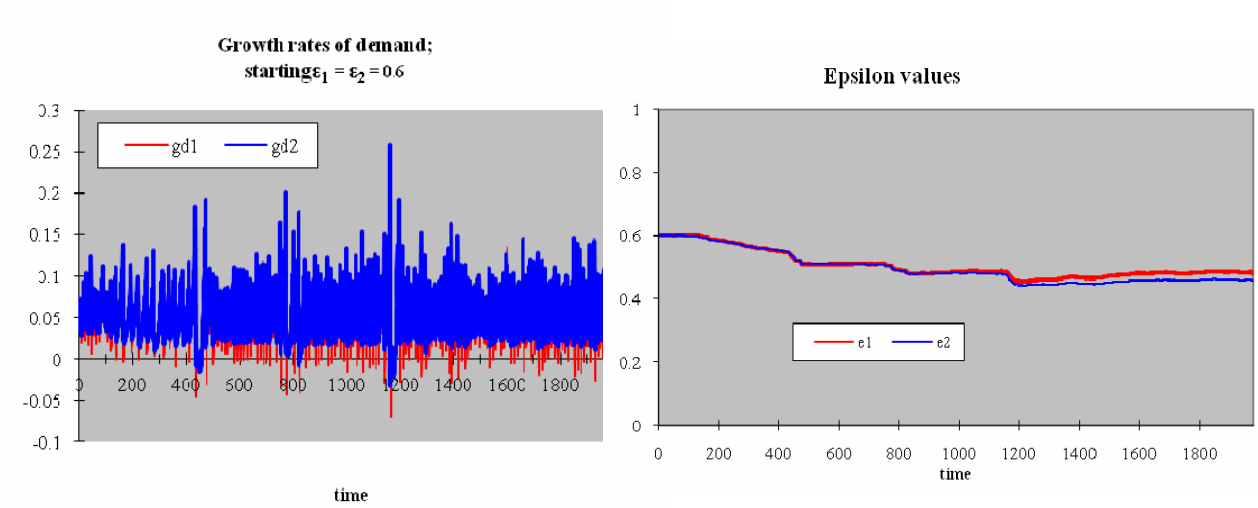
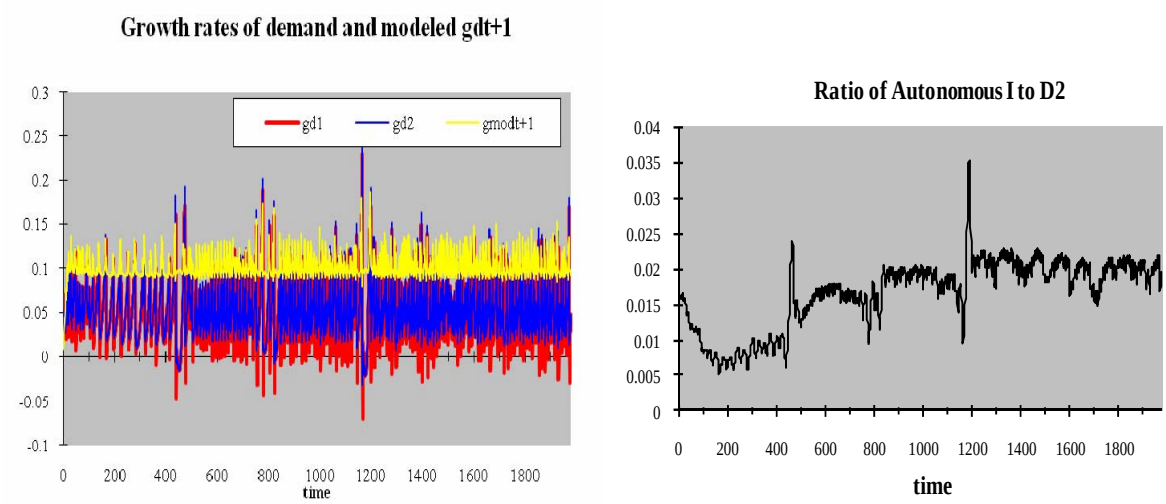



**Figure 3 (a) Learning only by a simple gradient-descent rule ($\eta$ = 0.5)**

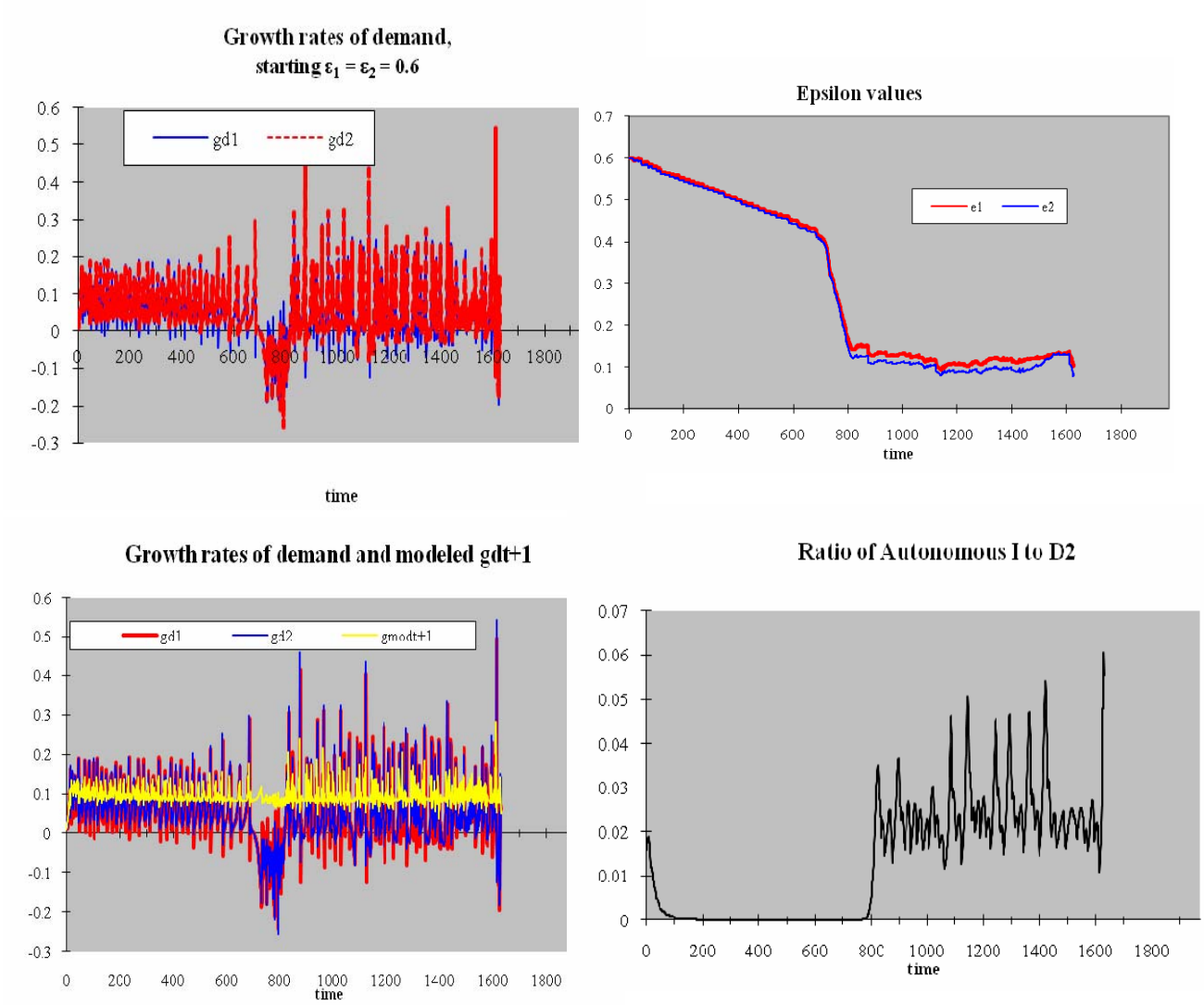
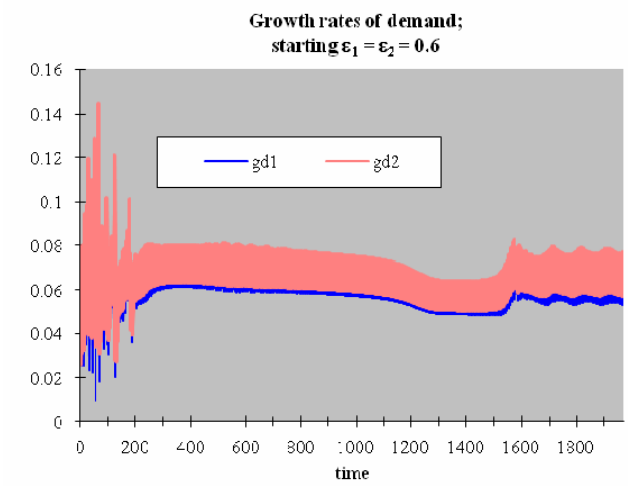
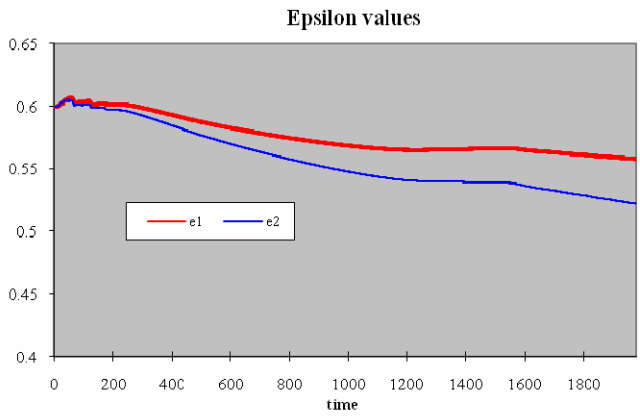
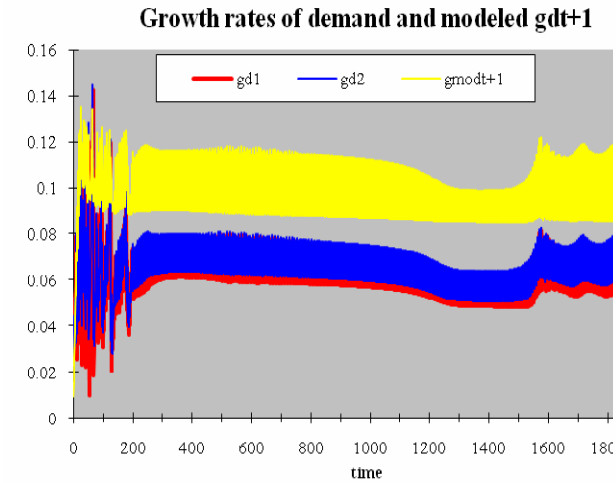
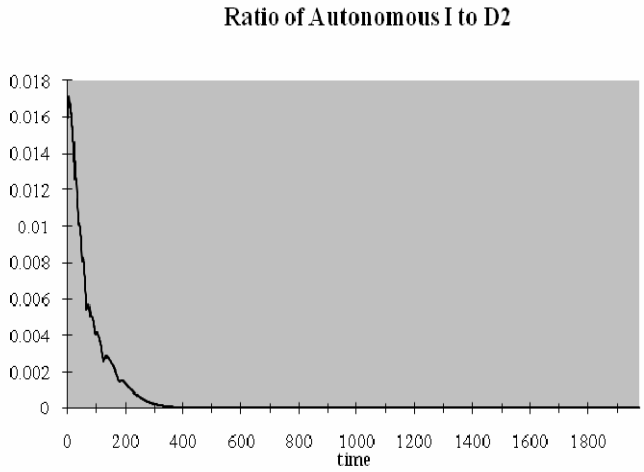



**(b) "True learning" ($\rho$ = 0.1, $\omega$ = 0.5, $\kappa$ = 1)**



**(c) "True learning" ($\rho$ = 0.5, $\omega$ = 0.5, $\kappa$ = 1)**



## (d) "True learning" ($\rho = 0.25$, $\omega = 0.5$, $\kappa = 1$)