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Abstract

The paper explores the implications of different autonomous demands, with differing rates of growth, in a demand-led growth model where policy makers are concerned about the ratios of public sector debt to income and external debt to income. The actual growth rate is explained in terms of the growth rate of aggregate demand, with emphasis in the formation of expectations about growth in the latter; and the relative importance in this regard of realised aggregate demand growth and autonomous demand growth, the latter being governed by export demand and public sector expenditure. Debt constraints - specifically, the ratio of public sector debt to output and the ratio of external debt to output – become relevant in the determination of the growth rate of government expenditure. The paper explores the likely interactions between debt constraints, the growth rate of aggregate demand and autonomous demand by means of dynamic simulations.

Classification codes: O41, B50, O43, E37

Keywords: Autonomous demand, growth, government expenditure, debt

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DEMAND-LED GROWTH WITH DEBT CONSTRAINTS[1]

1. Introduction

The aim of the present paper is to shed further light on the nature of demand-led growth, with explicit recognition of the constraints imposed by both public debt and foreign debt. It therefore takes as its starting point the non-marginalist position that growth is ultimately demand constrained; and, in particular, that growth is driven by the autonomous components of demand. For the purposes of this paper, we regard the key autonomous components of demand as export demand and government expenditure.

As is well known however, the extent to which both of these expenditures are "autonomous" is not uncontroversial. Even from a non-orthodox perspective, and quite aside from any consideration about the pros and cons of discretionary fiscal policy, public sector expenditure is clearly subject to the concerns of policy makers about sustainable public sector debt trajectories. Additionally, recent decades have witnessed increased concern about links between public sector balances and external balance and thus in turn with foreign debt trajectories.[2]

The focus of this paper is primarily on the dynamics of growth in the context of concern by policy makers about these two types of debt; while at the same time adopting demand-led approach to the explanation of growth, where the two key drivers of this growth are export demand and government expenditure.[3]

Section 2 begins the analysis by deriving an expression for the actual growth rate of demand at any time in terms of actual growth rate in the preceding period, the expected rate of growth of aggregate demand, the expected rate of growth of autonomous demand and ratio of autonomous demand to income. Sections 3 and 4 deal with the relationship between debt and the two autonomous components of demand; and this discussion in turn allows one to consider in a tentative way the complexities of demand-debt interaction. Section 5 outlines the nature of the steady state solutions to the model with debt constraints, while Section 6 considers alternative formulation for the determination of the rate of growth of government expenditure and the expected rate of growth of autonomous demand as a prelude to dynamic simulation of the model. Sections 7 and 8 provide a discussion of the results of these dynamic simulations. Section 9 provides some brief concluding notes.

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[1] I am indebted to participants of the Conference on "Institutional and Social Dynamics of Growth and Distribution" in Lucca, Italy, December, 2007 and to partcipants in seminars at the Centro Sraffa, Faculty of Economics, University of Rome, 3, in December 2007 and November, 2008, and participants at the Seventh Society of Heterodox Economists Conference, at University of NSW, in December, 2008, for helpful comments on earlier drafts of the present paper. Remaining errors and omissions are of course fully my own responsibility.

[2] A reasonable interpretation of Australian experience of the last two and a half decades would put the current account of the balance of payments as a proportion of GDP and the public sector debt to income ratio as the overriding concerns in relation to fiscal policy.

[3] The notion that the steady state outcome in a demand-led growth model corresponds to the rate of growth of autonomous demand is also not uncontroversial; see Park, 2000, Barbosa-Filho, 2000, White, 2006, Cesaratto, et.al., 2003, Palumbo and Trezzini, 2003.

2. The actual growth rate – a simple model

We seek a fairly simple explanation of the actual growth rate. Our analysis is formally conducted in terms of the growth rate of aggregate demand; the justification for this being that in the transition to a long-run equilibrium, the rate of growth of output will be converging on the rate of growth of demand. Production is assumed to take time, while circulation – when demands are expressed and supply is forthcoming – happens at the junction of two periods i.e. *t*/*t-1*. We start with the composition of aggregate demand at the end of period *t*, D_t

$$D_t = C_t + I_t + G_t + X_t - M_t \qquad \text{.....(1)}$$

Where C_t, I_t, G_t ,X_t and M_t refer respectively to consumption, investment, government expenditure and export demand expressed at the end of period *t*. With regard to consumption demand, this is properly a function of after-tax income inclusive of interest income on public debt and interest payments on external debt.[4] Thus

$$C_t = c.\{Y_t(1-t) + r.(B_{t-1} - E_{t-1})\} \qquad \text{.....(2)}$$

where *c* is the propensity to consume, Y_t is income/output in period *t*, *r* is the rate of interest, assumed the same for public debt *B* and external debt *E*.

With regard to output, Y_t, the immediate complication is that output levels have to be decided prior to the expression of demand, so that it is necessary for producers to formulate expectations about future demand. Of course this is also necessary for the formulation of investment decisions at the end of each period. We assume that producers attempt to forecast the growth rate of demand one-period ahead: in other words, for investment decisions at the end of period t, producers formulate an expectation about the growth rate of demand between *t+1* and *t*. However, since, demand at the end of period t is not known prior to the formulation of investment decisions, the most recent demand level on which producers' growth expectations can be based is that at the end of *t-1*. We therefore also assume that the growth rate expected between *t+1* and t on that between *t-1* and *t-2*. In particular, the expectation held at the end of period t concerning the level of demand at the end of *t+1* is given by

$$D^e_{t+1} = D_{t-1}\left(1 + g^{de}_{t+1}\right)^2 \qquad \text{.....}$$

As noted above, the expected growth rate of demand between t+1 and t is based in part on the rate of growth of demand most recently observed, viz., g^d_{t-1}. But we add to this component two other considerations: first an allowance for the fact that expectations about demand in the past may have been incorrect and in turn capacity may have been deficient or excessive in relation to actual demand; and, second, that growth in the economy is partly driven by the scale of government and by external demand and producers are aware of this. The implication of this latter consideration is

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[4] Of course the influence of interest on debt on consumption demand is also indirect, via their impact on the nature of the debt constraints and in turn on the growth rate of government expenditure and the growth of income.

that producers take some account of the rate of growth of the economy as a whole and by implication of the exogenous components of demand when forecasting the growth rate of demand in their own sector.[5] Putting these factors together gives as an expression for the expected rate of growth of demand as

$$g^{de}_{t+1} = \varepsilon.a^{exp}_{t-1} + (1-\varepsilon).\left[g^{d}_{t-1} + x.\left(g^{d}_{t-1} - g^{de}_{t-2}\right)\right] \qquad .....(4)$$

Where $a^{exp}{}_{t-1}$ refers to the expected rate of growth of autonomous demand between t and t_{+1}. The second term on the right hand side of expression (4) represents the part of the expectation of future demand growth based on recent demand growth in the producer's own sector; and this component includes an allowance for the extent to which expectations about demand growth in one's own sector were incorrect in the light of most recent experience.

Output in period t is given by

$$Y_t = D_{t-1}\left(1+g^{de}_{t+1}\right) = D_{t-1}.\left\{1+\varepsilon.a_{t-1} + (1-\varepsilon).\left[g^{d}_{t-1} + x.\left(g^{d}_{t-1} - g^{w}_{t-1}\right)\right]\right\} \qquad .....(5)$$

If we assume everlasting fixed capital so that all investment is net investment, we can represent the investment level decided on at the end of period t in terms of the increment of demand expected between periods t-1 and t+1. In other words,

$$I_t = v.Y_t.g^{de}_t \qquad .....(6)$$

where v is the desired capital to output ratio.

With regard to autonomous demand, we assume this consists entirely of government expenditure and expenditure on exports, so that autonomous demand at time t, A_t, is

$$A_t = G_t + X_t \qquad .....(7)$$

and, as ratios to income,

$$A^Y_t = \frac{A_t}{Y_t} = G^Y_t + X^Y_t \qquad .....(8)$$

where $G^Y{}_t$ and $X^Y{}_t$ represents the ratios of government expenditure and exports to income respectively.

Substituting equation (5) for Y_t in equation (2) allows one to express consumption as a function of demand; and substituting equation (5) for Y_t in equation (8) allows one to express aggregate autonomous demand, A_t, as a function of the ratio of autonomous demand to income, $A^Y{}_t$, and demand. In turn, combining these manipulations with combining with equations (1), (4) and (6), allows one to express the growth rate of

[5] This approach to the modeling of the expected growth rate is used for a two-sector model where fixed capital is taken account of explicitly in White, 2008.

aggregate demand, $g^d{}_t$ as a function of growth rates in the preceding period and the ratio of autonomous demand to income in the preceding. In other words,

$$g_t^d = g^d\left(g_{t-1}^d, g_{t-1}^{de}, a_{t-1}^{exp}, A_t^Y\right) \qquad \text{.....(9)}$$

Additionally, in view of equation (5), the rate of growth of output between period t and t-1, denoted $g^y{}_t$, can be written as

$$g_t^y = \frac{g_{t-1}^d.\left(1+g_t^{de}\right)+g_t^{de}-g_{t-1}^{de}}{1+g_{t-1}^{de}} \qquad \text{.....(10)}$$ [6]

Finally, it is worth also briefly clarifying the growth rate of autonomous demand. This can be expressed as

$$a_t = \frac{g_{Xt}+g_{Gt}.\alpha_{t-1}}{1+\alpha_{t-1}} \qquad \text{where} \qquad \alpha_t = \frac{G_t}{X_t} \qquad \text{.....(11)}$$

It follows that

$$\frac{da_t}{d\alpha_{t-1}} = \frac{g_{Gt}-g_{Xt}}{\left(1+\alpha_{t-1}\right)^2} \qquad \text{.....(12)}$$

Starting from a situation where $g_{Gt} = g_{Xt}$ and supposing that g_{Gt} then rises above g_{Xt}, with g_{Xt} remaining constant, the numerator of the expression will be positive and the differential $a_t'\left(\alpha_{t-1}\right) > 0$. α will be rising so that in the subsequent period (i.e.*t+1*), *a* will rise. If g_{Xt} then rises above g_{Gt}, , the differential $a_t'\left(\alpha_{t-1}\right) < 0$, and α will be falling so that in the subsequent period (i.e.*t+1*), *a* will also rise.

It also follows from the expression for a_t, that at will converge to whichever of the two rates of growth – g_{Xt}, g_{Gt} – is the higher:

$$\lim_{\alpha_{t-1}\to\infty}\left(\frac{g_{Xt}+g_{Gt}.\alpha_{t-1}}{1+\alpha_{t-1}}\right) = g_{Gt}$$
$$\text{.....(13)}$$
$$\lim_{\alpha_{t-1}\to 0}\left(\frac{g_{Xt}+g_{Gt}.\alpha_{t-1}}{1+\alpha_{t-1}}\right) = g_{Xt}$$

3. Policy constraints and autonomous demands

The more interesting question at this point concerns the determination of the two components of autonomous demand – government expenditure and export demand. For the purposes of the following discussion we assume that export demand is wholly

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[6] Hence, if the expected rate of growth of demand remains is unchanged between t and t-1, then the growth rate of output will track the growth rate of demand, with a one-period lag, i.e. $g_t^y = g_{t-1}^d$

exogenous; while government expenditure is ultimately governed by concern about debt, specifically, the behaviour of the ratio of public debt to income and the ratio of foreign debt to income.

Taking exchange rates also as exogenous and further assuming that the domestic economy has little or no influence over the interest rate on foreign debt the assumption that export demand is wholly exogenous implies that the only means by which the government could exert influence over the trade account and in turn over the current account and over the ratio of foreign debt to income is by manipulating the rate of growth of domestic demand. This in turn would affect the external debt to income ratio in at least two ways: indirectly through the trade account via a change in the rate of growth of imports; and directly via a change in the rate of growth of income.

However, we also assume, significantly, policy makers are not necessarily concerned to the same degree about the foreign debt to income and public sector debt to income ratios. In particular, the analysis proceeds on the assumption that while governments would accept some rise in the level of external debt as a proportion of income, they will be much less tolerant of a rise in the public debt to income ratio from its current level. We think this reflects the more recent position of governments, certainly with respect to public debt. Thus, while an exogenous growth of exports together with the current and past growth rates of aggregate demand may entail some rise in external indebtedness, the past growth rate of demand – to the extent that it governs the current rate of growth of output - may act as a binding constraint on the rate of growth of public sector expenditures.

In other words, we assume that the government is mindful of both the export to income ratio and the public sector expenditure to income ratio (to the extent that they have implications for foreign and public sector debt ratios), though, in the absence of control over interest rates on external debt and in the absence of considerable latitude for "cheap money" it is restricted to manipulating the public sector expenditure ratio. In contrast to an orthodox approach to growth however, we maintain that public sector expenditures will be an important driver of autonomous demand and thus of the rate of growth of output.[7]

Before proceeding further, it is appropriate to articulate the relationships between the autonomous components of demand and the two debt to income ratios referred to above, albeit in a simplified way.

Taking firstly the public debt to income ratio and assuming that any budget deficits are fully funded by the issue of government debt, one can write,

$$\Delta B_{t/t-1} = G_t - T_t + r.B_{t-1} \qquad \text{…..(14)}$$

where $\Delta B_{t/t-1}$ represents the change in public sector outstanding debt, B, between end of t-1 and end of t; T, represents total tax revenue and r is the interest rate payable on

[7] We leave aside the issue of whether monetary policy, but affecting on interest rates on public sector debt, thereby influences the relationship between the public sector expenditure to income ratio and the public sector debt ratio. To the extent that the objectives of monetary policy are elsewhere, viz., inflation control, this may seem such an heroic assumption.

public sector debt. In terms of ratios to income, the public debt ratio at the end of *t*, b_t^Y, is given by

$$b_t^Y = G_t^Y - t + \frac{b_{t-1}^Y.(1+r)}{1+g_t^y} \qquad \text{.....(15)}$$

where *t* is the income tax rate (there are no other taxes), assumed constant and G^Y is the ratio of government expenditure to income. Similarly, the change in foreign debt between *t* and *t-1*, $\Delta E_{t/t-1}$,can be expressed as

$$\Delta E_{t/t-1} = M_t - X_t + r.E_{t-1} \qquad \text{.....(16)}$$

Where *M* represents imports and *r* again is the relevant interest rate on foreign debt (hence, assumed the same as the interest rate on public debt) so that, the ratio of foreign debt to income at the end of *t* is given by

$$e_t^Y = m - X_t^Y + \frac{e_{t-1}^Y.(1+r)}{1+g_t^y} \qquad \text{.....(17)}$$

where *m* is the import propensity, also assumed constant, and X^Y is the ratio of export demand to income.

Bearing in mind the assumptions that *t*, *r* and *m* are constant, expressions (15) and (17) can be rewritten in the form of constraints respectively on the rate of growth of government expenditure consistent with a stable public debt to income ratio and the rate of growth of exports consistent with a stable foreign debt to income ratio, for any particular rate of growth of income. More precisely, expressing the ratio of government expenditure to income in period *t* as

$$G_t^Y = G_{t-1}^Y.\frac{1+g_t^G}{1+g_t^Y} \qquad \text{.....(18)}$$

and setting $b^Y{}_t = b^Y{}_{t-1}$, then solving for $g^G{}_t$, expression (15) becomes

$$g_t^G = \frac{b_{t-1}^Y.(g_t^Y - r) + t.(1+g_t^Y) - G_t^Y}{G_t^Y} \qquad \text{.....(19)}$$

The more pertinent form of this is as an inequality, viz.,

$$g_t^G \le \frac{b_{t-1}^Y.(g_t^Y - r) + t.(1+g_t^Y) - G_{t-1}^Y}{G_{t-1}^Y} \qquad \text{.....(20)}$$

The right-hand side of this inequality shows the maximum growth rate of government expenditure, given *r* and *t*, the growth rate of income and the ratio of government expenditure to income from the preceding period, consistent with a constant public debt to income ratio. If we were to further simplify and assume an unchanging expected growth rate of demand, then from equation (10) (see footnote 6), we may substitute the demand growth rate for the output growth rate in expression (20) so that the relevant constraint becomes

$$g_t^G \leq \frac{b_{t-1}^Y.\left(g_{t-1}^d - r\right) + t.\left(1 + g_{t-1}^d\right) - G_{t-1}^Y}{G_{t-1}^Y} \qquad \text{.....(21)}$$

We refer to this as the *public debt constraint*, hereafter *PDC*.

One can similarly rearrange expression (17): setting $e^Y_t = e^Y_{t-1}$, solving for g^X_t and bearing in mind

$$X_t^Y = X_{t-1}^Y.\frac{1 + g_t^X}{1 + g_t^Y} \qquad \text{.....(22)}$$

yields

$$g_t^X \geq \frac{e_{t-1}^Y.\left(r - g_{t-1}^d\right) + m.\left(1 + g_{t-1}^d\right) - X_{t-1}^Y}{X_{t-1}^Y} \qquad \text{.....(23)}$$

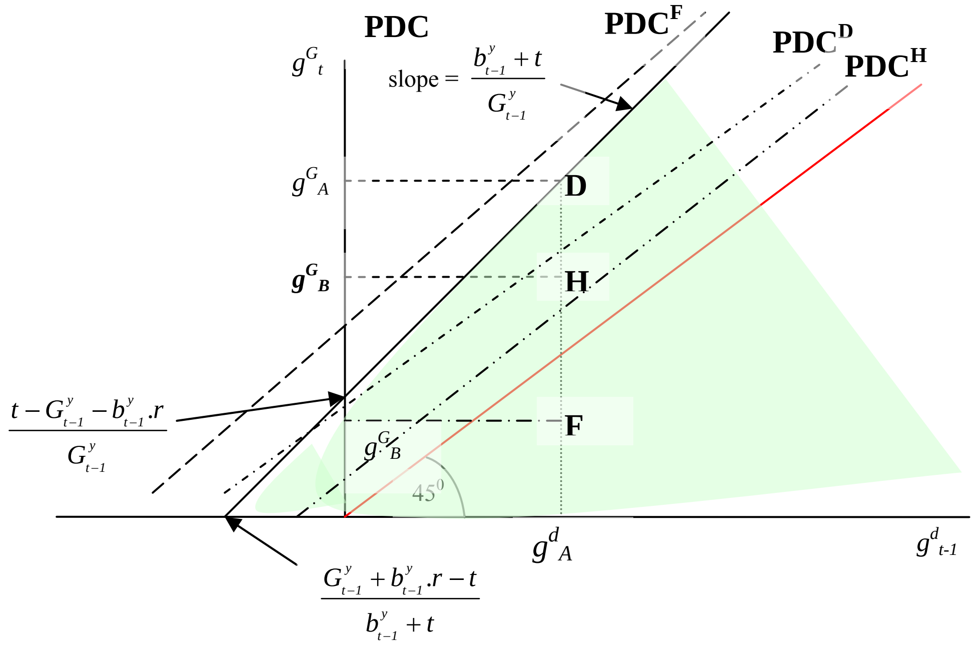
Expression (23) represents the *foreign debt constraint* or *FDC* and represents the minimum growth rate of exports, consistent with a constant foreign debt to income ratio, given *r*, *m*, growth in demand in the preceding period (assuming a constant expected growth rate of demand) and the export to income ratio of the preceding period.

Figure 1 below depicts the *PDC* which shows, for different levels of the growth rate of demand, g^d_{t-1}, the growth rate of government expenditure consistent with the constancy of the ratio of public debt to income, for a particular previous period value of that ratio, b^y_{t-1}, as well for the ratio of government expenditure to income, G^y_{t-1}. The area below the *PDC* represents combinations of g^d_{t-1} and the rate of growth of government expenditure g^G_t which would lead to a reduction in the ratio of public debt to income; in other words, $b^y_t < b^y_{t-1}$.

The diagram also includes a 45° line, points along which entail $g^d_{t-1} = g^G_t$ so that G^y_{t-1} is unchanged. Since the slope of the *PDC* is equal to $\frac{b^y_{t-1} + t}{G^y_{t-1}}$, a positive public debt to income ratio ($b^y > 0$) and a primary budget surplus ($t > G^y$), the slope of the *PDC* is therefore greater than unity and thus the PDC is steeper than the 45° line. With an overall budget surplus and thus $\frac{t - G^y_{t-1} - b^y_{t-1}.r}{G^y_{t-1}} > 0$, lies wholly above the 45° line. On the other hand, points off the 45° line, where $g^G \neq g^d_{t-1}$, will obviously entail changes in G^y and thus changes in the slope of the *PDC* and both its intercepts. Points off the *PDC* will involve changes in the public debt to income ratio, b^y, and thus also changes in the slope and in both intercepts of the *PDC*.[8]

[8] Thus, it is only at a point of intersection of the PDC and 45^0 line where both b^y and G^y are constant and the PDC is not shifting. But if the PDC is steeper than the 450 line, such an intersection – in

FIGURE 1 - DEMAND GROWTH AND THE PUBLIC DEBT CONSTRAINT



At a point such as ***D*** for example with $g^d{}_{t-1} = g^d{}_A$ and $g^G{}_t = g^G{}_A$, the ratio of government expenditure to income, G^y, will be rising. However, since ***D*** is on the PDC, this combination of demand growth and government expenditure growth will keep the ratio of public debt to income constant (i.e. $b^y{}_t = b^y{}_{t-1}$). Hence, with a constant tax rate, t, the *PDC* function will become flatter.

It can also be shown that with a total budget surplus (implied by the positive vertical intercept and negative horizontal intercept of the *PDC*), the horizontal intercept of the *PDC* increases and the vertical intercept decreases with a rise in $G^y{}_{t-1}$.[9] Hence for example, the combination $g^d{}_{t-1} = g^d{}_A$ and $g^G{}_t = g^G{}_A$, will lead to a shift in the *PDC* such as that represented by the dashed line in Figure 1.

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positive space – would require that the vertical intercept of the PDC be negative and thus that there is a total deficit (which is of course can coexist with a primary budget surplus).

[9] The horizontal intercept is given by $\frac{G^y_{t-1}+b^y_{t-1}.r-t}{G^y_{t-1}+t}$. Differentiating this with respect to $G^y{}_{t-1}$ yields $\frac{2t-b^y_{t-1}.r}{\left(G^y_{t-1}+t\right)^2}$ which is positive provided $t > \frac{b^y_{t-1}.r}{2}$, which is clearly the case with a total budget surplus (i.e. $t-G^y_{t-1}-b^y_{t-1}.r > 0$). With regard to the vertical intercept, $\frac{t-G^y_{t-1}-b^y_{t-1}.r}{G^y_{t-1}}$, the differential of this with respect to $G^y{}_{t-1}$ is $\frac{b^y_{t-1}.r-t}{\left(G^y_{t-1}\right)^2}$ which is in turn clearly negative with a total budget surplus.

By contrast, consider the combination of growth rates at point **F**, $g^d_{t-1} = g^d_A$ and $g^G_t = g^G_B$. In this case G^y will be falling; and b^y is also falling (**F** is in the shaded region). But it can be shown that, with a total budget surplus, the change in the horizontal intercept will be in the same direction, while the vertical intercept will move in the opposite direction. The precise shift in the PDC will depend on the change in its slope. This, it turns out, depends positively on the sign of the difference $g^d_{t-1} - g^d_A$. Hence the effect of a combination of growth rates such as that at point **F,** where $g^d_{t-1} > g^d_A$ and thus the differential of the slope with respect to a change in G^y and b^y both in the same direction is positive, the PDC shifts to something like the dotted line in Figure 1.

There remains the implications of points lying on the interval **DH**, where $g^G > g^d_{t-1}$, and thus where G^y is rising but where b^y is falling. The slope of the *PDC* must lessen for a given tax rate, *t*. The complexity in such cases concerns the movement in the intercepts of the *PDC*; though on "reasonable" values for parameters,[10] the movement of the *PDC* will be something like **PDC**$^{\mathbf{H}}$ in Figure 1.[11]

Figure 2 below depicts the *FDC*. We assume for the purposes of discussion that $m < e^Y_{t-1}$,[12] so that the *FDC* is negatively sloped; and that the current account is in deficit (i.e. $CAD^Y_{t-1} = m + e^Y_{t-1}.r - X^Y_{t-1} > 0$) so that both the vertical and horizontal intercepts of the *FDC* are positive (assuming a positive level of foreign debt). Analogously to the *PDC*, combinations of g^X_t and g^d_{t-1} lying on the *FDC* entail a constant foreign debt to income ratio. Points to the right (left) of the FDC entail a fall (rise) in e^Y.

As with the *PDC* depicted in Figure 1, Figure 2 provides an illustration of the three possible shifts of the *FDC*, depending on the combination of g^X_t and g^d_{t-1}.[13]

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[10] In particular, using long-run historical values for the Australian economy (see Table 1 below and also footnote 7 above).

[11] Differentiating $\dfrac{G^y_{t-1} + b^y_{t-1}.r - t}{G^y_{t-1} + t}$ with respect to both G^y and b^y gives $\dfrac{r.\left(G^y_{t-1} + t\right) db^y_{t-1} + \left(2t - b^y_{t-1}.r\right).dG^y_{t-1}}{\left(G^y_{t-1} + t\right)^2}$.

The sign of this differential depends on the relative size of db^y_{t-1} and dG^y_{t-1} as well as the relative size of the coefficients $r.(G^y_{t-1}+t)$ and $(2t-b^y_{t-1}.r)$ which are both positive.

[12] Again, which appears to accord with historical data for Australia.

[13] A comparison of the implications of points A and C is interesting in this regard. Since both points are below the 45° line, X^Y will be falling in both cases. However, point C entails a rising e^Y while point A entails a falling e^Y. But the *FDC* is shifting in the same direction – upwards; meaning that in order to prevent a rising debt in the future the rate of growth of exports has to be higher than previously for any given growth rate of demand. Interestingly the upward shift is larger in relation to point A, even though e^Y is falling. The most reasonable interpretation of this is that the larger apparent fall in the ratio of exports to income X^Y associated with point A (i.e. the excess of the growth rate of demand, and, by assumption, income over the growth rate of exports is larger at A) is sufficient to offset the fact that point C entails a rising e^Y while point A entails a falling e^Y.

FIGURE 2 – DEMAND GROWTH AND THE FOREIGN DEBT CONSTRAINT

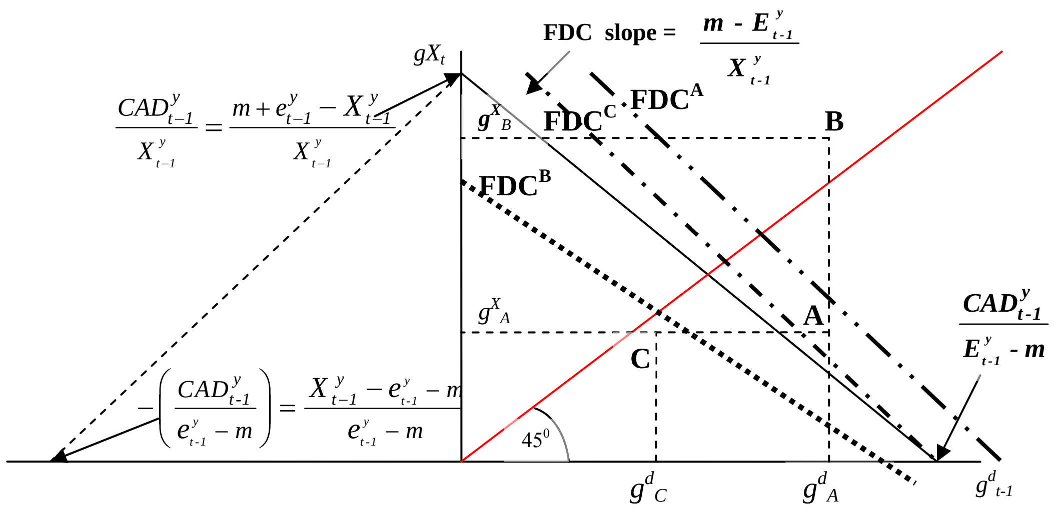
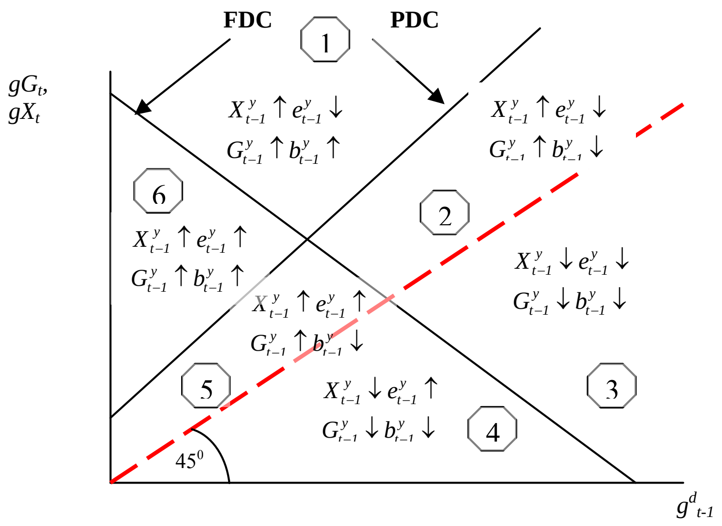


Figure 3 plots both the *PDC* and *FDC* together with a 45° line: the vertical axis measures values for both g^{G}_{t} and g^{X}_{t}, with g^{d}_{t-1} measured on the horizontal axis. Since, as noted above, the PDC lies wholly above the the 45° line then the intersection of the PDC and FDC will also lie above the 45° line.

Figure 3 also depicts indicates a number of different regions – combinations of growth rates of the autonomous components of demand and growth rates of aggregate demand – distinguished according to the implications for X^{Y}, G^{Y}, e^{Y} and B^{Y}. Arguably, the most desirable regions for policy makers would be regions 2 and 3, since only in these regions are both debt to income ratios declining (assuming of course that less debt relative to GDP is preferred to more by policy makers).

FIGURE 3 – COMBINED PUBLIC AND FOREIGN DEBT CONSTRAINTS



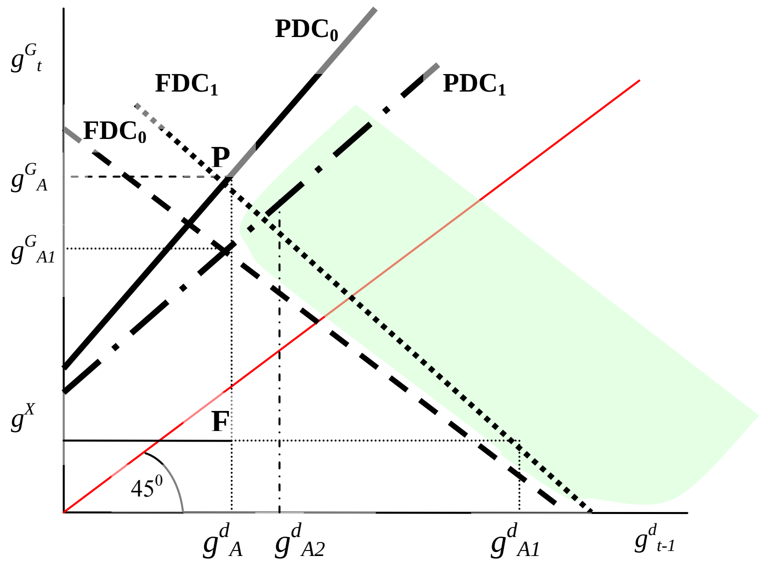
However, given the assumption made above (p. 5) regarding the relative concern on the part of policy makers about public sector debt as a proportion of income, one could argue that, at least temporarily policy makers would tolerate being in regions 4 and 5, where e^Y is rising but b^Y is falling; with regions 1 and 6 being the least desirable of all.[14]

4. One possible policy scenario

Figure 4 below suggests one possible scenario that may confront policy makers. It serves also to illustrate the complexity of the demand-debt dynamics under consideration. Assume that the growth in demand in *t-1* is equal to $g^d{}_A$ and an exogenously given growth rate of export demand equal to g^X. Since point F lies to the left of the initial FDC (FDC_0), this combination of aggregate demand (and income between *t* and *t-1*) and export growth rates entails a rising foreign debt to income ratio between *t* and *t-1*. Suppose initially that the growth rate of government expenditure is $g^G{}_A$ – i.e. point P, which, being on the initial PDC (PDC_0), implies a constant public debt to income ratio between *t* and *t-1*.

It is also clear that point P and F imply a rising G^Y and falling X^Y respectively. On the basis of the discussion so far, these changes, together with the unchanged public sector debt ratio and a rising foreign debt ratio, will lead to shifts of the debt constraints to something like PDC_1 and FDC_1. Ignoring any changes in the growth rate of demand, these shifts clearly imply that, first, with an unchanged export growth rate, the foreign debt problem will be exacerbated, in the sense of requiring an even higher rate of growth of income than previously to stabilize foreign debt as a proportion of income (e.g. $g^d{}_{A1}$, g^X); and second, to maintain an unchanged public debt ratio, either the rate of growth of income will have to rise or the rate of growth of government expenditure will have to be reduced (e.g. $g^G{}_{A1}$, $g^d{}_A$).

FIGURE 4 – ONE POSSIBLE POLICY SCENARIO



But there is a further complication here. Because the original points P and F entail that $g^G > g^X$, then α (equation (11)) will be rising, and, assuming the rate of growth of autonomous demand, *a*, is initially below g^G, a will also be rising (equations (12) and

[14] Of course, at any point in time policy makers may find themselves in two different regions, since the rate of growth of exports and the rate of growth of government expenditure will typically differ.

(13)). Equation (9) in turn implies that this will affect the rate of growth of aggregate demand, via a positive impact of rising a on g^d.

For the sake of argument and pursuing this scenario a little further, suppose the net impact of the rising rate of growth of autonomous demand on g^d is positive, so that, in terms of Figure 4, g^d is pushed to the right of $g^d{}_A$, e.g. something like $g^d{}_{A2}$. This would of course require less of a cutback in the rate of growth of government expenditure, consistent with a stable public debt to income ratio and it would also work to slow the rise in the foreign debt to income ratio.

Although crude and highly simplified one interesting possibility that suggests itself in the above discussion relates to the possibilities – including possible dilemmas - for policy makers concerning the rate of growth of government expenditure.

The economy may well find itself in a position where the rate of growth of exports and income are such that the foreign debt ratio will rise if the rate of growth of exports does not rise. If the latter cannot be manipulated systematically, the only way a rise in the foreign debt ratio can be avoided (given the assumptions made in the paper so far) is if the rate of growth of income rises. Yet, to the extent that the latter is itself driven by autonomous demand, then the only possibility to avoid a rising foreign debt ratio is through a rise in the rate of growth of government spending. The obvious question here is the extent to which such a change in the rate of growth of government spending is consistent with the desired public debt outcomes of policy makers?[15]

5. Demand growth and debt: steady states

Unfortunately however the framework above – specifically an analysis in terms of shifting PDC and FDC - is rather inadequate and cumbersome in addressing such questions; quite aside from the fact that for the most part the analysis so far has assumed that the expected rate of growth of demand has remained unchanged, so that the rate of growth of income mirrored that of demand, with a lag. Even with this assumption, the movements of both the PDC and FDC will be considerable even over a small number of periods.

For this reason we turn our attention in the following section to what might allow for a better grasp of the dynamics of demand and debt, viz., dynamic simulation of the model. For this purpose, however we need to complete the model outlined above with some proposition about the determinants of the rate of growth of government expenditure, $g^G{}_t$. As a prelude to this, we first identify the equilibria of the model, as specified so far.

In effect, the present model offers three different steady states, according to whether the rate of growth of government expenditure exceeds, is equal to or is less than the exogenously given rate of growth of export demand. Recalling from equations (13)

[15] Of course, a second type of policy scenario – not illustrated in Figure 4 – involves the reverse of that discussed above and is probably more fortuitous. Suppose for whatever reason g^X is persistently above g^G, so that the growth rate of autonomous demand and the growth rate of output tend over time towards the growth rate of exports, g^X. Hence G^Y will tend to fall acting in turn to relax the PDC facing policy makers.

the growth rate of autonomous demand will converge on the greatest of the two growth rates of exports and government expenditure; and that a steady state is characterized by constant ratios of autonomous demands to income, then a difference between the rates of growth of government expenditure and export demand requires that either G^Y or X^Y tends to zero as the economy approaches the steady state growth trajectory.

Equation (9) for the growth rate of aggregate demand can be written explicitly for the steady state case where realized and expected growth rates of aggregate demand, are all equal to the rate of growth of autonomous demand, so that

$$\begin{aligned}(1+a)(1+m+c.t)+c.r.\hat{e}^y &= (1+a)(c+a.v)+c.r.\hat{b}^y \\ &\quad +\left(1+\hat{g}^G\right).\hat{G}^Y+\left(1+g^X\right).\hat{X}^Y\end{aligned} \qquad \text{.....(9a)}$$

where the '^' script refers to the steady state value of the variable. Since the steady state should also be characterized by constant debt (both public and foreign) to income ratios, then in view of equations (15) and (17), one should have

$$\hat{b}^y=\frac{\left(\hat{G}^Y-t\right).(1+a)}{(a-r)} \qquad \text{.....(15a)}$$

and

$$\hat{e}^y=\frac{(1+a).\left(m-\hat{X}^Y\right)}{(a-r)} \qquad \text{.....(17a)}$$

respectively.

Rewriting the steady state public debt and government expenditure ratios (to income) in terms of their relation to their corresponding historical ratios, so that

$$\hat{b}^y=\gamma_b.\bar{b}^Y \quad \text{and} \quad \hat{G}^Y=\gamma_G.\bar{G}^Y,$$

one can then rewrite expressions 9(a) and 15(a) respectively as

$$\begin{aligned}(1+a)(1+m+c.t)+c.r.\hat{e}^y &= (1+a)(c+a.v)+c.r.\gamma_b.\bar{b}^y \\ &\quad +\left(1+\hat{g}^G\right).\gamma_G.\bar{G}^Y+\left(1+g^X\right).\hat{X}^Y\end{aligned} \qquad \text{.....(9b)}$$

$$\gamma_b.\bar{b}^y=\frac{\left(\gamma_G.\bar{G}^Y-t\right).(1+a)}{(a-r)} \qquad \text{.....(15b).}$$

In view of equations (13) we can also write for the steady state value of 'a',

$$\hat{a}=Max\left(g^X,\hat{g}^G\right) \qquad \text{.....(13a)}$$

where $\bar{b}^Y$ and $\bar{G}^Y$ are exogenously given historical average or historical peak values of the corresponding ratio.

Together, equations (9a), (15a), (17a) and (13a) contain 6 unknowns: a, $\hat{e}^y$, γ_b, $\hat{g}^G$, γ_G and $\hat{X}^Y$. Consider the case where $\hat{g}^G$ is set equal to the rate of growth of export by policy makers. The rate of growth of autonomous demand 'a' will therefore be equal to $\hat{g}^G = g^X$, by means of equation (13a), so that the remaining three equations (9a), (15a) and(17a) consist of four unknowns: $\hat{e}^y$, γ_b, γ_G and $\hat{X}^Y$. This case appears to leave open the possibility that one of either $\hat{b}^y$ or $\hat{G}^Y$ (via γ_G) could be set by policymakers exogenously. Hence, at least in this case, policymakers could set $\hat{g}^G = g^X$ as well as setting either γ_b or γ_G.

But how general is this case, specifically, in general is it possible for policymakers to set the steady state rate of growth of government expenditure as well as γ_b? The answer to this is no. Where $\hat{g}^G \neq g^X$ the steady state solution will involve either $\hat{G}^Y$ or $\hat{X}^Y$ equal to zero and thus, effectively, $\hat{g}^G$ and either $\hat{G}^Y$ or $\hat{X}^Y$ are the two exogenous variables.

Hence, if the steady state rate of growth of government expenditure differs from the exogenous rate of growth of export demand, the three equations (9a), (15a), (17a) and (13a) provide only *one* degree of freedom: The rate of growth of autonomous demand will be equal to either $\hat{g}^G$ or g^X, according to equation (13a) depending on which of these is largest. The remaining three equations would contain four unknowns: $\hat{e}^y$, γ_b, $\hat{g}^G$ along with one of γ_G or $\hat{X}^Y$, the other being equal to zero. Thus if the steady state rate of growth of government expenditure is set exogenously at a level different from g^X, steady state γ_b is endogenous and thus policymakers cannot set exogenously the steady state public debt to income ratio. On the other hand, policymakers could set this latter ratio exogenously, but they cannot also choose a particular steady state growth rate of government expenditure.

Hence an added degree of flexibility exists for policymakers, at least in relation to steady state trajectories, in setting a debt to income ratio as well as the rate of growth of government expenditure, only where the latter is set equal to the exogenous rate of growth of export demand.

Of course, another way to look at this is that by choosing a growth rate of government expenditure different from g^X, policymakers are effectively also choosing either a steady state export to income ratio $\hat{X}^Y$ equal to zero (for the case where $\hat{g}^G > g^X$) or a steady state government expenditure to income ratio $\hat{G}^Y$ equal to zero (for the case where $\hat{g}^G < g^X$). In either case, policymakers have to live with the resultant endogenously determined γ_b. Thus in the case above, where $\hat{g}^G$ is set by policymakers equal to g^X, a target and thus exogenous ratio of public debt to income means that policymakers have to live with a particular ratio of government expenditure to income.

6. Closing the model

The question arises at this point as to which of the three types of equilibria – corresponding respectively to $\hat{g}^G \gtreqless g^X$ - is the most relevant. This is essentially a question of the stability of the different equilibria. One way of approaching this is to consider, whether, starting from the steady state characterized by $g^G = g^X$, and for a reasonably intuitive hypothesis about the rate of growth of government expenditure, the economic system's response to a shock which pushes g^X temporarily away from g^G, leads to a growth rate equal to the new value of g^X.

Taking this last question a step further requires hypotheses about the determination of both g^G and the expected rate of growth of autonomous demand (equation (4)).

(i) *The rate of growth of government expenditure*: As a starting point, the approach adopted here is to suppose that the rate of growth of government expenditure reflects three concerns of policy makers. First, that there may be a limited counter-cyclical role for this expenditure. Hence we assert that if actual growth for example falls short of the expected rate of growth of demand, *ceteris paribus* policy makers will consider increasing g^G. Second, we suppose that concern about foreign debt has an influence on fiscal policy, via the belief that the current account deficit is influenced to some extent directly by the public sector budget balance.[16] Third, we suggest that the influence of these first two considerations is subject to an overriding concern that the ratio of public sector debt to income does not grow over time.

In putting together these three considerations in a compact form, suitable for a simulation exercise, we suppose that the rate of growth of government expenditure is determined at time *t* according to the following rule:

$$g_t^{G*} = g_t^{GB} + \mu.\left(g_{t-2}^{de} - g_{t-1}^{d}\right) - \phi.\left(e_t^Y - e_{t-1}^Y\right) \quad ..... (24(i))$$

$g^G{}_t$ is assumed to be set in each period on the basis of that growth rate of government expenditure, g_t^{GB}, which would generate - given the existing debt to income ratio, government expenditure to income ratio, and actual and expected growth rates of demand - a public debt to income ratio equal to a proportion γ_b of the historical average value, $\bar{b}^Y$; but modified by any existing gap between actual and expected rates of growth of demand (this being a proxy for under and overutilization of capacity);[17] and any growth in the foreign debt to income ratio. Substituting $\gamma_b.\bar{b}^y$ for b_t^Y, g_t^{GB} for g_t^G in equation (15) and solving for g_t^{GB} yields

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[16] In turn reflecting a view that the current account deficit is a reflection of the public sector budget balance, or more generally that it reflects a deficiency of national saving and thus, given private sector saving, a deficiency of public sector saving. Needless to say this is not a view which the present author accepts.

[17] Cyclical fluctuations in economic activity should see some tendency to counter-cyclical movements in the size of public outlays as a proportion of income, via the operation of automatic stabilizers. In effect, expression (24) assumes, with $\mu > 0$, that there will be in addition some discretionary changes in the rate of growth of government expenditure consistent with the direction of such automatic stabilizers.

$$g_t^{GB} = \frac{\left(1+g_t^y\right).\left(t+\gamma_b \bar{b}^y\right) - .\left(1+r\right).b_{t-1}^y - G_{t-1}^Y}{G_{t-1}^Y} \qquad .....(25)$$

The second term on the right-hand side of (24) is the counter-cyclical aspect in government activity: it is assumed that in a cyclical upturn, where $g^d > g^{de}$, policy makers will be prepared to enforce a fall in the ratio of government expenditure to income and conversely, in a cyclical downturn where $g^d < g^{de}$. The third term on the right-hand side of (24) represents the negative effect of a rising foreign debt to income ratio on the desired rate of growth of government expenditure.

The second determination of $g^G{}_t$ used for the purposes of simulation modifies the influence of g_t^{GB} and inserts what might be termed a "conservative bias" in fiscal policy. In particular, we assume that policymakers are prepared to lower $g^G{}_t$ below $g^G{}_t$ (ignoring the influence of counter cyclical concerns and concerns about foreign debt) to try and restore the public debt ratio to its desired level, but they feel no such obligation to raise $g^G{}_t$ when it falls below g_t^{GB}. Thus,

$$g_t^{Gp} = Min\left(g_t^{GB}, g_{t-1}^{Gp}\right) + \mu\left(g_{t-1}^{de} - g_{t-1}^{d}\right) - \theta\left(e_{t-1}^y - e_{t-2}^y\right) \qquad ..... (24(ii))[18]$$

Note that, according to both equation (24i) and (24ii), the rate of growth of government expenditure could only stabilize with an unchanging foreign debt to income ratio; normal utilization of productive capacity (to the extent capacity utilization is reflected in the divergence of actual and forecast demand growth) and the stabilization of the debt to income and government expenditure to income ratios.

Interestingly also, this determination of the rate of growth of government expenditure, together with the dynamics of demand and output embodied in equations (4), (9), (10) and (13), in principle suggests a mechanism by which the rate of growth of government expenditure would be regulated by the exogenous rate of growth of export demand. Consider for example the case where initially $g^G < g^X$. Since the rate of growth of autonomous demand is governed over time by the largest of g^G and g^X, if

[18] Strictly speaking, simulations were conducted in relation to expression (24(ii)) using the following rule:

$$g_t^{Gp} = g_{t-1}^{Gp} - \omega.Pos\left(g_{t-1}^{Gp} - g_t^{GB}\right) + \mu\left(g_{t-1}^{de} - g_{t-1}^{d}\right) - \theta\left(e_{t-1}^y - e_{t-2}^y\right)$$

Where the operator '*Pos*' means that where $g_{t-1}^{Gp} > g_t^{GB}$, then $\omega.Pos\left(g_{t-1}^{Gp} - g_t^{GB}\right) = \omega.\left(g_{t-1}^{Gp} - g_t^{GB}\right)$, otherwise, $\omega.Pos\left(g_{t-1}^{Gp} - g_t^{GB}\right) = 0$. Hence where $g_{t-1}^{Gp} > g_t^{GB}$, and assuming $\omega < 1$, the rate of growth of government expenditure is $g_t^{Gp} = g_{t-1}^{Gp}.\left(1-\omega\right) + \omega.g_t^{GB} + \mu\left(g_{t-1}^{de} - g_{t-1}^{d}\right) - \theta\left(e_{t-1}^y - e_{t-2}^y\right)$. In other words, for this case, that part of the rate of growth government expenditure associated with a desired public debt to income ratio, is effectively a weighted average of the most recent growth rate of government expenditure and the growth rate required to attain the desired long-run debt ratio. Alternatively put, where $g_{t-1}^{Gp} > g_t^{GB}$, and ignoring countercyclical and foreign debt impacts, the rate of growth of government expenditure is equal to last periods rate, but discounted by an amount in part detrmined by the extent to which recent growth in government expenditure exceeds the rate required to attain the long-run desired public debt to income ratio.

g^G persistently below g^X entails that 'a' will grow faster than g^G, and eventually demand, both expected and actual and in turn output will grow faster than government expenditure. G^Y will be falling; and if this fall pushes $b^y{}_t$ below $\gamma_b.\bar{b}^y$, then g^{GB} will rise and with it g^G.[19] Conversely, with g^G persistently above g^X, X^Y will be falling, and in so far as this pushes $e^y{}_t$ up the government expenditure function would entail some downward pressure on g^G.

(ii) *The expected rate of growth of autonomous demand*: Concerning the expected rate of growth of autonomous demand, we assume that producers form these expectations on the basis of a long-run historical trend component to autonomous demand and a part which reflects more recent changes in that demand. For simplicity, we therefore propose the following formulation for the expectation at time *t* of growth in autonomous demand between *t* and *t+1*:

$$a_t^{exp} = \delta.a^{Hist} + \left(1-\delta\right).a_t^{av} \qquad \text{…..(26)}$$

where a^{Hist} is an historical average rate of growth for total autonomous demand and a_t^{av} is a moving average of the total autonomous demand growth rate over the last *n* periods; and $0 < \delta < 1$. But we also endogeneise a^{Hist}, making it a function of the longer–run trend realized growth rate of autonomous demand. In this case

$$a^{Hist} = a_{t-1}^{Fcst} + \eta.\left(a_t^{LRav} - a_{t-1}^{Fcst}\right) \qquad \text{…..(27)}$$

where a_t^{av} in expression (26) represents the shorter-run trend in the growth rate of autonomous demand.

7. Demand and debt dynamics I: the local stability of $g^G = g^X$

Equations (4), (9)-(11), (15), (17)-(18), (22), (24) and (26)-(27) provide the basis for a simulation of the dynamics of demand and debt. In particular, they constitute a recursive dynamic system: period *t* growth rates of demand actual and expected, the growth rates of output, government expenditure and autonomous demand, together with ratios of autonomous demand, exports, government expenditure, and public and foreign debt to income can be derived from the values of the same variables in previous periods together with the values of the parameters, *t*, *m*, *r*, *c*, ε, *x*, *v*, μ, θ, γ_b, and $\bar{b}^Y$ together with the exogenous rate of growth of exports. Alternatively put, solved values of the endogenous variables for period *t*, together with parameter values and g^X, are sufficient to enable the solutions to the endogenous variables for period *t+1*.

The simulations discussed below are designed specifically to shed light on the stability of the equilibrium characterized by $g^G = g^X$. To this end, it is assumed that the system is initially moving along its steady state trajectory and is then subject to a shock which pushes the exogenously given growth rate of exports and the actual

[19] This of course assumes that any reduction in g^G due to an excess of g^d over g^{de} and/or a rising e^y is not be sufficiently large enough to offset the rise in g^{GB}.

growth rate of output apart. Unless otherwise specified, the shock in question pushes up the rate of growth of export demand.[20]

The key results of dynamic computer simulation of the model are depicted in Figures 5-8. Figure 5, panel (a), shows growth trajectories for the case where g^G is determined according to equation (24(i)).

Panel (b) shows the results for exactly the same model and parameters, except for a larger steady state public debt to income ratio (0.4 instead of 0.2) and a positive $\hat{e}^y$, that is, net steady state external liabilities. It is immediately noticeable that this change eliminates the tendency for damped cycles; and suggests that with the larger desired public debt to income ratio and one which is targeted by policymakers each period, fiscal policy contributes – i.e. exacerbates – cyclical fluctuations in growth rates of expenditure and output. Perhaps not surprisingly , particularly in view of 24(i), a higher value for the coefficient μ – which governs the extent of the "counter-cyclical" fiscal policy response – reduces the extent (i.e. amplitude) of fluctuations in the growth rate of demand.

It is interestingly to reflect a little further on this last result. Panel (c)(ii) compares the trajectory of the public debt to income ratio for the two cases – the trajectory associated with 5 (b) and the trajectory associated with 5 (c)(i) with the higher value of μ, and hence with what one might interpret as a more "activist" counter-cyclical fiscal policy.[21] Interestingly, the fluctuations in the public debt to income ratio are clearly less pronounced with the higher value of μ, as is the average ratio of public debt to income. Certainly, in this case, at least, a more activist fiscal policy, to the extent that it is able to "smooth" fluctuations in the growth rate of demand and income, does not entail larger public debt to income fluctuations or indeed a larger average public debt to income ratio.

The comparison of the results depicted in Panels (a) and (b) of Figure 5, and thus the result of having a larger desired public debt to income ratio, does the raise the issue of whether and to what extent the alternative government expenditure rule – represented by expression (24(ii)) might stabilize the system. Figure 6 illustrates the same case as in Figure 5 (b) but with this alternative rule for government expenditure growth. This change quite obviously makes a significant difference to the system's dynamics; with gradual convergence of growth rates to the new higher growth rate of export demand.

The remaining simulations for the model make use of this alternative government expenditure rule: responses of growth rates and of debt and expenditure ratios to an exogenous increase in g^X and separately an exogeneous decrease in g^X, are depicted in Figure 7 Panels (a) – (b). These simulations typically consider a larger desired public debt ratio, even compared with the case depicted in Figure 5, panel (c), but a smaller accelerator coefficient. This combination clearly stabilizes the system compared with earlier simulations and produces asymptotic stability in growth rates, in line with the

[20] Initial values for parameters and lagged endogenous variables for simulations are discussed in the Appendix.

[21] In the sense that for any given degree of capacity over or under-utilization, the response in terms of a change in the rate of growth of government expenditure is larger.

new steady state rate – i.e. equal to the new growth rate of export demand - debt ratios and ratios of autonomous demand to income.

A final set of simulation results are depicted in Figure 8 for the case where the steady state rate of growth, given by the rate of growth of export demand, both before and after the shock, is lower than the rate of interest on debt. As is evident from that figure, the alternative government expenditure function, along with a lower value for the accelerator coefficient allows for asymptotic stability in this case also.

8. Demand and debt dynamics II: expenditure and debt ratios

There are two particularly interesting features of the results for these latter simulations. First, the rate of growth of government expenditure tends over the long-run to come into line with the new rate of growth of exports. In the cases depicted in Figures 6 and 7 (a), where the economy responds to a positive shock to g^X, the growth rate of government expenditure consistent with the "desired" (maximum) public debt to income ratio exceeds the actual growth rate of government expenditure. Hence, from expression (24(ii)), g^{GB} becomes irrelevant and g^{Gp} is effectively governed by it's value in the most recent past together with a countercyclical and a component responding to movement in the foreign debt ratio.

In the case of Figure 7(a) the initial impact of the rise in g^X is positive on the rate of growth of demand and output; but this takes time to impact, via expectations about autonomous demand growth. Hence there is a slow reaction of expected demand growth, and actual demand growth and the rate of growth of output. For some time therefore g^y is below the new higher g^X, so that X^Y is falling and with it the foreign debt ratio, e^y.

With regard to government expenditure the lag between the expected growth rate of demand means that the difference $\left(g^{de}_{t-2} - g^{d}_{t-1}\right)$ in equation (24(ii)) is for the most part negative, this having a negative impact on the growth rate of government expenditure, at least initially. After a time however, the positive impact of a falling foreign debt ratio on the rate of growth of government expenditure begins to outweigh the former influence, so that the rate of growth government expenditure begins to rise. To the extent that the expected and actual growth rates of demand tend to become closer, the difference $\left(g^{de}_{t-2} - g^{d}_{t-1}\right)$, becomes smaller, and the relative impact of the falling foreign debt ratio on g^{Gp} becomes larger. Hence, while demand and income growth remain below the new higher level of g^X and X^Y rises and e^y falls, g^{Gp} is gradually pushed upwards in line with the new higher g^X.

In relation to public debt, the initial fall in the rate of growth of government expenditure, below the level consistent with the maintenance of the initial steady state and desired maximum value (i.e. $\gamma_b.\bar{b}^Y$). But while g^{Gp} is gradually rising to the new higher g^X, $g^G{}_{st}$ (the rate required to maintain debt constant) is gradually falling from a level above the new higher g^X.

The increase in the rate of growth of income resulting from the increase in g^X also results in a fall in G^y and in b^y. In turn this raises $g^G{}_{st}$, the growth rate of government expenditure consistent with a constant b^y; more importantly this rate is raised above the new higher g^X. As the actual growth rate of government expenditure rises towards the new higher g^X over time (for the reasons stated above), $g^G{}_{st}$ starts to fall towards g^X. In other words, part of the longer-run dynamics for the simulations depicted in Figures 6 and 7 (a) involves $g^G{}_{st}$ and g^{Gp} both converging on the new higher g^X, but the former from above and the latter from below. This dynamics also involves the public debt to income ratio stabilizing at a level lower than the "desired" maximum equal to $\gamma_b.\bar{b}^Y$ and which is relevant to g^{GB}. Hence, in the new steady state, the rate of growth of the economy is equal to the higher growth rate of export demand, and the rate of growth of government expenditure converges on that rate. But, $\hat{b}^y < \gamma_b.\bar{b}^y$ in the new steady state.

An alternative scenario, at least with respect to the evolution of the public debt ratio, can be seen in Figures 7 (b) and 8. Recall that the former deals with a negative shock to g^X, while the latter deals with a positive shock to g^X, but in a situation where $g^X < r$. In relation to the public debt ratio, these two cases differ from those of Figures 6 and 7 (a) in that the growth rate of government expenditure required to maintain the public debt to income ratio constant ($g^G{}_{st}$) in the long-run stabilizes at a level higher than the actual growth rate of government expenditure, yet the public debt ratio is constant at zero. This reflects a constraint built into the model, (which, as it turns out is binding only in the case of these two simulations) by way of lower bound at zero for this debt ratio.[22]

In the case of the negative shock to g^X (Figure 7(b)), one of the initial responses to that shock is a significant fall in the rate of growth of government expenditure. And this is a response to an initial rise in the public debt ratio, because of a significant initial drop in income growth (responding in turn to the fall in g^X) relative to government expenditure growth and in turn rising G^Y. This rise in by pushes g^{GB} below g^{Gp} and thus (via 24(ii)) constrains g^{Gp}. Over time, this fall in g^{Gp} reduces G^Y and in turn increases the rate of growth of government expenditure consistent with a constant b^y ($g^G{}_{st}$). With g^{Gp} below the growth rate of income, G^Y continues to fall, until the public debt to income ratio drops to zero. In the longer-run, $g^G{}_{st}$ continues to rise and only stabilizes when G^Y stabilizes, and hence when g^{Gp} adapts fully along with income to the new lower rate of growth of exports.

In the case depicted in Figure 8, the system's response involves a more pronounced cyclical fluctuation in growth rates, compared with the same type of shock for the cases depicted in Figures 6 and 7 (a). The fluctuation in the actual growth rate of government expenditure (g^{Gp}) and in the ratio of government expenditure to income (G^Y) are such that the latter is sufficiently low enough that the public debt to income ratio is reduced to zero before $g^G{}_{st}$ (the growth rate of government expenditure consistent with a constant by) is brought into line with the higher growth rate of exports.

[22] Looking at expression (14), this lower bound on b^y effectively means, that starting from a zero public debt, any primary budget surplus (i.e. $G < T$) does not entail an accumulation of government assets (ignoring seinorage), but rather a contraction in domestic liquidity. As such, the lower bound on public debt, assumes domestic monetary expansion, this being required to maintain (at least domestic) interest rates fixed, as is assumed throughout this paper. The other way to look at this is that with $b^y = 0$, and $g^{Gp} < g^G{}_{st}$, it is possible for policymakers to increase the rate of growth of government expenditure without increasing the public debt ratio. A more thorough and realistic analysis would relax this assumption and allow for policymakers in this case to increase g^{Gp} in line with $g^G{}_{st}$.

9. Conclusion

This paper has attempted to shed some light on the dynamics of growth and debt in terms of a demand-led growth model. In particular, it has focused primarily on the dynamics associated with a situation where one of the two autonomous components of demand is beyond control for policy makers; while their use of the component under their control – government expenditure – is subject to a public debt constraint and also reflects concerns about the evolution of foreign debt as a proportion of income.

Most significantly, the paper shows, via the results of computer simulations, that a set of parameters exist consistent with a stable set of dynamics for a demand-led growth model; where government expenditure growth reflects concerns over public debt and foreign debt ratios and arguably competing pressures for the use of fiscal policy as a countercyclical tool. In particular, these simulations demonstrates the existence of a set of parameters for which the steady state characterized by a rate of growth of government expenditure equal to the exogenous rate of growth of export demand is stable. Computer simulations of the model also demonstrate the existence of a mechanism pushing the growth rate of government expenditure in line with the exogenous growth rate of exports over time.

Clearly, the analysis above is preliminary in at least three respects: first, the assumption of an exogenously given and *constant* rate of growth of exports needs to be relaxed and consideration given to behaviour with an exogenous but fluctuating growth rate of exports. Secondly, further analysis should consider the relative weighting of autonomous demand and the economy's growth as a whole *vis a vis* growth in a producer's own sector, in the formation of expectations by producers about future growth, within an explicit multi-sectoral approach; viz., a multi-commodity, multi-industry approach.[23] Third, a more flexible government expenditure rule would seem appropriate in light of the cases where the public debt to income ratio falls close to zero and which would allow, realistically, for a different response from policymakers than that considered in the simulations discussed in this paper.

[23] I am indebted to a participant at the Society of Heterodox Economists conference (University of NSW, Dec. 2008) for impressing this point upon me.

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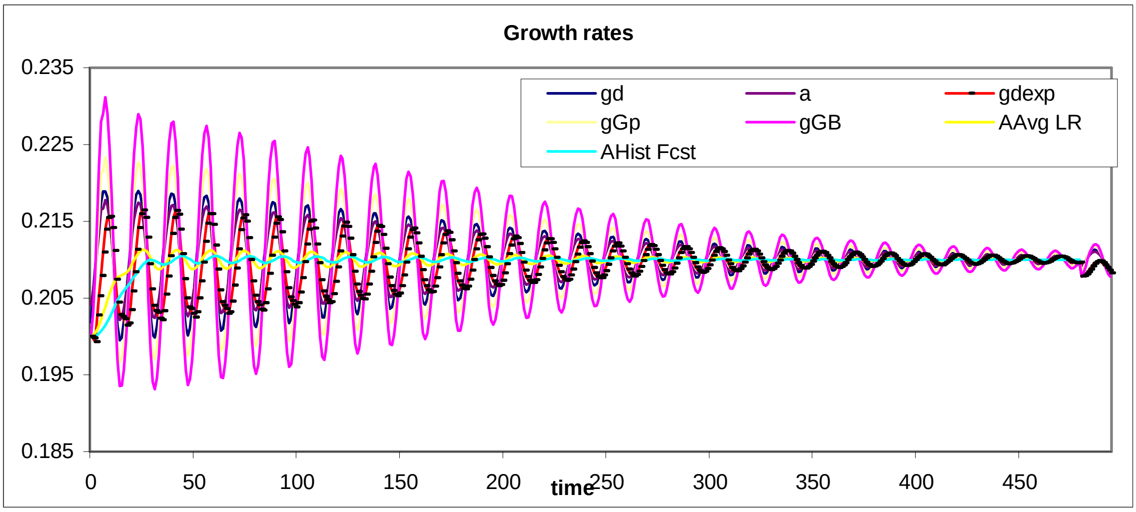
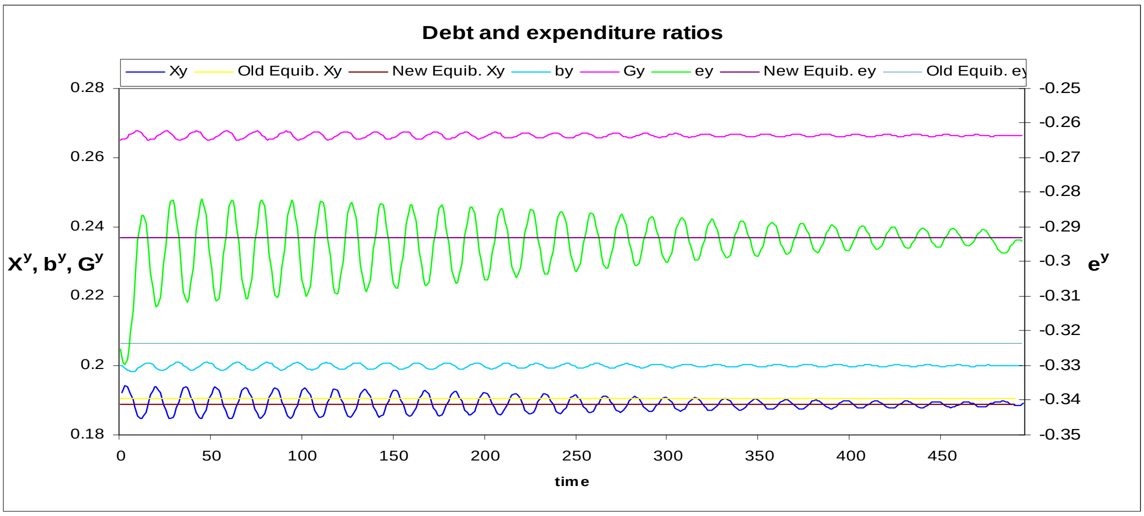
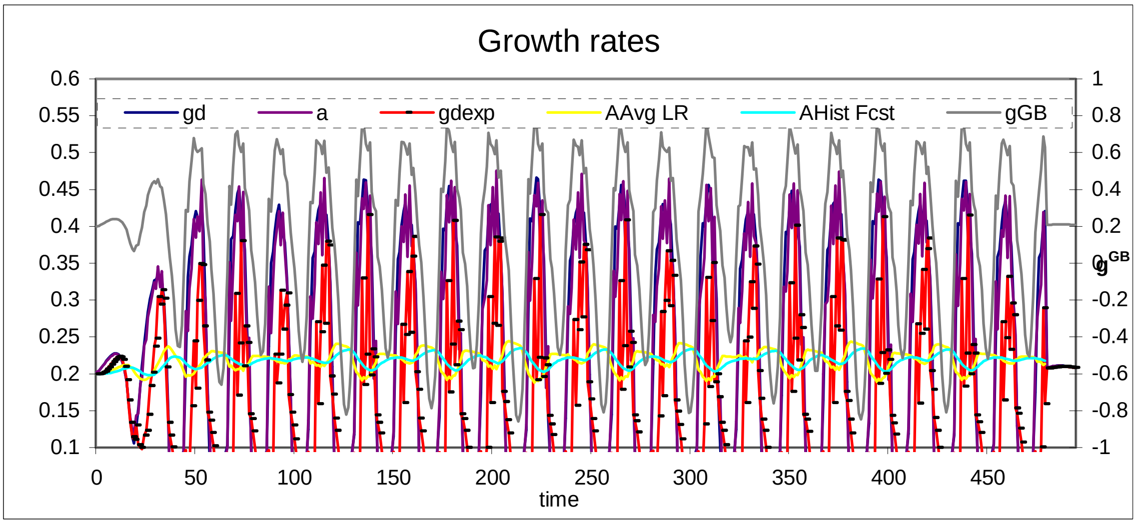
Figure 5

(a) $g_t^{G*} = g_t^{GB} + \mu.\left(g_{t-2}^{de} - g_{t-1}^{d}\right) - \phi.\left(e_t^{Y} - e_{t-1}^{Y}\right)$; **'lower'** $\gamma_b.\bar{b}^{y}$; $\hat{e}^{y} < 0$

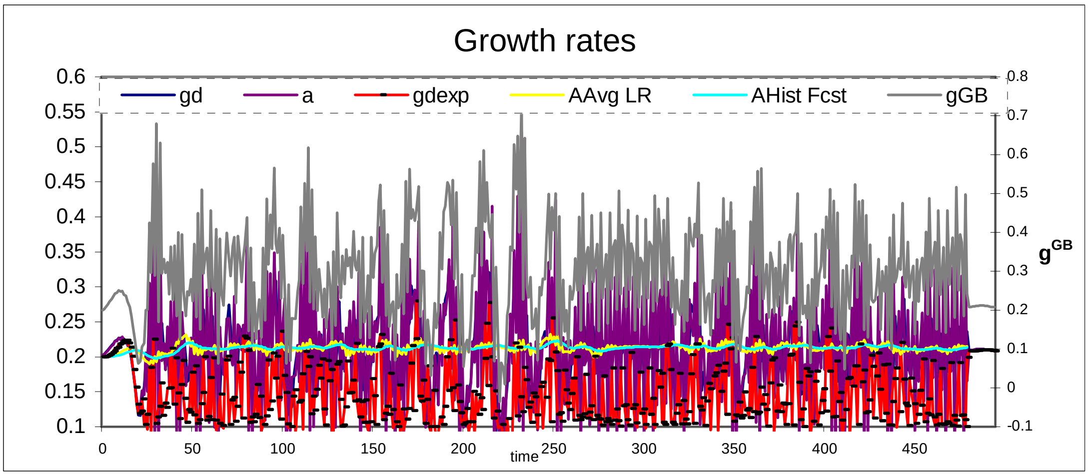




(b) $g_t^{G*} = g_t^{GB} + \mu.\left(g_{t-2}^{de} - g_{t-1}^{d}\right) - \phi.\left(e_t^{Y} - e_{t-1}^{Y}\right)$; **'higher'** $\gamma_b.\bar{b}^{y}$; $\hat{e}^{y} > 0$



(c) (i) Effects of a higher μ



(c) (ii) Effects of a higher μ

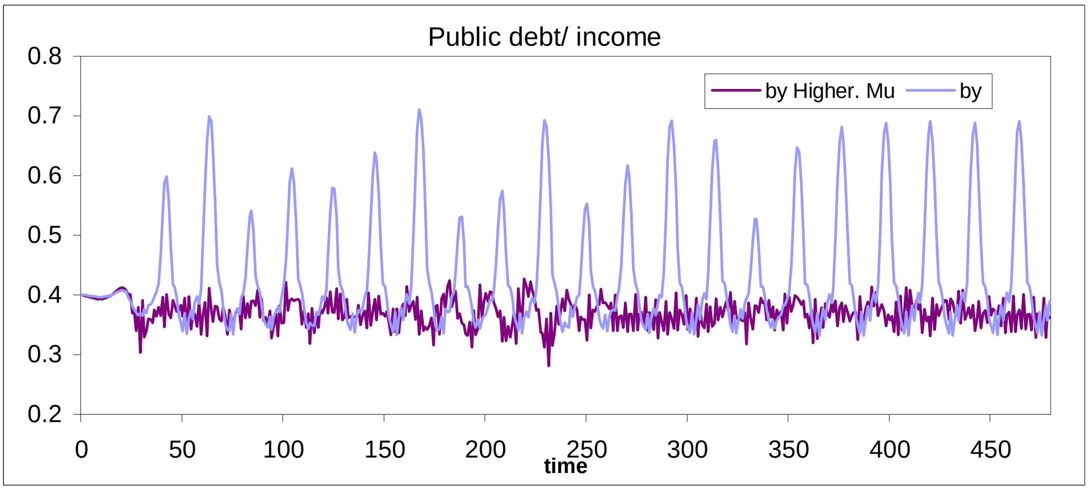


Figure 6

$g_t^{Gp} = Min\left(g_t^{GB}, g_{t-1}^{Gp}\right) + \mu.\left(g_{t-2}^{de} - g_{t-1}^{d}\right) - \phi.\left(e_t^{Y} - e_{t-1}^{Y}\right)$; **'higher'** $\gamma_b.\bar{b}^y$; $\hat{e}^y > 0$ **(same parameters as in 5 (b)); ω = 1**

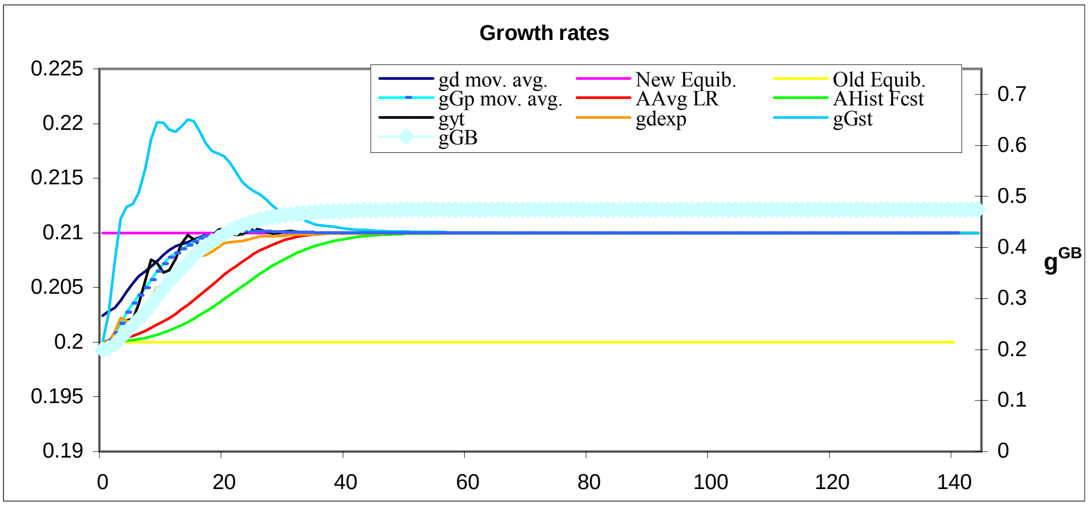
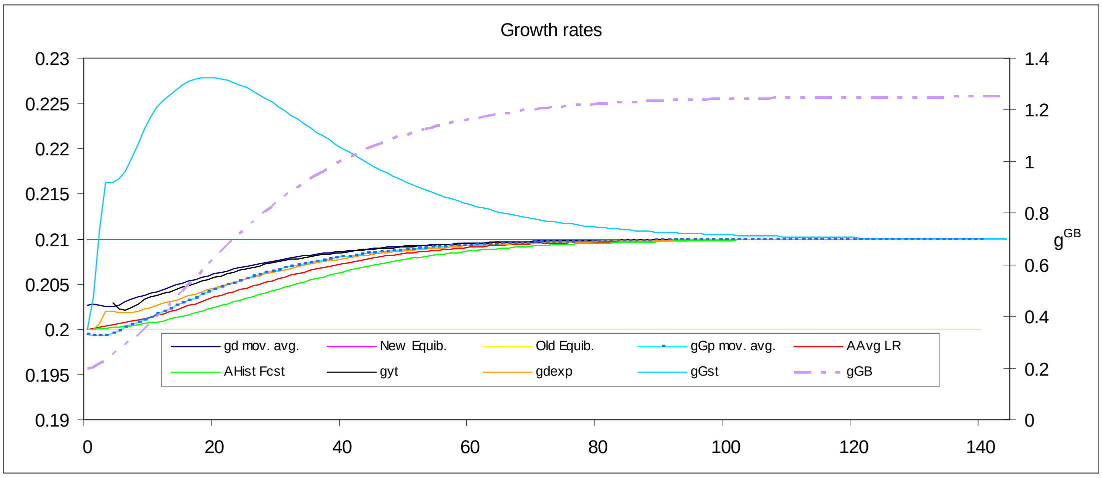
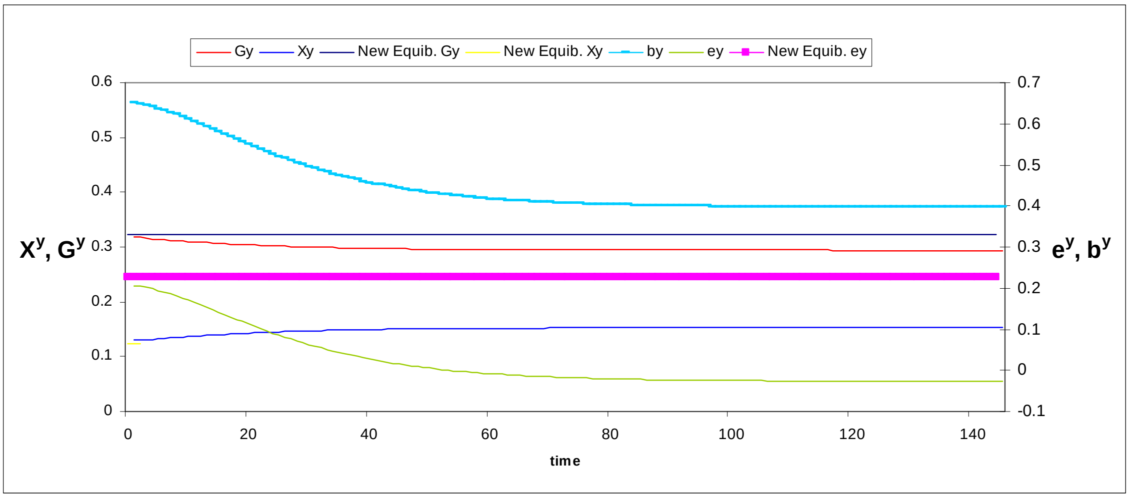


Figure 7

(a) $g_t^{Gp} = Min\left(g_t^{GB}, g_{t-1}^{Gp}\right) + \mu.\left(g_{t-2}^{de} - g_{t-1}^{d}\right) - \phi.\left(e_t^Y - e_{t-1}^Y\right)$ **; exogenous increase in** g^X





(b) $g_t^{Gp} = Min\left(g_t^{GB}, g_{t-1}^{Gp}\right) + \mu.\left(g_{t-2}^{de} - g_{t-1}^{d}\right) - \phi.\left(e_t^Y - e_{t-1}^Y\right)$ **; exogenous decrease in** g^X

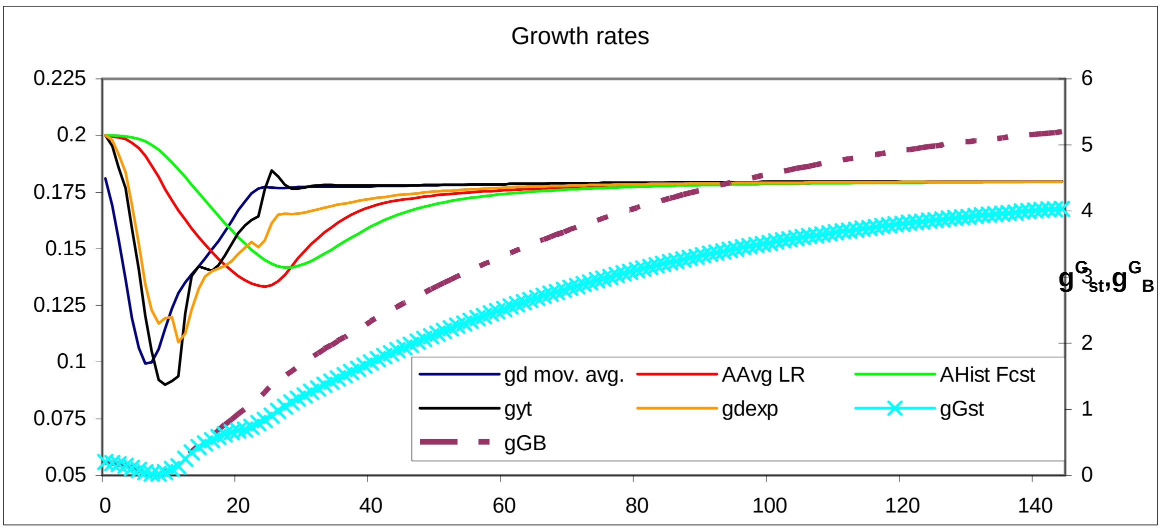


Figure 7 (b) con't.

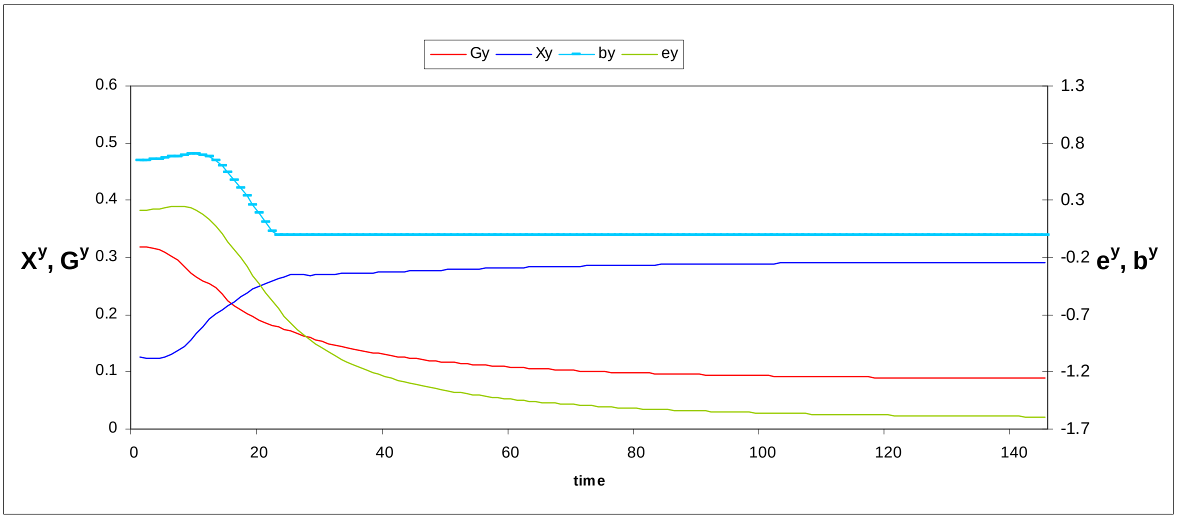
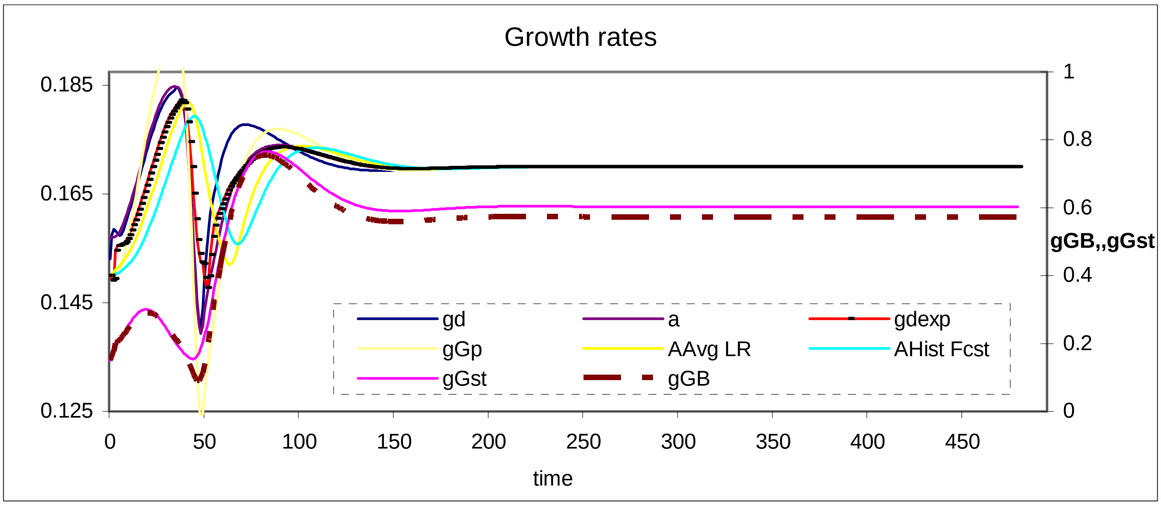
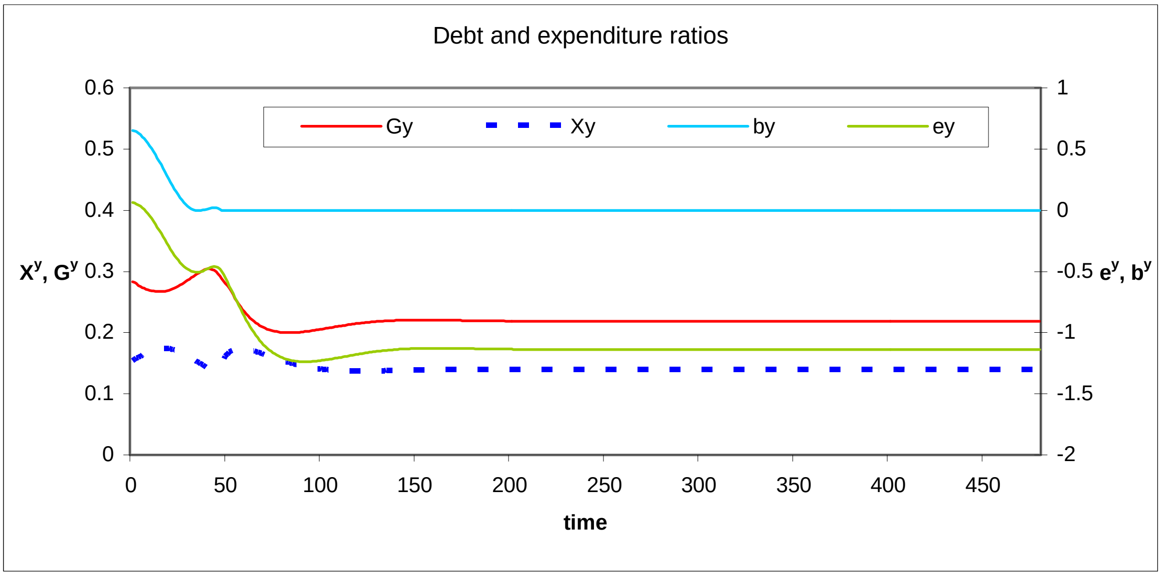


Figure 8

(a) $g_t^{Gp} = Min\left(g_t^{GB}, g_{t-1}^{Gp}\right) + \mu.\left(g_{t-2}^{de} - g_{t-1}^{d}\right) - \phi.\left(e_t^Y - e_{t-1}^Y\right)$ **; exogenous increase in g^X; $g^X < r$**





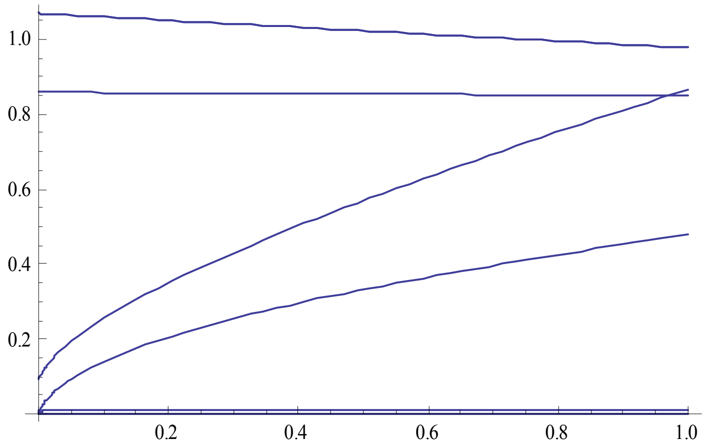
Appendix

The analysis of stability:

In principle, two methods are available for the study of stability of the system given by equations (4), (9)-(11), (15), (17)-(18), (22), (24) and (26)-(27). The first involves an examination of the local stability properties of the model's equilibria. As is well known, this entails examining the eigenvalues of the relevant Jacobian matrix for the difference equation system (4), (9)-(11), (15), (17)-(18), (22), (24) and (26)-(27), evaluated at equilibrium. For the purposes of the present paper, only limited use is made of this procedure: it is used exclusively as a means for determining "plausible" values of certain key coefficients, viz., the accelerator coefficient 'v' and the consumption propensity, 'c' and 'ε', the coefficient governing the relative weighting of expectations about autonomous demand growth in the expectation about overall demand growth.

More precisely, to arrive at plausible values for these two coefficients, an analysis of eigenvalues for different 'v', 'c', and 'ε' with all other reaction coefficients – θ, η, ω, μ and x– set to zero. For example, using parameter and steady state values for the simulation depicted in Figure 5(a), but with θ, η, ω, μ and x set to zero, the eigenvalues of the (8x8) Jacobian matrix can be represented as a function of the values of ε. Figure *A1* below, shows that all the eigenvalues are less than unity in absolute value for 0 < ε < 1, thus suggesting stability in the vicinity of the steady state given by $g^G = g^X$.

Figure A1: Eigenvalues for Jacobian as a function of 'ε'



The preferred method for the examination of stability employed here is computer simulation, albeit, bearing in mind the behaviour of the eigenvalues of the Jacobian for key coefficients, such as v, c and ε.

Parameter values and initial values for the lagged endogenous variables for simulations are provided in Tables A1 and A2 below. With regard to the latter, taking the first period of the simulation for which endogenous variables are solved as period 't', this requires initial values for the following lagged endogenous variables: $g^d_{t-2\to t-1}$, $g^{de}_{t-2\to t-1}$, $a_{t-20\to t-1}, g^X_{t-1}, g^G_{t-1}, a^{exp}_{t-1}, a^{Fcst}_{t-1}, a^{av}_{t-1}, e^Y_{t-2\to t-1}, G^Y_{t-1}, X^Y_{t-1}, b^Y_{t-1}, \alpha_{t-1}$.[24]

[24] Note that, since the model is started in its steady state, the values for g^X_{t-1} and b^Y_{t-1} are those listed in the third last and last rows respectively of Table A1.

TABLE A1: Parameters

	Fig. 5 (a)	Fig. 5(b)	Fig. 5 (c)	Fig. 6	Fig. 7 (a)	Fig. 7(b)	Fig. 8
c	0.85	0.85	0.85	0.85	0.9	0.9	0.9
v	0.15	0.15	0.15	0.15	0.02	0.02	0.02
t	0.24	0.4	0.24	0.25	0.25	0.25	0.3
m	0.15	0.15	0.15	0.15	0.15	0.15	0.15
r	0.05	0.05	0.05	0.05	0.075	0.075	0.18
x	0.05	0.05	0.05	0.05	0.025	0.025	0.025
ε	0.4	0.4	0.4	0.4	0.5	0.5	0.5
σ	2000	2000	2000	2000	2000	2000	2500
μ	0.5	0.5	1.25	0.5	0.5	0.5	0.075
θ	0.5	0.5	0.5	0.5	0.5	0.5	0.1
η	0.15	0.15	0.15	0.15	0.15	0.15	0.15
δ	0.3	0.3	0.3	0.3	0.5	0.5	0.5
γ	1	1	1	1	1	1	1
ω	1	1	1	1	0.5	0.5	0.2
g^X (before shock)	0.20	0.20	0.20	0.20	0.20	0.20	0.15
g^X (after shock)	0.21	0.21	0.21	0.21	0.21	0.18	0.17
$\bar{b}^y$	0.2	0.4	0.4	0.4	0.65	0.65	0.65

TABLE A2: Lagged endogenous variables

	Fig. 5 (a)	Fig. 5(b)	Fig. 5 (c)	Fig. 6	Fig. 7 (a)	Fig. 7(b)	Fig. 8
$g^d_{t\text{-}2\to t-1}$	0.2	0.2	0.2	0.2	0.2	0.2	0.15
$g^{de}_{t-2\to t-1}$	0.2	0.2	0.2	0.2	0.2	0.2	0.15
$a_{t-20\to t-1}$	0.2	0.2	0.2	0.2	0.2	0.2	0.15
g^G_{t-1}	0.2	0.2	0.2	0.2	0.2	0.2	0.15
a^{exp}_{t-1}	0.2	0.2	0.2	0.2	0.2	0.2	0.15
a^{Fcst}_{t-1}	0.2	0.2	0.2	0.2	0.2	0.2	0.15
a^{av}_{t-1}	0.2	0.2	0.2	0.2	0.2	0.2	0.15
$e^Y_{t-2\to t-1}$	-0.323636	0.025974	0.025974	0.025974	0.207403	0.207403	0.066288
G^Y_{t-1}	0.265	0.45	0.45	0.45	0.317708	0.317708	0.283043
X^Y_{t-1}	0.190455	0.146753	0.146753	0.146753	0.128396	0.128396	0.151729
α_{t-1}	1.391408	3.066372	3.066372	3.066372	2.474449	2.474449	1.865451