Chapter 7
Adaptive Multi-resolution Thresholding with Fuzzy Partition

As noticed in our previous experiments, the resulting images from our method depend on the fuzzy partition of the histogram of the original images. The proportion of the object pixels will affect the shape of the histogram. In some cases the background area of the original image is too large for the peaks representing the objects to show as being significant in the histogram. Consequently the optimal threshold cannot be achieved by our method. How to correct this bias is the issue that is discussed in this chapter.

7.1 Quadtree Scheme

A quadtree scheme is a data structure which represents the image in multiscale. The goal of the quadtree scheme is to segment the image into homogeneous and heterogeneous areas. A quadtree structure represents a two dimensional image hierarchically. The whole image is the root of the quadtree. If the block which the root node represents is not homogeneous, it is split into quadrants, each of which is called a 'child'. A quadrant is recursively split into sub-quadrants until homogenous, or when the size of the block is less than a predefined value R^*. The child blocks are merged if they satisfy the parity check.

There are many criteria that can be used as a homogeneity measure [59]-[56]. The Kolmogorov-Smirnov distance defined as the maximum (vertical) distance between the normalized gray scale distribution is used as the criterion by Geman, et al. [59] [98]. This may work efficiently for textured images but experiments on our images did not achieve satisfactory results. According to statistics theory, the test for homogeneity of a block is:

Calculate the gray scale standard deviation σ in this block as follows:

$$\sigma = \sqrt{\frac{1}{m \cdot n}\sum_{i=1}^{m}\sum_{j=1}^{n}[f(i,j) - \mu]^2} \tag{7.1}$$

If $\sigma < \sigma^*$, the block is homogeneous; otherwise, it is not homogeneous and subject to split. σ^* is an adjustable parameter. m and n are the height and width of the block; i and j are the coordinates of the pixels in the block. $f(i,j)$ is the gray scale of the pixel at (i,j); and μ is the average of the gray scales in this block. It is defined as follows:

$$\mu = \frac{1}{m \cdot n}\sum_{i=1}^{m}\sum_{j=1}^{n} f(i,j). \tag{7.2}$$

The homogeneity of the two blocks b_1 and b_2 is checked before merging. If they are both homogeneous then the disparity between the blocks is examined. The data sets of blocks b_1 and b_2 are B_1 and B_2, respectively. The difference between the two blocks is $|B_1 - B_2|$. The average $\mu(|B_1 - B_2|)$ and the standard deviation $\sigma(|B_1 - B_2|)$ of the difference of the two blocks are used to determine the disparity of the two blocks as follows:

$$\mu(|B_1 - B_2|) = |\mu_1 - \mu_2|\,, \tag{7.3}$$

$$\sigma(|B_1 - B_2|) = \sqrt{\sigma_1^2 + \sigma_2^2}. \tag{7.4}$$

If

$$\mu(|B_1 - B_2|) < d_1$$

and

$$\sigma(|B_1 - B_2|) < d_2\ ,$$

the parity check is true and the two blocks are merged. d_1 and d_2 are controllable predefined values.

Applying the quadtree scheme, an image is represented by homogenous and heterogeneous blocks. The homogeneous blocks are considered as information redundancy. Too much information redundancy will distort the histogram such that many features of the image, (such as the peaks and the valleys), cannot be shown correctly in the histogram. A histogram is the probability distribution of gray scales in an image. It reflects many features of the image. For an l-level image $D \equiv \{(i,j) : i = 0, 1, ..., M-1; j = 0, 1, ..., N-1\}$, the histogram is

$$h_k = \frac{n_k}{N \cdot M}(k = 0, 1, ..., l-1),$$

where M and N are two positive integers representing the size of the image in the x and y axis. n_k is the total number of pixels with level k. The histogram of an image plays an important role in the performance of the thresholding methods which select the threshold based on the histogram. These methods are usually derived based on some assumptions, such as the major peaks in the histogram represent different objects or the background. However, we know that the histogram represents global information and many details cannot be reflected. If the ratio of the number of pixels in the objects and the background is not moderate, the peak of the one with relatively small pixels will vanish. Therefore, the threshold cannot be selected correctly. We do not want the significant redundant gray scale information to be included in the histogram because they distort it. The homogeneous areas are considered as the repeated gray scale information and should be excluded from the histogram which we use for calculation. The new histogram is computed only in the heterogeneous areas. Figure 7.1(c) and (d) show the histograms of the whole image and the new one computed only in the heterogeneous areas. We can see that for the original image with a large area of background, the histogram of the whole image is just a very tall peak. It does not reflect the object information adequately. Many thresholding methods based on the histogram cannot produce a good threshold in this case.

Remark: The ratio between the height and the width of the images we are dealing with varies from image to image. The blocks in the quadtree structure however should be as close to the squares as possible. If the ratio between the adjacent two sides of the block is too large or too small, the block is not able to represent the image. Thus a reasonable ratio is required. The images with either too large or too small a ratio should be split into half parts until the ratio is reasonable. The reasonable ratio r is defined as:

$$\frac{1}{\sqrt{2}} \leq r \leq \sqrt{2}.$$

This is derived from the procedure detailed below.

It is supposed that the ratio between two adjacent sides of the block should be improved (closer to 1) after splitting the block into two parts at the long side. Mathematically it can be written as:

$$y/x > x/\frac{y}{2}$$

or

$$y/x > \sqrt{2}.$$

where y is the long side of the block and x is the short one.

7.2 New Histogram of the Quadtree Image

The histogram on which the searching is based is calculated on the heterogeneous areas of an image instead of the whole image. That is only the blocks which are considered as containing both objects pixels and background pixels are taken into account when calculating the new histogram. Those labelled as the homogeneous area are considered as containing only one kind (object or background) of pixels.

They are excluded from the new histogram. Doing so prevents the dominance of one kind of pixels in the histogram. As we have seen before, when there is a large area of background in the original image there will be a major peak at the gray scale which represents the background in its histogram. Sometimes this peak is so significant that it stops the object pixels showing a major peak in the histogram. This is the reason only one major peak is shown in many histograms which should show two peaks in ideal cases, one for the object, and one for the background. The resulting histogram is thus rather meaningless. Also it decreases the performance of many thresholding methods based on histograms.

Figure 7.1 demonstrates how the histogram will change if we only include the pixels in the heterogeneous blocks into the calculation of the histogram. Figure 7.1(a) is a gray scale fingerprint image. There is a good area of background around the object. Note there is some noise at the left edge and top of the image. Figure 7.1(b) is its quadtree image after the quadtree scheme is applied to the original image. All the homogeneous blocks are displayed as white and the heterogeneous ones black. We can see that most of the background is labelled correctly as homogeneous. The part containing noise is left as belonging to the opposite class, which is as expected. We can see most of the background will be cropped out of the histogram. Figure 7.1(c) shows the old histogram based on the whole image. Only one significant peak can be identified. The new histogram based on only heterogeneous blocks is shown in Figure 7.1(d). Two major peaks can be seen on it. Thus, we can say that the new

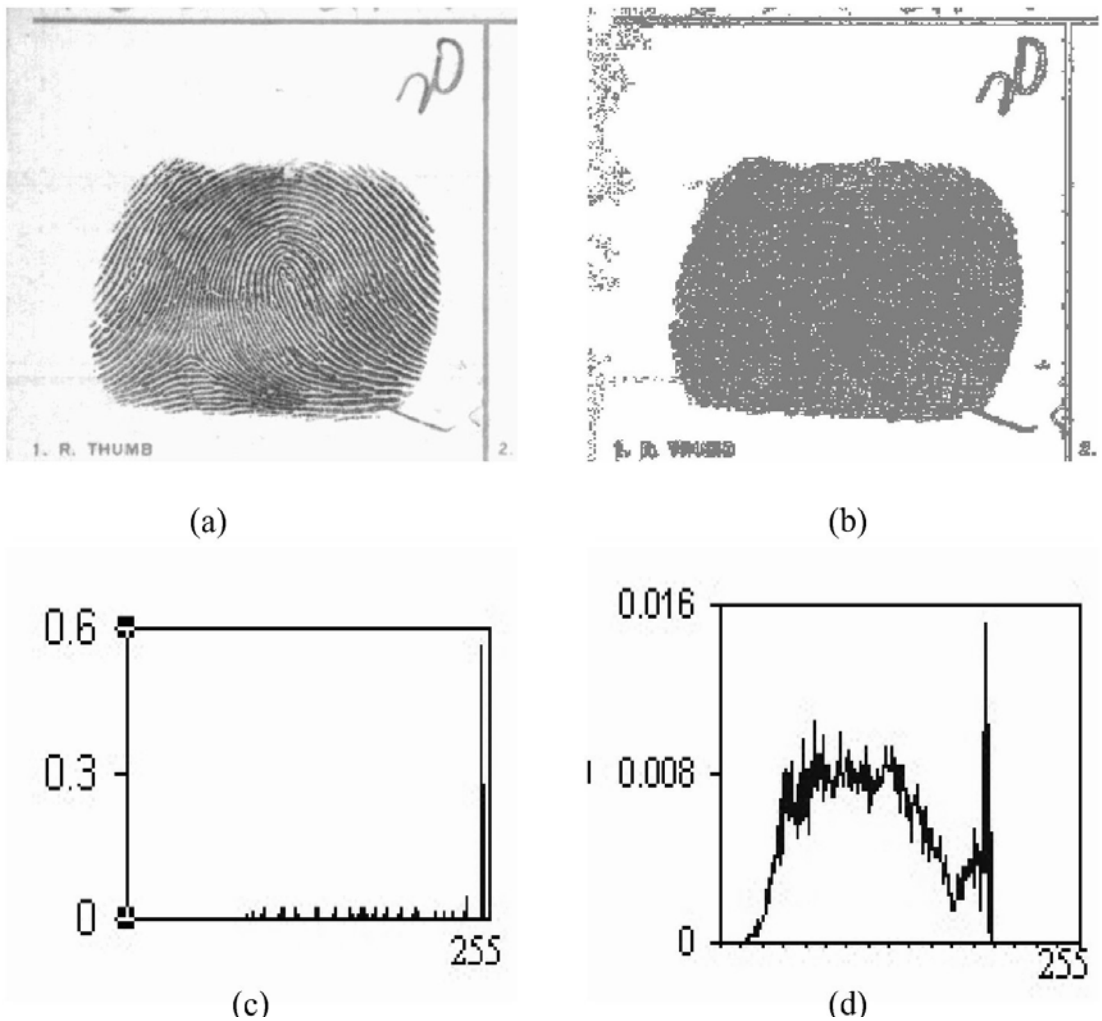

Figure 7.1: (a) Gray scale fingerprint image; (b) Quadtree image: white area is homogeneous and gray area is heterogeneous; (c) Histogram of the whole fingerprint image; (d) Histogram of the heterogeneous area.

histogram gives more information than the old one. From an entropy point of view we can see that this is right because $H = -\sum_{i=0}^{n} p_i \cdot \lg(p_i)$. That means the more diversified the signal the more entropy. The more entropy the more information. If all the signals have equal probability, the maximum entropy will be reached. Apparently, the new histogram in Figure 7.1(d) will give a greater entropy than the one in Figure 7.1(c). Thresholding methods based on histograms would make more sense if the new histogram is used in the computation.

7.3 Local Fuzzy Partition

7.3.1 Local Histogram

A local histogram is the histogram based on the square window area centered by the processed pixel. The window size is determined by its radius r. r is the distance between the processed pixel and the pixels located at both edges of the window and at the same row or column. (See Figure 2.3(b). $r = sw$ in this case.)

The pixels in the heterogeneous areas should be thresholded adaptively according to the local histogram of each pixel. The threshold for each pixel is selected based on the fuzzy partition of the local histogram. Like the new histogram used to derive the threshold for the homogeneous area, the local histogram is calculated using only the pixels in the heterogeneous area. The pixels in the heterogeneous area are labeled “1”, while those in the homogeneous areas are labeled “0”. The new method of calculating the local histogram is described as follows:

$$h_k^W = \frac{n_k}{N} \tag{7.5}$$

where h_k^W $(k = 0, 1, ..., 255)$ is the local histogram in window W; n_k is the total number of heterogeneous pixels (labeled "1") with gray scale k in the square window W centered at the processed pixel; N is the total number of the heterogeneous pixels in this square window W .

7.3.2 Local Fuzzy Partition

For a pixel located at coordination (i, j) with gray level g (denoted as $g(i, j)$, the local fuzzy partition scheme is described as follows:

1. Given the window radius r , locate the active calculation window centered at $(i, j) : W_{i,j} = \{(x, y) : (i - r) \leq x \leq (i + r), (j - r) \leq y \leq (j + r)\}$.
2. Compute the histogram h_k^W of the window $W_{i,j}$ using Equation 7.5.
3. Use the search schedule in Fuzzy-2 partition to search for the optimal partition based on h_k^W and calculate the threshold $T_{i,j}$ for pixel $g(i, j)$.
4. Threshold the pixel $g(i, j)$ with $T_{i,j}$.

$$f'(i, j) = \begin{cases} 1 & if \quad g(i, j) < T_{i,j}; \\ 0 & \quad otherwise. \end{cases}$$

$f'(i, j)$ is the value of pixel at (i, j) after thresholding.

7.4 The Adaptive Segmentation Method

In the proposed method, a quadtree scheme is applied to roughly segment the image into homogeneous and heterogeneous areas. The histogram of the heterogeneous areas is calculated and used to search the global threshold T_g. All pixels in the homogeneous areas are thresholded with this global threshold. The threshold for each pixel in the heterogeneous areas are selected by using local fuzzy partition as described in the previous section.

The radius r of the square window depends on the size of the block in which the processed pixel is located. If the width and the height of the block is denoted as w and h respectively, $r = 2 \cdot Max(w, h)$, where $Max(w, h)$ is the greater of w and h. Because the block is assumed to contain the objects pixels, its surrounding area is assumed to contain the background and needs to be taken into account in order to get a reasonable ratio between the number of object pixels and background pixels. Then, we search for the optimal fuzzy partition for *each pixel* inside the block based on the histogram of the square window centered at each pixel with radius r. Note the local histogram is calculated using Equation 7.5.

The proposed method is as described below:

1. Input image.

2. Generate the quadtree to represent the image with blocks by splitting and merging using the quadtree scheme proposed in Section 7.1. The image is

split into different sizes of homogeneous blocks and small sized heterogeneous blocks.

3. Calculate the new histogram with only the heterogeneous areas.

4. Search the optimal fuzzy partition of the new histogram. The obtained threshold is adopted as the global threshold T_g for all the homogeneous areas.

5. For each block: Check its homogeneity,

   (a) If homogeneous, classify every pixel in the homogeneous block with the global threshold T_g. For pixel $(i, j) \epsilon W$,

   $$f^{'}(i,j) = \begin{cases} 1 & if \quad f(i,j) < T_g \\ 0 & otherwise. \end{cases}$$

   (b) If heterogeneous, determine the width w and the height h of the block. Set the local calculated window radius $r = 2 * Max\{w, h\}$. For every pixel (i, j) in this block:

      i. Locate the calculated window W centered at (i, j) with radius r.

   $$W_{i,j} = \{(x, y) : ((i - r) \leq x \leq (i + r); (j - r) \leq y \leq (j + r))\}.$$

      ii. Compute the histogram h_k^W $(k = 0, 1, ..., 255)$ of the window $W_{i,j}$ with Equation 7.5.

      iii. Search the optimal fuzzy partition using h_k^W, and calculate the threshold T_{ij}.

iv. Threshold the pixel (i, j) with $T_{i,j}$.

$$f'(i,j) = \begin{cases} 1 & if \quad f(i,j) < T_{i,j} \\ 0 & otherwise \end{cases} .$$

7.5 Experiment Results and Discussion

Our method is tested on many images. To test the efficiency of the new histogram and the fuzzy partition, the global fuzzy partition based on the heterogeneous histogram is also carried out. The global fuzzy partition method selects a global threshold by searching the optimal fuzzy partition of the heterogeneous histogram and then segments the whole image with the global threshold. Although it is still a global threshold, the implementation of the fuzzy partition and the new calculation of the histogram improve the performance of the segmentation. A comparison between our method and Pun's method is also carried out. The parameters used in all these images were: $R^* = 25$; $d_1 = 10$; $d_2 = 5$; $\sigma^* = 10$. Figures 7.2-7.5 show some of the results.

Figure 7.2(a) shows the segmented fingerprint image shown in Figure 7.1(a) with global thresholding using fuzzy partition based on the new histogram based on the heterogeneous areas. The calculated threshold is obviously not appropriate for this image and the fingerprint appears with many broken lines in the segmented image. Figure 7.2(b) shows the segmented image by using the proposed method. We can see that the fingerprint is clearer in Figure 7.2(b) and the fringes are separated from each other. Pun's method failed on this image. We attempted to give a close examination of the reasons that Pun's method failed, and also that the global fuzzy thresholding resulted in an image with broken lines . We can see how the histogram affects the performance of the segmentation methods based on the histogram. In Figure 7.1(b), we can see that the background is the dominant part of the histogram which results in the vanishing of the object in the histogram. The partition of the background is so large that it produces an unreasonable value of the threshold. Therefore Pun's method fails to distinguish the object from the background. After the application of the quadtree scheme, the main part of the new histogram based on the heterogeneous areas now represents the object (See Figure 7.1(d)). Because the objects are so condensed at the middle of the image, the background is relatively so small in the heterogeneous area, which leads to a high threshold by the global fuzzy thresholding method (See Figure 7.2(a)). The proposed method combines the local and global information, therefore producing a better result (Figure 7.2(b)).

Figure 7.3(a) is a gray scale image of a building. The quadtree image is shown in Figure 7.3(b). The major outline of the building is reflected in the quadtree image. As we can see from the histogram of the whole image (shown in Figure 7.3(c)), there are two modes in the histogram. However, the two peaks represent water and sky. The pixels for the building are too few to be shown as a peak in the histogram. As a result, the building is overlapped by water in the segmented image using Pun's method (See Figure 7.3(e)). The histogram of the heterogeneous areas is shown in Figure 7.3(d),

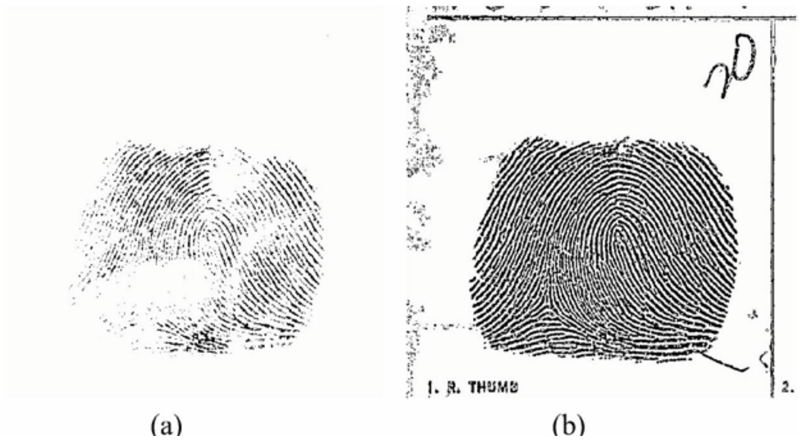

Figure 7.2: Multiresolution vs. global thresholding. (a) Global thresholded image using the heterogeneous histogram; (b) Segmented image using the proposed method.

in which the gray scale distribution of the object is magnified. The global segmented image using fuzzy partition based on the new histogram (Figure 7.3(f)) gives a better result than Pun's method. The details of the building are clearer in the segmented image using the proposed method than the global thresholding method and Pun's method.

Figure 7.4(a) shows a gray scale image of a baby. The homogeneous areas are moderate, which is different from the previous ones which contain a large amount of homogeneous areas. The quadtree image is shown in Figure 7.4(b). There is a little change in the histogram of the whole image compared to the histogram of the heterogeneous areas (See Figure 7.4(c) and (d)). The global fuzzy thresholding method produces a similarly segmented image as Pun's method (Figure 7.4(e) and (f)) due to the lack of change between the two histograms. The proposed method

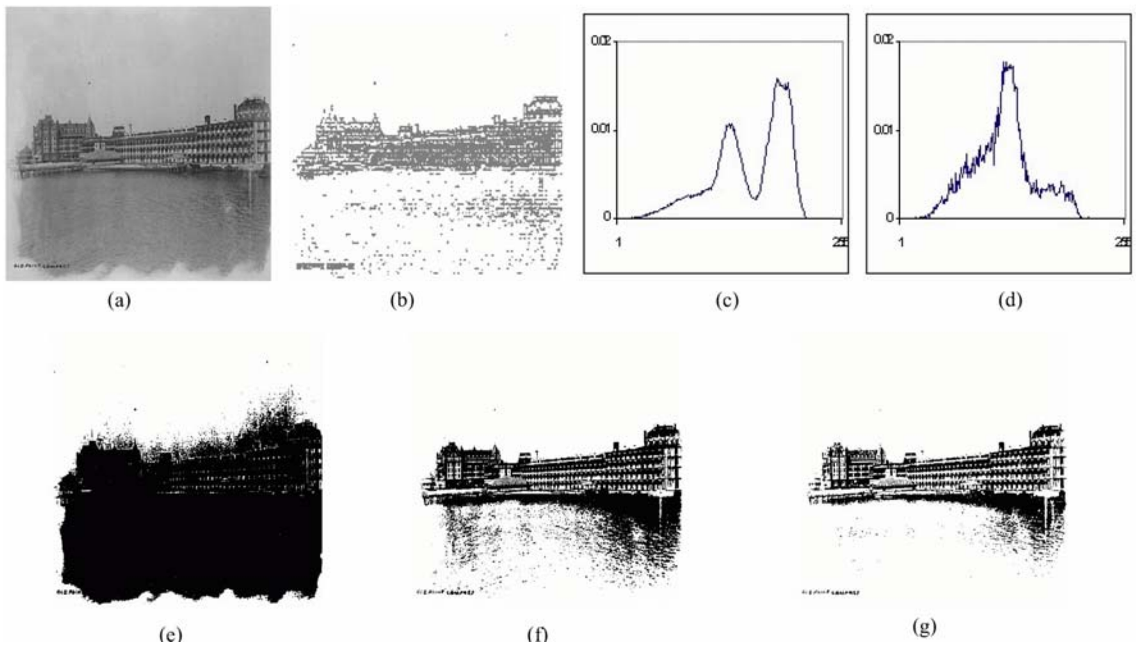

Figure 7.3: (a) Gray scale building image; (b) Quadtree image; (c) Histogram of the whole image; (d) Histogram of the heterogeneous areas of the image; (e) Segmented image using Pun's method; (f) Global thresholding image using the heterogeneous histogram; (g) Segmented image using the proposed method.

produces a better image retaining more details than the other two methods(Figure 7.4(g)).

Figure 7.5(a) shows a lady with a strawberry in hand. The homogeneous areas are not very large, except in the left corner of the image (See Figure 7.5(b)). The background pixels form a dominant part of the histogram because the gray scale of the objects are scattered in a wide range as indicated by a flatter part in the histogram. Based on this histogram, Pun's method produces a dark segmented image (See Figure 7.5(f)) in which half of the face is covered with black blocks which do not show the features of the face. The histogram of the heterogeneous area removed most of

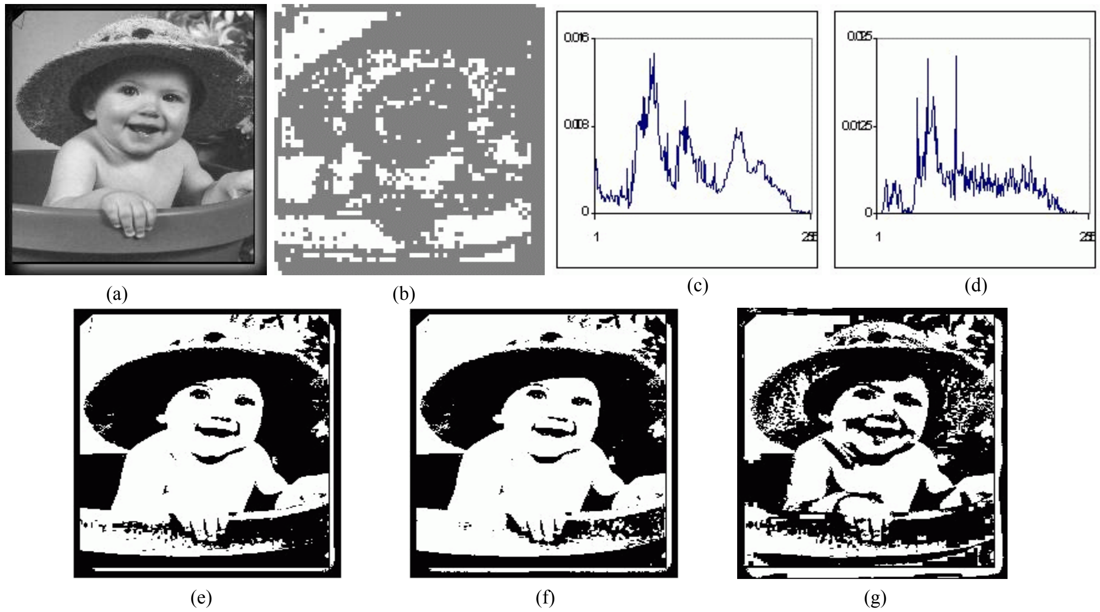

Figure 7.4: (a) Gray scale baby image; (b) Quadtree image; (c) Histogram of the whole image; (d) Histogram of the heterogeneous areas of the image; (e) Segmented image using Pun's method; (f) Global thresholding image using the heterogeneous histogram; (g) Segmented image using the proposed method.

the background pixels and the gray scale distribution in the objects and their close surroundings are the main part of the new histogram (See Figure 7.5(d)). The face is clear in the segmented image using the global thresholding method based on the new histogram (Figure 7.5(f)). However, the information of the hand is lost. The proposed method captures most of the features of the image as shown in Figure 7.5(g).

Tests on some images containing signatures were also carried out. The major feature of the signature images is that the partition of the background is much larger than that of the object, which is too small to be reflected in the histogram of the whole image (See Figure 7.6(c)). The segmentation based on this histogram results in a bad segmentation (Figure 7.6(e)). As in the above examples, by applying the

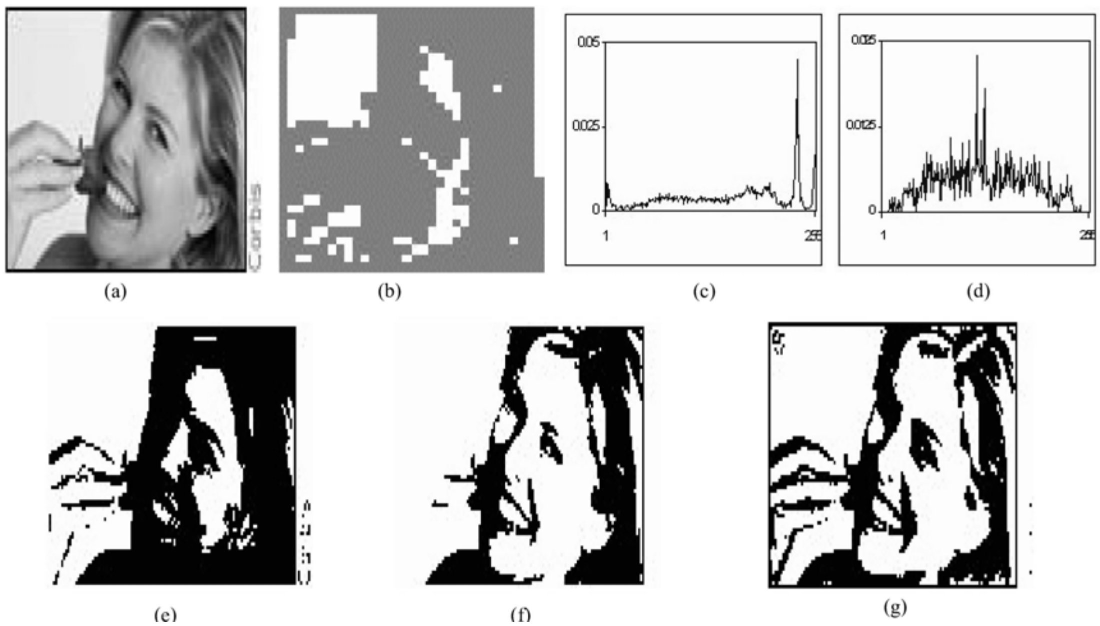

Figure 7.5: (a) Gray scale lady image; (b) Quadtree image; (c) Histogram of the whole image; (d) Histogram of the heterogeneous areas of the image; (e) Segmented image using Pun's method; (f) Global thresholding image using the heterogeneous histogram; (g) Segmented image using the proposed method.

Image	Pun's Method	Global Fuzzy Partition	Proposed Method
Fingerprint	Fail	5%	20%
Building	47%	21%	14%
Baby	50%	47%	50%
Lady	49%	31%	34%
Signature	72%	10%	8%

Table 7.1: Percentage of the black pixels in the segmented image by different methods

quadtree scheme, the image is separated into homogeneous and heterogeneous areas as shown in Figure 7.6(b). The homogeneous area which is mainly background is removed from the original histogram and the new histogram of the heterogeneous area is calculated (Figure 7.6(d)). In this, the gray scale distribution of the object pixels and their close neighbors is magnified and used to determine a good threshold. The segmented image using the global fuzzy partition thresholding method based on the new histogram is efficient at capturing most of the object except some areas between the strokes (Shown in Figure 7.6(f)). The proposed method produces a clear segmented image (Figure 7.6 (g)) of the original blurred signature image (Figure 7.6(a)).

Table 7.1 shows the percentages of black pixels in the segmented image using different methods. We can see that Pun's method produces segmented images with nearly equal parts. The percentage of the black pixels in the segmented images are all close to 50 percent. Both the global fuzzy partition and the proposed method, however, produce varied partitions. One interesting thing worth noting occurs in the baby image. Both Pun's method and the proposed method produce 50 percent of black pixels. However, the segmented images given by these two methods are different. The image produced by the proposed method retains more details of the object than does Pun's method which shows that the number of black pixels in the segmented image is not a good indicator.

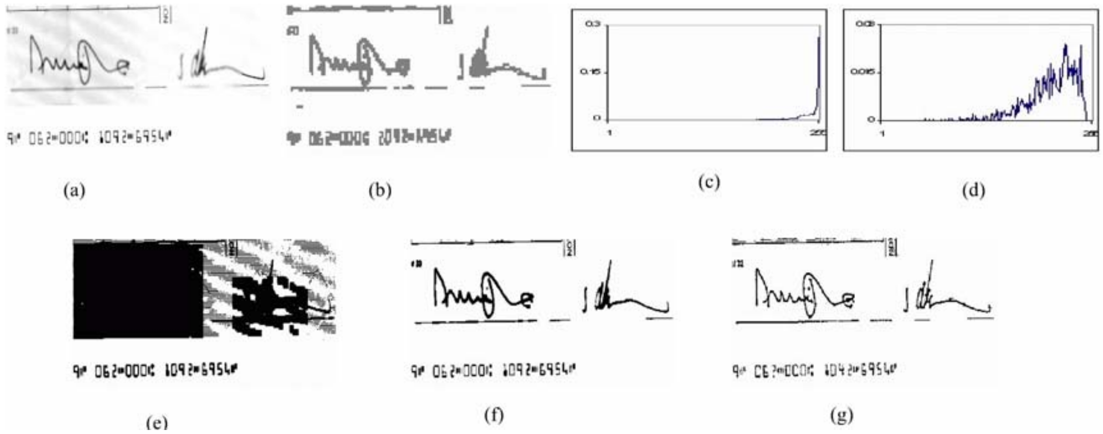

Figure 7.6: (a) Gray scale blured signature image; (b) Quadtree image; (c) Histogram of the whole image; (d) Histogram of the heterogeneous areas of the image; (e) Segmented image using Pun's method; (f) Global thresholding image using the heterogeneous histogram; (g) Segmented image using the proposed method.

From these experiments it can be seen that our method is efficient at keeping local details and at the same time is capable of repressing background noise. The quadtree scheme coarsely splits the image into homogeneous and heterogeneous blocks and it can show the coarse outline of the objects. This is helpful in determining whether local or global methods should be applied. Therefore the balance between detail preserving and noise repressing is achieved.

7.6 Summary

In this chapter, a multi-resolution image segmentation technique based on a fuzzy partition and entropy maximization is proposed. The proposed algorithm combines global and local methods and applies them adaptively. Fuzzy partition thresholding techniques are derived from the entropy approach. An efficient search procedure for optimal fuzzy partition is also given. This is derived based on the relationship between fuzzy partition and the probability partition. The quadtree structure is adopted to coarsely partition the image into homogeneous and heterogeneous blocks. The global and local fuzzy partition methods are applied adaptively to each block according to their homogeneity. The proposed method is efficient in retaining local details and repressing noise and the experiment results are encouraging.