Chapter 6

Effects of Moving Transmitters on Reference Antenna Cancellation

In this chapter some of the algorithms discussed so far are considered in greater detail. In particular, different reference antenna cancellation algorithms are compared in the presence of fringe rotation decorrelation. Equations (2.9) through (2.12) show that the phase difference of a wavefront at two antennae will change for different geometric delays, \( \tau_{jk} \) (e.g. for different source positions), or for different frequencies, \( \nu \) (e.g. for different spectral components of a signal). In either case the power measurement will oscillate through sinusoidal fringes if the phase difference is increased or decreased, and in chapter 2 it was shown that both cause decorrelation of the cross-correlation measurement. The proportion of the power remaining is \( F_{jk}^{(\tau)} \) for variable \( \tau_{jk} \), and \( F_{jk}^{(\nu)} \) for variable \( \nu \). In this section the decorrelation is examined with simulated examples, and the effect on different interference mitigation algorithms is investigated.

The chapter starts in section 6.1 with a description of the simulations and some of the decorrelation results. The ability of the adaptive filter weights to track changing delays of an interfering signal, as described in section 5.3, is then considered in 6.2, and techniques for altering the Wiener filter and post-correlation canceller weights to allow for any decorrelation are examined in 6.3. This includes simulated comparisons of the filters and an example from the Australia Telescope Compact Array.

6.1 Fringe Rotation Simulations

To investigate the quality of the reference antenna cancellation schemes for various fringe rates, simulations were carried out in MATLAB (The Mathworks, Inc., 1998), with the geometric delay of the main antenna increasing at each consecutive voltage sample while the reference antennae delays were held constant. Since each different spectral component of a signal experiences a different phase shift when the geometric time delay changes, the wide-band voltages were generated by adding together their
simulated Fourier components, i.e. averaging many monochromatic waves across the bandwidth with random initial phases (the frequencies used are described shortly when the Fourier transforms are discussed). The waves were generated in this fashion to allow each spectral component to be advanced in phase arbitrarily. Time was then advanced (with a step size of twice the bandwidth, i.e. at the Nyquist rate), and the synthesised voltages allowed to progress. This approximates Gaussian noise sources (see §7.2 of Press et al., 1986).

One voltage sequence represented the main voltage, $v_m(t)$, containing RFI but no receiver noise or astronomy signal, and two other voltage sequences represented the reference voltages, $v_{r1}(t)$ and $v_{r2}(t)$, containing RFI and receiver noise. All signals, that is the RFI in all three voltages and the system noise in the reference voltages, had equal power. All of the additive voltage components were generated as broadband noise running from a frequency of zero up to the bandwidth frequency (defined here for generality as a function of Nyquist samples). So the signals contain spectral components with widely different frequencies, which will experience widely different phase shifts for a given delay. For this reason, when inspecting input and output power levels, the power spectra will be investigated. After any required voltage domain processing had been applied (for example adaptive filtering), the voltages were Fourier transformed into the frequency domain. The spectra were then multiplied together and accumulated, as described in chapter 5. One should note that normally the baseband spectrum of a narrow-band signal is being investigated, which also runs from a frequency of zero up to the bandwidth. However in that case the bandwidth is much less that the original radio observing frequency, and so all of the spectral components are much closer together relative to zero frequency. This situation is shown in the example in section 6.3.4.

In section 2.2 the proportion of interference power remaining in the cross-correlation function of two voltage sequences in the presence of fringe rotation was investigated. For a narrow frequency band centred at frequency $\nu_0$, the proportion of power remaining in the cross-correlation of signals from main antenna, $m$, and reference antenna, $r$, is $F_{mr}(\tau) = \text{sinc}(\nu_0 \Delta \tau_{mr})$. The variable $\nu_0 \Delta \tau_{mr}$ is the number of interference fringes wrapped during the integration, that is, the phase difference caused by the geometric delay changing by $2\pi \nu_0 \Delta \tau_{mr}$ radians.

Recall that the optimal filter weight vector for removing RFI from the main signal path, given by the Wiener-Hopf solution, $\bar{w}^*$, is a function of the main-reference cross-correlated power, $\bar{P}_{mr}$. The RFI power in the reference signal is estimated and used with $\bar{P}_{mr}$ to deduce what the RFI power in the main signal must be. Since the cross-correlated power has been decorrelated, the filter will assume an insufficient level of main signal power and not subtract a correctly amplified reference signal (the interference in the auto-correlations will not decorrelate since $\Delta \tau_{mm} = \Delta \tau_{rr} = 0$). The

\[ \text{The statement that } \Delta \tau_{mm} = 0 \text{ may not hold if the main signal is the output of a phased array, in which case interference from a moving source may also decorrelate. Here, however, it is assumed that the main antenna is a single receiving element.} \]

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adaptive approach can attempt to keep up with $\Delta \tau_{mr}$, and if successful avoid the fringe rotation decorrelation drawback (if the fringe rotation is not too severe, one can also estimate and attempt to correct for the decorrelation, which is discussed in the section 6.3).

To investigate the effect of decorrelation on the filters, the results are given as a function of frequency, since each spectral component of the wide-band signal suffers a different amount of decorrelation (and the total effect would be difficult to interpret). The filtering was applied in the voltage domain and the various input and output voltage streams were Fourier transformed to investigate the spectra. In the frequency domain, for main and reference signals $V_m(\nu)$ and $V_r(\nu)$ respectively (where $V_r(\nu)$ represents either of the two references), the remaining power in a narrow frequency channel of the cross-power spectrum, $P_{mr}(\nu) = \langle V_m(\nu)V_r^*(\nu) \rangle$, is

$$ F_{mr}^{(c)}(\nu) = \text{sinc}(\nu \Delta \tau_{mr}) $$

where again $\nu \Delta \tau_{mr}$ is the number of interference fringes wrapped during the integration. Let $\Delta T = 2\nu \Delta \tau_{mr}$ be the change in delay measured in Nyquist samples per integration, and also let the frequency spectrum be split into $L_{fft}/2$ quasi-monochromatic frequency bands, $L_{fft}/2$ because the spectrum is complex (these were the $L_{fft}/2$ frequencies used to generate the input monochromatic voltages which formed the wide-band signals). Since the signals are correlated at baseband, there are $L_{fft}/2$ different frequencies running from 0 to $\Delta \nu$ Hz, all suffering different levels of decorrelation. The frequencies are denoted $\nu(n) = \frac{n}{L_{fft}/2} \Delta \nu$, where $n \in [0, L_{fft}/2 - 1]$, so $F_{mr}^{(c)}(\nu)$ becomes

$$ F_{mr}^{(c)}(\nu(n)) = \text{sinc}(\nu(n) \frac{\Delta T}{2\Delta \nu}) = \text{sinc}\left(\frac{n}{L_{fft}} \Delta T \right). $$

Figure 6.1 is an array of plots showing the normalised amplitude of the cross-power spectrum of the main and reference signals, $P_{mr}(\nu)$, for three fringe rates ($\Delta T = 0, 1, \text{and } 10$ Nyquist samples per integration). The solid lines are the theoretical curves given by $F_{mr}^{(c)}(\nu)$, and the points are from the simulation. Note again that the heavy frequency dependence is a result of this being wide-band interference. Apart from the left spectrum, $\Delta T = 0$, which suffered no fringe rotation, the decorrelated spectra in Figure 6.1 will not give the correct weights if used in (5.7), $\tilde{w}^* = \tilde{p}_r^{-1} P_{mr}$. The weights would give an underestimate of the RFI power in the main voltage sample and the algorithm would fail to accurately minimise the output power. Shorter integration lengths can be used to keep the decorrelation to a minimum, however the matrix inversion required in (5.7) becomes computationally demanding for systems with a large number of reference delays and the shorter the integrations the more often the inversions are required. Alternatively, one might attempt to reduce the length of the baseline between the main and reference receivers (which can obviously have practical limitations due to, for example, the size of the antennae).
Figure 6.1: Normalised amplitudes of simulated cross-power spectrum, $P_{ms}(\nu)$, of wide-band RFI for fringe rates of 0, 1, and 10 fringes per integration.

### 6.2 Adaptation of Time Domain Filters

An adaptive filter can attempt to keep up with the fringe rate, since it can change its weights at the sample rate (i.e. the weights are slightly modified at a rate equal to twice the bandwidth for Nyquist sampling). To demonstrate the adaptation the adaptive algorithm (5.15) was implemented for each of the fringe rates shown in Figure 6.1. The adaptation gain factor $\mu$ was set to be 1 percent of a typical delay change per integration (1% of 2.5 Nyquist samples), multiplied by the maximum value allowed by (5.26); $\mu = \frac{\Delta T}{100 (L+1)\sigma_\tau}$. This value was chosen from the following consideration. There are $N$ samples per integration, so the residual power surface minimum was changing by at most $\Delta T/N$ samples (i.e. lag space channels or weight positions), per voltage sample. $\mu$ is set to some fraction, $\mu_0$, of its maximum, that is $\mu = \frac{\mu_0}{(L+1)\sigma_\tau}$. The weights can be written

$$\tilde{w}_{k+1} = \left(1 - 2\mu_0^2\right) \tilde{w}_k = \left(1 - \frac{2\mu_0}{L + 1}\right) \tilde{w}_k.$$

In the simulations $L = 32$, i.e. 16 negative and 16 positive delays (the maximum possible being $L_{\text{fft}}$). This gives

$$\tilde{w}_{k+1} = \left(1 - \frac{2\mu_0}{32 + 1}\right) \tilde{w}_k \approx \left(1 - \frac{\mu_0}{16}\right) \tilde{w}_k.$$

A choice of $\mu_0 = \Delta T/100$ meant that convergence held even for large values of $\Delta T$, while at every time step the weights could move faster than the maximum rate of the optimal weight change $\Delta T/128000$ ($L_{\text{fft}} = 128$ voltage samples were used in each FFT,
and there were 1000 FFTs averaged in the integration, so that the number of Nyquist samples per integration, $N = 128000$).

An example weight vector for the Gaussian noise RFI is given in Figure 6.2. Since the RFI covers the entire spectrum with equal power, a suitable vector would be single peak at an appropriate delay (the negative of the geometric delay between the main and reference antennae) with an amplitude equal to the value the reference signal needs to be multiplied by to match the RFI in the main signal. This is indeed the weight vector that the algorithm gave – except for a small zero-mean noise floor (this “misadjustment” noise, due to the fact that the weights are being adapted on a noisy residual power surface, is discussed at length in Widrow and Stearns, 1985).

![Weight Vector](image)

Figure 6.2: Example weight vector. The reference vector is delayed by each of the delays along the horizontal axis, multiplied by the corresponding amplitude, then all of the weighted copies are added together to give the model voltage estimate.

Figure 6.3 is a grey-scale map of the adaptive weight vector (e.g. Figure 6.2) displayed vertically as the integration progresses through the $N$ samples. The grey value of each pixel indicates the amplitude of the weight. As the delay changes, the main peak in the weight vector follows it. As the fringe rate increases, the adaptive filter can easily keep up (as long as the stability requirements are still met). The weight vector shown in Figure 6.2 is the last column of the $\Delta T = 10$ image in Figure 6.3. The weights for either the Wiener filter or the post-correlation filter can only be updated once per integration, and would be smeared out.

Figure 6.4 shows the residual power across the output spectrum for the three different fringe rates. These plots do not contain any cosmic or main receiver noise power. The dashed lines show the theoretical level of reference receiver noise added during filtering; zero-mean noise which will continue to average towards zero as the integration length, $\tau_A$, is increased. The adaptive filter results (indicated by the dots), follow this theoretical limit across the entire spectrum for each of the fringe rates, indicating that the filter is indeed tracking the changing delays. The solid line represents the Wiener filter residual power level (weights calculated using the output of the correlated
voltages). The cross-correlations used to generate the Wiener weights (c.f. equation 5.7) are not accurate, as indicated by the decorrelation of the spectra shown in Figure 6.1. The excess power above the dotted line is residual interference which will not average down any further. When the delays are constant, the Wiener and adaptive filter residual power levels are the same, consisting of zero-mean noise from the reference signal.

Figure 6.3: Weight vectors as a function of integration number as the integration progresses through the $N$ samples. Grey-scale indicates the amplitude of the weights.

Figure 6.4: Residual power after filtering a wide-band RFI signal (normalised to unity for no filtering). The lines indicate the theoretical amount of residual power remaining; solid lines for Wiener filtering and dashed lines for adaptive filtering. Points indicate the simulation results, crosses for Wiener filtering and dots for adaptive filtering.
So the weights which optimise the noise canceller can adaptively track the changing geometric delays. If the geometric delays are constant, the theoretical weight solution can be determined from the statistics of the main and reference signals (i.e. with a Wiener filter). From section 5.9 we know that the statistically determined solution can also be applied directly to the power spectra (the post-correlation canceller), but any hope of tracking changing geometric delays faster that the integration length is lost (as is the case with the Wiener filter). However in Figures 6.1 and 6.4 it is clear that the level of decorrelation in the cross-correlation can be estimated quite well (of course knowledge of the fringe rate is required). This estimate can be used to amplify the decorrelated power back to the correct level for cancelling the interference, although any noise will also be amplified (the ability of the adaptive algorithm to track requires the weights to have the freedom to bounce around which also adds noise).

6.3 Correcting for Fringe Rotation

Up to this point all of the Wiener filter and post-correlation equations (which are all based on signal statistics) have assumed that the geometric delay terms are constant during the time integration, so that various phase terms cancel in the filtering process. Specifically, the algorithms require that all of the following phase terms are constant over the time integration: $\phi_m$, $\phi_r$, $\phi_{mr}$, $\phi_{m1}$, $\phi_{mr2}$ and $\phi_{r1r2}$. However if the RFI location or the baseline vector change during the integration, some or all of these phases will change, resulting in decorrelated cross-power spectra. If the rate at which a phase term is changing is known (which it always is for terrestrial RFI sources with stable RF paths to the array, since physically the phases are fixed but a negative fringe rotation is applied at the correlator to fringe stop for the astronomy signal) – and the decorrelation is not too close to total – then equation (2.25) can be used to account for the decorrelation (although amplifying the power back to the ideal level also amplifies the noise in the cross-power spectrum, particularly at the top end of the band).

6.3.1 Wiener Filter Corrections

When operating directly on voltages one can use a purely adaptive approach, as discussed in the previous section, or a statistical approach using the Wiener solution, with weights given by (5.30) and (5.33) for MK1 and MK2 filters respectively. In the presence of fringe rotation the two Wiener solution weights become

$$W_{mk1}^r = \frac{F_{mr}G_mG_{mr}^*\sigma_I^2}{G_r\sigma_I^2 + \sigma_N}, \text{ and}$$

$$W_{mk2}^r = \frac{F_{mr}G_mG_{mr}^*\sigma_I^2}{F_{r1r2}G_{r2}G_{r2}^*\sigma_I^2}, \text{ (6.1)}$$

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respectively. If one can calculate the $F^{(r)}$ terms — and they aren’t too close to zero — then one can attempt to reverse the decorrelation. Using a hat, $\hat{\cdot}$, to denote a weight that has been corrected for fringe rotation decorrelation, the terms in (6.1) become

$$\hat{W}_{mk1} = \frac{1}{F^{(r)}_{m1} W'_{mk1}} = \frac{G_m G_i^* \sigma_i^2}{|G_r| \sigma_r^2 + \sigma_N^2}, \text{ and}$$

$$\hat{W}_{mk2} = \frac{F^{(r)}_{m2} W'_{mk2}}{F^{(r)}_{m2}} = \frac{G_m}{G_r^*},$$

(6.2)

respectively, correctly resetting the weights. Since the post-correlation algorithms are actually applied in the frequency domain, the focus of the rest of the discussion will be on reversing the effect of decorrelation on the post-correlation correction spectra which were given in section 5.9.

### 6.3.2 Post-Correlation Corrections

The most general post-correlation algorithm to consider is the dual-reference algorithm (section 5.9.2), since it is possible for all of the cross-correlated components involved to experience fringe rotation decorrelation (other systems are limiting cases of this one). Recall that the spectrum which models the RFI in the main astronomy correlations is called the correction spectrum. If a prime, $'$, is again used to denote a decorrelated quantity, then each channel of the decorrelated correction spectrum for the cross-correlation of signals $V_i$ and $V_m$ given in (5.52) can be written in the more general form

$$C'_{pk2} = \frac{P_{r1r2}^r P_{mr2}^s}{P_{r1r2}^s} = \frac{\langle G_i G_i^* \sigma_i^2 e^{j\phi_{r1}} \rangle \langle G_m G_r^* \sigma_r^2 e^{j\phi_{mr2}} \rangle^*}{\langle G_r G_r^* \sigma_r^2 e^{j\phi_{r12}} \rangle^*}. \quad (6.3)$$

Recall that $C'_{pk2}$ is a complex scalar quantity which is subtracted from the equivalent spectral channel, $P_{lm}$, of the main cross-power spectrum (keep in mind that each spectral channel is processed independently). For the time being, assume that antennae gain towards the RFI is approximately constant over the integration (even if the assumption is not so good, the idea of this section is to focus on fringe rotation effects). Equation (6.3) becomes

$$C'_{pk2} = G_i G_m^* \sigma_i \langle e^{2\pi i (\tau_1 - \tau_{r1})} \rangle \langle e^{-2\pi i (\tau_m - \tau_{r2})} \rangle \langle e^{2\pi i (\tau_1 - \tau_{r2})} \rangle \langle e^{-2\pi i (\tau_m - \tau_{r1})} \rangle. \quad (6.4)$$

The RFI component of the correlation of interest (the main antennae cross-correlation), which will be denoted $P_{lm,r1}$, may also have suffered decorrelation, and is written
\[ P'_{im, ej} = G_i G_m^{*} \sigma_I^2 e^{2 \pi j (\eta - \eta_m)}. \] (6.5)

As was shown in section 5.9.2, (6.4) and (6.5) are equivalent when the delays are all constant. However, each of the quantities in angle brackets effectively become sinc functions if the delays vary approximately linearly, as given by (2.25):  

\[ \langle e^{j \phi_{jk}} \rangle = F_{jk}'(\tau) = \text{sinc}(\nu_0 \Delta \tau_{jk}), \]

for baseline \( jk \) at the central frequency, \( \nu_0 \), of the (quasi-monochromatic) spectral channel. Again, \( \Delta \tau_{jk} \) is the total amount that \( (\tau_j - \tau_k) \) changed during the integration. Using (2.25) in (6.4) and (6.5), one obtains an expression for the residual power:

\[ C'_{pc2} = \frac{F_{im}^{(\tau)} F_{mr}^{(\tau)}}{F_{r1r2}^{(\tau)}} G_i G_m^{*} \sigma_I^2, \]

\[ P'_{im, ej} = F_{im}^{(\tau)} G_i G_m^{*} \sigma_I^2 \]

\[ \Rightarrow R'_{pc2} = \left( F_{im}^{(\tau)} - \frac{F_{ir1}^{(\tau)} F_{mr2}^{(\tau)}}{F_{r1r2}^{(\tau)}} \right) G_i G_m^{*} \sigma_I^2. \]

From (6.6), it can be seen that one can modify (6.4) so that it takes the decorrelation into account

\[ \hat{C}_{pc2} = \frac{F_{ir1}^{(\tau)} F_{mr2}^{(\tau)}}{F_{r1r2}^{(\tau)}} C'_{pc2}, \] (6.7)

where again, \( \hat{\cdot} \), indicates that fringe rotation has been corrected for. If most of the RFI power has been decorrelated away, there is little that can be done to reverse the decorrelation (if most of the RFI power has also been decorrelated out of \( P_{im} \), then it might be better to do nothing).

To summarise, if one is attempting to cancel an interfering signal which is moving appreciably through interferometric fringes, then the resulting decorrelation needs to be taken into account during the cancellation (unless the interference has completely decorrelated out of the main astronomy correlation, in which case one need not continue with the cancellation). This is dependent on both having enough of the signal left to get an adequate measure of the remaining power, and on being able to estimate the fringe rate. This should become clearer after a few examples. The first example uses the simulation from earlier in this chapter, and the second uses genuine satellite RFI incident on the ATCA.

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6.3.3 Simulated Filter Comparisons

The simulation is identical to the one described in section 6.2, with a fringe rate of 1.5 Nyquist samples per integration, and post-correlation fringe rotation corrections. Consider the simplified situation in which we would like to remove RFI from a contaminated main signal power spectrum. Since this is an auto-correlation \( (l = m) \), the baseline length is zero giving \( F_{lm}^{(r)} = F_{mm}^{(r)} = 1 \) (i.e. no decorrelation). Furthermore, let’s assume that the reference antennae are collocated; specifically, two orthogonally polarised receivers of a single antenna. So long as the RFI is 100% polarised (and not completely orthogonal to the polarisation of one of the receivers), then \( F_{rr}^{(r)} = 1 \), \( F_{rr}^{(r)} = F_{mr}^{(r)} = F_{mr}^{(r)} \), and the various equations in 6.6 and 6.7 reduce to

\[
C_{pc2} = F_{mr}^{(r)} G_m G_m^* \sigma_I^2,
\]

\[
R_{pc2} = \left( 1 - F_{mr}^{(r)} \right) G_m G_m^* \sigma_I^2 \text{, and}
\]

\[
\widehat{C}_{pc2} = \frac{1}{F_{mr}^{(r)}} C_{pc2} = P_{mm, r},
\]

where the subscript \( r \) by itself indicates that the RFI signal arrives at both reference antennae at the same time and that \( F_{rr}^{(r)} = 1 \).

![Figure 6.5](image)

**Figure 6.5:** a. Cross-correlated RFI power, \( P_{mr}^{(r)} \), on the baseline between the main and reference antennae which has been decorrelated due to fringe rotation; b. The cross-correlated power, \( \tilde{P}_{mr} \), corrected using an estimate of the decorrelation, \( F_{mr}^{(r)} \).

Figure 6.5 shows RFI in the main-reference cross-correlation, and the effect of correcting for the decorrelation. The decorrelated spectrum, \( P_{mr}^{(r)} \), in Figure 6.5a is divided by \( F_{mr}^{(r)} \) (the dashed line), to give \( \tilde{P}_{mr} \), shown in Figure 6.5b, where the dashed line in
6.5b is the theoretically expected result (c.f. equation 6.3). The power level in each frequency channel has been correctly reset to be centred on the expected (decorrelation free) value.

The difference between cancelling with frequency channels from spectrum 6.5a and 6.5b is shown in Figure 6.6a (Figures 6.6b and 6.6c are zoomed and logarithmic versions of 6.6a respectively). The vertical axis has been scaled so that the input power, $P_{nn} = 1$. As the spectral channels move to higher frequencies and the amount of decorrelation in Figure 6.5a increases, the residual power after post-correlation cancelling using the correction spectrum, $\tilde{C}_{pc2}$, which has not accounted for fringe rotation, contains greater amounts of residual RFI (as indicated by the black curve which matches well to the theoretical dashed red line), and this power level will not average down as the integration is continued. The residual power after post-correlation cancelling using the correction spectrum, $\tilde{C}_{pc2}$, which has accounted for fringe rotation (Figure 6.5b) continues to average down towards the theoretical limit as the frequency channel and decorrelation increase, and is indicated by the magenta curve. The dark blue curve is the residual power after filtering the voltages with adaptive weights as described in section 6.2, and the dashed light blue line shows the theoretical noise injected by a dual-reference filtering process under stationary conditions (i.e. no fringe rotation), which is the best that one can do with these filters. The residual power in the adaptive and corrected post-correlation filtered spectra does not contain RFI, and will average down with the dashed light blue line, albeit with a larger noise floor where more fringe rotation correction is needed (i.e. the higher frequency channels in Figure 6.5).

![Figure 6.6](image.png)

Figure 6.6: Residual power levels for $P_{nn}$ in the presence of fringe rotation. All three plots show the same information with different vertical scales (b and c are zoomed and logarithmic versions of a respectively). The two dashed lines show the theoretical levels of the post-correlation residual power with and without fringe rotation.

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So while the post-correlation and Wiener filter cancellers can not be made to track an interfering signal’s geometric delay at a rate faster than the inverse of the integration length, the resulting decorrelation of the cross-correlations in the filter equations can often be accounted for. Before concluding the section, an example using real observed data will be given.

6.3.4 A Real-World Narrow-Band Example

Two ATCA antennae on a 4.4 km baseline were used to collect Global Positioning System (GPS) satellite interference (the X polarisation of antenna CA03 was used for main antenna \( m \), and the X and Y polarisations from antenna CA06 were used for references \( r_1 \) and \( r_2 \) respectively). The data used were from one of the SRTCA data sets summarised in section 3.2.1, and were from a 4 MHz wide frequency band centred at 1575 MHz. Figure 6.7a is a plot of \( F_{mr}^{(r)} \) for various integration times. The measured values of \( F_{mr}^{(r)} \) were calculated by dividing the measured cross-correlated power, \( P_{mr} \), by an estimate of \( P_{mr} \) (estimated by averaging together the amplitudes of many short integrations, where fringe rotation is minimised) for various integration lengths. The power spectra were initially smoothed with a 19 point running mean filter. The fringe rate used for the theoretical curve in figure 6.7a was \( 5.03 \times 10^{-4} \) fringes per second per metre of (East-West) baseline. (This value was a few percent different from an estimate made using the satellite trajectories\(^2\). This discrepancy needs further investigation.) The measured power follows the theoretical sinc function \( F_{mr}^{(r)} \) well. The proportion of interference power remaining in \( P_{mm} \) after cancellation is \( (1 - F_{mr}^{(r)^2}) \), as shown in figure 6.7b. As mentioned, the two orthogonal linear polarisations of antenna CA06 were used as references \( r_1 \) and \( r_2 \). It is clear that fringe rotation decorrelation has a large effect on interference excision as the integration time is increased. The observed effect of the decorrelation matches well to the theory.

Figure 6.7c is a plot showing the proportion of interfering power remaining after post-correlation cancellation using the corrected spectrum, \( \hat{C}_{mm} \), as \( \tau_A \) is increased. Even when the reference cross-correlation, \( P_{mr} \), has been decorrelated down to 25%, the amount of residual power is still close to zero. As the amount of decorrelation approaches 100%, the amount of residual power approaches infinity. This is because there is still the same amount of RFI power in \( P_{mm} \), but the RFI power in \( P_{mr} \) is zero. An intelligent algorithm would be needed to turn this process off if it were to be implemented successfully (possibly by inspecting the closure relations), so that no interference cancelling would be applied.

\(^2\)The estimated fringe rate was calculated in MATLAB using the N3EMO “ORBIT” C program – available from ftp.uu.net/usenet/comp.sources.misc/volume11/n3emo-orbit – which uses regularly updated satellite orbital information which can be downloaded from Celestrak, www.celestrak.com/NORAD/elements.
a. Proportion of correlated power, $F_{fr}$, remaining in $P(M,R)$

$b. Proportion of residual power, $1-F_{fr}^2$, remaining in $P(M,M)$

$c. Residual power in $P(M,M)$ when decorrelation has been accounted for.

Integration length (seconds)

Figure 6.7: a. Proportion, $F_{mr}^{(c)}$, of cross-correlated power remaining for a 4.4 km baseline as a function of integration time $\tau_A$. Black dots show the measured power of GPS interference and the line shows the theoretical estimate based on fringe rotation decorrelation. b. The proportion of residual RFI remaining in the output power spectrum after filtering as the integration length is increased. c. Proportion of residual RFI remaining in the output power spectrum when the scaled weights, $\tilde{C}_{mm}$, were used.

Figure 6.8a shows the residual spectra at three integration lengths when the decorrelated weights, $C'_{mm}$, were used, and figure 6.8b shows the same spectra when the scaled weights, $\tilde{C}_{mm}$ were used. The improvement achieved by scaling the weights is clear, however the noise in parts of the spectrum that are free from RFI has also been increased. While the decorrelation effect is obviously frequency dependent in Figure 6.5 (as in section 6.1), there are no obvious signs of frequency dependence in Figure 6.8. In the simulations the fringe rotation was applied at baseband, so the channel frequencies range from 0 to $\Delta \nu$; which obviously experience widely different fringe rates. On the other hand, the genuine observation of GPS data was for a 4 MHz wide band centred at 1575 MHz (a difference in frequency across the band of only 0.25%).

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Figure 6.8: Residual power spectra after three integrations of length 0.008, 0.311, and 1.229
seconds, for a. the standard post-correlation filter, $C'_f$, and b. the scaled post-correlation
filter $\tilde{C}'_f$.

6.4 Fringe Rotation Summary

So, in the presence of a noticeable level of fringe rotation, there is a mechanism for
putting the post-correlation (and Wiener) filters back on par with fully adaptive filters.
One should keep in mind the particular case of terrestrial interference (i.e. interference
coming from a fixed source which is always stationary relative to the array). In this
situation the fringe rate is the reverse of the sidereal fringe rate (the interferometer is
fringe tracking the celestial sphere). Since the correlator is inserting this phase term,
one always knows the fringe rate and can calculate $F^{(\tau)}$ exactly. The required level of
suppression and added complexity, however, may dictate that shorter integration times
are necessary to eliminate the need for fringe rotation decorrelation corrections. There
are many phenomena that can affect the interference mitigation algorithms, of which
fringe rotation decorrelation is an example (albeit an extremely important one). Since
all of the effects cannot be covered here in as much detail as fringe rotation, the next
chapter briefly describes some of the more important issues that are being addressed
or that will need to be addressed in the near future.