Equilibrium Price Dispersion: A Model of Intermediated Search with Repeated Interaction

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Abstract

This thesis develops a model in which homogeneous producers and merchants interact repeatedly in a search market. Merchants are able to reduce the cost of search by offering trading certainty to producers with whom they have a preexisting relationship. Equilibria are characterised in Markov strategies, and it is found that price-dispersed equilibria exist in asymmetric strategies. Conditions in which a price-dispersed equilibrium can be welfare improving compared to a single-price equilibrium are found, and two extensions to the basic model are provided.

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1 Introduction

In a model of a market in which exchange is costless, the canonical 'Law of One Price' must hold: identical goods must sell at the same price. Diamond (1971) famously showed that the law continues to hold even if buyers can only learn about prices through costly sequential search, provided that buyers have homogeneous valuations and sellers have homogeneous costs. Indeed, in such a model, an equilibrium must be characterised by all sellers charging the monopoly price. Yet, price dispersion—the sale of the same good for different prices in a market—is widely observed empirically (Baye, Morgan, and Scholten, 2006). This thesis seeks to partially explain these violations of the 'Law of One Price', incorporating previous work in the field of price dispersion and using the modelling techniques found in the intermediated search literature. The central finding of this thesis is that equilibrium price dispersion can be supported in a multi-period model with homogeneous agents. The possibility of repeated sales over time provides merchants with a novel incentive to charge a lower price, that is, to induce customers to return in subsequent periods. A price-dispersed equilibrium can exist when the profit from charging a price low enough to induce repeated patronage is the same as that from charging a higher price, which deters customers from returning.

Specifically, this thesis builds on a model presented in Bose and Sengupta (2007) of intermediated search and exchange in an environment of repeated interaction. This approach explicitly considers the role of specialist merchants in reducing search frictions, an aspect of trade that has often been neglected. However, the focus of Bose and Sengupta is on the endogenous emergence of merchants; the equilibria they consider all have a uniform market price. The objective here is to take as given a market in which specialist merchants exist, and find equilibria in which prices are dispersed.

Following Diamond (1982) and Bose and Sengupta (2007), one type of agent plays the role of both producer and consumer, and this agent must seek exchange in the search market. The other type of agent is a specialist merchant who does not produce, but rather acts solely in the search market as an intermediary. Each period a producer enters the search market and finds either another producer, a merchant or no-one. The advantage to a specialist merchant in this setting of repeated interaction is that her current clients can find her with certainty in the next period. This means that the client may be willing to continue returning to a merchant who charges a commission, in exchange for avoiding the possibility of not finding a trading partner in the following period and forgoing a consumption opportunity.

The producer's decision to return to a merchant or search anew in the following period forms the crux of this model. Given a client's price threshold for returning, the merchant can charge this price to induce return, or charge a higher price and forgo any future profit from that client. The central finding is that it is possible for both of these merchant strategies to coexist in equilibrium, resulting in price dispersion.

The rest of this thesis is organised as follows. Section 2 looks at the existing literature on price dispersion and intermediation models, and then frames the present thesis as drawing on the intermediation literature to investigate price dispersion. Section 3 sets out the model and Sections 4 and 5 describe optimal behaviour. Equilibria are characterised in Section 6. Section 7 provides an analysis of comparative statics and

Section 8 provides two extensions to the basic model. The modelling assumptions used are discussed in Section 9, before some conclusions are drawn in Section 10.

2 Literature Review

Classical microeconomic theory rests on the assumption that all possible gains from trade can be exhausted at zero cost; an assumption which makes 'The Law of One Price' a logical necessity. If all agents are implicitly aware of all prices in the market and trade is costless, then any equilibrium must necessarily have only one price for each good. Any other scenario would see all demand flowing to the lower priced good. This thesis builds on the literature that explicitly models the search process that must exist for trades in real markets to occur, which contrasts with the theoretical ideal of a disembodied Walrasian auctioneer.

The price dispersion literature has developed models that employ a variety of search frictions. An instructive lens through which to view this literature is provided by the Diamond paradox (1971). This arose from a model of costly sequential consumer search among vendors in a market. The inescapable conclusion is that for any arbitrarily small positive search cost, all vendors charge the monopoly price in equilibrium. Driving this result is the observation that for any price charged, a vendor can raise her price by an amount less than the search cost without losing clients. This process ratchets the market price up until the monopoly price is reached, at which point there is no longer an incentive to raise prices.[1] The remainder of this section reviews the price dispersion literature

[1] This result holds for markets modelled as having a continuum of agents, as employed in this thesis. For discrete models, there can be an incentive to undercut prices and equilibria may not exist.

that has developed since Diamond (1971), and outlines the modelling techniques used in the intermediated search literature. Then it describes the model used in this thesis as drawing on the intermediation literature to investigate price dispersion.

2.1 Price Dispersion

The ubiquity of price dispersion in actual markets for homogeneous goods led Stigler (1961) to carry out the first theoretical study of the phenomenon. In his seminal article, he looked at the problem from the consumer's perspective and characterised optimal consumer search behaviour under a variety of price distributions. However, he did not model firm behaviour. This approach led Rothschild (1973) to the criticism that such models were 'partial equilibrium' models that took price dispersion as given, so did not really explain the phenomenon. Most subsequent models have incorporated optimal behaviour on both sides of the market.

Two models of search are employed in the literature: sequential search and nonsequential search. Under sequential search, consumers visit one outlet at a time and must incur a cost for each additional search. Nonsequential search requires consumers to select the number of outlets to be searched before commencing search. The choice of modelling technique can influence theoretical outcomes. On the practical side, both types of search are plausible. A consumer searching for, say, a car is likely to search several vendors in turn, be it physically or in some other way. However, examples of parallel search are also available, such as the task of finding a contractor to renovate a house. Here, the consumer may need to ask for several quotes that take time to procure. With sufficient time preference, the consumer will decide how many quotes to request at the outset. Some models also consider the dissemination of information via non-search channels such as advertising, an aspect of search not considered in this thesis.

Price dispersion modelling has taken two broad approaches. Most models have incorporated some sort of ex ante heterogeneity into the consumer or producer populations, which can result in price dispersion. Some models have used ex ante homogeneous populations and still had features that generated price dispersion. Multi-period models of price dispersion are most closely related to the model presented in this thesis, but they are relatively rare in the price dispersion literature.

2.1.1 Ex Ante Heterogeneous Models

In response to Rothschild's criticisms of early models of price dispersion, heterogeneity was introduced into models to generate price dispersion. Reinganum (1979) showed that in a simple market model with elastic demand, homogeneous consumers and costly search, equilibrium price dispersion can be supported if producers have a continuum of marginal costs. The key assumption that avoids the Diamond paradox is producer cost heterogeneity.

Under these assumptions, a continuous distribution of prices is observed in equilibrium. The prices for each firm are given by the standard monopoly profit maximising condition, so a firm's market price increases with its marginal cost. With costly sequential search, optimal consumer behaviour is to continue searching until encountering a price below some threshold, which is determined by the distribution of prices in the market. The price distribution is thus truncated at the consumer search threshold. This result parallels Diamond's paradox in that costly search allows the exercise of monopoly power, but only up to a limit determined in the market.

Clearly if marginal costs are homogeneous across firms, Diamond's paradox returns and price dispersion disappears. Also, the assumption of elastic demand is crucial. Many consumer search models assume unitary demand (Baye, Morgan, and Scholten, 2006). This assumption would collapse the price distribution in this model to the threshold price because lowering the price does not increase sales, as would occur under elastic demand. An interesting corollary of this result is that the search process allows higher cost firms to continue making profits in the market.

MacMinn (1980) found a similar result in a non-sequential search environment. Again, search results in the firms facing a downward sloping expected demand curve. This means that optimal pricing will depend on firms' marginal costs, so heterogeneous costs yield equilibrium price dispersion.

Heterogeneity has also been incorporated in the consumer side of the market. A common approach (for example Varian, 1980) is to partition the consumers into two types. One type has access to perfect information, so can always find the lowest price in the market. The other type is uninformed, so buys from anyone charging a price below their threshold. In this model, the existence of two market segments creates a trade-off for producers. A low price can capture informed consumers, while a high price can extract profits from the uninformed consumers. Note that neither market segment could support price dispersion in isolation. If all consumers were informed, firms would compete solely on price, so Bertrand competition and marginal cost pricing would obtain. Alternately, if no consumers were informed then all firms would charge their (uniform) maximum willingness to pay.

If both types of consumer exist, neither of those equilibria can be supported. Bertrand pricing is not optimal because an increase in prices will yield positive profits from the uninformed consumer base. Uniform monopoly pricing cannot be optimal in equilibrium because a slight price reduction will capture the entire informed market which more than compensates for the loss in revenue from the uninformed segment. A symmetric pricing equilibrium cannot be supported in this model because there is always an incentive to undercut: to accrue the informed market. This results in price dispersion. Consumers can be partitioned in several ways. One alternative is to include a subset of consumers who can search at zero cost. Consumers that can search for free always inspect the whole market, while consumers for whom search is costly search less (Salop and Stiglitz, 1977). Another alternative is to include a central 'information clearinghouse' that contains all (or some) market information, and to which only some consumers have access (Baye and Morgan, 2001).

Diamond's paradox can thus be circumvented by imposing heterogeneous production costs or some form of heterogeneity among consumers. However, this is not fully satisfactory for two reasons. First, it is only applicable to markets that exhibit such heterogeneity so may not always explain price dispersion. Second, it would intuitively seem that the search process itself may be sufficient to generate price dispersion. A strand of the literature has pursued this avenue using models of homogeneous markets.

2.1.2 Ex Ante Homogeneous Models

The models discussed above rely on some kind of ex ante heterogeneity on one side of the market to generate price dispersion. Some other models can support equilibrium price dispersion with ex ante homogeneity. These models usually rely on the search process to generate some form of ex post heterogeneity in the market.

Burdett and Judd (1983) model processes of non-sequential search and of 'noisy sequential search' in which more than one price is observed in a single search with a positive probability. The example given is where a consumer buys a newspaper to obtain price information, but there may be either one or two advertisements in the paper. This search outcome is crucial to the model's result because an equilibrium with price dispersion requires that some consumers observe one price and some observe two (or more) prices. This essentially segments the (ex post) market. One segment has only observed one price so will purchase if the price is below their maximum willingness to pay. The other has observed several prices so will take the cheapest alternative.

This ex post result parallels the ex ante consumer segmentation in the models outlined above. Either segment of the market in isolation could not support price dispersion, but a mixture of the two can do so. The former segment of consumers in isolation results in the Diamond paradox, while the latter segment would bid prices down in Bertrand-style competition. A difference of this model is that the level of consumer search is endogenously determined. Search behaviour is optimal so in the non-sequential search version of the model, consumers must be indifferent between searching one or two options for price dispersion to obtain. The price distribution must be specified such that the expected reduction in price from searching an additional producer is exactly offset by the extra search cost. This model has multiple equilibria. For example, the Diamond result of uniform monopoly pricing sustains an equilibrium. A unique market price means that a single search is optimal so there is no incentive for a producer to undercut the competition.

Burdett, Shi, and Wright (2001) and Arnold (2000) assume producer capacity constraints and find that this can cause equilibrium price dispersion in models with ex ante homogeneity. In their models, consumers have full information over the prices and locations of firms but not over the intentions of other consumers. They cannot, therefore, coordinate their activities to ensure that no firm is capacity constrained, even if there is no binding capacity constraint on the market in sum. Consumers visit producers stochastically, which causes the crucial feature of these models: ex post heterogeneous levels of demand.

Burdett, Shi, and Wright (2001) impose symmetric price setting among producers, but allow prices to be renegotiated upon the realisation of demand. This renegotiation drives the price dispersion outcome. They also show that asymmetric capacity constraints can heighten price dispersion. Arnold (2000) does not impose symmetry on price setting and finds that price dispersion equilibria exist in pure strategies (for firms) for certain parameter values. This results from the dual effects of a price reduction. One effect is to attract more customers, but this then increases the probability of a stock outage. The second effect attenuates the first because consumers factor in the probability of stock outages when assigning probabilities to visiting different producers. The possibility of running out of stock dampens a firm's incentive to cut their price because it means they could have posted a higher price and achieved the same number of sales.

2.1.3 Multi-Period Models

The models discussed above all operate only in a single period. In many real markets, trade occurs repeatedly over time. Incorporating multiple periods may generate other mechanisms for price dispersion that depend on the possibility of repeated interaction or other features that arise only in multi-period models. Such models have been rare in the price dispersion literature.

Salop and Stiglitz (1982) incorporate a two-period structure into a model of homogeneous producers and consumers. Consumers live for two periods and want to consume in both. Search in each period is costly and consumers can purchase enough in the first period to consume in both periods, or just for the current period. A new generation is born each period and producers can not price discriminate between generations. For first period transactions, a consumer has a maximum price per unit (say, p_1^*) that she is willing to pay for one period's consumption, and a maximum price per unit (say, p_2^*) that she is willing to pay to buy enough to last for both periods. Both the distribution of prices in the market and the cost of storage affect these threshold prices.

In this model, consumers can only search once per period so producers only ever charge one of the two threshold prices mentioned above. Also note that if $p_2^* \geq p_1^*$ then all producers would charge p_2^* because it gives higher profits. In the case for which $p_1^* > p_2^*$, the level of p_2^* depends on the fraction of producers that are expected to charge p_1^* in the following period. For example, if a consumer expects all producers to be charging p_1^* in the next period, p_2^* will be higher than if the consumer expects only a fraction of producers to be charging p_1^*. This is because more producers charging p_1^* means a higher expected price for the consumer in period two. The positive effect of the number of producers charging p_1^* on the level of p_2^* means that there is a unique equilibrium fraction of producers charging p_1^*. To see this, consider the situation in which the profit from charging p_1^* is greater than the profit from charging p_2^*. There is now an incentive to switch to charging p_1^*, which will raise p_2^* and increase the profits from charging it. For a range of parameter values, the model converges to a unique equilibrium with price dispersion given by the two different prices.

The model presented in this thesis has some common features with Salop and Stiglitz (1982). In particular, it shares the multi-period structure as a mechanism for generating price dispersion. The pricing structure is also similar in both models—there are two possible optimal prices for sellers. In the present model, one (lower) price induces buyers to return, while the other (higher) price does not. Again, the fraction of sellers charging each price affects the threshold price of searchers. However, the present model does not employ a two-period structure or a constant inflow of new consumers. Rather, it generalises to an infinite-period setting and allows repeated interaction with a constant pool of buyers. Also the model in this thesis employs repeated interaction, rather than storage, to investigate price dispersion.

Bagwell and Ramey (1992) investigate the Diamond (1971) paradox in a dynamic setting. Their model shares the possibility of repeated interaction with the present thesis, but the focus is on a different aspect of the Diamond paradox. In Diamond's model, the unique equilibrium sustains the monopoly price for any positive search cost but jumps discontinuously to the competitive price if the search cost is zero. Bagwell and Ramsey show that in the class of consumer-optimal equilibria, the (common) market price decreases monotonically with search costs between the monopoly and competitive prices. However, price dispersion is not considered in their paper.

2.2 Intermediation in Search Markets

The models of price dispersion considered above include only consumers that search and producers that sell a product. In many search markets, intermediaries facilitate the search of market participants. This strand of the literature was initiated by Rubinstein and Wolinsky (1987). They modelled a search market made up of sellers, buyers and middlemen. The existence of an equilibrium with active middlemen (that is, where middlemen buy from sellers and sell to buyers) depends critically on the assumption that middlemen are relatively efficient at finding matches in the market. This increased efficiency allows them to charge a positive margin, in exchange for giving buyers and sellers an easier way of finding trading opportunities. This thesis does not explicitly draw on their model, aside from the inclusion of intermediaries in a search model.

The models considered above all examine two-sided markets, that is, markets with consumers and producers. An alternative method of modelling search markets is to recognise that economic agents both produce and consume, and, outside of a subsistence economy, must trade their produce for what is eventually consumed. Diamond (1982) constructs a simple model of a barter economy where agents produce a good, but are required to trade it prior to consumption. This modelling technique is designed to encapsulate the idea that most agents trade the fruits of their labour in order to consume, while avoiding the complexity of a multi-good model. Diamond's model does not involve intermediaries, but it introduced the coordination of trade problem in an environment of homogeneous agents that both produce and consume.

Some form of trading friction is required for intermediaries to operate profitably. If trade were costless, as in the case of a Walrasian auctioneer, intermediaries would be redundant. In Rubinstein and Wolinsky's (1987) model trading intermediaries extract a commission in exchange for expediting trade. The barter economy in Diamond's (1987) model does not allow a role for intermediaries but the notion of a coordination of trade problem in a one-sided search market has since become important in the intermediation literature.

The recent literature on intermediated search has focused on the role and utility of intermediaries in the exchange process. This aspect of intermediation is not the focus of this thesis. However, because the modelling technique is drawn primarily from this strand of literature, some similar models will be considered below.

Masters (2007) takes a Diamond-type model, and allows agents to choose to be specialist intermediaries rather than produce goods. Intermediaries do not have any explicit advantage in search, but by forgoing production they spend more time in the market so have an increased overall probability of finding trading partners. Agents have a continuum of production costs, and in equilibrium, agents with production costs above some threshold level choose to be intermediaries. This occurs because the opportunity cost of forgoing production is relatively small.

Shevchenko (2004) similarly allows agents to choose between production and consumption. Multiple goods are included in the model, such that the advantage to an intermediary is the ability to stock a range of goods to increase the probability of suiting the tastes of a randomly chosen agent. The focus of the paper is to examine the intermediation decision and their welfare implications. However, prices are determined by a bargaining process that depends on the relative abundance of different goods within each store, which in turn depends on the stochastic trading history of each intermediary. The model thus exhibits a form of price dispersion that is driven by inventory differences caused by stochastic trading histories.

The model in this thesis draws much of its structure from Bose and Sengupta (2007). The setting allows for repeated interaction between intermediaries and their clients over time. That is, if a producer trades with an intermediary in one period, she has the option of returning to the same intermediary in the following period, bypassing the search market in the process. Intermediaries and producers are treated symmetrically in the search market. However, the intermediary's ability to allow producers to avoid the costs of search is sufficient to make it profitable for some measure of agents to specialise as intermediaries under a range of parameter values.

The focus of Bose and Sengupta (2007) was on the endogeneity of the decision to become an intermediary. In doing this, only symmetric pricing strategies are considered. Two distinct classes of equilibria are characterised. One consists of intermediaries charging a high price and not inducing their clients to return in the following period, while the other has intermediaries charging a lower price that does induce client return.

2.3 The Present Synthesis

The price dispersion and intermediation literatures have rarely come together. Spulber (1996) partially marries the two strands by incorporating intermediaries into a model similar to that presented in MacMinn (1980). The focus of the paper is on how firms acting as intermediaries can create a bid-ask spread in contrast to a Walrasian equilibrium. However, it is also found that non-degenerate price distributions can exist in equilibrium with heterogeneous agents. The price dispersion in this model is driven by the heterogeneous agents as in MacMinn (1980), so it should be considered as more of a contribution to the intermediation literature rather than the price dispersion literature.

This thesis draws upon several aspects of the literature outlined above. The population of agents who produce, search and exchange mirrors Diamond's (1982) model. The exogenous population of merchants is imposed as in Rubinstein and Wolinsky (1987). Endogenous intermediation is not considered because it adds an extra layer of complexity to the model without improving the exposition of price dispersed equilibria. The search and intermediation process is modelled as in Bose and Sengupta (2007)—the introduction of asymmetric strategies allows the investigation of price dispersion. In bringing these disparate areas of the literature together this thesis explores a novel avenue to examine price dispersion in a theoretical model.

3 The Model

3.1 Overview

This thesis employs a stylised model of production, search, intermediation and exchange in a setting of repeated interaction to investigate price dispersion. The model draws heavily on Bose and Sengupta (2007), and owes its origins to Diamond (1982). The modelled economy operates over an infinite horizon with discrete time periods. There are two types of agents, both of whom are risk-neutral and infinitely-lived. The first type has the role of producer. Each period, producers make one unit of a homogeneous consumption good, the production cost of which is normalised to zero. The second type of agent is the merchant, who exists only in the search market. They cannot produce but can exchange goods with producers. Each class of agent is homogeneous within itself, and both types of agents can consume the single good in the model.

While there is only one type of good in the economy, an embargo is placed on the consumption of one's own production, following Diamond (1982). In any specialised economy, exchange is necessary in order for production and consumption to take place. Introducing multiple goods to make the model realistically represent this fact would lead to significant complexity. This may obscure the role of the search process, which is the focus of this thesis. Diamond's prohibition thus introduces the necessity of exchange into the model, while avoiding the complexity brought about by a range of goods. This results in a producer needing to enter the search market each period in order to exchange her produced good prior to consumption. An alternative approach to this problem is to introduce a role for money (for example Hellwig, 2002). This approach is not employed here in order to maintain the focus on search and exchange, rather than monetary phenomena.

A producer can search only once per period, resulting in one of three outcomes. First, she can meet another producer. If this occurs, the symmetric bargaining situation results in the goods being exchanged one-for-one so each producer consumes one unit. Second, she can find a merchant. This interaction is not symmetric in bargaining because the merchant can consume the good she holds but the producer cannot, which tips the bargaining power towards the merchant. When a producer finds a merchant, the merchant has a temporary local monopoly over the producer because the producer cannot consume her good and cannot search again in the market. The resultant bargaining situation is modelled with the merchant as a price-setter, which means that a merchant can extract the entire surplus from trade. The third possible outcome from search is not finding either type of agent. If this happens, exchange cannot occur and the producer's good is not consumed. Inventory cannot be carried in this model. This can be thought of either as goods expiring at the end of each period, or alternately as production being impossible if a unit of the good is already held.

The search process is stochastic, with a probability assigned to each of the three possible outcomes. The cost of search in this model is embodied by the probability of an unfavourable search outcome and the resulting missed consumption opportunity. However, this search cost can be circumvented by a producer that has a preexisting relationship with a merchant. Each period following trade with a merchant, a producer may choose to return with certainty to the same merchant she traded with in the previous period, provided she remembers the merchant's location. However, there is a certain probability that a producer will forget this information before the beginning of the next period. A producer that begins a period with knowledge of a merchant's location is called an *informed* producer. Note that a producer who begins a period as informed does not forget her merchant during that period, even if she searches afresh. This means that the probability of an informed producer remaining informed in the folowing period is independent of her current-period action. Producers who are not informed in a given period must enter the search market. This feature of merchant trading certainty creates the potential for a continued and mutually beneficial trading relationship between merchant and producer. The inclusion of a probability of forgetting a merchant's location is designed to encompass a variety of frictions that can impede a long-term trading relationship, such as an agent leaving the market area.

The ability to guarantee trade in the following period is imagined to be exclusively held by merchants. This ability could flow from merchants being able to dictate their location in the market with certainty. Also, if search is considered to have both spatial and temporal dimensions, this ability could come from the permanent residency of merchants within the search market, compared to producers transiently entering the market to trade when not engaged in production.

Agents in the economy act to maximise, at each date, the present value of their expected discounted payoff in terms of the consumption good. For producers, the choice variable amounts to the decision of whether or not to return to their merchant if they are informed at the start of a period. A merchant must choose the price to post in the market each period. The question of interest here is whether multiple prices can be supported in equilibrium in this model economy.

3.2 Producers and Merchants

The populations of agents are modelled as continua. The measure of merchants is normalised to 1, and the measure of producers is denoted as σ. Continuous populations mean that the probability of meeting any given agent in the search market is zero.

The model is treated as a dynamic stochastic game. For producers, each period unfolds as follows. Initially, the producer observes her information state, that is, whether she knows of a merchant. If she does, she also remembers the price that she observed the merchant charge. If informed, her decision is whether to return to the merchant she knows (choice R), or search afresh (choice S). Informed producers then either return to their merchant or search in the market along with all uninformed producers. Exchange and consumption then occur in accordance with the outcomes of search. Each producer who ends a period with knowledge of a merchant forgets that information with a probability γ before the beginning of the next period. This ensures that there are always producers in the search market. The set of actions available to a producer thus depends on her information state in the period. For an informed producer, $A^i = \{R, S\}$, and for an uninformed producer, $A^u = \{S\}$.

From the perspective of the merchant, each period proceeds as follows. The merchant begins the period with knowledge of the prices she has set, and the size of her clientele, in each previous period. She then posts a price and trades with all producers who meet her in that period. All customers, regardless of their history, must be charged the same price in each period. A merchant in period t thus chooses a price p_t from the action set $A^\mu = [0, 1]$.

Both types of agents have imperfect knowledge over the history of play. A producer knows the history of prices she has paid upon meeting merchants in the past and the outcomes of previous searches. A merchant knows her own personal sequence of prices and clientele sizes. An agent's strategy is defined by a function that prescribes an action in her action set that depends only on her own personal history and current information state.

The per-period payoff of an agent depends on the amount of the good that she consumes at the end of each period. A producer consumes one unit if she trades with another producer and zero units if she does not trade at all. The consumption that comes from a merchant trade depends on the price charged. For a trade that occurs at a price p_t, the producer consumes $1 - p_t$ units of the good, while the merchant consumes p_t. The total payoff to a merchant in a period is thus $p_t k_t$, where k_t is the clientele size in period t. The expected discounted continuation payoff is found by summing an agent's expected per-period payoffs along the infinite time horizon.

3.3 The Search Process

Search mechanics in the model are governed by the two matching functions $\lambda^p(s_t)$ and $\lambda^m(s_t)$, where s_t is the size of the population of producers who search in period t. This population consists of all uninformed producers added to informed producers that choose to search in that period. It can also be considered as the total population of producers σ less the informed producers that choose to return to their merchant.

The probability of search resulting in a producer meeting is given by $\lambda^p(s_t)$ and the probability of meeting a merchant is $\lambda^m(s_t)$. It is assumed that $\lambda^p(s_t)$ is increasing in s_t. The appropriate assumptions to make on $\lambda^m(s_t)$ are less obvious, as it is the probability of a given producer meeting a merchant. A reasonable baseline assumption would be that it is non-increasing in s_t. The following assumptions are also maintained on the matching functions.[2]

(i) $\lambda^p(s_t) + \lambda^m(s) < 1 \quad \text{for all } s_t \in [0, \sigma]$

(ii) $\lambda^p(0) = 0$

(iii) $\lambda^p(s_t) > 0$ and $\lambda^m(s_t) > 0$ for $s_t > 0$

3.4 Solution Concept

There are two classifications of strategies that will prove useful when finding equilibria in the model: *Markov* strategies and *symmetric* strategies. An agent's strategy is Markov if the action prescribed in period t depends solely on observations made in the previous period and the agent's information state in the current period. A further condition of a Markov strategy is time-invariance, in the sense that the function prescribing actions based on the previous period's observations must not change over time. For an informed producer adopting a Markov strategy, the return decision in period t can depend only on the price observed in period $t-1$. A Markov strategy for a merchant requires that her period t price depends only on the price she set in period $t-1$ and the size of her clientele in that period.

A strategy profile is symmetric if all agents within a class adopt the same strategy. Symmetry amongst producers requires that all producers set the same return decision rule, based on their observed price. Amongst

[2] The matching probabilities are functions of s_t, but for notational convenience will sometimes be denoted just as λ^p and λ^m.

merchants, a symmetric strategy profile requires all merchants to adopt the same pricing function, which sets their price dependent on their own p_{t-1} and k_{t-1}.

Symmetry of merchant strategies is particularly important when considering price dispersion. The clientele size of each merchant is stochastic, so it is possible for a symmetric strategy profile to lead to a variety of equilibrium market prices if p_t depends on the size of k_{t-1}. Pricing would then depend on the particular arrivals of producers at merchants, which is determined by the stochastic search of producers. However, the proceeding analysis shows that asymmetry in merchant strategies is a necessary criterion for equilibrium price dispersion in this model.

A further consideration is whether producers can condition their strategy on the identity of the merchant they meet. In the basic model, it is assumed that merchants are anonymous so producers cannot condition their strategies in this way. This means that producers must employ the same decision rule for any merchant they meet. This assumption is relaxed in a model extension in Section 8.2, where equilibria involving producers conditioning on merchant identity are examined.

The expected continuation payoff of informed producers and merchants in each period depends on the strategies of both types of agents. For a strategy profile to constitute an *equilibrium*, the strategies employed by each agent must be optimal given the strategies of other agents, for any possible realised personal history. In particular:

- the return decision of each informed producer must maximise her expected continuation payoff in every period; and
- the price p_t set by each merchant must maximise her expected continuation payoff at each period t.

The primary focus of this thesis is equilibria in which multiple prices coexist in the market. However, equilibria with a unique price are also characterised, for completeness. Equilibria are found in Markov strategies with symmetric producer strategy, while asymmetric producer strategies are considered in a model extension in Section 8.1. Note that while equilibria are sought in Markov strategies only, at any equilibrium the strategies of agents must be optimal within the class of all strategies, not just Markov strategies. Non-Markov strategy profiles may yield further equilibria, but are not considered in this thesis.

Bose and Sengupta (2007) characterise optimal pricing and return decisions under symmetric Markov strategies only. This enables characterisation of single-price equilibria, detailed in Section 6.1. This thesis extends their model to examine price dispersion by incorporating asymmetric merchant strategies in Section 6.2, considering comparative statics for price-dispersed equilibria in Section 7 and introducing producer strategy heterogeneity in Section 8.

4 Merchant Pricing

This section outlines optimal merchant pricing, given the strategies of informed producers. A Markov strategy for an informed producer must consist of a decision rule that dictates whether the producer searches or returns in period $t + 1$ given the observation of p_t. This can be represented as a function $f : [0, 1] \to \{R, S\}$. Under a Markov strategy this function must not change over time, and producer symmetry implies that all producers adopt the same strategy. Given that producers cannot condition their return decision on a merchant's identity, all merchants in the market face the same producer strategy.

A producer's strategy can be represented by constructing two time-invariant disjoint sets, P^R and P^S, such that $P^R \cup P^S = [0,1]$. P^R represents the set of prices that induce an informed producer to return to the merchant she traded with in the previous period. The set of merchant prices that do not induce return if observed in the previous period is denoted as P^S.

In any period, the only prices that can possibly be optimal for a merchant to charge are either the highest price that will induce producers to return or the maximum possible price. This is because any other price could be raised by a small amount to increase a merchant's current period payoff without affecting the producer's return decision and future period payoffs. A producer's strategy from the perspective of a merchant can thus be viewed as setting a cut-off value $\hat{p}$ such that the producer will return to $\hat{p}$, but not any price greater than $\hat{p}$, that is, $\hat{p}$ is the highest price in P^R.

A merchant chooses her optimal action by comparing the expected payoff that comes from charging 1 and from charging $\hat{p}$. Charging 1 gives a higher current-period payoff but causes the merchant to forgo future payoffs from repeat clients. A merchant's Markov strategy must determine the price set in period t by the price set, and the size of her clientele served, in period $t-1$. This can be expressed as a time-invariant function $f : [0,1] \times [0,\sigma] \to [0,1]$. That is, p_t is expressed as a function of p_{t-1} and k_{t-1}.

4.1 Optimal Pricing

The expected continuation payoff of a merchant in period t depends on the size of her clientele in that period k_t, which in turn depends upon k_{t-1}. The stochastic nature of search and client memory means that the size of a merchant's clientele can change over time, even in equilibrium.[3] Supposing that a merchant charges a price $p_{t-1} = \hat{p}$, then her expected clientele size at time t is given by:

$$E\big(k_t | k_{t-1}, p_{t-1}, s_t\big) = \gamma k_{t-1} + \lambda^m(s_t) s_t \tag{1}$$

The clientele size in period t is denoted k_t, the measure of producers searching is s_t and E is the expectation operator. The merchant retains the fraction γ of her clients from the previous period that remember her location and accrues a fraction of searching producers in the current period. Producers finding a merchant are expected to be evenly distributed among merchants, so this appears in the equation as the merchant matching probability $\lambda^m(s_t)$ multiplied by the population of searching producers.

A merchant chooses to charge $\hat{p}$ or 1 depending on which yields the higher payoff. The optimal strategy for merchants depends on the cut-off point $\hat{p}$ that producers set when searching. Intuitively, if $\hat{p}$ is set sufficiently high, merchants will find it worthwhile to set their price to $\hat{p}$ to induce return rather than setting a price of 1. Define a strategy of setting $p_t = 1$ as *bandit pricing*, and a strategy that sets $p_t = \hat{p}$ as *return pricing*. Equation (1) allows the calculation of a merchant's expected payoff from return pricing, which can be compared to the payoff from

[3] Judd (1985) shows that a law of large numbers does not necessarily hold in a matching environment where agents are modelled as continua.

bandit pricing. The following lemma describing optimal pricing is also in Bose and Sengupta (2007); its proof is supplied for completeness.

Lemma 4.1 (Optimal Pricing)**.** *A merchant's optimal pricing rule is to set price in every period t as follows:*

$$p_t = \begin{cases} \hat{p} & \textit{if producers set} \quad \hat{p} > 1 - \gamma\delta \\ \hat{p} \;\; or \; 1 & \textit{if producers set} \quad \hat{p} = 1 - \gamma\delta \\ 1 & \textit{if producers set} \quad \hat{p} < 1 - \gamma\delta. \end{cases} \tag{2}$$

Proof. In any given period the only possibly optimal prices a merchant can charge are $\hat{p}$ and 1. Any other price can be increased without altering the client's return decision so cannot maximise a merchant's continuation payoff. Therefore any optimal pricing strategy must only include the prices $\hat{p}$ and 1.

Consider a merchant that sets $p_t = \hat{p}$ in every period. The best one-shot deviation from this strategy would be to set $p_\tau = 1$ in some period τ. This would result in the merchant increasing her per client payoff by $(1 - \hat{p})$ in period τ, but losing the expected earnings from those clients that remain informed in future periods. The change in expected payoff from such a deviation can be expressed as follows, denoted the discount factor as δ.

$$\begin{aligned} \Delta_{\hat{p}} &= k_\tau(1 - \hat{p}) - \sum_{j=\tau+1}^{\infty} \hat{p} k_\tau (\gamma\delta)^{j-\tau} \\ &= k_\tau(1 - \hat{p}) - \frac{\hat{p} k_\tau \gamma\delta}{1 - \gamma\delta} \\ &= k_\tau \left(1 - \frac{\hat{p}}{1 - \gamma\delta}\right) \end{aligned} \tag{3}$$

Similarly, the optimal deviation for a merchant that sets 1 in every period is to set $p_\tau = \hat{p}$. This decreases her period τ payoff by $(1-\hat{p})$ but increases her next period payoff because it induces her clients to return. The change in expected payoff from such a deviation can be expressed as follows.

$$\begin{aligned}\Delta_1 &= -k_\tau(1-\hat{p}) + k_t\gamma\delta \\ &= k_\tau[\hat{p} - (1-\gamma\delta)] \end{aligned} \tag{4}$$

Examination of $\Delta_{\hat{p}}$ shows $\hat{p} < 1-\gamma\delta$ must hold for a one-period deviation from setting $\hat{p}$ in every period to be profitable. Similarly, for a one-shot deviation from setting 1 in every period to be profitable, $\hat{p} > 1-\gamma\delta$ must hold. If $\hat{p} = 1-\gamma\delta$, then a deviation from either strategy does not change the merchant's payoff. Using the one-shot deviation principle (Abreu, 1988), this shows that a price-path of $\hat{p}$ is optimal if $\hat{p} > 1-\gamma\delta$ and setting $p_t = 1$ in every period is optimal if the inequality is reversed. If $\hat{p} = 1-\gamma\delta$, then any combination of the two prices is optimal.[4] □

5 The Producer Return Decision

An equilibrium requires that informed producers choose their strategy over the return decision optimally. This section considers this decision when all merchants charge a uniform price. A producer optimally returns to her merchant if she expects the merchant to set a price low enough to make paying it with certainty less costly than searching in the market.

A producer who is informed in period t and chooses to search in that period will be informed again in period $t+1$ with probability γ. The probability of the producer being informed in period $t+1$ is also γ if she returns to her merchant in period t. The expected continuation payoff in

[4] The explicit proof for multi-period deviations can be found in Bose and Sengupta (2007).

period $t+1$ is thus identical for an informed producer who searches and one who returns in period t, regardless of the actual search outcome. This is the case because the producer will choose optimally in period $t+1$, and all merchants charge the same price so meeting a new merchant yields information with the same value as the knowledge of the old merchant. Denoting the expected continuation payoff from returning in period t as V_t^r, the payoff from search in period t as V_t^s, and the payoff in period $t+1$ as V_{t+1}, these payoffs can be expressed as follows.

$$V_t^r = 1 - E(p_t) + \delta V_{t+1} \tag{5}$$

$$V_t^s = \lambda^m(1 - E(p_t) + \delta V_{t+1}) + \lambda^p(1 + \delta V_{t+1}) + (1 - \lambda^m - \lambda^p)\delta V_{t+1} \tag{6}$$

If the producer returns to her merchant she expects a payoff of $1 - E(p_t)$ in the current period, followed by the discounted next-period payoff. A searching producer gets a current-period payoff of $1 - E(p_t)$ if she meets a merchant, which occurs with probability $\lambda^m(s)$. Her current-period payoff is 1 if she meets another producer, which occurs with probability $\lambda^p(s)$. A null search occurs with the complementary probability $(1 - \lambda^m - \lambda^p)$. In the following period after any search outcome the producer's discounted payoff is δV_{t+1}.

Lemma 5.1. *An informed producer returning to her merchant is optimal if* $1 - E(p_t) \geq \Omega(s_t)$*, where*

$$\Omega(s_t) = \frac{\lambda^p(s_t)}{1 - \lambda^m(s_t)} \tag{7}$$

Proof. For producers returning to be optimal, it must yield at least as high a payoff as search. This optimality condition can be expressed as follows, using equations (5) and (6).

$$V_t^r \geq V_t^s \tag{8}$$

$$1 - E(p_t) \geq \frac{\lambda^p(s_t)}{1 - \lambda^m(s_t)} \tag{9}$$

□

The left hand side of (9) shows the benefit of returning to a merchant charging p, which must be at least as great as the right hand side, which shows the benefit of search. For producers to be indifferent between search and returning to their merchant, the condition in equation (9) must hold with equality. Denoting this indifference price as $p^*(s_t)$, it is given by $p^*(s_t) = 1 - \Omega(s_t)$.

6 Model Equilibria

6.1 Single-Price Equilibria

Observation 6.1. In an equilibrium where all informed producers return to their merchants, the search market converges to a steady state size. At that size the measure of informed producers forgetting their merchants is the same as the measure of searching producers finding and remembering a merchant. Denote this steady state size as $\overline{s}$. If $s_t < \overline{s}$, then the search market increases in size because more informed producers forget their merchant than do searching merchants become informed. If $s_t > \overline{s}$, the reverse process occurs.

The description of optimal merchant and producer behaviour in Sections 4 and 5 is sufficient to fully characterise single-price equilibria in Markov strategies. Proposition 6.1 gives the three prices that can possibly be supported in a single-price equilibrium, while Proposition 6.2 finds the necessary and sufficient conditions for the existence of equilibria involving each price. These three classes of single-price equilibria can be defined as follows.

Definition 6.1.

- A *bandit equilibrium* consists of all producers setting $p_t = 1$ in all periods and producers never returning.
- A *monopoly equilibrium* consists of all producers setting $p_t = p^*(\overline{s})$ in all periods and producers always returning when informed.
- A *competitive equilibrium* consists of all producers setting $p_t = 1-\gamma\delta$ in all periods and producers always returning when informed.

In a bandit equilibrium, merchants appropriate the entirety of their client's good and do not plan on repeated interaction. The terms 'monopoly' and 'competitive' equilibrium are used to reflect the division of surplus between the producer and the merchant in each type of equilibrium. In a monopoly equilibrium the merchant receives the entire surplus from trade, while an informed producer who returns to a merchant receives the same expected payoff as an uninformed searching producer. In a competitive equilibrium the producer receives the entire surplus from trade. A merchant is indifferent between charging $1 - \gamma\delta$ and 1, but a returning producer can have a payoff that strictly exceeds that of a searching producer.

Proposition 6.1. *No price other than* $1 - \gamma\delta$, $p^*(\overline{s})$ *and* 1 *can be supported in any single-price equilibrium.*

Proof. Lemma 4.1 shows that in equilibrium, merchants charge either $\hat{p}$ or 1. In an equilibrium where producers do not return, all merchants charge 1. In an equilibrium where producers do return, the market price must be $\hat{p}$. For this price to be optimal for merchants, producers must set $\hat{p} \geq 1-\gamma\delta$. For returning to this price to be optimal for producers, Lemma 5.1 shows that $1 - \hat{p} \geq \Omega(\bar{s})$ must hold. Noting that $p^*(s) = 1 - \Omega(s)$, any return equilibrium must have $1 - \gamma\delta \leq \hat{p} \leq p^*(\bar{s})$.

Consider an equilibrium in which $1 - \gamma\delta < \hat{p} < p^*(\bar{s})$. This implies that $1 - \hat{p} > \Omega(\bar{s})$, so a producer strictly prefers to return to a merchant charging $p_{t+1} = \hat{p}$. Given that $\hat{p} > 1 - \gamma\delta$, a merchant will optimally charge $p_{t+1} = \hat{p}$. Therefore, after observing any p_t a producer expects to see $p_{t+1} = \hat{p}$. This means that a client will optimally return to any price, so her merchant will optimally set a price of 1 in period t. A producer returning to 1 cannot be part of an equilibrium because the search market yields a strictly greater payoff, so such a $\hat{p}$ cannot hold in equilibrium. □

Proposition 6.2.

(i) A bandit equilibrium always exists.

(ii) Both a monopoly and competitive equilibrium exist if and only if:

$$\Omega(\bar{s}) \leq \gamma\delta \tag{10}$$

Proof. Merchants charging 1 is optimal when $\hat{p} \leq 1 - \gamma\delta$ by Lemma 4.1. Given that all merchants are charging $p_t = 1$ in all periods, any producer strategy that does not induce return at that price is optimal. This proves part (i).

Merchants charge $p^*(\bar{s})$ in a monopoly equilibrium. Returning is thus optimal for producers because $p = p^*(\bar{s})$ implies $1 - p = \Omega(\bar{s})$. Charging this price is optimal for merchants if and only if $p^*(\overline{s}) = \hat{p} \geq 1 - \gamma\delta$, which gives the condition in (10).

In a competitive equilibrium merchants charge $1 - \gamma\delta$. Return is optimal for producers if and only if $\Omega(\overline{s}) \leq \gamma\delta$, by Lemma 5.1. Merchants charging $1 - \gamma\delta$ in every period is optimal under the following merchant strategy if $\hat{p} = 1 - \gamma\delta$.

$$p_t = \begin{cases} 1 - \gamma\delta & \text{if} \quad p_{t-1} \leq 1 - \gamma\delta \\ 1 & \text{if} \quad p_{t-1} > 1 - \gamma\delta \end{cases} \tag{11}$$

Merchants cannot profitably deviate from this strategy because charging 1 yields the same payoff as charging $1-\gamma\delta$ and charging $p_t \neq 1-\gamma\delta$ yields a lower payoff. This proves the existence of the competitive equilibrium, and completes part (ii).[5] □

In a bandit equilibrium, long term producer-merchant relationships do not develop, so search costs are not reduced in the market. In the monopoly and competitive equilibria, these long-run relationships do develop, so total search costs in the market are reduced. Proposition 6.2 shows that if $\Omega(\overline{s}) > \gamma\delta$, then only the bandit equilibrium exists. This occurs because if all merchants charge a price to induce return, the cost of search is not sufficiently large to make the return price high enough for charging it to be profitable for merchants. However, if some merchants were to charge the bandit price, the cost of search would increase because meeting a bandit-pricing merchant is effectively the same as a null search result. Suppose that a measure of bandit-pricing merchants increases

[5] Note that this type of strategy cannot form part of an equilibrium at any other price. If $\hat{p} > 1 - \gamma\delta$ then charging a price of 1 cannot be optimal so a merchant would optimally charge $p_{t+1} = \hat{p}$ even after charging $p_t > \hat{p}$. Producers would thus always return so merchants would optimally charge 1, which cannot occur in an equilibrium where informed producers return.

the cost of search enough to make mutually profitable long-term trading relationships possible between producers and the remaining merchants. The resultant price-dispersed equilibrium could reduce aggregate search costs, compared with the bandit equilibrium. This provides a motivation for investigating priced-dispersed equilibria, which is done below.

6.2 Price-Dispersed Equilibrium

In a price-dispersed equilibrium in which some merchants charge a return price and others set the bandit price, the payoff from both pricing strategies must be the same, because merchants are free to switch strategies. Lemma 4.1 shows that this can only occur if producers set $\hat{p} = 1 - \gamma\delta$. A price-dispersed equilibrium must then contain the two prices $1 - \gamma\delta$ and 1. Denote the measure of merchants charging a price $1 - \gamma\delta$ as $\alpha \in (0, 1)$, such that the rest of the merchant population charges 1.

Optimal producer behaviour in this scenario can be modelled by introducing α into the analysis in Section 5. First, note that a producer with knowledge of a merchant expected to charge 1 in period t can effectively be considered as not being informed because the producer will always optimally search. As in the single-price case, an informed producer in period t will expect the same payoff in period $t+1$, regardless of whether she searches or returns to her merchant in period t. This yields the following expected continuation payoff for return and search.

$$V_t^r = \gamma\delta + \delta V_{t+1} \tag{12}$$

$$V_t^s = \alpha\lambda^m(\gamma\delta + \delta V_{t+1}) + \lambda^p(1 + \delta V_{t+1}) + (1 - \alpha\lambda^m - \lambda^p)\delta V_{t+1} \tag{13}$$

These payoffs are the same as in the single-price case, but with $E(p_t) = 1 - \gamma\delta$ and with a reduced probability $\alpha\lambda^m(s)$ of meeting a merchant charging that price. Returning to a merchant who charges $1-\gamma\delta$ is thus optimal for a producer if the following condition is satisfied.

$$V_t^r \geq V_t^s \tag{14}$$

$$\gamma\delta \geq \frac{\lambda^p(s_t)}{1-\alpha\lambda^m(s_t)} \tag{15}$$

Redefine $\Omega(s_t, \alpha)$ as the right hand side of condition (15). The value of the benefit from search, $\Omega(s_t, \alpha)$, is increasing in α.[6] This shows how merchants switching from return pricing to bandit pricing increases the cost of search and so can induce producers to return to a price of $1-\gamma\delta$.

Proposition 6.3. *A price-dispersed equilibrium in which producers return to the $\alpha \in (0,1)$ measure of merchants charging $1-\gamma\delta$ and do not return to the remainder of the merchant population, who charge 1, exists if and only if*

$$\Omega(s_t, \alpha) \leq \gamma\delta \tag{16}$$

Proof. Producers optimally return to a merchant charging $1-\gamma\delta$ if and only if the condition in (15) holds, as set out above. If producers set $\hat{p} = 1-\gamma\delta$, merchants are indifferent between charging $1-\gamma\delta$ and 1. This $\hat{p}$ can thus support an equilibrium with a measure of merchants charging 1 and the remainder adopting the strategy detailed in equation (11), analogously to the competitive equilibrium. □

Observation 6.2. A special case of the price dispersed equilibrium where producers are indifferent between search and merchant return exists if there exists an $\hat{\alpha} \in (0,1)$ that satisfies $\Omega(s, \hat{\alpha}) = \gamma\delta$. This

[6]Changing α also has indirect effects on $\Omega(s_t, \alpha)$ via the matching probabilities. This is considered in Section 7.

yields the following value for $\hat{\alpha}$.

$$\hat{\alpha} = \frac{\gamma\delta - \lambda^p}{\lambda^m \gamma\delta} \tag{17}$$

6.3 The Steady State Search Market

In an equilibrium where the measure of return-pricing merchants is constant, the steady state size of the search market can be found. This steady state size s can be expressed as a function of α as follows.

$$s(\alpha) = (\sigma - s)(1-\gamma) + s(1 - \alpha\lambda^m(s)\gamma) \tag{18}$$

$$s(\alpha) = \frac{\sigma(1-\gamma)}{1-\gamma[1-\alpha\lambda^m(s)]} \tag{19}$$

At the start of each period a fraction $(1-\gamma)$ of producers who returned in the previous period forget the location of their merchant. The measure of such producers is given by $(\sigma - s)$. There are s producers searching in each period. A fraction $\lambda^m(s)$ of them meet a merchant, so $\alpha\lambda^m(s)$ meet a merchant that induces return. A fraction γ of those producers remember their merchant and so leave the search market. This yields equation (18), which simplifies to (19). This means that in equilibrium, the size of the search market s_t can be denoted by $s(\alpha)$. This means that $\Omega(s_t, \alpha)$ is a function of α alone in equilibrium.

There are two special cases that are pertinent to the equilibria found in Section 6.1. In an equilibrium where no merchants charge the return price, the search market is given by the producer population σ. This can be seen from equation (19) with $\alpha = 0$. In an equilibrium where all merchants charge the return price, all producers return when informed. This search market size, denoted $\overline{s}$, can be found from equation (19) with

$\alpha = 1$ as follows.

$$\overline{s} = \frac{\sigma(1-\gamma)}{1-\gamma[1-\lambda^m(\overline{s})]} \tag{20}$$

7 Equilibrium Existence and Comparative Statics

The existence of the equilibria characterised in Propositions 6.2 and 6.3 depends on the value taken by $\Omega(s_t, \alpha)$, which can be denoted as $\Omega(\alpha)$ in equlibrium. In particular, a bandit equilibrium always exists and monopoly, competitive and price-dispersed equilibria have the same existence criterion: $\Omega(\alpha) \leq \gamma\delta$. Equation (15) shows that the value of $\Omega(\alpha)$ depends on $\alpha, \lambda^m(s)$ and $\lambda^p(s)$. The matching probabilities are determined by $s(\alpha)$.

To determine the range of α that can be supported in equilibrium, the effect of changes in α on the value of $\Omega(\alpha)$ must be considered. This is because changing α can affect whether the necessary equilibrium condition of $\Omega(\alpha) \leq \gamma\delta$ continues to hold. In particular, the direction of the effect of changing α on is important for marginal cases.

The direct effect on $\Omega(\alpha)$ of an increase in α is negative because reducing the number of bandit-pricing merchants in the market increases the expected value of producer search, with all other factors held constant. This can also be seen by inspection of equation (15). There is also an indirect effect that works through the size of the search market and the matching probabilities. The total effect is investigated further in Appendix A.1, where it is found that for matching functions with no thick-market externalities the indirect effect opposes the direct effect in general.[7] However, using the linear matching functions the total effect of α on $\Omega(\alpha)$ is negative for all $\alpha \in (0,1)$ for a broad range of parameter values.

7.1 Specific Matching Functions and Closed-Form Equilibria

To reduce the ambiguity associated with the generic matching functions used above, a particular functional form is specified for use in the remainder of Section 7. The following linear matching functions with no thick-market externalities are employed, with the value of s given by equation (19).

$$\lambda^m(s) = \frac{\lambda}{1+s} \tag{21}$$

$$\lambda^p(s) = \frac{\lambda s}{1+s} \tag{22}$$

Equation (23) gives the existence criterion for a price-dispersed equilibrium with a given α using these matching functions (as in Proposition 6.3). The existence of the monopoly and competitive equilibria requires a special case of this condition to hold, namely with $\alpha = 0$ and $s = \overline{s}$.

$$\gamma\delta \geq \frac{\lambda s(\alpha)}{1 + s(\alpha) - \alpha\lambda} = \Omega(\alpha) \tag{23}$$

These matching functions also allow the specification of the aggregate cost of search in the market. In equilibrium, if a producer search results in meeting either another producer or a merchant, goods are traded and

[7] Matching functions with no thick-market externalities have constant returns to scale, which means that the total probability of finding a match is the same independent of the market size. Here this means, defining a constant $\lambda \in [0,1]$, that $\lambda^p(s) + \lambda^m(s) = \lambda$ for all s. Using this type of matching technology follows Bose and Sengupta (2007).

consumed. The division of consumption depends on the terms of trade, but the good is consumed regardless. From the perspective of aggregate welfare then, the search cost is only incurred if producer search results in finding no-one. The probability of this is given by a constant $(1-\lambda)$ using the matching functions specified above.

The aggregate search cost in equilibrium can thus be represented as a function of α, that is $C(\alpha) = s(\alpha)(1-\lambda)$. The steady state size of the search market is decreasing in α, so the aggregate cost function is also decreasing in α. This means that the aggregate welfare maximising equilibrium is given by the equilibrium with the maximum supportable α.

7.2 Equilibrium Existence

This section investigates the range of α that can be supported in equilibrium. Clear results can be found for two particular cases. First, where $\Omega(\alpha)$ is a monotonically increasing function of α, and second where it is monotonically decreasing.

Proposition 7.1. *If $\Omega(\alpha)$ is a monotonically increasing function of α, then the range of α that can be supported in equilibrium is given by:*

$$Range\ of\ \alpha = \begin{cases} 0 & if \quad \hat{\alpha} \leq 0 \\ [0, \hat{\alpha}] & if \quad \hat{\alpha} \in (0,1) \\ [0,1] & if \quad \hat{\alpha} \geq 1 \end{cases} \tag{24}$$

$$where\ \hat{\alpha}\ solves\ for \quad \hat{\alpha} = \frac{1+s(\hat{\alpha})}{\lambda} - \frac{s(\hat{\alpha})}{\gamma\delta} \tag{25}$$

Proof. The expression for $\hat{\alpha}$ in equation (25) comes from setting $\Omega(\hat{\alpha}) = \gamma\delta$ and substituting the liner matching functions. If $\Omega(\alpha)$ is monotonically increasing then $\Omega(\alpha) < \gamma\delta$ holds for any $\alpha < \hat{\alpha}$. Similarly $\Omega(\alpha) > \gamma\delta$ holds for any $\alpha > \hat{\alpha}$. Noting that $\Omega(\alpha) \leq \gamma\delta$ is the existence criterion is for an equilibrium where an α fraction of merchants charge the return price completes the proof. □

Corollary 7.1. By the same argument, if $\Omega(\alpha)$ is monotonically *decreasing* in α, then

$$\textit{The range of supportable } \alpha = \begin{cases} [0,1] & if \quad \hat{\alpha} \leq 0 \\ [\hat{\alpha},1] & if \quad \hat{\alpha} \in (0,1) \\ 1 & if \quad \hat{\alpha} \geq 1 \end{cases} \tag{26}$$

The welfare maximising equilibrium requires the maximum α that can be supported. If $\Omega(\alpha)$ is decreasing in α then the maximum supportable α must be either 0 or 1. In particular, if $\Omega(1) \leq \gamma\delta$ then an equilibrium where all merchants charge the return price exists and is optimal. If $\Omega(1) > \gamma\delta$ then only the bandit equilibrium exists.

However, if $\Omega(\alpha)$ is increasing in α and an equilibrium with $\alpha = 1$ does not exist, a price dispersed equilibrium exists if there exists an α such that $\Omega(\alpha) \leq \gamma\delta$. The welfare maximising price-dispersed equilibrium in this case requires the maximum supportable α, which is given by $\hat{\alpha}$ in equation (25). The comparative statics for this case are analysed below.

7.3 Comparative Statics

This section looks at comparative statics for the welfare-maximising equilibrium under the assumption that $\Omega(\alpha)$ is increasing in α. In particular, the case is considered in which there exists an $\hat{\alpha} \in (0,1)$ such that $\Omega(\hat{\alpha}) = \gamma\delta$ both before and after the exogenous parameter change. This ensures that the welfare-maximising equilibrium is price-dispersed.

The exogenous parameters that can change are the discount rate δ, the probability of producers remembering their merchants γ, the measure of producers σ as well as the total matching probability λ. Shifting these exogenous parameters can change the equilibrium level of $\hat{\alpha}$ and s, as well as affect the total welfare in the market as measured by aggregate search costs. The return price $1 - \gamma\delta$ also changes with γ and δ.

Consider an exogenous increase in δ to δ'. Given that before the change $\Omega(\hat{\alpha}) = \gamma\delta$, after the change $\Omega(\hat{\alpha}) < \gamma\delta'$ must hold because the value of $\Omega(\hat{\alpha})$ is unaffected by δ. This means that there must now exist an $\alpha > \alpha'$ that is supported in equilibrium w because $\Omega(\alpha)$ is increasing in α. This increase in $\hat{\alpha}$ reduces $s(\hat{\alpha})$ and so increases total welfare. The increased patience means that the return price does not need to be as high for merchants to optimally charge it. This lower price makes returning optimal for a producer in an equilibrium with a reduced search cost compared to the initial equilibrium, so producer welfare also increases.

An increase in γ to γ' has the same effect as increasing δ, via the change in the equilibrium return price. There is also a second effect. Increasing γ decreases s because fewer producers reenter the search market as a result of forgetting their merchant. A decreasing s reduces $\lambda^p(s)$ and increases $\lambda^m(s)$. The net effect of this is to increase the cost of search and reduce $\Omega(\hat{\alpha})$, as can be seen from the following partial derivative computed from equation (23).

$$\frac{\partial\Omega(\hat{\alpha})}{\partial s} = \frac{\lambda - \hat{\alpha}\lambda^2}{(1 + s - \hat{\alpha}\lambda)^2} > 0 \tag{27}$$

The combined effect of this results in $\Omega(\hat{\alpha}) < \gamma'\delta$ so an increased α can be supported in equilibrium. This further reduces the size of the search market and so increases total welfare. This parameter change works through the same price channel as changing δ, but also increases the size of the cost saving from long term merchant-producer relationships because they persist with a higher probability.

An increase in σ to σ' causes an increase in s, as shown in equation (19). This causes an increase in $\Omega(\hat{\alpha})$, as per equation (27). This causes $\Omega(\hat{\alpha}) > \gamma\delta$, so α must decrease to reestablish equilibrium. The decrease in α dampens the increase in s, but the overall increase in s causes an increase in aggregate search costs. However, a fairer analysis of welfare would also need to factor in the increase in the population.

Suppose the total matching probability increases from λ to λ'. This increases both $\lambda^m(s)$ and $\lambda^p(s)$. The increase in $\lambda^m(s)$ reduces the size of the search market (see equation 19), which reduces $\Omega(\hat{\alpha})$, as above. The change in λ also has a direct effect, as the following partial derivative shows.

$$\frac{\partial\Omega(\hat{\alpha})}{\partial\lambda} = \frac{s + s^2}{(1 + s - \hat{\alpha}\lambda)^2} > 0 \tag{28}$$

These two effects oppose each other so the direction of the total effect depends on which one dominates. If the direct effect dominates then λ' causes $\Omega(\hat{\alpha}) > \gamma\delta$ so α must decrease in equilibrium, which opposes the change in s. However, if the indirect effect dominates then $\Omega(\hat{\alpha})$ decreases so an increased α can be supported in equilibrium. This further decreases s and so increases overall welfare.

There is a qualitative difference between changes to δ or γ and to changes in σ or λ. Changing $\gamma\delta$ changes the range of values of $\Omega(\alpha)$ that can be supported in equilibrium, and in particular, alters the value of $\Omega(\alpha)$ that solves $\Omega(\alpha) = \gamma\delta$. On the other hand, a change in σ or λ does not change this value of $\Omega(\alpha)$. Rather, it only changes the $\hat{\alpha}$ required for the condition to hold. Because $\Omega(\alpha)$ is a measure of the benefit from search, this means that any change in the cost of search caused by an exogenous change in σ or λ is endogenously counterbalanced by endogenous changes in $\hat{\alpha}$ if a price-dispersed equilibrium is maintained. This means that these changes do not change the welfare of producers in the model. However, changes in γ or δ do change the cost of search and so affect producer welfare.

8 Extensions To The Basic Model

This section develops two extensions to the basic model set out above, both involving a broader set of possible producer strategies. First, mixed producer strategies are considered. The possible increase in heterogeneity of actions widens the set of prices that can be supported in equilibrium. The second extension considers a different form of producer strategy heterogeneity where strategies can be conditioned on the identity of the particular merchant encountered. Again this increases the degree of price dispersion that can be supported in equilibrium because multiple equilibria can essentially be supported simultaneously in different market segments.

8.1 Mixed Producer Strategy

In an equilibrium where producers are indifferent between search and return after observing a particular price, it is possible that producers sometimes search, and at other times return, in the period following meeting a merchant who charges that price. This can be modelled in at least two ways. First, producers could use a symmetric but mixed strategy, such that all producers return to the indifference price with a given probability. Second, producers could use pure but asymmetric strategies, such that a fraction always return at the indifference price p^* and the remainder never do. Asymmetric mixed strategies could also be used.

Modelling producer strategy in these different ways affects the price that makes merchants indifferent between charging the return price and charging 1, which is a necessary condition for price dispersion in this model. Taking producer strategies as symmetric and mixed requires a relatively minor modification to the optimal pricing description in Lemma 4.1. Denoting the probability of return to a price $\hat{p}$ as ω, the expected discounted payoff flowing from a merchant's current period clientele when charging $\hat{p}$ changes to become:

$$\sum_{j=\tau+1}^{\infty} \hat{p} k_\tau (\omega\gamma\delta)^{j-\tau}$$

From the merchant's perspective, the probability that a producer will not return that is dictated by strategy is no different to the exogenous producer memory parameter γ. The producer strategy that makes merchants indifferent between return pricing and bandit pricing is thus given by producers returning to $\hat{p} = 1 - \omega\gamma\delta$ with a probability γ.

Modelling producer strategy as asymmetric and pure presents a difficulty because of the stochastic nature of search. The indifference price for each merchant would depend on the realised fraction of current period clients that adopt each type of producer strategy. Moreover, merchants would not know with certainty which of their clients were using each type of strategy, so merchant beliefs would update each period. The indifference price is found under the simplifying assumption of deterministic search in Appendix A.2, but the symmetric mixed strategy approach is used here.

Proposition 8.1. *A price-dispersed equilibrium exists with:*

(i) A fraction α of merchants charging $1 - \omega\gamma\delta$;

(ii) Remaining merchants charging 1; and

(iii) Producers returning to a price of $1 - \omega\gamma\delta$ with probability ω.

if and only if

$$\Omega(s, \alpha) = \omega\gamma\delta \tag{29}$$

Proof. This equilibrium can be supported by the following strategy profile. Producers return with probability ω if $p_t = 1 - \omega\gamma\delta$ and never return to a different price. Merchants set their price as follows.

$$p_t = \begin{cases} 1 - \omega\gamma\delta & \text{if} \quad p_{t-1} = 1 - \omega\gamma\delta \\ 1 & \text{otherwise} \end{cases} \tag{30}$$

Given that merchants setting $p_t \neq 1 - \omega\gamma\delta$ will charge 1 in the next period, it is optimal for producers not to return to any such price. Given the producer strategies in use, merchants are indifferent between charging the two prices included in their strategy. This means it is (weakly) optimal to charge a price of 1. If a fraction ω of producers returned to a higher price, charging that price would be strictly preferred to charging 1 so could not maintain an equilibrium. Producers must be indifferent between search and return in order to employ a mixed strategy. The indifference price is unchanged from the basic model so the condition in equation (29) is found by setting $\Omega(s, \alpha) = 1 - \hat{p}$. □

Mixed producer strategy also makes possible an equilibrium that supports three distinct prices. Suppose $\hat{\alpha}$ and $\hat{\omega}$ constitute an equilibrium as described in Proposition 8.1. Consider a small measure of merchants that currently charge 1 switching to charge $1-\gamma\delta$ (keeping $\alpha = \hat{\alpha}$). Note that $1-\gamma\delta < 1-\hat{\omega}\gamma\delta$, so all producers return to the β measure of merchants charging $1-\gamma\delta$. This change decreases the producer-indifference price p^* because the search market yields an increased expected payoff. Equilibrium can be achieved if ω changes such that $p^* = 1-\omega\gamma\delta$. The producer-indifference price now depends on the merchant strategy parameters α and β. The value of p^*, shown below, is determined in Appendix A.3.

$$p^* = \frac{(1-\gamma\delta)[1-\lambda^p - \alpha\lambda^m + \beta\lambda^m\delta(1-\gamma)]}{(1-\gamma\delta)(1-\alpha\lambda^m)+\beta\lambda^m} \tag{31}$$

If $\beta = 0$, then equation (31) reduces to condition (29), which describes the case in which no merchants charge $1-\gamma\delta$. An equilibrium with the three prices $1-\gamma\delta$, $1-\omega\gamma\delta$ and 1 can thus be supported if α, β and ω are chosen such that $p^* = 1-\omega\gamma\delta$. Proposition 8.2 shows that this is the maximum number of prices that can be supported simultaneously in equilibrium.

Proposition 8.2. *No more than three prices can coexist in any equilibrium.*

Proof. The indifference price p^* is the same for all producers because it is determined by the prices of merchants in the market. This means that there can be only one price at which producers optimally employ a mixed return decision, call this price $\hat{p}^\omega$. A price such that $1-\gamma\delta < p_t < \hat{p}^\omega$ cannot induce all producers to return at equilibrium. If it did it would be strictly optimal for merchants to charge so producers would expect it to be charged and so always return to any price. All producers returning to $1-\gamma\delta$ can be supported at equilibrium because merchants are indifferent between charging $1-\gamma\delta$ and 1, so this can be supported as in Proposition 6.2. Merchants never set a price less than $1-\gamma\delta$ and producers never return to a price greater than p^* so the price 1 is the only price other than $\hat{p}^\omega$ and $1-\gamma\delta$ that can be supported in equilibrium. □

8.2 Identifiable Merchants

This section considers a version of the basic model in which producers can vary their actions depending on the particular merchant they meet in the market. This could be applicable to an environment where merchants are identified by name or location. To model this let $x \in [0,1]$ uniquely identify each merchant. Producer strategy is now a function of p_{t-1} and x_{t-1}, with $f : [0,1] \times [0,1] \to \{R, S\}$. A producer's strategy can thus be represented as setting a return rule for each merchant i, that is, setting a $\hat{p}_i$ for each $i \in [0,1]$. Optimal behaviour for any given merchant is unchanged from the preceding analysis, except that now for a merchant i, optimal pricing is determined by $\hat{p}_i$. Producers not conditioning on merchant identity is equivalent to them setting $\hat{p}_i = \hat{p}$ for all merchants.

Optimal behaviour for producers is also similar to the preceding characterisation. Now, $\hat{p}_i$ must be set optimally given the pricing strategy of each merchant i. This means that for any subset of merchants, any of the equilibria set out in Propositions 6.2 and 6.3 can be supported as long as their necessary and sufficient conditions are met.

For example, divide the population of merchants into three categories: $[0,\theta]$, (θ,ϕ) and $[\phi,1]$. Suppose producers (symmetrically) set $\hat{p}_i \leq 1-\gamma\delta$ for $i \in [0,\theta]$, $\hat{p}_i = p^*$ for $i \in (\theta,\phi)$ and $\hat{p}_i = 1-\gamma\delta$ for $i \in$

$[\phi, 1]$. As long as returning to p^* is optimal, a bandit, monopoly and competitive equilibrium can be supported in the three distinct merchant segments respectively. Each merchant segment can effectively support an equilibrium independently of the other market segments because the personal arbitrage condition between merchants using different strategies does not apply between market segments. The level of the producer indifference price p^* will depend on the fraction of merchants engaging in each of the different pricing strategies, as outlined in Section 8.1.

This kind of scenario may not be applicable to many real market situations. The assumption that producers can somehow identify their merchants is somewhat incongruous with the random search paradigm employed. It is possible that merchants may have characteristics that are observable only upon meeting the merchant. However, a proper incorporation of this into the present model would require some kind of ex ante merchant heterogeneity or the possibility of merchants endogenously controlling an observable characteristic, such as the appearance of their store.

9 Modelling Assumptions

The model presented in this thesis has several features that drive the result of equilibrium price dispersion. The key conceptual feature is the repeated interactions between merchants and producers, which enable the formation of ongoing relationships that reduce search costs. This section discusses whether other features and assumptions included in the model are also critical in generating this result.

The existence of a parallel search market has important consequences for behaviour in the model. If this market did not exist then producers returning to their merchants would be weakly optimal even if all merchants were to charge the bandit price of 1.[8] However, if prices are dispersed, the producer return decision is influenced by the distribution of prices in the market. This distribution creates a threshold price less than unity, above which return is less profitable than search. Parallel search markets are an important feature of intermediation models, but represent a departure from the price dispersion literature. The externality that comes from the decisions of agents influencing the size of this parallel search market, and thus the matching probabilities, drives much of the complexity of this model.

A positive probability of producers forgetting their merchant is included in the present model. This is required for the continued existence of the producer search market. If producers always remember their merchant, then any measure of merchants charging a return price would eventually accrue all producers as clients and collapse the market.[9] Perfect producer memory would also undermine the continued profitability of bandit pricing in a price-dispersed equilibrium. Again, the search market would empty of producers so merchant payoffs could only come from existing clients. The only way for bandit pricing to be optimal in this scenario would be for the payoff in earlier periods with a non-empty search market to be sufficiently large to make up for the zero payoff in later periods.

The market modelled in this thesis is one-sided. That is, a set of producers exchanges goods amongst themselves, as opposed to a two-sided market where consumers exchange with producers. The model could

[8]This case can be considered in this model by setting $\lambda^p(s)$ to zero. This means that $\Omega(s_t) = 1$, so returning is always optimal.

[9]This can be seen from the equation for the steady state search market in Section 6.3 with γ set to 1.

be adapted to a two-sided market in a fairly straight-forward manner. Consumers and producers would engage in search in a market containing merchants in the same manner as in the present model. Merchants with the ability to guarantee trade with preexisting clients would be able to command a commission in exchange for this service. The similarity of this scenario to the situation modelled here suggests that price dispersion would likely be an outcome of such a two-sided model.

The bargaining process here is modelled with merchants as price-setters. This may appear a restrictive assumption, but in essence it means that the cost of search is the risk of missing out on consumption as a result of an unfavourable search outcome. In this sense, this model does not differ greatly from models of costly sequential search. Modelling the bargaining process in a different way might present further insight into the problem, particularly into the importance of the specifics of the search and bargaining process for the equilibrium results. Possible alternative approaches would be to change the explicit bargaining process or including features into the search process that alter bargaining power. The latter route could involve including a probability of a producer observing the prices of two different merchants in one search, or a chance of being able to search a second time after declining trade. Another modification to the search process could include some channel of information dissemination amongst producers.

Agents in the present model are exogenously assigned to their occupations of producer or merchant, in keeping with the focus of this thesis. The exogeneity assumption could also be justified by the argument that some of the factors leading merchants to choose their occupation may, in fact, be outside of the model. For example, merchants may trade in several different markets simultaneously, meaning a comparison of payoffs within a single-market model may not tell the full story. However, the present model could be adapted to check whether the price dispersion result is robust to endogenous occupational choice. This would require equal payoffs from choosing the occupation of producer and from becoming a start-up merchant. Agents switching between occupations would be captured by changing σ, the ratio of producers to merchants.

10 Conclusion

The primary result of this thesis is that equilibrium price dispersion can be supported in a model where agents are homogeneous within each type. This departure from the existing price dispersion literature is driven primarily by the setting of repeated interaction, in which the long-term relationships between merchants and their clients reduce the costs of search. The present model describes a particular setting, which may not be broadly applicable to real-world markets. However, it highlights the potential importance of multi-period models in investigating price dispersion. A further result of this thesis is that a price-dispersed equilibrium can sometimes improve welfare compared to a single-price equilibrium.

As is often the case with repeated games, the model presented in this thesis exhibits multiple equilibria. The model can support a monopoly equilibrium, as described in Proposition 6.2, which echoes the Diamond paradox. However, the repeated interaction makes possible a broader set of equilibria, some of which are price-dispersed. This has been found in some previous single-period models of price dispersion, for example Burdett and Judd (1983). Multiple equilibria may indeed be a factor in the wide range of pricing outcomes that are observed in real markets.

Customers' expectations about the pricing policies that sellers will employ, and sellers' expectations about consumer search habits, could easily affect the equilibrium that is in fact achieved.

The modelling assumptions discussed in Section 9 could be relaxed or changed to further investigate their importance to the result of equilibrium price dispersion. Also, repeated interaction is only one aspect of what may cause price dispersion in real markets. Other factors thought to be important in price dispersion, such as ex ante agent heterogeneity, could be incorporated into a model similar to the one presented here. In the present model, allowing a greater degree of heterogeneity in agent strategy increased the range of prices that could be supported in equilibrium. It would be interesting to see the effect of ex ante agent heterogeneity on equilibrium price dispersion, particularly with respect to the result that at most three prices can be supported concurrently in equilibrium.

11 Appendix

A.1 The Total Effect of α on $\Omega(\alpha)$

The total effect of changes in α on the value of $\Omega(\alpha)$ consists of a direct effect and an indirect effect. An increase to α means a reduced number of bandit-pricing merchants in the market. This reduces the probability of a bad search outcome so would be expected to raise the expected payoff from search and increase $\Omega(\alpha)$, which is confirmed below.

The indirect effect works through the matching probabilities, which are altered by the change in the size of the search market in response to changes in α. More return-pricing merchants increases the number of producers that return to a merchant, and so reduces the size of the search market, given above in equation (19). This is intuitively clear, but proves analytically complicated to show. A reduction in the size of the search market reduces λ^p and would be expected not to reduce λ^m.[10] Increasing either matching function lowers the probability of a null search result, so increases $\Omega(\alpha)$ analogously to a reduction in the number of bandits. The combined effect can be expressed as the following total derivative.

$$\frac{d\Omega(\alpha)}{d\alpha} = \frac{\partial\Omega(\alpha)}{\partial\alpha} + \frac{\partial\Omega(\alpha)}{\partial\lambda^m(s)}\frac{d\lambda^m(s)}{ds(\alpha)}\frac{ds(\alpha)}{d\alpha} + \frac{\partial\Omega(\alpha)}{\partial\lambda^p(s)}\frac{d\lambda^p(s)}{ds(\alpha)}\frac{ds(\alpha)}{d\alpha} \quad \text{(A-1)}$$

The partial derivatives for $\Omega(\alpha)$ can be computed from equation (15), and are shown below.

$$\frac{\partial\Omega(\alpha)}{\partial\alpha} = \frac{\lambda^p\lambda^m}{(1-\alpha\lambda^m)^2} < 0 \quad \text{(A-2)}$$

$$\frac{\partial\Omega(\alpha)}{\partial\lambda^p(s)} = \frac{1}{1-\alpha\lambda^m} < 0 \quad \text{(A-3)}$$

$$\frac{\partial\Omega(\alpha)}{\partial\lambda^m(s)} = \frac{\alpha\lambda^p}{(1-\alpha\lambda^m)^2} < 0 \quad \text{(A-4)}$$

This confirms the direct effect of a change in α outlined above, and shows that $\Omega(\alpha)$ is also decreasing in both matching functions as expected. Supposing that $\lambda^m(s)$ is decreasing in s, that $\lambda^p(s)$ is increasing in s and that $s(\alpha)$ is decreasing in α as outlined above, the signs of the components of the total derivative can be expressed as follows.

[10] By assumption, see Section 3.3.

$$
\begin{aligned}
\frac{d\Omega(\alpha)}{d\alpha} &= \overbrace{\frac{\partial\Omega(\alpha)}{\partial\alpha}}^{+} + \overbrace{\frac{\partial\Omega(\alpha)}{\partial\lambda^m(s)}}^{+}\overbrace{\frac{d\lambda^m(s)}{ds(\alpha)}}^{-}\overbrace{\frac{ds(\alpha)}{d\alpha}}^{-} + \overbrace{\frac{\partial\Omega(\alpha)}{\partial\lambda^p(s)}}^{+}\overbrace{\frac{d\lambda^p(s)}{ds(\alpha)}}^{+}\overbrace{\frac{ds(\alpha)}{d\alpha}}^{-} \\
&= \overbrace{\frac{\partial\Omega(\alpha)}{\partial\alpha}}^{+} + \overbrace{\frac{\partial\Omega(\alpha)}{\partial\lambda^m(s)}\frac{d\lambda^m(s)}{ds(\alpha)}\frac{ds(\alpha)}{d\alpha}}^{+} + \overbrace{\frac{\partial\Omega(\alpha)}{\partial\lambda^p(s)}\frac{d\lambda^p(s)}{ds(\alpha)}\frac{ds(\alpha)}{d\alpha}}^{-} \qquad \text{(A-5)}
\end{aligned}
$$

Or equivalently,

$$
\frac{d\Omega(\alpha)}{d\alpha} = \overbrace{\frac{\partial\Omega(\alpha)}{\partial\alpha}}^{+} + \overbrace{\frac{ds(\alpha)}{d\alpha}}^{-}\left[\overbrace{\frac{\partial\Omega(\alpha)}{\partial\lambda^m(s)}\frac{d\lambda^m(s)}{ds(\alpha)}}^{-} + \overbrace{\frac{\partial\Omega(\alpha)}{\partial\lambda^p(s)}\frac{d\lambda^p(s)}{ds(\alpha)}}^{+}\right] \qquad \text{(A-6)}
$$

The sign of the total derivative thus depends on the size of each positive and negative term. This will of course depend on the exact form of the matching functions $\lambda^m(s)$ and $\lambda^p(s)$. However, restricting attention to matching functions that do not exhibit thick-market externalities (that is where $\lambda^m(s) + \lambda^p(s)$ is constant) allows for greater determination.[11] For such matching functions, the derivatives of the two functions with respect to α must be equal in magnitude but opposite in sign. Also, the following observation follows from equations (A-3) and (A-4):

$$
\begin{aligned}
\frac{\partial\Omega(\alpha)}{\partial\lambda^m(s)} - \frac{\partial\Omega(\alpha)}{\partial\lambda^p(s)} &= \frac{-\alpha\lambda^p}{(1-\alpha\lambda^m)^2} - \frac{-1}{1-\alpha\lambda^m} \\
&= \frac{\alpha(\lambda^p+\lambda^m)-1}{(1-\alpha\lambda^m)^2} < 0 \qquad \text{(A-7)}
\end{aligned}
$$

Using (A-7) in (A-6) allows the expression of the total derivative as:

$$
\frac{d\Omega(\alpha)}{d\alpha} = \overbrace{\frac{\partial\Omega(\alpha)}{\partial\alpha}}^{+} + \overbrace{\frac{d\lambda^m(s)}{ds(\alpha)}\frac{ds(\alpha)}{d\alpha}\left[\frac{\partial\Omega(\alpha)}{\partial\lambda^m(s)} - \frac{\partial\Omega(\alpha)}{\partial\lambda^p(s)}\right]}^{-} \qquad \text{(A-8)}
$$

[11] In fact, the conclusion here will hold whenever $\left|\frac{d\lambda^p(s)}{ds(\alpha)}\right| \geq \left|\frac{d\lambda^m(s)}{ds(\alpha)}\right|$.

The overall direction of the effect of changes in α depend on whether the direct or indirect effect dominates. The direct effect on $\Omega(\alpha)$ is positive due to the fewer bandits in the market caused by more merchants adopting the return price, as discussed above. The net indirect effect under matching functions with no thick-market externalities is positive because the smaller number of producers in the search market decreases $\lambda^p(s)$ by the same amount as the increase to $\lambda^m(s)$. The per-period payoff from meeting a producer is higher than from meeting a merchant so the net indirect effect of an increase in α reduces the value of search captured by $\Omega(\alpha)$.

Using the specific linear matching functions set out in Section 7.1, the total derivative for $\Omega(\alpha)$ with respect to α can be computed as follows.

$$\frac{d\Omega(\alpha)}{d\alpha} = \frac{\lambda[s'(\alpha)(1-\alpha\lambda) - s(\alpha)(1-\lambda)]}{(1+s(\alpha)-\alpha\lambda)^2} \tag{A-9}$$

$$\text{where} \quad s'(\alpha) = \frac{\partial s(\alpha)}{\partial \alpha}, \quad \text{and } s(\alpha) \text{ is given by equation (19).}$$

The overall direction of the derivative is in general undetermined, even using the linear matching functions. However, computation of the total derivative shows that $\Omega(\alpha)$ is monotonically increasing in α for a broad range of parameter values. For example, it holds for all combinations of parameters where the following conditions hold: $\gamma \in (0, .8]$; $\sigma \in [2, 10]$; and $\lambda \in (0, 1)$. It also holds for larger γ if σ is also increased, such as $\gamma = .95$ and $\sigma = 20$, with $\lambda \in (0, 1)$.

A.2 Optimal Merchant Pricing Under Asymmetric Pure Producer Strategies and Deterministic Search

Suppose that a fraction ω of informed producers always return to $\hat{p}$, and the remainder never do. This means that in the period after meeting a merchant for the first time, a fraction $\omega\gamma$ of producers return. In every subsequent period a fraction γ of those remaining clients return. This changes the merchant indifference price, making it higher because more producers return over time. To simplify calculations, suppose that consumer search is now deterministic. This indifference price can be found by examining optimal merchant deviation, as in the proof of Lemma 4.1. The optimal deviation from a constant price path $\hat{p}$ is shown here.

$$\begin{aligned} \Delta_{\hat{p}} &= k_\tau(1-\hat{p}) - \sum_{j=\tau+1}^{\infty} \omega\hat{p}k_\tau(\gamma\delta)^{j-\tau} \\ &= k_\tau(1-\hat{p}) - \frac{\omega\hat{p}k_\tau\gamma\delta}{1-\gamma\delta} \\ &= k_\tau\left(1 - \frac{\hat{p}[1-\gamma\delta(1-\omega)]}{1-\gamma\delta}\right) \end{aligned} \tag{A-10}$$

A combination of ω and $\hat{p}$ that makes $\Delta_{\hat{p}}$ equal to zero makes merchants indifferent between charging $\hat{p}$ and 1. Such combinations are given by the following equation, with $(\omega, \hat{p}) \in (0,1) \times (0,1)$.

$$\hat{p} = \frac{1-\gamma\delta}{1-\gamma\delta(1-\omega)} \tag{A-11}$$

A.3 The Producer Indifference Price with Three Market Prices

Suppose there are α merchants who charge $p > 1-\gamma\delta$ and β who charge $1-\gamma\delta$, with remaining merchants charging 1. Consider the decision of a producer who is informed in period t about a merchant who charges p. If the producer searches in period t and does not find a merchant charging $1-\gamma\delta$, then her expected payoff in period $t+1$ is the same as if she returned to her merchant. Denote this payoff as V_{t+1}^{p}. However, if the producer finds a merchant charging $1-\gamma\delta$, then her expected payoff will be different because knowledge of this merchant is valuable. Denote this payoff as $V_{t+1}^{1-\gamma\delta}$. The expected payoff for such an informed producer who returns to her merchant is set out in equation (A-12), and the payoff from search is shown in (A-13).

$$V_t^r = 1-p+\delta V_{t+1}^{p} \tag{A-12}$$

$$\begin{aligned} V_t^s =& \lambda^p\left(1+\delta V_{t+1}^{p}\right)+\alpha\lambda^m\left(1-p+\delta V_{t+1}^{p}\right)+\beta\lambda^m\left(\gamma\delta+\delta V_{t+1}^{1-\gamma\delta}\right) \\ &+\left[1-\lambda^p-(\alpha+\beta)\lambda^m\right]\delta V_{t+1}^{p} \\ =&\lambda^p+\alpha\lambda^m(1-p)+\beta\lambda^m\left(\gamma\delta+\delta V_{t+1}^{1-\gamma\delta}-\delta V_{t+1}^{p}\right)+\delta V_{t+1}^{p} \end{aligned} \tag{A-13}$$

Assume that a producer who is informed about more than one merchant will either remember all of the merchants or none of them in subsequent periods. Suppose, provisionally, that a producer is originally informed about a merchant charging p^*, such that returning to this merchant gives the same expected payoff as search. This means that the benefit to this producer if she finds a merchant who charges $1-\gamma\delta$ will be $\gamma\delta-(1-p)$ per period that the producer remains informed. This implies the following relationship.

$$\begin{aligned} V_{t+1}^{1-\gamma\delta}-V_{t+1}^{p^*} &= \sum_{j=t+1}^{\infty}(\gamma\delta+p^*-1)^{(j-t-1)} \\ &= \frac{\gamma\delta+p^*-1}{1-\gamma\delta} \end{aligned} \tag{A-14}$$

Substituting (A-14) into (A-13) yields:

$$V_t^s = \lambda^p + \alpha\lambda^m(1-p^*) + \beta\lambda^m\left[\gamma\delta + \delta\left(\frac{\gamma\delta + p^* - 1}{1-\gamma\delta}\right)\right] + \delta V_{t+1}^{p^*} \quad \text{(A-15)}$$

The producer indifference price can be found by setting $V_t^r = V_t^s$, and solving for p^*. Using equations (A-12) and (A-15) in this indifference condition gives the following determination of p^*.

$$\begin{aligned} 1 - p^* &= \lambda^p + \alpha\lambda^m(1-p^*) + \beta\lambda^m(\gamma\delta) + \beta\lambda^m\left[\gamma\delta + \delta\left(\frac{\gamma\delta + p^* - 1}{1-\gamma\delta}\right)\right] \\ p^* &= \frac{(1-\gamma\delta)\left[1 - \lambda^p - \alpha\lambda^m + \beta\lambda^m\delta(1-\gamma)\right]}{(1-\gamma\delta)(1-\alpha\lambda^m) + \beta\lambda^m} \end{aligned} \quad \text{(A-16)}$$

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