MODELLING PUBLIC TRANSPORT CORRIDORS WITH AGGREGATE AND DISAGGREGATE DEMAND

Sergio Jara-Díaz, Alejandro Tirachini and Cristián Cortés
Universidad de Chile

INTRODUCTION

Traditional microeconomic modeling of public transport (usually a single line) has been based on aggregated demand description, that is, total ridership along a line and average journey length. This way, aggregation is spatial rather than temporal, as most public transport systems are designed for the period of largest demand. With this approach, various authors have formulated optimization models whose goal is to maximize social benefit or minimize total costs (considering both users and operators), finding optimal levels for the decision variables such as frequency, vehicle size, number of bus stops and spacing between lines. Presently, it is feasible to capture more precise demand patterns in cases where either the payment device or the specialized infrastructure in buses and stations can provide not only passenger counts, but also exact identification of the origin and destination of the passenger journey (RFID cards along with wide-range devices on bus doors, cameras, etc.).

In this paper we examine the advantages of having different levels of aggregation regarding demand information in order to explore improvements on the recommendations and conclusions obtained from classical microeconomic models. Thus, we establish the optimal conditions for the relevant decision variables (frequency and vehicle size) on a public transport corridor with inelastic demand, in cases where the demand data is only available at an aggregated level (at the level of an entire line, or ridership per direction of movement) as well as cases in which it is feasible to obtain more detailed information on the demand structure, like origin-destination matrices at the level of bus stops or number of passenger who board and alight each bus at each stop.

In the following section analytical expressions for the optimal frequency and vehicle size are developed for each case. A theoretical comparison is presented in section 3 regarding optimal frequencies, and numerical experiments are conducted in section 4. We close with some relevant comments, conclusions and further research in the final section.

SINGLE LINE MODELS WITH AGGREGATE AND DISAGGREGATE DEMAND

Demand description

In what follows, a linear corridor is used as a representation of a generic public transport system, which can correspond to either a single isolated bus line or a line inserted within an existing network of fixed topology (number and position of bus routes). We assume that passengers arrive randomly to the stations, a reasonable assumption for high frequency corridors with headways of less than 10 minutes (Seddon and May, 1974; Danas, 1980). The demand is treated parametrically in the proposed formulations. Our purpose is to find the optimal value for the design variables (in these developments, optimal frequency f and vehicle size K) in order to maximize the social welfare of the system, which in case of inelastic demand is equivalent to minimizing the total cost, taking into consideration both users and operators. Users' costs will include both waiting and travel time of passengers, both very dependent of the line frequency, the latter because of the effect of boarding and alighting at stations. Access time to bus stops is not included in the system optimization since the number and location of bus stops along the corridor are assumed to be fixed.

As mentioned earlier, the major objective of this paper is to compare the analytical expressions obtained for the optimal design variables under different demand aggregation levels. First, a model based upon aggregated demand is formulated in two versions, one whereby only the total cycle demand y is know (denoted as Model 1 or M1) and a second model relying on aggregated demand information per direction of circulation, say y1 and y2 on direction 1 and 2 respectively (denoted as Model 2 or M2). In both cases the average journey length is assumed to be known.

Then, a model relying on disaggregated demand is presented (denoted as Model 3 or M3). Unlike M1 and M2, in this case we assume that a stop-to-stop OD matrix is available. From the more detailed demand data, the cost functions can be stated in a more comprehensive way, mainly the components that depend on the in-vehicle time, which is modeled just as an average value by the traditional aggregated demand models in the transit microeconomic literature. Besides, for the M3 specification, the line contains N stations in one direction (N-1 stretches), as shown in Figure 1. The operation directions are denoted direction 1 (from station 1 to N) and direction 2 (from station N to 1).

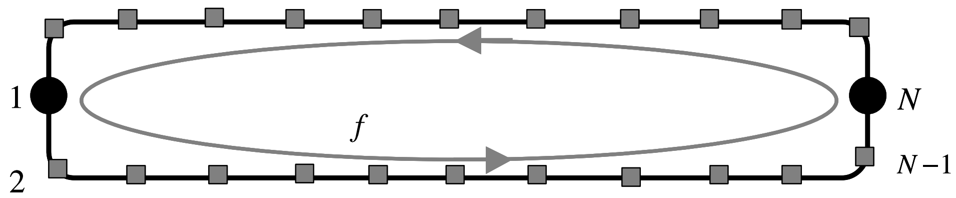

Figure 1: Generic public transport corridor

In the models we describe next, the following parameters are assumed known and fixed:
L: Length of the corridor [km]
R_k: Bus movement travel time under normal service between stations *k* and *k+1,* including acceleration and deceleration times at bus stops [min]
β: Marginal passenger boarding time [seg/pax]
λ_{kl}: Trip rate between stations *k* and *l* [pax/hour] (used for M3). This demand is assumed fixed over the studied period, defining a trip matrix of the form:

$$\begin{bmatrix} 0 & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \ddots & & \vdots \\ \vdots & & & \lambda_{N-1N} \\ \lambda_{N1} & \cdots & \lambda_{NN-1} & 0 \end{bmatrix}$$

Additionally, for M3 also, the following functions are defined:

- Passenger boarding rate at station k, whose destination is among stations l_1 and l_2 inclusive [pax/hour]: $\lambda_k^+(l_1, l_2) = \sum_{l=l_1}^{l_2} \lambda_{kl}$
- Passenger alighting rate at station k, whose destination is among stations l_1 and l_2 inclusive [pax/hour]: $\lambda_k^-(l_1, l_2) = \sum_{l=l_1}^{l_2} \lambda_{lk}$

Thus, from these functions we can define the following quantities:

- Passenger boarding rate at station k, direction 1: $\lambda_k^{1+} \equiv \lambda_k^+(k+1, N) = \sum_{l=k+1}^{N} \lambda_{kl}$
- Passenger alighting rate at station k, direction 1: $\lambda_k^{1-} \equiv \lambda_k^-(1, k-1) = \sum_{l=1}^{k-1} \lambda_{lk}$
- Passenger boarding rate at station k, direction 2: $\lambda_k^{2+} \equiv \lambda_k^+(1, k-1) = \sum_{l=1}^{k-1} \lambda_{kl}$
- Passenger alighting rate at station k, direction 2: $\lambda_k^{2-} = \lambda_k^-(k+1, N) = \sum_{l=k+1}^{N} \lambda_{lk}$

We assume that the boarding process dominates over the alighting process, and therefore, in the model only the first phenomenon (quantified through the parameter β) is considered.

From these definitions, we can relate the demand defined in the context of the three models as follows

$$y = y_1 + y_2 = \sum_{k=1}^{N} \left(\lambda_k^{1+} + \lambda_k^{2+} \right) \tag{1}$$

Besides, the vehicle arrival distribution to the stations is crucial for a correct computation of the passenger waiting time, which decreases as the headways become more regular. Following Delle Site and Fillipi (1998), two possible bus arrival patterns to stations are considered, namely scheduled service (regular headways) and random bus arrivals (Poisson process). The former is associated with systems with low variability in both running and passenger transfer times at bus stops, for instance special bus segregated corridors with efficient transfer operations at stops. In this case the average waiting time turns out to be half of the headway. Random arrivals are characteristic of networks with high variability in running times (poorly controlled bus systems); in this case the expected waiting time is equal to the average headway.

Recalling that the pursued objective is to minimize the total system cost expression with respect to the relevant design variables (frequency and vehicle size), next we define analytically the different cost components considered for the three models, from both the users and operator standpoints. The former comprises the waiting and in-vehicle time costs while the latter comprises two operational cost expressions associated with distance and time respectively.

Users' costs

The waiting time cost (C_w) is considered as the product between the total expected waiting time experienced by all customers and the subjective value of waiting time (P_w). If x is an auxiliary binary variable (equals to 1 if buses arrive Poisson, 0 if buses arrive at constant headway, as discussed earlier) we can compute the waiting time cost component as follows:

$$C_w = P_w \frac{1+x}{2} \frac{y}{f} \tag{2}$$

where f is the operational frequency, computed as the inverse of the headway. Expression (2) applies to all models, regardless of the demand aggregation level applying expression (1).

The in-vehicle time cost (C_v) is modelled as the product between the total expected in-vehicle time and the subjective value of the in-vehicle time (P_v).Unlike the waiting time component C_w, C_v adopts a different form depending on the demand aggregation level. For the M1 model, in-vehicle travel time is expressed as a fraction of the total cycle time, i.e. the ratio between the average journey length l and the total route length 2L (Mohring, 1972; Jansson, 1980; Jara Díaz and Gschwender, 2003). On the other hand, for the M2 model, it is possible to split this component per direction, as the average journey length on direction i (namely l_i) of journey over the route length (L). Moreover, the cycle time t_c is computed as the sum of the running time by direction (denoted as R_i) and the total stopping time at stations (computed as the total expected passenger boarding time). The latter component is computed as the product between the average number of passengers boarding a vehicle, y_i / f, and the marginal boarding time. Analytically, for M1:

$$C_v = P_v \frac{l}{2L}\left(\frac{y}{f}\beta + R_1 + R_2\right) y \tag{3}$$

and for M2:

$$C_v = P_v \left[\frac{l_1}{L}\left(\frac{y_1}{f}\beta + R_1\right) y_1 + \frac{l_2}{L}\left(\frac{y_2}{f}\beta + R_2\right) y_2\right] \tag{4}$$

For M3 no approximation is needed; travel time t_{kl} for each OD pair (k,l) is given by

$$t_{kl} = \begin{cases} \sum_{i=k}^{l-1}\left(R_i + \beta \frac{\lambda_i^{1+}}{f}\right) & si\, k < l \\ \sum_{i=l+1}^{k}\left(R_{i-1} + \beta \frac{\lambda_i^{2+}}{f}\right) & si\, l < k \end{cases} \tag{5}$$

Then, multiplying (5) times λ_{kl}, adding over all OD pairs and multiplying by P_v. the total in-vehicle travel time cost is obtained in monetary units. Analytically,

$$C_v = P_v \left\{ \sum_{k=1}^{N} \sum_{l=k+1}^{N} \left[\sum_{i=k}^{l-1} \left(R_i + \beta \frac{\lambda_i^{1+}}{f} \right) \right] \lambda_{kl} + \sum_{k=1}^{N} \sum_{l=1}^{k-1} \left[\sum_{i=l+1}^{k} \left(R_{i-1} + \beta \frac{\lambda_i^{2+}}{f} \right) \right] \lambda_{kl} \right\} \tag{6}$$

Operator cost

In this computation we take into account two components for the operator cost, as some items are better represented on a temporal basis (labor) and others over a spatial basis (running cost, maintenance, etc.). Following Jansson (1980) and Oldfield and Bly (1988), a linear dependency on the vehicle capacity K is assumed for the operator cost functions. Let us denote $c(K)$ as the cost per vehicle-hour (\$/veh-h) and $c^{\cdot}(K)$ as the cost per vehicle-kilometer (\$/veh-km). Analytically,

$$c(K) = c_0 + c_1 K \qquad c^{\cdot}(K) = c_0^{\cdot} + c_1^{\cdot} K \tag{7}$$

Therefore, the operator cost can be expressed as:

$$C_o = c(K)F + c'(K)vF \tag{8}$$

where v is the commercial speed and F is the fleet size given by frequency f times cycle time t_c discussed in section 2.2. Thus, (8) can be rewritten as a function of t_c and f as follows

$$C_o = c(K) f t_c + 2c'(K) f L \tag{9}$$

and finally,

$$C_o = f\left[c(K)\left(\frac{y}{f}\beta + R_1 + R_2\right) + 2c^{\cdot}(K)L\right] \tag{10}$$

which applies for the three models.

Optimal Value of the Frequency and the Vehicle Capacity.

The total cost minimization problem comprises the joint minimization of both users' and operators' costs, encompassing equations (2); (3), (4) or (6); and (10). In order to find the optimal value of the variables f and K, first order conditions (FOC) are applied. The vehicle capacity is adjusted in order to accommodate the demand of the most loaded segment along the corridor, q_{max}, which can be easily obtained from the OD matrix in M3. For models M1 and M2, this value must be assumed to be known (or accurately estimated). Then, by defining a safety factor $\eta \in (0,1]$, and computing $K = q_{\max}/\eta f$ the FOC yield the following values for the optimal frequency for M1, M2 and M3 respectively:

$$f^* = \sqrt{\frac{P_e \frac{1+x}{2} y + P_v \beta \frac{l}{2L} y^2 + c_1 \frac{q_{\max}}{\eta} \beta y}{c_0 (R_1 + R_2) + 2c_0^{\cdot} L}} \tag{11}$$

$$f^* = \sqrt{\frac{P_e \frac{1+x}{2} y + P_v \beta \left(\frac{l_1}{L} y_1^2 + \frac{l_2}{L} y_2^2\right) + c_1 \frac{q_{\max}}{\eta} \beta y}{c_0 (R_1 + R_2) + 2c_0^{\cdot} L}} \tag{12}$$

$$f^* = \sqrt{\frac{P_e \frac{1+x}{2} y + P_v \beta \left(\sum_{k=1}^{N} \sum_{l=k+1}^{N} \lambda_{kl} \sum_{i=k}^{l-1} \lambda_i^{1+} + \sum_{k=1}^{N} \sum_{l=1}^{k-1} \lambda_{kl} \sum_{i=l+1}^{k} \lambda_i^{2+} \right) + c_1 \frac{q_{\max}}{\eta} \beta y}{2\left(c_0 \sum_{k=1}^{N-1} R_k + c_0^{'} L \right)}} \quad (13)$$

ANALYTICAL COMPARISON

By looking at expressions (11), (12) y (13), we can observe that the only difference in the optimal frequency values appears in the term associated with the in-vehicle travel time within the square root because in-vehicle time cost is quadratic with the demand. Therefore, the relevant comparison involves

$$\frac{l}{2L} y^2 \quad (11a)$$

$$\frac{l_1}{L} y_1^2 + \frac{l_2}{L} y_2^2 \quad (12a)$$

$$\sum_{k=1}^{N} \sum_{l=k+1}^{N} \lambda_{kl} \sum_{i=k}^{l-1} \lambda_i^{1+} + \sum_{k=1}^{N} \sum_{l=1}^{k-1} \lambda_{kl} \sum_{i=l+1}^{k} \lambda_i^{2+} \quad (13a)$$

Note first that in systems where the stop time at stations is fixed, the three formulae (equations 11, 12 and 13) provide the same result. On the other hand, by writing the average journey time length as a function of the disaggregated quantities we obtain

$$l_1 = \frac{L}{y_1 (N-1)} \sum_{k=1}^{N} \sum_{l=k+1}^{N} \lambda_{kl} (l-k)$$
$$l_2 = \frac{L}{y_2 (N-1)} \sum_{k=1}^{N} \sum_{l=1}^{k-1} \lambda_{kl} (k-l)$$
$$l = \frac{L}{(y_1 + y_2)(N-1)} \left[\sum_{k=1}^{N} \sum_{l=k+1}^{N} \lambda_{kl} (l-k) + \sum_{k=1}^{N} \sum_{l=1}^{k-1} \lambda_{kl} (k-l) \right]$$

First, we focus our analysis in models M1 and M2. Let us d$l_i$ote D_i the total traveled distance along direction i, that is, $D_i = l_i y_i$ that yields

$$D_1 = \frac{L}{(N-1)} \sum_{k=1}^{N} \sum_{l=k+1}^{N} \lambda_{kl} (l-k)$$
$$D_2 = \frac{L}{(N-1)} \sum_{k=1}^{N} \sum_{l=1}^{k-1} \lambda_{kl} (k-l)$$

Then (11a) and (12a) can be rewritten as follows

$$\frac{l}{2L} y^2 = \frac{D_1 + D_2}{2L} (y_1 + y_2)$$
$$\frac{l_1}{L} y_1^2 + \frac{l_2}{L} y_2^2 = \frac{D_1}{L} y_1 + \frac{D_2}{L} y_2$$

Comparing these two expressions is equivalent to analyzing the sign of the expression $(y_1 - y_2)(D_2 - D_1)$. Then, expression (11) will be larger than (12) if either $(y_1 > y_2) \wedge (D_2 > D_1)$ or $(y_1 < y_2) \wedge (D_2 < D_1)$, which is equivalent to say that the traveled distance is the largest on the smallest demand direction of movement. On the other hand, if the largest demand direction matches the largest traveled distance (which is a very reasonable intuitive assumption), the most aggregated model (expression 11) underestimates the optimal frequency compared with that obtained from the model that differentiates both directions behind expression (12).

Now, let us compare formulae (12a) and (13a), i.e. M2 and M3. By writing (12a) in a disaggregated way we have:

$$\frac{l_1}{L}y_1^2+\frac{l_2}{L}y_2^2=\frac{D_1}{L}y_1+\frac{D_2}{L}y_2=\left(\sum_{k=1}^{N}\lambda_k^{1+}\right)\left(\sum_{k=1}^{N}\sum_{l=k+1}^{N}\lambda_{kl}\frac{l-k}{N-1}\right)+\left(\sum_{k=1}^{N}\lambda_k^{2+}\right)\left(\sum_{k=1}^{N}\sum_{l=1}^{k-1}\lambda_{kl}\frac{k-l}{N-1}\right)$$

The previous expression must be compared against (13a). A priori, it seems not possible to perform any comparison between both expressions under a generic situation. In order to approach to the solution, let us to examine some interesting particular cases,. First, let us examine the case of equal trip rates in each direction, that is,

$$\lambda_{kl}=\begin{cases}\lambda_1 & if\ k<l\\ \lambda_2 & if\ k>l\end{cases}$$

In such a case, (12a) and (13a) yield the same result given by

$$\frac{N^2\left(N^2-1\right)}{12}\left(\lambda_1^2+\lambda_2^2\right) \tag{14}$$

Moreover, expression (11a) becomes

$$\frac{N^2\left(N^2-1\right)}{24}\left(\lambda_1+\lambda_2\right)^2 \tag{15}$$

Note that (15) is always lower or equal than (14). Besides, in this case the average length of trip is the same in both directions.

Let us now see the case in which the number of stations equals three (N=3). In this case, (12a) and (13a) become, respectively (by simplicity, we analyze only direction 1):

$$\left(\lambda_{12}+\lambda_{13}+\lambda_{23}\right)\left(\frac{\lambda_{12}}{2}+\lambda_{13}+\frac{\lambda_{23}}{2}\right) \tag{12b}$$

$$\lambda_{12}\left(\lambda_{12}+\lambda_{13}\right)+\lambda_{13}\left(\lambda_{12}+\lambda_{13}+\lambda_{23}\right)+\lambda_{23}^2 \tag{13b}$$

The comparison then is reduced to

$$2\lambda_{12}\lambda_{23}+\lambda_{13}\lambda_{23} \tag{12c}$$

$$\lambda_{12}^2+\lambda_{23}^2+\lambda_{12}\lambda_{13} \tag{13c}$$

.

The relative value of the trip rates λ will determine the value of the optimal frequencies. Two particular cases:

a) If $\lambda_{12} = \lambda_{23}$, (12c) is equivalent to (13c) and both optimal frequencies turn out to be the same.
b) If $\lambda_{13} = \lambda_{23} \equiv \lambda_1$ and $\lambda_{23} \equiv \lambda_2$, then if $\lambda_1 > \lambda_2$, (12c) is larger than (13c) and consequently (12) is larger than (13); the other case is analogous.

Therefore, we can not establish *a priori* a ranking among optimal frequencies obtained from the different models. That ranking mostly depends upon the value of the matrix cells. Apparently, the more heterogeneous the matrix cells become, the higher the probability of getting different values for the optimal frequencies from the various proposed models. Therefore, it seems clear that the concentration of trips is a relevant issue when comparing the analytical recommendations obtained from the different aggregation level models. In the next section, we complement these analytical insights with the conclusions from some numerical examples.

NUMERICAL COMPARISON

In this section we conduct some numerical computations of the optimal design variables (frequency, vehicle and fleet size) obtained by applying the different demand aggregation models (M1, M2 and M3). We concentrate our analysis on two numerical cases. One is the study of a public transport corridor in Santiago, Chile, called Los Pajaritos, from where we have origin-destination demand matrix for the most demanded morning peak hour on a typical day of operation (MTT, 1998). Los Pajaritos is a corridor of 7 km., with 9 segments and 10 stations along each direction. The second example is the experiment proposed by Delle Site and Phillipi (1998), where also a detailed station to station origin destination demand matrix is available. The matrices used in both examples were properly generated from real data of affluence at the level of stations. In both examples, the assumed parameters are those shown in Table 1 next.

Table 1: Summary of the assumed model parameters

Parameter	Value
N	10
Running time between station [min]	1
Distance between stations [km]	0.5
Subjective value of waiting time[$/h]	2700
Subjective value of travel time[$/h]	900
c_0 [$/h]	1800
c_1 [$/h-seat]	30
c_0' [$/km]	400
c_1' [$/km-seat]	1
Boarding time β [sec/pax]	5
Safety factor η	0.9

The currency $ is Chilean Peso (CLP), AUD 1 ≈ CLP 425

Example 1: Corridor Los Pajaritos

The morning peak hour OD matrix and the associated load profile are shown in Table 2 and Figure 2 respectively. Load imbalance between both directions of movement is evident. Results are summarized in Table 3. Models M2 and M3 show similar results while M1 clearly underestimates not only the optimal frequency but also the optimal fleet size.

Table 2: OD matrix, Los Pajaritos corridor

	600	189	165	64	44	342	605	726	395
3620		11	10	4	3	20	35	42	23
790	38		5	2	1	10	18	22	12
1585	75	82		0	0	2	4	5	3
281	13	14	14		2	13	24	29	16
186	9	10	9	8		13	22	27	15
264	13	14	13	12	9		12	14	8
2631	125	135	130	117	86	107		36	19
337	16	17	17	15	11	14	18		67
4425	211	228	218	197	144	180	232	200	

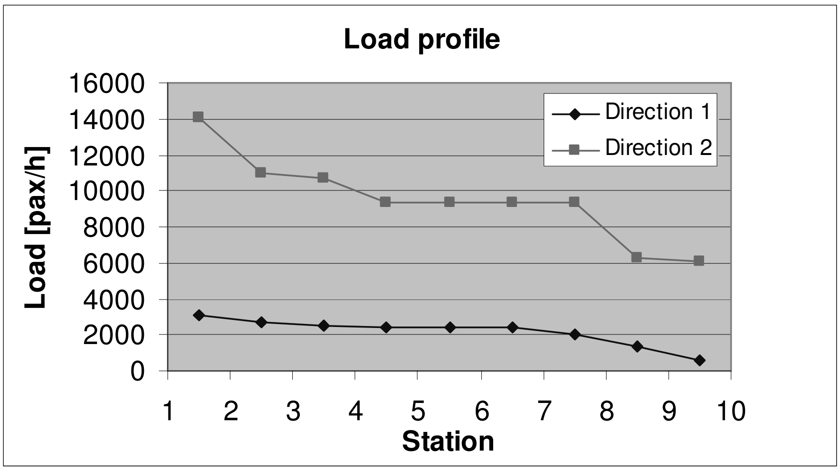

Figure 2: Load profile, Los Pajaritos corridor

Table 3: Optimal design variables, Los Pajaritos corridor

Model	Regime	C_w [$/min]	C_t [$/min]	C_o [$/min]	C_{tot} [$/min]	Frequency [veh/h]	Fleet size [veh]	Cap veh [pax/veh]
M1	Scheduled	2152	42594	23784	68530	215	94	74
M2		1876	40500	25834	68210	247	103	64
M3		1876	40500	25833	68209	247	103	64
M1	Poisson	4022	41525	24755	70302	230	98	69
M2		3561	39775	26701	70037	260	107	61
M3		3561	39775	26700	70036	260	107	61

Example 2: Delle Site and Phillipi (1998).

The morning peak hour OD matrix and its load profile are shown in Table 4 and Figure 3. In this case, demand imbalance between directions is concentrated in only a group of stations. Results for M1, M2 and M3 are summarized in Table 5, which shows that optimal frequency increases with the level of demand of information available, suggesting that lesser information will result in an underestimation of the optimal frequency. The fleet size remains unchanged. As the difference among models only happens in travel time, Table 6 shows the results of a sensitivity analysis increasing both travel time value *Pv* to 1800 $/h and travel time between stations to 3 minutes, which could be caused by traffic congestion. The difference in optimal frequency becomes larger and the optimal fleet size is now different for the three models.

Table 4: OD matrix, Delle Site and Phillipi (1998) example

	29	14	64	4	3	3	1	1	25
14		15	70	4	4	3	1	1	27
5	5		49	3	3	2	1	0	19
8	7	4		18	15	12	4	3	111
74	63	35	0		5	4	1	1	37
4	4	2	0	0		5	2	1	50
1	1	0	0	0	3		20	16	636
8	6	3	0	0	26	5		7	262
16	14	7	0	0	58	11	0		77
13	11	6	0	0	47	9	0	10	

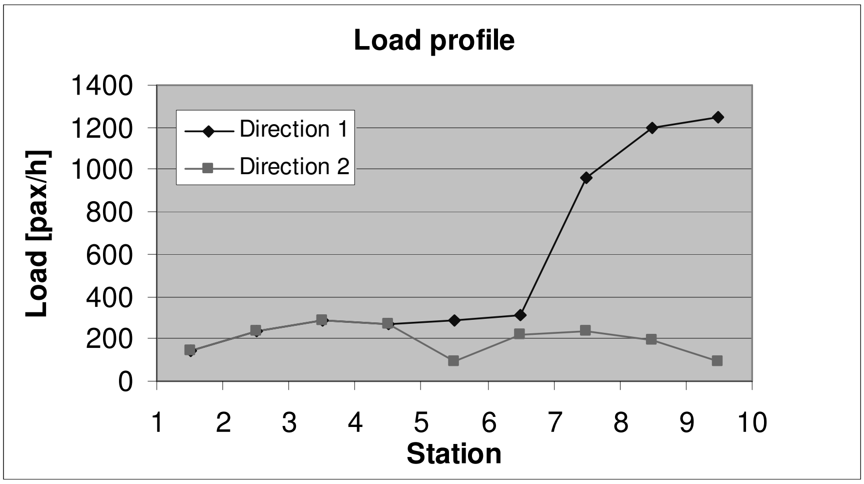

Figure 3: Load profile, Delle Site and Phillipi (1998) example

Table 5: Optimal design variables, Delle Site and Phillipi (1998) example

		C_w [\$/min]	C_t [\$/min]	C_o [\$/min]	C_{tot} [\$/min]	Frequency [veh/h]	Fleet size [veh]	Cap veh [pax/veh]
M1	Sched	1538	2563	2910	7011	31	13	45
M2	Sched	1491	2536	2975	7002	32	13	44
M3	Sched	1425	2499	3074	6998	34	13	42
M1	Poisson	2344	2354	3560	8258	41	16	35
M2	Poisson	2302	2341	3610	8253	42	16	34
M3	Poisson	2240	2324	3688	8252	43	16	33

Table 6: Modified optimal design variables, Delle Site and Phillipi (1998) example

		C_w [\$/min]	C_t [\$/min]	C_o [\$/min]	C_{tot} [\$/min]	Frequency [veh/h]	Fleet size [veh]	Cap veh [pax/veh]
M1	Sched	1545	11857	3869	17271	31	31	45
M2	Sched	1471	11773	4000	17244	33	33	43
M3	Sched	1375	11662	4194	17231	35	35	40
M1	Poisson	2460	11496	4541	18497	39	38	36
M2	Poisson	2384	11453	4646	18483	40	39	35
M3	Poisson	2278	11392	4806	18476	42	41	34

In both numerical examples the optimal frequencies obtained from M1 were systematically smaller than those obtained from M2 and M3; and these latter yield different results in the second example only. As a general rule, it seems that the better represented is transit demand, the larger the optimal frequency and the smaller vehicle size. This makes even more dramatic Jansson's (1984) observation regarding the underestimation of optimal frequency and overestimation of vehicle size when users' cots are not taken into account, considering that he was using an M1 type model. Both the analytical developments and the numerical examples suggest that the larger the cost associated with in-vehicle travel time, the more likely is that the various aggregated models predict different (smaller) values for the optimal frequencies.

CONCLUSIONS

In this paper we examine the advantages of having detailed demand information when using classical public transport microeconomic models. We have establish the optimal conditions for frequency on a public transport corridor with inelastic demand, in cases where the demand data is only available at an aggregated level (at the level of an entire line, or ridership per direction of movement) as well as cases in which it is feasible to obtain more detailed information on the demand structure, like origin-destination matrices at the level of bus stops or number of passenger who board and alight each bus at each stop.

We have developed two analyses, one which is purely analytical and another based on the result of applying the different aggregation level models to two examples in which real public transport data for peak periods are available. From the analytical part we clearly identified those terms in the optimal frequency expression that remarks the differences among the models. However, some conclusions with respect to the ranking among frequencies obtained in each model could only be obtained from the analysis of the empirical results. This comment motivates a deeper analysis of the analytical expressions, and also and more importantly, more numerical examples to validate our conclusions by discarding some possible cases that could be imposed numerically in the analytical developments, but not very likely to occur in reality. Overall, however, our results suggest that the underestimation of optimal frequency and overestimation of vehicle size when not accounting for users' costs is even more important than predicted by Jansson (1984).

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