Direct Simulation Studies of Suspended Particles and Fibre-filled Suspensions

A thesis submitted in fulfillment
of the requirements for the degree of
Doctor of Philosophy

School of
Aerospace, Mechanical and Mechatronic Engineering
The University of Sydney
August, 2003

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Dedication

> Of the second-rate rulers, people speak respectfully, saying, *'He has done this, he has done that'*. Of the first-rate rulers they do not say this. They say: *'We have done it all ourselves'*.
>
> Lao Tze 604 – 531 BC, *Tao Te Ching Ch.* 17

This work is dedicated to a mentor of my earlier years, John Van Opstal.

Declaration

I declare that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person where due reference is not made in the text.

Acknowledgements

I wish to thank Professors Nhan Phan-Thien and Xijun Fan for their effective and wise supervision in the early years of my candidature. I also wish to thank Professor Roger Tanner who supervised me for the remainder. Roger has always shown good humour and great understanding towards me. I appreciated his allowing me the indulgence to dictate my own pace and set my own course. This gave me the freedom to breath and explore which I value enormously. For the people and places I have met and seen, and the abilities and opportunities I have gained under their guidance, I am greatly in debt to my three supervisors.

I acknowledge and thank the financial support of the Australian Research Council (ARC), and Moldflow Inc. who arranged for the provision of the APA(I) scholarship awarded to me during my candidature.

I thank current and former members of the Rheology group who also acted in times as unofficial co-supervisors. In particular I acknowledge and thank Drs Yurun Fan, Hua Shu Dou and Shicheng Xue, who each helped substantially when I needed further technical guidance. I thank Dr Xiaolin Luo from CMIS CSIRO for his help with aspects in programming with *Fastflo*. I am grateful to Mark Waring and Optimus Pty Ltd, who have continued to support me while I finish my candidature, and have shown great interest in my career development.

The friendship of the Rheology group members were invaluable to me during my candidature. I appreciated the welcoming hand extended to me by Simin Nasseri, Surjani Uthayakumaran, Ping Jiang, Jane Liu, Marcus Newberry, Howard See, Damien Marik and Babatunde Fasheun. The many extracurricular activites enjoyed with 'the group' are the most memorable of my stay. Outside of the group, I am grateful for the great and loyal friends I have in Morteza Nateghi - the eternal student and Linux installer extraordinaire, and 'the captain', Darren Lysenko, my first choice on any away team. My friends kept me together during my candidature and so I thank them all once again for getting me over the line.

Finally my family took the burden of my years as a student with unending patience, understanding and good humour. To Wendy, Grace, Neal and my mum, I am eternally grateful.

Abstract

A new Direct Simulation fibre model was developed which allowed flexibility in the fibre during the simulation of fibre suspension flow. This new model was called the 'Chain-of-Spheres' model. It was hypothesised that particle shape and deformation could significantly affect particle dynamics, and also suspension bulk properties such as viscosity. Data collected from the simulation showed that *flexible* fibres in shear flow resulted in an order of $7-10\%$ bulk relative viscosity increase over the 'rigid' fibre result. Results also established the existence of a relationship between bulk viscosity and particle stiffness. In comparison with experimental results, other more conventional rigid fibre based methods appeared to underpredict relative viscosity. The flexible fibre method thus markedly improved the ability to estimate relative viscosity. The *curved* rigid fibre suspension also exhibited increased viscosity of the order twice that of the equivalent straight rigid fibre suspension. With such sensitivity to fibre shape, this result has some important implications for the quality of fibre inclusions used. For consistent viscosity, the shape quality of the fibres was shown to be important.

The 'Chain of Spheres' simulation was substantially extended to create a new simulation method with the ability to model the dynamics of arbitrarily shaped particles in the Newtonian flow field. This new '3D Particle' simulation method accounted for the inertial force on the particles, and also allowed particles to be embedded in complex flow fields. This method was used to reproduce known dynamics for common particle shapes, and then to predict the unknown dynamics of various other particle shapes in shear flow. In later sections, the simulation demonstrated inertia-induced particle migration in the non-linear shear gradient Couette cylinder flow, and was used to predict the fibre orientation within a diverging channel flow. The performance of the method was verified against known experimental measurements, observations and theoretical and numerical results where available. The comparisons revealed that the current method reproduced single particle dynamics with great fidelity.

The broad aim of this research was to better understand the microstructural dynamics within the fibre-filled suspension and from it, derive useful engineering information on the bulk flow of these fluids. This thesis represents a move forward to meet this broad aim. It is hoped that future researchers may benefit from the new approaches and algorithms developed here.

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