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Structural Evolution of Economy and Environment in Australia

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INTRODUCTION

Recent industrialisation of many of the earth regions has resulted in an increased stress being placed on the relationship between society and the natural environment. There has been an exponential growth of dependency on Earth's natural capital, whilst some of the earth's natural dissipative systems are approaching capacity. The material requirements of our livelihoods are far from abating and we are already confronted with surpassing limits to the atmospheric concentration of greenhouse gases.

Meanwhile, the engines of production of our societies – our economies - have been evolving to more and more complex entities. No longer is the link between production and consumption straightforward. Understanding the evolutionary tract of our economies from simple to multifaceted entities is essential to plan robust and sustainable futures. Whilst there is considerable depth of research on economic entities, less is known about the key changes for environmental flows. As public interest becomes more focussed on including environmental parameters in national planning decisions, the actors and processes crucial to driving change in our environmental impact need to be considered. The interpretation of the evolution of our economies in environmental terms is the central tenet of this research.

The purpose of this research is to help understand the key relationships in an evolving economic structure that are driving resource use and greenhouse gas emissions in Australia.

The approach involves looking at the factors and relationships that underpin economic growth in Australia. This research seeks to understand the changes in these factors by taking a historical perspective to the determinants of environmental impact through an investigation of structural changes over a period of 30 years. A detailed model is developed using the macro-economic tool of input-output analysis. This model makes it possible to investigate inter-relationships and intra-relationships between sectors of the environment, the economy and the population at disparate scales.

PRINCIPAL RESEARCH QUESTION

The principal aim of this thesis is to answer the research question:

- ❖ What are the principle drivers of environmental change in the structural relationships of Australia's economy?

This research is broken into two parts. The first section concerned with the set-up of the model, and the second section concerned with the application of the model.

PART I

Subsidiary research questions are:

- ❖ What are the modelling requirements needed to study the change in economy-environmental relationships?

These issues are addressed in Part I, Chapter 1 for the economy and Part I, Chapter 3 for the environment.

- ❖ What accuracy issues can affect current methods?

This issue is addressed in Part I, Chapter 2

Part I is designed to give a brief introduction to inter-industry or input-output modelling. Chapter 1 outlines the method of model construction in detail. The construction of the model required significant usage of optimisation and balancing techniques, which are discussed in Appendix 1-3. Chapter 2 investigates stability and predictability issues of the model developed in Chapter 1. The key purpose of Chapter 2 is to understand the stability and hence accuracy of the model under certain applications, which is integral to subsequent usage in Part II. Chapter 3 deals with environmental data generalised to the economic system – in particular greenhouse gas emissions and material flow.

PART II

Part II uses the model set up in Part I to investigate:

- ❖ What are applicable methods for studying the principle drivers of environmental change?

This issue is addressed in Part II, Chapter 1

- ❖ What macro-relationships in Australia have been driving changes in environmental-economic impacts?

This issue is addressed in Part II, Chapter 2 and Chapter 3

- ❖ What micro/entity-level structural relationships are driving environmental-economic impacts in Australia?

This issue is addressed in Part II, Chapter 5

- ❖ How can we interpret changing structural relationships within the evolution of Australia's economic and environment flows?

This issue is addressed in Part II, Chapter 6

Part II is designed to specifically answer the principal research question by the subsidiary research questions outlined above. An introduction to studying sustainable consumption is given in Chapter 1. This paper is set to establish the background for the subsequent chapters examining changing structural relationships. The decomposition of aggregate environmental

impact into macro-level components of the economic system is undertaken for material use and greenhouse gas emissions in Chapter 2 and 3 respectively. This analysis is taken in a temporal context to analyse the key drivers of change. Chapter 4 deals specifically with methodological issues of the analysis undertaken in Chapter 2 and 3. Chapter 5 returns to analysing changing structural relationships, but by considering a life-cycle focus in order to interpret entity-level changes in a direct production context. The overarching concerns of economic evolution and the implication for the environment are then drawn out in Chapter 6.

PART I

PART I - INTRODUCTION

In this section, the development of a time series of input-output tables is undertaken, with a generalisation for greenhouse gas emissions and material flow. This section contains 3 chapters, with 3 auxiliary papers included in an Appendix to Part I. The appendix includes published work of which the candidate is not principal author, but which refer to optimisation techniques that have been integral to the development of the model.

The first chapter, *A single-region input-output time-series for Australia* outlines the construction techniques used in developing a coherent temporal model on structural inter-relationships:

Chapter 1 - A Single Region Input-Output time series for Australia.

The reader is then referred to the optimisation process included in Appendix I. The stability of the model was then investigated:

Chapter 2 - Stability of Input-output coefficients and multipliers for Australia.

Chapter 1 and 2 have been submitted for publication as a single piece of work:

Wood, R. 2009. Construction and stability of an input-output time series for Australia. *Economic Systems Research*: Submitted.

The generalisation of the model for greenhouse gas emissions and material flow is undertaken in Chapter 3.

Chapter 3.1 - Embodied emission evolution in Australia.

Chapter 3.2 - Material Flow in Australia

Chapter 3.1 has been expanded and submitted in two parts as:

Wood, R and Dey, C. 2009. Australia's Carbon Footprint. *Economic Systems Research*: Submitted.

Wood, R and Dey, C. 2009. Embodied emission evolution in Australia *Energy Policy*: Submitted.

The model developed in Part I drew on a number of disparate data sources. A method was required to reconcile the data sources according to the specifications outlined in Chapter 1. As the derived model contained some 2.5 million variables and 500,000 constraints per year, existing methods were in need of refinement before their application to this problem. In the appendix to Part I, three papers are included that address the three major issues faced. Firstly, the variety of constraint data necessitated an adjustment to the traditionally used RAS method (Appendix 1).

Lenzen, M., B. Gallego, and R. Wood. 2006. A flexible approach to matrix balancing under partial information. *Journal of Applied Input-Output Analysis* 11 & 12.

Secondly, the effect of negative values is considered in the balancing process.

Lenzen, M., R. Wood, and B. Gallego. 2007. Some comments on the GRAS method. *Economic Systems Research* 19(4): 461-465.

Finally, the presence of conflicting information was a persistent issue in the model development. This issue is addressed in Appendix 3.

Lenzen, M., B. Gallego, and R. Wood. 2009. Matrix balancing under conflicting information. *Economic Systems Research*: 21(1): 23-44.

Statement of contribution

Part I of this thesis contains published work with co-authors within the appendix.

Appendix 1

Lenzen, M., B. Gallego, and R. Wood. 2006. A flexible approach to matrix balancing under partial information. *Journal of Applied Input-Output Analysis* 11 & 12.

The candidate is responsible for contributing to conceptualisation of the issues addressed, and various text.

Appendix 2

Lenzen, M., R. Wood, and B. Gallego. 2007. Some comments on the GRAS method. *Economic Systems Research* 19(4): 461-465.

The candidate is responsible for identifying target function issues (Section 2.2), the effect of the exponential in the target function (Section 4) and the iterative formulation (Section 5).

Appendix 3

Lenzen, M., B. Gallego, and R. Wood. 2008. Matrix balancing under conflicting information. *Economic Systems Research*: 21(1): 23-44.

The candidate is responsible for the organisation, contribution to the conceptualisation of the problem, and the case study for Australia.

Signed:



Richard Wood



Manfred Lenzen (principal co-author)

A SINGLE-REGION INPUT-OUTPUT TIME-SERIES FOR AUSTRALIA

Richard Wood

Wood, R. 2009. Construction and stability of an input-output time series for Australia. *Economic Systems Research*: Submitted.

Abstract

A time series of Australian input-output tables has been developed. This paper gives an overview of the construction techniques employed, including the major issues encountered. In order to delineate environmentally important processes, a range of fine detail commodity data was utilised in order to expand an initial input-output system from roughly 100 sectors into a temporally consistent 344 sectors. Confidential and inconsistent data was overcome using the KRAS method, a recent modification of RAS (Lenzen et al. 2006, 2008; Lenzen et al. 2007). Tables were estimated firstly from published IO data (17 tables over 30 years), secondly by adding in unpublished supply/use data (10 recent years), and finally by including National Accounts data (all years). This enabled a consistent time series to be constructed, whilst incorporating the highest level of detail from published and unpublished IO tables.

Keywords: Time series, input-output tables, RAS

1 Introduction

Due to the pervasive nature of environmental concerns, the repercussions of both impacts and mitigating policies are often felt throughout the whole of the economy and society. As such, a macro approach is often needed to capture the full impact of policy options (Pearson 1989; James 1978). Input-output analysis is such a macroeconomic tool and is designed to trace impacts across all sectors and markets. The strength of input-output analysis lies in its ability to capture the complex relationships underlying production and consumption processes within an economy, and its capability of encapsulating both direct and indirect impacts.

From the base representation of economic flows, the input-output framework can be expanded into an *open* model by incorporating other factors required for and produced by production. This can involve incorporating the inputs of material use into production, the output of emissions, or other quantities of concern. This enables the analysis of flows of these quantities into and between industries, and the effects of both industrial and final consumption.

Input-output analysis was first developed as a representation of economic flows by Wassily Leontief (1941), and was extended to allow quantitative analysis of environmental choices and flows in the late 1960s (see Daly 1968; Isard et al. 1967; Ayres and Kneese 1969; Leontief and Ford 1971). Various models were proposed, including integrated economic-ecological representations (Isard et al. 1967), but the most useful approach was taken by Leontief and Ford (1971) in which an open input-output framework was established in order to incorporate the forward and backward flows between economy and the environment.

Since that time, the use of the static input-output framework has been applied extensively to environmental problems, both as an investigative tool, and as a means in which to derive policy. Regional applications (see Hewings and Jensen 1989) were an initial focus, principally for planning objectives (Cumberland 1966; Beyars 1974), and resulted in a wide range of studies on levels of development (Miernyk 1965; Hewings 1981, 1982; Hewings and Romanos 1994; Jensen et al. 1987; Jackson et al. 1989); inter-regional linkages (Miernyk 1965; Batten 1982; Solomon and Rubin 1985; Sonis and Hewings 1998); and inter-sectoral linkages (Beyars 1974; Coffey 1996; Hughes and Holland D. 1994; Holland and Cooke 1992). Early integration of input-output analysis with environmental processes gained significant momentum in the 1970s when the oil crises hit, and demand grew for knowledge of the interdependency of the economy on energy resources. As a result, a number of early studies looked at the energy dependency of goods and services (Bullard and Herendeen 1975; Herendeen 1973; Bullard 1976; Bullard et al. 1978) and industrial energy use (Giarratani 1976).

Further development not only integrated consumption of environmental resources, but also included effects of residuals such as air pollution (see Chatterji 1975b; Leontief and Ford 1971; Chatterji 1975a). As time passed, and with scientific knowledge concerning anthropogenic global warming growing, greater emphasis has been placed on carbon dioxide, greenhouse gas and energy studies. The basic input-output framework has been applied on numerous occasions for analysing responsibility of energy use or emissions. Examples include sectoral responsibility for energy use in Australia (Shariful Islam and Morison 1992); industrial energy use in the UK (Jenne and Cattell 1983); household energy demand, (Pachauri and Spreng 2002), and the greenhouse gas emissions due to consumption patterns (Kim 2002; Lenzen 1998). Further applications include specific focuses of relationships between different sectors, and in the case of Machado and Miller (1997), the substitution of information for energy in economic terms. More recently the focus has been on assigning consumer responsibility for carbon emissions and other environmental impacts using the notion of life-cycle assessments or carbon and ecological footprint techniques (Wood and Garnett 2009; Wiedmann et al. 2008; Wiedmann et al. 2007; Hubacek and Giljum 2003; Bicknell et al. 1998).

With the increasingly complex problems being addressed in generalised input-output analysis, the data requirements have also increased. Two particular cases of these large data requirements now becoming popular are multi-regional input-output models and temporal studies. Statistical agencies originally relied wholly on survey techniques to update input-output tables. The expense and time for such surveys has resulted in a move to more mathematical approaches, especially since Stone's introduction of the RAS technique (Stone and Brown 1962). Now when national statistical offices publish updated input-output tables, they are usually based on a mix of survey and mathematical techniques. Originally, the RAS technique was a procedure for balancing row and column totals, but it was subsequently modified to incorporate data on superior elements of a new table. It has undergone significant development, most of which is outlined elsewhere (Lahr and de Mesnard 2004; Lenzen et al. 2006, 2008; Lenzen et al. 2007; Roberto Mínguez 2009; Jackson and Murray 2004).

In this work, a RAS variant is employed that allows for the consideration of any type of superior data that can be expressed as a linear combination of the table elements, and that act as constraints of the table to be balanced (Lenzen et al. 2006). The most innovative feature of the technique is that it can handle incompatibilities among these constraints (Lenzen et al. 2008).

The purpose of this paper is to outline the techniques employed and issues encountered in constructing a time series of input-output tables for Australia at a high level of detail. The main considerations are – creation of an initial estimate of the

system, incorporation of constraints at different levels of classification on the system and balancing or optimisation of the system.

The process followed is outlined below, beginning with an outline of the system structure (Section 2), a discussion on levels of aggregation (Section 3), the creation of the initial estimate (Section 4) and the application of constraint data (Section 5). The final balancing is briefly discussed (Section 6).

2 System structure

The structure of the open input-output system in basic prices utilised in this work is shown in Figure 1. Variable names are defined as per Table 1. In addition to the representation of the system in basic prices, additional margin and tax tables were required to be constructed to translate basic prices into purchaser's prices. These margin tables as well as a number of aggregate variables, such as GDP, are listed in Table 1. In principle they could be visualised as layers stacked on top of the basic price table. The definition follows that outlined in the UN handbook on Input-Output Accounting (United Nations Department for Economic and Social Affairs Statistics Division 1999).

3 Aggregation level

The more aggregated a dataset, the easier it is to construct, but the less useful it is for policy formation, and the less accurate it is for analysing specific flows. A reduction in industry resolution can significantly limit the resulting model for impact analysis and decision making, especially when non-economic indicators are integrated into the account. It is generally desirable to use the maximum level of detail in compiling the system, and aggregate results after impact analysis, if necessary.

The principal reason for working at a high level of detail is that physical data exogenous to the closed input-output system is often available at varying levels of detail for different industry sectors depending on the subject being investigated. For example, material data is detailed across mining and similar industries, energy data is detailed across manufacturing industries and employment data is detailed across service industries. In essence, we are seeking holistic accuracy rather than accuracy of individual elements (Gallego and Lenzen 2008; Jensen and West 1980; Hewings et al. 1988).

		Industry	Commodities	Final Demand	Total Output
		$1:n$	$1:m$	$1:b$	1
Industry	$1:n$	$\emptyset$	V_{ij}	$\emptyset$	x_j^I
Commodities	$1:m$	U_{ij}	$\emptyset$	y_{ij}	x_j^C
Primary Inputs	$1:a$	Q_{ij}^{ec}	$\emptyset$	y_{ij}^{Qec}	
Imports	$1:n$	M_{ij}	$\emptyset$	y_{ij}^M	
Total Inputs	1	$x_i^{I'}$	$x_i^{C'}$		
Factor inputs	$1:p$	Q_{ij}		y_{ij}^Q	

Figure 1 Structure of Australian open input-output system, basic prices

Variable	Dimension	Description
x^I	n	Gross output, industry
x^C	n	Gross output, commodity
V_{ij}	m,n	Make table, showing sales of Industry i of commodity j , basic prices, domestic flows.
U_{ij}	m,n	Use table, showing purchases of commodity j by industry i , basic prices, domestic flows.
y_{ij}	m,b	Final demand, showing purchases of commodity i by destination of final demand j , basic prices, domestic flows.
Q_{ij}^{ec}	a,n	Primary inputs into industry demand, economic, showing inputs into production of input type i into industry j , price invariant, domestic flows.
y_{ij}^{Qec}	a,b	Primary inputs into final demand, economic, showing inputs of input type i into final demand destination j , price invariant, domestic flows.
M_{ij}	m,n	Imports table, showing purchases of imported commodity i , by industry j , c.i.f. valuation, imported flows.
y_{ij}^M	m,b	Final demand of imported goods, showing purchases of imported commodity i , by final demand destination j , c.i.f. valuation, imported flows.
Q_{ij}	p,n	Factor (environmental) inputs into industry demand, showing environmental inputs into production of type i into industry j , domestic flows.
y_{ij}^Q	p,b	Factor (environmental) inputs into final demand, showing environmental inputs into production of type i into industry j , domestic flows.
T_{ij}	m+1,n+b	Total taxes tables, showing total taxes and subsidies paid on product i by industry j . Dimensions include m products and Re-exports, n industries and b categories of final demand.
T_{ij}^k	m+1,n+b	Individual taxes or subsidies, k , paid on product i by industry j . k tax tables include GST, Duties, Other taxes, Subsidies. Dimensions as above, Stack to T_{ij} .
P_{ij}	m+1,n+b	Total margins tables, showing total margins paid on product i by industry j . Dimensions as above.
P_{ij}^k	m+1,n+b	Individual margins, k , paid on product i by industry j . k margins tables include Wholesale, Retail, Hospitality, Road, Rail, Pipeline, Water, Air, Port, Marine, Gas. Dimensions as above. Stack to P_{ij} .
GDP	1x1	Overall sum of value added, a component of primary inputs.
U_{ij}^{pp}	m,n	Use table, showing purchases of commodity j by industry i , purchaser's prices, domestic flows.
y_{ij}^{pp}	m,b	Final demand, showing purchases of commodity i by destination of final demand j , purchaser's prices, domestic.

Table 1 Variables of open input-output system

To highlight this issue (compare Gallego and Lenzen (2008)), the greenhouse gas emissions allocated to beef cattle is considerably different to the greenhouse gas emissions allocated to sheep farming. If, as is common, the beef cattle industry is aggregated with sheep farming in a ‘livestock’ industry, there is no way to discern the difference in the greenhouse gas intensity of the two industries. The consumption of livestock products will embody a large quantity of greenhouse gases, even if in reality all consumption was from the sheep products and none from beef cattle. This is important in Australia’s situation, where large changes in sheep and cattle farming are occurring in opposing directions and are often destined for different levels of demand domestically and for export. In an aggregated framework, the specific detail of the different practices is not captured, whereas aggregation after analysis will include these differences. An empirical study demonstrating this effect is found in Lenzen et al. (2004).

In this work, the task has been undertaken of creating a standardised procedure to generate the most detailed generalised input-output framework possible. As a first step, choices of detail had to be made with respect to the industry classification. In Australia, industry classification detail is firstly limited by the input-output industry classification, which is generally around 100 industry sectors, and secondly limited by confidentiality of the input-output product classification (IOPC) of around 1000 products, as defined by the ABS (*Australian National Accounts, Input-Output Tables (Product Details), 2001-02, Cat. no. 5215.0.55.001* (Australian Bureau of Statistics 2006)). The product data differs from the industry data in that whilst there is greater detail in the supply by product, the use is still by the industry classification. The result is a rectangular Supply and Use matrix of dimension circa 1000x100. Further, there is a higher level of confidentiality in the product data, as well as a greater uncertainty with totals not always balanced.

Hence, in order to have a temporally consistent set of input-output data, a selection of 344 commodities was taken from the IOPC. This level was specific enough for most available data on primary and environmental inputs, whilst enabling tractability of classification changes and handling of confidential cells.

4 Creation of initial estimate

The Australian input-output tables have historically been subject to numerous classification changes, and the first step in creating an initial estimate of the full time series was to create a classification as stable as possible across the years. Due to both confidentiality issues and classification issues, data points for some commodities were only available for a select number of years. In order to obtain tables for years without real data points, an initial estimate was required. The options available to create this initial estimate were:

- Assume data points are constant from known years to unknown years
- Linear regression
- Using aggregated (industry) data
- Other form of regression

The first option, assuming data points are constant, increases the error when some commodity data is known and some commodity data is unknown. In essence, price and volume changes would be applied to the known commodity data, but no estimates of changes would be applied to the unknown commodity data. As some commodity data is confidential for periods up to 20 years, the lack of consideration of price and volume changes would cause considerable error. The second option of using linear regression was found to be lacking, as any rapid economic changes would be reflected in the regression by negative or infinite estimates. Limits could be placed on the regression, but in reality, very little economic growth is linear anyway. The third option, using aggregated data such as the more aggregated level industry data can only occur if the industry data is non-confidential, which is not always the case. Further, if the industry data is not confidential, it is will already be applied in the constraints of the balancing process (see next section).

In order to overcome these issues, the fourth option of applying some other form of regression was taken. Due to the issues previously identified, a stable, sign-preserving form of regression was desired. Assuming the economy has roughly undergone exponential growth, the exponential form was taken, with a limiting factor to prevent very large estimates. Hence, for known Z (known commodity data) at time t , we estimate a , such that $a < \alpha$, for some α – the limiting factor, chosen to reflects estimated limits to growth rates and to maintain stability over the time series. Calculation is done for each i flow in the input-output system, across all time periods j where Z_{ij} is known.

$$Z_{i,j=known} = \exp(a_i t_{j=known})$$

This is solved for a_i for each i flow in the input-output system using least squares for known Z_i , Z_0 is estimated across the whole time series j :

$$Z_{0i,j} = \exp(a_i t_j)$$

Z_0 is the initial estimate of the full input-output system. Rather than working in matrix formation, Z_0 is the vectorised form of the input-output system – that is, it is a one dimensional representation of every flow of the input-output system. Z_0 includes the Supply matrix, Use matrix, Imports matrix, and all margins and taxes matrices. The size of Z_0 is hence dependent on the number and size of the variables of the

input-output system. In this case, with 344 commodities, 344 industries, 5 primary inputs, 7 levels of final demand and 17 margin and tax tables as well as the Supply, Use and Imports tables, the size of $\mathbf{Z}_0$ is circa 2.5 million variables. $\mathbf{Z}_0$ is defined as:

$$\mathbf{Z}_0 = \text{vectorisation}(\mathbf{V}_0; \mathbf{U}_0; \mathbf{M}_0; \mathbf{P}_0^k; \mathbf{T}_0^k)$$

As previously mentioned, the detail on commodities only refers to sales of goods, not purchases. Data on purchases in the initial estimate are included in three ways: constraints on possible/impossible flows were included from a session utilising expert knowledge (Foran et al. 2003); other flows were pro-rated from total production; and homogenous production was assumed within an industry for each commodity. In essence, this is a diagonalisation of the Make matrix within each industry - compare Mastoris et al. (2002). This has the effect of aligning industry output closer to commodity output. Industry and commodity output is not matched, however, as no assumption is made on the homogeneity of production outside the commodity's respective industry (i.e. the off-diagonal elements of the Make matrix outside the industry).

5 Constraints

Once the initial estimate was fully populated, constraints were applied across the full time series. Two types of constraints were applied – 1) accounting or balancing constraints, to ensure such relationships such as gross output equals gross input were maintained; and 2) data constraints, such as industry, commodity and national accounts data. A list of constraints $\mathbf{C}_r$ is constructed, where r refers to a subset of constraints rather than individual constraints. $\mathbf{C}_r$ is constructed alongside a concordance matrix $\mathbf{G}$ that links the constraint values $\mathbf{C}_r$ with the initial estimate $\mathbf{Z}_0$. $\mathbf{G}$ is simply a sparse binary matrix showing the existence or non-existence of a relationship between $\mathbf{C}_r$ and each element of $\mathbf{Z}_0$. $\mathbf{G}$ is derived from *concordance* matrices $\mathbf{W}$. Concordance matrices are defined as a rectangular matrix showing the existence of a relationship between constraints and/or variables. The usual format of a concordance matrix is a matrix of binary elements with rows matching the external classification, and columns matching the internal classification. Concordance matrices are not applied as in general matrix multiplication, but instead in set theory. A matrix multiplication of a row concordance $\mathbf{W1}$ and a column concordance $\mathbf{W2}$ to $\mathbf{Z}$ (such as $\mathbf{W1}*\mathbf{Z}*\mathbf{W2}'$) would give the union of the indices of $\mathbf{Z}$ represented in $\mathbf{W1}$ and $\mathbf{W2}$. Instead we want the conjunction of the sets of indices of $\mathbf{W1}$ and $\mathbf{W2}$. That is, we want $\mathbf{W1}\wedge\mathbf{W2}$, and it is applied to $\mathbf{Z}$ as a means of indexing, i.e. $\mathbf{Z}_{\mathbf{W1}\wedge\mathbf{W2}}$.

5.1 Accounting Constraints

The accounting or balancing constraints of the input-output system are subsequently defined in this section.

Firstly define GDP relationships, by income approach - summation of value added (gross operating surplus, compensation of employees, taxes on production) and taxes on products:

$$C_1 \Rightarrow \sum_{i=1,a} \sum_{j=1,n} Q_{ij}^{ec} + \sum_{i=1,a} \sum_{j=1,b} y_{ij}^{Qec} = \mathbf{GDP}$$

And expenditure approach – summation of domestic final demand and exports, minus imports (excluding re-exports):

$$C_2 \Rightarrow \sum_{i=1,m} \sum_{j=1,b} y_{ij} + \sum_{i=1,a} \sum_{j=1,b} y_{ij}^{Qec} - \left(\sum_{i=1,m} \sum_{j=1,n} M_{ij} - \sum_{j=1,b} y_{REX,j}^M \right) = \mathbf{GDP}$$

Match column totals of supply and use matrices:

$$C_3 \Rightarrow \sum_{i=1,m} V_{i,1:n} = \sum_{i=1,m} U_{i,1:n} + \sum_{i=1,a} Q_{i,1:n}^{ec}$$

And match row totals of supply, imports and use matrices

$$C_4 \Rightarrow \sum_{j=1,n} V_{1:m,j} + \sum_{j=1,n} M_{1:m,j} + \sum_{j=1,b} y_{1:m,j}^M = \sum_{j=1,n} U_{1:m,j} + \sum_{j=1,b} y_{1:m,j}$$

Supply of imports is equal to competing imports into production (PIIMP)

$$C_5 \Rightarrow \sum_{j=1,n} M_{1:m,j} + \sum_{j=1,b} y_{1:m,j}^M = U_{PIIMP,1:n}$$

And for re-exports (REX),

$$C_6 \Rightarrow y_{REX,1:b}^M = y_{PIIMP,1:b}$$

Net taxes (less subsidies) on products are the aggregation of the respective tables on taxes, including taxes on re-exports.

$$C_6 \Rightarrow \sum_{k=1,4} \sum_{i=1,m+1} T_{i,1:n+b}^k = Q_{Tax,1:n}^{ec} + y_{Tax,1:b}^{Qec}$$

Net margins are not simply the aggregation of the d margins tables, but the redistribution of the margins. Using a concordance vector W^k for $k=1:d$ margins, which maps margin table totals to margin rows of P.

$$C_7 \Rightarrow \mathbf{P}_{1:m,1:n+b} = \sum_{k=1:d} \mathbf{P}_{1:m,1:n+b}^k - \sum_{i=1:m} \mathbf{P}_{\mathbf{w}^k,1:n+b}$$

Further, margin commodities are 100% related to each margin (i.e. there are no non-margin components):

$$C_8 \Rightarrow \bigvee_{k=1:d} \mathbf{P}_{\mathbf{w}^k,1:n+b} = 0$$

According to Australian input-output tables (Australian Bureau of Statistics 2008), margins on re-exports are excluded from each margin commodity and included in the total. i.e.

$$C_9 \Rightarrow \mathbf{P}_{REX,1:n+b} = \sum_{k=1:d} \mathbf{P}_{REX,1:n+b}^k$$

National accounts data is mainly available in purchaser's prices; hence, a Use table in purchaser's prices is constructed

$$C_{10} \Rightarrow \mathbf{U}_{1:m,1:n}^{PP} + \mathbf{y}_{1:m,1:b}^{PP} = \mathbf{U}_{1:m,1:n} + \mathbf{y}_{1:m,1:b} + \mathbf{T}_{1:m,1:n+b} + \mathbf{P}_{1:m,1:n+b}$$

Due to varying classifications used, the Supply matrix is kept as homogenous as possible. Hence, for the respective industry data, a concordance matrix $\mathbf{W}^{comm,ind}$ mapping commodities to industry units is used such that:

$$C_{11} \Rightarrow \mathbf{V}_{1:m,1:m} = \sum_{j=1,n} \mathbf{V}_{\mathbf{w}^{comm,ind}_{1:m,j},j}$$

5.2 Benchmark data

The 16 years of published IO data are referred to as the “benchmark” years. In these years, the data available includes balanced input-output tables as well as commodity data. Some conflicts are evident in the commodity data and within the input-output tables such that the system whilst close, is not fully in equilibrium. The commodity data and the industry level data of the input output tables is used as constraints on the initial estimates of the IO system, subject also to the accounting constraints above. Hence, we use $\mathbf{Z}_{Bm}$ to represent a component of the IO system (such as $\mathbf{U}$, $\mathbf{P}$, $\mathbf{T}$, $\mathbf{V}$) in original format from the benchmark years, and $\mathbf{W}_{Bm}^{n,ind}$ to represent the concordance matrix from the benchmark data to the estimated IO system ($\mathbf{Z}^*$). A dash ' refers to the transpose.

For square systems, such as the transactions of the Supply or Use matrix, where the classification of the products m is equal to that of industries n :

$$C_{12} \Rightarrow \mathbf{Z}_{i,j}^{*m,n} = \mathbf{Z}_{Bm, \mathbf{W}_{Bm}^{n,ind}}^{ind,ind} (\mathbf{W}_{Bm}^{n,ind})'$$

For rows (e.g. Value Added):

$$C_{13} \Rightarrow Z_{i,j}^{*a,n} = Z_{Bm,i,(\mathbf{w}_{Bm}^{n,ind})}^{a,ind}$$

For columns (e.g. Final Demand):

$$C_{14} \Rightarrow Z_{i,j}^{*m,b} = Z_{Bm,\mathbf{w}_{Bm}^{n,ind},j}^{ind,b}$$

And for commodity data (*Australian National Accounts, Input-Output Tables (Product Details), Cat. no. 5215.0.55.001*), using a concordance matrix $\mathbf{W}_{Bm}^{m,comm}$ to map between estimated product data and benchmark commodity data $Z_{Bmi,j}^{comm,b}$. As benchmark commodity data is only available by row, with columns still in industry classification, the benchmark data $Z_{Bmi,j}^{comm,b}$ is pre-multiplied by the commodity concordance, and post-multiplied by industry concordance:

$$C_{15} \Rightarrow Z_{i,j}^{*m,n} = \times Z_{Bm,\mathbf{W}_{Bm}^{m,comm},(\mathbf{w}_{Bm}^{n,ind})}^{comm,ind}$$

Categories of final demand (or other columns) are simply pre-multiplied:

$$C_{16} \Rightarrow Z_{i,j}^{*m,b} = Z_{Bm,\mathbf{W}_{Bm}^{n,comm},j}^{comm,b}$$

There is no additional row data for value added at the commodity level.

The constraints $C_{12} - C_{15}$ are calculated for each available table – i.e. for each of the Supply, Use, Imports, Margins and Taxes tables.

5.3 Supply/Use data

A time series of Supply Use data is available for Australia, updated according to current methodology, at the conclusion of each year¹. The most recent data is from 1994-5 to 2003-04. The data is published as a Supply table in basic prices and a Use table in purchaser's prices for 225 commodities and 52 industries. In addition, there are commodity totals for imports, subsidies, taxes, and for a breakdown into 10 margins.

Similar to the benchmark data, constraints are constructed using an industry $\mathbf{W}_{SU}^{n,ind}$ and commodity $\mathbf{W}_{SU}^{m,comm}$ concordance matrix on the original Supply/Use data $Z_{SU,i,j}^{comm,ind}$

¹ Currently under review by the ABS.

$$C_{17} \Rightarrow Z_{i,j}^{*m,n} = Z_{SU, \mathbf{w}_{SU}^{m,comm}}^{comm,ind}(\mathbf{w}_{SU}^{n,ind})$$

$$C_{18} \Rightarrow Z_{i,j}^{*a,n} = Z_{SU,i,(\mathbf{w}_{SU}^{n,ind})}^{a,ind}$$

$$C_{19} \Rightarrow Z_{i,j}^{*m,b} = Z_{SU, \mathbf{w}_{SU}^{m,comm},j}^{comm,b}$$

The Supply/Use constraints are constructed on the non-confidential points of the Supply and Use tables, the totals (row/column) of the Supply and Use tables and on the row totals of the imports, margins and taxes tables.

5.4 National Accounts data

Finally, National Accounts aggregates are available across the whole time series. Constraints are hence constructed from all available data. The applicable National Accounts data {Australian Bureau of Statistics, 2007 #3797} includes:

Value Added by 18 industries:

$$C_{20} \Rightarrow \sum_{i=1,a} Q_{i,j}^{ec} = Q_{a,(\mathbf{w}_{NA}^{n,18})}^{ec,NA}$$

where $\sum_{i=1,a} Q_{i,j}^{ec}$ is the value added by industry, $Q_{a,1:18}^{ec,NA}$ is the value added by industry in the National Accounts classification (18 sectors), and $(\mathbf{w}_{NA}^{n,18})$ is the concordance matrix between the 18 sectors of the National Accounts and the ISAPC.

GDP (1 data point)

$$C_{21} \Rightarrow GDP = GDP^{NA}$$

Total Final demand by destination (7 data points), in purchaser's prices

$$C_{22} \Rightarrow \sum_{i=1:n} \mathbf{y}_{ij}^{pp} = \mathbf{y}_{1:7}^{T,NA}$$

where the sum over final demand rows $\sum_{i=1,n+a+1} \mathbf{y}_{ij}^{pp}$ is equal to final demand category totals of the national accounts $\mathbf{y}_{1:7}^{T,NA}$.

Total Competing imports of the National Accounts $M^{T,NA}$ (1 data point) is the sum over all imports

$$C_{23} \Rightarrow \sum_{i=1:m} \sum_{j=1:n} \mathbf{M}_{ij} + \sum_{i=1:m} \sum_{j=1,b} \mathbf{y}_{ij}^M = M^{T,NA}$$

Total inputs into production by type (Compensation of employees, Gross Operating Surplus, Taxes on products, Taxes on production). (4 data points)

$$C_{24} \Rightarrow \sum_{j=1:n} \mathbf{Q}_{1:a,j}^{ec} + \sum_{j=1:b} \mathbf{y}_{1:a,j}^{Qec} = \mathbf{Q}_{1:a}^{ec,NA'}$$

Household final demand in purchaser's prices is provided by 38 commodities in the National Accounts $\mathbf{y}_{1:38,1}^{NA}$, and is related to estimated final demand $\mathbf{y}_{1:n,1}^{pp}$ by the concordance matrix $\mathbf{W}_{y,NA}^{n,38}$:

$$C_{25} \Rightarrow \mathbf{y}_{1:n,1}^{pp} = (\mathbf{W}_{y,NA}^{n,38}) \times \mathbf{y}_{1:38,1}^{NA}$$

Compensation of employees by industry (17 data points) is calculated similarly:

$$C_{26} \Rightarrow \mathbf{Q}_{1,1:n}^{ec} = \mathbf{Q}_{1,(\mathbf{W}_{Q,NA}^{n,17})}^{ec,NA}$$

And for gross operating surplus by industry (17 data points)

$$C_{27} \Rightarrow \mathbf{Q}_{2,1:n}^{ec} = \mathbf{Q}_{2,(\mathbf{W}_{Q,NA}^{n,17})}^{ec,NA}$$

Agricultural income in the National Accounts (8 data points) is defined in more detail for gross production:

$$C_{28} \Rightarrow \sum_{i=1:m} \mathbf{U}_{i,1:n}^{pp} + \sum_{i=1:a} \mathbf{Q}_{i,1:n}^{ec} = \mathbf{U}_{Ag,T,(\mathbf{W}_{Ag,NA}^{m,8})}^{pp,NA}$$

intermediate inputs:

$$C_{29} \Rightarrow \sum_{i=1:m} \mathbf{U}_{i,1:n}^{pp} = \mathbf{U}_{II,(\mathbf{W}_{Ag,NA}^{n,8})}^{pp,NA}$$

value added:

$$C_{30} \Rightarrow \sum_{i=1:a, i \neq TAX} \mathbf{Q}_{i,1:n}^{ec} = \mathbf{Q}_{VA,(\mathbf{W}_{Ag,NA}^{n,8})}^{ec,NA}$$

taxes on products:

$$C_{31} \Rightarrow \sum_{i=TAX} \mathbf{Q}_{i,1:n}^{ec} = \mathbf{Q}_{TAX,(\mathbf{W}_{Ag,NA}^{n,8})}^{ec,NA}$$

compensation of employees:

$$\mathbf{C}_{32} \Rightarrow \sum_{i=COMP} \mathbf{Q}_{i,1:n}^{ec} = \mathbf{Q}_{COMP,(\mathbf{w}_{Ag,NA}^{n,8})}^{ec,NA}$$

These 32 subsets of Constraints $\mathbf{C}_r$ make up the full list of constraints $\mathbf{C}$. $\mathbf{C}$ is constructed for each year of the time series. These constraints act on the initial estimate as outlined in the next section. In this system, and ideally, every value of the $\mathbf{Z}$ is constrained, as we utilise powerful constraints such as row/column sums, which apply to all values of the applicable row/column. Whilst such coverage is most necessary, we are also interested in how many values each constraint applies to (i.e. how specifically constrained are the values). Such an assessment is difficult to quantify when holistic accuracy is concerned, and where most effort is put on estimating sensitive values. Specific constraints to insensitive values are worthless if there are no specific constraints to sensitive values. I address this question empirically in the next chapter.

6 System Balancing

The initial estimate $\mathbf{Z}_0$ does not necessarily (and is unlikely to) satisfy the list of constraints $\mathbf{C}$. Hence, some type of balancing or optimisation process is required in order to obtain $\mathbf{Z}^*$ such that:

$$\mathbf{Z}^* = \mathbf{Z}_n \Leftrightarrow \mathbf{C} = \mathbf{G} \times \mathbf{Z}_n$$

Where $\mathbf{Z}_n$ is solved iteratively through balancing $\mathbf{C}$ and $\mathbf{G} \times \mathbf{Z}$ (such that overall information gain in $\mathbf{Z}_n$ is reduced). The most commonly used technique is that of RAS (Stone and Brown 1962) which has undergone significant development, and whose critical application in the form of KRAS to the Australian case is shown elsewhere (see Appendix 1-3 or Lenzen et al. 2006, 2008; Lenzen et al. 2007).

As shown elsewhere (Junius and Oosterhaven 2003; Lenzen et al. 2007; Oosterhaven 2005) RAS and variants are a form of mathematical optimisation process where a logarithmic form is used to minimise a target function t that represents the distance of $\mathbf{Z}_0$ from $\mathbf{Z}_n$:

$$t(\mathbf{Z}_n, \mathbf{Z}_0) = \sum_i |Z_{n,i}| S_i \ln(S_i)$$

With

$$S_i = \frac{Z_{0,i}}{e * Z_{n,i}}$$

As our data is from different sources, we are not always assured of being able to actually meet the criteria:

$$\mathbf{C} = \mathbf{G} \times \mathbf{Z}_n$$

We thus need to also minimise differences between $\mathbf{C}$ and $\mathbf{G} \times \mathbf{Z}_n$ according to estimates of the reliability of the constraints, which we call $\mathbf{sC}$. We thus define:

$$\mathbf{C} = \mathbf{G} \times \mathbf{Z}_n + \boldsymbol{\alpha} \cdot \mathbf{sC}$$

Where $\boldsymbol{\alpha}$ is a vector of the same size as $\mathbf{sC}$ and the dot shows scalar multiplication. We then modify our target function to

$$t(\mathbf{Z}_n, \mathbf{Z}_0) = \sum_i |Z_{n,i}| S_i \ln(S_i) + \sum_k |\alpha_k|$$

The implementation of this approach in an iterative fashion is the subject of an alternate paper (Lenzen et al. 2008).

This balancing process is undertaken for each year of the input-output system, in this case, covering the time period 1975-2005. System convergence was obviously very specific to correct definition of all balancing constraints. Most notably GDP has the strongest affect due to its simultaneous affect on row/column balances through the income and expenditure approaches of measuring GDP. Convergence (occurring when differences between $\mathbf{C}$ and $\mathbf{G} \times \mathbf{Z}$ are less than a specified threshold) was achieved after several thousand iterations of the KRAS algorithm. For more general results on the performance of the KRAS algorithm in these problems, the reader is referred to Lenzen et al. 2008. The balancing process is undertaken for each year of the input-output system, in this case, covering the time period 1975-2005. The characteristics of the resultant model are analysed in the next chapter.

6.1 Constant price tables

Constant price tables were also required, and an additional step was taken once a time series of current price tables was established. The double deflation method (United Nations Department for Economic and Social Affairs Statistics Division 1999) was used on the current price input-output tables to give a first estimate of constant price tables. National accounts data in the same format as that for current price tables was then applied in a secondary balancing step. This ensured consistency with aggregate data, as well as giving greater resolution from the application of the price indices.

7 Conclusion

In this work, the construction of a time series of Australian input-output tables was described. A range of fine detail commodity data was utilised in order to utilise a temporally consistent set of 344 sectors. Initial estimates were generated through an exponential regression, and subsequently balanced by applying a range of accounting constraints and data sources. Tables were constrained firstly from published IO data (17 tables over 30 years), secondly by adding in unpublished supply/use data (10 recent years), and finally by including National Accounts data (all years). This enabled a consistent time series to be constructed, whilst incorporating the highest level of detail from published and unpublished IO tables in order to delineate environmentally important processes.

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STABILITY OF INPUT-OUTPUT COEFFICIENTS FOR AUSTRALIA

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Abstract

This chapter analyses the stability over time of input-output coefficients. Similar to the work in Dietzenbacher and Hoen (2006), a time series of input-output tables is analysed. The issue of temporal stability of coefficients and multipliers was investigated under the Leontief and Ghosh models of supply/demand. The predictability of the models was then examined under updated final demand or primary input components. Finally, the predictability of the input-output model over varying time scales and under different updating techniques is also investigated.

Keywords: Input-output analysis; Stability; Predictability

1. Introduction

In order to understand the evolution of structural change in the economy, time series evaluation is essential. However analysts are turning to mathematical methods in order to update input-output tables in lieu of tables available by survey methods. Even when national statistical offices publish updated input-output tables, they are often based on a mix of survey and mathematical techniques. Analysts are also confronted with changing classifications and methodologies underlying the published input-output tables, so when time series analysis is performed, there is often no consistency across years.

Analysts are hence left with the task of estimating input-output tables across time, in a common classification, and, possibly, in different prices. Techniques in order to do this have evolved over the last 40 years, and are reviewed elsewhere (Lenzen et al. 2006). However, with this estimation comes uncertainty on the accuracy of the estimates. Error analysis can be undertaken by investigating distributions of uncertainty in model variables. A modern example is Percoco et al. (2006), who estimate uncertainty using an assumed log-normal distribution of stochastic error in input-output coefficients and final demand. However, whilst such a method allows for identifying the most sensitive sectors prone to error, it does not address non-stochastic change over time. Instead, in this paper, the reliability or predictability of dated and mathematically updated tables are investigated.

Numerous papers have been written on analysing the change over time. Bon (Bon 1984, 1986, 1988; Bon and Bing 1993) has concentrated particularly on the predictability of supply and demand multipliers. Whilst not updating tables himself, his work is of interest in comparing the accuracy of using older transactions matrices with updated data. Dietzenbacher and Hoen (2006), subsequently referred to as D&H, couple an analysis of the stability of model coefficients, with the predictability analysis of Bon.

The situation of table availability for Australia is such that a semi-survey input-output table is published by the Australian Bureau of Statistics circa every 2-4 years. In addition, Supply/Use tables were published for a time series of 10 years (1994/5-2003/4). Also, the National Accounts, published annually, provides data on some parts of the input-output tables. In this work, maximum information is sought to be incorporated in the estimation of the tables, and the accuracy of the estimated tables is compared against those published.

In this work, the following questions are investigated:

- How much change is expected in input-output coefficients over time for Australia?
- How accurate is the use of an old input-output table compared to a new input-output table?

- How accurate is the use of an old input-output table updated using various estimation techniques compared to a new input-output table?

After defining the model (Section 2) the stability of coefficients and multipliers in the time series of Australian IO tables are compared (Section 3). Following these stability tests, the predictability of the tables are investigated (Section 4). The predictability is measured from both prior data and estimated data compared to real data (Section 5).

2. Model choice

In the Leontief (demand driven) model, gross output $\mathbf{x}$ is defined as the summation of intermediate transactions $\mathbf{T}$ and final demand $\mathbf{y}$. $\mathbf{u}$ is defined as the summation vector (a vector of ones, size $n \times 1$):

$$\mathbf{x} = \mathbf{T}\mathbf{u} + \mathbf{y} \quad (1)$$

$\mathbf{A}$ is defined as the direct requirements or coefficients matrix, relating proportions of intermediate usage to gross output:

$$\mathbf{A} = \mathbf{T} * \hat{\mathbf{x}}^{-1} \quad (2)$$

The Leontief model then redistributes to:

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y} \quad (3)$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \quad (4)$$

In the Ghosh (supply driven) model,

$$\mathbf{x}' = \mathbf{u}'\mathbf{T} + \mathbf{v}' \quad (5)$$

$$\mathbf{B} = \mathbf{T}' * \hat{\mathbf{x}}^{-1} \quad (6)$$

$$\mathbf{x}' = \mathbf{x}'\mathbf{B} + \mathbf{v}' \quad (7)$$

$$\mathbf{x}' = \mathbf{v}'(\mathbf{I} - \mathbf{B})^{-1} \quad (8)$$

The dash represents transposition. The data sources for this analysis are defined elsewhere (Part I, Chapter 1).

3. Stability

As a precursor to investigating the predictability of the Leontief and Ghosh models, following D&H, the stability of the coefficients matrix is first investigated.

As per D&H, the coefficient of variation is utilised to see the change in model parameters over time. The coefficient of variation $CVar$ is defined as the average square root of the variance of any variable z over a time period $t=1..p$ i.e.:

$$CVar_i = \frac{\sqrt{var_i}}{\bar{z}_i} \quad (9)$$

Where mean $\bar{z}$ is calculated according to

$$\bar{z}_i = \frac{\sum_{t=1:p} z_i^t}{p} \quad (10)$$

And variance is calculated according to

$$var_i = \sum_{t=1:p} \frac{(z_i^t - \bar{z}_i)^2}{p} \quad (11)$$

The coefficient of variation across the transaction matrix $\mathbf{T}$ would be expected to be high due to the variation of both structural relationships and prices. This is evident in Figure 1, where $CVar$ is plotted against the values of $\mathbf{T}$.

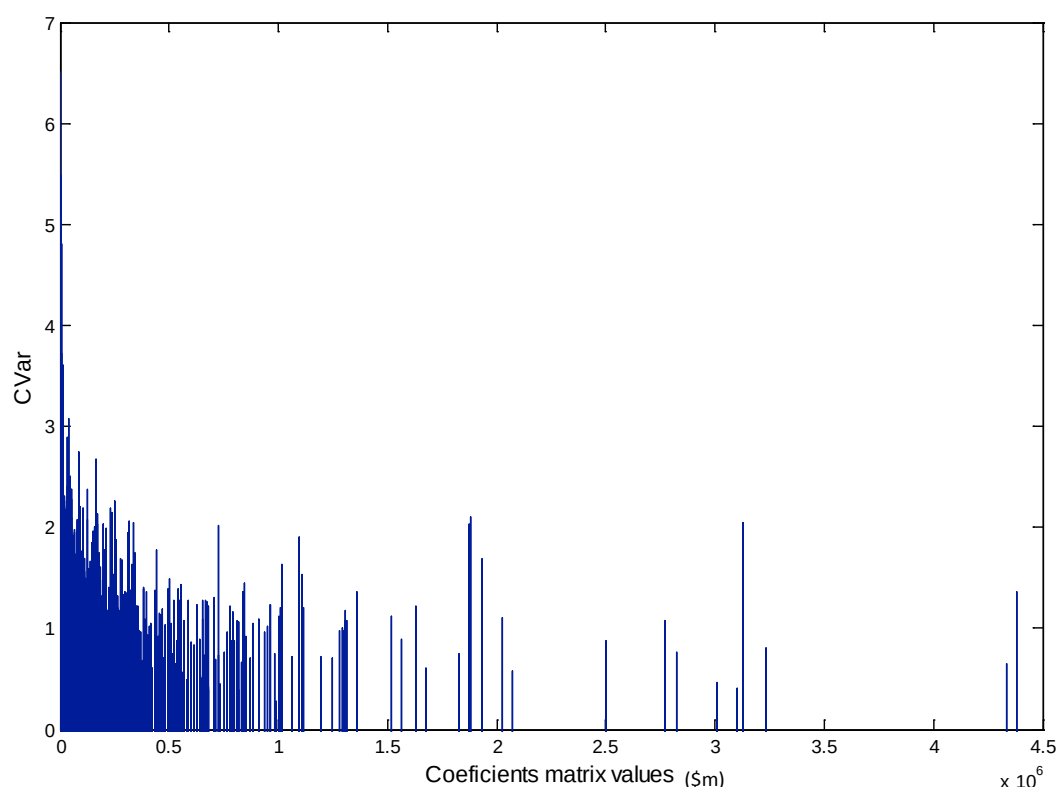


Figure 1 Coefficient of variation (CVar) of the transaction matrix T in \$m

In comparison, the coefficient of variation of the Leontief coefficients matrix **A** (Figure 2) only shows the changing relationships, and is independent of prices. As is expected from economic theory, the greater relative change has occurred within smaller coefficients. The magnitude of *CVar* is highly dependent on aggregation level, such that the results presented here with 109 sectors have a much larger *CVar* than those found in D&H with only 12 sectors. Comparing the Leontief model (Figure 2) with the Ghosh model (Figure 3), a much greater variation of coefficients is found in small coefficients than in larger coefficients. In fact the average coefficient of the Ghosh model was some 25% larger than the Leontief model, implying smaller primary inputs compared to final consumption, whilst the median coefficient was 15% larger in the Ghosh model, implying a greater concentration of inputs than outputs on average.

The coefficient of variation for both the Leontief and Ghosh models for large coefficients (coefficients greater than 0.1, or 10% of total inputs/outputs of the respective sectors) is generally in the range of 0.5 to 1.0, meaning the standard deviation of the coefficients across time are in the same order as the mean of the coefficients.

Of greater impact is the variation observed in multipliers rather than coefficients. Whilst the direct relationships of **A** and **B** may change depending on the presence or lack of presence of intermediate sectors, when mapping environmental flows, of greatest interest is the variability in total (direct and indirect) relationships. Again, both Leontief (Figure 4) and Ghosh (Figure 5) multipliers show increased variation on small multipliers, with the Ghosh this time showing much greater variation in comparison with the Leontief matrix. The clusters of multiplier values greater than 1 represent the diagonals of the multiplier matrices, which interpret the direct linkage between final consumption and primary inputs. As would be hoped, the coefficient of variation of these linkages is very minor. The relatively larger coefficients of variations of the Ghosh model imply this model is less accurate when predicting important interrelationships in the economy. The observed difference between the Ghosh and Leontief models can be explained by a greater variation in the primary inputs of key economic flows than the final consumption of these flows. In essence, in Australia, key economic flows have been subject to less variation in final consumption and greater variation in primary inputs.

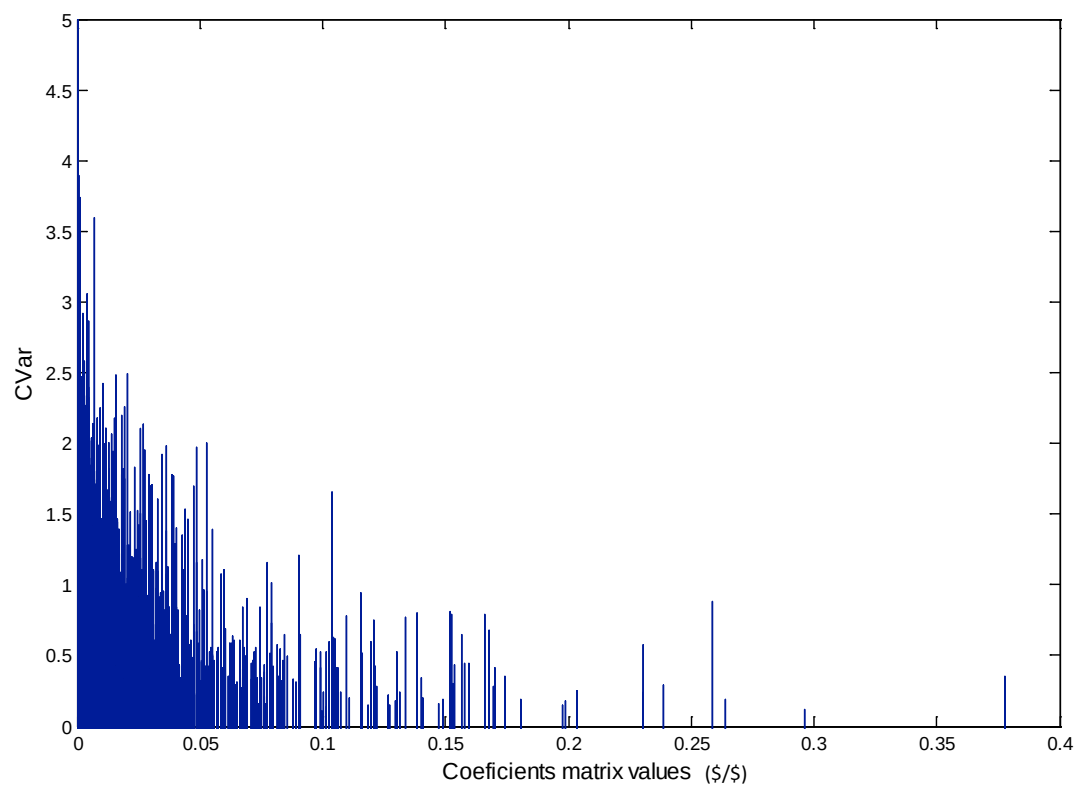


Figure 2 Coefficient of variation CVar of the Leontief coefficients matrix A

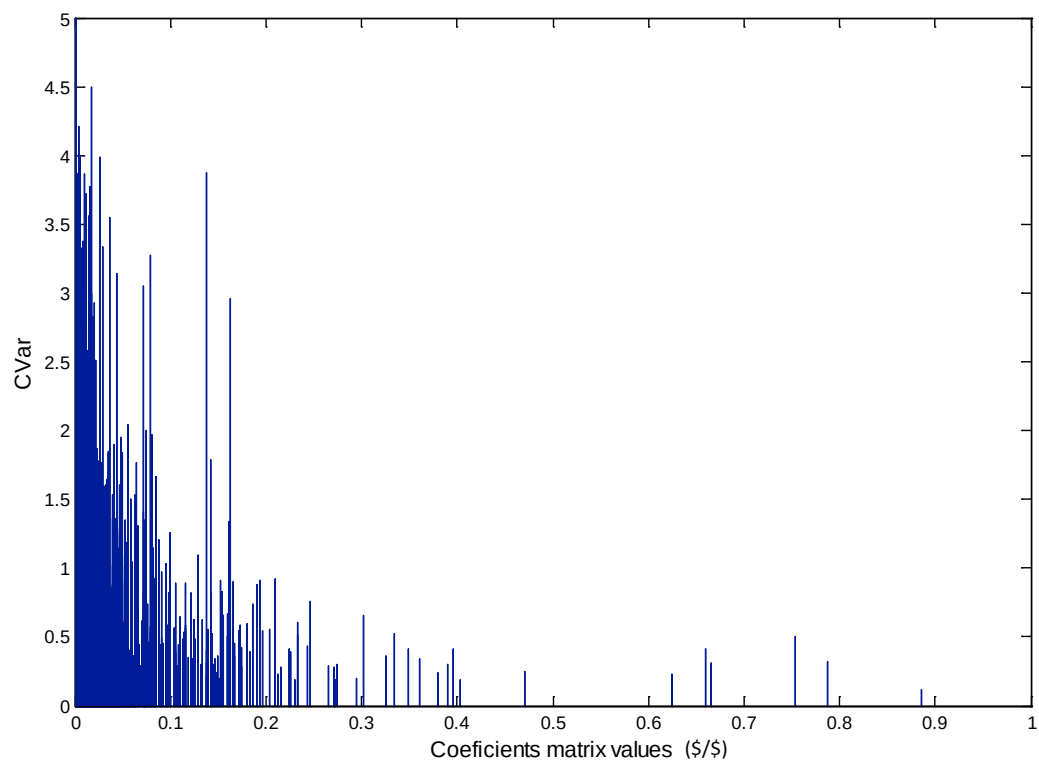


Figure 3 Coefficient of variation CVar of the Ghosh coefficients matrix B

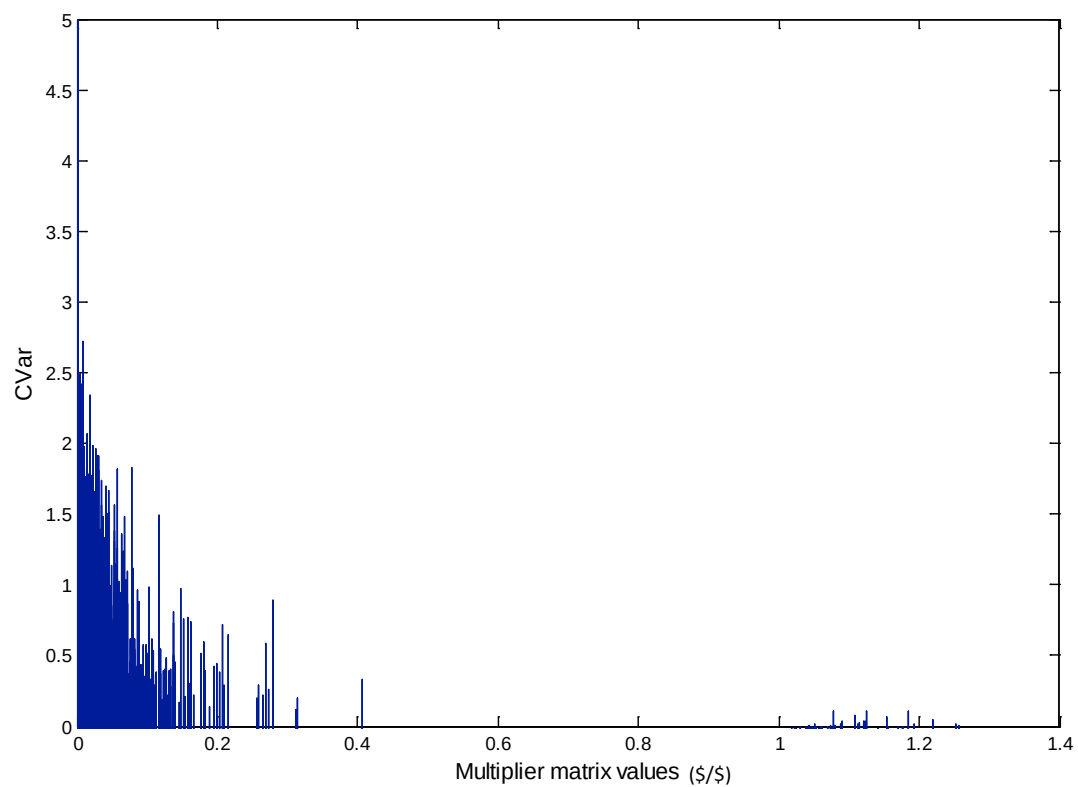


Figure 4 Coefficient of variation CVar of the Leontief multiplier matrix L

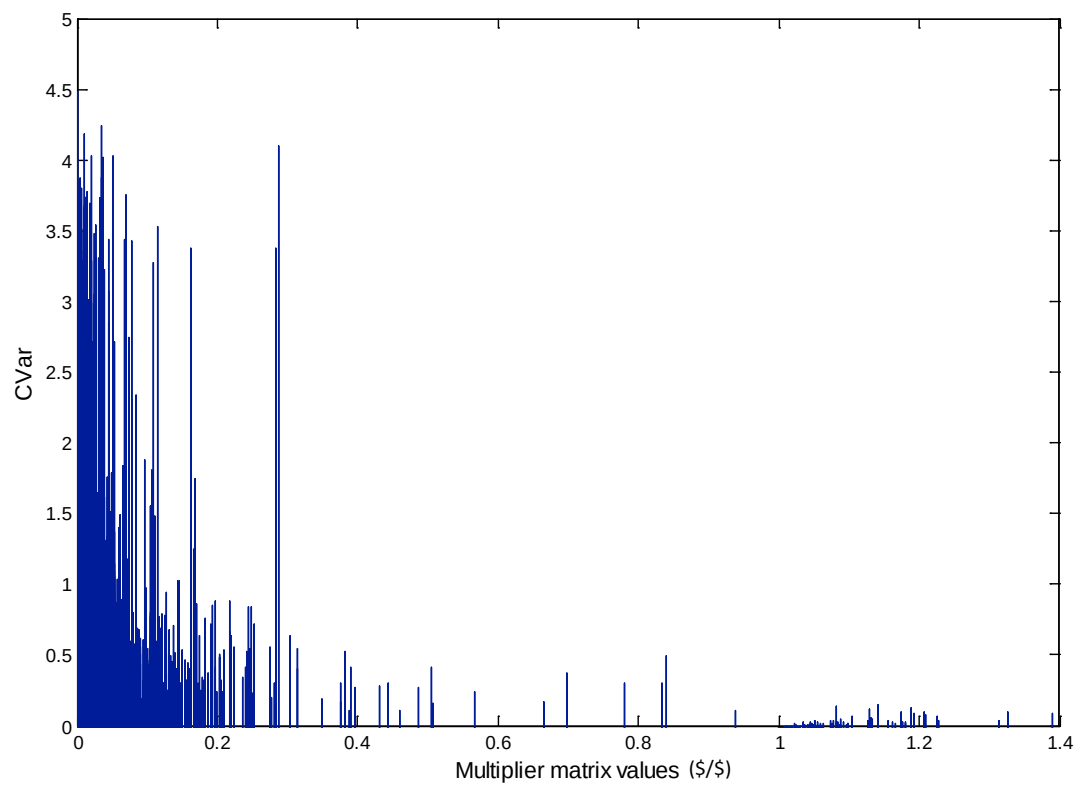


Figure 5 Coefficient of variation CVar of the Ghosh multiplier matrix G

4. Predictability (Joint stability of supply and demand multipliers)

In the next section, predictability of multipliers of the Leontief and Ghosh production functions is analysed. By predictability, the lead of D&H is followed in referring to what Bon named “stability”.

In this study, the predictability of the Leontief model is assessed under known final demand $\mathbf{y}_{t+1}$ by comparing known gross output $\mathbf{x}_t$ with estimated gross output $\tilde{\mathbf{x}}_{t+1}$ calculated according to:

$$\tilde{\mathbf{x}}_{t+1} = (\mathbf{I} - \mathbf{A}_t)^{-1} \mathbf{y}_{t+1} \quad (12)$$

The assumption being tested is that the coefficients matrix $\mathbf{A}$ is reasonably constant over time. For large variation in $\mathbf{A}$ a large difference in $\tilde{\mathbf{x}}_{t+1} - \mathbf{x}_t$ will be evident.

The predictability of the Ghosh model under known value added $\mathbf{v}_{t+1}$ is assessed by comparing known gross output $\mathbf{x}_t$ with estimated gross output $\tilde{\mathbf{x}}_{t+1}$ calculated according to:

$$\tilde{\mathbf{x}}_{t+1}' = \mathbf{v}_{t+1}' (\mathbf{I} - \mathbf{B}_t)^{-1} \quad (13)$$

In assessing predictability, five tests are used to determine the differences between the estimated gross output and the known gross output. Each test behaves slightly differently, and no test is deemed conclusive (Butterfield and Mules 1980). Refer to (Lenzen et al. 2008) for the source of these measures:

$$\text{The relative arithmetic mean of absolute differences } AMAD = \frac{\sum_i |x_{ti} - \tilde{x}_{ti}|}{\sum_i x_{ti}}; \quad (14)$$

$$\text{The relative geometric mean of absolute differences } GMAD = \frac{\sqrt{\sum_i |x_{ti} - \tilde{x}_{ti}|^2}}{\sqrt{\sum_i x_{ti}^2}}; \quad (15)$$

1- the Isard/Romanoff Similarity Index which has been used as a “Dissimilarity

$$\text{Index” by (Thissen and Löfgren 1998). } SIM = \frac{\sum_i \frac{|x_{ti} - \tilde{x}_{ti}|}{x_{ti} + \tilde{x}_{ti}}}{N^2}; \quad (16)$$

The arithmetic mean of relative differences $AMRD = \frac{\sum_i \frac{|x_{ti} - \tilde{x}_{ti}|}{x_{ti}}}{N^2}$; (17)

The relative mean square error $RMSE = \sqrt{\frac{1}{N} \sum_i \left(\frac{|x_{ti} - \tilde{x}_{ti}|}{x_{ti}} \right)^2}$; (18)

	AMAD	GMAD	SIM	AMRD	RMSE
1977	6%	1%	4%	8%	10%
1978	6%	1%	4%	7%	9%
1979	9%	3%	5%	9%	13%
1980	9%	2%	5%	10%	20%
1981	6%	2%	3%	5%	9%
1982	6%	1%	4%	7%	12%
1983	4%	1%	2%	4%	7%
1984	11%	3%	6%	12%	19%
1985	3%	1%	2%	4%	7%
1986	3%	1%	2%	3%	6%
1987	10%	3%	4%	8%	11%
1988	4%	1%	2%	4%	6%
1989	3%	1%	2%	4%	5%
1990	10%	4%	5%	9%	15%
1991	10%	4%	4%	11%	37%
1992	3%	1%	2%	4%	5%
1993	6%	2%	2%	5%	8%
1994	9%	3%	5%	10%	19%
1995	19%	4%	12%	32%	33%
1996	6%	1%	4%	7%	10%
1997	5%	1%	4%	9%	21%
1998	5%	1%	3%	6%	8%
1999	5%	1%	3%	6%	8%
2000	6%	1%	4%	8%	11%
2001	3%	1%	2%	4%	6%
2002	3%	0%	2%	5%	8%
2003	3%	0%	3%	5%	9%
2004	2%	0%	2%	4%	5%
2005	7%	1%	5%	9%	15%
2006	5%	1%	3%	6%	6%
Average	6%	2%	4%	8%	12%

Table 1 Predictability of Leontief coefficients, 1976-2006

	AMAD	GMAD	SIM	AMRD	RMSE
1977	8%	1%	4%	8%	9%
1978	7%	1%	4%	8%	13%
1979	5%	3%	3%	6%	9%
1980	6%	2%	3%	6%	12%
1981	6%	2%	3%	5%	7%
1982	5%	1%	3%	5%	7%
1983	4%	1%	3%	5%	10%
1984	8%	3%	5%	9%	13%
1985	3%	1%	2%	3%	4%
1986	2%	1%	1%	3%	4%
1987	4%	3%	2%	5%	7%
1988	3%	1%	2%	4%	6%
1989	3%	1%	2%	4%	5%
1990	7%	4%	4%	9%	14%
1991	12%	4%	6%	14%	17%
1992	4%	1%	2%	4%	6%
1993	4%	2%	3%	5%	7%
1994	9%	3%	5%	11%	18%
1995	20%	4%	11%	26%	31%
1996	6%	1%	4%	8%	12%
1997	6%	1%	4%	8%	11%
1998	5%	1%	4%	7%	12%
1999	4%	1%	2%	5%	9%
2000	6%	1%	5%	8%	14%
2001	4%	1%	3%	5%	8%
2002	3%	0%	3%	6%	15%
2003	4%	0%	3%	7%	10%
2004	3%	0%	2%	5%	7%
2005	6%	1%	5%	10%	14%
2006	5%	1%	3%	7%	7%
Average	6%	2%	3%	7%	11%

Table 2 Predictability of Ghosh coefficients, 1976-2006

One of the main differences is in the weighting of the differences, whether the difference is relative to the single variable or to the sum over the variables. The RMSE is relative to the single variable, and is more influenced by small flows. This is less than ideal in an input-output system when small flows have less importance. It is included it here for comparability to D&H.

Average results (Table 1 and Table 2) are in line with D&H and Bon, with the Leontief and Ghosh results being comparable in magnitude of error. A noticeable year is 1995, with larger errors evident in this year across both models. This is the first year the Time series of Supply/Use tables were published by the ABS, and mark a significant change in methodology.

The breakdown by industry is presented in Figure 6 and Figure 7. Sectors are arranged from 1 to 109, with primary industries on the left and tertiary industries on the right. Like D&H, there appears to be less variation in the Leontief model in the more established primary sectors, and considerably more variation in the Leontief model in tertiary sectors. This result, whilst consistent with D&H is opposite to the findings of Bon. D&H's explanation was that they were studying post-war reconstruction. The same explanation does not apply to this study, although there has been considerable change away from agricultural industries and to knowledge services in Australia.

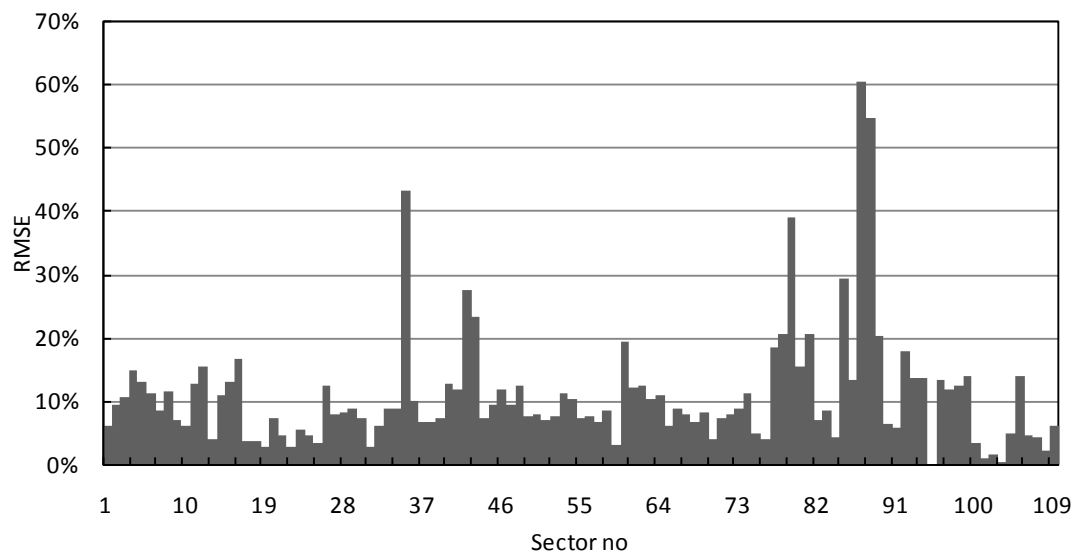


Figure 6 Annualised (average) RMSE of Leontief coefficients across full time series

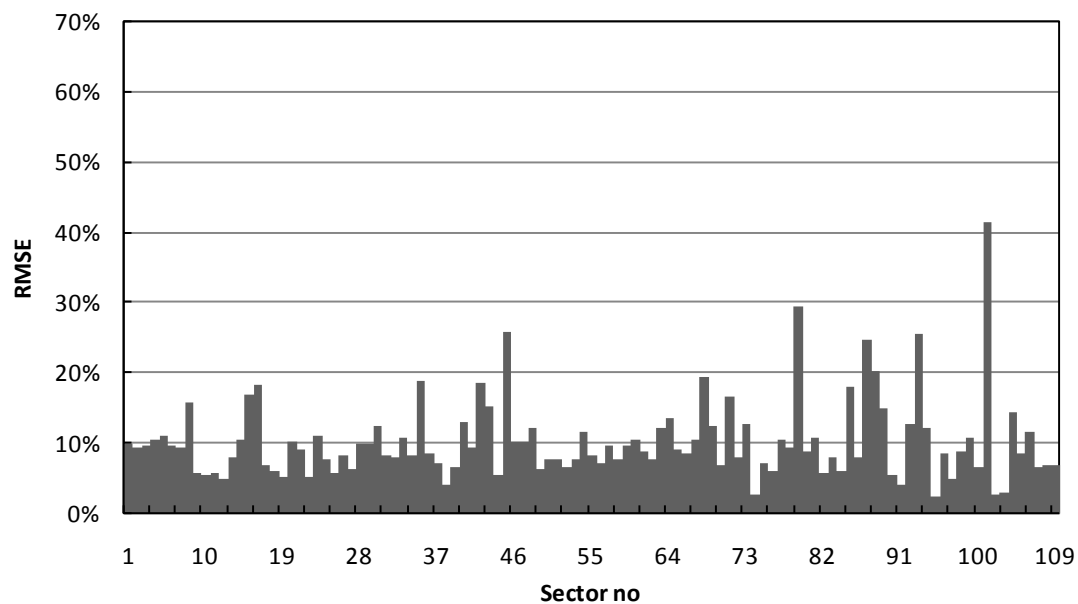


Figure 7 Annualised (average) RMSE of Ghosh coefficients across full time series

5. Analysis of Temporal Predictability

In section 4, the predictability of $\mathbf{x}_t$ was calculated from $\mathbf{L}_{t-1}$. In essence, the gross output was calculated given current final demand and the previous year's technological structure. A common occurrence, however, is that there is often a greater lag between available input-output tables and current final demand. Analysts often cite the stability of technological coefficients as justification for using dated structural data. In this section, this assumption is analysed for Australia for 2005 final demand. The series of equations investigated are represented by:

$$\forall_{t=1975:2006} \{\tilde{\mathbf{x}}_t = (\mathbf{I} - \mathbf{A}_t)^{-1} \mathbf{y}_{t=2005}\} \quad (19)$$

The measure of AMAD is used as it gave one of the better representations in Section 4 to map the evolution of $\tilde{\mathbf{x}}_t$

$$AMAD = \frac{\sum_i |x_{2005,i} - \tilde{x}_{t,i}|}{\sum_i x_{2005,i}} \quad (20)$$

The AMAD between the estimated gross output $\tilde{\mathbf{x}}_t$ to 2005 gross output $\mathbf{x}_{2005}$ is presented in Figure 8.

As expected, gross output estimates shown in Figure 8 are poor at a greater distance from the benchmark year (2005). A general trend ($R^2=0.89$) is found between the temporal distance from the benchmark and the measure of error. Using previous or subsequent year data gives an error in the range of 4-7% per year, as previously examined in Table 1.

Most statistical agencies have a lag time of 3-4 years in producing IO tables (Australia currently has a 3 year lag, the U.K.'s last published tables are from 1995). From the results presented here, it would be expected that using the dated tables would give an error of absolute differences in the range of 5-15%. However, as the measures employed present only the absolute error in industrial output, the error may be reduced if the uncertainty is stochastic (only somewhat likely, as there is economic evolution underlying the change) and normally distributed.

In order to reduce the magnitude of the error presented here, tables can be updated by a number of alternative means. This issue is now explored with reference to the two most recent input-output tables published for Australia in 2002 and 2005. The 2002 table is used as the base data source, and updating by various means is undertaken to attempt to replicate 2005 data.

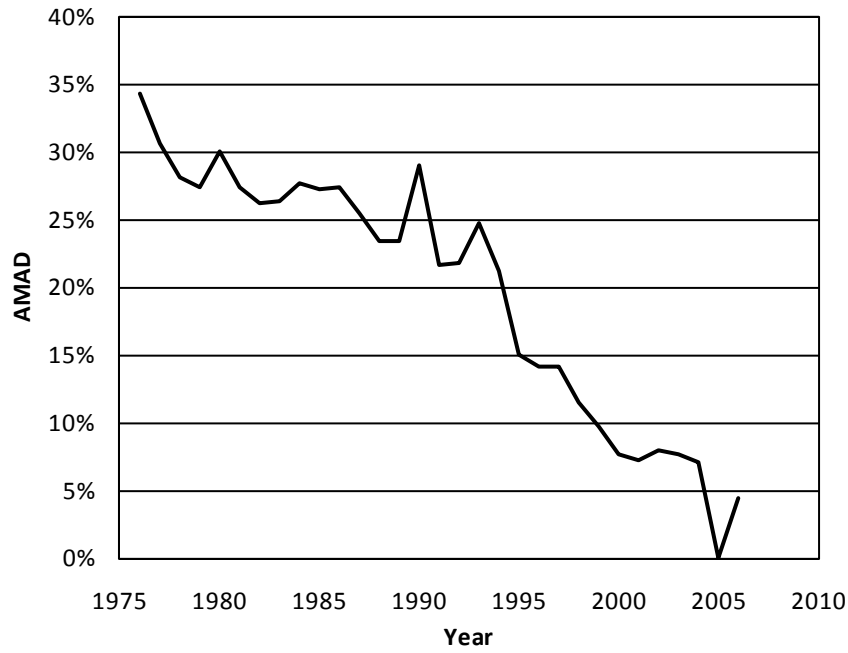


Figure 8 Evolution of gross output estimates $\tilde{x}$ compared to real x_{2005}

Four updating (or non-updating) options are explored particular for the Australian case, but likely to be encountered in other situations:

1. Not-updated
2. Updated according to National Accounts
3. Updated according to Supply/Use tables
4. Updated via change in value added from National Accounts, and according to National Accounts.

The first option, non-updated 2002 data is the base data from the 2002 input-output table, published by the ABS. The second option, update according to National Accounts, updates the final demand and primary input vectors, with subsequent RAS balancing of the transaction table $\mathbf{T}$. The third option updates the transaction tables, primary inputs and final demand according to published Supply/Use tables. These tables are in a more aggregated classification (roughly 50 industries) and in purchases prices (see Chapter 1 for a full explanation). Further, the publication of Supply/Use is also non-current, and the last available table was 2004. Tables are KRAS balanced (Lenzen et al. 2008) due to conflicting information in the Supply/Use tables. The fourth and final option explored was updating the 2002 data by the evolution of value added. That is, a ratio of 2005 to 2002 value added by sector was used and applied across the transaction table, final demand and primary inputs were updated according to National Accounts, and the system was subsequently KRAS balanced.

There are hence four estimates of the new transactions table $\tilde{\mathbf{T}}$, with the coefficients table $\tilde{\mathbf{A}}$ being estimated as per equation 2.

Results are presented in Table 3 according to:

$$\tilde{\mathbf{x}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1} \mathbf{y}_{t=2005} \quad (21)$$

AMAD is again used to measure distance according to: $AMAD = \frac{\sum_i |x_{2005,i} - \tilde{x}_i|}{\sum_i x_{2005,i}}$

Option #	Method	Error
1	2002 data not updated	8.1%
2	2002 data with National Accounts	12.3%
3	2002 data with Supply/Use	6.2%
4	2002 data with value added ratios +NA	8.1%

Table 3 Distance measures of gross output estimates $\tilde{\mathbf{x}}$ compared to $\mathbf{x}_{2005}$

No firm conclusions can be drawn from these results. The strongest result is the difference between addressing all components of the input-output table (option 3, by using Supply/Use tables) compared to addressing only value added/final demand components (option 2). The use of Supply/Use tables proffered a modest advantage over the original dataset (option 1). The most important reasons for this not being greater are likely to be: the Supply/Use table available referred to 2004, and some further change would be expected by 2005; the higher level of aggregation in the Supply/Use tables would only improve broader trends, and not competing (positive/negative) changes at the input-output industry level; the issue of inconsistency in the Supply/Use tables may have caused further destabilisation of the system. In comparison, however, the exclusive use of National Account data, whilst convenient, and an easy means to update according to available data, provides a decline in the data quality. This could be due to the transaction table $\mathbf{T}$ being only indirectly updated by accounting constraints (see Chapter 1 of this Thesis), whilst the primary inputs and final demand blocks were directly updated with real data. As row/column totals would only change by the differences in primary inputs and final demand, only very minor changes would be expected to occur on the transaction table $\mathbf{T}$. In essence, under this method, the price and quantity changes expected in economic evolution are directly applied to only part of the input-output system. This inconsistency shows the problems of this approach. It is important that analysts note this finding, as often confidential data is estimated in this way.

Option 4 provided no discernible improvement on the original dataset, but unlike option 2, by accounting for price and quantity effects in the full input-output table, the overall error in the calculated gross output was not increased.

6. Conclusions and Discussion

The stability and predictability of input-output coefficients is an important consideration for using input-output analysis to answer questions of temporal significance. Often the analyst has available current final expenditure data, but dated data on technical coefficients. Knowledge of the evolution of technical coefficients can help inform the validity of modelling exercises.

In this chapter, the evolution of the technical coefficients in Australia has been explored along the lines of D&H in order to give comparability for the Australian case. To begin with, the issue of stability of coefficients and multipliers was investigated under the Leontief and Ghosh models of supply/demand. As expected, there is considerably more variation in smaller coefficients than large coefficients. Multipliers were generally found to be generally more stable than coefficients. The interpretation from this finding is that adjustments to individual components of production chains are greater than adjustments to the corresponding full production chains. This may be due to increasing intermediary sectors or steps within a production chain.

A comparison between the Ghosh and Leontief models showed superior performance of the Leontief model, with the Ghosh being subject to increased variation in some large coefficients and multipliers. Hence in Australia, there has been more stability in demand side components than supply side components. This would be in line with economic theory that states demand driven economic growth rather than supply driven growth.

The predictability of the models was then examined under updated final demand or primary input components. The model results showed errors in the 0-15% range depending on choice of metric. The average root mean square error of 11-12% is higher than found in D&H, most likely due to the significantly higher disaggregation level used in this study (109 sectors compared to 12 sectors). Using error measures that weight out large changes in small values, the errors were a more reasonable 2-8%.

No significant difference between the Leontief and Ghosh models could be discerned at the whole of economy level. Whilst differences were observed in model performance at the industry level, no reliable trends were found according to economic maturity. This is in line with D&H, but contradictory to Bon's findings.

The predictability of the input-output model over varying time scales was also investigated. The results showed the importance of using up-to-date coefficient matrices, with errors subsiding from almost 35% on a 30 year time scale to nearer 5% for previous year tables. Simple methods to update input-output tables were investigated. The use of Supply/Use tables due to their greater explaining power produced the best result.

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PART I – CHAPTER 3.1

EMBODIED EMISSION EVOLUTION IN AUSTRALIA

Richard Wood

This chapter has been expanded and submitted in two parts as:

Wood, R and Dey, C. 2009. Australia's Carbon Footprint. *Economic Systems Research*: Submitted.

Wood, R and Dey, C. 2009. Embodied emission evolution in Australia *Energy Policy*: Submitted.

Abstract

A time series of Australian input-output tables has been developed and generalised for greenhouse gas emissions. This paper gives an overview of the construction techniques and methods used to generalise the tables for greenhouse gas emissions. The system was augmented with a time series of energy and non-energy related greenhouse gas emissions by a detailed mapping of production related emissions. A brief analysis of the change in greenhouse multipliers over the time series is given, extracting key industries in relation to both greenhouse emissions and economic importance.

Keywords: Australia, Input Output, Time series, Greenhouse.

1. Introduction

This chapter presents a detailed inventory of carbon dioxide equivalent greenhouse gas emissions for the Australian economy. This work goes beyond currently available emissions inventories by using detailed economic and physical data to generate a higher level of detail in the emissions inventory, and by relating these emissions to final consumption. Measures of emissions (CO₂-e) presented in this inventory are:

- Direct industrial greenhouse gas emissions by industry (Scope 1 under the Greenhouse Gas Protocol Corporate Standard (World Resources Institute and World Business Council for Sustainable Development 2004));
- *Embodied* greenhouse gas emissions by industry, i.e. direct emissions generated in the production of the industry's output and emissions generated in the production of inputs used by the industry, in terms of final demand;
- Scope 2 greenhouse gas emissions by industry, i.e. emissions generated in the production of electricity used by the industry; and
- *Embodied* electricity emissions by industry, i.e. electricity generation emissions generated in the production of inputs used by industry, allocated to final demand.

The objectives of this work require an approach that is consistent with national accounts data for both the economy and greenhouse gas emissions whilst being able to comprehensively cover all processes that embody greenhouse gas emissions in final demand. In order to accomplish this, the macroeconomic tool of input-output analysis has been employed. Australian input-output tables are consistent with national accounts data, whilst allowing the tracking of greenhouse gas emissions from source industries through the production system to types of final demand.

Input-output analysis is a macroeconomic technique that uses data on inter-industrial monetary transactions to account for the complex interdependencies of industries in modern economies. Since its introduction by Leontief (1936, 1941), it has been applied to numerous economic and environmental issues, and input-output tables are now compiled on a regular basis for most industrialised, and also many developing countries.

Emissions are allocated as inputs in physical amounts into disparate industries, and subsequently allocated through the production system by means of economic requirements – i.e. emissions are passed on proportionally to economic flows. The advantage of this approach is that price impacts associated with carbon caps will likely be conveyed in a way similar to the modelled allocation. That is, the addition of a cost of producing carbon will likely be transferred to consumers as an additional margin to current costs not volumes. Physical methods of allocation (through such methods as physical input-output analysis, material flow allocation), whilst perhaps

better at allocating environmental services to material throughput are not as closely aligned to the economic basis of decision making and policy development concerning carbon trading and carbon taxation.

This analysis is a static, ex-post analysis of embodied greenhouse gas emissions. It is not a dynamic analysis that includes projected changes in economic or greenhouse emissions if a cost to carbon was introduced. As such, its main use is in providing a detailed snapshot of the evolution of the economy and greenhouse gas emissions.

This chapter briefly outlines the required variables and calculations in Section 2, before detailing the methods and assumptions in constructing an integrated database in Section 3. A brief examination of major trends is presented in Section 4 before conclusions are drawn in Section 5.

2. Calculations of emissions

In this paper, only domestic emissions are included, and emissions embodied in imports are excluded. This is to align with the National Greenhouse Gas Inventory under the United Nations Framework Convention on Climate Change. Scope 1 (industrial) emissions from all sources are defined as $\mathbf{Q1}^{ind}$.

The Scope 1 *direct emissions intensity* of domestic production is defined as:

$$\mathbf{q1}^{ind} = \mathbf{Q1}^{ind} \hat{\mathbf{x}}^{-1} \quad (1)$$

Where $\mathbf{q1}^{ind}$ is the vector of *direct emissions intensity*. Its elements $\{\mathbf{q1}_j^{ind}\}_{j=1,\dots,n}$ describe emissions by industry sector j per A\$ value of total output $\mathbf{x}$ of industry sector j . $\mathbf{q1}^{ind}$ represents only emissions produced directly in each industry, but not in upstream supplying industries.

Total embodied emissions are defined as all greenhouse emissions emitted in the production of goods and services. It is the full life cycle emissions for the entire production chain and is referenced by category of final demand.

A vector of total embodied emissions by category of final demand $\mathbf{Q1}^{emb}$ is calculated according to the basic input-output relationship as

$$\mathbf{Q1}^{emb} = \mathbf{q1}^{ind} (\mathbf{I} - \mathbf{A}^d)^{-1} \hat{\mathbf{y}}_d \quad (2)$$

Where $\hat{\mathbf{y}}_d$ is a diagonalised matrix of the vector of domestic final demand, $\mathbf{A}^d$ is the (domestic) direct requirements matrix and $\mathbf{I}$ is the $n \times n$ identity matrix. Its elements

$\{I_{ij}\}_{i=1,\dots,n; j=1,\dots,n}$ are $I_{ij}=1$ if $i=j$, and $I_{ij}=0$ if $i \neq j$. $\mathbf{Q1}^{\text{emb}}$ refers to the total emissions from all sources embodied in the goods and services consumed within domestic final demand.

Scope 2 emissions $\mathbf{Q}^{\text{elec}}$ refer to emissions only from electricity supply. Electricity supply is distinguished so that it can be treated as a direct flow. This increases the accuracy of emissions allocation, as electricity related emissions are expressed in physical units rather than relying on distribution by economic units within the input-output model.

Scope 2 direct emission intensity is defined as:

$$\mathbf{q}^{\text{elec}} = \mathbf{Q}^{\text{elec}} \hat{\mathbf{x}}^{-1} \quad (3)$$

Total embodied electricity emissions are calculated as:

$$\mathbf{Q}_{\text{elec}}^{\text{emb}} = \mathbf{q}^{\text{elec}} (\mathbf{I} - \mathbf{A}^d)^{-1} \hat{\mathbf{y}}_d \quad (4)$$

$\mathbf{Q}_{\text{elec}}^{\text{emb}}$ refers to the total emissions from electricity generation embodied in the goods and services consumed within final demand.

In calculating embodied electricity emissions explicitly within the Scope 1 framework, it is necessary to redistribute $\mathbf{Q1}^{\text{ind}}$ of electricity using the physical allocation of emissions $\mathbf{Q}^{\text{elec}}$ across the economy. That is,

$$\mathbf{Q}^{\text{ind}} = \mathbf{Q1}_{i=1:n, i \neq \text{elec}}^{\text{ind}} + \mathbf{Q}^{\text{elec}}$$

Total embodied emissions with direct allocation of electricity emissions then proceeds as per normal:

$$\mathbf{q}^{\text{ind}} = \mathbf{Q}^{\text{ind}} \hat{\mathbf{x}}^{-1}$$

$$\mathbf{Q}^{\text{emb}} = \mathbf{q}^{\text{ind}} (\mathbf{I} - \mathbf{A}^d)^{-1} \hat{\mathbf{y}}_d$$

2.1. Calculations of emissions embodied in imports

The importance of imports in assessing the carbon footprint of a population has been a recent focus of research (Peters and Hertwich 2006a, 2006b; Wiedmann et al. 2007, 2008). For some countries, emissions embodied in imports are greater than domestic emissions. Australia has a much lower import demand, and thus the need for the use of multi-regional IO models to assess import requirements is lower.

Assuming domestic technology is similar to the overseas technology producing imported goods (such that domestic greenhouse intensity is used in place of overseas greenhouse intensity) we calculate imported emissions as (compare (Lenzen 2001) Eq10):

$$\mathbf{Q}^{imp} = \mathbf{q}^{ind} * (\mathbf{I} - \mathbf{A}^d - \mathbf{A}^m)^{-1} \mathbf{A}^m (\mathbf{I} - \mathbf{A}^d)^{-1} \mathbf{y}_d + \mathbf{q}^{ind} * (\mathbf{I} - \mathbf{A}^d - \mathbf{A}^m)^{-1} \mathbf{y}_m$$

where $\mathbf{A}^m$ is the technology matrix of imports relative to domestic output.

2.2. Calculations of emissions embodied in exports

As with imports, the knowledge of emissions going to export is essential for assigning a carbon footprint to the population of Australia. Whilst the notion of a carbon footprint includes responsibility for imported goods and services, it also implies that exported goods and services are the responsibility of the importing country, and hence should be subtracted from domestic footprints.

The estimation of emissions embodied in exports is straightforward due to the identification of exports as a destination of final demand. However, imported goods can also be subsequently exported, thus the principle equation becomes:

$$\begin{aligned} \mathbf{Q}^{exp} = & \mathbf{q}^{ind} * (\mathbf{I} - \mathbf{A}^d)^{-1} \mathbf{y}_{d,exp} + \mathbf{q}^{ind} * (\mathbf{I} - \mathbf{A}^d - \mathbf{A}^m)^{-1} \mathbf{A}^m (\mathbf{I} - \mathbf{A}^d)^{-1} \mathbf{y}_{d,exp} \\ & + \mathbf{q}^{ind} * (\mathbf{I} - \mathbf{A}^d - \mathbf{A}^m)^{-1} \mathbf{y}_{m,exp} \end{aligned}$$

where the subsets of domestic final demand $\mathbf{y}_d$ exported is denoted $\mathbf{y}_{d,exp}$ and the subset of imported final demand $\mathbf{y}_m$ to export is denoted $\mathbf{y}_{m,exp}$.

3. Data sources and alignment

3.1. Data sources

Three main data sources have been used in this study. Firstly, input-output data has been generated as per Part I, Chapter 1 of this Thesis. Secondly, the ABARE publication *Australian energy consumption and production 1974-75 to 2004-05* (Australian Bureau of Agricultural and Resource Economics 2006) was used to delineate fuel combustion energy emissions, by using higher levels of detail of energy data and by using industry specific equipment types, emission factors and fuel types. Thirdly, the *National Greenhouse Gas Inventory (NGGI) 2005* (Australian Greenhouse Office 2007) has been the key source of data on greenhouse gas emissions. Additional data sources have been used in the allocation of energy and emissions to economic sectors, and these are detailed below.

The NGGI database distinguishes some six categories of emissions (Fuel combustion, Industrial processes, Solvent and other product use, Agriculture, Land use change and

forestry and Waste - zero emissions are reported for Solvent and other product use, thus they are not considered here). Within these six categories, there are a varying number of sub-classifications, of which some conform directly to input-output classifications, and some of which report on processes common to a number of input-output classifications.

Fuel combustion emissions were allocated according to the using industry. Agricultural and land use change emissions were allocated mainly following the methodology employed by Wilkenfeld (George Wilkenfeld & Associates Pty Ltd and Energy Strategies 2002), but necessarily updated to a time series of allocation factors due to the not insignificant changes in agricultural practices over time. In addition, the NGGI publishes a database allocated to ANZSIC categories, along with allocation factors. Where these data were superior to other available data, these allocations were employed. Finally, where disaggregated data was not available, it was necessary to use other proxies (such as production value, employment, production volume, etc) in order to allocate emissions. These assumptions are described below. Reference is made to the NGGI classification of sectors as per Kyoto Accounting conventions.

3.2. Fuel Combustion

3.2.1. Energy data

Whilst the NGGI offers the most complete dataset on greenhouse emissions in Australia, the source data on energy consumption from the Australian Bureau of Agricultural and Resource Economics (ABARE) can provide additional detail for specific industries. ABARE no longer publishes additional detail for mining and other energy industries, and no longer publishes fuel use by equipment type. Hence data at this level of detail comes from the late 1990's. The final data hence has the relative breakdown of this highest level of detail from the late 1990's whilst conforming in absolute terms to the level of detail of the current year data.

The process followed was to create an initial estimate of energy use by input-output category. This was then optimised to account for additional data on equipment use by fuel and equipment use by industry.

The sales breakdown of the corresponding input-output category of each of the 28 fuels is used to create an initial estimate. The sales breakdown is taken direct from the input-output tables, and shows the purchases of the corresponding product to each industry. The input-output data is the best available detail on the use of fuels by industry, but unlike the fuel data, is in monetary terms. The effect is that there is an implicit assumption that within each category of the ABARE energy data (see Table 1), all industries pay the same price for the fuels. This assumption is expected to be valid for most types of energy consumption. Where a significant difference in price is

Current ABARE Classification	PJ
Div. A Agriculture, forestry, fishing	67
Div. B Mining	275
21 Food, beverages, tobacco	184
22 Textile, clothing, footwear and leather	13
23-24 Wood, paper and printing	61
251 Petroleum refining	1,780
252 Petroleum and coal products nec	2
253 Basic chemicals	157
254-256 Other chemicals, rubber and plastic	23
25 Petroleum, coal and chemical	1,973
261 Glass and glass products	11
262 Ceramics	25
263 Cement, lime, plaster and concrete	43
264 Non-metallic mineral products nec	13
26 Non-metallic mineral products	92
271 Iron and steel	397
272-273 Basic non-ferrous metals	339
274-276 Other metal products	8
27 Metal products	744
28 Machinery and equipment	20
29 Other manufacturing	0
Div. C Manufacturing	1,434
361 Total electricity generation	1,972
362 Gas production and distribution	15
37 Water, sewerage and drainage	6
Div. D Electricity, gas, water	1,992
Div. E Construction	30
Commercial and Services b	206
61 Road transport	935
62 Railway transport	29
6301 International bunkers	31
6302 Coastal bunkers	21
63 Water transport	52
Domestic air transport	78
International air transport	105
64 Air transport	183
65-67 Other Transport, services and storage	12
Div. I Transport & storage	1,210
Residential	384
Solvents, Lubricants, Greases & Bitumen	61
Total	7,339

Table 1: Current ABARE energy classification and gross fuel consumption data in Petajoules (PJ), 2002.

expected within an ABARE category, such as for aluminium production within non-ferrous metals, additional constraint data is sourced and applied. Further, due to the nature of the ABARE data, important energy flows were at a sufficient level of detail to allocate directly to input output categories, whilst less energy intensive sectors, particularly within the services sector, required more disaggregation. Thus inaccuracies caused by disaggregation are limited.

Unpublished data on mining of mineral sands, iron ore, gold, silver, etc, and briquette manufacturing, as well as the separation of aluminium from other non-ferrous metals were made available for the 1990's, and the relative breakdown was applied in the initial estimate for subsequent years.

In addition to the direct energy consumption data, energy consumption was previously available by equipment type (17 types) and fuel (27 fuels) (Australian Bureau of Agricultural and Resource Economics 1997) and equipment type shares were available by industry and fuel (National Greenhouse Gas Inventory Committee 1994). The importance of this data is highlighted for particular industries such as aluminium production, silver, lead, zinc, coal and uranium mining, mineral sands mining, and others. As a result, emission factors by industry, equipment type and fuel type were able to be used in order to be most precise in the estimation of greenhouse emissions by industrial sector. As a result, the final form of the energy consumption matrix detailed input-output category by equipment type by fuel use.

3.2.1.1. Transport

The ABARE data set on energy consumption aggregates all forms of road transport into the "road transport" sector – hence they do not distinguish between private use of automobiles, business use of automobiles and freight. Similarly for military purposes, all energy use within vehicular transport and air transport are aggregated into the road and air transport sectors. In order to allocate transport energy use to end use categories, military use is first extracted (National Greenhouse Gas Inventory Committee 2008), then the remaining fuel use is apportioned between private and business fuel use for road transport (Australian Bureau of Statistics 2003), and direct to air transport and water transport.

Table A 1 from the methodology workbook for energy (transport) (National Greenhouse Gas Inventory Committee 2008), provides allocation factors for military activities across road, water and air transport. These factors are used to delineate military from other uses, directly from ABARE energy data.

TABLE A 1 FACTORS USED TO ALLOCATE ABARE FUEL CONSUMPTION TO UNLISTED CATEGORIES

National Greenhouse Gas Inventory, 2006. Methodology Workbook for Energy (Transport), Department of Climate Change, Canberra, Tab. A1, p. 41

	General use	Military	Small marine craft ^a	Off-road vehicle ^a	Utility engines ^b
Road transport automotive gasoline	96.81%	0.06%	2.39%	0.1%	0.63%
Road transport ADO	99.50%	0.50%			
Water transport ADO	60.00%	40.00%			
Water transport fuel oil	99.95%	0.05%			
Air transport aviation gasoline	96.50%	3.50%			
Air transport aviation turbine fuel	92.00%	8.00%			

^a assumed residential^b unallocated

The remainder of the road transport energy consumption data (once military data is excluded) is split between residential and business use by means of the Survey of Motor Vehicle Use (Australian Bureau of Statistics 2003). Total fuel consumption by fuel and vehicle (Table 2) is apportioned by distance travelled by purpose and type of vehicle (Table 3) to give energy consumption by purpose and fuel (Table 4), aggregating vehicle types for non-business travel, and converting to energy units by including energy content of fuels (Australian Bureau of Agricultural and Resource Economics 2007). Finally, due to discordance between this estimate and the ABARE energy consumption for transport, final values are scaled to match ABARE data.

Type of fuel	TOTAL FUEL CONSUMPTION (million litres)						Total
	Passenger vehicle	Light commercial	Rigid trucks	Articulated trucks	Non-freight carrying trucks	Buses	
Petrol							
leaded	1,430	357	29	-	2	3	454
unleaded	12,513	2,035	15	-	5	17	14,585
Total	13,943	2,392	44	-	7	20	16,406
Diesel	627	1,263	1,977	2,913	49	438	7,267
LPG/CNG/dual fuel	1,830	490	20	9	2	39	2,390
Total	16,400	4,145	2,041	2,922	58	497	26,063

Table 2: (Australian Bureau of Statistics 2003), Table 1.

Type of vehicle	Total kilometres travelled (million)			Total
	All business	To and from work	Personal and other	
Passenger vehicles	33,712	36,151	74,813	144,676
Light commercial vehicles	19,677	5,527	6,145	31,349
Rigid trucks	6,879	156	45	7,080
Articulated trucks	5,417	5	2	5,424
Non-freight carrying trucks	221	2	2	225
Buses	1,641	28	106	1,775
Total	67,547	41,869	81,113	190,529

Table 3: (Australian Bureau of Statistics 2003), Table 5.

FUEL CONSUMPTION, BY TYPE OF FUEL AND PURPOSE OF TRIP (PJ)						
Type of fuel	Petrol		Total	Diesel	LPG/CNG/ dual fuel	Total
	leaded	unleaded				
Business - Passenger vehicles	11.4	99.7	111.1	5.6	11.0	127.7
Business - Light commercial vehicles	7.7	43.7	51.3	30.6	7.9	89.9
Business - Rigid trucks	1.0	0.5	1.5	74.1	0.5	76.1
Business - Articulated trucks	0.0	0.0	0.0	112.3	0.2	112.5
Business - Non-freight carrying trucks	0.1	0.2	0.2	1.9	0.1	2.1
Business - Buses	0.1	0.5	0.6	15.6	0.9	17.2
All business	20.2	144.6	164.8	240.2	20.6	425.5
To and from work	14.4	119.2	133.6	16.7	14.0	164.3
Personal and other	27.7	235.0	262.7	23.6	26.9	313.1
Total	62.3	498.8	561.1	280.5	61.4	903.0
Energy content (MJ/L; ABARE 99.4)	34.2	34.2	34.2	38.6	25.7	0

Table 4: Fuel Consumption, by type and purpose of trip (PJ), calculated from Table 2 and Table 3.

ABARE data aggregates railway passenger and freight transport services. In order to delineate passenger services, energy consumption statistics on fuel consumption in passenger and freight transport services (Apelbaum Consulting Group Pty Ltd 2007) were scaled to match aggregate ABARE data.

Fuel consumption of taxis was estimated by forecasting total taxi distance travelled from Australian Taxi Association data and ABS data on distance travelled (ABS Cat no 9311). Total taxi distance travelled was converted to energy units by using fuel consumption data on new cars with 85% of taxis assumed to be running on LPG.

3.2.2. Greenhouse Gas Coefficients

Greenhouse gas emissions from fuel combustion data were subsequently calculated from the estimation of energy consumption by industry, fuel type and equipment type. This maintains the level of detail available in the energy data, including specific emission factors for each industry by fuel and equipment type. Calculated emissions are subsequently constrained by published emissions of greenhouse emissions within the NGGI. The effect of this process is that the relative breakdown by input-output category within the energy data is carried across into the greenhouse data.

In order to translate energy data into greenhouse emissions, we consider CO₂ only emissions according to the emission factors in Table 5. In order to not double count primary and secondary fuels, and to enable tractability of important physical flows, within solid fuels only primary fuels were included. Only minor combustion of solid secondary fuels occurs for such fuels as coke and briquettes. For liquid fuels, only

secondary fuels were included, such that no emissions are associated with feedstock. In this way, gross, rather than net, emissions are estimated without double counting.

CO2 emission factors	(kt/PJ)
Black coal	90
Brown coal	95
Coke	-
Coal byproducts	-
Brown coal briquettes	-
Wood, woodwaste	94
Bagasse	97
Refinery feedstock	-
LPG	59
Auto gasoline-leaded	66
Auto gasoline-unleaded	66
Aviation gasoline	68
Aviation turbine fuel	68
Lighting kerosene	70
Power kerosene	70
Heating oil	70
ADO	70
IDF	70
Fuel oil	73
Petroleum products nec	69
Solvents	-
Lubricants & greases	-
Bitumen	-
Natural gas	51
Town gas	59
Solar energy	-
Electricity	-
Synthetics - biofuels	-

Table 5: CO₂ emission factors.

In addition, Non-CO₂ emissions are considered for the greenhouse gases CH₄ and N₂O. These emission factors are calculated by sector, by fuel type and by equipment type (Danish Environment Ministry 2004) and (National Greenhouse Gas Inventory Committee 1998a, 1998b, 1998c, 1998d). Other greenhouse gases are not directly considered due to uncertainty in emission factors. They are, however, included indirectly when included in the NGGI by the benchmarking of the greenhouse gases estimated from fuel combustion with the CO₂-equivalent greenhouse emissions of the NGGI. Non-CO₂ transport emissions are estimated from the energy data, with mobile sources of emissions from AGO Workbook 3.1 A.5.

3.2.3. Scope 2 emissions

Scope 2 emissions (emissions from the generation of purchased electricity) are estimated directly from ABARE energy consumption data, specifically for electricity use. The process used in this estimation is identical to the general fuel consumption data (allocation is undertaken per economic flows from the input-output table). Emissions are then constrained by the ANZSIC classification of Scope 2 emissions. Scope 2 emissions are subsequently generalised directly to the input-output tables.

3.3. Industrial Processes

For industrial processes, category 2.A Mineral Products are allocated directly to the cement and lime input-output category and to Iron and Steel semi-manufacturers for Limestone and Dolomite Use. Emissions from 2.B Chemical Industry and 2.D,F Production of Halocarbons and Sulphur Hexafluoride are allocated to Basic Chemicals and Other Chemical Products by production value. Emissions from 2.C Metal Production are allocated directly from the NGGI to the input-output categories Iron and steel semi-manufactures, Alumina, and to the three aluminium product sectors (Aluminium, Aluminium semi-manufactures, Aluminium foil – allocated within these three sectors by production value). Emissions from 2.E,F Consumption of Halocarbons and Sulphur Hexafluoride are principally due to Refrigeration and Air Conditioning Equipment, and make up almost 3 kilo-tonnes of emissions. The use of refrigeration and air conditioning is common to many economic sectors, hence making allocation difficult. The AGO provided allocation to ANZSIC divisions (Table 6). This provided a first step. Emissions were then allocated according to employment for other industries where it was assumed that air conditioning of employee space was the major cause of emissions, and according to production value for Trade and Road transport, as these sectors are margin industries with throughput assumed to reflect the volume of refrigeration of goods driving these emissions.

2.F.1 Refrigeration and Air Conditioning Allocation		%
Div M	Government Administration and Defence	1.529%
Div H,P,Q	Accommodation, Cultural and Personal	7.325%
DIV F, G	Wholesale and Retail Trade	8.985%
Div K, L	Finance, Insurance, Property and Business	1.529%
R NT	Residential (non transport)	20.750%
R T	Residential (Transport)	40.087%
Div A	Agriculture, Forestry, Fishing	1.690%
61	Road Transport	18.106%

Table 6: Allocation of refrigeration and Air Conditioning (AGO pers. Comm.).

Foam blowing was allocated to Other Chemical products, Fire extinguishers to Fire Brigade, Solvents to Machinery and Equipment and Other Electrical to Electrical Equipment. Emissions from “2.G Other” are confidential emissions but stem from soda ash, ammonia and nitric acid production. These chemicals are used within the sectors glass products, fertiliser and chemical products, pulp and paper and water supply/sewerage. The emissions are allocated to these sectors by production value in lieu of any other information.

3.4. Agriculture

For agriculture, a number of proxy variables are needed in order to allocate NGGI categories to Input-Output categories. The main proxies used are land use area, production volume and livestock head. The principal source data is the ABARE Commodity Statistics, along with ABS publication *Agricultural Commodities, ABS Cat. No. 7121.0*. Whilst data is available on total pasture land area, and land area of crops, land area data is not available for livestock. Hence, an estimation process is employed following Wilkenfield (George Wilkenfeld & Associates Pty Ltd and Energy Strategies 2002) where total natural and sown pasture area is allocated by head number, with a stocking ratio for sheep to cattle of 14.3 used (Table 7). In addition, dairy cattle are delineated, with a stocking ratio for dairy to beef cattle of 2.

Other land area data (e.g. for crops) and production volume (tonnes) are taken direct from ABARE Commodity Statistics and *Agricultural Commodities, ABS Cat. No. 7121.0*. In 7121.0, land area data is available by irrigated area as well as total area, corresponding to NGGI classification by irrigated and non-irrigated emissions.

	Equiv Head ^a (‘000)	Hectare ^b (‘000 ha)	Grazing Density ^c (ha/head)	Land Share (%)
Sheep and lambs	106,166	103,691	1.0	22%
Beef cattle	24,739	345,516	14.0	73%
Dairy cattle	3,131	21,865	7.0	5%
TOTAL	134,035	471,071		
Pastures and grasses		471,071		
Sown ^d		24,064		
Natural ^d		447,007		

^aABARE Commodity Statistics 2007

^b Estimated from Total Pastures and Head numbers

^c Grazing density ratio of 14.3 used from Wilkenfield 2002, Table 5.10.

Dairy assumed to have a grazing density twice that of beef

^d ABS 7121.0; 2001-02

Table 7: Estimation of Land Use for principal livestock.

Emissions from NGGI category 4.A Enteric fermentation and 4.B Manure management are provided within the NGGI by type of livestock. These emissions are hence allocated directly. Emissions from 4.C Rice cultivation are also allocated directly. Within NGGI category 4.D Agricultural Soils, Direct Soil Emissions; Synthetic fertilisers are allocated directly to discrete categories and by shares in fertiliser use (see Table 8) for non-discrete categories. The shares in fertiliser use is estimated as per Wilkenfield (George Wilkenfeld & Associates Pty Ltd and Energy Strategies 2002) and updated to current year land area data. The land share is then used to allocate any non-specific greenhouse gas emissions. This refers principally to fertiliser use, as methane and other emissions are allocated directly, as noted above.

Emissions from Synthetic Fertilisers for Pasture are estimated solely by land area. NGGI level 5 data, Animal Waste Applied to Soils, Nitrogen Fixing Crops, Cultivation of Histosols, and Crop Residue are allocated directly to input output categories. Crop residue data for Cereals was allocated to input-output sub-categories by production volume (tonne).

Synthetic Fertiliser allocation	Application Rates ^a (kg N ₂ O/ha)	Land area ('000 ha)	Estimated emissions ('000 kg N ₂ O)	Fertiliser emissions share (%)
Oats, sorghum and other cereal grains ^b	0.72	2,099	1,511	10%
Wheat ^b	0.72	11,529	8,301	57%
Barley ^b	0.72	3,707	2,669	18%
Rice	0.73	144	105	1%
Oilseeds	0.62	1,881	1,166	8%
Legumes	0.38	2,086	793	5%

^aWilkenfield (2002), Table 5.4, 1999 data.

^b Application rates of Cereals used

Table 8: Synthetic Fertiliser Allocation.

Emissions from NGGI 4.D Agricultural Soils; Animal production are aggregated but are due to nitrogen excretion via faeces and urine, and are hence allocated by animal production volume (tonnes). Indirect emissions from Atmospheric deposition from fertilisers are allocated as per Table 8. Manure is allocated directly according to identified livestock types. Emissions from Other; Field burning of agricultural residue are again allocated by production volume (tonne) for Cereals, and directly for other categories. Prescribed burning of Savannahs was fully allocated to beef cattle grazing, as per Wilkenfield (George Wilkenfeld & Associates Pty Ltd and Energy Strategies 2002).

Indirect emissions from Nitrogen leaching and run-off were allocated per fertiliser use (Table 8) for crops and by land use area for pasture. Nitrogen leaching from Manure

was allocated directly to livestock types. Other (Soil disturbance) emissions within Agricultural Soils were allocated by land use area for crops and pasture respectively.

NGGI category 4.E Prescribed burning of savannahs was allocated fully to beef cattle, as per Wilkenfield (George Wilkenfield & Associates Pty Ltd and Energy Strategies 2002). Category 4.F emissions from field burning of agricultural residues were allocated by land use area for cereals, and directly for pulses and sugar cane.

3.5. Land use change and forestry

Land use change and forestry is broken down within the NGGI to Afforestation and reforestation and Land use change (cropland/grassland). Afforestation and reforestation emissions were allocated according to number of seedlings planted. The ABS publication *Agricultural Commodities ABS Cat. No. 7121.0* detail forestry activities by purpose for both number of seedlings planted and area (Table 9). Number of seedlings planted was used as the proxy to allocate emissions in order to not subject the allocation to difference within planting practices (intensive verses extensive planting). As a result, this does not take into account differences in seedling mortality between purposes, and assumes uniform carbon absorption by seedling. Timber or pulp planting was allocated between softwoods and hardwoods by production volume (m3) of each. Proportion of seedlings planted for nature conservation and protection of land and water were allocated directly to Residents. Enhanced production and Fodder, plant products were allocated to agricultural sectors by land use area.

	ha	no
Timber or pulp production	9%	21%
Nature Conservation	15%	17%
Enhanced production (e.g. Shade, windbreaks, etc)	29%	17%
Fodder and plant products	3%	7%
Protection of land and water	39%	33%
Other plantings	3%	5%

Table 9: proportion of forestry activities by purpose by area(ha) and number of seedlings planted.

Land use change emissions from deforestation (land converted to cropland) were allocated to the various crops sectors by land use area. Emissions from land converted to grassland were allocated as per Wilkenfield (George Wilkenfield & Associates Pty Ltd and Energy Strategies 2002), which allocated 85% of total land use change emissions to beef, 4.5% to residential, and the remainder to crops – which are separately identified in the current NGGI.

3.6. Waste

Waste emissions are dominated by solid waste disposal on land, which are directly allocated to the Sanitary and Garbage disposal input-output sector. Emissions from wastewater handling for Industrial wastewater is allocated according to the AGO ANZSIC allocation (Table 10), distributed to input-output categories by production value. Domestic and commercial wastewater emissions were allocated directly to the water supply and sewerage sector.

6.B.1 Waste, Wastewater Handling, Industrial Wastewater	
Wood, Paper and Printing	19.92%
Basic Chemical Manufacturing	6.43%
Food, Beverages, Tobacco Manufacturing	73.65%

Table 10: Allocation of Emissions from industrial wastewater.

Waste emissions from Waste incineration for solvents were allocated to Iron and steel semi-manufacturers as per the AGO ANZSIC allocation, and other non-specified emissions were allocated to the Sanitary and garbage disposal sectors.

4. Results

4.1. Domestic emissions, 2005

The emissions allocated to producing sector are shown in Figure 1 for 2005. This represents Q^{ind} and is the emissions as reported by Australia under Kyoto Accounting. As expected the importance of primary industries is high, with over one-third of total emission. Manufacturing and utilities make up an additional third. Thus even with the almost complete reliance on coal for electricity generation in Australia, these emissions only make up less than 15% of the total. A significant issue for Australia is the weight of agricultural and mining emissions. These emissions occur in non-urban areas and are caused by industry supporting rural communities. As they often have significant social and economic benefits, the only politically palatable option to address these emissions is to seriously focus on efficiency improvements.

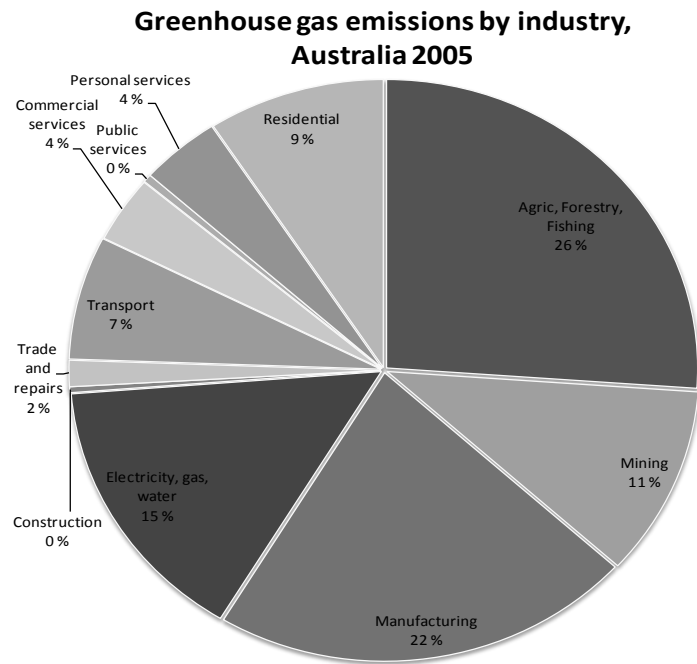


Figure 1 Emissions by producing sector, Australia 2005

The emissions allocated to producing sector over the time series 1990-2005 is shown in Figure 2.

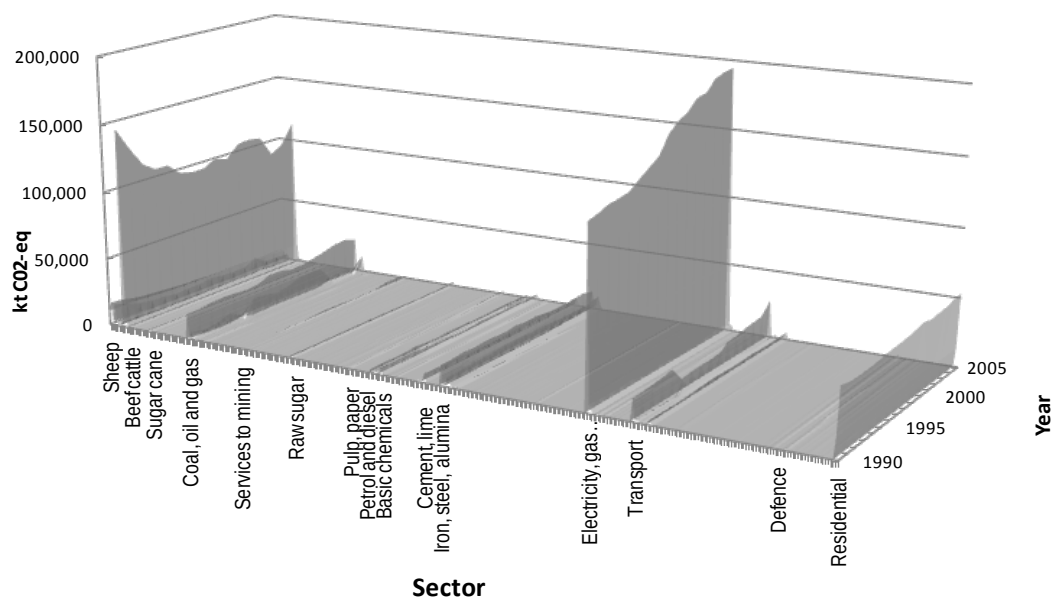


Figure 2 Emissions by producing sector, Australia 1990-2005, kt CO₂eq

Beef cattle emissions whilst large have stayed relatively stable over time. This is due to reductions in land clearing (most evident in the dip in the early 1990's) offsetting the overall expansion of the industry. In contrast, electricity use has grown substantially over time such that emissions are currently almost twice 1990 levels. A significant proportion of these emissions are caused by demand of electricity by other industries. Thus a distorted view could be gained of which industries are 'clean' or 'dirty' by examining only direct emissions, and ignoring the upstream emissions embodied in production. An alternative approach then, is to consider the driving forces underlying these emissions. The attribution of these emissions to levels of final demand provides such an approach. Calculation of embodied emissions going to final demand categories is known as the 'carbon footprint' of consumption and is undertaken as per Equation 2 and shown in Figure 3. Here, the emissions responsibility is shifted considerably towards the manufacturing and service sectors.

Carbon footprint by industry, Australia 2005

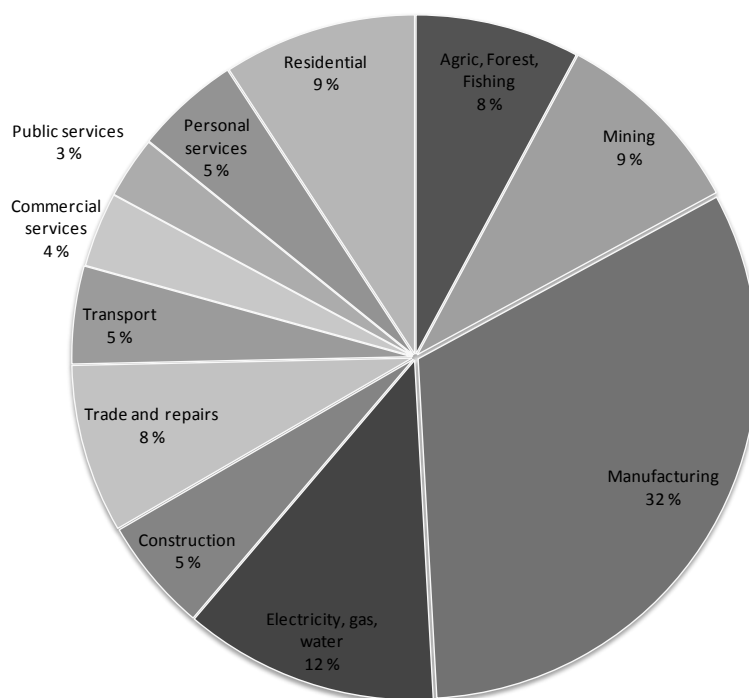


Figure 3 Carbon footprint of sectors, Australia, 2005

Manufacturing products contribute almost one-third of total emission. There are two main areas of impact within this sector (compare Table 7), with agricultural emissions (primarily associated with beef cattle) embodied in food production and industrial emissions (including electricity emissions) embodied in aluminium, alumina and other non-ferrous metal products. Electricity emissions (going direct to final demand) contribute roughly 12% of the total, whilst direct residential (transport, cooking, etc)

are around 9%. Together, these two parts are usually considered the main impact of the population, whilst in this approach, we see they miss 80% of the total impact.

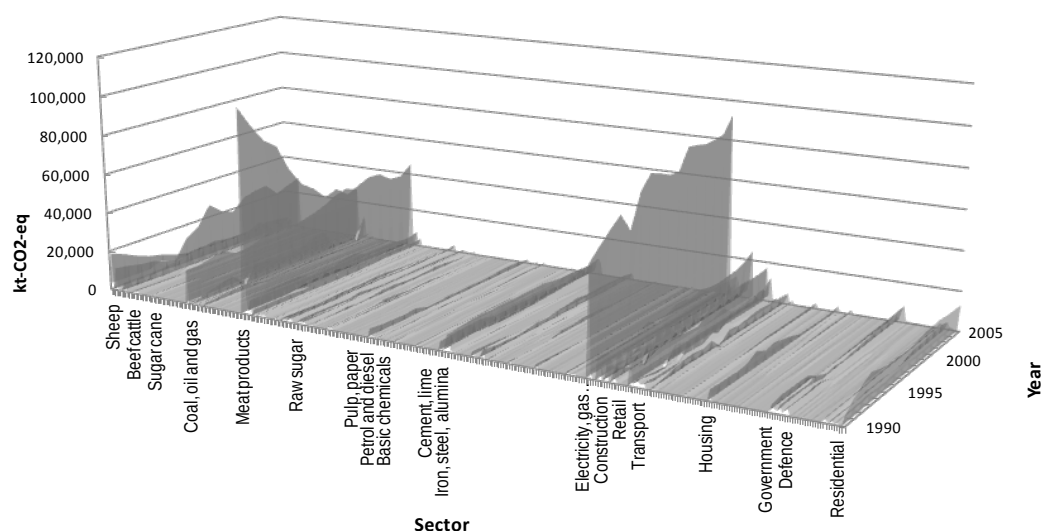


Figure 4 Emissions by sector of final demand, Australia 1990-2005

The first major component of Figure 4 from left to right is beef cattle (sector 4, leftmost). Whilst it was previously discussed that reduced land clearing had a negative effect on greenhouse gas emissions particularly in the early 1990's, this graph shows that demand related emissions have been growing. This is due to large expansion in the export of beef cattle from low levels in the early 1990's to significant levels now. The second most evident sector in Figure 2 is mining (sectors 40-50), with slowly growing emissions, again mainly driven by exports. The meat product sector is evident at sector 70. Demand for meat products has been more constant than that for beef cattle, resulting in the diminishing of the embodied emissions of the sector thanks to the reduction in the upstream land clearing (compare multiplier effects in Figure 5). Circa sector 170 are the non-ferrous metal industries, with the embodied emissions in aluminium products being most evident. Electricity (sector 240) is again strongly upward trending, and overshadows the construction industries just to the right, which have an important contribution to the domestic emissions. Service industries and household consumption (far right) are also responsible for significant industries, and both have been growing over time.

Whilst some of these sectors contribute significantly to greenhouse gas emissions, they also contribute significant amounts to the economic well being of Australia. If Australia is to move to a lower carbon society, it will want to maximise sectors with large social and economic contributions and small environmental impacts. Hence

using the income approach to measuring GDP, summing over value added (operating surplus, employment & taxes), we compare the relative contribution of each sector to overall GDP in Australia (Figure 5).

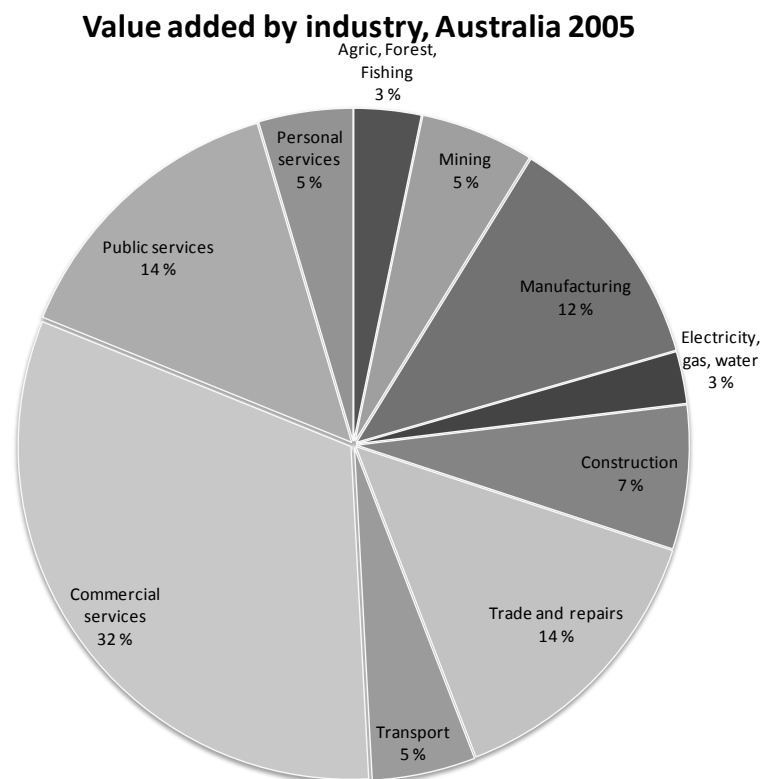


Figure 5 Value Added (% \$ - Profit, employment & taxes) by Sector

The Australian Government is using Value Added as one proxy for determining permit allocations and possible compensation under its forthcoming emissions trading scheme, known as the Carbon Pollution Reduction Scheme (CPRS). Although there are thresholds for the sizes of businesses included, under the draft CPRS, much of the manufacturing transport and primary industries will, as expected, require significant emissions permits. In addition to the emissions calculations, the Australian Governments is currently examining the level of trade exposure of Australian industries (Department of Climate Change 2009).

4.2. Imports, exports and capital effects

The calculation of the carbon footprint is not driven solely by the consumption of residents in Australia. As previously mentioned, there are export effects, as well as consumption for the common good (public/government consumption), and goods and services flowing into capital formation. These impacts are disaggregated in Figure 6.

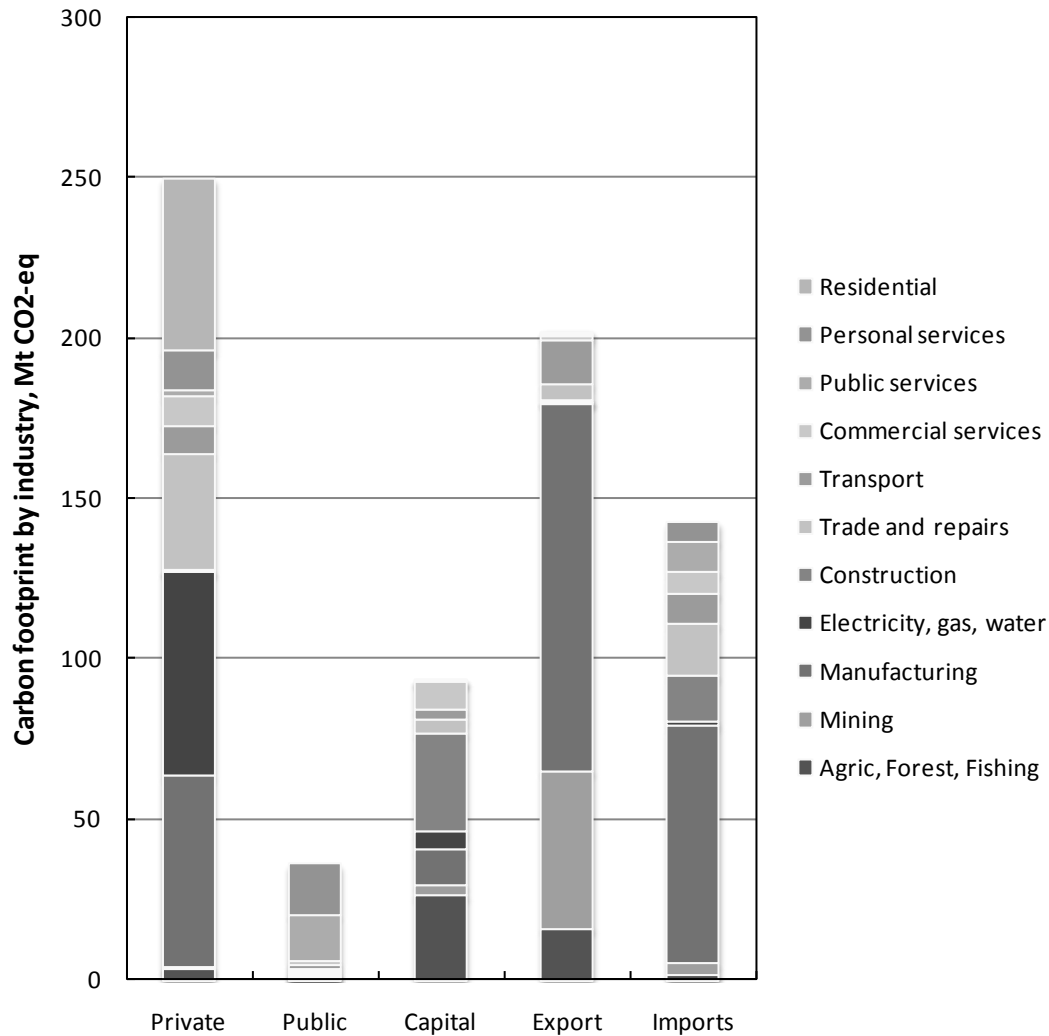


Figure 6 Emissions by type of demand, imports distinct, Australia 2005.

This gives a more accurate view of the sector disaggregation in a carbon footprint. The distortion caused by exports on primary industries is evident through the 80-90% of emissions embodied in exports being within primary and manufacturing industries. Often these goods are further processed before being made into consumer items in destination countries. Within private final consumption, we see manufactured goods and utilities almost equal contributors to the carbon footprint. However, considerable greenhouse gas emissions are embodied in manufactured goods within Australia. The per-capita effects are shown in Figure 7, with exports shown to be negative. Imported manufactured goods are responsible for almost six tonnes of CO₂-eq gases.

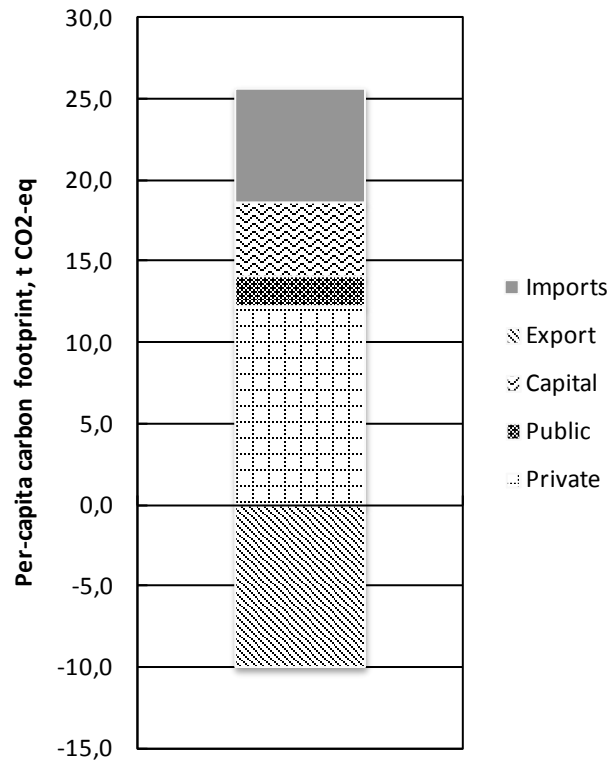


Figure 7 Per-capita carbon footprint, Australia, 2005

4.3. Domestic multipliers, 2005

The multipliers of final demand in Australia were calculated in order to show the sectors with the highest impact per \$ of consumption in Australia. Obviously these sectors are key for consumers to address in order to reduce their personal carbon footprint. In Figure 8, the final demand multipliers are mapped in an aggregated model of 30 sectors. This figure the importance of changes in direct and indirect production patterns in Australia over time. Lighter colours show high multiplier values (right axis), so darkening implies lesser indirect requirements. Again, beef cattle (leftmost) and electricity production (sector 23) are the two key sectors, with the changing land practices in beef cattle evident in the fluctuating multiplier values (denoted by changing colour intensity). The liberalisation of the electricity markets in the mid 1990's is evident in the change in that multiplier (sector 23).

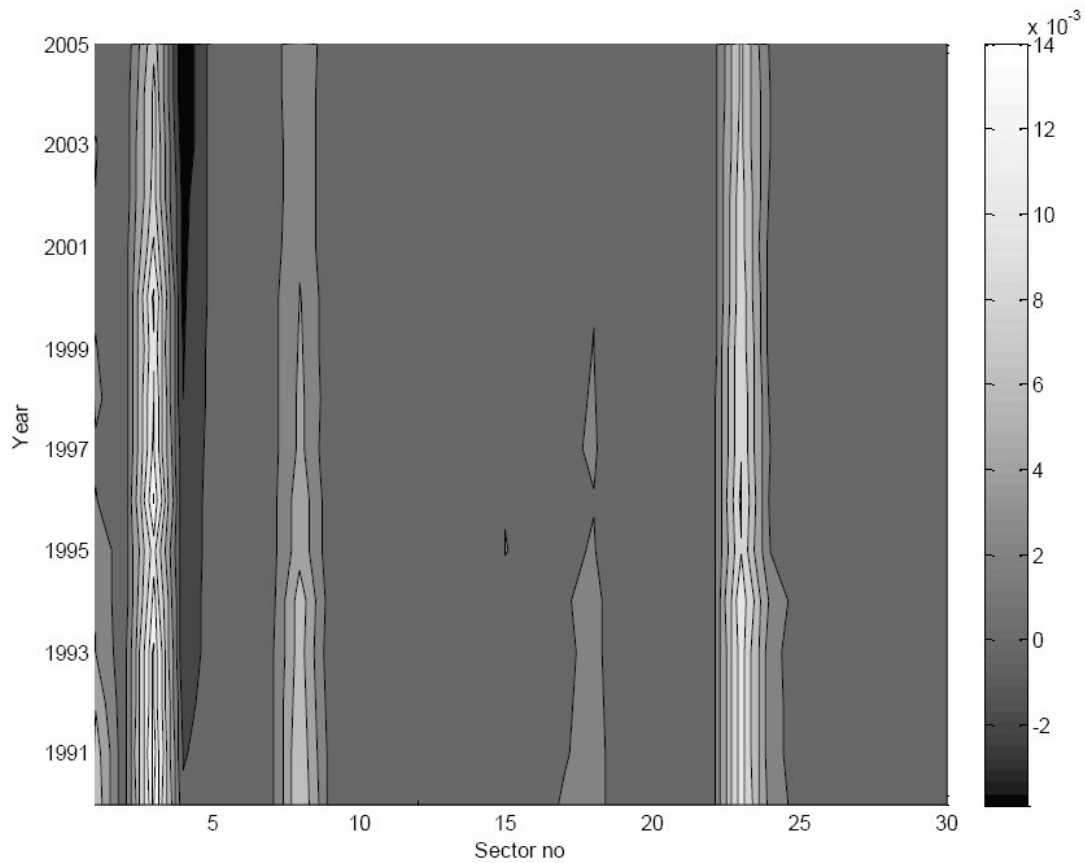


Figure 8 Multiplier evolution, Australia 1990-2005, 30 sectors.

5. Conclusions

This chapter has detailed the assumptions and methods used to estimate greenhouse gas emissions by input-output category. A range of proxy data was used in order to break down aggregate emissions into categories relevant to inter-industry analysis. A time series of these emissions was produced, and emissions embodied in final demand were calculated. A few key sectors, particularly beef and electricity produce the majority of Australia's domestic emissions, where as emissions embodied in final demand show the importance of meat products, mining and to a lesser extent, service industries. Very few of the emissions important industries contribute significantly to GDP.

In order to understand what has been driving the changes witnessed in this paper, analysis needs to be conducted on decomposing the main components of the production system (see Part II of this Thesis).

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MATERIAL FLOW ANALYSIS

Richard Wood

Abstract

In this note, Material flow analysis (MFA) is defined and construction techniques are outlined. Due to the simpler process involved in comparison to greenhouse gas emissions, the information is included as a note rather than a full chapter.

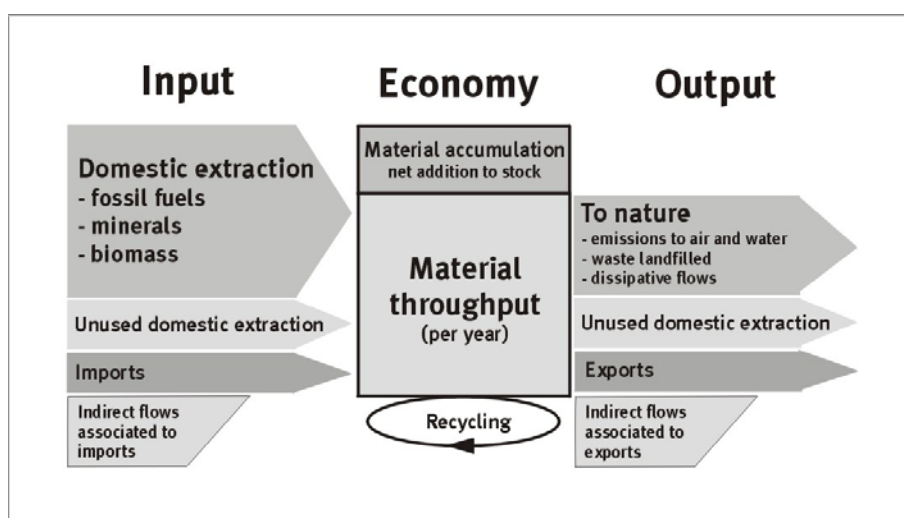


Figure 1: General scheme for economy-wide MFA, excluding water and air flows Source: EUROSTAT, 2001

1. Introduction

Material flow analysis has gained momentum since the early 1990s with the Concerted Action (Bringezu et al. 1997; Kleijn et al. 1999), followed by the World Resource Institute cross-country studies (Adriaanse et al. 1997) and (Matthews et al. 2000). Standardization was achieved through the publication of a methodological guidebook by the European Statistical Office (EUROSTAT 2001), and subsequent development in the field is summarized in (Weisz et al. 2007).

There are a number of definitions applicable to MFA (Figure 1). *Domestic extraction* (DE) and *Direct Material Consumption* (DMC) are calculated directly from production, import and export statistics (as defined in Weisz et al. 2007). Domestic extraction is the mass of material entering the economic system without including any unused extractions (domestic or imported) or the flows associated with imports. In comparison DMC is the actual material consumption of the population, including imports and excluding exports. In contrast to these direct indicators, a fourth indicator, *Total Material Requirements* (TMR) is defined as the summation of all material flows required for a population (EUROSTAT 2001). This includes material that enters the economic realm as well as “unused” flows – which are the necessary displacement of material for an auxiliary purpose (e.g. the creation of tailings associated with mining). Estimations of unused flows are important for three reasons. Firstly, in a macro-sense they define the physical metabolism of the whole economic system which in many circumstances defines an economy’s sophistication within globalised trade i.e. a price-taking rather than a price-making economy. Secondly, the unused flows in farming are important ecologically as the soil erosion component impacts negatively on long-term sustainability. Thirdly for mining, high levels of material flow often reflect declining ore grades and increasing requirements for mine rehabilitation services once the core product has been extracted.

Direct imports are included, and indirect impacts and unused flows associated with the imports also included in TMR. Poldy and Foran (1999) argue against the inclusion of unused flows in imports in order to avoid double counting in inter-country analysis. As the availability of data on unused extraction for Australia’s import partners is very low, in this study, the intensity of unused extraction embodied in imports is assumed to be the same as domestic production. For Australia, imports are reasonably minor, and of these imports, relatively few are from primary industries with materially large unused extraction associated, thus there is likely a negligible distortion on final results.

The only deviation we take from the current standards (Weisz et al. 2007) is that we exclude calculation of grazed biomass as a component of domestic extraction, however all fodder is still included. We find it hard to reconcile the fact that uneconomic flows of mining, forestry and other agriculture (tailings, wood off cuts, etc) are not counted in domestic extraction, whilst for livestock, natural growth of grasses must be counted as a material/economic input to livestock production. The argument for excluding plants in other sectors is stated as (Weisz et al. 2007):

“Statisticians would be forced to convert rather robust and valid data on annual agricultural and timber harvest to comparable weak estimates of the primary inputs needed to produce these plants. Moreover, all differentiation between different types of crops would be lost, as well as the conceptual link to the system of national accounts. It is hard to imagine how such data could possibly be interpreted in any meaningful way, given the limitations of a black box accounting system such as MFA.”

We do not see how this can be interpreted any differently to natural growth of rangelands for livestock production. By calculating grazed biomass rather than animal weight, we are making an inconsistency with national accounts boundaries, we are placing weak assumptions on uptake of dry matter from very different grazing lands and we are losing the differentiation in the type of livestock (we would see grazed biomass rather than sheep or cattle).

2. Data Sources

Material flow data were sourced from the Australian Bureau of Agricultural and Resource Economics (ABARE), which publishes annual historical data on commodity production (Australian Bureau of Agricultural and Resource Economics 2008). These data were complemented by the United Nations’ FAO database, which has more complete data for food commodities (FAO Statistics Division (FAOSTAT) 2008). Further data were available from the ABS on building approvals, dwelling size and road construction (Australian Bureau of Statistics 2008, 2005). Finally waste statistics were taken from state environmental reports, and the National Pollution Inventory (NPI) was used to calculate emissions to air, water and land (National Pollutant Inventory 2008). Only primary data was included so as not to double count manufactured goods.

3. Calculation of TMR

Product	Production (t product/head)	Manure factors (t manure/head)	TMR/DE Conversion (t manure/t product)
Sheep and lambs	0.005	0.6	112
Shorn wool	0.004	0.6	139
Beef cattle	0.23	9.2	41
Dairy Cattle	4.72	13.5	3
Pigs	0.14	1.5	11
Poultry	0.002	0.04	24

Table 1 Manure conversions DMI to TMR, after Poldy and Foran (1999)

Seven types of relationships for TMR are included. Firstly, biomass uptake and manure is calculated for all livestock. Production statistics are calculated from ABARE (2008), whilst manure factors are taken from Poldy and Foran (1999) to obtain a TMR/DMI conversion.

Biomass grazing is calculated according to estimated dry matter uptake. Grazing practices are significantly different in Australia than Europe, so industry data was used on dry matter intake by animal:

Product	t biomass/head
Sheep and lambs	0.548
Beef cattle	3.65
Dairy cattle	7.30

Manure and biomass uptake are in theory talking about the same thing – inputs and outputs of animal digestive processes. We use the lower of the two estimates where data is available, as it is not clear if manure is a dry matter weight or not.

Erosion coefficients are also calculated for various land types from Lu (2003), and applied to annual areas of land use.

Land type	Area 000 km²	Erosion t/ha/year
Forest	277	1
Woodland	2 180	2.8
National Park	184	7.4
Residual Native Pastures	4 232	5.4
Plantation	146	2.1
Improved pasture/legumes	200	1.1
Cereals excluding rice	193	2.7
Oilseeds	4	3.2
Other agricultural lands	16	11.1
Sugarcane	5	16.1

Table 2 Erosion coefficients, after Lu (2003)

Biomass and Forestry unused extraction are applied directly from Poldy and Foran (1999), whereas updated data from Mudd (2007, 2008) was used where available for mining commodities (Table 3).

Product Group	Product	Ratio (unused extraction/DE)	Reference
Biomass	Grains	1.37	Poldy&Foran
	Oilseeds	2.58	Poldy&Foran
	Legumes	2.33	Poldy&Foran
	Fruit	0.3	Poldy&Foran
	Sugar cane	0.14	Poldy&Foran
	Cotton	1.82	Poldy&Foran
Forestry	Softwoods	0.25	Poldy&Foran
	Hardwoods	2	Poldy&Foran
Metal ore	Iron ores	2	Mudd
	Bauxite	1.39	Poldy&Foran
	Copper	245	Mudd
	Gold	1800000	Mudd
	Other ores	20.9	Poldy&Foran
	Silver and zinc ores	10	Mudd
Energy products	Brown coal	1.04	Mudd
	Crude oil	0.0795	Poldy&Foran
	Natural gas	0.301	Poldy&Foran
	Uranium ores	2415	Mudd
Non-metallic ore	Construction materials	0.02	Poldy&Foran
	Limestone	0.71	Poldy&Foran
	Clays	3	Poldy&Foran
	Salt	0.14	Poldy&Foran

Table 3 Unused extraction ratio to used product

Mudd regresses overburden of black coal over time ($f(t)=0.08101(t)-156.51$, where t is year, p39 Mudd 2007), which we use to give a better temporal profile for this product.

Finally, excavation coefficients for construction activities followed Poldy and Foran (1999). Length of road construction and area of building construction was taken or estimated from ABS Yearbooks (e.g. Australian Bureau of Statistics 2005).

Activity group	Activity	Excavation Unit coefficient
Road surface	Bitumen or concrete	20 000 t/km
	Gravel, crushed stone or other improved surface	10 000 t/km
	Formed only	5 000 t/km
	Cleared only	5 000 t/km
Building	Houses	0.8 t/m ²
	Other Residential	0.5 t/m ²

Table 4 Excavation coefficient of construction activities, after Poldy and Foran (1999)

We have updated erosion coefficients from the National Land Water Audit (Lu (2003), Table 2 – reproduced below).

4. Total Material Flow

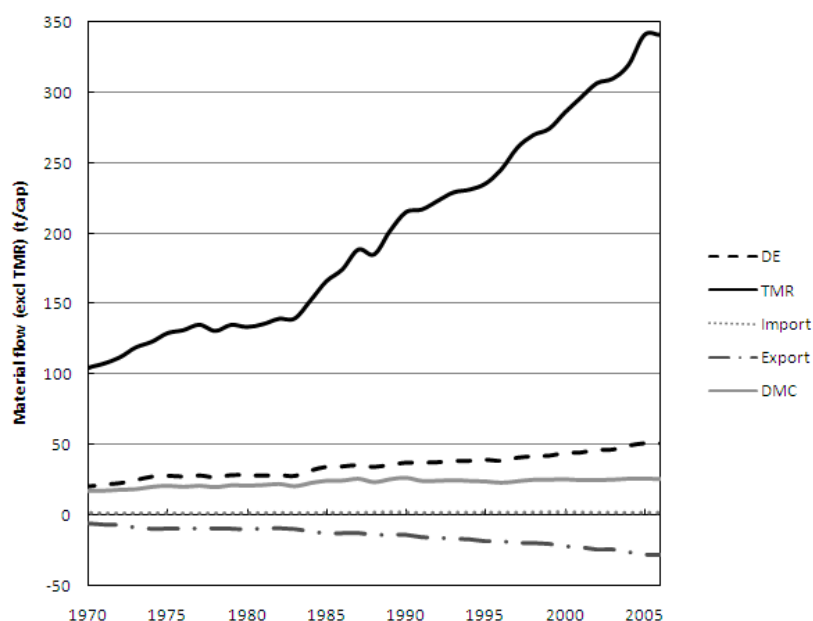


Figure 2: Per-capita material flow estimated for Australia.

Since the mid 1970's, DE has grown by a factor of 2.5, whilst TMR has grown by a factor of almost 3.5 (Figure 2). This growth, and the difference in the rates of growth, is mostly due to an expansion of the mining industries in Australia, principally due to export demand. This is reflected in the graph of DMC, which has been fairly stable since the late 1970's. Imports reflect only used flows, and generally represent only about 3-4% of domestic material production. Exports are much more significant, with up to 60% of total domestic material production being exported. The rapid rise following the mid-1980s is due mostly to black coal exports but also to expansion of gold and diamond mining which have low yields of final product per unit of mined ore (see Figure 3). In parallel, the material flows of the agriculture sector are reducing slightly due to declines in total sheep numbers and thus less soil erosion as the international demand for wool lessens.

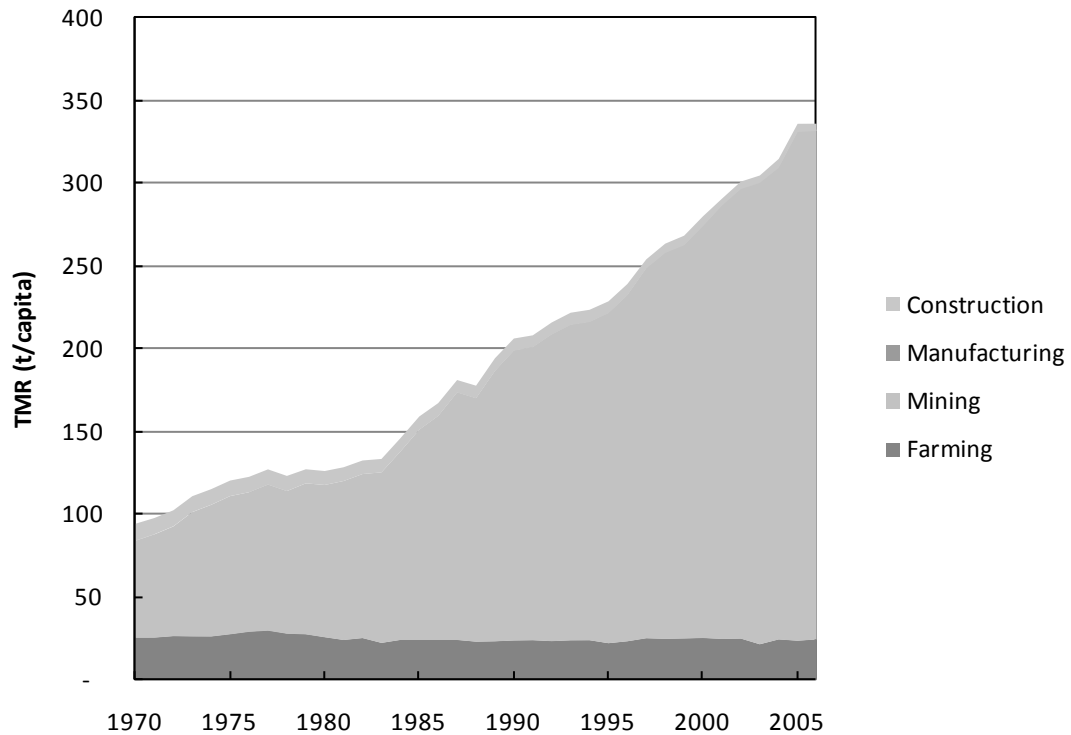


Figure 3: Components of per-capita TMR estimated for Australia.

The material allocated to producing sector is shown in Figure 4 for 1969-2005. The growth in material output of the mining sectors is most evident circa sector 50. Agricultural industries contribute smaller amounts (circa sector 0), whilst at the other end of the spectrum, construction activities are responsible for significant material flow (circa sector 250).

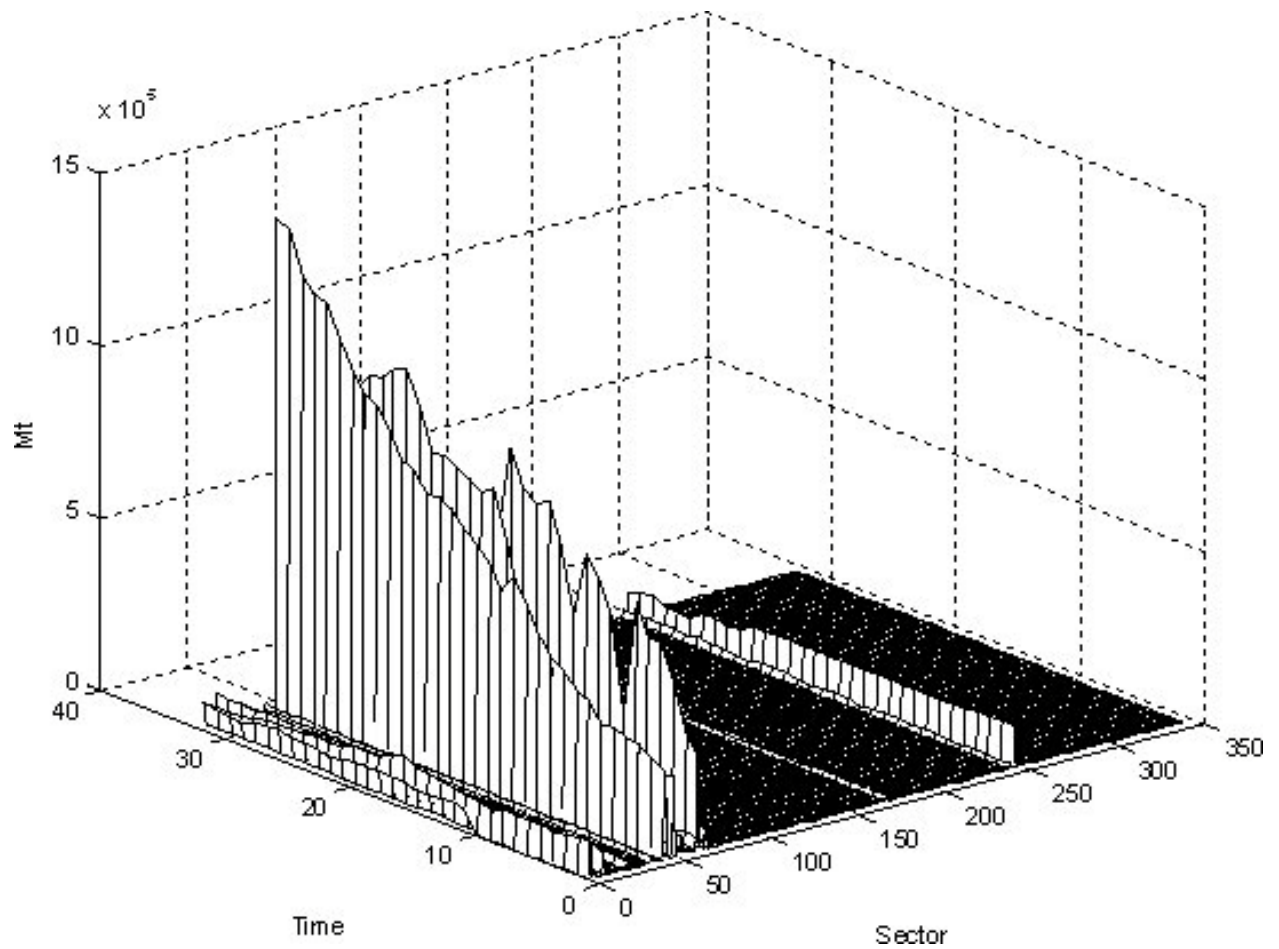


Figure 4 Material Flow by producing sector, Australia 1969-2005, sector number: primary ~0, tertiary ~ 350.

5. Conclusion

Material requirements in Australia have grown rapidly over the last 30 years. The underlying effects and cause of this change will be analysed in Part II of this thesis.

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A FLEXIBLE APPROACH TO MATRIX BALANCING UNDER PARTIAL INFORMATION

Manfred Lenzen, Blanca Gallego and Richard Wood

Lenzen, M., B. Gallego, and R. Wood. 2006. A flexible approach to matrix balancing under partial information. *Journal of Applied Input-Output Analysis* 11 & 12.

Abstract – We present an extension of the standard RAS algorithm, referred to as ‘constrained RAS’, or ‘cRAS’. In contrast to existing RAS variants employing additional partial information in a particular aggregated form, our technique makes use of partial information of any kind. This is achieved by imposing constraints on arbitrary-sized and –shaped sets of elements to be balanced. The characteristics of the cRAS method are investigated in detail.

Keywords: RAS matrix balancing, partial information

1 Introduction

A common problem in compiling and updating input-output tables is that of incomplete data. Missing matrix elements may be due to a variety of reasons such as costly and therefore incomplete industry surveys, or the suppression of confidential information. Unknowns almost always outnumber data points, resulting in the system being underdetermined, that is exhibiting too many degrees of freedom to be solved analytically, so that certain estimation, balancing or optimisation techniques have to be applied.

Despite having received mixed responses in relation to the accuracy of its results¹, a widely applied remedy in this situation is the *bi-proportional*, or RAS method (Stone and Brown 1962). Especially for the task of constructing or updating regional input-output tables from national data, the RAS method has been compared with other approaches such as location quotients and commodity balances (Schaffer and Chu 1969; Leontief 1953), gravity models (Leontief and Strout 1963), linear programming (Moses 1960; Matuszewski *et al.* 1964; Davis *et al.* 1977), or entropy and minimum information gain techniques (Uribe *et al.* 1966; Batten 1982; Golan *et al.* 1994). The origins of the method go back several decades (Deming and Stephan 1940); for a historical background, see Bacharach 1970, Chap. 1-3, and Polenske 1997.

The RAS method – in its basic form – bi-proportionally scales a matrix $\mathbf{A}_0$ of unbalanced preliminary estimates of an unknown real matrix $\mathbf{A}$, using $\mathbf{A}$'s known row and column sums. The balancing process is usually aborted when the discrepancy between the row and column sums of $\mathbf{A}_0$ and $\mathbf{A}$ is less than a previously fixed threshold. Bacharach 1970 has analysed the bi-proportional constrained matrix problem in great detail, in particular in regard to the economic meaning of bi-proportional change², the existence and uniqueness of the iterative RAS solution, its properties of minimisation of distance metric³, and uncertainty associated with errors in row and column sum data and with the assumption of bi-proportionality.

A special situation arises when some of the matrix elements of $\mathbf{A}$ are known in addition to its row and column sums, for example from an industry survey. The 'modified RAS' approach (Paelinck and Walbroeck 1963; Allen 1974; Lecomber 1975) deals with this partial information as follows: the preliminary estimate $\mathbf{A}_0$ has to be "netted", that is the known elements are subtracted, and $\mathbf{A}_0$ contains 0 at the corresponding positions. The net $\mathbf{A}_0$ is then

¹ See for example Uribe *et al.* 1966; Czamanski and Malizia 1969; Goldman 1969; Miernyk 1969; Malizia and Bond 1974; Morrison and Smith 1974; Miernyk 1976; Hinojosa 1978; Parikh 1979; Butterfield and Mules 1980; Morrison and Thuman 1980. Jensen *et al.* 1979 summarise the dilemma of regional input-output modeling as a trade-off between 'professional purity' and uncertain accuracy.

² When applied to the forecasting of monetary input-output matrices, bi-proportional changes have been interpreted as productivity, substitution or fabrication effects (Leontief 1941; Stone and Brown 1962) affecting industries over time. Miernyk's (1976) view however is that the RAS method "substitutes computational tractability for economic logic", and that the production interpretation loses its meaning when the entire input-output table is balanced, and not only inter-industry transactions (see also Giarratani 1975).

³ The RAS, Linear Programming and minimum information gain algorithms yield a balanced matrix estimate that is – in terms of some measure of multidimensional 'distance' – closest to the unbalanced preliminary estimate. When applied to temporal forecasting, this property is explained as a conservative hypothesis of attributing inertia to inter-industrial relations (Bacharach 1970, p. 26).

subjected to the standard RAS procedure, and the known elements are added back on after balancing. An operational example of this approach is the GRIT (Generation of Regional Inter-output Tables) method (Jensen *et al.* 1979; West *et al.* 1984; Bayne and West 1989), which involves 4 steps: 1) preparation of the national table, 2) preliminary estimate of the regional table using a non-survey approach such as commodity balances or location quotients⁴, 3) insertion of superior, survey-based data, and 4) identification of important elements, and subsequent collection and insertion of superior data. Within GRIT, aggregated superior data can only be inserted after aggregating the preliminary table (Jensen *et al.* 1979, Sec. 6.4). Another variant of the modified RAS method takes into account the uncertainty of the preliminary estimates, and contains the occurrence of perfectly known elements as a special case⁵ (Allen and Lecomber 1975).

In practice, situations can arise where, in addition to certain elements of $\mathbf{A}$, some aggregates of elements of $\mathbf{A}$ are known. For example, a published table $\mathbf{A}^G$ of national aggregates may constitute partial information when constructing a multi-regional input-output system, or a more disaggregated national table. Accordingly, Oosterhaven *et al.* 1986 add a “national cell constraint” to the standard row and column sum constraints. Similarly, Jackson and Comer 1993 use partition coefficients for groups of cells of a disaggregated base year matrix to disaggregate cells in an updated but aggregated matrix. Batten and Martellato 1985, p. 52-55) discuss further constraints structures, involving intermediate and final demand data. Gilchrist and St Louis 1999 propose a three-stage RAS for the case when aggregation rules exist under which the partial aggregated information $\mathbf{A}^G$ can be constructed from its disaggregated form $\mathbf{A}$. These rules take the form of matrix operators $\mathbf{P}$ and $\mathbf{Q}$, with $\mathbf{A}^G = \mathbf{PAQ}$. The essence of their TRAS method is that balancing is carried out alternately at the disaggregated and at the aggregated level, with the aggregated row and column scaling operators $\mathbf{R}^G$ and $\mathbf{S}^G$ being disaggregated via multiplication with the transposed operators $\mathbf{P}^t$ and $\mathbf{Q}^t$. Subjecting an input-output matrix to random censoring, Gilchrist and St Louis 1999 demonstrate that the inclusion of partial aggregated information into the RAS procedure leads to superior outcomes than applying the standard RAS method.

In the above approaches the partial information has to assume a particular aggregated form. A number of authors⁶ have generalised the method 1) by formulating it as a Lagrangian multiplier calculus, as an econometric estimation, or as an optimisation, 2) by enabling balancing of negative elements, and 3) by allowing for partial information of any form, by imposing constraints on arbitrary-sized and –shaped sets of elements of $\mathbf{A}_0$. Lahr and de Mesnard 2004 provide a recent overview.

⁴ For an overview of these techniques see Schaffer and Chu 1969, or Harrigan *et al.* 1980.

⁵ This is accomplished by introducing a matrix $\mathbf{E}$ of standard errors of the elements in $\mathbf{A}_0$, which is subsequently balanced in order to take up the difference between the preliminary and true totals. Where $E_{ij}=0$, the element in $\mathbf{A}_0$ remains unchanged. Allen and Lecomber 1975 also investigate the influence of errors in the “true” totals.

⁶ See for example Byron 1978, Morrison and Thuman 1980, van der Ploeg 1982 and Cole 1992 on the Lagrangian method, Gerking 1976; 1979a and Toh 1998 on econometric estimation, and Tarancon and Del Rio 2005 on optimization. See Junius and Oosterhaven 2003 and Jackson and Murray 2004 on the problem of negative entries. See Morrison and Thuman 1980, Cole 1992, Thissen and Löfgren 1998 and Tarancon and Del Rio 2005 on generalized constraints.

The aim of this work is to test the performance of approaches that feature arbitrary constraints. In particular we investigate whether

- 1) it is better to constrain a few large elements, or many small elements,
- 2) it is better to have a few constraints spanning many elements, or many constraints on single elements,
- 3) the shape of the constraints (adjacent elements or disjunct elements) influences the performance.

We enumerate performance using a range of common distance measures, and apply them to the flow matrix, the Leontief inverse, and some multipliers. We restrict ourselves to testing a RAS-type approach (referred to as ‘constrained RAS’, or ‘cRAS’), and ignore Lagrangian and optimisation approaches.

This article is organised as follows: We describe our proposed cRAS method, then present an application to a generalised regional input-output system for Australia, and conclude.

2 A representative cRAS method

In the standard RAS method, the preliminary estimate $\mathbf{A}_0$ is alternately row- and column-scaled using diagonal matrices $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$, so that after one round of balancing, $\mathbf{A}_1 = \hat{\mathbf{r}}\mathbf{A}_0\hat{\mathbf{s}}$. $\mathbf{A}_1$ is then subjected to the next scaling operation. One may start with either rows, $\mathbf{A}_1 = (\hat{\mathbf{r}}\mathbf{A}_0)\hat{\mathbf{s}}$, or columns, $\mathbf{A}_1 = \hat{\mathbf{r}}(\mathbf{A}_0\hat{\mathbf{s}})$. In the first round, $\hat{r}_{ii} = a_i / \sum_j A_{0,ij}$ and $\hat{s}_{jj} = a_j / \sum_i A_{0,ij}$, and so on, where the a_{ij} are row/column totals of $\mathbf{A}$.

Let C be the number of additional available data points to be used as constraints on the A_{ij} . Let E_c be the number of elements of $\mathbf{A}$ participating in constraint c . Let $(i_{c,e}, j_{c,e})$ be the coordinates of element e in constraint c . Let $B_c = \sum_{e=1}^{E_c} A_{i_{c,e}, j_{c,e}}$ be the value prescribed for the sum of elements participating in constraint c . A typical cRAS algorithm can then be described as a sequence:

- Generate preliminary estimate $\mathbf{A}_0$;
- scale rows of $\mathbf{A}_0$ by pre-multiplication with $\hat{\mathbf{r}}$;
- scale constraint 1 by multiplying all E_1 participating elements with
$$B_1 / \sum_{e=1}^{E_1} A_{0,i_{1,e}, j_{1,e}} = \sum_{e=1}^{E_1} A_{i_{1,e}, j_{1,e}} / \sum_{e=1}^{E_1} A_{0,i_{1,e}, j_{1,e}} ;$$

- scale constraint 2 by multiplying all E_2 participating elements with
$$B_2 \bigg/ \sum_{e=1}^{E_c} A_{0,i_{2,e},j_{2,e}} = \sum_{e=1}^{E_c} A_{i_{2,e},j_{2,e}} \bigg/ \sum_{e=1}^{E_c} A_{0,i_{2,e},j_{2,e}} ;$$
- and so on, until constraint C ;
- scale columns by post-multiplication with $\hat{\mathbf{s}}$;
- result of the above operations is $\mathbf{A}_1$;
- repeat scaling process for $\mathbf{A}_1$ until all scaling factors deviate from 1 by less than a specified threshold.

Any permutation of this sequence is also a valid cRAS algorithm. Constraints can include any number of elements, which may be fully, partly or non-adjacent.⁷ Constraints may also exclude some of the row and column totals (compare Thissen and Löfgren 1998, p. 1994).

3 Performance evaluation

In the following, we will examine the performance of the cRAS method for the example of estimating the use matrix $\mathbf{U}$ of the 1996-97 Australian input-output tables from its true row and column sums, as well as varying numbers of subsets of true elements, starting with different preliminary estimates. We will also appraise estimates for the Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{U}\hat{\mathbf{x}}^{-1})^{-1}$, and output and CO₂ multipliers $\boldsymbol{\xi} = \mathbf{1}\mathbf{L}$ and $\boldsymbol{\chi} = \mathbf{c}\mathbf{L}$, respectively, where $\mathbf{I}$ is a unity matrix, $\hat{\mathbf{x}}$ is the diagonalised gross output vector, $\mathbf{1}$ is a unity row vector, and $\mathbf{c}$ is a row vector of sectoral CO₂ intensities.

While a performance evaluation based on $\mathbf{U}$ emphasises the importance of the table elements themselves, evaluations based on $\mathbf{L}$, $\boldsymbol{\xi}$ and $\boldsymbol{\chi}$ recognise that these derived quantities are ultimately of interest in applied models. With each subsequent step of further processing $\mathbf{U}$ (through inversion to $\mathbf{L}$, summing to $\boldsymbol{\xi}$ and weighting to $\boldsymbol{\chi}$), the derived quantities embody more widely inter-linked information. Evaluations of these quantities (rather than table elements) are the subject of a more integrated and applied measure of overall, ‘holistic’, or operational accuracy (Jensen 1980). While table accuracy represents the conventional understanding of a measure of the deviation of all estimated matrix elements from their “true” values, holistic accuracy is concerned with the representativeness of a table of the synergistic characteristics of an economy. In this perspective, the accuracy of single elements may be unimportant, as long as the results of modeling exercises yield a realistic picture for the purpose of the analyst or decision-maker. Table accuracy implies holistic accuracy, but not vice versa; nevertheless, the former may not be achievable, while the latter may be more realistic in pragmatic terms. However, a drawback is that general criteria for holistic accuracy

⁷ Constraints with $E_c = 1$ need not be part of the scaling procedure, but could be “netted out” using the “modified RAS” method.

are hard to define, and hence are a characteristic of the model output, such as the derived quantities above.

3.1 Programming features

A number of stochastic experiments of up to 250 runs were carried out, in which the preliminary estimate, the number of constraints, the numbers E_c and coordinates $(i_{c,e}, j_{c,e})$ of constrained subset elements, and other parameters were varied.

3.1.1 Balancing threshold

It is in principle possible to terminate the balancing process based on absolute or relative criteria. For example, a threshold can be formulated as $\max_i (|u_i - u_{n,i}| / u_i)$, where $\mathbf{u}$ contains the row and column sums of $\mathbf{U}$, and $\mathbf{u}_n$ those of the n^{th} iteration of $\mathbf{U}_0$. However, industry sectors would receive equal weights independent of their size. Therefore, we adopted the criterion $|u_i - u_{n,i}| < \bar{U} \alpha$, for the row and column totals, and also for all additional constraints, where $\bar{U} = N^{-2} \sum_{ij} U_{ij}$ is the average absolute value of the true flow matrix, and α is a scaling factor. This way, the balancing result will exhibit an overall absolute proximity to constraints, which is better by a certain factor than the average magnitude of the matrix itself.

3.1.2 Preliminary estimate

We allowed three different types of preliminary estimates $\mathbf{U}_0$: 1) the 1994-95 Australian flow matrix $\mathbf{U}^{95}$, 2) a weighting $U_{ij}^l = (u_i u_j) / \sum_{mn} U_{mn}$ according to industry size, and 3) random numbers U_{ij}^R with mean $\bar{U}$. Option 1 represents a classical updating task. Option 2 resembles an estimation task (often of regional tables) in the absence of any data except prescribed row and column sums; the formulation is the most simple prediction for inter-industry flows. It is based on the assumption of independence, that is it implies that flows from industry i to industry j are large if i produces much and j uses much. Since the production recipe, transport distances or other parameters do not play any role, this estimate is generally quite unrealistic (Uribe *et al.* 1966). Note that in the absence of additional subset constraints, option 2 already satisfies row and column constraints, since $\sum_i (u_i u_j) / \sum_{mn} U_{mn} = u_j$, and therefore cannot precede simple RAS balancing.

3.1.3 Constraint size and structure

In principle, the elements belonging to any particular constraint can be an arbitrary-shaped and -sized subset of the elements of $\mathbf{U}$. This was simulated by randomly selecting the number and size of constraints as well as the coordinates of elements. The constraint value is the sum over values of the target matrix elements that belong to the respective constraint. In this work, all constraints are consistent by definition, because their values are determined from the 1996-97 $\mathbf{U}$ matrix.

The elements belonging to any constraint can be anything from completely disjunct (referring to completely different pairs of industries) to all adjacent (having industries in common). One common example for completely disjunct constraints are so-called “rectangular” constraints which arise out of constraining all elements U_{ij}^{rs} connecting industries i and j and regions r and s of a multi-regional input-output table, by the national estimate U_{ij} . More random disjunct constraints can occur if a table expressed in a certain industry sector classification is constrained using data expressed in another industry sector classification. In Australia, for example, superior data often exists classified as ANZSIC⁸, while input-output tables are classified in IOPC⁹. The most common versions of these, ANZSIC 4-digit and IOPC 4-digit, overlap in a sense that for some parts of the economy, ANZSIC is more detailed, for others IOPC. Moreover, while IOPC is structured to minimise the output of secondary products for all sectors, ANZSIC could include entire establishments with multiple output commodities in one sector.¹⁰ The result of constraining an IOPC system with ANZSIC data are constraints that are scattered over non-adjoining rows as well as columns (Fig. 1 left).

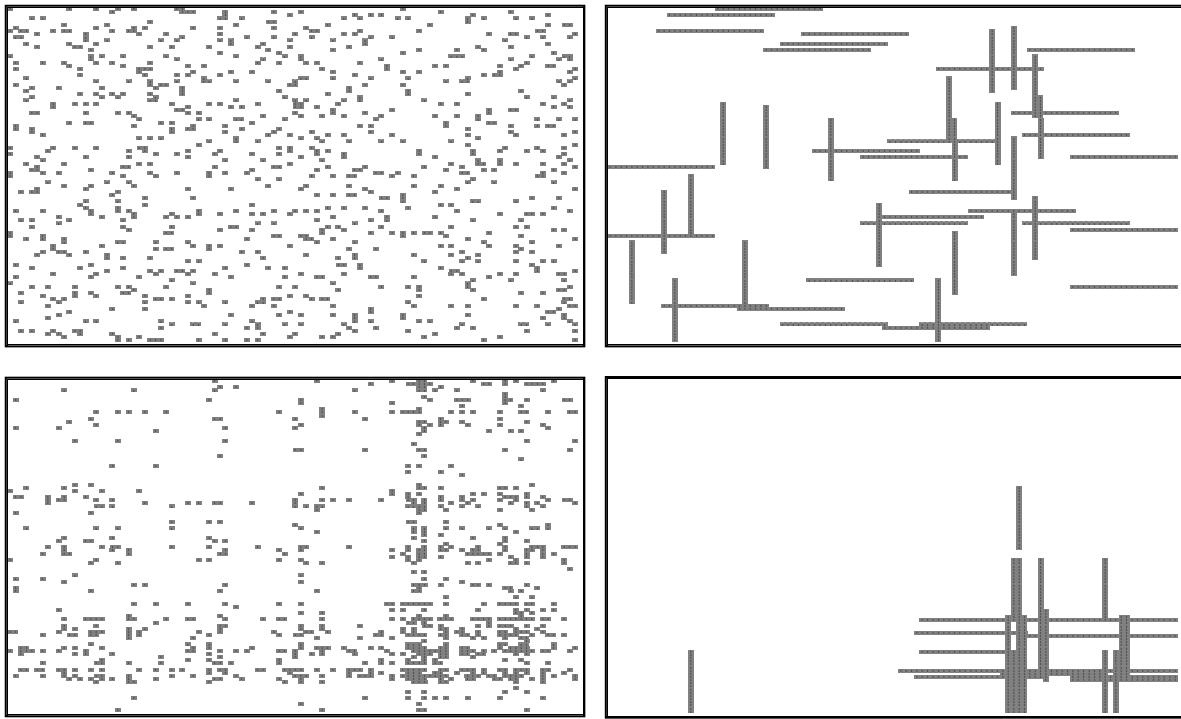


Fig. 1: Constraint structures and coordinate locations in the flow matrix U , for the example of distributing 50 constraints with 20 elements each. Set elements can be either disjunct (left) or adjacent (right). Their coordinates can assume either random positions (top) or be skewed towards top-ranking elements (bottom). Note that in the numerical experiments the number of set elements does not have to be constant.

⁸ Australian and New Zealand Standard Industry Classification.

⁹ Input-Output Product Classification.

¹⁰ An example is broadacre farming, where combined beef-wheat farms are classified in one ANZSIC sector, but split into two IOPC sectors.

It is equally likely to encounter constraints that incorporate adjacent elements, which mostly result from aggregated superior information, for example the intermediate input of a whole group of commodities, or the output of a whole group of industries. These will appear as either vertical (inputs) or horizontal (outputs) bands of the table to be estimated (Fig. 1 right). In general, subsets of randomly selected elements will be mostly disjunct. In order to generate adjacent-cell subsets, a seed element was first selected randomly, and then only adjacent cells were selected for the remaining elements up to the respective constraint size. In this work, only row- and column-segment-shaped constraints (with random orientation) were considered as adjacent-cell subsets.

With increasing number and/or size of constraints, overlaps and/or duplications become more likely. In the following we refer to the (gross) number of elements in all constraints as ‘set elements’, while a smaller number of elements (net of overlap) are actually ‘constrained’. If set elements are few, they are likely not overlapping (Fig. 2, set elements = constrained elements, triangles and solid line coincide; overlap ratio = 1). As the number of set elements increases, overlap/duplication increases: the number of actually constrained elements falls below the number of set elements contained in all constraints (Fig. 2, set elements > constrained elements, triangles and solid line deviate; overlap ratio >1).

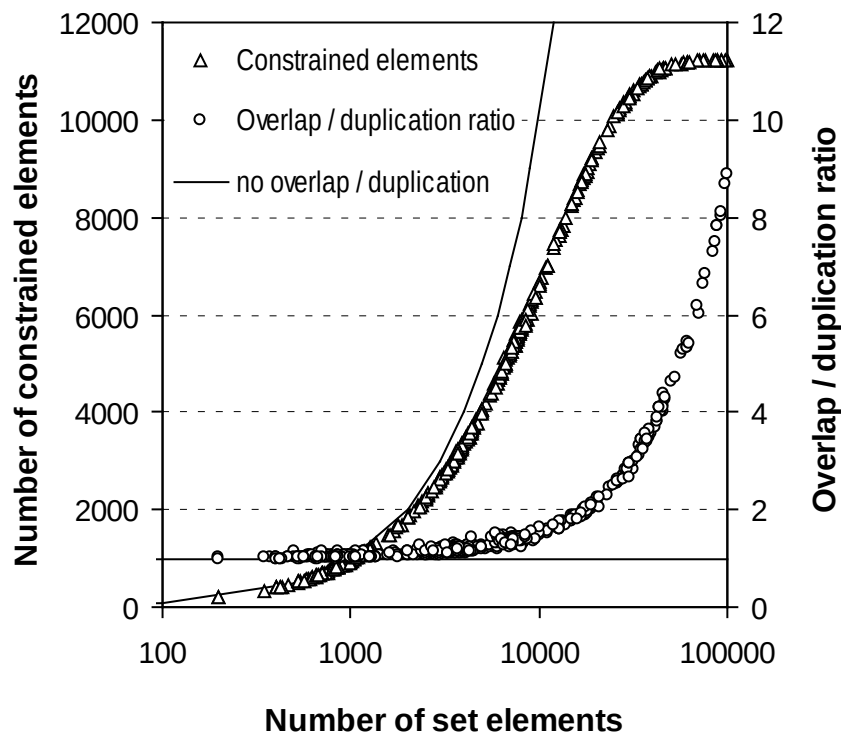


Fig. 2: Number of constrained elements (Δ , left y -axis) and overlap / duplication ratio ($\circ$, right y-axis) as a function of the number of set elements (all constraint structures, equal selection of constraint element coordinates).

3.1.4 Coordinate location

In principle, constraints can incorporate arbitrary elements, or focus on important elements. In order to cover these alternatives in our simulations, set elements (Fig. 1 left) or seed elements (right) were either selected equally (top) or preferentially (bottom) from a ranking of important elements. The latter were defined as those elements that have the largest value in the preliminary estimates $\mathbf{U}^{95}$ or $\mathbf{U}^I$.

Preferential selection (Fig. 1 bottom) can be realised by generating random rank numbers (in this case from 1 to $N^2 = 106^2$) with a larger probability for lower numbers (upper ranks) in order to ensure that these are always part of a constraint subset. Such a probability distribution can be constructed by repeatedly applying a random number $Rand \in [0,1[$ as the upper bound of a subsequent random number. An algorithm that carries out this task is quite simple:

$x = 1. : \text{For } k = 1 \text{ To } r : x = x \times Rand([0,1[) : \text{Next } k : rank = x \times N^2$

The mean of random values x generated in such a way decreases with the number of randomisations r . Based on once-randomised ($r = 1$) values in $[0,1[$, elements of rank $rank$ are selected with equal probability (curve 1). Since low numbers $x \in [0,x[$ occur with increasing frequency after repeated randomisations ($r > 1$), top-ranking elements are selected with increasing probability (Fig. 3).

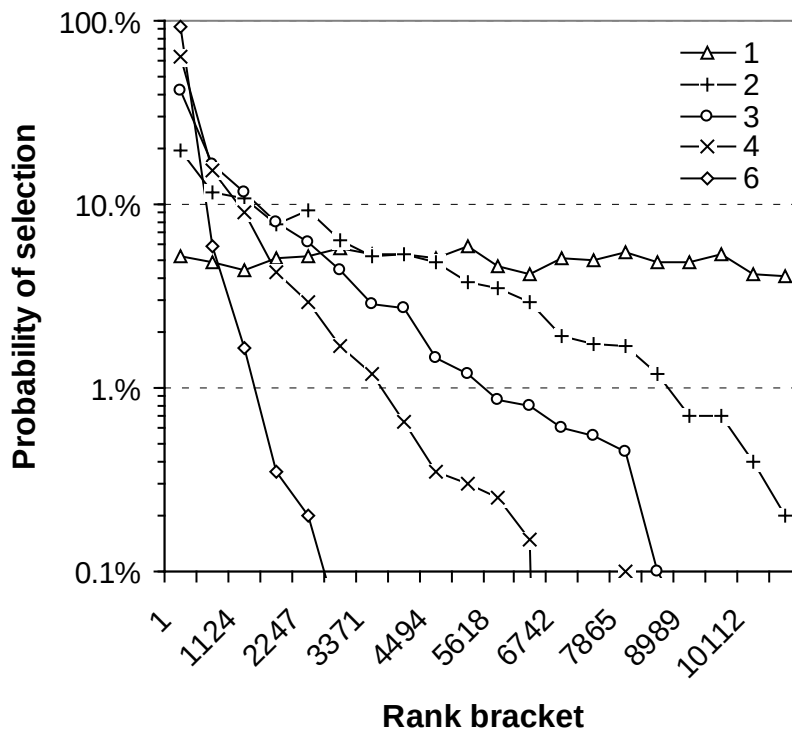


Fig. 3: Probabilities for selecting elements from a ranked list, obtained from 2000 runs. The curve parameter is the number r of randomisations. $r = 1$ selects all ranks equally.

A streamlined constraint selection is much in the spirit of ‘GRIT’ and similar techniques: Jensen and West 1980 report that a surprisingly large number of smaller elements in a flow matrix can even be removed before output multipliers show a significant change, because the value of these elements is often negligible compared to the combined value of a few large flow values (see Fig. 4). Hewings and Jensen (1989, p. 314) note that not necessarily the largest elements are the most important in terms of their effect on multipliers, but that both coefficient size and location matter. Jensen and West 1980, West 1981 and Hewings 1984 propose generating a ranking of matrix elements in terms of their effect on multipliers, in order to identify elements for which more accurate information should be collected. These considerations could determine selection procedures similar to the simple one chosen here.

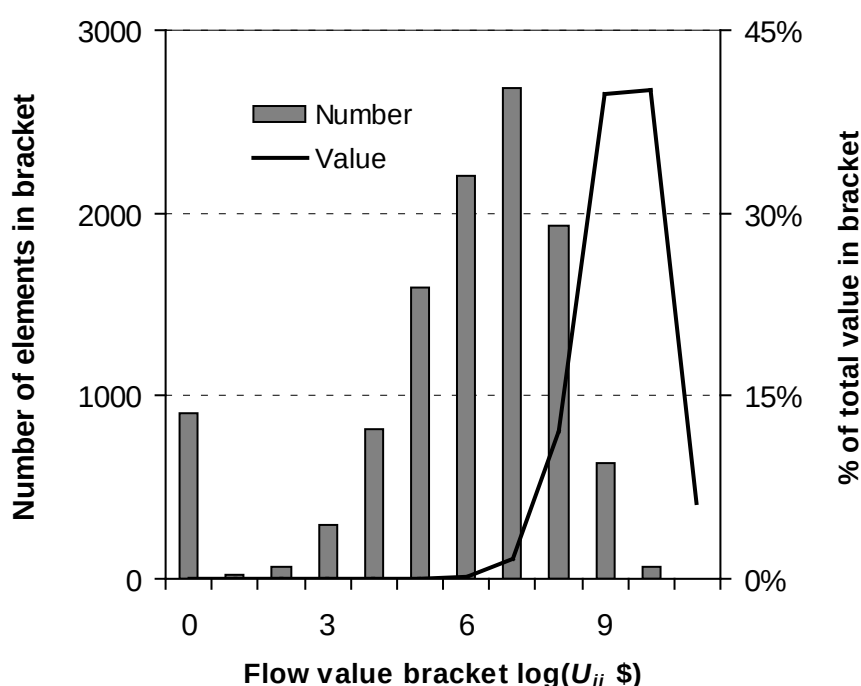


Fig. 4: Distribution of element numbers and values in the 1994/95 Australian flow matrix across size brackets. While there are large numbers of elements near zero and between A\$1m and A\$100m, most of the total intermediate transaction value is concentrated in a few elements of A\$1b and larger.

3.1.5 Distance measures

The results of the balancing process are compared with the published 1996-97 matrices. A number of authors¹¹ examine concepts of relative distance between two matrices in order to characterise the comparative performance of matrix balancing methods. According to Butterfield and Mules (1980, p. 293), "there exists no single statistical test for assessing the accuracy with which one matrix corresponds to another. Analysts working in this area have

¹¹ Czamanski and Malizia 1969; McMenamin and Haring 1974; Morrison and Smith 1974; Lecomber 1975; Harrigan *et al.* 1980; Jackson and Comer 1993

tended to use a number of [complementary] tests.” Accordingly, we chose seven measures (compare Harrigan *et al.* 1980, Günlük-Senesen and Bates 1988, and Lahr 2001):

- the relative arithmetic mean of absolute differences $AMAD = \frac{\sum_{ij} |U_{ij} - U_{n\ ij}|}{\sum_{ij} U_{ij}}$;
- the relative geometric mean of absolute differences¹² $GMAD = \frac{\sqrt{\sum_{ij} |U_{ij} - U_{n\ ij}|^2}}{\sqrt{\sum_{ij} U_{ij}^2}}$;
- the Isard/Romanoff Similarity Index¹³ $SIM = 1 - \frac{\sum_{ij} |U_{ij} - U_{n\ ij}|}{N^2}$;
- the χ^2 distribution of absolute differences $CHI = \sum_{ij} \frac{(U_{ij} - U_{n\ ij})^2}{U_{ij}}$;
- the arithmetic mean of relative differences $AMRD = \frac{\sum_{ij} \frac{|U_{ij} - U_{n\ ij}|}{U_{ij}}}{N^2}$;
- the information index $INFO = \sum_{ij} U_{n\ ij} \log_2 \frac{U_{n\ ij}}{U_{ij}}$; and
- the correlation coefficient $CORR = \frac{\text{Cov}(U_{ij}, U_{n\ ij})}{\sqrt{\text{Var}(U_{ij})} \sqrt{\text{Var}(U_{n\ ij})}}$,

where $\mathbf{U}_n$ is the n^{th} iteration of $\mathbf{U}_0$, and $N = 106$ the dimension of $\mathbf{U}$ and the $\mathbf{U}_n$.

3.1.6 Stochastic experiments

In the following, each experiment will be characterised by a selection of parameters, such as

- the preliminary estimate: 1994/95 flow matrix $\mathbf{U}^{95}$, independent flows $\mathbf{U}^I$, random numbers $\mathbf{U}^R$;
- the constraint structure: disjunct, adjacent;

¹² The square of this measure has also been referred to as ‘Theil’s inequality index’ (Lahiri 1984).

¹³ $1 - SIM$ has been used as a “Dissimilarity Index” by Thissen and Löfgren 1998.

- the selection of constrained elements, r : 1: equally, 2 and 3: increasing degree of preference for top-ranking flows;
- constraint size: number of elements contained in constraints;
- total \$ value of set elements: percentage of total \$ value of flow matrix $\mathbf{U}$;
- number of constraints;
- accuracy type (table $\rightarrow$ holistic): distance test performed on flow matrix $\mathbf{U}$, Leontief inverse $\mathbf{L}$, output multipliers ξ , CO₂ multipliers χ .

4 Results

The following results are a selection from thousands of balancing runs carried out on a contemporary desktop computer, programmed in simple Visual Basic. While in most runs the threshold was reached after 30 seconds or so, only a minority of the runs took up to 5 minutes computing time.

4.1 Influence of the balancing threshold

In order to illustrate the effect of varying the balancing threshold¹⁴, we have chosen an example with a potentially long balancing procedure, where an initially unrealistic estimate ($\mathbf{U}^1$) is substantially altered during balancing, based on a large number (2500) of additional data points. All distance measures reach a stationary state already when the balance criterion $\alpha_n = |u_i - u_{ni}| / \bar{U}$ falls below 10 (Fig. 5, left graph Δ). This may seem surprisingly large, however considering the size distribution of elements (Fig. 4), $10\bar{U}$ is in the order of 10^7 A\$, which is well below the value range that contains the bulk of the total transaction value.

As expected the correlation and similarity indices increase, while absolute differences and the information gap decrease (Fig. 5). Interestingly, the subsequent row and column balance improves the balance criterion (grey parts of Δ , left graph), but counteracts the distance measures by unbalancing the constraint subsets (black parts). This alternating process of constraint and row/column balancing causes all performance measures to perform a damped oscillation.

The absolute mean of relative differences (AMRD) is much higher than the absolute mean of absolute differences (AMAD), since the former is influenced by a large number of small elements that are not well approximated. Moreover, the AMRD as well as the χ^2 and information measures do not seem to recover from the effect of the second row/column

¹⁴ Polenske (1997, p. 79) already pointed out that “analysts should consider [... the issue of the] minimum level of discrepancy allowed between control totals and estimated totals”, and that “information concerning these issues is so limited”.

balancing just after step 2500. We could not understand and explain this feature in detail, but re-examining the formulae in Sec. 3.1.5 shows that these measures are the only ones that feature a single term U_{ij} in the denominator, and not a sum. It appears that because of this characteristic, these three measures are quite susceptible to suboptimal balancing outcomes of small (and potentially unimportant) elements, which was confirmed in further simulations. The GMAD behaves very similarly to the AMAD, but varies less. Similarly, CORR and SIM vary less over a single and between runs. The above results were confirmed when the constraint structure, size, numbers and value were changed, and as a consequence, the AMAD was chosen as the preferred performance measure in most of the analyses below.

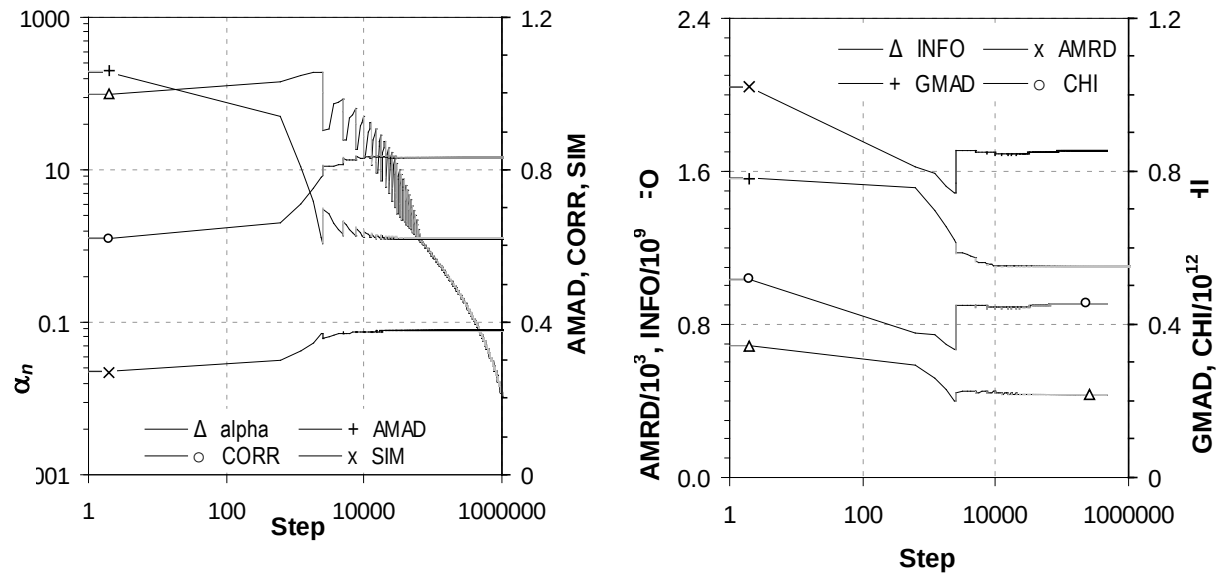


Fig. 5: Convergence of a typical cRAS run for the example of “initial estimate $\mathbf{U}^I$, 2500 disjunct constraints of size 1 and total \$ value 50% $\mathbf{U}$, selected with $r=2$ preference, distance test on $\mathbf{U}$ ”. The curves show a record of the balance criterion $\alpha_n = |u_i - u_{ni}|/\bar{U}$, and all distance measures. The intermittent steps (some visible in grey) show the log of row and column adjustments (counted as 1 step for all rows, and 1 step for all columns), while the remaining black parts represent the adjustments (1 step each) of the additional constraints.

4.2 Influence of the preliminary estimate

The importance of the preliminary (pre-RAS) estimate is well known from experiences in the estimation of regional input-output tables, and our results confirm this. If the 1994-95 flow matrix $\mathbf{U}^{95}$ is used as a start matrix in an updating exercise, the resulting balanced estimate is much closer to the 1996-97 flow table $\mathbf{U}^{97}$ (Fig. 6, left diagram) than in the case of the independence estimate $\mathbf{U}^I$: Similarity and AMAD are about 0.2 points up, and 0.6 points down, respectively, and the correlation is close to unity. At the other extreme, a random matrix with mean $\bar{U}$ (Fig. 6, right diagram) is quite a bad approximation, but this cRAS outcome performs similarly to that resulting from the independence estimate (Fig. 5), even

though the industry-size-weighted form makes more intuitive sense (compare Hewings 1977, p. 940). Varying the additional constraints yielded table accuracy AMADs of around 95%, 85% and 20%, for U^R , U^I , and U^{95} respectively. This finding confirms Polenske's (1997) review¹⁵, and also Miernyk's (1969; 1976) view that – at least in terms of table accuracy – regional input-output estimates can be an inadequate substitute for real data. In all cases the main impetus towards improving table accuracy are the additional constraints, and not the row and column constraints. Additional constraints carry much more information about U^{97} than both U^I and U^R , making them appear almost equal in comparison.

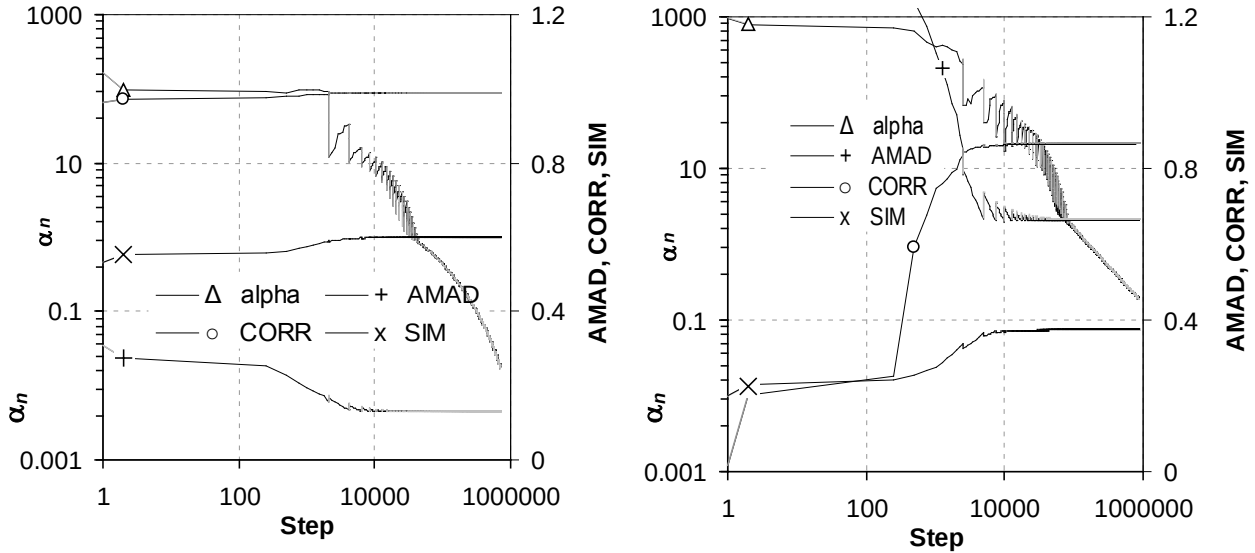


Fig. 6: Convergence of cRAS for the example in Fig. 5, except initial estimates U^{95} (left diagram) and U^R (right diagram). The curves show a record of the balance criterion $\alpha_n = |u_i - u_{ni}| / \bar{U}$, and the distance measures AMAD, CORR and SIM. The intermittent steps (some visible in grey) show the log of row and column adjustments (counted as 1 step), while the remaining black parts represent the adjustments (1 step each) of the additional constraints.

4.3 Influence of the constraint selection and size

In practice, the analyst is most likely to be confronted with finding data for a certain number of constraints, while limited in time and resources. In looking for superior information, the analyst can choose between constraints of different size and \$ value, subject of course to data availability. A crucial question is then which constraints would lead to good results in terms of approximating the target matrix: a few large constraints spanning many elements, or many

¹⁵ Polenske (1997, p. 81) finds that analysts often only report on comparative differences in balancing outcomes, but rarely state the absolute values of table inaccuracy, which are around 30%. Exceptionally “low errors are attributable to special situations, such as the a priori introduction of large amounts of actual data, and to favourable economic conditions, such as a small degree of structural change”.

small constraints? Is it efficient to focus on potentially large elements? In the following we investigate these issues by varying constraint selection and size. We test for disjunct structures only, since results for adjacent structures are quite similar (see also Sec. 4.4).

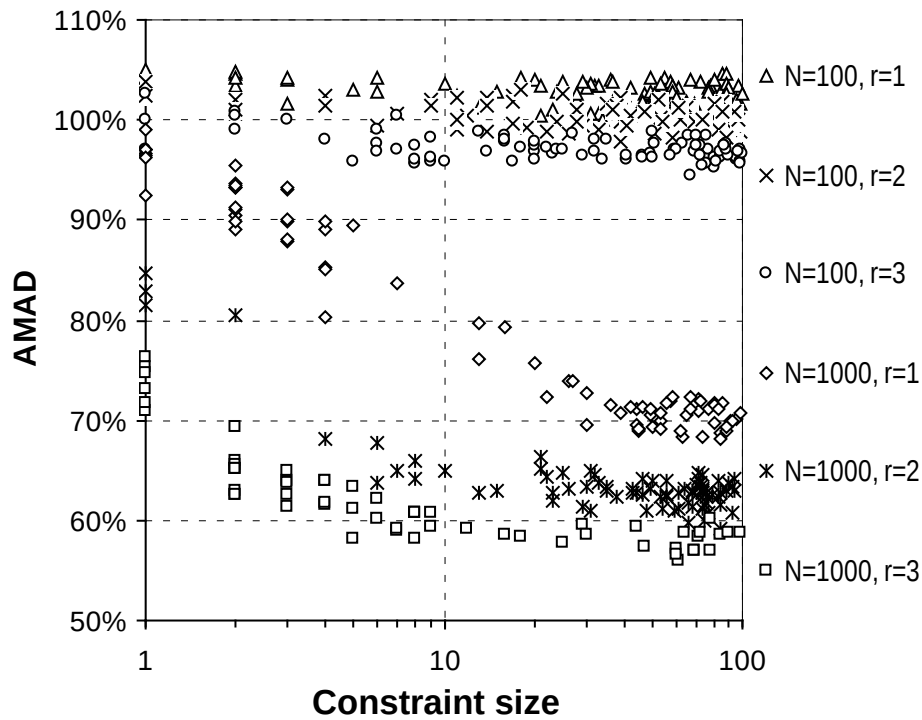


Fig. 7: Performance of cRAS runs for the example of “initial estimate U^l , N disjunct constraints of varying size and undefined total \$ value, selected with $r=1,2,3$ preference, distance test on U ”. Increasing the number of constraints has a larger impact on accuracy of U than increasing constraint size or selectivity.

Generally speaking, adding more constraints improves table accuracy, provided the data are not conflicting. In our example, an increase in the number of constraints from 100 to 1000 led to a 30-40% decrease in the AMAD (Fig. 7, compare curves $\Delta \times \circ$ for 100 constraints with curves $\diamond \ast \square$ for 1000 constraints). In the case of 100 constraints, altering the constraint size (1 – 100 elements) or selection (equally $r = 1$, increasing focus on large transactions $r = 2, 3$, see Fig. 3) did not change the AMAD significantly. The first finding is remarkable since as it says that for the estimation accuracy to be expected from 100 constraints, it does not matter much whether these contain 100 or 10,000 set elements.

With increasing number of constraints, the way of selection and their size become important (compare Hewings and Janson 1980, p. 848; Lynch 1986, pp. 280-282). First, focusing on large transactions is always beneficial, although $r = 3$ produces only slightly better results than $r = 2$. Constraint size increases are also beneficial, however exceeding 10 elements improves accuracy only marginally, at least for preferential selection. The declining impact of the preferential selection is due to the fact that already at $r = 2$, almost all top elements have been selected. The declining impact of constraint size increase can be understood in the sense

that the information contained in larger constraints is less specific to potentially important single elements within the constraint, and hence table accuracy is diminished.

Rather than constraint size and hence number of set elements, the total transaction value covered by the constraints appears to be a more important determinant (Fig. 8). 100 constraints can potentially cover 100% of the transaction value (and more for preferential selection, due to overlap/duplication), but only if the constraint size is near 100 (less for preferential selection). At these constraint sizes, the non-specificity prevents the constraints from having an impact on table accuracy. Selecting 1000 constraints can cover more than 100% transaction value at sizes of less than 10 elements, and hence improve the AMAD significantly.¹⁶ Above coverages of 1000% no more changes occur because of the high degree of duplication in the constraint information. Running more than 3000 low-sized constraints, the AMAD could be reduced to less than 30%.

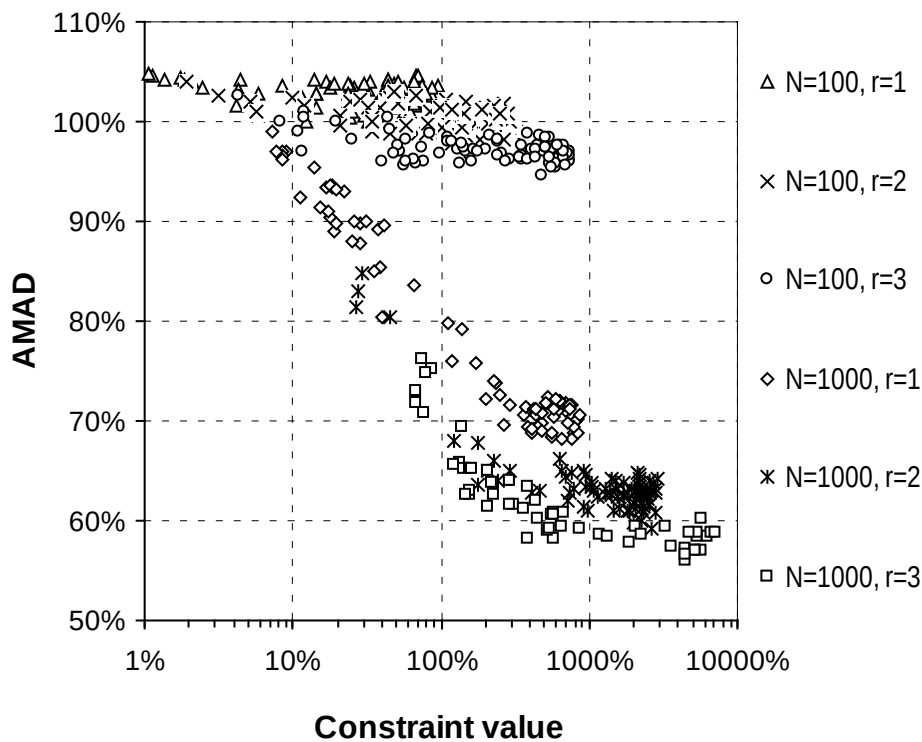


Fig. 8: Performance of cRAS runs for the example in Fig. 7, but with varying total constraint \$ value, and undefined size. Compared to covering more \$-value with constraints, selectivity makes a minor difference to accuracy of U .

4.4 Influence of the constraint structure

We now turn to variations of constraint structure, as illustrated in Fig. 1. Our first observation is that for single-element constraints (size = 1 in Fig. 9) the curves for disjunct and adjacent

¹⁶ A number of runs with fixed total constraint \$ values of 10%U, 30%U and 50%U were carried out, showing that many small constraints performed better than a few large constraints.

structures coincide for equal ($\square \blacktriangle$) and preferential ($\times \boxtimes$) selection. This is to be expected, since an adjacent structure of size 1 is essentially a disjunct structure. For constraint sizes of 30 and larger however, the adjacent constraint structures perform worse than the disjunct ones, and above size 90, their curves ($\blacktriangle \boxtimes$) join. This deterioration is due to the fact that the constraint length approaches the dimension of the flow matrix, so that long constraints more or less coincide with already existing row and column sum constraints, thus effectively reducing the numbers of constraints through increasing duplication.

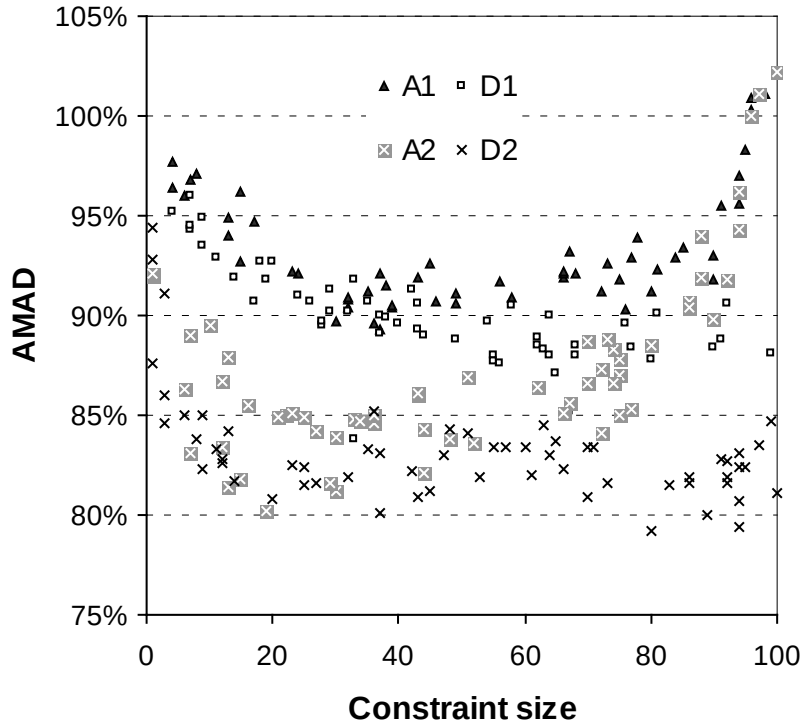


Fig. 9: Comparison of balancing adjacent (A) and disjunct (D) structures, under equal (1) and preferential (2) selection (U^I initial estimate, total $\$$ -value undefined, 500 constraints, test on U). The diagram is organised as in Fig. 7.

Increasing the focus on large elements ($\bigcirc$ in Fig. 10) improves the performance of adjacent structures at low constraint sizes, but not at large sizes. For $r = 3$, a U-shaped form becomes apparent, which is due to two effects for declining performance: 1) small numbers of set elements at low sizes, and 2) overlap with row/column sum constraints at large sizes. In order to concentrate constraints even more on large flows, we selected seed elements from the ranking list, starting with the top element for the first constraint, and then proceeding down the list, with all other set elements attached in random directions, as before (labeled A_∞). Once again, performance improves at low constraint sizes, but not at large sizes. In practice, however, it is unlikely that information can be found exclusively for top-ranking flows, so that in the following we will restrict ourselves to the preferential selections as described in Sec. 3.1.4.

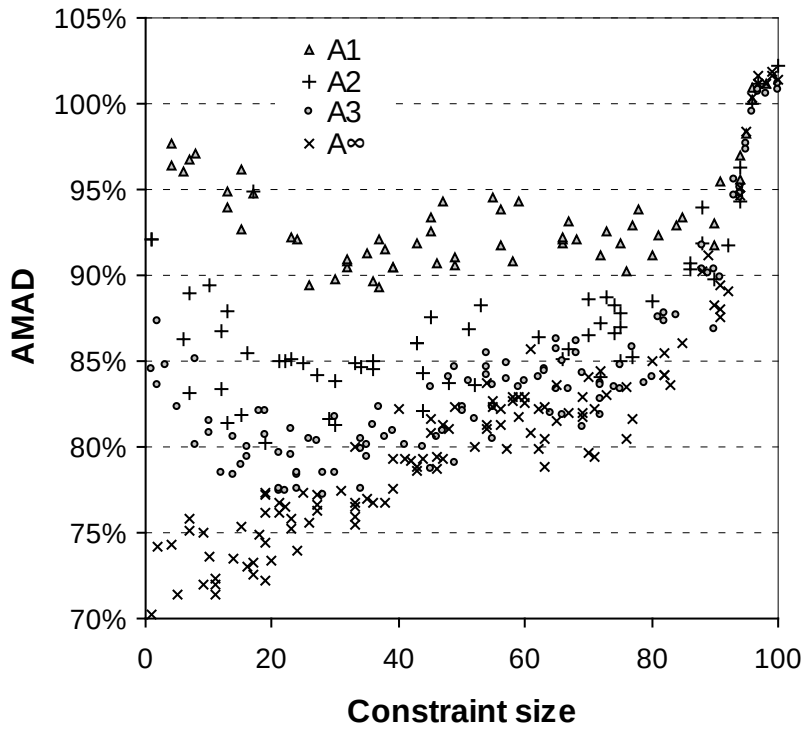


Fig. 10: Performance of balancing adjacent (A) structures under equal (1) and preferential (2, 3, ∞) selection ($\mathbf{U}^I$ initial estimate, total \$-value undefined, 500 constraints, test on $\mathbf{U}$). The diagram is organised as in Figs. 7 and 9.

4.5 Influence of the accuracy type

We will now compare the balancing effects on the flow matrix, the Leontief inverse, and multipliers. It is well known that the uncertainty of a sum of many numbers can be considerable lower than the uncertainties of any of the numbers. This effect can be observed when the flow matrix $\mathbf{U}$ is converted into a Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{U}\hat{\mathbf{x}}^{-1})^{-1}$, and – further increasing the degree of summation – into output and CO₂ multipliers $\boldsymbol{\xi} = \mathbf{1L}$ and $\boldsymbol{\chi} = \mathbf{cL}$ (Fig. 11).¹⁷ The table accuracy of $\mathbf{L}$ in terms of the AMAD is considerably lower at around 30% than that of $\mathbf{U}$, which is due to the stochastic error cancellations during the numerous additions and multiplications during the matrix inversion. However, multipliers (and not $\mathbf{L}$) are likely to be the objects of practical applications. Thus aggregating even further, the (holistic) accuracy of output multipliers $\boldsymbol{\xi}$ is extremely low at around 2%. CO₂ multipliers $\boldsymbol{\chi}$ are less accurate at around 20%, because in the case of Australia CO₂ intensities $\mathbf{c}$ are exceptionally high for only a few industries such as beef cattle grazing, beef products, aluminium or electricity generation, so that the overall uncertainty is largely determined by these few contributions, leading to a lower degree of error cancellation.

¹⁷ Probably first observed by Hewings 1977, p. 934).

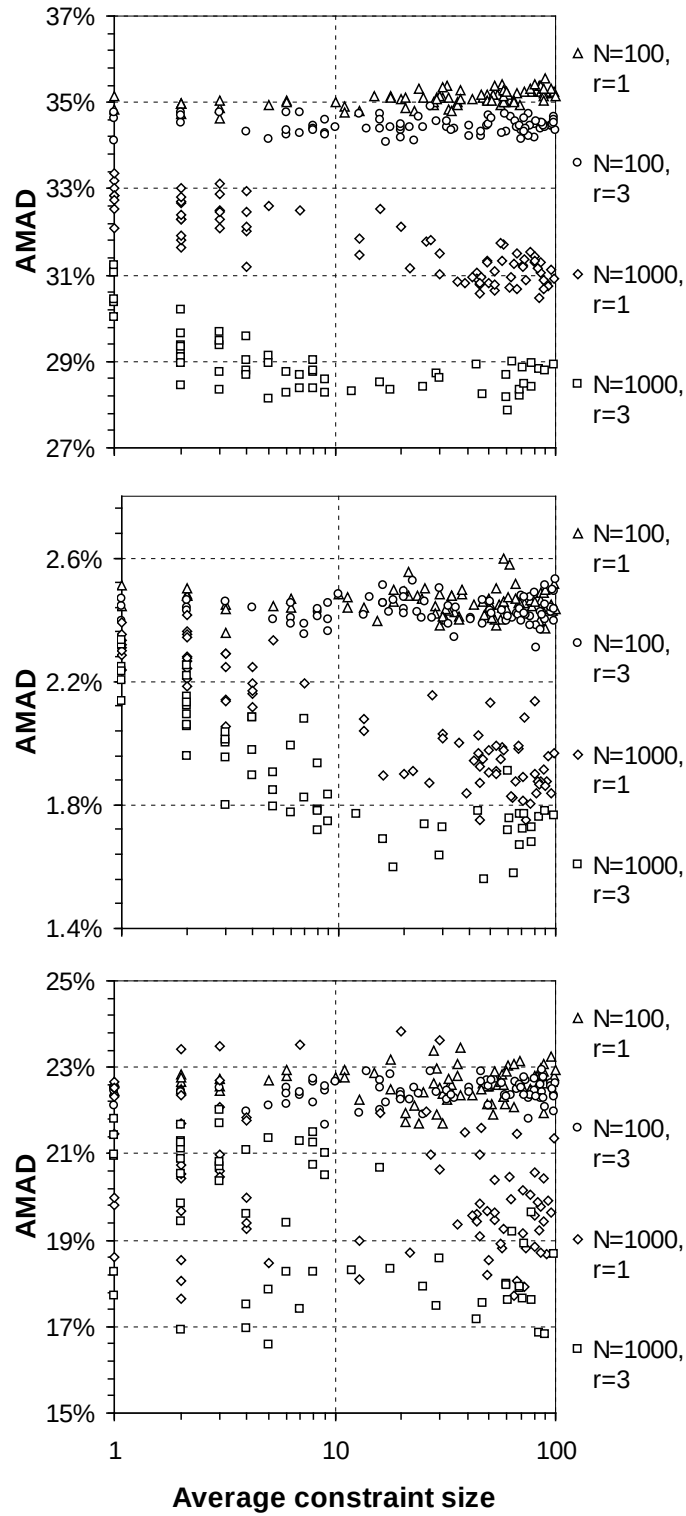


Fig. 11: Performance of cRAS runs for the initial estimate $\mathbf{U}^I$, N disjunct constraints of varying size and selectivity, and undefined \$ value, tested on the Leontief inverse $\mathbf{L}$ (top), output multipliers ξ (middle), and CO_2 multipliers χ (bottom). The diagrams are organised as in Figs. 7, 9 and 10.

We repeated the above numerical experiment for adjacent structures, and under variation of the initial estimate (Tab. 1). In summary, holistic accuracy is always higher than table

accuracy (improving from $\mathbf{U}$ via $\mathbf{L}$ to ξ and χ), and updating ($\mathbf{U}^{95}$) yields much better results than estimating ($\mathbf{U}^I$ and $\mathbf{U}^R$), with the assumption of independence performing only slightly better than a random preliminary estimate (compare Sec. 4.2).

Accuracy type	$\mathbf{U}$	$\mathbf{L}$	ξ	χ
Preliminary estimate				
$\mathbf{U}^R$	97.1±5.3	33.5±1.1	2.0±0.3	23.7±1.9
$\mathbf{U}^I$	85.9±9.5	30.8±3.3	1.8±0.3	22.4±2.9
$\mathbf{U}^{95}$	20.3±1.0	7.2±0.3	0.4±0.04	3.0±0.6

Tab. 1: AMAD of cRAS runs on 500 adjacent constraint structures of varying size and value, but with preferentially selected seed elements ($r = 2$), including its standard deviation over constraint sizes 1 to 100 (curve shapes as in Fig. 9), for different preliminary estimates and accuracy types.

4.6 Sequence-(in)variance

As already shown for ordinary RAS by Bacharach 1970, in order to fulfil the minimum information gain property, the cRAS balance outcome has to be unique, and in particular independent of the order of the constraint sequence. A large number of cRAS runs of the adjacent, slightly selective type ($r=2$) were carried out, with constraint numbers between 50 and 500, and constraint sizes between 5 and 100, and tested on $\mathbf{U}$. In all instances, the balancing result was unique.

4.7 Other issues

As with the Linear Programming and minimum information gain methods, (c) RAS preserves any preliminary zero values. Zero-value data in a preliminary matrix to be updated (for example $\mathbf{U}^{95}$) may sometimes reflect technological impossibilities but – as Bacharach (1970, p. 26) pointed out – sometimes actually be an approximation of a very small positive value¹⁸, in which case the zero should not be preserved. On the other hand, setting preliminary zero-value data to an arbitrarily small positive value may allow for positive updated values, but may also lead to non-zero transactions that in reality are technically impossible. We compared two runs of the adjacent, preferentially selected ($r = 2$) type, with one run starting with the original 1995 flow matrix $\mathbf{U}^{95}$, and the other having all zero elements in $\mathbf{U}^{95}$ set to a small value. We obtained no discernible differences in the AMAD of the cRAS estimates. This is probably due to the fact that zero elements in $\mathbf{U}^{95}$ are either zero in $\mathbf{U}^{97}$, or positive but small, and thus their contribution to the AMAD is negligible in any case.

In real-world applications, data for additional constraints may not actually superior, but even conflicting. Due to the nature of industry surveys proceeding as separate sales and purchasing (supply and demand) recordings, even corresponding row and column totals of gross output and input may be different. Even a small discrepancy between nominally equivalent

¹⁸ Or, in the case of temporal forecasting, technical innovation may turn a truly zero input into a positive input.

constraints can prevent convergence of the (c)RAS method, most obviously if the given balancing threshold is smaller than the discrepancy. Reconciliation is therefore necessary, possibly by using a RAS technique on the conflicting constraints themselves, or by using Lagrangian or optimisation technique with uncertainty or reliability terms embedded in the function to be minimised.¹⁹

5 Conclusions

We have described the performance of a RAS matrix balancing variant, called ‘constrained RAS’ or cRAS. In contrast to previous approaches for the estimation of input-output tables, cRAS allows incorporating constraints on arbitrary-sized and –shaped sets of elements into the preliminary estimate. These elements may be disjunct or adjacent, and exhibit a widely varying degree of overlap. We have tested cRAS’s performance for the example of the 1996-97 Australian input-output flow matrix, using a number of distance measures.

First, the preliminary estimate is of crucial importance for the accuracy of the balancing outcome. Our experiments yielded table accuracies of balanced matrices of around 90% for estimated preliminaries, and around 20% for the previous-year preliminary. They showed that a preliminary based on some distribution assumption and row and column sums may even not be much better than a random matrix.

Second, including more constraints into the balancing process always improves the balance outcome. This is especially the case if constraints are small, numerous, disjunct, and concentrated on important matrix cells. This result agrees with previous findings by Jensen and West 1980, in the sense that for many analytical purposes, it is more important to estimate a few strategic table elements as accurately as possible, than to estimate as many elements as possible. In this context, we also found it important to “pin-point” such strategic elements in disjunct, small constraints, rather than have the value of say one important element “buried” in an adjacent, large constraint. While the size of a constraint directly diminishes the ability to uncover one particular strategic element from its summed value, an adjacent structure means that this uncertainty affects the industry sector to which the strategic element belongs.

Third, the accuracy of derived quantities (such as inverse matrices and aggregates thereof) is always higher than the accuracy of the flow matrix itself. This is due to cancellation of stochastic error components during numerous additions and multiplications that lead to the derived quantities. Holistic accuracy – as this effect has been termed – is therefore certainly a concept to be kept in mind when compiling and using inter-industry tables. Finally, as ordinary RAS, cRAS produces a unique balancing outcome.

¹⁹ A number of authors discuss balancing techniques that include uncertainty and reliability, using either RAS (Jensen and McGaurr 1976; 1977; Lahiri 1984), variance minimisation (Gerking 1976; 1979a; b; Miernyk 1979), or Lagrangian minimization (van der Ploeg 1982). All methods have in common the problem that probabilities for conflicting data are mostly unavailable, so that subjective judgment has to be consulted.

Given the scarcity of superior data, especially when estimating regional or physical input-output tables, the main value of cRAS is that the analyst can make use of any kind of information they can find, irrespective of the degree of aggregation.

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SOME COMMENTS ON THE GRAS METHOD

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Abstract

Junius & Oosterhaven, 2003 present a RAS matrix balancing variant that can incorporate negative elements in the balancing. There are, however, a couple of issues in the approach described – the first being the handling of zeros in the initial estimate, and the second being the formulation of their minimum-information principle. We present a corrected exposition of GRAS.

Keywords: RAS matrix balancing, GRAS, optimisation

1 Introduction

Junius & Oosterhaven, 2003 (J&O) describe a RAS matrix balancing variant called GRAS, which deals with an efficient and theoretically sound way of balancing a matrix containing negative entries, to prescribed row and column totals. In their paper, J&O derive GRAS from a minimum-information principle (see also Oosterhaven, 2005)

2 Problems in GRAS

We believe there are some problems in Junius and Oosterhaven's approach.

2.1 Definition of z_{ij} for $a_{ij} = 0$

J&O's definition of the variable $z_{ij} = x_{ij} / a_{ij}$ as the ratio of the adjusted table elements x_{ij} and the initial estimate a_{ij} does not work with their target function $t(\mathbf{A}, \mathbf{Z}) = \sum_{ij} |a_{ij}| z_{ij} \ln(z_{ij})$. J&O set $z_{ij} \equiv 0$ for $a_{ij} = 0$. However, the target function cannot be evaluated for $z_{ij} = 0$ since $\ln(0)$ is not defined. That is, unless one defines inside the target function that $z_{ij} \ln(z_{ij}) \equiv 0$ for $z_{ij} = 0$. Assuming that the GRAS algorithm is to preserve zero values, one can assume that if $a_{ij} = 0$, then $x_{ij} = 0$ also. Therefore, it is better to set $z_{ij} \equiv \lim_{\varepsilon \rightarrow 0} \varepsilon / \varepsilon = 1$ for $a_{ij} = 0$.

2.2 The target function $t(\mathbf{A}, \mathbf{Z}) = \sum_{ij} |a_{ij}| z_{ij} \ln(z_{ij})$

J&O's target function $t(\mathbf{A}, \mathbf{Z}) = \sum_{ij} |a_{ij}| z_{ij} \ln(z_{ij})$ does not assume a minimum for $x_{ij} = a_{ij}$ as required for reflecting the distance between, or similarity of, $\mathbf{X}$ and $\mathbf{A}$. Fig. 1 illustrates this for the one-dimensional case $t(a, z) = |a| z \ln(z) = |a| x/a \ln(x/a)$ with $a = 2$, so that $t(x) = x \ln(x/2)$. Fig. 1 (dashed curve) clearly shows clearly a minimum for $x = 2/e$, and not for $x = 2$. If however the target function is modified to

$t'(\mathbf{A}, \mathbf{Z}) = \sum_{ij} |a_{ij}| z_{ij} \ln(z_{ij})$ with $z_{ij} = \frac{x_{ij}}{e a_{ij}}$, condensing to $t(x) = x/e \ln(x/2e)$ in the one-

dimensional case, a minimum occurs at $x = 2$ as required (solid curve). We will exploit this modification further on in Section 4.

One can construct a case where J&O's GRAS balancing algorithm leads to a solution $\mathbf{X}$ that is inferior to the initial estimate in terms of their target function. This can be seen for the trivial case of starting with an initial estimate $\mathbf{A}$ that already satisfies all prescribed row and column totals, that is $\sum_j a_{ij} = u_i$, and $\sum_i a_{ij} = v_j$. In this case $z^{\mathbf{A}}_{ij} = 1 \forall i, j$, and $t(\mathbf{A}, \mathbf{Z}^{\mathbf{A}}) = 0$ (compare Fig. 1). The initial estimate should be the optimal solution.

However, problems show up throughout J&O's exposition. First, J&O's solutions

$$z_{ij} = e^{\lambda_i} e^{\tau_j} e^{-1} \text{ for } a_{ij} \geq 0 \quad (1a)$$

$$z_{ij} = e^{-\lambda_i} e^{-\tau_j} e^{-1} \text{ for } a_{ij} < 0 \quad (1b)$$

cannot work, since in this trivial case *all* $z_{ij}^{\mathbf{A}}$ must equal 1, irrespective of the sign of a_{ij} . No selection of λ_i and τ_j can fulfil this.

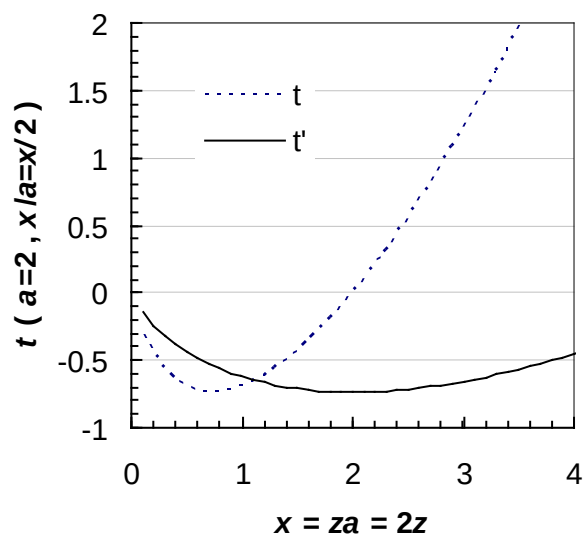


Fig. 1: J&O's target function $t(\mathbf{A}, \mathbf{Z})$, plus a modified target function $t'(\mathbf{A}, \mathbf{Z})$ for a one-dimensional case.

Second, using J&O's balancing algorithm (their Eqs. 20-25), but starting with a perfect initial estimate, will actually move *away* from that perfect solution. This is because J&O's GRAS algorithm actually balances any initial estimate towards scaled row and column totals $\mathbf{u}^* = e\mathbf{u}$ and $\mathbf{v}^* = e\mathbf{v}$, instead of prescribed totals $\mathbf{u}$ and $\mathbf{v}$. This scaling results directly out of the Lagrangean optimality condition for their target function. So, one would start with a perfect solution $\mathbf{A}$ satisfying totals $\mathbf{u}$ and $\mathbf{v}$, but then unbalancing it until it satisfies $\mathbf{u}^*$ and $\mathbf{v}^*$ (see J&O's solution conditions in their Eqs. 9 and 10), and then scaling it down by a factor e . That GRAS solution $\mathbf{X}$ may minimize J&O's target function t , but is clearly inferior to the initial estimate $\mathbf{A}$. This is also reflected in Fig. 1. The following Section will give a numerical example in order to demonstrate the magnitude of this effect.

3 A numerical example

Consider the following row-sum- and column-sum-constrained 2×2 sector table, and initial estimate:

$$\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix} ; \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}. \quad (9)$$

In contrast, J&O's GRAS algorithm would start with $\mathbf{A}$ and scale this to satisfy row and column sums $\mathbf{u}^* = e\mathbf{u} = (2.72, 13.59)$ and $\mathbf{v}^* = e\mathbf{v} = (2.72, 13.59)$. $\mathbf{X}$ is “adjusted away” from $\mathbf{A}$, thus increasing the value of the target function $t(\mathbf{A}, \mathbf{Z}) = \sum_{ij} |a_{ij}| z_{ij} \ln(z_{ij})$ (Tab. 1).

Step	Variables				J&O target function	Constraint	Constraint adjustment			
	x_1	x_2	x_3	x_4	$t(\mathbf{A}, \mathbf{Z})$		u_1	u_2	v_1	v_2
1	- 0.602	2.000	3.321	3.000	-2.776	1	2.72	5.0	1.4	6.32
1	- 0.602	5.437	3.321	8.155	-0.936	2	2.72	13.59	4.83	11.48
1	- 0.904	3.622	3.321	8.155	-1.509	3	2.42	11.78	2.72	11.48
1	- 0.904	3.622	3.933	9.659	-0.774	4	3.03	13.28	2.72	13.59
2	- 0.965	3.622	3.683	9.659	-0.835	1	2.72	13.28	2.66	13.34
2	- 0.965	3.707	3.683	9.884	-0.718	2	2.72	13.59	2.74	13.57
2	- 0.970	3.688	3.683	9.884	-0.722	3	2.71	13.57	2.72	13.57
2	- 0.970	3.688	3.690	9.902	-0.713	4	2.72	13.59	2.72	13.59
3	- 0.970	3.688	3.689	9.902	-0.714	1	2.72	13.59	2.72	13.59
3	- 0.970	3.689	3.689	9.903	-0.713	2	2.72	13.59	2.72	13.59
3	- 0.970	3.689	3.689	9.903	-0.713	3	2.72	13.59	2.72	13.59
3	- 0.970	3.689	3.689	9.903	-0.713	4	2.72	13.59	2.72	13.59

Tab. 1: J&O's GRAS algorithm for the constrained table in Eq. 9. One GRAS step is defined as one cycle through all four constraints (end of each cycle printed in bold in left part of Table). The variables x_i are scaled in order to satisfy row-sum- and column-sum-constraints u_i and v_i in turn (bold in right part of Table).

After downscaling by e , J&O's final solution is $\mathbf{X} = (-0.357, 1.357, 1.357, 3.643)$. This solution satisfies all row- and column-sum constraints, but its target function value of $t(\mathbf{A}, \mathbf{Z}) = -0.713$ shows that it is clearly inferior to $\mathbf{A} = (-1, 2, 2, 3)$, simply because of its distance to $\mathbf{A}$. A correct GRAS algorithm would stop during the very first step, and return $\mathbf{A}$ as the solution, since $\mathbf{A}$ satisfies all four constraints. This is the subject of the following two Sections.

4 A consistent solution

We believe that the target function must take the following form in order to avoid this unwanted effect (compare Bacharach, 1970, pp. 80, 85-86; see also Fig. 1):

$$t'(\mathbf{A}, \mathbf{Z}) = \sum_{ij} |a_{ij}| z_{ij} \ln(z_{ij}) \text{ with } z_{ij} = \frac{x_{ij}}{e a_{ij}}. \quad (2)$$

The derivative of the GRAS Lagrangean

$\mathcal{L}(\mathbf{Z}, \lambda, \tau)$

$$= \sum_{(i,j) \in P} a_{ij} z_{ij} \ln(z_{ij}) - \sum_{(i,j) \in N} a_{ij} z_{ij} \ln(z_{ij}) + \sum_i \lambda_i \left[u_i - \sum_j a_{ij} z_{ij} \right] + \sum_j \tau_j \left[v_j - \sum_i a_{ij} z_{ij} \right]$$

is then

$$\frac{\partial \mathcal{L}}{\partial z_{ij}} = a_{ij} \ln \frac{z_{ij}}{e} + a_{ij} - \lambda_i a_{ij} - \tau_j a_{ij} \text{ for } a_{ij} \geq 0 \quad (3a)$$

$$\frac{\partial \mathcal{L}}{\partial z_{ij}} = -a_{ij} \ln \frac{z_{ij}}{e} - a_{ij} - \lambda_i a_{ij} - \tau_j a_{ij} \text{ for } a_{ij} < 0. \quad (3b)$$

Imposing the optimality condition yields

$$\ln z_{ij} - \lambda_i - \tau_j = 0 \Leftrightarrow z_{ij} = e^{\lambda_i} e^{\tau_j} \text{ for } a_{ij} \geq 0 \quad (4a)$$

$$-\ln z_{ij} - \lambda_i - \tau_j = 0 \Leftrightarrow z_{ij} = e^{-\lambda_i} e^{-\tau_j} \text{ for } a_{ij} < 0. \quad (4b)$$

Calling $r_i = e^{\lambda_i}$ and $s_j = e^{\tau_j}$, the target matrix is then

$$x_{ij} = r_i a_{ij} s_j \text{ for } a_{ij} \geq 0 \quad (5a)$$

$$x_{ij} = r_i^{-1} a_{ij} s_j^{-1} \text{ for } a_{ij} < 0, \quad (5b)$$

with the diagonal matrices $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ solving

$$(\hat{\mathbf{r}} \mathbf{P} \hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1} \mathbf{N} \hat{\mathbf{s}}^{-1}) \mathbf{i} = \mathbf{u}, \text{ and} \quad (6a)$$

$$\mathbf{i} (\hat{\mathbf{r}} \mathbf{P} \hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1} \mathbf{N} \hat{\mathbf{s}}^{-1}) = \mathbf{v}, \quad (6b)$$

instead of J&O's solution, which leads to $\mathbf{u}^* = e\mathbf{u}$ and $\mathbf{v}^* = e\mathbf{v}$. In Eq. 6, $\mathbf{P}$ and $\mathbf{N}$ are a decomposition of $\mathbf{A} = \mathbf{P} - \mathbf{N}$ into a matrix $\mathbf{P}$ containing the positive elements of $\mathbf{A}$, and $\mathbf{N}$ containing the absolutes of the negative elements of $\mathbf{A}$.

5 Iterative Formulation

The iterative solution of Eqs. 6a and 6b becomes:

Step 1. Choose $\mathbf{r}(0) = \mathbf{i}$ (for justification, see J&O, p93)

Step 2. Calculate $\hat{\mathbf{s}}(1) = \sigma(\hat{\mathbf{r}}(0))$ using Eq 7a below

Step 3. Calculate $\hat{\mathbf{r}}(1) = \rho(\hat{\mathbf{s}}(1))$ using Eq 7b below

Step 4. Continue for $\hat{\mathbf{s}}(k) = \sigma(\hat{\mathbf{r}}(k-1))$ and $\hat{\mathbf{r}}(k) = \rho(\hat{\mathbf{s}}(k))$, for $k=1,2..n$

Step n . Exit when $\hat{\mathbf{s}}(n) - \hat{\mathbf{s}}(n-1) < \varepsilon$ for some chosen $\varepsilon > 0$.

Step $n+1$. $\mathbf{X} = \hat{\mathbf{r}}(n)\mathbf{P}\hat{\mathbf{S}}(n) - \hat{\mathbf{r}}(n)^{-1}\mathbf{N}\hat{\mathbf{S}}(n)^{-1}$
utilising

$$\rho_i(s) = \frac{\mathbf{u} + \sqrt{\mathbf{u}^2 + 4p_i(s)n_i(s)}}{2p_i(s)} \text{ and} \quad (7a)$$

$$\sigma_j(r) = \frac{\mathbf{v} + \sqrt{\mathbf{v}^2 + 4p_j(r)n_j(r)}}{2p_j(r)},$$

(7b)

where

$$p_i(s) = \sum_j p_{ij}s_j, \quad p_j(s) = \sum_i p_{ij}s_i, \quad (8a,b)$$

$$n_i(s) = \sum_j \frac{n_{ij}}{s_j}, \text{ and} \quad n_j(s) = \sum_i \frac{n_{ij}}{s_i}. \quad (8c,d)$$

6 Conclusion

Using the target function t' as in Eq. 2, the problems noted in Section 2 do not arise. First, for $\lambda_i = \tau_j = 0$, all z_{ij}^A equal 1, irrespective of the sign of a_{ij} . Second, using J&O's balancing procedure with t' , starting with a perfect initial estimate will terminate the algorithm.

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MATRIX BALANCING UNDER CONFLICTING INFORMATION

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Abstract – We have developed a generalised iterative scaling method (KRAS) that is able to balance and reconcile input-output tables and SAMs under conflicting external information and inconsistent constraints. Like earlier RAS variants, KRAS can:

- a) handle constraints on arbitrarily sized and shaped subsets of matrix elements;
- b) include reliability of the initial estimate and the external constraints;
- c) deal with negative values, and preserve the sign of matrix elements.

Applying KRAS in four case studies, we find that, as with constrained optimisation, KRAS is able to find a compromise solution between inconsistent constraints; this feature does not exist in conventional RAS variants such as GRAS. KRAS can constitute a major advance for the practice of balancing input-output tables and Social Accounting Matrices, in that it removes the necessity of manually tracing inconsistencies in external information. This quality does not come at the expense of substantial programming and computational requirements (of conventional constrained optimisation techniques).

Keywords: Matrix balancing, inconsistent constraints, constrained optimisation, RAS, Bregman's method, entropy

1 Introduction

A common problem in compiling and updating Social Accounting Matrices (SAM) or input-output tables is that of incomplete data. Missing matrix elements may be due to a variety of reasons such as costly and therefore incomplete industry surveys, or the suppression of confidential information. External data points can be used to formulate a system of equations that constrain the unknown matrix elements. However, unknowns usually outnumber external constraints, resulting in the system being underdetermined, that is exhibiting too many degrees of freedom to be solved analytically. The two most prominent numerical approaches for reconciling such an underdetermined system are probably the *RAS bi-proportional scaling* method, and mathematical programming methods summarised here under the term *constrained optimisation*.

During the past 40 years, both approaches have successfully tackled a number of challenges, leading to a number of useful features¹: Ideally, the technique should

- a) incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
- b) allow considering the reliability of the initial estimate;
- c) allow considering the reliability of external constraints;
- d) be able to handle negative values and to preserve the sign of matrix elements if required;
- e) be able to handle conflicting external data.

While all criteria have been addressed by constrained optimisation methods, there is currently no RAS-type iterative scaling technique that satisfies criterion e).² The inability of RAS to deal with conflicting external data represents a considerable drawback for practice, because for most statistical agencies such data are often rather the norm than the exception.

The most simple case of conflicting data is probably a situation in which two data sources are located that prescribe two different values for the same matrix entry, resulting in inconsistent constraints. When faced with such constraints, existing RAS variants adjust the respective matrix element alternatively to either of the conflicting values, and thus enter into oscillations without ever converging to a satisfactory solution.

More generally, sets of external data can be conflicting indirectly amongst each other. Cole 1992 mentions convergence problems, and gives a simple example as a matrix $\begin{pmatrix} a & b \\ c & 1 \end{pmatrix}$ with a ,

¹ Lahr and de Mesnard 2004 provide a recent overview of extensions to the classic RAS technique.

² Recently, Canning and Wang 2005 note that “another important advantage of the mathematical programming model over scaling methods is in its flexibility. Additional constraints can be easily imposed, such as [...] incorporating an associate term in the objective function to penalize solution deviations from the initial row and column total estimates when they are not known with certainty.”

$b, c \geq 0$, and with inconsistent row and column totals and $\{1, 3\}$. Möhr *et al.* 1987 provide an example where the RAS-infeasibility is brought about by the existence of unadjustable zero values in the matrix to be balanced.³ In practice, indirect conflict might present itself for example when on one hand, data on final demand and gross output of wheat suggest a certain intermediate demand of wheat, however on the other hand this intermediate demand is too large to be absorbed by the flour milling sector. Further examples involving conflicting external information are GDP measures⁴, and multi-national and regional input-output systems. Möhr *et al.* 1987 call such problems “RAS-infeasible”. In practice, such inconsistencies are often traced and adjusted manually by statisticians.⁵

This work has two aims: 1) We undertake a comparative review of the development of both RAS and constrained optimisation techniques. 2) We present a new generalised iterative scaling method that is able to handle all five criteria above, at once, and especially conflicting external data and inconsistent constraints. We achieve this capability by introducing standard error estimates for external data. We build on previous RAS variants that satisfy the remaining criteria, and thus arrive at a RAS-type method that matches the capabilities of constrained optimisation. We will refer to our method as KRAS (Conflicting RAS).

2 Literature review

The following review is organised according to the criteria list in the introduction: Section 2.1 deals with criterion a), Section 2.2 with b) and c), Section 2.3 with d), and Section 2.4 with e).

The RAS method – in its basic form – bi-proportionally scales a matrix $\mathbf{A}_0$ of unbalanced preliminary estimates of an unknown real matrix $\mathbf{A}$, using $\mathbf{A}$ ’s known row and column sums. The balancing process is usually aborted when the discrepancy between the row and column sums of $\mathbf{A}_0$ and $\mathbf{A}$ is less than a previously fixed threshold. Bacharach 1970 has analysed the bi-proportional constrained matrix problem in great detail, in particular with regard to the

³ Möhr *et al.* 1987 make this problem feasible by adding an ‘augmentation’ which ensures that matrix elements are non-zero where constraints require it. While this is a solution to these authors’ problem, it does not address constraint conflicts in general.

⁴ Barker *et al.* 1984 (p. 475) write: “... we observed that the income, expenditure, production and financial estimates of data are typically inconsistent. The presence of such accounting inconsistencies emphasises the unreliable nature of economic data.” See also Smith *et al.* 1998.

⁵ Barker *et al.* 1984 (p. 475) remark that “...trading off the relative degrees of uncertainty of the various data items in the system in order to adjust the prior data to fit the accounting identities [...] is essentially what national income accountants do during the last stages of compiling the accounts when faced with major discrepancies between data from different sources”. Dalgaard and Gysting 2004 (p. 170) from Statistik Denmark report that many analysts responsible for compiling input-output tables favour manual adjustment, because “based on the experience that many errors in primary statistics are spotted in the course of a balancing process that is predominantly manual, compilers are typically convinced that a (mainly) manual balancing process yields results of higher quality than those emanating from a purely automatic balancing of the accounts. From that point of view, the resources involved in manual balancing are justified as a very efficient consistency check on the accounts.”

economic meaning of bi-proportional change⁶, the existence and uniqueness of the iterative RAS solution, its properties of minimisation of a distance metric⁷, and uncertainty associated with errors in row and column sum data and with the assumption of bi-proportionality. The origins of the method go back several decades.⁸ Stone and Brown 1962, Bacharach 1970, and Polenske 1997 provide a historical background.

2.1 Constraints on arbitrarily sized and shaped subsets of matrix elements

The ‘modified RAS’ (MRAS) approach (Paelinck and Walbroeck 1963; Allen 1974; Lecomber 1975a) was developed for cases when some of the matrix elements of $\mathbf{A}$ are known in addition to its row and column sums, for example from an industry survey. MRAS “nets” the preliminary estimate $\mathbf{A}_0$, that is the known elements are subtracted, and $\mathbf{A}_0$ contains 0 at the corresponding positions. The net $\mathbf{A}_0$ is then subjected to the standard RAS procedure, and the known elements are added back on after balancing.

In addition to certain elements of $\mathbf{A}$, some aggregates of elements of $\mathbf{A}$ may be known. For example, a published table $\mathbf{A}^G$ of national aggregates may constitute partial information when constructing a multi-regional input-output system, or a more disaggregated national table. Accordingly, Oosterhaven *et al.* 1986 add a “national cell constraint” to the standard row and column sum constraints. Similarly, Jackson and Comer 1993 use partition coefficients for groups of cells of a disaggregated base year matrix to disaggregate cells in an updated but aggregated matrix. Batten and Martellato 1985 (p. 52-55) discuss further constraints structures, involving intermediate and final demand data. Gilchrist and St Louis 1999; Gilchrist and St Louis 2004 propose a three-stage “TRAS” for the case when aggregation rules exist under which the partial aggregated information $\mathbf{A}^G$ can be constructed from its disaggregated form $\mathbf{A}$. Cole 1992 describes the general TRAS type that accepts constrained subsets of any size or shape. Gilchrist and St Louis, as well as Lenzen *et al.* 2006a demonstrate that the inclusion of partial aggregated information into the RAS procedure leads to superior outcomes.

2.2 Reliability of the initial estimate and external information

Another variant of the MRAS method takes into account the uncertainty of the preliminary estimates, and contains the occurrence of perfectly known elements as a special case (Lecomber 1975a; Lecomber 1975b, with case studies in Allen 1974 and Allen and Lecomber

⁶ When applied to the forecasting of monetary input-output matrices, bi-proportional changes have been interpreted as productivity, substitution or fabrication effects (Leontief 1941; Stone and Brown 1962) affecting industries over time. Miernyk 1976 view however is that the RAS method “substitutes computational tractability for economic logic”, and that the production interpretation loses its meaning when the entire input-output table is balanced, and not only inter-industry transactions (see also Giarratani 1975).

⁷ The RAS, Linear Programming and minimum information gain algorithms yield a balanced matrix estimate that is – in terms of some measure of multidimensional ‘distance’ – closest to the unbalanced preliminary estimate. When applied to temporal forecasting, this property is explained as a conservative hypothesis of attributing inertia to inter-industrial relations (Bacharach 1970, p. 26). While the classic RAS method is aimed at maintaining the value structure of the balanced matrix, the closely related Cross-Entropy methods (Robinson *et al.* 2001) are aimed at maintaining the coefficient structure.

⁸ Deming and Stephan 1940; Kruithof 1937 as cited in Lamond and Stewart 1981; attributed to 1930s Leningrad architect Sheleikhovskii by Bregman 1967a.

1975). This is accomplished by introducing a matrix $\mathbf{E}$ containing “reliability information” about the elements in $\mathbf{A}_0$. $\mathbf{E}$ instead of $\mathbf{A}_0$ is then balanced in order to take up the difference between the preliminary and true totals:

$$\mathbf{A}^* = (\mathbf{A}_0 - \mathbf{E}) + \hat{\mathbf{r}}\mathbf{E}\hat{\mathbf{s}}. \quad (1)$$

$\mathbf{A}^*$ is the balanced estimate, and $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ are diagonal scaling matrices, as in the conventional RAS algorithm. Where $E_{ij} = 0$, A_{ij} remains unchanged during balancing. Lecomber 1975a; Lecomber 1975b also investigates the influence of errors in the “true” totals.

A shortcoming of Lecomber’s approach is that the elements of $\mathbf{E}$ cannot be interpreted as standard deviations. If we follow Lecomber in maintaining $0 \leq E_{ij} \leq A_{0ij}$, and consider that RAS preserves the positive signs in $\mathbf{E}$, then $A_{ij}^* \geq A_{0ij} - E_{ij} \forall i, j$. In other words if E_{ij} were the standard deviations of A_{0ij} , then the balanced estimate $\mathbf{A}^*$ could never go below the initial estimate $\mathbf{A}_0$ for more than one standard deviation. An upper limit for $\mathbf{A}^*$ does not exist however. Thus, as Lecomber points out, the elements of $\mathbf{E}$ must be sufficiently large to ensure the controlling vectors are non-negative – but there is no method to ensure this, whilst still interpreting the elements of $\mathbf{E}$ as standard errors. Thus considering that conflicting external information may well diverge by more than one standard deviation, it follows that MRAS will not reach a solution under sufficiently inconsistent constraints, unless more (unspecified) information on errors is obtained.

Lahr 2001 takes into account the uncertainties of external constraints in treating the tolerances of the RAS termination criteria as functions of the varying reliabilities of row and column sums. Dalgaard and Gysting 2004 incorporate information about the reliability of external constraints (again row and column totals) into the balancing process as “confidence factors” λ , and successively adjust the target totals $\mathbf{u}^{(n)}$ of the n th iteration as a weighted sum $u_j^{(n)} = \lambda_j^{(n-1)} u_j^{(0)} + (1 - \lambda_j^{(n-1)}) u_j^{(n-1)}$ of the initial unbalanced totals $u_j^{(0)}$ and the totals $u_j^{(n-1)}$ of the previous iteration. With subsequent iterations, the confidence factors $0 \leq \lambda_j^{(n)} \leq 1$ become smaller and smaller, thus gradually converging away from the unbalanced initial totals $\mathbf{u}^{(0)}$, towards the balanced totals $\mathbf{u}^{(\infty)}$. The innovation is that totals with high confidence ($\lambda_j \leq 1$) get adjusted away from the initial totals much slower than those totals with low confidence ($\lambda_j \geq 0$).

While both approaches consider the varying reliability of totals, they cannot deal with inconsistent totals. In applying conventional RAS scaling factors, Lahr’s algorithm would always end up balancing matrix elements to satisfy only one of a number of conflicting

external constraints. Similarly, for large enough n , Dalgaard and Gysting's algorithm would oscillate around those inconsistent totals $u_j^{(n-1)}$ with non-zero confidence.⁹

2.3 Negative elements

Junius and Oosterhaven 2003 derive a generalised RAS ("GRAS") algorithm that can balance negative elements, by splitting the matrix $\mathbf{A}$ into positive and negative parts $\mathbf{P}$ and $\mathbf{N}$, and balancing $\mathbf{A} = \mathbf{P} - \mathbf{N}$ according to

$$\begin{aligned} (\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1})\mathbf{i} &= \mathbf{u}^* \\ \mathbf{i}(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}) &= \mathbf{v}^* \end{aligned} \quad (2)$$

where $\mathbf{i}$ is the summation vector, and $\mathbf{u}^* = e\mathbf{u}$ and $\mathbf{v}^* = e\mathbf{v}$.¹⁰

2.4 Conflicting data – constrained optimisation

Neither MRAS, TRAS, nor GRAS deal satisfactorily with uncertainty and data conflict. Such problems are so far only solved by constrained optimisation methods. Bacharach 1970 has already shown that the conventional RAS technique is equivalent to the constrained minimisation of an information gain function $f = \sum_{ij} A_{ij} \ln(A_{ij}/eA_{0ij})$. Naturally, this circumstance lead to the parallel developments of both RAS and constrained optimisation techniques for the purpose of balancing input-output tables or SAMs. It is interesting to see that researchers working on either technique have faced almost the same challenges.

The basic structure of a constrained optimisation problem applied to SAMs is

$$\text{Minimise } f(\mathbf{A}, \mathbf{A}_0), \text{ subject to } \sum_i A_{ij} = x_j \text{ and } \sum_j A_{ij} = x_i, \quad (3)$$

where f is the objective function, and x_i and x_j are row and column totals. Morrison and Thumann 1980 minimise a weighted sum of squares of deviations $f = \sum_{ij} (A_{ij} - A_{0ij})^2 / w_{ij}$, where the w_{ij} are the weights. They also explicitly describe the incorporation of external information referring to general subsets of matrix elements, into a Lagrange multiplier approach.

Byron 1978 incorporates variances for the initial estimate into a quadratic Lagrange function, and uses the first-order conditions to solve for the Lagrange multipliers and the balanced SAM. van der Ploeg 1982; van der Ploeg 1984; van der Ploeg 1988 elegantly extends Byron's formulation by a) adding disturbances to the external constraint information, and b)

⁹ Dalgaard and Gysting 2004 do describe balancing matrices with "unequal net row and column sum" and "macro differences between supply and use". However, rather than inconsistencies in external information, this means correct differences in the sum over supply by *industry* and use by *product*, which naturally occur in asymmetric commodity-by-industry supply and use tables.

¹⁰ Using the trivial case of starting with an initial estimate $\mathbf{A}$ that already satisfies all prescribed row and column totals, Lenzen et al. 2006b construct a case where Junius' and Oosterhaven's GRAS balancing algorithm leads to a solution $\mathbf{X}$ that is inferior to the initial estimate in terms of their target function. They show that the factor e has to be taken out of Junius' and Oosterhaven's formulation in order to correct the problem.

combining the SAM vector with these disturbances, and solve the for both the SAM and the disturbances simultaneously. Since the solution for the Lagrange multipliers involves matrix inversion, computing times are strongly influenced by the sizes of the SAM and constraint system. Both Byron and van der Ploeg go to great lengths in exploiting the sparse structure of the coefficients matrix, and in devising efficient algorithms in order to be able to solve large SAMs. In effect, it is the introduction of the variance and disturbance terms that enables handling conflicting external data (van der Ploeg calls it “constraint violation”), because the solutions for the constraint values are allowed to deviate from its prescribed value.

Lecomber 1975a, Morrison and Thumann 1980, and Harrigan and Buchanan 1984 note that the conventional Langrange multiplier procedure (Eqs. 3a and 3b) does not guarantee non-negative solutions. This is undesirable because negative matrix entries can present problems in input-output analysis (Ten Raa and Van der Ploeg 1989). With the requirement of non-negativity, the constrained optimisation problem essentially becomes a *bounded* constrained optimisation. In general, one asks that the unknown SAM elements are within lower and upper bounds $l_i \leq a_i \leq u_i$.¹¹

The mixing of equality and inequality conditions requires quadratic programming methods, which renders the solution of the optimisation problem considerably more complicated, as the expositions of Harrigan and Buchanan 1984, Zenios *et al.* 1989, and Nagurney and Robinson 1992 may testify. This circumstance provides the motivation for searching for a RAS variant that is able to deal with conflicting external data.

Criterion	RAS-type technique	Constrained optimisation
a)	Gilchrist and St Louis 1999	Morrison and Thumann 1980
b)	Lecomber 1975a; Lecomber 1975b	Stone <i>et al.</i> 1942; Byron 1978
c)	Lecomber 1975a; Lecomber 1975b; Lahr 2001; Dalgaard and Gysting 2004	van der Ploeg 1982
d)	Junius and Oosterhaven 2003	Harrigan and Buchanan 1984
e)	This work	van der Ploeg 1982

Tab. 1: Recent extensions to RAS and optimisation techniques for balancing SAMs and input-output tables.

3 KRAS – an extension of GRAS

Compared to constrained optimisation techniques, RAS has enjoyed higher popularity, which is probably due to ease of programming. However, as Tarancon and Del Rio 2005 (p. 2) explicitly state, at present “... the RAS process cannot be developed with interval estimates of the margins. Hence, point estimates are used, which may carry an implicit error.”

¹¹ Tarancon and Del Rio 2005 present an interesting variant of the bounded optimisation problem, by deriving lower and upper bounds from criteria for the stable structural evolution of input-output coefficients, and introducing supplementary variables to take up the slack between the bounds and the matrix entries. If the model turns out to be inconsistent because some constraints cannot be met within those bounds, then the analyst manually chooses certain constraints to be relaxed, until no variable exceeds the bounds.

Considering that the use of RAS in statistical agencies requires the manual and therefore often tedious removal of inconsistencies in the constraint system, it would be desirable to have a RAS technique that deals with such common occurrences in a systematic and automated way. The description of such a RAS variant is the topic of this Section. We will base our derivation strongly on the GRAS method of Junius and Oosterhaven 2003. Our exposition proceeds in three steps: First, we generalise of GRAS to constraints on arbitrary subsets of matrix elements. Second, we relax the restriction of GRAS for constraint coefficients to be either 1 or -1 , and allow any real number. Third, we incorporate reliability and conflict of external data. While we present these extensions in a succession for the sake of clarity, each of them can be applied separately to GRAS. For example, it is possible to modify the original GRAS method only by incorporating the modifications in Section 3.3 in order to deal with reliability and conflict of external data.

Bacharach 1970 (pp. 79-86) show that the simple bi-proportional RAS algorithm can be derived from minimizing a minimum-information function

$$f(\mathbf{A}, \mathbf{A}_0) = \sum_{i,j} A_{ij} \ln \frac{A_{ij}}{e A_{0ij}}, \quad (4)$$

subject to constraints $\mathbf{u}$ and $\mathbf{v}$ on known row and column totals

$$\sum_j A_{ij} = u_i \text{ and } \sum_i A_{ij} = v_j, \quad (5)$$

where e is the basis of the natural logarithm. The GRAS method is derived in the same way. However, the preliminary estimate $\mathbf{A}_0$ (which becomes the solution $\mathbf{A}^{(0)}$ at step zero), is split into positive and negative parts according to $\mathbf{A}^{(0)} = \mathbf{P}^{(0)} - \mathbf{N}^{(0)}$, and alternately row- and column-scaled using diagonal scalar matrices $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$, so that after the n -th round of balancing, $\mathbf{A}^{(n)} = \hat{\mathbf{r}}^{(n-1)} \mathbf{P}^{(n-1)} \hat{\mathbf{s}}^{(n-1)} - [\hat{\mathbf{r}}^{(n-1)}]^{-1} \mathbf{N}^{(n-1)} [\hat{\mathbf{s}}^{(n-1)}]^{-1}$. $\mathbf{A}^{(n)}$ is then subjected to the next scaling operation. Junius' and Oosterhaven's GRAS derivation arrives at a second-order polynomial that defines scalars

$$r_i^{(n)} = \frac{u_i + \sqrt{u_i^2 + 4 \sum_j P_{ij}^{(n)} \sum_j N_{ij}^{(n)}}}{2 \sum_j P_{ij}^{(n)}}, \text{ with} \quad (6)$$

$$P_{ij}^{(n)} = P_{ij}^{(n-1)} s_j^{(n-1)} \text{ and } s_j^{(n-1)} = \frac{v_j + \sqrt{v_j^2 + 4 \sum_i P_{ij}^{(n-1)} \sum_i N_{ij}^{(n-1)}}}{2 \sum_i P_{ij}^{(n-1)}}.$$

The algorithm converges if $\|(\hat{\mathbf{r}} \mathbf{P} \hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1} \mathbf{N} \hat{\mathbf{s}}^{-1}) \mathbf{i} - \mathbf{u}\| < \delta \|\mathbf{u}\|$ and $\|\mathbf{i}(\hat{\mathbf{r}} \mathbf{P} \hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1} \mathbf{N} \hat{\mathbf{s}}^{-1}) - \mathbf{v}\| < \delta \|\mathbf{v}\|$ for a sufficiently small δ .

3.1 Incorporating constraints on arbitrary subsets of matrix elements

Consider now a generalised formulation of constraints as in $\mathbf{Ga} = \mathbf{c}$, where $\mathbf{a}$ is the vectorisation of $\mathbf{A}$ above. Such a formulation includes constrained row and column sums, constraint single elements, constrained subsets, and negative elements as special cases. Constraints can include any number of elements, which may be fully, partly or non-adjacent.¹² Constraints may also exclude some of the row and column totals (compare Thissen and Löfgren 1998, p. 1994). Junius' and Oosterhaven's minimisation problem then becomes

$$\text{minimise } f(\mathbf{a}, \mathbf{a}_0) = \sum_{i,j} |a_{ij}| \ln \frac{a_{ij}}{e a_{0ij}} \text{ subject to } \mathbf{Ga} = \mathbf{c} . \quad (7)$$

Let there be N_C constraints. Eq. 6 can then be generalised to

$$r^{(n)} = \frac{c_i + \sqrt{c_i^2 + 4 \sum_{j, a_j^{(n-1)} G_{ij} > 0} G_{ij} a_j^{(n-1)} \sum_{j, a_j^{(n-1)} G_{ij} < 0} G_{ij} a_j^{(n-1)}}}{2 \sum_{j, a_j^{(n-1)} G_{ij} > 0} G_{ij} a_j^{(n-1)}} \quad \text{and} \quad (8)$$

$$a_j^{(n)} = a_j^{(n-1)} \left[r^{(n)} \right]^{\text{sgn}(a_j^{(n-1)} G_{ij})}, \quad \text{with } i = n \bmod N_C.$$

In Eq. 8, the negative elements in Eq. 6 have been replaced with negative coefficients on positive elements, but otherwise the formulation is exactly the same. There is only one scalar r_i for each constraint i , and these scalars are applied consecutively for all $i = 1, \dots, N_C$.¹³ The r_i and a_j are calculated alternately. The GRAS feature of scaling negative elements by the inverse of the positive scalar is evident in the exponent $\text{sgn}(a_j^{(n-1)} G_{ij})$ in Eq. 8. The algorithm converges if

$$\|\mathbf{Ga} - \mathbf{c}\| < \delta \|\mathbf{c}\| , \quad (9)$$

for a sufficiently small δ .

3.2 Incorporating constraints with non-unity coefficients

Consider now non-unity constraint coefficients $G_{ij} \in \mathbb{R}$.¹⁴ For such constraints Junius' and Oosterhaven's GRAS will not work anymore. Allowing the a_i to be both positive and negative (as in GRAS), we write the Lagrangean as

¹² Single-element constraints need not be part of the scaling procedure, but could be “netted out” using the “modified RAS” method.

¹³ Here, the modulus function $a \bmod b$ refers to the remainder of the division of a by b .

¹⁴ Non-unity constraints can appear when there is knowledge of relative as opposed to absolute values for some matrix elements. For example, in the construction of multi-regional input-output systems we may use information regarding the fraction of value added or gross output allocated to a given region.

$$\mathcal{L}(\mathbf{a}, \mathbf{a}_0, \boldsymbol{\lambda}) = \sum_{i; a_i \geq 0} a_i \ln \frac{a_i}{e a_{0i}} - \sum_{i; a_i < 0} a_i \ln \frac{a_i}{e a_{0i}} + \sum_i \lambda_i \left[\sum_j G_{ij} a_j - c_i \right]. \quad (10)$$

The first-order minimum conditions are

$$0 = \frac{\partial \mathcal{L}(\mathbf{a}, \mathbf{a}_0, \boldsymbol{\lambda})}{\partial a_k} = \begin{cases} \ln \frac{a_k}{e a_{0k}} + 1 + \sum_i \lambda_i G_{ik} = \ln \frac{a_k}{a_{0k}} + \sum_i \lambda_i G_{ik} & \text{if } a_k \geq 0 \\ -\ln \frac{a_k}{e a_{0k}} - 1 + \sum_i \lambda_i G_{ik} = -\ln \frac{a_k}{a_{0k}} + \sum_i \lambda_i G_{ik} & \text{if } a_k < 0 \end{cases}. \quad (11)$$

The solution for $\mathbf{a}$ is hence of the form

$$a_k = \begin{cases} a_{0k} e^{-\sum_i \lambda_i G_{ik}} = a_{0k} \prod_i e^{-\lambda_i G_{ik}} & \text{if } a_k \geq 0 \\ a_{0k} e^{\sum_i \lambda_i G_{ik}} = a_{0k} \prod_i e^{\lambda_i G_{ik}} & \text{if } a_k < 0 \end{cases}. \quad (12)$$

This form reflects the fact that negative and positive matrix elements are proportionately balanced with respectively inverse scaling terms $\prod_i e^{-\lambda_i G_{ik}}$ and $\prod_i e^{\lambda_i G_{ik}}$. Inserting Eq. 11 into the condition $\mathbf{Ga} = \mathbf{c}$ yields

$$\sum_{j; a_{0j} \geq 0} G_{ij} a_{0j} \prod_k e^{-\lambda_k G_{kj}} + \sum_{j; a_{0j} < 0} G_{ij} a_{0j} \prod_k e^{\lambda_k G_{kj}} - c_i = 0. \quad (13)$$

The multi-proportional problem in Eq. 13 can be solved iteratively using Bregman's balancing method.¹⁵ Starting with the initial solution $\mathbf{a}^{(0)} = \mathbf{a}_0$, and with chosen $\{\lambda_k^{(0)}\}_{k=1, \dots, N_C}$, only one of the N_C Lagrange multipliers λ_i is adjusted at any one time, by first solving constraint 1

$$\begin{aligned} 0 &= \sum_{j; a_j^{(0)} \geq 0} G_{ij} a_j^{(1)} e^{-\lambda_i^{(1)} G_{ij}} \prod_{k \neq i} e^{-\lambda_k^{(0)} G_{kj}} + \sum_{j; a_j^{(n-1)} < 0} G_{ij} a_j^{(0)} e^{\lambda_k^{(1)} G_{ij}} \prod_{k \neq i} e^{\lambda_k^{(0)} G_{kj}} - c_i \\ &= \sum_{j; a_j^{(0)} \geq 0} G_{ij} a_j^{(0)} e^{-(\lambda_i^{(1)} - \lambda_i^{(0)}) G_{ij}} \prod_k e^{-\lambda_k^{(0)} G_{kj}} + \sum_{j; a_j^{(0)} < 0} G_{ij} a_j^{(0)} e^{(\lambda_k^{(1)} - \lambda_i^{(0)}) G_{ij}} \prod_k e^{\lambda_k^{(0)} G_{kj}} - c_i, \end{aligned} \quad (14)$$

then updating

¹⁵ See Bregman 1967b; Elfving 1980; Lamond and Stewart 1981. Elfving 1980 distinguishes “Gauss-Seidel type schemes”, where only one Lagrange multiplier is adjusted in every step, and “Jacobi type methods” where the non-linear system in Eq. 12 is solved simultaneously for all Lagrange multipliers. The latter methods require the Jacobian of Eq. 12 to have full rank, that is all constraint equations to be linearly independent (compare Eriksson 1980; Erlander 1981).

$$a_j^{(1)} = \begin{cases} a_j^{(0)} e^{-(\lambda_i^{(1)} - \lambda_i^{(0)})G_{ij}} \prod_k e^{-\lambda_k^{(0)}G_{kj}} & \text{for } a_j^{(0)} \geq 0 \\ a_j^{(0)} e^{(\lambda_i^{(1)} - \lambda_i^{(0)})G_{ij}} \prod_k e^{\lambda_k^{(0)}G_{kj}} & \text{for } a_j^{(0)} < 0 \end{cases}, \quad (15)$$

with $\lambda^{(1)}$ given by Eq. 14. This is followed by solving constraints $i \forall i > 1$

$$\sum_{j: a_j^{(n-1)} \geq 0} G_{ij} a_j^{(n-1)} e^{-(\lambda_i^{(n)} - \lambda_i^{(n-1)})G_{ij}} + \sum_{j: a_j^{(n-1)} < 0} G_{ij} a_j^{(n-1)} e^{(\lambda_i^{(n)} - \lambda_i^{(n-1)})G_{ij}} - c_i = 0 \quad (16)$$

for $\lambda_i^{(n)}$, iteratively, with known $\lambda_k^{(n-1)} \forall k$, and with $i = n \bmod N_C$.¹⁰ The new solution of each step (n) is computed via (compare Eqs. 2.5 and 2.6 in Elfving 1980)

$$a_j^{(n)} = \begin{cases} a_j^{(n-1)} e^{-(\lambda_i^{(n)} - \lambda_i^{(n-1)})G_{ij}} & \text{for } a_j^{(n-1)} \geq 0 \\ a_j^{(n-1)} e^{(\lambda_i^{(n)} - \lambda_i^{(n-1)})G_{ij}} & \text{for } a_j^{(n-1)} < 0 \end{cases}, \text{ and} \quad (17)$$

$$\lambda_i^{(n)} = \begin{cases} \text{given by Eq.(16) for } i = n \bmod N_C \\ \lambda_i^{(n-1)}, & \text{for } i \neq n \bmod N_C \end{cases}. \quad (18)$$

Calling $e^{(\lambda_i^{(n)} - \lambda_i^{(n-1)})} = r_i^{(n)}$, solving Eq. 16 is equivalent to calculating the roots of the generalised polynomials

$$P_i(\lambda_i^{(n)}) = \sum_{j: a_j \geq 0} G_{ij} a_j^{(n-1)} r_i^{(n) - G_{ij}} + \sum_{j: a_j < 0} G_{ij} a_j^{(n-1)} r_i^{(n) G_{ij}} - c_i = 0. \quad (19)$$

iteratively for all $\{r_i^{(n)}\}_{i=1, \dots, N_C}$, given known $p_j^{(n-1)}$ and $\lambda_k^{(n-1)}$.¹⁶

It is important to understand that Junius' and Oosterhaven's idea of inversely scaling positive and negative matrix elements applies in exactly the same way to elements that are added and subtracted in a constraint, respectively. This is reflected in the sign of the exponents G_{ij} of the r_i . For example, consider the constraint $a_1 + a_2 - a_3 = 2$, with $a_{01} = 3$, $a_{02} = 5$, and $a_{03} = 1$. Scaling a_1 and a_2 with $\frac{1}{2}$, and a_3 with $\frac{1}{2}^{-1} = 2$ yields the desired result $1.5 + 2.5 - 2 = 2$. If an element is both negative and subtracted, it has to be scaled like a positive added element. For example, consider the above constraint with $a_{01} = 3$, $a_{02} = 5$, and $a_{03} = -1$. Scaling all elements a_1 , a_2 and a_3 with $\frac{2}{9}$ yields the desired result $\frac{6}{9} + \frac{10}{9} - (-\frac{2}{9}) = 2$.

Note that the G_{ij} are not necessarily unity integers as in GRAS, but $G_{ij} \in \mathbb{R}$. Since in general no analytical solution exists, Erlander 1981 and Eriksson 1980 suggest Newton's method for

¹⁶ The problem can be restated so that all exponents are positive, by multiplying Eq. 14 with $r_i^{(n) \max_{j: p_j \geq 0} \{G_{ij}\}}$.

an iterative solution.¹⁷ Applying Newton's method to Eq. 14 requires calculating the derivative

$$\begin{aligned} \frac{\partial P_i(\lambda_i^{(n)})}{\partial \lambda_i^{(n)}} = & \sum_{j; a_j^{(n-1)} \geq 0} G_{ij} a_j^{(n-1)} e^{-\left(\lambda_i^{(n)} - \lambda_i^{(n-1)}\right) G_{ij}} \prod_k e^{-\lambda_k^{(n-1)} G_{kj}} (-G_{ij}) \\ & + \sum_{j; a_j^{(n-1)} < 0} G_{ij} a_j^{(n-1)} e^{\left(\lambda_k^{(n)} - \lambda_i^{(n-1)}\right) G_{ij}} \prod_k e^{\lambda_k^{(n-1)} G_{kj}} G_{ij} \end{aligned} \quad (20)$$

and iterating over k

$$\lambda_i^{(n,k)} = \lambda_i^{(n,k-1)} - \frac{P_i(\lambda_i^{(n,k-1)})}{\frac{\partial P_i(\lambda_i^{(n,k-1)})}{\partial \lambda_i^{(n)}}} \quad (21)$$

3.3 Incorporating reliability and conflict of external data

In cases of inconsistent constraints brought about by conflicting external data, the termination condition in Eq. 9 may never be met, and GRAS has to be terminated if the distance function between the constraints $\mathbf{c}$ and their realisations $\mathbf{Ga}$ does not improve anymore, that is if for two subsequent iterations $n-1$ and n

$$\|\mathbf{Ga} - \mathbf{c}\|^{(n)} - \|\mathbf{Ga} - \mathbf{c}\|^{(n-1)} < \delta \quad (22)$$

for a sufficiently small δ . Following this termination, we propose a GRAS-type algorithm that modifies the constraints $\mathbf{c}$ as well:

$$\begin{aligned} r^{(n)} &= \frac{c_i^{(n)} + \sqrt{c_i^{(n)2} + 4 \sum_j G_{ij}^+ a_j^{(n-1)} \sum_j G_{ij}^- a_j^{(n-1)}}}{2 \sum_j G_{ij}^+ a_j^{(n-1)}}, \\ c_i^{(n)} &= c_i^{(n-1)} - \text{Sgn} \left(c_i^{(n-1)} - \sum_j G_{ij} a_j^{(n-1)} \right) \times \text{Min} \left(\left| c_i^{(n-1)} - \sum_j G_{ij} a_j^{(n-1)} \right|, \alpha \sigma_i \right) \text{ with } c_i^{(0)} = c_i, \quad (23) \\ a_j^{(n)} &= a_j^{(n-1)} \left[r^{(n)} \right]^{\text{sgn}(G_{ij})}, \quad \text{and } i = n \bmod N_C, \end{aligned}$$

where $0 \leq \alpha \leq 1$ and the σ_i are the standard deviations of the c_i .¹⁸ We refer to this algorithm as KRAS ('Conflicting RAS'). The essence of this idea is that once GRAS terminates in

¹⁷ Using Newton's method, a root r of a function $f(x)$, in the vicinity of x_0 , is approached by first truncating the Taylor expansion around x_0 : $f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 + \dots$, to $0 = f(r) \approx f(x_0) + f'(x_0)(r - x_0)$, and then iteratively solving $x_n = x_{n-1} - f(x_{n-1}) / f'(x_{n-1})$.

¹⁸ If external data are not normally distributed, the adjustment of constraints $\alpha \sigma_i$ in Eq. 23 could be adapted to the characteristics of the alternative distribution. For example, the shape parameter of a Weibull distribution could take the place of the standard deviation σ in Eq. 23. Further, if conflicting values were distributed uniformly, adjustments could be made in proportion to the uniform ranges. In the case of subjective reliability scores, one could in a similar fashion, that is, by taking normalised scores as step-wise adjustments.

oscillations without reaching convergence, the original external constraints c_i can clearly not all be satisfied simultaneously, and either some of them or all of them must be erroneous. Just as in constrained optimisation, the c_i must be modified “towards” their realisations $\{\mathbf{Ga}\}_i$ in order to achieve convergence. Since each constraint is known to a higher or lower degree of accuracy. Therefore, an amount $\alpha\sigma_i$ is added or subtracted from each $c_i^{(n-1)}$, depending on the sign $\text{Sgn}(c_i^{(n-1)} - \sum_j G_{ij}a_j^{(n-1)})$. The constant α can be chosen freely: The higher its value, the more rapid the adjustment process, but also the more inaccurate the adjustment. Note that in order to prevent overshoot in situations where the realisation $\{\mathbf{Ga}\}_i$ is closer to the c_i than σ_i , the maximum adjustment allowed is $|c_i^{(n-1)} - \sum_j G_{ij}a_j^{(n-1)}|$. With constraint values modified as in Eq. 23, the termination criterion of KRAS is equal to that in Eq. 9.

4 Applications

In the following, we will apply KRAS to RAS-infeasible balancing problems documented in the literature.

4.1 Cole 1992

The most simple RAS-infeasible problem that we found in the literature was posed by Cole 1992; vectorising Cole’s matrix to vector $\mathbf{a}$, it can be represented as

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 3 \\ 1 \end{pmatrix}. \quad (24)$$

The first four constraints are the column and row sums, respectively, and the fifth constraint fixes a_4 . This problem is infeasible for $a_i > 0 \forall i$, because in requiring $a_4 = 1$, constraint 5 imposes on constraints 2 and 4 that both $a_2 = 2$ and $a_3 = 2$, which conflicts with constraints 1 and 3. Starting with the initial estimate $\mathbf{a}_0 = (1, 1, 1, 1)$, and setting the standard deviation¹⁹ $\boldsymbol{\sigma} = (0.01, 0.01, 0.01, 0.01, 0.01)$, KRAS produces the solution $\mathbf{a} = (0, 1^{1/3}, 1^{1/3}, 1^{1/3})$ with the realisation $\mathbf{Ga} = (1^{1/3}, 2^{2/3}, 1^{1/3}, 2^{2/3}, 1^{1/3})$. The difference between imposed and realised constraint values is $\mathbf{Ga} - \mathbf{c} = (1^{1/3}, -1^{1/3}, 1^{1/3}, -1^{1/3}, 1^{1/3})$, and thus constant across all constraints. This is a direct consequence of the standard deviation $\boldsymbol{\sigma}$ being equal for all constraints.

¹⁹ Cole does not give any information on reliability. All standard deviations were set by the authors.

σ	0.01	0.01	0.01	0.01	0.001
$\mathbf{a}$	0.00	1.48	1.48	1.05	
$\mathbf{Ga}$	1.48	2.52	1.48	2.52	1.05
$\mathbf{Ga} - \mathbf{c}$	0.48	-0.48	0.48	-0.48	0.05
$(\mathbf{Ga} - \mathbf{c})\hat{\sigma}^{-1}$	47.57	-47.62	47.61	-47.67	47.66
σ	0.01	0.01	0.01	0.01	0.1
$\mathbf{a}$	0.00	1.08	1.08	1.83	
$\mathbf{Ga}$	1.08	2.92	1.08	2.92	1.83
$\mathbf{Ga} - \mathbf{c}$	0.08	-0.08	0.08	-0.08	0.83
$(\mathbf{Ga} - \mathbf{c})\hat{\sigma}^{-1}$	8.27	-8.31	8.32	-8.36	8.34
σ	0.05	0.04	0.03	0.02	0.01
$\mathbf{a}$	0.00	1.37	1.62	1.13	
$\mathbf{Ga}$	1.62	2.50	1.37	2.75	1.13
$\mathbf{Ga} - \mathbf{c}$	0.62	-0.50	0.37	-0.25	0.13
$(\mathbf{Ga} - \mathbf{c})\hat{\sigma}^{-1}$	12.46	-12.49	12.50	-12.56	12.56

Tab. 1: KRAS solutions for Cole's RAS-infeasible problem (Cole 1992). Vectors are presented as table rows.

Setting $\sigma_5 = 0.001$ and $\sigma_5 = 0.1$, respectively, more or less accuracy is put on the fifth constraint (which determines a_4) compared to the column and row sum constraints (Tab. 1). Placing more accuracy on constraint 5 yields a solution $a_4 = 1.05$ which is closer to the required value ($\{\mathbf{Ga} - \mathbf{c}\}_5 = 0.05$), while placing less accuracy on constraint 5 yields a solution $a_4 = 1.83$ further away ($\{\mathbf{Ga} - \mathbf{c}\}_5 = 0.83$). The opposite holds for constraint 1 to 4. It is interesting to observe though that the distance $(\mathbf{Ga} - \mathbf{c})\hat{\sigma}^{-1}$ in terms of the number of standard deviations is about constant across all constraints in both cases.

Choosing the standard deviation to diminish from constraint 1 through to 5 results in the absolute difference to decrease as well, with constraint 5 satisfied best. Once again, the distance in terms of the number of standard deviations is about constant across all constraints. This is a direct consequence of the constraint adjustment mechanism described around Eq. 23, which adjusts constraint values $c^{(n)}$ relative to $c^{(n-1)}$ by adding or subtracting a fixed portion α of a one standard deviation. In this case, the final solution is about 12 standard deviations away from the initially prescribed constraint values.

For example, since in the third case the standard deviations of constraints are all different, constraint values are adjusted differently (in absolute terms) throughout the KRAS run. In this case, constraint 1 is assigned the highest standard deviation and its value is therefore altered most (Fig. 1).

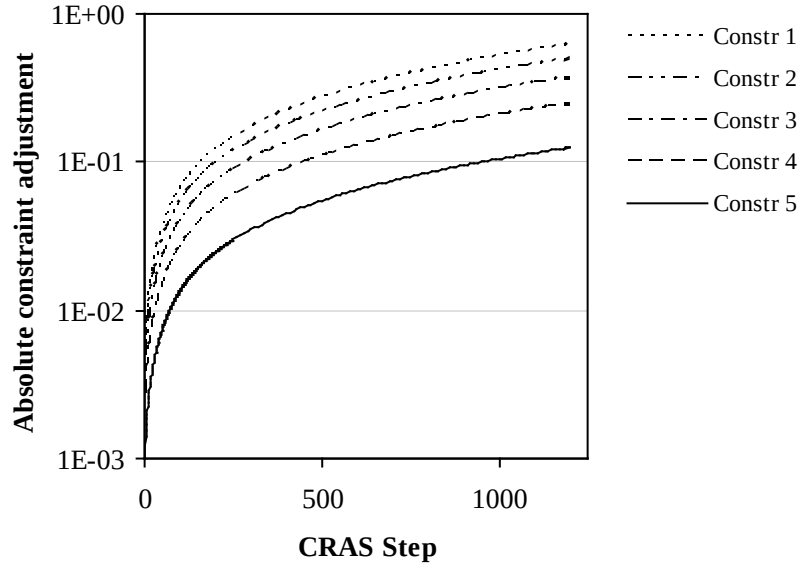


Fig. 1: Evolution of $\Delta \mathbf{c}^{(n)} = \mathbf{c}^{(n)} - \mathbf{c}^{(0)}$ throughout KRAS for Cole's problem.

Once again, as a consequence of the balancing mechanism in Eq. 23, the adjustment in terms of the number of standard deviations is constant across all constraints (Fig. 2). Throughout the KRAS balancing run, the adjusted constraints move away evenly from the initially prescribed constraints until at about 12 standard deviations, where no more conflict occurs.

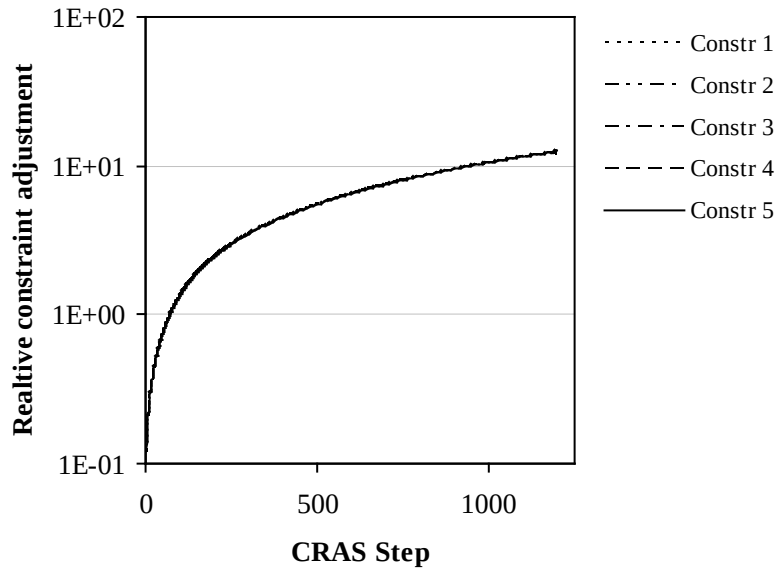


Fig. 2: Evolution of $\Delta \mathbf{c}^{(n)} \hat{\boldsymbol{\sigma}}^{-1}$ throughout KRAS for Cole's problem.

In one final numerical experiment on Cole's problem, we increased the range of the σ_i to span 2 orders of magnitude, from 0.001 to 0.1 (Tab. 2). We observed that the least stringent constraint (#1) is more closely matched ($9\sigma_1$) than other constraints ($16\sigma_i$).

The reason for this phenomenon is that because of its relatively “lax” standard deviation, constraint 1 starts to become well balanced already from about step 200 in the KRAS run. In other words the realisation $\{\mathbf{Ga}\}_1$ of constraint 1 starts to continuously fall within the interval $[c_1^{(n)} - \alpha\sigma_1, c_1^{(n)} + \alpha\sigma_1]$. This means that the term $\text{Min}(|c_1^{(n-1)} - \sum_j G_{1j}a_j^{(n-1)}|, \alpha\sigma_1)$ in Eq. 23 yields $\Delta c_1^{(n)} = c_1^{(n-1)} - \sum_j G_{1j}a_j^{(n-1)}$, and the constraint value needs to be adjusted by less than $\alpha\sigma_1$ in order to prevent overshoot (compare Section 3.3). As a consequence $\Delta \mathbf{c}^{(n)}$ and $\Delta \mathbf{c}^{(n)} \hat{\boldsymbol{\sigma}}^{-1}$ “flatten out” (Fig. 3).

$\boldsymbol{\sigma}$	0.10	0.05	0.01	0.005	0.001
$\mathbf{a}$	0.00	1.16	1.90	1.02	
$\mathbf{Ga}$	1.90	2.18	1.16	2.92	1.02
$\mathbf{Ga} - \mathbf{c}$	0.90	-0.82	0.16	-0.08	0.02
$(\mathbf{Ga} - \mathbf{c})\hat{\boldsymbol{\sigma}}^{-1}$	9.02	-16.39	16.39	-16.41	16.40

Tab. 2: KRAS solutions for Cole’s RAS-infeasible problem (Cole 1992).

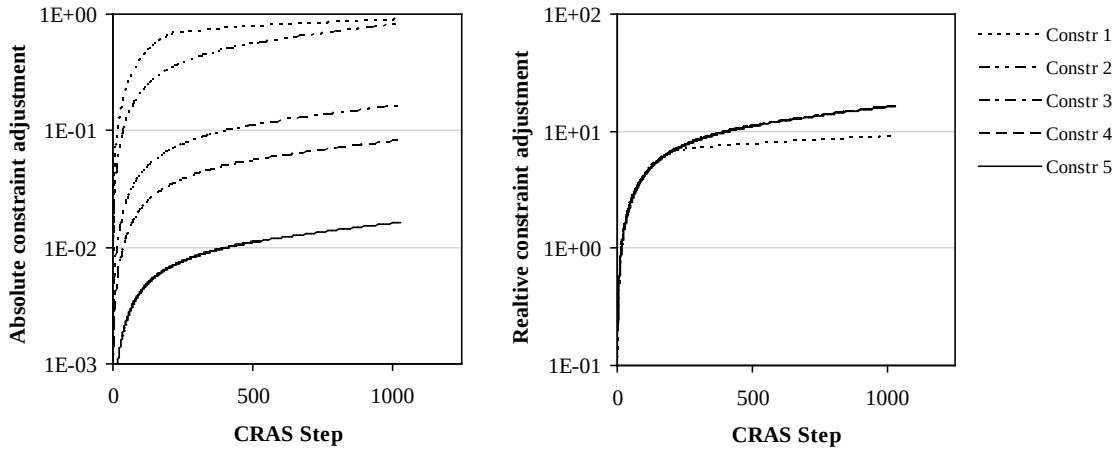


Fig. 3: Evolution of $\Delta \mathbf{c}^{(n)}$ and $\Delta \mathbf{c}^{(n)} \hat{\boldsymbol{\sigma}}^{-1}$ throughout KRAS for Cole’s problem.

4.2 Möhr *et al.* 1987

Another RAS-infeasible problem was posed by Möhr *et al.* 1987; it can be represented as

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 1 & 1 & & & & & \\ & & & 1 & 1 & 1 & 1 & & \\ & & & & & & 1 & 1 & 1 & 1 \\ & & & & & & & & 1 & 1 & 1 & 1 \\ 1 & & & 1 & & & 1 & & & 1 & & & \\ & 1 & & & 1 & & & 1 & & & 1 & & \\ & & 1 & & & 1 & & & 1 & & & 1 \\ & & & 1 & & & 1 & & & & 1 & & \\ & & & & 1 & & & 1 & & & & 1 \\ & & & & & 1 & & & 1 & & & & 1 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} 301 \\ 104 \\ 105 \\ 10 \\ 100 \\ 220 \\ 100 \\ 100 \end{pmatrix}. \quad (25)$$

The first four constraints are the row sums, the last four constraints are the column sums of the variable matrix

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{pmatrix}. \quad (26)$$

This problem is RAS-infeasible for the initial estimate

$$\mathbf{A}_0 = \begin{pmatrix} 90 & 0 & 95 & 95 \\ 5 & 101 & 2 & 2 \\ 5 & 101 & 2 & 2 \\ 0 & 18 & 1 & 1 \end{pmatrix}, \quad (27)$$

because on the one hand a_1 , a_2 and a_4 belong to columns whose sums are required to equal 100 by constraints 1, 2 and 3, hence $a_1 \leq 100$, $a_2 \leq 100$, and $a_4 \leq 100$, but on the other hand $a_1 + a_2 + a_4 = 301$ is imposed by the first constraint. Strictly speaking, this problem is RAS-infeasible not because of conflicting constraints, but because of ill-set initial estimates. Setting all standard deviations to $\sigma_i = 0.1$, KRAS produces the solution

$$\mathbf{A} = \begin{pmatrix} 100.16 & 0 & 100.26 & 100.26 \\ 0.09 & 104.18 & 0.03 & 0.03 \\ 0.09 & 105.18 & 0.03 & 0.03 \\ 0 & 10.31 & 0.01 & 0.01 \end{pmatrix} \quad (28)$$

with $\mathbf{Ga} = (300.67, 104.33, 105.33, 10.33, 100.33, 219.67, 100.33, 100.33)$, $\mathbf{a}$ being the vectorisation of $\mathbf{A}$. All realisations $\{\mathbf{Ga}\}_i$ are 0.33, or 3.3 standard deviations away from the imposed constraints $\mathbf{c}$.

4.3 Updating the Australian Supply-Use tables for 1993-94

A third example was trialled for a “real-world” scenario that may confront analysts.

Currently the Australian Bureau of Statistics (ABS) provides data within the input-output framework that is balanced between the supply and use tables. This has not always occurred, however, and the 1993-94 and earlier supply-use tables are not properly balanced across row and column sums. These discrepancies are due to inconsistent source data. For 1993-94, there is a discrepancy of row totals of up to 11% for 56% of the rows. The final published tables still contain these discrepancies, and the analyst is left with unbalanced tables to use. Further to this issue, are the occurrences of confidential cells, which must also be estimated (and is possible with the normal RAS procedure).

The scale of this problem is at the level of 107 products and 107 industries, with 7 categories of final demand, and 6 categories of value added. In this example, the balance is performed across the separate supply and use tables. The data used for both initial estimate and constraints were the 1993-94 IO tables, published by the ABS (ABS1997). Estimates for the standard deviations of published data were obtained from Australian business operations and industry performance statistics (ABS1995), and from financial operating data of Australian manufacturing industries (ABS2005). Additional ‘balancing constraints’ were constructed so as to ensure the balance of row totals, column totals, final and intermediate demand totals, value added totals, and respective row and column totals between the supply and use tables. Standard deviations were set to zero for the balancing constraints so as to ensure that these constraint algorithms were satisfied exactly. The total number of variables is 25,353, and the total number of constraints, $N_c = 26,191$.

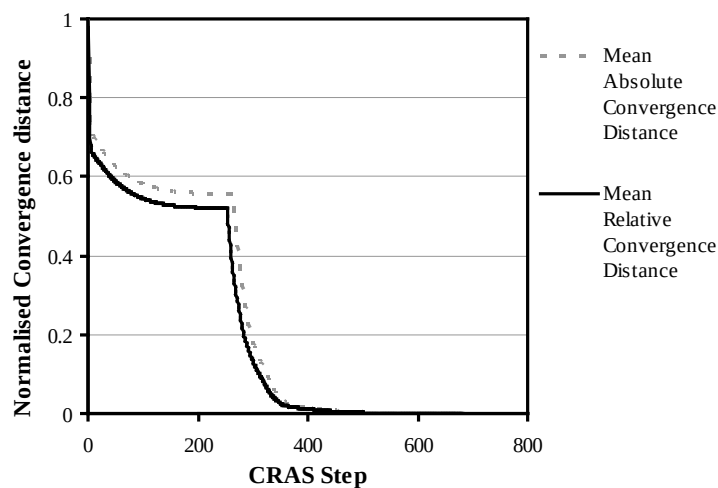


Fig. 4. Mean absolute convergence distance, $\sum_i |\mathbf{Ga} - \mathbf{c}^{(n)}|_i / N_c$, and Mean relative convergence distance $\sum_i |\mathbf{Ga} - \mathbf{c}^{(n)}|_i / ([\sigma_i + \delta] N_c)$, normalised, for the 1993-94 IO tables.

The balancing run consists of two steps: in a first step a RAS procedure is run until $\mathbf{Ga}$ stops converging towards $\mathbf{c}^{(n)} = \mathbf{c}^{(0)}$, which occurs when adjustment cannot continue with the given conflicting constraint values (here after 264 iterations); in a second step a KRAS procedure allows the constraints to change according to their assigned deviations. The initial RAS step is implemented in order to estimate confidential cells and other imbalances in the table before KRAS is applied. Convergence is assumed when the maximum deviation of any $(\mathbf{Ga}^n)_i$ from $c_i^{(n)}$ is less than \$1 (here after 681 iterations). Figure 4 shows the evolution of the average distance between $\mathbf{Ga}$ and $\mathbf{c}^{(n)} : \sum_i |\mathbf{Ga}^n - \mathbf{c}^{(n)}|_i / N_C$ and of the weighted average: $\sum_i |\mathbf{Ga} - \mathbf{c}^{(n)}|_i / ([\sigma_i + \delta] N_C)^{20}$ throughout the RAS and KRAS steps (up to iteration number 1500). Values have been normalised to their respective maxima.

Figure 4 shows the initial step during normal RAS convergence, with preliminary rapid gains diminishing at approximately half of the required convergence distance to the final solution. This ‘premature’ convergence is entirely due to constraint conflict. At the 265th iteration, KRAS allows further convergence, with rapid initial gains (corresponding to the adjustment of the most problematic constraints) again slowing as true convergence is reached, and finer adjustments are required.

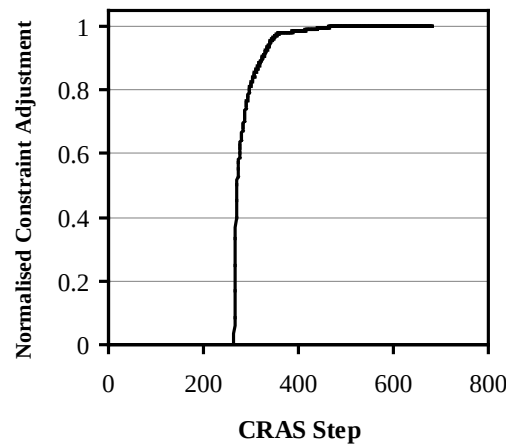


Fig. 5. Mean constraint adjustment by iteration, $\sum_i |\mathbf{c}^{(n)} - \mathbf{c}^{(0)}|_i / N_C$, normalised to the maximum adjustment, for the 1993-94 IO tables.

In Figure 5, the mean deviation of $\mathbf{c}^{(n)}$ from $\mathbf{c}^{(0)}$, $\sum_i |\mathbf{c}^{(n)} - \mathbf{c}^{(0)}|_i / N_C$, is plotted, again, normalised to the maximum deviation.

²⁰ The small δ value has been incorporated to handle the case of $\sigma_i = 0$.

The initial 264 iterations follow the normal RAS procedure, and hence do not require constraint adjustment. The ensuing iterations, however, show large initial changes, corresponding to the changing of the most inaccurate constraints. The speed at which this adjustment takes place (and hence the slope of the graph) can be controlled through changing the precision of the adjustment parameter, α . The run time for this example was in the order of several seconds for complete convergence, using the intel fortran90 compiler v.7.1 on a RHEL-WS24 linux Kernel with a Xenon 32 bit and 2x3.1 GHz CPU and 4Gb RAM.

4.4 Australian multi-regional input-output system

In a fourth case study, we have used KRAS to balance a detailed multi-regional input-output (MRIO) framework of Australia. This framework covers 344 industrial sectors across the eight Australian States and Territories²¹ and makes use of a large and diverse amount of the most recent survey data available (for details see Gallego and Lenzen 2007). To our knowledge, this is the first such detailed framework estimated in Australia. Furthermore, this MRIO system has been complemented with physical data on more than a thousand social and environmental parameters, such as employment, water use or greenhouse gas emissions.²² This generalisation allows tracing social and environmental impacts along interregional supply chains. The trade specification in this model has a strong influence on evening out regional differences of environmental intensities of commodities bought by consumers. For example, due to Australia's variable climate, land and water use intensities in agriculture vary significantly across regions. These intensities become "mixed" in a final consumer's commodity basket due to interregional trade. The extent of this dilution, and hence the magnitude of land and water intensities of regionally purchased commodities, depends critically on the specification of the trade model.

A preliminary estimate of the Australian 1998-99 economic MRIO system, which contains 8,792,675 variables, has been balanced to satisfy $N_c = 462,909$ constraints using KRAS. These constraints take different shapes and sizes ranging from 1 coefficient constraints to constraints containing more than 8 million coefficients. The coefficients, in turn, include both negative as well as non-unity real values. Furthermore, many of the constraints come from survey data such as the national input-output tables, state accounts, and other commodity statistics and contain errors which make them conflicting. Therefore they have been assigned standard deviations $\{\sigma_i\}$ according to their origin²³. A smaller fraction of the constraints, such as the internal accounting constraints are to be satisfied exactly and are assigned deviations $\sigma_i = 0$.²⁴

²¹ New South Wales, Victoria, Queensland, South Australia, Western Australia, Tasmania, Northern Territory, Australian Capital Territory.

²² See http://www.isa.org.usyd.edu.au/research/ISA_TBL_Indicators.pdf.

²³ It has been assumed that these $\{\sigma_i\}$ represent the standard deviations of normally distributed random errors in the survey data. Possible systematic errors in the data sources have not been captured in this model.

²⁴ A small δ value was applied in cases of $\sigma_i = 0$.

As per the Australian 1993-94 tables, the balancing run consists of two steps: the initial RAS procedure until $\mathbf{Ga}$ stops converging towards $\mathbf{c}^{(n)} = \mathbf{c}^{(0)}$ (here after 526 iterations); and the subsequent KRAS procedure. Convergence is assumed when the maximum deviation of any $(\mathbf{Ga}^n)_i$ from $c_i^{(n)}$ is less than \$1000 (here after 3328 iterations). Figure 6 shows the evolution of the average distance between $\mathbf{Ga}$ and $\mathbf{c}^{(n)} : \sum_i |\mathbf{Ga}^n - \mathbf{c}^{(n)}|_i / N_C$ and of the weighted average: $\sum_i |\mathbf{Ga} - \mathbf{c}^{(n)}|_i / ([\sigma_i + \delta] N_C)$ throughout the RAS and KRAS steps (up to iteration number 1500). Values have again been normalised to their respective maxima.

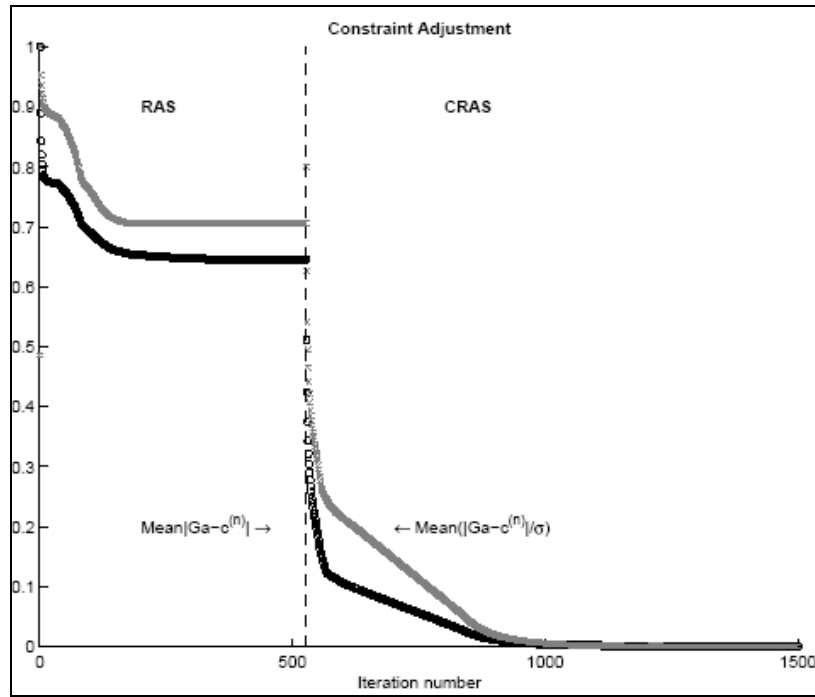


Fig. 6: Evolution of $\sum_i |\mathbf{Ga} - \mathbf{c}^{(n)}|_i / N_C$ and $\sum_i |\mathbf{Ga} - \mathbf{c}^{(n)}|_i / ([\sigma_i + \delta] N_C)$ (normalised to their respective maxima) throughout RAS and the first two thirds of KRAS during the balancing of the Australian MRIO system.

Both balancing steps are characterised by a rapid initial adjusting phase (first third of the full step) followed by a slow adjustment towards the final values which extends over the remaining iterations. The evolution of the normalised average change in constraint values $\sum_i |\mathbf{c}^{(n)} - \mathbf{c}^{(0)}|_i / N_C$ during the first 1500 KRAS iterations is shown in Figure 7. The initial RAS iterations took up to 5 min of run time each due to the time spent on the Newtown algorithm looking for a solution of Eq. 16 when far from the initial estimate. However, this time shortened as the run advanced and the average run time for a KRAS iteration was about 5 sec.

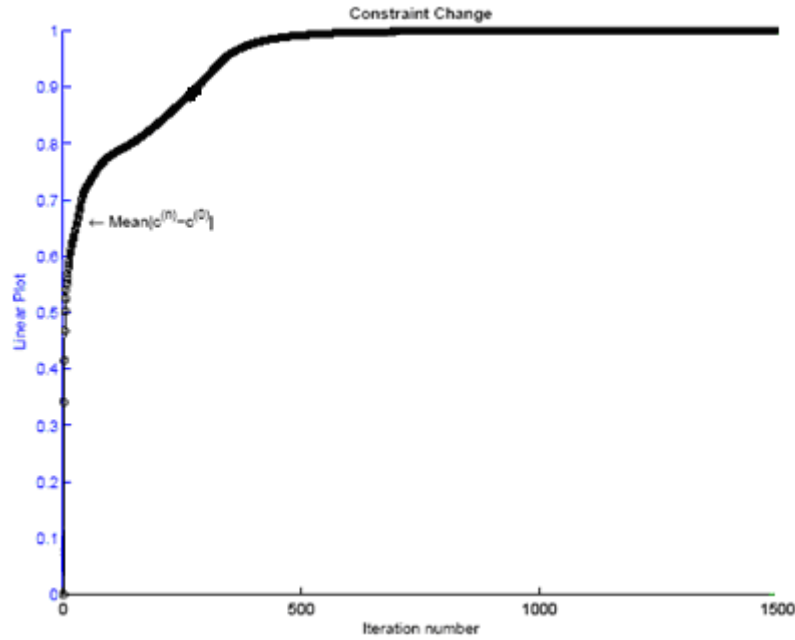


Fig. 7: Evolution of $\sum_i |\mathbf{c}^{(n)} - \mathbf{c}^{(0)}|_i / N_C$ (linear scale) normalised to the maxima during the balancing of the Australian MRIO system.

5 Conclusions

We have developed a RAS variant, generalised iterative scaling method (KRAS) that is able to balance and reconcile input-output tables and SAMs under conflicting external information and inconsistent constraints. In addition, KRAS fulfils all requirements of earlier RAS variants such as GRAS, and of constrained optimisation techniques:

- a) constraints on arbitrarily sized and shaped subsets of matrix elements;
- b) consideration of the reliability of the initial estimate and the external constraints;
- c) ability to handle negative values and to preserve the sign of matrix elements.

Applying KRAS in four case studies, we find that, as with constrained optimisation, KRAS is able to find a compromise solution between inconsistent constraints; this feature does not exist in conventional RAS variants such as GRAS.

KRAS can constitute a major advance for the practice of balancing input-output tables and Social Accounting Matrices, in that it removes the necessity of manually tracing inconsistencies in external information. In contrast to constrained optimisation, this quality does not come at the expense of substantial programming requirements, and long run times.

While KRAS appears to be able to solve these problems, it should not be forgotten that collecting data requires skilled manual input. As Barker *et al.* 1984 (p. 475) emphasise, automated balancing of national accounts “is not a replacement for knowledge of the data and its sources but an enhancement of it, allowing us to produce fully balanced accounts with the adjustments reflecting the quality of the data.”

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PART II

INTRODUCTION

In this section, the model developed in Part I is applied to study structural changes in the economy and the implications for material flow impacts and greenhouse gas emissions.

This section contains six chapters.

The first chapter, *Principal Methodological Approaches to Studying Sustainable Consumption*:

Scenario Analysis, Ecological Footprints and Structural Decomposition Analysis was a contribution to the *Handbook on Input-output economics for Industrial Ecology*. The paper is designed as an introductory to various methods of studying change in environmental impact.

Wood, R. and M. Lenzen. 2009. Principal methodological approaches to studying sustainable consumption: Scenario Analysis, Ecological Footprints and Structural Decomposition Analysis. In: Suh S, *Handbook on Input-Output Economics for Industrial Ecology*, Elsevier.

Concentration is then applied to focusing on inter-relationships within the economic system, and the effect on material use and greenhouse gas emissions. Two chapters are presented applying the concept of structural decomposition analysis, introduced in Chapter 1, to these two issues. Chapter 2 looks at material flows in Australia, it has been submitted to the *Journal of Industrial Ecology*:

Wood, R., M. Lenzen, and B. Foran. 2009. A material history of Australia: Evolution of material intensity and drivers of change. *Journal of Industrial Ecology*: In press.

The next chapter applies a similar methodology to greenhouse gas emissions. Previous work (outside of the candidature) was undertaken on this subject. The work presented here is a completely new and updated analysis, utilising the full model developed in Part I, incorporating a complete analysis of all greenhouse gas emissions (i.e. including non fuel combustion emissions), and incorporating methodological advancements (see Chapter 4).

Wood, R. 2009. Structural decomposition analysis of Australia's greenhouse gas emissions. *Energy Policy*: Accepted.

Applying the methodologies to the detailed model developed in Part I raised issues with previously accepted methods in decomposition studies. The fourth chapter investigates the issue of zero values in the logarithmic mean divisia index, a particular, and most widely used,

index in decomposition studies. The results of this work fed into the previous two papers, but due to its more technical nature, it is included subsequent to these papers.

Wood, R. and M. Lenzen. 2006. Zero-value problems of the logarithmic mean divisia index decomposition method. *Energy Policy* 34(12): 1326-1331.

The application of structural decomposition analysis raised further inquiry about interpretation and usefulness of results in policy formation. The Fifth paper in this section develops a methodology to apply structural decomposition analysis in a life-cycle context. This paper combines two well established techniques to give insight into key structural changes in the economy, and within production processes. An example application is undertaken for greenhouse gas emissions in Australia.

Wood, R. and M. Lenzen. 2009. Structural Path Decomposition. *Energy Economics*: 31(3): 335-341.

The final chapter in this section draws from the previous studies in investigating the evolution of the Australian economy by applying notions of complexity to the changing interrelationships driving material use and greenhouse gas emissions. The broad body of work covered in this thesis is drawn together for interpretation of economic-environmental evolution.

Wood, R. and M. Lenzen. 2009. Aggregate measures of complex economic structure and evolution – a review and case study. *Journal of Industrial Ecology*: 13(2): 264-283.

Conclusions are then drawn.

PART II - STATEMENT OF CONTRIBUTION

Paper 1. *Principal Methodological Approaches to Studying Sustainable Consumption: Scenario Analysis, Ecological Footprints and Structural Decomposition Analysis*

The candidate is responsible for the structure of the paper, the introduction, the section on structural decomposition analysis and wrote the conclusion in conjunction with the second author.

Paper 2 *A material history of Australia: Evolution of material intensity and drivers of change.*

The candidate is responsible for the structure of the paper, the introduction, the methodology, the data sources, the results, and the conclusion.

Paper 3 *Structural decomposition analysis of Australia's greenhouse gas emissions.*

The candidate is wholly responsible for this paper.

Paper 4 *Zero-value problems of the logarithmic mean divisia index decomposition method.*

The candidate is responsible for the conceptualization of this problem. The methodology was written in conjunction with the second author. The candidate is responsible for the empirical example, the results and the conclusions.

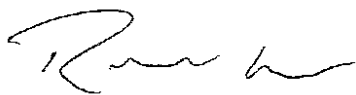
Paper 5 *Structural Path Decomposition.*

The candidate is responsible for the majority of this paper. The second author contributed to the conceptualization of the problem and in the initial mathematical derivation.

Paper 6 *Aggregate measures of complex economic structure and evolution – a review and case study.*

The candidate is responsible for all of this paper excepting the literature review.

Signed:



Richard Wood



Manfred Lenzen (principal co-author)

PRINCIPAL METHODOLOGICAL APPROACHES TO STUDYING SUSTAINABLE CONSUMPTION: SCENARIO ANALYSIS, ECOLOGICAL FOOTPRINTS AND STRUCTURAL DECOMPOSITION ANALYSIS

Richard Wood, Manfred Lenzen

Wood, R. and M. Lenzen. 2009. Principal methodological approaches to studying sustainable consumption: Scenario Analysis, Ecological Footprints and Structural Decomposition Analysis. In: Suh S, *Handbook on Input-Output Economics for Industrial Ecology*, Elsevier.

Abstract

The environmental impact of a person, whether measured in terms of average energy consumption, specific CO₂ emissions, or a person's occupation of ecological space, has received sustained interest at least since the 1970s. The need for quantifying drivers, key impact segments and leverage points from a consumption perspective lead to the formulation of various indicator concepts and analysis techniques, amongst which are scenario analysis, the ecological footprint, and structural decomposition analysis. Since sustainable consumption has become a key interest of environmental policy makers, not at least through the 2002 World Summit on Sustainable Development, there is an increased interest in such investigations. This paper reviews the quantification of the environmental impacts of consumption and sketches different options for analysis.

Keywords: sustainable consumption, scenario analysis, ecological footprint, structural decomposition analysis

1. Introduction

The environmental impact of a person, whether measured in terms of average energy consumption, specific CO₂ emissions, or a person's occupation of ecological space, has received sustained interest at least since the 1970s. The need for quantifying drivers, key impact segments and leverage points from a consumption perspective lead to the formulation of various indicator concepts and analysis techniques, amongst which are scenario analysis, the ecological footprint, and structural decomposition analysis. Since *sustainable consumption* has become a key interest of environmental policy makers, not at least through the 2002 World Summit on Sustainable Development, there is an increased interest in such investigations.

Rather than providing a broad literature review of the issue, the purpose of this paper is to concurrently demonstrate example applications of the three input-output based methods mentioned above - scenario analysis, the ecological footprint, and structural decomposition analysis - and by so doing, provide a means for comparison and critique. Whilst all three analysis techniques are concerned with the overall theme of the study of sustainable consumption, they each provide a different focus for assessment and interpretation, and are thus suited to diverging purposes and research questions.

The distinguishing facet of *scenario analysis* is that it is principally used in assessment by describing hypothetical states – generally about the future, and often for purposes relating to identifying bottlenecks, potential problems and opportunities, and for analysing comparative benefits/losses between hypothetical states, or between hypothetical and present states.

In contrast, the *ecological footprint* is principally used to analyse the effect of current (or past) consumption. By converting all impacts into a single index (land area), the ecological footprint also allows comparison to a finite upper limit of available land space/biocapacity, and provides a more easily interpretable result for use in education and communication.

Structural decomposition analysis (SDA), however, extends beyond the basic static model in order to reveal the driving forces for changes in the economy over time. For this purpose, decomposition techniques can be applied “by means of a set of comparative static changes in key parameters” (Rose and Miernyk 1989). This provides results on progress towards/away from goals of sustainable consumption, but due to its greater complexity, often only considers one impact indicator type at a time.

The main aim of the applications demonstrated in this paper is to study some of the effects of consumption in terms of the goal of ‘sustainable consumption’. In order to illustrate the different foci of the three methods, we present an Australian case study for each. Since the research questions addressed by each method differ to a certain degree, it is difficult to find a unifying ‘theme’ to compare them. Firstly, the scenario analysis investigates the effects of a current and hypothetical diet. The ecological footprint investigates the land and energy use of the current Australian consumption pattern, and the structural decomposition analysis focuses

on contributing factors to greenhouse gas emissions induced by the consumption of the last 30 years.

Having sketched the principal commonalities and distinctions in this section, we describe the three techniques separately in Section 2, with an introduction and critique, and the methodology for an application of each technique. The results of these applications are presented in Section 3, along with brief analysis, once again methodologically separately. Finally, in Section 4, the three techniques are once again merged, in being compared and critiqued against each other.

2. Methodology

2.1.1. Scenario analysis

Both consumers and policy-makers often face conflicts between economic and environmental issues arising from apparent compromises between, for example, the environmental pollution from industries and the employment and revenue those industries provide. For example, potential employment reductions are often used as poorly substantiated threats in public debates on greenhouse gas emissions. A reliable scenario analysis methodology that can assess competing factors such as these is desirable for making the inherent compromises more transparent, and thus better informing decision-making.

Historically, attempts to employ scenario analysis were stimulated by the oil price shocks of the early 1970s to resolve conflicting energy and employment issues of consumption. Numerous energy analyses of specific tasks were carried out¹, using process or input-output analysis (Chapman 1974), or hybrid methods (Bullard et al. 1978). Based on theoretical work by Leontief and Ford (1970), Hannon and co-workers extended input-output-based energy analysis in order to investigate so-called *dollar-energy-labour (DEL) impacts* of personal consumption decisions. These DEL impacts characterise energy and labour inputs, which are, directly or indirectly, required to produce the output that is needed to meet consumer demand. For example, Bezdek and Hannon (1974) examined the effects of re-investing government spending on road infrastructure into six other types of governmental programs such as health insurance, education, water and waste treatment, tax relief, and law enforcement. One of the alternatives was to divert the funding to railway and mass-transit spending which was considered a direct substitute for the road construction. Five of six alternative spending options were associated with reduced energy use, while all alternatives required an increase

¹ (Herendeen 1973); (Berry and Fels 1973); (Chapman et al. 1974); (Bullard and Herendeen 1975a); (Rotty et al. 1975); (Chapman 1975); (Pilati 1976); (Herendeen and Tanaka 1976); (Perry et al. 1977); (Herendeen 1978).

in employment. In these and other studies², DEL impacts were calculated by multiplying monetary expenditure data with *energy and labour intensities*, which were obtained from input-output analysis. While the studies mentioned above were mainly motivated by the substantial increase in energy prices at that time, recent research is more concerned about the environmental consequences of energy use, such as climate change. Laitner et al. (1998) use input-output-based multipliers to determine the emissions, employment, and other macroeconomic benefits of innovation-led investments in energy-efficient and low-carbon technologies. They conclude that such re-directing of investments provides economic efficiency and productivity gains directly, and that the associated “shift in spending pattern away from fossil-fuel-based energy supply has an inherently employment-enhancing effect, since these traditional energy supply sectors are substantially less labour-intensive than the rest of the US economy, and one of them (petroleum) is heavily import-dependent”. Another recent study of future scenarios was completed by Duchin and Lange (1994). They evaluated the feasibility of the recommendations of the Brundtland Report, and found them unrealistic. Using an input-output model of the world economy, closed for international trade, they found that the considerable savings from technological improvements were more than offset by both population increases, and increases in standards of living, particularly in the developing world.

As an example application, in this work, input-output based multipliers are applied to examine the impacts of alternative dietary options for the Australian population in terms of energy and employment, and in addition, greenhouse gas emissions, income, imports and taxes. A current and an environmentally motivated diet are investigated. Both alternatives were chosen to satisfy the perfect-substitutes criterion.

Input-output analysis shall be applied here to the following exogenous *production factors*: primary energy (E , in units of mega joules, MJ), greenhouse gas emissions (G , in units of kilograms of carbon dioxide equivalent, kg CO₂-e), employment (or “labour”, L , in units of full-time equivalent employment-hours (emp-h), as well as income W (wages and salaries), indirect taxes less subsidies T , and imports M (all in units of Australian dollars, A\$). For a detailed description of the respective data sources, closure type, and treatment of margins and imports see Lenzen (2001), and Lenzen and Dey(2002).

In order to evaluate the two alternative diets, a set of production factor multipliers (Hafkamp and Nijkamp 1983) are derived from the basic input-output identity and are applied to data on food consumption baskets $\{c_k\}$ of $k=1, \dots, N$ commodities expressed, in this case, in the Australian input-output classification. The overall factor embodiment C_f in terms production factor f (i.e. the overall energy, labour, etc) contained in each option is $C_f = \sum_k c_k f_k$.

² (Hannon 1972; Folk 1972; Herendeen and Sebald 1973; Hannon and Puleo 1974; Bullard and Herendeen 1975b; Herendeen and Sebald 1975; Hannon et al. 1975; Hannon et al. 1978; Hannon et al. 1980)

Changes in consumption, whether initiated by voluntary consumer action or as a result of government policy, have associated changes in household monetary expenditure. For example, it can be assumed that money saved in one area will be spent elsewhere. The opposite holds for increased household costs, where money has to be withdrawn from some purposes. These spending increases or cuts will have further impacts in addition to those of the initial change. We refer to these impacts as rebounds. Within consumer rebounds, we examine three different household income quintiles³ (lowest 20%, middle 20%, and highest 20%). This differentiation is useful because household consumption patterns vary considerably with household income. Therefore, different rebound effects should be expected depending on whether the impact analysis is applied to a rich or a poor household.

Cuts or savings are most likely to affect only the marginal consumption that is items bought after all necessities are satisfied. Therefore, rebounds have to be calculated using factor multipliers of household expenditures at the upper expenditure limit. In a first step, production factor budgets $B_f(H) = \sum_k y_k(H) f_k$ were calculated using expenditure data $y_k(H)$ of households with different income H .⁴ Second, these factor budgets were regressed as a function of per-capita household expenditure y

$$B_f(y) = r_f y^{\eta_f}, \quad (1)$$

where r_f is a regression constant and

$$\eta_f = (\partial B_f / \partial y) / (B_f / y) \quad (2)$$

is called the elasticity of production factor f with regard to per-capita household expenditure y .⁵ It was then assumed that rebounds occur at marginal factor multipliers

$$\partial B_f / \partial y = \eta_f B_f(y) / y = \eta_f r_f y^{\eta_f - 1}. \quad (3)$$

When using static input-output analysis, scenario analyses are associated with certain limitations: it is impossible to precisely quantify changes in production factors, which would occur under real-economy shifts in consumption. The difference between production factor embodiments of two alternative diets calculated as above would be equal to real production factor changes caused by the corresponding shift from the current to the alternative diet only if during that shift (1) all commodity prices stayed constant, (2) there were no changes in technology, no input substitution, and hence no change in production factor intensities, (3) there were no constraints on production factors, such as a rigid labour supply, and (4) production costs were linear functions of production output (as inherently assumed in input-

³ Household incomes are 9880 A\$ for the lowest quintile, 11315 A\$ for the middle quintile, and 18813 A\$ for the highest quintile (Australian Bureau of Statistics 1995a).

⁴ Based on the 1993-94 Australian Household Expenditure Survey (Australian Bureau of Statistics 1995a).

⁵ Compare (Wier et al. 2001).

output analysis). The latter condition applies to production situations where there are no economies of scale, and where average costs equal variable costs, that is, fixed costs are zero. Since none of the above conditions are satisfied in practice, the difference in production factor embodiments, calculated for the alternative diets, is only indicative of the effect that real-economy demand or supply shifts would have on production factors (compare (Laitner et al. 1998), p. 431). Nevertheless, taken as a proxy, the scenario differences illustrate compromises between various impacts of alternative consumer choices. The results demonstrate that, at least in principle, shifts between these choices may for example lead to reductions in energy use whilst increasing employment. Similarly, other comparisons may suggest the possibility of reducing greenhouse gas emissions or imports.

2.1.2. The Ecological Footprint

The ecological footprint was originally conceived as a simple and elegant method for comparing the sustainability of resource use among different populations (Rees 1992). The consumption of these populations is converted into a single index: the land area that would be needed to sustain that population indefinitely. This area is then compared to the actual area of productive land that the given population inhabits, and the difference is calculated between available and required land. Ecological footprints calculated according to the original method have become important educational tools in highlighting the unsustainability of global consumption (Costanza 2000). It was also proposed that ecological footprints could be used for policy design and planning (Wackernagel et al. 1997; Wackernagel and Silverstein 2000).

Since the formulation of the ecological footprint, however, a number of researchers have criticised the method as originally proposed.⁶ The criticisms largely refer to the oversimplification in ecological footprints of the complex task of measuring sustainability of consumption, or comparisons among populations being misleading⁷. In addition, the aggregated form of the final ecological footprint makes it difficult to understand the specific reasons for the unsustainability of the consumption of a given population, and to formulate appropriate policy responses. In response to the problems highlighted, the concept has undergone significant modification (Bicknell et al. 1998), (Simpson et al. 2000), (Lenzen and Murray 2001). Development of, and debate about the method is continuing. The applications of input-output methodology to the calculation of ecological footprints are growing (Bicknell et al. 1998; Ferng 2001; Bagliani et al. 2002; Ferng 2002; Hubacek and Giljum 2003; Wiedmann et al. 2006; Wood and Garnett 2009). Debate about the method is ongoing (Lenzen et al. 2007; Venetoulis and Talberth 2007; Wiedmann and Lenzen 2007).

⁶ (Levett 1998); (van den Bergh and Verbruggen 1999); (Ayres 2000); (Moffatt 2000); (Opschoor 2000); (Rapport 2000); (van Kooten and Bulte 2000).

⁷ For example, as a result of calculations by (Wackernagel 1997), some countries with extremely high land clearing rates (Australia, Brazil, Indonesia, Malaysia) exhibit a positive balance between available and required land, thus suggesting that these populations are using their land at least sustainably.

The original ecological footprint is defined as a land area that would be needed to meet the consumption of a population and to absorb all their waste (Wackernagel and Rees 1995). Consumption is divided into 5 categories: food, housing, transportation, consumer goods, and services. Land is divided into 8 categories: energy land degraded or built land, gardens, crop land, pastures and managed forests, and 'land of limited availability', considered to be untouched forests and 'non-productive areas'. The ecological footprint is calculated by compiling a matrix in which a land area is allocated to each consumption category. In order to calculate the per-capita ecological footprint, all land areas are added up, and then divided by the population, giving a result in 'global hectares' per capita.

In the original ecological footprint method, the land areas included were mainly those directly required by households, and those required by the producers of consumer items. Beyond this boundary however, these producers draw on numerous input items themselves, and the producers of these inputs also require land. Bicknell et al. (1998) were the first to apply a boundary-free input-output-based ecological footprint method in their study of the New Zealand population. Using Australian data, input-output-based ecological footprints have been calculated for more than 100 industry sectors and product groups, for states, local areas and cities, and for companies and households (Lenzen and Murray 2001). Another advantage of using input-output analysis is that imports and exports can be easily accounted for.

In the original method, the areas of forest, pasture and crop land do not represent local land, but areas that would be needed to support the consumption of the population, if local farming and forestry was conducted at 'world average productivity'.⁸ Proceeding as such makes it easy to compare ecological footprints of different countries or populations (Wackernagel et al. 1999). However, the loss in detail through the conversion to world-average productivity precludes formulating regional policies, because the latter always involve region-specific economic, political, technological, environmental and climatic aspects (Lenzen and Murray 2001).

Furthermore, ecological footprints addressing the question of human demand on global productivity does not reflect the intensity of human-induced changes to land (compare (van den Bergh and Verbruggen 1999). Intensive land use patterns such as cropping – no matter how productive – drastically alter ecosystems, whereas land used, for example, for non-intensive grazing or native forestry may be only slightly altered. This land condition, or the deviation from a pristine state is not captured in the productivity-based approach. Assuming equal productivity, sustainable growing of organic wheat (and for example, not causing salinity), would attract the same ecological footprint in the original method as wheat grown conventionally and causing salinity.

⁸ In order to express an ecological footprint at world-average productivity, the consumption of the reference population is assessed in weight units, which are subsequently translated into units of area by multiplication with local yield factors. Different land types (pasture, crop land, forest) are then converted into land at world-average biomass productivity by multiplication with an equivalence factor (Wackernagel et al. 2002).

For this reason, (Lenzen and Murray 2001) have argued that a better approach is to base the ecological footprint on land condition, using actual areas of land used by the respective population. They derive a list of disturbance weightings (see Tab. 1) for different types of land use ranging from 0 (undisturbed or slightly disturbed) to 1 (completely disturbed).⁹ Similarly, the emissions component of an ecological footprint can be determined as the projected land disturbance due to climate change caused by the greenhouse gas emissions. To obtain a disturbance-based ecological footprint, each area of land is multiplied by its land condition factor. For extensions and further details see (Lenzen and Murray 2001).

Land use type	Land condition
CONSUMED	1.0
Built	
DEGRADED	0.8
Degraded pasture or crop land	
Mined land	
REPLACED	0.6
Cleared pasture and crop land	
Non-native plantations	
SIGNIFICANTLY DISTURBED	0.4
Thinned pasture	
Urban parks and gardens	
Native plantations	
PARTIALLY DISTURBED	0.2
Partially disturbed grazing land	
SLIGHTLY DISTURBED	0.0
Reserves and unused Crown land	
Slightly disturbed grazing land	

Table 1. Basic weighting factors for land use patterns, reflecting land condition in Australia (after (Lenzen and Murray 2001)).

⁹ The measurement of land condition forms a field of investigation in itself, and a number of approaches have been made in studies incorporating land use into life-cycle assessment. Amongst others (see (Lindeijer 2000b, 2000a), ecosystem biodiversity and bioproductivity measures (Swan and Pettersson 1998) as well as species diversity of a particular group of plants (Köllner 2000); (van Dobben et al. 1998) have been proposed as suitable indicators. For Australia, the degree of landcover disturbance may be a useful proxy for land condition at a very broad scale, as it indicates processes such as biotic erosion that lead to land degradation. A comprehensive survey of landcover disturbance over the Australian continent has been conducted by (Graetz et al. 1995) using satellite imagery to compare the current coverage of vegetation with the 'natural' state, taken to be that of 1788.

In this work, an example application is given for the population of Australia, based on consumption data of the national accounts. National Accounts provide a systematic annual summary of national economic activity. They map key economic flows: production, income, consumption, investment, imports and exports. Currently, the most important measure of overall economic performance is the Gross Domestic Product, or GDP. One way of measuring GDP is the expenditure approach, in which GDP is described as

$$\text{GDP} + M = \text{GNE} + X = \text{PFC} + \text{GFC} + \text{GFCE} + \text{CS} + X, \quad (4)$$

where M are imports, GNE Gross National Expenditure, X exports, PFC private final consumption, GFC government final consumption, GFCE gross fixed capital expenditure, and CS are changes in stocks.

These tables are of course expressed in monetary terms, but after calculating ecological footprint multipliers ef_k for commodities k in the usual way (Lenzen and Murray 2001), the financial National Accounts can be converted into a National Ecological Footprint Account with values EF_k in area units (Tab. 4), according to $EF_k = y_k ef_k$.

2.1.3. Structural Decomposition Analysis (SDA)

Two strains of time series decomposition can be found in the literature: Index Decomposition Analysis (IDA) and SDA. Both are comparative static methods (i.e. comparing differences between two sets of data) that use either aggregate sector-level or country-level data (IDA) or input-output data (SDA). IDA is generally less data-intensive, but also less detailed since indirect inter-industry effects cannot be resolved. Even though IDA and SDA have developed independently, their techniques and indices can be mutually applied (Hoekstra and van den Bergh 2003); (Liu and Ang 2003). IDA and SDA studies have been carried out on a wide range of variables, such as energy, CO₂ emissions, labour, productivity, imports, and many others. A number of comprehensive literature reviews have been completed¹⁰.

The principle purpose of SDA (or IDA) is to break down the total changes in an indicator of interest (in the example of this study, it will be greenhouse gas emissions) into a number of independent contributing factors – for example, an SDA is designed to express changes in emissions by changes in industrial structure, energy efficiency, consumption levels, or other factors.

¹⁰ For example (Ang 1995b), (Ang 1995a)b, p. 41), (Hoekstra and van den Bergh 2002), Sec. 6), and (Liu and Ang 2003). (Rose and Casler 1996), summary in (Rose 1999) discuss additional issues such as trade changes, price changes, model closure, hybrid units, projection and forecasting, and the relationship to neoclassical production functions. Ang (1999) deals with method selection, the zero-values problem, periods versus years, and sector aggregation. Ang (2000) gives a historical overview, lists 124 studies, and classifies these according to application area, indicator type and decomposition scheme. (Ang 2004) reviews selection criteria and application areas, and attempts a taxonomy of methods.

In this work we present a Structural Decomposition Analysis of greenhouse gas emissions for 11 factors - fuel mix, energy intensity, industrial interdependence, final demand commodity mix, final demand destination, final demand level, population, export commodity mix, export level, residential fuel mix and residential energy consumption. This is done for a time series combining data on greenhouse gas emissions from energy use, and input-output tables of the Australian economy from 1968 to 1997. Constant price tables were required for the SDA, and these were generated using price indices and constant price data sets (Australian Bureau of Statistics 2007). See Part I for more details.

2.1.4. Factorisation

As for any given time t , total industrial greenhouse gas emissions C_{tot} can be represented by the fundamental input-output relation $C_{tot} = f(\mathbf{I} - \mathbf{A})^{-1}Y$, which can subsequently be decomposed into a range of determinants:¹¹

$$C_{tot} = (\mathbf{cF})\mathbf{E}(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{uv}YP + \mathbf{bZ}) + \mathbf{c}_{res}\mathbf{F}_{res}E_{res}P \quad (5)$$

with the determinants being:

emission coefficients c_{ij} ($f \times n$): emission of greenhouse gases in CO₂-equivalents resulting from the usage of fuels $i=1, \dots, f$ in industries $j=1, \dots, n$ per unit of total usage of fuels i ; $\mathbf{c}$ is a weighted sum over three gases with different global warming potentials w : $\mathbf{c} = \mathbf{c}_{CO_2} + w_1 \mathbf{c}_{CH_4} + w_2 \mathbf{c}_{N_2O}$; there is a variation of c_{ij} across industries because of equipment-specific emission factors for non-CO₂ gases, and industry-specific equipment mix;

fuel mix F_{ij} ($f \times n$): usage of fuels i in industries j per unit of total usage of energy in industries j ;

energy intensity E_{ij} ($n \times n$): diagonalised vector of total energy usage per unit of gross output of industry sectors j ;

industrial interdependence $\{(\mathbf{I} - \mathbf{A})^{-1}\}_{jk}$ ($n \times n$): gross output required from industry sectors j by industry sectors k per unit of final demand from industry sectors k ;

final demand commodity mix u_{kl} ($n \times d$): final demand of commodities from industry sectors k in final demand destinations¹² $l=1, \dots, d$ per unit of total final demand from industry sectors k ; this matrix excludes exports;

¹¹ In this work we decompose absolute levels, because we only deal with one country, Australia. In cross-country comparisons, decompositions of relative changes, intensities, elasticities or other coefficients may be more appropriate (Ang and Lee 1996). Similarly, in cases where level effects overly dominate other structural effects, a decomposition of intensities or elasticities yields better information (Ang 1994), for an example see (Skolka 1977).

final demand destination mix v_l ($d \times 1$): final demand of commodities from destinations l per unit of total final demand;

final demand level Y (1×1): total final demand (all commodities, industry sectors destinations), per capita;

population P (1×1): Australian population;

export commodity mix b_l ($n \times 1$): export of commodities by industry sectors l per unit of total exports;

export level Z (1×1): total exports (all commodities).

Similarly, residential emissions are described by $c_{res}F_{res}E_{res}$, where c_{res} ($1 \times f$) contains the emission coefficients for residential fuels, F_{res} ($f \times 1$) the residential fuel mix, and E_{res} (scalar) the per-capita residential energy consumption. Note that changes in economic structure, that is the input-output coefficients can be a result of technological change as well as changes in the product mix within sector aggregations (Bezdek and Dunham 1976); (Afrasiabi and Casler 1991); (Dietzenbacher and Los 1997). These effects can however only be distinguished if corresponding disaggregated information is available.

In (temporal¹³) SDA, we are interested in the changes over time, hence the analysis employed is really a derivative of equation 5 with respect to time, solved for the time period bordered

by the years with data available. i.e. $\Delta C_{tot} = \int_{C_0}^{C_1} dC$. The complication in this calculation, is

that the path the integral takes is not known (i.e. whether all the changes, in, say, fuel mix, occur at the start of the period, the end of the period, gradually in between, or along some other random path - in application, this refers to whether the comparison in changes is made between the terminal value, the initial value, or other values in between). As a result, a large array of different methods of decomposition according to different integral paths have been published. Most simply, constant values can be assumed along the integral path. Using base weights (initial values) is known as the Laspeyres method, using mid-point weight (average values) is the Marshall-Edgeworth, and using terminal weights (final values) is the Paasche method. The Laspeyres and Paasche decompositions, however, are never exact, leaving residuals due to joint terms of the derivative. The Marshall-Edgeworth decomposition is only exact for $n = 2$, but produces a residual for $n > 2$. Zero residuals however are a desirable

¹² In the case of Australia, but also for many other countries, final demand destinations are 1) private final consumption, 2) government final consumption, 3) gross fixed capital formation, 4) changes in inventories, and 5) exports.

¹³ Rather than temporal, a structural comparison is also possible across countries or regions. However, this approach is difficult to implement for SDA because of the requirement of aligning input-output table classifications (Zhang and Ang 2001); (De Nooij et al. 2003); (Alcántara and Duarte 2004).

property of decompositions, in addition to linear homogeneity (invariance under determinants scaling, see (Hoekstra and van den Bergh 2002), Note 6), invariance under time reversal, and zero-value robustness (see (Hoekstra and van den Bergh 2003), p. 6-7 and Tab. 2, and (Ang 2000), Tab. 6).

An *exact* decomposition results when the integral follows a path where all changes in the determinants are proportional to each other. A number of more complex decomposition methodologies exist: the Mean Rate-of-Change method, (Chung and Rhee 2001); the Logarithmic Mean Divisia method, (Ang and Choi 1997; Ang and Liu 2001; Ang et al. 1998); the $n!$ -comminatoric integral path method, see (Dietzenbacher and Los 1997, 1998; Albrecht et al. 2002).

2.1.5. Application to Australian greenhouse gas emissions

In this work we will give an example of applying the Marshall-Edgeworth SDA variant. Carrying out the decomposition of the factorisation of Australian greenhouse gas emissions yields

$$\begin{aligned}
\Delta C_{tot} = & \Delta(\mathbf{cF})\mathbf{E}(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{uvYP} + \mathbf{bZ}) \\
& + \mathbf{cF}\Delta\mathbf{E}(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{uvYP} + \mathbf{bZ}) \\
& + \mathbf{cFE}\Delta(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{uvYP} + \mathbf{bZ}) \\
& + \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\Delta\mathbf{uvYP} \\
& + \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{u}\Delta\mathbf{vYP} \\
& + \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{uv}\Delta\mathbf{YP} \\
& + \Delta(\mathbf{c}_{res}\mathbf{F}_{res})\mathbf{E}_{res}P \\
& + \mathbf{c}_{res}\mathbf{F}_{res}\Delta\mathbf{E}_{res}P \\
& + \left((\mathbf{cF})\mathbf{E}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{uvY} + \mathbf{c}_{res}\mathbf{F}_{res}\mathbf{E}_{res} \right) \Delta P \\
& + \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\Delta\mathbf{bz} \\
& + \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}\Delta\mathbf{Z} \\
& + R
\end{aligned} \tag{6}$$

where R is the residual term due to joint effects in higher order terms. The interpretation of residuals is difficult, but fortunately their values have been found to be small. In order to simplify the presentation of the results, the components of Eq. 16 are given simplified notations (Table 2).

Finally, the SDA model of course shares all shortcomings of the static input-output framework, that is, it does not allow for dynamic forecasts and is also limited by the assumption of linearity (ignores economies of scale), not accounting for constraints on production factors (such as limited capital and labour), the assumption of constancy of commodity prices, and the assumption of fixed input structure in each industry.

Parameter and simplified notation	Description
$\Delta \mathbf{cF} = \Delta(\mathbf{cF})\mathbf{E}(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{uvYP} + \mathbf{bZ})$	ΔC due to changes in relative fuel mix in industrial production.
$\Delta \mathbf{E} = (\mathbf{cF}\Delta \mathbf{E}\ell)(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{uvYP} + \mathbf{bZ})$	ΔC due to changes in the absolute level of industrial consumption of energy.
$\Delta \mathbf{L} = \mathbf{cFE}\Delta(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{uvYP} + \mathbf{bZ})$	ΔC due to the change in the industrial structure.
$\Delta \mathbf{u} = \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\Delta \mathbf{uvYP}$	ΔC due to the change in the mix of final demand of goods and services from the industries.
$\Delta \mathbf{v} = \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{u}\Delta \mathbf{vYP}$	ΔC due to the relative change in destination of final demand.
$\Delta Y = \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{uv}\Delta YP$	ΔC due to the change in the per-capita final demand.
$\Delta \mathbf{cF}_{res} = \Delta(\mathbf{c}_{res}\mathbf{F}_{res})\mathbf{E}_{res}P$	ΔC due to the change in the residential fuel mix.
$\Delta \mathbf{E}_{res} = (\mathbf{c}_{res}\mathbf{F}_{res})\Delta \mathbf{E}_{res}P$	ΔC due to the change in total residential energy consumption.
$\Delta P = ((\mathbf{cF}\Delta \mathbf{E}\ell)(\mathbf{I} - \mathbf{A})^{-1}\mathbf{uvY} + (\mathbf{c}_{res}\mathbf{F}_{res})\mathbf{E}_{res})\Delta P$	ΔC due to the change in population.
$\Delta \mathbf{b} = \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\Delta \mathbf{bZ}$	ΔC due to the change in the mix of exports.
$\Delta \mathbf{Z} = \mathbf{cFE}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}\Delta \mathbf{Z}$	ΔC due to the change in the total level of exports.
ΔC_{tot}	Total change in greenhouse gas emissions (summation of above terms)

Table 2 Description of structural decomposition components; ΔC = Change in greenhouse gas emissions.

3. Australian case studies

In the following, the results of case studies are presented for the methods and applications introduced in Section 2. In order to provide some common ground, we let all studies deal with the production and consumption of food in the Australian economy.

As the underlying input-output data set, the 1994-95 Australian input-output tables (ABS 1999b) were used. These are the first set of tables to be based on the System of National Accounts 1993 (SNA93). Within SNA93, National Income, National Expenditure and National Product are now benchmarked on input-output tables by employing the commodity flow method, which is an input-output approach for compiling National Accounts (Barbetti and De Silva 1998). The characteristic feature of the commodity flow method is that it balances total supply and use for each commodity while simultaneously balancing total production and input for each industry. In practice, the reconciliation of the three GDP estimates based on income, expenditure and production is achieved by an iterative confrontation and balancing process involving approximately 1000 commodities and 107 industries. As a result of this approach, previously common discrepancies within the National Accounts and between input-output tables and the National Accounts no longer occur. Furthermore, an Economic Activity Survey incorporating taxation statistics has been specifically designed by the ABS (Australian Bureau of Statistics 1999) to support the input-

output approach from 1994-95 onwards by expanding and detailing the industry data collection, and by facilitating the production of annual input-output tables (previously triennial).

3.1.1. Scenario analysis

As an example for the first approach illustrated in this work, we present a scenario analysis of the typical Australian diet. The provision of food to Australians in 1994 entailed the consumption of 280 PJ of primary energy (7% of the Australian total) and the emission of 85 Mt CO₂-e of greenhouse gases (15% of the Australian total; see (Lenzen 1998). However, over the last century, food intake has changed markedly, with cereal and vegetable intake decreasing in favour of a marked increase in consumption of animal products and refined sugar (Department of Health 1984). As a result, the nutritional energy intake in 1993 was 40% in excess of the recommended dietary intake (RDI), while the protein intake was 118% in excess of the RDI (Australian Bureau of Statistics 1995b). These changes in the provision of food have had important implications for environmental factors as diverse as land and water use, greenhouse gas emissions, soil erosion and desertification, logging of rainforests, and pesticide use (Pimentel and Hall 1989); (EarthSave Foundation 1992). The problem of increasing energy requirements for food production was already recognised in the 1970s (Cambel and Warder 1976), and has been studied more recently (Organisation for Economic Co-operation and Development 1982; Stout 1992; Coley et al. 1998).

In light of these problems, a worthwhile research question would ask what are the differences between the impacts of the current diet and a hypothetical diet, based on the RDI. Utilising scenario analysis to answer this question, Table 3 shows the results of such an analysis for a current Australian diet and a recommended diet. The columns contain various items concerning annual food intake and expenditure on food, as well as the associated factor embodiments for both scenarios. The rows in the lower third of the table contain the difference between the current Australian and the recommended diet, and the rebound impact for households in three different income quintiles. The consumption data were taken from (Australian Bureau of Statistics 1995b) for food intake and from (Australian Bureau of Statistics 1995a) for expenditure. A recommended diet was derived from figures given by (Watt 1979) and (Thomas and Corden 1977). The expenditure on the recommended diet was calculated using the ratio of expenditure to food intake of the current Australian diet. The net effect of a diet change is given as the sum of the scenario difference and the rebound impact. Net effects are shown as a percentage of the current Australian scenario.

	Consumption		Factor embodiments					
	Food intake (g day ⁻¹ per capita)	Expenditure (A\$ day ⁻¹ per capita)	Energy (GJ)	Greenhouse (t CO2-eq)	Labour (emp-h)	Income (\$A)	Imports (A\$)	Taxes (A\$)
<i>Current Australian diet</i>								
Meat and meat products	305	308	3.26	2.8	14.3	163	37	54
Dairy products	368	193	2.44	0.5	7.0	91	26	35
Fruit and vegetable products	743	272	3.30	0.4	8.3	133	39	52
Oils and fats	52	26	0.32	0.1	0.7	12	5.6	4.2
Flour mill products and cereal foods	248	63	0.75	0.1	2.2	31	8.7	11
Bakery products	149	189	1.81	0.3	8.3	108	23	36
Sugar and confectionery	125	139	1.36	0.2	4.9	75	20	33
Other food seafood, jam, honey.	36	253	2.64	0.6	8.3	123	37	46
Soft drinks, cordials and syrups	275	171	2.11	0.2	4.6	77	24	42
Beer and malt	267	184	0.69	0.1	2.5	35	7.7	127
Wine and spirits	53	162	0.88	0.1	3.6	49	11	92
Meals out and take-away		583	5.5	0.8	22.2	304	77	146
Sum	2621	2541	25.0	6.0	87	1201	316	679
<i>Recommended diet</i>								
Meat and meat products	155	157	1.66	1.4	7.3	83	19	28
Dairy products	214	112	1.42	0.3	4.1	53	15	21
Fruit and vegetable products	570	209	2.53	0.3	6.4	102	30	40
Oils and fats	30	15	0.19	0.0	0.4	7	3.3	2.4
Flour mill products and cereal foods	175	44	0.53	0.1	1.6	22	6.2	8
Bakery products	105	134	1.28	0.2	5.9	76	16	25
Sugar and confectionery	58	59	0.57	0.1	2.1	32	9	14
Other food seafood, jam, honey.	28	202	2.11	0.5	6.7	99	29	37
Soft drinks, cordials and syrups	116	72	0.89	0.1	2.0	32	10	18
Beer and malt	128	88	0.33	0.0	1.2	17	3.7	61
Wine and spirits	26	78	0.42	0.1	1.7	23	5	44
Meals out and take-away		583	5.5	0.8	22.2	304	77	146
Sum	1602	1753	17.4	3.8	61	850	224	444
Difference between scenarios	-1019	-788	-7.6	-2.2	-26	-350	-92	-236
Rebound, lowest income quintile		788	9.4	1.2	23	373	103	201
Net effect		0	1.7	-1.1	-3	23	10	-35
(%)			-7	(-18)	(-3)	(2)	(3)	(-5)
Rebound, middle income quintile		788	9.2	1.1	23	372	103	200
Net effect		0	1.6	-1.1	-3	22	10	-35
(%)			(6)	(-18)	(-3)	(2)	(3)	(-5)
Rebound, highest income quintile		788	8.5	1.0	23	370	101	199
Net effect		0	0.9	-1.2	-3	19	9	-37
(%)			(4)	(-20)	(-4)	(2)	(3)	(-5)

Table 3. Factor embodiment comparison between the current average Australian diet and a recommended diet (after (Lenzen and Dey 2002). The effects of consumer rebounds are shown in the lower part of the table where the net effect of the change is also shown as a percentage.

Table 3 is self-explanatory, so that here only a few features are discussed specifically. It can be seen that an adjustment of their eating pattern would save the average Australian about 790 A\$ per year. Compared with the current diet, the recommended diet generates ~20% fewer greenhouse gas emissions, with all other factors generally unaffected, when rebounds are included. It is interesting to note, that differences in the rebound effect between the quintiles, were found to actually be small.

3.1.2. A national ecological footprint account for Australia

The results of the second case study, The National Ecological Footprint Account of Australia with values EF_k in area units is shown in Table 4. Important footprints can be observed within the private final consumption of meat products, clothing, retail goods, and meals out, and the export of wool and meat products. Note that multipliers of domestically produced commodities and imports are assumed to be identical. As a consequence, embodiments in imports of yarns, fabrics and other textile products are probably overestimated. This is because processed wool is classified together with yarns and fabrics, which, when produced overseas, are likely to require much less land than for their production in Australia. Moreover, the domestic consumption of meat is based on more densely stocked pastures in the South-East of Australia, while the majority of pastures in the ELZ produce meat for exports. Hence, the embodiments in domestic consumption as given in Table 4 are probably too high, and those in exports too low. Nevertheless, Australia is a net ‘disturbance exporter’.

Amongst other features, this Ecological Footprint Account shows that almost half of Australia’s national ecological footprint is caused by producing exports of products from sheep and beef cattle grazing. The total export-related footprint amounts to about 110 million hectares. The fact of Australia exploiting a considerable part of its natural assets for exports is the result of a history of economic planning: since the 1980s, Australia has sought to escape from increasing foreign debt and falling primary commodity prices by expanding the volume of meat, wool and other primary exports in order to maintain total export revenues and living standards (Daniels 1992; Muradian and Martinez-Alier 2001). Daly (1993) paraphrased this strategy as entering into an environmental ‘race to the bottom’. Moreover, primary exports generally neither promote technological innovation and development of labour skills, nor positively influence economic growth, mainly because the respective producing industries are poorly linked back to other domestic, value-adding sectors (Fosu 1996; Lenzen 2003), and thus exert little economic impetus. In this respect, Australia is sharing the predicament of many developing countries that are locked into an environmental-economic dilemma through increasing dependency on environmentally degrading production and further erosion of environmental quality (Daniels 1992; Muradian and Martinez-Alier 2001). The environmental degradation associated with this situation is now slowly emerging, and will be exacerbated unless the national income is drawn from more sustainable production and trade, for example by internalising environmental cost into export prices, or by shifting towards alternative production structures, that is establishing strong value-adding secondary sectors through fostering education and research (Muradian and Martinez-Alier 2001).

	Private Final Consumption	+ Government Final Cons.	+ Changes in Stocks	= GNE	+ Exports	= GNT	- Consumer Imports	- Imports into Stocks	- Industrial Imports	= GDP
Sheep and shorn wool	0.23	0.00	-5.41	-5.2	35.18	30.0	0.00	0.00	0.00	30.0
Grains	0.33	0.00	-1.01	-0.7	4.91	4.2	0.01	0.02	0.37	3.8
Beef cattle	0.92	0.00	2.58	3.5	3.94	7.4	0.00	0.00	0.00	7.4
Other agriculture	1.39	0.04	-0.07	1.4	0.33	1.7	0.05	-0.01	0.11	1.5
Forestry	0.11	3.50	-0.01	3.6	0.11	3.7	0.02	0.00	0.12	3.6
Fishing	0.21	0.02	0.00	0.2	0.11	0.3	0.01	0.00	0.00	0.3
Coal mining	0.00	0.00	-0.08	-0.1	1.71	1.6	0.00	0.00	0.00	1.6
Crude oil, natural gas and LPG	0.09	0.00	-0.11	0.0	0.49	0.5	0.00	-0.05	0.53	0.0
Other mining	0.00	0.01	-0.07	-0.1	1.35	1.3	0.00	0.09	0.06	1.1
Meat products	38.30	0.00	0.97	39.3	40.37	79.6	0.59	0.03	0.33	78.7
Dairy products	2.09	0.00	-0.02	2.1	1.05	3.1	0.12	0.02	0.05	2.9
Sugar	3.75	0.00	0.01	3.8	2.25	6.0	0.84	0.03	0.10	5.0
Other food products	4.58	0.00	0.04	4.6	0.73	5.4	0.53	0.00	0.45	4.4
Alcohol and tobacco	0.84	0.00	0.09	0.9	0.36	1.3	0.24	0.03	0.20	0.8
Processed wool, yarns, fabrics	3.75	0.00	-0.09	3.7	6.11	9.8	1.22	-0.10	11.52	-2.9
Textile products and footwear	19.83	0.00	0.44	20.3	4.36	24.6	7.75	0.19	2.59	14.1
Saw mill products	0.01	0.00	0.01	0.0	0.28	0.3	0.01	0.00	0.44	-0.2
Paper products	0.79	0.02	-0.06	0.7	0.13	0.9	0.18	-0.02	1.40	-0.7
Refinery products	0.58	0.00	0.11	0.7	0.17	0.9	0.07	0.02	0.35	0.4
Chemical products	1.60	0.62	0.20	2.4	1.04	3.5	0.48	0.10	4.03	-1.1
Non-metal construction materials	0.06	0.00	0.02	0.1	0.05	0.1	0.03	0.00	0.22	-0.1
Basic iron and steel	0.00	0.00	0.04	0.0	0.44	0.5	0.00	0.00	0.47	0.0
Aluminium	0.00	0.00	0.02	0.0	1.02	1.0	0.00	0.00	0.01	1.0
Other non-ferrous basic metals	0.08	0.00	-0.01	0.1	1.09	1.2	0.01	0.01	0.21	0.9
Metal products	0.10	0.00	0.01	0.1	0.11	0.2	0.04	0.00	0.34	-0.2
Motor vehicles	0.83	0.00	0.13	1.0	0.18	1.1	0.34	0.06	1.11	-0.4
Other transport equipment	0.03	0.00	-0.01	0.0	0.10	0.1	0.01	0.00	0.21	-0.1
Other manufacturing	2.02	0.00	0.15	2.2	0.66	2.8	0.82	0.10	3.54	-1.6
Electricity supply	3.85	0.14	0.00	4.0	0.02	4.0	0.00	0.00	0.00	4.0
Gas supply	0.10	0.00	0.00	0.1	0.00	0.1	0.00	0.00	0.00	0.1
Water supply	0.52	0.01	0.00	0.5	0.00	0.5	0.00	0.00	0.00	0.5
Construction	0.00	0.25	0.00	0.3	0.07	0.3	0.00	0.00	0.02	0.3
Wholesale and retail trade	13.34	0.00	0.04	13.4	1.09	14.5	0.09	0.00	0.00	14.4
Accommodation and restaurants	9.00	0.00	0.00	9.0	0.98	10.0	0.37	0.00	0.28	9.3
Road transport	0.43	0.10	0.01	0.5	0.19	0.7	0.04	0.00	0.01	0.7
Rail transport	0.28	0.07	-0.01	0.3	0.19	0.5	0.03	0.00	0.00	0.5
Water transport	0.05	0.00	0.00	0.0	0.47	0.5	-0.04	0.00	0.39	0.2
Air transport	0.81	0.00	0.00	0.8	0.60	1.4	0.29	0.00	0.28	0.8
Ownership of dwellings ^b	4.42	0.00	0.00	4.4	0.00	4.4	0.00	0.00	0.00	4.4
Commercial services	3.33	0.57	0.00	3.9	0.56	4.5	0.10	0.00	0.50	3.9
Public administration and services	2.51	6.59	0.00	9.1	0.13	9.2	0.03	0.00	0.02	9.2
All industries	121.2	11.9	-2.1	131.0	112.9	244.0	14.3	0.5	30.3	198.9
Natural processes ^a	-5.3			-5.3		-5.3				-5.3
Residential Usage	2.0			2.0		2.0				2.0
Private Vehicle Usage	2.5			2.5		2.5				2.5
Ecological footprint	120.4	11.9	-2.1	130.2	112.9	243.1	14.3	0.5	30.3	198.1
Ecological footprint / cap	6.7	0.66	-0.11	7.2	6.25	13.5	0.8	0.0	1.7	11.0

Notes: all figures in Mha; ^a includes CO₂ sequestration, soil carbon uptake, prescribed burning, and wildfires; ^b output of industry is rent for dwellings, GNE = Gross National Expenditure, GNT = Gross National Turnover, GDP = Gross Domestic Product.

Table 4. A National Ecological Footprint Account (after (Lenzen and Murray 2001). The table is organised in order to reflect the National Accounting Identity (Eq. 17).

3.1.3. Structural Decomposition Analysis

Results from the third methodological approach, SDA, applied to the total changes in Australian greenhouse gas emissions between 1968 and 1997 (according to Eq. 16) are presented in Table 5. The results at the whole-economy scale show that the most important determinants of overall greenhouse gas emissions increase (+2.3%/y) across the whole economy are: per-capita final consumption (affluence, +1.5%/y); energy intensity improvements (−1.3%/y); population growth (+1.1%/y); structural economic change (+1.0%/y); and export growth (+0.8%/y). All other determinants contributed less than 0.5%/y to the total increase rate.

$\Delta \mathbf{cF}$	$\Delta \mathbf{E}$	Industrial			$\Delta \mathbf{Y}$	Residential		Population	Exports		Total
		$\Delta \mathbf{L}$	$\Delta \mathbf{u}$	$\Delta \mathbf{v}$		$\Delta \mathbf{cF}_{(res)}$	$\Delta \mathbf{E}_{(res)}$		$\Delta \mathbf{b}$	$\Delta \mathbf{Z}$	
-0.3%	-1.3%	1.0%	-0.2%	-0.2%	1.5%	0.0%	0.1%	1.1%	-0.1%	0.8%	2.3%

Table 5. Decomposition of changes in total industrial greenhouse gas emissions, Australia, 1968-1997; nomenclature in Tab. 3).

The above values are of course averages over the analysis period, and there have been fluctuations in trends over the years (Fig. 1). Changes in fuel mix ($\Delta \mathbf{cF}$, top left diagram) generally had a negative forcing on aggregate emissions (averaging -0.3%). During the analysis period the fuel share of natural gas increased across the economy from 0.2% in 1969 to 26% in 1997, and the fuel shares of the emissions-intensive fuel oil and leaded petroleum decreased from 10% to 1%, and from 10% to 2%, respectively. The largest negative point covers the years of the second oil price shock of 1978-79, when the shares of the primary fuel refinery feedstock decreased by 1.5%, which in turn affected the share of secondary oil fuels, with diesel and other petroleum products decreasing by roughly 0.5%. More recently, a sharp increase in $\Delta \mathbf{cF}$ occurred in the period 1995-97, which was mainly driven by an increase in the share of brown coal and a decrease in the share of natural gas. Also during this period, a conservative stance on greenhouse issues was being taken by the Australian Government in the years leading up to the Kyoto conference. The graph shows that significant reductions in greenhouse emissions can be achieved quite quickly when responding to pricing factors (for example during the oil price shocks), and can conversely increase quite quickly when there is no impetus for change (as in the most recent trends).

Industrial interdependence ($\Delta \mathbf{L}$) has shifted to be more energy intensive over the period 1969-1997. Once again, a notable dip occurs during the 1978-79 oil price shock, reflecting the structural response towards lower requirements of oil. More recently, there has been a significant positive forcing on emission levels by changes in industrial structure. Interestingly, these have occurred during negative fuel mix and energy intensity trends. This would tend to suggest that substitution between energy and non-energy inputs can be

achieved within the Australian economy.¹⁴ This is an important result, as considerable credit is given to industries that reduce their fuel use, yet based on these historical results, this credit may be unfounded, as the overall effect on emissions levels is diminished by the need to source other non-energy inputs but which have higher embodied energy/emissions content.

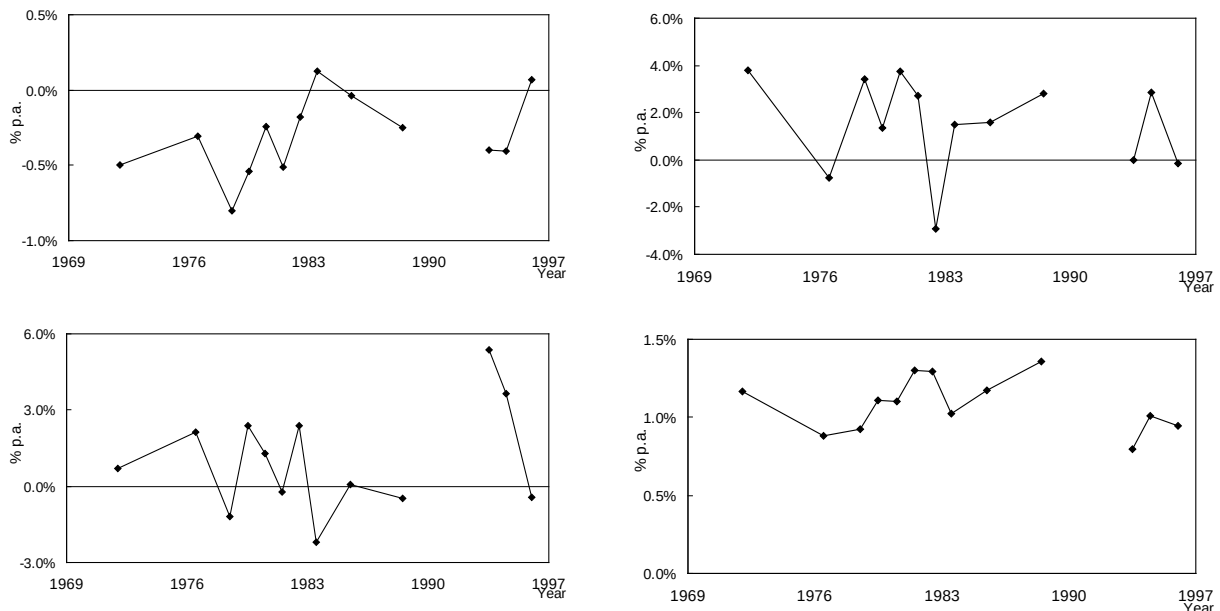


Figure 1 Trends in contributions to the change rate of greenhouse gas emissions in Australia, 1968-1997 (ΔC top left, ΔL bottom left, ΔY top right, ΔP bottom right). A gap in the time series appears for the period 1990-1993. This is due to a significant change in data classification by the Australian Bureau of Statistics between the 1990 and 1993 input-output tables which was not reconcilable.

A general positive forcing on emission levels by Y (affluence) is discernible, except for 1975-78 and 1982-83 (recessions in the Australian economy). The effect of a further recession in the early 1990's is unfortunately not captured in this analysis due to the data gap. The magnitude of this determinant reiterates the importance of the link between affluence, economic growth and emissions (Lenzen and Smith 2000). As expected, population growth over the study period has resulted in positive contributions to emission levels, averaging about 1.1% p.a. The variability is almost wholly due to changes in the level of immigration into Australia.

¹⁴ A more complete description is that when ΔE causes a downturn in emissions (caused by a reduction in industrial fuel use), ΔL causes a contrary upward forcing on emissions. As ΔE reflects the direct use of energy in industrial production, and ΔL reflects the embodied emissions in the industrial inputs into production, this converse forcing by the two factors suggests an inverse relationship between the fuel and industrial inputs.

A further interesting analysis is the inclusion of export level ΔZ into the private final demand variable ΔY , allowing the investigation of the effect of total economic activity. The result is a reduction in the variability of the trends, which implies that when domestic demand decreases, surplus product (with its embodied emissions) is exported in order to maintain a moderately stable level of production. The notable exceptions to this are for the recession years.

1994-95	ΔcF	ΔE	ΔL	Δu	Δv	ΔY	ΔP	Δb	ΔZ	ΔC
Meat and meat products		-0.17%						-0.19%	0.11%	0.05%
Beer and malt				-0.29%						-0.26%
Motor vehicles and parts					0.05%					0.07%
Electricity supply	-0.08%	-0.23%	0.69%	1.06%	-0.53%	0.64%	0.19%			1.74%
Water,sewerage and drainage				0.42%						0.29%
Residential building		-0.57%	0.79%		0.26%	0.19%	0.06%			0.50%
Other construction		-0.46%			0.26%	0.16%				-0.01%
Wholesale trade		-0.16%	0.81%	0.51%		0.08%		0.14%	0.05%	1.26%
Retail trade		-0.23%	0.85%		-0.12%	0.14%		0.10%		0.75%
Restaurants, hotels, clubs		-0.18%		0.53%	-0.09%	0.10%		0.28%		0.40%
Road transport		-1.57%	0.62%	-0.90%	-0.13%	0.22%	0.07%	-0.19%	0.18%	-1.70%
Water transport	-0.06%	0.32%						-0.28%	0.08%	0.09%
Ownership of dwellings		-0.14%		-0.27%	-0.06%	0.07%				-0.60%
Technical and computer				0.43%						0.31%
Government administration	0.08%		-1.82%	-0.41%	0.06%	0.13%				-1.94%
Defence			-0.31%	-0.26%						-0.48%
Education				0.90%						0.81%
Health services			-1.53%			0.09%				-1.67%
Community services			0.49%							0.42%
Total changes	-0.41%	-5.20%	3.64%	0.12%	-0.65%	2.85%	0.86%	-1.36%	2.25%	1.21%

Table 6. Structural decomposition of changes in 1993/4-94/5 Australian greenhouse gas emissions into industry contributions; top 19 industries with contributions of $|\Delta| > 0.5\%$.

A more specific breakdown into contributing industries (Table 6) is shown for 1994-95, a period fairly representative of trends of consistent dominance by specific sectors of the economy that is noticeable over the whole time series. First, changes due to primary industries such as agriculture, forestry, fishing and mining are small. This is due to low end uses of the products of these sectors, and the emissions incurred during the production of these commodities being embodied in intermediate inputs of other industries.

Second, the manufacturing sector has a mixed history, with little representation in later years, whilst in earlier years the effect of the industry was considerably larger. The prominent industries include meat products, petroleum and coal products, and motor vehicles and machinery. In terms of exports driving emissions, meat products are significant. Third, amongst the utilities sectors, electricity supply is always represented, consistently representing a large contribution to change in overall emissions for each period. Construction is also consistently represented, as are the transport sectors. Fourth, within the service sector, government administration, defence, education, health and community (or welfare) services have consistently contributed to change, often forcing emissions upwards.

4. Conclusions

In this paper, demonstrations of the application of scenario analysis, the ecological footprint and structural decomposition analysis have been presented for Australian consumption patterns.

In all three applications, the desired objective was the options for moving towards a sustainable level of consumption. Utilising scenario analysis, a comparison was made between the impacts of current diet, and the impacts of an environmentally motivated diet. Results were able to show the compromises that would be made if the Australian diet was suddenly changed. In this example, significant savings were found in terms of greenhouse gases and slight increases in levels of income and imports, but at the expense of decreases in employment and taxes.

Applying the ecological footprint to similar data, but this time encompassing all forms of consumption in Australia, was able to show the absolute land impacts, and the relative importance of differing forms of consumption. As a result, impacts from agricultural products, particularly from meat products, were found to be the largest contributors to both domestic and exported land disturbance.

Finally, the application of structural decomposition analysis compared the level of impacts over time, thus allowing to separate likely factors influencing greenhouse emission levels, and thus to gain insights into the underlying causes of changes. As a result, the upward trends of population growth and GDP were distinguished from industrial effects, such as industrial structure and industrial energy efficiency. Whilst the overall impacts of all effects has unsurprisingly been towards increased emissions over the last 30 years, the variability in some factors, and the average reducing effect of other factors has mitigated increases to an extent, and provides hope for future improvements.

Thus the three methodologies, whilst all focusing on levels of Australian consumption, have provided a variety of results – compromises from the scenario analysis, space impacts from the EF, and disaggregated trends from the SDA. An important question then, is if there is value in using these methodologies in conjunction – whether they are complementary approaches or substitutes for each other.

Firstly, the disadvantage of using more than one approach simultaneously is obviously the increased time/costs involved. The first two approaches, scenario analysis and the ecological footprint, however, have the advantage of similar base data and similar application. Once impact indicator and land disturbance data is available, the calculation of the same consumption data sets may be performed for each method. This provides the advantage of an easily interpretable result from the ecological footprint, coupled with the possible compromises of environmental, social and economic indicators and the effects of rebounds in a scenario analysis of response strategies.

Structural decomposition analysis does not share the advantage of significantly overlapping data sets; it relies on historical data and as such provides two important complementary facets to the other methodologies. Firstly, it is concerned with historical trends, an important input into the construction of hypothetical scenarios. Secondly, pure consumption-induced impacts can be differentiated from other factors. For example, if the result of an ecological footprint study incites action, it may be more poignant to target industrial processes rather than household consumption if the results of an SDA show (for arguments sake) that final demand induced impacts have been decreasing whilst industrial structure induced impacts have been increasing.

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A MATERIAL HISTORY OF AUSTRALIA: EVOLUTION OF MATERIAL INTENSITY AND DRIVERS OF CHANGE

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Abstract

This article presents an analysis of the material history of Australia in the period 1975-2005. The values of economy wide indicators of material flow roughly trebled since 1975, and we identify the drivers of this change through structural decomposition analysis. The purpose of this work is to delve beneath the top level trends in material flow growth to investigate the changes in the economy that have been driving the change. The major positive determinants for this change were found to be the export level, export mix, industrial structure, affluence and population. Only improvements in material intensity offered retardation of growth in material flow. Other structural components had only small effects at the aggregate level. At a more detailed level, however, the importance of the mineral sectors became apparent. Improvements in mining techniques have reduced material requirements, but increased consumption within the economy and increased exports have offset these reductions. The full roll-out of material flow accounting through Australian society and business and a systematic response to its implications will require change in the national growth focus of the last two human generations, with serious consideration needed to recantation of volume focused growth of the economy and also to neo-liberal and globalised trade policies that have dominated the globe for the past decades.

Keywords: material flow analysis, structural decomposition analysis

1. Introduction

It is commonly believed that the increasing activities of a consumption-hungry industrial system have been driving the increases in Australia's environmental impact. Indeed, in Australia's economic structure, the natural environment is linked to the industrial ecology, but ultimately - via quite complex production chains – to the consumption of an affluent population. Changes in any part of the production structure will result in changes in overall environmental impact. For example, even if consumption increased, net environmental impact may not worsen, if industrial efficiency of natural resource use improved at the same time. Therefore, the knowledge of these environment-economy linkages, and their historical trends, can direct policy to be most effective.

This study seeks to understand changes in the factors that influence material flows by taking a historical perspective to the determinants of growth in the material intensity of Australia. We undertake an investigation of structural changes in the economy over a period of 30 years (1975-2005). A detailed structural decomposition model is developed using input-output (IO) analysis at a sector level of 344 industries. This level of detail was selected in order to enable continuity over the series of input-output tables and disaggregation of environmentally important industries. Such disaggregation is very important in order to not lose track of changes when otherwise industries with very different extractive processes are grouped together.

The use of input-output analysis in studying environmental effects stems from the late 1960s (see Daly 1968; Isard et al. 1967; Ayres and Kneese 1969; Leontief and Ford 1971). Its subsequent use in structural decomposition analysis (SDA) initially focused on energy use, (Proops 1988; Rose and Chen 1991; Chen and Rose 1990), and greenhouse gas emissions (Casler and Rose 1998; Common and Salma 1992; Proops et al. 1993; Wier 1998; Chang and Lin 1998; De Haan 2001; de Nooij et al. 2003).

The work documented in this article breaks new ground: First, it updates MFA studies on Australia by Poldy and Foran (1999) and complements the work of Schandl et al. (2008). Further, to the knowledge of the authors, the only published applications of SDA in the Material Flow field is the total material requirements analysis for West Germany¹ and an analysis of Chile (Pablo Muñoz and Hubacek 2008). Hoffrén et al. (2001) and Hashimoto et al. (2008) analyze material flows, but instead of SDA, these authors use Index Decomposition Analysis, a decomposition variant that does not make use of input-output tables. Our data set is the most detailed extended input-output data set that we know of in Australia, and the only existing time series. We extend Moll's methodology by incorporating recent work on index decomposition (see for an introduction, see Ang et al. 1998; Ang 2004). We use the adaptive weighting of the logarithmic mean divisia index (LMDI) to enable a perfect decomposition (i.e. leaving no residuals), with correct handling of zeros in a SDA context (Ang and Liu 2007; Wood and Lenzen 2006).

¹ A body of work has been published, with a publication accessible in English by (a body of work has been published, with an English accessible publication in Moll et al. 1998).

As the purpose of this work is to delve beneath the obvious trends in material flow growth to investigate the changes in the economy that have been driving the change, we perform a decomposition analysis in order to reveal the influence of material intensity changes, affluence and population growth, structural changes in the economy, and the effects of final demand level, mix and destination. The results of our work demonstrate to policy-makers the practical utility of combining MFA with SDA. For example, we explicitly take into account changes due to variations in exports, so that policy-makers can consider material consequences of international trade in their decisions.

The basic methodology and sources of data of this study is included in section 2. Results of the decomposition for Australia's material flow from 1975-2005 are presented in section 3. In Section 4 we then discuss these results by describing tangible and relevant implications for business or policy. Conclusions are drawn in section 5.

2. Methodology

2.1. Material Flow analysis

The reader is referred to Part I chapter 3 for full detail on construction of a material flow data set for Australia.

2.2. Input-output analysis

Input-output analysis is a top-down macroeconomic technique, which uses sectoral transaction data in order to account for the complex interdependencies of industries in modern economies. Generalization of the basic formulation of input-output analysis for physical factors (such as material flow) yield so-called *factor multipliers* that are embodiments of the physical factor per unit of final demand of commodities.

Mathematically, a multiplier matrix $\mathbf{M}$ is calculated from a $1 \times n$ matrix $\mathbf{F}$ containing sectoral physical factor usage, and from a $n \times n$ direct requirements (input-output) matrix $\mathbf{A}$ according to

$$\mathbf{M} = \mathbf{F} (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{F} \mathbf{L} , \quad (1)$$

where $\mathbf{I}$ is the $n \times n$ unity matrix. $\mathbf{A}$ comprises requirements from intermediate demand of domestically produced and imported commodities. $\mathbf{L}$ is used as an abbreviation for the Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$.

The factor inventory F of a given functional unit represented by a $n \times 1$ final demand vector $\mathbf{y}$ is then simply

$$F = \mathbf{M} \mathbf{y} . \quad (2)$$

M y represents the indirect usage of factors embodied in all inputs into the functional unit. Further mathematical details on the input-output formalism for Australia can be found in Lenzen (2001) and Wood (2009).

In this study, we are interested in decomposing the overall factor usage F into temporally and structurally important determinants, or *drivers*. We are also very interested in the role exports have to play. Hence $\mathbf{y}$ is decomposed into $\mathbf{uvYP} + \mathbf{bZ}$, where $\mathbf{u}$ is the $(n \times d)$ matrix of d categories of final demand relating individual components of final demand to absolute levels of final demand for each category ($u_{n,d} = y_{n,d}/y_d$); $\mathbf{v}$ is the vector of length d relating categories of final demand to gross national expenditure (GNE) ($v_d = y_d/Y$); Y is the per-capita measure of economic activity (GNE); and P is the population. The term $\mathbf{bZ}$ refers to exports, and is hence independent of domestic population or expenditure measures. This term shows the mix of products going to export $\mathbf{b}$ and the overall level of exports Z . Thus, we are able to distinguish changing export orientation, shift to service sectors, personal affluence (expenditure per person), and population, amongst many other potential drivers, in their effect on the overall material metabolism of Australia over the past 30 years. Equation 2 becomes:

$$F = \mathbf{FL}(\mathbf{uvYP} + \mathbf{bZ}). \quad (3)$$

In the input-output applications of this study, we focus on handling imports and exports as flows within the environmental-economic system; hence we embody domestic, imported and exported material flow within the SDA calculation. We thus talk about *direct material input* (DMI) rather than pure domestic extraction (DE) or exogenously corrected consumption (DMC). Imports are modeled implicitly in the calculation (that is, if the imports are material in nature, as part of $\mathbf{F}$, and if they are immaterial in nature, as part of $\mathbf{L}$), and exports share the burden of domestic extraction and processing by linking them to $\mathbf{FL}$. We also analyze two separate representations of F , being DMI as defined above, and the total material requirement TMR which includes unused flows.

2.3. Structural decomposition analysis

The basic approach to additive (see Hoekstra and van den Bergh 2003; Choi and Ang 2003) structural decompositions of a function $y(x_1, x_2, \dots, x_m)$ of m determinants is through its total differential

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \dots + \frac{\partial y}{\partial x_m} dx_m. \quad (4)$$

In this case $y(x_1, x_2, \dots, x_m) = x_1 \cdot x_2 \cdot \dots \cdot x_m$ (with the x_i being scalars, vectors or matrices),

$$dy = \prod_{j=1, j \neq 1}^m x_j dx_1 + \prod_{j=1, j \neq 2}^m x_j dx_2 + \dots + \prod_{j=1, j \neq m}^m x_j dx_m = \sum_{i=1}^m \left(\prod_{j=1, j \neq i}^m x_j dx_i \right). \quad (5)$$

Analysing discrete time series with a Divisia decomposition approach, differences Δy are obtained by integrating infinitesimal changes dy :

$$\Delta y = \int_{y_0}^{y_1} dy = \int_{x_{1,0}}^{x_{1,1}} \prod_{j=1, j \neq 1}^m x_j dx_1 + \int_{x_{2,0}}^{x_{2,1}} \prod_{j=1, j \neq 2}^m x_j dx_2 + \dots + \int_{x_{n,0}}^{x_{n,1}} \prod_{j=1, j \neq m}^m x_j dx_m = \sum_{i=1}^m \left(\int_{x_{i,0}}^{x_{i,1}} \prod_{j=1, j \neq i}^m x_j dx_i \right). \quad (6)$$

In order to compute the integral one has to know what average values y assumes while the x_i change from $x_{i,0}$ to $x_{i,1}$ (the “integral path”). Conventional Parametrical Divisia methods assume a parametrical average $\bar{y}^\alpha = y_0 + \alpha(y_2 - y_1) = y_0 + \alpha\Delta y$, with $0 \leq \alpha \leq 1$. Searching for a non-parametric method, Ang and Choi (1997) and Ang and Liu (2001) propose the logarithmic mean $\bar{y}^\lambda = \Delta y / \Delta(\ln y)$. The resulting Logarithmic Mean Divisia Index (LMDI) formulation

$$\Delta y^L = \sum_{i=1}^n \frac{\Delta y}{\Delta(\ln y)} \ln \frac{x_{i,1}}{x_{i,0}} \quad (7)$$

is non-parametric, exact, and time-reversible (for proofs see Ang and Choi (1997), and Ang et al (1998)). A number of different indexing techniques have been proposed in the literature, but as there is no single ‘correct’ method, Ang (2004) recommends the LMDI in these types of applications (for a more complete overview see Lenzen (2006), Hoekstra and van den Bergh (2002) and Janssen et al.(2001)). Issues were found when applying the original formulation of the LMDI to input-output tables (Wood and Lenzen 2006), where the high number of zero values influenced results due to the handling of the logarithmic term. Hence, when applying the LMDI formulation, limits, rather than tiny values were used for the logarithmic term (see Ang and Liu 2007).

Applying equation (6) to equation (3), gives:

$$\begin{aligned} \Delta F &= \Delta F(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{u}\mathbf{v}YP + \mathbf{b}Z) \\ &+ \mathbf{F}\Delta(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{u}\mathbf{v}YP + \mathbf{b}Z) \\ &+ \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\Delta\mathbf{u}\mathbf{v}YP \\ &+ \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{u}\Delta\mathbf{v}YP \\ &+ \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{u}\mathbf{v}\Delta YP \\ &+ \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{u}\mathbf{v}Y\Delta P \\ &+ \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\Delta\mathbf{b}Z \\ &+ \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}\Delta Z \end{aligned} \quad (8)$$

where each factor change (for example ΔF) is calculated according to equation 7, not simply as, for example, $F_1 - F_0$ (which is a type of conventional parametric divisa index, as per equation 6).

Factor	Description
	The change in total material flow reflected in the changes in:
ΔF	material intensity
ΔL	industrial structure
Δu	the mix of final demand of goods and services from the industries
Δv	the relative change in destination of final demand
ΔY	affluence - the change in the per-capita GNE
ΔP	population
Δb	the relative change in type of goods and services to export
ΔZ	export level - the change in overall volume of exports
ΔF	total change in material flow (summation of above terms)

Table 1. Description of structural decomposition components

The terms of Equation 8 are named by their difference factor (Table 1). Material intensity ΔF is TMR or DMI per dollar of gross production by sector. Industrial structure ΔL reflects the technological or input-output relationships, and shows the total inputs by sector for a unit of final product by sector. Changes in this factor would show substitution between one production chain with another production chain. The mix of final demand Δu is the type of product per destination of final demand. Changes in this factor would show changes in the consumption habits of the populations and other types of final demand (e.g. infrastructure, etc). Destination of final demand Δv shows the effect of changes in the amount of private and government current and capital expenditures. Affluence ΔY is the per-capita expenditure (GNE) of the population ΔP (GNE differs from GDP in that imports are included, but exports are not). Type of export Δb is the relative mix of export, whilst overall export volume ΔZ is self-evident.

Source for economic data (L, u, v, Y, b, Z) was the Australian Bureau of Statistics (Australian Bureau of Statistics 2004, 2005, 2006), especially publications 5209.0 and 5215.0. Eighteen sets of tables have been published by the ABS, with the years being year 1968-69, 1974-75, 1977-78, 1978-79, 1979-80, 1980-81, 1981-82, 1982-83, 1983-84, 1986-87, 1989-90, 1992-93, 1993-94, 1994-95, 1996-97, 1998-99, 2001-02, 2004-05. Extensive reclassification was required due to changes in classification over time. Economic data was also required to be in constant prices. In order to convert the original current price tables to constant prices, we applied price indices by sector, and reconciled the inflated figures with available constant price national account data. A complete description of the data processing steps is available in (Wood 2009).

2.4. Limitations

The limitations of this study can be summarized as:

- Lack of detail on data on unused extraction – including comprehensive coverage of all materials by individual industry, and data on foreign trading partners.
- Data stability of input output data; it is difficult to obtain an idea of how stable the final constant price data is, and how much an effect classification changes have. This is analyzed elsewhere (Wood 2009). We minimize these effects in this study by applying rolling averages to each combination of input-output tables.
- Using monetary input-output tables instead of physical input-output tables to analyze structural changes (Dietzenbacher 2005). As such, sectors that pay different prices for the same material will show up differently in results. Overall, the average is the same.

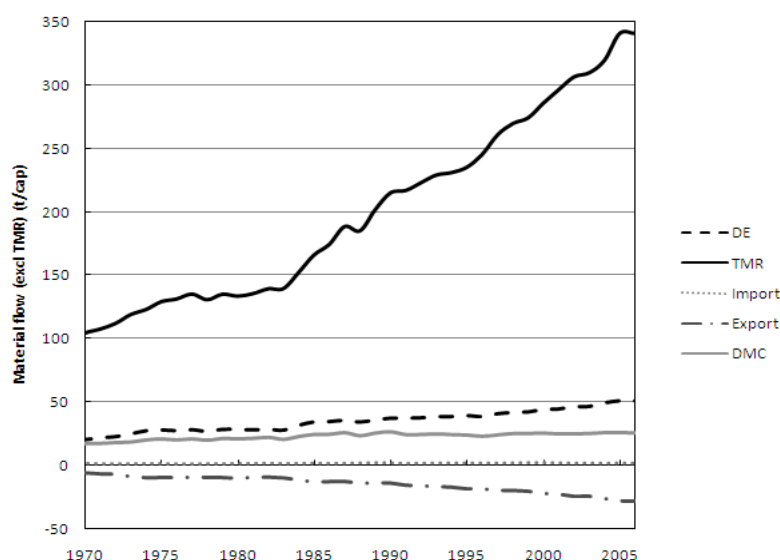


Figure 1: Per-capita material flow estimated for Australia.

3. Results and Analysis

Since the mid 1970's, DE has grown by a factor of 2.5, whilst TMR has grown by a factor of almost 3.5 (Figure 2). This growth, and the difference in the rates of growth, is mostly due to an expansion of the mining industries in Australia, principally due to export demand. This is reflected in the graph of DMC, which has been fairly stable since the late 1970's. Imports reflect only used flows, and generally represent only about 3-4% of domestic material production. Exports are much more significant, with up to 60% of total domestic material production being exported. The rapid rise following the mid-1980s is due mostly to black coal exports but also to expansion of gold and diamond mining which have low yields of final product per unit of mined ore (see Figure 3). In parallel, the material flows of the agriculture sector are reducing slightly due to declines in total sheep numbers and thus less soil erosion as the international demand for wool lessens.

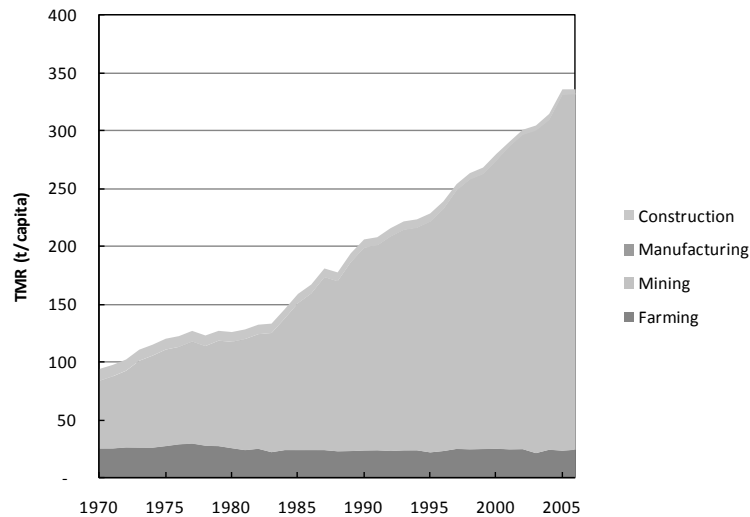


Figure 2: Components of per-capita TMR estimated for Australia.

The evolution of the efficiency of production is shown in Figure 4 for TMR. This is the estimated material flow per \$ GDP. DMC is not shown, as GDP includes export valuation, which is excluded in DMC. Results for DE show identical trends to TMR. The principle result from this figure is the reasonably large efficiency improvements across time. Hence, whilst more material is being processed, particularly for export, economically, it is being done in a more efficient manner. Two explanations are available for this – either the economy has become more material-efficient, or we are seeing inflationary effects. In order to separate these impacts, we also plot evolution in constant prices.

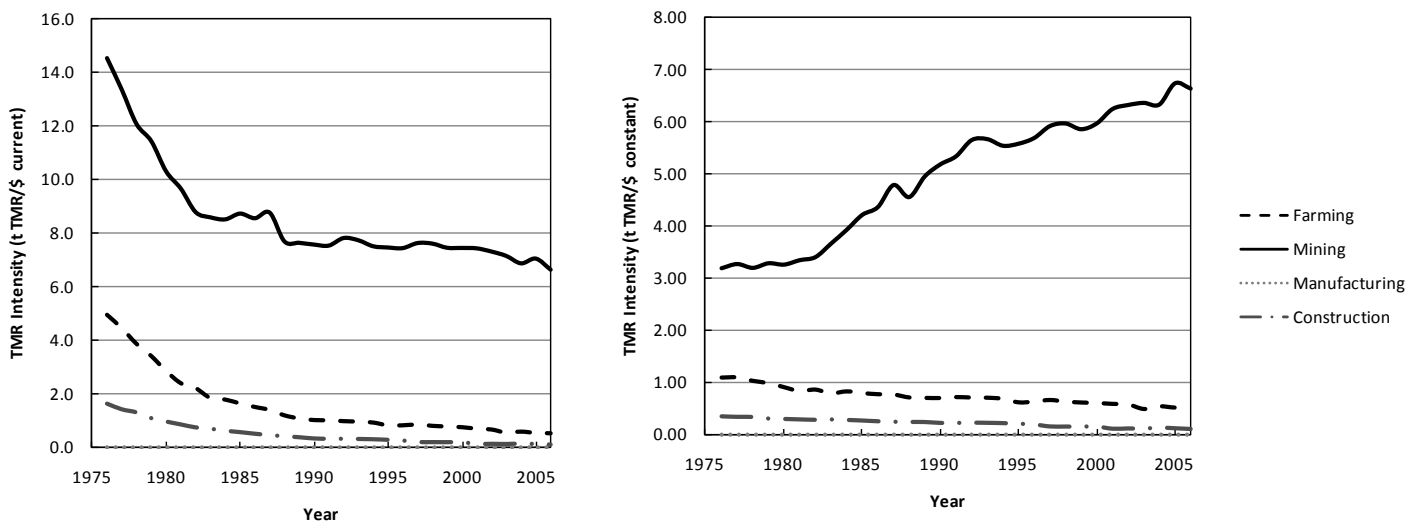


Figure 3: Evolution of TMR intensity (tones) per \$ GDP for current and constant (2005) prices.

Constant-price time series (Figure 4) demonstrate that most intensity improvements are more to do with price increases than real efficiency measures. Farming and construction

have both managed to achieve real efficiency measures, but mining intensity changes have largely occurred due to price increases. TMR of mining even increases, which is consistent with declining ore grades as reported by Mudd (2007). The most important component of these results is that of coal mining, with overburden increasing around a factor over 30 years. In fact, the overburden of the coal industry is responsible for over 50% of Australia’s TMR, and is so important that changes in it overwhelm other material changes. Schandl et al. (2008) discuss the geographic and economic importance of some of these sectors, and we refer the reader to this work for this interpretation.

3.1. Structural decomposition analysis

The decomposition of these material flows according to equation 7 and 8, averaged over the time series, is presented in Figure 3. As previously mentioned results of this part of the analysis refer to domestic, imported and exported flows endogenous to the variables shown. Whilst imports could be handled explicitly like exports in the presentation of results, we do not do the same, due to their very minor contribution to final outcomes. We present results for both TMI and DMR, as outlined in the method, in order to give a direct comparison between the measures.

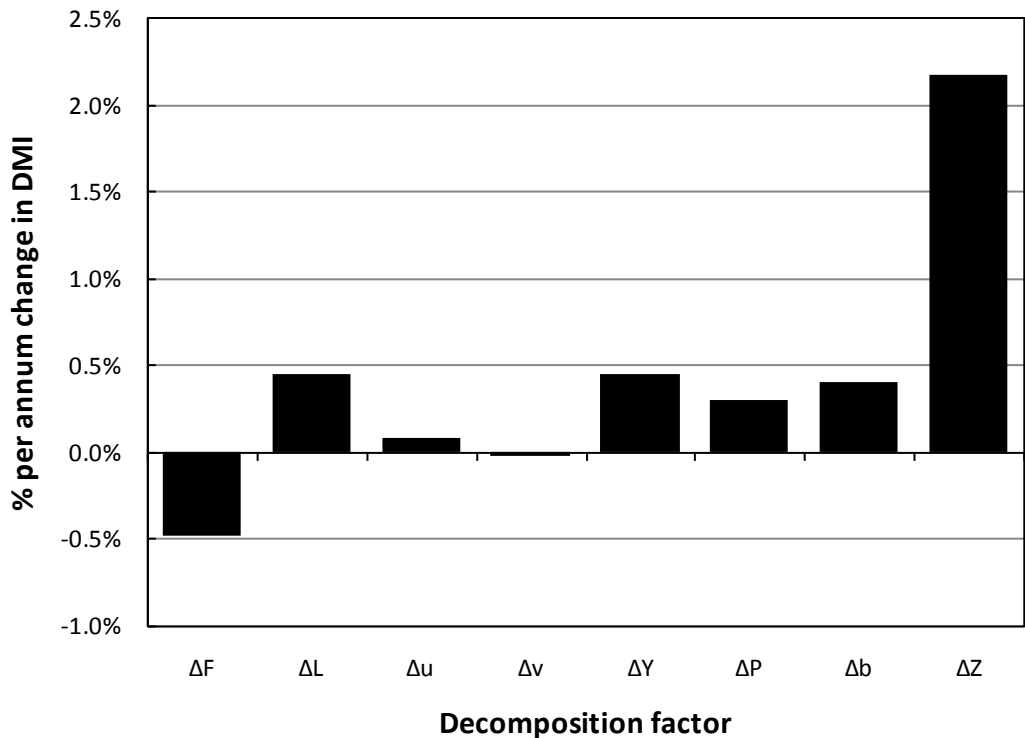


Figure 5. Average p.a. affect of each determinant across the time series on direct material intensity. Values are calculated relative to gross DMI.

The key drivers for increasing material usage of both DMI (Figure 5) and TMR (Figure 6) are firstly, not surprisingly, volume of exports ΔZ , as we have already discussed. The mix of exports Δb is also shifting to more material intensive products; in particular, this occurs for items with high unused flows, as shown by the larger impact of Δb in TMR compared to DMI. Other affects include structural changes ΔL in the economy, which shows just less than a 0.5% p.a. shift towards more resource intensive flows. This is to say that the technology used in Australia production is becoming more reliant on material extraction – probably at the expense of labor and capital. Domestic final demand mix Δu and destination Δv show limited change in type of consumption (i.e. there has been no ‘greening’ of consumption in Australia), as well as limited change in capital verse current consumption (Δv). As expected, increases in domestic population ΔP and affluence ΔY have passed through the economy with roughly a 0.2-0.4% p.a. increase each.

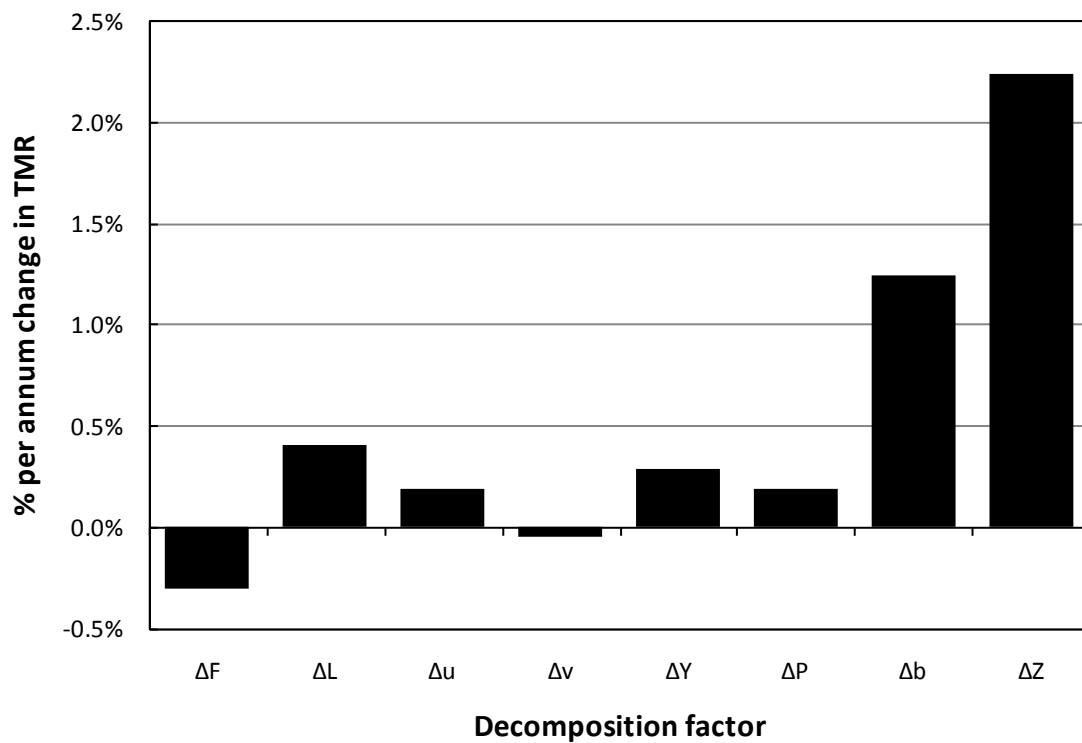


Figure 5. Average p.a. affect of each determinant across the time series on total material requirements. Values are calculated relative to gross TMR.

The notion of a steady state system where resource efficiency improvements allow improving affluence in parallel with declining resource consumption (the ‘factor 4’ or ‘factor 10’ of von Weizsäcker et al. 1997) has not been observed in the Australian economy to date (see UNGASS - United Nations General Assembly Special Session 1997). Small improvements in efficiency have been more than offset by large economic demands.

	Extractive industry	ΔF	ΔL	Δu	Δv	ΔY	ΔP	Δb	ΔZ
TMR	Beef cattle	-0.2%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%
	Black coal	0.1%	0.1%	0.1%	0.0%	0.1%	0.0%	1.2%	1.3%
	Iron ores	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.4%
	Copper concentrates and ores	-0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%
	Gold	0.2%	0.0%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%
DMI	Sugar cane	0.2%	-0.2%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.1%
	Black coal	-0.3%	0.1%	0.0%	0.0%	0.0%	0.0%	0.6%	0.7%
	Brown coal	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	Crude oil	-0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%
	Iron ores	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.8%
	Bauxite	-0.1%	0.2%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.2%
	Gravel	-0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	Construction materials	-0.3%	0.1%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%

Table 2. Decomposition at the level of material type – average factor change by relative to total average material flow.

	Consumptive industry	ΔF	ΔL	Δu	Δv	ΔY	ΔP	Δb	ΔZ
TMR	Black coal	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	1.2%	1.3%
	Iron ores	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.4%
	Gold	0.2%	0.0%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%
	Fresh meat	-0.1%	0.1%	-0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
	Copper, silver, lead, zinc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%
DMI	Black coal	-0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	0.7%
	Crude oil	-0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%
	Iron ores	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.8%
	Raw sugar	0.2%	-0.1%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.1%
	Alumina	-0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	-0.1%	0.2%
	Electricity supply	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Table 3. Decomposition at the point of output from industry – average factor change by industrial output relative to total average

These results show the obvious effects of export volume, population and affluence having large aggregate effects on material input. We also know that there are some important changes occurring within our export mix $\Delta \mathbf{b}$, technological structure $\Delta \mathbf{L}$ and in material intensity $\Delta \mathbf{F}$. In order to understand these effects further, the aggregated decomposition shown in Figures 5 and 6, can be further decomposed at the level of each individual flow. We hence present results by extractive industry (that is, by the type of goods entering the economy) and by consumptive industry (that is, the type of goods and services leaving the economy). Only the major extractive industries contributing to overall change in DMI and TMR are included (a material having a greater than 1% change from average material flow for any decomposition factor).

The agricultural and mining industries become obvious as the main drivers for change. Within agriculture, there are decreases in material intensity $\Delta \mathbf{F}$ for beef cattle, offset directly by its increased use within the economy ($\Delta \mathbf{L}$). Sugar cane shows the opposite – more inputs, but less relative use. Black coal is the key material in terms of exports – with higher overall volumes $\Delta \mathbf{Z}$ and higher relative volumes $\Delta \mathbf{b}$ compared to other types of exports. One interesting result is the fact that material intensity $\Delta \mathbf{F}$ of black coal decreased for DMI but increased for TMR. This result reflects the increasing amounts of overburden being produced in coal mining relative to ore as miners go to greater depths (Mudd 2007). The importance of other mining industries is evident in Table 2; with large per annum changes in export volume $\Delta \mathbf{Z}$ for metal ores. Improvements have generally been seen in the material intensity $\Delta \mathbf{F}$ of these industries, excluding gold, which interesting enough has become more popular with Australian consumers ($\Delta \mathbf{u}=+0.2\%$). Comparing results from TMR and DMI, it is evident that, as would be expected, high value materials are evident in TMR, as these often have large unused flows associated with them. DMI, in contrast, has a number of low value products such as gravel and construction materials, which embody very little unused flows. This result reinforces the importance of at least being aware of unused flows in high value products.

The decomposition by consumptive industry (Table 3) shows the major categories of final consumption that have been driving change. These results show where material leaves the economic system and flows to final consumption (or export). We use ‘industry’ rather than product, as the classification of goods and services is in industrial rather than product classification, and the values hence represent the type of output from each industry. The presence of a similar range of industries in Table 2 to those shown in the extractive breakdown (Table 2), is further evidence that key changes have been occurring in industries with minimal processing – that is, it is the same industries which extract high volumes of material that deliver it to final demand. Little change has occurred in increasing the circulatory production of material intensive resources (by value adding) in Australia. The exceptions to this are fresh meat (in TMR) and in DMI, raw sugar, alumina (which is a processing industry of bauxite) and electricity supply. All of these industries are fairly minor in terms of value adding. There has been greater use of alumina products (indirectly through demand for aluminum) in the economy (shown in $\Delta \mathbf{L}$) and slightly elevated exports

ΔZ of this processed product. In comparison, electricity has grown most through material intensity effects ΔF , which would be due to the almost complete reliance on coal for electricity production. Note, in these results, ΔF shows changes in material intensity rather than material efficiency, hence whilst electricity might be using less material inputs to produce a kWh of electricity, it is actually using more material inputs per \$ of constant price production.

4. Discussion

While Australia's performance in some environmental sectors such as native forests and specific export fisheries can be seen as relatively good, other broader areas of environmental performance such as greenhouse gas emissions, mammal extinctions, stressed river systems and ecological footprint indicators present more complex challenges for policy roundtables (Morton et al 2009; WWF 2008). Accepting MFA as a key systems indicator for policy is thus problematic because its reduction would mean a conflict with activities that drive economic growth and international trade. At least historically, we have seen nothing in the drivers of change that would allow us to think differently. Whilst Australia's decision-makers promote environmental policies for a developed world country, the country has maintained or even accentuated environmental and resource indicators typical of resource rich and/or developing countries (Daniels 1992). Our results clearly show a minimum shift away from low value adding industries and that exports of raw product are still dominant.

Of course, Australia's unique geographic and demographic circumstances form part of this dilemma: A relatively small population, a large land area, and rich mineral deposits currently consign it to an industrial structure skewed towards extracting primary industries, which support other countries' material requirements, and this explains part of the lack of developed-world MFA indicators. This is demonstrated by the significant role energy and mineral exports play in Australia's material flow and the increasing role exports have been shown to play in Australia's material extraction. In Australia's case, and perhaps generally in a regional context, the general assumption that TMR is a proxy for environmental pressures does not necessarily hold. Australian extractive industries adhere to stringent environmental standards for each new development, and have relatively low areal requirements (for example compared to agriculture), so that a local mine does not necessarily mean local environmental problems (Driussi and Jansz 2006). This can be seen by the dominance of coal overburden in Australia's TMR in comparison to more polluting activities. Also, a significant environmental legacy does exist however of mine developments from previous eras (see for example Mudd (2005)). In a global context, however, Australia's TMR does matter, because

Australia's mineral exports are globally significant,

- its expected exports of black coal and natural gas out to 2050 could cause 56 billion tons of carbon dioxide emissions if combusted without carbon capture and storage technologies

- much of these exports are used in overseas production processes where TMR is a good proxy for many other environmental impacts (for example steel making), and
- much of these exports end up embodied in consumer items that are imported back into Australia.

This means that were Australia to take environmental policy seriously, and thus promote material flow indicators to equal standing with GDP and employment growth rates, decision-makers would have to adopt a more systematic viewpoint of the flows and feedbacks involved in the global materials cycle. Japan for example is developing industrial 'sound material cycle' that aspire to give an economy with negligible inputs of virgin materials particularly metals (Moriguchi 2007).

Were MFA implemented in a globally-scoped new environmental policy framework, a daily analysis of material trends would almost certainly lead to three profound changes in national policy and action. These would affect: First growth policies, second the focus on efficiency, and third extend the liability of our trade policies.

As for the first item: In most national forums, Figure 5 and 6 would show the resounding success of a two-generation policy focus on population and economic growth, and the promotion of affluence in society. However were material flow data seen as environmental performance indicators, they would show abject policy failure. Mainly through material-intensive exports that provide for manufactured imports, changing technology, population and affluence together have accelerated DMI and TMR by 3.9% and 4.5% per annum respectively, while efficiency policies could only retard them by 0.3% and 0.5% per annum respectively, an almost 10 fold difference in overall long-term trends. The tensions here between economic and environmental world views and aspirations are almost insoluble in a resource-rich country so dependent in global trade. To address these material flow trends, policy would need to constrain affluence and population growth or at least the links of these factors to the primary industries. In the process this could reveal a serious deficiency in the secondary and tertiary sectors expected in an advanced economy. Constraining physical affluence in a material sense would perhaps even entail fundamental societal reorientation, here and in most economies, as with few exceptions, affluence expansion has been the driving rationale for human civilization.

Secondly: The drive for efficiency, be it based on financial budgets or industrial processes, usually generates the rebound effect (Holm and Englund 2009; Hanley et al. 2009; Hertwich 2005; see also the link between efficiency- and affluence-driven rebounds in Alcott 2008) by decreasing price, but increasing availability and finally overall volumetric throughput. Thus efficiency, the centerpiece of many government and business policies, can have the perverse effect of stimulating material flow. Whilst our results do not prove this rebound effect, we are seeing increasing volumes accompanying the improvements in efficiency. Korhonen (2007) makes a good case for systematically implementing inefficiency as a feature in national policy to ensure that uniqueness and diversity are maintained throughout most regions and countries of the world. Illogical as this may seem, inefficiency for some sectors, is enshrined in all countries in cultural (animal slaughter protocols for Islamic

countries) or resilience terms (protection of high cost agriculture in Japan). Land-use policies in the EU reward more biological or organic forms of agriculture because of ecosystem advantages and local employment generation. A transition to low-carbon and renewable energy is inherently inefficient in narrow economic terms, as it requires higher infrastructure volumes and cost per unit of energy service, but its rewards are global, in eventual lowered atmospheric concentrations of greenhouse gasses.

Third and finally, since Australia's TMR is driven both by bulk and precious minerals, policies to decrease it would require that minerals exporters actively engage with destination countries to ensure that their products become the catalyst for lower material flows in those countries, and the product volumes themselves progressively decline. Hypothetically, iron ore could only be exported to give low-impact suburbs and rapid mass transit systems, rather than as feedstock for motor cars and consumer trinkets. Similarly coal and natural gas exports could contribute mainly to building renewable energy infrastructure, thus possibly requiring a more massive physical capital to ensure lower physical flows for subsequent generations. Significant potential could also exist for metals such as gold, which are mined in small quantities but represent high value, have enjoyed continued growth, and find sophisticated uses in high-tech equipment (Huleatt and Jaques 2005). Material flows from uranium mining require stringent global management at this level and similar approaches are being trialed for other environmentally sensitive metals such as lead. However it would be an anathema to the realm of global business that it should engage with its customers' futures to ensure that product volumes, both in origin and destination countries actually decrease over time.

That globalization, and its attendant inter-country commodity flows, poses the biggest challenge to institutionalizing the material flows concept and has been a regular theme in the literature since the inception of MFA methodology. Giljum et al. (2008) note the political tension between dematerialization scenarios for Europe, and trade policies for other OECD countries such as Canada, USA and Australia where material extraction and basic industries are more important. Should every country move to a 'more circular' lower impacting economy, then trade would virtually cease. Developing countries would be then stranded with little capacity, save aid monies, to progressively step out of currently constrained livelihoods. Weisz et al. (2006) come to similar conclusions in an analysis of the EU-15 where they note a number of 'natural' distinctions between even adjoining countries due to resource endowment and national area per capita. Behrens et al. (2007) cut closest to the essential issue by highlighting that the "*wealthiest regions of the world [...] are responsible for a disproportionate share of global resource extraction and use in relation to their population*". Unless wealthy countries contain their desire to growing economy and per-capita affluence, then material flows must continue to grow. In the absence of a systemic revolution in why economies exist, material flow analysis remains an excellent way to measure the sickness of the patient, but measurement alone of course does little to change the patient's behavior and then generate a trend towards robust good health.

5. Conclusion

This article has presented a structural decomposition analysis of the material flow in Australia for the period 1975-2005. Through the application of this method we are able to distinguish changing export orientation, material intensity, personal affluence, and population growth, amongst many other potential drivers, and their effects on the overall material metabolism of Australia over the past 30 years.

Between 1975 and 2005 Australian TMR and DMI has more than tripled. The major determinants for this change were found to be in order of importance: export growth; affluence growth; increasing material flows within economic structure; and population. Only material intensity was seen to have a significant retarding effect on material flow. Other structural components had small effects at the aggregate level.

At a more detailed level, however, the importance of the energy and mineral sectors in extracting mineral resources is apparent. Improvements in mining techniques have reduced material requirements, but increased exports and consumption within the economy has offset these reductions. From a consumption perspective, energy and minerals are still important, as well as some secondary production. No major impacts occur from changing demand in tertiary sectors. The lack of any significant shift in the Australian economy from minimum processed export goods to secondary and tertiary production could be seen as a failure in moving towards a knowledge based society.

The full roll-out through Australian society and business of material flow accounting and a systematic response to its implications will require change in the national growth focus of the last two human generations, the re-embrace of inefficiency in particular economic sectors, and the recantation of neo-liberal and globalised trade policies that have dominated the globe for the past decades. Because it poses such visceral challenges to global economic values, it is perhaps unlikely that material flow analysis will be institutionalized in national policy making quickly. In the interim, it would be good if greater emphasis is placed on improving the role of MFA in Australia.

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STRUCTURAL DECOMPOSITION ANALYSIS OF AUSTRALIA'S GREENHOUSE GAS EMISSIONS

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Abstract

A complex system of production links our greenhouse gas emissions to our consumer demands. Whilst progress may be made in improving efficiency, other changes in the production structure may easily nullify global improvements. Utilising a structural decomposition analysis, a comparative-static technique of input-output analysis, over a time period of around 30 years, net greenhouse emissions are decomposed in this study into the effects due to changes in industrial efficiency, forward linkages, inter-industry structure, backward linkages, type of final demand, cause of final demand, population affluence, population size, and mix and level of exports.

Historically, significant competing forces at both the whole of economy and industrial scale have been mitigating potential improvements. Key sectors and structural influences are identified that have historically shown the greatest potential for change, and would likely have the greatest net impact. Results clearly reinforce that the current dichotomy of growth and exports are the key problems in need of address.

Keywords: Australia, greenhouse gas emissions, structural decomposition analysis

1. Introduction

A complex system of production embedded in our economic structure links our levels of greenhouse gas emissions to our consumer demands. Whilst progress may be made in curtailing consumption or improving efficiency, elsewhere changes within the economy may have a nullifying effect. Thus policy formed from a whole of production/consumption perspective will likely be most effective. The macro-economic tool of input-output analysis is utilised in this study in order to capture the economy in these terms, whilst incorporating greenhouse emissions, and decomposing net impacts into different structural effects.

This study seeks to understand the changes in the factors that influence greenhouse gas emissions by decomposing overall emissions into a number of key determinants over a period of almost 30 years. A detailed model is developed using the macro-economic tool of input-output analysis. This model makes it possible to investigate inter-relationships and intra-relationships between and within sectors of the economy, levels of consumption, exports, emissions production and population. An important facet of the model is the micro-level breakdown of economic structure and energy use, with a disaggregation to 344 industries and a residential sector. This enables insight to be gained on the sections of the economy which are most responsible for emissions growth, and which could be easily addressed to reduce aggregate emissions.

The use of input-output analysis in studying environmental effects stemmed from the late 1960s (see Daly 1968; Isard et al. 1967; Ayres and Kneese 1969; Leontief and Ford 1971). Its subsequent use in structural decomposition studies initially focussed on energy studies (Proops 1988; Rose and Chen 1991; Chen and Rose 1990), but has been extended to greenhouse studies as the importance of climate change has become apparent (Casler and Rose 1998; Common and Salma 1992; Proops et al. 1993; Wier 1998; Chang and Lin 1998; De Haan 2001; de Nooij et al. 2003; Roca and Serrano 2007; Llop 2007; Wachsmann et al. 2008; Lim et al. 2009; Wachsmann et al. 2009; Weber 2009; Zhang 2009). The point of departure of this study is a level of decomposition investigating 11 different factors – including industrial efficiency, forward linkage, industrial structure, backward linkage, mix of final demand, and destination of final demand, affluence, and per-capita residential emissions, population, export mix and export level. This is a novel separation of forward and backward linkages, and affluence and population growth, for a temporal study – (compare de Naoi et al. 2003 for an inter-country study); and over a 30 year time frame. Investigation is performed at a highly detailed level for the economy (breakdown to 344 industrial sectors before subsequent aggregation).

The basic methodology and sources of data of this study is included in section 2. Results of the decomposition for Australia's emissions from 1976-2005 are presented in section 3, with subsequent analysis, before conclusions are drawn in section 4.

2. Methodology

2.1. Input-output analysis

Input-output analysis is a top-down macro-economic technique, which uses sectoral transaction data to account for the complex interdependencies of industries in modern economies. Generalisation of the basic form of input-output analysis for factors external to inter-industry transactions results in a $1 \times n$ vector of factor multipliers that is embodiments of a production factor (greenhouse emissions) per unit of final consumption of commodities produced by n industry sectors. A multiplier matrix $\mathbf{M}$ is calculated from a $1 \times n$ matrix $\mathbf{c}$ containing sectoral production factor usage, and from a $n \times n$ direct requirements matrix $\mathbf{A}$ according to

$$\mathbf{M} = \mathbf{c} (\mathbf{I} - \mathbf{A})^{-1} , \quad (1)$$

where $\mathbf{I}$ is the $n \times n$ unity matrix. $\mathbf{A}$ comprises requirements from current as well as capital intermediate demand of domestically produced and imported commodities. $(\mathbf{I} - \mathbf{A})^{-1}$ is known as the Leontief Inverse, $\mathbf{L}$, i.e.

$$\mathbf{M} = \mathbf{c} \mathbf{L} , \quad (2)$$

The factor inventory C of a given functional unit represented by a $n \times 1$ commodity inputs vector $\mathbf{y}$ and a scalar C_d of direct factor usages is then simply

$$C = \mathbf{M} \mathbf{y} + C_d . \quad (3)$$

$\mathbf{M} \mathbf{y}$ represents the indirect usage of factors embodied in all inputs into the functional unit.

An introduction into the input-output method and its application to environmental problems can be found in (Choi and Ang 2003; Leontief and Ford 1971). The mathematical formalism used to derive Equations (1) and (2) are described in detail elsewhere.

In this study, we are interested in decomposing the overall factor C into structurally important factors for overall greenhouse emissions. Hence, beginning with the original formulation:

$$C = \mathbf{c} \mathbf{L} \mathbf{y} + C_d . \quad (4)$$

$\mathbf{L}$ is decomposed into forward linkages, $\mathbf{f}$, which represent the sales of a producer to its industrial consumers, $\mathbf{S}$, industrial structure, and $\mathbf{b}$, backward linkages, or purchases of a producer from its suppliers (Lenzen 2003). Hence, defining,

$$\mathbf{f} = \widehat{\sum_{i=1:n} L_{ij}} , \quad (5)$$

$$\mathbf{S} = \frac{\mathbf{L}}{\sum_{i=1:n} L_{ij} * \sum_{j=1:n} L_{ij}}, \quad (6)$$

$$\mathbf{b} = \widehat{\sum_{j=1:n} L_{ij}} \quad (7)$$

$\mathbf{L}$ is decomposed as:

$$\mathbf{L} = \widehat{\sum_{i=1:n} L_{ij}} * \frac{\mathbf{L}}{\sum_{i=1:n} L_{ij} * \sum_{j=1:n} L_{ij}} * \widehat{\sum_{j=1:n} L_{ij}}, \quad (8)$$

So,

$$\mathbf{L} = \mathbf{fSb} \quad (9)$$

A hat signifies diagonal. $\mathbf{y}$ is decomposed into $\mathbf{uvYP} + \mathbf{gZ}$, where $\mathbf{u}$ is the $(n \times d)$ matrix of d categories of final demand (excluding exports) relating individual components of final demand to absolute levels of final demand for each category ($u_{n,d} = y_{n,d}/y_d$); $\mathbf{v}$ is the vector of length d relating categories of final demand (excluding exports) to gross national expenditure (GNE) ($v_d = y_d/Y$); Y is the per-capita measure of economic activity (GNE); P is the population; $\mathbf{g}$ is the mix of exports (length n) and Z is the magnitude of total exports. Finally, direct emissions, C_d , representing residential emissions, is decomposed into the factor $c_{res}P$, where c_{res} represents the per-capita greenhouse emission factors for residential use. In Summary Equation 3 becomes:

$$\mathbf{C} = \mathbf{cfSb}(\mathbf{uvYP} + \mathbf{gZ}) + c_{res}P \quad (10)$$

2.2. Structural decomposition analysis

The basic approach to additive (Hoekstra and van den Bergh 2003; Choi and Ang 2003) structural decompositions of a function $y(x_1, x_2, \dots, x_m)$ of m determinants is through its total differential

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \dots + \frac{\partial y}{\partial x_m} dx_m. \quad (11)$$

In this case $y(x_1, x_2, \dots, x_m) = x_1 \cdot x_2 \cdot \dots \cdot x_m$ (with the x_i being scalars, vectors or matrices),

$$dy = \prod_{j=1, j \neq 1}^m x_j dx_1 + \prod_{j=1, j \neq 2}^m x_j dx_2 + \dots + \prod_{j=1, j \neq m}^m x_j dx_m = \sum_{i=1}^m \left(\prod_{j=1, j \neq i}^m x_j dx_i \right). \quad (12)$$

Analysing discrete time series with a Divisia decomposition approach, differences Δy are obtained by integrating infinitesimal changes dy :

$$\Delta y = \int_{y_0}^{y_1} dy = \int_{x_{i,0}}^{x_{i,1}} \prod_{j=1, j \neq 1}^m x_j dx_1 + \int_{x_{2,0}}^{x_{2,1}} \prod_{j=1, j \neq 2}^m x_j dx_2 + \dots + \int_{x_{n,0}}^{x_{n,1}} \prod_{j=1, j \neq n}^m x_j dx_n = \sum_{i=1}^m \left(\int_{x_{i,0}}^{x_{i,1}} \prod_{j=1, j \neq i}^m x_j dx_i \right) \quad (13)$$

In order to compute the integral one has to know what average values y assumes while the x_i change from $x_{i,0}$ to $x_{i,1}$ (the “integral path”). Conventional Parametrical Divisia methods assume a parametrical average $\bar{y}^\alpha = y_0 + \alpha(y_1 - y_0) = y_0 + \alpha \Delta y$, with $0 \leq \alpha \leq 1$. Searching for a non-parametric method, (Ang and Choi 1997) and (Ang and Liu 2001) propose the logarithmic mean $\bar{y}^\lambda = \Delta y / \Delta(\ln y)$. The resulting Logarithmic Mean Divisia (LMD) formulation

$$\Delta y^L = \sum_{i=1}^n \frac{\Delta y}{\Delta(\ln y)} \ln \frac{x_{i,1}}{x_{i,0}} \quad (14)$$

is non-parametric, exact, and time-reversible (for proofs see (Ang and Choi 1997), and (Ang et al. 1998), for overview see (Lenzen 2006) and (Hoekstra and van den Bergh 2002) and (Janssen et al. 2001). The issue of handling zero and negative values in the logarithmic term has been addressed in (Wood and Lenzen 2006; Ang and Liu 2007a, 2007b).

Applying equation (13) to equation (3), gives:

$$\begin{aligned} \Delta C &= \Delta \mathbf{cfSb}(\mathbf{uvYP} + \mathbf{gZ}) \\ &+ \mathbf{c} \Delta \mathbf{fSb}(\mathbf{uvYP} + \mathbf{gZ}) \\ &+ \mathbf{cf} \Delta \mathbf{Sb}(\mathbf{uvYP} + \mathbf{gZ}) \\ &+ \mathbf{cfS} \Delta \mathbf{b}(\mathbf{uvYP} + \mathbf{gZ}) \\ &+ \mathbf{cfSb}(\Delta \mathbf{uvYP}) \\ &+ \mathbf{cfSb}(\mathbf{u} \Delta \mathbf{vYP}) \\ &+ \mathbf{cfSb}(\mathbf{uv} \Delta \mathbf{YP}) \\ &+ \mathbf{cfSb}(\Delta \mathbf{gZ}) \\ &+ \mathbf{cfSb}(\mathbf{g} \Delta \mathbf{Z}) \\ &+ \Delta c_{res} P \\ &+ (\mathbf{cfSbu} \mathbf{vY} + c_{res}) \Delta P \end{aligned} \quad (15)$$

where each factor change (the delta term) is calculated according to equation 14.

The terms of Eq (14) are named by their difference factor (i.e. the first term, is hence named $\Delta \mathbf{c}$), and described in Table 1.

Factor	Description
	The change in total greenhouse gas emissions reflected in the changes in:
Δc	Greenhouse gas efficiency of industrial production.
Δf	Forward linkage.
ΔS	industrial structure.
Δb	Backward linkage.
Δu	in the mix of final demand of goods and services from the industries.
Δv	the relative change in destination of final demand.
ΔY	the change in the per-capita GNE.
Δc_{res}	per-capita direct greenhouse gas emissions
ΔP	population.
Δg	the mix of exports.
ΔZ	the total level of exports.
ΔC	The total change in emissions (summation of above terms)

Table 1. Description of structural decomposition components

2.3. Data Sources

Data sources for economic data (f, S, b, u, v, g, Z) was the Australian Bureau of Statistics publication 5209.0. Seventeen sets of tables have been published by the ABS, with the years being 1974-75, 1977-78, 1979-79, 1979-80, 1980-81, 1981-82, 1982-83, 1983-84, 1986-87, 1989-90, 1992-93, 1993-94, 1994-95, 1996-97, 1998-99, 2001-02, 2004-05. These tables were moulded into a true time series by combining Supply/Use information and National Accounts data. This was done to smooth out any changes in classification and methodology. A full description of the process is outlined in Wood (2009). The implementation of Supply/Use tables in 1994-5, and the change in SNA for the 1994-5 tables resulted in quite a significant change that introduced too much error, and this change has been omitted from this analysis. Price Indices by industry were needed to deflate subsequent years data, and national accounts data was further used to balance estimates (Wood 2009). The source for the data on greenhouse gas emissions was the National Greenhouse Gas Inventory (NGGI) (Australian Greenhouse Office 2007), and the application to the input-output system has previously been defined. A description of allocation techniques used is available elsewhere (Wood and Dey 2009). Population data was obtained from the ABS (Australian Bureau of Statistics 2008).

3. Results and Analysis

A summary of the decomposition of the total change in greenhouse gas emissions over time according to Eq 7 is presented in Figure 1, and for a rolling average coverage of years, and under accounting schemes of fuel combustion only and Kyoto in Table 2. Each column corresponds to the parameters of the decomposition described in Table 1.

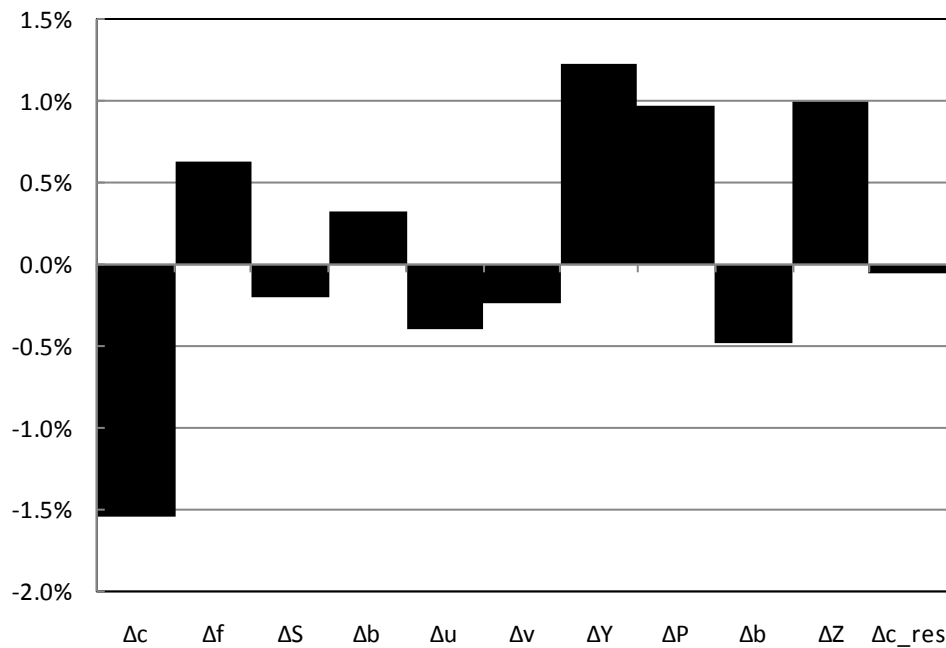


Figure 1 Decomposition parameters contributing to total change in emissions, both fuel combustion and other emissions, Australia, 1976-2005

At this macro-level, changes in industrial efficiency, final demand mix, final demand destination and export mix were found to generally have a negative forcing on emissions. The change in industrial efficiency was the strongest component of this group particularly when only referring to fuel combustion emissions. In contrast, changes in structural interrelationships, overall economic growth, population and export level had large positive forcing on emission levels. The effect of the oil crisis in the late 1970s was seen to have a significant reduction in the direct energy use by industry (Δc). The residential effect over the whole time series was less pronounced, with per-capita emissions slightly decreasing. Overall, the influence of population on emission levels was a fairly consistent 1%. Diverging results occur for some factors between fuel combustion results and all types of greenhouse gases (Table 2). These differences occur mainly in the structural component of the economy, and in the type of final demand and are due to the changing importance of agricultural and land use change emissions across certain years. Where negative changes are seen under Kyoto accounting but not fuel combustion, this shows less demand for the goods and services with high embodiments of non-energy emissions.

A further decomposition was performed at the final demand level over the time series. A graphical representation of the all factors by industry is included in Figure 2 for the last decade (1995-2005). Industries are ordered such that primary industries are left most, and service (tertiary) industries are rightmost. Industries are aggregated to 30 sectors.

		Industrial				Expenditure			Population	Export		Residential
		Δc	Δf	ΔS	Δb	Δu	Δv	ΔY	ΔP	Δg	ΔZ	Δc_{res}
Fuel Combustion Emissions only	1976-1981	-4.5%	7.8%	-4.8%	3.2%	0.2%	-0.2%	0.6%	1.0%	-0.1%	-0.1%	-0.1%
	1978-1983	-2.4%	2.4%	-0.3%	-0.1%	-0.9%	0.2%	1.1%	1.1%	0.4%	0.1%	-0.1%
	1980-1985	-3.6%	-0.2%	0.1%	1.1%	1.8%	-0.3%	1.5%	1.2%	0.1%	0.0%	-0.1%
	1982-1987	-0.4%	-0.1%	0.4%	0.1%	0.3%	-0.2%	0.5%	1.1%	-0.7%	0.9%	0.0%
	1984-1989	-1.1%	2.6%	-0.5%	-0.1%	-2.2%	0.0%	2.6%	1.3%	-0.2%	1.2%	0.0%
	1986-1991	0.0%	1.4%	-0.3%	-0.7%	-1.1%	0.4%	1.2%	1.2%	-0.1%	1.3%	0.0%
	1988-1993	0.2%	-2.7%	0.7%	-0.7%	2.5%	-0.6%	0.6%	1.1%	0.2%	0.9%	0.1%
	1990-1995	-0.8%	-0.6%	-1.6%	2.1%	3.6%	-0.2%	0.8%	0.9%	0.2%	0.9%	0.1%
	1992-1997	-0.5%	1.9%	-2.7%	2.1%	-0.8%	-0.4%	2.2%	0.9%	-0.9%	1.2%	0.0%
	1994-1999	-3.5%	4.2%	-4.5%	3.4%	-0.4%	0.4%	2.5%	0.9%	-1.2%	1.2%	0.0%
	1996-2001	-0.9%	0.7%	-0.8%	0.0%	-1.7%	-0.4%	1.8%	0.9%	-0.3%	1.4%	0.0%
	1998-2003	-1.0%	0.4%	-1.2%	0.4%	-0.6%	-0.1%	1.7%	0.9%	0.3%	1.1%	-0.1%
	2000-2005	0.0%	0.5%	-0.6%	0.2%	-1.1%	0.1%	1.4%	0.7%	0.2%	0.4%	-0.1%
All types of GHG emissions (Kyoto Accounting)	1990-1993	0.0%	-5.6%	2.5%	-1.0%	2.9%	-2.4%	-0.6%	0.8%	-0.4%	1.5%	-0.2%
	1992-1995	2.6%	-1.4%	1.9%	-2.0%	-2.9%	-1.8%	1.2%	0.5%	-0.7%	0.8%	0.0%
	1994-1997	1.5%	-2.8%	-0.6%	0.2%	-1.8%	-0.5%	0.8%	0.6%	-0.5%	1.7%	0.0%
	1996-1999	-1.2%	2.8%	-0.7%	-0.6%	-3.6%	0.4%	2.3%	0.8%	-0.6%	1.2%	0.0%
	1998-2001	-3.3%	0.7%	-0.7%	0.5%	2.7%	-1.2%	1.1%	0.9%	-1.0%	2.3%	-0.1%
	2000-2003	-0.4%	0.8%	-1.2%	0.8%	-1.6%	0.2%	1.1%	0.9%	-0.8%	0.9%	-0.1%
	2002-2005	-1.5%	-4.0%	4.0%	0.3%	-5.8%	4.8%	2.8%	0.8%	-0.8%	-0.1%	0.1%
Average Fuel Comb		-1.7%	1.0%	-1.0%	0.8%	0.1%	-0.1%	1.4%	1.0%	-0.2%	0.8%	0.0%
Average ALL GHG		-0.5%	-2.1%	1.6%	-0.4%	-1.1%	-0.3%	1.3%	0.8%	-0.7%	1.4%	-0.1%

Table 2 Decomposition of parameters contributing to total change in emissions, Australia, 1976-2005. Negative changes highlighted.

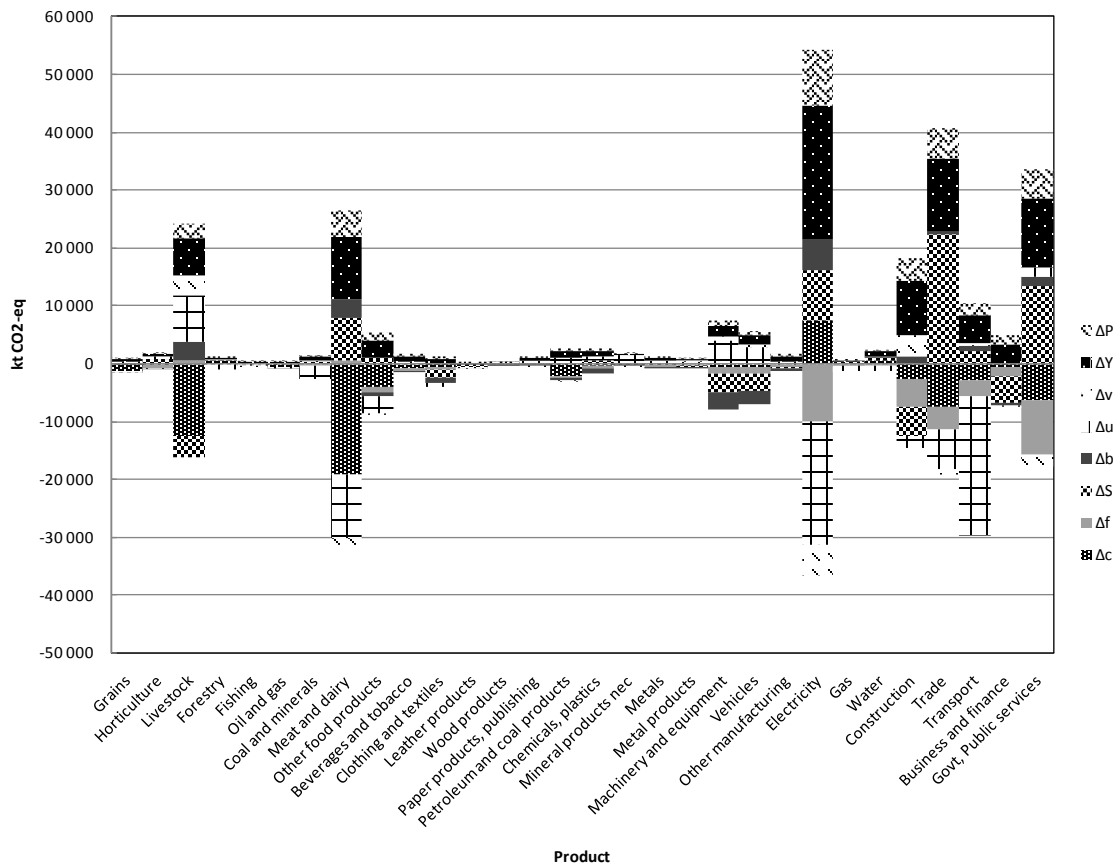


Figure 2 Summary of contribution to change in emissions by industry, 1995-2005, domestic final demand only.

One of the main results out of the analysis is that the majority of these changes (in terms of both number and magnitude) are occurring in ‘end use’ sectors such as services and transport, depicted on the right of Figure 2. Thus to achieve easy emission reductions, policy focus should be placed on these sectors, both in terms of the inputs into production for the industries (particularly the structural effects of f and S), and in terms of final demand for their products (u), as these parameters contributed most to change, independent of the affluence and population effects.

In the utilities sector, electricity supply is strongly represented, consistently representing the largest contribution to change in overall emissions across each period of the full time series. Construction is also consistently represented, as are the transport sectors. Within the service sector, government administration, defence, education, health and community (or welfare) services have consistently contributed to change, often forcing emissions higher.

For exports, the significant industries are meat products, which in earlier years had a negative forcing on emissions, and in later years had more of a positive forcing. Petroleum and coal products had a perceptible but intermittent effect, whilst the transport industries, particularly road transport consistently influenced the levels of emissions. Industries that were more primary in production showed their importance in this decomposition, due to the nature of most of Australia’s exports.

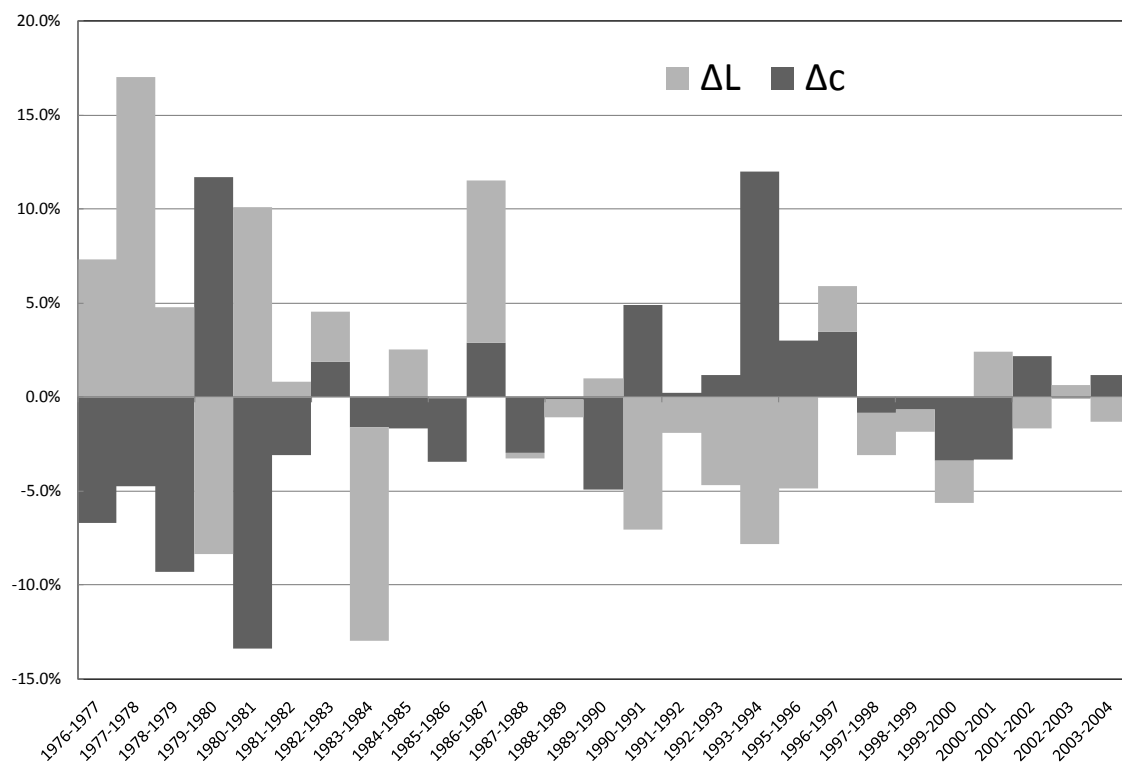


Figure 3 Percentage contributions of structural change and industrial efficiency to overall emissions, by year.

It was also found that variances in the influence of the factors at the industrial level were considerable over the time series, implying that considerable gains could be made in reducing emissions by developing policy that exploits the maximum influence of the different parameters at the same time.

A final important result was that the influence of the industrial efficiency (ϵ) and overall industrial structure ($\Delta L = \Delta f + \Delta S + \Delta b$) parameters often showed large and opposite impacts in the same period (specifically when looking at fuel combustion emissions only - Figure 3). This is a result of the substitution of direct energy inputs for non-energy inputs, which still embody high levels of emissions. In order to have an aggregate negative impact on emission levels, it is important that both these considerations are taken into account. This result also shows the importance of using the methodology employed, which is able to account for embodied emissions in the production process, whereas a normal intensity analysis of emissions production would not capture this effect.

4. Discussion

This analysis adds further weight to the fact that efficiency measures are not making up for our increasing demands from the economy. If we want to stabilise emissions, we cannot continue growing population and consumption levels at a rate that the economy cannot adapt to. Improvements in emissions intensity of 0.5-1.7% p.a. (depending on accounting) have not so far been sufficient to make up for our evolving economy and levels of consumption. The current dichotomy of pushing products with high resource and greenhouse gas emissions embodied straight to export must be changed for Australia to move towards any semblance of sustainability (compare Wood et al. 2009).

Further, these results show how important it is to act in a life cycle context. Action on greening energy technologies will only have limited benefit if industry substitutes from greener energy providers to dirtier industrial providers. This was most evident in the oil crisis of the 1970's, but also in most other years, when consistent improvements in greenhouse gas intensity closely matched opposite effects in 'burden shifting' within the technology of the economy (represented here by the Leontief matrix). Whilst cap and trade policies should prevent this type of activity from happening, we need to be observant of not only burden shifting from one industry to another, but also to other environmental and social impacts not explicitly included in carbon frameworks.

5. Conclusion

In this study, the extent to which structural economic change has influenced levels of greenhouse gas emissions has been investigated for Australia's recent history. A disaggregation of total changes by inter-industrial and environmental-industrial interrelationships was undertaken through the development of a model based on input-output and structural decomposition analysis.

At a macro-level, changes in industrial efficiency, final demand mix, final demand destination and export mix were found to decrease emissions. The change in industrial efficiency was the strongest component of this group, particularly for fuel combustion emissions. In contrast, changes in structural interrelationships, affluence and export level strongly increased emissions. The residential effect was less pronounced, with emissions per-capita slightly reducing. Overall, the influence of population growth on emission levels was around a fairly consistent +1% p.a.

At the disaggregated level, the importance of the electricity supply, construction, and transport and service industries to changing emission levels was confirmed, but no industry showed a constant effect over the full time series. A further important result was that there appeared to be a high level of substitution between energy and structural inputs, with diminished effect on total emissions. This suggests that the real gains in energy efficiency would not be as great as an energy intensity analysis would show, and that the consideration of impacts embodied in production was needed to capture all effects.

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ZERO-VALUE PROBLEMS OF THE LOGARITHMIC MEAN DIVISIA INDEX DECOMPOSITION METHOD

Richard Wood and Manfred Lenzen

Wood, R. and M. Lenzen. 2006. Zero-value problems of the logarithmic mean divisia index decomposition method. *Energy Policy* 34(12): 1326-1331.

Abstract

Recently, the Logarithmic Mean Divisia Index (LMDI) approach to energy decomposition has been espoused as the preferred indexing method. Whilst the LMDI method provides perfect decomposition, and is time-reversal invariant, its strategy to handle zero-values is not necessarily robust. In order to overcome this problem, it has been recommended to substitute a small value $\delta = 10^{-10}$ to 10^{-20} for any zero values in the underlying data set, and allow the calculation to proceed as usual. The decomposition results are said to converge as δ approaches zero. However, we show that under this recommended procedure the LMDI can produce significant errors if applied in the decomposition of a data set containing a large number of zeroes and/or small values. To overcome this problem, we recommend using the analytical limits of LMDI terms in cases of zero values. These limits can be substituted for entire computational loops, so that in addition to providing the correct decomposition result, this improved procedure also drastically reduces computation times.

Keywords: Logarithmic Mean Divisia Index, Structural Decomposition Analysis, zero values

1. Introduction

In recent articles by Ang and colleagues (Ang 2004a; Ang et al. 2003; Ang 2004b; Ang et al. 1998), the Logarithmic Mean Divisia Index (LMDI) approach to energy decomposition analysis has been espoused as the preferred indexing method. Whilst the LMDI provides perfect decomposition, and is time-reversal invariant, its strategy to handle zero-values is not necessarily robust. In order to overcome this problem, it has been recommended to substitute a small value $\delta = 10^{-10}$ to 10^{-20} for any zero values in the underlying data set, and allow the calculation to proceed as usual (Ang 2004a; Ang et al. 1998, 4th page). The decomposition results are said to converge as δ approaches zero (Ang and Choi 1997, p68).

In this article we show that this procedure can lead to significant errors in decompositions of data sets containing a large number of zeroes and/or small values.¹ In the following, we first briefly review the LMDI methodology, then demonstrate the zero-value problem, and finally offer a remedy.

2. Methodology

The basic approach to additive² structural decompositions of a function $y(x_1, x_2, \dots, x_n)$ of n determinants is through its total differential

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \dots + \frac{\partial y}{\partial x_n} dx_n. \quad (1)$$

In case $y(x_1, x_2, \dots, x_n) = x_1 \cdot x_2 \cdot \dots \cdot x_n$ (with the x_i being scalars, vectors or matrices),

$$dy = \prod_{j=1, j \neq 1}^n x_j dx_1 + \prod_{j=1, j \neq 2}^n x_j dx_2 + \dots + \prod_{j=1, j \neq n}^n x_j dx_n = \sum_{i=1}^n \left(\prod_{j=1, j \neq i}^n x_j dx_i \right). \quad (2)$$

Analysing discrete time series with a Divisia decomposition approach, differences Δy are obtained by integrating infinitesimal changes dy :³

¹ This statement can refer to either Index Decomposition Analysis (IDA) or Structural Decomposition Analysis (SDA). We note that the LMDI was originally conceived for applications in IDA, where the number of zero values is generally less than in SDA.

² For a comparative review see (Hoekstra and van den Bergh 2003) and (Choi and Ang 2003).

³ We denote $y_0 = y(t=0)$, $y_1 = y(t=1)$, $x_{i,0} = x_i(t=0)$, and $x_{i,1} = x_i(t=1)$.

$$\begin{aligned}\Delta y &= \int_{y_0}^{y_1} dy = \int_{x_{i,0}}^{x_{i,1}} \prod_{j=1, j \neq 1}^n x_j dx_1 + \int_{x_{2,0}}^{x_{2,1}} \prod_{j=1, j \neq 2}^n x_j dx_2 + \dots + \int_{x_{n,0}}^{x_{n,1}} \prod_{j=1, j \neq n}^n x_j dx_n = \sum_{i=1}^n \left(\int_{x_{i,0}}^{x_{i,1}} \prod_{j=1, j \neq i}^n x_j dx_i \right) \\ &= \sum_{i=1}^n \left(\int_{x_{i,0}}^{x_{i,1}} \prod_{j=1}^n x_j \frac{dx_i}{x_i} \right) = \sum_{i=1}^n \left(\int_{x_{i,0}}^{x_{i,1}} y(\dots, x_i, \dots) \frac{dx_i}{x_i} \right) = \sum_{i=1}^n \bar{y}([x_{i,0}, x_{i,1}]) \ln \frac{x_{i,1}}{x_{i,0}}.\end{aligned}\quad (3)$$

In order to compute the integral one has to know what average values y assumes while the x_i change from $x_{i,0}$ to $x_{i,1}$ (the “integral path”). Conventional Parametrical Divisia methods assume a parametrical average $\bar{y}^\alpha = y_0 + \alpha(y_1 - y_0) = y_0 + \alpha\Delta y$, with $0 \leq \alpha \leq 1$. Searching for a non-parametric method, Ang and Choi (1997) and (Ang and Liu 2001) propose the logarithmic mean $\bar{y}^\lambda = \Delta y / \Delta(\ln y)$. The resulting Logarithmic Mean Divisia (LMD) formulation

$$\Delta y^L = \sum_{i=1}^n \frac{\Delta y}{\Delta(\ln y)} \ln \frac{x_{i,1}}{x_{i,0}} \quad (4)$$

is non-parametric, exact, and time-reversible (for proofs see Ang and Choi (1997), and Ang et al., (1998)).⁴ It is also said to be zero-robust if a small value $\delta = 10^{-10}$ to 10^{-20} is substituted for any zero values (Ang and Choi 1997, p68; Ang 2004a; Ang et al. 1998, 4th page). However, in our opinion while the latter statement is theoretically valid, it does not necessarily work under any circumstance.

3. Zero-value issues

Case	y_0	y_1	x_0	x_1	$\Delta y^L = \sum_{i=1}^n \frac{\Delta y}{\Delta(\ln y)} \ln \frac{x_i}{x_0}$
1	0	PN	0	PN	y_1
2	PN	0	PN	0	$-y_0$
3	0	PN	PN	PN	0
4	PN	0	PN	PN	0
5	0	0	PN	PN	0
6	0	0	0	0	0
7	0	0	PN	0	0
8	0	0	0	PN	0

⁴ Even compared to its main non-parametrical competitor, the Dietzenbacher/Sun/Albrecht method (Albrecht et al. 2002; Dietzenbacher and Los 1998; Sun 1998), it has the advantage of being computationally less demanding. The Mean-Rate-of-Change non-parametrical decomposition method suggested by (Chung and Rhee 2001) is equivalent to the LMDI once logarithmic weights are used (Lenzen 2004).

Table 1: Limits of the LMDI weights for eight possible cases of zeros in the underlying data set. PN = positive number (from (Ang et al. 1998), p. 492).

Ang et al. (1998, p. 492) give eight situations in which zero values can occur in Eq. 4 (see Tab. 1).⁵ Evaluating Ang et al.’s case 1 for the example of $y_0 = x_0 = 0$, $x_1 = 1$, and $y_1 = \{2, 10, 100\}$ shows that – because of the slow convergence of the term in Eq. 4 – errors in the Δy^L term are not negligible, even if δ is reduced to near the limit of double precision (approximately 10^{-323}), and especially if the difference between x_1 and y_1 is large (Fig. 1). Similar results can be produced for the remaining cases.

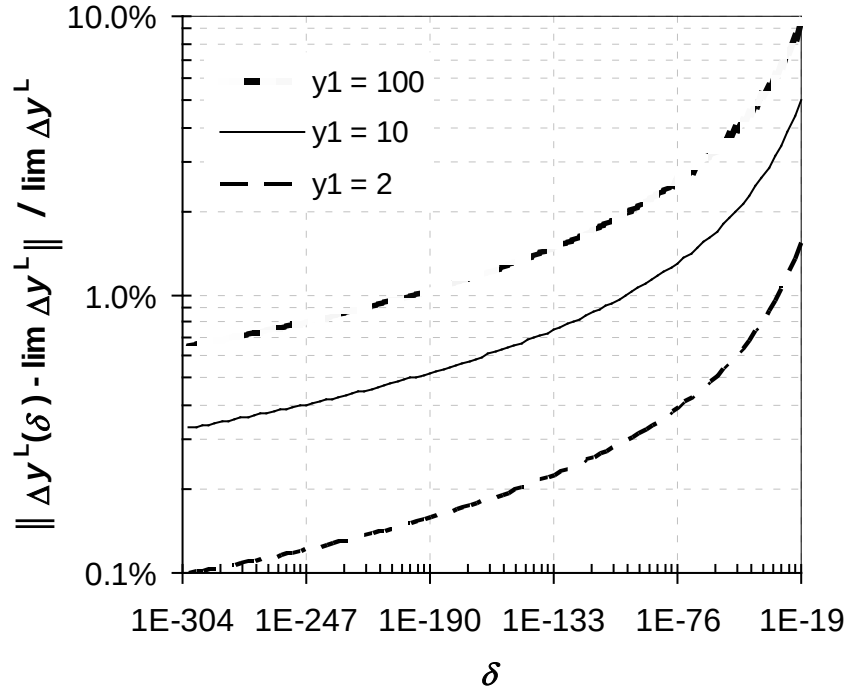


Fig. 1: Relative deviation $\| \Delta y^L(\delta) - \lim_{\delta \rightarrow 0} \Delta y^L \| / \lim_{\delta \rightarrow 0} \Delta y^L = \| \Delta y^L(\delta) - y_1 \| / y_1$, for Ang et al.’s case 1: $y_0 = x_0 = 0$, $x_1 = 1$, and $y_1 = \{2, 10, 100\}$, as a function of “small values” δ below Ang et al.’s recommended setting. Even with δ at the limit of double precision, errors can be significant.

This problem may not be discernible when analysing data sets that contain only a small number of zero values. However, in SDA it is often the case that the data contain a sufficient number of zero values. Under this circumstance, discrepancies will show up as a kind of “background noise” (essentially the cumulative errors in the Δy^L). This noise can constitute a significant fraction of results from a correct structural decomposition, especially when

- a) zero values outnumber non-zero values;

⁵ The remaining possible combinations can be excluded, since $x_0 = 0 \Rightarrow y_0 = 0$, and $x_1 = 0 \Rightarrow y_1 = 0$.

b) apart from the zero values, the data set contains many small values, the

contribution of which gets buried in the “noise”;

c) there are large changes in the data set over the period of study (Fig. 1).

We came across inconsistencies while carrying out an SDA of CO₂ emissions E from energy consumed in the Brazilian and Australian economies, factorised as $E = \mathbf{c} \mathbf{F} \mathbf{L} \mathbf{y}$ and $E = \mathbf{c} \mathbf{F} (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \dots) \mathbf{y}$, where $\mathbf{y}$ is final demand, $\mathbf{A}$ is the direct requirements matrix calculated from the input-output tables, $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse, $\mathbf{F}$ is the industrial fuel intensity matrix, and $\mathbf{c}$ is the greenhouse gas content of fuels (basically a summation vector of constant weights). These and other factorisations can contain a large number of zero and small values, when

- 1) industries use only a limited number of fuel types ($\mathbf{F}$),
- 2) industries do not – at least directly – interact much with each other ($\mathbf{A}$), and
- 3) $\mathbf{A}$ and $\mathbf{L}$ are in their make-use formulation.

The latter point refers to production and consumption of commodities and in industries being represented in one matrix, containing two separate, off-diagonal, non-zero blocks and two diagonal zero blocks (Fig. 2). In addition, the make matrix is usually similar to a unity matrix and therefore sparse.

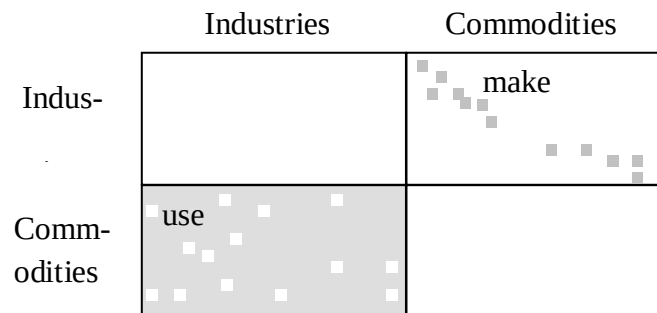


Fig. 2: Schematic of a make-use framework. White fields represent zero values.

4. An empirical example

In order to support Fig. 1 with an empirical example we will examine the effect of the zero elements in the 1994-95 and 1996-97 Australian fuel use matrices $\mathbf{F}_1$ and $\mathbf{F}_2$, and direct requirements matrices $\mathbf{A}_1$ and $\mathbf{A}_2$. For example in the $\mathbf{A}_i$, a surprisingly large number of coefficients are zero, and also most of the coefficient value is concentrated in a few elements, causing most elements in $\mathbf{A}_i$ to be considerably smaller than their average (Fig. 3).

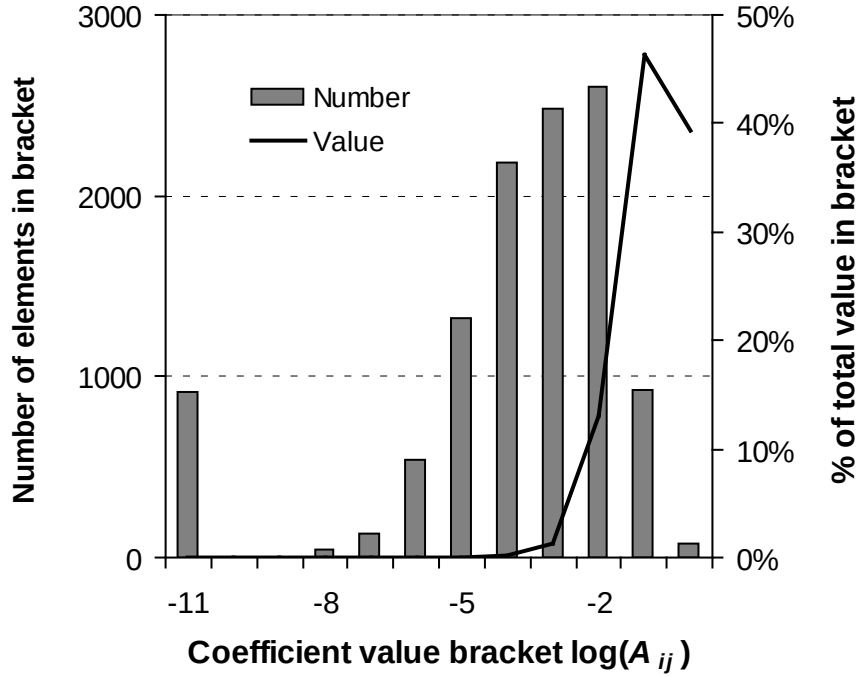


Fig. 3: Distribution of element numbers and values in the 1994/95 Australian direct requirements matrix A , across size brackets. Almost 1000 out of 11,236 elements are smaller than 10^{-10} (vertical bars, left y-axis). Most of the total coefficient value is concentrated in about 1000 elements of size 0.01 and larger (solid curve, right y-axis).

Complementing the direct requirements matrix A with CO₂ intensities c F , we carry out a structural decomposition of $E = c F A y$, which is the second term in the Taylor expansion two paragraphs above. We calculate

$$\Delta E(F) = \int \sum_{lm} \frac{\partial E}{\partial F_{lm}} dF_{lm} = \sum_{lm} \int \frac{\partial \left(\sum_{ijk} c_i F_{ij} A_{jk} y_k \right)}{\partial F_{lm}} dF_{lm} = \int \sum_{lmk} c_i F_{lm} A_{mk} y_k \frac{dF_{lm}}{F_{lm}}$$

which after integration is

$$= \sum_{lmk} \overline{c_i F_{lm} A_{mk} y_k}^\lambda \ln \frac{F_{lm,1}}{F_{lm,0}} = \sum_{lmk} \overline{E_{lmk}}^\lambda \ln \frac{F_{lm,1}}{F_{lm,0}}, (5)$$

and similarly

$$\Delta E(A) = \sum_{imn} \overline{c_i F_{im} A_{mn} y_n}^\lambda \ln \frac{A_{mn,1}}{A_{mn,0}} = \sum_{imn} \overline{E_{imn}}^\lambda \ln \frac{A_{mn,1}}{A_{mn,0}}, (6)$$

and

$$\Delta E(\mathbf{y}) = \sum_{ijn} \overline{c_i F_{ij} A_{jn} y_n}^\lambda \ln \frac{y_{n,1}}{y_{n,0}} = \sum_{ijn} \overline{E_{ijn}}^\lambda \ln \frac{y_{n,1}}{y_{n,0}}. \quad (7)$$

Since $\mathbf{c}$ is constant, $\Delta E(\mathbf{c}) = 0$. The total effect is $\Delta E = \Delta E(\mathbf{F}) + \Delta E(\mathbf{A}) + \Delta E(\mathbf{y})$.

In our analysis we pursue two parallel strategies: First, we substitute any zero value with a “small value” δ , ranging from 10^{-10} and 10^{-50} , and calculate various ΔE_δ . Second, we shortcut all loops over i, k, l, m , and n in Eqs. 5-7 that contain any of the possibilities listed in Tab. 1, and directly use the analytical limit (right column of Tab. 1) in calculating the correct ΔE_0 .

We quantify the error caused by using Ang et al.’s “small-value” strategy in terms of ten measures: Four are the relative errors of the aggregate structural decomposition results ($\Delta E_{\text{rel}}(\mathbf{F}) = [\Delta E_\delta(\mathbf{F}) - \Delta E_0(\mathbf{F})] / \Delta E_0(\mathbf{F})$, and analogous expressions for $\Delta E(\mathbf{A})$, $\Delta E(\mathbf{y})$, and ΔE). The remaining measures aim at establishing a metric of relative distance between the ΔE_δ and

ΔE_0 in matrix form, defined as $\{\Delta E_\delta(\mathbf{F})\}_{lm} = \sum_k \overline{E_{lmk}}^\lambda \ln \frac{F_{lm,1}}{F_{lm,0}}$, and analogously for the terms

in Eqs. 6 and 7. According to Butterfield and Mules (1980), p. 293), “there exists no single statistical test for assessing the accuracy with which one matrix corresponds to another. Analysts working in this area have tended to use a number of [complementary] tests.” Accordingly, we chose two recommended metrics (Lenzen et al. 2004; Harrigan et al. 1980):

$$\text{the relative arithmetic mean of absolute errors } AMAE = \frac{\sum_{ij} |\Delta E_{0ij} - \Delta E_{\delta ij}|}{\sum_{ij} \Delta E_{0ij}}, \text{ and}$$

$$\text{the arithmetic mean of relative errors } AMRE = \frac{\sum_{ij} \frac{|\Delta E_{0ij} - \Delta E_{\delta ij}|}{|\Delta E_{\delta ij}|}}{N},$$

where N is the number of summands with $\Delta E_{ij} \neq 0$.

The $AMAE$ is still a measure of *overall* error within the ΔE_δ , since it describes the importance of all cumulative errors relative to the total effect ΔE_0 . The difference between the $AMAE$ and the aggregate relative errors ΔE_{rel} is that in the relative errors positive and negative contributions cancel out, while the $AMAE$ accumulates the absolute value of the errors; hence $AMAE > \Delta E_{\text{rel}}$.

The $AMRE$ reflects the relative error at the level of single constituents $\Delta E_{\delta ij}$, but averaged over the entire matrix. Note that we have chosen ΔE_δ as the reference in the denominator; this was done in order to be able to capture the very errors due to substituting zero values with δ .

These errors would not have shown up if ΔE_0 had been the denominator, since cases with $\Delta E_0 = 0$ must be excluded from the sum.

5. Results

In 1994-95 and 1996-97 the total amounts of emissions $E = \sum_{ijk} c_i F_{ij} A_{jk} y_k$ were 83.0 Mt CO₂ and 90.2 Mt CO₂, respectively, yielding a change of $\Delta E = 7.15$ Mt CO₂. A correct decomposition using our shortcut strategy yields $\Delta E(\mathbf{F}) = 112$ kt, $\Delta E(\mathbf{A}) = 26$ kt, and $\Delta E(\mathbf{y}) = 7,015$ kt, so that most of the change was actually driven by final demand $\mathbf{y}$, and to minor degrees by changes in energy intensity and fuel mix ($\mathbf{F}$) and economic structure ($\mathbf{A}$). Inserting $\delta = 10^{-10}$ for all zeroes, the decomposition yields $\Delta E(\mathbf{F}) = 103$ kt, $\Delta E(\mathbf{A}) = 29$ kt, and $\Delta E(\mathbf{y}) = 7,022$ kt.

Varying δ , we found that – as expected from Fig. 1 – errors did not converge rapidly enough to zero, and significant errors remained even at $\delta = 10^{-50}$ (Fig. 4). The largest relative errors – around 10% – were naturally registered for the smallest change term $\Delta E(\mathbf{A})$, followed by $\Delta E(\mathbf{F})$ with ΔE_{rel} and $AMAE$ around 1%, but $AMRE$ near 100%. Only the largest contributor to overall change $\Delta E(\mathbf{y})$ is represented with reasonable accuracy, errors being between 0.001% and 0.1%.

The crucial finding is that none of these errors decreases sufficiently rapidly as δ approaches zero, so that in cases where data sets contain a sufficient number of zeroes and/or small values, Ang et al.’s “small-value” strategy will always yield flawed results, no matter how small δ .

In addition to providing the correct decomposition result, this improved procedure also drastically reduces computation times for the LMDI, especially when zero values are numerous and/or when the function to be decomposed is a product of many factors and/or large matrices. In the latter case, many deeply nested multiple summations can be skipped whenever one of the eight cases in Tab. 1 occurs, and simply shortcut by using the respective limit directly. Reductions in computing time can be estimated as follows: A decomposition involving n determinants, each of row size R_i , then the number of loops λ in a SDA of the product of these n determinants is $\lambda = \prod_{i=1}^n R_i$. Inside the innermost loop, n terms (one Δ -terms for each factor) have to be evaluated. Each Δ -term requires $2(n - 1) + 8$ operations⁶, so

⁶ In terms of Eq. 5: $n - 1 = 3$ multiplications for $E_{lmk} = c_l F_{lm} A_{mk} y_k$, and $2(n - 1)$ operations for evaluating $E_{lmk,0}$ and $E_{lmk,1}$. One operation each for the difference ΔE , the logarithms $\ln E$, and the difference of the

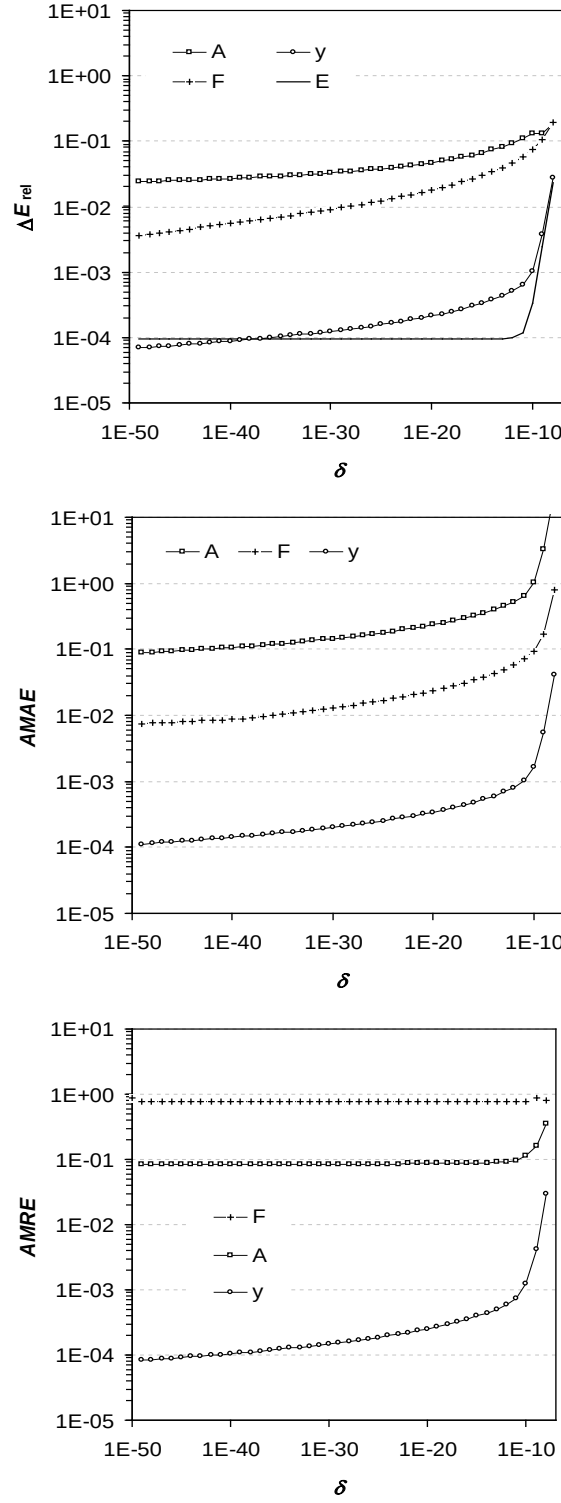


Fig. 4: ΔE_{rel} , $AMAE$ and $AMRE$ of decomposition terms for F , A and y , as a function of the “small value” δ .

logarithms $\Delta \ln E$ increases the total to $2(n - 1) + 4$. Calculating $\overline{E_{lmk}}^\lambda$ requires a further division, calculating $\ln F_{lm,1}/F_{lm,0}$ requires two operations, and including the final multiplication, the total number of operations is $2(F - 1) + 8$.

that the total number of operations is $\omega = \lambda \times n \times 2(n - 1) + 8$. For the example in this paper, $n = 3$, $R_1 = 30$, $R_2 = 107$, and $R_3 = 107$, so that $\lambda = 343,470$ and $\omega = 12,364,920$. Note that this calculation neglects accesses to memory, and sub-operations for evaluating logarithms. Assuming that a desktop computer carries out about 10^8 such operations per second, the computing time for an SDA as in this paper is below one second. However, SDAs involving more factors and larger matrices are not uncommon: In recent case studies of decomposing a CO₂ calculation involving Brazilian Social Accounting Matrices (Wachsmann, 2005) or Australian make-use matrices (Wood 2009), $\omega_{Br} \approx 9 \times 10^{10}$ and $\omega_{Au} \approx 10^{17}$, resulting in computing times of $\frac{1}{2}$ h and 10^5 h, respectively. Considering that in CO₂ and energy compositions, fuel mix, energy use and final demand matrices are sparse, skipping zero loops can reduce these computing times significantly: We calculated that if in three matrices 50% of elements are zero, the Brazilian case reduces by a factor of almost 10, and if in five matrices 90% of elements are zero, the Australian case reduces by a factor of 10^5 .

6. Conclusions

The Logarithmic Mean Divisia Index (LMDI) decomposition – at least under the recommended procedure – may be theoretically zero-value robust, but cannot handle zero values if the data set to be decomposed contains a large number of zeroes and/or small values. Substituting a small value $\delta = 10^{-10}$ to 10^{-20} for any zero values, and allowing the calculation to proceed as usual can lead to significant errors.

To overcome this problem, we recommend using the limits published by Ang et al. (1998, reproduced here) for the eight possible cases of zero values (see Tab. 1, right column). These limits can be substituted for entire computational loops representing multiple summations as in Eqs. 5-7. In addition to providing the correct decomposition result, this improved procedure also drastically reduces computation times for the LMDI, especially when zero values are numerous and/or when the function to be decomposed is a product of many factors and/or large matrices.

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STRUCTURAL PATH DECOMPOSITION

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Abstract

We combine Structural Decomposition Analysis (SDA) and Structural Path Analysis (SPA) in order to examine the temporal changes within a full production chain perspective. To our knowledge this work constitutes the first formulation of what we call Structural Path Decomposition (SPD). SPD provides noteworthy insight in two instances: First it extracts and ranks those interactions within an economy that are most important in driving change; second it provides a temporal perspective to standard input-output-based Life-Cycle Assessment. In this paper, we develop the mathematical model of SPD and provide two case studies of the most important changes in structural paths in Australia from 1995 to 2005.

Keywords: Input-output analysis; Structural Decomposition Analysis; Structural Path Analysis; Life-Cycle Assessment

1. Introduction

In this article, we combine two well-established techniques in input-output analysis, Structural Decomposition Analysis (SDA) and Structural Path Analysis (SPA). The motivation for this work is twofold – 1) to aid policy applications from SDA, by enabling the tracing of the change in key production chains over time; 2) to aid input-output life-cycle analysis techniques by modelling temporal developments in production chains.

Aggregate indicators often hide many competing trends within an economy, and improvements in some sectors are often offset by deteriorations in others. It is generally not possible to appreciate the scale of these competing trends, unless a more detailed breakdown is undertaken. The application of what we call Structural Path Decomposition (SPD) is designed to act at the production chain level such that aggregate results are informed by the changing relationships at the industry level.

1.1. Structural Decomposition Analysis

The application of Structural Decomposition Analysis (SDA) is aimed at identifying the driving factors for change in key variables over time. All variants of SDA are static comparative methods that examine time series of either sector-level and/or country-level data. In essence, SDA formulates an explained variable, say energy use, as a sum or product of explanatory determinants, say energy efficiency, technology, per-capita consumption and population. A pair-wise comparison of changes at two points in time is undertaken, by each determinant and the explanatory variable.

Predecessors to the technique of SDA can be found in the work on structural economic changes by (Leontief 1941, 1953), and were first applied to environmental issues by Leontief and Ford (1971) when they analysed structural effects on air pollution and projected future emission scenarios. Following Leontief's pioneering work, energy analysis became the focus of SDA investigation (Proops 1988; Chen and Rose 1990; Rose and Chen 1991), and has been examined in terms of output mix (Pløger 1984), and also technology and demand change (Gowdy and Miller 1987). This was later extended to SDAs of carbon dioxide and other impacts within economies (Common and Salma 1992; Proops et al. 1993; Casler and Rose 1998; Chang and Lin 1998; Wier 1998; de Haan 2001; Roca and Serrano 2007; De Nooij et al. 2003; Hoekstra and van den Bergh 2002; Wilting et al. 2006; Llop 2007; Peters et al. 2007; Guan et al. 2008; Pablo Muñoz and Hubacek 2008; Wachsmann et al. 2008).

Whilst SDA generally concerns itself with the macro-indicators of change within an economy, its application to policy formulation usually requires a breakdown at higher levels of detail, such as by production sector (e.g. Alcántara and Duarte 2004; Chang and Lin 1998; De Nooij et al. 2003; Dietzenbacher and Los 1998, 2000; Jacobsen 2000; de Haan 2001; Wier 1998). In this paper, we seek to extend the relevance to policy formulation with the integration of SPA. By so doing, we are able to extract and rank the actual production paths contributing most to changes in the dependent variable

(energy/carbon etc). This is particularly relevant when analysing changes within the Leontief representation of the structure of the economy.

1.2. Structural Path Analysis

The use of input-output tables in such applications as Life Cycle Assessment (LCA) has greatly benefited from the technique of Structural Path Analysis (SPA), introduced in the early 1980s (Defourny and Thorbecke 1984). Recently, this has seen increasingly widespread use, both in an LCA context (Treloar et al. 2001; Lenzen 2002, 2003; Suh 2004; Wood et al. 2006; Llop 2007; Wood and Lenzen 2003; Treloar 1997; Strømman et al. 2009), and in more general areas such as trade modelling (Peters and Hertwich 2006a, 2006b; Lenzen et al. 2007) and in trophic systems (Suh 2005; Lenzen 2006b).

The basic idea behind a SPA is the unravelling of the Leontief inverse by means of a series expansion of the direct requirements matrix (Vaughan 1950). This allows the analyst to investigate impacts that are caused directly by final consumption (such as emissions from gas cooking) to those caused in the first order away from the consumer (such as emissions in electricity generated for the consumer) to those in higher orders (for example, emissions in electricity for steel for a train for a train journey taken by the consumer).

The use of SPA has only been applied statically - often for extracting the main upstream impacts of products or organisations. In this paper, by applying SDA techniques, we seek to a) investigate the effects of changes within the production chain over time and b) decompose production chains to a level of interest for analysis of change.

1.3. This work: Structural Path Decomposition (SPD)

This paper proceeds with a mathematical description of the method in Section 2. The dual benefits of the methodology is shown, by a) decomposing SDA into ranked production chains in a case study in Section 3, and by b) providing a temporal aspect to SPA through a case study in Section 4. Application notes are briefly discussed in Section 5 before conclusions are brought in Section 6.

2. Mathematical Description

In this section, we derive the mathematical basis for the combination of SDA and SPA. We start from the basic Leontief production function, and derive the SDA equations before applying the decomposition into structural paths (SPA).

2.1. Structural Decomposition Analysis

Beginning from the basic Leontief model, total output can be expressed as:

$$\mathbf{x} = \mathbf{L}\mathbf{y}, \quad (1)$$

where $\mathbf{x}$ is a vector of total output, $\mathbf{L}$ is the Leontief inverse $(\mathbf{I}-\mathbf{A})^{-1}$ of the direct requirements matrix $\mathbf{A}$, and $\mathbf{y}$ is a vector of final demand (Leontief (1966) or comparable). Whilst decomposition can be applied to this purely monetary equation, it is often performed for a physical production factor such as greenhouse gas emissions or energy consumption. Hence, as an example, we expand equation 1 to:

$$C = \mathbf{c}\mathbf{L}\mathbf{y} = \mathbf{c}\mathbf{L}\boldsymbol{\psi}\boldsymbol{\delta}YP = \mathbf{c}(\mathbf{I}-\mathbf{A})^{-1}\boldsymbol{\psi}\boldsymbol{\delta}YP \quad (1a)$$

where:

- C : Total CO₂-eq emissions (1×1)
- $\mathbf{c}$: Greenhouse gas intensity of production n sectors ($1 \times n$)
- $\mathbf{L}$: Leontief inverse ($n \times n$)
- $\mathbf{I}$: Unity matrix ($n \times n$)
- $\mathbf{A}$: Direct requirements matrix ($n \times n$)
- $\boldsymbol{\psi}$: Commodity structure of final demand ($n \times d$)
- $\boldsymbol{\delta}$: Destination structure of final demand ($d \times 1$)
- Y : Per-capita final demand (1×1)
- P : Population (1×1)

The decomposition of eq 1a then becomes (compare Proops et al. 1993)

$$\begin{aligned} dC = & d\mathbf{c} \mathbf{L} \boldsymbol{\psi} \boldsymbol{\delta} Y P + \mathbf{c} d\mathbf{L} \boldsymbol{\psi} \boldsymbol{\delta} Y P \\ & + \mathbf{c} \mathbf{L} d\boldsymbol{\psi} \boldsymbol{\delta} Y P + \mathbf{c} \mathbf{L} \boldsymbol{\psi} d\boldsymbol{\delta} Y P + \mathbf{c} \mathbf{L} \boldsymbol{\psi} \boldsymbol{\delta} dY P + \mathbf{c} \mathbf{L} \boldsymbol{\psi} \boldsymbol{\delta} Y dP \end{aligned} \quad (1b)$$

Eq 1b represents a typical SDA of macro-variables, here for seven components (change in emissions due to: change in greenhouse gas intensity of production $d\mathbf{c}$; change in industrial structure $d\mathbf{L}$; change in commodity structure of final demand $d\boldsymbol{\psi}$; change in destination of final demand, $d\boldsymbol{\delta}$; change in expenditure dY ; and change in population dP).

As a continuous time series of data is often not available, in order to apply equation 3, differences ΔC , are obtained by integrating changes in dC over discrete time intervals (corresponding to years of IO data). As the integral path within each interval is not known, it is approximated using a form of indexing. In this paper, we use the logarithmic mean divisia index (LMDI), the mathematics of which are explained elsewhere (Wood and Lenzen 2006; Ang et al. 2003; Ang and Lee 1994) For an overview of alternative indices see Lenzen (2006a).

The result of the interval approximation and indexing gives us a series of equations:

$$\begin{aligned}
\Delta C &= \Delta c + \Delta L + \Delta \psi + \Delta \delta + \Delta Y + \Delta P \\
\Delta c &= dc L \psi \delta Y P \\
\Delta L &= c dL \psi \delta Y P \\
\Delta \psi &= c L d\psi \delta Y P \\
\Delta \delta &= c L \psi d\delta Y P \\
\Delta Y &= c L \psi \delta dY P \\
\Delta P &= c L \psi \delta Y dP
\end{aligned} \tag{1}$$

where the differential is calculated (dc for example) according to the LMDI:

$$dc = \left(\frac{C_2 - C_1}{\log(C_2 / C_1)} \right) * \log\left(\frac{c_2}{c_1}\right) \tag{2b}$$

Subscripts refer to Year 1 and Year 2 - the endpoints of the analysis period. For choice of index and uniqueness issues, we refer the reader to (Ang 2004; Dietzenbacher and Los 1998). C and c are calculated as per equation 1a.

2.2. Applying Structural Path Analysis

From equation 1b, using the Taylor series expansion familiar to SPA,

$$L = I + A + A^2 + A^3 + \dots$$

greenhouse content can be expressed along the actual production pathway:

$$C = c(I + A + A^2 + A^3 + A^4 + \dots) \psi \delta Y P \tag{2}$$

Such that equation 1b can be summarised as:

$$dC = d(c \psi \delta Y P) + d(c A \psi \delta Y P) + d(c A^2 \psi \delta Y P) + d(c A^3 \psi \delta Y P) + \dots \tag{4}$$

Or in expanded form:

$$\begin{aligned}
dC &= dc \psi \delta Y P + cd\psi \delta Y P + c\psi d\delta Y P + c\psi \delta dY P + \dots \\
&+ c\psi \delta Y dP + dcA \psi \delta Y P + cdA\delta Y P + cAd\psi \delta Y P + \dots \\
&+ cA\psi d\delta Y P + cA\psi \delta dY P + cA\psi \delta Y dP + dcAA\psi \delta Y P + \dots \\
&+ cdAA\psi \delta Y P + cAdA\psi \delta Y P + cAA d\psi \delta Y P + \dots \\
&+ cAA\psi d\delta Y P + ceAA\psi \delta dY P + ceAA\psi \delta Y dP + \dots
\end{aligned} \tag{5}$$

Where the first line of equation 5 is the decomposition of first-order impacts, the second line is the decomposition of second-order impacts and the third and fourth line are the decompositions of third-order impacts. Fourth and above orders follow the same form.

Some examples for single-path structures of the first four orders are:

1st-order: $dc_i \psi_{jk} \delta_k YP$, or $c_i \psi_{jk} \delta_k YdP$;

2nd-order: $dc_i A_{ik} \psi_{kl} \delta_l YP$, or $c_i dA_{ik} \psi_{kl} \delta_l YP$;

3rd-order: $dc_i A_{im} A_{mk} \psi_{kl} \delta_l YP$, or $c_i A_{im} dA_{mk} \psi_{kl} \delta_l YP$;

4th-order: $dc_i A_{im} A_{mn} A_{nk} \psi_{kl} \delta_l YP$, or $c_i A_{im} dA_{mn} A_{nk} \psi_{kl} \delta_l YP$,

The first example: $dc_i \psi_{jk} \delta_k YP$, is the change brought about by the change in greenhouse content (Δc) of fuel i , used in sector j , for destination of final demand k , whereas the second 1st-order example, $c_i \psi_{jk} \delta_k YdP$, is the change in the same production chain brought about by population change (ΔP). The higher order examples read the same, but with intermediate production steps shown through the introduction of the technical co-efficient matrix $\mathbf{A}$. In summary, our differential terms are (compare equation 2 and 5):

$$\Delta C = \Delta c + (\Delta \mathbf{A} \mathbf{1} + \Delta \mathbf{A} \mathbf{1} \mathbf{A} \mathbf{2} + \mathbf{A} \mathbf{1} \Delta \mathbf{A} \mathbf{2} + \Delta \mathbf{A} \mathbf{1} \mathbf{A} \mathbf{2} \mathbf{A} \mathbf{3} + \mathbf{A} \mathbf{1} \Delta \mathbf{A} \mathbf{2} \mathbf{A} \mathbf{3} + \mathbf{A} \mathbf{1} \mathbf{A} \mathbf{2} \Delta \mathbf{A} \mathbf{3} + \dots) + \Delta \psi + \Delta \delta + \Delta Y + \Delta P \quad (6)$$

ΔC	Change in total greenhouse gas emissions
Δc	Change due to greenhouse intensity of production
$\Delta \mathbf{A} \mathbf{1}$	Change due to first order technical co-efficient
$\Delta \mathbf{A} \mathbf{2}$	Change due to second order technical co-efficient
$\Delta \mathbf{A} \mathbf{3}$	Change due to third order technical co-efficient
...	So forth for higher orders of $\mathbf{A}$
$\Delta \psi$	Change due to type of final demand (product mix)
$\Delta \delta$	Change due to destination of final demand (household, capital, export, etc)
ΔY	Change due to affluence
ΔP	Change due to population

Table 1 Summary of differentials of SPD

3. Case Study, Australia

In our case study, we use the Australian IO tables for two years - 1995 and 2005 (Australian Bureau of Statistics 1999, 2008a) converted to constant prices using price indices and optimising the dataset against chain volume national account aggregates (Australian Bureau of Statistics 2008b, 2007, 2008c). For the purpose of illustrating this method, we have aggregated the tables to 30 sectors. We use indirect allocation of imports to reflect the true technology of the economy (i.e. analysis is performed on the combined domestic and imports matrices). Greenhouse gas emissions are sourced from the National Greenhouse Gas Inventory (Australian Greenhouse Office 2007).

The decomposition equation is:

$$C = c(\mathbf{I} - \mathbf{A})^{-1} \psi \delta YP \quad (7)$$

Variables are per Table 1. Results presented in Table 2 are changes in total emissions (ΔC) due to changing greenhouse gas intensity Δc , changes in technology ΔL , changes in type of final demand $\Delta \psi$, changes in destination of final demand $\Delta \delta$, changes in affluence ΔY and changes in population ΔP . Percentages are the change in each variable with respect to change in total emissions. Results are annualised.

Δc	ΔL	$\Delta \psi$	$\Delta \delta$	ΔY	ΔP	ΔC
-7,544	-982	-6,539	110	14,675	5,944	5,663
-133%	-17%	-115%	2%	259%	105%	100%

Table 2 SDA of carbon emissions in Australia kt CO₂-eq, 1995-2005.

Reasonably large increases in greenhouse efficiency occurred in Australia, with Δc being the one of the largest drivers of change across all variables (Table 2). ΔL and $\Delta \psi$ show likewise decreasing effect on emissions. ΔL represents the production recipes used within the economy - the flows between different industries in intermediate production. A decrease in ΔL is often a result of shifts to using lesser quantities of emissions intensive industries such as electricity or transport, as will become clearer in our later analysis (Table 3). $\Delta \psi$ shows a diminishing effect on emissions due to relative changes in final demand categories (see Table 3 for more detail). In essence, the results are showing consumption changing from products with high embodied carbon to products with low embodied carbon. Offsetting these improvements, were large changes in affluence ΔY , population ΔP , and to a lesser extent, final consumption destination $\Delta \delta$. $\Delta \delta$ reflects the changing importance of domestic, capital and exported consumption. Over the 10 year time period, the effect of $\Delta \delta$ is minimal. Shorter time periods often show larger (fluctuating) capital effects.

However, apart from the scalar effects of ΔY and ΔP , these changes are not consistent across all production chains within the economy. There are generally many competing sectoral trends within an economy, where improvements in emissions in some sectors are offset by deterioration in others. It is simply not possible to appreciate the scale of these competing trends, unless a more detailed breakdown is undertaken. Thus, in order to analyse those production chains driving the aggregate changes, the SPD formulation derived in Section 2 is applied to extract the production paths with the greatest impacts (Table 3).

To firstly explain these results, Table 3 presents the top 30 production chains that have contributed most to change in Australia's greenhouse gas emissions over the period 1995-2005. Results are annualised such that the kT CO₂-eq figure is the average annual change from each production chain over the time period. The "order" of the path represents how many steps occur in the production chain. A path of order 1 shows a direct flow from production to consumption – for example electricity production to household consumption. A path of order 2 shows an intermediary step, for example, electricity

production to trade to household consumption. Paths of order 0 (for example household combustion of gas) are not included in this analysis, as they are not strictly structural economic changes. The differential refers to the variable of change within the production chain (see Table 1). Sector 1, 2, 3, 4 refers to the agents in the production chain. Sector 1 will always be the source of emissions – the producing industry. Sector 2 will be the final sector of consumption in the case of a first order path, or in the case of higher order paths, will be an intermediary sector in the production chain. For example, path 21 shows emissions produced in the livestock industry, embodied in the inputs to the dairy and meat industry, which is subsequently required as an input into the trade sector, which is destined for household consumption. The final demand component is disaggregated into destination of final demand (see equation 1a).

It is clear that the key drivers are mainly first or second order production chains, and that paths stemming from the electricity and livestock sectors are most important (Table 3). The top paths show an effect of up to 1.4 MT CO₂-eq, or 25% of the total annualised change in greenhouse gas emissions across the economy. The competing nature of these paths is evident, with the top 6 paths offsetting each other to have close to zero net impact. The first and third paths, show the changes that have occurred from the changing mix of products going to export ($\Delta\psi$). These paths show reduced emissions from a reduction in the proportion of livestock and meat products (which both have high embodied greenhouse gas emissions) exported. This is mainly due to the contraction of the Australian sheep industry in the global marketplace. It should however be noted that in this small-scale example we present, sheep and beef cattle are aggregated, and hence share the same greenhouse gas intensity. This is not true in practice, as sheep and cattle have considerably different greenhouse gas emissions. Hence a more disaggregated model would need to be applied before policy is derived from the results we present.

Shifts in the structure of final demand ($\Delta\psi$) are responsible for many of the top paths, and from the analysis, we can see that household demand is having an increasing impact on emissions through household consumption habits incorporating a greater relative reliance on transport (path 4), and a decreasing impact through lower relative electricity consumption (path 7; -896 kT CO₂-eq) and meat and dairy (a second order path, path 9). Overall, however, impacts from household electricity consumption are responsible for increasing emissions, with the decrease in relative household consumption offset by the affluence effect (path 2 – representing overall increased expenditure on this path), the population effect (path 15) and the intensity effect (path 20; Δc ; 468 kT CO₂-eq – representing decreases in greenhouse efficiency within the electricity sector).

Significant change has occurred in livestock practices in Australia over the 1995-2005 period, both in the changing structure of exported demand as previously mentioned, and for domestic relationships. Due to reductions in direct land clearing by the industry (decreasing Δc), path 5 and 8 show the implications of this for household and export demand for meat products, with an annual saving of close to 1MT CO₂-eq each. Unfortunately this saving has been offset by the increase in overall demand of meat products due to affluence and population effects (paths 6 and 26). Similar to Electricity

Rank	kT CO2-e	Order	Differential	Sector 1	Sector 2	Sector 3	Sector 4
1	-1,374	1	$\Delta\psi$	Livestock	Exports		
2	1,370	1	ΔY	Electricity	Household demand		
3	-1,154	2	$\Delta\psi$	Livestock	Meat and dairy	Exports	
4	1,139	1	$\Delta\psi$	Transport	Household demand		
5	-1,078	2	Δc	Livestock	Meat and dairy	Household demand	
6	911	2	ΔY	Livestock	Meat and dairy	Household demand	
7	-896	1	$\Delta\psi$	Electricity	Household demand		
8	-888	2	Δc	Livestock	Meat and dairy	Exports	
9	-862	2	$\Delta\psi$	Livestock	Meat and dairy	Household demand	
10	806	1	$\Delta\psi$	Livestock	Changes in inventories		
11	751	2	ΔY	Livestock	Meat and dairy	Exports	
12	-704	1	Δc	Livestock	Exports		
13	-642	1	$\Delta\psi$	Transport	Private capital		
14	596	1	ΔY	Livestock	Exports		
15	555	1	ΔP	Electricity	Household demand		
16	-511	1	Δc	Grains	Exports		
17	-500	1	$\Delta\psi$	Electricity	Public enterprise capital		
18	-495	1	$\Delta\delta$	Electricity	Public enterprise capital		
19	478	2	$\Delta A1$	Livestock	Trade	Household demand	
20	468	1	Δc	Electricity	Household demand		
21	414	3	$\Delta A2$	Livestock	Meat and dairy	Trade	Household demand
22	-404	1	Δc	Livestock	Private capital		
23	387	2	$\Delta A1$	Livestock	Meat and dairy	Household demand	
24	378	1	ΔY	Metals	Exports		
25	373	1	$\Delta\psi$	Livestock	Private capital		
26	369	2	ΔP	Livestock	Meat and dairy	Household demand	
27	-366	1	Δc	Govt, Public services	Government demand		
28	365	1	ΔY	Coal and minerals	Exports		
29	342	1	ΔY	Livestock	Private capital		
30	319	2	$\Delta A1$	Livestock	Meat and dairy	Exports	
TOTAL	5,663						

Table 3 SPD of carbon emissions in Australia, 1995-2005. Top 30 ranked paths, "Differential" refers to the variable of change (refer Eq 2), "Order" refers to the length of the production chain.

consumption, we are at least seeing a decrease in the relative consumption of meat products in a household basket of goods (path 9; $\Delta\psi = -862 \text{ kT CO}_2\text{-eq}$).

Changes in the production structure of the economy, represented by paths with differential $\Delta\mathbf{A}$, show the effects of changes of inter-relationships of industries in the economy. Path 19 and 21 are the highest ranked paths, and they show that for the given household consumption of trade (which refers to retail and wholesale trade, and incorporate hospitality) there have been higher requirements directly from livestock (path 19, $\Delta\mathbf{A1}$) and also via the meat and dairy industry ($\Delta\mathbf{A2}$ indicates the second linkage in the production chain). Hence, comparing to path 9, we are seeing a decrease in the household consumption of meat and dairy, but an increase in livestock and meat utilised in the trade industry. In simple terms, there is a greenhouse saving due to the population eating less meat at home, but this is offset by more meat purchases at restaurants.

Other important structural changes within the economy are shown by path 23 and 30, the increase in livestock required for the average meat and dairy product. Lower rankings (not shown in Table 2) include the reduction in animal inputs into clothing manufacture, which saves almost 289 kT CO₂-eq a year; and the increasing requirements of electricity of the trade sector for household demand, most probably due to the increased uptake of air-conditioning systems, producing 173 kT CO₂-eq more a year.

Other interesting paths in the top 30 include several effects stemming from improvements in the greenhouse gas intensity of production (Δc). Apart from those already identified, we are seeing improvements in efficiency in paths stemming from the grains sector (path 16), and government and business services (path 27). Paths 17 and 18 show the decreased emissions stemming from reduced relative electricity needs of public enterprise capital, and a shift away from public enterprise capital relative to total final demand. Finally, the overall expenditure/affluence effects ($\Delta\mathbf{Y}$) show strongly for key export sectors of metals (path 24) and coal and minerals (path 28).

In summary, this case study has been selected to illustrate the application of SPD to an SDA study. The analysis of an SDA is broken down to look at the competing forces underlying the aggregate results. Unlike a traditional SDA undertaken at the sector level, this allows a true consumption/life-cycle viewpoint to be used, and enriches the detail extracted, particularly when analysing production chains within the Leontief production structure of the economy.

4. Case Study, SPD of a Wood products IO-LCA

We now turn to applying SPD in a more conventional Life-Cycle context. In this case study, we utilise the same methodology and context for the analysis of the environmental impact of timber products as done by (Lenzen and Treloar 2002). We analyse only fuel combustion related emissions, as per the energy analysis of Lenzen and Treloar. This case study is

designed to show the application of the SPD methodology to Life-Cycle Assessment, where temporal factors within the production process are sought.

As such, we investigate the principal causes in the changes in the production process for a unit of timber products in Australia between 1995 and 2005. As we are applying our analysis on a single unit of final demand, equation 5 becomes,

$$dC = d(c\gamma) + d(cA\gamma) + d(cA^2\gamma) + d(cA^3\gamma) + \dots \quad (7)$$

Where γ is the final demand vector with unitary demand (AUD\$1 million) for timber products and zero demand for other product categories, *i. i.e.*:

$$\gamma_{i=\text{timber}} = 1,$$

$$\gamma_{\forall i, i \neq \text{timber}} = 0$$

A standard SPA for wood products is presented in Table 4, with the top 15 paths ranked.

Rank	kT CO2	Order	%	Sector 1	Sector 2	Sector 3
1	137	1	38.6%	Wood products		
2	73	2	20.6%	Electricity	Wood products	
3	18	2	5.2%	Transport	Wood products	
4	15	2	4.2%	Forestry	Wood products	
5	11	2	3.1%	Wood products	Wood products	
6	10	3	2.8%	Electricity	Electricity	Wood products
7	6	3	1.7%	Electricity	Wood products	Wood products
8	4	3	1.2%	Electricity	Transport	Wood products
9	3	2	0.8%	Chemicals, plastics	Wood products	
10	3	3	0.8%	Electricity	Chemicals, plastics	Wood products
11	3	3	0.8%	Electricity	Trade	Wood products
12	2	2	0.7%	Metals	Wood products	
13	2	2	0.6%	Mineral products	Wood products	
14	2	3	0.6%	Forestry	Forestry	Wood products
15	2	3	0.4%	Electricity	Business	Wood products
TOTAL		355				

Table 4 SPA of wood products in the Australia, 2005. Top 15 ranked paths

The standard SPA allows us to investigate the principle areas of impact within the manufacture of wood products. The top 4 paths are straightforward, with the first path showing direct (first order) emissions from fuel combustion in the wood products industry. The second, third and fourth paths show second order production paths for the supply of electricity, transport and forestry respectively. These are thus emissions caused in the direct suppliers to the wood products industry. Overall, these top 4 paths account for 69% of the total emissions. Path 5 Wood products embodied in wood products shows the intra-sector purchases between different establishments. For example, within the wood products sector, goods such as joinery products have inputs from other establishments, such as those producing undressed timber. This intra-sector circulatory is also evident in the electricity

and forestry sectors (paths 6 and 14) and is manifested more in aggregated data sets such as in use here. The existence of these paths, particularly in the third order paths reflects both the importance of the key sectors of electricity, forestry and wood products, as well as the diverse manufacturing base (large numbers of establishments) within the sectors. Other production chains of note are second and third order chemical paths (paths 9 and 10), metals and minerals (paths 12 and 13), and finally, the significance of the impact of electricity used in the business and finance sector servicing wood products (rank 15).

Such an analysis as this can be extremely important in identifying carbon bottlenecks in the supply chain of a product. However, often we are not only interested in this static analysis, but we would like to know how things are changing over time, and whether there is any improvement in the processes under study, especially if some kind of policy has or can be applied. In order to do this, we apply the decomposition to the 1995 and 2005 results with SPD.

Rank	kT CO ₂ -e	Order	Differential	Sector 1	Sector 2	Sector 3
1	2.7	1	Δc	Wood products		
2	-1.4	2	$\Delta A1$	Electricity	Wood products	
3	-1.0	2	$\Delta A1$	Wood products	Wood products	
4	0.8	2	Δc	Electricity	Wood products	
5	-0.7	2	$\Delta A1$	Transport	Wood products	
6	-0.6	3	$\Delta A2$	Electricity	Wood products	Wood products
7	-0.5	2	Δc	Forestry	Wood products	
8	-0.5	2	Δc	Transport	Wood products	
9	0.4	3	$\Delta A1$	Electricity	Electricity	Wood products
10	0.3	2	Δc	Wood products	Wood products	
11	-0.2	3	$\Delta A2$	Transport	Wood products	Wood products
12	0.2	2	$\Delta A1$	Chemicals, plastics	Wood products	
13	-0.2	3	$\Delta A2$	Electricity	Trade	Wood products
14	-0.2	3	$\Delta A1$	Electricity	Wood products	Wood products
15	-0.2	2	Δc	Chemicals, plastics	Wood products	
TOTAL	-3.3					

Table 5 SPD of wood products in Australia, 1995-2005. Top 15 ranked paths

By applying the SPD method (Table 5), we see that the top path from the SPA in Table 4 (forestry inputs into wood products) is undergoing significant change over time. Decreases in efficiency of the sector (Δc - path rank 1) have increased emissions by about 2.7 kT CO₂-eq per year. This is equivalent to over 80% of the total change (-3.3 kT CO₂-eq). Structural changes are next most important in the ranking, with changes in the inter-sector flow between electricity and wood products (path 2) and the intra-sectoral flows of wood products (path 3) both reducing emissions in the order of 1 kT CO₂-eq. The second path is showing that the production chain is becoming less electricity intensive (ΔA - path rank 2), meaning that a lesser proportion of electricity is required in producing the wood products.

The results for the structural change differentials thus imply that the manufacturing process has increased direct fuel use (path 1), whilst reducing the need for other inputs (paths 2,3,5,6,11,13,14).

Two other structural changes show increased impact on emissions – that of chemicals (second order, path 12) and the upstream requirements of electricity supply (third order, with change occurring in the intra-sector electricity flows – path 9). This means that even whilst the wood products sector is reducing emissions through reduced electricity demand (path 2), the benefit is being reduced by the higher upstream requirements of the electricity sector itself (path 9). The SPD also allows us to observe the decreasing efficiency of electricity supply ($+\Delta c$) flowing into wood products (path 4), and the converse increases in efficiency ($-\Delta c$) of the second order paths forestry, transport and chemicals into wood products (paths 7,8 and 15). The chemicals path is a good example of no net effect of increased chemical use ($\Delta A = 0.2 \text{ kT CO}_2\text{-eq}$, path 12) due to savings in efficiency of the sector ($\Delta c = -0.2 \text{ kT CO}_2\text{-eq}$, path 15).

5. Application notes

We found the most useful approach was to begin with an SPA algorithm in order to trace the branches of the production tree, with ranked sectors and higher orders only being traced when they are greater than threshold. For a system of 30 sectors, it is feasible to cover all production chains and up to 10 orders, however, for larger models, thresholds and rankings are recommended to reduce computational time. For the application in SPD, three concurrent SPAs are performed – one for the initial year, one for the final year, and one for the index term. Instead of calculating the value of the path as per normal SPA, the differential of the path is calculated for the n variables, as per the examples in Section 2.2.

6. Conclusions and Discussion

6.1. Summary

In this paper we have elucidated a means for investigating the temporal changes in production chains by combining the two techniques of Structural Path Analysis and Structural Decomposition Analysis. The amalgamation of the two techniques enables a bi-fold advantage to be gained - firstly such that SPA and its application in LCA can be employed in a temporal setting giving insight into changes and trends occurring within a production chain - and secondly by allowing the SDA analyst the ability to derive more policy applicable results, extricating the production pathways (and in particular the changes in the Leontief production structure) from the aggregate results where competing forces often subsume the changes occurring on the ground.

To illustrate the method, two case studies were undertaken - firstly for an SDA of Australia between 1995 and 2005, and secondly for an IO-LCA of wood products. Key results show

strong competing forces in the top ranked paths, with shifts in demand structure being highly significant. There was a general decrease in the greenhouse gas intensity across the economy, and some key shifts in the production paths of intermediate demand. The analysis provided particularly for wood products highlighted these results, with generally lower greenhouse gas requirements from intermediate production paths offset by higher source emissions.

6.2. Discussion

Knowledge of the carbon liability of products and within countries is becoming increasingly important in a carbon constrained world. Identifying the evolution of production practices in a country can significantly help in the development of policy to curb rampant growth in consumption or production behaviour inducing high levels of emissions. Whilst the methods introduced in this paper do not further the benchmarking of environmental impact, they do help us to measure whether progress is being made on the way to sustainability. As almost all climate policy has been developed around relative reductions in carbon emissions, rather than absolute goals, these measures on progress within an economy are essential to sound out the areas in most need of address.

One of the weaknesses of conventional SDA, in the authors' opinion, is the difficulty interpreting the changes in the Leontief production structure for meaningful policy. Previous analysis would be able to decipher that overall production recipes of a sector were getting better or worse, but had no convenient means to explain why. The advantage of the method introduced in this paper, is the ability to breakdown the Leontief production structure into individual production chains. This then allows the extraction of the links in the chain which are most important in the evolution of a country's emissions.

The development of policy based on the aggregate indicators of an SDA can miss significant opportunities, when, as identified in Section 3, there are competing forces in different production chains counterbalancing each other at an industry level. The study of Australia found exactly this - with significant changes occurring both in type of consumption and technology of production overshadowed in the aggregate analysis.

In an LCA context, the method adds to the practitioner's ability to identify trends in the sustainability of products - whether - and which - practices are moving towards low emissions, and which practices are potentially blowing out carbon or other environmental liabilities.

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AGGREGATE MEASURES OF COMPLEX ECONOMIC STRUCTURE AND EVOLUTION – A REVIEW AND CASE STUDY

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Abstract

It is perhaps in the nature of complex systems that they call for aggregate measures that enable analysts to grasp their structure and evolution without being overwhelmed by their very complexity. Complex inter-industry theory and models are a typical case, where the underlying database – an input-output table – routinely contains thousands of data points for a single year. Within input-output analysis, quantitative measures have been developed that describe and characterize inter-industry interactions, and that have been used to compare economies, both in a static taxonomy and in tracing their evolution over time. First, we review, and critically discuss a number of concepts that have been proposed and applied to inter-industry systems, such as interconnectedness, interrelatedness, linkages, and economic landscapes. Second, we apply these concepts and the quantitative measures to a case study of the Australian economy between 1975 and 1999, in terms of environmental headline indicators. Our results enable the reader to judge the usefulness and ability of aggregate measures of economic structure in capturing at a glance, the key structural elements and evolutionary processes governing the interaction between the economy and the environment.

Keywords economic systems, input-output tables, industrial interdependence, structural evolution

1. Introduction

Industrial ecology examines interactions between the environment, the economy and technology. Whilst particular interactions or life cycles are often a focus, holistic measurements are also key to understanding the metabolism of our societies. A holistic approach needs to assume a top-down view of the entire complex system of interactions within our society.

It is perhaps in the nature of complex systems that they call for aggregate measures that enable analysts to grasp their structure and evolution without being overwhelmed by their very complexity. This is particularly true for the economic sciences, where researchers have over the past five decades sought measures that described in a condensed form the structure and functioning of national economies (see the classical work by Chenery and Watanabe (1958)).

The sub-discipline within Economics that perhaps deals most with the complex structure of economies is *inter-industry*, or *input-output* theory. Based on path-breaking work by Nobel Prize laureate Wassily Leontief (1966), input-output analysis examines the interdependence of productive sectors amongst each other, as well as their dependence on primary inputs such as labor and capital, and their output for final demand. Leontief was always interested in aspects of natural resource use and repercussions of economic activity for the environment (Leontief and Ford 1970), and so it is no surprise that his input-output framework is ideally suited to integrate the economic system with resource and pollution variables as additional primary inputs into production. By *generalizing* the input-output theory to incorporate links between the economy and the environment, we are able, in one way, to analyze the symbiosis of productive industries, an analogy that sits at the heart of the discipline called Industrial Ecology.

Because of its analytical elegance, input-output analysis is widely applied to economic and environmental issues. For example, every modern General Equilibrium model has an input-output model at its core (Rose 1995). Input-output tables are compiled regularly in many countries around the world, and accounting standards are governed by guidelines published by the (United Nations Department for Economic and Social Affairs Statistics Division 1999). They also form the core of the System of National Accounts.

Today, most input-output tables break down the economy into at least 50, but up to 500 sectors of production. Thus, they routinely contain thousands of data points for a single year, with each data point representing a monetary transaction between either industries, or in terms of a commodity flows¹. These monetary tables have been complemented with satellite accounts describing resource and environmental impacts (de Haan and Keuning 1996; United Nations Statistics Division 2003). It is because of this wealth of information that researchers have developed quantitative measures that

¹ For a distinction of industries and commodities in input-output analysis, see (Gigantes 1970) and (Ten Raa and Rueda-Cantuche 2007). In this work, we will refer to both entities as *sectors*.

encapsulate the structure of complex inter-sectoral interactions into a few descriptors, which in turn have been used to characterize economies, both in a comparative taxonomy, and in tracing their evolution and state of development over time.²

This article's aim is twofold: First, it critically discusses a number of concepts that have been proposed and applied to input-output systems, such as (inter-)connectedness, (inter-)relatedness, linkages, and economic landscapes. It thus updates older reviews by (Hamilton and Jensen 1983), (Szyrmer 1985b), and (Jensen and West 1985).³ These measures are designed to give an aggregate indication of complexity, and when applied in a temporal context, can elucidate trends towards or away from complexity of the economic system. Generally, we would expect the measures to show an evolution towards an increasingly complex economic system. This might be thought of as an evolution towards more circuitous or longer production paths, more reliance of primary industries on secondary and tertiary industries and so forth. Secondly, we want to interpret complexity in terms of environmental indicators. We ask, for example: Is an increased complexity of economic flows equated with increased complexity of environmental flows, and is this associated with increasing or decreasing absolute flows? Are we seeing a dispersion of economic flows, and an increased density of environmental flows? Finally, as growing economies are more intertwined (complex), does it follow that greater environmental impacts are consistent with greater complexity of environmentally important flows?

We apply these concepts to the Australian economy for 1975-1999. The case study includes environmental headline indicators, and therefore yields insights for Industrial Ecology: Our results attempt to enable the reader to judge the usefulness and ability of aggregate measures of economic structure in capturing at a glance, the key structural elements and evolutionary processes governing the interaction between industries and the environment.

2. Review of concepts to measure complexity in economic systems

2.1. A taxonomy of measures

Numerous measures of complexity have been proposed in the literature, and they can be classified in a number of ways. In the following section we present Szyrmer's (1985b, 1985a) unifying taxonomy of classification of the measures.

Firstly, we distinguish between measures of *connectivity* and *connectedness*. By connectivity, we are referring to the Boolean existence or presence of connections within the system. In comparison, connectedness refers to metrics that consider the magnitude of these connections. A comparison of both types of measure can give an

² See (Jensen et al. 1987) for a discussion of the possibility and usefulness of a taxonomy of economies.

³ Further reviews have been undertaken by (Hewings and Romanos 1981) and (Asami 1985) (as cited by (Jensen and West 1985)); however these are not readily accessible. Short overviews can be found in (Hewings and Jensen 1989) and (Rose and Miernyk 1989).

indication of whether the system is diversifying in structure and/or whether the system is becoming more reliant on key sectors.

Secondly, we distinguish measures that deal with direct flows in the form of a *transactions* matrix ($\mathbf{T}$ or $\mathbf{A}$ – see Section 2.2 for definition of variables), or which also consider indirect flows by using a *multiplier* matrix ($\mathbf{L}$ and $\mathbf{G}$). If the complexity measures are consistent, we should see similar results between these categories.

A third distinction is made between measures dealing with *inputs* ($\mathbf{T}\hat{\mathbf{x}}^{-1}$, $\mathbf{L}$) and those dealing with *outputs* ($\hat{\mathbf{x}}^{-1}\mathbf{T}$, $\mathbf{G}$).

Most of the measures are defined for input-driven quantities such as the Leontief inverse (Tab. 1), which has probably to do with the Ghoshian calculation and interpretation being less well known. The majority of measures describe connectedness, obviously because of its greater explanatory power. Consequently, as the number of empty fields in Tab. 1 indicates, many of the measures proposed in the literature have been defined for only one particular combination of characteristics, and corresponding alternative definitions pertaining to different categories are readily possible.

Within the following review and subsequent case study, we have unearthed all complexity measures we could find. Occasionally, we slightly modify the measures in order to be able to use a common notation. These modifications do not alter the essential characteristics of the measure, and are merely incorporated to facilitate comparison. Because Szyrmer's categorization succeeds well in structuring all approaches, we have adopted his taxonomy in our case study in Section 3. Unfortunately, the literature generally has not followed Szyrmer's terminology, and theoretically related measures often analyze either both transactions and multipliers or inputs and outputs. Therefore we will present the review in this Section as it has evolved historically.

		Connectivity	Connectedness
Transactions	Inputs		$\omega, I^m, \det(\mathbf{I}-\mathbf{A}),$ $\sigma, \kappa, \lambda_{\max}, \tau,$ H^T
	Outputs		
Multipliers	Inputs	K^{**}	$U_j, \mathbf{S}, \gamma^*, H^L,$ I^c, I^s
	Outputs		$U_i, \mathbf{S},$

Tab. 1: Classification of measures of economic complexity.

2.2. Input-output theory

The following review deals with concepts and measures that were initially founded in input-output theory, but since had numerous applications in Industrial Ecology (Duchin 1992). It is therefore appropriate to briefly recapitulate the basic input-output relationships. The main ingredient is a transactions matrix $\mathbf{T}$ with elements $(T_{ij})_{i=1,\dots,N}$ describing the intermediate demand by sector j of commodities produced by sector i . The transactions matrix connects primary inputs $\mathbf{v}$ (wages, salaries, surplus, taxes) and final demand $\mathbf{y}$ (household consumption, government consumption, fixed capital expenditure, exports) in the *National Accounting Identity*

$$v_k + \sum_i T_{ik} = \sum_j T_{kj} + y_k = x_k ,$$

where $\mathbf{x}$ is called gross output. Re-arranging the right-hand side of the above equation to

$$x_k = \sum_j \frac{T_{kj}}{x_j} x_j + y_k =: \sum_j A_{kj} x_j + y_k ,$$

$$\Leftrightarrow \mathbf{x} = \mathbf{Ax} + \mathbf{y}$$

where elements of the matrix $\mathbf{A}$ are called *direct requirements*, yields the well known input-output relationship

$$(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{y} \Leftrightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{y} = \mathbf{Ly} ,$$

where $\mathbf{L}$ is the famous *Leontief inverse*, also referred as the *total requirements* matrix.

Alternatively in the Ghosh model, $\mathbf{A}^*$ is the *direct sales* matrix, such that $A^*_{ij} = T_{ij}/x_i$ allows the interpretation of supply side effects:

$$\mathbf{x}'(\mathbf{I} - \mathbf{A}^*) = \mathbf{v} \Leftrightarrow \mathbf{x}' = \mathbf{v}'(\mathbf{I} - \mathbf{A}^*)^{-1} = \mathbf{v}'\mathbf{G} ,$$

with $\mathbf{G}$ being the Ghoshian inverse, and the prime denoting transposition.

In its most basic form, the transactions and direct requirements matrices describe only the relationships between domestic industries. There exist a number of variants to the basic input-output formulation, with capital, imports, or households endogenized into intermediate demand. The endogenization of imports leads to an $\mathbf{A}$ matrix describing technical coefficients which, in contrast to the domestic requirements, describe the production recipe of industries.

Input-output analysis as introduced above has been the topic in the *Journal of Industrial Ecology* for many years (for example (Joshi 2001; Lenzen 2001; Nakamura et al. 2007)), and usually involves the generalization of the input-output system from being expressed in purely monetary terms to incorporate some ecological or similar measure:

$$\mathbf{Q} = \left(\hat{\mathbf{q}} \mathbf{L} \right) \mathbf{y} ,$$

with $\mathbf{Q}$ being total impact/resource requirements (for example, total greenhouse gas emissions), $\mathbf{q}$ being direct intensity (e.g. kt-CO₂-eq/\$) and the hat signaling diagonalization. We combine $\mathbf{q}$ with the Leontief inverse $\mathbf{L}$ in order to be able to perform our analysis of complexity measures directly on the square matrix of multipliers $\mathbf{qL}$. In essence, we use a physical input-output table, estimated from a generalized input-output table. Most measures reviewed below were developed purely for economic measurement, and the implementation of the generalized system does not occur until Section 3.

2.3. Production recipes and productivity

The first attempt to reduce the complex structure of inter-industry transactions to a manageable number of numbers was probably undertaken by Leontief (1953) himself, who examined input-output tables the US economy compiled for the years 1919, 1929, and 1939. Leontief was able to interpret a decrease of the share of intermediate inputs in the production recipe amounted as an increase in sectoral productivity.

Chenery and Watanabe (1958) attempted the first inter-country comparison of production structure in their analysis of Japan, Italy, Norway and the United States. These authors used industry-specific ratios of intermediate to total output as a characteristic of overall utilization of production factors.

Both Leontief (1953) and Chenery and Watanabe (1958) were able to present their results in graphs and tables featuring each single sector, because their databases contained only 13 and 29 sectors, respectively. For larger databases such as the one examined in our case study of Australia, such detail will be overwhelming. Accordingly, we define a more aggregate measure

$$\omega = \frac{\sum_i v_i}{\sum_i x_i} = \frac{v}{x} = \frac{\sum_i y_i}{\sum_i x_i} = \frac{y}{x}$$

as the *proportion of value added* $v = \sum_i v_i$ within total output $x = \sum_i x_i$. The equality

between the ratios of value-added v/x and final demand y/x holds because of the National Accounting Identity. It should be noted that in the SNA output is dependent on the boundary of a producer unit (an establishment of enterprise) rather than a process of production. Hence, in our measurements of complexity, we are investigating the interconnectedness of establishments, not processes within the economy.

A related summary measure proposed by (Hamilton and Jensen 1983) is the normalized sum of intermediate coefficients

$$I^m = \frac{\sum_{ij=1}^N A_{ij}}{N} .$$

Basically, the larger the intermediate coefficients, the higher the degree of internal dependence in the economy, and the lower the degree of openness, both towards exogenous (primary) inputs, and exogenous outputs (final demand). A general interpretation is that the higher the degree of internal dependence, the higher the division of labor, indicating in turn a more developed economy.⁴ In other words, “an economy is more ‘complex’ if it churns more to achieve a given final demand” (Robinson and Markandya 1973). This general principle also holds for many of the measures introduced in the following.

2.4. Distribution of coefficients

It is not unusual for an input-output table to contain a large number of small elements, and a small number of elements embodying most of the monetary flow. Peacock and Dosser (1957) suggested the number of non-zero coefficients in **A** as a measure of complexity. Presumably, the larger this number, the more connected the economy. While in the early days of input-output table compilation, zero elements were a common occurrence, improvements in data collection and the usage of mathematical balancing techniques have resulted in tables that are entirely occupied with non-zero, albeit sometimes very small elements, so that non-zero element counts are not really applicable anymore. Instead, we examine two measures for the distribution of coefficients: The *skewness* σ of a distribution characterizes the asymmetry around its mean, while the *kurtosis* κ characterizes its peakedness. The less connected an economy, the higher the concentration of monetary flows in a few large elements, and the higher both skewness and kurtosis.

2.5. Linkages and economic landscapes

Rasmussen (1956) introduced the concept of forward and backward inter-industry *linkages* (written $U_{i\cdot}$ and $U_{\cdot j}$) as measures of structural interdependence.⁵ $U_{i\cdot} > 1$ indicates strong forward linkages, or “sensitivity of dispersion”, of sector i , meaning that a change in all sectors’ final demand would cause an above-average production increase in sector i , that is sector i ’s products would be in greater demand. $U_{\cdot j} > 1$ indicates strong backward linkages, or “power of dispersion”, of sector j , meaning that a change in the final demand of sector j would create an above-average increase in the activity of the whole economy, that is sector j would draw more heavily on the

⁴ The same is said about ecosystems (Finn 1976; Ulanowicz 1983): The higher the degree of internal cycling of mass and/or energy, the more mature the ecosystem.

⁵ Their use for the identification of key sectors was subsequently suggested by (Hirschman 1958), who postulated that economic development and structural change proceed predominantly along above-average linkages, so that a relatively small number of industries accelerate and amplify initially small changes, which eventually affect the whole economy.

rest of the system. A key sector is characterized by $U_{i\cdot} > 1 \wedge U_{\cdot j} > 1$, and exhibits both above-average dependence and influence on other sectors.

Rasmussen's initial formulations were subsequently modified by a number of authors⁶, mostly in terms of weighting and normalization. The consensus appears to be that forward linkages $U_{i\cdot}$ are measured using the Ghosh inverse $\mathbf{G}$, weighted by value added $\mathbf{v}$

$$U_{i\cdot} = \frac{G_{i\cdot}}{\bar{G}} = \frac{\sum_j \frac{v_i}{v} G_{ij}}{\frac{1}{N} \sum_{ij} \frac{v_i}{v} G_{ij}},$$

and that backward linkages $U_{\cdot j}$ are measured using the Leontief inverse $\mathbf{L}$, weighted by final demand $\mathbf{y}$

$$U_{\cdot j} = \frac{L_{\cdot j}}{\bar{L}} = \frac{\sum_i L_{ij} \frac{y_j}{y}}{\frac{1}{N} \sum_{ij} L_{ij} \frac{y_j}{y}}.$$

The weighting ensures that the linkages describe the effects of percentage rather than unit (\$) changes in final demand. The $U_{i\cdot}$ and $U_{\cdot j}$ are normalized by the global intensities of the Ghosh and Leontief inverses so that $\sum_i U_{i\cdot}/N = 1$ and $\sum_j U_{\cdot j}/N = 1$.

Rasmussen's concept of inter-industry linkages was further elaborated by Sonis and co-workers⁷ in their studies on linkage hierarchies, fields of influences, and economic landscapes. The field of influence of a transaction A_{ij} in the direct requirements matrix $\mathbf{A}$ measures the change in the Leontief inverse $\mathbf{L}$ as a result of a change in A_{ij} . The field of influence is hence an $N \times N$ matrix for each A_{ij} , but it can be reduced to a scalar *field-of-influence intensity*. (Lenzen 2003) shows that this intensity can be calculated from forward and backward linkages according to

$$S_{ij} = \frac{U_{\cdot i} T_{ij} U_{\cdot j}}{\sum_{k,l} U_{\cdot k} T_{kl} U_{\cdot l}}.$$

The matrix $\mathbf{S}$ has also be called a *Multiplier Product Matrix* (MPM; (Sonis et al. 1995a; Sonis and Hewings 1999). Sonis et al. (2000) shows that the MPM has minimum information properties in that it is the most homogeneous distribution of

⁶ (Hazari 1970; Bharadwaj 1966; Jones 1976; Heimler 1991; Cuello et al. 1992; Schultz 1977; Clements and Rossi 1991; Rao and Harmston 1979; Beyers 1976; Laumas 1976); for a summary see (Lenzen 2003).

⁷ (Sonis and Hewings 1991; Sonis et al. 1995b; Sonis et al. 1996; Sonis and Hewings 1999). These authors use only the Leontief inverse.

row and column multipliers of the inverse matrix.⁸ A time series of economic landscapes based on the MPM is presented for example in West (1999).

Spatial and temporal extensions of the linkage concept have been presented by Dietzenbacher and van der Linden (1997) and West (1999), respectively. Strassert (2001) argues that conventional Rasmussen linkages exclude a number of inter-industry circuits, and instead suggest a hypothetical extraction method⁹. Soofi (1992) uses backward and forward linkages to define backward and forward *concentration indices*

$$G_{.j} = \sqrt{N \left(1 - \sum_i \left(\frac{L_{ij}}{L_{.j}} \right)^2 \right)} \quad \text{and} \quad G_{i.} = \sqrt{N \left(1 - \sum_j \left(\frac{G_{ij}}{G_{i.}} \right)^2 \right)}.$$

These measures were applied by the New Zealand Treasury (Claus 2003), where it was already pointed out that – unintuitively – these indices become *small* for a *high* degree of concentration in intermediate transactions. They are similar to normalized coefficients of variation of the Rasmussen linkages, also called backward and forward *spread* (West 1999).

2.6. Finn's flow indices

Input-output theory underwent a development in theoretical biology and ecological modelling similar to economics (for a review see Suh (2005)). Finn (1976) defines the *cycling index* of ecosystems as

$$I^c = \frac{\sum_i (1 - L_{ii}^{-1}) x_i}{x},$$

where the numerator describes the portion of total ecosystem output that returns to the same sector one or more times. Here we apply this index to an economic system, as well as the *straight-through flow index*

$$I^s = \frac{x - \sum_i (1 - L_{ii}^{-1}) x_i}{x} = 1 - I^c.$$

2.7. Determinants and eigenvalues

Based on the identity $(\mathbf{I} - \mathbf{A})^{-1} = \text{adj}(\mathbf{I} - \mathbf{A}) / \det(\mathbf{I} - \mathbf{A})$, (Wong 1954) suggested the *determinant* of $\mathbf{I} - \mathbf{A}$

⁸ Strictly speaking, (Sonis et al. 2000) prove this for an MPM purely based on the Leontief inverse, and not for $\mathbf{S}$.

⁹ See also (Dietzenbacher and van der Linden 1997; Miller and Lahr 2001). Strassert already makes an – albeit qualitative – connection to the total flow concept (Section 2.11). (Gallego and Lenzen 2005) show that hypothetical extractions and total flow are mathematically equivalent.

$$\det(\mathbf{I} - \mathbf{A}) = \frac{\text{adj}(\mathbf{I} - \mathbf{A})_{ij}}{(\mathbf{I} - \mathbf{A})_{ij}^{-1}} \text{ for any } i, j$$

to measure complexity: The smaller the determinant, the larger the Leontief multipliers, indicating a greater degree of complexity.

Lorenzen (1981) interprets the dominant eigenvalue γ^* of the Leontief inverse as the *gross quota*¹⁰

$$\gamma^* = \frac{\mathbf{x}^*}{\mathbf{y}^*} = \frac{\sum_{i=1}^N x_i^*}{\sum_{i=1}^N y_i^*}$$

between the eigenvector $\mathbf{y}^*$ belonging to γ^* and the gross output $\mathbf{x}^*$ required to produce $\mathbf{y}^*$. The higher γ^* the higher the interdependence of sectors in the economy. γ^* can be compared with the actual gross quota of the economy

$$\gamma_g = \frac{\mathbf{x}}{\mathbf{y}} = \frac{\sum_{i=1}^N x_i}{\sum_{i=1}^N y_i} = \frac{\sum_{i=1}^N \sum_{j=1}^N L_{ij} y_j}{y} = \sum_{j=1}^N \sum_{i=1}^N L_{ij} \frac{y_j}{y} = \sum_{j=1}^N L_{.j}.$$

If key sectors – as measured by $L_{.j}$ – are strongly represented in the actual final demand $\mathbf{y}$, then $\gamma_g > \gamma^*$, and vice versa. The actual gross quota is also identical to the *mean path length* used by Finn (1976), and is also strongly related to the indices described in Section 2.3.

Dietzenbacher (1992) suggests the dominant eigenvalues of the input and output coefficients matrices to be used as a general indicator for backward and forward inter-industry linkages, respectively. This author presents an elegant calculus connecting the left-hand and right-hand eigenvectors of the input and output matrices, respectively, to the weighted and unweighted linkages (Section 2.5). Lenzen (2006) uses the dominant eigenvalue $\lambda_{\max}$ of $\mathbf{A}$ to characterize feedback in ecosystem networks: The higher $\lambda_{\max}$, the more internal feedback exists within the system, and the longer it takes the system to adjust to an initial exogenous shock. The same logic can be applied to economic systems.

2.8. Order matrices

Yan and Ames (1965) examine the decomposition of the Leontief inverse into production rounds (see also (Suh and Heijungs 2007))

¹⁰ “Bruttoquote” in German. Lorenzen shows that $\mathbf{x}^*$ is also eigenvector of $\mathbf{A}$, and that $(1 - 1/\gamma^*)$ is its eigenvalue.

$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \dots$$

The existence of zero values A_{ij} in $\mathbf{A}$ prevents changes in one sector j to directly affect certain other sectors i . However, indirect effects between such sectors can propagate through the economy via a number of intermediate sectors, for example $A_{im} A_{mn} A_{nj}$. The elements K_{ij} of an order matrix $\mathbf{K}$ measure how many production rounds are necessary to connect two sectors i and j . For example, assuming that $\mathbf{A}^2 \ni A_{im} A_{mj} = 0 \forall m$ but that $\exists m, n$ so that $\mathbf{A}^3 \ni A_{im} A_{mn} A_{nj} \neq 0$, then $K_{ij} = 3$, because at least three production rounds are necessary for a change in sector j to affect sector i . Yan and Ames (1965) suggest the reciprocal of the harmonic mean of elements of $\mathbf{K}$ as measure of economic complexity.

Blin and Murphy (1974) object that this measure reflects the existence of relations but not their magnitude, and that it is sensitive only to the shortest existing path between two sectoral nodes (their “first meeting”), but blind to all other, longer paths linking the same nodes. Blin and Murphy (1974) show that weighting Yan and Ames’ order matrix with the size of coefficients A_{ij} simply reproduces the series expansion in the above equation, and that the harmonic mean becomes a backward linkage.

Further, and once again, with modern tables often being filled with small but entirely non-zero elements, such an order matrix is not applicable anymore.¹¹ This obstacle can be circumvented by a procedure proposed by Robinson and Markandya (1973), who calculate the number of rounds k necessary for the (cumulative) effect of a change in sector j on sector i to become less (more) significant than a chosen cut-off point, that is for $A_{ij}^k (\sum_{l=0}^k A_{ij}^l)$ to become smaller (larger) than that cut-off. They assemble the (K_{ij}) into a *Transactions Round Matrix* $\mathbf{K}$ showing the number of times sectors supply each other. According to Robinson and Markandya (1973), the total number of transactions

$$K^{**} = \sum_{i,j=1}^N k_{ij}$$

“measures the total amount of information required by the system to reach equilibrium (and thus is perhaps a good measure of the complexity of the system).” This measure was also used by Bosserman (1981) to describe ecosystems, and by Schnabl and co-workers (Holub and Schnabl 1985; Weber and Schnabl 1998; Schnabl 1994) in qualitative input-output analysis and Minimal Flow Analysis (MFA).

2.9. Triangulations

Chenery and Watanabe (1958) were the first to point out the use of input-output table triangulations for determining the degree of *one-way interdependence* within total

¹¹ The same is true for the graph-theoretic version of Yan and Ames’ argument, put forward by (Campbell 1975, 1972). (Fontela et al. 2000) suggest applying a threshold criterion for the existence of a relation.

interdependence.¹² Accordingly, we write the *degree of linearity* (Helmstädter 1965b) as

$$\tau = \frac{\sum_{i>j} T_{ij}^*}{\sum_{ij} T_{ij}^*},$$

where $\mathbf{T}^*$ is a triangulated version of the transaction matrix $\mathbf{T}$. $\mathbf{T}^*$ is obtained from $\mathbf{T}$ by re-ordering the sectors so that the sum of transaction on one side of diagonal is maximized, and each sector delivers more to than it receives from the subsequent sector. In a hypothetical economy where primary industries such as agriculture, fishing, forestry and mining supply secondary industries (for example manufacturing) and tertiary industries (for example services), and secondary industries supply tertiary industries, but not vice versa, τ would assume zero. Any departure from a zero value of one-way interdependence characterizes the dependence of primary industries on secondary and tertiary output, and so on.

Ghosh and Sarkar (1970) and Roy (1971) formulate an optimal ordering problem of an input-output matrix by minimizing a weighted sum of intermediate transactions, where the weights d_{ij} are a pair wise “distance” between industry i and j . This distance can be understood in a spatial sense, but also in a sense of distance in an ordered list. Blin (1973) puts this approach into a general context of combinatorial optimization models.

2.10. Entropy

Proops (1988, 1983) used the well-known entropy

$$H^L = -\sum_{ij} p_{ij} \ln p_{ij}, \quad p_{ij} = \frac{L_{ij}}{\sum_{ij} L_{ij}}$$

to describe how sectors of an economic system become more enmeshed with each other over time. The larger the entropy, the more equal the coefficients of the Leontief inverse, and hence the more equal the sectors of the economy are participating in production. (Zucchetto 1981) and (Jackson et al. 1989) apply the same principle to the transactions matrix $\mathbf{T}$

$$H^T = -\sum_{ij} c_{ij} \ln c_{ij}, \quad c_{ij} = \frac{T_{ij}}{\sum_{ij} T_{ij}}.$$

¹² This concept was taken further soon after by a number of German researchers (Helmstädter 1965a; Korte and Oberhofer 1969; Lamel et al. 1972; Wessels 1981), and extended to international transactions by (Helmstädter 1969), and to block-triangularity by (Simpson and Tsukui 1965). [Nakamura, 2007 #3624] apply triangulations in their Waste-Input-Output approach to material flows.

Jackson et al. (1989) additionally define the *information gain* H' between subsequent transaction matrices $\mathbf{T}$ and $\mathbf{T}_0$ as

$$H' = \sum_{ij} p^*_{ij} \ln \frac{p^*_{ij}}{p^*_{0,ij}}, \quad p^*_{ij} = \frac{T_{ij}}{\sum_{ij} T_{ij}}.$$

However, this would require constant price matrices. Ulanowicz (1980, 1981) defines a number of additional measures based on the *system uncertainty*

$$H^L = -x \sum_i g_i \ln g_i, \quad g_i = \frac{x_i}{\sum_j x_j},$$

which is further decomposed into four constituents: *ascendency*, *tribute*, *dissipation*, and *redundancy*. These measures have only limited interpretations for economic systems (see Szyrmer (1985b)).

Finally, Soofi (1992) applies the entropy formula to forward and backward linkages. In this view, a high entropy value brought about by small variations in linkages indicates a high degree of integration in the economy. This measure was applied by the New Zealand Treasury (Claus 2003).

2.11. Total flow

Szyrmer (1985a; 1992) introduces *total flow* as an alternative to the traditional concepts of direct and indirect flows as measured by $\mathbf{A}$ and $\mathbf{L}$. The basic idea is that each flow originating from an intermediate sector i , proceeding via intermediate sectors j and k , and terminating in final demand l , not only contributes to connectedness through its direct transfer ($i \rightarrow j$; $\mathbf{A}$) or its supply to final demand ($i \rightarrow j \rightarrow k \rightarrow l$; $\mathbf{L}$), but also via indirect intermediate paths ($i \rightarrow j \rightarrow k$). General measures for system connectedness can hence be leveraged off the total flow matrix

$$\mathbf{Z}^{\text{total}} = \mathbf{A} \hat{\mathbf{L}}^{-1} \hat{\mathbf{x}} = (\mathbf{L} - \mathbf{I}) \hat{\mathbf{L}}^{-1} \mathbf{L} \mathbf{y},$$

where $\hat{\mathbf{L}}$ is the diagonal matrix of diagonal elements of the Leontief inverse $\mathbf{L}$. Each element z_{ij}^{total} measures the total output between sector i and j , not just the portion destined for final demand.¹³ Szyrmer labels as *transit flow* those purely intermediate flows that make up the difference between total flow and the Leontief inverse. Total flow coefficients can be defined as

¹³ Note that the total flow matrix does not support Leontief-type impact analyses anymore, since unlike in the Leontief system, where final demand components have mutually exclusive but collectively exhaustive own supply-chain network, total flow is characterised by non-additivity. For discussions surrounding this issue, see (Heimler 1991; Milana 1985; Gallego and Lenzen 2005; Lenzen et al. 2007).

$$h_{ij} = \{\mathbf{H}\}_{ij} = \{\hat{\mathbf{x}}^{-1} \mathbf{Z}^{\text{total}}\}_{ij} = \begin{cases} \frac{L_{ii}x_j}{L_{jj}x_i} & i \neq j \\ \frac{L_{ii}x_j - x_j}{L_{jj}x_i} & i = j \end{cases}.$$

$\mathbf{Z}^{\text{total}}$ and $\mathbf{H}$ support the same range of measures as $\mathbf{A}$ and $\mathbf{L}$ (Szyrmer 1985a): sums of transactions, sums of coefficients, average coefficients, and entropies. For example, average coefficients and entropies are defined as

$$\frac{\sum_{ij} h_{ij}}{N^2} \quad \text{and} \quad -\sum_{ij} \frac{z_{ij}^{\text{total}}}{x} \ln \left(\frac{z_{ij}^{\text{total}}}{x} \right), \text{ respectively.}$$

Applications of the total flow concept can be found in (Szyrmer and Walker 1983; Szyrmer and Ulanowicz 1987).

3. The Australian economy 1975 – 1999: a case study

We are most interested in using the measures to analyze changes in complexity of our economic system in terms of what we would like to call key performance indicators (KPIs). To give a practical example, we draw on twenty five years of development in Australia. We begin by analyzing the economic development in Australia with reference to all measures surveyed above. We then move to selecting the most appropriate measures in order to interpret the evolution of complexity in Australia's economy according to our KPIs. Our KPIs include: economic flows, greenhouse gas emissions (from fuel use), material flow and employment. We included these indicators, as they are used by the Australian government to cover the three tenants of economic, social and environmental performance at the national level (see for example Foran et al (2005)). Employment provides a simple way to measure social outcomes in an input-output context, and greenhouse gas emissions and material flow give two complementary views of environmental issues.

In a survey of measures by (Hamilton and Jensen 1983), they found accounting conventions and levels of aggregation affecting most measures. Hence, in this study, we use consistent conventions of accounting, including indirect allocation of imports to properly study the true structure of the economy. (Results were calculated both for indirect or direct allocation of imports, and in this case for Australia, there was limited effect, probably due to low import levels). Also, three levels of aggregation were studied, an enlarged 344 sector database, an industry level database of 105 sectors, and an aggregated database of 30 sectors. This was accomplished so as to ascertain aggregation effects. Ideally, we want measures of complexity that are aggregation invariant or at least not inconsistent at different levels of aggregation. We finally studied relative changes (by normalizing all results), as we are interested in measures

that are not necessarily constant in absolute terms, but which give consistent results over time.

3.1. Data sources

Many of the aggregate measures reviewed in Section 2 vary with the structure and aggregation level of the sector classification in the input-output tables. Comparative studies – whether over time or inter-country – thus require these tables to be expressed in a common classification. Therefore, for the purpose of this analysis, the input-output tables compiled between 1975 and 1999 by the Australian Bureau of Statistics were aligned with regard to their sector classifications. As numerous authors attest to, this alignment is not a trivial undertaking, because it is generally hampered by a lack of information about the relationships, or *concordances* between changing classifications. This issue was already encountered by Chenery and Watanabe (1958) when faced with the task of consolidating four countries' input-output databases into one common format.¹⁴ In the framework used in this study, we have used a range of commodity data to complement published input-output tables to create a series of tables of consistent classification at a level of detail of 344 sectors. Fifteen input-output tables were published during this time series which are made use of. Data on greenhouse gas emissions from fossil fuel combustion was extracted from an energy database (Australian Bureau of Agricultural and Resource Economics 2006) and had emission factors applied. Material flow data was taken from (Australian Bureau of Agricultural and Resource Economics 2004; Australian Bureau of Statistics 2004a), (Australian Bureau of Statistics 2001) and employment from input-output tables (Australian Bureau of Statistics 2004b) and earlier. Current price tables are used for all measures in this paper, as all variables are determined within a single year.

3.2. Review of all measures

As there is no generally accepted measure of complexity (Hamilton and Jensen 1983), in order to be able to interpret the range of measures surveyed, we firstly give an abridged analysis of all measures. Measures that are found to be inconsistent across aggregation, or which provide no discernibly useful information are subsequently excised.

In order to capture the temporal development, we analyze trends, and normalize all results to 1999 levels. In all graphs, we also weight by expected complexity such that an upwards trend is cognizant with increasing complexity. For reference, a pictorial summary of the measures across all indicators (Figure 1) is provided, and needs some initial explanation. Measures are classified by type (see Section 2.1); according to aggregation (three levels of aggregation are provided); and trends are colour coded according to: an observance in agreement with increasing complexity (grey), an

¹⁴ In particular for inter-country comparisons there exist a number of issues pertaining to the varying treatments of capital and imports, to sector aggregation, and to definitions of economic establishments and intra-industry transactions (Chenery and Watanabe 1958; Sonis and Hewings 1999; Sonis et al. 2000). Since this work is about an inter-temporal comparison, these issues do not arise here.

		K**			K1**			K2**			K3**			K4**			K5**			K6**		
Connectivity/ Multipliers	Aggregation	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30
	Economic																					
	Greenhouse																					
	Material																					
	Employment																					

		ω			σ			κ			lm			$\det(I-A)$			λ_{max}			τ			HT		
Connectedness / Transactions	Aggregation	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30
	Economic																								
	Greenhouse																								
	Material	n/a	n/a	n/a																					
	Employment	n/a	n/a	n/a																					

		y^*			y_g			HL			lc			ls		
Connectedness / Multipliers	Aggregation	344	105	30	344	105	30	344	105	30	344	105	30	344	105	30
	Economic															
	Greenhouse															
	Material															
	Employment															

Increased complexity

Decreased complexity

No trend

Figure 1 Summary of all measures across all indicators. Refer to text for explanation.

observance in agreement with decreasing complexity (black) and no trend (white) – refer back to section 2 for reasoning.

In order to provide an objective rather than subjective gauge of trends, we apply a linear regression to the results, with the independent variable being time, and the dependent variable being the results of the measure. We state an existence of a trend if the calculated relative standard error of the linear co-efficient is less than 50%. Whilst such a method is somewhat arbitrary and imprecise, we only wanted a reasonable reflection of the results, and the reader is referred to the subsequent figures in this section for their own interpretation.

As an example, for skewness, σ , of economic flows, there was no reliable trend found for the full system (344 sectors) and for the system aggregated to 105 sectors (both indicated white). However, at an aggregation of 30 sectors, a trend consistent with increasing complexity is found (indicated grey). For the greenhouse gas flows, a trend of increasing complexity was found for 344 sectors, and a trend of decreasing complexity (black) was found for the other levels of aggregation. We now proceed with a more in-depth review of these results.

3.2.1. Connectivity Measures of Multipliers

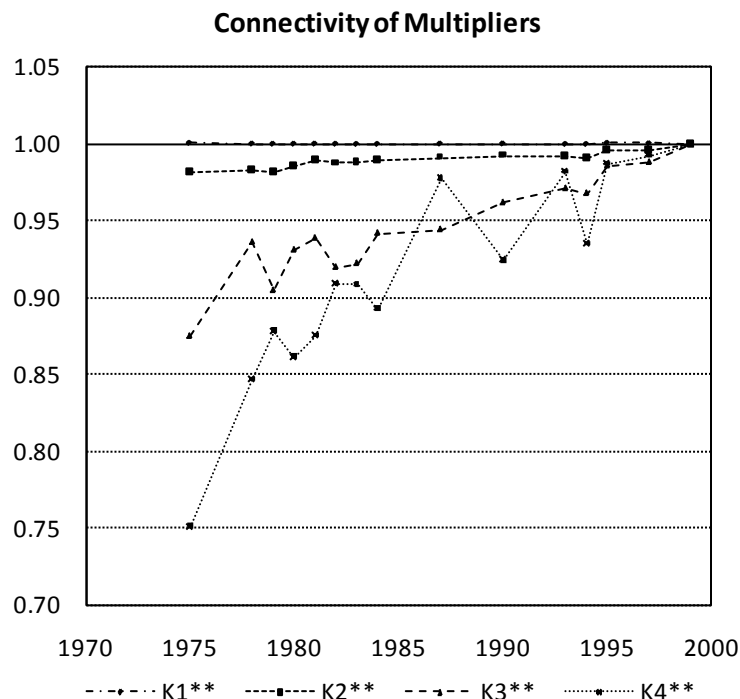


Figure 2 Evolution of relative values of order matrices, Australia 1975-1999, 344 Sectors.

The only connectivity measure reviewed is K , the Transactions Rounds Matrix (Section 2.8). The reader is reminded that connectivity measures assess the Boolean existence of connections within a system. The summary of trends found for K are classed according to different cut-offs values (see Section 2.8 and (Robinson and Markandya (1973)). As

previously mentioned, a cut-off of zero can put the analyst at the mercy of ‘manufactured’ data points from computational table creation. In contrast, placing the cut-off too high in disaggregated systems will cause the measure to not pick up on changes in connectivity. In Figure 1 and Figure 2 we define the variables; K** cut-off =0, K1** cut-off =0.1, K2** cut-off=0.01, K3** cut-off = 0.001, K4** cut-off = 0.0001, K5** cut-off = 0.00001).

As expected, the cut-off choice had an obvious effect on results. Around a cut-off of 0.001 (K3** in Figure 2) minimal “noise” was found across all aggregations (see Figure 1). This corresponds to just below the mean of the Leontief (~0.005 in 1999), whereas at 0.0001 and below, results start fluctuating greater (compare Figure 1).

Our results clearly show a trend which is interpreted as a decrease in the overall sum of the Transaction Rounds Matrix. This indicates a shortening of the average path length connecting two sectors and would be expected in a maturing economy. In essence, more intermediate sectors are taking on the delivery of goods and services and a greater number of feedback loops are occurring within the economy. We found aggregation to affect results only for high and low cut-off values (see Figure 1), and judge this measure with an average cut-off value to be a good indicator of changing complexity, and henceforth use K3**.

3.2.2. Connectedness Measures of Transactions

The connectedness measures of transactions measure the presence and magnitude of connections in a system. They include the proportion of value added (ω), the proportion of intermediate transactions (I^m); the determinant; the dominant eigenvalue ($\lambda_{\max}$); degree of linearity (τ); the entropy of transactions (H^T); skewness (σ) and kurtosis (κ). Of these measures, the determinant gave highly variable results over time with no consistent trend. The dominant eigenvalue of the transactions matrix showed high initial variability followed by no consistent trend and the degree of linearity also shows no trend or consistency under aggregation. In contrast, ω , I^m and H^T show clear trends.

The dominant eigenvalue in interdependent systems determines how many rounds of interactions are needed in order for the system to return to a stationary state after a shock and hence determines the asymptotic behaviour of the system. The lack of a trend in the results implies there is no convergence towards an asymptotic or limited state being observed in the Australian economy.

The degree of linearity (τ) is weighted not by the total economic production, but by the total industrial production. Our other measures (ω or I^m) mentioned below show increasing industrial production relative to economic production. Hence the measure of the degree of linearity is somewhat subsumed by larger scale efficiency effects, and its exclusive interpretation is not so useful.

The proportion of value added or final demand in total output (ω) shows a small decrease over time (approximately 6%). This equates to an increasing dominance of intermediate transactions (shown also by I^m) within the economy, as one would expect with an increasingly complex production system. Chenery and Watanabe (1958) find average ω and

I^m in the range of 30%-50% in a cross-country analysis. They found increasing share of intermediate goods in total output (I^m) with the developing economies and expected increased disaggregation to sharpen distinctions between intermediate and final use. Similar results are found for Australia, with intermediate component of gross output growing from 35% in 1975 to 42% in 1999. In contrast the proportion of value added (including imports) shrank from 62% to 58%; excluding imports, this dropped from 56% in 1975 to 50% in 1999.

H^T , the entropy of the transactions matrix, has a consistent trend upwards over time, reflecting an increasing enmeshing of sectors, consistent with increasing complexity. Proops (1988) generally found an increase in entropy over time, consistent with the expectation of increased complexity. However, much like the findings in this study, increases were not always consistent, and comparisons between countries did not always find more developed countries having higher entropies.

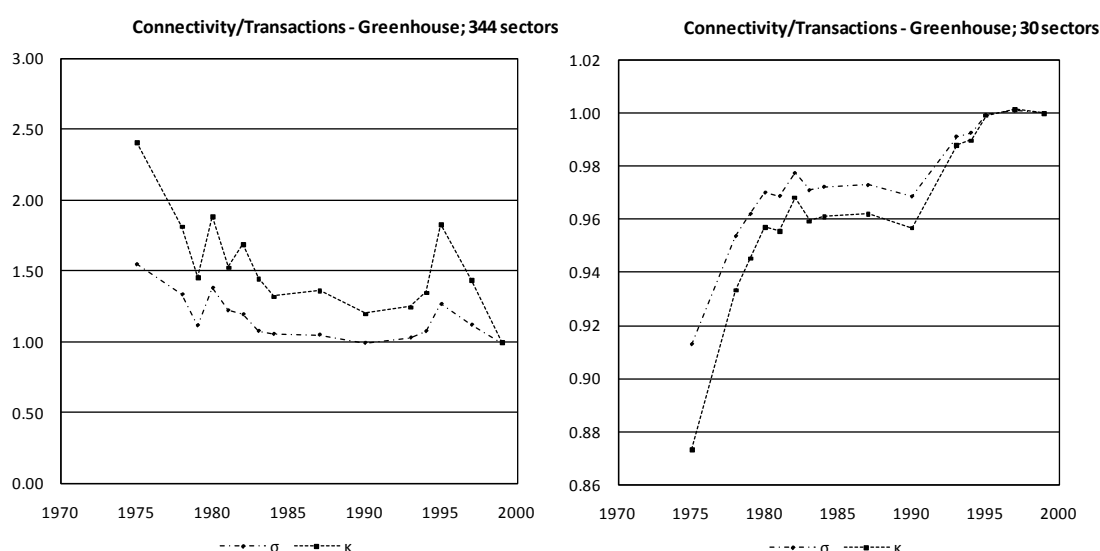


Figure 3 Skewness and kurtosis of structure of greenhouse flows. Australia, 344 sectors (left), 30 sectors (right).

Skewness and kurtosis (σ and κ) within the Australian economy show relative distribution of the connections of the system. This measure was found to vary with aggregation (see Figure 1) across both economic and environmental indicators. Absolute results show the skewness for 1999 was 17.5, indicating, as for all years, that most of the data is greater than the mean. A similar result was found for the kurtosis, which shows the data being very outlier prone ($\kappa \sim 320$, compared to a normal distribution with $\kappa=3$). These measures are perhaps not suitable for detailed datasets with large variation in cell values. Skewness and kurtosis are sensitive to element size, and aggregation exactly influences this - an obvious shortcoming. The results for greenhouse emissions are shown in Figure 3, where we found actual inversion of results under aggregation. We consider these measures inappropriate for such analysis, and no longer contemplate their use.

3.2.3. Connectedness Measures of Multipliers

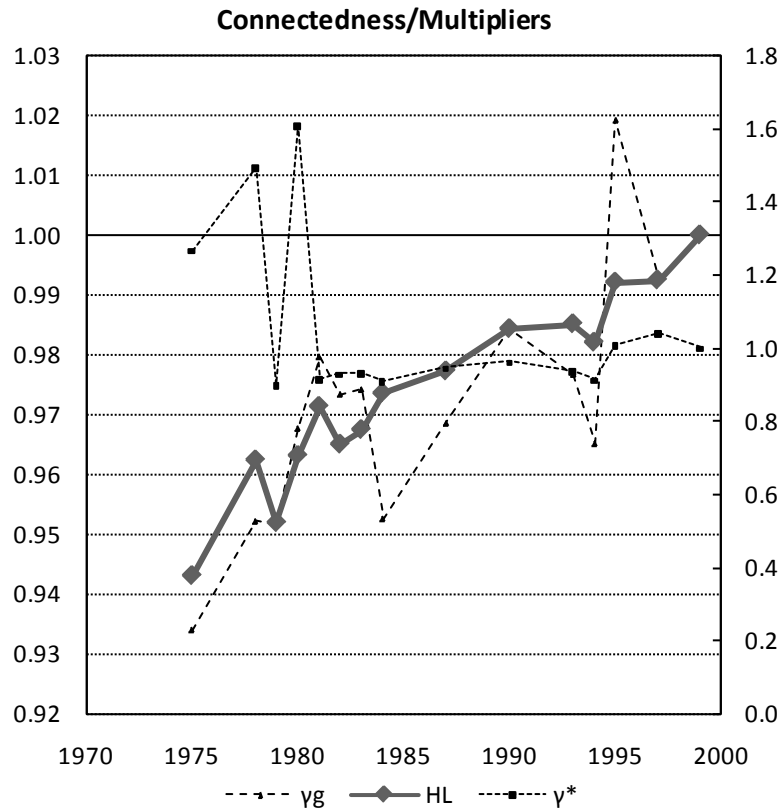


Figure 4 Connectedness measures of multipliers, 344 sectors.

Of the range of connectedness measures of multipliers, we again saw a number of measures providing indeterminate results. Of the five aggregate measures, only γ^g and H^L showed useful trends (Figure 4) and consistency under aggregation. Gross quota γ_g generally gradually increases over time, which is in line with expected developments of complexity. γ_g is consistently less than γ^* , showing key sectors are not strongly represented in final demand (See Section 2.7), as would be expected in a developing economy, where key sectors are subject to greater intermediate processing. The dominant eigenvalue of $\mathbf{L}$, γ^* , showed almost identical results to the dominant eigenvalue of $\mathbf{A}$, $\lambda_{\max}$, implying the measure, whilst consistent across transactions and multipliers, suffers the same shortcomings as identified in 3.2.2. Finn's flow indices (I^c , I^s) were related to each other, and dependent on the diagonal of the multiplier matrix. The increasing dispersal of other flows in the economy is not captured by this index, and this is perhaps why it has not provided useful results in this context.

3.2.4. Summary of review

Not all measures of economic development reviewed were expected to provide useful application in a holistic study of complex system evolution. A number were found to be too greatly influenced by either aggregation (σ and κ), or by not being completely holistic in their coverage (τ , I^c , I^s). The determinant can be interpreted as a measurement of the volume

of the economic system, but has not been found to provide useful insight into the inter-relatedness of the system.

The measures that were found to be useful in tracking the evolution of the Australian economy were: Transactions Rounds Matrices K^{**} , the proportion of value added (ω), the normalized sum of intermediate transactions (I^m); the entropy of transactions (H^T); the gross quota (γ^g); and the entropy of multipliers (H^L). The entropy measures could be considered similar to skewness and kurtosis (σ and κ), as both seek to estimate the equality of the system variables. The entropy, however, is weighted by element size resulting in greater invariance under aggregation. The dominant eigenvalue γ^* did not show clear trends in the economic analysis, but this may be due to its construction and interpretation. Hence, we maintain its use in the study.

In the survey of six of the measures by Hamilton and Jensen (1983), they found all measures but ω to be affected by aggregation. We do not find such great effects here. They also found I^m to be the most useful measure, consistent with findings here.

We further group the measures into those related to overall economic/environmental efficiency (ω , I^m , γ^g). These measures operate on a summation of system variables, and generally relate to impacts occurring inside/outside the economic system compared to the overall economic output. Secondly, we group entropy measures and transactions rounds matrices (H^T , H^L , K^{**}) to signify *enmeshing* and equality of system variables. These measures relate less to efficiency, and more to how connected the system is. Finally, we have the measure of *resilience* – the dominant eigenvalue of the multiplier matrix (γ^*) which can be interpreted as a measure of asymptotic behaviour of the system.

3.3. Analysis of complexity with Key Performance Indicators

With a reduced set of measures, we now turn to analyzing the implications of increased complexity for Industrial Ecology through our KPIs. Each economic measure of complexity that showed a trend was consistent with the hypothesis of increased complexity over time in Australia. We now focus on determining whether this is mirrored in terms of greenhouse gas emissions, material flow and employment. We would like to determine whether we are seeing more enmeshing of flows of KPIs in our economy, and whether this is happening in line with, or opposite to economic development. Trends for the six aggregate measures are presented in Figure 5. Enmeshing measures (H^T , H^L , K^{**}) are presented on a secondary axis for the KPIs due to the relatively higher changes in the other measures, which are presented on a logarithmic scale.

Moderate trends towards increased complexity are evident for the economic measures. This includes our efficiency measures (ω – decreasing proportion of value added, I^m – larger intermediate flows, γ^g – larger ratio of output to final consumption) and our enmeshing measures (H^T , H^L , K^{**}).

For greenhouse gas emissions, we are seeing trends of decreasing complexity. This is mainly due to efficiency measures, however, with lower greenhouse impacts occurring in the industrial structure (measured by I^m , γ^g). ω for greenhouse emissions measures the weighted residential greenhouse gas emissions, which has been fairly constant over time. Our indicators of enmeshing do not show significant changes, despite a rapid change in the mid 1990's, which may have been due to the early 1990's electricity liberalization in Australia. Excluding this event, there was a slight reduction in the enmeshing over time, as indicated by (H^T, K^{**}) , but which was not reflected in H^L (entropy of multipliers). This indicates that specialization in greenhouse emitting activities occurred within the economy such that direct links to emissions were reduced, but indirect links were maintained.

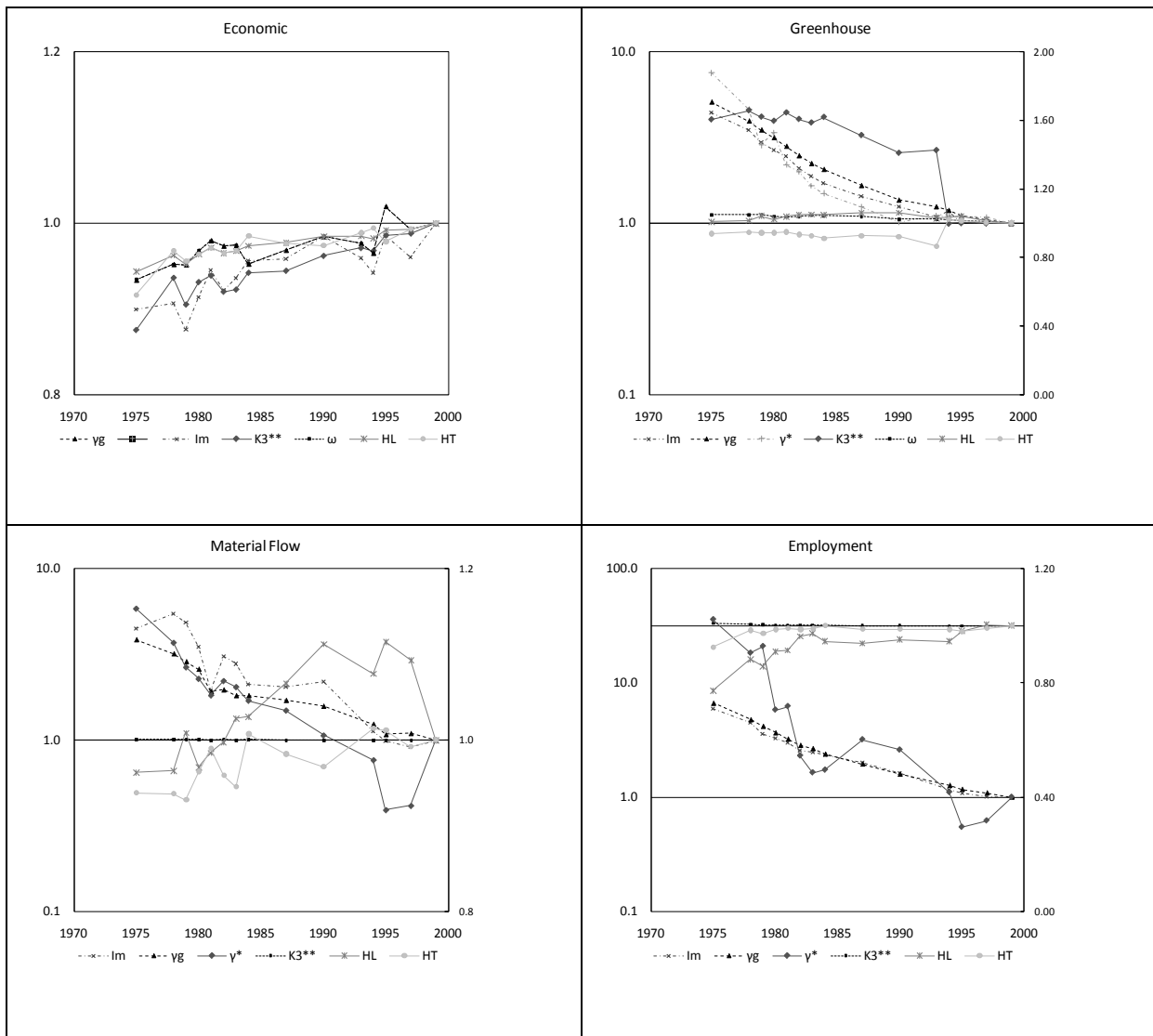


Figure 5 Evolution of measures of complexity for Australia, 1975-1999 (logarithmic scale)

The material flow and employment indicators show similar results – large efficiency improvements (I^m , γ^g) of the system, with smaller changes in enmeshing. K^{**} shows almost no change over time for both indicators. The entropy measures showed mixed results, but

were generally showing a trend of increased connectedness – i.e. there is an increased equality in the distribution of the flows of these KPIs.

The resilience measure (γ^*) had high variability and an indeterminate trend for economic flows (see section 3.2.3). This is not the case for our KPIs where a strong downward trend is observed. γ^* indicates the system's asymptotic behaviour, hence we would expect it to reduce as we are converging on a limiting factor of growth.. For the KPIs, we are seeing a clear trend towards reduced γ^* , indicating the system is converging towards a more stable state, with the expectation that this implies there is less abundant or excess use of the KPIs.

3.4. Linkages and economic landscapes

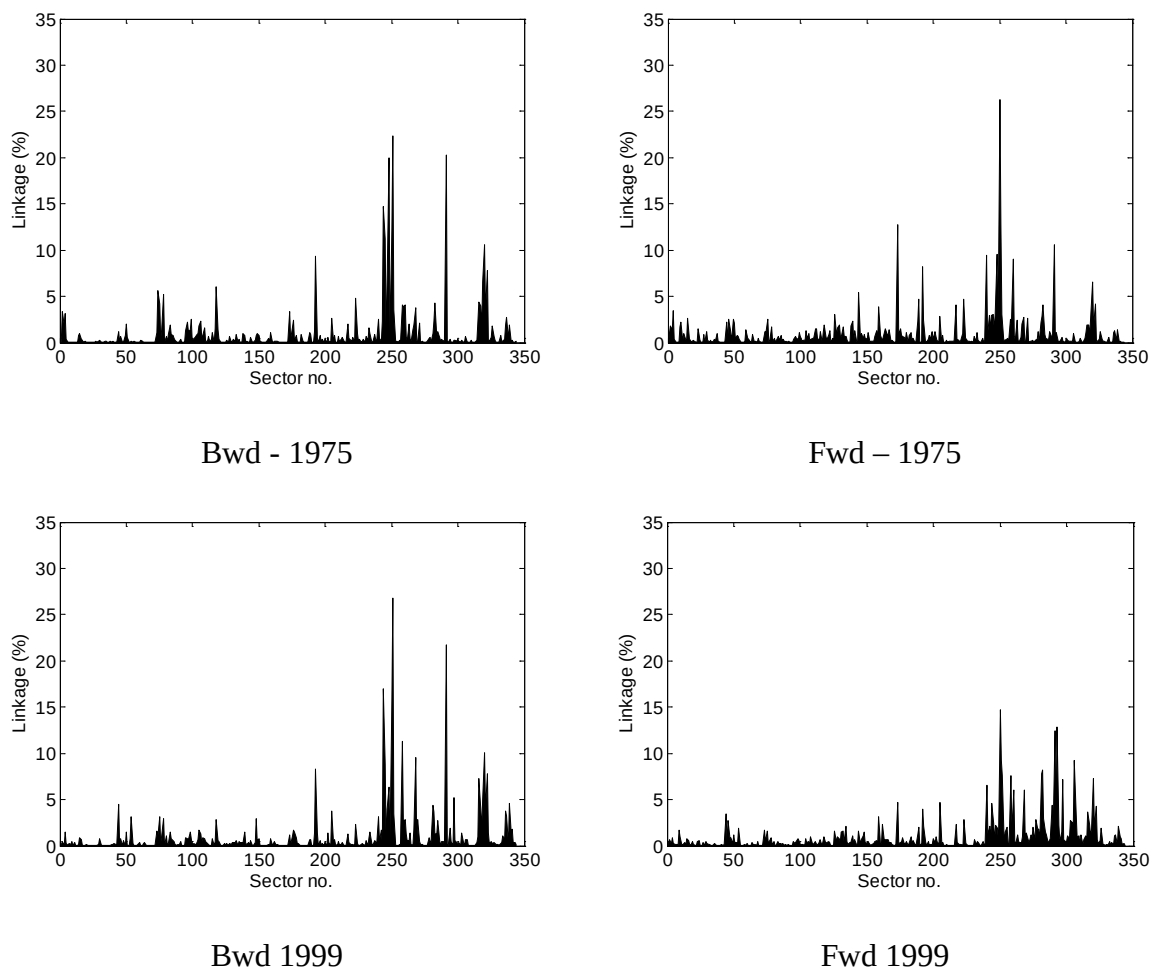


Figure 6 Backward and Forward linkages, Australia, 1975 and 1999, 344 sectors. X-axis shows primary industries on left and service industries on the right.

The preceding measures have all been aggregate measures that amass the changes in an economy into a single figure. At a more detailed level, we can look at linkages and ‘economic landscapes’ (see Section 2.5). These measures retain the sectoral breakdown, and

as such, do not provide the single measure desirable in interpreting the evolution of complex systems. However, they are useful in gaining an impression of the structural changes occurring in an economy over time, and as such, can inform and validate the aggregate results of the previous measures. We present these results separately here in order to assess and corroborate the findings in Section 3.3.

Due to the complexity of the figures, we present results only for economy (Figure 6) and greenhouse gas emissions (Figure 7) and for the beginning and end years of the analysis.

We can immediately see from Figure 6 that the evolution of the economy results in both a much greater dispersion of significant backward linkages as well as an increase in the size of backward linkages. This is consistent with our aggregate measures of complexity, where we saw an increase in the complexity of the economy over time. Forward linkages also show an evolution to a greater dispersion of key sectors, particularly in the trade, transport and services section of the economy. Thus, in terms of forward linkages, we are seeing a flattening in the relative importance of sectors over time.

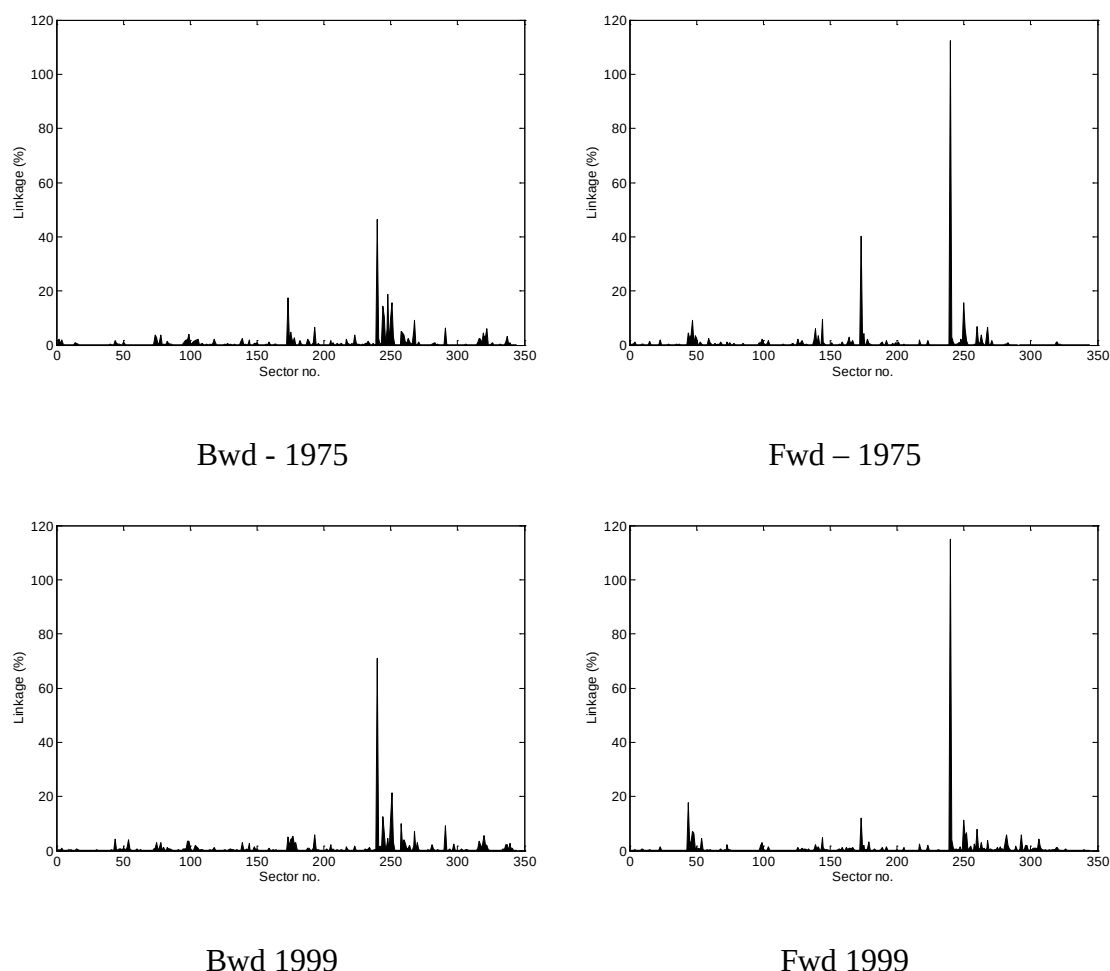


Figure 7 Backward and Forward linkages of Greenhouse Gas Emissions, Australia, 1975 and 1999, 344 sectors. X-axis shows primary industries on left and service industries on the right.

For greenhouse gas emissions (Figure 7), changes are less evident, but we are seeing fewer sectors having large backward linkages, and a decrease in the relative size of most important forward linkage sectors. The importance of electricity supply is most evident. Manufacturing (middle sectors of the graph) has decreased both in forward and backward linkages, implying that we are seeing less direct and indirect resource flows coming and going out of this sector.

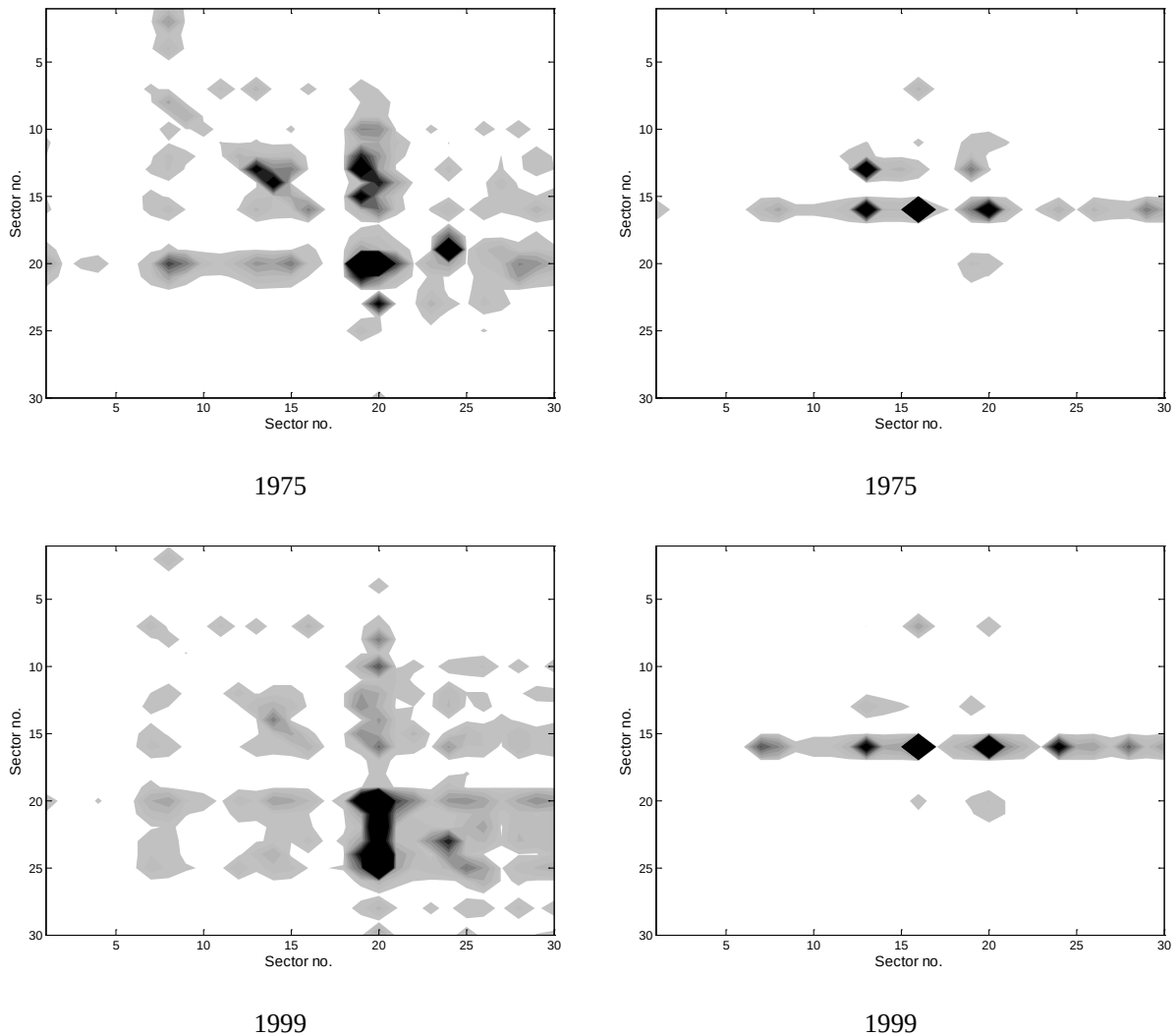


Figure 8 Contour plots showing evolution of economic structure (left) and greenhouse emissions (right) in Australia by field of influence intensity, S . Calculations performed at 344 sectors before aggregating to 30 sectors. Primary industries upper left, service industries lower right.

The economic landscape is perhaps best shown by the contour plots of multiplier product matrix, S (Figure 8). From these plots, the economic evolution is apparent, with a greater number and spread of key sectors over time, particularly in the secondary and tertiary sectors of the economy. This shows an almost classic economic evolution with the fading of primary and manufacturing sectors being replaced by a concentration of tertiary activity. The effect for greenhouse emissions is also apparent, with a decreased complexity of important greenhouse flows shown by the reduction in fill of the graphs, with concentration

centering on a few key sectors. Most evident of this is the decrease in emissions from manufacturing (shown in sectors 8-15), whereas electricity requirements are becoming more widespread across the economy (sector 16).

West (1999) in a study of Queensland, and later in study of Taiwan, found a 'hollowing out' of the economies using measures of relative change of intermediate requirements and key sector maps. This appears to be corroborated by the reduction in manufacturing shown in Figure 7 and Figure 8.

4. Discussion

4.1. Summary of findings

Out of fourteen measures of complexity, we concentrated on seven measures, which we used in order to interpret the development of linkages within the economy and aspects of the environment of Australia. We classed these measures into categories describing efficiency, enmeshing and resilience. Our aggregate measures clearly showed increasing complexity in economic terms within the system for efficiency and enmeshing. These findings were reinforced by the disaggregate measures shown in the forward and backward linkages and the fields of influence.

Across the measures, material use and greenhouse gas emissions generally showed a reduced complexity of flows within the economy. The connectedness measures, K^{**} , showed only slight reductions in complexity, implying that the number of environmental flows within the economy is not changing significantly. More pronounced were the broad scale efficiency measures relating resource use within the economy to gross production. As has often been the case, these measures are showing reductions in environmental intensity of production.

Both connectivity and connectedness measures of enmeshing generally showed reductions for greenhouse and increases for material flow and employment. This indicated that our economy is actually reducing the number of flows associated with greenhouse gas emissions, and that the magnitudes of the remaining flows were generally reducing. However, when we sought to capture indirect effects by examining multipliers rather than transactions, we found almost no change in the measures. As a result, it appears there is greater specialization occurring for greenhouse gas emissions in Australia, with overall requirements changing little. This was re-inforced by the contour plots, which showed a transfer of emissions from manufacturing sectors to electricity.

The dominant eigenvalue within economic measures showed no asymptotic behaviour. Within the environmental measures, however, there was a clear trend for greenhouse gas emissions and a reasonable trend for material flow. In industrial ecology terms, this means we are seeing less free and abundant use of resources and energy, and a move closer to specialization and efficiency.

5. Conclusion

This paper explores the use of measures to describe the evolving complexity of environmental-economic systems. Our analysis takes advantage of the detailed description of economic structure provided by input-output tables, generalized for environmental and social factors. We analyze a time series of input-output tables over twenty-five years for Australia in terms of a variety of measures suggested in the literature.

In our case study, we find almost a classic case of economic evolution, with complexity in the Australian economic system growing reasonably consistently over time. From linkage and contour plots, we can see this has come about from the reduction in the poorly linked primary and manufacturing sectors, to an increase in the tertiary and service sectors.

The impact this has had on environmental and social flows is almost opposite to the economic case. Whilst we have seen a decrease in economic specialization, there has been a corresponding increase in the efficiency of resource use and employment, resulting in lower levels of these factors necessary for economic function.

One of the major findings of our work is that whilst our economy is diversifying, our resource flows are becoming more specialized. Trade within the economy has been increasing, and the diversity of our economic sectors has increased. However, the industries with large environmental impacts have become more specialized and more important to the functioning of the economic system. In essence, the flourishing diversity of our economy is coming at the expense of the outsourcing of the environmental impact.

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PART III

CONCLUSION

The purpose of this work was to delve into the structural evolution of Australia's economy by investigating the implications for two indicators of environmental sustainability – material usage and greenhouse gas emissions.

The key components of this work were:

Part I – Model development

- Establishing a model of structural relationships for Australia
- Establishing a temporal framework of the model in order to investigate evolutionary pressures
- Establishing links between the Australian economy and environmental indicators

Part II – Application

- Analyse key influences on change in material use and greenhouse gas emissions
- Analyse key inter-industry relationships driving environmental impact
- Determine the overall economic evolution of the Australian economy with insight into material use and greenhouse gas emissions

In establishing the detailed model of structural relationships, a range of data sources were utilised. These data sources required significant reconciliation in order to have a temporally consistent database of input-output tables over a 30 year time-frame. A flexible and integrated mechanism for handling data under different classifications and pricing was implemented. A consequence of establishing the necessary detail was the resulting size of the model, and issues with existing optimisation techniques. New techniques were thus utilised in order to produce consistent and balanced tables describing the structural relationships.

Analysis of the stability of the resulting model showed annual variation in the order of 5% - in line with other work that has investigated stability of multipliers. Further results showed the importance of using up-to-date coefficient matrices, with errors subsiding from almost 35% on a 30 year time scale to nearer 5% for previous year tables. Simple methods to update input-output tables were investigated. The use of Supply/Use tables due to their greater explaining power produced the best result.

The generalisation of the model for environmental effects – notably greenhouse gas emissions and material flow required a range of proxy data in order to break down aggregate impacts into categories relevant to inter-industry analysis. A time series of these impacts was produced, and impacts embodied in final demand were calculated. In terms of greenhouse gases, a few key sectors, particularly beef and electricity produce the majority of Australia's domestic emissions, whereas emissions embodied in final demand show the importance of meat products, mining and to a lesser extent, service industries. Very few of the emissions important industries contributed significantly to GDP.

Part II began with a review of three methods for studying sustainable consumption. The key result from this review was the usefulness of structural decomposition analysis (SDA) in studying historical trends; an important input into the construction of hypothetical scenarios. SDA enabled pure consumption-induced impacts to be differentiated from other factors. The utility of such an approach is such that it may be more poignant to target industrial processes rather than household consumption if the results of an SDA show (for arguments sake) that final demand induced impacts have been decreasing whilst industrial structure induced impacts have been increasing.

Hence SDA was applied to the issues of material usage in the Australian economy and to greenhouse gas emissions. The major determinants of change in material flows were found to be the material efficiency within the economy, the level of economic activity and the mix of final demand. Other structural components had smaller effects at the aggregate level. At a more detailed level, however, the importance of the mineral sectors became apparent. Their improvements in mining techniques have necessitated less material requirements, but their increased use within the economy has had an offsetting effect. Export growth had also been responsible for significant per annum increases.

For greenhouse gas emissions, at a macro-level, changes in industrial efficiency, final demand mix, final demand destination and export mix were found to decrease emissions. The change in industrial efficiency was the strongest component of this group, particularly for fuel combustion emissions. In contrast, changes in structural interrelationships, affluence and export level strongly increased emissions. The residential effect was less pronounced, with emissions per-capita slightly reducing. Overall, the influence of population growth on emission levels was around a fairly consistent +1% p.a.

At the disaggregated level, the importance of the electricity supply, construction, transport and service industries to changing emission levels was confirmed, but no industry showed a constant effect over the full time series. A further important result was that there appeared to be a high level of substitution between energy and structural inputs, with diminished effect on total emissions. This suggests that the real gains in energy efficiency would not be as great as an energy intensity analysis would show, and that the consideration of impacts embodied in production was needed to capture all effects.

From these two studies, two issues became apparent – firstly a methodological note on the implementation of decomposition analysis with respect to zero values, and secondly, that

greater explaining power could be derived by looking at the above effects in a life-cycle view. Both issues were addressed, and in an application on Australia's emissions, the importance of disaggregating the competing forces of different production chains found important policy insights - with significant changes occurring both in type of consumption and technology of production overshadowed in the aggregate analysis.

Finally, the evolution of the Australian economy was investigated with respect to material flow and greenhouse gas emissions. One of the major findings of the work was that whilst the economy was diversifying, resource flows were becoming more specialized. Trade within the economy had been increasing, and the diversity of economic sectors had increased. However, the industries with large environmental impacts had become more specialized and more important to the functioning of the economic system. In essence, the flourishing diversity of the economy was coming at the expense of the outsourcing of environmental impact.

In order to bring this thesis to a conclusion, I revisit the principle research question posed at the beginning.

What are the principle drivers of environmental change in the structural relationships of Australia's economy?

Australia's economy has evolved significantly over the last 30 years. In this evolution, population, affluence and exports have been consistent drivers of increased greenhouse gas emissions and material flow. Underpinning these macro-effects, however, is the reliance on changing industrial structure or technology to bring about movement towards a steady state system. Overall, it was found these effects to be lacking. Increased diversity in the technological structure of the economy has had the opposite effect on environmental flows, where increased specialisation is seen. This has been reflected in large changes occurring in the individual production relationships that constitute our economic system, and does provide hope that if environmental flows were appropriately valued, these structural relationships could be harnessed to implement rapid change.

APPENDIX

SUMMARY OF PUBLISHED WORK

Publications

Wood, R. and S. Garnett. 2009. "An assessment of environmental sustainability in Northern Australia using the ecological footprint and with reference to Indigenous populations and remoteness." *Ecological Economics* 68(5): 1375-1384.

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Wood R and Lenzen M, Institutional ecological footprinting, Second Annual Conference of Sustainable Universities "Windows to a Greener Campus", The University of Melbourne, RMIT and Monash University, Melbourne, Australia,

Other

Awarded Australian Endeavour Research Fellowship (not taken up)

Executive committee organising PhD Conference: *Environmental Research Event*, Cairns, Dec 2007.

Executive committee organising PhD Conference: *Environmental Research Event*, Sydney, Dec 2006.