Evolutionary Structural Optimisation as a Robust and Reliable Design Tool

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Abstract

Evolutionary Structural Optimisation (ESO) is a relatively new design tool used to improve and optimise the design of structures. It is a heuristic method where a few elements of an initial design domain of finite elements are iteratively removed. Such a process is carried out repeatedly until an optimum design is achieved, or until a desired given area or volume is reached.

There have been many contributions to the ESO procedure since its conception back in 1992. For example, a provision known as Bi-Directional ESO (BESO) has now been incorporated where elements may not only be removed, but added. Also, rather than deal with elements where they are either present or not, the designer now has the option to change the element's properties in a progressive fashion. This includes the modulus of elasticity, the density of the material and the thickness of plate elements, and is known as Morphing ESO. In addition to the algorithmic aspects of ESO, a large preference exists to optimise a structure based on a selection of criteria for various physical processes. Such examples include stress minimisation, buckling and electromagnetic problems.

In a changing world that demands the enhancement of design tools and methods that incorporate optimisation, the development of methods like ESO to accommodate this demand is called for. It is this demand that this thesis seeks to satisfy. This thesis develops and examines the concept of multicriteria optimisation in the ESO process. Taking into account the optimisation of numerous criteria simultaneously, Multicriteria ESO allows a more realistic and accurate approach to optimising a model in any given environment.

Two traditional methods – the Weighting method and the Global Criterion (Min-max) method have been used, as has two unconventional methods – the Logical AND method and the Logical OR method. These four methods have been examined for different combinations of Finite Element Analysis (FEA) solver types. This has included linear static FEA solver, the natural frequency FEA solver and a recently developed inertia FE solver. Mean compliance minimisation (stiffness maximisation), frequency maximisation and moment of inertia maximisation are an assortment of the specific objectives incorporated. Such a study has provided a platform to use many other criteria and multiple combinations of criteria.

In extending the features of ESO, and hence its practical capabilities as a design tool, the creation of another optimisation method based on ESO has been ushered in. This method concerns the betterment of the bending and rotational performance of cross-sectional areas and is known as Evolutionary Moment of Inertia Optimisation (EMIO). Again founded upon a domain of finite elements, the EMIO method seeks to either minimise or maximise the rectangular, product and polar moments of inertia. This dissertation then goes one step further to include the EMIO method as one of the objectives considered in Multicriteria ESO as mentioned above.

Most structures, (if not all) in reality are not homogenous as assumed by many structural optimisation methods. In fact, many structures (particularly biological ones) are composed of different materials or the same material with continually varying properties. In this thesis, a new feature called Constant Width Layer (CWL) ESO is developed, in which a distinct layer of material evolves with the developing boundary. During the optimisation process, the width of the outer surrounding material remains constant and is defined by the user.

Finally, in verifying its usefulness to the practical aspect of design, the work presented herein applies the CWL ESO and the ESO methods to two dental case studies. They concern the optimisation of an anterior (front of the mouth) ceramic dental bridge and the optimisation of a posterior (back of the mouth) ceramic dental bridge. Comparisons of these optimised models are then made to those developed by other methods.

Declaration

Candidate's Certificate

This is to certify that the work presented in this thesis was carried out by the candidate in the Discipline of Structural Optimisation, The School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, and has not been submitted to any other university of institution for a higher degree.

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Table of Contents

Nomenclature

A, A_0	cross-section area and original cross-section area
C	mean compliance
C_{all}	mean compliance prescribed limit
$crit$	criterion
d	distance
d_j^+	measure of under-achievement
d_j^-	measure of over-achievement
$\boldsymbol{d}$	global nodal displacement
e	subscript of element
E	Young's modulus
$f(\boldsymbol{x})$	vector of objective functions
$f(\boldsymbol{x}^*)$	utopia point
$f_j(\boldsymbol{x}_j^{min})$	vector of minimum objective functions
$F_{multicrit}^{i}$	weighted criterion sensitivity number
$G_{multicrit}^{i}$	global criterion sensitivity number
i	element number
I_x, I_y, I_z	moment of inertia about x, y and z-axis
I_{xy}	product of inertia
j	criterion number
k_x, k_y, k_z	radius of gyration about x, y, and z-axis
k_{xy}	product radius of gyration
K	stiffness
$\boldsymbol{K}$	global stiffness matrix
m_n	modal mass
M	applied moment or number of elements
$\boldsymbol{M}$	global mass matrix
n	natural frequency subscript
N	number of criteria
OC_e^{crit}	element criterion sensitivity number

$OC_{\max}^{crit}$	maximum value of criterion sensitivity number
p	constant of global criterion method
$\boldsymbol{P}$	nodal load vector
r	tooth notch radius
t	thickness
u^i	element displacement vector
$\boldsymbol{u}_n{}^i$	element eigenvector
$U(f(\boldsymbol{x}))$	utility function
$\boldsymbol{u}_n$	eigenvector
V	volume
w	pontic connector width
w_j	weighting preference
$\boldsymbol{x}$	design variable vector
$\boldsymbol{x}^*$	optimum design variable vector
X_c, Y_c	centroid coordinates

Greek Symbols

α^i	strain energy sensitivity number
α_e^{crit}	element sensitivity number of *crit* criterion
$\alpha_{\max}^{crit}$	maximum value of element sensitivity number of *crit* criterion
$\alpha_{average}^{crit}$	average value of element sensitivity number of *crit* criterion
α_{MOI}	moment of inertia sensitivity number
$\left\{\alpha_n^i\right\}^{new}$	linearly adjusted frequency sensitivity number
$\left\{\alpha_n^i\right\}^{old}$	original frequency sensitivity number
$\left\{\alpha_n^*\right\}^{old}$	original maximum value frequency sensitivity number
ε_j	criterion constraint
ν	Poisson's ratio
σ_e^{vm}	von Mises stress of element
$\sigma_{\max}^{vm}$	maximum value of von Mises stress

ω_n	natural frequency
ρ	density

Abbreviations

2-D	two dimensional
3-D	three dimensional
BESO	bi-directional evolutionary structural optimisation
CWL	constant width layer
EMIO	evolutionary moment of inertia optimisation
ESO	evolutionary structural optimisation
FE	finite element
FEA	finite element analysis
FG	fixed grid
GA	genetic algorithm
ICC	intelligent cavity creation
MOI	moment of inertia
RoG	radius of gyration
RR	ESO rejection ratio
SS	ESO steady state

List of Publications

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