PREFORMED ORTHODONTIC HELICAL COMPRESSION SPRINGS :-

SELECTION AND MECHANICAL PROPERTIES

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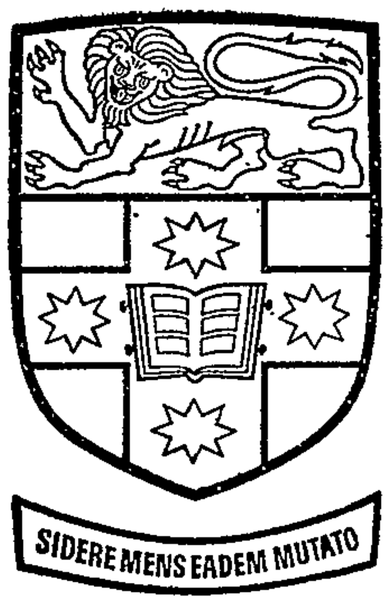

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INTRODUCTION

The vast diversity of preformed orthodontic helical compression springs manufactured by various dental suppliers has created a need to discriminate between the practical and the impractical and to choose the efficient from the inefficient. To name a few, Rocky Mountain Dental Products Company offers 35 types of open coil helical springs with different combinations of wire and core sizes, Unitek Corporation 30 and Dentaurum 18. Besides manufacturing preforms, Unitek Corporation also makes springs to individual specifications.

In addition to the variations in wire and core dimensions, different manufacturers make their springs from different materials. Preformed helical compression springs supplied by Rocky Mountain Dental Products Company are made of "Red Elgiloy" wire; Unitek manufactures their springs in 15-5 precipitation-hardening ("Hi-T"), and 18-8 type 302 ("Permachrome Standard") stainless steel wires; Dentaurum springs are made from a type of 18-8 stainless steel wire (Personal communication).

To achieve a systematic method of selection, a theoretical analysis using the established formulae for the determination of load, deflection and stress in a helical spring should locate which of

these preformed springs have low load-deflection rates.

The experimental study was designed to evaluate the mechanical properties of the five types of preformed orthodontic helical compression springs with potentially the lowermost load to deflection rates.

The consistency of the stated wire and core dimensions, the load-deflection curves on loading and unloading, the load-percentage deflection rates, the maximum elastic force limits and the effects of compressing the springs solid were to be investigated.

The results from the experimental study were intended to be used to develop a method of using the available space for the spring to determine its length for activation to its elastic limit.

REVIEW OF LITERATURE

CHAPTER 1

ORTHODONTIC HELICAL COMPRESSION SPRINGS

Development

"The spring was introduced to orthodontics by John DeHaven White, who described its uses in the Dental News in October, 1850. He called it a "spiral spring" as did such man as Thomas W. Evans (1854) and Edward Arnold (1926), who further elaborated upon its application. At some subsequent time the word "coil" came into use. Although it is obviously a misnomer, usage of this term is common practice. A true coil spring is composed of a number of turns that begin at a central core; each successive turn lies over the previous one in the same plane in circles of ever-increasing diameters. It expresses its energy across its diameter (e.g., a watch spring).

The turns of a spiral spring are all of the same diameter and lie parallel to each other. The length of the spring, whether wound closely or openly, is determined by the number of turns. Its energy is expressed in line with its length.

Early springs were made of gold wire wrapped over a central core of stiff wire and tempered to the desired hardness. Springs made of stainless steel and Elgiloy have entirely surpplanted the use of gold for this purpose and are available commercially" (Renfroe,1975)

In the following review, the terms helical compression spring and open coil spring will be regarded as synonymous. As various investigators used different systems of units of measurements, a mixture of units, British and Metric, will be used.

Further applications of open coil springs were reported by Anderson (1931) and Arnold (1934). Four cases were presented by Arnold showing expansion of maxillary buccal segments with coil springs. He activated the coil springs every 6 weeks by 2 to 3 millimeters.

Johnson (1934) mentioned the use of coil springs of 0.010 inch diameter exerting four ounces of force on end tubes to move molars distally. For opening or closing spaces in the anterior region, coil springs of 0.009 inch diameter stainless steel wires were used. In his 1941 article, he changed to half inch length coil springs made of 0.009 inch 18-8 stainless steel wire with a core diameter of .036 in. to effect distal movements of molars with three ounces force. To move incisors mesially or distally, quarter inch length coil springs of .0056 in. wire with a .020 in. core diameter were used to deliver forces of two and one half ounces.

Additional applications of open coil springs as expansion devices were suggested by Burke, 1960; Softley, 1960; Greenwood, 1962; Signorella, 1968; Stoner, 1969 and Adams, 1971, among others.

Previous Studies of Orthodontic Coil Springs

In 1941, Johnson published an article explaining some of the factors concerning coil springs in regard to the force they exert. They are briefly as follows:-

1. Precious metal spring is about one half as efficient as a steel spring made from 18-8 steel alloy.

2. Although wrapped on the same size core, springs of smaller wire diameter exert less force.

3. Springs of the same wire diameter wrapped on a smaller core exert more force.

4. If compressed to the same amount, the force exerted by a spring is inversely proportional to the length of the spring.

5. A spring that fits a wire too snugly will bind and will exert less force.

In order to obtain a coil spring that fitted snugly on his end tubes, Johnson (1941) wrapped his springs on a mandrel .004 inch smaller than the desired final core size. He fabricated open coil spring by winding a closed spring and then pulling it to twice its length. The spring was then compressed as much as possible before cutting it to the desired length. Enough resiliency in the spring still allowed him to activate the spring to one half its length.

Bell (1951) investigated the amount of force delivered by various brands of elastics and coil springs. Close and space wound coil springs of 40, 30 and 20 millimeter length of 3 types of material (UT, Ret Alter and Tru-chrome) of .008 in., .009 in. and .010 in. wire diameters wound on .022 in., .028 in., .030 in., .032 in. and .036 in. arbor diameters were loaded and unloaded in tension. The average force from loading and unloading was expressed in terms of per millimeter deflection of the spring. He noted that more forces were exerted by springs of larger wire diameter or of smaller arbor size. The efficiency of the steel springs was unaffected by oral fluid or long periods of use. He advised "either to compress or to elongate the spring by one half its length, whether it is close or space wound, before placing it in the mouth" so as to obtain a spring which would exert"a dependable amount of force and may be relied upon to act constantly in the interval between appointments".

Although the overall length of the springs tested was specified, the number of turns in the spring was not mentioned. The actual force applied to compress open space wound UT 189 .008 in. on .036 in. to half its original length of 20 millimeters was 200 grams. The force required to compress the spring per millimeter was 20 grams. The mandrel size used to support the springs tested in compression was not mentioned.

The relationship between the length of the spring, diameter of coil lumen, diameter of the spring wire, amount of compression and the force developed were investigated by Born (1955). Open coil springs were tested in compression first on 0.018 in. arch wire and then on archwires of the same diameter as the spring lumen. ¼-, ½- and 1-inch length springs were compressed 1/16-, 1/8- and ¼-inch. Results from 30 types of combinations of wire diameter (.006 in., .007 in., .008 in., .009 in. and .010 in.) and arbor diameter (.020 in., .022 in., .030 in., .032 in., .036 in. and .040 in.) substantiated the findings of Johnson (1941) and Bell (1951). There was no difference in force output when a spring made of .006 in. or .007 in. wire was compressed on an archwire .004 in. smaller than its lumen size. For springs made from .008 in., .009 in. and .010 in. wire, the loss in force output when tested on an archwire .018 in. undersize from the lumen diameters varied from one to one and a half ounces. Such loss in force output due

to buckling was not constant. He regarded that "the best spring would be one where ¼ inch of compression is used to attain the pressure desired, but in as long a spring as is convenient".

The number of active turns (coils) in the springs tested and how they were fabricated were not mentioned. Hence comparisons with the results of other investigators were impossible.

To date, no additional investigations into the mechanical properties of orthodontic coil springs have been reported.

Mechanical Properties of Orthodontic Helical Compression Springs

"All orthodontic mechanisms are designed to store forces and deliver them to the teeth in the proper direction and at the proper rate. Each adjustment puts force into storage and sets up the mechanism for delivery". (Thurow , 1972).

In designing any orthodontic appliance, Burstone (1975) mentioned three "spring characteristics" which have to be evaluated. These properties are (1) the moment-to-force ratio, (2) the load-deflection rate and (3) the maximal force or moment of any component of an appliance.

(1) The moment-to-force ratio establishes the centre of rotation of the tooth or a group of teeth. This centre of rotation in part controls the stress distribution in the periodontal membrane and the ease of tooth movement.

(2) The load-deflection rate involves with force delivery per unit activation of the appliance. A low load-deflection rate maintains a relatively more constant force as the tooth is displaced. As a long range of activation is required to build up the desired force value, greater control in the magnitude of force becomes possible.

An active (flexible) member of any appliance should therefore have a low load-deflection rate. As for a reactive (anchorage) member, where rigidity is an advantage, it should have a high load-deflection rate.

(3) Maximum elastic load or moment is the greatest force or moment that can be applied to a member without producing permanent deformation of the material. Maximum elastic load should include a safety margin beyond the optimal force level to be reached to take into account any possible accidental overloading.

From the above consideration, it would appear that the 2 mechanical properties affecting the clinical performance of orthodontic helical compression springs are their maximum elastic loads and the load to deflection rates.

When a helical compression spring is used for active tooth movement, it is desirable to have a low load to deflection rate. This rate will provide the clinician with a greater control in the magnitude of the force input. The spring will also maintain a relatively more constant force as the tooth is displaced. Otherwise frequent adjustments of the spring force for efficient tooth movement will be required.

The next criterion to be considered is the capability of the spring to deliver a specific amount of force. The amount of force required for specific tooth movements were studied in several clinical investigations.

Efficient retraction of upper canines were reported with ranges of force varying from 30 to 50 grams (Crabb and Wilson, 1972), 50 to 70 grams (Paulson et al, 1970), 55 to 140 grams (Boester and Johnson, 1974), 150 to 250 grams (Reitan, 1957), 150 to 260 grams, (Lee, 1965) and around 300 grams by Hixon et al (1969).

The observed rates of maxillary canine movement varied in different studies from 0.125 to 0.45 mm. per week (Hixon et al, 1969); 0.18 to 0.6 mm. per week (Paulson et al, 1970) and from 0.33 to 0.55 mm. per week (Boester and Johnson, 1974).

For the retraction of lower canines, the optimal range of forces observed by Storey and Smith (1952) varied from 150 to 250 grams. Reitan (1957) suggested a range of 100 to 200 grams. Hixon et al (1969) found that with forces of 300 grams or less, the average rate of canine movement increased as the load per unit area of the periodontal membrane increased. The observed rates of lower canine movements varied from 0.14 to 0.31 mm. per week (Hixon et al, 1969).

As for the tipping of anterior teeth, Burstone and Grove (1960) found the optimum force to range from 50 to 75 grams. Efficient movements of premolars with 70 gram force were observed by Reitan (1957) and Atherton (1964).

Histologically, Reitan (1975) observed that 50 to 70 gram force is the optimum force for a tipping movement of incisor teeth and premolars. A tipping movement of canines and molars would not require a much stronger force. For movement of a small tooth, it would be favourable to apply an initial force as light as 20 to 30 grams. A light initial force increases cellular activity without causing undue tissue compression and prepares the tissues for further changes. Both strong forces above 200 to 300 grams and light forces 30 to 70 grams for tipping movement, 80 to 150 grams for bodily movement cause hyalinization and undermining resorption to occur during the initial period of tooth movement. Within the light force levels these tissue reactions will be less extensive and of a shorter duration. With heavy forces there is a tendency to a longer initial hyalinization period and also to the formation of secondary hyalinization zones. The tooth will not move again until the bone subjacent to the hyalinized tissue has been eliminated by undermining resorption. The duration of this resorption is more or less proportional to the extent of the hyalinized zone. Thus a light force acting through a certain distance will move a tooth more rapidly than a strong force.

He also suggested that for tipping tooth movement it is favourable to maintain throughout the secondary period of tooth movement the same amount of force applied during the initial period. For bodily tooth movement, the force during the secondary period of tooth movement may be increased to offset the resistance from stretched fibre bundles. During final closure of extraction space, this force must be significantly increased.

He regarded that the amount of force acting at the moment that tooth movement is initiated and the working range of this force determine the distance through which the tooth will be moved.

With this wide range of force for specific tooth movement, the mechanical properties of a variety of helical compression springs will have to be examined so that a spring can be selected according to the requirements of a given situation. The crux of the problem of proper selection from the vast diversity of preformed orthodontic helical compression springs would seem to be one of locating springs with a high percentage deflection within a force range that is physiologically acceptable for optimal tooth movement.

As the dental literature dealing specifically and solely with coil springs is very limited, I turned to the mechanical engineering literature for more information concerning load, deflection and stress characteristics of helical compression springs. A brief review is in order.

CHAPTER 2

HELICAL ROUND WIRE COMPRESSION SPRINGS

A typical helical round wire compression spring is shown in Fig. 1.

The spring is wound from a spring material of a circular cross-sectional diameter, d, into a true helical form. Each turn (coil) of the helix has the same diameter. Due to the thickness of the wire diameter, d, each turn of the helix has an outside coil diameter and an inside coil diameter. The mean diameter of the spring is the average of the outside and inside diameters of the coil. The ratio of the mean coil diameter to the wire diameter gives the spring index of the spring.

Within the body of the spring, each turn is completely free to deflect. These turns are known as the active coils of the spring. When a spring is compressed, there is a progressive seating of the ends of the spring onto their abutments. These turns at the end of the spring are known as end turns or coils as distinct from the active coils which are completely free to deflect. (Wahl, 1949).

The pitch of the spring is the distance between corresponding points on two adjacent coils measured along the axis of the spring. The ratio of the pitch to the mean circumference of the spring is the

tangent of the pitch angle of the spring. (Finniecome, 1949).

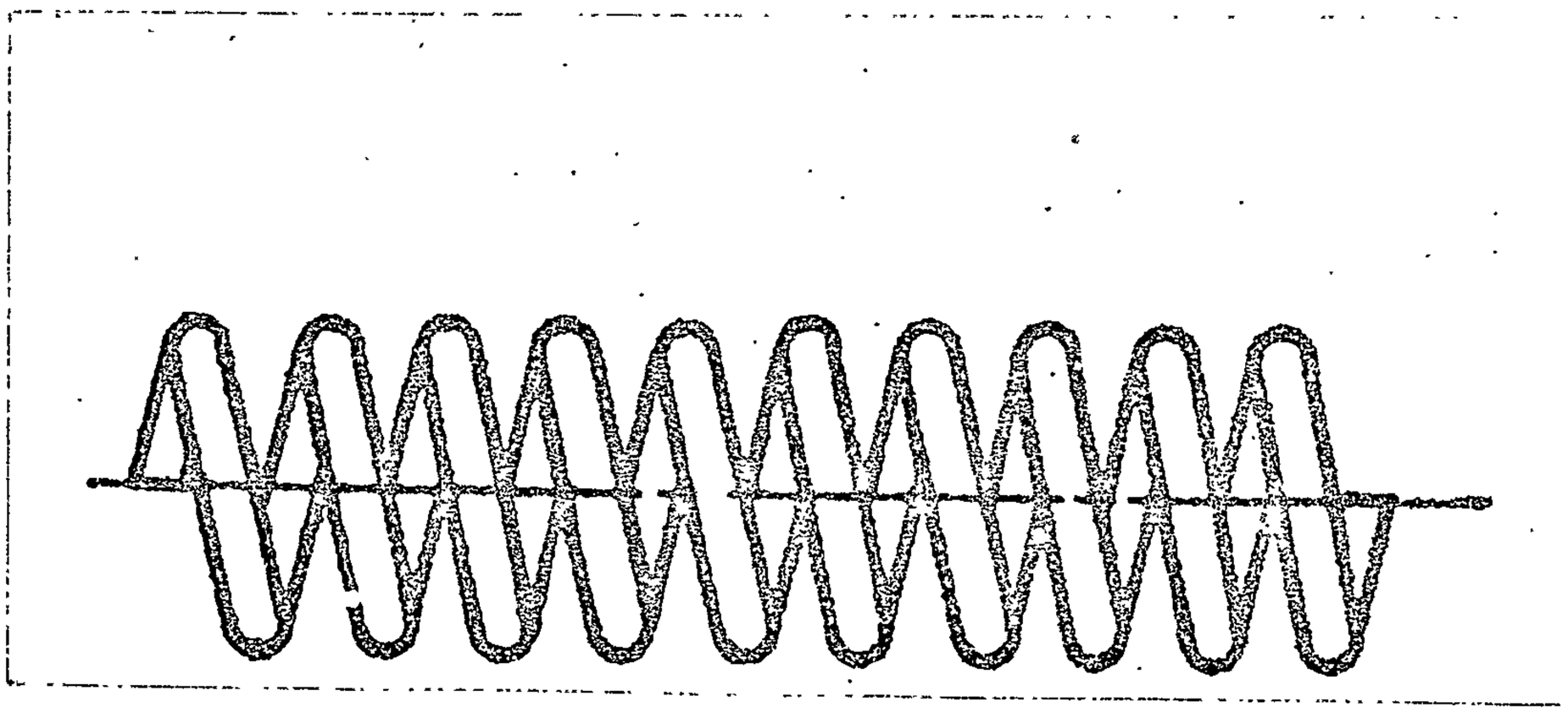

Fig. 1. Helical compression spring.

SYMBOLS

A summary of the symbols used in this review are as follows:-

SYMBOL		DEFINITION
P	=	Axial load.
R	=	Mean radius of the spring.
D	=	Mean diameter of the spring.
d	=	Diameter of the wire.
n	=	Number of active coils.
E	=	Modulus of elasticity.
G	=	Modulus of rigidity.
δ	=	Deflection of the spring.
c	=	$\frac{D}{d}$ is the spring index.
L	=	Free length of the spring.
Ph	=	Pitch of the spring.
α	=	Pitch angle.
Tan α	=	Ratio of the pitch to mean circumference of the spring. ($Ph/\pi D$).
I	=	Moment of inertia of area.
I_p	=	Polar moment of inertia.
K_w	=	Wood's deflection factor.

SYMBOLS (cont'd.)

SYMBOLS		DEFINITIONS
$\tau_{max.}$	=	Maximum shear stress.
$\sigma_{max.}$	=	Maximum bending stress.
τ_e	=	Equivalent shear stress.
K	=	Maximum shear stress factor.
K_σ	=	Maximum bending stress factor.
K_{st}	=	Stress factor.
p.s.i.	=	pounds per square inch.
in. (")	=	inch.
mm	=	millimetre.

A. ENERGY STORAGE

The essential function of a spring is to store energy elastically. Elasticity is that "property of matter by virtue of which it tends to return to its original shape and dimensions when the applied forces are removed." (Berry, 1961). However, when the applied forces are greater than the elastic limit of the matter, it will not return to its original shape and dimensions with the removal of the applied forces. It is therefore essential to know the value of the stress at the elastic limit of any material in order to utilize its elastic properties.

The elastic energy stored in a spring is imparted by external work in deflecting the spring. This elastic energy of deformation for ordinary helical compression springs subjected to torsional stress is given by Wahl (1949) as:

$$\text{Stored energy, } U = \frac{\tau_{max}^{2} \; V}{KG}$$

where τ_{max} = max. torsional shear stress in the spring material,

V = volume of active material in the spring,

G = modulus of rigidity,

K = a constant depending on the cross-section of the material and the type of spring.

for helical compression springs K=4 modified by a curvature correction.

The formula for the stored energy (resilience) in the spring indicates that the maximum amount of stored energy depends on three factors:

1. The square of the maximum stress at the elastic limit of the material.

2. The volume of the active material present in the spring.

3. The modulus of rigidity of the spring material.

The maximum permissible stresses of the material for a compression spring are dependent on the types of loading to which the spring is subjected in service. In general, springs may be subjected to two types of loading, viz., static and dynamic.

Static Loading

Static loading implies a slow application of load so that the spring will not be deflected beyond the final equilibrium position. According to Berry (1961), in these springs the physical properties of the

material which has to be considered in calculating maximum stress are the elastic limit of the material, the hysteresis effect and creep of the material at elevated temperatures.

Dynamic Loading

Dynamic loading implies that the spring reaches an equilibrium position after some oscillations with large initial deflections which cause higher maximum stress. This class may again be sub-divided into impact loading and variable or fatigue loading.

In orthodontics, the elastic energy from recoil of deflected helical springs has long been used to supply the desired force. In this present study, the main interest is in the static loading of helical compression springs, particularly in its load-deflection characteristics. An understanding of the deflection of the spring and the induced stress in the spring is therefore required.

B. DEFLECTION

The springiness of a helical spring is connected with the deformation of the material from which it is made. The extent of deflection is influenced by the geometrical configuration and dimensions of the spring and the modulus of elasticity and regidity of the spring material. Specifically the factors found affecting the deflection are the spring index, the initial pitch angle, the extent of deflection in relation to mean coil radius, the degree of freedom of movement of the end coils and the modulus of elasticity and rigidity of the spring material.

Effects of Geometry and Dimension on Small Deflection

For springs with initial pitch angle less than 10 degrees and deflection per coil less than half the mean coil radius, the coil radius and pitch angle during deflection of the spring may be considered unaffected by the deflection. For such a spring compressed under an axial load, P, the deflection, δ_o , of the spring within its elastic range, neglecting the effects of the end coils, is given by Wahl (1949) as:

$$\delta_o = \frac{64\ PR_o^{\ 3} n}{G\ d^4} \qquad \text{(1)}$$

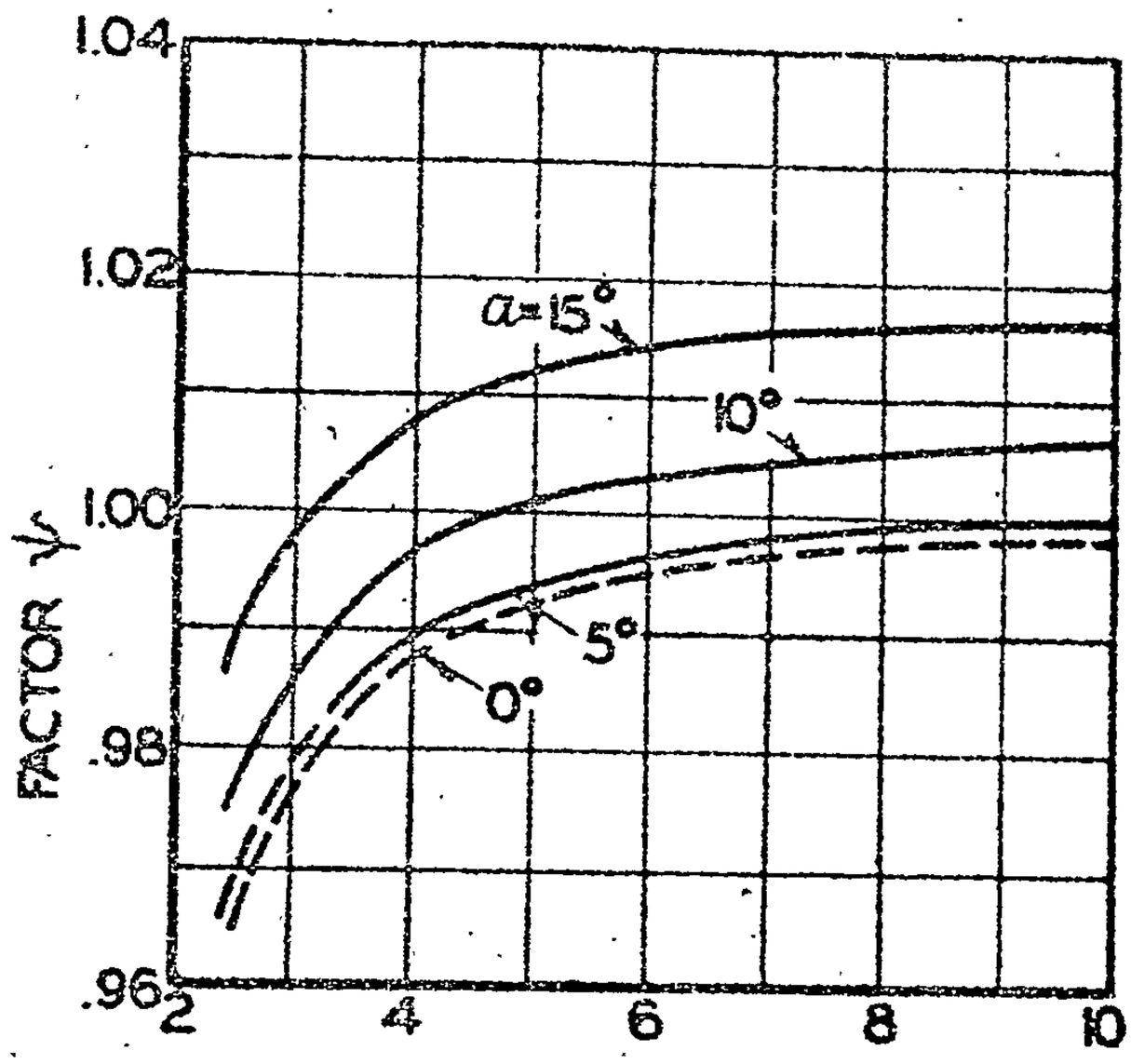

Fig. 2. Curves for finding ψ for various spring indexes C and pitch angles α .

where R_o = mean coil radius at zero load,

n = number of active coils,

d = diameter of wire.

This deflection formula is based on an assumption that the wire behaves as a straight bar under torsion (Berry, 1961).

If the curvature effect of the spring expressed by the spring index, C, and the pitch angle, α, are to be taken into consideration, the deflection of the spring as calculated from Equation (1) has to be multiplied by a correction factor. The correction factor, ψ, given by Goehner (1932) is expressed as follows:-

$$\psi = \frac{\cos \alpha}{1 + \left(\frac{3}{16}\right)\left[\cos^4 \alpha/(C^2 - 1)\right]} + \frac{2G}{E} \sin\alpha \tan\alpha \qquad (2)$$

in which α = initial pitch angle

$$C = \frac{2R}{d}$$

ψ is a constant depending on the spring index C and on the initial pitch angle, α. Values of this constant have been plotted by Wahl (1949), Fig. 2, as a function of the spring index, C, and various pitch angles, α. In making the calculation the ratio $\frac{G}{E} = 0.38$ was taken. According to Wahl (1949), a relatively large change in this ratio $\frac{G}{E}$ will have little effect on the value of ψ.

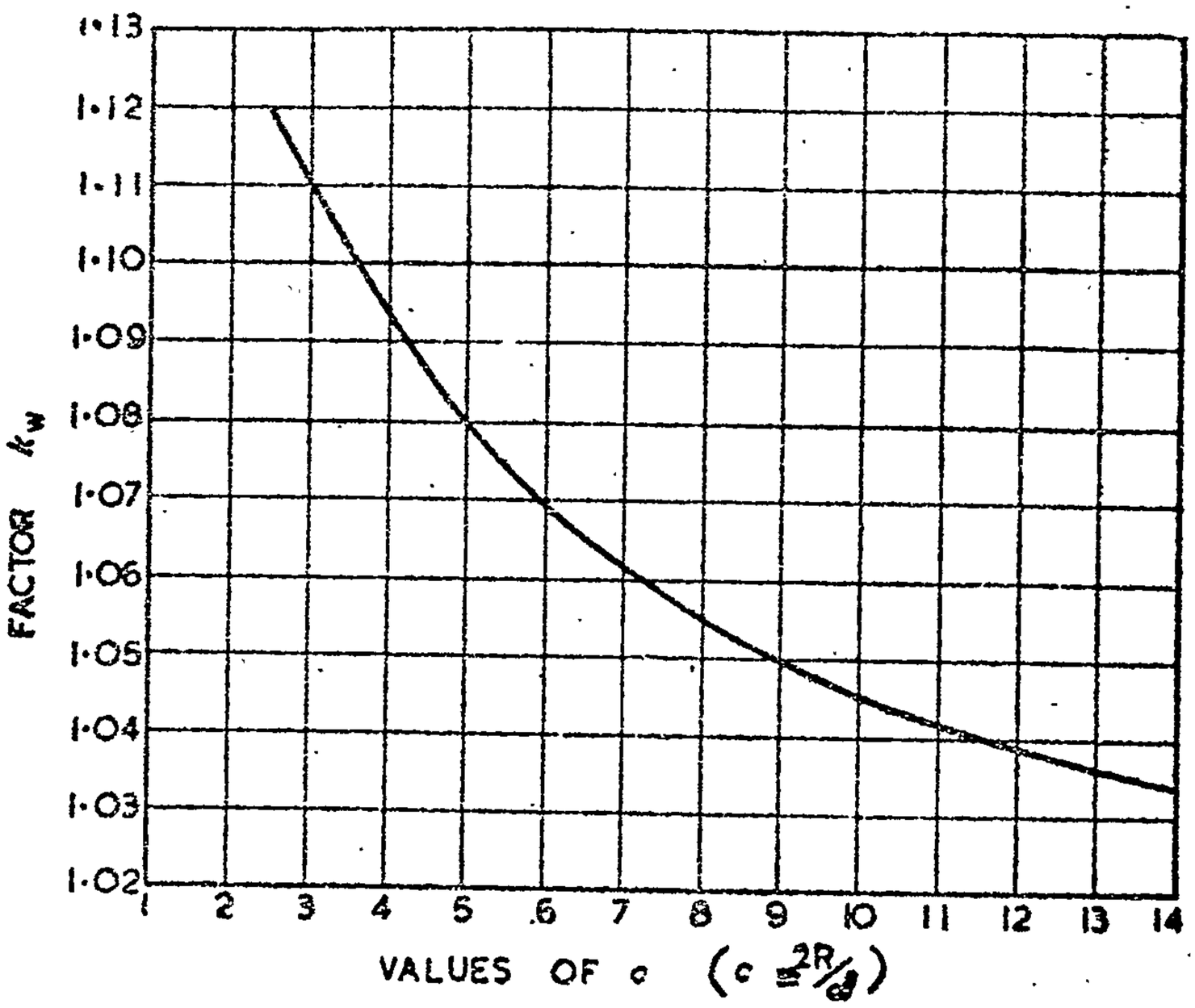

Fig. 3. Wood's deflection factor, $K_w = \frac{2c^2 + c - 1}{2c^2}$

(Finniecome)

The differences in deflection for springs with indexes C > 2.5 and pitch angles α > 15 calculated with Goehner's correction factor and that from Equation (1) are under 2½%. For springs where α < 10 degrees and C > 4, the deviations in deflection with and without using the correction factor are under one per cent. These small differences in deflection are considered by Wahl (1949) as negligible in practice, for other factors such as the effects of variation in wire and coil sizes, shape of end coils and the modulus of rigidity would normally be greater than this. Hence he recommended using Equation (1) to calculate deflection, provided the deflection per coil is less than half the mean coil radius.

As Equation (1) is derived from a straight bar stressed torsionally, Wood in the discussion of Wahl's 1929 paper recommended that it should be multiplied by a correction factor to take into account the effects of curvature of the spring on its actual deflection.

This deflection correction factor, K_w, is given as (Fig. 3):

$$K_w = \frac{2C^2 + C - 1}{2C^2} \qquad (3)$$

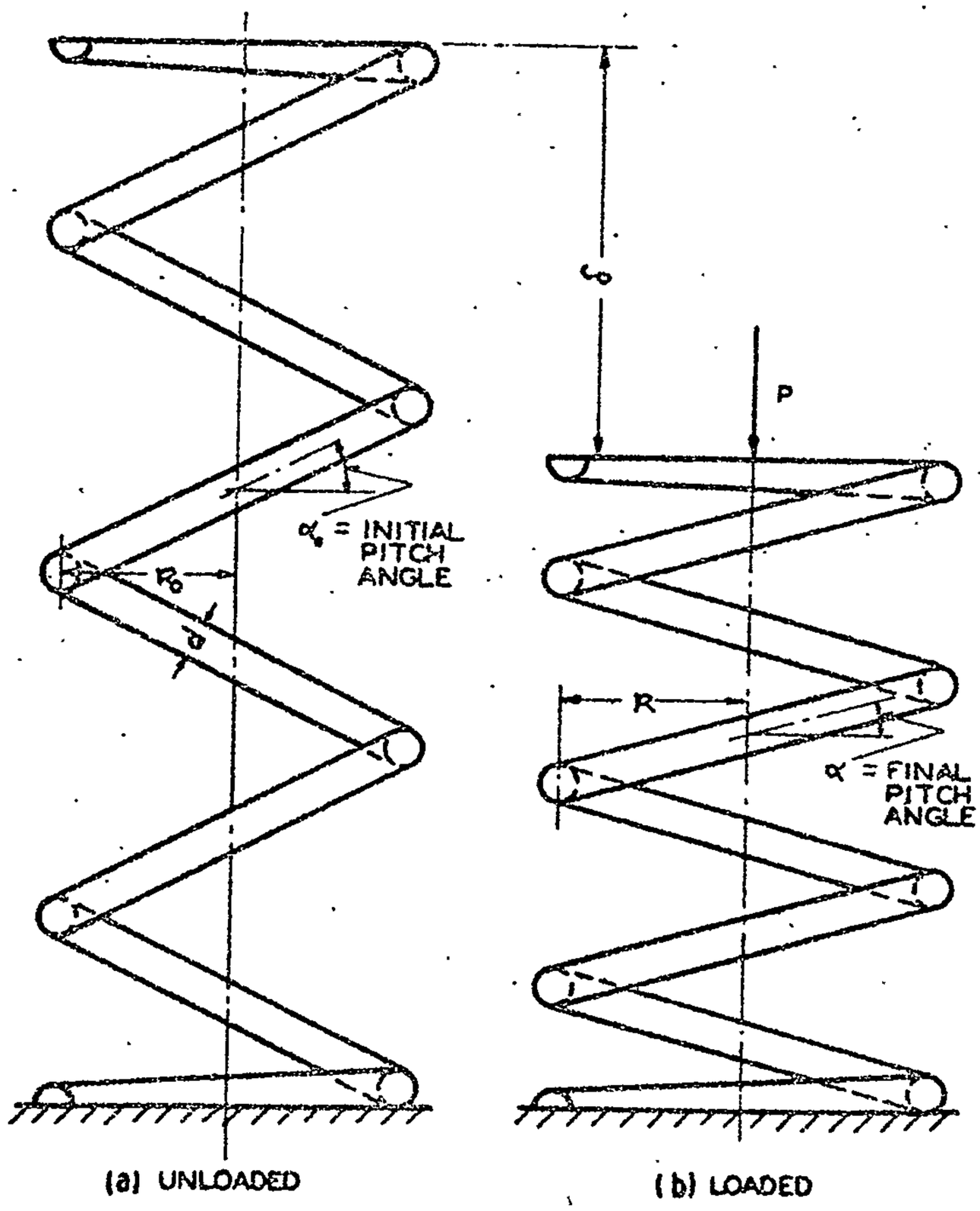

Fig. 4. Open-coiled spring with large deflection showing changes in pitch angle α and mean coil radius R. (Wahl)

In determining the torsional elastic limit of Patented Plain Carbon Spring steel, Berry (1938) validated that the inclusion of Wood's deflection correction factor in the calculations of deflection gave a more accurate modulus of rigidity of the tested material.

Finniecome (1949) also recommended the inclusion of the Wood's deflection correction factor, K_w, into Equation (1) for the calculation of the actual deflection of springs of different spring indexes.

When C = 3, the correction for deflection due to curvature effect is 11.1 per cent. Even so, Berry (1961) considered that, for springs of C = 5 to C = 10, the Wood's correction factor of 8 per cent to 4½ per cent can be neglected. He maintained that, due to difficulties in control, springs themselves will vary considerably in manufacture from the designed data.

Effects of Geometry and Dimension on Large Deflection

When a compression spring is compressed to a large deflection, the mean coil radius and the pitch angle change significantly. Thus when a compression spring is compressed from the initial position, Fig. 4 (a), to that shown in Fig. 4 (b), the mean coil radius increases from R_o to R. As the deflection of a spring is proportional to the cube of the mean coil radius, the spring will become

more flexible as it is compressed. This occurence requires further modifications to Equation (1) for determining large deflections in a spring under axial load.

To further complicate the matter, Equation (1) also neglects the effects of end coils on deflection. When a spring is compressed under an axial load, there is a tendency for one end of the spring to rotate with respect to the other about the spring axis. In compression springs, friction can develop at the interface between the spring ends and their abutments, forming end restraining moments which tend to prevent such rotational tendencies. It is therefore necessary to distinguish further whether the ends of the spring are free to rotate about the spring axis or are fixed so that rotation about the spring axis is prevented. As minor movements of the teeth occurs during function, the ends of compression springs used in orthodontics are probably free to rotate except when tied down by ligature wires.

Deflection of Spring with End Coils Free to Rotate

Movements of end coils assume importance in springs with large spring indexes and initial pitch angles greater than 10 degrees. For springs with small and moderate indexes, excessive stresses are set up when the deflection per coil approaches the mean coil radius. In these springs the proportional limit would have been

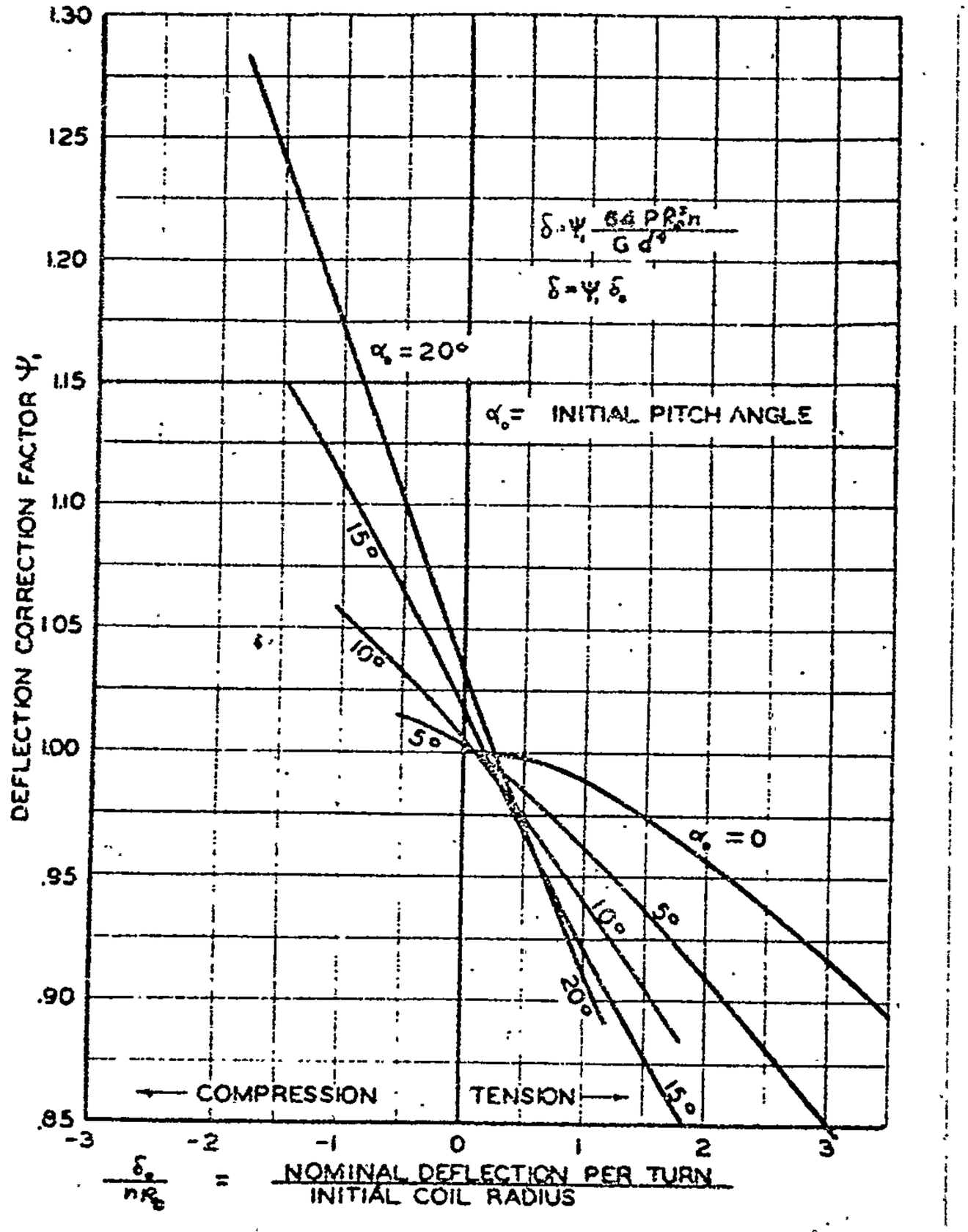

Fig. 5. Curves for finding deflection correction factor ψ_1

(Wahl)

exceeded. Once the spring has deflected beyond its elastic range, Equation (1) is not applicable for the calculation of its deflection (Wahl, 1949).

When a helical spring having a large spring index with an initial pitch angle more than 10 degrees and the ends free to rotate is compressed and deflected to a large extent under an axial load, P, the deflection, δ , is given by the following formula:-

$$\delta = \frac{64\ PR_o^3 n}{Gd^4} \times \psi_1 = \delta_o \psi_1 \qquad \text{——— (4)}$$

where δ_o = nominal deflection using Equation (1).

ψ_1 = a factor depending on the initial pitch angle α_o, the ratio $\frac{E}{G}$ between bending and shear moduli and on the ratio $\frac{\delta_o}{nR_o}$ between nominal deflection per turn and initial coil radius.

Values of ψ_1 had been calculated for a ratio $\frac{E}{G} = 2.6$ and the results plotted by Wahl (1949) is shown in Fig. 5 for various values of α and $\frac{\delta_o}{nR_o}$. Wahl maintained that a considerable change in this $\frac{E}{G}$ ratio would only cause an insignificant change in the final results.

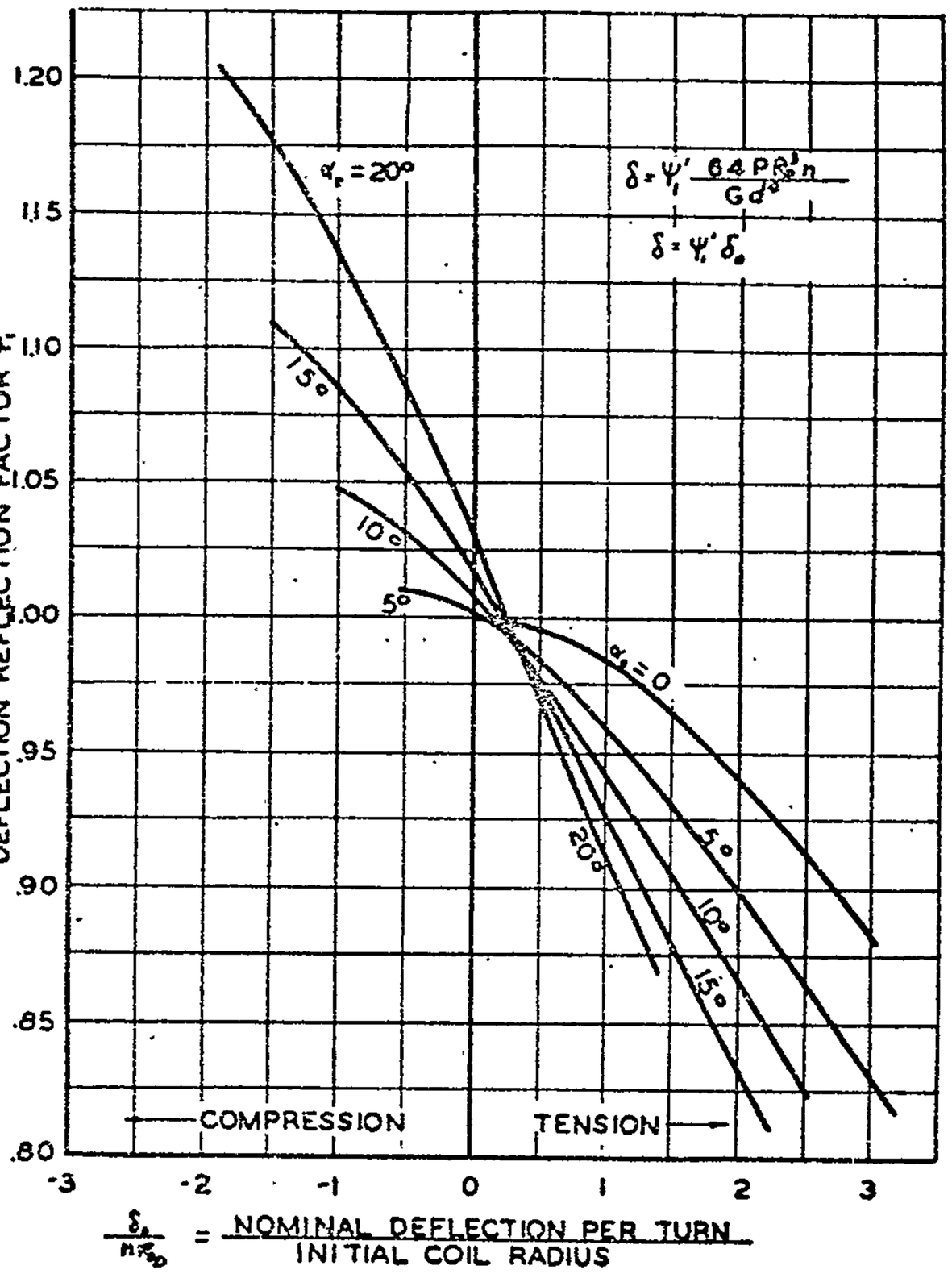

Fig. 6. Curves for finding deflection correction factor ψ_1' (Wahl).

To use Fig. 5 for practical calculation, use formula (1) to determine δ_o for a given load P. The radio $\frac{\delta_o}{nR_o}$ may be found. Knowing this and the initial pitch angle α_o, the factor ψ_1 may be read from Fig. 5 . The maximum deflection under load P will then be equal to $\psi_1 \delta_o$.

Deflection of Spring with End Coils Fixed

If the ends are fixed due to friction or ligation between the ends of a compression spring and their abutments so that the relative rotation of the spring ends is prevented, the deflection δ under an axial load, P, is given by:-

$$\delta = \frac{64\ PR_o^{\ 3}n}{G\ d^4} \times \psi_1' = \delta_o \times \psi_1' \qquad \text{——(5)}$$

$$\psi_1' = \frac{\sin\alpha - \sin\alpha_o}{\cos^3\alpha_o\left(\sin\alpha - \sin\alpha_o + \frac{E\ I}{G\ I_p}\tan\alpha\,(\cos\alpha_o - \cos\alpha)\right)} \qquad \text{——(6)}$$

The values of ψ_1' for various values of α and $\frac{\delta_o}{nR_o}$ given by Wahl (1949) are as shown in Fig. 6.

Curves for ψ_1 and ψ_1' shown in Figs. 5 and 6 are applicable to compression springs subjected to loads less than the buckling load or when these external movements are prevented by guides. Buckling load is the load that starts to cause a sudden sidewise buckling of an axially loaded spring.

Even where large deflections are involved, for springs with initial pitch angle less than 15 degrees and the calculated deflection per turn less than half of the mean coil radius, the correction factor, ψ_1 and ψ_1' are less than 6½ per cent as a maximum. Berry (1961) considered that, as these factors reduce very rapidly with the reduction in either the initial pitch angle or the ratio of calculated deflection-per-turn to mean radius, "for most practical purposes such corrections can be neglected".

Effects of the Physical Properties of the Spring Material on Deflection

Equations (1), (2), (4) and (5) indicate the mode of dependence of the deflection of any helical compression spring under axial load on the moduli of elasticity and of rigidity of the spring material.

Craig et al (1965) using wires of 0.014 and 0.022 inches in diameter compared the physical properties of 17-7 precipitation-hardenable stainless steel, 18-8 stainless steel and cobalt-chromium-nickel alloy (Elgiloy) produced by the National Standards Co., Unitek and

Rocky Mountain Metal Products Co., respectively. He observed that the moduli of elasticity of these materials were within the range of 27 - 30 x 10^6 p.s.i.

Firma Dentaurum gave the value of the modulus of elasticity for their 18-8 stainless steel spring material No. 4310 as 20,000 Kg./mm^2 which is approximately 28.5 x 10^6 p.s.i. (Personal Communication).

The modulus of rigidity for material No. 4310 given in DIN 17224 (1968) is 10.6 x 10^6 p.s.i.

The modulus of elasticity and of rigidity given by Rocky Mountain Co. for their coil spring material ("Red Elgiloy"), which is a cobalt-chromium-nickel alloy, are 28.5 x 10^6 p.s.i. and 11.2 x 10^6 p.s.i. respectively. (Personal Communication).

As for their 18-8 type 302 and 15-5 precipitation-hardenable stainless steelspring materials, Unitek Co. gave the modulus of elasticity and of rigidity as 29 x 10^6 p.s.i. and 10.5 x 10^6 p.s.i. respectively. (Personal Communication).

If the modulus of rigidity of 10.5 x 10^6 p.s.i. is taken as a base line, Elgiloy has a higher modulus of rigidity over the other materials by 6.6 per cent.

The modulus of elasticity for these materials stated by their manufacturers corresponded very well with the values found by Craig et al in 1965.

The ratio of $\frac{G}{E}$ for 18-8 stainless steel spring material No. 4310 is 0.37, for "Red Elgiloy" is 0.39 and for 18-8 type 302 and 15-5 precipitation-hardenable stainless steel is 0.36. The ratio of $\frac{E}{G}$ for the corresponding materials are 2.7, 2.56 and 2.77 respectively. Hence the graphs drawn by Wahl (1949) shown in Figs. 2, 5 and 6 are applicable to these materials.

In summary, for practical application, the basic deflection formula, Equation (1), multiplied by the Wood's correction factor, defined by Equation (3), can be used to calculate the actual deflection in any helical compression spring if the axial load, the modulus of rigidity, the mean coil radius, the wire diameter and the number of active coils are known. The deflection per coil for springs under a given load and for a given spring material is therefore proportional to $\frac{R^3}{d^4} \left(\frac{2C^2 + C - 1}{2C^2} \right)$.

Significance of the Review on Deflection in Relation to the Present Project

The terms in the formula for calculating deflection:-

$$\delta = \frac{64\ PR^3 n}{G\ d^4} \left(\frac{2\ C^2 + C - 1}{2\ C^2} \right)$$

can be rearranged to give the load-deflection ratio of the spring.

Thus,

$$\frac{P}{\delta}\ n = \frac{G\ d^4}{64R^3} \left(\frac{2\ C^2}{2\ C^2 + C - 1} \right)$$

The load-deflection ratio per coil of the spring for any given material is proportional to

$$\frac{d^4}{R^3} \cdot \left(\frac{2\ C^2}{2\ C^2 + C - 1} \right)$$

As the nominal wire diameter and internal core diameter are specified in the manufacturer's catalog, the load-deflection ratio, in ascending order, for the whole range of preformed open coil springs can be calculated.

The moduli of rigidity for 18-8 stainless steel spring material No. 4310 (Dentaurum "super spring hard wire"), 18-8 type 302 stainless steel (Unitek "Permachrome Standard wire") and 15-5 precipitation-hardenable stainless steel (Unitek "Hi-T wires") are essentially the same. "Elgiloy" (Rocky Mountain cobalt-

TABLE 1 SPRING CONFIGURATION WITH LOW LOAD-DEFLECTION RATIO PER COIL OF THE SPRING

	Company	Spring configuration wire diameter	internal core diameter	$\frac{R^3}{d^4}$	Spring Index c	$\frac{2c^2+c-1}{2c^2}$	$\frac{\text{Load}}{\text{deflection}}$ per coil *	Load/deflection ratio per coil using .007" x .036" as base line
1.	Unitek	.006	.036	7145.84	7	1.0612	1.3187×10^{-4}	.583
2.	Rocky Mountain	.007	.036	4139.271	6.142	1.0681	2.2618×10^{-4}	1
3.	Rocky Mountain	.0056	.022	2672.296	4.928	1.0808	3.4623×10^{-4}	1.531
4.	Dentaurum and Rocky Mountain	.007	.030	2637.08	5.285	1.0767	3.5219×10^{-4}	1.557
5.	Unitek and Rocky Mountain	.008	.036	2599.609	5.5	1.0743	3.5806×10^{-4}	1.583

* Effects of higher modulus of rigidity for Elgiloy were not included in the calculation of load-deflection ratio.

chromium-nickel wire) has a modulus of rigidity 6.6 per cent higher than the other three materials.

Keeping in mind this small difference in moduli, open coil springs manufactured from these 4 types of materials may be considered to have nearly the same moduli so that they may be pooled and a list in ascending order of load-deflection ratio per coil may be calculated. If required, the load-deflection ratio for springs made from "Elgiloy" may be readjusted by multiplying the calculated ratio with the constant 1.066.

After calculating all of the 35 types of helical compression springs manufactured by Rocky Mountain Company, 15 types from Unitek and 18 types from Dentaurum, Table 1 shows the 5 combinations of wire and internal core diameter which will give low load-deflection ratios per coil starting with the lowest possible load-deflection ratio.

C. STRESS

The second criterion in the selection of a helical compression spring is its maximum range of deflection. This range is dependent on the stress at the elastic limit of the spring material and the distance between the active coils of the spring. An analysis of the stresses in an axially loaded helical spring is in order.

Stress Calculations

The formula for calculating stress in an axially loaded helical spring under a static load, P, within the elastic limit of the spring is derived initially from the straight bar in torsion theory. (Finniecome, 1949). The maximum shear stress, τ, is determined by:-

$$\tau = \frac{8PD}{\pi d^3} \qquad (7)$$

where P = axial load

D = mean diameter of the spring

d = diameter of the wire

τ = maximum shear stress due to torsion only.

This formula is based on the assumption that the spring wire behaves as a straight bar under torsion subjected to a torque moment $\frac{PD}{2}$ only

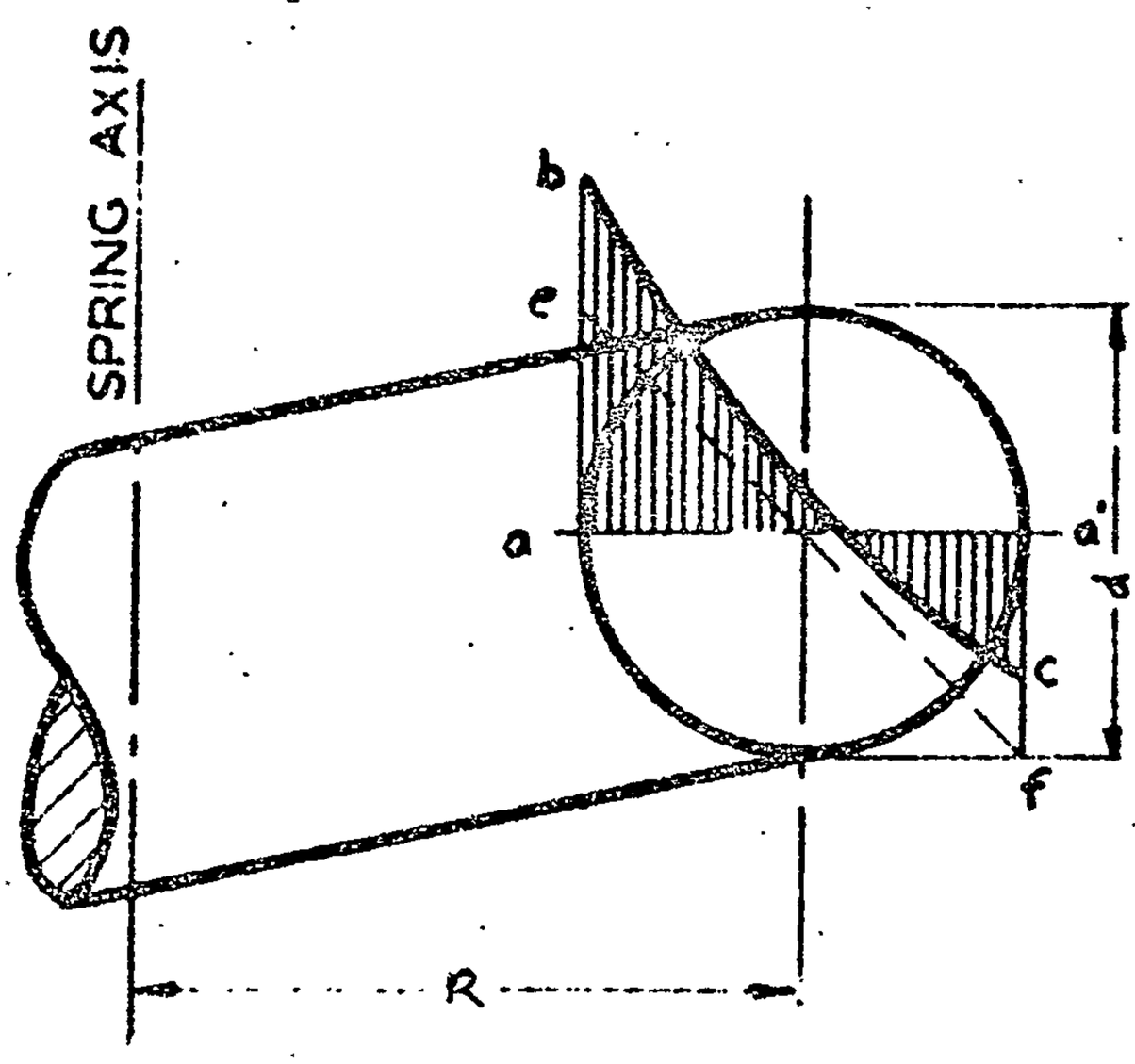

Fig. 7. Distribution of stress along transverse diameter of the wire of a helical spring. (Wahl)

(Berry, 1961). However, examination of a spring with a pitch angle, α , under an axial load, P, reveals that the stresses that are present include a torsional moment $\frac{PD}{2}$ Cos α , a direct shear force P cos α from the axial load, both producing a direct stress at the inside fibre; a bending moment $\frac{PD}{2}$ sin α which gives a normal stress and a direct tension or compression stress due to the direct load P sin α (Wahl, 1949).

When the axial load is within its elastic limit and the attendant deflection per turn is less than the mean coil radius, the stress distribution along a transverse diameter is shown approximately by the line bc in Fig. 7 . The stress a b on the inside of the coil is much greater than the stress a' c on the outside of the coil. This condition of stress concentration into a relatively small area is diagramatically shown in Fig. 8.

There are 2 reasons for this stress concentration. Firstly the fibre length along the inside of the coil is much less so that a higher shearing stress is present for any angular rotation of adjacent cross-sections (Fig. 9). Secondly, the torque moment and shear stress resulting from the direct axial load act in the same direction along the inside of the coil (Wahl, 1949; Finniecome, 1949 and Berry, 1961).

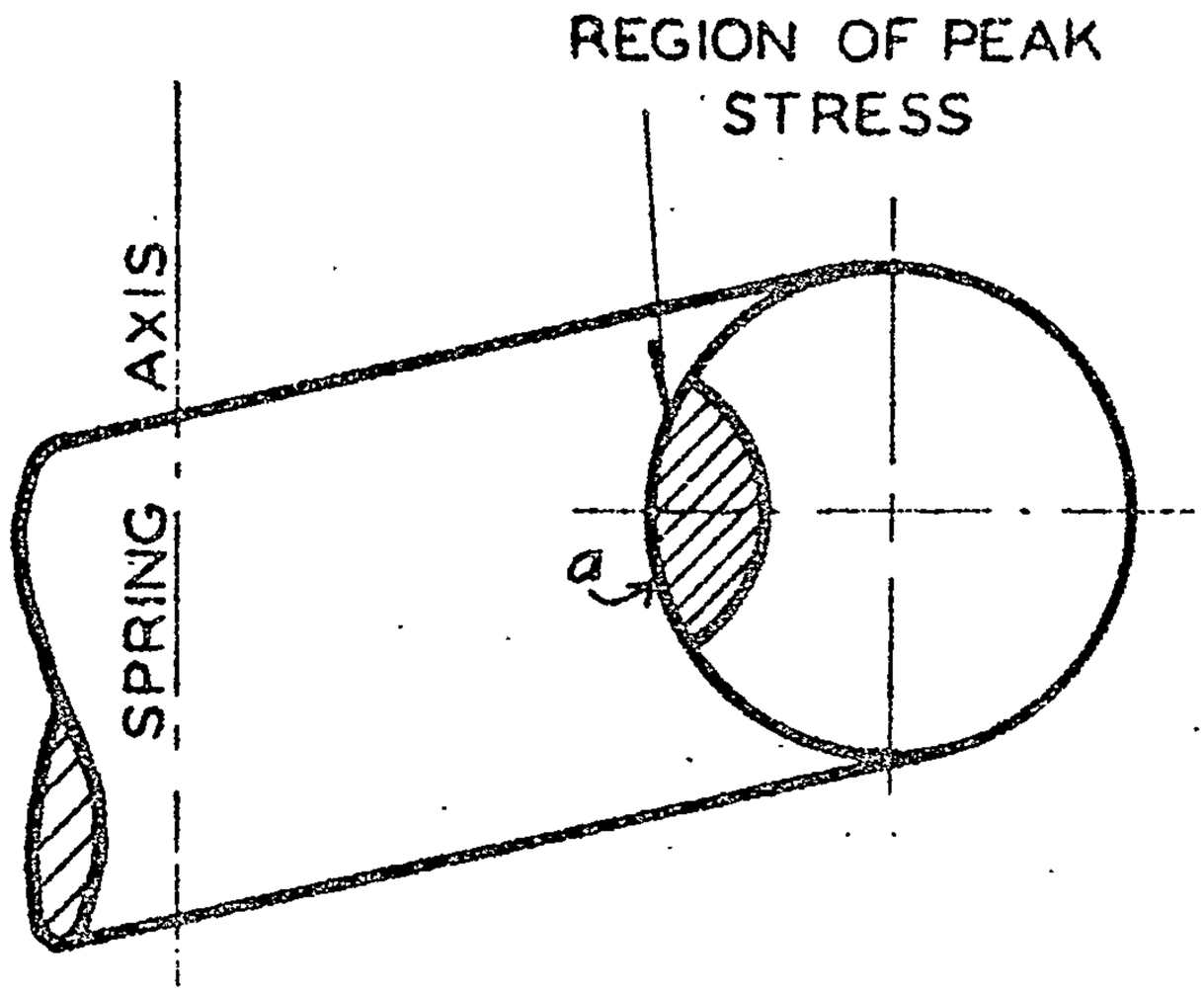

Fig. 8. Stress concentration near the inside of the coil at "a" of a spring with a small index. (Wahl)

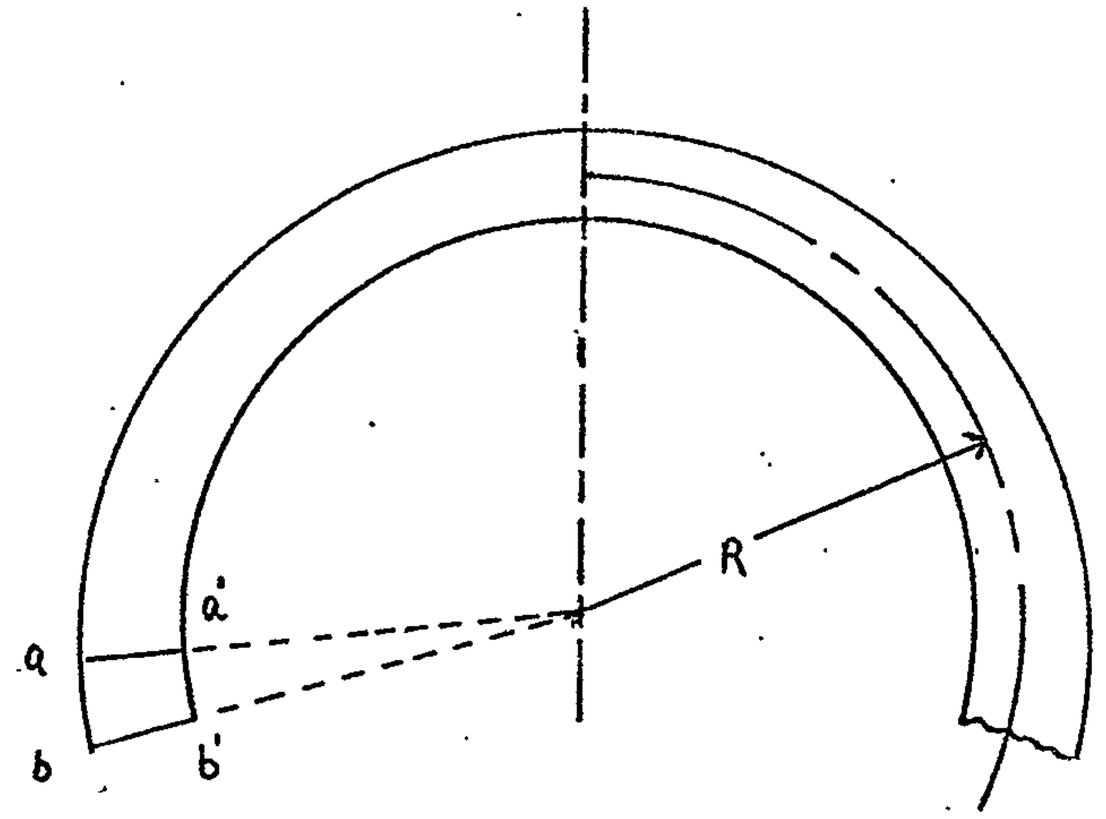

Fig. 9. Shorter fibre length at inside of coil. (Wahl).

Rover (1913), recognizing the inadequacy of the formula for calculating stress derived from the simple torsion theory, advanced some pioneer correction factors to be used with Equation (7). His correction factors took into consideration the effects of the curvature of the spring, the cross-section of the wire and the pitch angle in the calculation of the maximum shear and bending stresses. His formula for maximum shear stress, τ_{max}, is

$$\tau_{max} = \frac{8PD}{\pi d^3} \times K \qquad \text{(8)}$$

$$\text{where} \quad K = \left[\frac{C}{C - \cos^2 \alpha} + \frac{1 + \sin^2 \alpha}{4C} \right] \cos \alpha \qquad \text{(9)}$$

The maximum bending stress, σ_{max}, is given as

$$\sigma_{max} = \frac{16\ PD}{\pi d^3} \times k_\sigma \qquad \text{(10)}$$

$$\text{where} \quad K_\sigma = \left[\frac{C}{C - \cos^2 \alpha} + \frac{\sin^2 \alpha}{4C} \right] \sin \alpha \qquad \text{(11)}$$

Combining these principle stresses acting at any given point, the resultant equivalent shear stress, τ_e, is given by the maximum-shear

theory as:-

$$\tau_e = \tau_{max} \sqrt{1 + \frac{1}{4}\left[\frac{\sigma_{max}}{\tau_{max}}\right]^2} \qquad \text{——— (12)}$$

"Maximum-shear theory states that failure (Yielding) is determined by the maximum shearing stress at any point in a stressed body. Unless the pitch angle is unusually large, σ_{max} will be small compared to τ_{max} so that in general for most practical springs, τ_e will differ but slightly from τ_{max} " (Wahl, 1949).

The maximum shear stress acts along a plane inclined 45° to the planes of the principle stresses (Finniecome, 1949).

Rover assumed that the rotation due to torsion took place about the geometrical centre of the wire. His formulae are used mainly to determine the corrected stress for an existing spring. For springs with spring indexes, c, between 5 to 10, the stress calculated without using the correction factor can be anything from 14 per cent to 30 per cent too low. When c = 3, the stress calculated from the straight bar in torsion theory can be approximately 60 per cent too low (Berry, 1961).

Wahl in 1929 in America again emphasized the importance of correction factors for the calculations of stress using the formula derived from

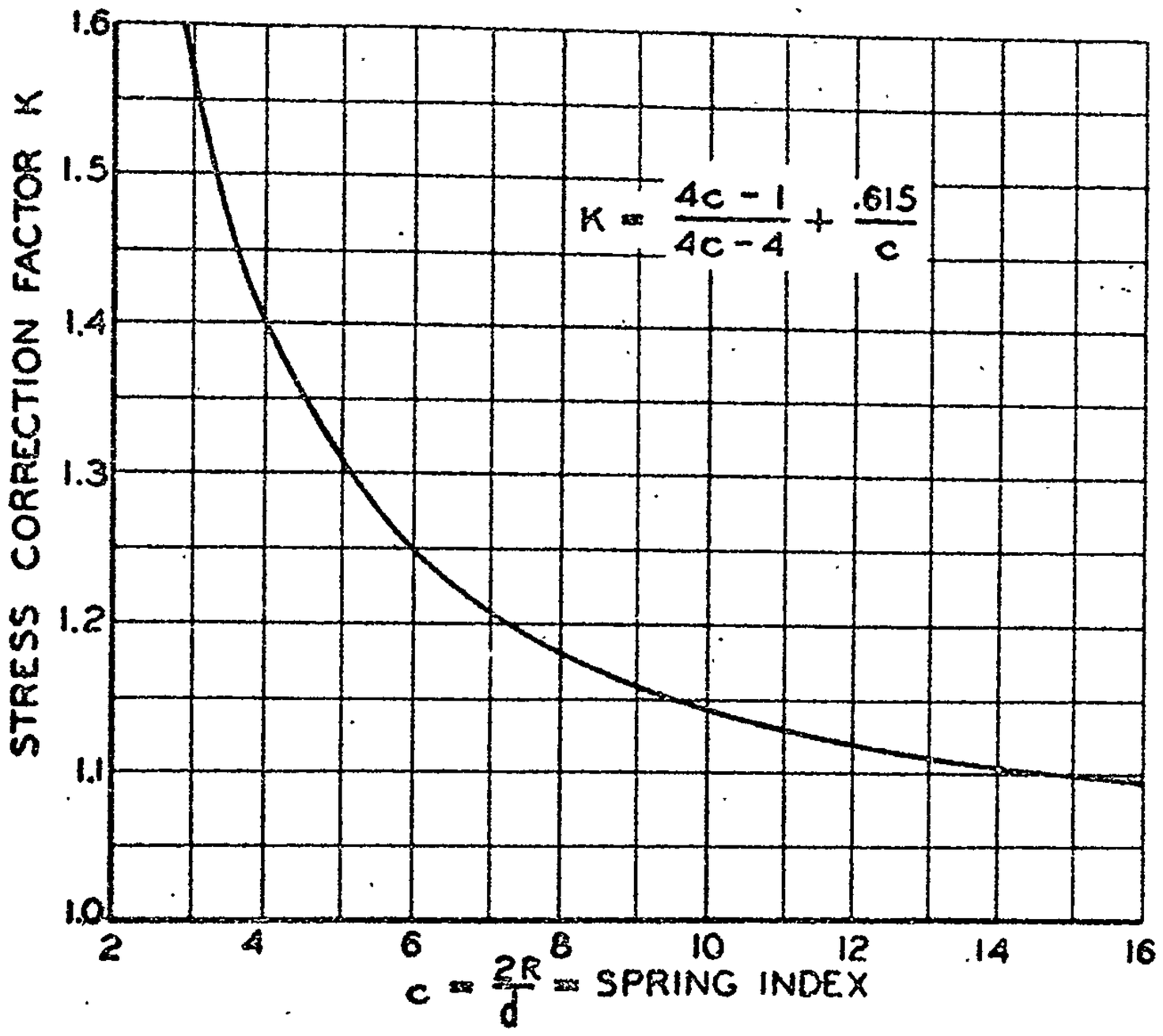

Fig. 10 Stress correction factor K for helical round wire compression springs. (Wahl).

the simple torsion theory, i.e, Equation (7).

He published additional formulae for a pitch angle equal to zero. His formula for the maximum shear stress τ_{max} at the inside curvature of the wire is:-

$$\tau_{max} = \frac{8\,PD}{\pi d^3}\,K \qquad \text{(13)}$$

where $$K = \frac{4C - 1}{4C - 4} + \frac{.615}{C} \qquad \text{(14)}$$

Values of K have been plotted as a function of the spring index, C, in Fig. 10.

He was the first to carry out tests on actual springs to determine the stress by measurements. Using springs with indexes of 3.14 and 2.89 respectively, he found that these test factors agreed with the values calculated by his formulae

Berry (1961) compared the curvature corrections for stress using the correction factors (for zero pitch angle) derived by Rover and Wahl. For springs with indexes of 3, 5, 10 and 20, the results from these formulae differed by less than one per cent. Finniecome (1949) found that the correction values calculated from Rover's formula were slightly lower than Wahl's for $C > 3$.

The displacement of the actual centre of rotation of the fibres from the geometrical cross-section of the wire during the loading and compression of helical springs were analysed by Honegger in 1931. His formulae included a correction for this displacement. The forumla for maximum shear stress, τ_{max}, is given as:-

$$\tau_{max} = \frac{8PD}{\pi d^3} \cdot K \qquad \text{(15)}$$

$$K = \cos\alpha \frac{\sqrt{c^2 - 1}}{c - \cos^2\alpha} \left[\frac{1}{1 + \frac{1}{6c^2} + \frac{1}{16c^4}} \right] + \frac{0.616}{c} \cos\alpha$$

$$\qquad \text{(16)}$$

The maximum bending stress σ_{max} is

$$\sigma_{max} = \frac{16\ PD}{\pi d^3} \cdot K_\sigma \qquad \text{(17)}$$

$$K_\sigma = \sin\alpha \left\{ \frac{c}{c + \sin^2\alpha} \left[\frac{1}{1 + \frac{1}{6c^2} + \frac{1}{16c^4}} \right] + \tfrac{1}{4} \sin^2\alpha \right\} \qquad \text{(18)}$$

Honegger also checked and confirmed his formulae with actual tests on springs with values of c = 3 and 3.9.

In 1930 and 1932, Goehner published lengthy mathematical analysis concerning stress in helical springs. His formulae for maximum shear stress τ_{max} is :-

$$\tau_{max} = \frac{8PD}{\pi d^3} \cdot K \qquad \text{——————} \quad (19)$$

$$K = \cos\alpha\left[\frac{\frac{c}{c-\cos^2\alpha} + \frac{1}{4}\frac{\cos^2\alpha}{c} + \frac{1}{16}\frac{\cos^4\alpha}{c^2} + \frac{\left\{0.62\cos^2\alpha\,(c-\cos^2\alpha)+.53\cos^4\alpha\right\}\tan^2\alpha}{c\,(c-\cos^2\alpha)}}{c^2 - \frac{13}{16}\cos^4\alpha}\right]$$

——————— (20)

for maximum bending stress, σ_{max}, is:-

$$\sigma_{max} = \frac{16\ PD}{\pi d^3} \cdot K_\sigma \qquad \text{——————} \quad (21)$$

$$K_\sigma = \sin\alpha\left[1 + 0.87\,\frac{\cos^2\alpha}{c} + \frac{0.64\ \cos^4\alpha}{c(c-\cos^2\alpha)} + \frac{1}{4c}\right] \quad \text{——} \quad (22)$$

Bergstrasser in 1933 simplified mathematically the formula derived by Goehner for maximum shear stress, τ_{max}, into:-

$$\tau_{max} = \frac{8PD}{\pi d^3} \cdot K_a \qquad \text{——————} \quad (23)$$

$$K_a = \frac{c + 0.5 + \sin^2\alpha}{c - 0.75 + 1.51\sin^2\alpha} \qquad \text{——————} \quad (24)$$

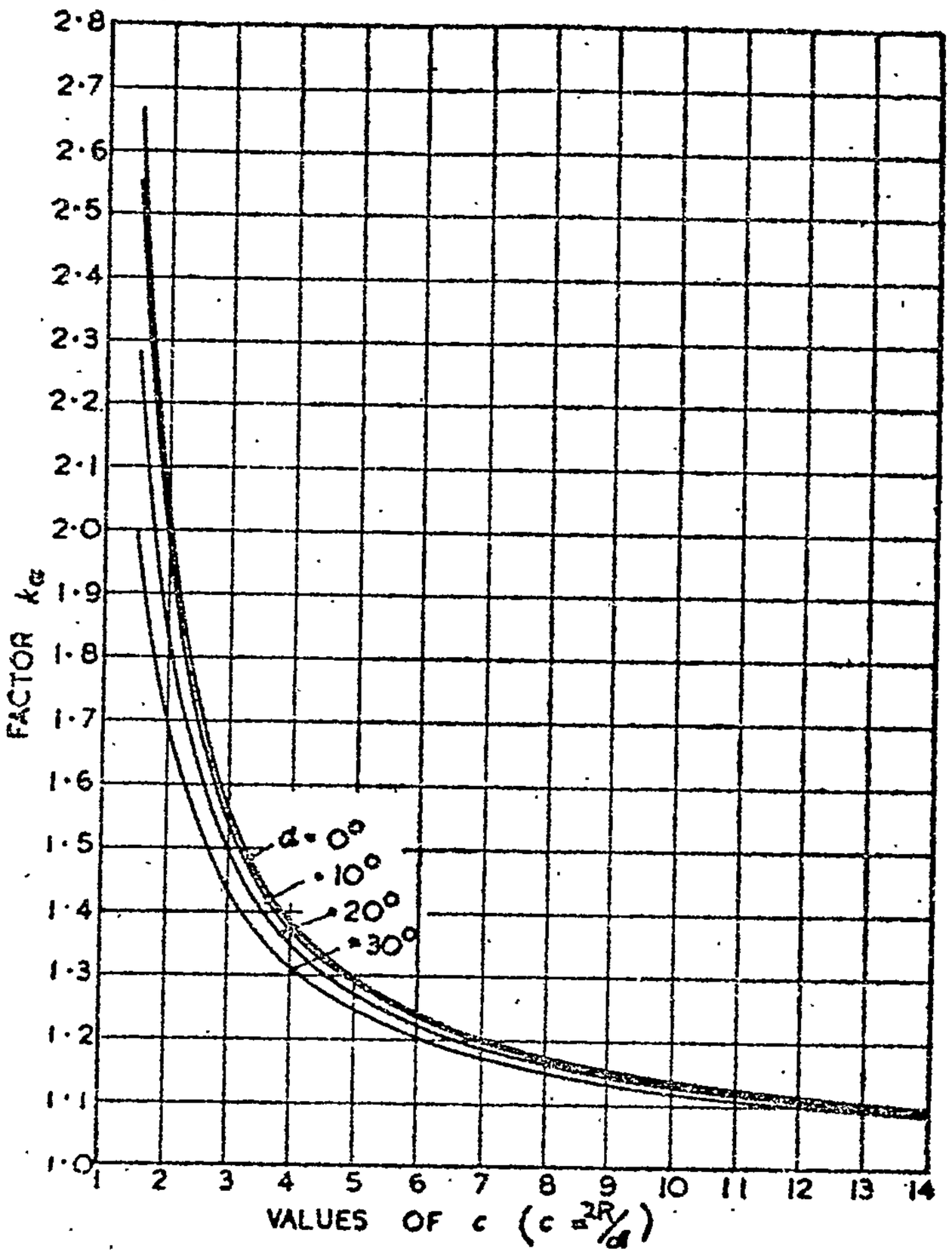

Fig. 11 Maximum shear stress factors derived by Bergstrasser.

Wahl (1949) compared the results obtained by using his formula, Equation (13), with that using Equation (12) combining Equation (19) and (21) derived by Goehner. The results derived from his formula for springs with indexes equal or greater than 3 are within 2 per cent of that of Goehner's. He indicated that Equation (13) is accurate enough for most purposes.

Berry (1961) found that the maximum divergence in the curvature corrections for stress in round wire compression springs between Bergstrasser's and Wahl's formulae, for c =3, is 1½%.

The difference in stress corrections using Goehner's and Bergstrasser's formulae as studied by Finniecome (1949) is only noticeable in the third decimal.

After comparing the numerical values of the maximum shear stress derived from the correction factors of Rover, Wahl, Honegger, Goehner and Bergstrasser, Finniecome concluded that "the formulae presented by these authors were practically identical. The difference being negligible". The formula derived by Bergstrasser was shown to give the least variation of the maximum shear stress factor with different pitch angles (Fig. 11).

Effects of Stress Concentration

In passing, a point needs emphasizing is the stress concentration effect at the inside of the coil in springs with small indexes. As the maximum stress concentrates in a relatively small portion of the cross-section, more unaffected material is left to withstand further stress until yielding occurs across the whole cross-section of the wire. Hence springs of smaller indexes are capable of withstanding a higher load than that calculated from the formulae for maximum shear stress (Wahl, 1949). This phenomenon indicates a reverse relationship between the spring index and force output from the spring.

Stress in Springs with Large Deflection

The formula, Equation (13), derived by Wahl (1949) for the calculation of the maximum shear stress is applicable with sufficient accuracy as long as the deflection per turn is small so that the coil radius and pitch angle may be considered to be constant.

With large deflections, the mean coil radius and pitch angle change significantly and the end coils tend to rotate about the spring axis. These changes have to be taken into consideration. The correction factors so far considered require further modifications.

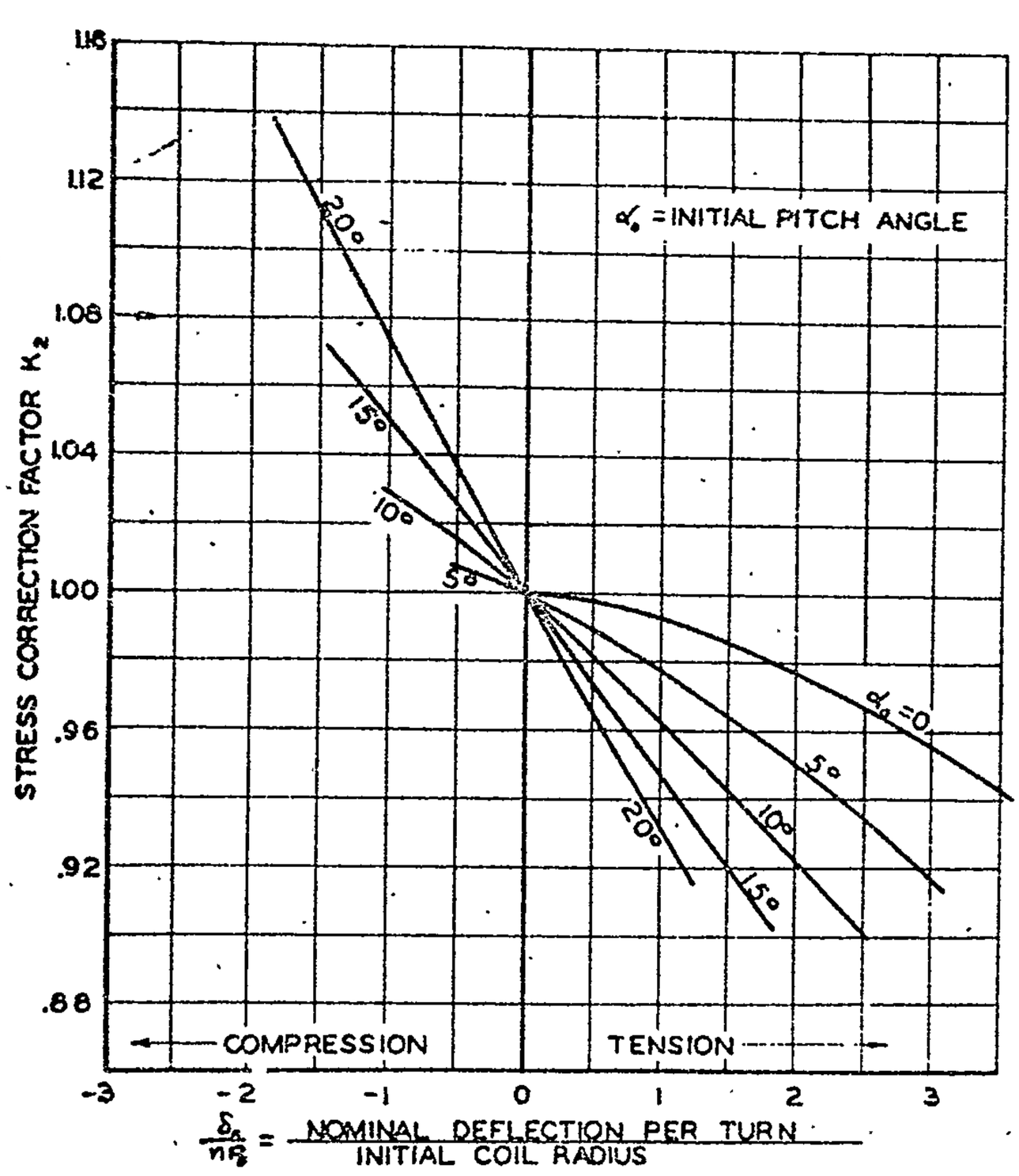

Fig. 12. Stress correction factor K_2 for springs with ends free to rotate. (Wahl).

Springs With Ends Free to Rotate (Wahl, 1949)

The formula for the maximum equivalent stress, τ_e, in an axially loaded open coil spring having large deflections and with the ends of the spring free to rotate, becomes:-

$$\tau_e = \frac{8PD}{\pi d^3} \times K \times K_2 \qquad \text{(25)}$$

where K is the stress correction factor for bar curvature and direct shear defined by Equation (14).

K_2 is a function of the ratio $\frac{\delta_o}{nR_o}$ (between nominal deflection per turn and initial coil radius) and of the initial pitch angle, α_o, at zero load. The nominal deflection δ_o is obtained by using Equation (1).

Values of K_2 for various α_o and $\frac{\delta_o}{nR_o}$ are shown in Fig. 12.

From the curves of Fig. 12 it may be seen that when $\alpha_o < 15$ degrees and $\frac{\delta_o}{nR_o} < .5$, the errors in the stress formula due to changes in pitch angle are less than 3 per cent. Under such conditions, Wahl (1949) suggested dropping the factor K_2 for most practical purposes.

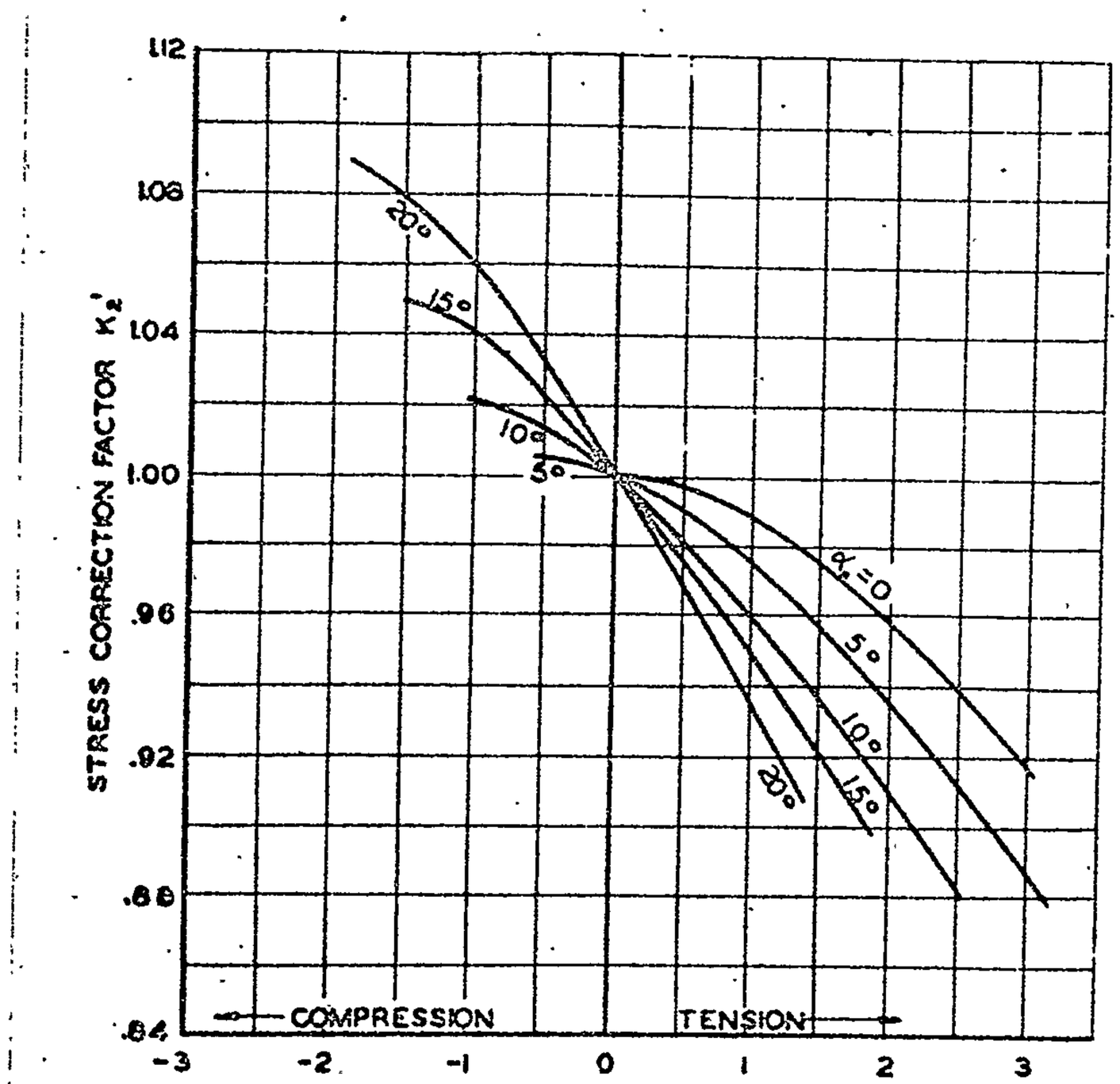

Fig. 13. Stress correction factor K_2' for springs with ends fixed. (Wahl)

Springs With Ends Fixed Against Rotation

Where the ends of the spring are prevented from rotation, the maximum equivalent stress in the spring becomes:-

$$\tau_e = \frac{8PD}{\pi d^3} \times K \times K_2' \qquad \text{(26)}$$

Values of K_2' plotted by Wahl (1949) as a function of α_o and $\frac{\delta_o}{nR_o}$ are shown in Fig. 13.

Berry (1961) concurred with Wahl (1949) in favour of neglecting the additional corrections to the stress formula for different initial pitch angles and changes in coil diameter from large deflections provided that the initial pitch angle is less than 15 degrees and the calculated diflection per turn is less than a quarter of the mean coil diameter. Under such conditions, the stress correction is less than 3 per cent as a maximum and the factor reduces rapidly with the reduction in either the initial pitch angle or the ratio of calculated-deflection-per turn to mean coil diameter. He recommended using the stress correction factor of Wahl, i.e., Equation (14),

$$K = \frac{4c - 1}{4c - 4} + \frac{.615}{c}$$

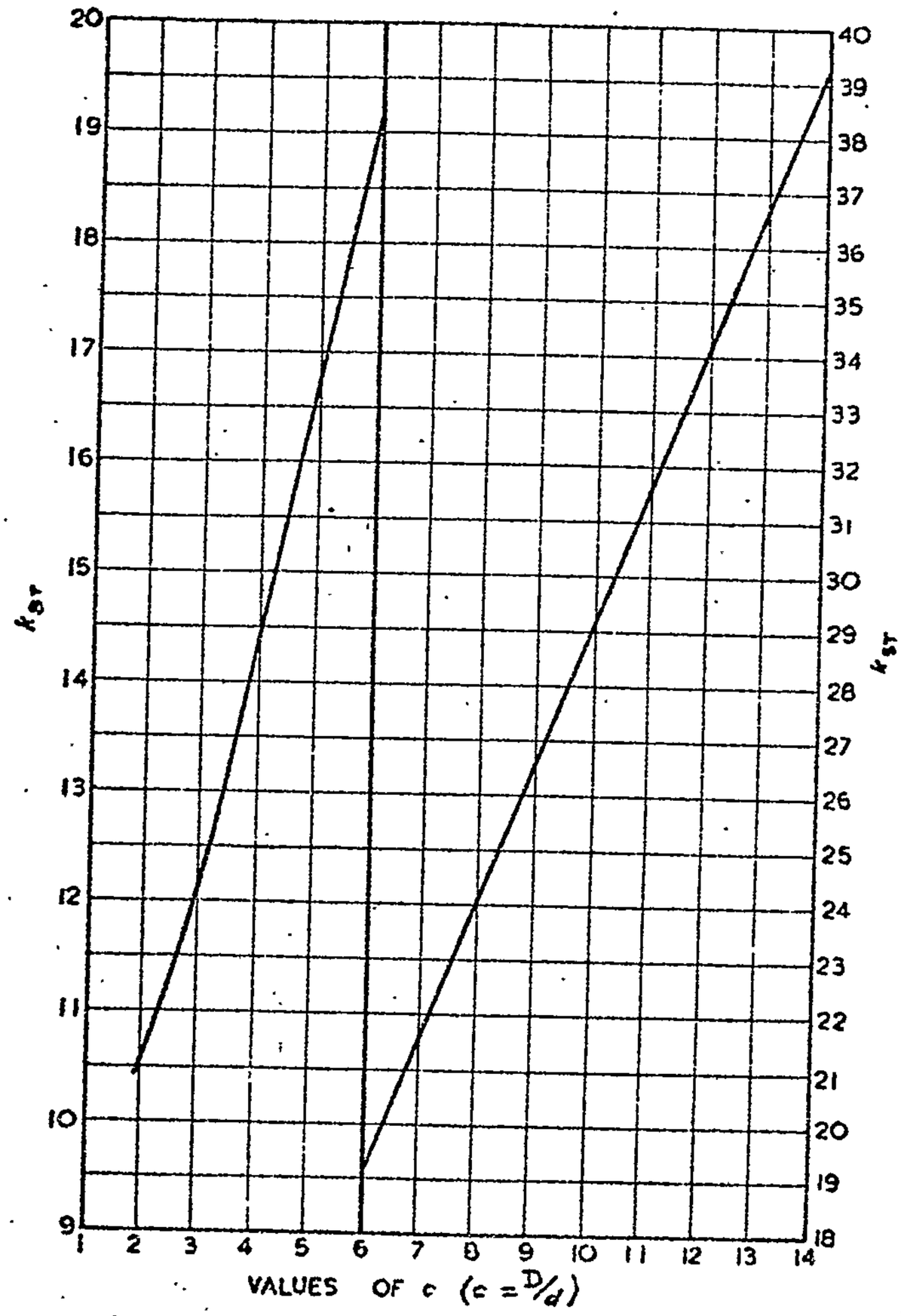

Fig. 14 : Values of K_{ST} plotted by Finniecome.

or Bergstrasser, i.e., Equation (24) with zero pitch angle,

$$K = \frac{4c + 2}{4c - 3} \qquad (27)$$

for calculating the maximum shear stress. He found that the formula derived by Bergstrasser does not over-correct the calculated stress in springs of small index as Wahl's formula tends to.

Finniecome (1949) simplified the formula derived by Wahl, Equation(13), into:-

$$\tau_{max} = K_{ST} \times \frac{P}{d^2} \qquad (28)$$

$$K_{ST} = \frac{8}{\pi}\left[c \left(\frac{4c - 1}{4c - 4} \right) + 0.615 \right]$$

Values of K_{ST} as a function of the spring index plotted by Finniecome are shown in Fig. 14.

With Equation (28) and Fig. 14 , the maximum shear stress can be easily determined.

In summary, in order to utilize the elastic properties of the spring material, the stress at its proportional limit has to be ascertained. Formulae for calculating shear stress in open coil springs have been developed by Rover, Wahl, Honegger, Gohner and Bergstrasser and for

calculating bending stress by Rover, Honegger and Gohner. For most practical springs the resultant equivalent stress, from combining the shearing and bending stresses, differs but slightly from the maximum shear stress. These formulae by the five authors for calculating shearing stress had been reviewed by Finniecome who concluded that the formulae were practically identical. The values of the maximum shear stress factors calculated with the formula by Bergstrasser show the least variation with different initial pitch angles. This formula is also recommended by Berry for it does not over-correct the calculated stress in springs with small indexes.

Significance of the Review of the Formulae for Calculating Stress in Relation to the Present Project

Following the recommendations of Finniecome and Berry, the formula derived by Bergstrasser is used to calculate the maximum shear stress in any helical compression spring under an axial load. Thus we get:-

$$\tau_{max} = \frac{8PD}{\pi d^3}\left(\frac{4c+2}{4c-3}\right)$$

This formula indicates that the maximum shearing stress is independent of the number of active coils in the spring. The load required to

stress a spring of any type of material to its proportional limit is proportional to :-

$$\frac{d^2}{c}\left(\frac{4c - 3}{4c + 2}\right)$$

As the nominal wire and internal coil diameters are specified in the manufacturer's catalog, the maximum force delivered by any coil spring stressed to the proportional limit of the spring material can be calculated.

The maximum forces capable of delivery by the springs showing the lowermost load-deflection ratios (Table 1) have been calculated. The results are shown in Table 2 . The force delivered by a spring of .007 in. wire with .036 in. internal coil diameter is used as a base line for comparison with springs of other configurations.

TABLE 2

Comparison of load for stressing the spring wire to its proportional limit

Spring configuration and dimension		Spring index	Max. force Proportional To $\frac{d^2}{c}\frac{(4c-3)}{(4c+2)}$	Max. force-ratio compared with that of .007- .036
wire size in	internal core diameter in	c	in^2	
.006	.036	7	4.2857×10^{-6}	0.66
.007	.036	6.142	6.4765×10^{-6}	1
.0056	.022	4.928	4.8982×10^{-6}	0.75
.007	.030	5.285	7.2682×10^{-6}	1.12
.008	.036	5.5	9.2121×10^{-6}	1.42

D. THE EFFECT OF COIL PITCH ON THE ELASTIC RANGE OF DEFLECTION

The coil pitch determines the amount of deflection possible for each coil. If the coil pitch is small, the coils will close very early before the spring material is stressed to any great extent. The elastic range of the spring is therefore limited. On increasing the pitch, larger deflections per coil are made possible and the stress induced will rise along its stress-strain curve. This situation continues until a pitch is reached where the spring material is stressed to its elastic limit when the spring is compressed solid. Further increase in the coil pitch will allow the spring to be stressed beyond the elastic limit of the spring material before it reaches its solid length. On compressing to solid length. a permanent set is introduced and the spring will have a new load-deflection characteristic (Wahl 1949).

For practical springs with small or moderate indexes, Wahl (1949) found that "excessive stress are set up where the deflections per turn approach the mean coil radius". In other words, the maximum elastic range of deflection per coil should be below its mean coil radius.

German Standard DIN 2095 (1973) on quality specifications for cold coiled helical compression springs made of round wire specified that

at the maximum permissible test load there should be a minimum space between the active coils. The sum of the minimum spaces between the active coils, S_a, should be:-

$$S_a = x.d.n$$

where d is the diameter of wire in mm. and n is the number of active coils. If the spring is compressed to within this space, S_a, the load-deflection ratio may show a strongly rising trend.

For a given spring index, C, the corresponding value of x is found from figure 15 .

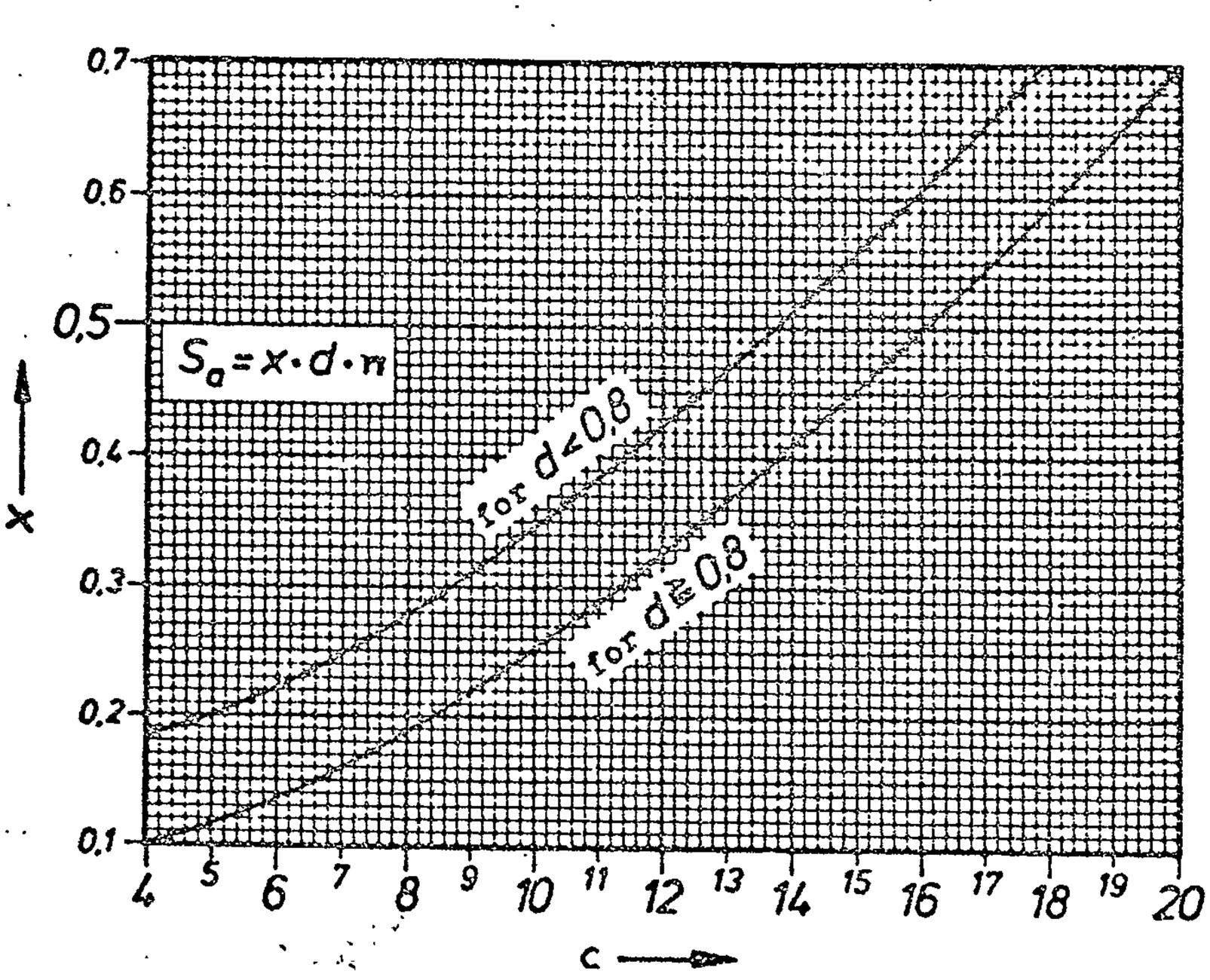

Fig. 15. Value of x as a function of spring index C.

Thus the minimum space between each coil is a function of the spring index and the diameter of the wire of the spring. The optimum pitch, ph, would be one which allows the deflection per coil, ph - (d+x), to stress the spring material nearly to its proportional limit.

In preformed springs, the coil pitch is predetermined by the manufacturers and is not specified in the catalog. Hence differences in coil pitch in springs with the same wire and core diameters made from different materials can over-shadow the differences in the physical properties of the materials. The differences in coil pitch can also account for differences in load-deflection ratio and elastic ranges of the deflection of springs with the same wire and core diameters.

If the coil pitch in the as-received condition is too large, it can be reduced by compressing the spring solid. Such setting operation increases the mean coil radius of the spring and the number of coils per unit length of the spring. Both of these changes increase the elastic range of the spring.

CHAPTER 3

INHERENT VARIATIONS IN SPRING DIMENSIONS

Compression springs wound cold from materials with high elastic limits introduces manufacturing difficulties and variations in the dimension of the cold-formed springs. A number of factors responsible for these variations was pointed out by Berry (1961). The spring back from wires with high elastic limit causes alterations in the dimension of the spring when the winding tension is released. This introduces slight variations in the pitch and the diameter of the individual coils of the spring.

Rockefeller (1937) reported that the comparatively small variations in hardness from Rockwell B92 to 97.5 found in consecutive lengths of commercially annealed wire caused changes in the outside diameter of springs coiled on the same mandrel. These variations in diameter could change the deflection of the spring by as much as 3 per cent. The effect would be more marked with hard drawn materials.

The varying hardness of the wire also makes it practically impossible to maintain a strictly accurate pitch for the coils. It is also impossible to purchase wires to exact dimensions on diameter due to

the normal wear on the tools employed in manufacturing the wire. Lastly, the number of turns in a spring will vary due to the uncoiling and coiling during heat treatment where this is carried out. Such variation is difficult to control.

The Effects of Variations in Spring Dimensions on Deflection

The effects of variations in spring wire and coil diameters on the deflection characteristic of compression springs as expressed by Wahl (1949) is shown with the help of the deflection formula, equation (1).

Equation (1) is given as:-

$$\delta_o = \frac{64PR_o^3 n}{Gd_o^4}$$

where R_o = nominal mean coil radius

d_o = nominal wire diameter.

If the measured mean coil radius and wire diameter are $R_o = R(1+\varepsilon)$ and $d_o = d(1+\lambda)$ where ε and λ are small deviations from their corresponding nominal values, relative to unity, the true deflection, δ_1, then becomes:-

$$\delta_1 = \frac{64PR^3 n}{Gd^4} (1+3\varepsilon - 4\lambda) \qquad (29)$$

This formula indicates: (1) a one per cent difference in mean coil diameter causes approximately a 3 per cent change in deflection and (2) a one per cent difference in the wire diameter results in approximately a 4 per cent change in the deflection characteristics of the spring. A cumulative variation in both coil and wire diameter of only 1 per cent will change the load-deflection characteristic of the spring by 7 per cent. Thus the importance of uncontrollable variables causing changes in spring dimensions as mentioned by Berry should be appreciated.

The Effects of Variations in Spring Dimensions on Stress

As for variations in stress due to variations in spring wire and coil diameters, Wahl (1949) found that a one per cent variation in the wire diameter causes approximately a 3 per cent variation in the maximum shear stress. Changes in coil diameter by one per cent cause approximately a one per cent change in the maximum shear stress.

Allowable Variations on Wire Diameter

The tolerance on the diameter of hard drawn steel wire for springs specified in B.S. 1408:1947 is $\pm$ 0.00025 inch for wire diameters from 0.0078 inch to 0.018 inch. (Berry 1961). This allowable variation on wire diameter could account for $\pm$ 12.8 per cent to $\pm$10 per cent variations in the deflection of the spring made from wires of 0.0078 inch to 0.010 inch diameter respectively. The same

allowable variation changes the stress by $\pm$ 9.6 per cent and $\pm$ 7.5 per cent respectively.

In the Australian Standard T32, the tolerance reported by Ware (1967) on commercial spring wires up to 0.012 inch diameter is +0.0002 inch. For wires of 0.0056 inch to 0.010 inch, this tolerance respectively allows for $\pm$ 3.57 per cent to $\pm$ 2 per cent variations in wire diameter which could account for $\pm$14.28 per cent to $\pm$ 8 per cent variations in deflection and $\pm$ 10.71 per cent to $\pm$ 6 per cent variations in stress developed in the spring.

Manufacturing Tolerances (Berry, 1961)

The recommended tolerances for coil diameter specified by the Coil Spring Federation Research Organization in June, 1949 to compensate for the variations in strength due to allowable tolerances on wire diameter as specified in B.S. 1408:1947 is as follows:-

$$\text{Tolerance} = \pm 1 + \frac{(c-4)}{10} \%$$

The minimum tolerance for coil diameter is $\pm$0.003 inch. The tolerance for load in springs with more than 10 active coils is from $\pm$3 per cent to $\pm$6 per cent, depending on grade.

The recommended dimensional tolerance for compression springs of indexes between 4 and 9 as specified by the S.A.E. limit for outside or inside coil diameter up to 0.0625 inch is ±0.004 inch. For springs with more than 15 active coils the tolerance for load and deflection rate is ±7% and ±5% respectively. For every 10 active coils the tolerance is ±½ coil. It also recommends that tests to check spring rate should be carried out between 10 and 50 per cent of the total deflection.

Permissible variations for coil diameter of compression springs with indexes between 4 and 8 are specified in Din 2095 (1973) as ±0.05 mm (0.002 inch) to ± 0.1 mm (0.004 inch) for nominal mean coil diameters from 0.63 mm (0.025 inch) to 1 mm (0.040 inch). The spring rate must be tested in the range from 0.3 to 0.7 of the smallest permissible test length.

In effect these specifications for permissible variations in coil diameter are very much the same.

Theoretically, it is possible to keep the load and deflection characteristics to fairly constant values by adjustments in coil diameter within the specified tolerances to compensate for variations in wire diameter. However, one still has to keep in mind the uncontrollable variables, mentioned by Berry (1961),

introduced during the manufacture of the springs.

It is not known whether the spring makers have made these adjustments for orthodontic helical springs. If no compensatory adjustments were made to keep the variations in load and deflection of the spring to narrow limits, tests on either the stress or deflection of the spring under an axial load would show a considerable scatter in the results.

Wahl (1949) reported that stresses and load-deflection characteristics of commercial springs deviated easily by 5 to 10 per cent from calculated values.

Tests carried out by Berry (1961) for stress at the torsional elastic limit of tension springs wound from carbon spring steel wire substantiated this scatter in the observed results. He regarded such scatter as "generally occurs with commercial materials". Springs to him are random commercial products.

CHAPTER 4

DESIGN CONSIDERATIONS IN HELICAL COMPRESSION SPRINGS

From the formula of stored elastic energy in a deflected spring, Wahl (1949) and Berry (1961) pointed out that the essential physical properties of various materials to be considered in designing springs are the maximum permissible working stress and the modulus of rigidity.

A. WORKING STRESS

In statically loaded helical spring, if loaded with a high working stress, as time goes on, there will be a slow deflection. Such changes in deflection with time known as "settage" are due to the plastic flow or creep of the material. The corresponding loss in load due to settage of the material is known as the "relaxation" of the material. These effects are more marked at elevated temperatures. (Wahl, 1949). The physical properties of the spring materials acquired through cold work will gradually reduce as the temperature rises. This situation continues until stability is reached where the maximum permissible stress will generally be so low as to be impracticable. The elastic limit of many spring materials also falls off fairly rapidly as the temperature rises. "Consequently, springs designed to operate satisfactorily at normal temperature will undergo a slow plastic deformation and show a permanent set

of greater or less extent depending on whether the operating stresses are relatively high or low and on the length of time during which the load is applied". (Berry, 1961).

To avoid creep or relaxation, maximum permissible working stress suggested by Wahl for a statically loaded spring is equal to the torsional yield stress of the spring material divided by a factor of safety which varies from 1.25 to 1.5. If the yield stress in tension is known, the torsional yield may be taken as 57.8 per cent of that in tension. In choosing the factor of safety he recommended considering the following:

(1) The consequence of failure of the spring in maintaining load or deflection.

(2) The uniformity and grade of the wire forming the spring.

(3) The degree of control during manufacturing process.

(4) Any available test data that approximates service condition.

(5) Peak load in service.

(6) Corrosion or temperature effects.

If a small amount of creep or load loss can be tolerated, then a higher working stress may be used than would be the case otherwise.

Zimmerli (1941) tested the effects of temperature on the relaxation of spring materials. He concluded that usual spring steels are

reliable when stressed to not more than 80,000 pounds per square inch at temperatures up to 350 degrees Fahrenheit. Specifically the results obtained by Zimmerli on stainless steel (18.2% Cr. 9.2%Ni) are summarized in Table 3.

Table 3. Percentage Loss in Load for Helical Springs

At Elevated Temperatures

Diameter of Wire inch	Temperature (F)	Optimum Stress Relieving Temperature	% Loss in Load At 80000 lb/in^2 stress	100000 lb/in^2 stress	120000 lb/in^2 stress
.148	250	800	1.6	2.2	3.2
.062	250	800	1.3	2	3.5

German Standard DIN 2089 (1963) on calculation and design of compression helical springs made from circular section wire and bar, specified that the maximum permissible shear stress must not exceed 50 per cent of the minimum value of ultimate tensile strength.

Yield and Tensile Strength

Tests on tensile and bending properties of 18-8 stainless steel by Delgado and Anderson in 1963 indicated that the ultimate tensile strength and proportional limit for 0.010 inch diameter wire are 206 kilograms per square millimeter (2.93×10^5 pounds per square inch) and 181 kilograms per square millimeter (2.57×10^5 pounds per square inch) respectively.

Tensile properties of 0.014- and 0.022-inch diameter orthodontic wires of 18-8 stainless steel (from Unitek Co.) and Red Elgiloy (from Rocky Mountain Co.) tested by Craig et al (1965) are summarized in Table 4.

Table 4.

Tensile Properties of 18-8 Stainless Steel & Red Elgiloy

MATERIAL	Yield Strength (0.1 Per Cent Offset) p.s.i.x10^5	S.D.	Tensile Strength p.s.i.x10^5	S.D.
18-8 Stainless Steel	1.76	0.16	2.84	0.23
Red Elgiloy	1.75	0.08	3.09	0.05

Their results indicated that yield and tensile strength of these two materials are much the same.

German Standard DIN 17224 (1968) on quality specifications for stainless steel wire and strip for springs listed the tensile strength obtainable for material No. 4310 (18-8 stainless steel used by Dentaurum) as follows:-

Wire Diameter	Tensile Strength
Up to 0.2mm(.008 inch)	235-to260-kg/mm^2 (3.34 to 3.69 x10^{5}p.s.i.)
Over 0.2mm to 0.4mm (.016 inch)	225-to250-kg/mm^2 (3.2 to 3.55 x10^{5}p.s.i.)

Mechanical properties of 0.016 inch diameter 15-5 precipitation hardening (PH) stainless steel (HiT wires from Unitek Co.) were tested by Masson in 1969. His results showed that the ultimate tensile strength and 0.02 per cent proof stress are $4.04 \times 10^5 \pm 0.025$ pounds per square inch and $3.17 \times 10^5 \pm 0.061$ pounds per square inch respectively.

In 1975, samples of orthodontic wires supplied by Dentaurum, Rocky Mountain, Unitek and Wilcock were tested by the Australian Dental Standards Laboratory. Their results are summarized in Table 5.

In summary, maximum permissible working stress can be calculated from either the ultimate tensile strength or the yield strength.

TABLE 5

Tensile strength and proof stress for Red Elgiloy, 18-8 and 15-5 PH Stainless Steel Wires

Material	Property	Diameter of Wire .014 inch	.016 inch	.018 inch	.020 inch
Red Elgiloy					
	Tensile strength Psi. x 10^5		3.16		
	0.1% Proof stress Psi. x 10^5		1.83		
Unitek Standard (18-8 type Stainless Steel)					
	Tensile strength Psi. x 10^5	3.35	3.22	3.28	3.02
	0.1% Proof stress Psi. x 10^5	2.97	2.32	2.90	2.60
Unisil (15-5PH Stainless Steel)					
	Tensile strength Psi. x 10^5	4.12	4.06	3.84	3.87
	0.1% Proof stress Psi. x 10^5	3.07	3.33	2.86	2.68

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The ultimate tensile strength of "Red Elgiloy", spring material No. 4310 and 18-8 type 302 stainless steel are about the same. 15-5 precipitation hardenable stainless steel has a higher ultimate tensile strength. The yield strength of these materials in descending order are 15-5 PH stainless steel, 18-8 type 302 stainless steel and finally "Red Elgiloy".

However, as the diameters of the wires of the coil springs tested in this project are smaller than those investigated by Delgado and Anderson (1963), Craig et al (1965), Masson (1968) and the Australian Dental Standards Laboratory (1975), the ultimate tensile and yield strengths reported should only be used as a guide. Delgado and Anderson (1963) reported that "one might assume that the smaller diameter wires would show a higher tensile strength as they had been drawn to a greater degree during manufacture. This was not found to be true for all the wires tested, though in some groups of wire from the same manufacture, the value of ultimate tensile strength increases as diameter decreases. Since frequent annealing is necessary when drawing wires to these fine diameters, one can only assume that the ultimate tensile strength is affected more by the amount of drawing since the last anneal, rather than the total amount of drawing necessary to produce a thin wire". Evidence to this effect is also presented in Table 5.

B. MODULUS OF RIGIDITY (Wahl, 1949)

In calculating deflection in a helical spring, the value of modulus of rigidity, G, used in the formula for deflection, Equation (1), is the effective torsional modulus of the spring material. This value may differ from that obtained from torsion tests on straight bars of the same material with ground and polished surfaces. The reasons for this difference include the effects of overstraining of the material and the working temperature of the spring.

Effect of Overstraining

Platta, Smith and Harrison found that the torsional modulus decreases by 1 to 5 per cent depending on the amount of overstraining of the helical springs. There is a tendency for the modulus to recover partially its initial value after the spring has stood for some time.

Effect of Temperature

Kenlegan and Hauseman (1933) found a decrease in the modulus of rigidity of various spring materials with an increase in temperature. The modulus of rigidity of most materials from -50°C to

$+50^{\circ}$ C could be represented by

$$G = G_o \ (1 + mt)$$

where G_o = modulus of rigidity at 0° C,

m = temperature coefficient of modulus, and

t = temperature in centigrade.

For 18-8 hard drawn stainless steel the coefficient, m, per degree centigrade is given as 0.00040.

The rough average value for the modulus of rigidity of 18-8 stainless steel is 10.6×10^6 pounds per square inch. Wahl (1949) found that individual test values may deviate from these average by several per cent.

The significance of the modulus of rigidity on the selection of performed coil springs has been studied under deflection in Chapter 2.

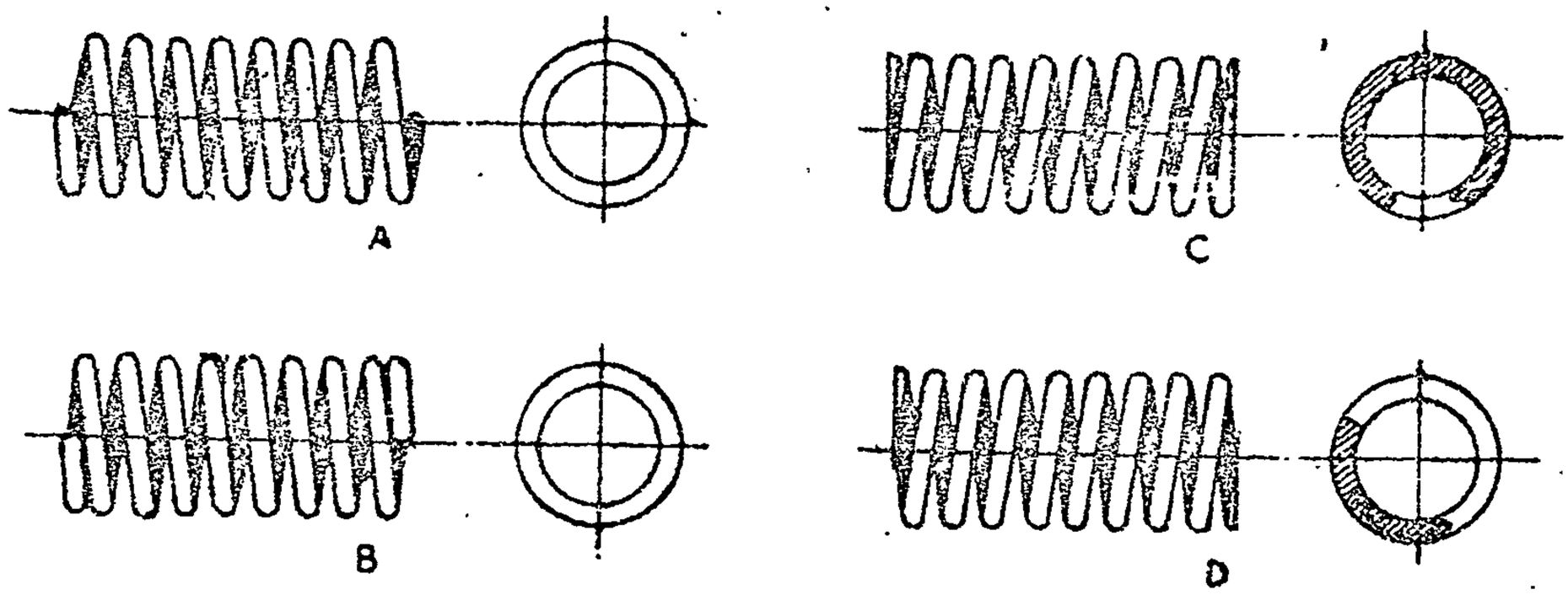


Fig. 16. Four general types of end coils as used in helical compression springs: A) Plain ends; B) Closed ends, not ground; C) Squared and ground ends; D) Plain and ground ends. (Berry, 1961)

C. EFFECTS OF END COILS ON DEFLECTION

The usual types of end coils employed in helical compression springs are shown in Fig. 16.

The most common type used in Orthodontics is that shown in Fig. 16A. The ends are left plain without any grinding. Springs with such end coils when compressed will cause the resultant load to be displaced from the spring axis by a small amount. This eccentricity of loading increases the stress on one side of the spring and decreases the stress on its diametrically opposite side. These unequal lateral stresses cause the spring to buckle particularly if its length is too long relative to its coil diameter, such as typically applied in orthodontic practice.

In determining deflection, an accurate estimate of the number of active coils which are completely free to deflect is required. When a spring is compressed, there is some progressive seating of the end coils so that the number of completely free coils decreases with increases in load. Progressive seating of the coils also produces a slight curvature in the load-deflection curve of the spring. The approximate value for the active coils with plain ends (Fig. 16A) is equal to the total number of coils in the spring minus half a coil (Wahl, 1949; Berry, 1961).

The total deflection of a compressed spring is shown by Equation (1) to be proportional to the number of active coils present in the spring. In clinical practice without a travelling microscope, it is difficult to count the number of coils present in any length of the spring. If the active coils of the spring are of an uniform pitch, the deflection of the spring is then proportional to its free length. In order to compare the deflection capacity of different springs, their individual deflections can be expressed as a percentage of their original free length. This percentage deflection enables comparison of deflection in springs with different original free lengths.

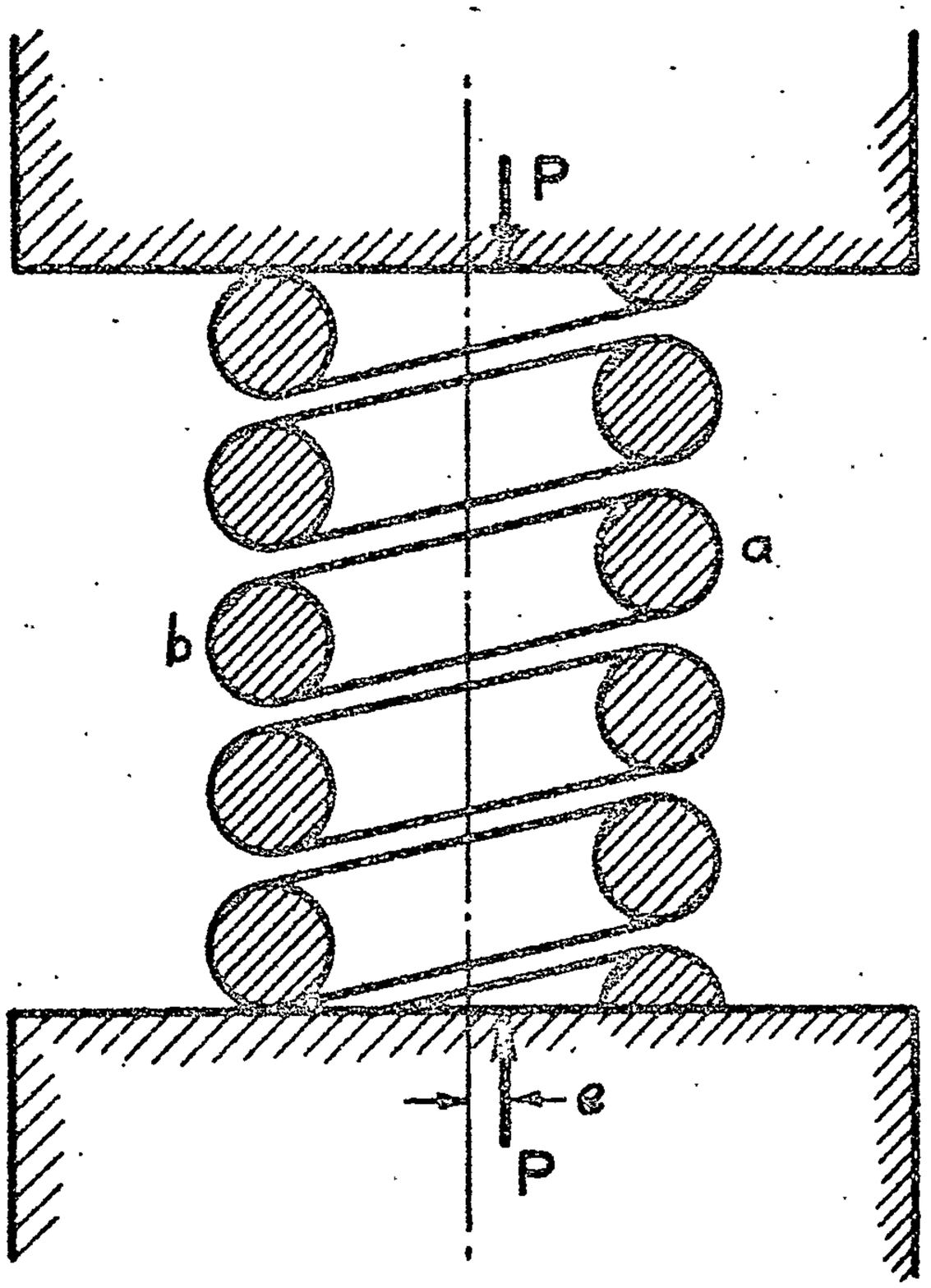


Fig. 17. Resultant eccentricity of loading with stress at b greater than a. (Wahl)

D. EFFECTS OF ECCENTRICITY OF LOADING

If a compression spring with plain ends is compressed between two parallel plates, it will be found that the resultant load is displaced from the spring axis by a small amount e shown in Fig. 17. The resultant eccentricity of loading increases the stress on one side of the spring and decreases the stress on its diametrically opposite side.

The relationship between eccentricity (e) to the mean coil radius, R, is given by Keysor (1940) as:

$$\frac{e}{R} = 1.123\ (Z - 1)$$

$$Z = 1 + \frac{.5043}{N} + \frac{.1213}{N^2} + \frac{2.058}{N^3}$$

where N is the total number of coils minus half a coil.

Where the spring index is fairly large, the stress will be increased approximately in the ratio $1 + \frac{e}{R}$ as compared with stress for purely axial load. Wahl (1949) suggested that for best results this effect should be taken into account particularly when the number of active coils is small.

E. STRESS AT SOLID COMPRESSION

If a compression spring with a sufficiently large initial coil pitch is statically loaded beyond the elastic limit of the spring material before the coils are compressed solid, the outermost fibres of the wire deform plastically and take a permanent set. After removal of this working stress, the elastically deformed inner fibres try to return to their original position. However, the plastically deformed outer fibres tend to remain in their new position. Thus residual stresses develop in the outer fibre in the opposite direction to the originally applied stress. This residual stress is balanced by a corresponding residual stress in the direction of the original applied stress at some points between the outer fibre and the neutral axis where there is no stress.

On subsequent reloading of the spring with the same working stress, the effective stress in the outer fibres of the spring wire is reduced by the amount of the residual stress present (Wahl, 1949; Berry, 1961.)

Berry (1961) also considered that "at some points between the outer fibres and the neutral axis the actual stress is increased by the amount of the residual stress at that position. The applied stress at this internal position is normally considerably less than the

calculated stress at the outer fibres and consequently the increase will not raise the stress above the elastic limit: in fact, it cannot do so as otherwise the fibre concerned would have yielded during the previous setting and the residual stress in that fibre would have been in the opposite direction."

The net result is that although a working load beyond its elastic limit would have caused the spring to take a permanent set when first applied, it cannot do so upon reloading. On subsequent loading,the spring behaves as if the wire had a higher elastic limit. This process of cold setting or overstressing increases the permissible working stress. Tests on springs compressed solid by the National Physical Laboratory reported by Sopwith (1947) indicated a 60 per cent increase in the elastic range of the springs. The general tendency is to press the spring solid several times after all heat treatment has been completed. B. S. STA 2 suggests compressing the spring solid for five times. Dr. Sopwith suggests twelve times. However, compressing the spring 5 times would set it to at least 95 per cent of that of the twelve times.

The phenomenon of removing some of the potential permanent set in the spring with each successive compression to close the spring solid suggests a progressive work hardening of the spring wire. Such inward progressive work hardening of the spring wire contradicts

the reasoning given by Berry (1961) of what happens to the internal portion of the wire when the spring is compressed solid for the second time. It is more likely that with each solid compression of the spring, the actual stress increased by the amount of residual stress present at that position causes an additional thin layer of the wire to deform plastically. Thus the spring undergoes a further permanent set. Hence the suggestion by B.S. STA 2 is to compress the spring solid for five times.

If a compression spring is pressed solid in service during installation, due to some plastic deformation this spring will acquire a different load to deflection characteristics from the original spring. If the stress built up in the spring material by settage operation is too high, upon standing for some time, its free length will increase slightly from that immediately following the operation. This "recovery" effect again changes the load-deflection characteristic of the spring. Wahl (1949) regarded that "it is desirable to choose the coil pitch such that when the spring is compressed solid, no appreciable permanent set will occur."

Wallace Barnes Company in 1941, cited by Wahl (1949) suggested torsion stess at solid compression to be 85,000 pounds per square inch for hard drawn 18-8 stainless steel wires of diameters up to 0.125 inch. At this level of stress, it is not necessary to remove

set. If stressed to 140,000 pounds per square inch, all set should be removed with repeated solid compression of the spring.

Berry (1961) recommended that the maximum working stress should be 70 per cent of the "original or primitive" torsional elastic limit of the spring. This working stress may be increased to 112 per cent of the original elastic limit if the spring has undergone cold "setting" operation.

In summary, moderate cold "setting" of statically loaded compression spring improves its maximum permissible working stress and removes some of the potential permanent set with attendant reductions in the initial coil pitch and free length of the spring. On subsequent loading, the same maximum load used to compress the spring solid must be reached before the spring will again start to yield appreciably. Where the accuracy of loading is not very critical, a spring with a proper over-sized coil pitch to allow for the necessary changes during moderate cold setting may therefore have an advantage.

CHAPTER 5

HYSTERESIS IN SPRINGS

If the elastic portion of a stress-strain curve of a spring material is determined with sufficient accuracy, deviations from the linear relation of strain to stress as expressed by the Hooke's law will become apparent. The ascending and descending lines of the curve form closed loops, indicating a lag of strain in relation to stress. This phenomenon of elastic hysteresis was first reported by Lord Kelvin in 1865.

Carson (1933) classified elastic hysteresis into statical and hereditary hysteresis.

Statical Hysteresis

The first type, statical hysteresis, is believed to be a property of the molecular structure of the material and is independent of the time of loading. It forms a thin hysteresis loop as illustrated in Fig. 18.

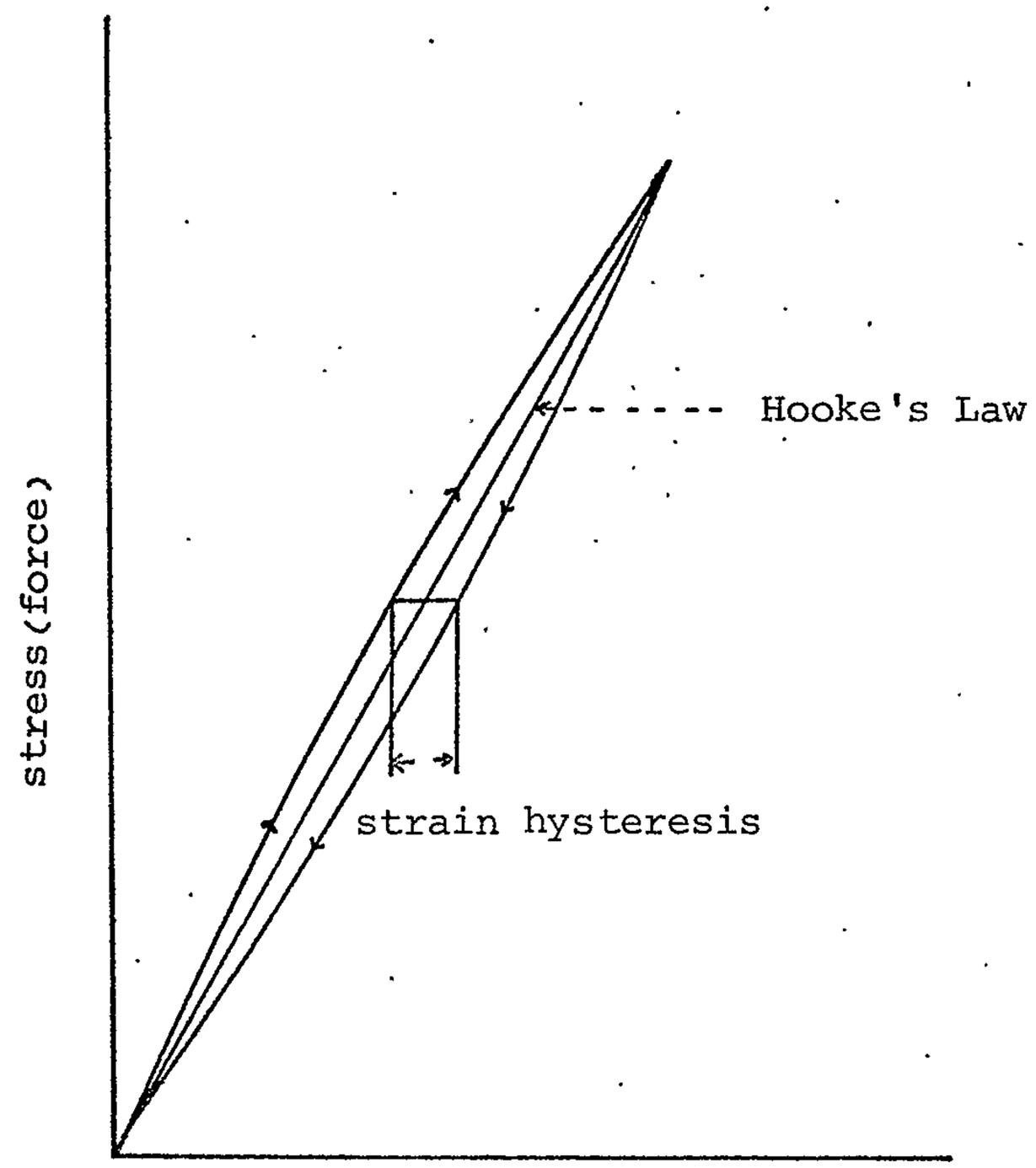


Fig. 18. Typical hysteresis loop. (Carson)

This effect is also known as internal friction, damping capacity and dynamic ductility.

Von Heydekampf (1931) listed some spring materials in ascending order of increasing hysteresis: chrome-vanadium steel, chrome-nickel steel, brass and then carbon steel. The statical hysteresis for a 1 per cent carbon steel worked at a maximum torsional stress of 20,000 pounds per square inch is approximately 4 per cent of the load deflection. Chrome-vanadium steel stressed likewise shows a hysteresis of 0.2 per cent.

The amount of statical hysteresis increases rapidly as the working stress is increased. In general, when the working stress is doubled, statical hysteresis increases to 3 times its original amount. However, Sayre (1931) found that the energy loss in statical hysteresis increases with stress at a higher power than the square of its original amount. "In successive loops, starting from the same nominal initial load and rising to successively greater peak loads, the ascending lines essentially coincide, while the descending lines differ for the different loops." With successive increase in the stress ranges, the area delineated by the closed hysteresis loops increases sharply. The area within these loops represents the energy lost in internal friction in the corresponding cycle. However, a point worth noting is that the slopes of the descending

graphed lines representing unloading become progressively less steep.

Sayre (1931) also noted that as the hysteresis loop became wider it was more and more difficult to distinguish between true elastic elongation and the collateral effects. Collateral effects mentioned were: (a) "permanent set", which was an irregular function of time starting and stopping with little apparent causes;

(b) elastic or pseudo-elastic after-effect (hereditary hysteresis) which was the gradual elongation on loading followed by an as gradual recovery to original length on unloading and

(c) statical hysteresis.

Sweetser in the discussion of Sayre's paper presented 2 curves showing hysteresis loops which occurred at either end of a stress-strain curve of mild steel in torsion. In curve A, the stress was increased gradually up to a maximum of about 52,000 pounds per square inch (lbs./in.^{2}) and then reduced to zero stress, giving a complete hysteresis loop. It was then stressed to 35,000 lb./in.^{2} and returned to 0 stress, giving a narrow hysteresis loop contained in the first loop. The up curves in the two cases coincide and the down curve of the smaller loop is steeper than that of the bigger one. This finding is the same as Sayre's tensile test on spring wires. The second curve, B, of Sweetser showed a hystersis loop

formed by reducing the load from the maximum down to a stress of about 16,400 lbs./in.^{2} and then returning to the maximum stress. This curve coming down coincides with the original down side of the bigger loop of curve A.

Their findings indicated that the slope of the unloading curve is dependent on and varies with the level of stress imparted in the wire during loading. A low working stress ensures a closer proportionality between load and deflection. Higher working stress, although accompanied by a larger inevitable loss of energy to internal friction, results in a slope which is less steep. The second hysteresis loop formed by unloading from a maximal stress and then reloading to the maximal as shown by Sweetser, approximates the condition where orthodontic helical springs are reactivated in service. The slope of the unloading curve remains the same as the original slope.

Timoshenko (1968) explained this deviation from perfect elasticity and the time effect in the elastic range in terms of the thermo-elastic property of the material. "If a tensile test involving very slow increase of the load is carried out, the temperature of the specimen remains equal to that of its surroundings and the stress-strain relation may be represented by the straight line OA (Fig. 19). The slope of this line gives the magnitude of the

modulus E under isothermal conditions.

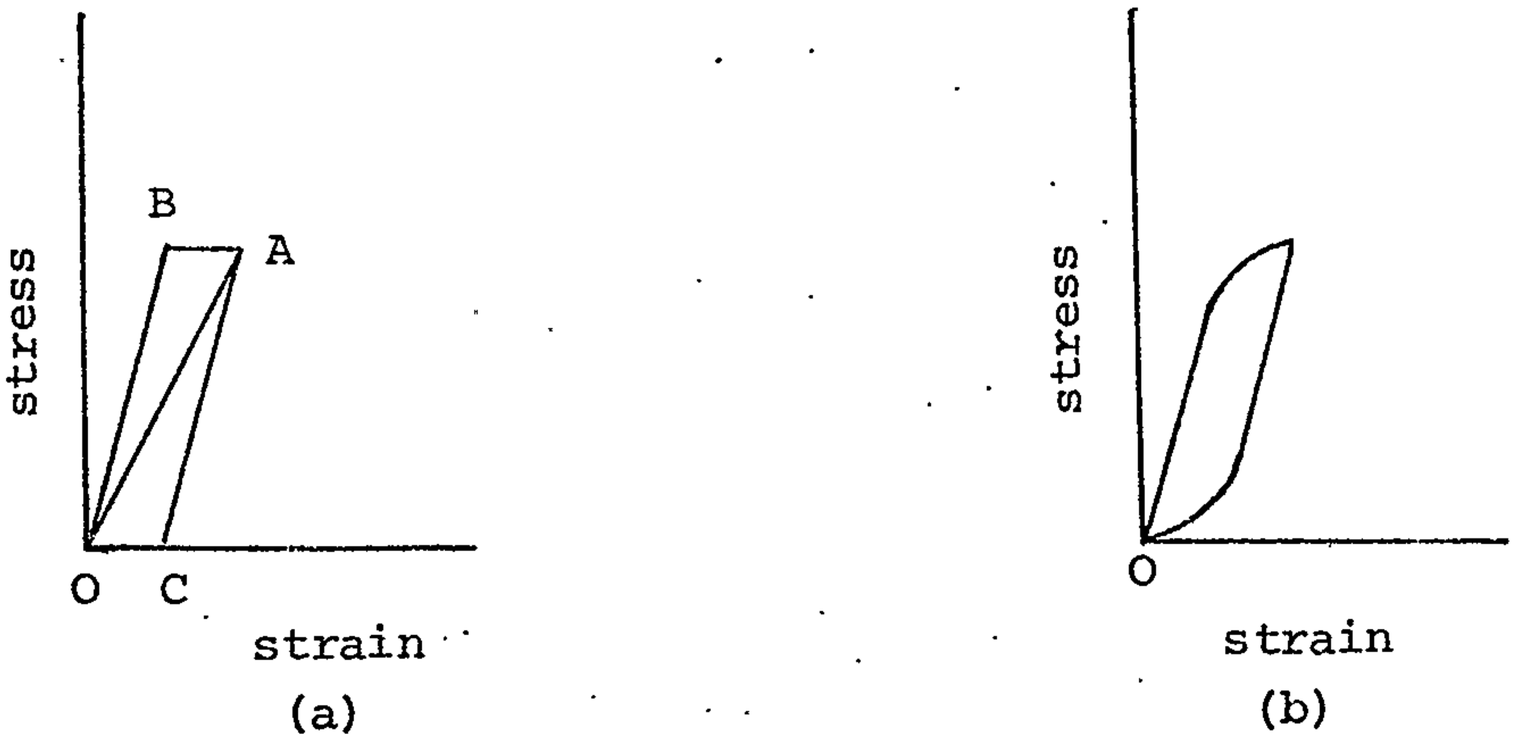


Fig. 19. Hysteresis Loop

If the tensile load is applied rapidly, so that there is not enough time for heat exchange to take place, the straight line OB is obtained instead of OA. The modulus E under adiabatic condition is larger than that obtained isothermally. Because of its sudden stretching, the specimen is cooled below room temperature. If the specimen remains under constant load for a sufficient length of time, it gradually warms up until it reaches room temperature. As a result, an additional elongation of the specimen is brought about (represented in Fig. 19 by the horizontal line BA). This is the elastic after-effect and is due to the thermoelastic property of the material. If, after complete equalization of the temperature, the specimen is suddenly unloaded, its adiabatic contraction will be represented in Fig. 19 by the line AC parallel to OB. Owing to this sudden shortening, the specimen warms up, and the subsequent

process of cooling to room temperature will give rise to a further contraction, represented by the length CO. It is seen that by deforming the specimen adiabatically and then allowing enough time for temperature equalization, a complete cycle, represented by the parallelogram OBAC is described. The area of this parallelogram represents the loss of mechanical energy per cycle. We have assumed adiabatic deformation in our reasoning, whereas in practice there is always some heat exchange during the cycle. Thus, instead of a parallelogram, a loop such as shown in Fig.19 (b) is obtained. (This loop is similar to that described by Carson).

The difference between the adiabatic and the isothermal moduli is usually small. For steel it is about 1/3 of 1 per cent. The loss of mechanical energy per cycle is also very small."

In a polycrystal specimen of structural steel, the heat generated in each grain is proportional to its volume and the heat exchange is dependent upon its surface area. A material with small grain size will show a larger hysteresis loop as temperature equalization is facilitated and the losses of mechanical energy increased.

Hereditary Hysteresis (Carson, 1933)

The second type, hereditary hysteresis, is the time lag of the spring deflection under a steady load or after removal of a steady load. The deflection of the spring continues to increase as long as the steady load is maintained. With the removal of the load, a similar effect takes place as the spring slowly recovers its former unloaded position. If the deflection is kept constant, the creep tends to reduce the load on the spring.

The magnitude of the hereditary hysteresis depends on the working conditions, the physical condition of the spring material and the previous history of the spring. It may introduce deviations of several per cent in the spring load or deflection.

Working conditions which influence the amount of hereditary hysteresis are the duration of the application of the load, the working stress and the operating temperature. Creep in a spring under a steady load continues with time at a decreasing logarithmic rate. Increased working stress increases the rate of creep and recovery. Working stresses that approach the elastic limit of the spring material cause an increasing amount of permanent set. Increasing the working temperature has the effect of lowering the elastic limit of the spring materials. Consequently, with increased operating temperature, the amount of creep, recovery and permanet set are increased.

Internal stresses introduced in cold working, cold forming or cold bending of the spring material also increase the hysteresis of the material.

Effects of hereditary hysteresis can be temporarily removed by repeated loading and unloading of the spring. Such cyclical loading superimposes a number of different creep and recovery actions so that the total effect is negligible. After a sufficient rest period, the normal hereditary hysteresis will again appear.

Brombacher in the discussion of Sayre's paper (1931) pointed out that the terms "hereditary hysteresis", "elastic after-effect", "after-working", "drift" or "creep" and "recovery" are synonymous. They refer to the effect "which is dependent on the time of application of the load as well as on the fibre stress and the material". The terms "mechanical hysteresis", "statical hysteresis", "elastic hysteresis" and "elastic lag due to elastic hysteresis" are synonymous. "These terms usually refer to the effect which is independent of the time of the application of the load, but of course may depend on the fibre stress and the material".

Timoshenko (1968) regarded that in addition to the losses in mechanical energy due to heat exchange, with larger working stresses plastic deformation within certain grains also contributes to the losses of energy during loading and unloading

of the specimen. The elastic properties of a single crystal vary with the orientation of the crystal axes with respect to the axis of the specimen. As the crystals are randomly orientated, the stresses in a test specimen are not uniformly distributed. Plastic deformation occurs in the most unfavourably oriented individual crystals before average stress reaches the elastic limit of the material. Such yielding of the individual crystals contributes to the losses in mechanical energy and will increase the area of the hysteresis loop. This explanation given by Timoshenko agrees with the findings of Sayre (1931) and Carson (1933) that the amount of statical and hereditary hysteresis increases with increased working stress.

Carson (1933) recommended that for precision springs the maximum working stress should not be more than 20 per cent of the elastic limit of the material if both types of hysteresis are to be controlled. Stress relief anneal for stainless steel at 400 degrees Fahrenheit for 5 hours should be carried out after all forming or shaping operations. These precautions reduce the elastic hysteresis of the spring to less than 0.1 per cent. Whether this precision in deflection with load is of clinical significance in orthodontic helical springs as a method of applying force is doubtful.

Berry (1961) considered that if a spring were to be subjected to long continuous loading at maximum stress, the effects of hysteresis

should be taken in consideration. The maximum permissible stress for alloy steel under such condition should be about 50 per cent of the probable elastic limit of the material.

Since plastic deformation of the unfavourably orientated crystals occurs within the elastic limit of the spring material and contributes to the loss of mechanical energy, moderate cold "setting" of the spring before service would minimise such loss during service. It would appear that moderate cold "setting" of helical compression springs besides improving its elastic limit also ensures a close proportionality between load and deflection by reducing the area of the hysteresis loop.

HEAT TREATMENT OF STAINLESS STEEL AND COBALT CHROMIUM WIRES

A review of the literature concerning the effects of heat treatment on stainless steel wires shows much detailed analysis.

Zimmerli (1941) examined the effects of various stress-temperature combinations on steel springs and the percentage loss in load in helical springs at elevated temperatures. His findings included the following:-

Loaded 18-8 stainless steel springs took 10 days to reach a stable situation such that further exposure to the elevated temperature would not cause any additional loss in load carried by the spring with any combination of stress up to 120,000 pound per square inch and temperature below 350°F.

The loss in load due to temperature was not governed by the length of the spring wire in the spring.

Springs compressed solid after a stress-relieving heat treatment gave a more uniform result than those compressed solid before the heat treatment.

Among his conclusions were the following:-

The optimum stress relieving temperature is the highest one that will not cause any lowering of the hardness or other physical properties of the material. (The optimum temperature used for 18-8 stainless steel springs was 800° F).

Springs heat treated (hardened and tempered) after coiling have no advantage over those made of properly stress relieved pretempered wire.

The effects of low temperature heat treatment on 0.020 inch and 0.021 x 0.025 inch stainless steel wires were evaluated by Backofen and Gales (1951). Their results from tensile tests of straight length of wires and expansion of U-shaped loops made from 0.021 x 0.025 inch rectangular wires under tension indicated a marked increase in the elastic limit of the wire when heat treated for 10 minutes at 750° F or 820° F. They attributed the results to the elimination of residual stresses from "cold working" the wires.

Funk (1951) examined the effects of heat treatment on "Tri-state", "Unitex" and "Tru-chrome" rectangular stainless steel wires. The wires were tested by bending straight sections and subjecting a series of V-shaped loops bent in the wires to tension. With heat treatment he found a marked improvement in the elastic properties of the wires tested in tension and a very slight increase in stiffness of the wires subjected to the bending test. Despite precautions

of selecting " a tension below the limit of elasticity ", the elastic properties of these wires after different heat treatment temperatures and time were judged by the extent of permanent set induced by the constant tensile forces. He concluded that the optimim heat treatment for these types of wires was at 850 degrees Fahrenheit for three minutes. The range of temperature for good results was 200 degrees. The stainless properties were not lost during heat treatment which should be carried out after all bending operations. The reason for this increase in elastic properties was thought to be the beneficial rearrangement of the internal structures.

His experiments indicated that the proportional limit of the wires had been increased by heat treatment.

Kemler (1956) found that optimum heat treatment five to fifteen minutes at 700 to 800 degrees Fahrenheit caused an increase in the proportional limit, modulus of elasticity and modulus of resilience of type 302 stainless steel orthodontic wires of 0.020 and 0.025 inch diameters. At higher temperatures up to 1,500° F the proportional limit and the modulus of resilience decreased although the modulus of elasticity continued to increase. The wires were tested on a static flexure machine, before and after heat treatment, with expansion of uniform round loops. Kemler also attributed the observed changes in properties to the relief of internal stresses incurred in the metal during cold working.

The effects of high temperature on the mechanical properties of hard drawn austeritic stainless steel orthodontic arch wire were investigated by Wilkinson (1960). 0.0198 inch diameter, 18.2 per cent chromium and 9 per cent nickel stainless steel wires were heat treated at temperatures ranged from 500°C to 700°C (932°F to 1292°F). His results indicated that heat treatment at 525°C (977°F) for 30 seconds did not cause any lowering of the tensile strength of the wire. Tensile strength of 318 x 10^3 pounds per square inch was the same as that in the as-received condition.

Delgado and Anderson (1963) examined the effects of low temperature heat treatment on the ultimate tensile strength and ductility of round 18-8 stainless steel wires manufactured by C. K. Smith Ltd., Fried. Krupp, and Stainless Steel Wire Co. as available in the British Isles. Heat treatments for 5 minutes at temperatures between 200°C (392°F) and 600°C (1,112°F) indicated that accurate temperature control was essential to improve the ultimate tensile strength of only some of the wires tested. Similarly the ductility of some wires was increased by stress-relief anneal at 450°C (842°F) for 3 minutes. As accurate temperature control was essential and the improvements did not always occur, the authors concluded that the benefits from low temperature heat treatment were not great enough to be used in routine clinical practice. However, as the ultimate tensile strength and ductility of stainless steel wires do not have direct clinical application, their conclusions do not seem justified.

The principles for the changes in the elastic limit with heat treatment were investigated by Ingerslev in 1965 and 1966. 18-8 stainless steel austenitic stainless steel wires of 0.40 mm (.016 inch) and 0.45 mm (.018 inch) diameter were bent into archwires. These formed archwires were tested before and after heat treatment at temperatures from 20° to 500° C for 2 to 60 minutes in expansion and compression. Straight segments of the same wire were tested by deflection. When the wires were tested in expansion and deflection, elastic limit was found to increase with heat treatment. With compression of the archwires the elastic limit decreased with heat treatment. This difference in reaction of the elastic limit to the direction of testing was attributed to the removal of internal stresses introduced during cold working of the wires.

His investigation indicates that heat treatment removes the Bauschinger effect whereby "the load that can be resisted by a plastically deformed metal object without undergoing further permanent deformation is greater if applied in the same directions as the force responsible for the original plastic deformation than if applied in the opposite direction".

The modulus of elasticity was found unaffected by heat treatment. Optimal temperatures varied from 350° to 375° C (660° to 707° F) for 20-25 minutes. 70 per cent of the total effect was achieved

after 4 minutes of heat treatment. At temperatures of 400°C (752°F) the corrosion resistence of 18-8 stainless steel was considerably reduced.

Howe et al (1968) found that with stress relief heat treatment at 700° to 900°F, the yield strengths of type 302 and 316 stainless steel orthodontic wires were improved by 10 per cent. For type 302 wire, there was about 8 per cent increase in modulus of elasticity and ultimate tensile strength. The modulus of elasticity of type 316 wire was increased by about 3 to 4 per cent. Heat treatment at 750° to 950°F for 30 seconds to 15 minutes apparently imparted optimal mechanical properties to these materials but relieved only 20 to 40 per cent of the residual stresses present in type 302 wires and 12 to 30 per cent in type 316 wire. With the large amount of residual stresses that remained after heat treatment, they maintained that "distortion of the clinical device will still be experienced to the extent that residual stresses remain". They also found considerable variations in the mechanical properties of the wires in the as-received condition from one manufacturer's lot to another.

The increase in resistence to permanent deformation of Elgiloy by heat treatment was quantified by Fillmore and Tomlinson (1976). Straight wires of .016" x .022" soft Elgiloy bent into a series of

U-loops were heat treated for 5 minutes at temperatures from 600° F to 1,500° F. After heat treatment the specimens were loaded in tension causing stress to develop in the wire in the opposite direction to that formed during the initial bending of the U-loops.

Their results showed that the resistance to permanent deformation of cobalt chromium wires increased to a maximum with heat treatment temperatures up to 1,200° F and declined rapidly with temperatures above 1,200° F. The percentage increases in resistance to permanent deformation when heat treated at 1,200° F and 900° F were 174 per cent and 95 per cent respectively.

As different tempered cobalt-chrome wires react differently to heat treatment and the "stiffer" the wire in its raw state, the "stiffer" will it be after heat treatment (Rocky Mountain 1957), results obtained by Fillmore and Tomlinson (1796) on soft Elgiloy are not directly applicable to "Red Elgiloy".

Stephens (1971) had already shown that the elastic properties of "Red Elgiloy" increased remarkably with heat treatment. However, after heat treatment it had little capacity for permanent deformation.

Masson (1969) also found that Red Elgiloy heat-treated to the recommended temperature of 900° F for seven minutes resulted in a brittle wire.

Hence heat treatment of Red Elgiloy coil springs may introduce problems during the activation of these springs.

In summary, heat treatment is a function of time and temperature. The major change in the mechanical properties of 18-8 stainless steel and chrome-cobalt wires with low temperature heat treatment is the marked increase in the elastic limit of the wires under reverse stress situations. The optimum time-temperature combination for 18-8 stainless steel is between 700°F and 800°F for 5 to 15 minutes. Most of the gain is achieved rapidly. Corrosion resistance of 18-8 stainless steel is not affected by temperatures below 800 F.

In coil springs which have been compressed solid before put to service, the residual stresses in the outer fibres are in the opposite direction to that formed during service (Wahl 1949, Berry 1961). This method of using the Bauschinger effect of the wire to raise its elastic limit would be lost with stress relieving heat treatment (Iugerslev 1965, 1966). Hence if such "cold setting" operation is to be used to increase the performance of coil springs, they should not be subjected to any subsequent heat treatments. As for Red Elgiloy coil springs, heat treatment would only introduce new problems.

CHAPTER 7

ELASTICITY AND STRENGTH OF MATERIALS

In order to explain the behaviour of springs tested in this research, it is appropriate to present the terminology generally involved in the testing of the elastic and plastic properties of wires.

When a force is applied to a material, Thurow (1972) defines that force as a stress. The major types of mechanical stresses experienced within a material are in response to tension, compression, shear and torsion force as illustrated in Fig. 20.

Phillips (1973) defined stress as the force per unit area in a body which resists an external force. The following are definitions given by Phillips for tensile, compressive and shearing stress.

Tensile Stress

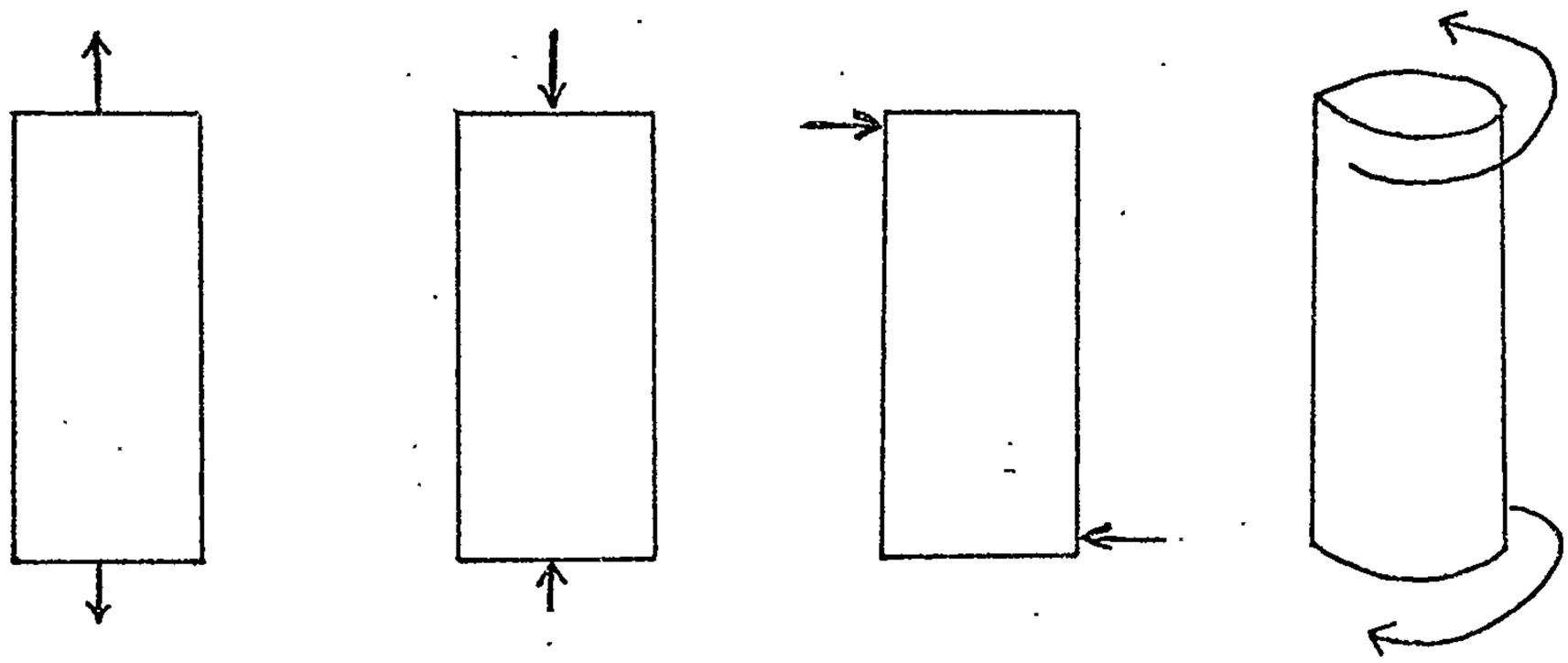
A tensile stress is any induced force which resists a deformation caused by a load which tends to stretch or elongate a body.

Compressive Stress

If a body is placed under a load which tends to compress or shorten it, the internal forces which resist such a load are called compressive stress.

Shear Stress

A stress which tends to resist a twisting motion or a sliding of one portion of a body over another, is a shear or shearing stress.



(a) TENSION (b) COMPRESSION (c) SHEAR (d) TORSION

Fig. 20. Basic types of load application. (Greener et al).

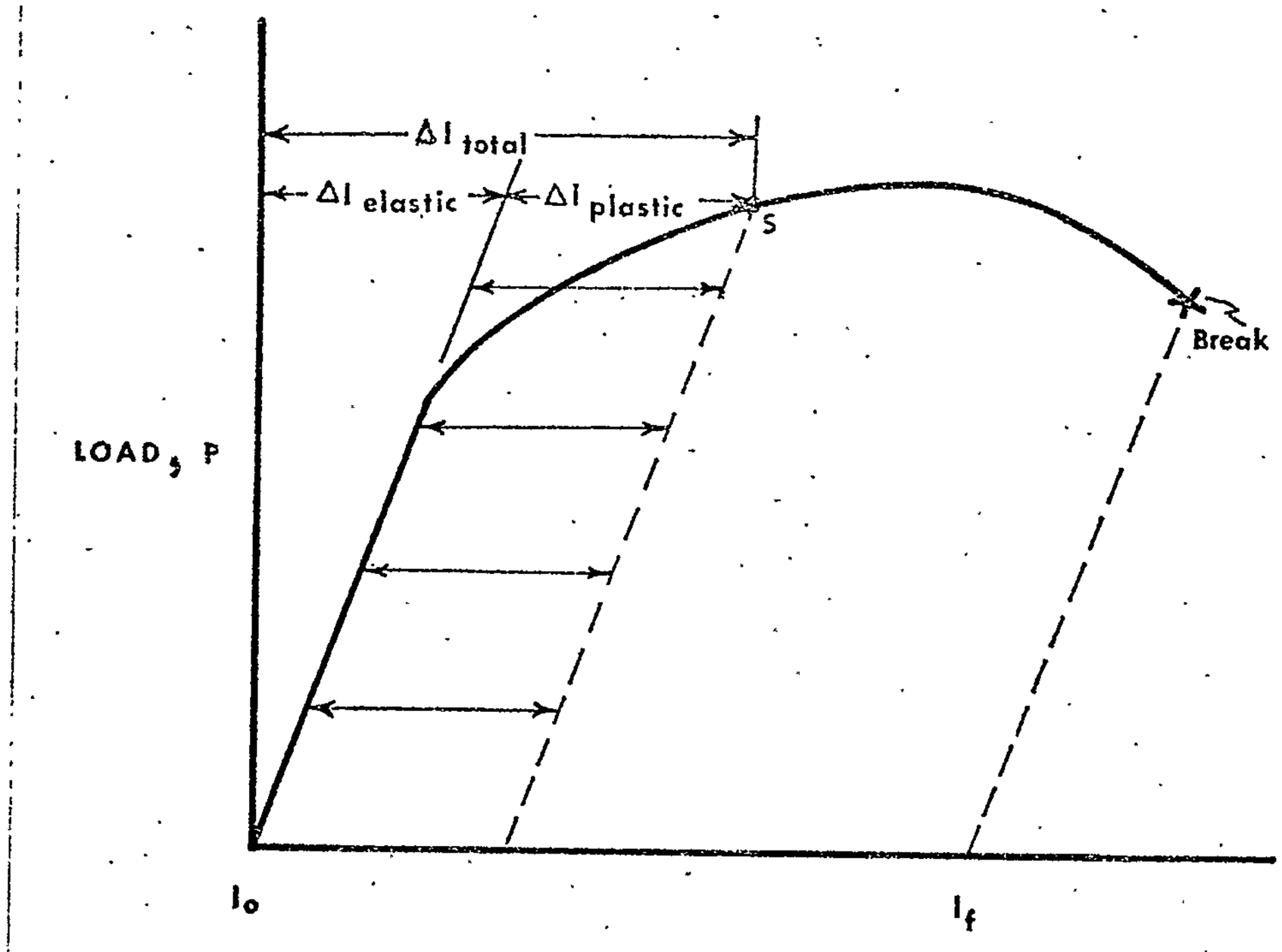


Fig. 21. Load vs. extension length curve for a specimen under tensile test. (Greener et al).

Whenever one or more stresses is placed on a material, the material will tend to deform. The relation between the stress and deformation can be expressed in a load vs. deflection curve. With reference to such a curve, a number of terms used to describe elastic and plastic behaviours of the material may be defined. The most widely used of all mechanical tests of structural materials is the tensile test. A uniaxial tension load vs. length curve shown in Fig. 21 is used to define these terms. Similar phenomena occur with the other types of stresses and then corresponding deformations.

The following is the stress-strain relationship under uniaxial tension described by Greener et al (1972), unless otherwise specified.

The nominal (engineering) stress, σ , is given by the applied load P divided by the original cross-sectional area A_o , namely,

$$\sigma = \frac{P}{A_o}$$

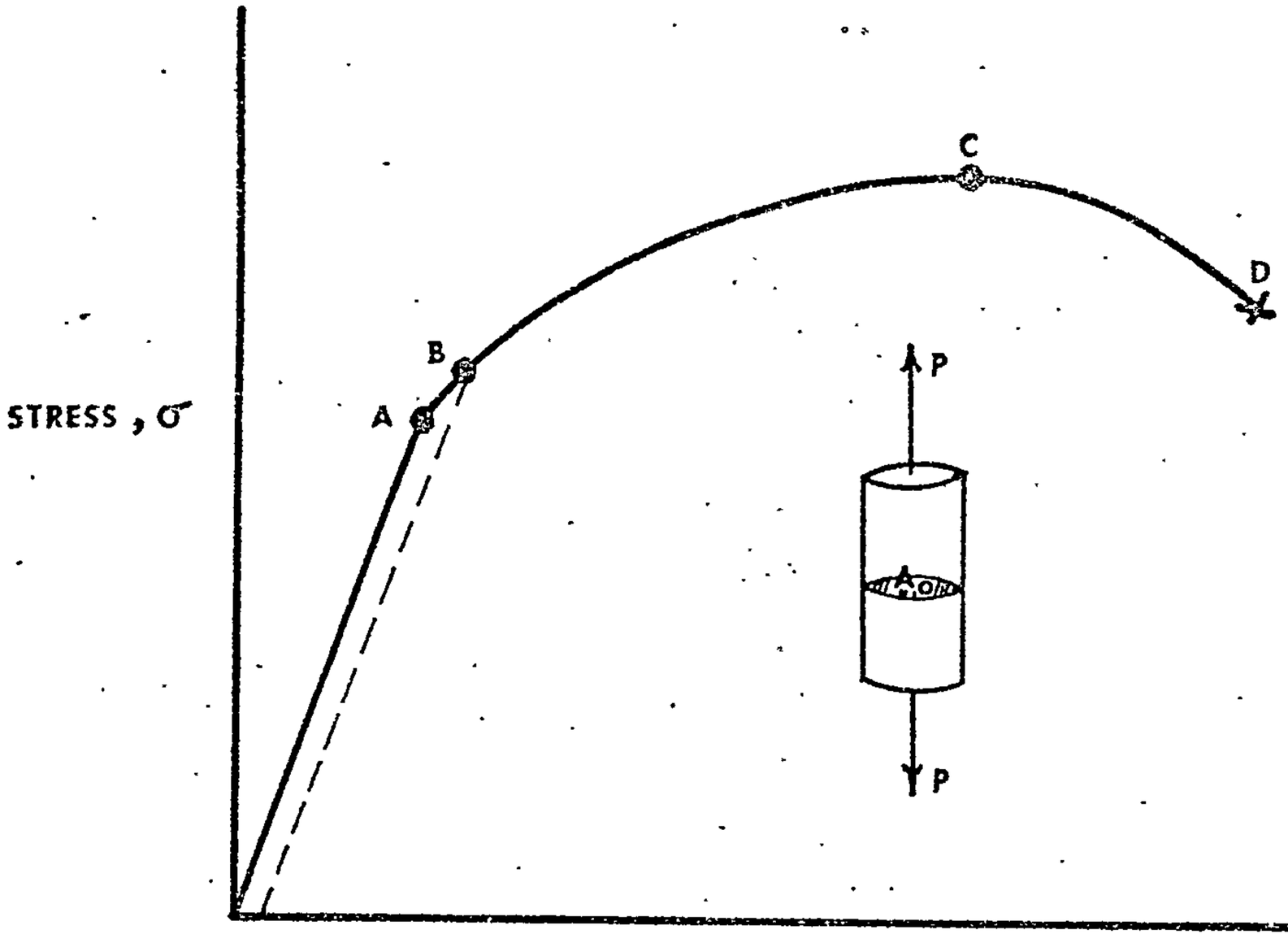


Fig. 22. Stress-strain curve for specimen under tensile test. (Greener et al).

The nominal (engineering) strain e is measured by the change in specimen length, δ , under loading divided by the original length, l_o, or:-

$$e = \frac{\delta}{l_o} \quad ..$$

The load vs. length curve (Fig. 21) can therefore be changed into a stress-strain curve shown in Fig. 22 by performing the appropriate mathematical operations.

Mechanical behaviour is usually discussed with reference to the stress-strain curve rather than the load-extension curve, as specimen geometry is incorporated into the expression for stress and strain by means of the above-mentioned equations.

During the initial stages of loading, the material deforms in direct proportion to the stress as seen by the straight line relation between stress and strain. The equation of this line becomes:-

$$\sigma = Ee$$

The parameter E is called the Young's modulus or the modulus of elasticity and this equation is known as Hooke's law.

The maximum stress for which this straight line relationship holds is called the proportional limit, σ_{pl}. A material loaded to stress levels less than or equal to σ_{pl} will completely return to its original shape upon removal of the stress. Thus up to the proportional limit it behaves elastically. Therefore σ_{pl} is also commonly referred to as the elastic limit.

The International Congress for Testing Materials at Brussels (1906) defined the proportional limit as the tensile stress at which the permanent set is 0.001 per cent (Timoshenko, 1968).

Some authors make a minor differentiation between elastic limit and proportional limit. The elastic limit is determined by cyclically loading in ever increasing increments and then removing the load. The elastic limit is defined as the greatest stress the material can withstand without showing any permanent set when the load is removed. The proportional limit is measured as the deviation from linearity in the stress-strain curve of a continuously increasingly loaded specimen. Depending upon the sensitivity of the strain measuring devices employed in these two different tests, there is sometimes a small difference between proportional limit and elastic limit with the latter usually being slightly larger.

Loading beyond the proportional limit produces deformation. The

region represented by stresses beyond σ_{pl} is called the plastic region. Loading to any stress in the plastic region produces both permanent and elastic deformation. As the stress is removed the elastic portion recovers along a line parallel to the original elastic slope (Figs. 21 and 22 dashed lines), leaving only the permanent set.

For engineering purposes, the limit of usable elastic behaviour is described by the yield point, yield strength or proof stress. The yield point is defined as the stress at which a small amount of permanent set (plastic elongation) usually 0.1 per cent or 0.2 per cent is produced. This yield point is then referred to as the 0.1 per cent or 0.2 per cent yield point or proof stress.

The yield point defined in this manner does not represent a definite physical characteristic of the material but depends on the arbitrarily chosen permanent set (Timoshenko, 1968).

As loading is continued into the plastic region, the stress increases with increasing strain but not usually in a linear fashion. This is known as work hardening or strain hardening. If the specimen is not brittle, the specimen will continue to elongate upon continued application of the load with accompanying work hardening. In time, with further additional load, a maximum load is reached, at which

point some section of the specimen is unable to strain harden rapidly enough to continue supporting the load. This load divided by the original cross-sectional area (Fig. 22 point C) gives the ultimate strength or tensile strength of the material. After the maximum load is exceeded, some localized section of the specimen will elongate rapidly, decreasing in cross-sectional area by necking down until fracture occurs.

The ability of metals to undergo plastic deformation once the proportional limit has been exceeded is known as ductility. A material which has good ductility displays appreciable elongation before fracture, while a material without appreciable elongation lacks ductility and is said to be brittle.

Toughness of a material is the ability to absorb energy. It is represented by the area under the stress-strain curve up to the point of fracture. This characteristic property of the material depends on both the strength and ductility of the material.

The modulus of resilience is the energy stored in a unit cube of a material in the elastic region. It is represented by the area under the stress-strain curve in the elastic region up to the proportional limit.

$$\text{Modulus of Resilience} = \tfrac{1}{2}\left(\sigma_{PL} \times \delta_{PL}\right) = \frac{\sigma_{PL}^2}{2E}$$

Load-deformation curves for specimen in compression, shear and torsion can similarly be transformed into stress-strain curves with the appropriate formulae. The stress-strain diagrams for these stresses are similar to that for tension.

For shear stress within the elastic region, the shear modulus, G, replaces the Young's modulus.

Experimental determinations for yield point stress, yield point strain, ultimate strength and total elongation show that they are dependent upon the rate of strain. In general, these quantities increase as the rate of strain increases. (Timoshenko, 1968).

Timoshenko (1968) cited the work of Ewing (1914) on tensile test and stated that "the time which elapses between unloading and reloading is of great influence on the stress-strain curve during reloading. If reloading begins immediately after unloading, accurate measurements show that there are deviations from the straight-line law at very low stress, and the proportional limit

is greatly lowered. But if a considerable interval of time elapses between unloading and reloading, the material recovers its elastic properties completely..... Experiments also show that if the material is subjected to mild heat treatment after unloading, say in a bath of 100°C, the recovery of elastic properties occurs in a much shorter interval of time".

In the present evaluation, the springs will be loaded and unloaded in a upright position threaded on a 0.016 inch diameter wire which will act as a mandrel. The load versus percentage deflection curve from such load application represents a composite response of the spring material to a combination of different types of stresses. It simulates the actual response of a spring under clinical stress.

According to Thurow (1972), in these curves the region of the elastic limit and plastic deformation is different from that of a stress-strain curve derived from a uniaxial tensile test of the spring wire. In tensile test, the entire specimen is subjected to the same strain so that the entire piece reaches the elastic limit at the same time.

In a compressed helical spring, the outer fibres of the wire are strained more than the inner fibres, so that these areas reach the

elastic limit first. The straight line portion of the load-deflection curve ends where the outer fibres begin to deform plastically, even though the part near the centre is still behaving elastically and following a straight line response. Hence the transition into the upper curved portion is much more subtle than it would be in the tensile test where the entire piece reaches the elastic limit at the same time.

The predominant torsional stress in a compressed coil spring first affects the largest part of the round wire farthest away from the centre of the wire. The outer fourth of the diameter includes 44 per cent of the entire wire material. The next fourth includes another 31 per cent, so that when half of the diameter of the wire is over-stressed in torsion, it will affect 75 per cent of the material. Hence there is a rapid depletion of reserve strength in torsion. Each successive increment of overload affects a smaller volume of material and therefore has a progressively larger effect.(Thurow, 1972). However, the range of deflection in a coil spring is limited by the distance between the individual coils as previously discussed. (P. 62).

From the above discussion, the straight line portion of a load-deflection curve thus represents an apparent proportionality between load and deflection of the spring. The limit of this

straight line portion of the curve may or may not represent the actual proportional limit of the spring material.

When the intercoil space is sufficiently large as to allow the deflection of the spring to overstress the spring material, the limit of the straight line portion of the load-deflection curve will then indicate the actual proportional limit of the spring material. The region represented by loads beyond the proportional limit is the plastic region. Deflections in this region includes both elastic and permanent deformations.

When the distance between the individual coils is small, the spring will be compressed solid with loads less than that required to stress the spring material to its actual proportional limit. The limit of the straight line portion of the load-deflection curve would then indicate the limit of an apparent proportionality between load and deflection. This limit is henceforth referred to as the <u>apparent proportional limit</u> of the spring in axial loading.

The subtle transition of the straight line portion of a load-deflection curve of a compressed coil spring into the upper curved part is difficult to estimate. In the present investigation, a method similar to that used to locate proof stress will be employed. The straight line portion of the load-deflection curve is first

determined by the line of best fit for a series of load and corresponding deflection of the spring. An offset line drawn parallel to and at either +3 or -3 standard deviation of the load from this regression line will intercept the upper curved portion of the load-deflection curve at a fairly definite point (see Appendix II). The load obtained at this point is taken as the apparent proportional limit of the spring. In clinical terms this load is the maximum load a spring can withstand with complete return to its original shape upon removal of the stress.

ORIGINAL WORK

AIM

The purpose of this study is:-

(a) To determine experimentally the load-deflection rates of some of the preformed orthodontic helical compression springs which theoretically have lowermost load-deflection rates.

(b) To determine the maximum elastic force limit (apparent proportional limit) of these springs.

(c) To develop a method of estimating the maximum free (uncompressed) lengths of these springs which when compressed into a given space will exert forces at their respective apparent proportional limits.

(d) To determine the effects of compressing these springs solid.

(e) To determine the consistency of the stated dimensions of the spring.

MATERIALS

HELICAL COMPRESSION SPRINGS TESTED

Preformed helical compression springs from 3 types of stainless steel wires and one type of cobalt-chromium alloy by three manufacturers were tested. The types of springs tested are listed in the following table.

Manufacturer	WIRE	SIZE				Coils of each type of spring tested
		Diameter of wire		Internal diameter of coil		
		mm	Inch	mm	Inch	
Unitek	Hi-T	.152	.006	.914	.036	1
Rocky Mountain	Eigiloy	.178	.007	.914	.036	1
Rocky Mountain	Elgiloy	.142	.0056	.559	.022	1
Rocky Mountain	Elgiloy	.178	.007	.762	.030	3
Dentaurum	Remanit	.178	.007	.762	.030	3
Rocky Mountain	Elgiloy	.203	.008	.914	.036	3
Unitek	Hi-T	.203	.008	.914	.036	3
Unitek	Permachrome Standard	.203	.008	.914	.036	3

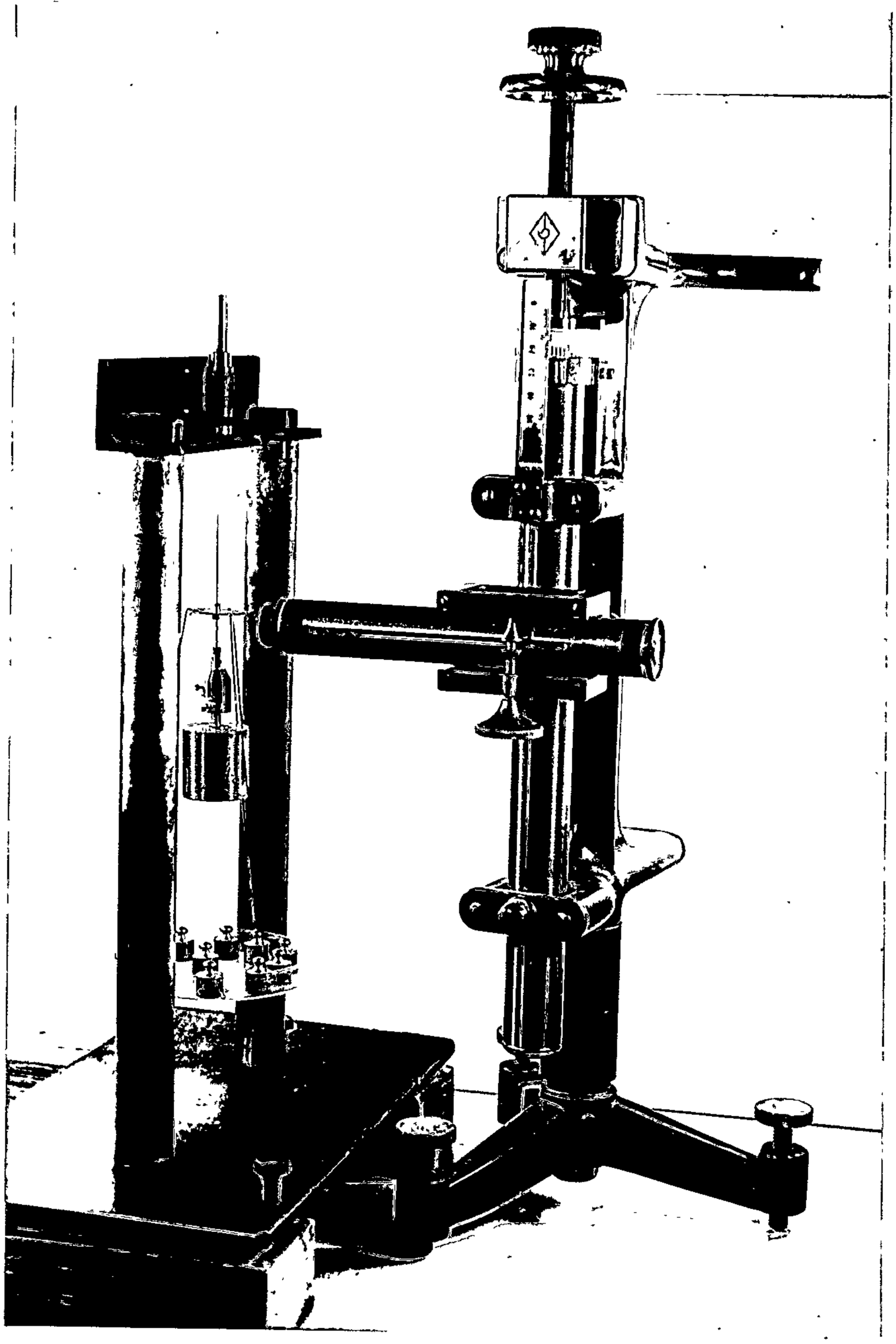


Fig. 23. Apparatus for compression test.

The springs were purchased from the agents of their manufacturers in Australia. They were tested in the as-received condition.

COMPRESSION TEST EQUIPMENT

The testing apparatus was constructed at the Dental Materials Laboratory, Faculty of Dentistry, University of Sydney (Fig. 23). It consisted of a scale-pan suspension arrangement hanging from a rigid framework. The framework had an L-shaped horizontal steel bar supported by two solid aluminium columns from a steel plate platform with three levelling screws.

The scale-pan suspension arrangement consisted of three basic parts:

(1) scale pan - collar unit

(2) central core assembly

(3) stops

(1) The scale pan was a 90 mm diameter Petri dish, 13.2 grams in weight. It was suspended freely from a brass collar by three straight lengths of Tru Chrome 0.021 in. x 0.025 in. (0.53 mm x 0.64 mm) wire (E-90), each 17 cm in length. The ends of the wires were bent into U-shaped loops which engaged the scale pan and collar at three holes, evenly spaced around the circumference of the scale pan and the collar. The centre of the collar had a 0.022 in. (0.56 mm) diameter hole. (Fig. 24).

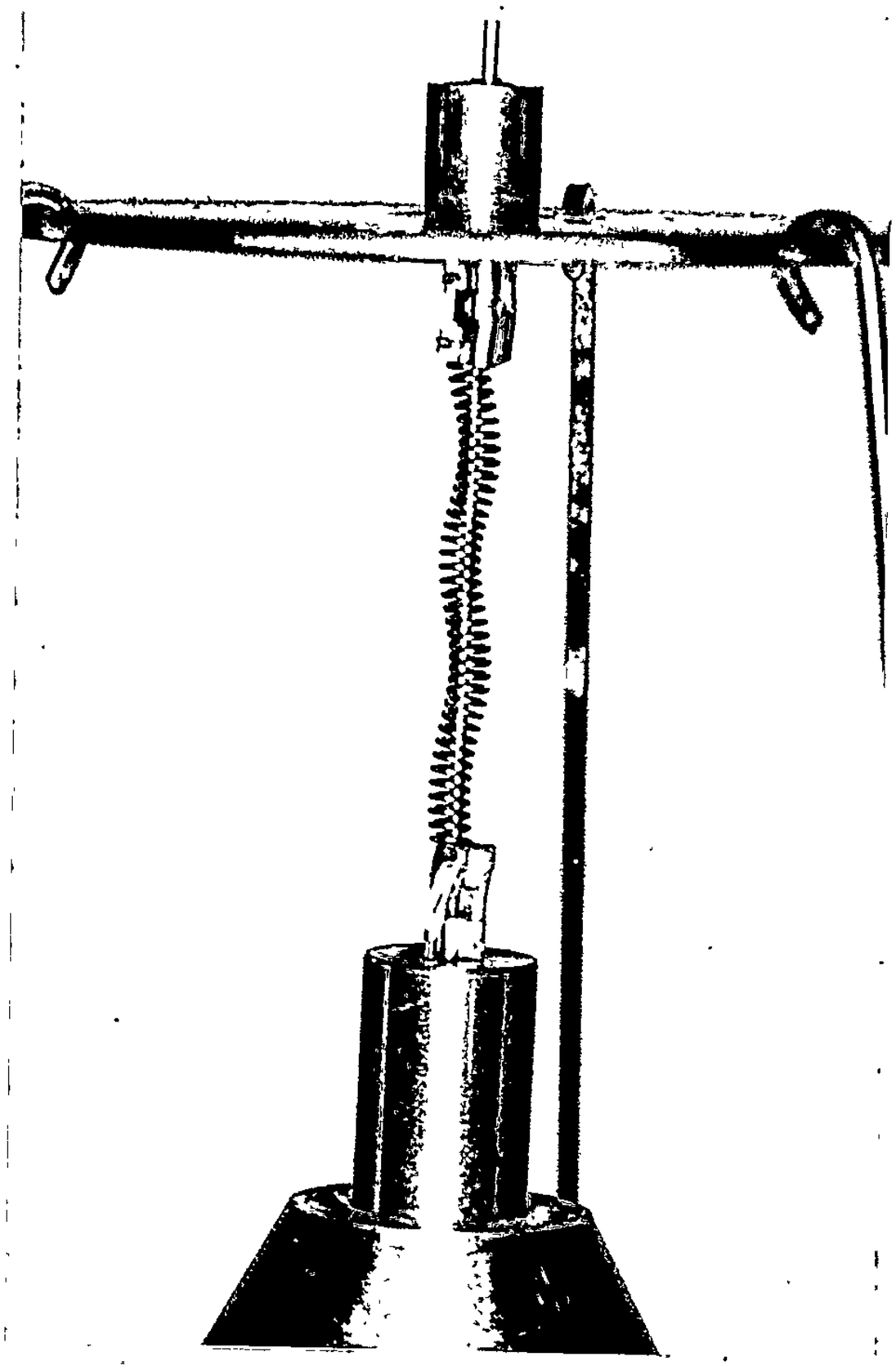


Fig. 24. Illustrating collar of the scale pan-collar unit and stops.

by the Pierre Roch Micrometer. Four readings D 1, D 2, D 3 and D 4, (Fig. 26) were taken at about the middle portion of the sample spring. The average of these 4 readings was taken as the external diameter of the coils for the sample.

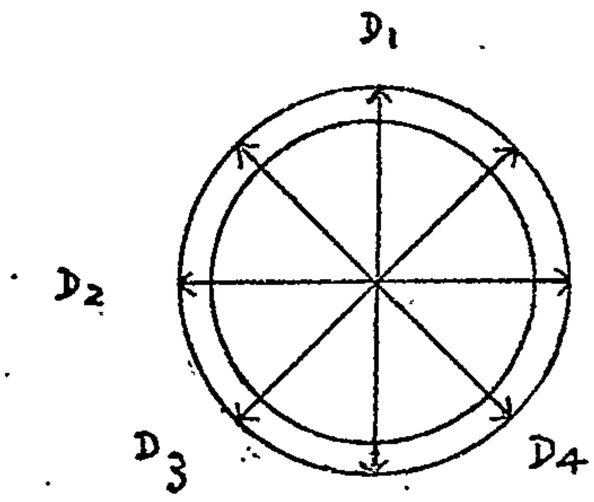


Fig. 26. End on view of coil.

Stage (2) Measurement of the Free (Uncompressed) Length and the Total Number of Coils in the Sample Spring.

The rigid framework of the compression test apparatus and the Cambridge cathetometer were checked for level with a circular bubble level. Any required adjustments were made with the levelling screws.

A stop and the sample spring were then threaded onto the suspension wire of the central core assembly. The uppen end of the suspension wire was then threaded through the collar at the L-shaped horizontal steel bar of the rigid framework, and was securely gripped by the

Cutting the Sample to a Standardized Length

A piece of 0.559 mm "Red Elgiloy" wire was first cut down to 2 centimeters length. This standard 2 cm length wire served as a gauge for determining the length of the test samples.

The first 2 centimeters of each section of each type of coil springs was discarded. The standard 2 cm. gauge length "Red Elgiloy" wire was then threaded into the core of the spring. With this visual aid the spring was cut 4 to 5 coils longer than the standard gauge length. It was trimmed down by cutting half a coil at a time with the tip of the cutting instrument (Hard wire cutter R.M. i-263) in line with the long axis of the spring. The objectives in this procedure was (a) to maintain the number of coils in the test sample spring to as near to a complete turn as possible, (b) to trim it down to as near to the standard gauge length as possible and (c) to avoid any possible widening of the intercoil distance by the blades of the cutter.

Collection of Data from the Sample Spring

This procedure was divided into Five stages.

Stage (1) Measurement of the External Coil Diameter of the Sample Spring

The external diameter of the coils in the sample spring was measured

METHOD

(I) PREPARATION OF TEST SPECIMEN

System of Identification

The coils of each type of spring were first labelled. When there was only one coil for that particular type of spring, that coil was labelled as Coil A. When there were three coils for a particular type of spring, the three coils were labelled as coil A, coil B and coil C.

Sampling

The spring was removed from its plastic container. It was cut into 4 sections of equal length. The test sample was taken from about the second 2 centimeter length of each section. Each section of Rocky Mountain helical compression springs was 6 inches long, Unitek springs 9 inches long and Dentaurum springs 10 inches long.

Naming of Samples

For ease of reference, each sample tested was named in the following manner:- type of wire, nominal diameter of wire, nominal internal diameter of coil (arbor size), name of the coil (A, B or C) and finally from which section (1, 2, 3 or 4). For example, a test sample spring made of Elgiloy wire 0.008 in. nominal diameter with an internal diameter of coil of 0.036 in. from the third section of coil B would be named as:- Elgiloy .008" - .036", B, 3. Data collected or related to each sample would be presented under its name.

cathetometer drum is 0.01 mm.

EQUIPMENT FOR MEASURING DIAMETERS

A Pierre Roch Micrometer (Rolle Switzerland) reading to 0.002 mm was used to measure the diameter of the wire and coil.

CALCULATING AND PLOTTING MACHINE

Calculations were completed on a Micro Statistician Calculating Machine. Load-deflection regressions were plotted by a Hewlett Packard 9810A Calculator and Plotter.

(2) The central core assembly consisted of a 15 cm straight length 0.016 in. (0.40 mm) "Red Elgiloy" suspension wire (E-216). The lower end of the wire was threaded through a 0.020 in. (0.51 mm) hole, 18 mm in length, at the centre of a brass unit which had provision for the attachment of a detachable 200 gram plumb weight. The wire was secured in this brass unit by a brass screw. (Fig. 25).

The upper end of the 0.016 in. (0.40 mm) suspension wire was threaded through the central hole in the collar of the scale pan-collar unit. It was then suspended through another brass collar at the L-shaped horizontal steel bar of the rigid framework by a 4-tooth pin vice. The pin vice was held by a clamp and turned by a crowbar applied through its body to ensure a secure grip of the vice on the uppermost 28 mm of the suspension wire.

(3) The springs were tested in compression between 2 stops, one at each end of the spring. The stops were made of Junior Siamese anterior brackets 2 mm in length with a 0.022 in. slot (Rocky Mountain A-215). The open face of the slot was closed by welding two layers of 0.004 inch thick band material (Rocky Mountain B-301). The mesio-distal base of the bracket was ground flat with the mesio-distal surfaces of the bracket. (Fig. 24).

A Cambridge cathetometer (magnification x24) was used to measure the deflection after each increment in load. Each scale division on the

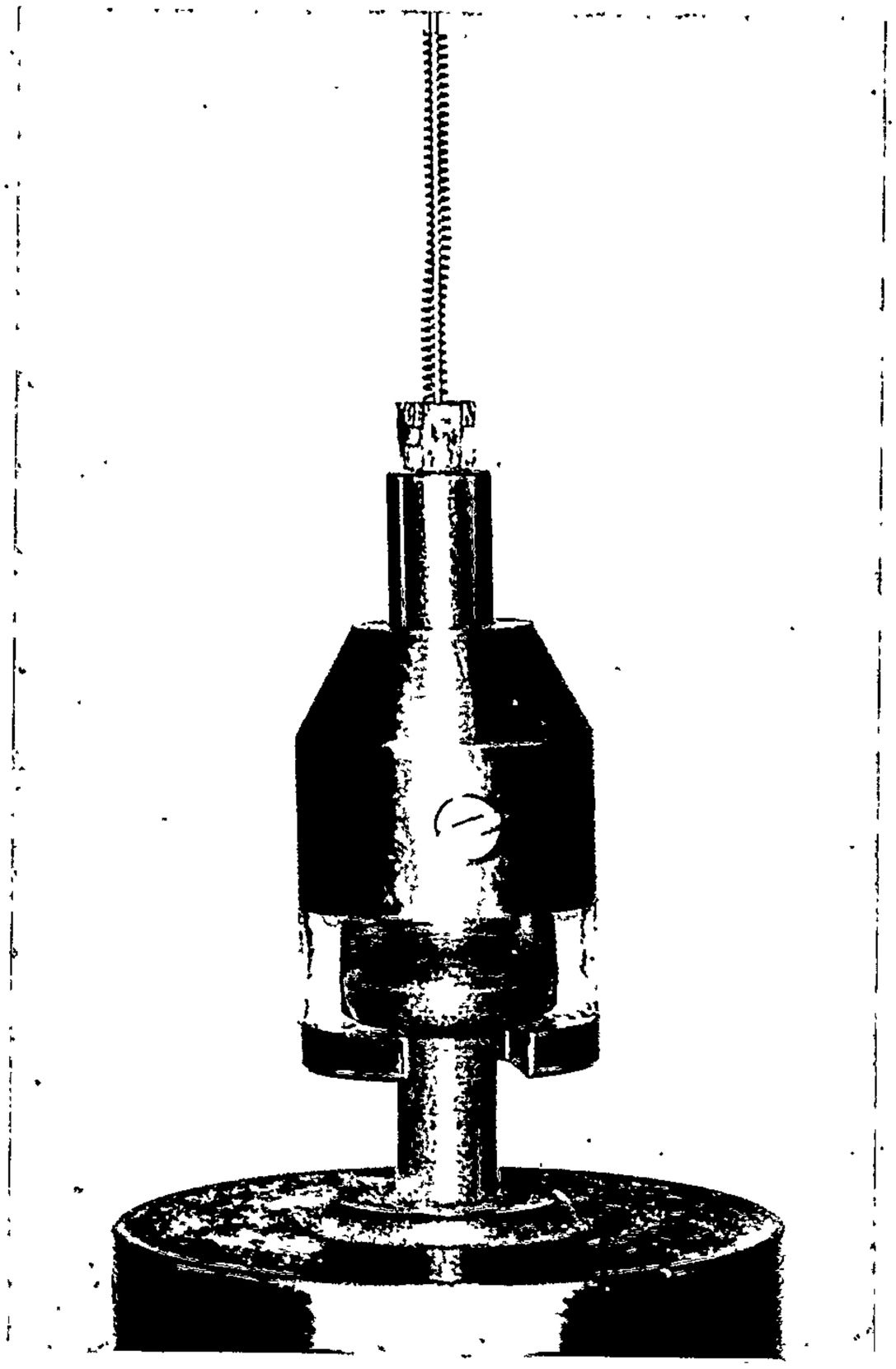


Fig. 25. Illustrating plumb weight attachment.

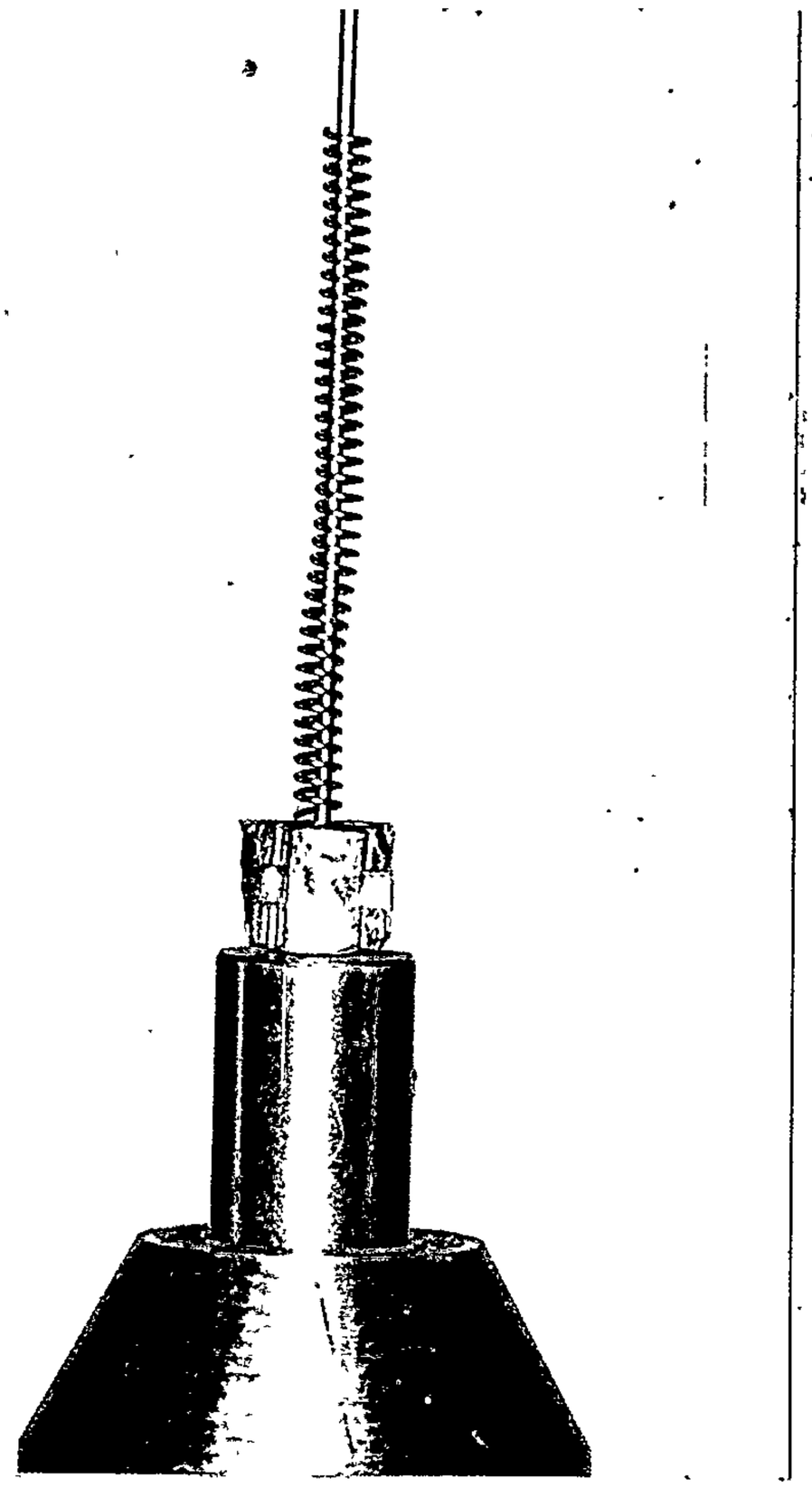


Fig. 27. Illustrating Stage (2) of the collection of data from sample spring.

pin vice. A 200 gram plumb weight was attached to the base of the brass unit of the central core assembly. At this stage the suspension wire was stretched firmly under tension in an up-right position. (Fig. 27).

The upper and lower ends of the sample spring were measured by the Cambridge cathetometer. The difference between the upper and lower readings was the free length of the sample spring.

The total number of coils in the sample spring was also counted.

Stage (3) Measurement of Load and Deflection of the Sample Spring

Step (i) The plumb weight was first removed from the central core assembly. The suspension wire was then released from the pin vice. The second stop and the scale pan-collar unit was threaded onto the suspension wire, (Fig. 28). The suspension wire was again introduced into and gripped by the pin vice after passing through the collar at the L-shaped horizontal steel bar of the rigid framework. The plumb weight was then re-attached to the central core assembly.

Step (ii) The upper and lower ends of the sample spring under the weight of the empty scale pan was measured with the cathetometer. The difference in the free length

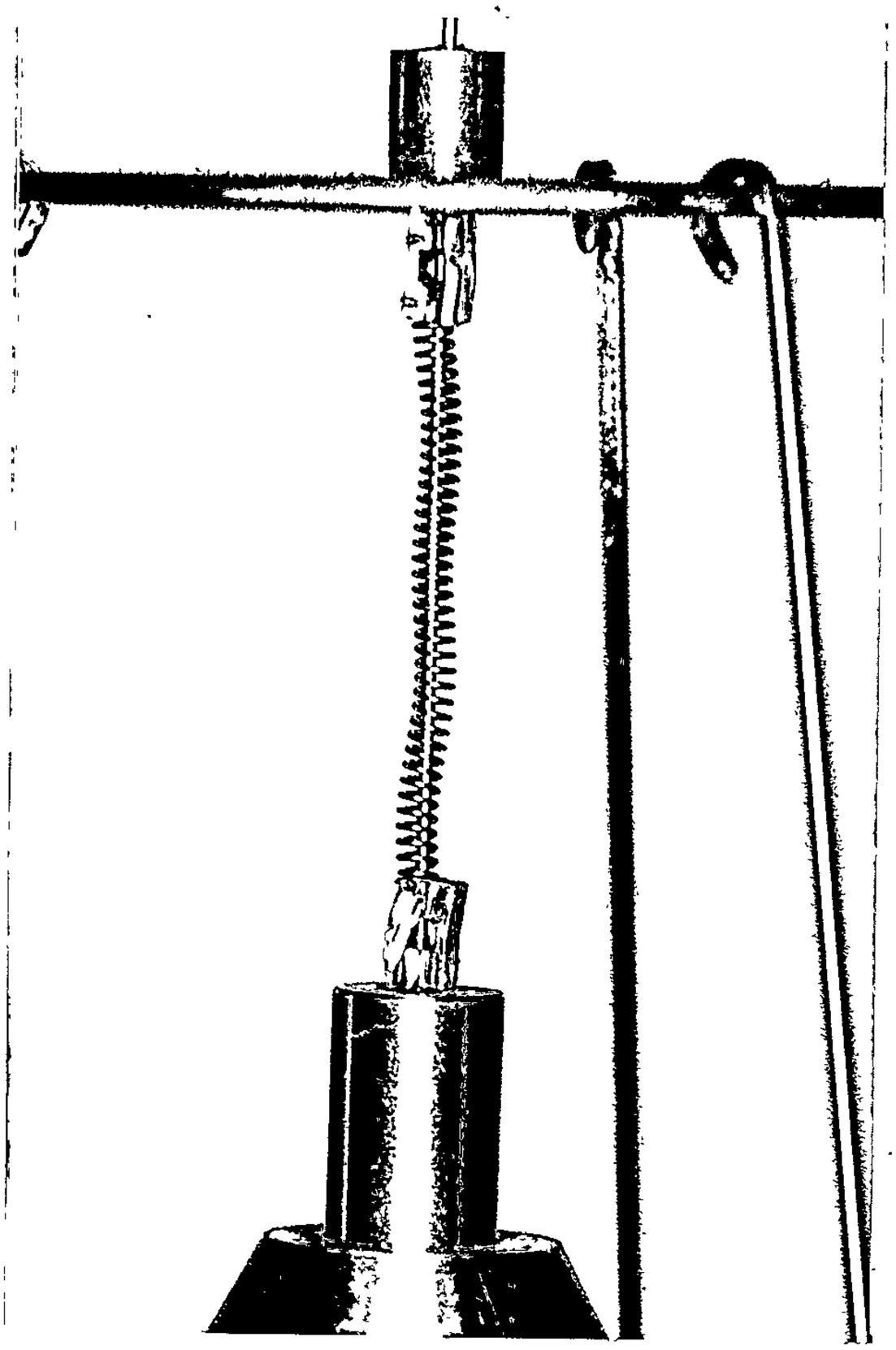


Fig. 28. Illustrating Stage (3) of the collection of data from sample spring.

of the spring and that under the weight of the empty scale pan (13.2 gm) was the deflection of the spring with a starting load of 13.2 grams.

Step (iii) To determine the load-deflection rate, the apparent proportional limit and permanent deformation of the sample spring, 4 consecutive loading and unloading cycles were employed.

First Loading and Unloading Cycle

In the first loading cycle, standard 10 gram weights were added in 10 gram increments to the scale pan up to a maximum of 80 grams. The movement of the upper end of the sample spring, being the deflection, was read and noted. To apply the load, adjust the cathetometer and take a reading, took slightly under 30 seconds per reading. Every effort was made to keep this time constant throughout the four loading and unloading cycles. The spring was then unloaded in 10 gram steps and the corresponding deflections of the spring were recorded. This first loading and unloading cycle furnished the data for calculating the loading and unloading loads to percentage deflection rates of the spring within its elastic limit in the as-received condition.

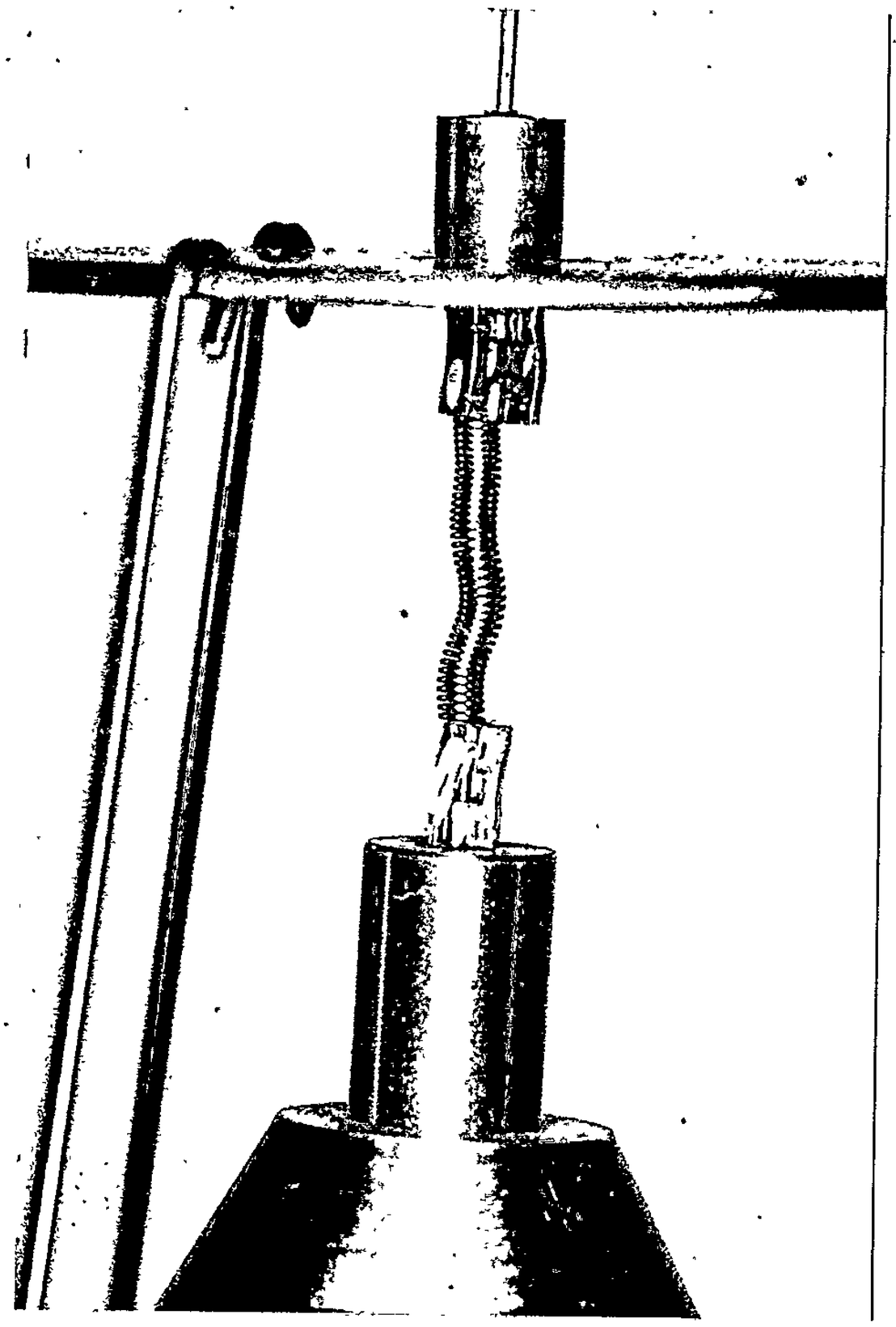


Fig. 29. Appearance of spring when compressed solid.

Second Loading and Unloading Cycle

In this loading cycle the spring was compressed solid (Fig. 29). The objectives in this loading cycle was to determine (a) the load-to-percentage-deflection rate, (b) the apparent proportional limit of the spring and (c) any permanent deformation of the spring if compressed solid.

Standard 10 gram weights were again added in 10 gram intervals. The corresponding deflection of the spring were recorded. (During the early portion of the experiment, an attempt was made to define the apparent proportional limit of the spring more clearly by increments of 3 gram loads when observed deflections suggested large deviation from linearity with load increments. This attempt was found to be unsuccessful and was left out in the latter part of the experiment.) When the spring had been compressed to nearly solid, 15 grams, 20 grams and 50 grams increments were used to induce appreciable deflections in different types of spring. Heavier increments in load were used in springs with larger wire diameters during the final stages of compressing the springs solid.

The spring was unloaded, after solid compression, in the same load steps as used in its loading cycle. The corresponding deflections of the spring with each change in load were recorded. Usually each reading took approximately 30 seconds. At certain loads different for each type of spring one minute or more was required for the spring to achieve an equilibrium position before the deflective reading could be recorded.

The values obtained for load and deflection were plotted. The load-to-percentage-deflection rate on loading and permanent deformation due to cold settage of the spring were calculated. Where indicated the apparent proportional limit of the spring was determined from its load to percentage deflection curve.

Third Loading and Unloading Cycle

The procedure of loading and unloading for this cycle was the same as the first cycle of loading and unloading. The objective of this cycle was to determine the unloading load to percentage deflection rate of the spring from a load within its elastic limit after it had been compressed solid once.

Fourth Loading and Unloading Cycle

In this cycle of loading and unloading the procedure was the same as the second cycle of loading and unloading. The objectives were to determine (a) the loading load to percentage deflection rate, (b) the apparent proportional limit and (c) any further permanent deformation of the spring although it had been compressed solid once.

Stage (4) Measurement of the Lower End of the Sample Spring

In order to determine whether the whole scale-pan suspension arrangement had slipped in the pin vice, the lower end of the spring was measured with the cathetometer. This reading was checked with that obtained in Step (ii) of Stage (3).

Stage (5) Measurement of the Diameter of the Wire

The plumb weight was detached, the suspension wire released from the pin vice, the scale pan-collar unit and the upper bracket stop removed before the sample spring was removed from the suspension wire. The spring was cut into two halves.

A coil from the cut edge was cut out so that the diameter of the wire could be measured with the Pierre Roch Micrometer. It was measured with the grip of the micrometer biting minimally on the coil at 4 points along the circumference of the coil.

By this time all the necessary measurements on the sample spring had been completely recorded. It took roughly one and a half to two hours for each sample spring. With the exception of Elgiloy .008" - .036", four sample springs from one coil of each type of spring were tested in this manner. As for Elgiloy .008" - .036", four sample springs from each of the three coils were tested in this way. The four sample springs from the remaining 2 coils of Elgiloy .007" - .030", Remanit .007" - .030", Unitek Hi-T .008" - .036" and Unitek Permachrome Standard .008" - .036" were not fully tested. Only those factors that determine the amount of deflection in any helical spring were measured. Those factors that were measured were (a) the external diameter of the coil, (b) the free length of the sample spring, (c) the total number of coils in the sample spring and (d) the diameter of the wire of the spring.

EXPERIMENTAL MEASUREMENT ERRORS OF DEFLECTION OF THE SPRING WITH THE CATHETOMETER

On two occasions, 1 week apart, the lower end of a sample spring was measured 10 times. After a reading was taken, the vertical-traverse microscope was moved in either direction for a varying distance before it was again focused on the end of the spring. The difference of the highest and lowest measured valve of the fixed landmark was 0.03 mm.

RESULTS AND DISCUSSION

A. CONSISTENCY OF DIAMETER OF WIRE AND COIL

The observed diameter of wire, outside coil diameter, number of turns in the measured uncompressed length (free length) of each of the tested sample springs are shown in Tables 6 to 13 in Appendix III. Pitch height was obtained by dividing its free length by the number of coils present.

Using the whole coil of each type of spring as a unit for assessment, Table 14 shows the percentage difference of the measured diameter from the stated diameter of the wire and coil of each type of spring.

A minus value indicates that the measured diameter is smaller than that stated. The measured diameter of the wire and coil was the averaged value of 16 readings.

Discussion

Manufacturing Tolerances for Wire Diameter

The manufacturing tolerance for the wire diameter of Elgiloy wire stated by its manufacturer is -0.0002 inch. This implies that for

TABLE 14 CONSISTENCY OF DIAMETER OF WIRE AND COIL

Column	1	2	3		4	5	6		7	8	9
Type of Spring Manufacturer	Material	Sample	Stated Wire diameter in	mm	Actural Wire Diameter mm	Percentage difference	Stated coil internal diameter in	mm	Actual internal coil diameter mm	Percentage difference	Actual deflection greater than nominal deflection by %
Unitek	Hi-T	Coil A	.006	.152	.1509	-0.72	.036	.914	1.0223	11.85	38.4
Rocky Mountain	Elgiloy	Coil A	.007	.178	.1735	-2.53	.036	.914	1.0045	9.90	39.8
Rocky Mountain	Elgiloy	Coil A	.0056	.142	.1382	-2.68	.022	.559	0.6082	8.80	37.1
Rocky Mountain	Elgiloy		.007	.178			.030	.762			
		Coil A			.1761	-1.07			0.8017	5.21	19.9
		Coil B			.176	-1.12			0.7831	2.77	12.8
		Coil C			.176	-1.12			0.7785	2.17	11.0
Dentaurum	Remanit		.007	.178			.030	.762			
		Coil A			.174	-2.25			0.787	3.28	18.8
		Coil B			.1748	-1.80			0.7861	3.16	16.7
		Coil C			.1746	-1.91			0.7878	3.39	17.8

TABLE 14 (cont'd)

Column	1	2	3		4	5	6		7	8	9
Type of Spring			Stated Wire diameter		Actual Wire Diameter	Percentage difference	Stated internal coil diameter		Actual internal coil diameter	Percentage difference	Actual deflection greater than nominal deflection by %
Manufacturer	Material	Sample	in	mm	mm		in	mm	mm		
Unitek	Hi-T		.008	.203			.036	.914			
		Coil A			.2065	1.72			1.0122	10.74	25.3
		Coil B			.2064	1.67			1.0093	10.43	24.6
		Coil C			.2068	1.87			1.0121	10.73	24.7
Unitek	Perma-chrome Standard		.008	.203			.036	.914			
		Coil A			.2063	1.63			.9863	7.91	17.2
		Coil B			.2064	1.67			.9887	8.17	17.8
		Coil C			.2062	1.58			.9922	8.56	19.4
Rocky Mountain	Elgiloy		.008	.203			.036	.914			
		Coil A			.1981	-2.41			0.9831	7.56	32.3
		Coil B			.1981	-2.41			0.9340	2.19	16.2
		Coil C			.198	-2.46			0.9265	1.37	14.0

0.0056 inch wire it could be under normal size by -3.57 per cent; for 0.007 inch wire by -2.85 per cent and 0.008 inch wire by -2.5 per cent. As for the Unitek wires, the stated tolerance is ±0.00025 inch. Thus for 0.006 and 0.008 inch wires, the actual sizes may vary by ±4.17 and ± 3.13 per cent respectively. Dentaurum did not give any value for their manufacturing tolerance on wire diameter.

The measured diameters of the wires of the various types of Rocky Mountain and Unitek springs are within the tolerance limit stated by their manufacturers (Table 14). They are also within the tolerance limit stated by Australian Standard T32 (Ware 1967) and by B.S. 1408 (1947). It also indicates that the limits of variation in wire size for any stated size can be predicted from the stated tolerance limits.

Manufacturing Tolerance for Coil Diameter

The tolerances on the coil inside diameter stated by Unitek for their coil springs are ± 0.001 inch for sizes 0.020 to 0.22 inch and ± 0.002 inch for sizes 0.030 to 0.039 inch. This implies that the inside coil diameter of springs with stated diameter of 0.036 inch may actually be larger by 5.56 per cent. The tolerance on the coil inside diameter stated by Dentaurum is ± 0.02 mm.

For Dentaurum coil springs with stated coil inside diameter of 0.030 inch or 0.762 millimeter the actual diameter of the coil may be larger by 2.62 per cent. Rocky Mountain did not give any value for their manufacturing tolerance for coil diameter.

The manufacturing tolerances for coil diameter stated by Unitek and Dentaurum are within the tolerance limit specified by B.S.1408 (1947), the S.A.E. limit, and German Standard DIN 2095 (1973). (See Section on Manufacturing Tolerances).

Results shown in Table 14 indicates that the inside coil diameters of the tested Dentaurum springs are slightly larger than their stated tolerance limit. For Unitek springs the difference in the measured coil inside diameter and that stated is larger than their tolerance limit. Thus the limits of variation in coil diameter for any stated size cannot be predicted from the stated tolerance limits. However, variations in the measured coil inside diameter between the 3 coils of Remanit .007" - .030", Unitek Hi-T .008" - .036" and Unitek Permachrome Standard .008" - .036" are small. For the 3 coils of Elgiloy .007" - .030", the between coil difference in coil inside diameter is slightly larger. The largest variation in coil inside diameter occurs in the 3 coils of Elgiloy .008" - .036".