AN APPLICATION OF THE

INVERSE SCATTERING TRANSFORM

TO SOME NONLINEAR SINGULAR

INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT

The quest to model wave propagation in various physical systems has produced a large set of diverse nonlinear equations. Nonlinear singular integro-differential equations rank amongst the intricate nonlinear wave equations available to study the classical problem of wave propagation in physical systems. Integro-differential equations are characterized by the simultaneous presence of integration and differentiation in a single equation.

Substantial interest exists in nonlinear wave equations that are amenable to the Inverse Scattering Transform (IST). The IST is an adroit mathematical technique that delivers analytical solutions of a certain type of nonlinear equation: soliton equation. Initial value problems of numerous physically significant nonlinear equations have now been solved through elegant and novel implementations of the IST.

The prototype nonlinear singular integro-differential equation receptive to the IST is the Intermediate Long Wave (ILW) equation, which models one-dimensional weakly nonlinear internal wave propagation in a density stratified fluid of finite total depth. In the deep water limit the ILW equation bifurcates into a physically significant nonlinear singular

water limit of the ILW equation is the famous Korteweg-de Vries (KdV) equation. Both the KdV and BO equations have been solved by dissimilar implementations of the IST.

The Modified Korteweg-de Vries (MKdV) equation is a nonlinear partial differential equation, which was significant in the historical development of the IST. Solutions of the MKdV equation are mapped by an explicit nonlinear transformation

Miura transformation manifested the intimate connection between solutions of the KdV

equation.

versions of the ILW and BO equations. Solutions of each modified nonlinear singular integro-differential equation should be mapped by an analogue of the original Miura

cases of the ILW equation, the modified version of the ILW equation should reduce to the MKdV equation in the shallow water limit and to the modified version of the BO equation in the deep water limit.

The Modified Intermediate Long Wave (MILW) and Modified Benjamin-Ono (MBO) equations are the two nonlinear singular integro-differential equations that display all the required attributes. Several researchers have shown that the MILW and MBO equations exhibit the signature characteristic of soliton equations. Despite the significance of the MILW and MBO equations to soliton theory, and the possible physical applications of the MILW and MBO equations, the initial value problems for these equations have not been solved. In this thesis we use the IST to solve the initial value problems for the MILW and MBO equations on the real-line. The only restrictions that we place on the initial values for the MILW and MBO equations are that they be real-valued, sufficiently smooth and decay to zero as the absolute value of the spatial variable approaches large values.

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