Bifurcation problems in chaotically stirred reaction-diffusion systems

Shakti Narayana Menon

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School of Mathematics and Statistics
University of Sydney

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Abstract

A detailed theoretical and numerical investigation of the behaviour of reactive systems under the influence of chaotic stirring is presented. These systems exhibit stationary solutions arising from the balance between chaotic advection and diffusion. Excessive stirring of such systems results in the termination of the reaction via a saddle-node bifurcation. The solution behaviour of these systems is analytically described using a recently developed nonperturbative, non-asymptotic variational method. This method involves fitting appropriate parameterised test functions to the solution, and also allows us to describe the bifurcations of these systems. This method is tested against numerical results obtained using a reduced one-dimensional reaction-advection-diffusion model. Four one- and two-component reactive systems with multiple homogeneous steady-states are analysed, namely autocatalytic, bistable, excitable and combustion systems. In addition to the generic stirring-induced saddle-node bifurcation, a rich and complex bifurcation scenario is observed in the excitable system. This includes a previously unreported region of bistability characterised by a hysteresis loop, a supercritical Hopf bifurcation and a saddle-node bifurcation arising from propagation failure. Results obtained with the nonperturbative method provide a good description of the bifurcations and solution behaviour in the various regimes of these chaotically stirred reaction-diffusion systems.

Declaration of originality

The work presented in this thesis is original except where stated otherwise. No part of this thesis has been submitted for the award of any other degree or diploma at this or any other university.

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Shakti Narayana Menon
The University of Sydney

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