6. Conclusion

I have to run so far to find a clear spot.

Don Van Vliet, Clear Spot, Reprise Records, Cat # K 54007, 1972.

6.1 Final Comments

Four new algorithms form the principal results of this thesis. Four techniques for the demodulation of one, two, and three-dimensional patterns have been derived and investigated. The foremost objective of this thesis is to show that not only do alternative methods of demodulation exist, but also that such methods are often preferable to the existing multidimensional methods. A secondary objective is to uncover some of the formerly unacknowledged connections between algorithms both new and old. There has been some success in meeting both objectives.

6.1.1 Chapter 2

In one dimension, the Hilbert transform and the analytic signal are the vital concepts in a unified description of modulation and demodulation. In Chapter 2, a discrete algorithm is developed for envelope detection. This five sample adaptive (FSA) algorithm is derived from a conventional phase-shifting algorithm. The formally discrete derivation revealed the undersampling property of the algorithm. This property is overlooked in the conventional work based on the continuous energy operator. The FSA can be considered to be a self-calibrating phase-shifting algorithm. Self-calibrating phase-shifting algorithms are revisited in Chapter 5. Chapter 2 shows quite clearly that accuracy and efficiency can be combined together in one algorithm even though they are usually considered mutually exclusive properties. The FSA compares very favourably with three other conventional algorithms. Subsequent papers by other researchers have assumed that the FSA is fundamentally limited to one twentieth of a sample accuracy. This limit is in fact only valid for a detuning of 50%, which is unrealistically large. In many cases the mean wavelength of the white light is known with high accuracy. If the detuning is reduced to a more realistic 1%, then the FSA has an inherent accuracy of the order one ten thousandth of a sample.

Although numerous algorithms for white light interferometry have been proposed over the last decade, a comprehensive comparison of algorithms and their performance has not yet been published, so the field is still open.

6.1.2 Chapter 3

The direct phase retrieval method gives good results with the examples tried, even with significant noise levels (unlike many other phase retrieval methods). At present the method only goes as far as deriving (what Papoulis calls) the spectral correlation function. The next step should be to generate the complex pupil function from this information and then calculate the corresponding complex field. The intensity predicted can then be compared with the original intensity. Any discrepancy may be ascribed to errors in the underlying model (approximations, resampling etc) and components in the original intensity which do not satisfy the Helmholtz equation.

Currently there is much interest in the extensions of the work to 3-D. Unraveling the connection between the intensity, the optical transfer function, the high angle Wigner distribution function, and the high-angle ambiguity function would seem to be a very fruitful area of research.

The direct phase retrieval problem that is solved may seem, on first reading, somewhat unconnected to the chapters before and after. The problem is to recover the underlying phase of the field given the intensity. If, however, the problem were to find the phase given the real part of the field, then a more conventional Hilbert half-plane approach could be used. The Hilbert method would only work if the symmetry were known beforehand. If the symmetry is unknown then an isotropic method is called for, but until recently there has been no direct (i.e. non-iterative) 2-D demodulation method that is also isotropic. An isotropic method has been proposed which performs remarkably well in simulations, but its derivation in not rigorous; the asymptotic method of stationary phase provides rigour in the following chapter.

6.1.3 Chapter 4

Chapter 4 formalises the isotropic approach to quadrature transforms in 2-D. A detailed literature review shows that the proposed transform is a special case of the Riesz transform, which is known to pure mathematicians, if not to physicists and engineers. The stationary phase method shows an important new result: deviations from the ideal quadrature are proportional to the relative fringe curvature. The deviations from the ideal are analogous in 1-D where deviations of the Hilbert transform from quadrature are displayed by non-bandlimited functions.

The question of how to define analyticity in more than one dimension is implicit in Chapter 4. The question is deliberately limited to fringe patterns so that a strong argument can be put for the special case, even though it seems incompatible with the (non-isotropic) definition of 2-D analyticity accepted in the area of phase retrieval. The discussion of a general definition for analyticity in 2-D is beyond the scope of this thesis.

6.1.4 Chapter 5

The final algorithm proposed in Chapter 5 competes with several others that have been published in the last few years. So far, the algorithm accuracy has not been fully investigated, although initial simulations show remarkably good performance. Only a few algorithms are known to work with less than five interferograms or frames. One of these is Kreis' interpretation of Lai and Yatagai's generalised phase-shifting interferometry. The method has never been clearly stated; rather it is implied in Kreis' excellent book. Kreis' method is similar to the method proposed in Chapter 5, except that it uses convention FTM methods on small (hand-picked) regions of each interferogram to estimate the relative phase of each frame. In contrast, using the spiral phase demodulator to estimate the relative phase each interferogram requires no manual intervention. It may be expected that the spiral phase algorithm uses more of the global information in the interferogram sequence, and is nearer to optimal, but this remains to be confirmed.

6.2 Bibliography

A number of publications not referred to in the main body of the thesis have been invaluable in stimulating my thoughts on demodulation, symmetry and pattern analysis. These publications a listed below.

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7. Appendix: Document lodged in support of thesis