1. Introduction

Modulation in everything is the path to happiness…

John Dickinson[2] (circa 1768).

1.1 Overview

Modulation and demodulation are familiar concepts in communication theory. In communications the theory is purely one-dimensional (1-D) and is to a significant extent well understood. In multiple dimensions demodulation theory is a rather immature subject in that the extension from one to many dimensions is not simple, nor well understood. Many attempts to define demodulation in two dimensions (2-D) for example are just one-dimensional methods applied along one-dimensional sections of the 2-D signal (or image). Such definitions often have unwanted or unexpected artefacts with symmetry unrelated to the underlying signal. The main objective of this thesis is to develop methods of demodulation which make sense in two or more dimensions.

The theory of amplitude modulation (AM) and frequency modulation (FM) developed during the first half of the twentieth century primarily for wireless (radio) communications. Many ingenious methods of demodulation were devised during that period. A Canadian named Reginald Fessenden received a US patent for an AM radio transmitter in 1901. It wasn't until 1920, when a sufficiently efficient transmitter design was available, that the first commercial AM station began transmitting. In contrast FM had first been suggested and rejected in the 1920s[3], but it wasn't until 1933 that the inspired Edwin Armstrong succeeded in making wideband FM work, and moreover display its inherent superiority.[4] In the following decades the theory of communication for continuous signals (now referred to as analog signals) developed to a high degree, with the works of Gabor,[5] Ville,[6] and Shannon[7] being particularly notable. It is scarcely possible to overestimate the importance of communication technologies in the remaining years of that century.

A significant change began to occur in the late 1970s with the widespread introduction of digital signal processing (DSP) hardware and software. DSP allows the direct implementation of sophisticated mathematical operations upon sampled signals. Until that time mathematical operations were limited to those realisable in devices and discrete components such as the diode, the transistor, the capacitor, and the resistor. But with all this sophistication the technology is firmly limited to 1-D signals.

The theory of modulation and demodulation in multiple dimensions is now in its infancy. There is a significant research focus on multi-dimensional demodulation in a number of separate disciplines such as image processing, (texture analysis, spectral analysis, pattern analysis), image restoration and recovery (especially phase retrieval), interferometry, fringe analysis, visual psychophysics, and synthetic aperture radar. The theory of modulation rather than demodulation is important for holography and diffractive optics. Multi-dimensional demodulation theory and practice will continue to develop in an ad hoc manner until a unified approach emerges. One of the great unifying principles in 1-D is the idea of the analytic signal introduced by Gabor in 1947. It is the intention of this thesis to find corresponding principles which connect problems in multi-dimensional demodulation.

1.2 Selection of Multi-dimensional Demodulation Problems

The approach taken has been to select a number of interesting optical problems in the broad area of multi-dimensional demodulation, and then to approach the problems with a small number of mathematical tools based primarily on Fourier analysis and symmetry. Such an approach is not guaranteed to reveal broad connections between problems, but the risk is outweighed by my deep fascination with the problems selected.

Each problem is presented in a separate chapter of the thesis. Emergent connections are suggested at the end of each chapter.

1.2.1 A neat algorithm for the demodulation of white light interferograms

The chosen problems begin with a question posed at a conference in 1994, when my doctoral work commenced. Do algorithms exist for the efficient analysis of the 3-D datasets obtained from (recently introduced) devices known as white-light profilometers? Although the data is collected over 3-D, there is a strong asymmetry that makes the problem essentially 1-D. Traditional 1-D methods can be used in this case, but a novel approach is required to obtain computationally efficient methods with sufficient accuracy. The chapter covers many issues not published prior to 1996.

1.2.2 Demodulation of 2-D Helmholtz wavefields

The second problem arose from my more general work on Fourier methods for simplifying the understanding of 3D diffraction theory (and especially imaging theory). A specific question about how to interpret the quality of a beam emitted from a planar integrated optical component using intensity measurements alone was posed by a colleague (Carol Cogswell). The problem is essentially 2-D in character and is, in many ways, similar to the well-known 2-D phase retrieval problem. Phase retrieval can in turn be considered as a particularly perverse demodulation problem. In this instance the highly constrained circular-arc symmetry of the optical field spectrum is perfect for testing broad arguments based on Fourier symmetry.

1.2.3 Natural demodulation of 2-D fringe patterns

Since 1982 researchers have been trying to extend the powerful Fourier Transform Method[8] of fringe analysis from 1-D to 2-D with mixed results. As recently as 1984 fringe analysis consisted of visually comparing actual pattern with tables of pattern types. For example figure 1.1 shows a sequence of patterns used to evaluate spherical aberration in an interferometer. Similarly figure 1.2 shows the recommended method for quantitative interferogram analysis.[9] The introduction of highly linear image sensors, like CCDs, and the widespread availability of image digitising hardware has allowed sophisticated mathematical techniques to be applied to fringe patterns.

Inspired partly by the simple symmetry properties of the Helmholtz field spectrum in the preceding chapter, a new look at the problem of 2-D fringe pattern

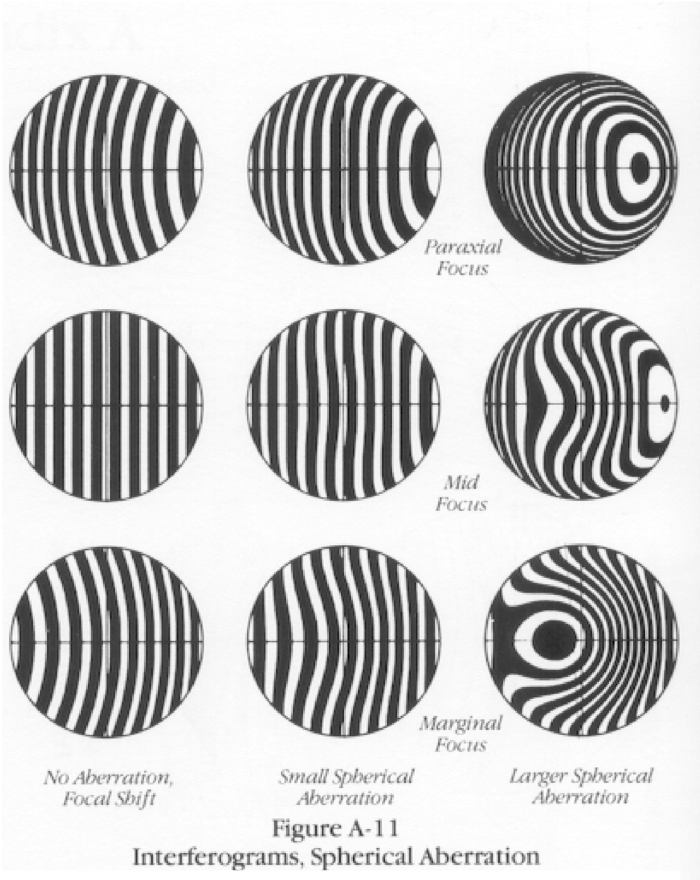

Figure 1.1
Comparison table for spherical aberration.
[Published in 1980 by Zygo Corporation.]

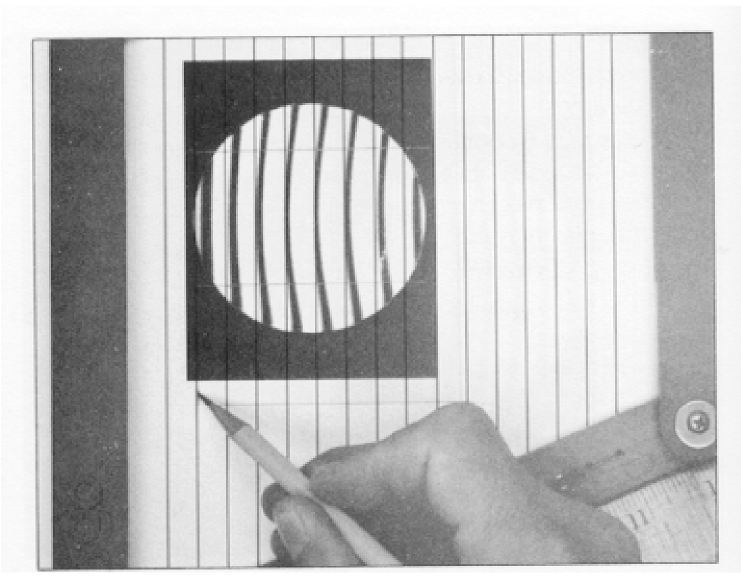

Figure 1.2
Manual fringe pattern analysis.
[Published in 1980 by Zygo Corporation.]

analysis finally yielded an intuitively justified Fourier transform method of demodulation using a spiral phase function. The research in this case was a group effort and inappropriate for an individual doctoral thesis. However, the group research did not yield a theoretical justification for the spiral phase demodulation. Instead, the challenge of developing a rigorous mathematical justification of the spiral phase transform was undertaken for the next stage of the thesis. Many approaches were tried to pin down the exact effects of the proposed transform, but only the stationary phase method revealed the solution and a possible explanation of the residual errors of the transform.

1.2.4 The ultimate phase-shifting algorithm?

The methodology and results developed for "natural" demodulation are shown to have direct application in another longstanding problem of 3-D fringe analysis. Since the very first phase-shifting algorithms were applied to sequences of phase-shifted interferogram, researchers have consider the possibility of phase-step errors. Consequently a significant number of error-correcting schemes or algorithms have been developed. I have been fascinated by these schemes since 1989 and the question of whether or not a scheme could be developed to correct for arbitrary errors? This chapter is a final opportunity to develop a quite individual solution to the problem.

1.3 Scope of the thesis

The main chapters try to solve topical problems in optical pattern analysis, which may have implications for more general pattern analysis. In a sense chapters 2 and 3 are the preamble to the new 2-D quadrature transform which is validated in chapter 4. Chapter 5 is then a successful new application of the transform. A description of the origin of the new 2-D quadrature function is outside the scope of the thesis, but is attached as an external publication to support the central theme.

1.4 References

1 The originator of this witty observation remains anonymous.

2 This is a misquote from J. Dickinson, "Letters from a Farmer in Pennsylvania" Pennsylvania Chronicle, 1768. The correct quotation is:
" Moderation in everything is the source of happiness. Too much writing, too much reading, too much idleness, too much loving, too much continence, too much law, or religion, all equally throw us from the balance of real pleasure. This has been said a thousand times, always believed and practiced against, it is still true."

3 J. R. Carson, "Notes on the theory of modulation", Proceeedings of the IRE **10**, 57-64, (1922).

4 E. H. Armstrong, "A method of reducing disturbances in radio signaling by a system of frequency modulation", Proc. IEEE **24**, (5), 689-7440, (1936).

5 D. Gabor, "Theory of communications", Journal of the Institution of Electrical Engineers, **93**, 429-457, (1947).

6 J. Ville, “Theorie de applications de la notion de signal analytique”, Cables et transmissions **2A**, (1), 61-74, (1948).

7 C. E. Shannon, “Communication in the presence of noise”, Proc. Institute of Radio Engineers **37**, 10-21, (1949).

8 M. Takeda, H. Ina, and S. Kobayashi, “Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry”, J. Opt. Soc. Am. **72**, (1), 156-160, (1982).

9 Zygo, “Interferogram Interpretation and Evaluation Handbook”, Zygo Corp, 1980, reprinted 1984.