

# **Topics in Multi-dimensional Signal Demodulation**

A Thesis

Submitted to

The Faculty of Science

in

The University of Sydney

by

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Supervised by

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In Fulfilment of the Requirements for the Degree

of

Doctor of Philosophy

December 2000

## ACKNOWLEDGMENTS

I would like to thank Professor Colin Sheppard for spreading his infectious enthusiasm for all things optical, especially the exquisite Fourier theory of image formation. Before meeting Colin I thought the Sine Condition was a form of hay fever.

A very special thank you is due to the delectable Anne-Marie Barnett who has encouraged, supported, cajoled, and inspired my writing of this thesis from the very beginning.

Special thanks go to Dr Bob Oreb, the co-author of my first scientific paper, for giving me the chance to rediscover my love of research whilst working at CSIRO, and for finding so much concealed inside a phase-shifting algorithm.

Dr Michael Oldfield provided numerous amusing anecdotes and many dire warnings about the use of MS Word for writing a thesis. He was quite correct. Michael also provided his superb image processing program, originally called Image32, which allowed me to rapidly investigate the effects of Fourier transforms upon the phase and magnitude of vast 2-D arrays of data.

Dr Phil Robertson and Dr Ian Gibson at CISRA encouraged and supported my PhD research over that last four years; thanks guys. The small fortune I spent on my university library copy card may finally be vindicated. As Professor Bob Gonsalves (Tufts University) recently remarked "several years hard work in the laboratory can easily save you fifteen minutes in the library".<sup>1</sup>

Carol Cogswell, in the Department of Physical Optics, encouraged me to have faith in my research ideas. Noni McIntosh provided much needed secretarial support, and Vicky Moore helped me obtain a number of difficult to find publications. A number of others have given me support in more subtle ways, these include Don Bone, Zoltan Hegedus, David Farrant, Nick Brown, Matthew Arnison, John Quartel, and Miguel Alonso.

**DEDICATION**

*I dedicate this work to my mother Kathleen Larkin,  
and to the memory of my father Joseph Larkin.*

## **DECLARATION**

The work presented in this thesis is my own, except where otherwise acknowledged. I am sole author of the work presented in chapters 1-2, and 4-6. The original idea for unravelling the autocorrelation appearing in chapter 3 was developed in conjunction with my supervisor, Colin Sheppard. Michael Oldfield at CISRA wrote the software framework for the Fourier analysis of complex images. An essential precursor to my own work in chapter 4 is the experimental development of a new transform by myself, Donald Bone, and Michael Oldfield. The precursor is necessary for a clearer understanding of chapter 4 and is lodged as a document in support of this thesis.

Kieran G. Larkin

## PUBLISHED ARTICLES

The work presented in various chapters has appeared in journals and conference proceedings during the last few years or has been submitted for publication.

### Chapter 2

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### Chapter 3

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### Chapter 4

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K. G. Larkin, "Natural demodulation of two-dimensional fringe patterns: II. Stationary phase analysis of the spiral phase quadrature transform.," Accepted for publication in the *Journal of the Optical Society of America, A*, (to appear circa July 2001).

### Chapter 5

K.G. Larkin, "The ultimate phase shifting algorithm?," In preparation.

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## ABSTRACT

Problems in the demodulation of one, two, and three-dimensional signals are investigated. In one-dimensional linear systems the analytic signal and the Hilbert transform are central to the understanding of both modulation and demodulation. However, it is shown that an efficient nonlinear algorithm exists which is not explicable purely in terms of an approximation to the Hilbert transform. The algorithm is applied to the problem of finding the envelope peak of a white light interferogram. The accuracy of peak location is then shown to compare favourably with conventional, but less efficient, techniques.

In two dimensions (2-D) the intensity of a wavefield yields to a phase demodulation technique equivalent to direct phase retrieval. The special symmetry of a Helmholtz wavefield allows a unique inversion of an autocorrelation. More generally, a 2-D (non-Helmholtz) fringe pattern can be demodulated by an isotropic 2-D extension of the Hilbert transform that uses a spiral phase signum function. The range of validity of the new transform is established using the asymptotic method of stationary phase. Simulations of the algorithm confirm that deviations from the ideal occur where the fringe pattern curvature is larger than the fringe frequency.

A new self-calibrating algorithm for arbitrary sequences of phase-shifted interferograms is developed using the aforementioned spiral phase transform. The algorithm is shown to work even with discontinuous fringe patterns, which are known to seriously hamper other methods. Initial simulations of the algorithm indicate an accuracy of 5 milliradians is achievable.

Previously undocumented connections between the demodulation techniques are uncovered and discussed.

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