Topics in Multi-dimensional Signal Demodulation

A Thesis

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DEDICATION

I dedicate this work to my mother Kathleen Larkin,

and to the memory of my father Joseph Larkin.

DECLARATION

The work presented in this thesis is my own, except where otherwise acknowledged. I am sole author of the work presented in chapters 1-2, and 4-6. The original idea for unravelling the autocorrelation appearing in chapter 3 was developed in conjunction with my supervisor, Colin Sheppard. Michael Oldfield at CISRA wrote the software framework for the Fourier analysis of complex images. An essential precursor to my own work in chapter 4 is the experimental development of a new transform by myself, Donald Bone, and Michael Oldfield. The precursor is necessary for a clearer understanding of chapter 4 and is lodged as a document in support of this thesis.

Kieran G. Larkin

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The work presented in various chapters has appeared in journals and conference proceedings during the last few years or has been submitted for publication.

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ABSTRACT

Problems in the demodulation of one, two, and three-dimensional signals are investigated. In one-dimensional linear systems the analytic signal and the Hilbert transform are central to the understanding of both modulation and demodulation. However, it is shown that an efficient nonlinear algorithm exists which is not explicable purely in terms of an approximation to the Hilbert transform. The algorithm is applied to the problem of finding the envelope peak of a white light interferogram. The accuracy of peak location is then shown to compare favourably with conventional, but less efficient, techniques.

In two dimensions (2-D) the intensity of a wavefield yields to a phase demodulation technique equivalent to direct phase retrieval. The special symmetry of a Helmholtz wavefield allows a unique inversion of an autocorrelation. More generally, a 2-D (non-Helmholtz) fringe pattern can be demodulated by an isotropic 2-D extension of the Hilbert transform that uses a spiral phase signum function. The range of validity of the new transform is established using the asymptotic method of stationary phase. Simulations of the algorithm confirm that deviations from the ideal occur where the fringe pattern curvature is larger than the fringe frequency.

A new self-calibrating algorithm for arbitrary sequences of phase-shifted interferograms is developed using the aforementioned spiral phase transform. The algorithm is shown to work even with discontinuous fringe patterns, which are known to seriously hamper other methods. Initial simulations of the algorithm indicate an accuracy of 5 milliradians is achievable.

Previously undocumented connections between the demodulation techniques are uncovered and discussed.

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