

Transmission Line Based Readout for Cryogenic Systems

BRENDAN HARLECH-JONES



THE UNIVERSITY OF
SYDNEY

Supervisor: David Reilly
Associate Supervisor: Stephen Bartlett

A thesis submitted in fulfilment of
the requirements for the degree of
Doctor of Philosophy

School of Physics
Faculty of Science
The University of Sydney
Australia

November, 2023

Preface

The research comprising this thesis was performed in the Quantum Nanoscience Laboratory (QNL) in the school of Physics at the University of Sydney. The QNL is one of several labs partnered with Microsoft. The focus of the lab is on developing scalable readout of quantum computers. The works covered in this thesis cover a fairly broad range of topics linked by the need to readout at radio frequencies. Chapter 4 focuses on the physics of edge magnetoplasmons in the quantum Hall regime. Whereas chapter 5 and chapter 6 are more focused on high frequency readout techniques. Chapter 5 uses radio frequency measurements in order to sense the reactance of a Josephson junction. In chapter 6 a three terminal selective area grown nanowire is used to explore the benefits that full S-parameter measurements may provide.

All the data was collected by me. All figures in the thesis were produced by me with the exception of figure 2.3 and figure 2.6 . Steven Waddy provided assistance with figure 2.4, figure 6.4, figure 6.5, and figure 6.6, as well as producing the drawing of the VAN used in figure 3.12 that was modified for figure 3.10 and figure 3.11. James Witt produced the data for figure 4.15 and assistance with figure 6.2. David Reilly produced the initial sketches for figure 6.14. Where relevant the authors have been appropriately cited.

I did not fabricate all of the devices used in this thesis. In chapter 4 the first device discussed was fabricated by Alice Mahoney using the Australian National Fabrication Facilities (ANFF) at the University of New South Wales. In chapter 5 the device discussed was fabricated by James Witt using the Research and Prototyping Foundry, an ANFF facility located at the University of Sydney. In chapter 6 the nanowires were fabricated by a team at Microsoft Quantum Copenhagen. All of the materials used were originally provided by Mike Manfra's team at Microsoft Quantum Purdue.

Declaration

This is to certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes. If not otherwise acknowledged, the work presented in this thesis is my own.

Signed: _____

Authorship attribution statement

In addition to the statements above, in cases where I am not the corresponding author of a published item, permission to include the published material has been granted by the corresponding author.

Signed: _____

As supervisor for the candidature upon which this thesis is based, I can confirm that the authorship attribution statements above are correct.

Signed: _____

Abstract

This thesis presents a series of experiments on different mesoscopic systems: a quantum Hall system, a Josephson junction, and a semiconductor-superconductor three-terminal device. Measuring these different systems allows for the power and versatility of radio frequency (RF) readout to be showcased. None of these experiments utilise deliberate matching structures. Each experiment shows a progression in the sophistication of the methods used to perform RF readout, culminating in a new approach – the transmission line resonator (TLR) method – which shows benefits over low frequency methods and over other current RF approaches.

The measurements of the quantum Hall system focuses on transmission measurements of the collective excitations called edge magnetoplasmons (EMPs). The links between EMPs and the quantum Hall effect are discussed in detail. Current models for EMPs and the quantum Hall effect are applied to predict the frequency response of EMPs as a function of magnetic field; the need for a new model is demonstrated. A new phenomenological model which contains no free parameters is produced.

A single gated Josephson junction (JJ) is measured in reflection. The reflection data is utilised in order to describe inductive changes that occur in this JJ. An equivalent circuit description which seeks to quantitatively describe these changes is presented.

Building on the previous measurements, the semiconductor-superconductor device is measured using the TLR approach. This allows the device to be measured simultaneously in transmission and reflection. This system serves as a test-bed for this measurement approach, demonstrating the utility of this method.

Acknowledgements

Completing a PhD is hard. Completing it during a global pandemic even harder. There are many people who have helped me in various ways throughout the course of my PhD. Here are a few of the people I wish to thank for their support during this time. Working in the QNL has generally been a wonderful experience; the personnel, equipment and knowledge are all exceptional.

To begin with I would like to thank my supervisor David Reilly. I am really appreciative of the opportunities he has provided me with. Although getting a hold of him can be a struggle, whenever anything is needed, no matter the cost, the answer has always been “go for it”. This brings me to Alexis George. I honestly don’t know how the lab would run without you or how anything would be organised. I don’t know if anyone knows an easy way to get a hold of David except through you.

With David not being available on a day-to-day basis I was still able to receive exceptional daily supervision. This was provided by my de facto supervisor James Witt. We unfortunately never did manage to get James recognised as an official co-supervisor – his fault. I really enjoyed working with James on both his and my experiments; even when he tricked me in to taking over (what I believed to be) his experiments. I really appreciate the guidance provided in experimental work, talks, writing and literally everything else to do with the completion of a PhD.

To my ‘comrades’ Steven Waddy and Abdallah El Kass, thank you. Coming in to work is made more enjoyable by working with the two of you. Steven is a master of electronics and everything to do with measurement, and Abdallah a master of the cleanroom amongst other things. I have worked with Steven very closely on many experiments, and much to Steven’s surprise, I have enjoyed the experience. The feedback on my writing has also been very helpful, I appreciate not looking ignorant; thank you.

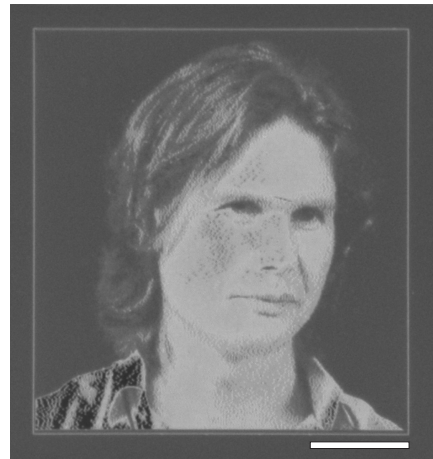


Figure 1: Black and white image of David. It was fabricated as part of a gold deposition test on an InAs substrate. The frame was created by etching. The gold dots had radii between 30 nm and 90 nm. The scale bar is 20 μm .

I spent a lot of time in the cleanroom throughout my PhD. I wish to thank the RPF staff who helped out with everything cleanroom. I also wish to thank Lim Sim and Maxime Pozzi, both very knowledgeable people as well as fun to be in the cleanroom (and to play volleyball) with. The tips and tricks they were able to share were invaluable.

To everyone else in the QNL I have not mentioned, do not think I have forgotten you. I appreciate each and every one of you. The lab is a wonderful place and this would not be the case without you.

I spent two years of this PhD locked down, unable to access the laboratory. I honestly struggled quite a bit through this period. I wish to thank my partner, my family, friends and my pets for helping me through this and the other tough times that occur during the course of life.

As a final shout out I would like to thank my hockey teams. Although they didn't provide any sort of academic support, the people in these teams are a amazing group of people. Without the sport and the people in it I would probably have gone (even) craz(ier) in trying to complete this thesis.

Publications

Listed below are details of the papers included in this thesis.

Chapter 4 – “Modelling the Frequency Response of Edge Magnetoplasmons” **B. Harlech-Jones**, J. D. S. Witt, and D. J. Reilly *In Preparation*, (2023).

Chapter 6 – “Local and Non-local Microwave Impedance of a Three-Terminal Hybrid Device” **B. Harlech-Jones**, S. J. Waddy, J. D. S. Witt, D. Govender, L. Casparis, E. Martinez, R. Kallaher, S. Gronin, G. Gardner, M. J. Manfra, D. J. Reilly (In review)

Current publications not included in this thesis where I am an author.

“Repairing the surface of InAs-based topological heterostructures” *Journal of Applied Physics*, **128**, pg. 114301 (2020). S. J. Pauka, J. D. Witt, C. N. Allen, **B. Harlech-Jones**, A. Jouan, G. C. Gardner, S. Gronin, T. Wang, C. Thomas, M. J. Manfra, J. Gukelberger, J. Gamble, D. J. Reilly, and M. C. Cassidy.

Acronyms

Listed below are the acronyms, alphabetically ordered, that can be found in this thesis. In the digital version of this thesis when an acronym appears it is hyperlinked to its definition.

1D – One-dimensional	IQHE – Integer quantum Hall effect
2D – Two-dimensional	IS – Incompressible strip
2DEG – Two-dimensional electron gas	LL – Landau level
3D – Three dimensional	LNA – Low noise amplifier
3T – Three-terminal	MAR – Multiple Andreev Reflection
ABS – Andreev bound states	MBS – Majorana bound state
AEMP – Acoustic edge magnetoplasmon	MBE – Molecular beam epitaxy
AFM – Atomic force microscope	MIBK – Methyl isobutyl ketone
ALD – Atomic layer deposition	MLA – Maskless Aligner
BCS – Bardeen, Cooper, and Schrieffer	MZM – Majorana zero mode
CPW – Co-planar waveguide	NMP – n-methyl-2-pyrrolidone
CS – Compressible strip	NTP – Non-topological phase
CSG – Chklovskii, Shklovskii and Glazman	PCB – Printed circuit board
CW – Continuous wave	PMMA – Polymethyl methacrylate
DAC – Digital to analog converter	QDS – Quantum dot state
DC – Direct current	QH – Quantum Hall
DI – De-ionised	QHE – Quantum Hall effect
DMM – Digital multimeter	QNL – Quantum nanoscience laboratory
DOS – Density of states	QPC – Quantum point contact
DUT – Device under test	RCSJ – Resistively and capacitively shunted junction
EBL – Electron beam lithography	RF – Radio frequency
EM – Electromagnetic	Rx – Return
EMP – Edge magnetoplasmon	SAG – Selective area growth
FQHE – Fractional quantum Hall effect	ScS – Superconductor-constriction-superconductor
FWHM – Full width half maximum	SdH – Shubnikov-de Haas
IEMP – Inter-edge magnetoplasmon	SEM – Scanning electron microscope
IF – Intermediate frequency	
IPA – Isopropanol	

SI – Système Internationale	TE – Transverse electric
SIS – Superconductor-insulator-superconductor	TLR – Transmission line resonator
SNR – Signal to noise ratio	TM – Transverse magnetic
SNS – Superconductor-normal metal-super-conductor	TP – Topological
SQUID – Superconducting quantum interference device	Tx – Transmission
	VNA – Vector network analyser
	ZBP – Zero bias peak

Contents

1	Introduction	1
2	Background	3
2.1	RF	3
2.1.1	Types of Measurements	4
2.1.1.1	S-parameters	5
2.1.1.2	Data representation	7
2.1.2	Impedance matching	9
2.2	Dilution Refrigerators	10
2.2.1	$^3\text{He}/^4\text{He}$	10
2.2.2	Fridge design	11
3	Methods	15
3.1	Sample Design	15
3.1.1	Quantum Hall device (Chapter 4)	15
3.1.2	Josephson junction device (Chapter 5)	16
3.1.3	Three-Terminal device (Chapter 6)	18
3.1.4	InAs materials	18
3.2	Low frequency readout	20
3.2.1	Quantum Hall devices (Chapter 4)	20
3.2.2	Josephson junction device (Chapter 5)	21
3.2.3	Three-Terminal device (Chapter 6)	21
3.3	High frequency readout	23
3.3.1	Quantum Hall device (Chapter 4)	23
3.3.2	Josephson junction device (Chapter 5)	24
3.3.3	Three-Terminal device (Chapter 6)	25
4	Modelling the Frequency Response of Edge Magnetoplasmons	27
4.1	Abstract	27
4.2	Introduction	27
4.2.1	Classical Hall effect	28
4.2.2	Quantum Hall Effect	30
4.2.2.1	Shubnikov-de Haas Oscillations	31

4.2.2.2	Single Electron Picture	32
4.2.2.3	Landau gauge description	32
4.2.2.4	Edge Channels	34
4.2.3	Edge Magnetoplasmons	37
4.3	Results and Discussion	40
4.3.1	Global Dispersion	41
4.3.2	Plateau Dispersion	43
4.3.2.1	Modelling the Incompressible Strips	44
4.3.2.2	Edge side model – position determined by incompressible strip	46
4.3.2.3	Edge side model – position fit	46
4.3.2.4	Johnson and Vignale Model	47
4.3.2.5	Resonance Fitting	48
4.3.2.6	Development of a Phenomenological ν Symmetric Model . .	50
4.4	Conclusions	53
5	Sensing the Reactance of a Josephson Junction	54
5.1	Abstract	54
5.2	Introduction	54
5.2.1	Superconductors	54
5.2.2	Josephson Junction	56
5.3	Results and Discussion	59
5.3.1	Measurement	59
5.3.2	Modelling	64
5.4	Conclusions	66
6	Local and Non-local Microwave Impedance of a Three-Terminal Hybrid Semiconductor-Superconductor Nanoscale Device	68
6.1	Abstract	68
6.2	Introduction	68
6.3	Methods	70
6.4	Results and Discussion	71
6.4.1	Key	71
6.4.2	Transmission Line Resonance	72
6.4.3	Spatial Resolution	75
6.4.4	Magnetic field response	77
6.4.5	Sub-gap features	78
6.4.6	Sensitivity	80
6.5	Summary	82
7	Conclusion and Outlook	84

Appendix	86
A Fabrication	86
B Modelling filling factors $\nu > 2$	95
C Additional information for chapter 5	99
D Additional information for chapter 6	106
E Other Devices	116
Bibliography	119

List of Figures

1	Dose Test	v
2.1	Representation of an arbitrary N -port network	4
2.2	Power flow in a 2-port network	5
2.3	Smith Chart	8
2.4	Picture of dilution refrigerator	10
2.5	$^3\text{He}/^4\text{He}$ Phase diagram	11
2.6	Schematic of <i>Leiden</i> dilution refrigerator	12
3.1	Quantum Hall device	15
3.2	Hall bar design	16
3.3	Josephson junction device	17
3.4	InAs 2DEG layer stack	17
3.5	False colour SEM of three-terminal device	18
3.6	Cross section of SAG nanowire	19
3.7	DC Setup for Hall bar measurement	20
3.8	DC Setup for Josephson junction measurement	21
3.9	DC Setup for three-terminal measurements	22
3.10	Transmission set-up	23
3.11	Reflection setup	24
3.12	Full S -parameter setup	25
4.1	Classic Hall effect	28
4.2	Typical Hall bar configuration	30
4.3	DOS of 2DEG in perpendicular magnetic field	31
4.4	Bending of Landau levels bands	32
4.5	Hall and longitudinal resistivity	34
4.6	Electron density at the edge of a sample	34
4.7	Edge channel picture	35
4.8	Incompressible strip development over a quantum Hall plateau	37
4.9	Surface plasmon	38
4.10	Edge magnetoplasmon motion	38
4.11	Magnetoplasmon dispersion spectra	40
4.12	EMP dispersion and Hall resistance measurements	42
4.13	Fit for the EMP dispersion relationship	43

4.14	Density profile at the edge of a sample and position change of incompressible strip	44
4.15	Calculated area of incompressible strip	45
4.16	n determined by incompressible strip position	47
4.17	Comparing fit n to calculated n	47
4.18	$\nu = 2$ fit by varying τ	48
4.19	Change in ΔS across the $\nu = 2$ plateaux	48
4.20	τ_Q of generic resonator	49
4.21	Normalised transmission data - plotted against B	50
4.22	Normalised transmission data - plotted against ν	51
4.23	ν symmetric model	52
5.1	Discovery of superconductivity	55
5.2	Meissner effect	56
5.3	Wavefunctions in the Josephson effect	57
5.4	Shapiro steps	58
5.5	RCSJ model of a JJ	58
5.6	Josephson junction lock-in bias measurement	60
5.7	Josephson junction RF bias measurement	61
5.8	Change of resonant frequency	62
5.9	I/Q projection of the Josephson junction bias	63
5.10	I/Q projection of the Josephson junction with top gate	64
5.11	Model for the RF response	65
5.12	Fit for the resonance position	66
5.13	Inductance of the JJ	66
6.1	Schematic of a three-terminal device	69
6.2	Andreev reflections and elastic co-tunnelling	70
6.3	Measurement paths for the three-terminal device	72
6.4	Formation of transmission line resonance	72
6.5	Experimental and simulated resonant mode effects	73
6.6	Experimental and simulated resonant mode effects	73
6.7	Benefit of resonance	74
6.8	DC cutter response	75
6.9	RF cutter response	76
6.10	Superconducting gap closing with magnetic field	77
6.11	Superconducting gap closing with magnetic field	78
6.12	Local and non-local DC measurement of sub-gap features	79
6.13	Sub-gap features biased from the left and right terminal measured at 4.795 GHz	80
6.14	Illustration of pathways for signal in RF measurement	81
6.15	Comparison of sensitivity of RF and DC cutter signals	81

6.16	Comparison of sensitivity of RF and DC signals	82
A.1	Example metal deposition	86
A.2	First hard mask test	92
A.3	PCB	94
B.1	$\nu = 3$ plateau, with n determined by incompressible strip position	95
B.2	$\nu = 4$ plateau, with n determined by incompressible strip position	95
B.3	$\nu = 5$ plateau, with n determined by incompressible strip position	96
B.4	$\nu = 3$ plateau, fit with JV model	96
B.5	$\nu = 4$ plateau, fit with JV model	96
B.6	$\nu = 5$ plateau, fit with JV model	97
B.7	$\nu = 2 - 5$ plateaux, fit with JV model	97
B.8	Line cut at $B=2.67$ T	97
B.9	EMP Q factor across $\nu = 2$ plateaux	98
C.1	Uncalibrated Josephson junction lock-in bias measurement	99
C.2	Shapiro steps measured at 3.758 GHz	100
C.3	Shapiro steps and antenna power	100
C.4	Positive and negative bias sweeps on JJ	100
C.5	Frequency selection	101
C.6	Frequency response with bias	101
C.7	Resonance shape	102
C.8	Gaussian fit parameters as a function of applied voltage	103
C.9	Response of Josephson junction to top gate	104
C.10	RF response of Josephson junction to top gate	104
C.11	Switching current size	104
C.12	Smith chart with Josephson junction top gate data	105
D.1	Power broadening	107
D.2	Frequency response of cutter-cutter measurement	108
D.3	Non-local measurement of the gap closing	109
D.4	Gap closing with parallel magnetic field	109
D.5	Local and non-local DC measurement of an ABS state	110
D.6	S -matrix of an ABS state	110
D.7	Sub-gap state of 500 nm device	111
D.8	Explicit units version of figure 6.9	112
D.9	Explicit units of left biased matrices in figure 6.13	113
D.10	Explicit units of right biased matrices in figure 6.13	114
D.11	DC and RF data presented in figure 6.16	115
E.1	Image of QH gyrator	116
E.2	Change in transmission of EMPs	117
E.3	Image of two droplet device	117

E.4	EMP dispersion of two droplets	118
E.5	Image of combined two droplet device	118
E.6	EMP dispersion of combined droplet	119
E.7	SEM of 3-Terminal 2DEG device	120

Chapter 1

Introduction

Many quantum states cannot be measured, or do not even exist, unless at cryogenic temperatures. This is because the thermal energy must be less than the level spacing (or other relevant energies) of the system [1–3]. For this reason, cooling apparatus that are able to reach temperatures below 4 K, such as dilution refrigerators, are often needed [4].

Low frequency and DC techniques are used fairly ubiquitously in measurements made at cryogenic temperatures. However, there exist states that cannot be measured using these type of measurements, either due to being separated from charge paths or because they only respond at higher frequencies. Even for those states that can be measured at low frequencies, doing so is not always the ideal choice. For instance, measuring at higher frequencies often results in lower noise, increased bandwidth, and increased sensitivity. In addition, high frequency measurements are sensitive to additional characteristics and can be better suited for scalability [5–7].

Breakthroughs in science often corresponded to entering new measurement regimes, be it temperature [8, 9], pressure [10], dimension [11, 12], material properties [13], or parts of the electromagnetic spectrum [14]. As such, engineering new ways to measure is an important cornerstone of science.

In this thesis several different quantum systems (based on semiconductors) are measured at radio frequencies (RF). The methods employed to measure these systems become progressively more sophisticated, leading ultimately to the development a new way of measuring systems at cryogenic temperatures in a dilution refrigerator.

Chapter 2 introduces principles of RF engineering. The various representations of RF data used throughout the thesis are explored. In addition, the operating principles of dilution refrigerators are discussed in this chapter.

Chapter 3 discusses the physical characteristics of the semiconductor devices used in each of the experimental chapters. The different low frequency (lock-in amplifier) set-ups used for each condensed matter system are examined. Each RF set-up is examined and the progression of each measurement is highlighted.

In **chapter 4** an RF excitation of the quantum Hall effect is studied using transmission measurements. This excitation (edge magnetoplasmon) cannot be measured using low frequency techniques. The changes in the frequency response as a function of magnetic field are studied and modelled. A new phenomenological model is produced in order to predict the observed trends.

In **chapter 5** a Josephson junction is measured in reflection, without the use of any deliberate matching networks. The responses of the junction are compared with low frequency measurements. Using these measurements, the inductance of the Josephson junction is modelled as a function of bias.

Chapter 6 covers the measurements made on a hybrid semiconductor-superconductor three-terminal nanowire device. In this chapter the techniques used in previous chapters are extended so that all transmission and reflection paths can be measured nearly simultaneously. The RF responses are compared with low frequency measurements. The benefits and shortcomings of the RF technique are discussed with respect to the device.

The final chapter, **chapter 7**, reflects on this body of work as a whole and offers suggestions for future directions.

Additional information is included in the **Appendices**.

Chapter 2

Background

2.1 RF

I am by no means an electrical engineer; I am a physicist who utilises radio frequency (RF) engineering to garner insights about interesting quantum systems. With that in mind, the discussion within this thesis will focus on providing enough understanding of RF engineering in order to allow a physicist not experienced with these concepts to interpret the results presented in later chapters.

To begin it is important to understand what is meant by radio frequencies. A generic definition of RF is electromagnetic (EM) waves in the radio domain – a somewhat circular definition. The limits of this domain can be a little blurred, but are generally considered to be between 30 MHz and 300 GHz¹ [15, 16]. The frequencies considered in this thesis will occupy a narrower range – somewhere in the realm of 300 MHz to 30 GHz ($\lambda_{vac}=1\text{ m}-1\text{ cm}$)² – partially due to the frequency responses of the devices tested and partially due to the frequencies accessible with our equipment.

A common feature of RF is that the wavelengths are generally of the same scale as the elements – such as inductors and capacitors – used [17]. This means that the phase of the wave significantly changes across an element. As a consequence, lumped element approximations are not always valid; instead distributed elements are used. This is in contrast to lower frequencies where lumped element approximations can almost always be used. The other extreme is when the wavelength is much smaller than the elements; at these frequencies geometric optics theory can be used.

DC and quasi-DC measurements are in general easier to interpret than the equivalent measurements in RF, because RF measurements are inherently complex³. DC measurements are sensitive to resistive changes whereas RF measurements are sensitive to resistive, in-

¹Bands such as Low Frequency, Medium Frequency, High Frequency etc. are defined such that the wavelengths span a single order of magnitude. This is why the bands in frequency have a slightly odd looking initial digit of three.

²Strictly this is the lower end of the microwave regime, containing the UHV and SHV bands (Ultra-High Frequency and Super High Frequency).

³Here I use complex in both the mathematical sense and as a measure of the difficulty.

ductive, capacitive and phase changes. This means that insights can be gained from paths completely inaccessible via DC [18]. Additionally, RF measurements offer other benefits such as increased speed [19], increased bandwidth [20], increased sensitivity [21], and smaller footprint [5]; these are all important aspects for scale-up of quantum systems [5, 6, 19–21].

2.1.1 Types of Measurements

When the wavelength is comparable to the electrical length, voltage and current measurements are difficult both to make and interpret whereas instantaneous power measurements are not [16, 17]. In these situations it is more convenient to apply network analysis theory. In this case, network analysis allows the regions of interest to be treated as a black box of connected impedances with a number of ports. This reduces the problem, from one where Maxwell's equations must be explicitly solved for a geometry, to one involving transmission lines and equivalent lumped element circuits. The equivalent circuit representation will use lumped elements that give the same impedance.

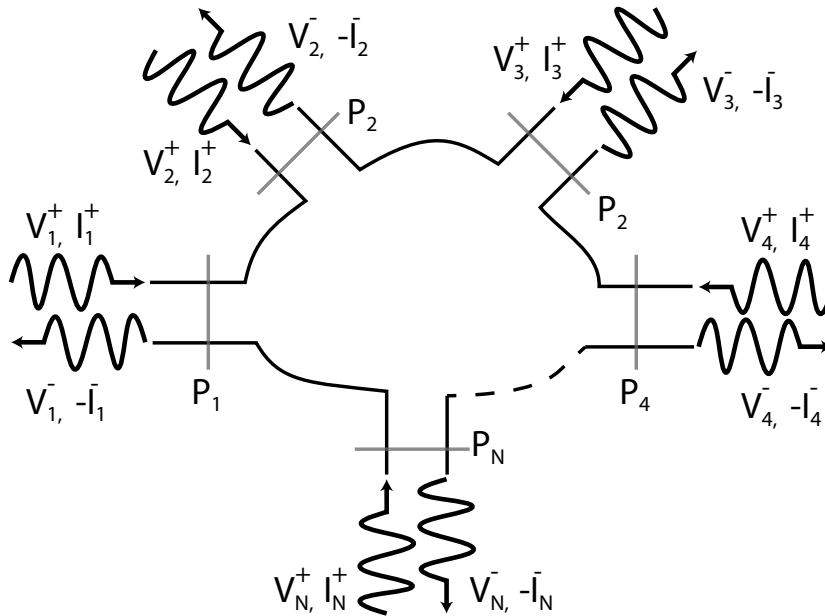


Figure 2.1: Representation of an arbitrary N -port network.

The term electrical impedance was coined by Oliver Heaviside⁴ [17]. It was used as a way to quantify the complex ratio of voltage to current in AC circuits. This concept was extended by Schelkunoff in the 1930's to the way we use it today [23]; that is, impedance is a characteristic of fields and media alike. For reference, the characteristic impedance of free space is $Z_0 \approx 377 \Omega$. Impedance is a complex quantity described by the following equation,

$$Z = R + jX = |Z|e^{j\theta} \quad (2.1)$$

where R is the resistance, X the reactance and j the imaginary unit⁵.

For the frequencies of interest, all of the equipment and cables in our laboratory have a

⁴Heaviside was an interesting character whose contributions to electromagnetic theory are still extremely relevant today. If you are unaware of who he is it is worth reading [22].

⁵ j is used in electrical engineering as i is reserved for instantaneous current.

characteristic impedance of $50\ \Omega$. For the rest of this thesis if the value of the characteristic impedance is not explicitly mentioned, it should be assumed to be $Z_0 = 50\ \Omega$.

2.1.1.1 S-parameters

A network of impedances can logically be represented by an impedance matrix, however, as the impedance matrix relies on open circuits to define parameters, it is not particularly easy to measure. Most of the time signals are guided to the device under test (DUT) using transmission lines with a specific characteristic impedance. Thus a more natural parameter to use is one constructed using matched load at ports. Parameters constructed from a matched network are called scattering parameters, with the short hand S used. For an arbitrary network with N ports the S -parameters are given by the S -matrix given below.

$$S = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ S_{NN} & \cdots & \cdots & S_{NN} \end{bmatrix} \quad (2.2)$$

Each S -parameter is a complex parameter that encodes both the phase and magnitude of a wave. An intuitive way to think of an S -parameter is as a measurement of power (figure 2.2). S_{ij} represents the ratio of power out of port i relative to the power incident on port j , i.e. a measure of the power flow $j \rightarrow i$. Reflections are the diagonal elements ($i = j$) of the S -matrix and transmissions are the off-diagonal ($i \neq j$).

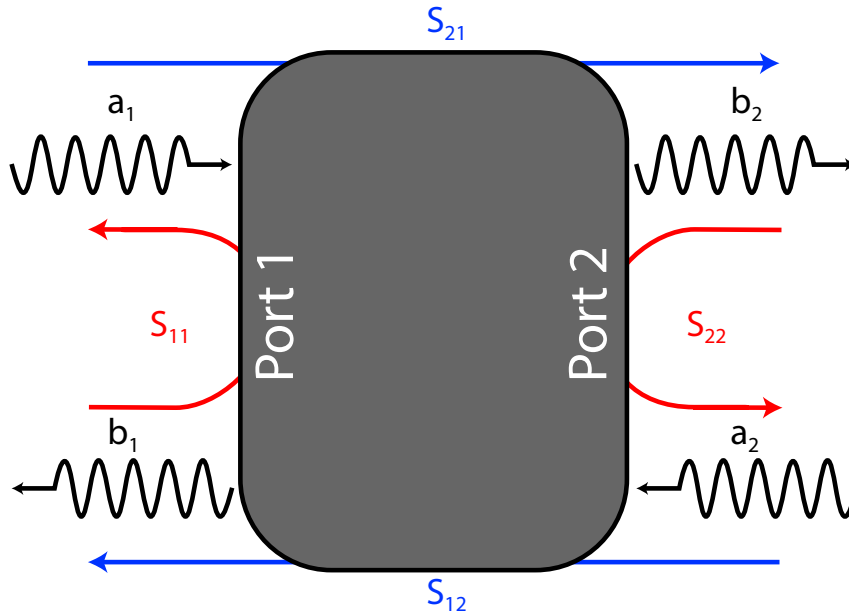


Figure 2.2: Power flow in a black box 2-port network defining S -parameters. Power incident on port n is denoted by a_n ; power out of port n is denoted by b_n . S -parameters (S_{ij}) are defined by the ratio of power out of port i relative to the amount of power incident on port j . Red shows the S -parameters that measure reflection and blue shows transmission. This same naming scheme can be naturally scaled for any arbitrary sized networks with N ports.

The units of S parameters are usually expressed by the “unitless unit” of decibels (dB).

This is a particularly convenient unit as loss is a division of signal, and gain is a multiplication of signal; this now becomes subtraction and addition. In terms of powers (P_i) or voltages (V_i), dB can be expressed as the following.

$$10 \log \frac{P_1}{P_2} \text{ dB} = 20 \log \frac{V_1}{V_2} \text{ dB} \quad (2.3)$$

The other unit often seen in conjunction with RF measurements is dBm. This is a measurement of absolute power rather than relative power. The power is in reference to 1 mW, i.e. set $P_2 = 1 \text{ mW}$ in equation (2.3).

The S -matrix can be a powerful tool to quickly identify properties of a network. For instance, reciprocal circuits will meet the condition $S_{ij} = S_{ji}$. A perfectly matched circuit – one where $Z = Z_0$ – has no reflections, so the diagonal terms are equal to zero, $S_{ii} = 0$. In a lossless circuit the power in must be the same as power out; this means that the S -matrix is unitary. A quick test for a lossless network is that the terms meet the following condition.

$$\sum_{i=1}^N |S_{ij}|^2 = 1, \quad \text{for all } j \quad (2.4)$$

Loss results in this sum being less than one, whereas gain results in this number being greater than one.

To measure S -parameters, a vector network analyser (VNA) is used. A VNA generates a continuous wave at a frequency that can be swept. Attenuators are used to control the output power level. The signal is split by a splitter; half the signal is used as the reference (assumed to be the same as a_1 in figure 2.2)⁶, the other half goes to the device via a directional coupler (actually a_1). The directional coupler separates the incident and reflected signal, allowing the reflected signal (b_1) to be measured. The signal through the device (b_2) enters a different port on the VNA and passes through another directional coupler before being measured. One needs to be aware of the path length difference between the reference path and the path to and from the device. If the frequency is swept too quickly (determined by the path length), the reference and return signals are at sufficiently different frequencies to cause amplitude and phase errors.

In order to measure the amplitude and phase of the reflected and transmitted signals, the signals are initially converted to a particular intermediate frequency (IF) by frequency mixing with a tuneable oscillator. This signal is amplified before being filtered, and then digitised. Further signal processing is performed digitally, before the signals are presented in whatever format the user desires. One of the peculiar features of VNAs, at least those used throughout this thesis [24, 25], is that the IF bandwidth that you set is not, by default, the IF bandwidth

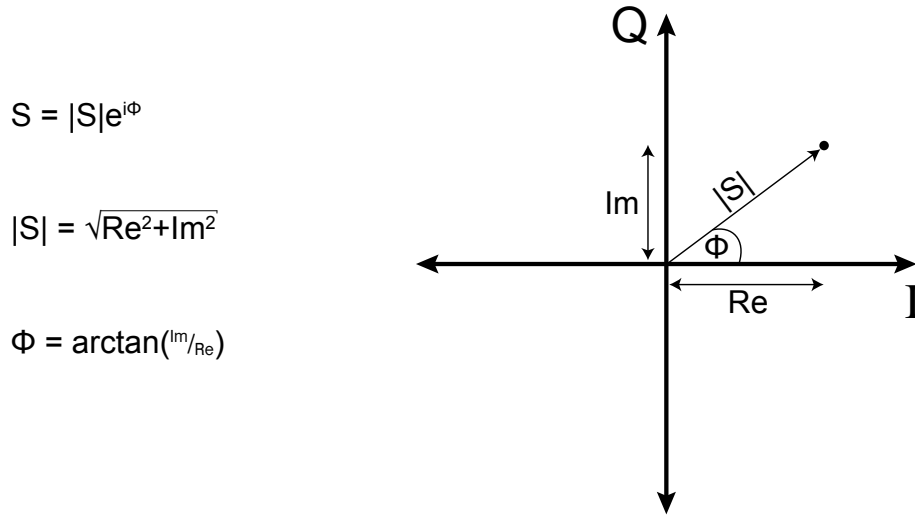
⁶Throughout this explanation I am assuming it is a two port VNA with the signal coming from Port 1. Each port of a VNA should be the same. On larger VNAs there many more internal switches to ensure correct signal routing.

actually used⁷, especially for low frequencies [26].

As with essentially all types of equipment, VNAs suffer from systematic errors, random noise, and drift. Systematic errors occur due to the imperfect nature of the components, e.g. the directional couplers are not 100% efficient which means that the assumption of the reference being the same as a_1 is not valid [27]. In general, systematic errors can be dealt with very well by calibration; unfortunately this is not the easiest thing to do within the constraints of a dilution refrigerator [28]⁸. Random noise is the noise emitted from the test equipment and drift is the variation in test equipment over time.

2.1.1.2 Data representation

There are different ways to represent the information obtained from a VNA. The most relevant ones for quantum systems and in this thesis are magnitude/phase, Smith chart, and in-phase/quadrature (I/Q) representation. Magnitude/phase representation is essentially the polar representation of S -parameters; it will be seen throughout all the experimental chapters in this thesis. I/Q representation is the rectangular representation of S -parameters.



The Smith chart, although initially daunting to view (figure 2.3), is essentially a polar plot representation of the reflection coefficient (equation (2.6)). The most common type of Smith chart used is the impedance Smith chart. In this chart, lines of constant normalised impedance ($z = Z/Z_0$) are plotted. Converting between input impedance and the reflection S -parameters is straightforward as

$$Z = Z_0 \left(\frac{1 + S_{ii}}{1 - S_{ii}} \right). \quad (2.5)$$

⁷By default, the system assumes that the user wants to maintain a similar amount of noise – not a constant IF – regardless of the frequency. For this reason it automatically multiplies the IF by a value less than one for ‘low’ frequencies. This is why the trace appears to take a long time at low frequencies.

⁸The calibration cannot be made with the fridge open and warm as the temperature changes from 300 K to <1 K makes the calibration invalid. The calibration needs to be performed while the fridge is operating, in a vacuum at extremely low temperatures. Commercial electronic calibration kits do not work at these temperatures [29].

Circles that pass through the centre line give the normalised resistance, whereas the arcs give the normalised reactance [30]. If something lands in the top half of the plot it is more inductive, if it is in the bottom half it is more capacitive, and if it is on the centre line it is purely real – either made up of something purely resistive, or something where the inductive and capacitive components cancel.

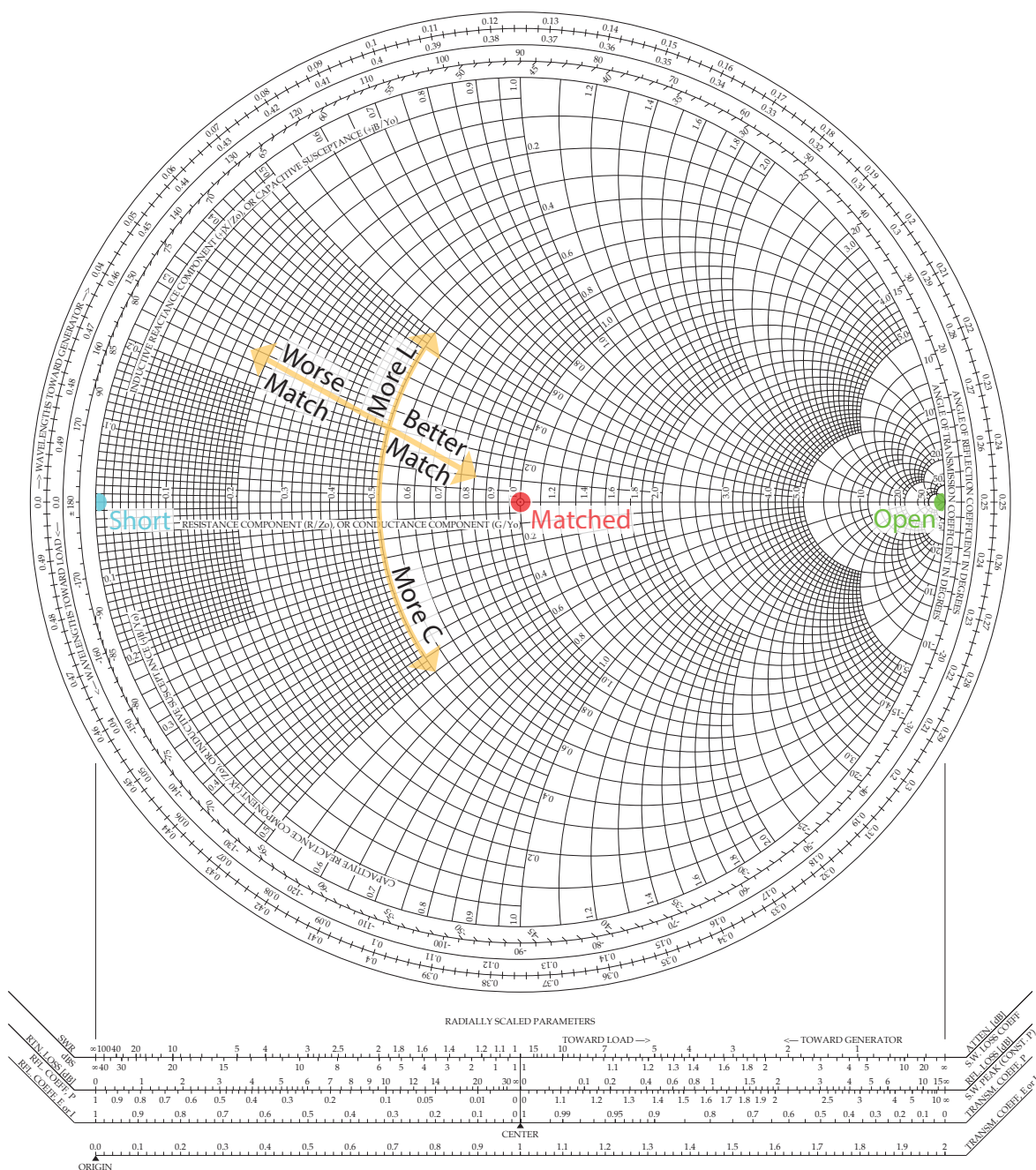


Figure 2.3: The impedance Smith chart, modified from [31]. Admittance Smith charts are essentially the mirror image of this.

The Smith chart is still used to build up an understanding and intuition about transmission line problems and impedance matching problems [16]⁹. For instance if something were perfectly matched, it would be expected to be a dot centred on $z = 1$; a perfect short exists as a spot on the left and a perfect open a spot on the right. Real networks are not perfect. In this

⁹Adding a lumped element inductor in series causes a clockwise rotation around a resistance circle. Adding it in parallel causes movement counterclockwise around a conductance circle. Capacitors have the opposite rotations.

thesis when a Smith chart is used, multiple frequencies are generally plotted, and it is the way that the frequencies ‘move’ across the chart that gives insights into what is happening physically. When the distance from the centre increases, the amount of reflection increases – there has been a increase in impedance mismatch. Rotations indicate changes in reactance – moving towards the top indicates an increase in inductance, whereas moving towards the bottom an increase in capacitance.

2.1.2 Impedance matching

Propagating waves that cross from one medium to another will experience only partial transmission, with the remaining energy being reflected. Generally, the impedance of a device will not be the same as the impedance of the transmission line, so not all energy will reach the device. Impedance matching is the process of transforming the impedance of one part of a network to be the same as another. The term that describes the amount of reflection that occurs is the reflection coefficient (Γ), which depends on frequency (f) and is given by

$$\Gamma(f) = \frac{V^-}{V^+} = \frac{Z_L(f) - Z_0}{Z_L(f) + Z_0} \quad (2.6)$$

where Z_L is the load impedance, V^+ and V^- are the incident and reflected voltages, respectively. The proportion of power that is reflected is given by $|\Gamma|^2$; this is related to the reflection S-parameters in the following way.

$$S_{ii} = -20 \log(\Gamma) \quad (2.7)$$

Maximum power is delivered to the load when its impedance matches the line impedance. If we are interested in power being delivered to a quantum system or to be sensitive to certain changes, matching can be very important [32–34]. For instance if one were interested in delivering RF power to a quantum Hall system, without matching, almost all the power is reflected. To see this the impedance of the load needs to be known; in this case the resistance will dominate – it is sufficient as a rough estimate to ignore the effect of reactances [34]. The load impedance is then proportional to the von Klitzing constant $R_k = \frac{h}{e^2} \sim 25.8 \text{ k}\Omega$. Inserting this into equation (2.6) gives $|\Gamma|^2 \sim 0.992$, only a tiny proportion of the power would be delivered to the QH state.

There are many different ways to impedance match a network. So long as Z_L has some non-zero real part, a matching network can always be found [16]. Regardless of the type of matching circuit used – quarter wave transformer, inductive or capacitive matches, stubs – the network will be perfectly matched at a single frequency (and depending on matching type, its harmonics), with decent matching occurring over a particular bandwidth.

2.2 Dilution Refrigerators

Some interesting phenomena, including many quantum states, only occur at temperatures near absolute zero [4]. Getting something cold and keeping it cold can be a lot of hard work [35]. Dilution refrigerators are one of the main experimental tools used to reach temperatures below 1 K [4, 9]. In this section, the principles of a *dry* dilution refrigerator will be outlined. During the course of my PhD I have worked with several 4 K systems, as well as dilution units from two different companies – *Bluefors* and *Leiden*. One of these is much more user friendly (automated) than the other. The less user friendly one is very useful to learn about vacuum systems and how dilution refrigerators work. The *Leiden* fridge will be used as a specific example of dilution refrigerators.

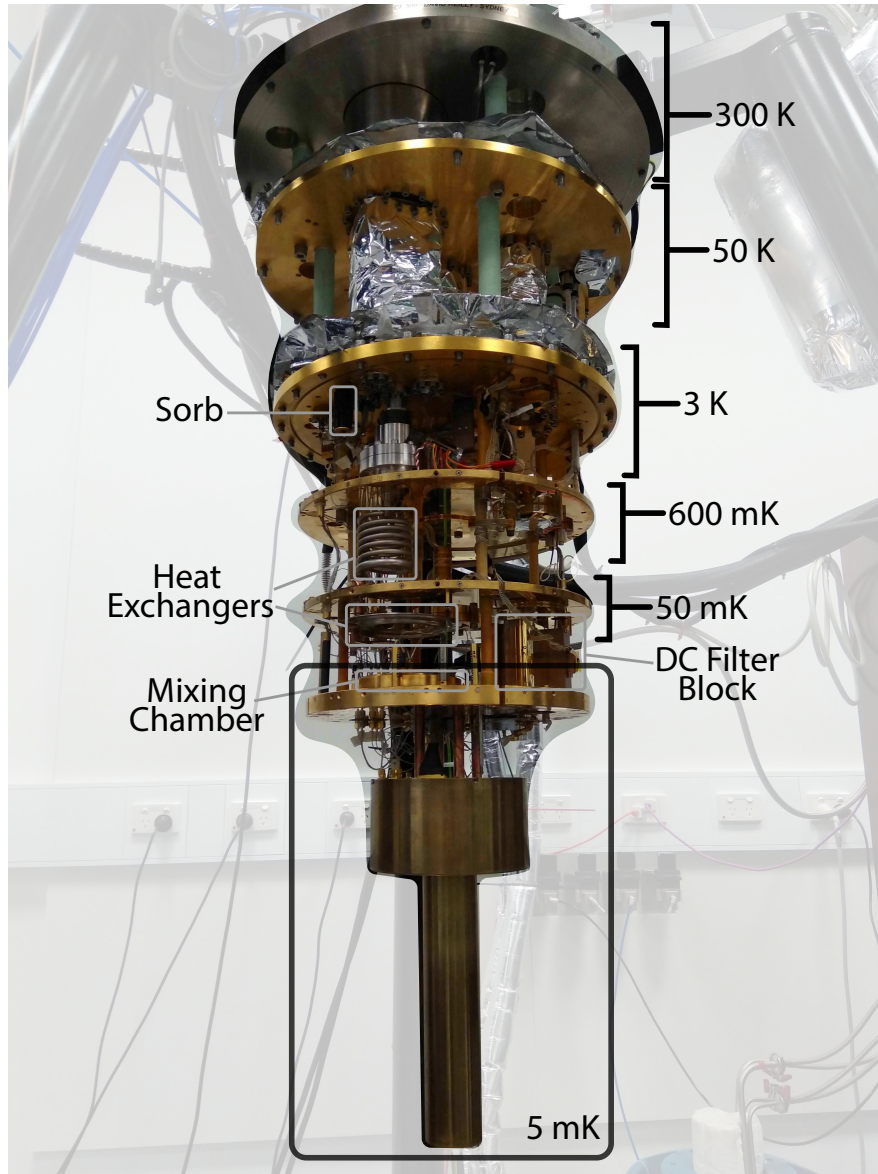


Figure 2.4: Picture of an open *Leiden* dilution refrigerator. The different temperature stages, as well as some important features are indicated.

2.2.1 $^3\text{He}/^4\text{He}$

The coldest part of a dilution refrigerator – the mixing chamber – exploits an interesting physics phenomenon that occurs between the helium ^3He and ^4He isotopes at cold temperatures. Below about 0.7 K certain concentrations of $^3\text{He}/^4\text{He}$ are energetically unfavourable

and two phases with different ^3He concentrations will form [9, 35] (figure 2.5). As ^3He is the lighter isotope, the ^3He -rich phase floats on the dilute phase. By forcing ^3He atoms to cross the phase boundary, cooling is achieved. Temperatures as low as 1.75 mK are possible [36]; however, for temperatures below 2 mK other cooling methods are preferred [4].

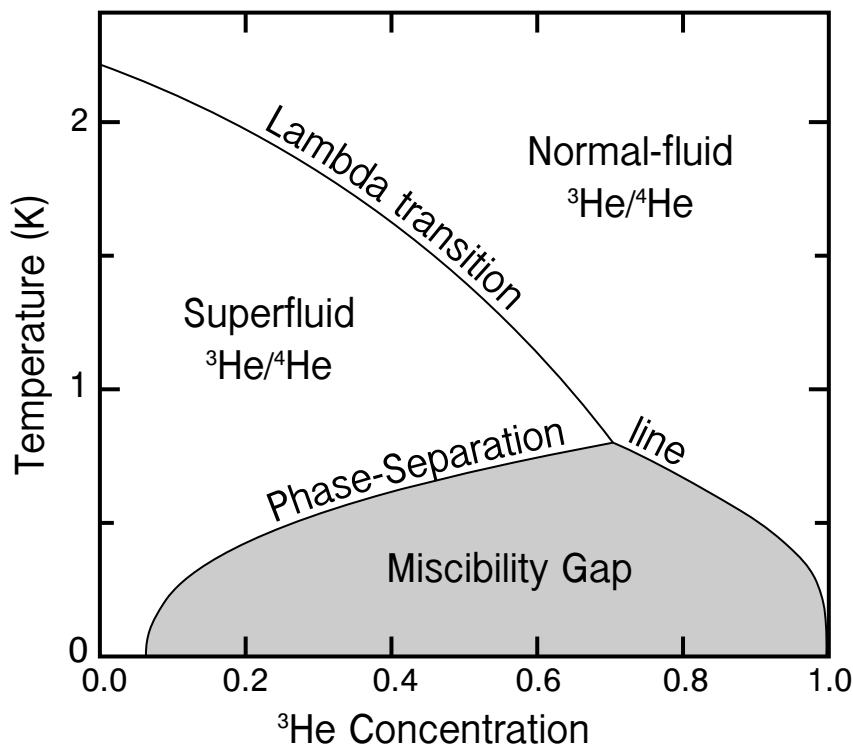


Figure 2.5: Constant pressure $^3\text{He}/^4\text{He}$ phase diagram based off [37, 38]. For dilution refrigerators the important part of this diagram is the miscibility gap.

The miscibility gap occurs due to the difference in zero point energies [9, 39]. The Van der Waals potentials of both isotopes are the same, but ^3He has a smaller mass which results in a larger zero-point motion. As ^3He atoms can be physically closer to ^4He than ^4He to ^4He , the ^3He - ^4He binding is more favourable than ^4He - ^4He . As a consequence of being fermions, the kinetic energy of ^3He increases with number density. This means the effective binding energy reduces as the concentration increases, leading to a point where it is more energy favourable to separate into two phases – producing the miscibility gap.

2.2.2 Fridge design

In order to get cooling from the miscibility gap, ^3He needs to cross the phase boundary. This movement is promoted by pumping on the mixture in the still. Even though the ^3He concentration is about 1%, its higher vapour pressure means that it is the primary component in the vapour phase [9]. Removing ^3He preferentially leads to a build up of osmotic pressure which drives more ^3He in to the dilute phase.

The pumped gas passes through the outer jacket of a Joule-Thomson heat exchanger, cooling the incoming gas and warming the out-flowing gas (figure 2.6). The mixture – primarily pure ^3He – circulates through liquid nitrogen cold traps before entering high impedance (condensing) lines. The mixture is thermally anchored to the 3 K plate which

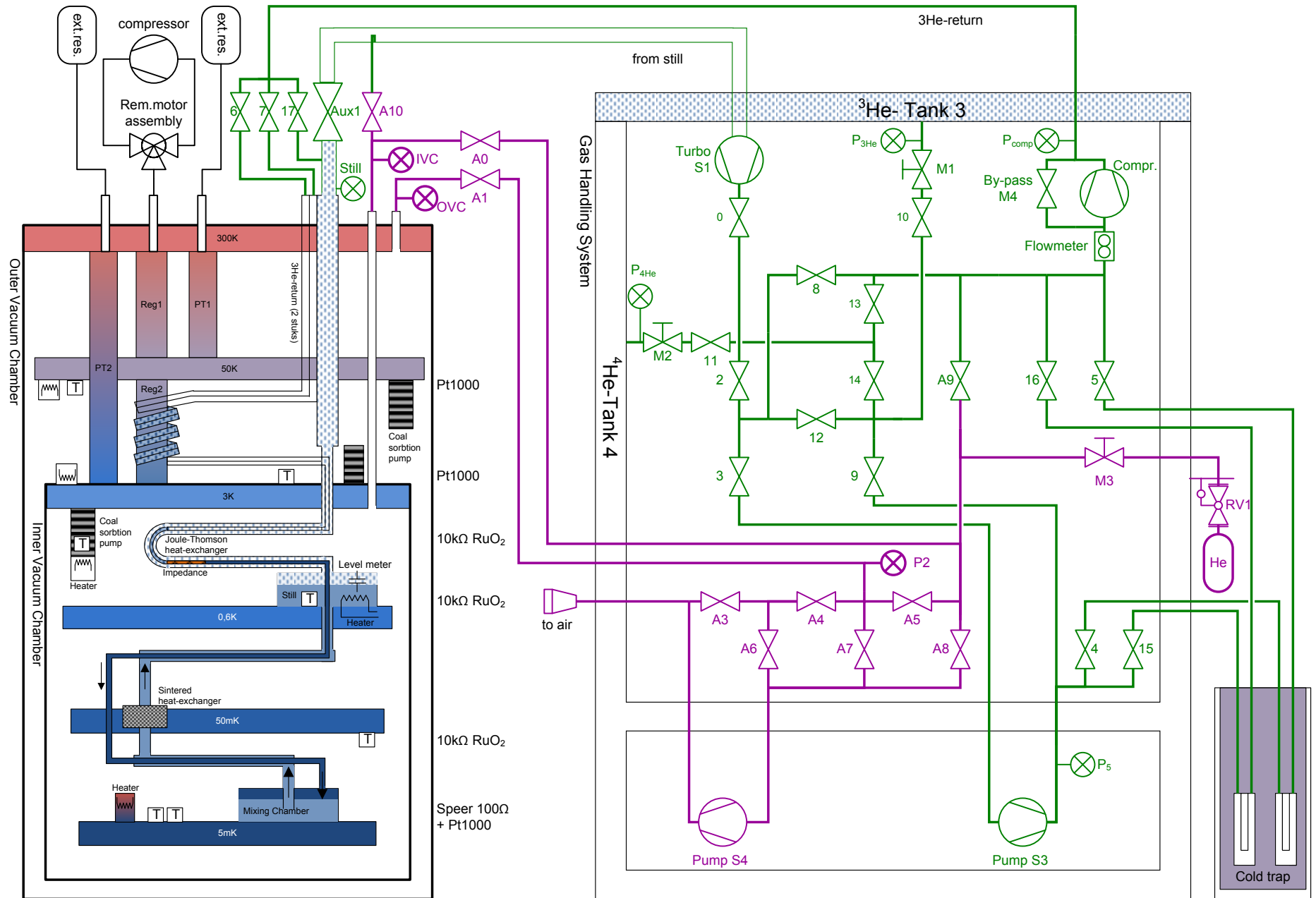


Figure 2.6: Schematic of *Leiden* dilution refrigerator provided by Leiden [41]. The helium is stored in the frame of the gas handling unit. The green circuit is the part of the dilution refrigerator that handles mixture. The purple circuit indicates the “dirty” part of the fridge. This part pumps out the vacuum cans and is used to introduce exchange gas.

cools the mixture to liquid ^4He temperatures. The mixture is further cooled through the Joule-Thomson heat exchanger. Before entering the mixing chamber the temperature is further reduced using a sintered heat exchanger; this type of heat exchanger is necessary as the area requirements for heat exchange are increased below 100 mK due to the Kapitza resistance [4, 40].

The liquid enters the mixing chamber on the ^3He -rich side. From the dilute side of the mixing chamber, liquid is brought up to the still where the cycle continues. Increasing the flow rate of the circulating mixture causes a decrease in base temperature. This leads to a counter-intuitive way of reducing the temperature that a device experiences – applying heat to the still. Applying heat to the still increases the flow rate, thereby reducing the mixing chamber temperature. This works up to a point; above about ~ 0.7 K proportionally more ^4He begins to circulate, which increases the heat load without causing additional cooling.

Generally speaking the different stages of the dilution refrigerator need to be as thermally isolated from each other as possible. For this reason, the inside of a dilution refrigerator will be a vacuum. *Leiden* dilution refrigerators use two vacuum chambers separated by a radiation shield. The vacuum chambers are pumped using the purple part of figure 2.6¹⁰. Within the inner vacuum chamber, there are an additional couple of radiation shields. The stages need to be physically connected to each other. In order to reduce the thermal link, the connection is made by rods with small cross sections produced from materials with low thermal conductivity.

Thermal isolation, while important, is counter-productive when initially cooling the dilution refrigerator. Before the mixture can be circulated, the dilution refrigerator needs to already be very cold. The temperature of the lowest stages – the 3 K plate, still, 50 mK and mixing chamber – should be below 4 K. To thermally connect these stages a small amount of helium is added to the inner vacuum chamber; this acts as an exchange gas. Turning off the heater on the carbon adsorption (sorb) pump causes the gas to be adsorbed to the carbon, allowing the stages to be thermally connected or isolated as needed. In order to cool the bulk of the fridge down to around 4 K a pulse tube cryocooler is used. Pulse tubes have no moving parts in the cold region of the cryocooler; this is important as lubrication at cryogenic temperatures is extremely difficult [9].

The thermometry used in dilution refrigerators is usually based on secondary thermometers. Secondary thermometers need to be calibrated in order to measure the temperature whereas primary thermometers do not. Secondary thermometers are usually used because after calibration, they are normally easier to use than primary thermometers [9]. Three types of thermometers are used in the dilution refrigerator – Pt1000, 10 k Ω RuO₂ and Speer1000¹¹; all of these are types of resistors (figure 2.6). Pt1000 are platinum resistors that have a resist-

¹⁰The inner vacuum can is attached to to 3 K plate. A special type of o-ring made from kapton is needed at these temperatures as regular o-rings tend to break.

¹¹These are carbon-based resistors made by the *Speer* company.

ance of $1000\ \Omega$ at 0°C . Above 50 K these resistors have an almost linear change in resistance as a function of temperature; below 20 K the resistance change is too small to be used for thermometry. For temperatures below 5 K, RuO_2 and carbon resistors are used [42, 43].

The general operation of a dilution refrigerator has been outlined. One thing that has not been mentioned is the way that signals are actually brought to the sample and what consideration needs to be made; specifications on attenuation, filtering, and set up will be covered in chapter 3. The main considerations for wiring are thermal load, thermal anchoring, and thermal expansion rates. Wiring connects the device inside the dilution refrigerator to the equipment at room temperature, meaning that every piece of wiring increases the thermal load of the system. In order to reduce thermal conductivity, DC wires are made of loom wires. In our dilution refrigerator we use constantan (55% copper, 45% nickel), a material whose high electrical conductivity is stable over large temperature extremes. In order to reduce thermal load on the sample, the wires need to be thermally anchored to the stages so they cool down to the same temperature. Since colder temperature stages have less cooling power than the warmer temperature stages, the wires should be anchored to each temperature stage. For the DC wiring, this is achieved by tightly wrapping the wire around thermalising rods. For RF cables the thermal anchoring occurs mainly through the attenuators¹². The temperature change from room temperature (sample loading) to the base temperature in a dilution refrigerator is very large. This means that thermal expansion rates need to be considered. In terms of wiring, this means that tension relieving loops need to be included so that contractions caused by the temperature reducing does not cause damage to the wiring. Thermal expansion rates also need to be considered when designing components for a dilution refrigerator, especially if the component are made up of different materials.

¹²The attenuation between each stage (on the Tx lines) should, for noise reasons, approximately reflect the ratio change in temperature. For Rx lines 0 dB sapphire attenuators are used as any attenuation prior to amplification reduces the signal to noise ratio.

Chapter 3

Methods

The work presented in this thesis focuses on three distinct topological systems: edge magnetoplasmons (EMPs), Josephson junctions (JJs) and nanowires. In this chapter the technical details of the devices and the measurement set-ups will be outlined. Of particular note is the progression in the RF set-up from measuring solely transmission, to measuring reflection and finally to measuring all components of the S -matrix. All of the measurements presented were performed in a *Leiden* dilution refrigerator with base temperature $T < 30$ mK.

3.1 Sample Design

In this section the material parameters and device details are outlined. Technical details of device fabrication are omitted; these can be found in appendix A.

3.1.1 Quantum Hall device (Chapter 4)

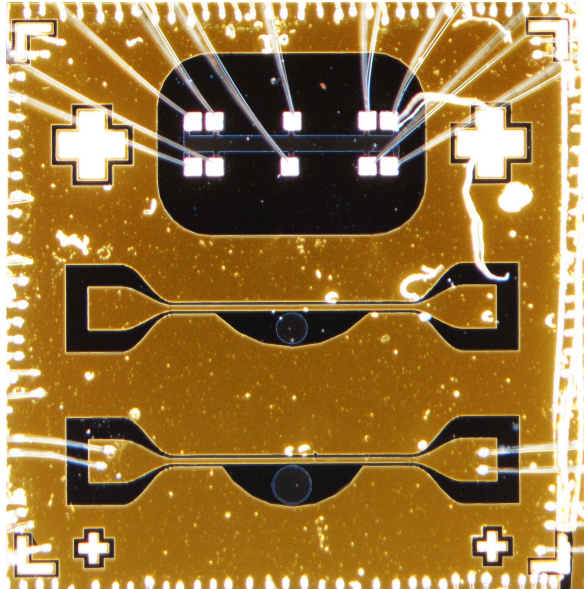


Figure 3.1: Quantum Hall device after bonding. Two droplets were available on this device for testing; one with a radius of $r = 175 \mu\text{m}$, the other with a radius of $r = 145 \mu\text{m}$. The results of measuring the $r = 175 \mu\text{m}$ droplet are presented.

The sample used in chapter 4 consists of two distinct devices: a Hall bar and a quantum Hall droplet coupled to a transmission line. On the device there was a second droplet, however this has not been measured (figure 3.1). The quantum Hall droplets were each

positioned near a different transmission line so that EMPs could be measured. The Hall bar was fabricated in order to verify the sample's electron density and electron mobility.

The circular quantum Hall droplets were defined by etching the GaAs substrate. They are nominally identical, differing only in size – one has a radius of $r = 175 \mu\text{m}$, the other a radius of $r = 145 \mu\text{m}$. The results of measuring the larger Hall droplet are presented in chapter 4. The droplet design – circular droplet next to a transmission line – was based on the Cano configuration [44]. The distance of the droplet from the transmission line affects the strength of the capacitive coupling: closer to the transmission line increases coupling, but can make fabrication more difficult. A compromise was found where the closest approach was $\sim 25 \mu\text{m}$. The transmission line was designed with RF matching in mind. Tapered launch pads were included in order to minimise the effect of impedance mismatches. The distance from the ground plane to the central part of the transmission line was chosen such that the transmission line was matched to 50Ω .

The GaAs material¹ used to make this device has a fairly deep 2DEG (270 nm below the surface)². The etch used to define the structures was similarly deep (350 nm). The extra depth of the etch allows for a relatively thick gold transmission line that remains in the plane of the 2DEG. A thicker film reduces thin film effects; these effects are more pronounced at cryogenic temperatures [45, 46].

For EMP experiments it is helpful to have an independent measurement for the electron density. The electron properties can be altered by fabrication steps and are not consistent across a wafer. For these reasons, the Hall bar was included on this device³. For a Hall measurement a minimum of five ohmic⁴ contacts are required, although typically six contacts are made [47]. The Hall bar on this device contained ten ohmic contacts (figure 3.2). This is because the least successful fabrication step, at least in the QNL, is producing ohmic contacts. The measurement of longitudinal resistance contained nine squares.

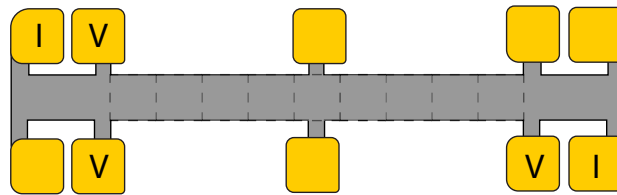


Figure 3.2: Design of the Hall bar. Multiple ohmic contacts were included as a form of redundancy. There were 9 squares over which the resistance was measured.

3.1.2 Josephson junction device (Chapter 5)

The Josephson junction (JJ) used in chapter 5 is a superconductor-normal metal-superconductor (SNS) style junction⁵. It was made from an InAs 2DEG proximitised by superconducting

¹This material was grown by the Manfra group.

²The 'standard' GaAs material used in the lab has a 2DEG depth of 90 nm

³The electron properties of the Hall bar were expected to be similar to the Hall droplet as the chip size was $\sim 1 \text{ cm}^2$ and all parts of the device went through the same fabrication processes.

⁴Ohmic contacts are those that follow Ohm's law. i.e. they form resistive contacts, not diodes.

⁵Fabricated by James Witt and grown by the Manfra group.

aluminium. The gap between the superconducting aluminium was $d = 110$ nm, and the width of the junction was $w = 21$ μm (figure 3.3). A gold gate was positioned directly above the junction; this was used to control the electron density in the 2DEG. The four bond pads allowed four-point measurements to be performed.

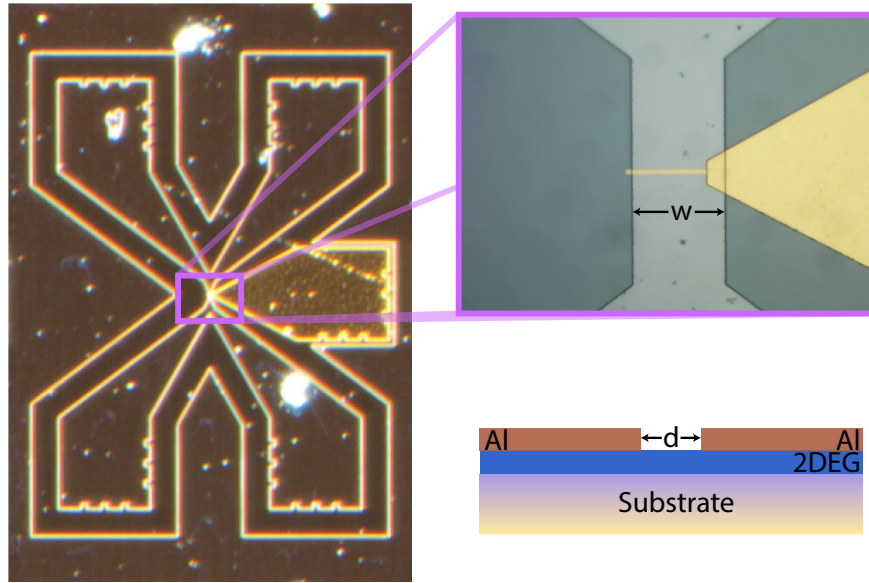


Figure 3.3: Optical image of Josephson junction device. The gold top gate is located directly over the junction. The junction had a width of $w = 21$ μm and a gap of $d = 110$ nm.

The quantum well of the InAs is located 10 nm below the surface (figure 3.4). This distance has been found to provide both high electron mobility and control over the induced gap [48, 49]. The aluminium on top of the InAs 2DEG was grown *in situ* by molecular beam epitaxy (MBE). Epitaxially grown aluminium in conjunction with a shallow 2DEG ensures a clean, nearly transparent interface that allows the proximity effect to occur [48, 50, 51]. Having such a shallow 2DEG means that surface effects become even more important than with regular 2DEGs [49, 52].

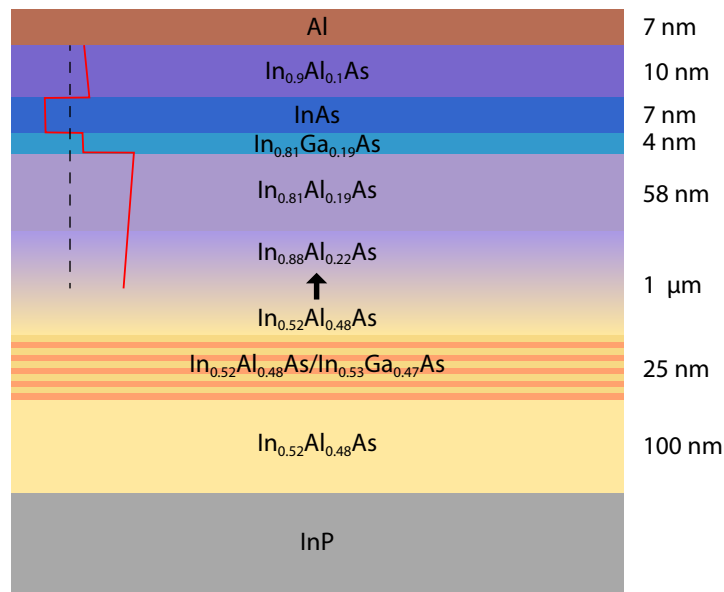


Figure 3.4: InAs 2DEG layer stack (not to scale). The relevant section of the band diagram is indicated in red; the quantum well exists where the red line falls below the black dashed line.

An antenna was used in the experiments to illuminate the JJ. This antenna consisted of a long loop of bond wire on the printed circuit board (PCB). This is not considered good antenna design [16]. However, it was sufficient to provide a small (unknown) amount of RF power across a wide range of frequencies to the device.

3.1.3 Three-Terminal device (Chapter 6)

The measurements presented in chapter 6 are from Al-InAs three-terminal (3T) nanowire devices⁶. Ohmic connections to the two normal terminal are made by Ti-Au contacts. The third (middle) terminal is made of aluminium which both proximitises the InAs – defining the active region – and acts as the third terminal (figure 3.5). The device presented has an active region of $L \sim 1 \mu\text{m}$ and $W \sim 40 \text{ nm}$. Tunnel barriers and a top gate, made from gold deposited on hafnium oxide, are used to provide control over the nanowire.

The material stack is shown in figure 3.6. The InAs nanowire was grown on a GaAs substrate and the shape was defined using selective area growth (SAG) by molecular beam epitaxy (MBE) [53]. Aluminium was deposited *in situ* immediately after semiconductor growth to produce a clean interface [53–55]. The excess aluminium was selectively removed by a wet etch to leave only the aluminium that makes up the superconducting terminal. The ohmic contacts were added before the HfO_2 layer was deposited. The oxide separates the tunnel barriers, and top gate, from the semiconductor.

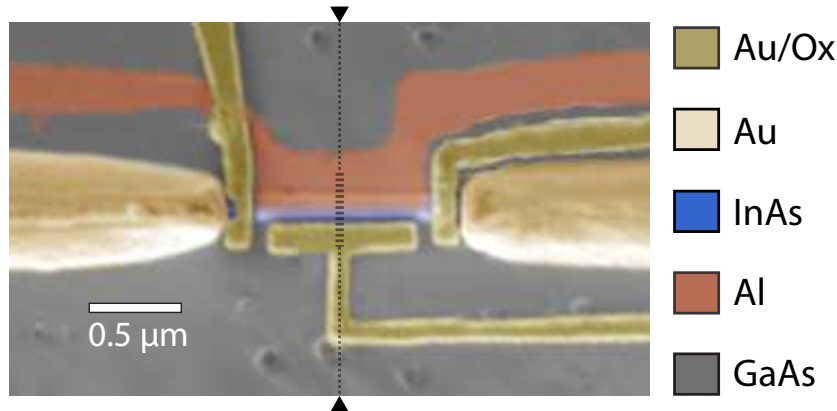


Figure 3.5: False colour SEM of SAG 3T nanowire device. The aluminium both proximitises the semi-conductor and acts as the third terminal.

Despite the device being designed solely for DC, it was successfully measured both at low frequencies (quasi-DC) and at RF. Notably there is no specific RF ground defined on the chip. Also, the traces that lead up to the device vary in size, and have sharp corners – principles to avoid when considering the routing of RF signals [16].

3.1.4 InAs materials

The InAs material used in this thesis were developed to meet the conflicting⁷ material requirements needed to host Majorana quasiparticles [54–60]. Majoranas are particles that

⁶Grown by the Manfra group and fabricated by the Microsoft HCO team.

⁷The main conflict is between the need for a large magnetic field to split the spins (so as to create an effectively spinless material) and the need for superconductors. Typically, large magnetic fields destroy the superconducting state.

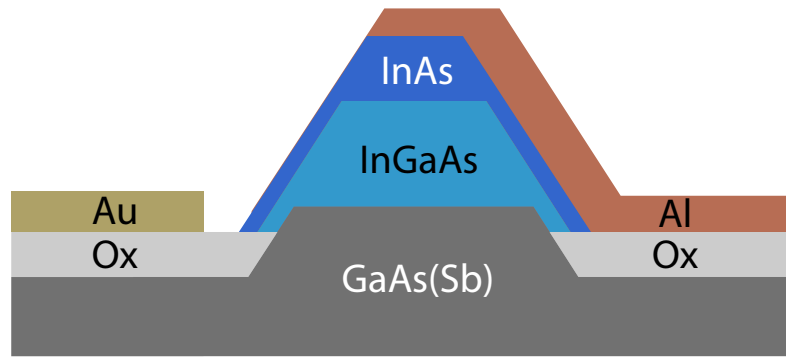


Figure 3.6: Cross section of the Al-InAs SAG nanowire; the line in figure 3.5 indicates where the cross section relates to. The InAs nanowire was grown by MBE on a GaAs substrate. The aluminium was deposited *in situ* at an angle. A hafnium oxide layer was added so that gates could be deposited.

are their own antiparticle; this imposes certain conditions including charge neutrality and spinlessness [61]. Majorana bound states are of particular interest as a building block of high fidelity quantum computers [62, 63]. Specifically, to be able to host Majorana bound states (MBS) a system must be in an effectively spinless, p-wave, superconducting topological phase [58, 59, 64]. The large Landé g -factor and strong spin orbit coupling of InAs, in addition to induced superconductivity of the epitaxially grown Al, means that when these InAs materials are restricted to 1D they may be able to reach this topological phase. However, this thesis does not focus on Majorana experiments, but does utilise InAs materials.

3.2 Low frequency readout

This section details the low frequency (0–100 Hz) setups used in the following chapters. For the most part, true DC measurements were not used⁸; instead the quasi-DC measurement obtained using lock-in amplifiers are presented. The DC filtering was the same in all set-ups: the lines are low-passed filtered by a combination of RC low-pass filters, and *minicircuits* LC low-pass filters with thermalising sapphire meanders [65]. The low-pass cutoff is sufficiently high (> 1 kHz) such that it does not impact low frequency measurements, as such the filtering is not explicitly shown in the following figures.

3.2.1 Quantum Hall devices (Chapter 4)

DC measurements were made of the Hall bar in order to determine the electron mobility, electron density, and when the system was on a quantum Hall plateau. No DC connections were present on the EMP part of the device. A simplified schematic of the measurement setup is shown in figure 3.7.

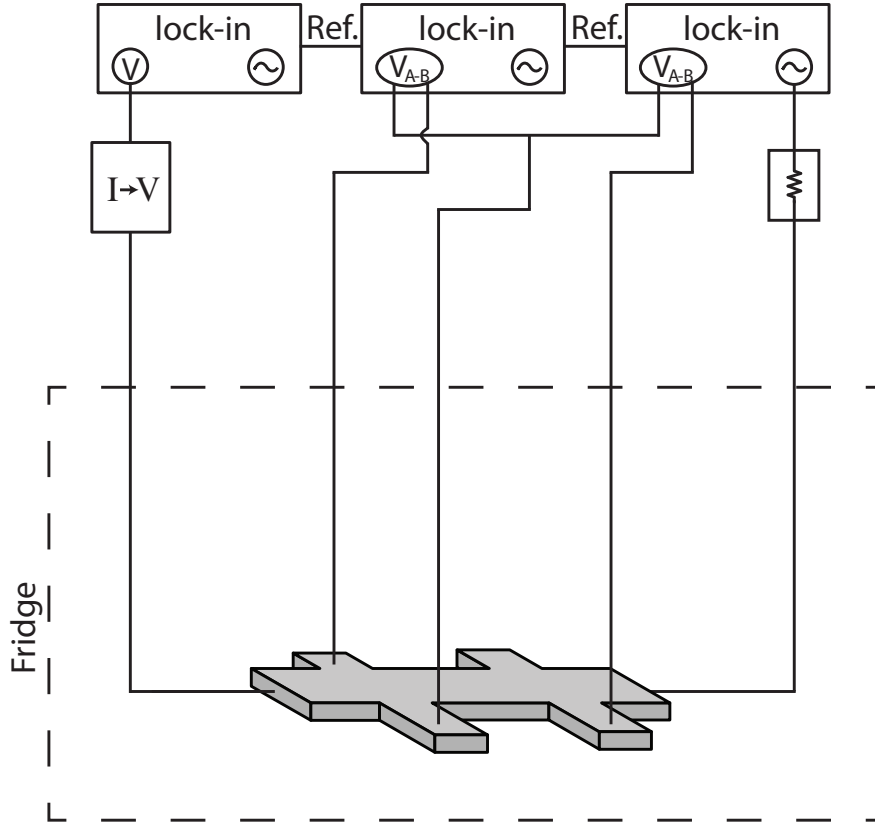


Figure 3.7: Low frequency setup for Hall bar measurements. Three lock-in amplifiers were referenced to the same trigger. One lock-in measured the current, after passing through a current to voltage pre-amplifier. The others measured V_{xx} and V_{xy} . An AC current with frequency < 40 Hz was sourced as a voltage through a $1\text{ M}\Omega$ resistor.

The measurements were made as a function of magnetic field. Three lock-in amplifiers were used to measure the current, longitudinal voltage, and Hall voltage. These lock-ins were phase locked. An AC voltage with a frequency less than 40 Hz was sourced from one of the lock-ins. A $1\text{ M}\Omega$ resistor was used to convert this voltage to a current. The device current

⁸True DC measurements were made, usually as verification that the set-up worked, but are not presented in this thesis.

was converted to a voltage by a current-to-voltage pre-amplifier before being measured. The Hall resistance and longitudinal resistance were calculated from these measurements.

3.2.2 Josephson junction device (Chapter 5)

A simplified schematic of the low-frequency setup used to measure the Josephson junction is shown in figure 3.8. An AC signal of approximately 17 Hz was provided by a lock-in amplifier. This oscillating signal was combined with a DC bias via a voltage divider. The divider had a resistance to ground of $100\ \Omega$, and division ratios of 10:1 and 1000:1. All of the DC voltages, the DC bias, and the gate voltage were provided by an in-house developed multi-channel digital to analog converter (DAC) – called the MDAC [66]. The control line for the top gate had a bias tee; this is where the RF and DC signals were combined.

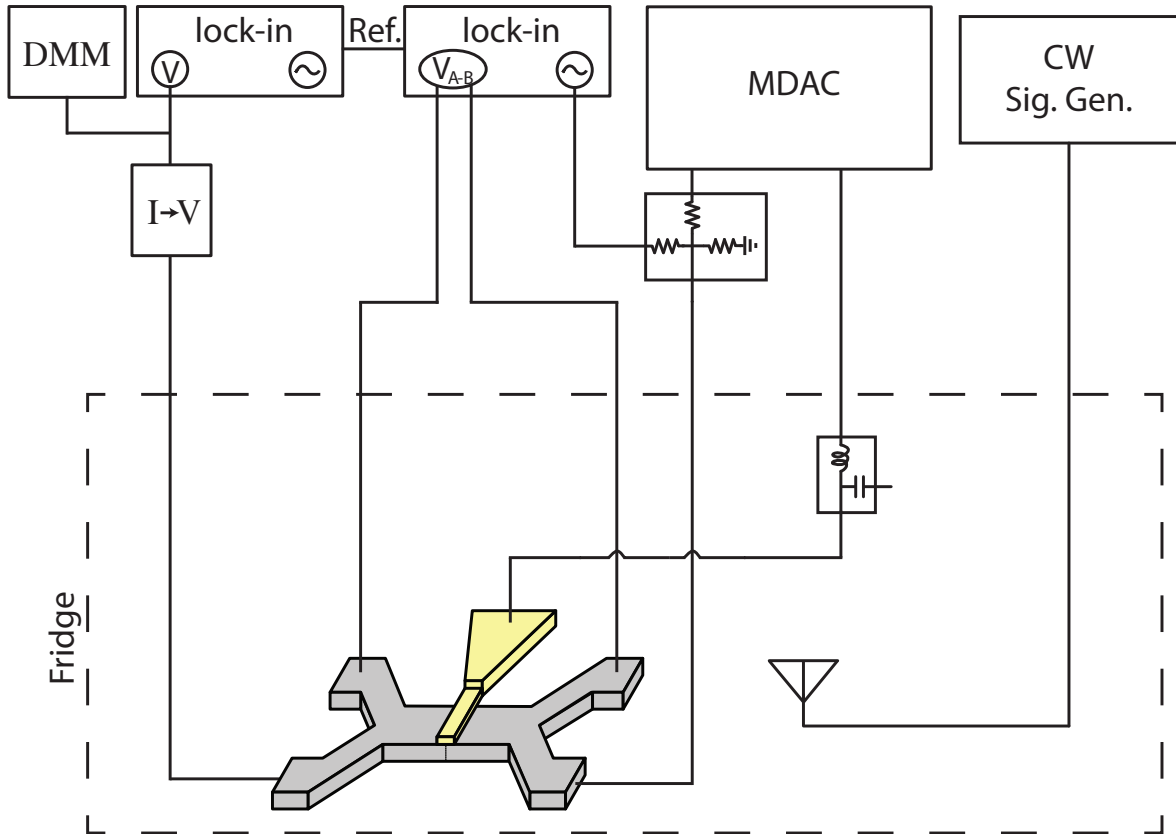


Figure 3.8: Low frequency setup for Josephson junction measurements. The AC signal was combined with DC bias via a voltage divider. The voltage and current were measured by frequency locked lock-in amplifiers. A signal generator was used to illuminate the JJ with RF power.

The device design allowed for four-point measurements to be made of the JJ. Lock-in amplifiers were used to measure the oscillating current and voltage. The lock-ins were both referenced to the same signal, and both measured in voltage mode. The current was converted to a voltage via a current-to-voltage pre-amplifier. A digital multimeter (DMM) was used to measure the DC current. A continuous wave (CW) signal generator was used to provide RF power to the antenna.

3.2.3 Three-Terminal device (Chapter 6)

Transport measurements of the 3T device were made at frequencies DC ($< 40\text{ Hz}$). A generalised setup is shown in figure 3.9. The terminal shown to be measured in the figure is

one configuration, as all of the terminals were excited at various points in the experiment. The bias voltage for the top gate, cutter gates, and DC transport bias were provided by an MDAC. A lock-in amplifier was used to provide the AC signal. This was combined with the DC bias using a resistive voltage divider. Current pre-amplifiers were used to amplify and convert the signals into voltages. These differential voltages were measured by pair of lock-in amplifiers.

For the magnetic field measurements shown, a 1 T-1 T-5 T vector magnet was used⁹. The main axis (5 T) of the magnet was aligned with the long axis (left-to-right) of the nanowire. One of the 1 T components was oriented perpendicular to the plane of the sample; the other 1 T component was in the plane of the sample and perpendicular to the long axis of the device.

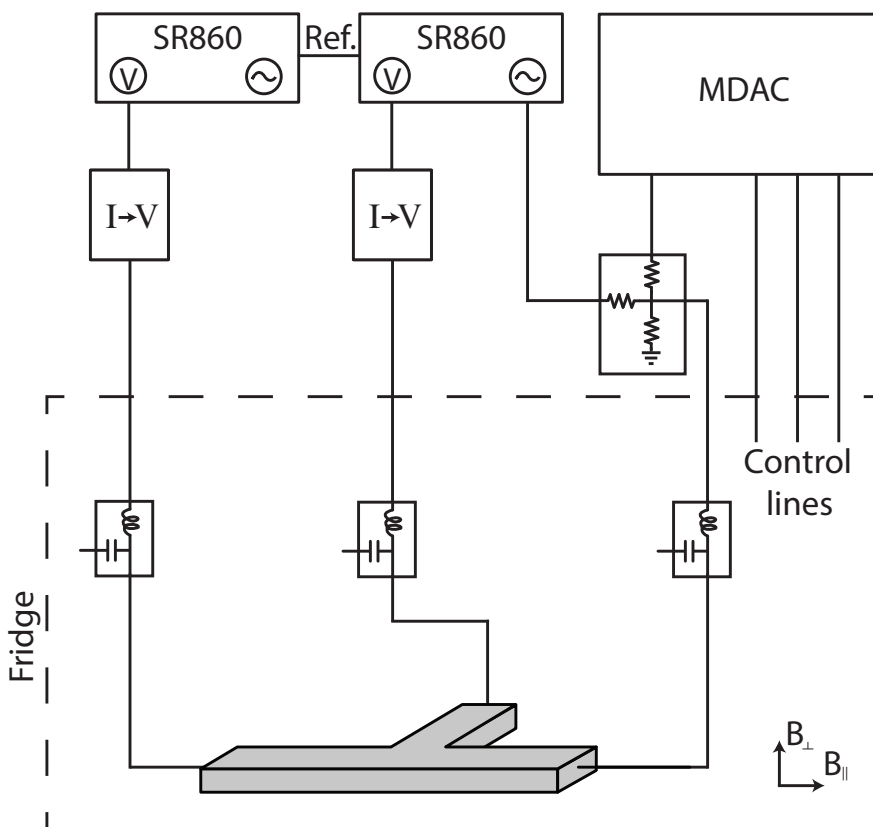


Figure 3.9: Representation of the low frequency setup for the 3T measurements. The AC signal was combined with the DC bias using a voltage divider. The oscillations were measured by lock-in amplifiers after amplification by current pre-amplifiers. All DC signals were provided by an MDAC.

⁹Not all of the vector magnet components were connected or used. One of the 1 T components was not connected to a power supply.

3.3 High frequency readout

This section highlights the high frequency (RF) measurement set-ups used in this thesis. The measurements progress from transmission and reflection to measuring all of the S -parameters of a multi-terminal device. The figures in this section omit the 0 dB sapphire (thermalising) attenuators on the return lines, as well as the DC blocks located on the device side of the amplifiers. Similarly the figures simplify the temperature stages; attenuation is present at all temperature stages, not just at the 4 K and mixing chamber stage.

3.3.1 Quantum Hall device (Chapter 4)

A simplified measurement setup is shown in figure 3.10. The setup is such that only one path could be measured, S_{21} (transmission). The attenuation on the transmit (Tx) path was approximately -40 dB. The return path (prior to the amplifier) consisted of superconducting coaxial cables that were thermalised at each temperature stage by 0 dB attenuators. The cryogenic amplifier provided ~ 35 dB of gain, across a frequency range of 1-15 GHz with a ~ 10 K noise temperature. A vector network analyser (VNA) was used to provide RF power and to measure the return signal; the power after attenuation (at the sample PCB) was ~ -80 dBm¹⁰.

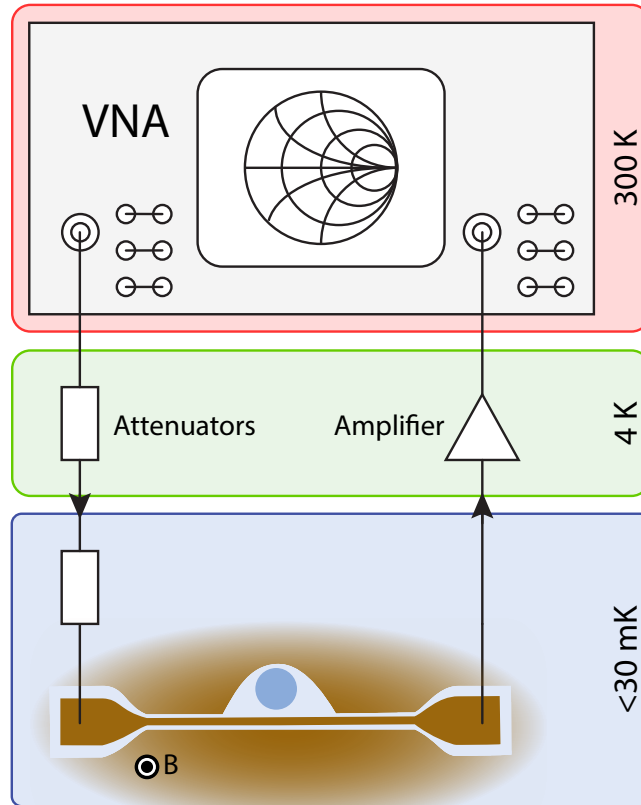
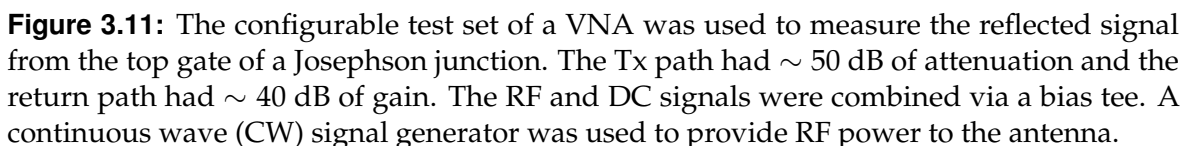


Figure 3.10: High frequency transmission set-up for the measurement of a Hall droplet coupled to a transmission line. The transmitted signal was attenuated at each temperature stage; 20 dB, 10 dB, 3 dB, and 6 dB for the 4 K, still, 50 mK, and mixing chamber respectively. The return path consisted of superconducting coaxial cables thermalised by 0 dB attenuators. A VNA was used to measure the transmitted signal.

¹⁰The power at the device itself would be less than this due to the impedance transformations that occur between it and the coaxial lines.

The JJ was measured in reflection (S_{11}) by a VNA (figure 3.11). The 9 dB coupled arm of a directional coupler was used as part of the transmission (Tx) path. This coupler orientation meant that the returned signal experienced less loss (only the ~ 1 dB¹¹ main line loss), improving the SNR. The RF signal was combined with the DC bias using a bias tee.



¹¹The measurement of loss is for room temperature. The directional coupler is a stripline based directional coupler (*Marki Microwave C09-0R412*); presumably the insertion loss reduces at lower temperatures.

3.3.3 Three-Terminal device (Chapter 6)

The set-up used in order to measure the SAG nanowire device can be seen as a natural extension of the previous setup. The configurable test set of a VNA was used to measure both the transmitted and reflected signals (figure 3.12). Each port of the device was connected to a bias tee – to combine RF and DC signals – and a directional coupler to direct the RF signals. The Tx path was attenuated at each temperature stage with a total attenuation of ~ 50 dB, including the 9 dB coupling loss. All of the coaxial cables on the receive (Rx) path were made of superconducting materials and were thermalised to each temperature stage by 0 dB attenuators.

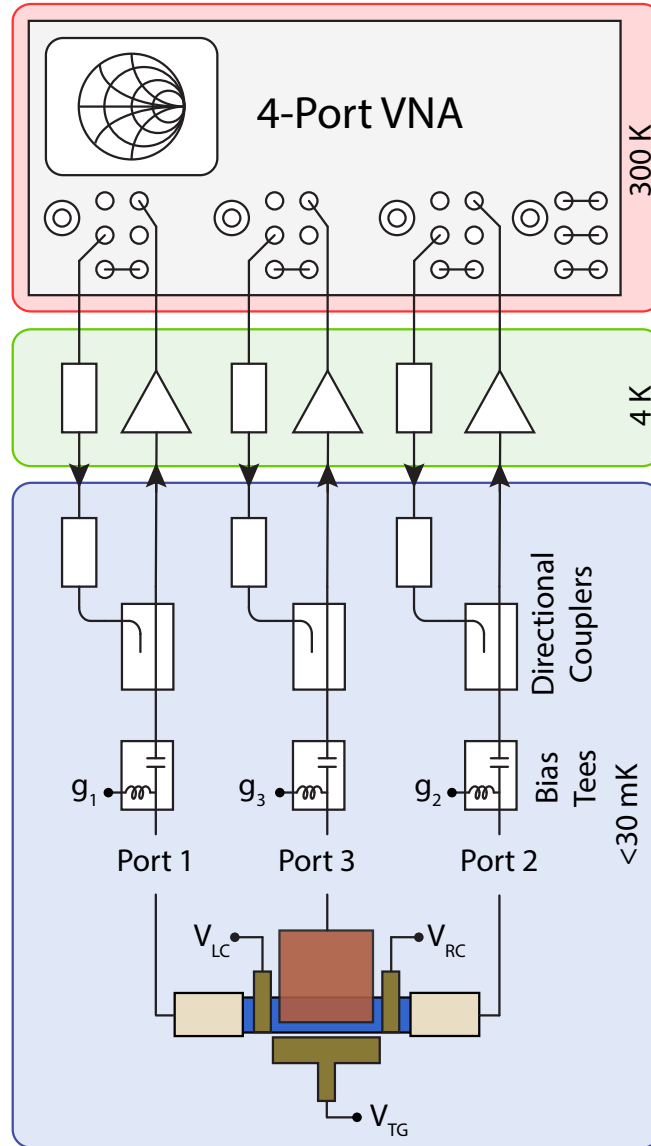


Figure 3.12: The configurable test set of a VNA was used to measure all the S -parameters of a 3T device. The Tx paths had ~ 50 dB of attenuation and the return paths ~ 40 dB of gain. RF and DC signals were combined via a bias tees.

The device-coaxial cable-amplifier system forms a transmission line resonator (TLR). The TLR transforms the device impedance periodically, with a period corresponding to integer multiples of $\lambda/4$ – this corresponded to frequency multiples of ~ 55 MHz. Slight differences in path lengths meant that the resonant frequency of each path was slightly different. The

Q-factor of the TLRs was approximately 30.

Chapter 4

Modelling the Frequency Response of Edge Magnetoplasmons

4.1 Abstract

In this chapter, the different regimes in the dispersion of edge magnetoplasmons (EMPs) are explored using transmission (S_{21}) measurements. The change in frequency over a single quantum Hall plateau is systematically fit using various models from the literature. These models link the change in density and damping that may be present across a plateau; it is found that these models are unable to predict reasonable fits to the data. Finally, a phenomenological model that is able to predict the shape of EMPs on *all* plateaux is presented.

4.2 Introduction

The edge of a Hall fluid can be excited to produce a collective excitation called an edge magnetoplasmon (EMP). Understanding the physical properties of EMPs has led to the development of interesting devices. The chiral nature of EMPs allows non-reciprocal electronic devices such as gyrators, circulators, and isolators to be developed; not only can these be produced, but, due to the low group velocity of EMPs, these devices are inherently smaller than alternate solutions [67–69]. Other interesting applications that have been realised include Mach-Zehnder interferometers [70], frequency converters [71], quantum state transfers [72], and entanglement [73]. Proposals for as-yet unrealised devices such as high impedance transmission lines and RF meta-materials also exist [74, 75].

Although EMPs do not require the quantum Hall effect (QHE) to exist – it is sufficient for a 2D system to be in the classical Hall regime – the weak damping and low group velocity are properties that depend on the system being in the Shubnikov-de Haas (SdH) or quantum Hall (QH) regime. In order to understand EMPs it helps to have a working understanding of the classical Hall effect and quantum Hall effect; the following sections aim to provide this understanding.

4.2.1 Classical Hall effect

To understand the QHE and the experimental results presented in this chapter, it helps to have an understanding of the classical Hall effect. An intuitive way to understand the Hall effect is as follows. In a magnetic field, moving charged particles experience the Lorentz force, which causes them to follow cyclotron orbits. At the edge of a sample, the charged particles cannot complete full circles and charge builds up. This creates a deficit of charge on one side of the sample and an excess of charge on the other, resulting in an electric field (figure 4.1). The electric field builds up until the force of the electric field is equal to the force of the magnetic field and the motion of the charged particles is no longer affected.

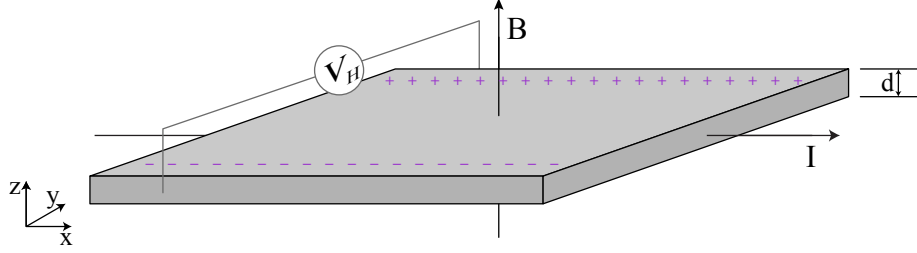


Figure 4.1: Classic Hall effect.

Realistically, there is no ‘build up’ of charge; it happens almost instantaneously. The voltage drop across the sample (perpendicular to the direction of current flow) is the Hall voltage, named after Edwin Hall, who first described the effect in 1879 [76]. The Hall effect is seen in both conductors and semiconductors.

The force experienced by a particle is described by the Lorentz force:

$$\mathbf{F} = e (\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (4.1)$$

When there is no electric field, the force is perpendicular to the motion of the particle; i.e. the same as a centripetal force. The frequency at which the particles trace out a circle is known as the cyclotron frequency, ω_c .

$$\omega_c = \frac{eB}{m} \quad (4.2)$$

In materials, the mass m is replaced by m^* , the effective mass.

The simplest model of charge transport, the Drude model, is able to predict the (classic) Hall effect. A drag term is added to (4.1) which includes a scattering time τ . This can be thought of as the average time between collisions.

$$\mathbf{F} = e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{m\mathbf{v}}{\tau} \quad (4.3)$$

The Hall effect is a steady state solution, so we are interested in the solutions where $\mathbf{F} = 0$.

$$\mathbf{v} - \frac{e\tau}{m} \mathbf{v} \times \mathbf{B} = \frac{e\tau}{m} \mathbf{E} \quad (4.4)$$

This equation can be written in matrix form. Using the definition for the current density $\mathbf{J} = n_0 e \mathbf{v}$ and substituting (4.2), this simplifies to

$$\begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix} \mathbf{J} = \frac{n_0 e^2 \tau}{m} \mathbf{E} \quad (4.5)$$

Identifying the zero field conductivity as $\sigma_0 = \frac{n_0 e^2 \tau}{m}$ and inverting the matrix gives

$$\mathbf{J} = \frac{\sigma_0}{1 + (\omega_c \tau)^2} \begin{pmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{pmatrix} \mathbf{E} = \sigma \mathbf{E} \quad (4.6)$$

and the conductivity matrix

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}^{-1} \quad (4.7)$$

This gives equations that are important experimentally. It provides the relationship between conductivity and resistivity, expanded in (4.8).

$$\begin{aligned} \sigma_{xx} &= \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, & \& \quad \sigma_{xy} &= -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \\ \rho_{xx} &= \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}, & \& \quad \rho_{xy} &= -\frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \end{aligned} \quad (4.8)$$

Explicitly, the conductivity is given by

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1 + (\omega_c \tau)^2}, \quad \& \quad \sigma_{xy} = -\sigma_{yx} = \frac{-\sigma_0 \omega_c \tau}{1 + (\omega_c \tau)^2}, \quad (4.9)$$

and the resistivity by

$$\rho_{xx} = \frac{m}{n_0 e^2 \tau}, \quad \& \quad \rho_{xy} = -\rho_{yx} = \frac{B}{n_0 e}. \quad (4.10)$$

When measuring a Hall bar, these quantities are not directly measured. The current, Hall voltage, and longitudinal voltage are explicitly measured (figure 4.2). One of the very pleasing things about the Hall effect is that the Hall resistance and Hall resistivity are the same quantity.

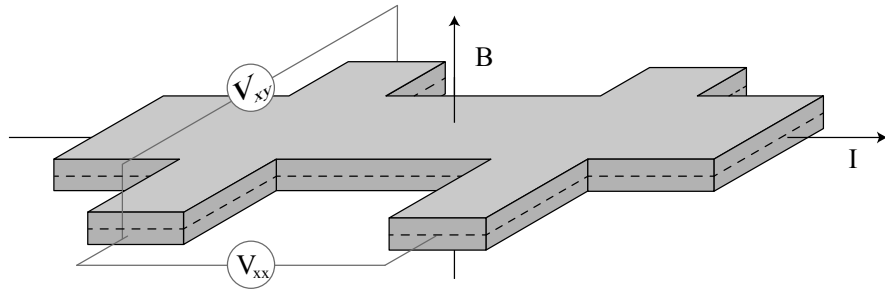


Figure 4.2: Typical Hall bar configuration. The dashed line indicates the 2D electron gas.

4.2.2 Quantum Hall Effect

The integer quantum Hall effect was discovered in 1980 [77]. It occurs in 2D electron gasses (2DEGs), and is characterised by a vanishing longitudinal conductivity ($\sigma_{xx} = 0$) that occurs at the same fields as a quantised Hall conductivity (4.11).

$$\sigma_{xy} = \nu \frac{e^2}{h} \quad (4.11)$$

A remarkable feature of the QHE, apparent in equation (4.11), is that the quantisation does not depend on the material, its shape, size, or in the way the contacts are made [78, 79]. It depends on fundamental physical constants – Planck’s constant h , and the elementary charge e , as well as the filling factor ν . What the filling factor represents should become clear in the following sections. The quantisation is of a microscopic property, and requires disorder to exist. Unsurprisingly, the integer quantum Hall effect has become an integral part of many areas in semiconductor physics [78]. The most obvious applications have been in metrology where – without needing to know the microscopic details of a system – the QHE is able to reproduce the SI unit Ohm, and give high precision measurements of both Planck’s constant and the fine structure constant [78, 80–84].

In the integer quantum Hall effect (IQHE), the filling factor takes on integer values, $\nu = 1, 2, 3, \dots$. In the fractional quantum Hall effect (FQHE) – as the name suggests – the filling factor takes on precise fractional values, $\nu \rightarrow f = \frac{p}{q}$ with $p = \{1, 2, 3, \dots\}$ and $q = \{3, 5, 7, \dots\}$, with some exceptions such as the $f = p/2$ series [82, 85–87]. Although EMPs exist in the fractional quantum Hall regime, and other quantum Hall regimes (such as the quantum anomalous Hall regime [69]) this is not the focus of this work. Here only the IQHE will be discussed in detail.

There are different ways to describe the quantum Hall effect; these include the single electron picture (semi-classical description), edge channels, and gauge symmetry arguments [85, 88, 89]. These models have different levels of complexity. When it comes to EMPs, the edge channel description is the most relevant.

4.2.2.1 Shubnikov-de Haas Oscillations

In a 2D system, prior to entering the IQHE regime, deviations from the classical predictions (equation (4.10)) of the Drude model appear. These deviations appear as oscillations in magneto-resistance; the oscillations are known as Shubnikov-de Haas (SdH) oscillations. Not only are SdH oscillations relevant to EMPs, understanding the origin of these oscillations forms a good starting point for developing an intuitive understanding of the QHE.

Electrons restricted to a 2D plane with a perpendicular magnetic field have an equally spaced energy spectrum given by

$$E_n = \hbar\omega_c \left(n + \frac{1}{2} \right), \quad (4.12)$$

where n defines the Landau level (LL). The density of state (DOS) of such a system is shown in figure 4.3. When spin is included this becomes,

$$E_n = \hbar\omega_c \left(n + \frac{1}{2} \right) \pm \frac{1}{2}g^*\mu_B B, \quad (4.13)$$

where g^* is the effective Landé g -factor and μ_B is the Bohr magneton.

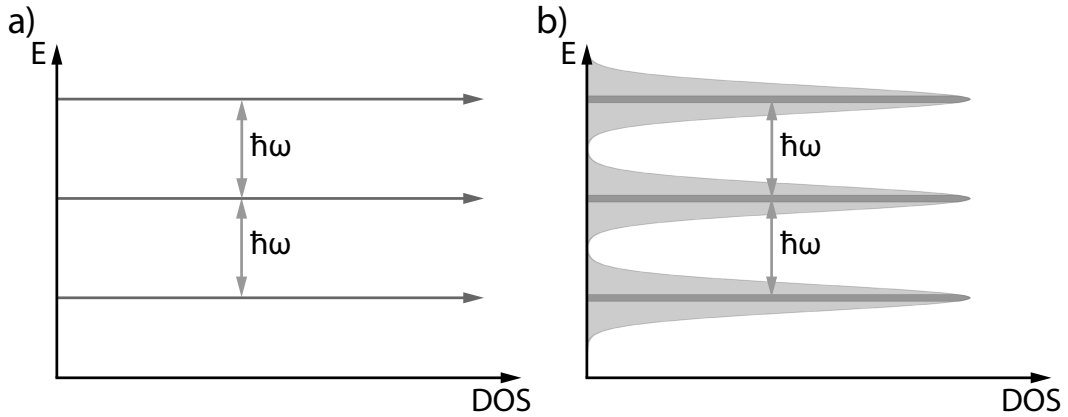


Figure 4.3: a) Ideal DOS of a 2DEG in a perpendicular magnetic field. b) DOS of a 2DEG in a perpendicular magnetic field with impurities. The darker areas represents extended states and the lighter areas localised states.

Oscillations in magneto-resistance occur when the separation of the Landau levels exceeds the broadening of the levels due to scattering ($\omega_c\tau \sim 1$). These oscillations are periodic in $1/B$. This can be seen by equating the parabolic dispersion relationship for free electrons with the DOS of the Fermi surface at zero magnetic field [88].

$$\frac{1}{B} = \frac{n_s n_v}{n_0} \frac{e}{h} \left(n + \frac{1}{2} \right) \quad (4.14)$$

Here n_s and n_v are the spin and valley degeneracy respectively ($n_s = 2$ and $n_v = 1$ in spin degenerate GaAs).

4.2.2.2 Single Electron Picture

The QHE can be viewed as a limit of the SdH effect where the scattering approaches zero, $\tau \rightarrow \infty$. Putting this into equation (4.10) does result in the correct resistivity values for a plateau.

When the Fermi level is in between Landau levels, states are localised everywhere except at the edge of the sample, where due to band bending, there exist extended states (figure 4.4). These extended states are often thought of as quantum analogues of skipping orbits. A more detailed explanation of the origin of edge states will be provided in section 4.2.2.4.

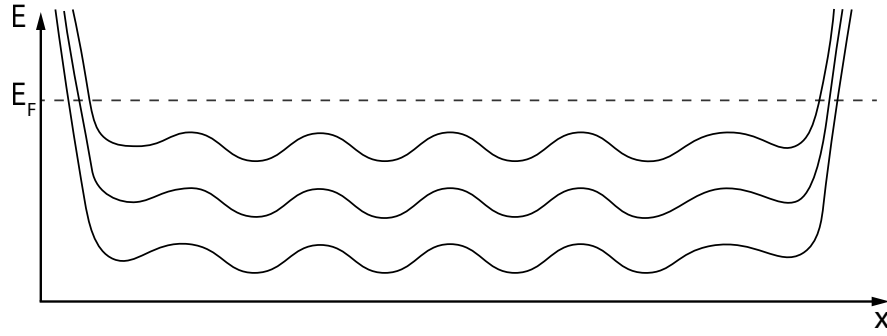


Figure 4.4: Bending of the Landau levels at the edge of a sample. Extended states will exist at the edge of the sample. Disorder in the bulk is represented by oscillations below the Fermi energy.

Whilst the view provided here is something of a simplification, it does help develop an intuitive understanding of what is going on in the IQHE. Scattering across the sample is suppressed and current carrying extended states exist at the edge of the sample. In order to have a plateau, disorder must be invoked; this will be addressed below.

4.2.2.3 Landau gauge description

Mathematically, to describe the above picture one invokes Hamiltonian mechanics. The equations of motion are formulated in terms of potential rather than directly as fields. The gauge chosen must satisfy

$$\nabla \times \mathbf{A} = \mathbf{B} \quad (4.15)$$

The choice of the vector potential $\mathbf{A}$ is not unique; it allows gauge transformations of the form $\mathbf{A} \rightarrow \mathbf{A} + \nabla V(\mathbf{R})$, as $\nabla \times \nabla V \equiv 0$. We are interested in solving for a 2DEG in the xy -plane with a perpendicular applied magnetic field $\mathbf{B} = (0, 0, B)$ (figure 4.2). A usual choice of gauge for solving this is the Landau gauge, $\mathbf{A} = (0, Bx, 0)$ [80, 90–94]. This choice of vector potential breaks rotational symmetry, and translational symmetry in the x direction; this means that although the physics will be invariant under all symmetries, the intermediate calculations will not be. The other particularly convenient gauge is the rotationally invariant symmetric gauge. This choice is more useful when discussing interacting electrons (FQHE) [85, 95, 96].

The Hamiltonian being solved is the usual isotropic effective mass Hamiltonian [90, 91],

$$H = \frac{1}{2m^*} (\mathbf{p} + e\mathbf{A})^2 + V(z) \quad (4.16)$$

where $\mathbf{p}$ is the momentum and $V(z)$ is the confining potential in the z direction. The Shrödinger equation for this system is separable as $\psi(\mathbf{r}, z) = \chi(x, y)\varphi(z)$. $\chi(x, y)$ satisfies

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + \frac{m^* \omega_c}{2} (x - x_0)^2 \right] \chi(x, y) = E_n \chi(x, y), \quad (4.17)$$

where $x_0 = \frac{\hbar k_y}{eB}$ is the center coordinate. Substituting the magnetic length $l_m = (\hbar/eB)^{1/2}$, the solution corresponds to the equation of a harmonic oscillator (4.12) centered on x_0 .

These states are highly degenerate, with the degeneracy increasing as the magnetic field increases. The total number of electrons in an area S is given by $N_e = n_0 S$. The total number of allowable states in a given area is given by $N_\phi = \phi / \phi_0$, where ϕ is the total flux BS and ϕ_0 is the flux quantum. The number of filled Landau levels is given by the filling factor ν .

$$\nu = \frac{N_e}{N_\phi} = \frac{n_0}{B} \phi_0 = \frac{\hbar n_0}{eB} \quad (4.18)$$

Although very many electrons occupy the exact same energy, they do not violate the Pauli exclusion principle. Within each energy band, the position of each electron is unique. The magnetic length introduced earlier is the radius of the area that an electron occupies.

It is generally accepted that disorder is necessary for QH plateaux to exist [82, 95–102]. The disorder gives rise to the extended states, near the centre of the of LLs, and localised states in the tails (figure 4.3 b). The type and amount of disorder is important; too much disorder and no quantum Hall plateaux exist. Long-range disorder tends to lead to a broadening and stabilizing of quantum Hall plateaus, while an increase of short-range disorder leads to an increase of the Landau level width – decreasing the Landau gaps and diminishing the size of the plateau [100]. The precise shape of the broadening is generally described by a Gaussian model [90, 92], Lorentzian model [96, 99], or by numerical models, which generally lead to similar results as the analytical solutions [92, 98]. It is not, however, universally accepted that disorder is necessary to explain plateaux; for example within the self-consistent picture (section 4.2.2.4) the edge-dominant side (low field) of a plateau can be explained solely by the changing density at the edge of a sample [89, 103–105]. It is possible that depletion of the 2DEG in front of ohmic contacts may be sufficient [82], or that disorder is not needed in Wigner crystals [102]. These views that do not require disorder to explain plateaux are, with the exception of the self-consistent picture, irrelevant to the discussion of EMPs.

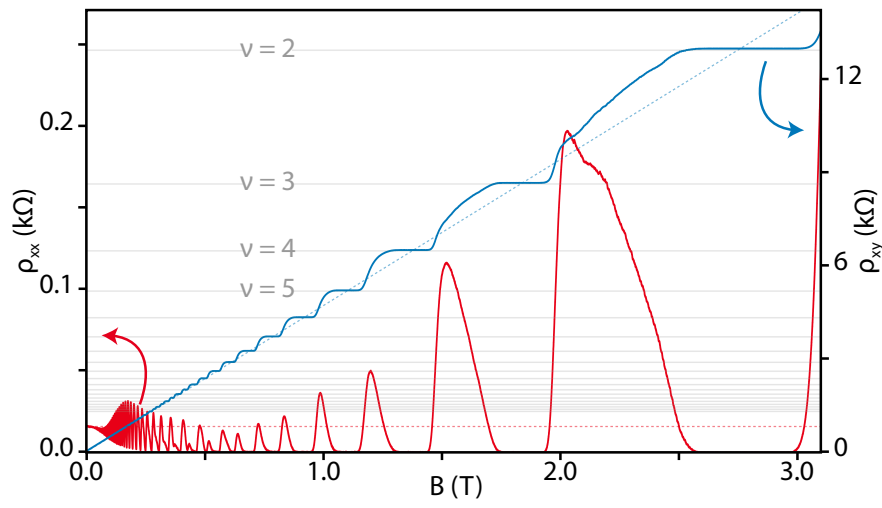


Figure 4.5: Longitudinal (red) and Hall resistivity (blue). Integer filling factors are indicated in grey. The predicted curves of the classic Hall effect (equations (4.10)) are plotted as dotted lines. The classic Hall, Shubnikov–de Haas and integer quantum Hall regime are all visible. There are hints of the fractional quantum Hall effect occurring slightly past 2 T. Below ~ 0.4 T the oscillations are not spin resolved.

4.2.2.4 Edge Channels

The role of the edge of a sample in Hall effect, classical or quantum, cannot be overstated. The Hall effect *cannot* exist without edges. This is also true for the quantum Hall effect; the band bending – mentioned in section 4.2.2.2 – required for extended states to exist occurs at the edge of a sample.

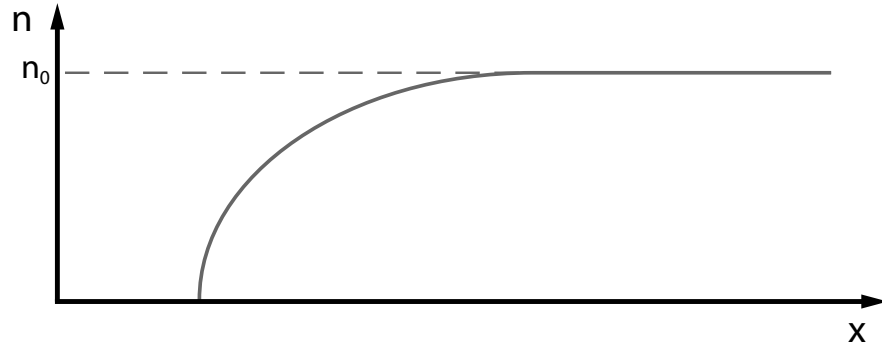


Figure 4.6: Electron density at the edge of a sample at zero magnetic field. Outside of the sample there is no electron density. The electron density should smoothly approach the density of the bulk, n_0 .

The edge of a sample can either be gate defined or be defined by an etch. For a gate defined 2DEG, the electron concentration should go to zero at the edge. In the bulk – ignoring the minor fluctuations from donor concentration – it should reach the constant value n_0 (figure 4.6). Due to the presence of surface charges, this picture can be more complicated at an edge defined by an etch [94, 106]. However, the concept of the density increasing from some low value to the bulk value is valid. The electron density profile changes when in a magnetic field. Not including screening effects, which the previous sections ignore, results in an unphysical, and unstable, step-like density profile [89, 94, 104]. When electron-electron interactions are ignored it becomes difficult to explain many experimentally observed features

of the QHE [100, 107, 108]¹.

Screening results in the concept of compressible and incompressible strips (ICs). Compressible strips (CSs) are regions where electrons are able to redistribute, whereas incompressible strips (ISs) are regions where electrons are unable to redistribute. ICs occur when the Fermi energy falls between two LLs. As the electrons are unable to redistribute, they do not contribute to screening (i.e they act like an insulator). CSs occur when the Fermi energy is pinned within a LL. The electrons are able to redistribute; they have metallic properties. This representation is shown in figure 4.7.

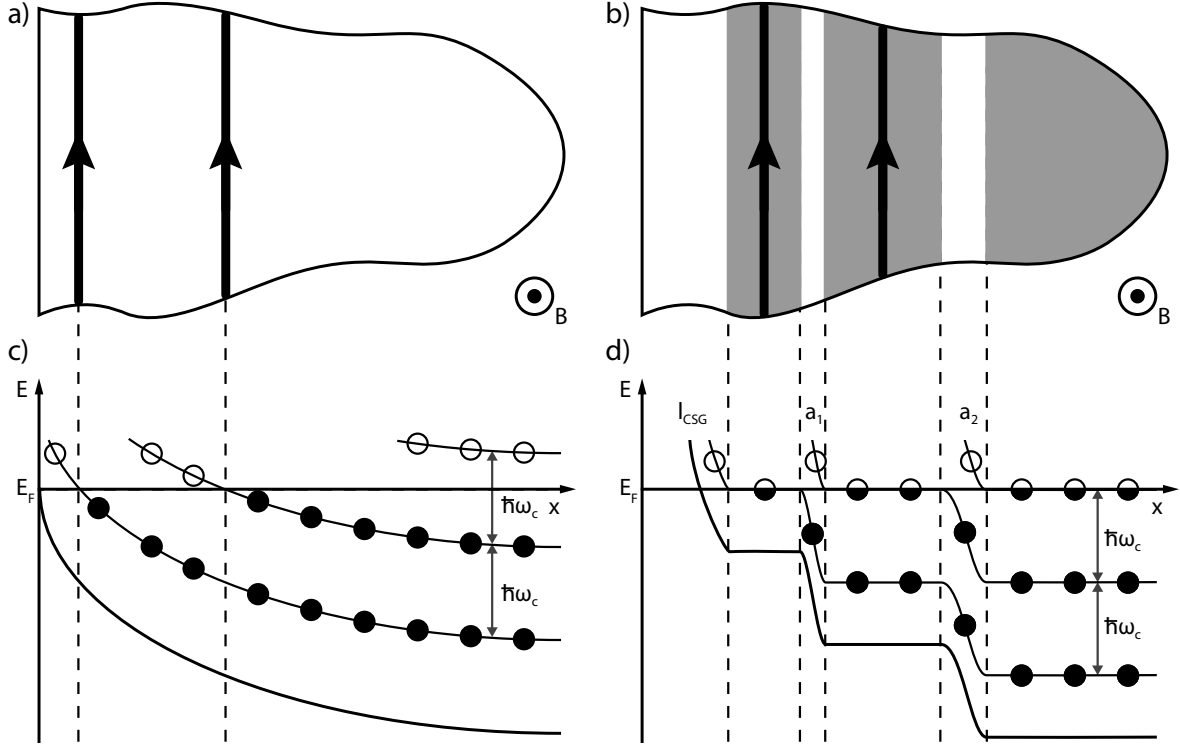


Figure 4.7: Comparison between the single electron picture (a & c) and the self-consistent picture (b & d). Both representations shown here ignore the effects of spin. a-b) shows a top down view of the sample edge. Arrows indicate the direction of electron flow. Shaded areas indicate the compressible regions. c-d) shows energy levels at the edge of the sample. The dots represent the occupied (solid dots), unoccupied (empty dots), and partially filled (partially filled dots) electronic states.

Although the concept of screening was not introduced by Chklovskii, Shklovskii and Glazman (CSG) [89] – the first works are [109, 110] – the analytical electrostatic model they produced is generally cited as *the* model for understanding edge channels. There have been a plethora of models that have come out after CSG; these have either challenged some of the assumptions used, or used more sophisticated methods to calculate the edge profile [80, 92, 94, 103, 105, 106, 111, 112]. Generally speaking, these models agree qualitatively with the CSG model but disagree quantitatively. Therefore the CSG is sufficient to explain the concepts of edge channels, even if some of the values it predicts are no longer accepted.

The conceptual starting point for the CSG has already been mentioned. In a gate-defined 2DEG, the electron concentration should smoothly transition from the bulk value n_0 to $n = 0$

¹Additionally, in order to make sense of the FQHE electron-electron interactions are required.

near the edge. In the CSG model the unmodified ($B = 0$) electron density, beyond the depletion depth ($x > l_{CSG}$), is given by

$$n(x) = \sqrt{\frac{x-l}{x+l}} n_0 \quad (4.19)$$

The region where the electron density is zero is the depletion depth (4.20), which depends linearly on V_g , the gate voltage used to define it. For an etched sample this is assumed to be the mid-gap energy (-0.75 V for GaAs).

$$l_{CSG} = \frac{V_g \epsilon}{(2\pi)^2 n_0 e} \quad (4.20)$$

Although the assumption used for etched samples, as well as the linear scaling with gate voltage, has been shown not to hold [94, 106], the concept of the density increasing from some low value to the bulk value is accepted. At strong magnetic fields alternating compressible and incompressible regions form. The position of the IS is determined by finding the position where the $n(x)$ would give an integer filling factor (rearranging equation (4.18)). The size of the k^{th} IS is given by

$$a_k^2 = \frac{2\epsilon\Delta E}{\pi^2 e^2 \left. \frac{dn}{dx} \right|_{x=k}} \quad (4.21)$$

where ΔE is the energy gap; in the CSG model spin is ignored so this is $\hbar\omega_c$. $\left. \frac{dn}{dx} \right|_{x=k}$ is the spatial derivative of electron density profile evaluated at the centre of the strip. It has been found that the CSG model tends to over-estimate the size of these strips; this is because more realistic profiles for the electron edge density tend to the bulk value more quickly [80, 92, 94]. Again, the trends observed in the CSG model are reasonable.

As the magnetic field increases towards a (bulk) integer filling factor, the innermost IS moves towards the centre of the sample and increases in size. In the absence of disorder, when the integer filling factor is reached the k^{th} IS leaves the system (figure 4.8). Only the position and width of the innermost (highest k value) need be considered, as ICs narrower than the Fermi wavelength, λ_F , are expected to become transparent due to scattering between neighbouring compressible regions [82, 100, 108]. Experimental evidence showing the movement of the k^{th} incompressible strip correlates extremely well with these predictions [82, 113–117]. These experiments have typically used scanning force microscopy on narrow Hall bars.

Multiple ICs will be present in the system when $\nu > 1$ (or $\nu > 2$ for systems where spin is not resolved). Only the IC furthest from the edge – corresponding to the largest filling factor – moves significantly across a plateau, and needs to be considered. This is because ICs that are narrower than the Fermi wavelength are transparent [108].

The low field side of a plateau is often referred to as the edge-dominant side [82, 103].

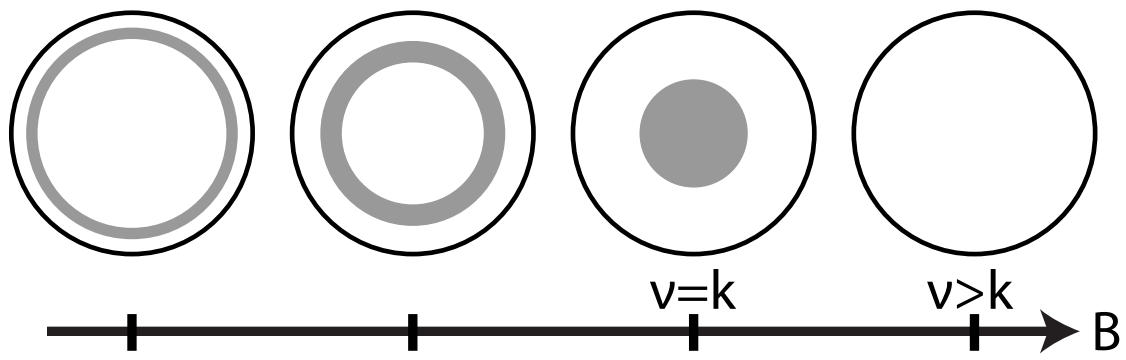


Figure 4.8: CSG picture of incompressible strip development over a quantum Hall plateau. As the magnetic field increases, so too does the size of the incompressible strip, and the centre of the strip moves away from the edge. This continues until the bulk filling factor reaches an integer value; beyond this point the incompressible strip leaves the system.

The high magnetic field side of a plateau is referred to as the bulk-dominant side, as the persistence of incompressible regions is in the bulk and relies on disorder. The onset of the plateau in the edge-dominant side is independent of size of the sample; this is not the case for the bulk-dominant side of the plateau [103]. Widths of ICs – and as a consequence the plateau width – decrease as temperature increases [100].

4.2.3 Edge Magnetoplasmons

In physics we are used to waves having quanta of oscillation. Light wave quanta are photons, sound wave quanta phonons, and for plasma oscillations they are called plasmons. Simply put plasmons are “just” the quantum of oscillation of a plasma. Unsurprisingly, this simple definition hides a wealth of interesting phenomena and contains an entire field of research – plasmonics [118].

The three main types of plasmons – volume/bulk, surface, and localised surface – can be thought of as being charge density waves allowed to propagate with different dimensionality [119]. The surface plasmon, the 2D plasmon, is the most similar to EMPs [120]. It is a charge fluctuation that propagates along the interface between a dielectric and a metal (figure 4.9). The strength of the fields decay exponentially with distance from the interface. In lossy media, the amplitude of the wave also exponentially decays with propagation distance. Only one propagation mode is allowed the transverse magnetic (TM) mode; the transverse electric mode (TE) is not allowed.

A magnetoplasmon by definition is a plasmon that depends on the magnetic field; i.e. a collective oscillation that in some way depends on the cyclotron frequency (equation (4.2)). EMPs are a special type of magnetoplasmon that occur at the edge of a 2D electronic system. The motion of charges that lead to an EMP can be understood in the following way. When a charge density fluctuation occurs, a restorative electric field is produced. The movement of electrons subject to this field causes an imbalance of charge in the opposite direction. This leads to a periodic density wave that moves around the edge of a sample (figure 4.10). This moving charge density is the edge magnetoplasmon.

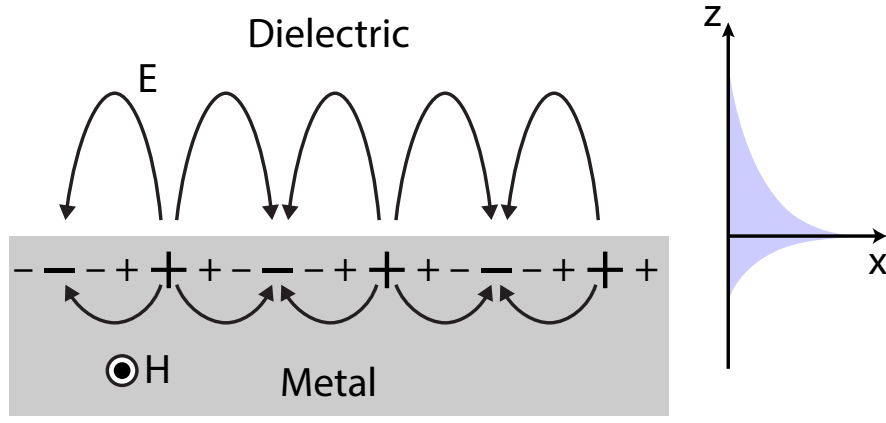


Figure 4.9: A surface plasmon is an electron density wave that occurs at the interface of a dielectric and a metal. The electric field density away from the interface is shown on the right. It has an exponential decay with the field penetrating further into the dielectric.

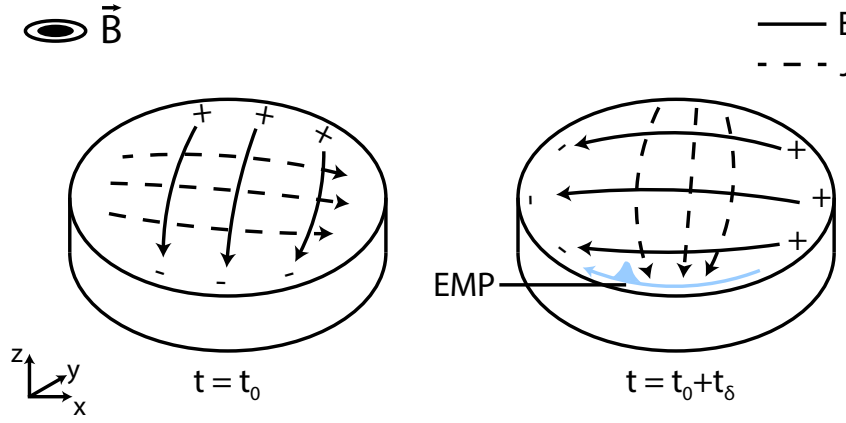


Figure 4.10: Distribution of charges, electric field E , and current density J of the fundamental EMP mode. At time $t = t_0$ a charge perturbation occurs. At $t = t_0 + t_\delta$, due to the drift velocity of electrons in a magnetic field, the charge perturbation rotates.

The speed of propagation of EMPs is dependent on the drift velocity of the electrons. At the edge of the sample this is approximately E/B . This means that the speed of propagation can be very slow, up to $1000\times$ less than the speed of light in vacuum [121, 122]. The speed is modified by screening; a gated EMP travels slower than an ungated one [121]. In the Hall effect electrons only travel in a single direction, so EMPs are inherently chiral.

Strictly speaking EMPs require only the Hall effect, not the QHE, in order to be observed. However, the damping of EMPs is proportional to the Hall conductivity [120]. As such the extremely small damping associated with EMPs is only observed when $\sigma_{xx} < \sigma_{xy}$; during SdH oscillations and on QH plateaux. This means that for large devices, only EMPs in these regimes are observed.

The frequency of oscillation of all types of magnetoplasmons depends on the magnetic field. For bulk magnetoplasmons, a gap opens in the spectrum equal to the cyclotron frequency [120, 123]. The dispersion relationship for bulk magnetoplasmons is given by

$$\omega_{MP} = \sqrt{\omega_p^2 + \omega_c^2} \quad (4.22)$$

where ω_p is the plasma frequency given by

$$\omega_p = \sqrt{\frac{n_0 e^2}{\epsilon_0 m^*}} \quad (4.23)$$

ϵ_0 is the vacuum permittivity. For semiconductors this is often multiplied by the material's effective permittivity ϵ^* .

EMPs have a different spectrum (figure 4.11). The dispersion relationship of EMPs was originally derived in [120]; it has a logarithmic dependence which accounts for the electrostatic screening [123]. The dispersion relationship is given by

$$\omega_{EMP} = \frac{q_y \sigma_{xy}}{2\pi \epsilon_0 \epsilon^*} \left(\ln \frac{2}{|q_y| l} + C \right) \quad (4.24)$$

where q_y is the quantised wavenumber. This can be expressed by

$$q_y = \frac{2\pi}{P} n \quad (4.25)$$

where P is the perimeter of the sample and n is a quantum number that defines the number of whole wavelengths that fit around the sample (i.e. it is the mode number). A circle is a particularly convenient shape to use in this equation as the perimeter of a circle is $2\pi r$, meaning q_y for a circle is simply $q_y = n/r$. It is worth noting that q_y appears both as part of, and in front of, the logarithmic term. This means that the harmonic frequencies are not quite integers of each other. l defines how wide the strip that the EMP exists in is. It is often considered to be the distance of the EMP from the edge [68, 121, 124, 125], but this is true only under certain conditions. It is related to the plasma and cyclotron frequencies, and is given by

$$l = \frac{n_0 m^*}{2\epsilon_0 \epsilon^* B^2} = \frac{\omega_p^2}{2\omega_c^2} \quad (4.26)$$

The constant C is a term which relates to the 'edge hardness' of the sample. Essentially this describes the steepness of the sample edge; it depends on how the sample is defined (gate or etch defined) [126]. This constant always takes on a value close to 1 [120, 126].

It is worth noting that although EMPs are supported by the 1D edge channels of a quantum Hall fluid, and only exist in 2D systems, the fields exist in 3D. This can be seen in equation (4.24) where the effective permittivity (ϵ^*) is explicitly a spacial average of the surrounding media.

When an EMP propagates between the boundary of two regions with different conductivities, it is called an inter-edge magnetoplasmon (IEMP) [127]. The dispersion relationship for an IEMP is essentially equation (4.24), where σ_{xy} is replaced by $\Delta\sigma_{xy}$ and n_0 by Δn_0 . The acoustic edge magnetoplasmon (AEMP) is fundamentally different, as it oscillates not only

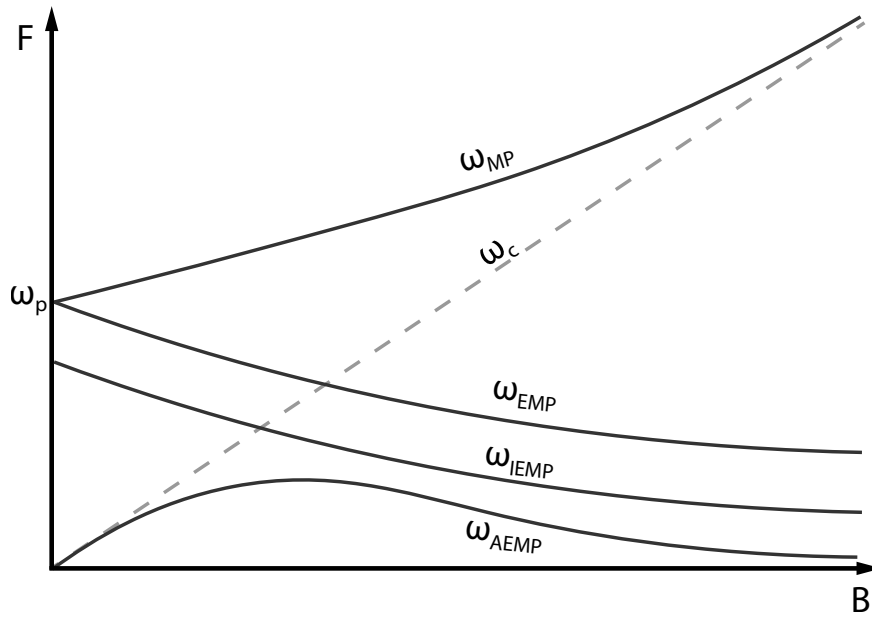


Figure 4.11: Dispersion spectrum of the lowest order spectra of various types of magnetoplasmons. Bulk magnetoplasmons have a frequency higher than the cyclotron frequency (ω_c) and in the long wavelength limit asymptotically approach ω_c . The AEMP and IEMP typically are lower in frequency than than the EMP.

along, but also across, the edge of a 2D system [128–131]. The dispersion of these different types of magnetoplasmons is summarised in figure 4.11.

EMPs and ISs are sometimes linked; this has primarily been in reference to time-of-flight measurements [121, 122, 125, 132–137]. The distinct crescent-like shape of the EMPs and the link to incompressible strips in the frequency domain has not been discussed (although mentioned in ref. [67]).

In this chapter, different regimes of the dispersion of EMPs are explored. The way that frequency and damping changes on a single QH plateau is observed and the possible links to ISs are discussed.

4.3 Results and Discussion

A quantum Hall droplet was defined in a GaAs wafer by etching the mesa. The droplet was in the shape of circle and had a radius of $r = 175 \mu\text{m}$. This was capacitively coupled to a co-planer waveguide (CPW) with characteristic impedance of $Z_0 = 50 \Omega$. RF signals were produced and measured by a vector network analyser (VNA). Dips in the magnitude of the S -parameters were expected to coincide with energy being coupled in to the Hall droplet.

A Hall bar was fabricated on the same chip; this was used to extract the electron density and mobility. The longitudinal and Hall resistances were measured and used to extract the electron density and electron mobility. The density was extracted using equation (4.10); this resulted in a value of $n_0 = 1.33 \times 10^{11} \text{ cm}^{-2}$. The peak mobility was calculated to be $\mu = 3 \times 10^6 \text{ cm}^2/\text{Vs}$. The Hall bar measurement used can be seen in figure 4.12 b. The electron density of the QH droplet itself was found to differ slightly from the Hall bar measurement, $n_0 = 1.36 \times 10^{11} \text{ cm}^{-2}$.

For more details and device images see the methods chapter (chapter 3.1.1).

4.3.1 Global Dispersion

The S_{21} transmission spectrum of the CPW as a function of magnetic field is shown in figure 4.12 (a). These data were taken on an uncalibrated VNA; in order to remove the frequency dependent response of the set-up (> 10 dB over the frequency range shown), the zero-field line ($B = 0$ T) was subtracted. Without this subtraction, the small changes (< 0.5 dB) in transmission caused by the changing magnetic field are not visible.

The Hall resistance (R_{xy}) and longitudinal resistance (R_{xx}) are shown in figure 4.12 (b). The values of Hall resistance where the filling factor is an integer are indicated by faded grey lines. As expected for the IQHE, clear plateaux with integer filling factors are observed to coincide with regions where there is no longitudinal resistance. Hints of fractional quantum Hall states are seen in these data at $B \sim 2.1$ T. In these data there is a clear progression from the SdH regime to the quantum Hall regime.

Dips in transmission (blue colours) occur when the frequency of the applied microwave excitation is resonant with the EMP mode. In the QH regime, these dips only occur when the DC data shows a plateau. Between plateaux – where $R_{xx} > 0 \Omega$ – energy does not couple into the Hall droplet. In the SdH regime there are no clear regions where the transmission is unaffected.

The dispersion relationship is plotted over the data in red; the experimental values of σ_{xy} extracted by the Hall bar measurements were used. The equation used in the fit is a slightly modified version of equation (4.24). This is because the equation derived in [120] does not fit well across the entire regime explored. Depending on the fit parameter ϵ^* , the fit can be made to fit well either in the low field regime or the high field regime, but not both simultaneously. In order fit well across the entire data, l needs to be replaced by $\sqrt{l^2 + h^2}$, where h is the lateral depletion length [138].

$$\omega_{EMP} = \frac{q_y \sigma_{xy}}{2\pi\epsilon_0\epsilon^*} \left(\ln \frac{2}{|q_y| \sqrt{l^2 + h^2}} + C \right), \quad (4.27)$$

l is often considered to represent both the strip width *and* the extent of the EMP away from the edge of the sample. Usually there is little need to distinguish between these descriptions; however as discussed in section 4.2.2.4, there exists a region away from the edge of a sample where there is no electron density. This depletion region has a constant width, regardless of magnetic field; it does however depend on sample details. Typical lengths are of order hundreds of nanometers to a few micrometers. This distinction has not always been recognised in literature where some authors continue to use equation (4.24) in regions where it is not completely appropriate. The different regimes can clearly be seen in figure 4.13. At high magnetic fields $l \ll h$; at low magnetic fields the opposite is true. The different regimes were used in order to separately extract h and ϵ^* .

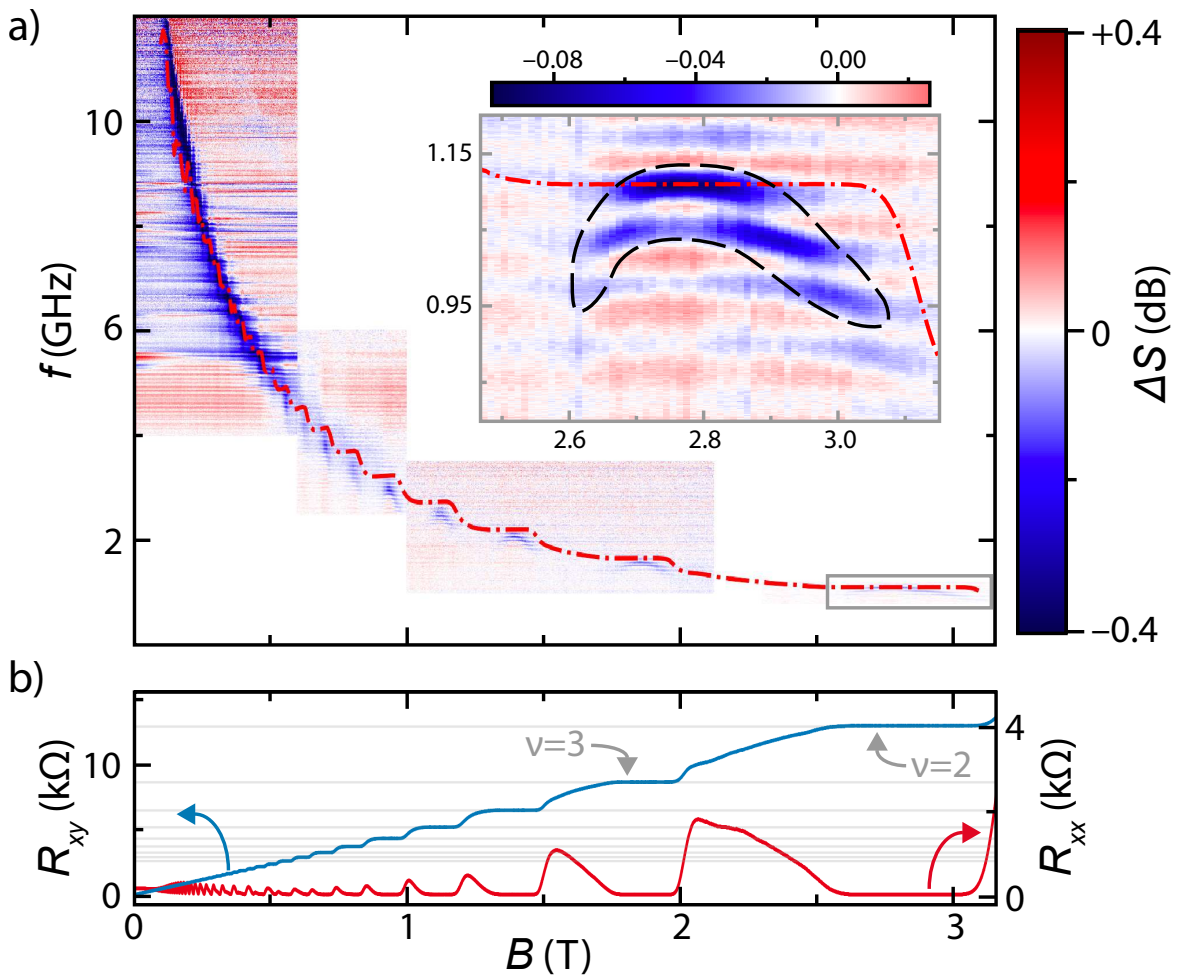


Figure 4.12: a) Transmission spectrum with the $B=0$ trace subtracted. EMPs are visible as dips in this spectrum. The red dot-dash line is a fit to equation (4.27); the values of the parameters are provided in the text. (Inset) Detail of the $\nu = 2$ plateau dispersion. The black dashed line is a guide to the eye. b) Longitudinal and transverse resistance measurements of the Hall bar. Light grey lines indicate the expected quantised Hall resistance values. The $\nu = 2, 3$ quantum Hall plateaux are indicated.

The parameters extracted from the fit are $h = 0.95 \pm 0.3 \mu\text{m}$, and $\epsilon^* = 7.9 \pm 0.5$. The constant C was not fit; it was assumed to be 1. The fit is shown as a red dash-dot line in figure 4.12 and figure 4.13. The limits of the errors are shown in figure 4.13 as the blue overlay. The σ_{xy} used in this fit was the experimental σ_{xy} . By using the classic equation for σ_{xy} , a slightly different value for the electron density is calculated.

A single higher order harmonic ($n = 2$) is visible in figure 4.12. Inherently, less energy is coupled into higher harmonics. Due to the detection method and power used, this harmonic was visible only at low frequencies. With such sparse data it was decided not to fit this higher harmonic.

It is interesting to note that there appears to be a correspondence between the field at which the Landau levels become spin-separated and the two different regimes in the EMP dispersion relationship. The spin-splitting can be seen in figure 4.13 (b); the shaded areas relate to the same field regimes in both panels of figure 4.13. This is something that warrants further experimental and theoretical investigation – the spin degree of freedom and its relation to EMPs has not been widely discussed in the literature.

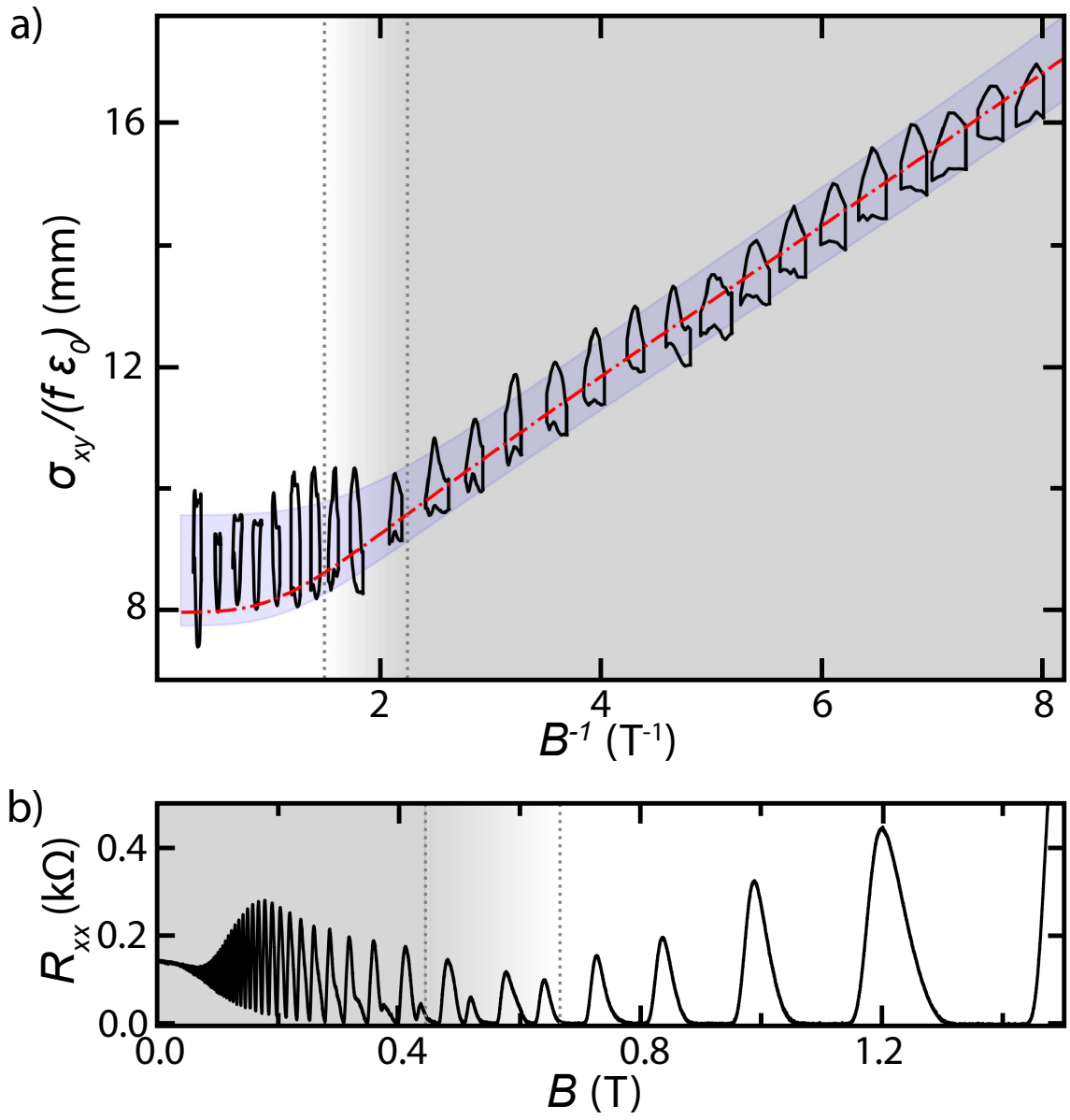


Figure 4.13: The grey regions indicate where the Landau levels are not spin resolved. a) EMP shapes with the blue overlay being the fits from equation (4.27). b) The measured longitudinal resistance of the Hall bar.

4.3.2 Plateau Dispersion

Although the overall shape of the dispersion is well described by equation (4.27), there is finer structure in the data which is not accounted for by this model. The EMPs are only excited at specific magnetic fields and have a distinctive dependence in frequency-magnetic field space – the most intensely absorbed frequencies display a crescent-like shape [68, 123, 139]. This can easily be seen in the inset of figure 4.12².

From the data, it can be seen that for an EMP to be observed (for energy to couple into the droplet from the transmission line), the 2DEG system has to be in a state exhibiting quantised conductance; that is, the presence of EMPs coincides in field with the presence of quantum Hall plateaux (in R_{xy}); see figure 4.12. The quantised conductance of the QHE is dependent upon the formation of edge-channels, due to the intersection of the Landau levels with the

²The ripples in frequency result from an impedance mismatch in the transmission line and do not relate to the interaction with the droplet.

changing electron density at the edge of the sample. This results in a spatial separation of regions with finite and zero density of electronic states – the compressible and incompressible strips, respectively. It is apparent that a well-defined incompressible strip *must be present* in the droplet for an EMP to be observed. Equivalently, the edge-channels must be separated (by an incompressible strip) from the ‘bulk’ of the 2DEG for an EMP to be observed.

The following models will be applied to $\nu = 2$; however, they can be applied to any quantum Hall plateau. Examples of other filling factors can be found in appendix B.

4.3.2.1 Modelling the Incompressible Strips

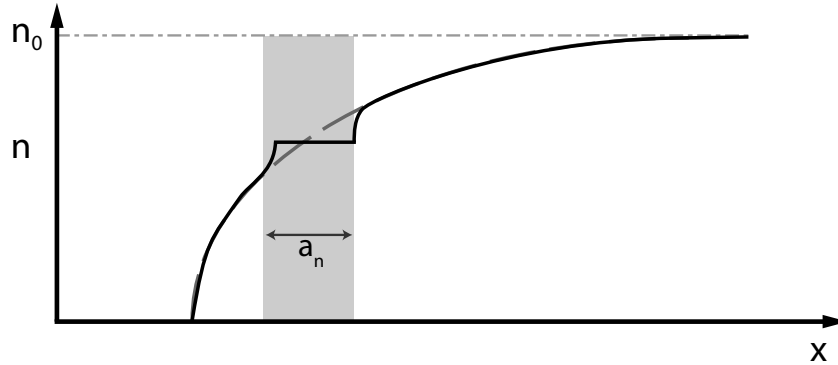


Figure 4.14: Density at the edge of a sample. The dashed grey line is the electrostatic density profile and the solid line is the modification that occurs when an incompressible strip forms.

EMPs occur on QH plateaux and have a distinct crescent-like shape. The question then becomes: what is changing to lead to this shape? Something that is known to change across a QH plateau is the position and size of the incompressible strips [89, 94]. Additionally, it has been shown by several authors that the speed of propagation changes across an individual QH plateau [121, 128, 132]; yet what the physical reason is for this has not been satisfactorily explained.

The models describing the position of the incompressible strips have been very well verified experimentally [82, 115, 140]. The general idea of these models is similar regardless of the complexity [89, 94, 105, 111] – the electron density cannot be the bulk density everywhere; there must be a smooth transition from the bulk to the (etch- or gate-defined) edge of the sample, where the electron density must be zero (figure 4.14).

The density of the incompressible strip needs to be known in order to calculate its position and size. Finding the density of the incompressible strip does not rely on details of the edge. The centre of the incompressible strip can be found by calculating the density that would give the conductance of the plateau, equation (4.28). σ_{xy} in this equation should take on a value of the quantum Hall plateau. As the electron density will not increase significantly beyond the bulk value, these models will only be able to model the edge-dominant side of a plateau.

$$n = \frac{\sigma_{xy} m^*}{e^2} \quad (4.28)$$

Finding the area of the incompressible strip is more complicated, as it is determined by the rate of change of density with distance. This relies on knowledge of the edge profile. Here the assumptions used by Aleiner and Glazman are followed [111]. The electron density profile as a function of distance, $n(x)$, across the diameter of the droplet was assumed to be,

$$n(x) = \frac{2n_0}{\pi} \arctan \left(\sqrt{\frac{x}{a}} \right) \quad (4.29)$$

where n_0 is the bulk density and a is parameter which effectively sets the hardness of the edge. The width of the incompressible strip, a_k , is determined electrostatically and is dependent upon the gradient of the density profile at the position of the strip [111],

$$a_k = \sqrt{\frac{2\epsilon_0\epsilon_r\hbar B}{\pi^2 e m^* \frac{dn}{dx} |_{x=x_k}}} \quad (4.30)$$

The areas calculated using this method are shown in figure 4.15. Although the way that the area of the incompressible strip changes with respect to magnetic field is reminiscent of the uptick in absorbed frequencies seen on the low field side of plateaux in figure 4.12, we have not found a way to quantitatively include this in a model. It is apparent that this model is only applicable on the low-field side of the plateau – up until the calculated bulk filling factor crosses the integer value figure 4.15. This is expected (see section 4.2.2.4), and demonstrates the limit of applicability of the ‘edge-mode’ model. Once the annulus has coalesced, a different model is required. This can be seen from equation (4.30); when $x_k = r$ the width of the strip, a_k , would diverge.

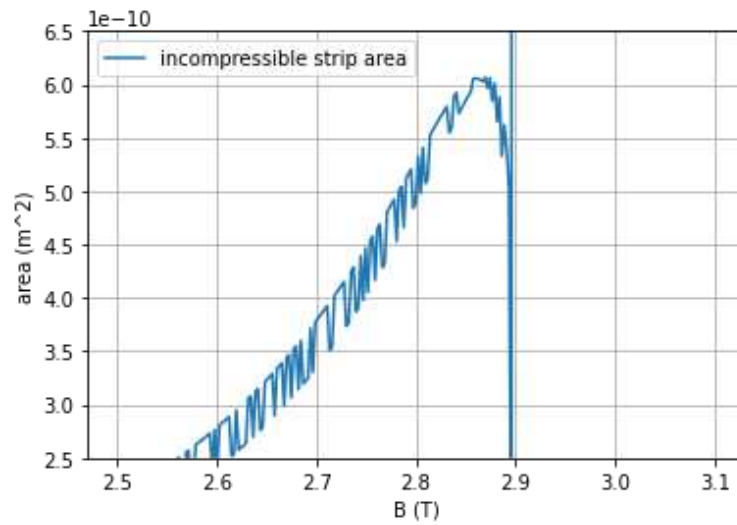


Figure 4.15: ‘Edge-side’ width of the incompressible strip as a function of applied magnetic field, B . The dashed line indicates where the model is no longer applicable.

4.3.2.2 Edge side model – position determined by incompressible strip

Although there was no clear way to use the area of an IC to determine the frequency, examination of equation (4.27) shows that the electron density n effects the frequency of the EMP. Before being able to include the change in n to equation (4.27) there is one more ingredient that needs to be included: the Hall conductivity (σ_{xy}) needs to be modified. This can be achieved by considering the classical Drude Hall conductivity [141], which includes an electron scattering time, τ ,

$$\sigma_{xy}(\omega) = \frac{e^2 n_0}{m^*} \frac{\omega_c}{\omega_c^2 - (\omega + i/\tau)^2} \quad (4.31)$$

In the limit in which the scattering rate greatly exceeds the EMP mode frequency, $1/\tau \gg \omega$, this relation can be approximated as,

$$\sigma_{xy} = \frac{e^2 n_0}{m^*} \frac{\omega_c}{\omega_c^2 + 1/\tau^2} \quad (4.32)$$

In employing the experimental transverse conductivity, $\sigma_{xy,E}$, in our model, to include the effects of scattering in the way described above, the experimental value used in the fit to the data is modified according to,

$$\sigma_{xy} = \sigma_{xy,E} \frac{\omega_c^2}{\omega_c^2 + 1/\tau^2} \frac{n}{n_0} \quad (4.33)$$

n appears in two areas in equation (4.27); the width determined by l , and the value of σ_{xy} . Incorporating these changes, the best fit occurs when $\tau \approx 2.27 \times 10^{-13}$ s; this can be seen as the red line in figure 4.16. Comparing this scattering time with the scattering time determined by the DC measurements can give some insight in to what a reasonable scattering rate might be. The DC scattering time can be calculated using the following equation,

$$\sigma = \frac{ne^2\tau}{m^*} \quad (4.34)$$

The DC scattering time for the $\nu = 2$ plateau is $\tau_{DC} = 1.38 \times 10^{-13}$ s. These scattering times differ by less than a factor of 2, which is not unreasonable. However, if one repeats the same analysis for higher filling factors the discrepancy increases. There is no clear pattern in these increases (appendix B).

4.3.2.3 Edge side model – position fit

Calculating the position of the incompressible strip gives a reasonable value for τ and a not-terrible fit for the shape. However, clearly a straight line does not properly capture the shape of an EMP. One might wonder how much the value of n needs to be modified to give the proper shape.

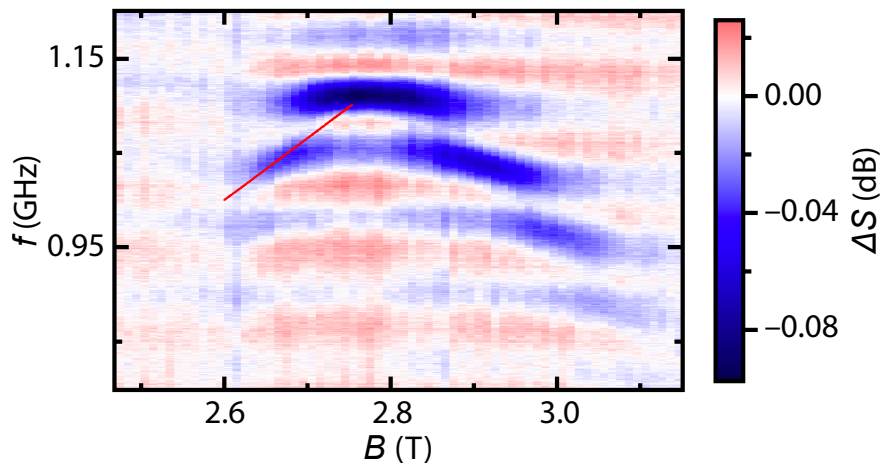


Figure 4.16: The EMP on the $\nu = 2$ plateau; the red line is the fit with n determined by the incompressible strip value.

Answering this question is a little vague as τ is still a fitting factor. However, taking the same value as before ($\tau \approx 2.27 \times 10^{-13}$ s), the density does not need to be significantly modified (figure 4.17). One could speculate that the change is caused by the change in area and/or position of the incompressible strip. The deviation is not large; however, trying to determine a link that quantitatively gives the right answer and can be justified physically is difficult.

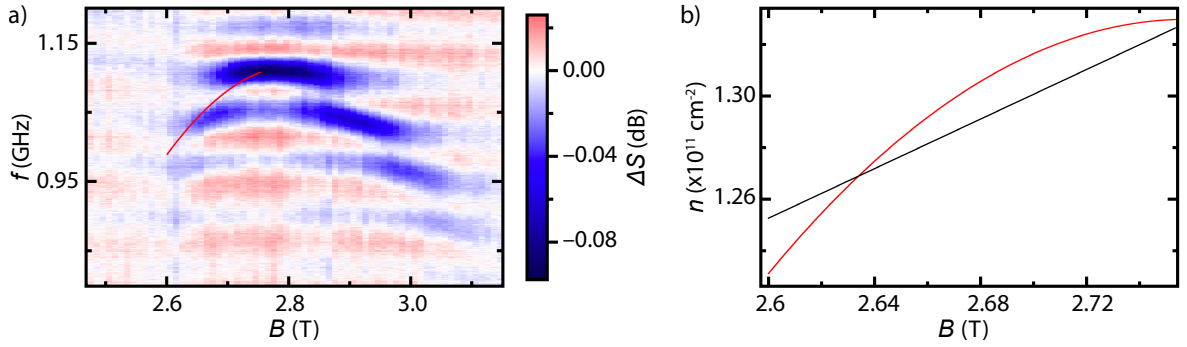


Figure 4.17: a) The EMP on the $\nu = 2$ plateau; the red line is a fit with n as a fit parameter. b) Comparison of the fit (red) and the calculated (black) density of the incompressible strip, with $\tau = 2.7 \times 10^{-13}$ s.

4.3.2.4 Johnson and Vignale Model

Johnson and Vignale have argued that the position of the incompressible strip plays no significant role in altering the frequency of EMPs [141]. Instead they argue that dissipation is what causes the change in frequency. In their model, dissipation is included phenomenologically via equation (4.32). Notably in this model there is no scattering on plateaux and so no change in frequency is expected; this is clearly in contrast to our experimental results. Here, motivated by the fact EMPs do not solely exist in the dissipationless edge channels, the phenomenological model presented in [141] is applied on plateau with the assumption that there is some dissipation.

In this section we work under the hypothesis that there is a change in dissipation across

the plateau, and that on the edge side the change in dissipation is caused by the change in incompressible strip position. The energy of the EMP will be focused primarily at the edge, falling away exponentially inwards, becoming zero at the bulk [141]. In the incompressible strip there will be no excitation; the EMP will be able to couple across the strip through long range interactions. Crossing the incompressible strip will lead to some loss in signal i.e. dissipation. As the incompressible strip moves towards the bulk, the amount of energy lost reduces (figure 4.18).

The incompressible strip will leave the system around the point of lowest dissipation; this occurs at the transition point between edge and bulk-dominant sides of a quantum Hall plateau [82, 141]. On the bulk side, dissipation increases, which is the expected behaviour [82].

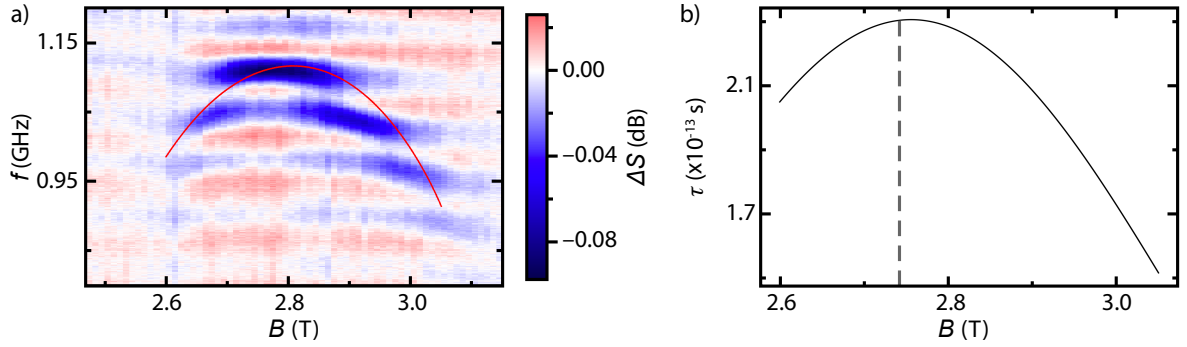


Figure 4.18: a) The EMP on the $\nu = 2$ plateau; the red line is a fit with τ fit as a Gaussian. b) Variation in τ ; the Hall bar measurement gave $\tau_{DC} = 1.38 \times 10^{-13}$ s. The dotted line separates the 'edge' (low field) and 'bulk' (high field) sides, it is where ν equals an integer value (from classic equations).

4.3.2.5 Resonance Fitting

We wanted to verify if the change in dissipation calculated by the Johnson and Vignale model reflects some physical damping. One method to do this is to treat the EMPs as being in a resonator and to calculate its Q factor. This is well motivated within the literature [74, 142, 143]. A resonator would have a Lorentzian shape, however, in this case as this is an average of noisy data a Gaussian – described by equation (4.35) – may be more appropriate.

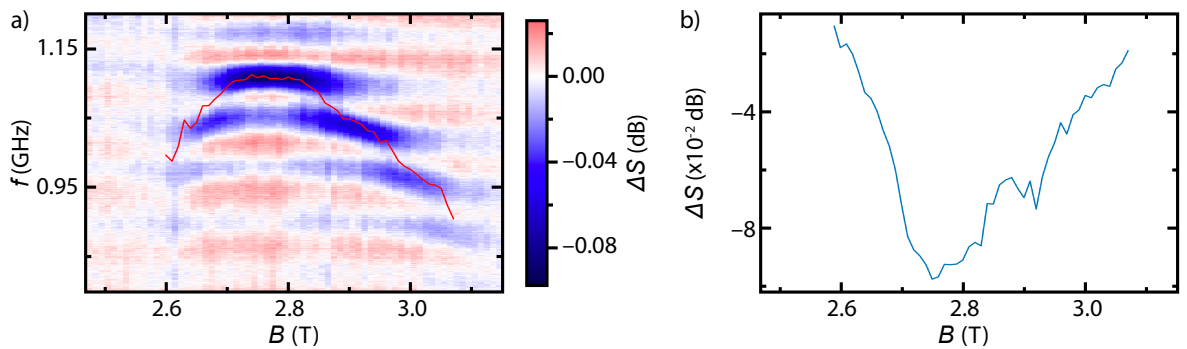


Figure 4.19: a) Centre of the Gaussian fits. b) The depth (change in ΔS) of the Gaussian fit following the trajectory of the red line in (a).

$$f(x) = a \cdot e^{-\frac{(x-b)^2}{2c^2}} \quad (4.35)$$

The Gaussian fits enable the dip depth (a) and position (b) to be determined (fig. 4.19), as well as giving a means to determine the Q factor of the resonator. The Q factor is defined as,

$$Q \equiv \frac{f}{\Delta f} = \frac{\omega}{\Delta \omega} \quad (4.36)$$

Where Δf is the FWHM, which can be calculated from,

$$\text{FWHM} = 2\sqrt{2 \ln 2} c \quad (4.37)$$

A generic resonator has a damping factor that is given by equation (4.38). The damping value that was calculated using this method is shown in figure 4.20. The shape is reminiscent of what was calculated by the Johnson and Vignale model; however, the calculated values for the relaxation time differ by several orders of magnitude. This is not an issue as we have used a generic resonator which does not account for the details of the system.

$$\tau_Q = \frac{2Q}{\omega} \quad (4.38)$$

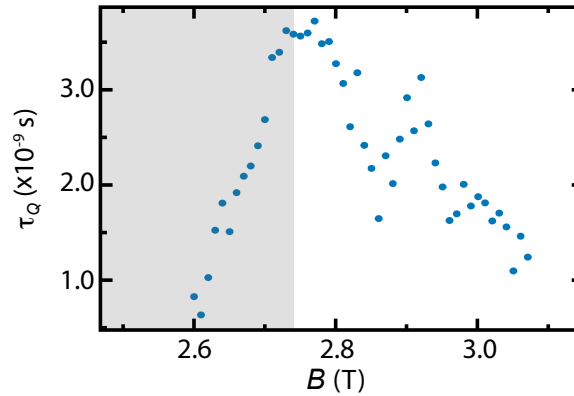


Figure 4.20: τ_Q across the $\nu = 2$ plateaux calculated assuming a general resonator. The grey indicates the side that is edge dominant.

It is interesting to determine by how much the resonator relaxation time, τ_Q , and the single electron scattering time, τ , differ. The constant that these differ by is calculated as $c = \tau / \tau_Q$. This results in a factor of $c = (10 \pm 4) \times 10^{-5}$. This is interesting as it falls very close to the value of $\sigma_{xy}(\nu = 2) = 7.7 \times 10^{-5}$ (ignoring units). A number of authors have indicated that the resonant loss term is proportional to σ_{xy} [74, 120, 142], although this is not universally accepted [135]. These data would support that it is indeed proportional to σ_{xy} .

If σ_{xy} plays a role in the resonance of the system, then in order for the units to make sense there needs to be another component present. Whatever the component is, it needs to have an impedance of order 1Ω . It is justifiable to treat the EMP resonant system as either capacitive [75, 136, 142], or resistive [74, 120], meaning that σ_{xy} is multiplied by the impedance of the

capacitor or ρ_{xx} respectively. For this frequency, in order for the impedance to be of order one, the capacitance would have to be of order 150 pF, a not unreasonable value³.

4.3.2.6 Development of a Phenomenological ν Symmetric Model

One of the major problems with all of the models that have been explored thus far is that there is no predictive power. All the models have been applied to the $\nu = 2$ plateau. Knowledge of the fit parameters used for this plateau does not give any information about how to fit other plateaux; to fit to other plateaux the models need to be reapplied. The analysis was repeated for other filling factors (see appendix B); however, no clear trends were observed with these data. In this section, a phenomenological model is developed; the intention of this model is that by fitting a single plateau the shape and position of all plateaux can be predicted.

Given the plethora of experimental support and how well equation (4.27) generally fits the global trend, it seems reasonable to work with the assumption that it is essentially correct, and that the EMP shape on plateaux can be fit by some modification to this equation. All of the previous models also used this assumption. In order to get an idea of how much modification may be necessary, the experimental data was plotted – normalised by dispersion relationship (figure 4.21). The shapes are not symmetric; however the depth in normalised frequency seems to be consistent.

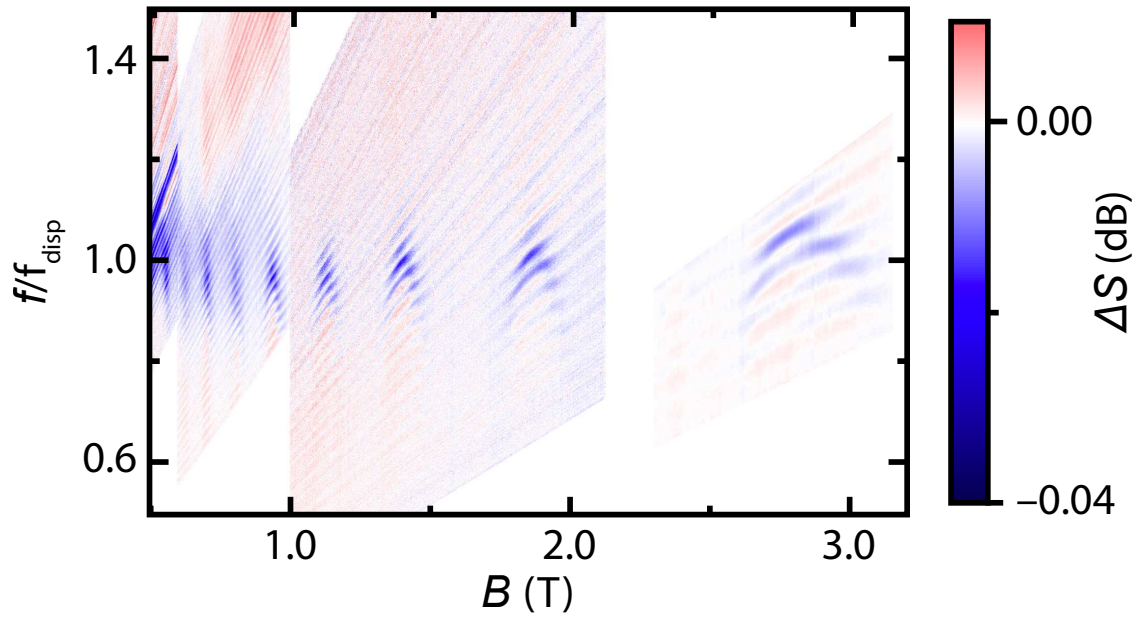


Figure 4.21: The same data as is seen in figure 4.12 normalised by the dispersion relationship (equation (4.27)).

Changing the abscissa axis to the bulk ν value using equation (4.18), an interesting trend becomes visible. The data appears to be symmetric in this space (figure 4.22), although this can be a little difficult to see given the standing waves. The $\nu = 2$ plateau is given as an inset with a Gaussian blur applied to limit the effect of the standing waves. Not only are the EMPs symmetric about integer filling factors, they appear to have the same extent in ν , as well as

³Using limiting views of the geometry places estimates the capacitance to be of order 100 pF

similar depths in normalised frequency.

The symmetry about ν is a little surprising given that EMPs are often linked to ECs, and the clearly asymmetric response of ECs in many experiments [113–117]. These experiments have primarily been conducted on small samples; more recently it has been shown that the degree of asymmetry diminishes as the sample size increases [103]. In wider samples the long-range potential fluctuations become more important [100].

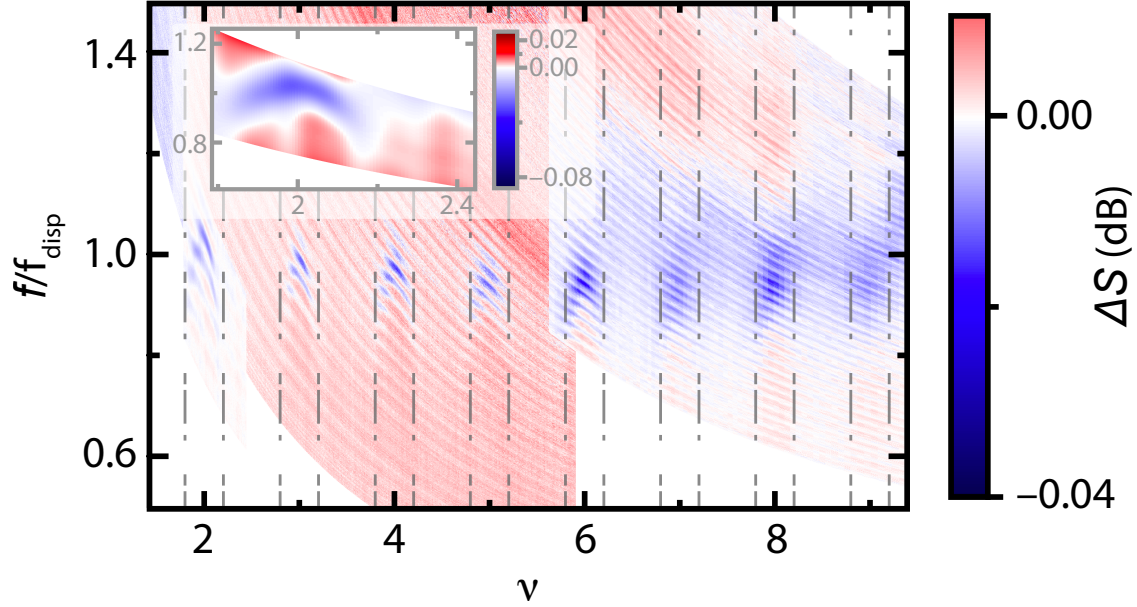


Figure 4.22: Normalised data of filling factors $\nu = 2$ to $\nu = 9$; this is equivalent to $B = 4$ T to $B = 0.6$ T. The dashed lines are $\nu_k \pm 0.2$, where ν_k is an integer. The inset shows the $\nu = 2$ plateau with the data smoothed by a Gaussian blur in order to eliminate the effect of the standing waves. The symmetric nature of the EMP is evident in these data.

A modification to equation (4.27) that gives a good fit to the dips observed in figure 4.22 is,

$$\frac{f_{norm}}{1 + 6.3(\nu - \nu_k)^2} \quad (4.39)$$

where ν is the bulk filling factor and ν_k is the integer filling factor of the plateau. It is worth noting that this modification is by definition unitless. One could ask if it would be better to use a different function for the modification. A Gaussian, for example, could be used and it would give a direct link to the probable shape of LLs. A quadratic is a reasonable approximation of a Gaussian – particularly to its peak, where the extended states exist – in first order and uses fewer parameters.

This modification to the dispersion relationship appears to be able to predict the shape for all the EMPs visible in this device, regardless of whether they are spin separated; in the SdH regime, or in the IQHE regime (figure 4.23 a.). It is important to check that this modification works for other samples. Previous data⁴ from an EMP gyrator device was used to check if this modification worked for another sample. This device was fabricated from a different GaAs wafer, which had different values for the electron density. It also had a different effective permittivity, lateral depletion depth, and radius – it would not have been

⁴Data was collected by the author for a previous experiment.

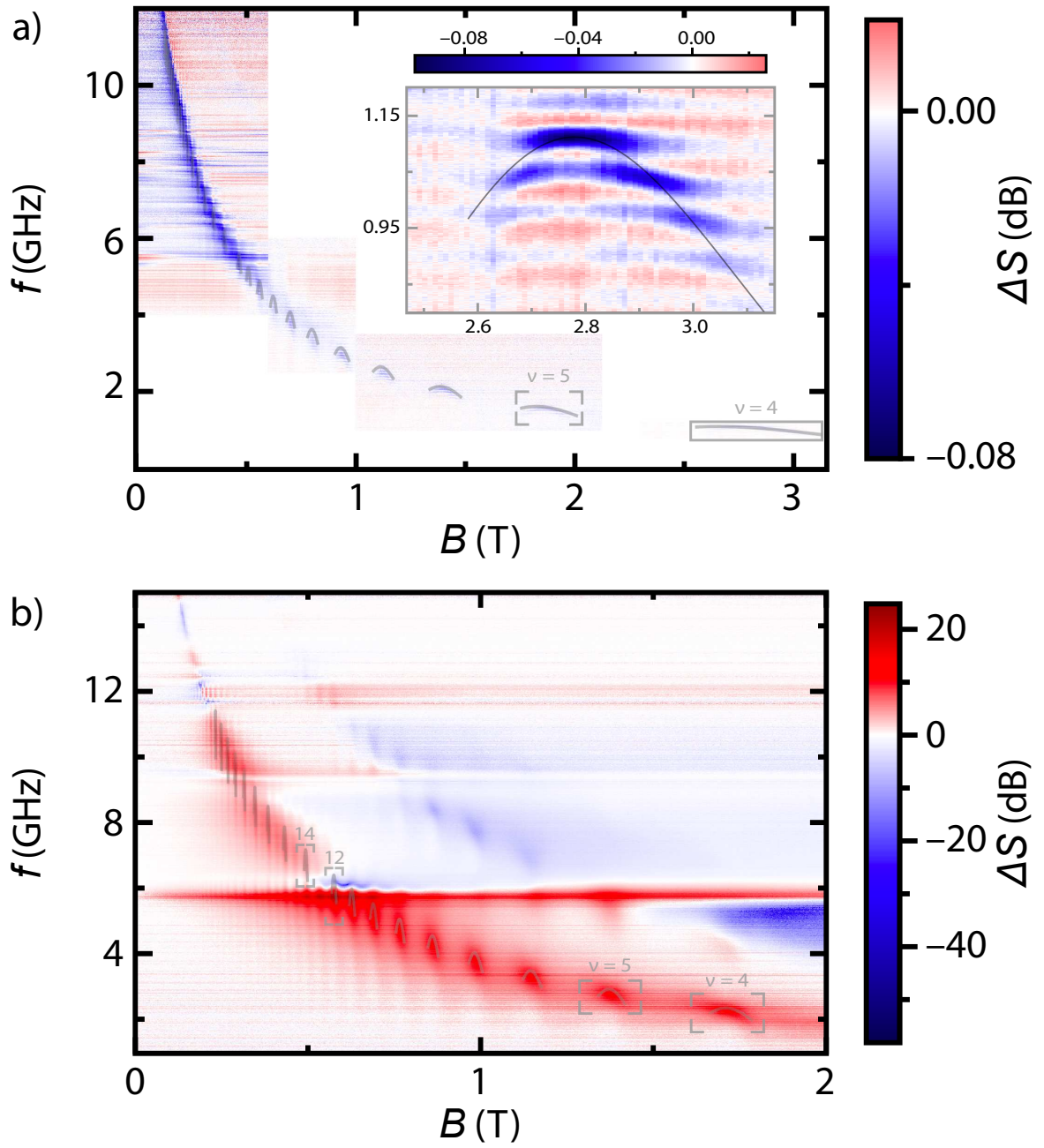


Figure 4.23: Transmission data of a) the transmission line device and b) an EMP gyrator device. The modification (4.39) is plotted in grey. This fits well for all the data shown. (Inset) Details of the $\nu = 2$ plateau with the modified dispersion relationship over-plotted. For the gyrator device transmission is increased on plateaux. There is a board resonance a little below 6 GHz which slightly shifts the frequencies. This is particularly obvious with the $\nu = 12$ plateau.

useful as a verification process if these were all the same. The device details and images can be found in appendix E. The results are shown in figure 4.23 b.; unlike the transmission line, the transmission increases when on a QH plateau. Despite the differences in the device, the modification is still able to give reasonable fits to these data.

Much like [141], here the modification is incorporated into the Hall conductivity σ_{xy} . This is justifiable as the Hall conductivity differs as a function of position and frequency [98, 144]. Beyond these considerations, EMPs are plasmons, which means they occur on boundaries and they are not truly 1D in nature, whereas the measured current in DC measurements exists solely within the 1D channels. In other words, the EMP is unlikely to experience the

same conductivity as a DC current would. A pleasing effect of incorporating the modification into σ_{xy} means that no new terms are added to the dispersion relationship (4.27); instead σ_{xy} on plateau becomes,

$$\sigma_{xy} = \nu \frac{e^2}{h} \rightarrow \frac{\nu}{1 + 6.3 (\nu - \nu_k)^2} \frac{e^2}{h}. \quad (4.40)$$

It is possible to justify the modification of the Hall conductivity. It needs to be investigated if this modification is universal. By determining when this phenomenological model can be used, insights may be gained that lead to this model being developed into a mechanistic model. For example, because small samples are known to show asymmetry about integer filling factors, does this symmetric phenomenological model fit when the size of the droplet is small ($< \mu\text{m}$)? Both samples used have used circular droplets; is this important? The symmetry about ν suggests that the shapes of the EMPs may be linked to the shape of the DOS; if this is the case, it gives another method to verify how the DOS is broadened in the QHE.

4.4 Conclusions

Motivated by literature, several models were used to fit the shape of edge magnetoplasmons on a quantum Hall plateau. It was found that these models, at best, could be applied to fit a single plateau, and at worst could be made to fit only the edge-dominant side of a plateau. None of these models were able to successfully *predict* the shape of any other plateau. A new phenomenological model was developed; this model is symmetric about integer filling factors ν_k . It contains no fit parameters⁵. This model was able to successfully predict the shape of edge magnetoplasmons on all quantum Hall plateaux, and even on Shubnikov-de Haas oscillations, for two different devices.

⁵The constant 6.3 is likely to be a fit parameter but for both devices tested this was the same value.

Chapter 5

Sensing the Reactance of a Josephson Junction

5.1 Abstract

In this chapter, a Josephson junction (JJ) is measured at low frequencies and at radio frequencies (RF). The RF data is measured without any deliberate matching networks or calibration. Despite this, the RF data reproduces many of the features observed from the low frequency measurements. The RF data is used to qualitatively describe how the resistance and inductance of the JJ changes with bias. An equivalent circuit model is produced in order to determine the quantitative changes that occur in the JJ. However, a more sophisticated model will be required in order to accurately capture the dynamics of the low bias regime.

5.2 Introduction

5.2.1 Superconductors

One of the requirements needed for the Josephson effects to occur is superconductivity [145]. The theories and equations – such as the London equations [146, 147], Ginzburg-Landau theory [148], and BCS theory [149] – that describe how superconductivity works will not be explored in-depth in this section; here a more phenomenological approach is taken. Insights from the theories will be used, however, the focus is on phenomena and how these relate to Josephson junctions (JJs).

The first superconductor discovered was mercury; this was discovered in 1911 by Heike Kamerlingh Onnes at Leiden [150]. What characterised something as a superconductor¹, as apposed to a ‘good conductor’, is that below a critical temperature (T_c), a superconductor has zero DC resistance (figure 5.1).

If the only effect of superconductors was that they posses zero DC resistance² it would be possible to discuss superconductivity solely within the descriptions provided by classical

¹At the time and for some time afterwards they were called “supraconductors”.

²For 22 years this, and a material dependent second order transition that occurs at the critical temperature, were the only effects of note known in superconductors.

electromagnetism. It was the discovery of perfect diamagnetism – the Meissner effect (by Meissner and Ochensfeld in 1933) – that distinguished superconductors as something more than just a perfect conductor [151, 152].

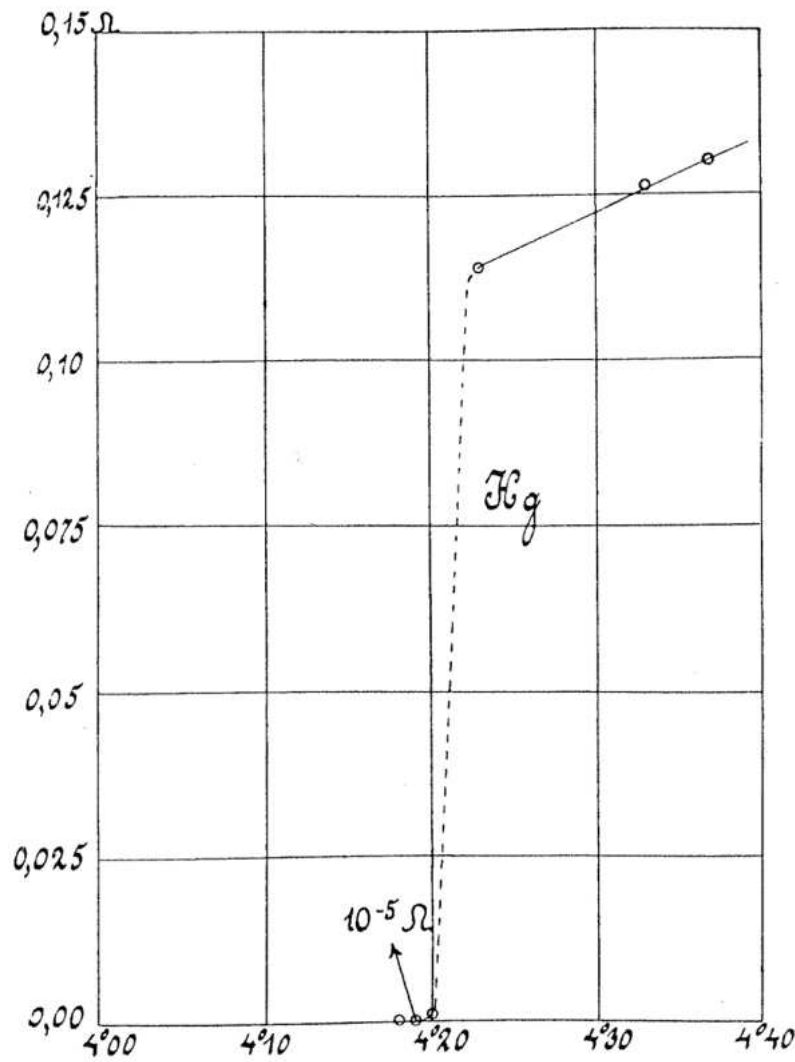


Figure 5.1: The first measurement of a superconductor. The measurement was of a mercury wire. The critical temperature (T_c) is 4.2 K³. Figure is reproduced from [150].

When a weak⁴ magnetic field is applied to a superconductor (or perfect conductor) the naïve assumption is that the magnetic field does not penetrate. In contrast, when a magnetic field is applied *before* the material becomes a perfect conductor, the magnetic field lines become frozen in place. However, this is not what happens with a superconductor; after transitioning through T_c the magnetic field is expelled from the superconducting material so that there is zero magnetic field within the superconductor (figure 5.2). Strictly speaking the magnetic fields are not completely expelled from superconducting materials; they penetrate to a certain depth as described by the London equations [146, 147].

In explaining the Meissner effect, a very important concept in superconductivity was introduced – coherence [147]. Superconductors maintain amplitude and phase coherence over macroscopic distances; it is this many-particle condensate wavefunction which is critical

³The axis says °; this is because prior to 1968 the unit Kelvin was degree Kelvin [153].

⁴The magnetic field needs to be less than critical field (B_c). If it is higher than this it destroys superconductivity.

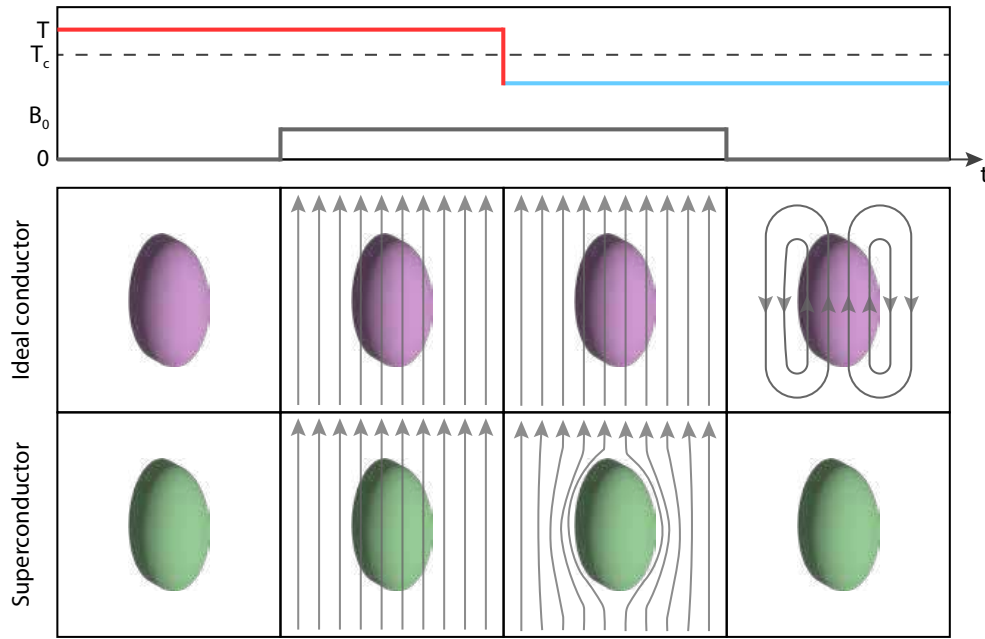


Figure 5.2: Thought experiment that differentiates an ideal conductor from a superconductor. Magnetic field is applied before cooling below T_c . An ideal conductor would pin the magnetic field, whereas magnetic fields become expelled from the superconductor (Meissner effect). When the magnetic field is switched off, the ideal conductor produces eddy currents to retain a magnetic field.

in explaining the properties of superconductors⁵ [9, 149, 155, 156]. The Josephson effect is a direct consequence of the phase of superconductors [145].

5.2.2 Josephson Junction

When two superconductors are separated by a ‘weak link’ – that is, a barrier region where the wavefunctions of the superconductors are able to overlap (figure 5.3) – the Josephson effects occur. The weak link allows the phases of the superconductors to differ regardless of the materials used, or the physical arrangement; configurations where a weak link exists between superconductors is called a Josephson junction⁶. There are two Josephson relations, one which describes the DC effect and another which describes the AC effect. Both of these effects depend on the phase difference of the superconductors on either side of the junction, $\varphi = \phi_1 - \phi_2$.

Even when the two superconductors are at the same voltage, a supercurrent will still flow between them. Intuitively this can be understood as Cooper pairs tunneling from one side of the junction to the other. The amount of current (I_S) that flows can take values from $-I_c$ to I_c , where I_c is the critical current. Equation (5.1) describes the process in an ideal junction.

⁵It is also the basis of defining what a superconductor is. Simplistic definitions define superconductors as materials that exhibit perfect conduction and are perfectly diamagnetic. However, type II superconductors complicate matters, as they contain a superconducting phase which exhibits both DC resistance and partial diamagnetism [154]. Type II superconductors will not be discussed further in this thesis.

⁶The geometry and the type of materials that make up the weak link classify the type of junction. Typical materials include superconductor-insulator-superconductor (SIS), superconductor-normal metal-superconductor (SNS), superconductor-constriction-superconductor (ScS), superconductor-2DEG-superconductor (S-2DEG-S), and superconductor-ferromagnet-superconductor (SFS) [157].

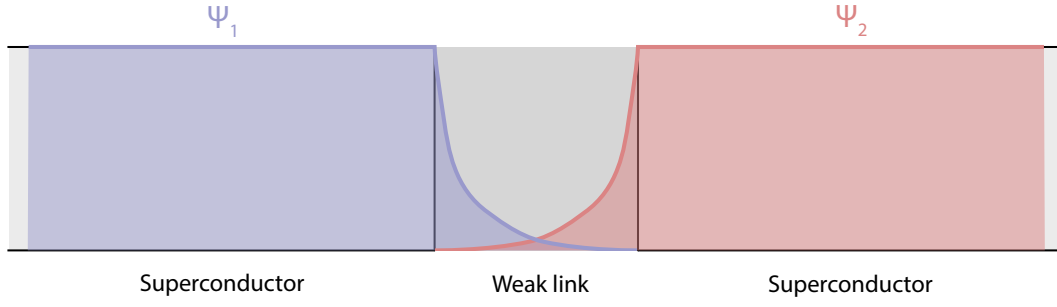


Figure 5.3: For the Josephson effect to occur, the macroscopic wavefunction ($\Psi_n = |\Psi_n|e^{i\phi_n}$) of each superconductor overlaps in the barrier region.

$$I_S = I_c \sin \varphi \quad (5.1)$$

More generally, the current phase relationship can be anything that meets the following conditions [157]:

1. A change of 2π in any electrode is not accompanied by a change to the physical state; i.e. the current phase relationship should be 2π periodic, $I_S(\varphi) = I_S(\varphi + 2\pi)$.
2. Changing the sign of φ should be associated with a change in the direction of current flow; $I_S(\varphi) = -I_S(-\varphi)$ ⁷.
3. In the absence of a phase difference ($\varphi = 0$) there should be no supercurrent; $I_S(2\pi n) = 0, \quad n = 0, \pm 1, \pm 2, \dots$
4. From conditions 1. and 2. it holds that the supercurrent should also be zero at $\varphi = \pi n$; $I_S(\pi n) = 0, \quad n = 0, \pm 1, \pm 2, \dots$

When a voltage difference is present across the junction, the phase difference between the superconductors evolves with time. The phase evolves linearly with respect to the voltage and fundamental constants. The fundamental constants appear due to the energy difference of Cooper pairs moved across a junction.

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \quad (5.2)$$

An external signal of frequency F is able to lock in to the frequency of this oscillation [158–161]. This results in steps at specific voltages. These steps are often referred to as Shapiro steps in honour of Sidney Shapiro, who first demonstrated the effect in 1963 [158]. The voltages where steps occur are determined by the following equation.

$$V_n = n \left(\frac{h}{2e} \right) F = n \phi_0 F \quad (5.3)$$

⁷This condition does not hold for SFS JJs and those made with superconductors that possess broken time-reversal symmetry.

Where n takes on both positive and negative integer values ($n = 0, \pm 1, \pm 2, \dots$). Shapiro steps are not visible for all RF powers (figure 5.4); the variation in the widths of the voltage steps ideally follows Bessel functions [159, 162].

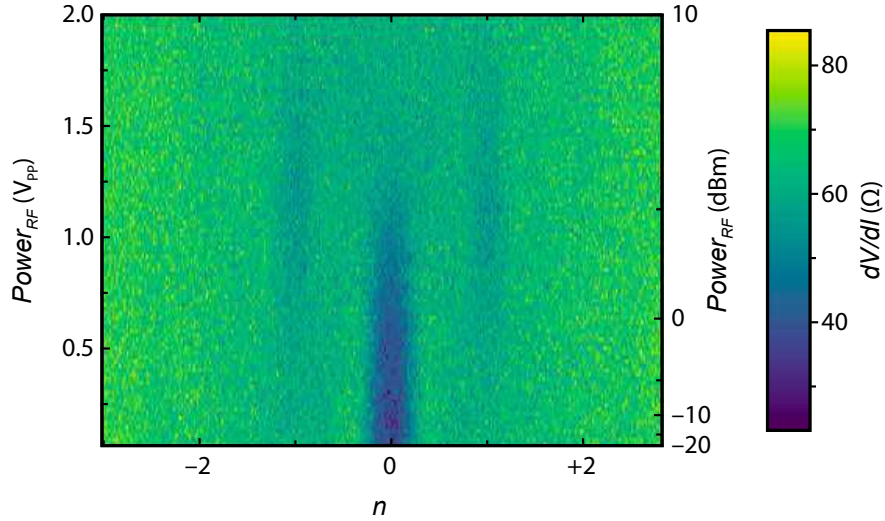


Figure 5.4: Shapiro steps as a function of RF power. The power is in reference to the power applied to the antenna (pre-attenuation). The applied frequency was 1.5 GHz.

The changing current with a fixed voltage means that an inductance can be associated with the junction. Inductance is defined as $L \equiv V \left(\frac{dI}{dt} \right)^{-1}$, so the ideal inductance of a JJ is given by equation (5.4).

$$L_J(\varphi) = \frac{\phi_0}{2\pi I_c \cos \varphi} \quad (5.4)$$

To model the dynamics of a junction away from DC, a simple equivalent circuit model is often used – the resistively and capacitively shunted junction (RCSJ) model [156, 163–165]. In this model the JJ is represented as an ideal junction shunted by a resistor and a capacitor (figure 5.5). The resistance represents the dissipation in the finite voltage regime. The capacitance represents the geometric capacitance between the two superconducting electrodes.

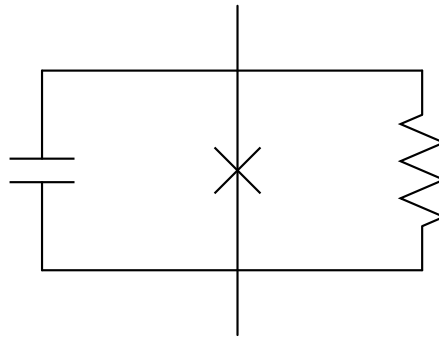


Figure 5.5: The resistively and capacitively shunted junction (RCSJ) model.

The inductance of a JJ is a very interesting quantity to measure, as it provides information about the current phase relationship [166]. Deviations from the ideal current phase relation-

ship (equation (5.1)) can provide important information about the underlying microscopic processes occurring in a junction [157, 166–170]. DC measurements do not provide information about this value [166, 171]. Previous measurements of this quantity have relied on superconducting quantum interference devices (SQUIDs) [167–170], cavities with quantum point contacts [172], or even used an RF tank circuit connected to thousands of JJs [166]. Additionally, recent simultaneous RF-reflectometry and DC measurements of a JJ have shown that the RF readout may be more sensitive to the transport features than the DC [171].

In this chapter a single (Al-InAs-Al) SNS JJ is measured in RF without the addition of matching circuits. Quasi-DC measurements were taken as a point of comparison, and so that the normal state resistance was well characterised. The goal was to be able to directly sense the Josephson inductance and determine its value using a single JJ.

5.3 Results and Discussion

5.3.1 Measurement

The quasi-DC measurements of a SNS (Al-InAs-Al) Josephson junction (JJ) was measured using lock-in amplifiers (see methods chapter 3.1.2 for more details). The device was voltage biased; in all of the following graphs the voltage shown is the voltage at the device level. The offset and scale were determined using the position of the Shapiro steps (see appendix C1). No significant hysteresis was observed in these measurements.

An example of the quasi-DC bias response of the JJ is shown in figure 5.6. No RF power, either via the antenna or via the gate, was applied during these measurements. The DC Josephson effect occurs in the region where the differential resistance measures $\sim 8 \Omega$. The normal state resistance of the junction at this particular gate voltage ($V_{TG} = 0$ V) was measured to be around 75Ω . It should be noted that, as expected, the measured resistance continues to change as a function of bias. The changes in normal state resistance, and the size of the super-current as a function of top gate voltage, are provided in appendix C5.

The response, as measured by the lock-in amplifiers, was fairly noisy. In order to smooth the data a Savitzky-Golay filter was applied. This filter is particularly useful as it increases the signal to ratio whilst maintaining peaks in the data [173]. The smoothing reveals dips that occur around $\pm 5 \mu\text{V}$; these are associated with multiple Andreev reflections (MAR) [174, 175].

The S_{11} measurement of the bias response of the device is shown in figure 5.7. For details of the measurement set-up, refer to methods chapter 3.3.2. For ease of comparison the filtered quasi-DC measurement is included on the plot with the magnitude of the reflected signal. The RF signal used to measure the response had a frequency of 3.767 GHz. This frequency is on the low frequency side of a resonance (see appendix C4).

Interpreting the RF measurements is more complicated as both the phase and magnitude of the response should be considered. Considering just the magnitude of the reflection,

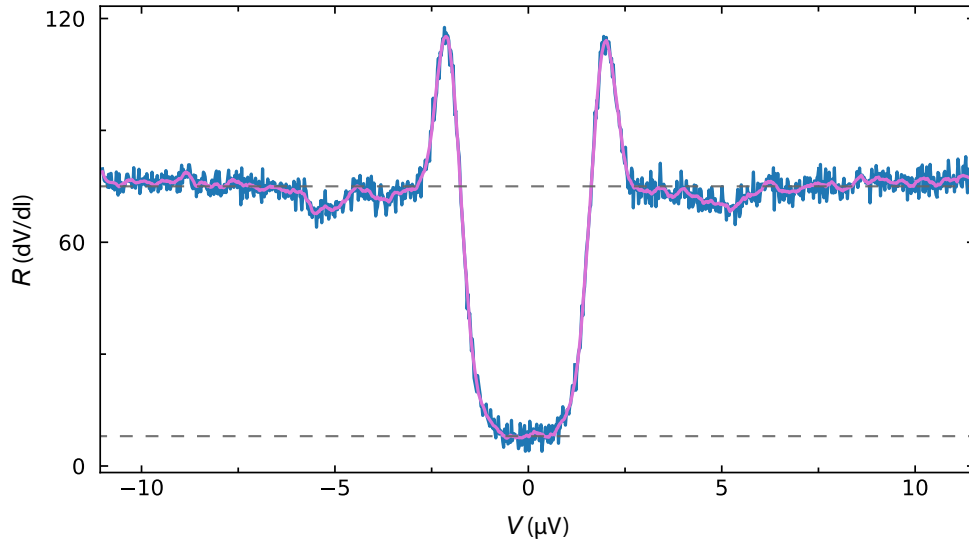


Figure 5.6: Differential measurement of the bias response of the Josephson junction. The purple is the data smoothed by a Savitzky-Golay filter [173]. The grey dashed lines indicate the resistance values of $8 \, \Omega$ and $75 \, \Omega$.

there is a feature that occurs near $0 \, \mu\text{V}$ and another that occurs at $\sim \pm 7.8 \, \mu\text{V}$. Comparing the magnitude of the RF response with the quasi-DC measurements, the $0 \, \mu\text{V}$ feature is consistent with the super-current; the change in slope occurs at the same place in both measurements. The $\pm 7.8 \, \mu\text{V}$ features do not appear in the lock-in measurements. These features are consistent with the $n = \pm 1$ Shapiro steps for a $3.767 \, \text{GHz}$ tone, the readout tone of this measurement. There is no clear change in magnitude associated with the MAR at $\pm 5 \, \mu\text{V}$.

The phase information over the same regions gives a similar picture. The super-current region causes the largest shift in frequency. It is interesting to note that this shift in frequency is in the opposite direction to the shift caused by the Shapiro steps. Additionally, for $|V| > 7.8 \, \mu\text{V}$, the phase is much higher than on the other side of the Shapiro step. From these data alone it is not clear why this is the case.

Measuring Shapiro steps with the same tone used to excite the Shapiro steps is a unique measurement; such measurements do not appear in the literature previously. It would be interesting to observe how the signatures of the Shapiro steps change with the applied power of the readout tone. The width of the Shapiro steps should follow Bessel functions [159, 162], so ideally there should be powers where the $n = \pm 1$ Shapiro steps are not visible in the measurement. Changing powers, however, will also cause a change in the SNR as this is also the readout tone. Additionally, with sufficient power, power broadening would also occur. Whilst it could be difficult to disentangle the various changes caused by changing power, it may be worth pursuing such a measurement.

In order to get a more complete picture it is useful to examine how the resonance changes (figure 5.8). When the frequency of the resonance changes, it results from a change in reactance, whereas a change in the Q factor is associated with a change in resistance. From

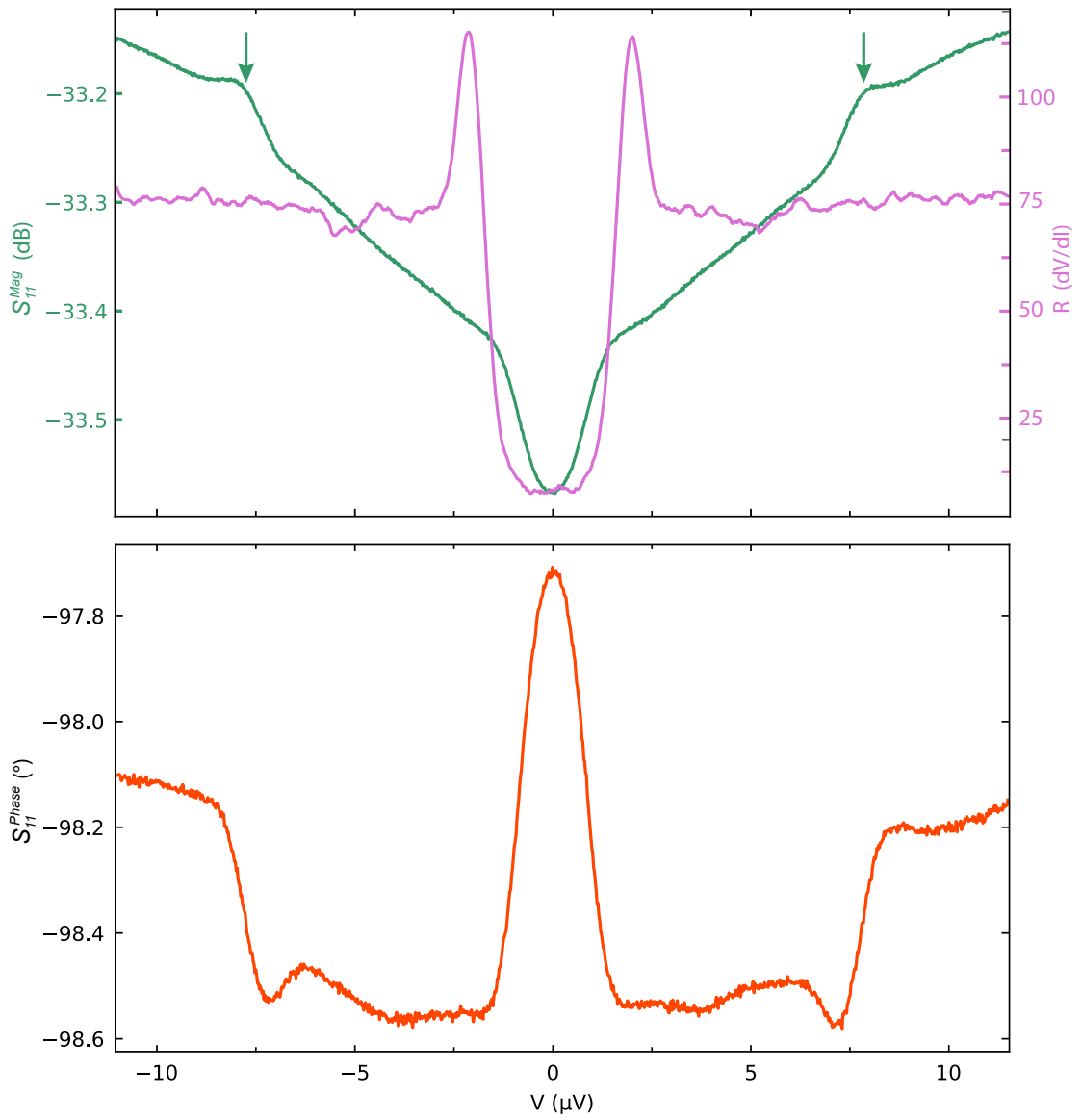


Figure 5.7: (Top) Green – Magnitude of the reflection coefficient at 3.767 GHz. The green arrows indicate the position of the $n = \pm 1$ Shapiro steps. Purple – smoothed lock-in data. (Bottom) Phase of the reflection coefficient at 3.767 GHz.

the RCSJ model it is reasonable to assume the capacitance – the capacitance between the superconducting electrodes – is constant, and that the changes in resonance frequency can be associated with a change in inductance. From the general expression for a LC resonance, $f_0 = \frac{1}{2\pi\sqrt{LC}}$, an increase in frequency is associated with a decrease in the inductance.

A current biased JJ is expected to have a minimum inductance at zero applied current. As the applied current increases (or is decreased from zero⁸) the inductance increases; the highest inductance occurs when the critical current is applied [156, 166]. In the dissipative regime, as the applied current is increased, the inductance decreases. The rate of change associated with the inductance changes as the current continues to be increased; the rate of change slows. The voltage bias regime does not probe the dissipationless regime in the same way; the expectation is that the highest inductance occurs near $V = 0$ V and the inductance should decrease as the voltage is increased. In terms of frequency, the converse would be expected; moving away from 0 V results in an increase in frequency – this is what is observed

⁸The response is completely symmetric. For this discussion the positive side will be discussed.

in figure 5.8.

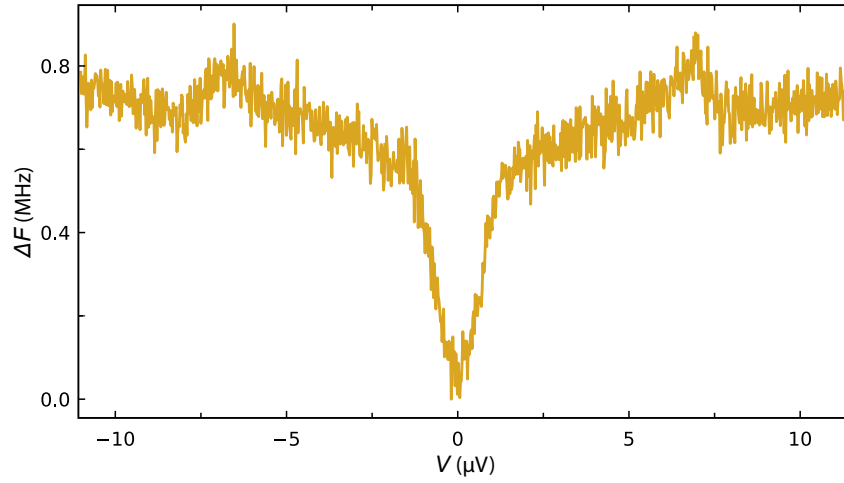


Figure 5.8: Change of resonant frequency as a function of bias. The change is taken with respect to the resonant frequency at zero bias (3767.98 MHz).

The Smith chart would be an ideal representation to use to quickly determine reactive and resistive values. Unfortunately, due to the losses in the system, only a small amount of signal returns to the VNA; in reflection this appears as if the system is well matched (see appendix C.6). Everything appears close to 50Ω , even though this is not the case. Additionally, the phase of the system is unknown, meaning that the data is arbitrarily rotated around the Smith chart. Identifying the values of inductive and capacitive values becomes extremely difficult. These problems could be avoided with a calibrated system; if these measurements were to be retaken the system would be calibrated.

Another useful way to represent the data is to view it in the I/Q plane, such as is shown in figure 5.9. The in-phase component is the real part of the reflection coefficient $I \equiv \Re(\Gamma)$; the imaginary part is represented by the quadrature component $Q \equiv \Im(\Gamma)$. However, without calibration the rotation about this plane is arbitrary; i.e. what is real and what is complex may be mixed. Nevertheless, lines of constant phase show resistive changes, either a real change of resistance or a balanced change of inductive and capacitive components. Circles of constant magnitude (centred on the origin) show phase changes that occur without changes to the matching conditions such as occur with changes in electrical length.

Several different regimes have been identified in figure 5.9. The positive and negative sides of the bias are mostly symmetric, indicating that there is little hysteresis in this JJ. The region associated with the super-current follows a straight non-radial path. This would suggest that there is a change in reactance occurring over this region. This, and the associated change of resonant frequency over this region, gives an indication that it is likely to be the inductance that is decreasing.

The region associated with the normal state resistance of the device nominally follows a radial line of constant phase. The slight deviation from the radial path indicates that there is a small reactive change occurring over this region. Given the slight leftwards tilt and the associated resonant frequency data, it is likely this is a small inductive decrease. From the

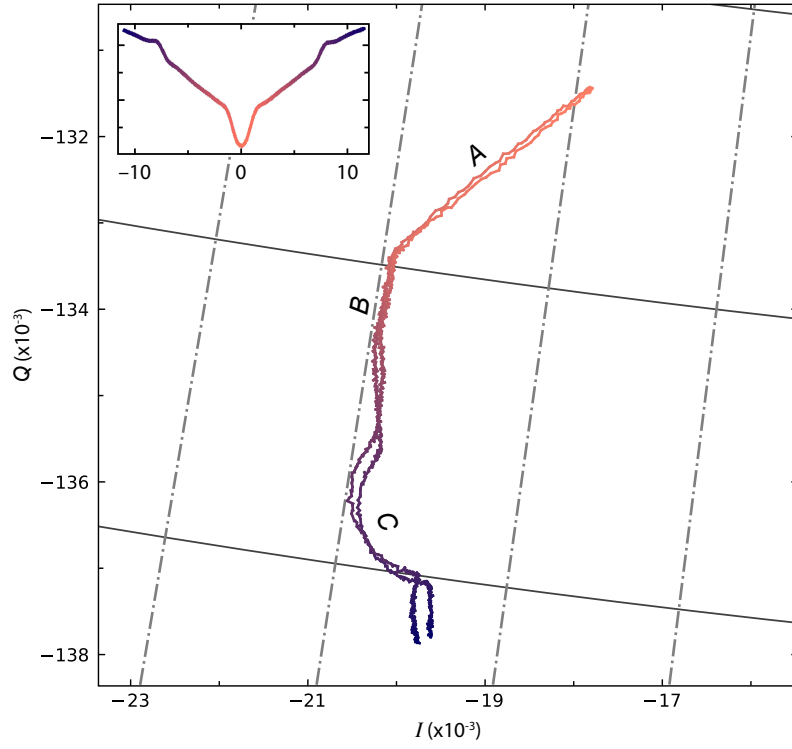


Figure 5.9: I/Q projection of the bias measurement of the Josephson junction. The letters indicate different bias regimes; A – the dissipationless Josephson bias regime, B – the bias exploring the normal state resistance of the junction, C – the Shapiro step associated with the 3.767 GHz readout tone. Dashed lines indicate radials of constant phase and the solid lines indicate circles of constant radius (magnitude). The inset is the same as the magnitude of the RF data in figure 5.7. It should be used as a guide for what voltages the colours represent.

DC data the resistive change is expected to be of order a few Ohms; nonetheless, this appears to be the dominant effect.

The region associated with the Shapiro steps does not follow a simple straight line. Much of this region is associated with a move away from the radial line indicating a potential increase in inductance. The Shapiro step itself follows a curved path.

The top gate provides control over the properties of the 2DEG. As the voltage is made more negative, the 2DEG becomes depleted and the normal state resistance increases. Figure 5.10 shows the I/Q response, averaged over bias, of the JJ in response to the top gate. The data does not follow a radial line as would be expected for a change solely in the resistance of the device. The direction of change indicates that as the applied voltage decreases, there is either a corresponding increase in inductance or an decrease in capacitance. This occurs at the same time that the resistance increases.

There are two distinct gradients to the slope present in the data in figure 5.10, separated by a step. These different regions are probably caused by the different sub-bands that are present in these types of heterostructures [52].

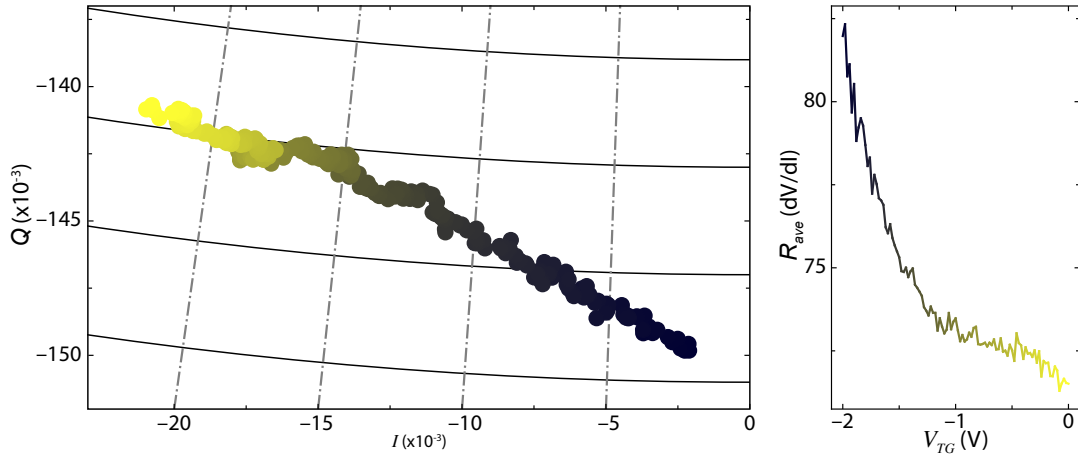


Figure 5.10: (Left) I/Q projection of the response of the Josephson junction to top gate. Each dot represents the average value of traces such as those shown in figure 5.9. Dashed lines indicate radial lines and the solid lines indicate circles of constant radius. (Right) Average differential resistance measured in DC. The colours are the same in top gate value in each plot.

5.3.2 Modelling

The descriptions used thus far have assumed the the resonance is formed solely by the JJ, modelled by the RCSJ model. This description is incomplete; a more complete picture needs to account for the system that the JJ is embedded within. In this case, this means taking into account the coupling from the gate to the JJ as well as taking into account the bond wire and trace inductance (figure 5.11).

The model in figure 5.11 has many free parameters; some of these parameters can be assigned with knowledge of the system and by making some approximations. Approximate values for the capacitances can be found by assuming the materials act like parallel plate capacitors, $C = \epsilon\epsilon_0 A/d$. Making this assumption, the shunt capacitance of the JJ is approximately 2 fF⁹. Similarly, the capacitance between the top gate and the JJ is of order 20 fF; the value for the gate capacitance was chosen to be 22 fF. From previous measurements the bondwire inductance is known to be in the tens of nH; a value of 80 nH was used for the modelling. The value of R was assigned to be the same value as that measured in DC.

Using these values the inductance was changed such that the position of the (smoothed) resonance position was fit¹⁰. Figure 5.12 shows the fit resonance position. The fit is in excellent agreement, except in the region where resistance measured 8 Ω .

Figure 5.13 shows the values of inductance produced by fitting the resonance. Although the resonance was symmetric about bias, both the negative and positive sides of the bias were fit. The resulting inductance values do not show the expected symmetry; the peak values appear at different bias values. Furthermore, the inductance values on the negative bias side

⁹The “plates” are of order $21 \mu\text{m} \times 100 \text{ nm}$, the distance $\sim 110 \text{ nm}$, through InAs which has a dielectric constant of order 12.

¹⁰To reduce the calculation time not every data point was fit, every 10th data point was fit.

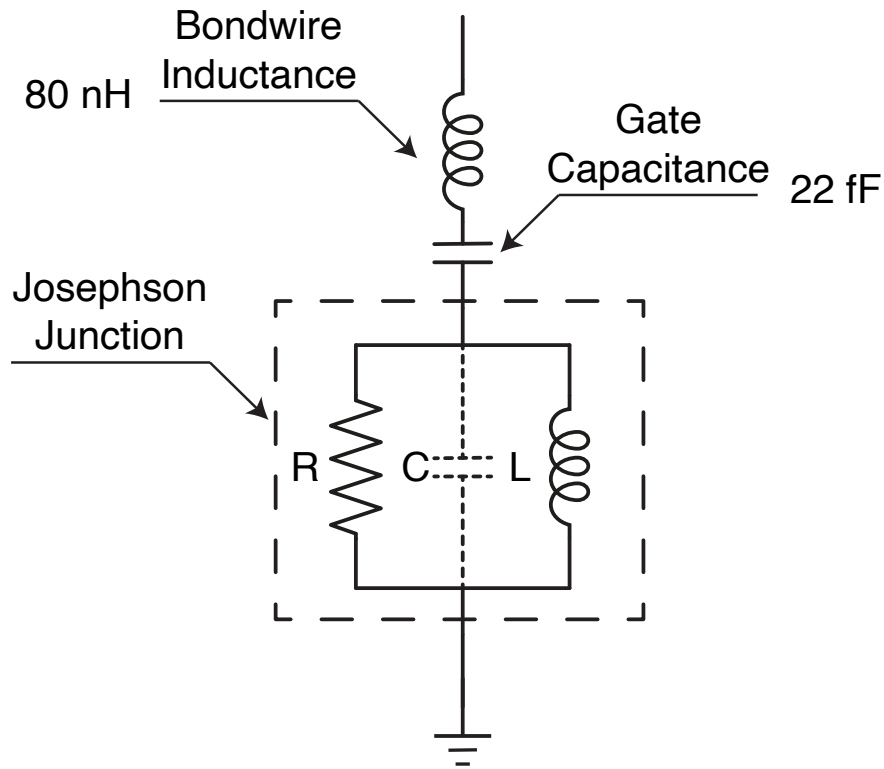


Figure 5.11: Simplified equivalent circuit used to model the RF response of the JJ. The fit parameters used are the resistance and inductance of the JJ.

are almost two orders of magnitude higher than the inductance values on the positive bias side. This is due to the starting value used¹¹, and that for this model the inductance values that give the same resonance are not unique.

The trends observed in the inductance follow the expected shape for a current-biased JJ; smallest at the lowest bias, peaking at the critical current, and reducing beyond this point. [156, 166]. Although this JJ was nominally voltage-biased, the applied current was changing with the applied voltage; i.e. the bias is provided by a finite source impedance so it retains some current bias characteristics. With this in mind, the trend does not seem completely unreasonable. Nonetheless, even if one accepts that this model may have produced the right trends, the numerical values are suspect. It is unclear from the model alone whether the inductance has values around 30 nH or 0.4 nH.

Although the model did not fit in the resonance low bias regime, it is interesting to note the trends in inductance, and even the values obtained, are similar to what has been seen in previous current bias measurements [166]¹².

It is not clear why the model failed in the low bias regime. The RCSJ model is intended to model the dynamics *away* from DC [156, 163–165], using this model in the low bias regime may not be appropriate. It is also possible that with the dynamics in this regime changing, the electrical characteristics also change, meaning that the values used for the other free

¹¹The initial guess was 100 nH. Each subsequent point used the previous points final answer as it's initial guess. Going from positive to negative bias the same plot was recovered but with which side was higher reversed.

¹²When taking into account that 1000s of JJs were used in the measurement in [166].

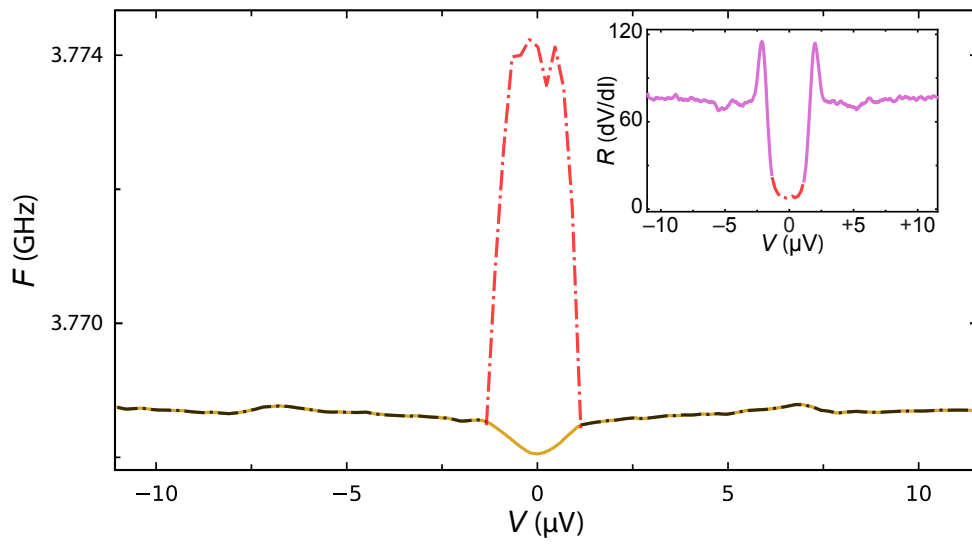


Figure 5.12: The gold is the resonant frequency extracted from the experimental data and smoothed by a Savitzky-Golay filter. The dashed line is the fit created using the equivalent circuit in figure 5.11. The red line indicates the region that did not fit the experimental data. (Inset) The differential resistance values used. The red line indicates those values where inputting these resistance values and changing solely the inductance was unable to give a reasonable fit.

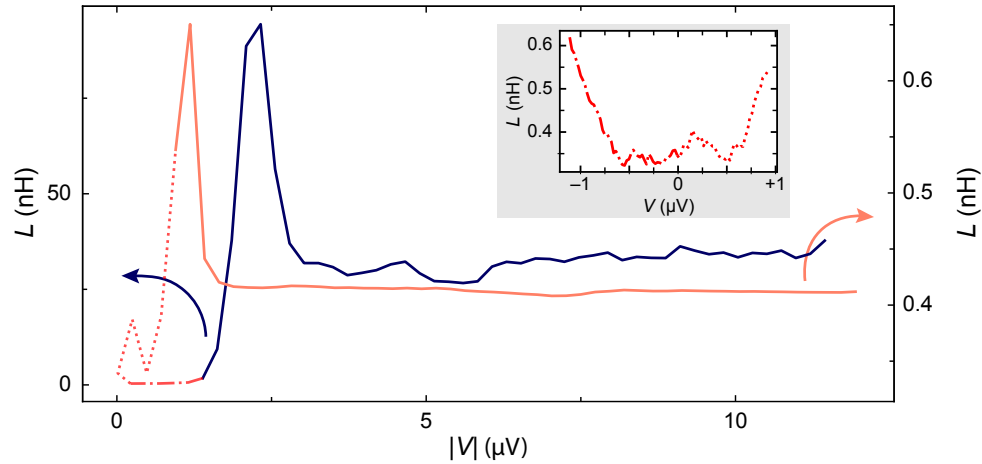


Figure 5.13: Fit inductance values of the JJ. Data was fit from the negative voltage values (blue) to the positive values (orange). The red (dotted and dash-dot) traces indicate the regions where the resonance did not match; this region is shown in the inset.

parameters (such as the gate capacitance) would also need to be modified in this regime. Whatever the reason, this model has not been able to produce values for the inductance of the JJ in such a way that the results could be used with confidence.

5.4 Conclusions

A single Josephson junction was measured both at low frequencies and using RF. The RF data in conjunction with the DC data was used to describe the qualitative changes in the resistance and Josephson inductance with respect to bias. An equivalent circuit based on the RCSJ model was used to model the junction; this model predicted the same qualitative trends. However, the validity of the quantitative results is suspect, particularly in the low bias regime.

The RF measurements are sensitive to the resistive and reactive changes that occur in a JJ. In order to extract the values, a different equivalent circuit model is needed or a different method. Simplifying the measurement so that the RF path is the same as the DC path would be a good first step. Calibrating the measurement would also serve to help in determining how the various parameters change.

Chapter 6

Local and Non-local Microwave Impedance of a Three-Terminal Hybrid Semiconductor-Superconductor Nanoscale Device

6.1 Abstract

In this chapter, the local and non-local responses of an Al-InAs three-terminal (3T) nanowire are explored at low frequencies, as well as at RF frequencies. The quasi-DC measurements are compared with the RF measurements. The RF technique used allows the full scattering matrix to be measured. The ability of this technique to resolve quantum features, to determine their probable location, and to provide extra information in the form of phase is discussed. The origins of the impedance transformations are also identified. The difference in sensitivity between the low frequency measurements and the RF measurements is discussed.

6.2 Introduction

The focus of this chapter is on the RF technique, not on measuring or modelling a particular quantum state. Additionally, in previous chapters many of the concepts needed to understand this chapter have already been introduced. As such, this introduction does not focus on providing specific in-depth information, instead taking a more general overview.

One of the major interests in superconductor-semiconductor hybrid devices, such as the one used in this chapter, is in meeting the requirements of generating Majorana quasiparticles. Majorana bound states (MBS) are predicted to be particularly useful in quantum computation as they are non-Abelian anyons¹, meaning they have topological protection – they are immune

¹Following neither fermionic nor bosonic spin-statistics upon exchange. Exchanges result in unitary rotations

to weak local perturbations [63]. Superconductor-semiconductor materials can be engineered such that they exhibit an effectively spinless, p -wave, 1D superconducting topological phase – the conditions where a Majorana may exist [57–59, 64].

Tunnelling conductance spectroscopy is the main technique used for probing sub-gap structure and identifying low-lying bound states of proximitised hybrid devices [60, 177–179]. However, the identification of MBSs has proven to be very challenging. The main challenge that has arisen is that other trivial bound states, such as Andreev bound states (ABS) and quantum dot states, can have very similar signatures [60, 62, 180–184]. A proposed experimental circumnavigation of this limitation is to probe the non-local properties of the phase or the correlation of sub-gap states between the two ends of a wire [185–187]. The three-terminal device structure (figure 6.1) was designed to achieve this [188, 189].

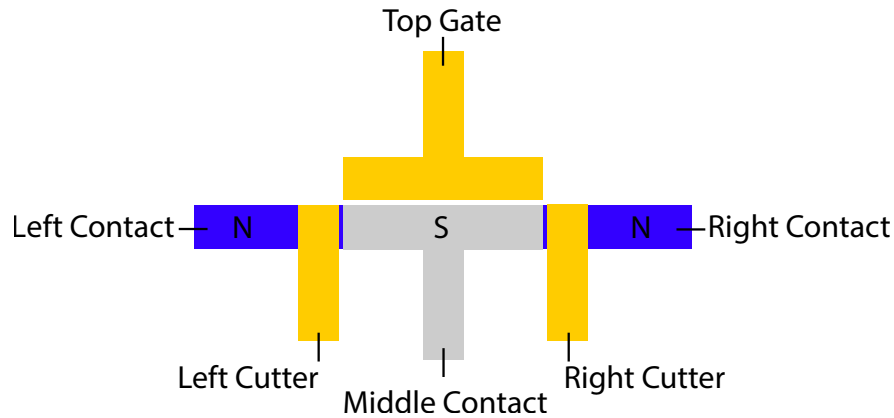


Figure 6.1: Idealised 3T superconductor-semiconductor device. The middle terminal is made up of a superconductor that proximitises the semiconductor. The cutters allow tunneling spectroscopy measurements to be made. The gate is used to change the chemical potential. The middle terminal allows for the discrimination of local and non-local measurements.

Somewhat ironically, ABS – what one wishes to distinguish from MBS – have served as test-beds for the non-local 3T techniques, serving as proxies for MBS. The technique itself can be validated this way because ABS are extended coherent states (as opposed to truly separated ‘non-local’ states), which will display correlated features in a sufficiently short wire². Conversely, MBS must be spatially distinct to retain their properties [190].

In this chapter Majorana particles are not sought; the regimes where they could exist in this particular device are not explored. Some of the states that could be measured are shown in figure 6.2. Andreev reflections would be expected to give a local signal with the electron and reflected hole that form the Cooper pair originating on the same terminal. Crossed Andreev reflection (CAR) and elastic co-tunnelling would provide a non-local signal; a signal that would be measured from one normal lead to the other. Local signals for these states are expected to be orders of magnitude greater than the non-local signals [190, 191]. The non-local correlations for these states are suppressed as the length of the wire increases.

of state vectors in a degenerate Hilbert space [176].

²Although the exact nature and location of the ABS formed in the wire are debated, evanescent states decay within the superconducting gap on the order of the coherence length, ξ_S , which would be indicative of the scale of an ‘extended’ state.

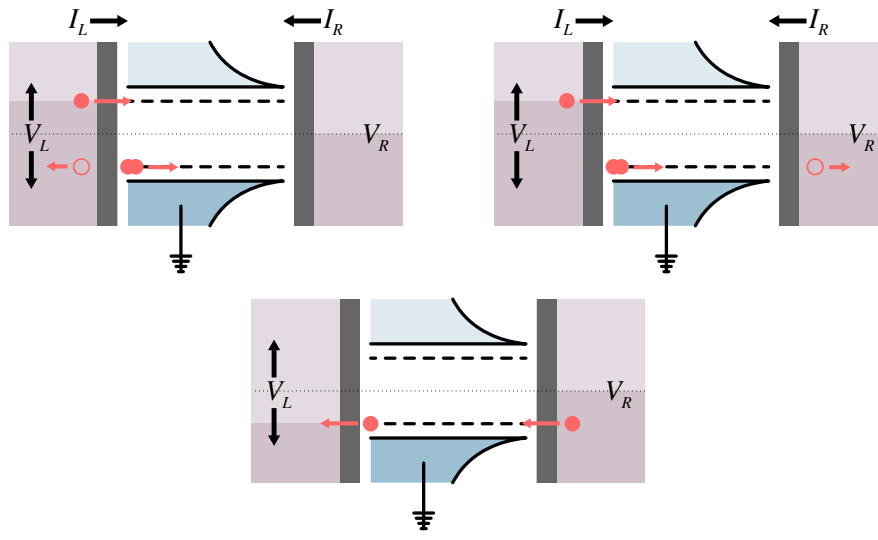


Figure 6.2: Schematic DOS of a 3T device biased from the left, showing electron and hole processes. The left voltage, V_L is defined as the potential difference between the left terminal and ground; there is no potential difference between the right and middle (superconducting) terminal and ground, $V_R = V_S = 0$. The positive current direction is defined as towards the superconducting gap.

(Left) Positive bias picture showing a local Andreev reflection process. The injected electron (filled circle) forms a Cooper pair with a reflected hole (empty circle) on the same terminal. (Right) Positive bias picture showing a non-local (Crossed) Andreev reflection (CAR) process. The injected electron forms a Cooper pair with a reflected hole on the other side of the superconductor.

(Middle) Negative bias picture showing elastic co-tunnelling. The electron tunnels into, then travels through, the superconducting region before tunnelling out into the normal region.

RF-measurements have proven to be a valuable tool in probing the state of delicate quantum systems, offering advantages of both increased sensitivity and speed [5, 171, 192, 193]. Typically, RF-readout utilises deliberate matching structures – either lumped element LC or co-planar waveguide (CPW) resonators [20, 193, 194]. Here, an RF technique is proposed and analysed that forgoes deliberate matching structures, instead relying on path specific reactances to form resonances.

6.3 Methods

A superconductor-semiconductor three-terminal (3T) device was measured at RF frequencies, as well as at low frequencies using standard lock-in amplifier techniques. The 3T device is an Al-InAs nanowire defined using selective area growth (SAG). The aluminium which proximitises the semiconductor also comprises the third terminal. This device had an active region of $L \sim 1 \mu\text{m}$ and $W \sim 40 \text{ nm}$. More device and set-up details can be found in chapter 3.

The amplitude of the AC oscillations applied in the lock-in measurements was of similar order to those used in RF. At the device level the AC oscillation was $V_{RMS} \sim 10 \mu\text{V}$. The RF power after attenuation but before the PCB was $\sim -80 \text{ dBm}$, which is equivalent to $V_{RMS} \sim 22 \mu\text{V}$; this gives a maximum size of the RF oscillation. Due to impedance transformations, the exact amplitude of the RF oscillation at the device depends on the frequency and the S-parameter. For example, for -30 dBm^3 at 4.795 GHz , the amplitude of the oscillation

³Applied at the top of the dilution refrigerator. This is the power used in subsequent measurements.

at the device was $\sim -15 \mu\text{V}$ for S_{11} , S_{12} and S_{13} , and $\sim 4 \mu\text{V}$ for all other parameters (see appendix D1 for relevant fits).

For the quasi-DC measurements the frequency used was usually $\sim 22 \text{ Hz}$; figure 6.10 is an exception where the frequency was $\sim 39.6 \text{ Hz}$. Many frequencies were used for the RF measurements. In order to discuss the results, a selection of frequencies has been chosen.

The majority of the RF data shown in this chapter is shown with arbitrary units. This was chosen as each S matrix consists of nine elements, each of which has a different offset and scale. Using an arbitrary scale in this case does not adversely affect the discussion, as the point was to determine whether changes could be identified and not to identify by what amount features cause change. Plots showing the explicit values for each S -parameter have been included in appendix D.5.

6.4 Results and Discussion

6.4.1 Key

As discussed in the background chapter, each S -parameter, S_{ij} , can be thought of as measuring the flow of energy from port j to port i . It is a measurement of the power at port i with respect to the power that was incident at port j . The device measured here consists of three ports. The usual way to express the S -matrix for such a device is given in equation (6.1).

$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \quad (6.1)$$

Throughout this chapter the RF measurements will be presented in a grid like the one depicted in figure 6.3. This figure can be used as a quick guide to understanding what each S -parameter measures. The left terminal has been designated as port 1, the right terminal as port 2, and the superconducting (middle) terminal as port 3.

The definition of local and non-local measurements used here is consistent with the definition used in literature [185–190, 195, 196]. That is, a local measurement measures between a normal terminal and the superconducting terminal, whereas a non-local measurement measures between the normal ends of the device. These measurements are named ‘local’ and ‘non-local’ as they are intended to distinguish between local and non-local states that exist in the device. However, in order for a state to be completely non-local, the size of the device needs to be longer than the superconducting coherence length [185, 186, 197]; this detail is ignored when defining the measurement type. For the RF measurements, S_{12} and S_{21} are the non-local measurements (figure 6.3).

The quasi-DC lock-in measurements will be referred to throughout the rest of this chapter as DC measurements. The DC measurements are presented as differential conductances defined as $g_{ij} = \frac{di_{ij}}{dv_{ij}}$, where di_{ij} is the differential current measured at terminal i injected at

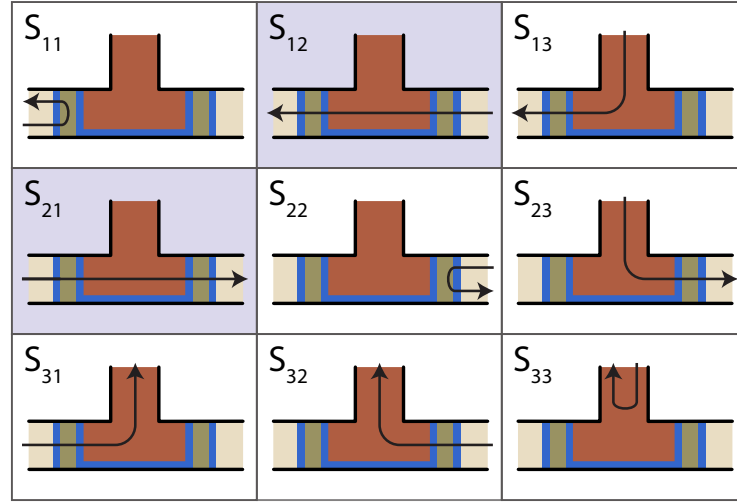


Figure 6.3: Measurement paths for the three-terminal device. Port 1 refers to the left terminal, port 2 to the right terminal, and port 3 the superconducting terminal. Reflection measurements are the diagonal terms – $S_{ij|i=j}$ – and transmission measurements the off-diagonal terms – $S_{ij|i \neq j}$. The non-local paths are highlighted in purple.

terminal j and dv_{ij} is the differential voltage measured between terminals i and j . Each port is defined such that it is consistent with the definition used in defining the RF measurements; port 1 = left, port 2 = right, port 3 = middle. For example, a local measurement would be defined as a g_{13} or g_{23} .

6.4.2 Transmission Line Resonance

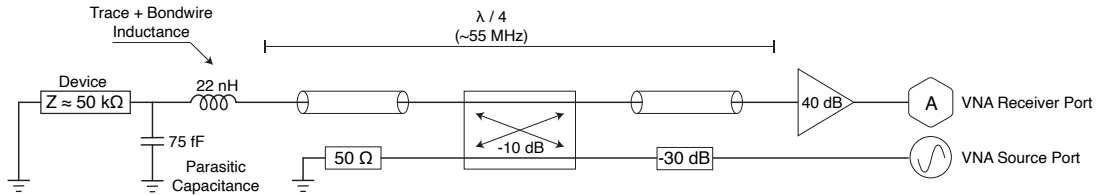


Figure 6.4: Simplified single-port circuit diagram of the transmission line resonator (TRL) approach, omitting the bias-tee⁴. The small impedance mismatch between the cryogenic amplifier and the cables is exploited to transform the device impedance.

Standard RF-reflectometry techniques generally use lumped element resonators in order to create impedance transformations [20, 194, 198]. In contrast, the set-up used here – dubbed distributed transmission-line resonator (TLR) – implements impedance matching between the device and the superconducting cable without the use of lumped elements (figure 6.4). Instead, standing waves are created between the device and the front end of the amplifier. This acts to transform the impedance of the device towards Z_0 at integer multiples of $\lambda/4$. For the particular length of cables used in this set-up, $\lambda/4$ corresponds to 55 MHz and the resonance has a Q -factor of approximately 30.

The periodic response of the transmitted and reflected RF signals can be viewed analog-

⁴The bias-tee was ignored as its effect on the spectra is small. The bias-tee is not quite 50 Ω ; as such, it does introduce an additional small ripple on the spectra.

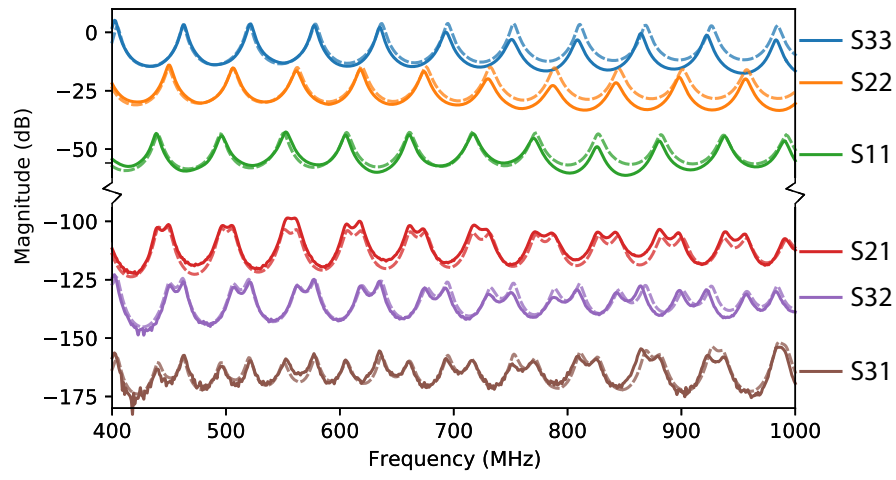


Figure 6.5: Measured (solid line) and simulated (faded dashed line) resonant mode effects. Transmission windows occur when harmonic modes of each cable overlap. Each successive trace has been offset for clarity by -20 dB.

ously to a Fabry-Pérot interferometer, with the semi-silvered mirrors replaced by the device impedance mismatch at one end and the amplifier mismatch at the other end. The length of the coaxial-cables differ slightly on each of the three ports, yielding slightly different frequency combs for each port. The spectra are consistent with the different measured physical length of the cables (figure 6.5).

The TLR produces a harmonic spectrum of resonance responses that enables impedance measurements at many different frequencies. The range of useful frequencies spanned from a few 100 MHz up to 7 GHz. In this configuration, where all of the parts are attached to TLRs, the entire S matrix can be nearly simultaneously obtained; a necessary condition if one were to to completely characterise a system or to acquire both the local and non-local response. This is in contrast to lumped element approaches, where the requirement that all resonators be matched to the same frequency may lead to cross-talk from shared parasitic elements. Alternatively, using lumped element resonators with separate resonance frequencies for multi-port measurements limits measurements to only reflection measurements; i.e. only probing the local response of the system.

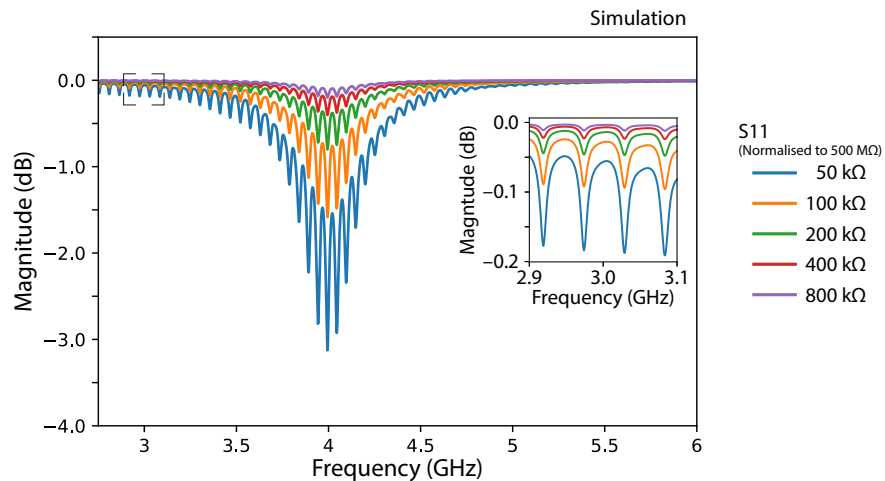


Figure 6.6: Single port simulation reflection of the TLR including series (bond-wire) inductance and parasitic capacitance.

In addition to the periodic response of the TLRs, there are parasitic LC contributions to the response. The inductance is primarily attributed to the inherent series inductance of the bond-wires, and the capacitance is identified as primarily coming from planar structures in conjunction with parasitic device capacitance. The combination of inductances and capacitances results in an additional resonance; for the expected device parameters the resonance occurs around 4 GHz (figure 6.6). The superconducting terminal of the device was sensitive at substantially lower frequencies than the other two terminals, likely as a result of the wider width of these on-chip lines (increased C_p) and the additional kinetic inductance from the superconductor itself.

The best SNR for reflections occurs at the peaks of the standing waves. There is a gain at these frequencies when compared with a response without standing waves. The best SNR of the transmission occurs when the peaks of the standing waves of the terminals overlap. It is still possible to measure in regions where there is no resonance; however this is detrimental to the size of the response (figure 6.7). It is worth keeping in mind that calibration would normalise these responses, but the modulation of the SNR would remain.

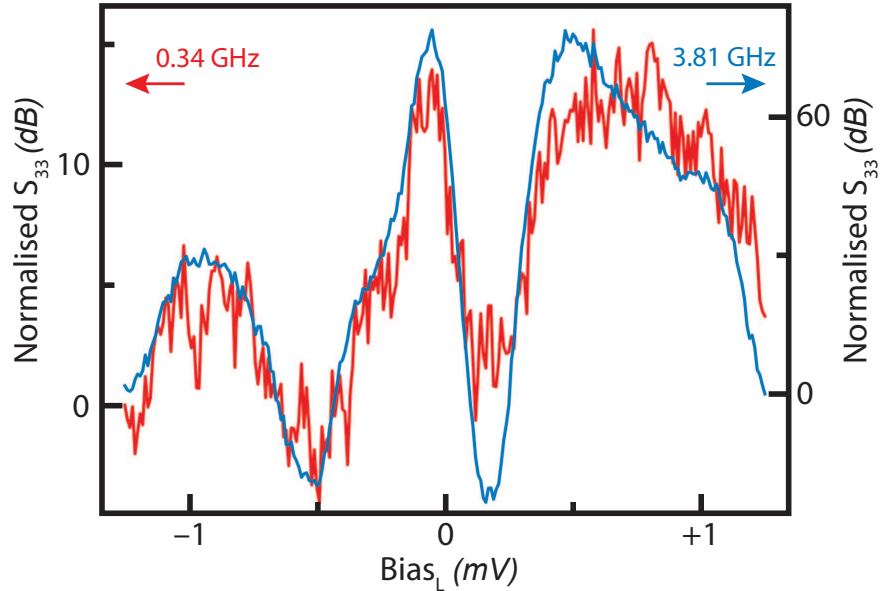


Figure 6.7: On- (blue) and off-resonance (red) measurements. The signal is present both on- and off-resonance, however the SNR is much higher when on resonance.

When uncalibrated, small variations between the cryogenic amplifiers, directional couplers, cables, and connectors introduce phase and amplitude offsets which lead to non-reciprocal responses. What is measured is a change in the S -parameter response as a function of applied device voltage or magnetic field. The changes are attributed solely to changes in the RF-response of the device, as the response of all other components remain constant for a given frequency.

The various S -parameters will not be uniformly sensitive at the same frequencies. This is an issue with the current set-up. However, an extension to the set-up may be possible which overcomes this drawback whilst maintaining the benefits of TLR. It should be possible to include a tuneable electrical length so that the position of the resonances could be modified,

such that they overlap for all ports. This would open up more opportunities where the RF contribution of specific ports could be selectively isolated. Furthermore, the frequency could be swept at the same time as the electrical length to allow the TLR benefits to occur at every frequency.

6.4.3 Spatial Resolution

Figure 6.8 shows the local conductances, g_{13} and g_{23} , measured as a function of left and right cutter gate voltage V_{LC} and V_{RC} . g_{13} probes local transport of the (normal) left lead, measuring between the left lead and the superconductor; g_{23} is a local probe of the (normal) right lead, measuring the charge path between the right lead and the superconducting terminal. As the gates pinch off, the data exhibits peaks that are likely due to charge state resonances associated with the disorder potential. Vertical features appear in the local probe of the left, g_{13} , whereas horizontal features appear in the data that probes the right, g_{23} . When measuring from the left the vertical features are strongly dependent on the left cutter gate voltage, but exhibit little dependence on the right cutter gate voltage. The opposite is true for the horizontal features when probing from the right. Taken together, these patterns suggest that vertical features are related to states associated with the left tunnel barrier and horizontal features relate to states near the right barrier. Such patterns are well-known in the quantum dot literature [199].

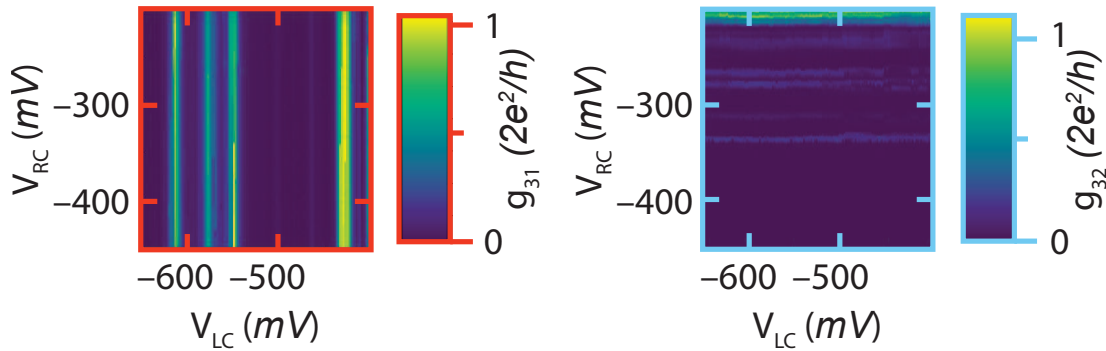


Figure 6.8: Local differential conductance measurements of the DC cutter-cutter response. The measured conductance between the left and middle terminal (left) is sensitive only to the left cutter, whereas the measured conductance between the right and middle terminal (right) shows features only due to the right cutter. Red and blue boxes indicates the equivalent measurements to the local conductance measurements shown in figure 6.9.

The same measurement was taken in RF; figure 6.9 shows the full S -matrix measured at 3.2 GHz. The magnitude of the S_{31} and S_{32} measurements (and their reciprocal elements) strongly resembles the local DC measurement. Vertical lines dominate the S_{31} measurement, much as they do in the g_{31} , and horizontal lines dominate the S_{32} measurement, much like the g_{32} measurement. The same trends are seen at other frequencies measured at the same time (see appendix D.2.). Unlike the magnitude response, the phase response shows both vertical and horizontal lines, particularly in the S_{32} measurement. Although both cutters cause phase shifts, the direction in which the phase changes is in opposite directions depending on the cutter causing the change.

The reflection parameters are equivalent to those made using a standard reflectometry set-up. The S_{11} measurement shows a strong dependence on the left cutter, whereas the S_{22} shows a strong dependence on the right cutter. It would be expected that the reflection would show both local and non-local dependencies as in a lossless system $|S_{11}|^2 = 1 - |S_{21}|^2 - |S_{31}|^2$. However, in general the local response is orders of magnitude stronger than the non-local [190, 191]⁵, see also figure 6.12; this implies $|S_{21}|^2 \ll |S_{31}|^2$. It is then perhaps not too surprising that such measurements only show clear features relating to the local measurement as $|S_{11}|^2 \approx 1 - |S_{31}|^2$. The same arguments can be made for the S_{22} response. The S_{33} measurement, however, is unique in that it is effectively a simultaneous local probe of both the left and the right. The data reflects this, showing both features due to the left and the right cutter.

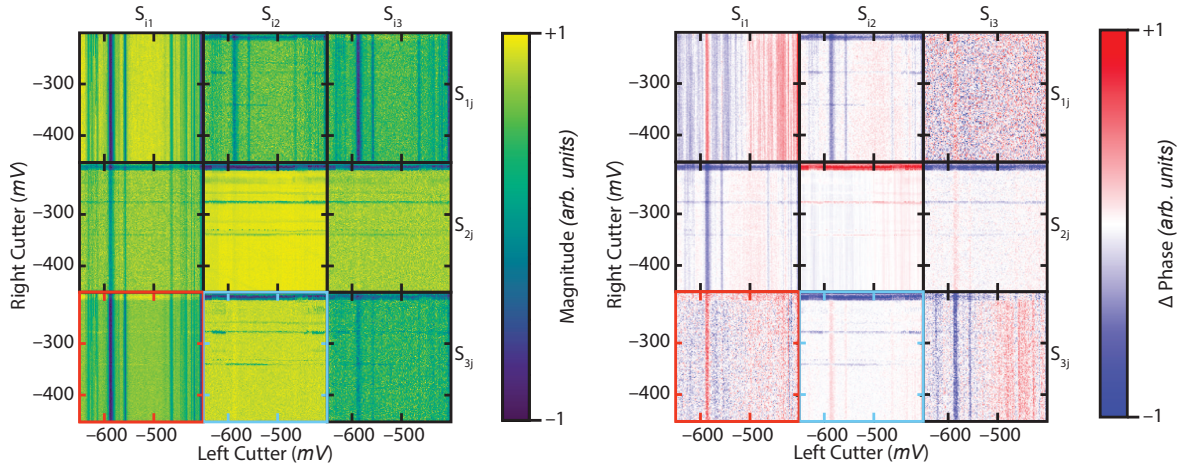


Figure 6.9: Magnitude (left) and difference in phase (right) response of S -parameters measured at 3.2 GHz. The red and blue boxed parameters are the RF equivalent of g_{13} and g_{23} respectively, the measurements shown in figure 6.8.

Perhaps more interesting are the S -matrix elements that relate to the non-local response – S_{12} and S_{21} . The bias for these measurements was set to 0 V; the expectation is that the power should all be terminated in to a well-matched system via the middle superconducting terminal. Instead, both vertical and horizontal features are present in both the magnitude and phase responses of these data, consistent with measuring both tunnel barriers in series. The measured change in both magnitude and phase – what appears as a feature – is much less in these parameters than in the other transport S parameters (see figure D.8 in appendix D.5. for individual scales and offsets). It is interesting to note that although the phase response of S_{32} showed both horizontal and vertical features (much like the non-local measurements), the direction of change depended on which cutter caused the feature. In contrast, phase change in the non-local measurement was only in a single direction for both left and right cutter features.

From these data it would appear that both the local and non-local measurements may be simultaneously measured with different elements of the S matrix. It should be possible to

⁵Strictly, many of the measurements in literature rely on this to define their ‘local’ measurement, which is actually a combination of local and non-local. Given the large difference in scale between the local and non-local responses, a combination of the two responses will usually appear to be a local measurement.

determine in which area of the device a state exists by examining which S parameters show a response. The complex nature of the RF measurements potentially affords extra insights in the form of phase information.

6.4.4 Magnetic field response

The concept that the different S -parameters may simultaneously reveal both the local and non-local response of the device was further tested by performing bias spectroscopy of the superconducting gap as a function of applied magnetic field. The magnetic field was applied in the plane of the device, perpendicular to the major axis of the nanowire; see the inset to figure 6.10.

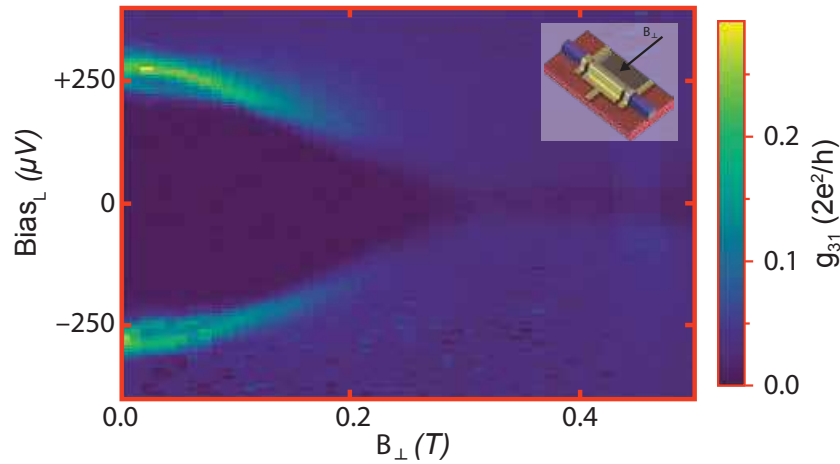


Figure 6.10: Local response of the superconducting gap with magnetic field applied in-plane and perpendicular to the wire. The inset indicates the direction of the magnetic field.

In a differential measurement, it is expected that peaks appear on either side of the region that is superconducting. The peaks are associated with the BCS density of state, which defines the superconducting gap. When the superconductor is subject to a magnetic field the size of the superconducting gap decreases. Given a strong enough magnetic field, the superconducting gap will completely close. A reopening of the superconducting gap is a signature of a topological phase [57, 200, 201].

The local differential measurement is shown in figure 6.10. The non-local measurement was also taken, but showed only noise (see appendix D.3.). No topological phase was observed; the gap was observed to close around 0.3 T and remain closed. When no magnetic field is applied, the induced gap and the parent gap appear to be very similar to each other. This is consistent with similar transport measurements [195].

The corresponding microwave measurement is the red bordered element in figure 6.11. In the microwave response, the magnitude of the S_{31} measurement shows features that correspond to the DC measurement. Evidence of a normal-superconducting tunnel barrier, observed as the peaks in bias, is present. The size of the superconducting gap and its dependence on magnetic field, determined by these peaks, are the same. The peaks in bias appear for each matrix element that involves the left terminal. This makes sense, as for this

measurement the bias is from the left.

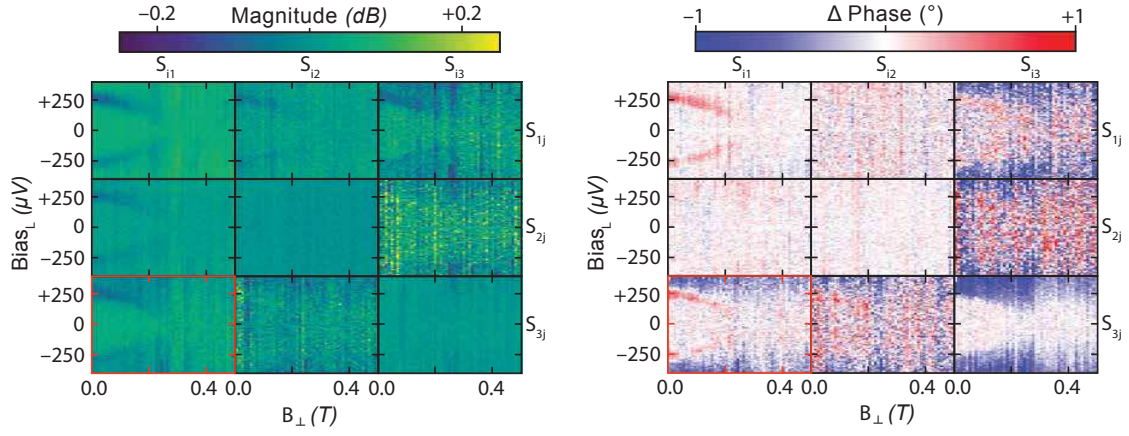


Figure 6.11: Magnitude (left) and phase (right) response of the S -parameters measured at 4.54 GHz. The background offset (0 bias) has been subtracted. The S_{31} response, highlighted in red, is equivalent to the DC measurement shown in figure 6.10.

The phase of the S_{31} parameter also shows the peaks associated with the closing of the superconducting gap. These appear as positive phase shifts (with respect to 0 bias); this also appears in the reflection measurement S_{11} . No clear shift is seen in the non-local measurement. In addition to the positive phase shifts associated with the induced superconducting gap, there are some features that appear as a negative (blue) phase shift. These appear in the S_{31} parameter as well as all other parameters which involve the third terminal. These features appear to be less affected by the magnetic field than the induced gap. The fact that this feature only appears on terminals that involve the superconducting terminal, and is affected by the magnetic field but less than the induced gap, suggests that this may be a direct measurement of the parent (Al) gap. Further data is needed to test this hypothesis.

6.4.5 Sub-gap features

The TLR technique is able to identify the superconducting gap. This is useful; however, it would be a more useful technique if it also is able to identify states that exist within the superconducting gap. Here, the interest is in showing that the technique is able to measure sub-gap states, not necessarily in finding a specific state such as a ZBP or MBS. The data presented here was taken by varying gate biases at zero magnetic field. Sub-gap features are frequently observed under these conditions in disordered devices [188, 189, 195].

In DC, a region with significant sub-gap structure was found (figure 6.12). This region had sub-gap features when biased from either the left or the right terminal. The sensitivity with top gate bias suggests that these features result from Andreev bound states (ABS) and unintentional quantum dots associated with disorder. The sub-gap features appear distinct when biasing from the left or the right. This implies that the state on the left is distinct from the one on the right; i.e. these are distinct local states.

The non-local signal is much weaker than the local signal, which is to be expected for these types of devices [190]. Only small regions around $\pm 260 \mu\text{V}$ show signal. Non-local transport occurs in the window between the induced gap and the parent gap. Given the

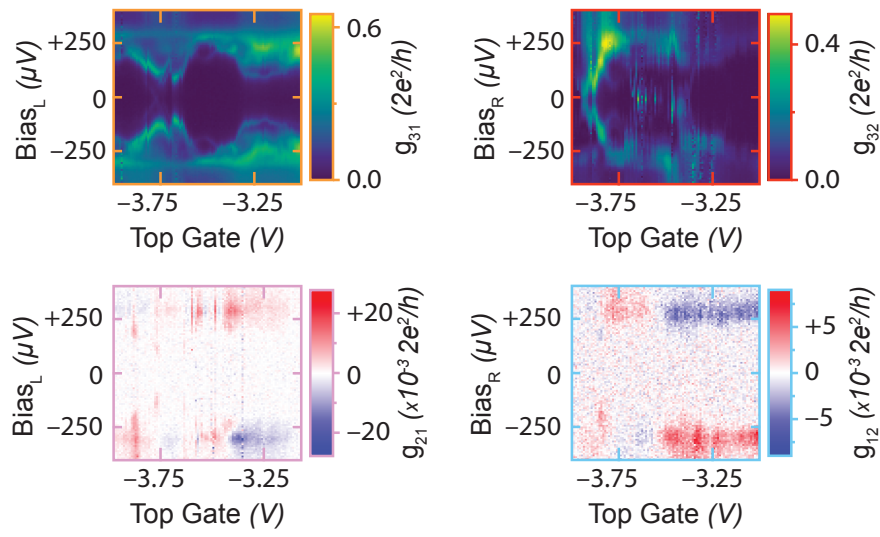


Figure 6.12: Local (top) and non-local (bottom) measurement of sub-gap features. The same gate space was examined biased from both the left lead (left) and the right lead (right).

small difference between the gaps observed in the magnetic field measurements, it would be unexpected if there was significant transport within the induced gap.

The corresponding bias measurements in RF are more complicated (figure 6.13). When biased from the left, the local responses associated with the left terminal – S_{11} and S_{31} – show the same sub-gap structure as was measured in DC. The local response for the normal non-biased (right) terminal – S_{22} and S_{23} – shows almost no signal, at least in terms of magnitude. A small response in phase is apparent. When biased from the right, a consistent picture forms: a sub-gap structure that resembles the DC measurement appears in the local responses associated with the side being biased – S_{22} and S_{32} . The local response associated with the non-biased (left) side does, however, show similar sub-gap structure to the DC measurements.

The non-local RF response does not resemble the non-local DC response. Instead it resembles the local conductance measurements. Again, much like the local response associated with the normal terminal not being biased, the signal is weaker than the local measurements. The mechanism behind why signal appears in all of these S -parameters is likely to be the same. RF measurements do not rely on conductance pathways; they are able to probe states that are accessible via tunneling or via capacitive pathways.

Whilst nominally a benefit, as it should allow states that are isolated from transport paths to be probed, here it is something of a disadvantage as the local response is dominating the non-local response. Figure 6.14 shows the various energy levels present in the system. Microwave signals are able to directly tunnel from a lead to a sub-gap localised state at a rate of Γ . Additionally, there exists capacitive paths from each lead to localised states; this is possible from both normal terminals. Essentially, this provides a mean of coupling the left and right port, allowing what would otherwise be a non-local measurement; for example S_{21} , to display local signal from the local state. This also would explain the weak signal that appears in the local measurement of the non-biased side; S_{31} for example in the bias from the

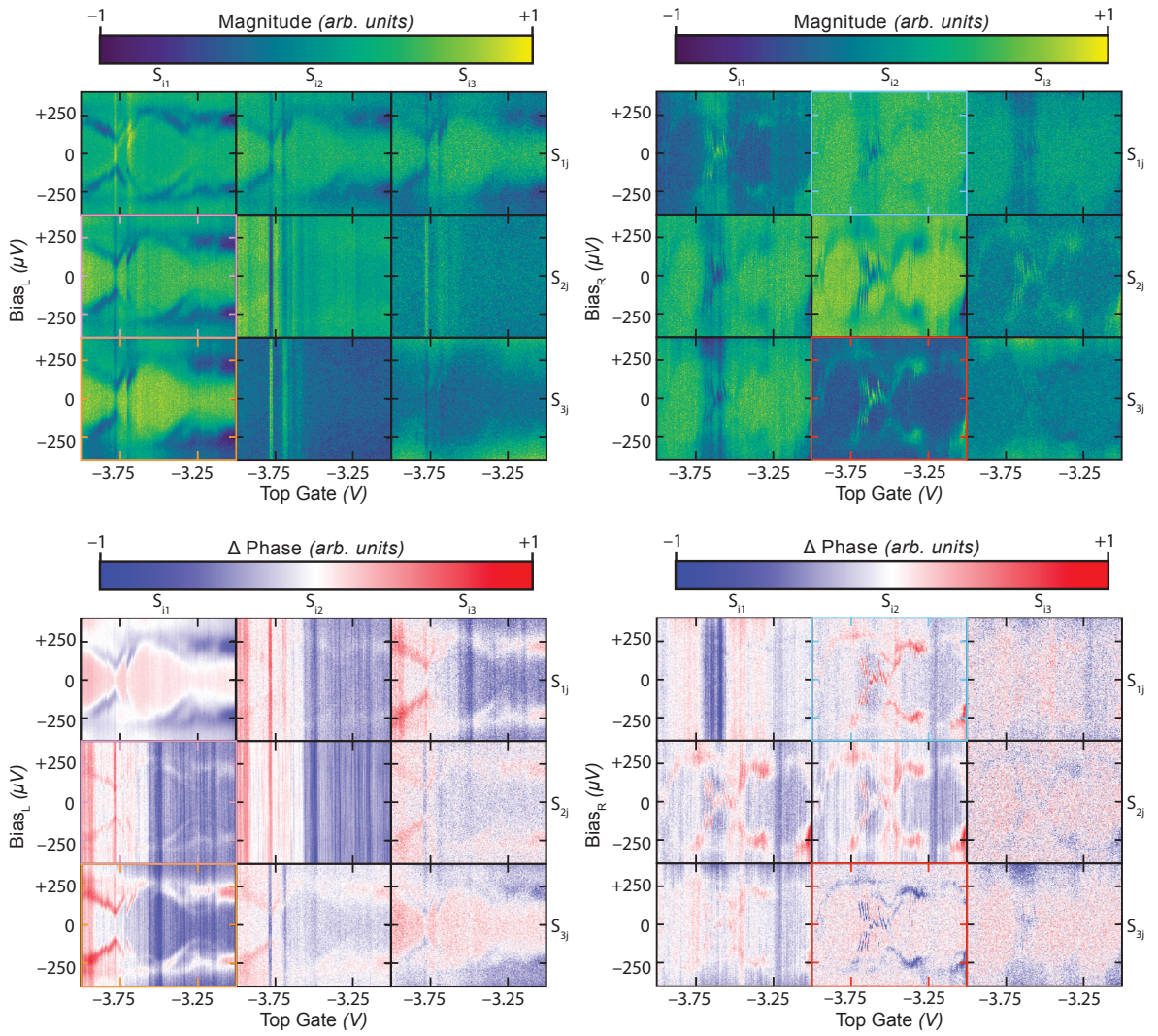


Figure 6.13: Magnitude (top) and phase (bottom) component of the S -parameters measured at 4.795 GHz. (Left) Left bias response and (right) is the right bias response. The coloured S -parameters are the equivalent measurements to those shown in figure 6.12.

right. In both cases this signal should be weaker than the direct local measurement.

6.4.6 Sensitivity

One claim often made in respect to RF measurements is that it is faster than the corresponding measurement in DC. Alternatively, this is presented as a higher SNR (for a given sampling duration); these statements are equivalent. Figure 6.15 shows a comparison of the response of the cutters, essentially a 1D slice of data similar to the figures shown in section 6.4.3. No clear peaks are found in the non-local DC response for voltages less than ~ -375 mV, whereas in RF many more peaks are seen. This is not a completely fair comparison as it is a comparison of the non-local response – the weaker DC response – to a local RF response. Comparing the local DC response to the RF, there are additional peaks that appear in RF that occur when the DC is below its noise floor. These have been indicated by green arrows.

The data shows more features; however, this does not necessarily mean the one with more features is more sensitive. In order to constitute an advantage, this should be true when the signal strength is comparable *and* when the sampling is similar. As noted in the methods section the size of the RF oscillation, although different for each frequency and each

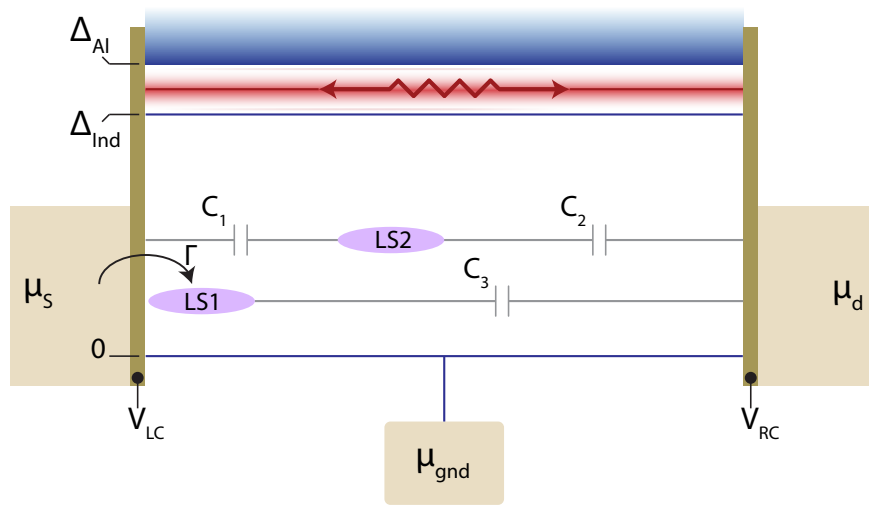


Figure 6.14: Illustration depicting the energy levels of a three-terminal device with localised states (LSs). Here, μ_S , μ_d , and μ_{gnd} are the chemical potentials of the terminals. Δ_{Al} and Δ_{Ind} represent the parent gap and the induced gap respectively. Tunnel barriers are defined by the gate voltages V_{LC} and V_{RC} . The microwave response between ports involves contributions from both tunneling and capacitive coupling to localised states. Red arrows indicate non-local transport.

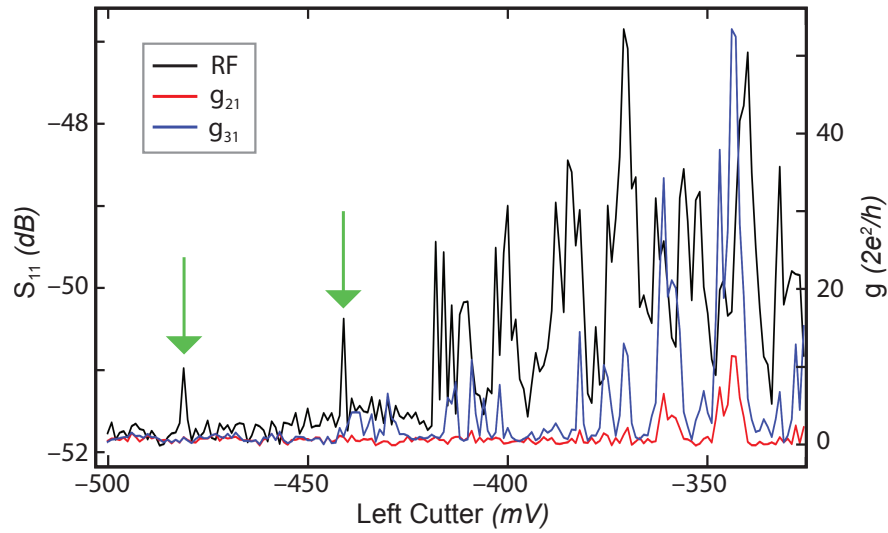


Figure 6.15: Comparison of sensitivity of RF and DC cutter signals. The RF shows extra peaks (indicated by green arrows) when the DC is in the noise floor.

S-parameter⁶, was similar to the AC oscillation at the device level. An upper estimate puts the RF oscillation at $2 \times$ the AC oscillation; in reality for most frequencies and S-parameters this will be a significant overestimate.

In determining the speed of the measurement certain simplifications will be made. The data handling, transfer, and processing shall be ignored⁷. In RF each path with the same source (each row in the S-matrix) is measured simultaneously; this will also be ignored. A simplified comparison of the speed is to examine the bandwidth used to capture the data. With the lock-in amplifiers, given the filtering used, in order for each point to be independent a bandwidth of ~ 10 Hz was used. In RF the bandwidth was not the same for each measurement. The bandwidth used was 0.1-1 kHz; both figure 6.15 and figure 6.16 used

⁶Depending on the signal path and impedance transform, the oscillation size at the device differs.

⁷These overheads depend primarily on the way that the measurement code is structured.

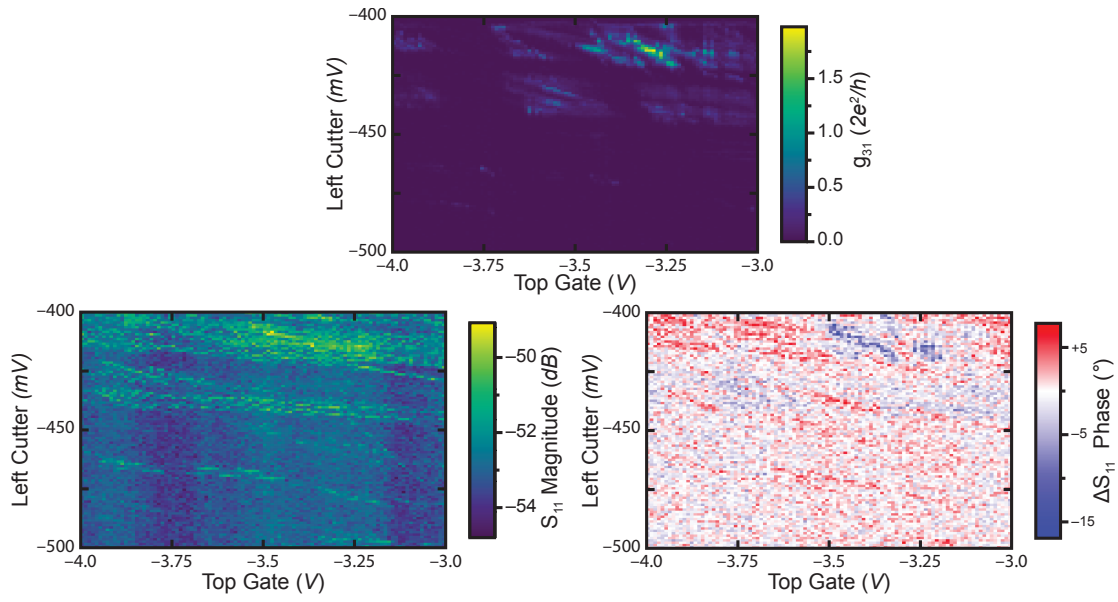


Figure 6.16: Comparison of local sensitivity of RF and DC zero bias signals. (Top) shows the local DC response. (Bottom) S -parameters measured at 5.61 GHz (Left) shows the magnitude and (right) the phase of the local response, S_{11} . The RF shows extra signals when the DC is in the noise floor. As the data is sparse, the phase response was normalised to the median value.

a bandwidth of 1 kHz. Averaging was not applied to any of the measurements.

The RF measurement had at most an oscillation that was twice as large as the AC oscillation. Each individual point in RF was obtained at least $10\times$ faster than each point in DC. Taken together⁸, the time spent per point in RF was substantially less than that used to capture the DC data, yet the RF measurements show features that occur when the DC was below the noise floor⁹. From this, it is reasonable to conclude that the RF measurement is more sensitive than the DC. Measuring at the same speed, with the same duration, will result in a much higher SNR.

6.5 Summary

Using the full scattering matrix, by determining which S -parameters are sensitive to a feature, the location of the individual feature can be determined. If a feature appears in the scattering parameters that involve the left terminal, it is likely to be located on the left hand side of the device. This was demonstrated in the cutter-cutter scans. Further evidence of this was shown in the measurements of superconducting gaps and sub-gap features.

The technique is also able to measure both the local and the non-local path simultaneously. Unfortunately, the non-local measurements, S_{21} and S_{12} contain, and are in fact dominated by, the local signal. The mixing is caused by direct capacitive coupling across the device, compounded by the vastly different signal strengths. As the size of the devices increases (which is believed to be needed anyway to measure truly non-local states [190]), the effect of the capacitive coupling should diminish. It does, however, demonstrate that the TLR

⁸Increasing integration time (or averaging) has a square root improvement on SNR, increasing signal scales linearly. Nonetheless, if both techniques were equally sensitive the lock-in measurement had the more favourable conditions and would show a higher SNR.

⁹This is true irrespective of the scale used, an additional plot is available in section D.6. where both data sets are plotted on logarithmic scales.

technique is able to measure paths not accessible via transport measurements, which would be useful in searching for ‘dark’ localised states that are cut off from the leads.

Understanding measurements taken in RF can be more complicated than understanding low frequencies, due its complex nature. The complex nature does however offer more information. Here, for instance, a positive phase shift was correlated with the induced gap, whereas a negative phase shift was attributed to the parent gap. More data is needed so that a concrete conclusion can be made.

In general, it has been shown that the TLR technique is able to measure the same responses as the low frequency technique. The technique also offers extra information in the form of phase information, as well as providing access to paths that are inaccessible via transport. Additionally, the RF technique is more sensitive; at similar power and with less time taken, the TLR measurements shows additional features.

With the current set-up, the way that the resonances forms means that the peaks in sensitivity do not necessarily match for each S -parameter. An extension to the TLR would be to add a variable electrical length to the path. This should allow the peaks in sensitivities to be tuned to completely overlap, or separated if desired. This could provide a mechanism where the increased sensitivity offered by TLR can be used at all frequencies.

Chapter 7

Conclusion and Outlook

This thesis presented a series of experiments on different mesoscopic systems. Each experiment showed a progression in the sophistication of the methods used to perform RF readout without the use of deliberate matching networks. This culminates in a approach – the TRL method – which shows certain benefits over low frequency methods and other current state of the RF approaches.

The measurement performed on the quantum Hall system showed that current models are unable to predict the shape (in magnetic field/frequency space) of EMPs when measured on a quantum Hall plateau. A new phenomenological model was produced that showed excellent predictive powers. This model contains no free parameters. The changes are incorporated into the Hall conductivity, using only the quantised and local filling factors.

It is worth pursuing if this model is universal; if not where and why it fails. Understanding when this model does not work should help in illuminating the mechanisms that underpin it, possibly providing a means to create a mechanistic model. This would help increase the understanding of EMPs, the quantum Hall effect, and the DOS of these systems.

The reflection measurements of the Josephson junction (JJ) showed that it is possible to directly measure a single junction using radio-frequencies reflecting off the top gate. This may be useful in compactly measuring single junctions, particularly in an array of junctions. The changes in the inductance as a function of bias were qualitatively determined, in order to quantitatively determine these changes a more sophisticated model is necessary.

These type of measurements may be improved by calibration, allowing more direct means of determining the inductance to be used, such as the Smith chart. Measuring using the same path as the DC – via the leads; should also simplify the equivalent circuit model representation needed to describe the junction, reducing the number of free parameters, and increasing confidence in the results.

A three-terminal (3T) semiconductor-superconductor device was measured both at low frequencies and in RF. The high frequency method used to measure the 3T device – the transmission line resonator (TLR) technique – is not only able to identify the same features as

low frequency measurements, it does so with more sensitivity. Using different S -parameters it is possible to use this technique to determine where a quantum state is localised within the device. Additionally, the technique was shown to be sensitive to states that are isolated from charge paths, this is important in identifying 'dark' states. Unfortunately, this was at the cost of mixing the local and non-local features such that they both appear in the non-local S -parameters. The effect of mixing should be reduced by using longer devices – which are needed anyway to measure truly non-local states.

The maximum SNR of the TLR technique, as currently used, is different for each terminal due to the different resonance path lengths. By extending the set-up to include a tuneable electrical length, the frequencies of highest SNR should be modifiable. This would allow the resonances to be made to overlap with each other. Additionally, this would allow the benefits of the resonances to be accessed at all frequencies. Or in cases where greater discrimination between ports is required, the resonances could be detuned.

Appendix A

Fabrication

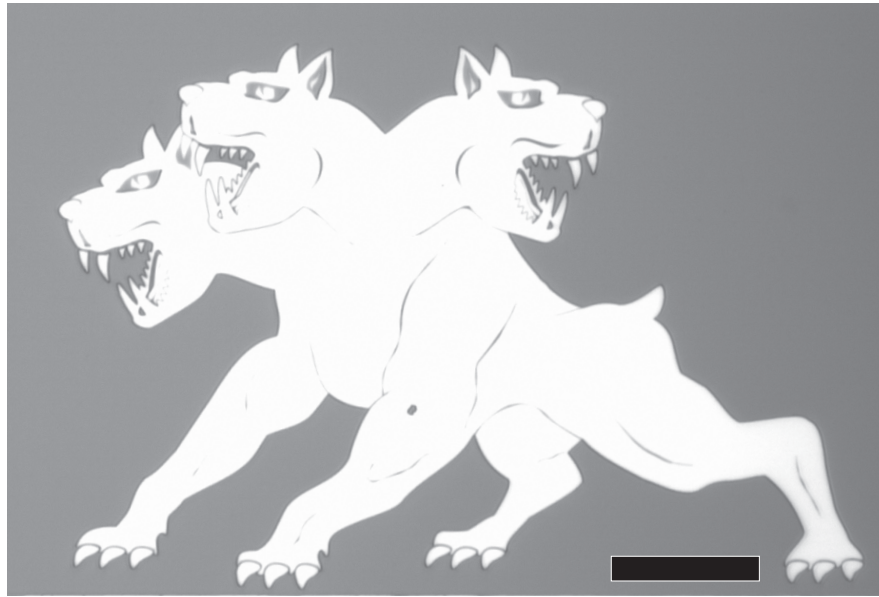


Figure A.1: Example of structures after lift-off. Bright areas are the gold, and the dark areas are heterostructure. EBL was used to define the features. The scale bar is 25 μm .

A.1. EMP Device Fabrication Steps

The first step of fabricating any device is to make a **design**. For me, this process occurs in Autocad. Generally, I like to design elements using blocks with adjustable scale sliders to allow for quick modification of designs.

The wafers we receive are generally 2", or parts thereof. The total size of a chip that we typically work with will be no larger than 1 cm^2 , and more typically is of order 25 mm^2 . **Cleaving** is the process by which the wafer is cut to the appropriate size for the device we want to make. Before cleaving, it is worthwhile to have spun resist on the wafer so that the surface is protected. The cleaving process involves marking the wafer with a diamond scribe. The wafer is placed on a glass slide so that the scribed mark is aligned with the edge of the slide. I generally cover the wafer with a cloth and use a ruler to apply light, even pressure. Pressure is applied to the edge of the sample that is hanging off the slide; this will cause the wafer to cleave along the scribed edge.

The wafers of GaAs we use typically come with Ga on their backs to aid in the MBE

growth process. This must be removed before processing further. To **remove** the **Ga**, the device is put on a glass slide face down into resist – usually PMMA A3, although any resist will do. To set the resist, the glass slide is baked at 180° C for 1 minute. To remove the Ga, the slide is heated on a 90° C hotplate and a cotton bud is used to remove the Ga¹. The process of heating and scraping with cotton buds is repeated until enough Ga has been removed. At this point the chip must be cleaned. To be extra sure no moisture is left on the surface, the chip can be heated to 180° C for several minutes.

Cleaning is repeated many times over the course of fabricating a device. Every time resist is removed, the chip will be cleaned. For the most thorough clean, the chip is placed in NMP and sonicated, followed by acetone, and then IPA. At times sonication and/or NMP will not be used. It is important to blow dry the chip to ensure the surface is clear of IPA.

Which **resist** one uses for optical lithography changes depending on what the goal of the step is. For etches, I typically use *AZ1512*, whereas for metal deposition steps I tend to use two resists in combination, *LOR-3B* and *SPR660*. The resists used are positive resists. For *AZ1512* with a thickness of 1006 ± 24 nm, the following recipe is used:

- 2 s @ 500 rpm/s
- 5 s @ 9000 rpm/s
- 60 s @ 6000 rpm/s
- Bake for 1 minute @ 100° C

The combined thickness of *LOR-3B* and *SPR660* is 850 ± 20 nm, with *LOR-3B* making up ~200 nm. The spin steps of the *LOR-3B* are:

- 5 s @ 500 rpm/s
- 5 s @ 9000 rpm/s
- 45 s @ 3000 rpm/s
- Bake for 3 minutes @ 180° C

Followed directly by the *SPR660*

- 5 s @ 500 rpm/s
- 5 s @ 9000 rpm/s
- 45 s @ 4000 rpm/s
- Bake for 1 minute @ 90° C

¹The melting point of Ga is 30°C.

The first spin step in each case is to give a chance to make sure that the device is centred. The fast spin is referred to as the 'edge bead removal' step - the goal of this step is to minimise edge beads, something that will be present on small samples, especially those that have a large aspect ratio. The last step in each case sets the height of the resist.

Exposing for optical lithography at the University of Sydney uses a *HeidelBerg MLA100*, a maskless aligner. The lights have a fixed wavelength of 365 nm. The two main parameters that can be changed are the dose and the defocus. The defocus allows an undercut to be developed; however, with lift-off resists, the undercut is provided by the different development rates of the resists. If *AZ1512* is used for lift-off, I will use a defocus of -4. Generally for *AZ1512* a typical dose is $\sim 90 \text{ mJ/cm}^2$; although I have a tendency to overexpose with this resist, exposing at 140 mJ/cm^2 . For the lift-off resist combination, a dose of 600 mJ/cm^2 is typical; no defocus should be used. When the layer is not the first, it will need to be aligned to known alignment marks. The amount of rotation between layers that the MLA can normally manage is 5 mrad. Anything beyond that and it will likely be misaligned.

Development of the optical resists mentioned here uses the same developer – *AZ726*. I develop the *AZ1512* resist for 30 s^2 , followed by 20 s in water and then blow dry with nitrogen. The lift-off resists are developed for 60 s, followed by 20 s in water and then blow dry with nitrogen. This typically leaves an offset between resists (undercut) of $\sim 3\text{-}4 \text{ }\mu\text{m}$.

The chip should be **post baked** if it is going to undergo an etch. Otherwise, it should *not* be post-baked, as this will tend to remove undercuts and can even cause the resist to develop an overcut. For *AZ726*, the post bake occurs at 115° C for a minute.

The mesa **etch** is performed using a phosphoric acid solution. It is made up of DI water, phosphoric acid, and hydrogen peroxide in a ratio of 50:5:1. I have found that the etch rate is $18.8 \pm 0.7^3 \text{ }\text{\AA}/\text{s}$. The length of time the etch should proceed depends on the desired depth of the etch. The etch should be deeper than the 2DEG.

The **resist strip** process is very similar to the cleaning of the sample. The chip will be placed in acetone for a few minutes so that the resist(s) dissolve. The chip is placed in IPA and then blown dry with nitrogen.

Metal **deposition** occurs after resist development. The tool used is an *AJA ATC-1800-E*. I like to stick the chip down on a glass slide with PMMA so that the entire sample will have metal deposited on it, rather than using tape to hold the device down. The metal thickness and deposition rates depend on the metal and the desired outcome. For the ground plane and transmission line, 10 nm of Ti and 150 nm of Au would be common. When depositing metals that go over a height difference, such as the mesa etch, the metals deposited need to be thicker than the difference so that after lift-off a continuous metal pattern is present.

Lift-off for optical processes involves putting the device in NMP for a long time ($>30 \text{ min}$

²Others at QNL will develop for 40 s.

³The etch rate is different for different layers in the mesa. This is an average across the parts that have been etched.

for lift-off resist; >1.5 hr for AZ1512). When the metal with resist under it appears to curl up, the device should be sonicated. For optical features, sonicating on the lowest setting for a minute should not cause any problems. The glass slide is useful here as it keeps the chip on its edge, so that when metal is removed, it does not fall back on the device. The chip is then pushed off the glass slide (if it didn't fall off earlier) and placed in acetone. A short sonication in acetone can help if loose metal has fallen on the device. The chip is then placed in IPA, then blown dry with nitrogen.

The **ohmics** layer occurs between the mesa etch and any other deposition⁴. After the resist is developed there are some extra steps over a “normal metal deposition”. The chip will be plasma-ashed using oxygen at 50 W for a minute, followed by an acid clean in HCl for 20 s. The ohmic metal stack used is:

- Ni – 5 nm – controls diffusion
- Ge – 35 nm – forms a eutectic with gold
- Au – 72 nm – forms a eutectic with Ge
- Ni – 18 nm – to stop the capping layer from participating in the eutectic
- Au – 50 nm – capping layer

The total eutectic height can be changed if a thicker ohmic is required, such as when going over a deep etch, so long as the ratio is maintained (88:12 Au:Ge atomic mass [47]). The capping layer thickness can also be changed.

After lift-off the ohmics need to be **annealed**. This is the process that forms the ohmic contact. I have used several recipes that resulted in working ohmics. The most important step is the high temperature step: I have used both 430° C for 130 s, and 470° C for 110 s, both which resulted in working ohmic contacts.

An oxide is then deposited on the chip using **ALD**. This is to protect the surface and allows a dielectric for the gates. I tend to use aluminium oxide as the grown dielectric. This has a growth rate of 0.781 ± 0.035 Å/pulse.

Top gates are a specific type of metal deposition. The metals that are deposited will generally go over a mesa etch. These metals do not strictly need to be taller than the mesa etch – so long as the etch profile is reasonably shallow – however, I tend to deposit metals so that they are taller than the mesa etch. This ensures that regardless of the etch profile, the metals will not break off when the rest of the metal is lifted off.

I have written the above sections in a way that describes each step, mostly in the order that they appear. However, it is written in a general way, so the slight variations between major processing steps is not explicitly mentioned.

⁴Strictly the ohmic layer can be deposited before the mesa etch, or after any other metal deposition, provided no ALD layer has been deposited.

A.2. JJ Device Fabrication

The lithography of the JJ device used in chapter 5 was exclusively electron beam lithography (EBL). Many of the other steps such as cleaving, cleaning⁵, metal deposition, resist strip, and mesa etching are lithography agnostic; what is written in section A.1 is entirely relevant.

Using EBL rather than optical lithography means that different resists must be used. In this case, the positive resist *PMMA A3* was used. With the following recipe the resist thickness was typically 85 ± 3 nm. When thicker resists were required, a double layer of resist was used⁶.

- 5 s @ 500 rpm/s
- 5 s @ 9000 rpm/s
- 45 s @ 4000 rpm/s
- Bake for 1 minute @ 180° C

The best dose used for these exposures is typically of order $800 \mu\text{C}/\text{cm}^2$. The resist is developed in a solution of MIBK:IPA in a ratio of 1:3 for 40 s. It is then rinsed in IPA for 20 s. I have found that a post bake is detrimental for metal depositions; otherwise, a post bake at 150° C for 45 s can be helpful.

An additional process is required when using a 2DEG which has epitaxially grown aluminium on top. In order to expose the 2DEG, the aluminium needs to be removed. This process was performed using “transene aluminium etchant D”. For this device, the aluminium etchant was warmed to 47.5° C. This etchant is meant to be highly selective and not etch into the semi-conductor. 11 s was found to be sufficient to remove 10 nm of the aluminium.

A.3. Nanowire Device Fabrication Steps

In the course of my PhD, I attempted to create a “2DEG nanowire”; essentially the device seen in chapter 6, but made from 2DEG and designed with RF in mind. This was a substantial amount of work which did not culminate (yet⁷) in any results worth reporting. There were many problems with the processing, the main issue being leakage. It was suspected that the InAs would leak if any processing occurred between the mesa etch and the ALD layer. Originally, the steps before the ALD layer were Al removal, mesa etch, and then gold deposition (RF ground and transmission line). The successful manufacture of the device had the gold deposition as the first “major step”.

⁵Being a shallow 2DEG, any surface contaminants have a more pronounced effect than in deeper 2DEGs so cleaning is even more important. Similarly it is important not to ash these samples as ashing can damage the 2DEG

⁶Instead, the copolymer for PMMA could be used; this would provide a good undercut

⁷Hopefully.

For the nanowire device only one of the steps used the MLA. The majority of processing used the 125k keV EBL. This requires resists that are sensitive to the electrons. I used three different resists: *PMMA A3*, *PMAA A4*, and *ma-N 2403*.

Both *PMMA* resists use the same spin and bake recipe (see above). With this recipe, *PMMA A3* has a thickness of 85 ± 3 nm and *PMMA A4* has a thickness of 121 ± 2 nm.

ma-N 2403 is a negative resist. With the recipe used the resulting thickness is 162 ± 9 nm.

- 3 s @ 8000 rpm/s
- 30 s @ 4000 rpm/s
- Bake for 1 minutes @ 90° C

The best dose was found to be about $270 \mu\text{C}/\text{cm}^2$. This resist is developed in **AZ726** for 20 s and rinsed in water for 30 s. This caustic solution does etch aluminium, so it is important to be aware of this as the epitaxial Al will be etched during development. Post-baking was found to be detrimental to the features.

The lift-off process for EBL processes is much quicker than for MLA processes. Generally, having the chip in acetone for 5 min and sonicating for 10 s will suffice for a successful lift-off.

The aluminium for this device was nominally 5 nm thick. Using “transene aluminium etchant D” in any way was found to be too aggressive, leaving pot-holes in the surface which would damage the 2DEG. Instead of using the etchant designed for aluminium etching, I used the **AZ726** caustic developer; this was found to be much gentler. A further 30 s beyond the development time was found to be sufficient to remove the aluminium entirely, without going through the protective layer. It was later realised that the aluminium etchant had expired; to date we have not yet determined if this is the reason for the pitting.

The mesa etch is very similar to that used for the EMP device. The acid solution is the same; the difference is the material etched. The InAs tends to etch slightly slower than the GaAs; however, I have found it to be less consistent. The etch seemed to continue for a short while after being rinsed in water, although much more slowly. Removing the resist straight after rinsing in water was a way to regain the consistency.

Although the fabrication of this device was very complicated, ohmic contacts were very simple. The 2DEG is so close to the surface that any metal deposited automatically forms an ohmic connection. In order to not form an ohmic connection, the ALD layer is very important. To better contact the Al, I used a 30 s Ar ion mill before deposition, removing ~ 1 nm of metal/metal oxide.

We suspect the leakage problems are partly caused by dangling bonds and impurities added by the resist. To try and help with these problems, I (with the aid of Abdallah) worked on making a hard mask. This should, in principle, leave the surface of the material in

the cleanest state and overcome the leakage problems. This technique can be used for all semiconductors, but it is particularly relevant for shallow 2DEGs.

The **hard mask** process we developed used 5 nm of ALD-grown AlO_x to create the mask. To create the patterns in the mask, resists were exposed and developed. Then the chip was processed in the caustic developer *AZ726*; this removes the AlO_x at a rate $\sim 0.5 \text{ \AA/s}$. This process is highly selective; the etch rate of the heterostructure was not measurable. After cleaning off the resists, the mask was used to define the etch. With this process, the surface of the heterostructure never needs to have resist in contact with it.

A.4. Metrology

Fabrication is often considered something of a “dark art”, often surrounded by mysticism and rituals; consider how many rituals and magic numbers are in many recipes, often for no better reason than it worked once. Fabricating a working device can be a very long process; every step and every tool will usually require optimisation. Metrology is how we can check if each step seems to have worked correctly. For me, viewing the device and keeping track of the numbers – such as resist heights, ALD growth rates, etch rates etc. – helps increase fabrication success rates, or at least gives the means to know when something has changed.

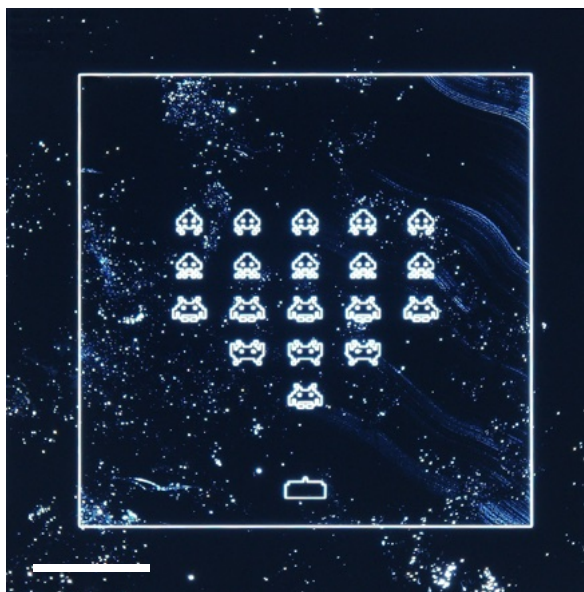


Figure A.2: Dark field image of the first hard mask etch structures. Scale bar is $400 \mu\text{m}$.

The most basic metrology is viewing the device. Usually the problems, like the features we are interested in, are not visible to the unaided eye⁸. **Optical microscopy** is the solution. Making sure the areas exposed are the ones that should be exposed, making sure that the alignment is right, checking lift-off worked, checking the size of (large) features, and even checking that the surface is clean enough are all things that optical microscopy is good for. Dark field is better for viewing the edges, which is particularly useful for viewing surface contamination and etches (figure A.2). A reminder – when using photo-resist the microscope lights can expose areas of resist; do not view before developing!

⁸Unless you really screw up.

When the feature sizes are too small to be viewed optically, the **SEM** is the tool to be used – the *Zeiss Crossbeam 550 XL* is the one I am familiar with. Not only is it good for determining feature sizes and creating images, it is also useful for element analysis, such as energy-dispersive X-ray spectroscopy (EDS), and milling – although this is a feature I have never used. The SEM will deposit carbon on the areas viewed and can inject charge into the material. For that reason I tend not to view active areas of a device unless it has already been measured. It can be helpful to have test structures and dummy devices (with different steps of the processing performed) on the chip so that diagnosis can be performed before measurements.

When a step height needs to be determined a **stylus profilometer** (*Bruker DekTak XT*) is used. This tool uses a force stylus that is dragged across the surface of the device to determine differences in heights. This is the tool used to determine many of the rates and heights mentioned in the previous sections.

At times the DekTak does not give enough accuracy. In that case an **AFM** (*Bruker Ion*) is used. The AFM rasters across the sample, building up a height map; how it does this depends on the mode of operation. Typically I use tapping mode, which limits the contact between the sample and the tip. The tip oscillates near its resonance frequency; the sinusoidal motion is modified by interactions with the sample. The map is useful not only for height maps and determining surface roughness but also in determining widths. It is usually not worth using this tool for determining widths, as it is rather slow in building up an image.

To determine the height of transparent materials the **ellipsometer** (*JA Woollam M2000*) is used. This uses a laser that scans over a range of frequencies. It measures the change in polarisation; by comparing this with models, the height of specific layers can be determined. It does not rely on step heights. In determining the ALD oxide growth it is much simpler to use a spare piece of Si for determining the height.

A.5. Bonding and PCBs

After the device is complete, it needs to be tested. To do this, electrical connections must be made. In our case Al bond wires serve to make these connections. In our lab we have both a manual and an auto bonder; I am far more familiar using the manual bonder. It is not worth noting the bonding parameters used as these change whenever the bonding wedge is changed – however, these are parameters that I keep record of.

The previous PCB that fit in the bore of the magnet used mini SMP connectors for RF and a soldered 37 pin for DC. Due to the way it needed to be soldered, the copper plate used for thermalisation was not reusable. For this reason⁹ and to make it compatible with wiring across all our fridges¹⁰, I helped design a new board (figure A.3). This used an Ardent connector for RF ports – an expensive but reusable connector. For DC, a 51 pin nano-D

⁹And the fact that I have a strong dislike of mini-SMPs.

¹⁰The newer fridges use two 25-pin connectors whereas the older fridges use 37-pin connectors. These will soon change to 25s as well.

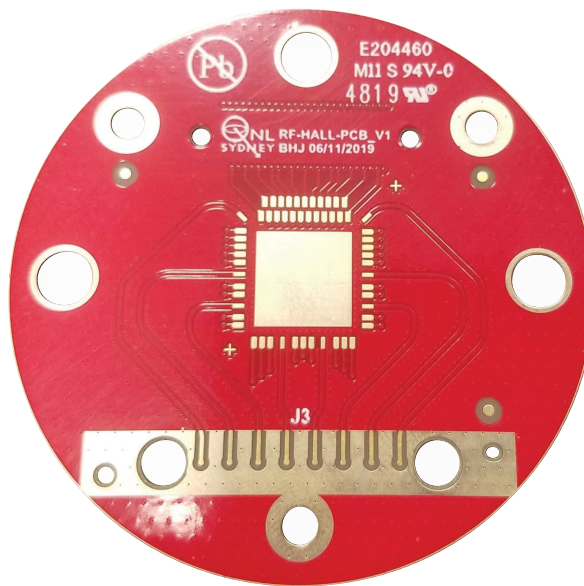


Figure A.3: PCB designed to fit in the bore of the magnet.

connector was soldered in place. More recently we discovered a reusable DC connector, and more boards have been designed to make use of this feature. With reusable connectors, PCBs now become very cheap and can be designed with a specific experiment in mind.

Appendix B

Modelling filling factors $\nu > 2$

B.1. Edge side model – position determined by incompressible strip

Several other filling factors from chapter 4. For filling factor $\nu = 3$, the dissipation from DC Hall measurements was $\tau_{DC} = 2.1 \times 10^{-13}$ s whereas the fit used $\tau \approx 3.7 \times 10^{-13}$ s.

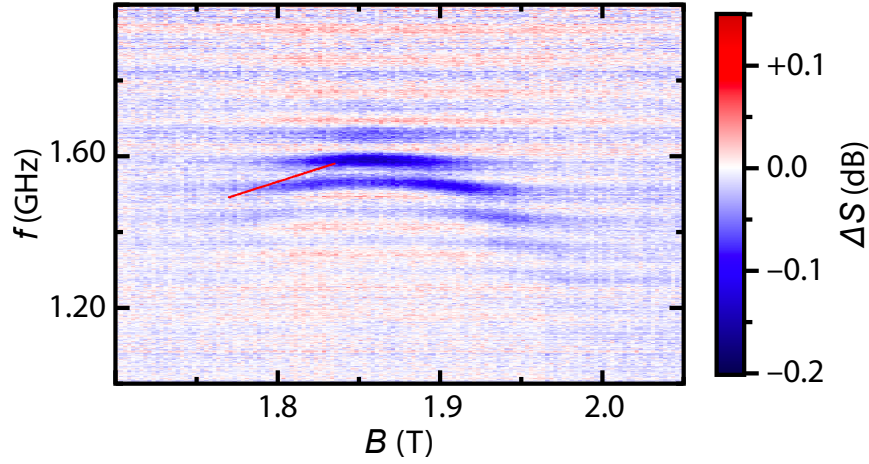


Figure B.1: The EMP on the $\nu = 3$ plateau; the red line is the fit with n determined by the incompressible strip value.

For filling factor $\nu = 4$, the DC dissipation was $\tau_{DC} = 2.8 \times 10^{-13}$ s, and the dissipation used in the fit was $\tau \approx 5.5 \times 10^{-13}$ s.

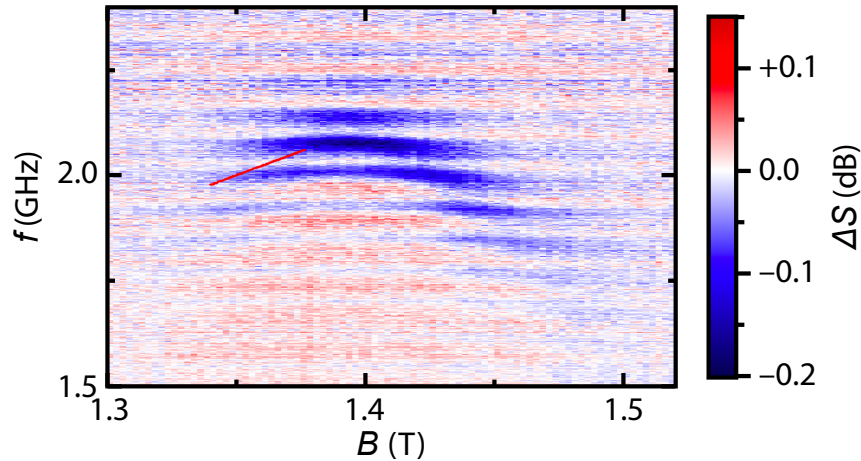


Figure B.2: The EMP on the $\nu = 4$ plateau; the red line is the fit with n determined by the incompressible strip value.

For filling factor $\nu = 5$ the DC dissipation was $\tau_{DC} = 3.4 \times 10^{-13}$ s, and the dissipation used in the fit was $\tau \approx 7.2 \times 10^{-13}$ s.

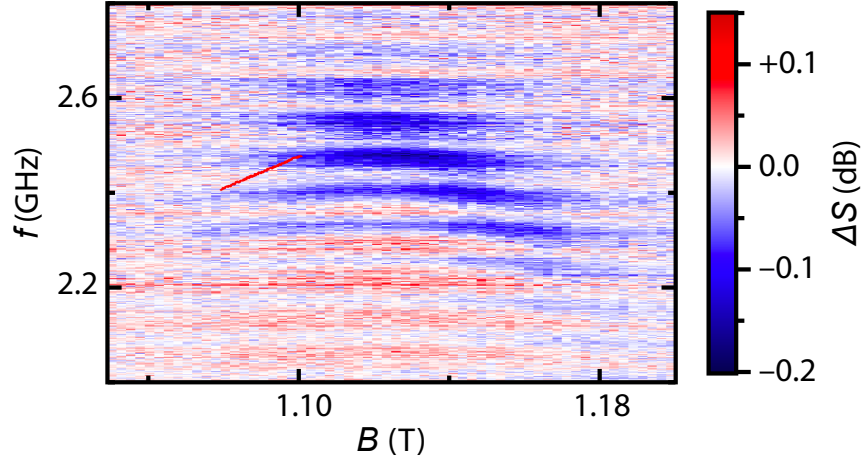


Figure B.3: The EMP on the $\nu = 5$ plateau; the red line is the fit with n determined by the incompressible strip value.

B.2. JV model

With the JV model, τ is what is fit. Here n was set to the bulk value $n_0 = 1.33 \times 10^{11} \text{ cm}^{-2}$. The calculated τ is shown for each filling factor. For filling factor $\nu = 3$, the Hall measurement gave $\tau_{DC} = 2.1 \times 10^{-13}$ s.

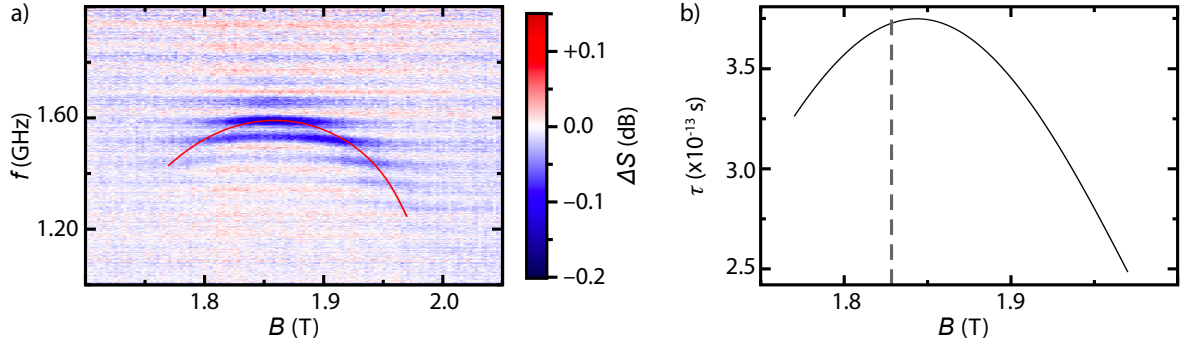


Figure B.4: a) The EMP on the $\nu = 3$ plateau; the red line is the JV fit. b) Variation in τ in this model. The dashed line indicates the magnetic field where $\nu = 3$.

For filling factor $\nu = 4$, the DC dissipation was $\tau_{DC} = 2.8 \times 10^{-13}$ s.

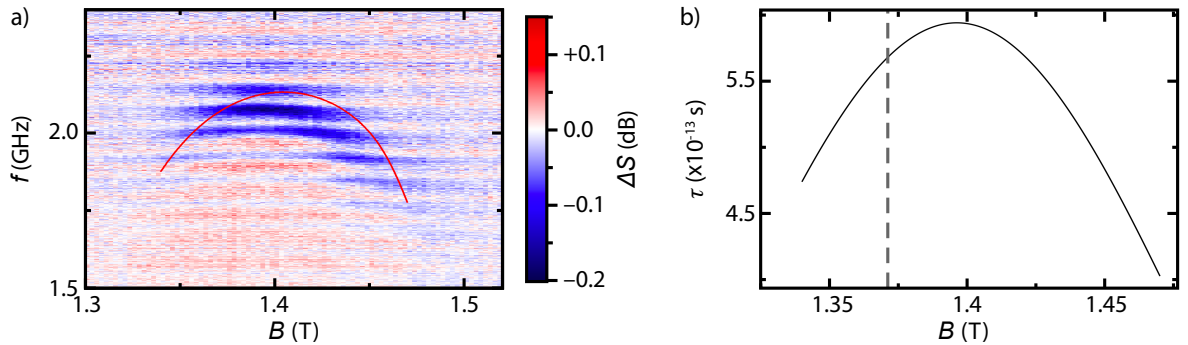


Figure B.5: a) The EMP on the $\nu = 4$ plateau; the red line is the JV fit. b) Variation in τ in this model. The dashed line indicates the magnetic field where $\nu = 4$.

For filling factor $\nu = 5$ the DC dissipation was $\tau_{DC} = 3.4 \times 10^{-13}$ s.

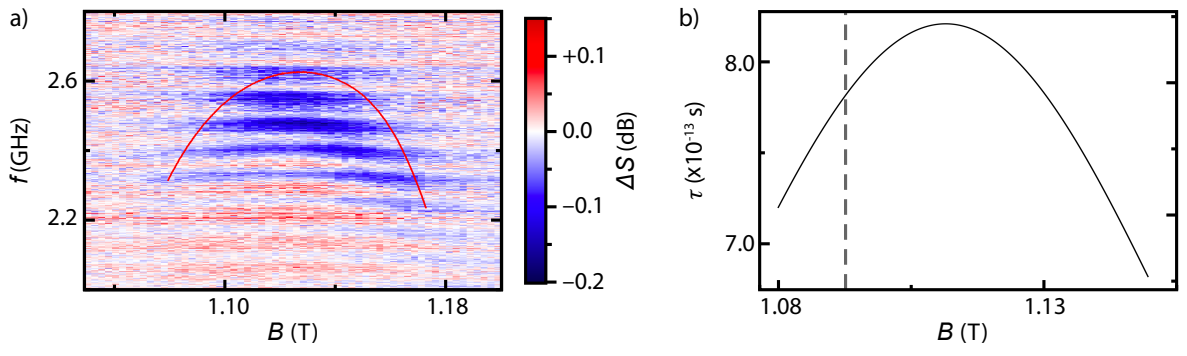


Figure B.6: a) The EMP on the $\nu = 5$ plateau; the red line is the JV fit. b) Variation in τ in this model. The dashed line indicates the magnetic field where $\nu = 5$.

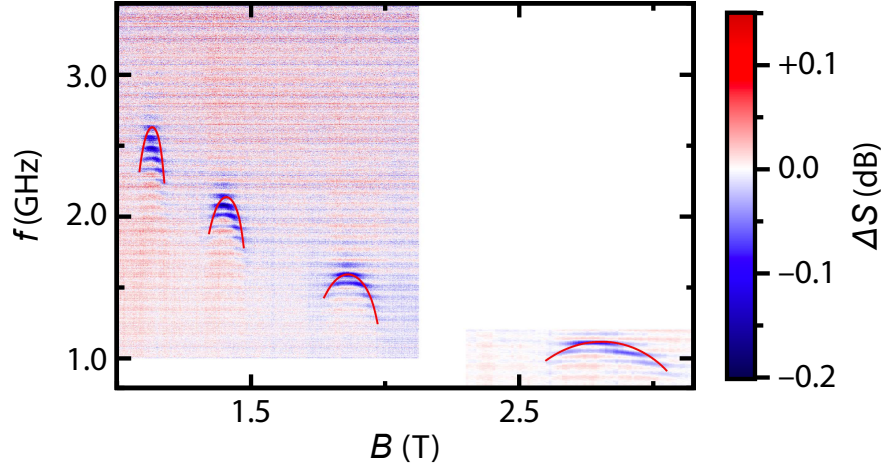


Figure B.7: Plateaux $\nu = 2 - 5$ fit with JV model.

B.3. Gaussian fit

An example of the Gaussian fit is given in figure B.8. This was repeated for each field value. The units were converted to linear units, and inverted so that the changes were positive. The Gaussian was also fit without converting to linear units. Due to the smallness of ΔS they end up being nearly identical.

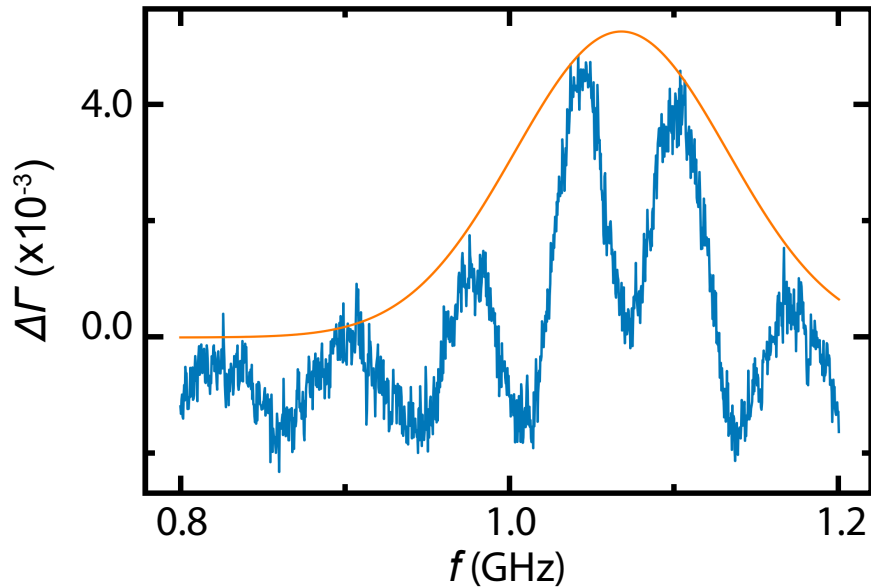


Figure B.8: Gaussian fit for the line cut at $B=2.67$ T. The change in S has been converted to linear units.

The calculated Q factor is given in figure B.9.

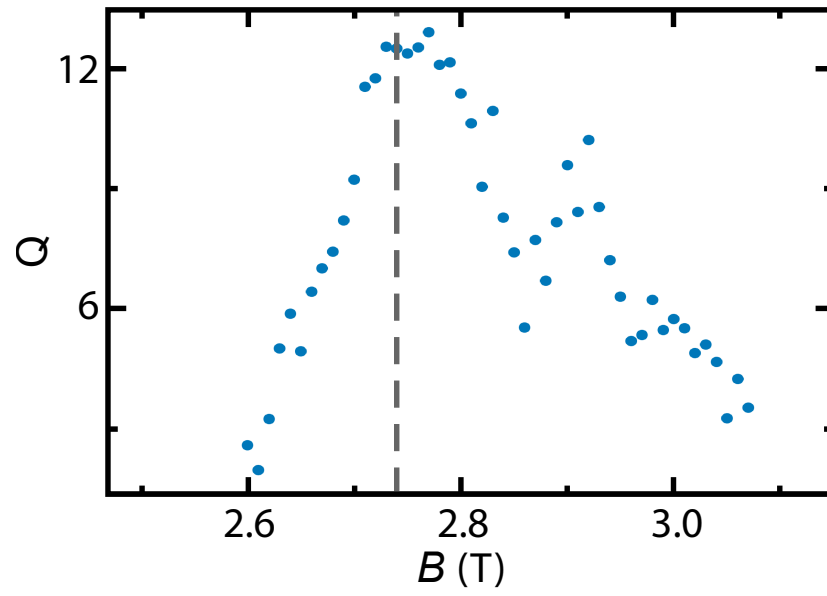


Figure B.9: Calculated EMP Q factor across the $\nu = 2$ plateaux. The dashed line indicates the boundary between the edge and the bulk dominant sides of a QH plateaux.

Appendix C

Additional information for chapter 5

C.1. Calibrating the voltage

The voltage values shown in the figures in chapter 5 are the values at the device level. The voltage at the device is both at a different scale and shows the effects of a bias offset somewhere in the dilution refrigerator (figure C.1).

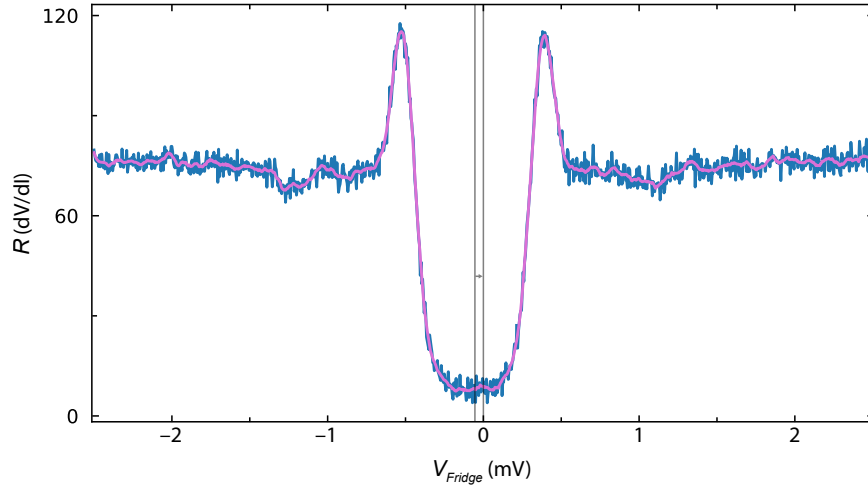


Figure C.1: The same data as figure 5.6, however voltages shown are those applied at the top of the dilution refrigerator. The lines indicate the bias offset.

In order to determine the voltage at the device level, Shapiro steps were used. This is well motivated as Shapiro steps have been used as voltage standard [160, 202–204]. The constant steps (Shapiro steps) depend only on fundamental constants and the driving frequency (F).

$$V = n \cdot \phi_0 \cdot F \quad (\text{C.1})$$

Φ_0 is the flux quantum ($\Phi_0 = 2e/h$), and n takes on integer values.

The JJ was illuminated with different frequencies using a ‘bond-wire antenna’. The positions where the steps appeared were then used to convert the applied voltage at the top of the dilution fridge to the voltage that the device experienced (figure C.2). The voltage of the fridge was offset by $-53 \mu\text{V}$ then scaled by 4.5×10^{-3} to give the voltage at the device.

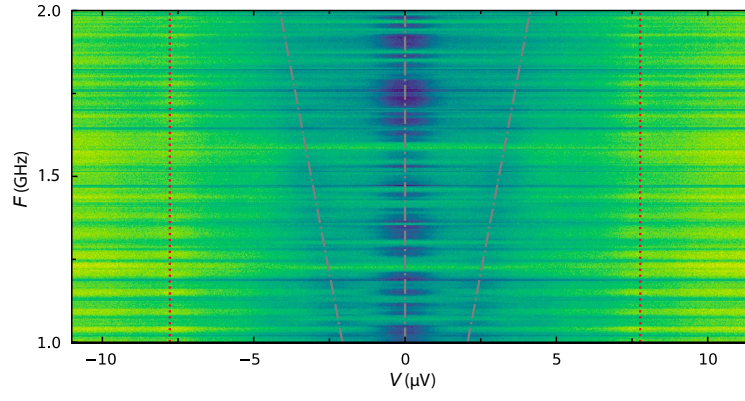


Figure C.2: Shapiro steps measured at 3.758 GHz. The frequency is the applied antenna frequency and the voltage is the voltage at the device level. The grey dash-dot lines indicate the expected position of $n = 0, \pm 1$ Shapiro step. The red dotted line is the expected position of the $n = \pm 1$ Shapiro step caused by the readout tone.

The reason the Shapiro steps appear inconsistently is the inconsistent power delivered to the device. The bond wire antenna was unable to provide consistent power to the device. A measurement of the power delivered to the top gate from the antenna (a good proxy for the power delivered to the JJ) is plotted over the Shapiro steps in figure C.3. Changes in power correlate well with the appearance and disappearance of Shapiro steps.

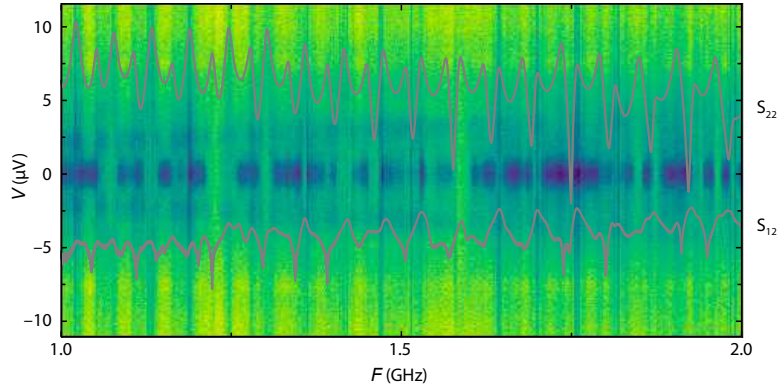


Figure C.3: The transmission (S_{12}) measurement from the antenna to the top gate and reflection of the antenna. Inconsistent power delivery resulted in the disappearance of the Shapiro steps.

C.2. Hysteresis

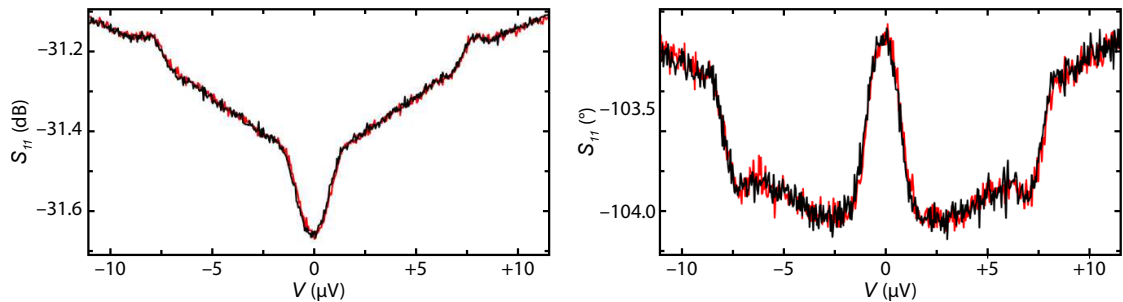


Figure C.4: Positive (red) and negative (black) direction bias sweeps on JJ device. The positive and negative sweeps appear the same within noise for both the magnitude and phase i.e. no significant hysteresis.

No significant hysteresis was observed in the device. Figure C.4 shows the lack of difference observed when sweeping in different directions.

C.3. Frequency Selection

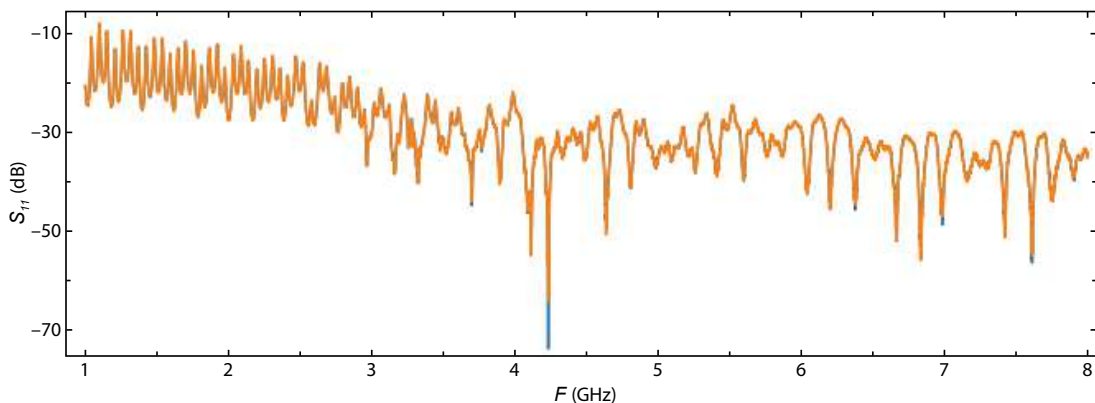


Figure C.5: Frequency response of the gate. The orange trace is at high bias and the blue trace is low bias.

To select the frequencies used to measure the Josephson junction (JJ), the reflection off the top gate was measured (figure C.5). Two different regions were used: one at zero bias and one at high bias ($>10 \mu\text{V}$ on the device). These regions were expected to have different impedances. This was done without calibration; this means that the standing waves present

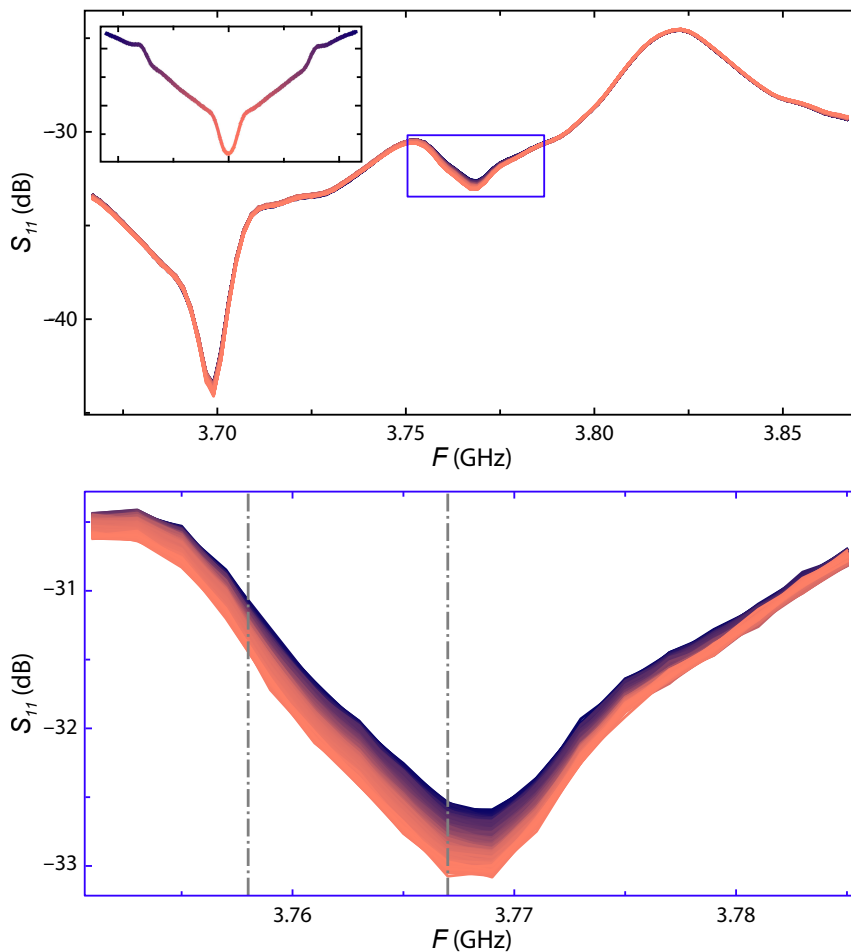


Figure C.6: Frequency response with bias. The inset shows the magnitude of S_{11} at 3.767 GHz, the colours in each figure represent the same bias. The dashed lines indicate the main frequencies used in the experiment.

in the transmission line and the overall losses in the system are clearly visible.

Figure C.5 shows only the magnitude of the response. This was used to give some initial frequencies to examine. This is not the ideal way to select frequencies; what should be used is the maximum change in the total response; i.e. maximal change in the IQ plane. Using just the magnitude potentially leads to a selection bias where the information is carried primarily in the magnitude.

The main frequencies used were not in the centre of a resonance, but on the lower side of the resonance (figure C.6). How this resonance moves as a function of bias is something that is of interest in determining how the system behaves. In order to determine this precisely, the data was fit with a Gaussian to determine the central position.

C.4. Resonance Frequency

The data in figure C.6 had a resolution of 2 MHz. This resolution was not enough to be able to directly extract the resonance shift. In order to determine how much the resonance shifted a fit was needed. In this case a Gaussian was fit to a restricted region of the data (figure C.7).

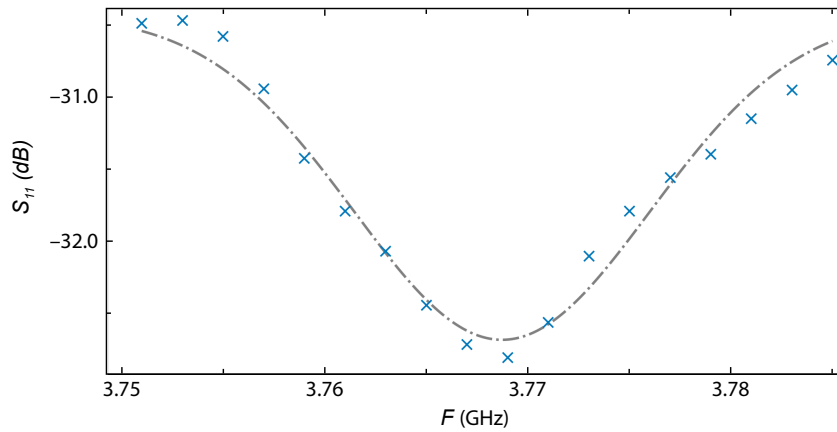


Figure C.7: An example of the shape of the resonance. The blue crosses are the measured data and the grey dash-dot line is a Gaussian fit.

A Gaussian is not the ideal way to fit an LC resonance. However, in this case the main feature of interest was the central frequency. A Gaussian gives a very good fit for the central frequency. The other feature of interest is the Q factor of the resonance. The change in the width of the Gaussian can be used to determine the change in Q factor (figure C.8). A definition for Q factor is $Q \equiv \frac{f_0}{\Delta f}$, where Δf is the full width at half maximum ($\Delta f = 2\sqrt{2\ln 2}\sigma$).

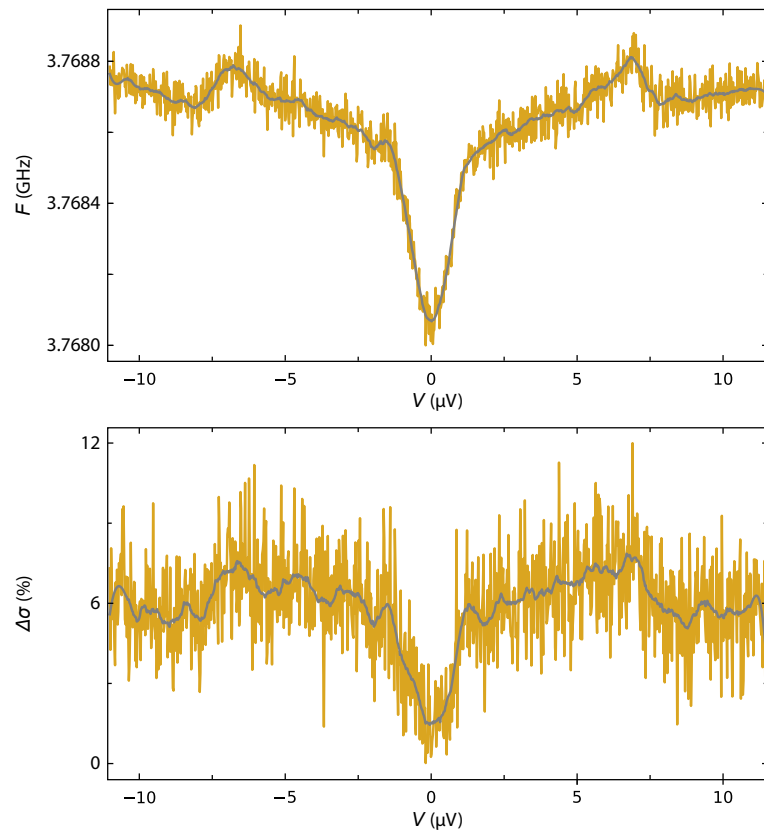


Figure C.8: (Top) The position of the central frequency of the resonance as determined by a Gaussian fit. (Bottom) Change in the standard deviation of the Gaussian fit. This gives an indication of the Q factor change occurring in the device. The inverse changes would be observed. The grey lines are the data smoothed with a Savitzky-Golay filter.

C.5. Top Gate response

The normal state resistance of the JJ was expected to increase as the voltage on the top gate was decreased. The change in normal state resistance can be seen in figure C.9. In this figure the resistance change shown is restricted to $500 \, \Omega$; at voltages lower than $-3 \, \text{V}$ the change is much more.

Due to the the way the device was biased, the amount of current that the passes through the device changes significantly when the resistance change is significant. To explore nominally the same region in current/voltage space, the data in RF was restricted to voltages between $-2 \, \text{V}$ and $0 \, \text{V}$ (figure C.10).

In figure C.11, three regions can be identified; a drop, followed by a plateau (or possibly a rise), followed by another drop. On the same chip a Hall bar was measured [205]; these trends match very well to the change in coherence length in the semiconductor. The trends observed are associated with filling of a second sub-band [205]. The distinct regions are also observed in figure 5.10; the plateau region is the kink in the line. The data collection of the $3.767 \, \text{GHz}$ trace was stopped early as the dilution refrigerator warmed significantly after that point.

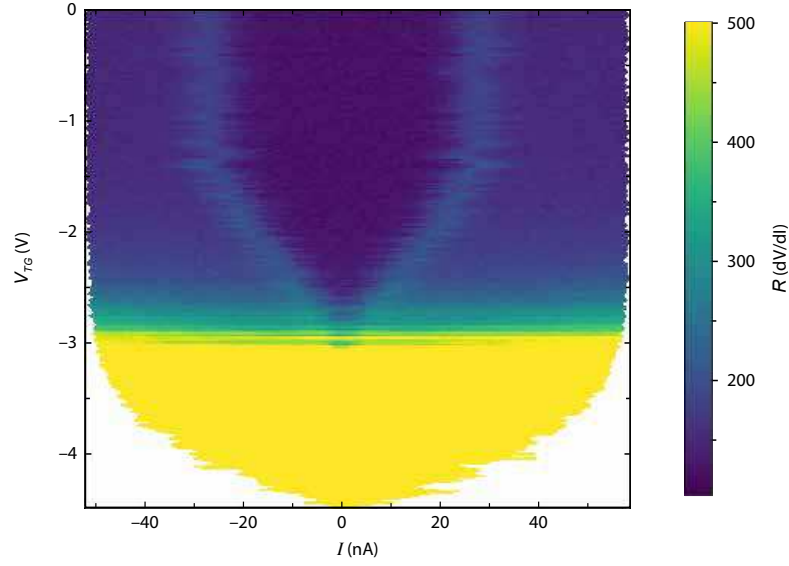


Figure C.9: Top gate response of Josephson junction measured via lock-in amplifiers.

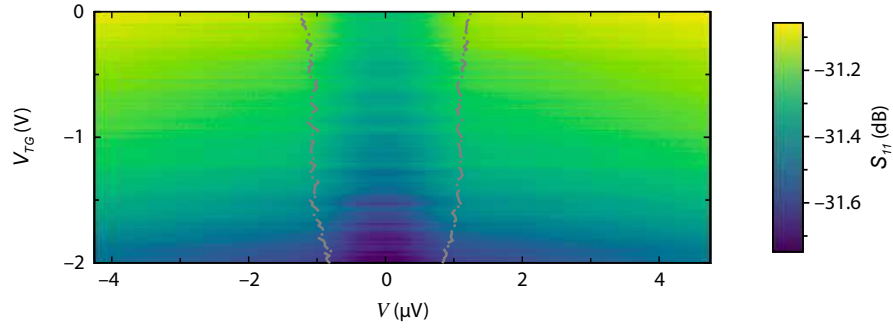


Figure C.10: Top gate response of Josephson junction measured at 3.758 GHz. The dash-dot lines are fits to the change in slope – a proxy for the size of the switching current¹.

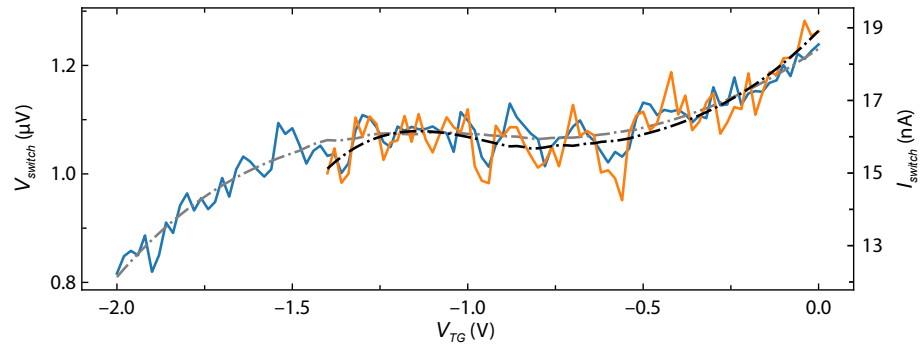


Figure C.11: Size of the switching current measured at 3.758 GHz (blue) and 3.767 GHz (orange). The dash-dot lines are smooths to the data.

C.6. Smith Chart

Without calibration, a Smith chart interprets small return signals as something being nearly perfectly matched. The data used is both uncalibrated and small ($S_{11} \approx -31$ dB). This is why the Smith chart (figure C.12) was not used to aid interpretation of the data in chapter 5.

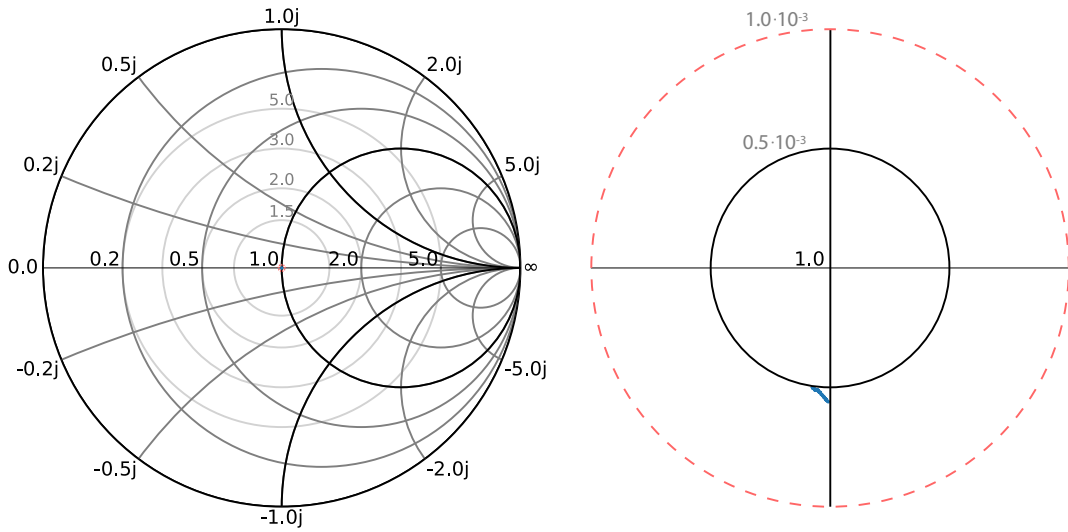


Figure C.12: (Left) Normalised Smith chart. The reflection data appears nearly perfectly matched. The characteristic impedance of the system is $Z_0 = 50 \Omega$. (Right) Zoomed-in section of the Smith chart. The way the top gate causes changes becomes more visible.

Appendix D

Additional information for chapter 6

D.1. Power Response

Power broadening gives a means to determine the size of the voltage oscillation that occurs at the sample level. Figure D.1 shows fits to the amount of power broadening observed. From the fits at -30 dBm (the power used in the experiments), the size of the oscillation for this frequency – 4.795 GHz – was $V_{RMS} \sim 15\mu\text{V}$ for S_{1j} , $V_{RMS} \sim 4\mu\text{V}$ for S_{2j} , and $V_{RMS} \sim 4\mu\text{V}$ for S_{3j} .

D.2. Frequency Response

The trends observed in the cutter-cutter response were observed across the frequencies measured. The vertical features associated with the left cutter were dominant in S -parameters that involved the left terminal (port 1), whereas horizontal features were dominant in S -parameters that involved the right terminal (port 2).

Additionally, figure D.2 shows frequency regions where one or more S -parameters loses sensitivity. This is particularly apparent in the S_{33} response, where between about 3.5–4.2 GHz, the SNR visibly decreases. This response is the reflection of power incident on the superconducting terminal, which was sensitive at lower frequencies than the other two terminals as a result of the wider width of the traces and the kinetic inductance from the superconductor.

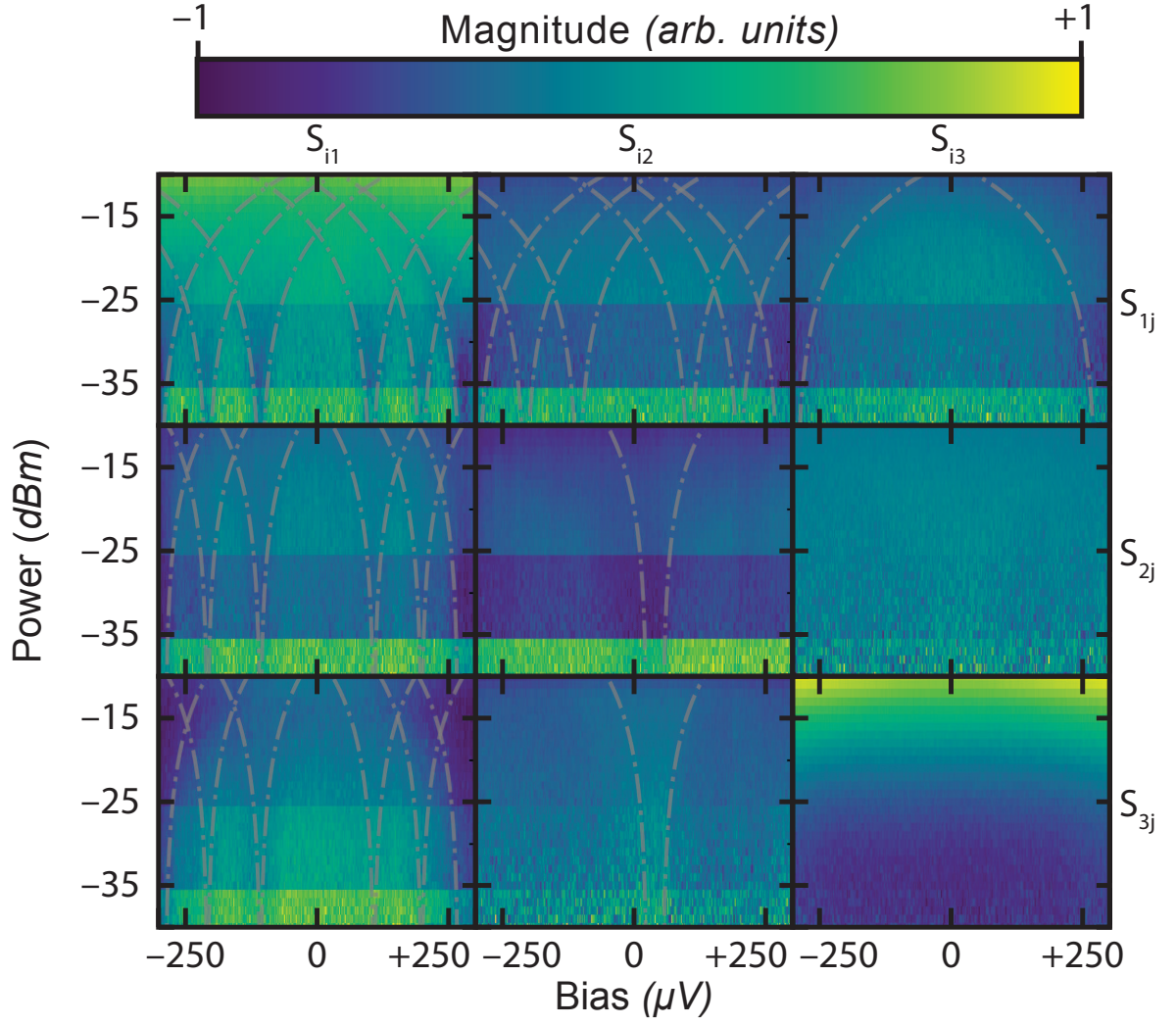


Figure D.1: Power broadening; the dashed lines indicate the fits for the power broadening. These fits are used to calculate the power at the device level for a specific applied power. The power shown is the power at the top of the dilution refrigerator.

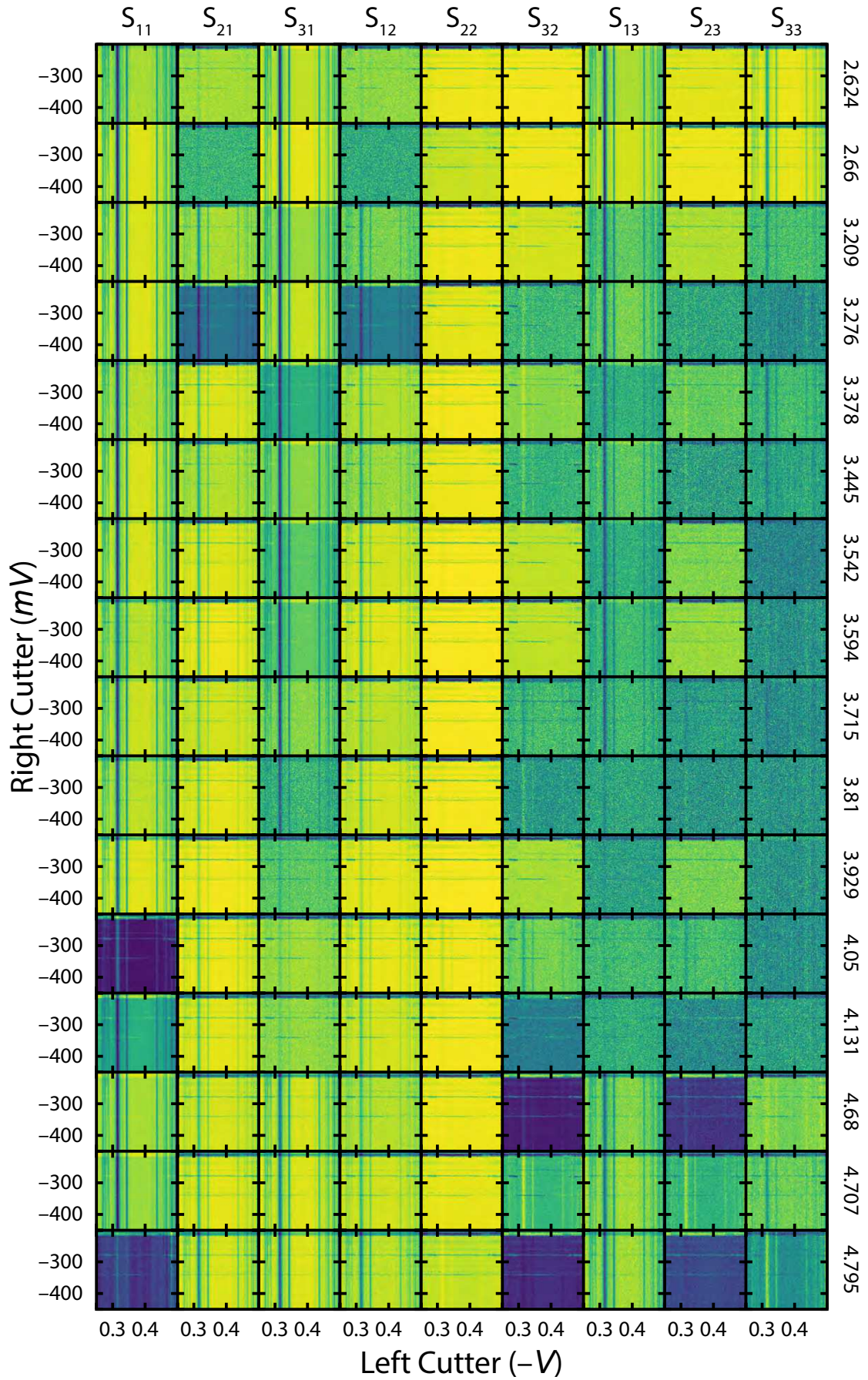


Figure D.2: Magnitude of S-parameters of additional frequencies; same parameters as figure 6.9. Each column represents an S-parameter and each row a frequency in GHz. The frequency increases in each row.

D.3. Additional magnetic field

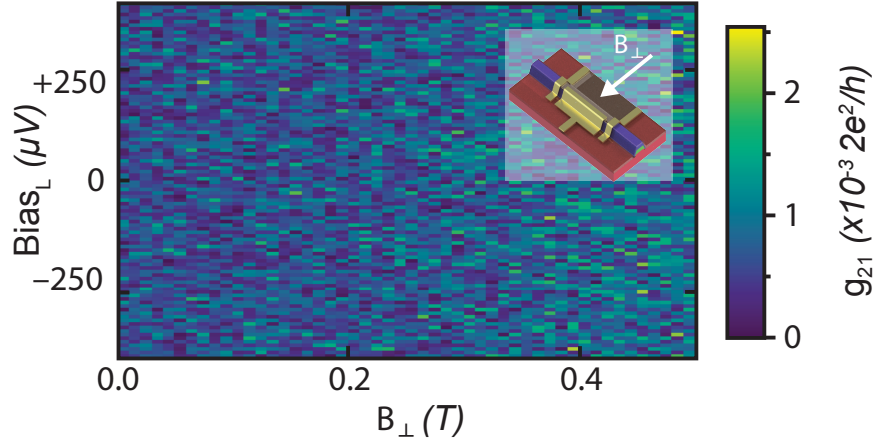


Figure D.3: Non-local measurement of the gap closing. The measurement only shows noise.

The DC non-local measurement of the gap closing, g_{21} equivalent of figure 6.10, was not shown in the main text as it contained only noise. Figure D.3 shows this measurement.

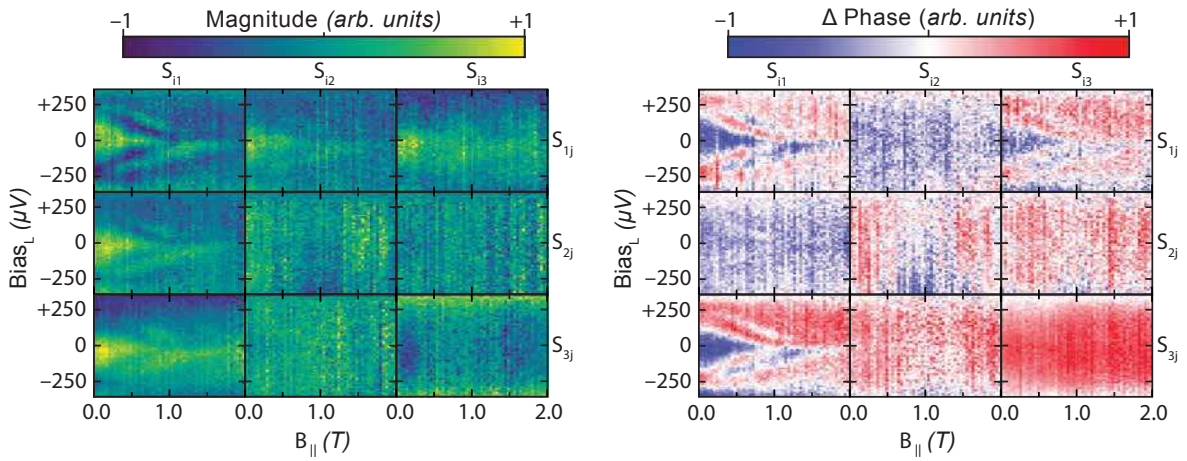


Figure D.4: 4.795 GHz difference in the magnitude (left) and phase (right) response of the S-parameters as a function magnetic field applied parallel with the long axis of the nanowire. The high bias trace was subtracted in order to remove some of the background effects.

Two of the three orthogonal directions of the vector magnet were connected. The magnetic fields available were aligned with the plane of the device¹. Applying the magnetic field parallel with the device resulted in the induced gap closing at a higher magnetic field – around 1.2 T – as would be expected. From the phase data it is unclear if the parent gap moved in a different direction to the induced gap.

D.4. Additional Sub-gap States

Many other sub-gap states were measured in the nanowires examined; figure D.5 and figure D.6 show an Andreev bound state (ABS) measured at zero magnetic field. Between the DC and the RF measurement, a switch occurred and the ABS occurred at slightly different top gate voltages. In the local conductance measurement, the strongest response was measured at a bias of 0 μV , something that has lead in the past to misidentifying these states as Majorana

¹Separate cooldowns were used so that fields greater than 1 T could be obtained in both orthogonal directions in the plane of the device.

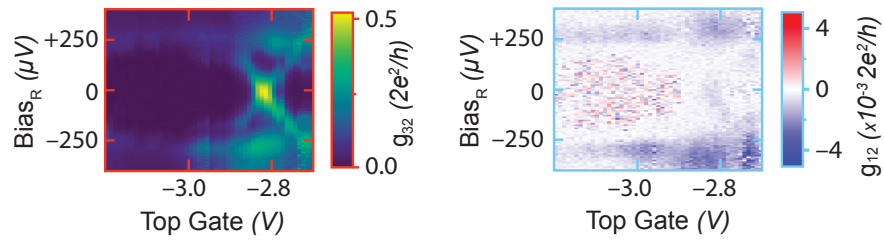


Figure D.5: Local (left) and non-local (right) measurement of an Andreev bound state (ABS). The non-local signal is more than $100\times$ weaker than the local signal.

bound states (MBS) [180–182]. Interestingly, in the RF measurement there appears to be an avoided crossing that occurs at zero bias.

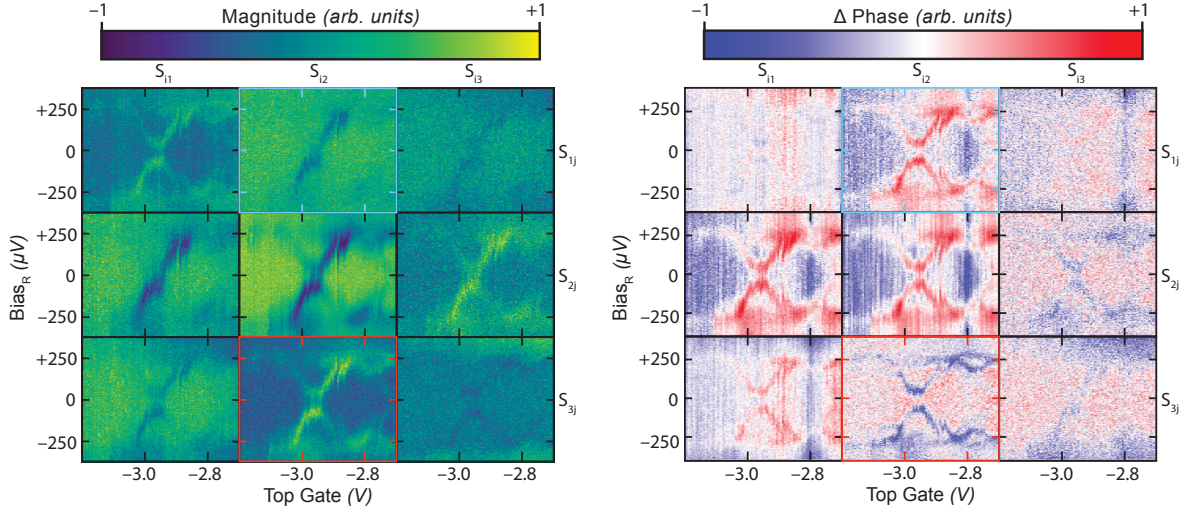


Figure D.6: Magnitude (left) and phase (right) response of the S-parameters measured at 4.795 GHz. The S-parameters highlighted in red and blue are equivalent to the measurements in figure D.5.

In addition to the $\sim 1 \mu\text{m}$ long nanowire, a shorter nanowire with an active region of $L \sim 500 \text{ nm}$ and width of $W \sim 90 \text{ nm}$ was also tested. Being a shorter device, states that would be otherwise be localised are more likely to extend on to the other side of the device. Figure D.7 shows one of the types of sub-gap states that were found in this device.

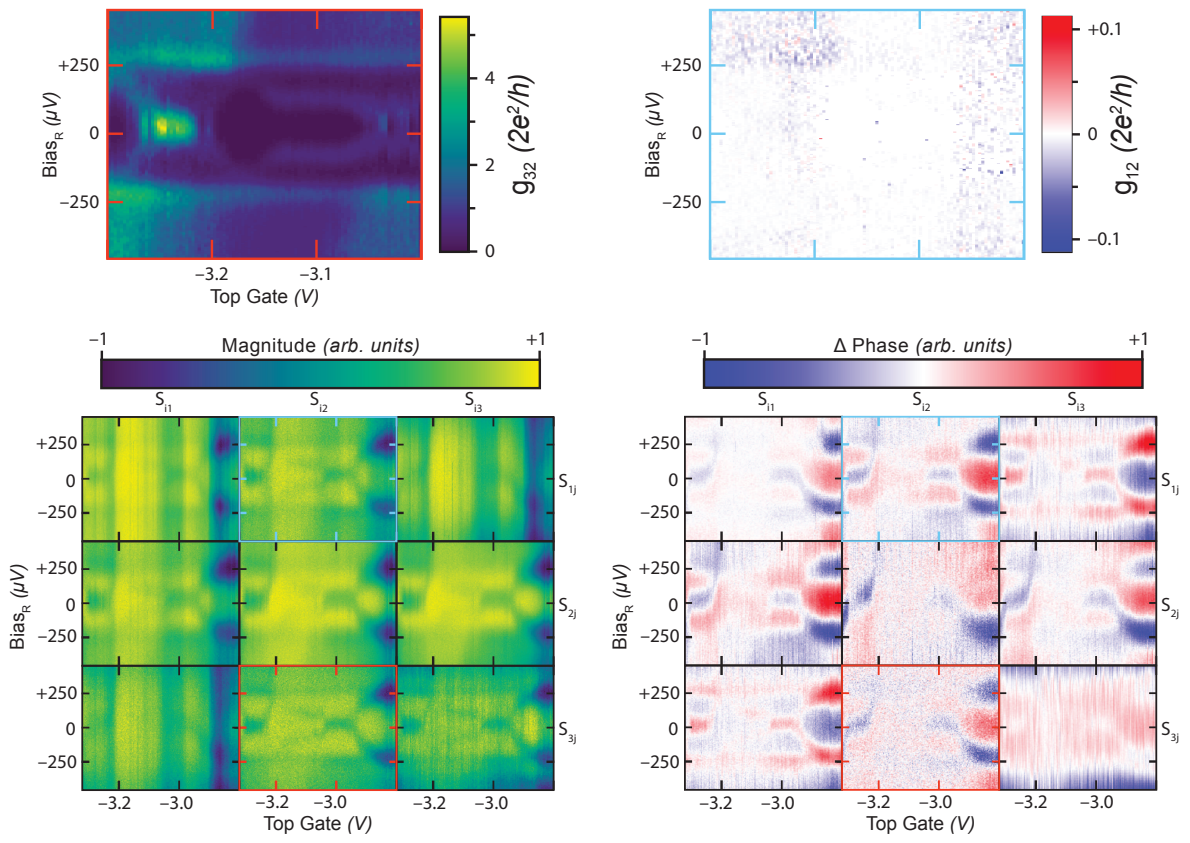


Figure D.7: (Top) Local (left) and non-local (right) measured at a low frequency <40 Hz. (Bottom) S-parameters measured at 6.05 GHz.

D.5. Offset and Scale

The data in the chapter was presented using an arbitrary scale for visual clarity. Here the scale and offset of raw RF data is shown. The offset for reciprocal parameters differs due to variations between the cryogenic amplifiers, directional couplers, cables, and connectors. However, the scale on reciprocal parameters has been set to be the same value. Some of the differences are a direct result of the differences in the amplifiers; each row is amplified by a different amplifier.

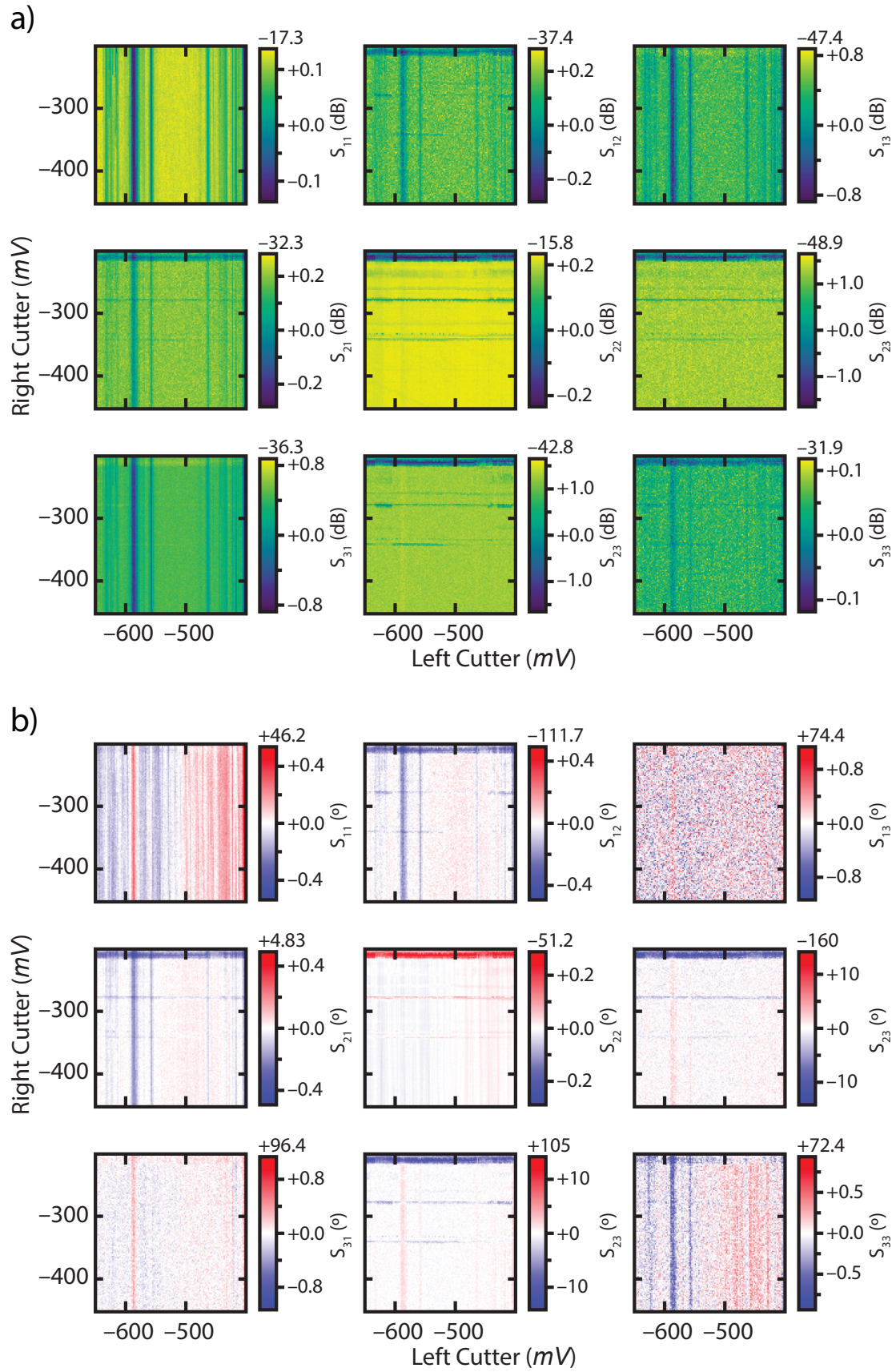


Figure D.8: Version of figure 6.9 without subtractions or normalisations. (a) Magnitude and (b) phase response.

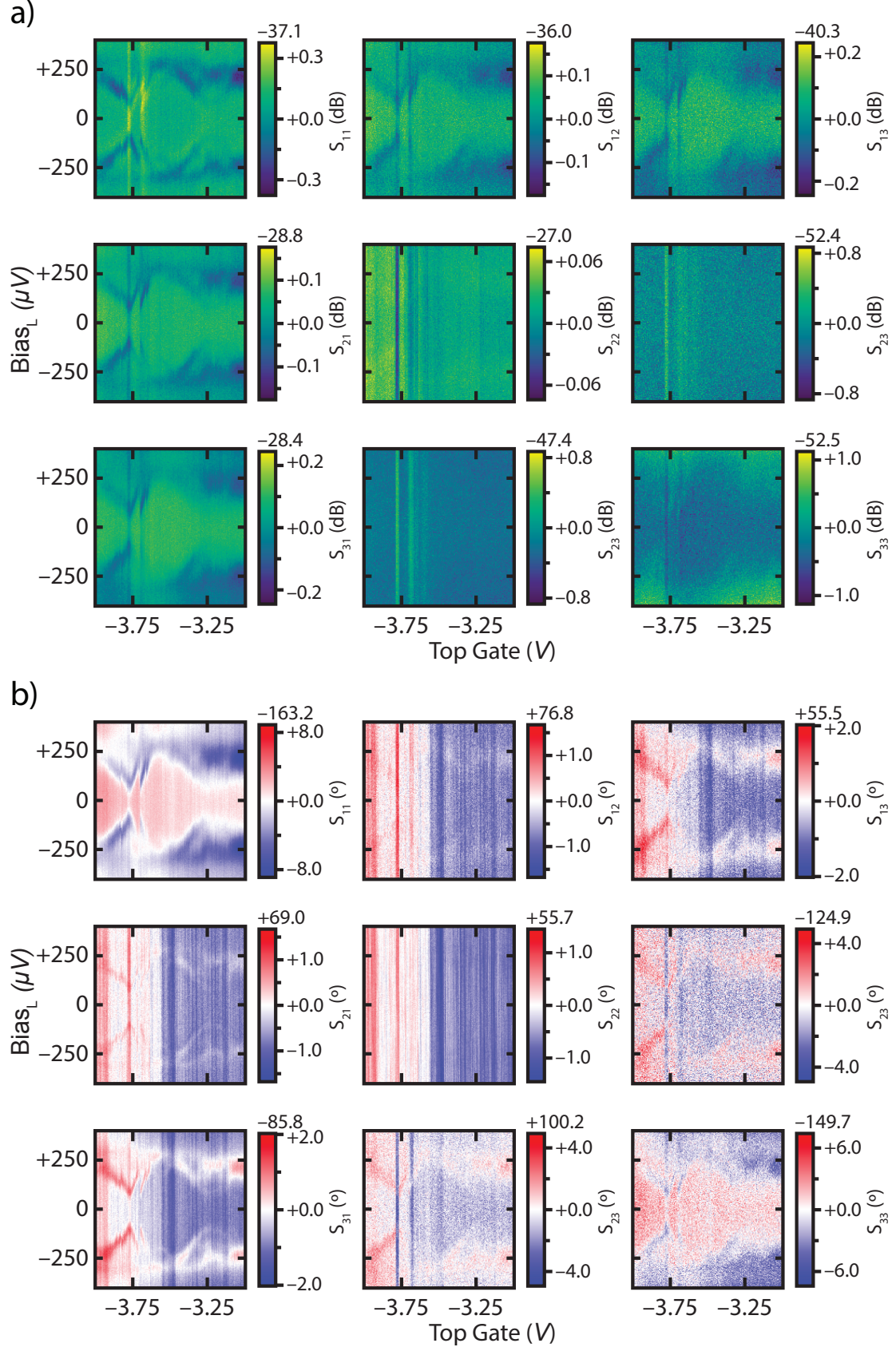


Figure D.9: Left biased part of figure 6.13 without subtractions or normalisations. (a) Magnitude and (b) phase response.

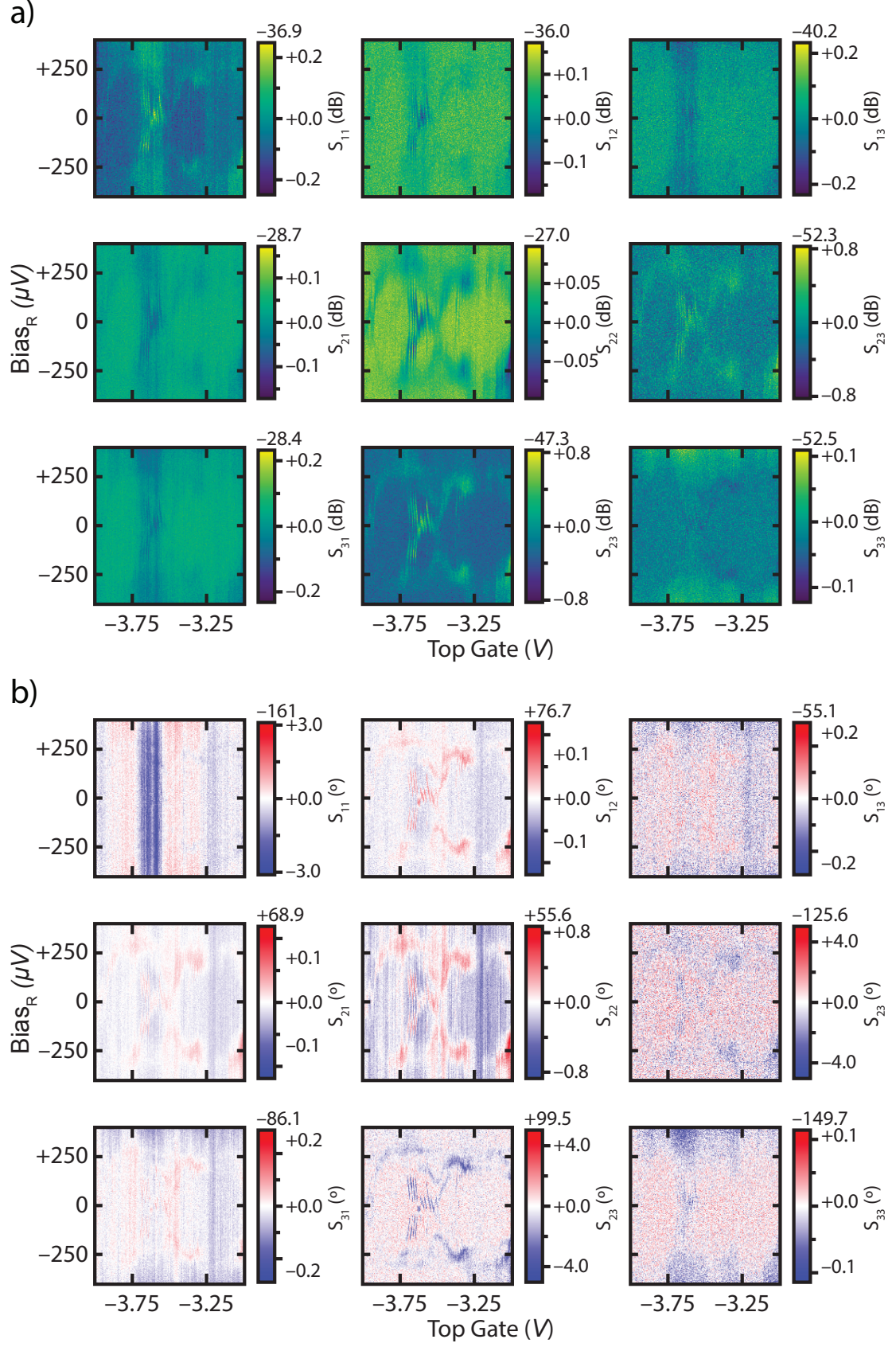


Figure D.10: Right biased part of figure 6.13 without subtractions or normalisations. (a) Magnitude and (b) phase response.

D.6. Comparison on same scales

Visually comparing the DC and RF can be viewed as unfair as one is presented on a linear scale and the other a log scale. Figure D.11 presents the same data as figure 6.16, however both are in log scales. Even though the RF data is collected at least $10\times$ faster than the DC, additional, more clearly defined features are visible in the RF data.

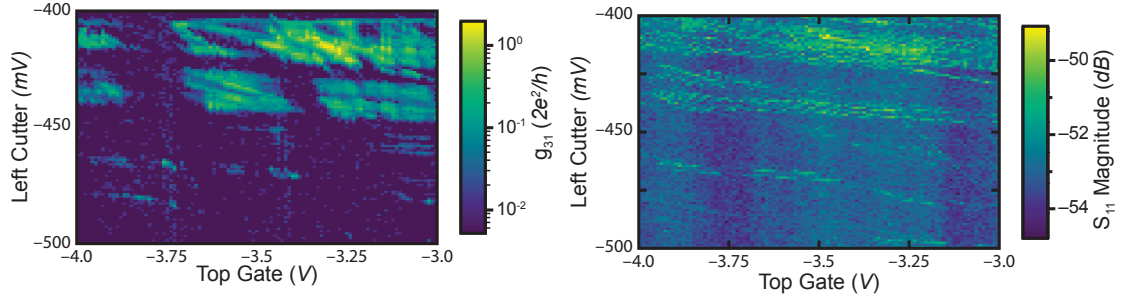


Figure D.11: Data from figure 6.16 presented such that they are both on logarithmic scales.

Appendix E

Other Devices

E.1. QH Gyrator Device

The gyrator device used as a verification of the ν symmetric model (section 4.3.2.6) was made on a different GaAs wafer. Being a gyrator, the design was very different. It was a two port device; there was a third port which was connected to RF ground. The QH droplet had a radius of $r = 150 \mu\text{m}$. No Hall bar was present on this device, however the position of the plateaux were used to calculate the electron density – which was calculated to be $n_e = 1.66 \times 10^{15} \text{ m}^{-2}$. The fitting parameters of lateral depletion depth and effective permittivity were calculated to be $h = 1.6 \mu\text{m}$ and $\epsilon^* = 7.93$.

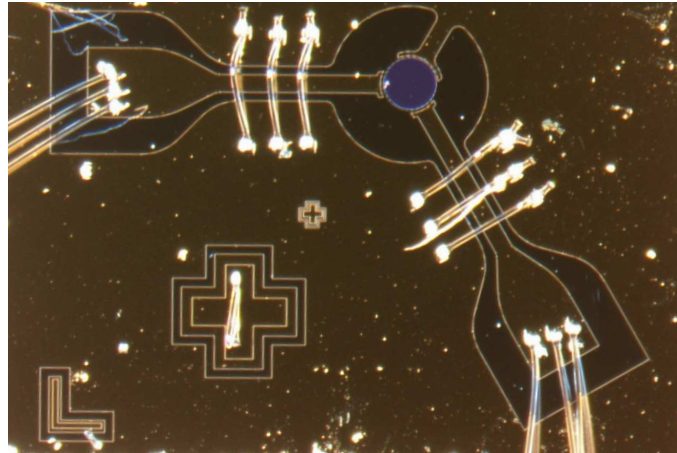


Figure E.1: Image of the QH gyrator device. The etched $r = 150 \mu\text{m}$ droplet is highlighted in blue. The 50Ω transmission line is comprised of Ti/Au. The ground terminal spreads twice as far around the droplet compared with a signal terminal.

A QH droplet located next to a transmission line such as the one discussed in chapter 4 loses power into the droplet when an EMP is present in the system. In contrast, a device where the QH is part of the transmission path – such as this gyrator – will generally have the transmission increase (with respect to $B = 0$) when an EMP is present in the system. This is demonstrated in figure E.2.

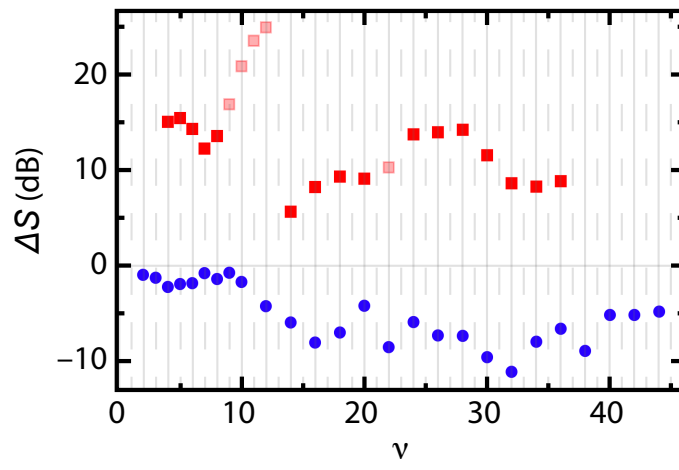


Figure E.2: (blue circles) $10\times$ the change in transmission on integer filling factors for the QH droplet located next to a transmission line. The effect of the standing waves has not been taken into account. (red squares) The change in transmission on integer filling factors for the QH gyrator. The faded red squares are the filling factors which were affected by the board resonances. The faded grey lines are present to help distinguish between the odd and even filling factors. The solid lines indicate even, and the dashed lines odd filling factors.

E.2. Two Droplet EMP Device

The two droplet device consisted of two nominally identical droplets positioned near a CPW (figure E.3). The droplets were circles with radii of $r = 165 \mu\text{m}$. The droplets were separated from each other by $30 \mu\text{m}$. The nearest approach to the transmission line was $55 \mu\text{m}$.

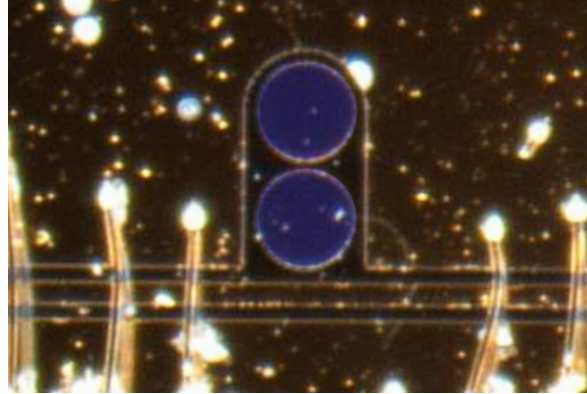


Figure E.3: Image of the two droplet device. The etched $r = 165 \mu\text{m}$ droplets (where 2DEG remains) are highlighted in blue. The point of closest approach of the droplets was $30 \mu\text{m}$.

Figure E.4 shows the transmission measurement of the two droplet device. The background was subtracted; the background was taken to be the median value of each frequency. No independent measure for the electron density was made on this device, so the ratio n_e/ϵ^* was calculated instead $n_e/\epsilon^* = 0.55 \times 10^{15} \text{ m}^{-2}$. This seems reasonable given that the gyrator had a ratio of $n_e/\epsilon^* = 0.21 \times 10^{15} \text{ m}^{-2}$ and the single droplet had a ratio of $n_e/\epsilon^* = 0.17 \times 10^{15} \text{ m}^{-2}$. The value of h has little influence in this part of the magnetic field; the value used in the fit was $h = 0.09 \mu\text{m}$. In reality it is probably larger than this, and the ratio n_e/ϵ^* , should be smaller, but larger magnetic fields (lower filling factors) would need to be observed to confirm this. The fundamental and first harmonic were clearly visible. The

second harmonic was barely visible.

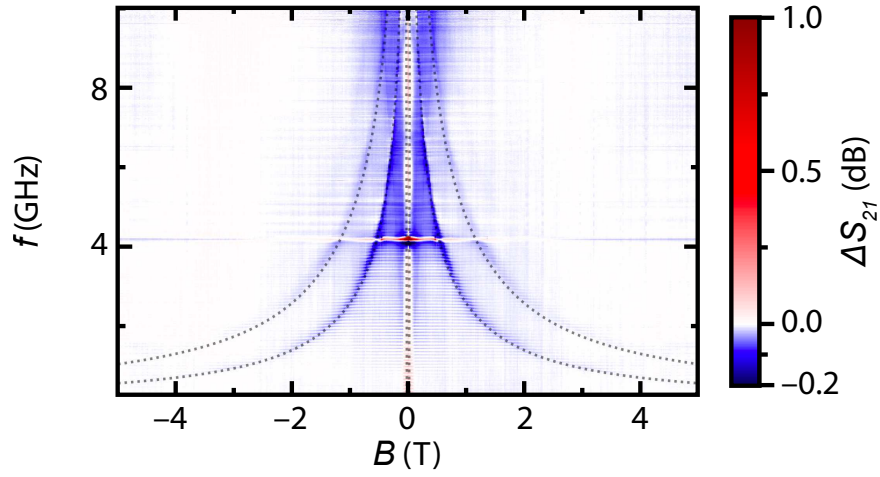


Figure E.4: Transmission spectrum with the background subtracted of two droplets. EMPs are visible as dips in this spectrum. The black dotted lines are the fundamental and first harmonic ($n = 1, 2$) fits to equation (4.24), the values of the parameters are provided in the text.

E.3. Combined Droplet EMP Device

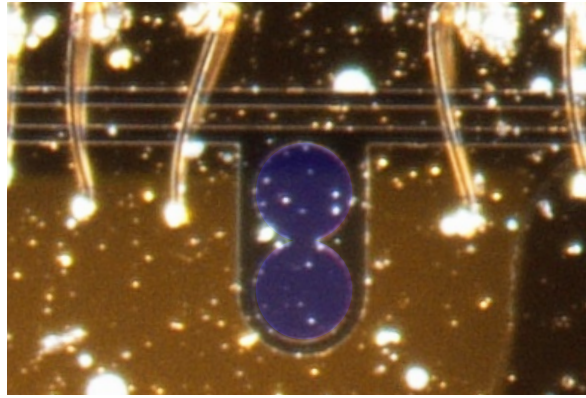


Figure E.5: Image of the combined two droplet device. Highlighted in blue is the etched droplet (where 2DEG remains) which a perimeter of $p = 1990 \mu\text{m}$.

The combined EMP droplet device was made on the same wafer as the two droplet device. It was fit with the same value for the ratio, $n_e/\epsilon^* = 0.55 \times 10^{15} \text{ m}^{-2}$, and $h = 0.09 \mu\text{m}$. The combined droplet was designed to consist of two $r = 165 \mu\text{m}$ circles, with centres separated by $2r$. Two arcs (141°) of a $r = 10 \mu\text{m}$ circle were used to combine the two droplet into one. This gave a total perimeter of $p \sim 1898 \mu\text{m}$.

This device had many more modes than I had seen in any other device. Throughout my PhD I had only ever observed a maximum of two obvious modes with hints of a third. In this device, four modes were clearly visible (figure E.6), $n = 1, 2, 3, 4$. The device having more absorption peaks aligns with the prediction in [44], where a constriction is meant to enhance the spectrum cone due to phonon coupling. However, what was more surprising was the modes that appeared much higher in frequency (green dotted lines in figure E.6). These could be fit with the same parameters and $n = 19, 30$; this however does not seem reasonable.

I speculated that these higher order modes could instead be acoustic EMPs or IEMP; however, both of these types have lower frequencies than EMPs [127–129]. My current hypothesis is that lower frequency modes follow the edge of the sample, whereas the the higher frequency modes do not. Instead of tracing a figure eight, they trace out a single circles' path, with the two circles beating. This would essentially give the (0,1) and (1,1) drum modes. The (0,1) is the fundamental, and the (1,1) a mode with a single nodal diameter, which has a frequency $1.54\times$ the fundamentals [206]. By changing the perimeter to $\sim 100\ \mu\text{m}$ and keeping all other parameters the same, these higher order modes can be fit. It would be expected that other parameters should also be affected; however more data is needed.

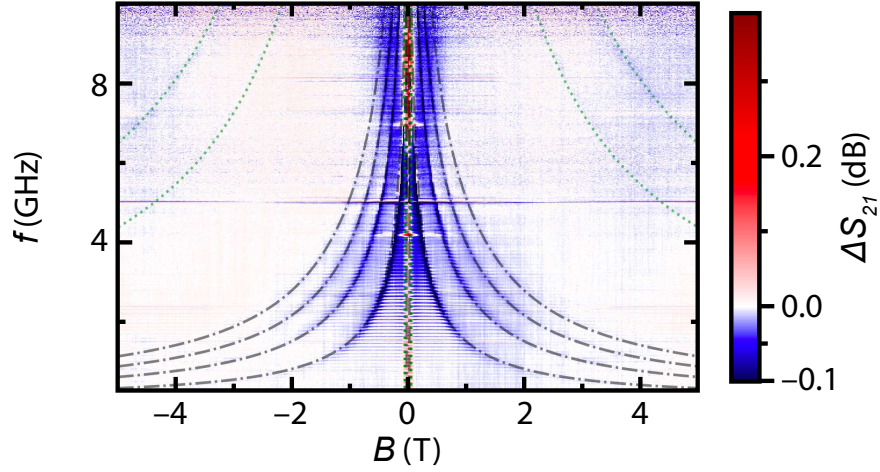


Figure E.6: Transmission spectrum of combined droplet with the background (median) subtracted. EMPs are visible as dips in this spectrum. The black dash-dot lines are fits to equation (4.24) with $n = 1, 2, 3, 4$. The green dotted lines are fits with $p = 100\ \mu\text{m}$ and $n = 1, 1.59$, consistent with drum modes [206]. The values of the other parameters are provided in the text.

E.4. 3-Terminal 2DEG Device

Although a 3-terminal 2DEG device has not as-yet been successfully measured, there has been a lot of progress in the fabrication (figure E.7). If my current understanding of where the leakage comes from is correct, using a hard mask should overcome this issue. Using an aluminum etchant that is not expired should also increase the quality of the remaining aluminium and give greater control over the feature sizes.

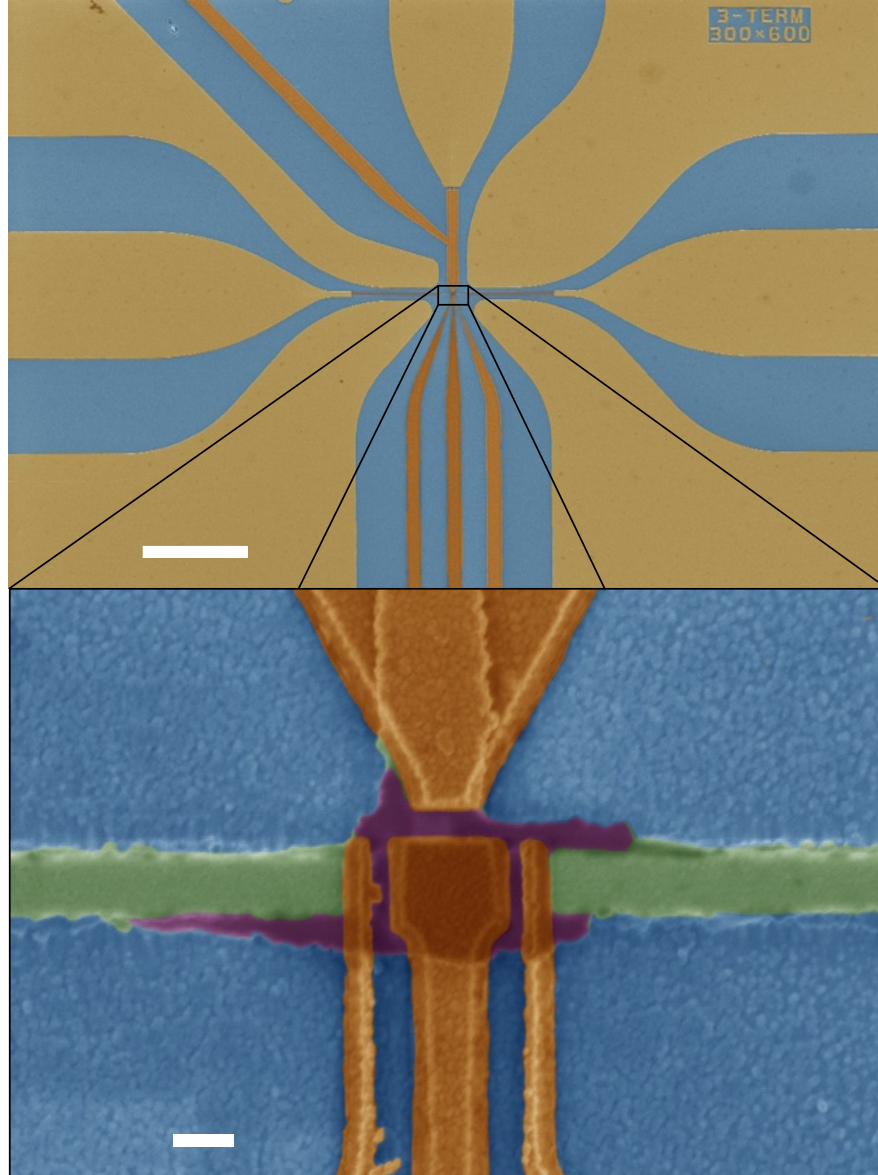


Figure E.7: False colour SEM of a 3-terminal 2DEG device. The active region was designed to be $300 \text{ nm} \times 600 \text{ nm}$. (Top) Gold – Ti/Au ($Z_0=50 \Omega$) waveguide, orange – Ti/Au gates (deposited on AlO_x), blue – etched region. The scale bar is $20 \mu\text{m}$. (Bottom) Zoomed-in area of the active region of the device. Gold, orange, and blue represent the same as above; the additional colours represent: green – InAs 2DEG, purple – remaining epitaxial aluminium. The scale bar is 200 nm .

Bibliography

- [1] J. Hook and H. E. Hall. ‘Low-dimensional systems’. In: *Solid State Physics*. 2nd ed. West Sussex, England: John Wiley & Sons, 1991. Chap. 14, pp. 399–416.
- [2] V. Corato, E. Esposito, C. Granata, B. Ruggiero, M. Russo and P. Silverstrini. ‘Macroscopic Quantum Phenomena in Underdamped Josephson Junctions’. In: *Macroscopic Quantum Coherence and Quantum Computing*. Ed. by D. V. Averin, B. Ruggiero and P. Silverstrini. 1st ed. New York: Springer Science, 2001. Chap. 2.3, pp. 45–54. DOI: [10.1007/978-1-4615-1245-5](https://doi.org/10.1007/978-1-4615-1245-5).
- [3] M. R. Geller, E. J. Pritchett, A. T. Sornborger and F. K. Wilhelm. ‘Quantum Computing With Superconductors I: Architectures’. In: *Macroscopic Quantum Coherence and Quantum Computing*. Ed. by M. E. Flatté and Trifea. 1st ed. Vol. 244. NATO Science Series II: Mathematics, Physics and Chemistry. Netherlands: Springer Dordrecht, 2007. Chap. 5, pp. 171–194. DOI: [10.1007/978-1-4020-6137-0](https://doi.org/10.1007/978-1-4020-6137-0).
- [4] H. Zu, W. Dai and A. de Waele. ‘Development of dilution refrigerators—A review’. In: *Cryogenics* 121 (2022), p. 103390. DOI: [10.1016/j.cryogenics.2021.103390](https://doi.org/10.1016/j.cryogenics.2021.103390).
- [5] D. J. Reilly, C. M. Marcus, M. P. Hanson and A. C. Gossard. ‘Fast single-charge sensing with a rf quantum point contact’. In: *Appl. Phys. Lett* 91 (16 2007), p. 162101. DOI: [10.1063/1.2794995](https://doi.org/10.1063/1.2794995).
- [6] J. I. Colless, A. C. Mahoney, J. M. Hornibrook, A. C. Doherty, H. Lu, A. C. Gossard and D. J. Reilly. ‘Dispersive Readout of a Few-Electron Double Quantum Dot with Fast rf Gate Sensors’. In: *Phys. Rev. Lett.* 110 (4 2013), p. 046805. DOI: [10.1103/PhysRevLett.110.046805](https://doi.org/10.1103/PhysRevLett.110.046805).
- [7] D. J. Reilly. ‘Challenges in Scaling-up the Control Interface of a Quantum Computer’. In: *2019 IEEE International Electron Devices Meeting (IEDM)*. 2019, pp. 31.7.1–31.7.6. DOI: [10.1109/IEDM19573.2019.8993497](https://doi.org/10.1109/IEDM19573.2019.8993497).
- [8] D. van Delft and P. Kes. ‘The Discovery of superconductivity’. In: *Physics Today* 63 (9 2010), pp. 38–43. DOI: [10.1063/1.3490499](https://doi.org/10.1063/1.3490499).
- [9] C. Enss and S. Hunklinger. *Low-Temperature Physics*. 1st ed. Springer, 2005.
- [10] H.-K. Mao, B. Chen, J. Chen, K. Li, J.-F. Lin, W. Yang and H. Zheng. ‘Recent advances in high-pressure science and technology’. In: *Matter and Radiation Extremes* 1.1 (2016), pp. 59–75. DOI: [10.1016/j.mre.2016.01.005](https://doi.org/10.1016/j.mre.2016.01.005).

- [11] D. Akinwande, C. J. Brennan, J. S. Bunch, P. Egberts, J. R. Felts, H. Gao, R. Huang, J.-S. Kim, T. Li, Y. Li, K. M. Liechti, N. Lu, H. S. Park, E. J. Reed, P. Wang, B. I. Yakobson, T. Zhang, Y.-W. Zhang, Y. Zhou and Y. Zhu. 'A review on mechanics and mechanical properties of 2D materials – Graphene and beyond'. In: *Extreme Mechanics Letters* 32 (2017), pp. 41–77. DOI: [10.1016/j.eml.2017.01.008](https://doi.org/10.1016/j.eml.2017.01.008).
- [12] T. Zhai, X. Fang, M. Liao, X. Xu, H. Zeng, B. Yoshio and D. Golberg. 'A Comprehensive Review of One-Dimensional Metal-Oxide Nanostructure Photodetectors'. In: *Sensors* 9 (8 2009), pp. 6504–6529. DOI: [10.3390/s90806504](https://doi.org/10.3390/s90806504).
- [13] K. von Klitzing. *Nobel Lecture: The Quantized Hall Effect*. www.nobelprize.org, Nobel Foundation. 1985.
- [14] R. Durrer. 'The cosmic microwave background: the history of its experimental investigation and its significance for cosmology'. In: *Classical and Quantum Gravity* 32 (2015), p. 124007. DOI: [10.1088/0264-9381/32/12/124007](https://doi.org/10.1088/0264-9381/32/12/124007).
- [15] A. Sabban. 'Electromagnetic Wave Propagation and Applications' & 'RF Measurements'. In: *Wideband RF Technologies and Antennas in Microwave Frequencies*. 1st ed. Hoboken, New Jersey: John Wiley & Sons, 2016. Chap. 1 & 14, pp. 1–18 & 400–420. DOI: [10.1002/9781119048640](https://doi.org/10.1002/9781119048640).
- [16] D. M. Pozar. *Microwave Engineering*. 2nd ed. John Wiley & Sons, 1998.
- [17] A. Oliner. 'Historical Perspectives on Microwave Field Theory'. In: *IEEE Transactions on Microwave Theory and Techniques* 32.9 (1984), pp. 1022–1045. DOI: [10.1109/TMTT.1984.1132815](https://doi.org/10.1109/TMTT.1984.1132815).
- [18] H. Liu, M. Lu, Y. Wu, J. Liu and X. C. Xie. 'Non-Hermiticity-stabilized Majorana zero modes in semiconductor-superconductor nanowires'. In: *Phys. Rev. B* 106 (6 2022), p. 064505. DOI: [10.1103/PhysRevB.106.064505](https://doi.org/10.1103/PhysRevB.106.064505).
- [19] J. Heinsoo, C. K. Andersen, A. Remm, S. Krinner, T. Walter, Y. Salathé, S. Gasparinetti, J.-C. Besse, A. Potočník, A. Wallraff and C. Eichler. 'Rapid High-fidelity Multiplexed Readout of Superconducting Qubits'. In: *Phys. Rev. Applied* 10 (3 2018), p. 034040. DOI: [10.1103/PhysRevApplied.10.034040](https://doi.org/10.1103/PhysRevApplied.10.034040).
- [20] J. M. Hornibrook, J. C. Colless, A. C. Mahoney, X. G. Croote, S. Blainvillain, H. Lu, A. C. Gossard and D. J. Reilly. 'Frequency multiplexing for readout of spin qubits'. In: *Appl. Phys. Lett.* 104 (2014), p. 103108. DOI: [0.1063/1.4868107](https://doi.org/10.1063/1.4868107).
- [21] M. F. Gonzalez-Zalba, S. Barraud, A. J. Ferguson and A. C. Betz. 'Probing the limits of gate-based charge sensing'. In: *Nature Communications* 6.1 (2015), p. 6084. DOI: [10.1038/ncomms7084](https://doi.org/10.1038/ncomms7084).
- [22] B. J. Hunt. 'Oliver Heaviside: A first-rate oddity'. In: *Physics Today* 65 (11 2012), pp. 48–54. DOI: [10.1063/PT.3.1788](https://doi.org/10.1063/PT.3.1788).

- [23] S. A. Schelkunoff. 'The impedance concept and its application to problems of reflection, refraction, shielding and power absorption'. In: *The Bell System Technical Journal* 17.1 (1938), pp. 17–48. DOI: [10.1002/j.1538-7305.1938.tb00774.x](https://doi.org/10.1002/j.1538-7305.1938.tb00774.x).
- [24] *Keysight 2-Port and 4-Port PNA-X Microwave Network Analyzers*. Keysight Technologies.
- [25] *PNA Family Microwave Network Analyzers (522x/3x/4xB)*. Keysight Technologies.
- [26] *Printed Version of PNA Help*. Ailent Technologies.
- [27] H. Zhang, S. Krooswyk and J. Ou. 'Chapter 5 - Measurement and data acquisition techniques'. In: *High Speed Digital Design*. Ed. by H. Zhang, S. Krooswyk and J. Ou. Boston: Morgan Kaufmann, 2015, pp. 199–219. DOI: <https://doi.org/10.1016/B978-0-12-418663-7.00005-8>.
- [28] M. Stanley, S. De Graaf, T. Hönigl-Decrinis, T. Lindström and N. M. Ridler. 'Characterizing Scattering Parameters of Superconducting Quantum Integrated Circuits at Milli-Kelvin Temperatures'. In: *IEEE Access* 10 (2022), pp. 43376–43386. DOI: [10.1109/ACCESS.2022.3169787](https://doi.org/10.1109/ACCESS.2022.3169787).
- [29] *Technical Overview: Electronic Calibration (ECal) Modules for Vector Network Analyzers*. Keysight Technologies.
- [30] P. H. Smith. 'Transmission Line Calculator'. In: *Electronics* 12.1 (1939), pp. 29–31.
- [31] Le Leivre. *Smith Charts*. URL: https://leleivre.com/downloadables/smith_chart.pdf. (accessed: 10-2022).
- [32] N. Ares, F. J. Schupp, A. Mavalankar, G. Rogers, J. Griffiths, G. A. C. Jones, I. Farrer, D. A. Ritchie, C. G. Smith, A. Cottet, G. A. D. Briggs and E. A. Laird. 'Sensitive Radio-Frequency Measurements of a Quantum Dot by Tuning to Perfect Impedance Matching'. In: *Phys. Rev. Applied* 5 (3 2016), p. 034011. DOI: [10.1103/PhysRevApplied.5.034011](https://doi.org/10.1103/PhysRevApplied.5.034011).
- [33] M. C. Jarratt, S. J. Waddy, A. Jouan, A. C. Mahoney, G. C. Gardner, S. Fallahi, M. J. Manfra and D. J. Reilly. 'Detection of the Quantum Capacitance of a Point Contact via Dispersive Gate Sensing'. In: *Phys. Rev. Applied* 14 (6 2020), p. 064021. DOI: [10.1103/PhysRevApplied.14.064021](https://doi.org/10.1103/PhysRevApplied.14.064021).
- [34] B. Placke, S. Bosco and D. P. DiVincenzo. 'A model study of present-day Hall-effect circulators'. In: *EPJ Quantum Technology* 4.1 (2017), p. 5. DOI: [10.1140/epjqt/s40507-017-0057-9](https://doi.org/10.1140/epjqt/s40507-017-0057-9).
- [35] G. Nunes jr. and K. A. Earle. 'Dilution Refrigerators'. In: *Experimental Techniques in Condensed Matter Physics At Low Temperatures*. Ed. by R. C. Richardson and E. N. Smith. 1st ed. Ithaca, New York: Cornell University, 1998. Chap. 2.3, pp. 45–54.

- [36] D. J. Cousins, S. N. Fisher, A. M. Guénault, R. P. Haley, I. E. Miller, G. R. Pickett, G. N. Plenderleith, P. Skyba, P. Y. A. Thibault and M. G. Ward. ‘An Advanced Dilution Refrigerator Designed for the New Lancaster Microkelvin Facility’. In: *Journal of Low Temperature Physics* 114.5 (1999), pp. 547–570. DOI: [10.1023/A:1021862406629](https://doi.org/10.1023/A:1021862406629).
- [37] T. A. Alvesalo, P. M. Berglund, S. T. Islander, G. R. Pickett and W. Zimmermann. ‘Specific Heat of Liquid He^3/He^4 Mixtures near the Junction of the λ and Phase-Separation Curves’. In: *Phys. Rev. A* 4 (6 1971), pp. 2354–2368. DOI: [10.1103/PhysRevA.4.2354](https://doi.org/10.1103/PhysRevA.4.2354).
- [38] G. Chaudhry and J. G. Brisson. ‘Thermodynamic properties of liquid $^3\text{He}/^4\text{He}$ mixtures between 0.15 K and 1.5 K’. In: *AIP Conference Proceedings* 1218.1 (2010), pp. 1562–1569. DOI: [10.1063/1.3422337](https://doi.org/10.1063/1.3422337). eprint: <https://aip.scitation.org/doi/pdf/10.1063/1.3422337>.
- [39] J. C. Wheatley, O. E. Vilches and W. R. Abel. ‘Principles and methods of dilution refrigeration’. In: *Physics Physique Fizika* 4 (1 1968), pp. 1–64. DOI: [10.1103/PhysicsPhysiqueFizika.4.1](https://doi.org/10.1103/PhysicsPhysiqueFizika.4.1).
- [40] J. C. Wheatley, R. E. Rapp and R. T. Johnson. ‘Principles and methods of dilution refrigeration. II’. In: *Journal of Low Temperature Physics* 4.1 (1971), pp. 1–39. DOI: [10.1007/BF00628435](https://doi.org/10.1007/BF00628435).
- [41] Leiden Cryogenics BV.
- [42] W. C. Black *jr.*, W. R. Roach and J. C. Wheatley. ‘Speer Carbon Resistors as Thermometers for Use Below 1°K ’. In: *Review of Scientific Instruments* 35.5 (1963), pp. 587–591. DOI: [10.1063/1.1718880](https://doi.org/10.1063/1.1718880).
- [43] R. W. Willekers, F. Mathu, H. C. Meijer and H. Postma. ‘Thick film thermometers with predictable R-T characteristics and very low magnetoresistance below 1 K’. In: *Cryogenics* 30.4 (1990), pp. 351–355. DOI: [10.1016/0011-2275\(90\)90315-4](https://doi.org/10.1016/0011-2275(90)90315-4).
- [44] J. Cano, A. C. Doherty, C. Nayak and D. J. Reilly. ‘Microwave absorption by a mesoscopic quantum Hall droplet’. In: *Phys. Rev. B* 88 (16 2013), p. 165305. DOI: [10.1103/PhysRevB.88.165305](https://doi.org/10.1103/PhysRevB.88.165305).
- [45] G. Brändli and J. L. Olsen. ‘Size effects in electron transport in metals’. In: *Materials Science and Engineering* 4.2 (1969), pp. 61–83. DOI: [10.1016/0025-5416\(69\)90046-9](https://doi.org/10.1016/0025-5416(69)90046-9).
- [46] N. Laman and D. Grischlowsky. ‘Terahertz conductivity of thin metal films’. In: *Applied Physics Letters* 93 (5 2008), p. 051105. DOI: [10.1063/1.2968308](https://doi.org/10.1063/1.2968308).
- [47] R. Williams. *Modern GaAs Processing Methods*. 1st ed. Artech House, Inc., 1999.

- [48] J. Shabani, M. Kjaergaard, H. J. Suominen, Y. Kim, F. Nichele, K. Pakrouski, T. Stankevic, R. M. Lutchyn, P. Krogstrup, R. Feidenhans'l, S. Kraemer, C. Nayak, M. Troyer, C. M. Marcus and C. J. Palmstrøm. 'Two-dimensional epitaxial superconductor-semiconductor heterostructures: A platform for topological superconducting networks'. In: *Phys. Rev. B* 93 (15 2016), p. 155402. DOI: [10.1103/PhysRevB.93.155402](https://doi.org/10.1103/PhysRevB.93.155402).
- [49] K. S. Wickramasinghe, W. Mayer, J. Yuan, T. Nguyen, L. Jiao, V. Manucharyan and J. Shabani. 'Transport properties of near surface InAs two-dimensional heterostructures'. In: *Applied Physics Letters* 113.26 (2018), p. 262104. DOI: [10.1063/1.5050413](https://doi.org/10.1063/1.5050413).
- [50] M. Kjaergaard, F. Nichele, H. J. Suominen, M. P. Nowak, M. Wimmer, A. R. Akhmerov, J. A. Folk, K. Flensberg, J. Shabani, C. J. Palmstrøm and C. M. Marcus. 'Quantized conductance doubling and hard gap in a two-dimensional semiconductor-superconductor heterostructure'. In: *Nature Communications* 7.1 (2016), p. 12841. DOI: [10.1038/ncomms12841](https://doi.org/10.1038/ncomms12841).
- [51] W. Mayer, J. Yuan, K. S. Wickramasinghe, T. Nguyen, M. C. Dartiailh and J. Shabani. 'Superconducting proximity effect in epitaxial Al-InAs heterostructures'. In: *Applied Physics Letters* 114.10 (2019), p. 103104. DOI: [10.1063/1.5067363](https://doi.org/10.1063/1.5067363).
- [52] S. J. Pauka, J. D. S. Witt, C. N. Allen, B. Harlech-Jones, A. Jouan, G. C. Gardner, S. Gronin, T. Wang, C. Thomas, M. J. Manfra, J. Gukelberger, J. Gamble, D. J. Reilly and M. C. Cassidy. 'Repairing the surface of InAs-based topological heterostructures'. In: *Journal of Applied Physics* 128.11 (2020), p. 114301. DOI: [10.1063/5.0014361](https://doi.org/10.1063/5.0014361).
- [53] F. Krizek, J. E. Sestoft, P. Aseev, S. Marti-Sanchez, S. Vaitiekėnas, L. Casparis, S. A. Khan, Y. Liu, T. š. Stankevič, A. M. Whitar, A. Fursina, F. Boekhout, R. Koops, E. Uccelli, L. P. Kouwenhoven, C. M. Marcus, J. Arbiol and P. Krogstrup. 'Field effect enhancement in buffered quantum nanowire networks'. In: *Phys. Rev. Materials* 2 (9 2018), p. 093401. DOI: [10.1103/PhysRevMaterials.2.093401](https://doi.org/10.1103/PhysRevMaterials.2.093401).
- [54] P. Krogstrup, N. L. B. Ziino, W. Chang, S. M. Albrecht, M. H. Madsen, E. Johnson, J. Nygård, C. M. Marcus and T. S. Jespersen. 'Epitaxy of semiconductor-superconductor nanowires'. In: *Nature Materials* 14.4 (2015), pp. 400–406. DOI: [10.1038/nmat4176](https://doi.org/10.1038/nmat4176).
- [55] S. Vaitiekėnas, A. M. Whitar, M.-T. Deng, F. Krizek, J. E. Sestoft, C. J. Palmstrøm, S. Marti-Sanchez, J. Arbiol, P. Krogstrup, L. Casparis and C. M. Marcus. 'Selective-Area-Grown Semiconductor-Superconductor Hybrids: A Basis for Topological Networks'. In: *Phys. Rev. Lett.* 121 (14 2018), p. 147701. DOI: [10.1103/PhysRevLett.121.147701](https://doi.org/10.1103/PhysRevLett.121.147701).
- [56] A. Y. Kitaev. 'Unpaired Majorana fermions in quantum wires'. In: *Physics-Uspeski* 44.10S (2001), p. 131. DOI: [10.1070/1063-7869/44/10S/S29](https://doi.org/10.1070/1063-7869/44/10S/S29).

- [57] J. D. Sau, R. M. Lutchyn, S. Tewari and S. Das Sarma. ‘Generic New Platform for Topological Quantum Computation Using Semiconductor Heterostructures’. In: *Phys. Rev. Lett.* 104 (4 2010), p. 040502. DOI: [10.1103/PhysRevLett.104.040502](https://doi.org/10.1103/PhysRevLett.104.040502).
- [58] Y. Oreg, G. Refael and F. von Oppen. ‘Helical Liquids and Majorana Bound States in Quantum Wires’. In: *Phys. Rev. Lett.* 105 (17 2010), p. 177002. DOI: [10.1103/PhysRevLett.105.177002](https://doi.org/10.1103/PhysRevLett.105.177002).
- [59] R. M. Lutchyn, J. D. Sau and S. Das Sarma. ‘Majorana Fermions and a Topological Phase Transition in Semiconductor-Superconductor Heterostructures’. In: *Phys. Rev. Lett.* 105 (7 2010), p. 077001. DOI: [10.1103/PhysRevLett.105.077001](https://doi.org/10.1103/PhysRevLett.105.077001).
- [60] R. M. Lutchyn, E. P. A. M. Bakkers, L. P. Kouwenhoven, P. Krogstrup, C. M. Marcus and Y. Oreg. ‘Majorana zero modes in superconductor–semiconductor heterostructure’. In: *Nature Review Materials* 3 (2018), p. 52. DOI: <https://doi.org/10.1038/s41578-018-0003-1>.
- [61] E. Majorana and L. Maiani. ‘A symmetric theory of electrons and positrons’. In: *Ettore Majorana Scientific Papers: On occasion of the centenary of his birth*. Ed. by G. F. Bassani. Translated from “Il Nuovo Cimento”, vol. 14, 1937, pp. 171–184, by Luciano Maiani in “Soryushiron Kenkyu”, 63 (1981) 149–462, additional edits by Giuseppe Franco Bassani. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006, pp. 201–233. DOI: [10.1007/978-3-540-48095-2_10](https://doi.org/10.1007/978-3-540-48095-2_10).
- [62] T. D. Stanescu and S. Tewari. ‘Majorana fermions in semiconductor nanowires: fundamentals, modeling, and experiment’. In: *J. Phys.: Condens. Matter* 25 (2013), p. 233201. DOI: [10.1088/0953-8984/25/23/233201](https://doi.org/10.1088/0953-8984/25/23/233201).
- [63] S. Das Sarma, M. Freedman and C. Nayak. ‘Majorana zero modes and topological quantum computation’. In: *npj Quantum Information* 1.1 (2015), p. 15001. DOI: [10.1038/npjqi.2015.1](https://doi.org/10.1038/npjqi.2015.1).
- [64] F. von Oppen, Y. Peng and F. Pientka. ‘Topological superconducting phases in one dimension’. In: *Topological Aspects of Condensed Matter Physics: Lecture Notes of the Les Houches Summer School: Volume 103, August 2014*. Ed. by C. Chamon, M. O. Goerbig, R. Moessner and L. F. Cugliandolo. Oxford University Press, 2017. Chap. 9, pp. 387–450. DOI: [10.1093/acprof:oso/9780198785781.003.0009](https://doi.org/10.1093/acprof:oso/9780198785781.003.0009).
- [65] Mini-Circuits. *LFCN-series*. URL: <https://www.minicircuits.com/WebStore/modelSearch.html?model=LFCN&search=1> (visited on 10/2022).
- [66] S. J. Waddy. ‘Quantum Readout and Control: Scalable Hardware and Techniques for Solid State Quantum Systems’. Available at <https://hdl.handle.net/2123/30281>. PhD thesis. Sydney, Australia: University of Sydney, 2023.

- [67] S. Bosco and D. P. DiVincenzo. ‘Nonreciprocal quantum Hall devices with driven edge magnetoplasmons in two-dimensional materials’. In: *Phys. Rev. B* 95 (19 2017), p. 195317. DOI: [10.1103/PhysRevB.95.195317](https://doi.org/10.1103/PhysRevB.95.195317).
- [68] A. C. Mahoney, J. I. Colless, S. J. Pauka, J. M. Hornibrook, J. D. Watson, G. C. Gardner, M. J. Manfra, A. C. Doherty and D. J. Reilly. ‘On-Chip Microwave Quantum Hall Circulator’. In: *Phys. Rev. X* 7 (1 2017), p. 011007. DOI: [10.1103/PhysRevX.7.011007](https://doi.org/10.1103/PhysRevX.7.011007).
- [69] A. C. Mahoney, J. I. Colless, L. Peeters, S. J. Pauka, E. J. Fox, X. Kou, L. Pan, K. L. Wang, D. Goldhaber-Gordon and D. J. Reilly. ‘Zero-field edge plasmons in a magnetic topological insulator’. In: *Nat. Commun.* 8 (1 2017), p. 1836. DOI: [10.1038/s41467-017-01984-5](https://doi.org/10.1038/s41467-017-01984-5).
- [70] N. Hiyama, M. Hashisaka and T. Fujisawa. ‘An edge-magnetoplasmon Mach-Zehnder interferometer’. In: *Applied Physics Letters* 107 (14 2015), p. 143101. DOI: [10.1063/1.4932111](https://doi.org/10.1063/1.4932111).
- [71] M. Hashisaka, R. Murata, K. Muraki and T. Fujisawa. ‘Frequency conversion of radio-frequency edge magnetoplasmons using a quantum point contact’. In: *Applied Physics Letters* 100 (23 2012), p. 233501. DOI: [10.1063/1.4726136](https://doi.org/10.1063/1.4726136).
- [72] T. M. Stace, C. H. W. Barnes and G. J. Milburn. ‘Mesoscopic One-Way Channels for Quantum State Transfer via the Quantum Hall Effect’. In: *Phys. Rev. Lett.* 93 (12 2004), p. 126804. DOI: [10.1103/PhysRevLett.93.126804](https://doi.org/10.1103/PhysRevLett.93.126804).
- [73] S. J. Elman, S. D. Bartlett and A. C. Doherty. ‘Long-range entanglement for spin qubits via quantum Hall edge modes’. In: *Phys. Rev. B* 96 (11 2017), p. 115407. DOI: [10.1103/PhysRevB.96.115407](https://doi.org/10.1103/PhysRevB.96.115407).
- [74] S. Bosco and D. P. DiVincenzo. ‘Transmission lines and resonators based on quantum Hall plasmonics: Electromagnetic field, attenuation, and coupling to qubits’. In: *Phys. Rev. B* 100 (3 2019), p. 035416. DOI: [10.1103/PhysRevB.100.035416](https://doi.org/10.1103/PhysRevB.100.035416).
- [75] S. Bosco, D. P. DiVincenzo and D. J. Reilly. ‘Transmission Lines and Metamaterials Based on Quantum Hall Plasmonics’. In: *Phys. Rev. Applied* 12 (1 2019), p. 014030. DOI: [10.1103/PhysRevApplied.12.014030](https://doi.org/10.1103/PhysRevApplied.12.014030).
- [76] E. H. Hall. ‘On a New Action of the Magnet on Electric Currents’. In: *Am. J. Math.* 2.3 (1879), pp. 287–292.
- [77] K. von Klitzing, G. Dorda and M. Pepper. ‘New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance’. In: *Phys. Rev. Lett.* 45 (6 1980), pp. 494–497. DOI: [10.1103/PhysRevLett.45.494](https://doi.org/10.1103/PhysRevLett.45.494).
- [78] K. von Klitzing. ‘Developments in the quantum Hall effect’. In: *Phil. Trans. R. Soc. A* 363.1834 (2005), pp. 2203–2219. DOI: [10.1098/rsta.2005.1640](https://doi.org/10.1098/rsta.2005.1640).

- [79] H. L. Störmer. ‘Nobel Lecture: The fractional quantum Hall effect’. In: *Rev. Mod. Phys.* 71 (4 1999), pp. 875–889. DOI: [10.1103/RevModPhys.71.875](https://doi.org/10.1103/RevModPhys.71.875).
- [80] A. Siddiki and R. R. Gerhardt. ‘Incompressible strips in dissipative Hall bars as origin of quantized Hall plateaus’. In: *Phys. Rev. B* 70 (19 2004), p. 195335. DOI: [10.1103/PhysRevB.70.195335](https://doi.org/10.1103/PhysRevB.70.195335).
- [81] W. Poirier, F. Schopfer, J. Guignard, O. Thévenot and P. Gournay. ‘Application of the quantum Hall effect to resistance metrology’. In: *Comptes rendus. Physique* 12.4 (2011), pp. 347–368. DOI: [10.1016/j.crhy.2011.04.008](https://doi.org/10.1016/j.crhy.2011.04.008).
- [82] J. Weis and K. von Klitzing. ‘Metrology and microscopic picture of the integer quantum Hall effect’. In: *Phil. Trans. R. Soc. A* 369 (1953 2011), p. 3954. DOI: [10.1098/rsta.2011.0198](https://doi.org/10.1098/rsta.2011.0198).
- [83] P. J. Mohr, D. B. Newell and B. N. Taylor. ‘CODATA recommended values of the fundamental physical constants: 2014’. In: *Rev. Mod. Phys.* 88 (3 2016), p. 035009. DOI: [10.1103/RevModPhys.88.035009](https://doi.org/10.1103/RevModPhys.88.035009).
- [84] K. von Klitzing. ‘25 Years of Quantum Hall Effect (QHE) A Personal View on the Discovery, Physics and Applications of this Quantum Effect’. In: *The Quantum Hall Effect: Poincaré Seminar 2004*. Ed. by B. Douçot, V. Pasquier, B. Duplantier and V. Rivasseau. Basel: Birkhäuser Basel, 2005. Chap. 1, pp. 1–21. DOI: [10.1007/3-7643-7393-8_1](https://doi.org/10.1007/3-7643-7393-8_1).
- [85] R. B. Laughlin. ‘Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations’. In: *Phys. Rev. Lett.* 50 (18 1983), pp. 1395–1398. DOI: [10.1103/PhysRevLett.50.1395](https://doi.org/10.1103/PhysRevLett.50.1395).
- [86] D. C. Tsui, H. L. Störmer and A. C. Gossard. ‘Two-Dimensional Magnetotransport in the Extreme Quantum Limit’. In: *Phys. Rev. Lett.* 48 (22 1982), pp. 1559–1562. DOI: [10.1103/PhysRevLett.48.1559](https://doi.org/10.1103/PhysRevLett.48.1559).
- [87] R. Willett, J. P. Eisenstein, H. L. Störmer, D. C. Tsui, A. C. Gossard and J. H. English. ‘Observation of an even-denominator quantum number in the fractional quantum Hall effect’. In: *Phys. Rev. Lett.* 59 (15 1987), pp. 1776–1779. DOI: [10.1103/PhysRevLett.59.1776](https://doi.org/10.1103/PhysRevLett.59.1776).
- [88] ‘The quantum Hall effects’. In: *Transport in Nanostructures*. Ed. by D. K. Ferry, S. M. Goodnick and J. Bird. 2nd ed. The Edinburgh Building, Cambridge CB2 8RU, UK: Cambridge University Press, 2009. Chap. 4, pp. 193–247.
- [89] D. B. Chklovskii, B. I. Shklovskii and L. I. Glazman. ‘Electrostatics of edge channels’. In: *Phys. Rev. B* 46 (7 1992), pp. 4026–4034. DOI: [10.1103/PhysRevB.46.4026](https://doi.org/10.1103/PhysRevB.46.4026).
- [90] R. B. Laughlin. ‘Quantized Hall conductivity in two dimensions’. In: *Phys. Rev. B* 23 (10 1981), pp. 5632–5633. DOI: [10.1103/PhysRevB.23.5632](https://doi.org/10.1103/PhysRevB.23.5632).

- [91] M. Büttiker. ‘Absence of backscattering in the quantum Hall effect in multiprobe conductors’. In: *Phys. Rev. B* 38 (14 1988), pp. 9375–9389. DOI: [10.1103/PhysRevB.38.9375](https://doi.org/10.1103/PhysRevB.38.9375).
- [92] R. R. Gerhardts. ‘The effect of screening on current distribution and conductance quantisation in narrow quantum Hall systems’. In: *physica status solidi (b)* 245.2 (2008), pp. 378–392. DOI: [10.1002/pssb.200743344](https://doi.org/10.1002/pssb.200743344).
- [93] D. Tong. *Lectures on the Quantum Hall Effect*. 2016. DOI: [10.48550/ARXIV.1606.06687](https://doi.org/10.48550/ARXIV.1606.06687).
- [94] A. Salman, M. B. Yucel and A. Siddiki. ‘Edge electrostatics revisited’. In: *Physica E* 47 (2013), pp. 229–236. DOI: [10.1016/j.physe.2012.10.035](https://doi.org/10.1016/j.physe.2012.10.035).
- [95] B. I. Halperin. ‘Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential’. In: *Phys. Rev. B* 25 (4 1982), pp. 2185–2190. DOI: [10.1103/PhysRevB.25.2185](https://doi.org/10.1103/PhysRevB.25.2185).
- [96] R. E. Prange. ‘Introduction’. In: *The Quantum Hall Effect*. Ed. by R. E. Prange and S. M. Girvin. 1st ed. New York: Springer-Verlag, 1990. Chap. 1, pp. 1–36. DOI: [10.1007/978-1-4612-3350-3](https://doi.org/10.1007/978-1-4612-3350-3).
- [97] R. Joynt and R. E. Prange. ‘Conditions for the quantum Hall effect’. In: *Phys. Rev. B* 29 (6 1984), pp. 3303–3317. DOI: [10.1103/PhysRevB.29.3303](https://doi.org/10.1103/PhysRevB.29.3303).
- [98] B. Huckestein. ‘Scaling theory of the integer quantum Hall effect’. In: *Rev. Mod. Phys.* 67 (2 1995), pp. 357–396. DOI: [10.1103/RevModPhys.67.357](https://doi.org/10.1103/RevModPhys.67.357).
- [99] S. Das Sarma. ‘Localization, Metal-Insulator Transitions, and Quantum Hall Effect’. In: *Perspectives in Quantum Hall Effects*. Ed. by A. P. Sankar Das Sarma. 1st ed. Buch Bücher dd ag, Birkach, Germany: Wiley-VCH, 1996. Chap. 1, pp. 1–36. DOI: [10.1002/9783527617258](https://doi.org/10.1002/9783527617258).
- [100] A. Siddiki and R. R. Gerhardts. ‘Range-dependent disorder effects on the plateau-widths calculated within the screening theory of the IQHE’. In: *International Journal of Modern Physics B* 21 (2007), pp. 1362–1371. DOI: [10.1142/S0217979207042847](https://doi.org/10.1142/S0217979207042847).
- [101] O. A. Tkachenko, V. A. Tkachenko, D. G. Baksheev and O. P. Sushkov. ‘Simulation of the Quantum Hall Effect in Samples with Weak Long-Range Disorder’. In: *JETP Lett.* 112 (2020), pp. 186–192. DOI: [10.1134/S0021364020150114](https://doi.org/10.1134/S0021364020150114).
- [102] K.-S. Kim and S. A. Kivelson. ‘The quantum Hall effect in the absence of disorder’. In: *npj Quantum Materials* 6 (1 2021), p. 22. DOI: [10.1038/s41535-021-00321-w](https://doi.org/10.1038/s41535-021-00321-w).
- [103] P. Haremski, M. Mausser, A. Gauß, K. von Klitzing and J. Weis. ‘Electrically induced breakdown of the quantum Hall effect at different Hall bar widths: Visualizing the edge- and bulk-dominated regimes within a quantum Hall plateau’. In: *Phys. Rev. B* 102 (20 2020), p. 205306. DOI: [10.1103/PhysRevB.102.205306](https://doi.org/10.1103/PhysRevB.102.205306).

- [104] D. B. Chklovskii, K. A. Matveev and B. I. Shklovskii. ‘Ballistic conductance of interacting electrons in the quantum Hall regime’. In: *Phys. Rev. B* 47 (19 1993), pp. 12605–12617. DOI: [10.1103/PhysRevB.47.12605](https://doi.org/10.1103/PhysRevB.47.12605).
- [105] K. Lier and R. R. Gerhardts. ‘Self-consistent calculations of edge channels in laterally confined two-dimensional electron systems’. In: *Phys. Rev. B* 50 (11 1994), pp. 7757–7767. DOI: [10.1103/PhysRevB.50.7757](https://doi.org/10.1103/PhysRevB.50.7757).
- [106] B. Y. Gelfand and B. I. Halperin. ‘Edge electrostatics of a mesa-etched sample and edge-state-to-bulk scattering rate in the fractional quantum Hall regime’. In: *Phys. Rev. B* 49 (3 1994), pp. 1862–1866. DOI: [10.1103/PhysRevB.49.1862](https://doi.org/10.1103/PhysRevB.49.1862).
- [107] S. Ilani, J. Martin, E. Teitelbaum, J. H. Smet, D. Mahalu, V. Umansky and A. Yacoby. ‘The microscopic nature of localization in the quantum Hall effect’. In: *Nature* 427.6972 (2004), pp. 328–332. DOI: [10.1038/nature02230](https://doi.org/10.1038/nature02230).
- [108] E. M. Kendirlik, S. Sirt, S. B. Kalkan, N. Ofek, V. Umansky and A. Siddiki. ‘The local nature of incompressibility of quantum Hall effect’. In: *Nature Communications* 8.1 (2017), p. 14082. DOI: [10.1038/ncomms14082](https://doi.org/10.1038/ncomms14082).
- [109] A. Chang. ‘A unified transport theory for the integral and fractional quantum hall effects: Phase boundaries, edge currents, and transmission/reflection probabilities’. In: *Solid State Communications* 74.9 (1990), pp. 871–876. DOI: [10.1016/0038-1098\(90\)90447-J](https://doi.org/10.1016/0038-1098(90)90447-J).
- [110] C. W. J. Beenakker. ‘Edge channels for the fractional quantum Hall effect’. In: *Phys. Rev. Lett.* 64 (2 1990), pp. 216–219. DOI: [10.1103/PhysRevLett.64.216](https://doi.org/10.1103/PhysRevLett.64.216).
- [111] I. L. Aleiner and L. I. Glazman. ‘Novel edge excitations of two-dimensional electron liquid in a magnetic field’. In: *Phys. Rev. Lett.* 72 (18 1994), pp. 2935–2938. DOI: [10.1103/PhysRevLett.72.2935](https://doi.org/10.1103/PhysRevLett.72.2935).
- [112] S. Ihnatsenka and I. V. Zozoulenko. ‘Spatial spin polarization and suppression of compressible edge channels in the integer quantum Hall regime’. In: *Phys. Rev. B* 73 (15 2006), p. 155314. DOI: [10.1103/PhysRevB.73.155314](https://doi.org/10.1103/PhysRevB.73.155314).
- [113] P. Weitz, E. Ahlswede, J. Weis, K. von Klitzing and K. Eberl. ‘Hall-potential investigations under quantum Hall conditions using scanning force microscopy’. In: *Physica E: Low-dimensional Systems and Nanostructures* 6.1 (2000), pp. 247–250. DOI: [10.1016/S1386-9477\(99\)00136-8](https://doi.org/10.1016/S1386-9477(99)00136-8).
- [114] E. Ahlswede, P. Weitz, J. Weis, K. von Klitzing and K. Eberl. ‘Hall potential profiles in the quantum Hall regime measured by a scanning force microscope’. In: *Physica E* 298 (1-4 2001), pp. 562–566. DOI: [10.1016/S0921-4526\(01\)00383-03](https://doi.org/10.1016/S0921-4526(01)00383-03).
- [115] E. Ahlswede, J. Weis, K. von Klitzing and K. Eberl. ‘Hall potential distribution in the quantum Hall regime in the vicinity of a potential probe contact’. In: *Physica E* 12 (1-4 2002), pp. 165–168. DOI: [10.1016/S1386-9477\(01\)00267-3](https://doi.org/10.1016/S1386-9477(01)00267-3).

- [116] R. R. Gerhardts, K. Panos and J. Weis. ‘Current-induced asymmetries of incompressible strips in narrow quantum Hall systems’. In: *New Journal of Physics* 15.7 (2013), p. 073034. DOI: [10.1088/1367-2630/15/7/073034](https://doi.org/10.1088/1367-2630/15/7/073034).
- [117] K. Panos, R. R. Gerhardts, J. Weis and K. von Klitzing. ‘Current distribution and Hall potential landscape towards breakdown of the quantum Hall effect: a scanning force microscopy investigation’. In: *New J. Phys.* 16 (11 2014), p. 113071. DOI: [10.1088/1367-2630/16/11/113071](https://doi.org/10.1088/1367-2630/16/11/113071).
- [118] A. A. Maradudin. ‘Introduction: Plasmonics and its Building Blocks’. In: *Modern Plasmonics*. Ed. by A. A. Maradudin, J. R. Sambles and W. L. Barnes. 1st ed. Vol. 4. Amsterdam, Netherlands: Elsevier Science & Technology, 2014. Chap. 1, pp. 1–36. DOI: [10.1016/B978-0-444-59526-3.00001-X](https://doi.org/10.1016/B978-0-444-59526-3.00001-X).
- [119] I. R. Hooper and W. L. Barnes. ‘The Basics of Plasmonics’. In: *Modern Plasmonics*. Ed. by A. A. Maradudin, J. R. Sambles and W. L. Barnes. 1st ed. Vol. 4. Amsterdam, Netherlands: Elsevier Science & Technology, 2014. Chap. 2, pp. 37–74. DOI: [10.1016/B978-0-444-59526-3.00001-X](https://doi.org/10.1016/B978-0-444-59526-3.00001-X).
- [120] V. A. Volkov and S. A. Mikhailov. ‘Edge magnetoplasmons: low frequency weakly damped excitations in inhomogeneous two-dimensional electron systems’. In: *Zh. Eksp. Teor. Fiz.* 94 (1988), pp. 217–241.
- [121] N. Kumada, H. Kamata and T. Fujisawa. ‘Edge magnetoplasmon transport in gated and ungated quantum Hall systems’. In: *Phys. Rev. B* 84 (4 2011), p. 045314. DOI: [10.1103/PhysRevB.84.045314](https://doi.org/10.1103/PhysRevB.84.045314).
- [122] H. Kamata, T. Ota, K. Muraki and T. Fujisawa. ‘Voltage-controlled group velocity of edge magnetoplasmon in the quantum Hall regime’. In: *Phys. Rev. B* 81 (8 2010), p. 085329. DOI: [10.1103/PhysRevB.81.085329](https://doi.org/10.1103/PhysRevB.81.085329).
- [123] E. Y. Andrei, D. C. Glatli, F. I. B. Williams and M. Heiblum. ‘Low Frequency Collective Excitations in the Quantum-Hall System’. In: *Surface Science* 196 (1 1988), pp. 501–506. DOI: [10.1016/0039-6028\(88\)90732-7](https://doi.org/10.1016/0039-6028(88)90732-7).
- [124] N. Q. Balaban, U. Meirav, H. Shtrikman and V. Umansky. ‘Observation of the logarithmic dispersion of high-frequency edge excitations’. In: *Phys. Rev. B* 55 (20 1997), R13397–R13400. DOI: [10.1103/PhysRevB.55.R13397](https://doi.org/10.1103/PhysRevB.55.R13397).
- [125] N. Kumada, P. Roulleau, B. Roche, M. Hashisaka, H. Hibino, I. Petković and D. C. Glatli. ‘Resonant Edge Magnetoplasmons and Their Decay in Graphene’. In: *Phys. Rev. Lett.* 113 (26 2014), p. 266601. DOI: [10.1103/PhysRevLett.113.266601](https://doi.org/10.1103/PhysRevLett.113.266601).
- [126] C. Dahl, S. Manus, J. P. Kotthaus, H. Nickel and W. Schlapp. ‘Edge magnetoplasmons in single two-dimensional electron disks at microwave frequencies: Determination of the lateral depletion length’. In: *Applied Physics Letters* 66 (17 1995), p. 2271. DOI: [10.1063/1.113189](https://doi.org/10.1063/1.113189).

- [127] S. A. Mikhailov and V. A. Volkov. 'Inter-edge magnetoplasmons in inhomogeneous two-dimensional electron systems'. In: *Journal of Physics: Condensed Matter* 4.31 (1992), pp. 6523–6538. DOI: [10.1088/0953-8984/4/31/005](https://doi.org/10.1088/0953-8984/4/31/005).
- [128] G. Ernst, R. J. Haug, J. Kuhl, K. von Klitzing and K. Eberl. 'Acoustic Edge Modes of the Degenerate Two-Dimensional Electron Gas Studied by Time-Resolved Magnetotransport Measurements'. In: *Phys. Rev. Lett.* 77 (20 1996), pp. 4245–4248. DOI: [10.1103/PhysRevLett.77.4245](https://doi.org/10.1103/PhysRevLett.77.4245).
- [129] I. V. Andreev, V. M. Muravev, D. V. Smetnev and I. V. Kukushkin. 'Acoustic magnetoplasmons in a two-dimensional electron system with a smooth edge'. In: *Phys. Rev. B* 86 (12 2012), p. 125315. DOI: [10.1103/PhysRevB.86.125315](https://doi.org/10.1103/PhysRevB.86.125315).
- [130] I. V. Andreev, V. M. Muravev and I. V. Kukushkin. 'Observation of acoustic edge magnetoplasmons near the filling factor $\nu = 1$ '. In: *JETP Letters* 96.8 (2012), pp. 536–540. DOI: [10.1134/S0021364012200040](https://doi.org/10.1134/S0021364012200040).
- [131] V. Shikin and S. Nazin. 'Acoustic edge magnetoplasmons and the quantum Hall effect'. In: *Low Temperature Physics* 43.1 (2017), pp. 121–124. DOI: [10.1063/1.4975106](https://doi.org/10.1063/1.4975106).
- [132] G. Ernst, N. B. Zhitenev, R. J. Haug and K. von Klitzing. 'Dynamic Excitations of Fractional Quantum Hall Edge Channels'. In: *Phys. Rev. Lett.* 79 (19 1997), pp. 3748–3751. DOI: [10.1103/PhysRevLett.79.3748](https://doi.org/10.1103/PhysRevLett.79.3748).
- [133] I. Petković, F. I. B. Williams, K. Bennaceur, F. Portier, P. Roche and D. C. Glatli. 'Carrier Drift Velocity and Edge Magnetoplasmons in Graphene'. In: *Phys. Rev. Lett.* 110 (1 2013), p. 016801. DOI: [10.1103/PhysRevLett.110.016801](https://doi.org/10.1103/PhysRevLett.110.016801).
- [134] N. B. Zhitenev, R. J. Haug, K. von Klitzing and K. Eberl. 'Linear and nonlinear waves in edge channels'. In: *Phys. Rev. B* 52 (15 1995), pp. 11277–11283. DOI: [10.1103/PhysRevB.52.11277](https://doi.org/10.1103/PhysRevB.52.11277).
- [135] V. I. Talyanskii, M. Y. Simmons, J. E. F. Frost, M. Pepper, D. A. Ritchie, A. C. Churchill and G. A. C. Jones. 'Experimental investigation of the damping of low-frequency edge magnetoplasmons in GaAs-Al_xGa_{1-x}As heterostructures'. In: *Phys. Rev. B* 50 (3 1994), pp. 1582–1587. DOI: [10.1103/PhysRevB.50.1582](https://doi.org/10.1103/PhysRevB.50.1582).
- [136] A. Endo, K. Koike, S. Katsumoto and Y. Iye. 'Frequencies of the Edge-Magnetoplasmon Excitations in Gated Quantum Hall Edges'. In: *J. Phys. Soc. Jpn.* 87 (6 2018), p. 064709. DOI: [10.7566/JPSJ.87.064709](https://doi.org/10.7566/JPSJ.87.064709).
- [137] M. N. Khannanov, A. A. Fortunatov and I. V. Kukushkin. 'Manifestations of incompressible strips in the edge magnetoplasmon spectrum in the quantum Hall effect regime'. In: *Jetp Lett.* 93 (11 2011), pp. 736–739. DOI: [10.1134/S0021364011110051](https://doi.org/10.1134/S0021364011110051).
- [138] S. A. Mikhailov and V. A. Volkov. 'Theory of electromagnetic response and collective excitations in antidots'. In: *Phys. Rev. B* 52 (24 1995), pp. 17260–17268. DOI: [10.1103/PhysRevB.52.17260](https://doi.org/10.1103/PhysRevB.52.17260).

- [139] I. Grodnensky, D. Heitmann and K. von Klitzing. ‘Nonlocal dispersion of edge magnetoplasma excitations in a two-dimensional electron system’. In: *Phys. Rev. Lett.* 67 (8 1991), pp. 1019–1022. DOI: [10.1103/PhysRevLett.67.1019](https://doi.org/10.1103/PhysRevLett.67.1019).
- [140] M. E. Suddards, A. Baumgartner, M. Henini and C. J. Mellor. ‘Scanning capacitance imaging of compressible and incompressible quantum Hall effect edge strips’. In: *New Journal of Physics* 14 (2012), p. 083015. DOI: [10.1088/1367-2630/14/8/083015](https://doi.org/10.1088/1367-2630/14/8/083015).
- [141] M. D. Johnson and G. Vignale. ‘Dynamics of dissipative quantum Hall edges’. In: *Phys. Rev. B* 67 (20 2003), p. 205332. DOI: [10.1103/PhysRevB.67.205332](https://doi.org/10.1103/PhysRevB.67.205332).
- [142] M. Hashisaka, H. Kamata, N. Kumada, K. Washio, R. Murata, K. Muraki and T. Fujisawa. ‘Distributed-element circuit model of edge magnetoplasmon transport’. In: *Phys. Rev. B* 88 (23 2013), p. 235409. DOI: [10.1103/PhysRevB.88.235409](https://doi.org/10.1103/PhysRevB.88.235409).
- [143] V. I. Talyanskii, A. V. Polisski, D. D. Arnone, M. Pepper, C. G. Smith, D. A. Ritchie, J. E. Frost and G. A. C. Jones. ‘Spectroscopy of a two-dimensional electron gas in the quantum-Hall-effect regime by use of low-frequency edge magnetoplasmons’. In: *Phys. Rev. B* 46 (19 1992), pp. 12427–12432. DOI: [10.1103/PhysRevB.46.12427](https://doi.org/10.1103/PhysRevB.46.12427).
- [144] V. Dziom, A. Shuvaev, A. V. Shchepetilnikov, D. MacFarland, G. Strasser and A. Pimenov. ‘High-frequency breakdown of the integer quantum Hall effect in GaAs/AlGaAs heterojunctions’. In: *Phys. Rev. B* 99 (4 2019), p. 045305. DOI: [10.1103/PhysRevB.99.045305](https://doi.org/10.1103/PhysRevB.99.045305).
- [145] B. D. Josephson. ‘Possible new effects in superconductive tunnelling’. In: *Physics Letters* 1.7 (1962), pp. 251–253. DOI: [10.1016/0031-9163\(62\)91369-0](https://doi.org/10.1016/0031-9163(62)91369-0).
- [146] F. London and H. London. ‘The electromagnetic equations of the supraconductor’. In: *Proc. Roy. Soc. London A* 149 (1935). Text was harvested from a scanned image of the original document using optical character recognition software and published online in 1997, pp. 71–88. DOI: [10.1098/rspa.1935.0048](https://doi.org/10.1098/rspa.1935.0048).
- [147] A. B. Pippard and W. L. Bragg. ‘An experimental and theoretical study of the relation between magnetic field and current in a superconductor’. In: *Proc. Roy. Soc. London A* 216 (1127 1953). Text was harvested from a scanned image of the original document using optical character recognition software and published online in 1997, pp. 547–568. DOI: [10.1098/rspa.1953.0040](https://doi.org/10.1098/rspa.1953.0040).
- [148] L. D. Landau and V. L. Ginzburg. ‘On the theory of superconductivity’. In: *Zh. Eksp. Teor. Fiz.* 20 (1950), p. 1064.
- [149] J. Bardeen, L. N. Cooper and J. R. Schrieffer. ‘Theory of Superconductivity’. In: *Phys. Rev.* 108 (5 1957), pp. 1175–1204. DOI: [10.1103/PhysRev.108.1175](https://doi.org/10.1103/PhysRev.108.1175).
- [150] H. Kamerlingh Onnes. *Further Experiments with liquid helium*. Leiden Communications. **120b**, **1220b**, **124c**. Koninklijke Akademie van Wetenschappen te Amsterdam, 1911.

- [151] W. Meissner and R. Ochsenfeld. ‘Ein neuer Effekt bei Eintritt der Supraleitfähigkeit’. In: *Naturwissenschaften* 21.44 (1933), pp. 787–788. DOI: [10.1007/BF01504252](https://doi.org/10.1007/BF01504252).
- [152] A. M. Forrest. ‘Meissner and Ochsenfeld revisited’. In: *Eur. J. Phys.* 4.117 (1983). Translation and commentary of ‘Ein neuer Effekt bei Eintritt der Supraleitfähigkeit’, in *Naturwissenschaften* 21.44 (1933), pp. 787–788, pp. 117–120. DOI: [10.1088/0143-0807/4/2/011](https://doi.org/10.1088/0143-0807/4/2/011).
- [153] H. Barrell. ‘Thirteenth General Conference of Weights and Measures’. In: *Physics Bulletin* 20.2 (1969), p. 61. DOI: [10.1088/0031-9112/20/2/006](https://doi.org/10.1088/0031-9112/20/2/006).
- [154] Y. B. Kim, C. F. Hempstead and A. R. Strnad. ‘Flux-Flow Resistance in Type-II Superconductors’. In: *Phys. Rev.* 139 (4A 1965), A1163–A1172. DOI: [10.1103/PhysRev.139.A1163](https://doi.org/10.1103/PhysRev.139.A1163).
- [155] K. K. Likharev. *Dynamics of Josephson Junctions and Circuits*. 1st ed. CRC Press, 1986.
- [156] M. Tinkham. *Introduction to Superconductivity*. 2nd ed. Mineola, New York: Dover Publications, Inc., 2004.
- [157] A. A. Golubov, M. Y. Kupriyanov and E. Il’ichev. ‘The current-phase relation in Josephson junctions’. In: *Rev. Mod. Phys.* 76 (2 2004), pp. 411–469. DOI: [10.1103/RevModPhys.76.411](https://doi.org/10.1103/RevModPhys.76.411).
- [158] S. Shapiro. ‘Josephson Currents in Superconducting Tunneling: The Effect of Microwaves and Other Observations’. In: *Phys. Rev. Lett.* 11 (2 1963), pp. 80–82. DOI: [10.1103/PhysRevLett.11.80](https://doi.org/10.1103/PhysRevLett.11.80).
- [159] S. Shapiro, A. R. Janus and S. Holly. ‘Effect of Microwaves on Josephson Currents in Superconducting Tunneling’. In: *Rev. Mod. Phys.* 36 (1 1964), pp. 223–225. DOI: [10.1103/RevModPhys.36.223](https://doi.org/10.1103/RevModPhys.36.223).
- [160] R. Behr, O. Kieler, J. Kohlmann, F. Muller and L. Palafox. ‘Development and metrological applications of Josephson arrays at PTB’. In: *Measurement Science and Technology* 23.12 (2012), p. 124002. DOI: [10.1088/0957-0233/23/12/124002](https://doi.org/10.1088/0957-0233/23/12/124002).
- [161] F. Tafuri. ‘Introductory Notes on the Josephson Effect: Main concepts and Phenomenology’. In: *Fundamentals and Frontiers of the Josephson Effect*. Ed. by F. Tafuri. Springer Series in Materials Science 286. Switzerland: Springer, 2019. Chap. 1, pp. 1–61. DOI: [10.1007/978-3-030-20726-7_15](https://doi.org/10.1007/978-3-030-20726-7_15).
- [162] L. P. Rokhinson, X. Liu and J. K. Furdyna. ‘The fractional a.c. Josephson effect in a semiconductor-superconductor nanowire as a signature of Majorana particles’. In: *Nature Physics* 8.11 (2012), pp. 795–799. DOI: [10.1038/nphys2429](https://doi.org/10.1038/nphys2429).
- [163] W. Stewart. ‘Current-voltage characteristics of Josephson junctions’. In: *Appl. Phys. Lett.* 12.8 (1968), pp. 277–280. DOI: [10.1063/1.1651991](https://doi.org/10.1063/1.1651991).

- [164] D. McCumber. 'Effect of an ac Impedance on dc Voltag-Current Characteristics of Superconductor Weak-Link Junctions'. In: *Journal of Applied Physics* 39.7 (1968), pp. 3113–3118. DOI: [10.1063/1.1656743](https://doi.org/10.1063/1.1656743).
- [165] J. A. Blackburn, M. Cirillo and N. Grønbech-Jensen. 'A survey of classical and quantum interpretations of experiments on Josephson junctions at very low temperatures'. In: *Physics Reports* 611 (2016), pp. 1–33. DOI: [10.1016/j.physrep.2015.10.010](https://doi.org/10.1016/j.physrep.2015.10.010).
- [166] C. Baumgartner, L. Fuchs, L. Frész, S. Reinhardt, S. Gronin, G. C. Gardner, M. J. Manfra, N. Paradiso and C. Strunk. 'Josephson Inductance as a Probe for Highly Ballistic Semiconductor-Superconductor Weak Links'. In: *Phys. Rev. Lett.* 126 (3 2021), p. 037001. DOI: [10.1103/PhysRevLett.126.037001](https://doi.org/10.1103/PhysRevLett.126.037001).
- [167] R. Haller, G. Fülöp, D. Indolese, J. Ridderbos, R. Kraft, L. Y. Cheung, J. H. Ungerer, K. Watanabe, T. Taniguchi, D. Beckmann, R. Danneau, P. Virtanen and C. Schönenberger. 'Phase-dependent microwave response of a graphene Josephson junction'. In: *Phys. Rev. Res.* 4 (1 2022), p. 013198. DOI: [10.1103/PhysRevResearch.4.013198](https://doi.org/10.1103/PhysRevResearch.4.013198).
- [168] S. Hart, Z. Cui, G. Ménard, M. Deng, A. E. Antipov, R. M. Lutchyn, P. Krogstrup, C. M. Marcus and K. A. Moler. 'Current-phase relations of InAs nanowire Josephson junctions: From interacting to multimode regimes'. In: *Phys. Rev. B* 100 (6 2019), p. 064523. DOI: [10.1103/PhysRevB.100.064523](https://doi.org/10.1103/PhysRevB.100.064523).
- [169] A. Murani, A. Kasumov, S. Sengupta, Y. A. Kasumov, V. T. Volkov, I. I. Khodos, F. Brisset, R. Delagrangé, A. Chepelianskii, R. Deblock, H. Bouchiat and S. Guéron. 'Ballistic edge states in Bismuth nanowires revealed by SQUID interferometry'. In: *Nature Communications* 8.1 (2017), p. 15941. DOI: [10.1038/ncomms15941](https://doi.org/10.1038/ncomms15941).
- [170] A. Murani, B. Dassonneville, A. Kasumov, J. Basset, M. Ferrier, R. Deblock, S. Guéron and H. Bouchiat. 'Microwave Signature of Topological Andreev level Crossings in a Bismuth-based Josephson Junction'. In: *Phys. Rev. Lett.* 122 (7 2019), p. 076802. DOI: [10.1103/PhysRevLett.122.076802](https://doi.org/10.1103/PhysRevLett.122.076802).
- [171] M. Mergenthaler, F. J. Schupp, A. Nersisyan, N. Ares, A. Baumgartner, C. Schönenberger, G. A. D. Briggs, P. J. Leek and E. A. Laird. 'Radio-frequency characterization of a supercurrent transistor made of a carbon nanotube'. In: *Materials for Quantum Technology* 1.3 (2021), p. 035003. DOI: [10.1088/2633-4356/ac1d57](https://doi.org/10.1088/2633-4356/ac1d57).
- [172] V. Chidambaram, A. Kringhøj, L. Casparis, F. Kuemmeth, T. Wang, C. Thomas, S. Gronin, G. C. Gardner, Z. Cui, C. Liu, K. Moors, M. J. Manfra, K. D. Petersson and M. R. Connolly. 'Microwave sensing of Andreev bound states in a gate-defined superconducting quantum point contact'. In: *Phys. Rev. Res.* 4 (2 2022), p. 023170. DOI: [10.1103/PhysRevResearch.4.023170](https://doi.org/10.1103/PhysRevResearch.4.023170).

- [173] A. Savitzky and M. J. E. Golay. ‘Smoothing and Differentiation of Data by Simplified Least Squares Procedures’. In: *Analytical Chemistry* 36.8 (1964), pp. 1627–1639. DOI: [10.1021/ac60214a047](https://doi.org/10.1021/ac60214a047).
- [174] H. A. Nilsson, P. Samuelsson, P. Caroff and H. Q. Xu. ‘Supercurrent and Multiple Andreev Reflections in an InSb Nanowire Josephson Junction’. In: *Nano Letters* 12.1 (2012). PMID: 22142358, pp. 228–233. DOI: [10.1021/nl203380w](https://doi.org/10.1021/nl203380w).
- [175] P. San-Jose, J. Cayao, E. Prada and R. Aguado. ‘Multiple Andreev reflection and critical current in topological superconducting nanowire junctions’. In: *New Journal of Physics* 15.7 (2013), p. 075019. DOI: [10.1088/1367-2630/15/7/075019](https://doi.org/10.1088/1367-2630/15/7/075019).
- [176] K. Shtengel. ‘A Home for Anyon?’ In: *Nature Physics* 3.11 (2007), p. 763. DOI: [10.1038/nphys767](https://doi.org/10.1038/nphys767).
- [177] S. Russo, M. Kroug, T. M. Klapwijk and A. F. Morpurgo. ‘Experimental Observation of Bias-Dependent Nonlocal Andreev Reflection’. In: *Phys. Rev. Lett.* 95 (2 2005), p. 027002. DOI: [10.1103/PhysRevLett.95.027002](https://doi.org/10.1103/PhysRevLett.95.027002).
- [178] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum and H. Shtrikman. ‘Zero-bias peaks and splitting in an Al–InAs nanowire topological superconductor as a signature of Majorana fermions’. In: *Nature Phys.* 8 (2012), pp. 887–895. DOI: [10.1038/nphys2479](https://doi.org/10.1038/nphys2479).
- [179] C.-K. Chiu, J. D. Sau and S. Das Sarma. ‘Conductance of a superconducting Coulomb-blockaded Majorana nanowire’. In: *Phys. Rev. B* 96 (5 2017), p. 054504. DOI: [10.1103/PhysRevB.96.054504](https://doi.org/10.1103/PhysRevB.96.054504).
- [180] D. Bagrets and A. Altland. ‘Class D Spectral Peak in Majorana Quantum Wires’. In: *Phys. Rev. Lett.* 109 (22 2012), p. 227005. DOI: [10.1103/PhysRevLett.109.227005](https://doi.org/10.1103/PhysRevLett.109.227005).
- [181] G. Kells, D. Meidan and P. W. Brouwer. ‘Near-zero-energy end states in topologically trivial spin-orbit coupled superconducting nanowires with a smooth confinement’. In: *Phys. Rev. B* 86 (10 2012), p. 100503. DOI: [10.1103/PhysRevB.86.100503](https://doi.org/10.1103/PhysRevB.86.100503).
- [182] J. Liu, A. C. Potter, K. T. Law and P. A. Lee. ‘Zero-Bias Peaks in the Tunneling Conductance of Spin-Orbit-Coupled Superconducting Wires with and without Majorana End-States’. In: *Phys. Rev. Lett.* 109 (26 2012), p. 267002. DOI: [10.1103/PhysRevLett.109.267002](https://doi.org/10.1103/PhysRevLett.109.267002).
- [183] C.-X. Liu, J. D. Sau, T. D. Stanescu and S. Das Sarma. ‘Andreev bound states versus Majorana bound states in quantum dot-nanowire-superconductor hybrid structures: Trivial versus topological zero-bias conductance peaks’. In: *Phys. Rev. B* 96 (7 2017), p. 075161. DOI: [10.1103/PhysRevB.96.075161](https://doi.org/10.1103/PhysRevB.96.075161).
- [184] P. Yu, J. Chen, M. Gomanko, G. Badawy, E. P. A. M. Bakkers, K. Zuo, V. Mourik and S. M. Frolov. ‘Non-Majorana states yield nearly quantized conductance in superconductor-semiconductor nanowire devices’. In: *Nature Physics* (2021). DOI: [10.1038/s41567-020-01107-w](https://doi.org/10.1038/s41567-020-01107-w).

- [185] B. M. Fregoso, A. M. Lobos and S. Das Sarma. ‘Electrical detection of topological quantum phase transitions in disordered Majorana nanowires’. In: *Phys. Rev. B* 88 (18 2013), p. 180507. DOI: [10.1103/PhysRevB.88.180507](https://doi.org/10.1103/PhysRevB.88.180507).
- [186] T. Ö. Rosdahl, A. Vuik, M. Kjaergaard and A. R. Akhmerov. ‘Andreev rectifier: A nonlocal conductance signature of topological phase transitions’. In: *Phys. Rev. B* 97 (4 2018), p. 045421. DOI: [10.1103/PhysRevB.97.045421](https://doi.org/10.1103/PhysRevB.97.045421).
- [187] J. Danon, A. B. Hellenes, E. B. Hansen, L. Casparis, A. P. Higginbotham and K. Flensberg. ‘Nonlocal Conductance Spectroscopy of Andreev Bound States: Symmetry Relations and BCS Charges’. In: *Phys. Rev. Lett.* 124 (3 2020), p. 036801. DOI: [10.1103/PhysRevLett.124.036801](https://doi.org/10.1103/PhysRevLett.124.036801).
- [188] G. L. R. Anselmetti, E. A. Martinez, G. C. Ménard, D. Puglia, F. K. Malinowski, J. S. Lee, S. Choi, M. Pendharkar, C. J. Palmstrøm, C. M. Marcus, L. Casparis and A. P. Higginbotham. ‘End-to-end correlated subgap states in hybrid nanowires’. In: *Phys. Rev. B* 100 (20 2019), p. 205412. DOI: [10.1103/PhysRevB.100.205412](https://doi.org/10.1103/PhysRevB.100.205412).
- [189] G. C. Ménard, G. L. R. Anselmetti, E. A. Martinez, D. Puglia, F. K. Malinowski, J. S. Lee, S. Choi, M. Pendharkar, C. J. Palmstrøm, K. Flensberg, C. M. Marcus, L. Casparis and A. P. Higginbotham. ‘Conductance-Matrix Symmetries of a Three-Terminal Hybrid Device’. In: *Phys. Rev. Lett.* 124 (3 2020), p. 036802. DOI: [10.1103/PhysRevLett.124.036802](https://doi.org/10.1103/PhysRevLett.124.036802).
- [190] H. Pan, J. D. Sau and S. Das Sarma. ‘Three-terminal nonlocal conductance in Majorana nanowires: Distinguishing topological and trivial in realistic systems with disorder and inhomogeneous potential’. In: *Phys. Rev. B* 103 (1 2021), p. 014513. DOI: [10.1103/PhysRevB.103.014513](https://doi.org/10.1103/PhysRevB.103.014513).
- [191] A. Ranni, E. T. Mannila, A. Eriksson, D. S. Golubev, J. P. Pekola and V. F. Maisi. ‘Local and Nonlocal Two-Electron Tunneling Processes in a Cooper Pair Splitter’. In: *Phys. Rev. Lett.* 129 (20 2022), p. 207703. DOI: [10.1103/PhysRevLett.129.207703](https://doi.org/10.1103/PhysRevLett.129.207703).
- [192] A. Laucht, F. Hohls, N. Ubbelohde, M. F. Gonzalez-Zalba, D. J. Reilly, S. Stobbe, T. Schröder, P. Scarlino, J. V. Koski, A. Dzurak, C.-H. Yang, J. Yoneda, F. Kuemmeth, H. Bluhm, J. Pla, C. Hill, J. Salfi, A. Oiwa, J. T. Muhonen, E. Verhagen, M. D. LaHaye, H. H. Kim, A. W. Tsen, D. Culcer, A. G. J. A. Mol, V. Mohan, P. K. Jain and J. Baugh. ‘Roadmap on quantum nanotechnologies’. In: *Nanotechnology* 32 (2021), p. 162003. DOI: [10.1109/JMW.2020.3034071](https://doi.org/10.1109/JMW.2020.3034071).
- [193] J. C. Bardin, D. H. Slichter and D. J. Reilly. ‘Microwaves in Quantum Computing’. In: *IEEE Journal of Microwaves* 1.1 (2021), pp. 403–427. DOI: [10.1109/JMW.2020.3034071](https://doi.org/10.1109/JMW.2020.3034071).

- [194] G. Hammer, S. Wuensch, M. Roesch, K. Ilin, E. Crocoll and M. Siegel. ‘Superconducting coplanar waveguide resonators for detector applications’. In: *Supercond. Sci. Technol.* 20 (11 2007), p. 408. DOI: [10.1088/0953-2048/20/11/s21](https://doi.org/10.1088/0953-2048/20/11/s21).
- [195] D. Puglia, E. A. Martinez, G. C. Ménard, A. Pöschl, S. Gronin, G. C. Gardner, R. Kallaher, M. J. Manfra, C. M. Marcus, A. P. Higginbotham and L. Casparis. ‘Closing of the induced gap in a hybrid superconductor-semiconductor nanowire’. In: *Phys. Rev. B* 103 (23 2021), p. 235201. DOI: [10.1103/PhysRevB.103.235201](https://doi.org/10.1103/PhysRevB.103.235201).
- [196] A. Melo, C.-X. Liu, P. Rožek, T. Ö. Rosdahl and M. Wimmer. ‘Conductance asymmetries in mesoscopic superconducting devices due to finite bias’. In: *SciPost Phys.* 10 (2 2021), p. 37. DOI: [10.21468/SciPostPhys.10.2.037](https://doi.org/10.21468/SciPostPhys.10.2.037).
- [197] Y.-H. Lai, J. D. Sau and S. Das Sarma. ‘Presence versus absence of end-to-end nonlocal conductance correlations in Majorana nanowires: Majorana bound states versus Andreev bound states’. In: *Phys. Rev. B* 100 (4 2019), p. 045302. DOI: [10.1103/PhysRevB.100.045302](https://doi.org/10.1103/PhysRevB.100.045302).
- [198] F. Vigneau, F. Fedele, A. Chatterjee, D. J. Reilly, F. Kuemmeth, M. F. Gonzalez-Zalba, E. Laird and N. Ares. ‘Probing quantum devices with radio-frequency reflectometry’. In: *Applied Physics Reviews* 10 (2 2023), p. 021305. DOI: [10.1063/5.0088229](https://doi.org/10.1063/5.0088229).
- [199] W. G. van der Wiel, S. De Franceschi, J. M. Elzerman, T. Fujisawa, S. Tarucha and L. P. Kouwenhoven. ‘Electron transport through double quantum dots’. In: *Rev. Mod. Phys.* 75 (1 2002), pp. 1–22. DOI: [10.1103/RevModPhys.75.1](https://doi.org/10.1103/RevModPhys.75.1).
- [200] T. D. Stanescu, S. Tewari, J. D. Sau and S. Das Sarma. ‘To Close or Not to Close: The Fate of the Superconducting Gap Across the Topological Quantum Phase Transition in Majorana-Carrying Semiconductor Nanowires’. In: *Phys. Rev. Lett.* 109 (26 2012), p. 266402. DOI: [10.1103/PhysRevLett.109.266402](https://doi.org/10.1103/PhysRevLett.109.266402).
- [201] A. Grivnin, E. Bor, M. Heiblum, Y. Oreg and H. Shtrikman. ‘Concomitant opening of a bulk-gap with an emerging possible Majorana zero mode’. In: *Nature Communications* 10.1 (2019), p. 1940. DOI: [10.1038/s41467-019-09771-0](https://doi.org/10.1038/s41467-019-09771-0).
- [202] B. N. Taylor, W. H. Parker, D. N. Langenberg and A. Denenstein. ‘On the Use of the AC Josephson Effect to Maintain Standards of Electromotive Force’. In: *Metrologia* 3.4 (1967), pp. 89–98. DOI: [10.1088/0026-1394/3/4/001](https://doi.org/10.1088/0026-1394/3/4/001).
- [203] J. Kohlmann and R. Behr. ‘Development of Josephson voltage standards’. In: *Superconductivity - Theory and Applications*. Ed. by A. Luiz. 2011. Chap. 11, pp. 239–260. DOI: [10.5772/17031](https://doi.org/10.5772/17031).
- [204] S. P. Benz. ‘Josephson Junctions for Metrology Applications’. In: *Fundamentals and Frontiers of the Josephson Effect*. Ed. by F. Tafuri. Springer Series in Materials Science 286. Switzerland: Springer, 2019. Chap. 15, pp. 587–609. DOI: [10.1007/978-3-030-20726-7_15](https://doi.org/10.1007/978-3-030-20726-7_15).

- [205] J. D. S. Witt, B. Harlech-Jones, A. Jouan, S. J. Pauka, G. C. Gardner, S. Gronin, M. J. Manfra and D. J. Reilly. 'Controlled Evolution of Geometric Phase in InAs 2DEG'. Unpublished (In preparation).
- [206] R. Worland. 'Normal modes of a musical drumhead under non-uniform tension'. In: *J. Acoust. Soc. Am.* 127 (2010), pp. 525–533. DOI: [10.1121/1.3268605](https://doi.org/10.1121/1.3268605).