

Experiments on nonlinear extreme waves in complex configurations

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Declaration

This is to certify that, to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes. All the assistance received in preparing this thesis and sources has been acknowledged.

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Abstract

Nonlinear rogue or freak waves are rare but utmost events in the ocean with at least twice the neighbouring wave heights. Understanding the properties of these events is crucial for ocean industry safety and expanding knowledge of the nonlinear hydrodynamic wave process. In addition to the dispersive focusing, i.e. the superposition of linear or nonlinear waves, it has recently been shown that both nonlinear focusing mechanisms play an important role in the description of rogue wave formation. In fact, the modulation instability (MI) is known to be active for the dimensionless depth value $kh > 1.363$, whereas the nonlinear wave shoaling takes place below this threshold value, and waves still being dispersive.

The universal modulation instability (MI) is also responsible for the formation of strong wave localizations in other nonlinear media, such as optical Kerr media, plasma, and Bose-Einstein condensate. These wave-focusing dynamics can be described by breather solutions of the nonlinear Schrödinger equation (NLSE), such as the time-periodic Akhmediev breather (AB), space-periodic Kuznetsov-Ma breather (KB), and the space and time doubly-localized Peregrine breather (PB), which can seemingly "appear from nowhere and disappear without a trace".

While the classical exact solutions of the NLSE have been extensively studied theoretically and experimentally, the integrability of the NLSE framework normally restricts them to the one-dimensional level and exact envelope shapes. Thus, the NLSE solutions differ from ordinary sea states, where wave shapes and directions are more arbitrary. In the current thesis, the role of wave nonlinearity in extreme wave formation beyond the analytical NLSE framework, i.e. under more complex wave configurations, was mainly investigated in two aspects: the effect of phase-shift and one particular case of wave field multi-directionality.

The variation of wave amplitude plays a significant role in the formation of rogue waves, as it affects the wave energy level and steepness. However, the phase evolution of the wave packet can also be critical, although it is often overlooked due to its implicit nature. In the

first part of the investigation into unidirectional wave properties, the importance of phase dynamics on rogue wave occurrence was emphasized. This can be demonstrated through a change in the NLSE framework, an overall phase-shift, or a localized phase manipulation.

Two different NLSE analytical frameworks can be used, depending on whether the research focuses on the temporal or spatial evolution of waves. These include the time- and space-NLSE frameworks. While different analytical frameworks show similar patterns in rogue wave amplitude, they exhibit differences in the phase evolution of the rogue wave packet. In Chapter 4, fundamental differences beyond the shape of the wave envelope that arise in the phase evolution are addressed. The Peregrine breather (PB) was used as a reference wave envelope model to create and track an unstable wave group in a wave flume using two boundary conditions, as defined by the time- and space-NLSE frameworks. It was found that, regardless of which boundary condition was used, the corresponding local shape of wave localization was very similar during the evolution, but the local phase evolution differed. The phase dynamics followed the predictions from the respective NLSE framework adopted in each case.

While the phase-shift studied above is strictly subjected to exact solutions, an arbitrary global phase-shift transformation can be achieved through the so-called Galilean transformation (GT), a universal operation connecting the coordinates of a dynamical system, which moves relative to each other at a constant speed. In the context of exact solutions of the universal NLSE framework, inducing a Galilean velocity (GV) to the pulse involves a frequency shift to satisfy the symmetry of the wave equation. As such, the Galilean transformation has been deemed to be not applicable to wave groups in nonlinear dispersive media. In Chapter 5, the Galilean-transformed solitons and breathers in a wave tank show clear variations from their respective dynamics on the water surface. The type of deviations depends on the sign of the GV and can be captured by the modified NLSE or the Euler equations. Moreover, it is shown that positive Galilean-translated envelope soliton pulses exhibit self-modulation. While designated GV and wave steepness values expedite multi-soliton dynamics, the strong focusing of such higher-order coherent waves inevitably leads to the generation of supercontinua due to soliton fission.

The phase can be shifted not only globally but also locally. In Chapter 6, it is shown that phase-shift localization applied to the background, excluding any amplitude modulation excitation, can indeed trigger extreme events. Such rogue waves can be, for instance, generated by considering the parametrization of fundamental breathers and thus by seeding only the local phase-shift information to the regular carrier wave. The wave tank experiments show an excellent agreement with the expected NLSE hydrodynamics and confirm that even though delayed in their evolution, breather-type extreme waves can be generated from a wave train with constant amplitude.

As mentioned above, MI is key in forming extreme waves in the unidirectional wave field. In contrast, on the multi-directional wave level, it has been suggested that the MI relevance weakens when the wave field becomes broadband or directional. In the second part of the investigation, the rogue wave properties within a particular case of multi-directional wave fields were investigated. Chapter 7 provides deterministic experimental evidence of nonlinear and distinct PB-type focusing in standing water waves describing the scenario of two counter-propagating wave trains. The collected collinear wave measurements are in excellent agreement with the hydrodynamic coupled NLSE (CNLSE) and suggest that MI can undisturbedly prevail during the interplay of several wave systems, emphasizing the potential role of exact NLSE solutions in extreme wave formation beyond the formal narrow-band, unidirectional limits.

To understand the extreme wave occurrence in more complex conditions under wave reflections, in Chapter 8, the extreme wave probabilities in random JONSWAP wave fields are studied under various spectrum bandwidths and water depths, combined with different levels of wave reflections as a consequence of different types of beach materials. It is shown that the occurrence of rogue waves inclines towards the nonlinear prediction even under complex wave conditions. Thus, wave nonlinearity is still relevant when the wave reflections are at play. In both deep water and finite depth regimes, a larger wave reflection leads to a general decrease in extreme wave occurrence. It was also observed in this study that, at an intermediate angle, the extreme wave occurrence increases near a more permeable beach compared to a less permeable one, which is due to the decrease of reflected wave energy. A very small or large angle can distort this effect either due to excessive wave breaking on a smooth beach surface

or inadequate beach-wave interaction regardless of the beach material. As the wave field becomes narrower-banded, the extreme occurrence increases even under reflections, which is in accordance with the author's research in Chapter 7. The experimental results are in good alignment with the numerical results based on the coupled weakly-nonlinear framework, suggesting the validity of the coupled NLSE framework for extreme wave prediction in a multi-directional wave system.

Authorship attribution statement

Chapter 4 of this thesis contains material published in reference [1]. Chapter 5 of this thesis contains material published in reference [2]. Chapter 6 of this thesis contains material published in reference [3]. Chapter 7 of this thesis contains material published in reference [4]. Chapter 8 of this thesis contains material that will be submitted in the future. The Abstract and Chapters 1, 2, and 3 of this thesis also contain material in references [1, 2, 3, 4].

As the first author, I primarily designed the studies above with my supervisors. I carried out experiments and simulations unless otherwise indicated, analysed the data, and wrote the drafts of the manuscripts together with the co-authors. For validation purposes regarding the experimental and weakly-nonlinear simulation results, high-order spectral method simulations were conducted by Guillaume Ducrozet and Alexey Slunyaev, as mentioned in the references [2] and [4], respectively.

In addition to the statements above, in cases where I am not the corresponding author of a published item, permission to include the published material has been granted by the corresponding author.

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16th August 2023

As the supervisor for the candidature upon which this thesis is based, I can confirm that the authorship attribution statements above are correct.

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16th August 2023

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Nomenclature

A	Amplitude perturbation function
A_p	Peregrine breather solution of the space-NLSE (2.29)
B	Constant in Equation (2.10)
B_p	Peregrine breather solution of the time-NLSE (2.34)
G	Wave elevation gauge
H	Wave height
H_s	Significant wave height
K	Perturbation wavenumber
N	Soliton order, $N \in \mathbb{N}$
$P(H)$	Exceedance probability of the wave height H
Q_p	Peakedness parameter [5] used to estimate the BFI
R	Real constant
R_0	Function related to the wave directionality [6]
$S(f)$	Spectrum power as a function of the frequency f
T	Wave period
a	Carrier wave amplitude
a_c	Characteristic amplitude
c	Galilean velocity (GV)
c_g	Group speed or speed of the wave envelope
c_p	Phase speed or speed of the wave crest
f	Carrier frequency
g	Gravitational acceleration
h	Water depth
i	Unit imaginary number
k	Carrier wavenumber

$k^{(1)}$	Incident wavenumber
$k^{(2)}$	Reflected wavenumber
m_0	Zero-th moment of the spectrum
r	Reflectivity
t	Dimensional time
u	Velocity field
x	Horizontal x-axis of the Cartesian coordinate
x_g	Elevation gauge location in experiments
y	Horizontal y-axis of the Cartesian coordinate
z	Vertical z-axis of the Cartesian coordinate
$\mathcal{H}$	Hilbert transform
$\Im$	Imaginary part
$\Re$	Real part
$\mathfrak{a}$	Parameter in Equation (2.40) determining the type of breather
$\mathfrak{b}$	Parameter in Equation (2.40) calculated from $\mathfrak{a}$
$\mathfrak{c}$	Parameter in Equation (2.40) calculated from $\mathfrak{a}$
Δ	Laplacian operator
Λ	Constant of the JONSWAP spectrum (2.52)
∇	Gradient operator
Ω	Perturbation frequency
Φ	Phase angle
α	Wave steepness defined as $\alpha = ak$
α_P	Phillips constant affecting the JONSWAP spectrum intensity
α_2	Constant for calculating BFI_{2D} [6]
α_c	Characteristic steepness
δ_ω	Normalized spectrum width
ϵ	Long-wave amplitude perturbation in the stability analysis [7]
η	Surface elevation
γ	Peak enhancement factor of the JONSWAP spectrum
κ	Kurtosis

λ	Wavelength
μ	Dispersion parameter in the space-NLSE (2.29)
μ_3	Third central momentum
μ_4	Fourth central momentum
ν	Nonlinear parameter in the space-NLSE (2.29)
ω	Carrier wave frequency
ω_p	Wave peak frequency
ϕ	Velocity potential
ψ	Wave envelope
$\psi^{(1)}$	Incident wave envelope
$\psi^{(2)}$	Reflected wave envelope
σ	Standard deviation
σ_h	Variable replacing $\tanh(kh)$
σ_ω	Spectrum width
θ	Beach inclination angle
2D	Two dimensional
AB	Akhmediev breather solution
BFI	Benjamin-Feir index
BFI _{2D}	2D Benjamin-Feir index in directional sea state
CNLSE	Coupled nonlinear Schrödinger equation
GT	Galilean transformation
GV	Galilean velocity
HOSM	High-order spectral method
JONSWAP	Joint North Sea Wave Project spectrum
KB	Kuznetsov-Ma breather solution
MI	Modulation instability
MNLSE	Modified nonlinear Schrödinger equation
NLSE	Nonlinear Schrödinger equation
PB	Peregrine breather solution
PSP	Phase-shift perturbation

List of Relevant Publications and Presentations

Journal Papers

- (1) He, Y., Witt, A., Trillo, S., Chabchoub, A., & Hoffmann, N. (2022). Extreme wave excitation from localized phase-shift perturbations. *Physical Review E*, 106 (4), L043101
- (2) He, Y., Slunyaev, A., Mori, N., & Chabchoub, A. (2022). Experimental evidence of nonlinear focusing in standing water waves. *Physical Review Letters*, 129(14), 144502
- (3) He, Y., Ducrozet, G., Hoffmann, N., Dudley, J. M., & Chabchoub, A. (2022). Galilean-transformed solitons and supercontinuum generation in dispersive media. *Physica D: Nonlinear Phenomena*, 133342
- (4) He, Y., Suret, P., & Chabchoub, A. (2021). Phase Evolution of the Time-and Space-Like Peregrine Breather in a Laboratory. *Fluids*, 6, 308
- (5) Eeltink, D., Branger, H., Luneau, C., He, Y., Chabchoub, A., Kasparian, J., van den Bremer, T. S., & Sapsis, T. P. (2022). Nonlinear wave evolution with data-driven breaking. *Nature communications*, 13(1), 1-11

Conference Paper

- (1) He, Y., Vila-Concejo, A., Babanin, A. V., & Chabchoub, A. (2020). Experiments on extreme waves in quasi-linear seas near reflective beaches. 22nd Australasian Fluid Mechanics Conference AFMC2020

Conference Presentation Abstracts

- (1) He, Y., Vila-Concejo, A., Babanin, A. V., Slunyaev, A., Mori, N., & Chabchoub, A. (2022). On the probability of higher-order nonlinear extreme waves in the presence of wave reflection. 37th International Conference on Coastal Engineering ICCE2022
- (2) He, Y., Ducrozet, G., Hoffmann, N., Dudley, J. M., & Chabchoub, A. (2022). A numerical and experimental study of Galilei-transformed nonlinear wave groups. EGU General Assembly 2022
- (3) He, Y., Vila-Concejo, A., Babanin, A. V., & Chabchoub, A. (2022). Extreme wave statistics near reflective beach installations. KOZWaves 2022
- (4) He, Y., Vila-Concejo, A., Babanin, A. V., & Chabchoub, A. (2021). Experiments on extreme wave events in the vicinity of reflective beaches. Dynamics Days Europe 2021
- (5) Chabchoub, A., He, Y., Vila-Concejo, A., & Babanin, A. V. (2021). Experiments and simulations on extreme waves near reflective beaches. EGU General Assembly 2021

CHAPTER 1

Introduction

Occupying approximately 70% of the Earth's surface, the ocean is a veritable treasure trove of natural resources and a vital artery of commerce. As a result, it has become a linchpin of sustainable development for humanity. However, the ocean is also a domain of great unpredictability and potential hazard due to its constantly shifting conditions, including inclement weather and the likelihood of resultant extreme wave events. Accordingly, it is of critical importance to enhance our understanding of the characteristics of extreme ocean waves.

While excluding any external effects on a wave system, extreme wave formations in unidirectional waveguides can be explained either as a result of unstable nonlinear wave interaction [8, 9, 10] or linear superposition principle [11, 12]. Both mechanisms have been intensively studied in the laboratory and the ocean environments [13, 14]. Examples of the linear and nonlinear wave focusing process are shown in Figure 1.1. On the left, waves at different wavelengths travel at various speeds and then overlap. The example on the right shows a nonlinear extreme wave emerging from a long-wave perturbation of a regular wavetrain and triggering modulation instability. Following the pioneering experimental studies on nonlinear water wave focusing [15, 16], there have recently been emerging research interests concentrating on the role of wave nonlinearity in extreme wave formation [17, 19, 20, 21, 22, 23, 24, 25, 13, 18]. One typical nonlinear wave framework is the so-called nonlinear Schrödinger equation (NLSE), which is a comprehensive framework describing the dynamics of wave pulses within the surface gravity water waves and other nonlinear media [26, 10], for instance, the optical Kerr media, plasma, and Bose-Einstein condensates [27, 17, 28]. The hydrodynamic NLSE we consider can be derived from the fully-nonlinear Euler equations [8]

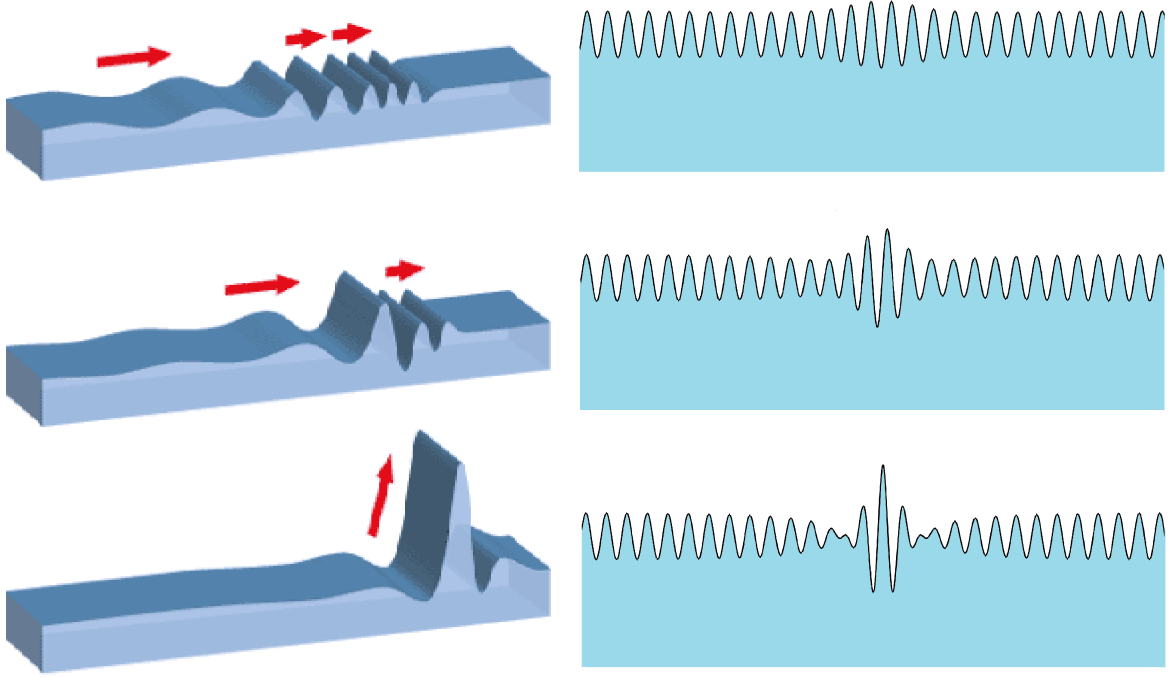


FIGURE 1.1. Example of dispersive wave focusing (left) and nonlinear wave focusing in deep-water (right) .

using the multiple scales approach while assuming that the flow is inviscid, incompressible, and irrotational [29, 30, 31]. Since it is integrable [32], several key wave envelope solutions have been derived and discussed within the context of modulation instability (MI) [33, 34, 35, 31, 36, 19, 13]. One way to create and control such rogue waves in a laboratory environment is using fundamental breather solutions in the form of time-periodic, doubly-localized, and space-periodic forms, also known as the Akhmediev breather (AB) [37], Peregrine breather (PB) [38], and Kuznetsov-Ma breather (KB) [39], respectively. Such solutions are based on a slow modulation of a wave envelope with a precisely dedicated amplitude modulation and phase-shift of the carrier wave at a specific location and instant. Such wave groups can appear in the ocean when the wave field is unidirectional and narrowband [40].

Similar to the Euler equations, the first-derived hydrodynamic NLSE describes the evolution of the wave field in time [8], implying that initial conditions are required to start the wave motion. This type of NLSE is also referred to as space-like NLSE since the spatial wave field evolves in time. The time-like NLSE, which rather describes the evolution of the

temporal wave field in space, can be obtained from the space-NLSE through a straightforward spatiotemporal coordinate transformation [10]. Nonetheless, boundary conditions in the form of temporal surface elevations can be determined by an exact NLSE solution as parameterized by either the time- or space-like framework of NLSE to drive the wave maker. As has already been discussed, there are substantial differences in the wave dynamics for solutions, which have the same type of periodicity as ABs [37, 41, 42]. Nevertheless, even though doubly localized solutions, such as the PB, exhibit comparable wave displacements when evolving, the phase dynamics concerning time- and space-like NLSE frameworks strongly differ from each other. Recently, the phase dynamics during the formation of extreme waves have been explicitly discussed in [43, 44]. Recall that the PB has been observed in a wide range of nonlinear dispersive media such as water, Kerr media, and plasma [20, 16, 45, 46] and can evolve on regular or irregular wave fields [25, 47, 48]. The formulation and evolution of the PB are illustrated in detail in Chapter 2.2.4. Using the PB as a referenced model, in Chapter 4, an experimental study addressing the temporal evolution of the phase dynamics along the spatial coordinate is reported. The phase jump features from the laboratory measurements are compared, as initiated by either the time- or space-NLSE boundary conditions. It is shown that despite the water surface elevation is almost identical, there are differences in the phase behaviour, especially at the location of maximal breather compression. These key features in the phase pattern might be useful for extreme wave detection in random and irregular wave states.

The NLSE framework has certain invariant transformations allowing the NLSE solution to remain unchanged after the transformation. One such transformation is the scaling transformation, which controls the amplitude of the solution [33]. In addition to the time-reversal symmetry [49], which has been demonstrated to have practical applications [50], another invariant operation is the Galilean transformation (GT) [33, 51, 52]. Such symmetry is, however, intrinsic to the approximative weakly-nonlinear framework and cannot be generalized to the full water wave problem [53, 54]. As a matter of fact, introducing a Galilean velocity (GV) to an NLSE pulse is colligated to a required carrier frequency-shift to satisfy symmetry [55]. As such, the GT has been legitimately considered not applicable and relevant for dispersive physical systems [33, 56, 54]. In Chapter 5, experimental observations of Galilean NLSE solitons

for a wide range of GVs without an external flow forcing [57] are reported. The experimental data show considerable deviations from hydrodynamic NLSE predictions. Nevertheless, a very good agreement with the modified NLSE (MNLSE) [51] and the numerically-solved Euler equations using the high-order spectral method (HOSM) [58, 59] has been achieved. It is shown that the type of deviations in the wave envelope depends on the sign of GV. For the case of the envelope soliton, the pulse's spectrum undergoes a strong broadening when GV is negative, whereas a strong self-focusing of wave envelope is observed when the GV is positive [60]. Furthermore, it is shown that for predetermined carrier wave steepness parameters, there are exact GV values for which a Galilean-transformed envelope soliton corresponds to an exact multi-soliton solution [61]. In fact, the higher the value of GV, the higher the order of the multi-soliton. As a result, supercontinuum generation is an unavoidable process, which follows as a result of substantial wave focusing and subsequent soliton fission events [62, 63, 64]. The hydrodynamic supercontinuum generation from Galilean envelope solitons in the form of an irreversible and severe spectral broadening genesis is confirmed. The influence of the GT on the PB dynamics is also discussed, even though the limited fetch of the experimental facility does not allow the study of emergent and long-ranging complex wave patterns as a result of intrinsic and higher-order MI processes [65, 66].

While GT induces an overall phase-shift on the carrier wave, localized phase evolution within the wave envelope has also drawn much attention [67, 44, 1, 68, 69]. A previous experimental study has revealed that locally the same type of maximal breather compression can be achieved by just starting from the solution's amplitude modulation of the carrier wave only and ignoring any local phase-shift in the wave field for the determination of the experiment's boundary conditions [68]. On the contrary, the aim of Chapter 6 is to show that the initial phase-shift profile contains the leading information that allows the breathers to undergo the peak growth and focusing process even when the input amplitude modulation is suppressed. Indeed, the current experimental investigation provides evidence that extreme wave events can be solely triggered by a localized phase-shift perturbation (PSP) of a breather solution while ignoring the amplitude modulation information. That said, the experimental boundary condition in the current study consists of a regular wave train with a localized PSP as determined by a fundamental breather wave envelope. Accordingly, the wave maker's motion can be controlled

in such a manner to locally adjust its motion to accurately generate such initial and small perturbations in a regular carrier. To achieve this, specific phase-shifts proper to the three fundamental NLSE breather solutions are adopted, as will be mentioned in Sections 2.2.4 and 2.2.8. It is shown that the extreme localization achieved in the wave flume resembles those of pure breathers, even though spatially delayed concerning the exact solution. All observations are in reasonable agreement with the NLSE prediction despite the very strong wave focusing reached. These results suggest that the analysis of ocean rogue waves should also comprise the local phase information to fully understand the early stage of formation.

Despite extreme waves within the unidirectional wave system, the formation of which has also been extensively investigated regarding multi-directional wave system [70, 71, 72] and, as a particular case of the latter scenario, the standing wave system [73, 74, 75, 76, 72, 77, 78]. A coupled nonlinear Schrödinger (CNLSE) framework was already derived for water waves in the '80s [73] and is nowadays considered a fundamental dynamical system for the study of complex, coherent, directional, and rogue wave dynamics in various physical media [79, 80, 81, 77, 82]. In fact, the MI has been proven to exist in complex sea states, such as crossing seas [70, 71, 72], however, it is less dominant compared to wave systems with a single wave vector due to the violation of both critical assumptions, namely unidirectionality and narrow-band energy level [83, 6]. Then again, it has been recently conjectured that there is an increase in the probability of rogue wave formation in coupled two-wave systems compared to an uncoupled directional wave field [84]. In addition to studying the phase dynamics of nonlinear rogue waves in unidirectional wave systems as mentioned above, this thesis also examines the formation of rogue waves in coupled wave fields using both deterministic and statistical approaches, with a focus on the special case of counter-propagating or standing wave fields.

Deterministically, Chapter 7 presents experimental evidence of nonlinear wave focusing in standing wave fields, using the analytical PB solution [38] as a referenced nonlinear rogue wave and special MI model evolving in one of the two counter-propagating wave systems. The study demonstrates that PB-type unstable packets on finite and zero backgrounds can evolve distinctly in the presence of a counter-propagating regular wave field without any

noticeable disturbance or loss of coherence during wave focusing and defocusing. The experimental results are in good agreement with the hydrodynamic CNLSE framework [73], and the findings are further supported by HOSM simulations of the governing water wave equations. This research demonstrates that nonlinear focusing can occur in such a simplified but representative particular case of multi-directional wave configuration.

Statistically, pioneers such as Longuet-Higgins [85, 86], Forristall [87], and Boccotti [88] initially investigated the probabilities of ocean waves. For more recent progress and theory on extreme wave statistics, the reader can refer to Section 2.3. In Chapter 8, to mimic complex coastal conditions, the rogue wave probability under various wave reflections [89, 30, 90] and beach materials were investigated, ranging from quasi-unidirectional to full-reflection [73, 79] with the water depth varying from deep water to finite depth regimes. The spectrum width, which also plays an essential role in extreme wave formation [6], was also incorporated into the current experimental study. From the observation, the introduction of wave reflections leads to a general decrease in extreme wave occurrence while the probability still remains drifted from the linear expectations. Nevertheless, the effect of beach material is complex under different angles, which is not only related to the beach permeability. As the spectrum width narrows, the rogue wave occurrence becomes more pronounced even under reflections, echoing the observation of nonlinear focusing in standing and quasi-standing waves [4] mentioned above. The experimental results are in perfect alignment with numerical predictions based on the CNLSE framework [73, 79, 40]. Note that the parameters of all experimental cases tested in the current statistical study are given in the Appendix.

Overall, in Chapter 2, the theoretical foundations for each study mentioned above are introduced. Chapter 3 provides an overview of the experimental and numerical methods employed in these studies, including wave generation, beach installations, numerical methods, data analysis, and filtering approaches. The experimental and numerical results for each topic mentioned above are presented in Chapters 4-8. Finally, the findings of these studies are summarized in Chapter 9.

CHAPTER 2

Theory of surface gravity water waves and research scope

2.1 Governing equations for free surface water waves

In this study, the property of waves at the water surface was investigated. By considering an irrotational, incompressible, and inviscid fluid, and unidirectional surface waves propagating on the horizontal x -direction only, the velocity potential $\phi(x, z, t)$ and wave elevation $\eta(x, t)$ at the horizontal water surface $z = \eta(x, t)$ can be described by the following simplified form of the Euler equations [26, 91]

$$\phi_t + \frac{1}{2}(\nabla\phi)^2 + gz = 0, \quad (2.1)$$

$$\eta_t + \nabla\phi \cdot \nabla\eta - \phi_z = 0, \quad (2.2)$$

whereas for $-h < z < \eta(x, t)$ with h being the water depth,

$$\Delta\phi = 0. \quad (2.3)$$

Assuming no flux is permitted at the bottom, the condition at $z = -h$ is

$$\phi_z = 0. \quad (2.4)$$

Here, x and z are the spatial dimensions of the Cartesian coordinates with z pointing upwards and x being horizontal, t is the time, $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial z})$ is the gradient operator, the Laplacian operator is defined by $\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$, and $g = 9.81 \text{ m s}^{-2}$ is the gravitational acceleration. Note that the fluid velocity u is the gradient of ϕ , i.e., $u = \nabla\phi$. Since the unidirectional wave condition is considered, all components in the y direction above are ignored. The

Laplace Equation (2.3) follows the law of mass conservation within an incompressible fluid. The Bernoulli Equation (2.1), also called the dynamic boundary condition, determines the pressure of water in terms of the potential ϕ at the free surface. Equation (2.2) is known as the kinematic free surface boundary condition [91].

Next, assuming the water waves have a small steepness ak , Equations (2.1) and (2.2) can be linearized by considering small disturbances at the water surface, so that the elevation at the mean water level $z = 0$ and the velocity potential at the surface are small. Thus, the following linearized equations at the fixed level $z = 0$ instead of $z = \eta(x, t)$ can be obtained from Equation (2.1) and Equation (2.2) by neglecting the nonlinear terms [26]. Therefore, the set of Equations (2.1)-(2.4) become [26]

$$\eta_t - \phi_z = 0, \quad z = 0, \quad (2.5)$$

$$\phi_t + g\eta = 0, \quad z = 0, \quad (2.6)$$

$$\Delta\phi = 0, \quad -h < z < 0, \quad (2.7)$$

$$\phi_z = 0, \quad z = -h. \quad (2.8)$$

2.1.1 Free surface gravity waves

As next, Equation (2.6) is substituted into Equation (2.5) to eliminate η and obtain

$$\phi_{tt} + g\phi_z = 0, \quad z = 0. \quad (2.9)$$

As a result, a unidirectional solution to the Laplace Equation (2.7) and Equations (2.8)-(2.9) reads [26, 91]

$$\phi(x, z, t) = B(z) \sin(kx - \omega t), \quad (2.10)$$

where k is the wavenumber and ω is the wave frequency. By substituting Equation (2.10) into the linearized dynamic boundary condition (2.6), the following expression for the surface elevation η at $z = 0$ [92] is obtained to be

$$\eta(x, t) = a \cos(kx - \omega t), \quad (2.11)$$

where a is defined as the wave amplitude [26]. If a is known, the function B from Equation (2.10) in finite water depth reads [92, 91]

$$B(z) = \left(\frac{\omega a}{k}\right) \frac{\cosh[k(z+h)]}{\sinh(kh)}. \quad (2.12)$$

In addition, substituting Equations (2.10) and (2.11) into the linearized kinematic boundary condition (2.5) gives the relationship between ω and k

$$\omega^2 = gk \tanh(kh), \quad (2.13)$$

which is known as the dispersion relation [91, 26, 93]. As a result, from Equations (2.11) and (2.13), the propagation speed of the wave crest, also known as the phase speed, can be expressed as

$$c_p = \frac{\omega}{k} = \sqrt{\frac{g}{k} \tanh(kh)}. \quad (2.14)$$

Note that in the deep water regime, that is when $h \rightarrow \infty$, B becomes purely exponential as a function of depth

$$B(z) = \frac{\omega a}{k} \exp(kz), \quad (2.15)$$

and the deep-water dispersion relation can be written as

$$\omega^2 = gk, \quad (2.16)$$

thus, the phase speed in deep water is

$$c_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}}. \quad (2.17)$$

2.1.2 Wave groups

The energy/wave envelope of water waves travels in wave groups with a particular speed. Such a wave group is not only a regular sinusoidal wave train but also the superposition of multiple wave trains at different frequencies. To derive the travelling speed of such slowly-varying amplitude, it is assumed that two regular wave trains with the same amplitude and frequencies

close to each other are superimposed [26, 91]

$$\eta(x, t) = a \cos(kx - \omega t) + a \cos[(k + \Delta k)x - (\omega + \Delta\omega)t], \quad (2.18)$$

which, through the trigonometric sum, gives the following form

$$\eta(x, t) = \psi(x, t) \cos \left[\left(k + \frac{1}{2} \Delta k \right) x - \left(\omega + \frac{1}{2} \Delta\omega \right) t \right]. \quad (2.19)$$

Here, $\Delta\omega$ and Δk are assumed to be small compared to the main frequency ω or wavenumber k , and $\psi(x, t) = 2a \cos(\frac{1}{2} \Delta k x - \frac{1}{2} \Delta\omega t)$ is the slowly varying amplitude of the wave train. Thus, similar to the calculation of the phase speed, the travelling speed of ψ is $\frac{\Delta\omega}{\Delta k}$, when considering the limiting case, that is $\Delta k \rightarrow 0$, this becomes

$$c_g = \frac{d\omega}{dk}, \quad (2.20)$$

which is referred to as the group velocity. Thus, by substituting Equation (2.13) into Equation (2.20), the group velocity in finite depth writes

$$c_g = \frac{1}{2} c_p \left(1 + \frac{2kh}{\sinh(2kh)} \right). \quad (2.21)$$

Again, in deep water when $h \rightarrow \infty$, c_g becomes

$$c_g = \frac{1}{2} c_p = \frac{\omega}{2k}, \quad (2.22)$$

which is half the phase speed.

2.1.3 Stokes wave theory

The so-called Stokes wave theory must be employed when considering higher-order effects at the water surface [94, 91]. It is also well-known that MI originates from a range of long-wave perturbations of second-order Stokes wave [7]. By taking the nonlinear terms in Equation (2.1) into account and applying an asymptotic approach to the velocity potential ϕ and surface elevation η with the expansion parameter, i.e., wave steepness $\alpha = ak \ll 1$, Stokes proved

that the second-order surface wave elevation is [94]

$$\eta(x, t) = a \cos(kx - \omega t) + \frac{3 - \sigma_h^2}{4\sigma_h^3} ka^2 \cos[2(kx - \omega t)], \quad (2.23)$$

where $\sigma_h = \tanh(kh)$, and the corresponding dispersion relation in finite depth is thus

$$\omega^2 = gk\sigma_h \left(1 + \frac{9 - 10\sigma_h^2 + 9\sigma_h^4}{8\sigma_h^4} \alpha^2 \right). \quad (2.24)$$

In deep-water regime when $h \rightarrow \infty$ and thus $\sigma_h \rightarrow 1$, the Stokes wave elevation in Equation (2.23) becomes

$$\eta(x, t) = a \cos(kx - \omega t) + \frac{1}{2} ka^2 \cos[2(kx - \omega t)], \quad (2.25)$$

and Equation (2.24) becomes

$$\omega^2 = gk(1 + \alpha^2). \quad (2.26)$$

2.2 Modulation instability and the hydrodynamic nonlinear Schrödinger equation

2.2.1 Complex wave envelope and surface elevation

The complex wave envelope function $\psi(x, t)$ is a function describing the slow variation of the wave amplitude and phase. To the leading order, the connection between ψ and η can be expressed as the following [93]

$$\eta(x, t) = \Re(\psi(x, t) \exp[i(kx - \omega t)]), \quad (2.27)$$

where $\Re$ denotes the real part, k and ω are referred to as the carrier wavenumber and frequency, $\exp[i(kx - \omega t)]$ is the complex carrier wave component. Since $\psi(x, t)$ is complex, its absolute value $\sqrt{\Im(\psi)^2 + \Re(\psi)^2}$ reflects the slowly-varying wave amplitude, while the phase variation of the envelope is calculated by

$$\Phi(x, t) = \tan^{-1} \left[\frac{\Im(\psi)}{\Re(\psi)} \right]. \quad (2.28)$$

2.2.2 The hydrodynamic nonlinear Schrödinger equation

The NLSE is a classical equation that describes the weakly-nonlinear wave group evolution in nonlinear media. The hydrodynamic NLSE was initially derived by Zakharov using the spectral method [8] to describe the Hamiltonian structure of water waves, which can also be derived using the Whitham's method [26]. The NLSE for arbitrary depth was derived from the Euler equations using the multiple scales method [29, 95]. Due to its integrability, the hydrodynamic NLSE has a variety of exact solutions for localized wave structures [93]. Thus, it is a simple but powerful model describing the stable and unstable extreme wave groups. The NLSE evolving in time at an arbitrary water depth [93, 29, 96] writes

$$i(\psi_t + c_g \psi_x) + \mu \psi_{xx} + \nu |\psi|^2 \psi = 0, \quad (2.29)$$

where

$$\begin{aligned} c_g &= \frac{\partial \omega}{\partial k} = \frac{c_p}{2} \left[1 + \frac{(1 - \sigma_h^2)kh}{\sigma_h} \right], \\ \mu &= \frac{1}{2} \frac{\partial^2 \omega}{\partial k^2} = -\frac{g}{8k\sigma_h\omega} \{ [\sigma_h - kh(1 - \sigma_h^2)]^2 + 4k^2h^2\sigma_h^2(1 - \sigma_h^2) \}, \\ \nu &= -\frac{k^4}{2\omega} \left(\frac{c_p}{2\sigma_h} \right)^2 \left\{ \frac{1}{c_g^2 - gh} [4c_p^2 + 4(1 - \sigma_h^2)c_p c_g + gh(1 - \sigma_h^2)^2] + \frac{9 - 10\sigma_h^2 + 9\sigma_h^4}{2\sigma_h^2} \right\}. \end{aligned} \quad (2.30)$$

Here, $\sigma_h = \tanh(kh)$, $c_p = \omega/k$ is the linear phase speed, and ψ is the unidirectional complex wave envelope function. Note that for deep water where $h \rightarrow \infty$ and thus $\sigma_h \rightarrow 1$, the parameters for dispersive and nonlinear terms become

$$\mu = -\frac{\omega}{8k^2}, \quad \nu = -\frac{1}{2}\omega k^2. \quad (2.31)$$

In water wave experiments, one usually measures the evolution of wave elevation time series at several fixed locations, instead of taking a series of "snapshots" of the waves that evolve in time due to accuracy and technical difficulties. Thus, an evolving-in-space NLSE (time-NLSE) framework is more suitable to be compared with experimental results. Through an analogy operation according to reference [93], the leading order in Equation (2.29) renders

$$\psi_t + c_g \psi_x \approx 0, \quad (2.32)$$

so that the following approximations can be obtained

$$\psi_x \approx -\psi_t/c_g, \quad \psi_{xx} \approx \psi_{tt}/c_g^2. \quad (2.33)$$

By substituting the above into the space-NLSE, i.e. Equation (2.29), the so-called time-NLSE can be obtained below

$$i(\psi_x + \frac{1}{c_g}\psi_t) + \frac{\mu}{c_g^3}\psi_{tt} + \frac{\nu}{c_g}|\psi|^2\psi = 0, \quad (2.34)$$

which is mainly used to numerically validate the experimental results in the current thesis.

In addition, the higher-order NLSE or modified NLSE (MNLSE) was derived by Dythe [51] to address the unfavourable comparison between the theory and realistic wave propagation data [97]. The spatial version of the MLNSE was derived by Stansberg and Trulsen [98] Kit and Shemer [99]

2.2.3 Modulation instability

The instability of second-order Stokes waves to side-band perturbations, also known as modulation instability (MI) or the Benjamin-Feir instability, is characterized by the exponential growth of the side-bands in the frequency domain and the disintegration of the weakly-nonlinear Stokes wave train with a slowly-varying amplitude modulation in the time domain [7, 93, 100]. The MI can also be equivalently derived from the weakly-nonlinear wave envelope approach [26]. Taking the regular envelope solution of the deep-water NLSE (2.31) as an example, which writes

$$\psi(t) = a \exp(-\frac{1}{2}i\alpha^2\omega t). \quad (2.35)$$

Not explicitly considering the case of unstable bichromatic wave groups [101], here we assume a long wave perturbation in the wave envelope of a single peaked wave train. The following can be obtained [26]

$$\psi(x, t) = [a_0 + \epsilon_+ \exp(i\Omega t + iK(x - c_g t)) + \epsilon_- \exp(i\Omega t - iK(x - c_g t))] \exp(-\frac{1}{2}i\alpha^2\omega t), \quad (2.36)$$

where the initial perturbation amplitude $\epsilon_{\pm} \ll 1$, K is the perturbation wavenumber, and Ω is the perturbation frequency. If Ω is imaginary, the amplitude will experience exponential growth according to Equation (2.36). By linearizing about a_0 and solving the eigenvalue problem gives the following dispersion relation [26]

$$\Omega^2 = \frac{\omega^2}{8k^2} (\frac{K^2}{8k^2} - \alpha^2) K^2, \quad (2.37)$$

indicating that when $(\frac{K^2}{8k^2} - \alpha^2) < 0$, i.e. $0 < K < 2\sqrt{2}k^2a$, Ω is imaginary. At $K = 2k^2a$, the maximum growth rate appears, which is $\Im(\Omega)_{\max} = \frac{1}{2}\omega\alpha^2$. Here, $\Im$ stands for the imaginary part of the complex function. The normalized growth rate of the normalized perturbation as a

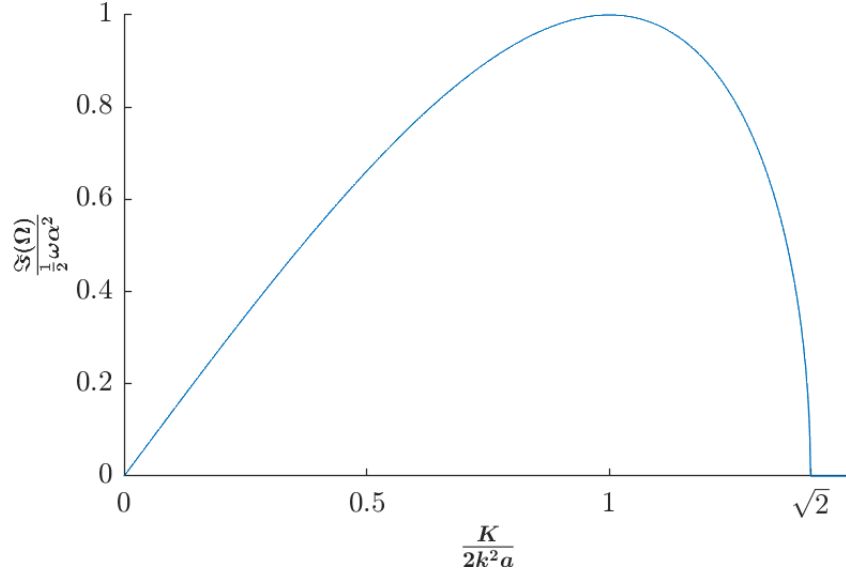


FIGURE 2.1. Instability diagram indicating the perturbation growth rate as a function of the normalized perturbation wavenumber K , drawn from Equation (2.37).

function of perturbation wavenumber is shown in Figure 2.1. The corresponding amplification factor $a_{\max}/a$ of the perturbed wave train is [26]

$$\frac{a_{\max}}{a} = 1 + 2\sqrt{1 - \left(\frac{K}{2\sqrt{2}k^2a}\right)^2}, \quad (2.38)$$

where the amplification factor $a_{\max}/a$ reaches the maximum value of 3 at $K = 0$, while $a_{\max}/a = 1 + \sqrt{2}$ when $K = 2k^2a$, corresponds to the maximum growth rate. Following Equation (2.38), this amplification factor is shown in Figure 2.2.

2.2.4 Solutions to the nonlinear Schrödinger equation

The NLSE is the simplest framework to describe the wave propagation of nonlinear dispersive waves [31]. For the deep-water wave problem, the slowly-varying wave envelope $\psi(x, t)$ around the wave number k satisfies the NLSE framework as written in Equation (2.34) [8, 93]. As a result of integrability, a variety of steady and pulsating solutions have been derived [32, 102]. The parameterization of fundamental breathers on a finite-amplitude carrier has been discussed in multiple previous works [103, 20, 104, 41].

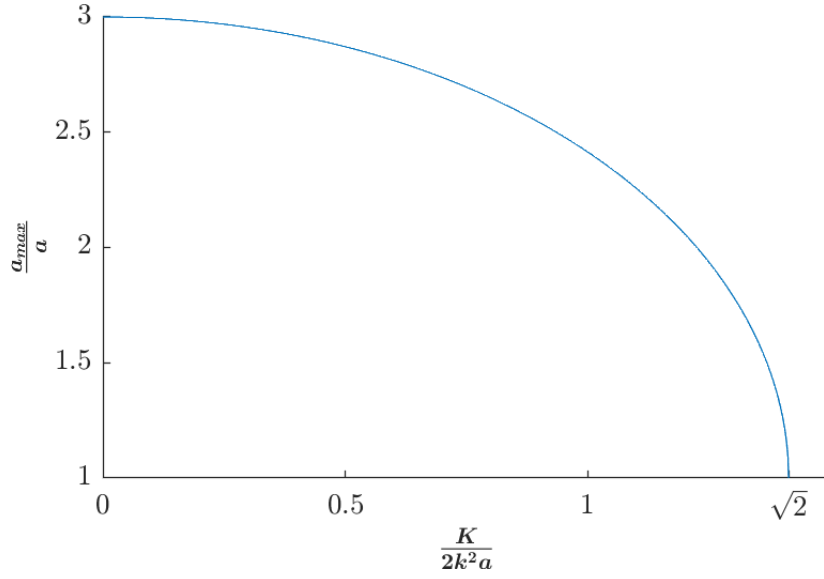


FIGURE 2.2. The amplification factor a_{max}/a due to growth saturation as a function of the normalized perturbation wavenumber K .

The exact NLSE solutions are strongly connected to MI. For example, the AB family describes the nonlinear stage of MI [37, 105], and the PB is a limiting case of zero modulation frequency [38], i.e., the case of infinite modulation period. Recall that the PB is a doubly-localized solution of the unidirectional NLSE [38], which has been so far observed in optics, water waves, and plasma [20, 16, 45] with a rational growth [33, 93], as is going to be discussed in details below. The PB can be approached by either AB or KB [33, 39]. The KB family, however, is the space-periodic solution (within the time-NLSE framework), which starts its evolution from a solitonic perturbation on the carrier and does not follow the MI process. In fact, the KB never converges to a uniform amplitude. All these solutions have been observed in a variety of nonlinear dispersive media. For more details, the reader can refer to [106, 103, 20, 16, 104, 17, 41, 13].

Besides, recall that the concept of soliton was adopted to explain the recurrence of the Fermi–Pasta–Ulam–Tsingou problem [107] in the Korteweg–de Vries (KdV) system [108, 109]. Within the NLSE framework, this process can be described by ABs, which will be discussed later. Given the NLSE framework, envelope solitons are steady solutions resulting from the balance between dispersion and nonlinearity. They can be derived from the so-called

inverse scattering transform (IST) [93]. The reader can refer to references [33, 110] for details regarding solitons. Here, the dimensional form of the envelope soliton of amplitude a satisfying Equation (2.34) reads

$$\psi_s(x, t) = a \operatorname{sech}\left(-\sqrt{2}\alpha k c_g\left(t - \frac{x}{c_g}\right)\right) \exp\left(-\frac{i\alpha^2 k x}{2}\right). \quad (2.39)$$

An example of the NLSE soliton wave elevation $\eta_s(t)$ and its upper envelope $|\psi_s(t)|$ at $x = 0$ m is shown in Figure 2.3. Similarly, given an amplitude of background a , the hydrodynamic

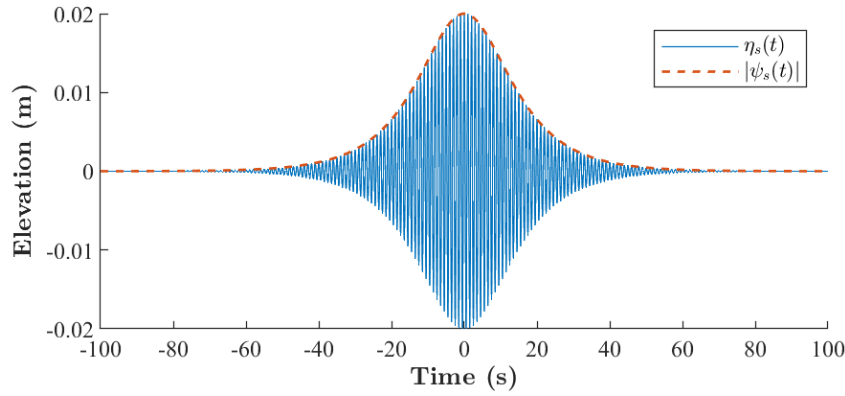


FIGURE 2.3. Example of the wave elevation $\eta_s(t)$ in blue curve and upper envelope $|\psi_s(t)|$ in red dashed line of an NLSE soliton solution at $x = 0$ m. Here, $a = 0.02$, m and $f = 1$ Hz.

deep-water AB, PB, or KB ψ can be parameterized as the following:

$$\begin{aligned} \psi_{\text{AB/KB/PB}}(x, t) = & a \left(1 + \frac{2(1 - 2\alpha) \cosh(a^2 k^3 \mathfrak{b} x) - i\mathfrak{b} \sinh(a^2 k^3 \mathfrak{b} x)}{\sqrt{2\alpha} \cos\left[\frac{ak\omega}{\sqrt{2}} \mathfrak{c} \left(t - \frac{x}{c_g}\right)\right] - \cosh(a^2 k^3 \mathfrak{b} x)} \right) \\ & \times \exp(-ia^2 k^3 x), \end{aligned} \quad (2.40)$$

where $\mathfrak{b} = \sqrt{8\alpha(1 - 2\alpha)}$ and $\mathfrak{c} = 2\sqrt{1 - 2\alpha}$. For $0 < \alpha < 0.5$, $\psi(x, t)$ is the AB while for $\alpha > 0.5$, $\psi(x, t)$ is the KB. When $\alpha \rightarrow 0.5$, $\psi(x, t)$ converges to the rational PB solution [104]. Examples of the exact AB, PB, KB solutions according to Equation (2.40) are shown in Figure 2.4.

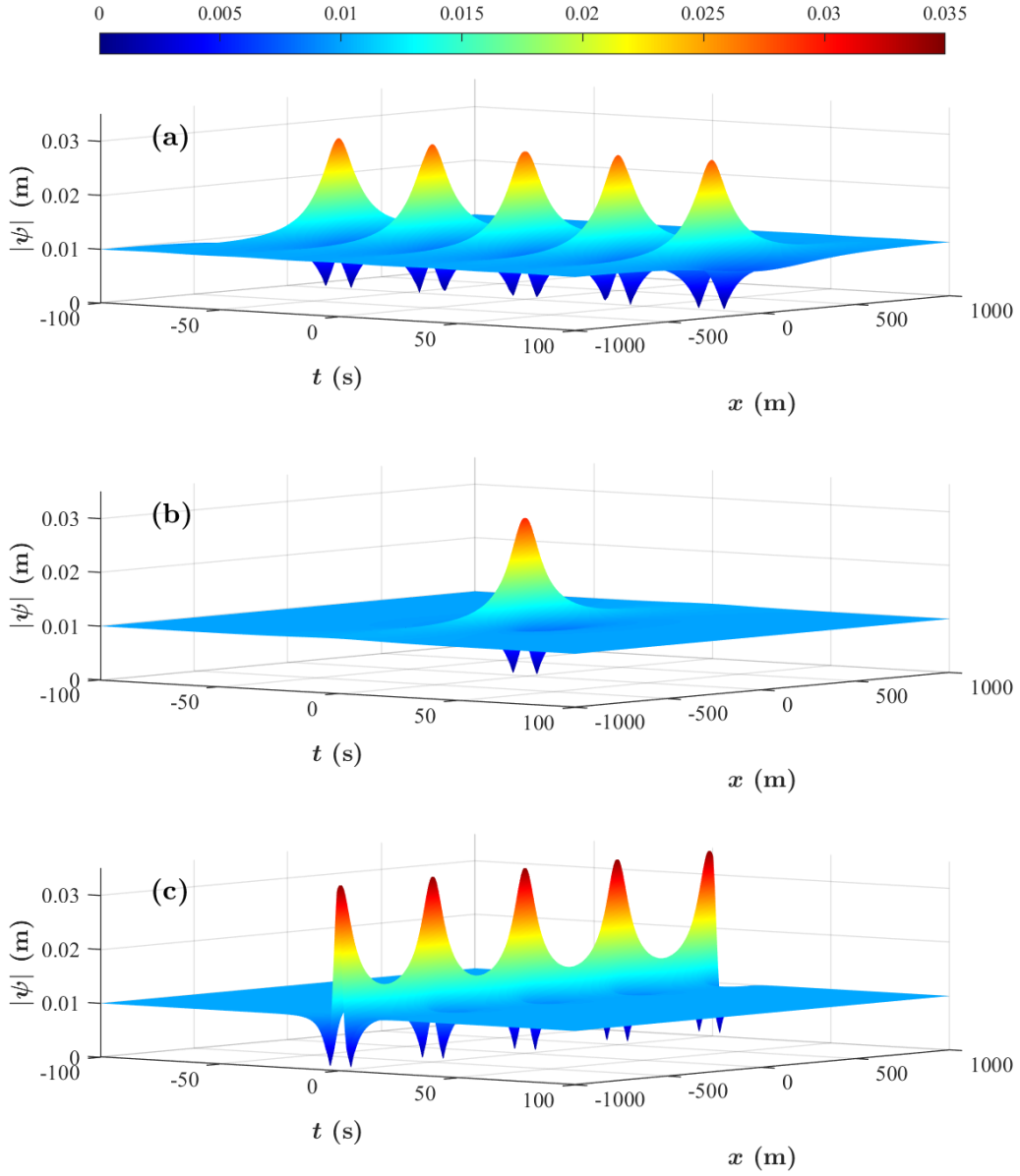


FIGURE 2.4. Examples of the exact NLSE solutions for (a) AB for $\alpha = 0.4$ (b) PB (c) KB for $\alpha = 0.8$. In all three plots, $a = 0.01$ m, $f = 1$ Hz.

With different parameterizations applied, the analytical NLSE solutions mentioned in this subsection are adopted in the experimental studies in Chapters 4-7 to deterministically investigate the properties of nonlinear extreme waves under various circumstances.

2.2.5 Phase dynamics of the space- and time-Peregrine breathers

As mentioned in Chapter 1, both space- and time-NLSE frameworks, i.e. Equations (2.29) and (2.34) are integrable and admit exact solutions, while the differences between the two frameworks are not trivial, especially regarding phase evolution. To understand the difference between time- and space-NLSE exact solutions, the PB is taken as an example in the following. $A(x, t)$ denotes the complex envelope in the space-NLSE framework and $B(x, t)$ denotes that in the time-NLSE framework to distinguish the two different frameworks. From the space-NLSE Equation (2.29), the dimensional representation of the PB solution of the space-NLSE can be easily written as the following:

$$A_P(x, t) = a \left(-1 + \frac{4(1 - i\alpha^2\omega t)}{1 + 8\alpha^2k^2(x - c_g t)^2 + \alpha^4\omega^2 t^2} \right) \exp \left(-\frac{i\omega\alpha^2 t}{2} \right), \quad (2.41)$$

while the dimensional PB solution of time-NLSE from Equation (2.34) can be written as

$$B_P(x, t) = a \left(-1 + \frac{4(1 - i\alpha^2\omega x/c_g)}{1 + 8\alpha^2k^2(x - c_g t)^2 + \alpha^4\omega^2 x^2/c_g^2} \right) \exp \left(-\frac{i\alpha^2\omega x}{2c_g} \right). \quad (2.42)$$

The distinctions between A_P and B_P solutions lead to an intriguing topic concerning that two rogue wave exact solutions under the same scale (amplitude and frequency) may coexist in one physical reality.

It is underlined that the amplitudes functions, i.e. $|A|$ and $|B|$ of the dimensional time- and space-like PB slightly differ. Deviations are obviously the largest at $x = 0$. The respective temporal intensity functions are

$$|A_P(x = 0, t)|^2 = a^2 \left| -1 + \frac{4(1 - i\alpha^2\omega t)}{1 + 8\alpha^2k^2(c_g t)^2 + \alpha^4\omega^2 t^2} \right|^2, \quad (2.43)$$

and

$$|B_P(x = 0, t)|^2 = a^2 \left| -1 + \frac{4}{1 + 8\alpha^2k^2(c_g t)^2} \right|^2. \quad (2.44)$$

In reference [41], the authors have discussed the differences in the wave envelope propagation of the quasi-identical time- and space-like PB wave group. Here, the investigation focuses on

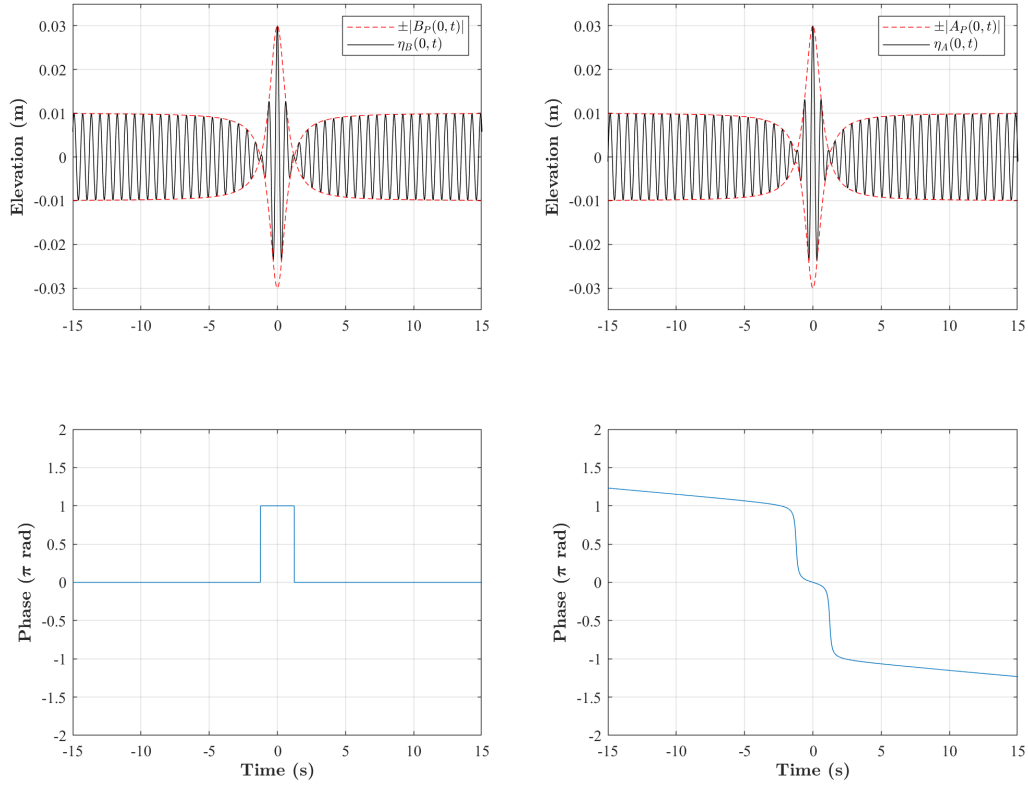


FIGURE 2.5. Theoretical surface elevation (solid black lines), wave envelope (dashed red lines), and the corresponding phase (blue lines) of the PB at its peak location $x = 0$ m from time-NLSE (left) and space-NLSE (right). The steepness adopted is $ak = 0.1$ and the carrier wave amplitude is $a = 0.01$ m. Same parameters for the following.

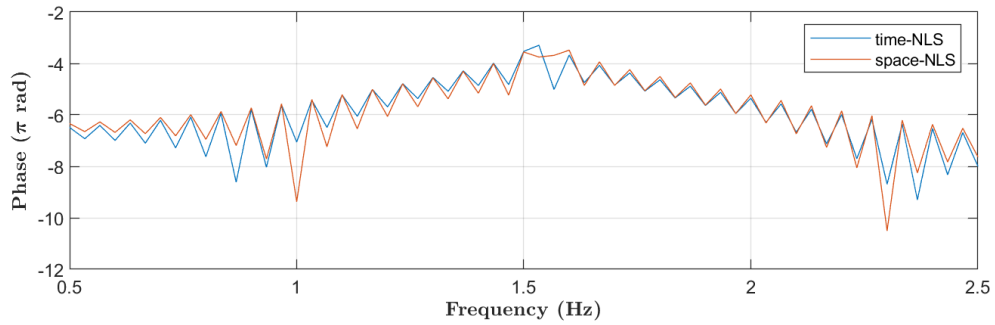


FIGURE 2.6. Theoretical frequency spectrum of the PB surface elevation time series at its peak time $t = 0$ s from time-NLSE (left) and space-NLSE (right). The sampling window size of the time series is 30 seconds.

their corresponding phase evolution. To initiate the current investigation, the boundary conditions for the two PB models are considered, namely the space-like and time-like formulations, $A_P(0, t)$ and $B_P(0, t)$, respectively, at maximal compression location $x = 0$ m and analyze the corresponding phase jumps in the respective temporal profiles. Figure 2.5 shows both the time series and the corresponding characteristic phase jump for each of the two cases. It

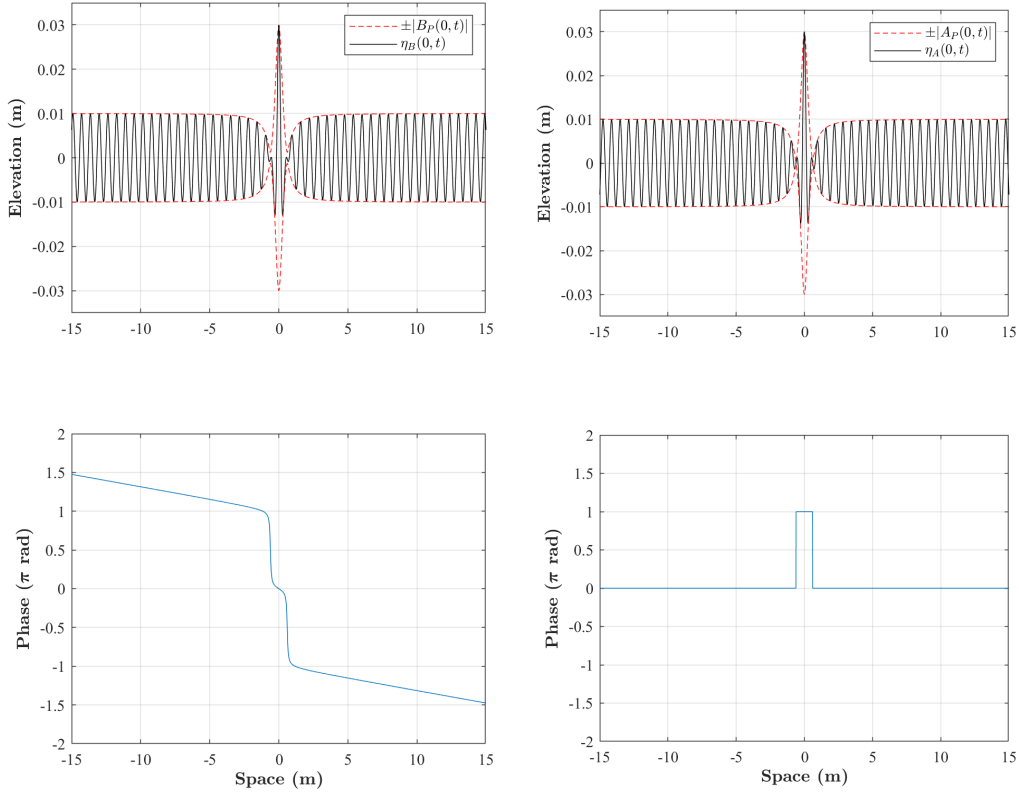


FIGURE 2.7. Theoretical surface elevation (solid black lines), wave envelope (dashed red lines), and the corresponding phase (blue lines) of the PB at its peak time $t = 0$ s from time-NLSE (left) and space-NLSE (right).

is noticeable that the phase diagram is different even though the envelope/surface elevation profiles look almost similar. As shown in Figure 2.6, the theoretical frequency spectra of the two solutions at $x = 0$ m are also similar. On the other hand, both phases experience a sharp transition of π .

This motivates a quantitative experimental investigation of the phase evolution when considering one boundary condition or the other to investigate the physical relevance of breather

models. Interestingly, when considering the initial conditions of the time- and space-like PB at the maximal focusing time $t = 0$ s, the same phase jump patterns can be noticed; however, the characteristic phase variations are interchanged compared to the previous case—see figure 2.7.

This suggests once again that when using two different boundary conditions to trigger the PB evolution, the wave envelope may look almost identical in both cases; nevertheless, the characteristic phase-shift dynamics may differ during the propagation.

When considering a reasonable choice of carrier steepness $\alpha = ak$ below the breaking threshold found to be at 0.12 for the PB, the difference in wave amplitude is very small and almost negligible in a laboratory experiment. Note that the real steepness of the waves is slightly higher than three times 0.12, which is larger than the onset of breaking for dispersive focused wave groups [111] The difference in intensity for $ak = 0.1$ and $a = 0.01$ m at $x = 0$ m is shown in figure 2.8.

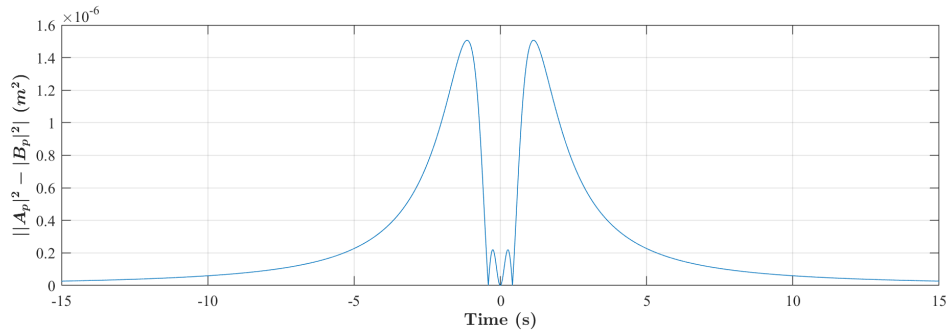


FIGURE 2.8. The absolute value of the amplitude difference between the time- and space-like PB for the carrier wave parameters $ak = 0.1$ and $a = 0.01$ m at $x = 0$ m.

The experimental results and discussions on the space- and time-Peregrine breathers in a wave tank are shown in Chapter 4.

2.2.6 The Galilean-invariance of the nonlinear Schrödinger equation

For a solution $\psi(x, t)$ of the dimensional NLSE Equation (2.34), note that the Galilean symmetry (GT) [33] reads

$$\psi_G(x, t) = \psi \left(x, t - \frac{\sqrt{2}c\alpha}{4c_g}x \right) \exp \left(\frac{i\sqrt{2}c\alpha k(x - c_g t)}{2} + \frac{ic^2\alpha^2 kx}{8} \right), \quad (2.45)$$

also known as the "gauge" transformation [112], which was recently investigated in the context of nonlinear optics [52], showing theoretically that Galilean invariance is not valid for optical pure-quadratic solitons under higher-order dispersion relation. Applying the latter GT

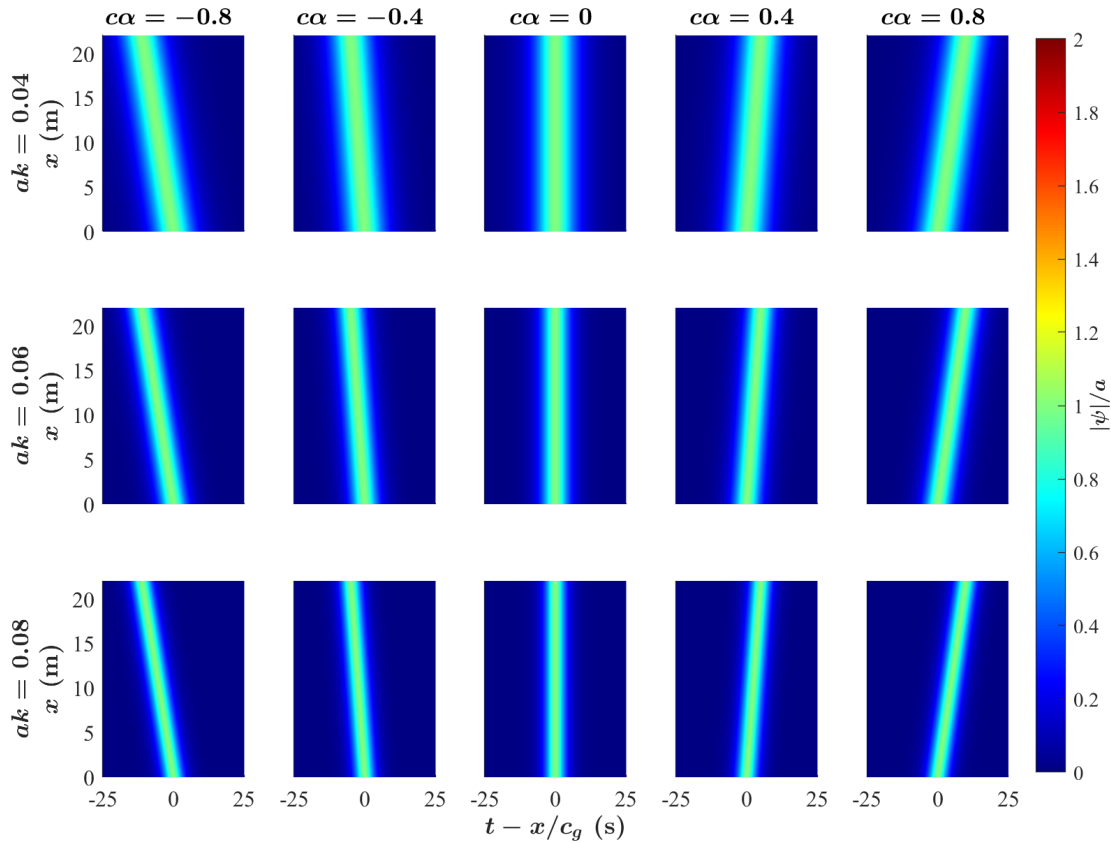


FIGURE 2.9. Evolution of an NLSE envelope soliton (middle panels), slower analogues with negative GV's (left panels), and faster counterparts with positive GV's (right panels) for three wave steepness values 0.04, 0.06, and 0.08.

to the sech-soliton in Equation (2.39) yields the following parametrization

$$\begin{aligned} \psi_{\text{GS}}(x, t) = & a \operatorname{sech} \left(-\sqrt{2}\alpha k c_g \left(t - \frac{x}{c_g} \right) + \frac{c}{2}\alpha^2 k x \right) \exp \left(-\frac{i\alpha^2 k x}{2} \right) \\ & \times \exp \left(\frac{i\sqrt{2}c\alpha k (x - c_g t)}{2} + \frac{ic^2\alpha^2 k x}{8} \right), \end{aligned} \quad (2.46)$$

which also satisfies the NLS Equation (2.34). The reader can refer to [33, 113] for GT details, including more convenient theoretical representations in dimensionless physical quantities. Note that the GT compromises a frequency shift of the carrier and is different from the Tajiri-Watanabe-type construction [114]. Examples of dimensional envelope soliton evolution for different GVs and wave steepness values are shown in figure 2.9.

Note that when the GV is negative, i.e. $c < 0$, the wave packet propagates faster, whereas when $c > 0$, the coherent wave groups evolve slower compared to the pure and non-Galilean soliton. The Galilean-transformed hydrodynamic PB [38] is expressed as

$$\begin{aligned} \psi_{\text{GP}}(x, t) = & a \left(-1 + \frac{4(1 - 2i\alpha^2 k x)}{1 + 4 \left(\frac{1}{2}c\alpha^2 k x + \sqrt{2}k\alpha(x - c_g t) \right)^2 + 4\alpha^4 k^2 x^2} \right) \\ & \times \exp(-i\alpha^2 k x) \exp \left(\frac{i\sqrt{2}c\alpha k (x - c_g t)}{2} + \frac{ic^2\alpha^2 k x}{8} \right). \end{aligned} \quad (2.47)$$

The Galilean-transformed solitons were tested in a water wave tank with boundary conditions defined by Equation (2.27) at the initial location x_{ini} . The experimental and numerical results of Galilean-transformed wave groups are reported and discussed in Chapter 5.

2.2.7 The higher-order modified nonlinear Schrödinger equation framework

Recall that whereas the NLSE can be derived from the Euler equations at third-order in steepness $\mathcal{O}(\alpha^3)$ [29], the modified nonlinear Schrödinger equation (MNLSE) [51, 115, 98, 116, 117] is an improvement at the next order $\mathcal{O}(\alpha^4)$, which accounts for higher-order dispersive and nonlinear effects [51, 118, 119]. In the current study, the spatial evolutive form

as described in [117] is adopted

$$i \left(\psi_x + \frac{1}{c_g} \psi_t \right) - \frac{k}{\omega^2} \psi_{tt} - k^3 |\psi|^2 \psi - i \frac{k^3}{\omega} (6 |\psi|^2 \psi_t + 2 \psi (|\psi|^2)_t - 2i \psi \mathcal{H}[(|\psi|^2)_t]) = 0, \quad (2.48)$$

where $\mathcal{H}$ denotes the Hilbert transform and is used to approximate the mean-flow term. Essentially, the MNLSE is known to well-describe the motion of unstable water wave packet focusing, which involves a severe broadening and higher-energetic spectrum [98, 120, 121].

The higher-order framework is adopted to validate the experimental results also in Chapter 5.

2.2.8 Phase and amplitude modulations of the nonlinear Schrödinger equation solutions

As mentioned above in Section 2.1.2, a side-band-perturbed plain wave elevation can be regarded as the product of the amplitude perturbation function $A(x, t)$ and a carrier wave component $\cos[(\omega + \frac{1}{2}\Delta\omega)t - (k + \frac{1}{2}\Delta k)x]$ containing the phase perturbation, both in the real regime. Similarly, the authors in reference [68] have shown that the exact NLSE solutions $\psi(x, t)$ can also be written as amplitude modulation function $\psi(x, t)$ and a respective phase-shift $\Phi(x, t)$, both evolving in time and space. This can be written as

$$\psi(x, t) = A(x, t) \exp[i\Phi(x, t)]. \quad (2.49)$$

It has also been shown in reference [68] that wave breather-like focusing can develop when starting from boundary conditions at the beginning of the flume, involving the amplitude modulation only and without consideration of the breather-specific local phase-shift.

In Chapter 6, an extreme wave focusing mechanism is proposed, which originates in a regular and amplitude-modulation-free wave train with localized phase-shift perturbations (PSPs).

2.2.9 Standing wave system and the coupled-nonlinear Schrödinger equation

The above introduces the MI and NLSE model for unidirectional weakly-nonlinear unidirectional waves. However, the wave properties in crossing seas, i.e. the co-existence of wave trains in multiple directions, remain ambiguous compared to the unidirectional scenario. The complete directional coupled NLSE (CNLSE) derived in [40] shows that the MI may increase within the directional sea in some cases, suggesting an increase of extreme events. The higher-order CNLSE, derived in reference [122], is more sophisticated and allows for the study of MI under different crossing angles.

The standing wave field is a particular type of crossing sea, in which the crossing angle between the two wave systems is π . The MI and a CNLSE-type framework in the standing

wave field were first derived in reference [73] through the Zakharov equation approach [8] based on the narrow-banded assumption, which indicates a variation of wave instability, followed by extensive investigations on the standing wave instability [79, 123, 124, 125, 126, 127].

The nonlinear interaction of standing waves involves an incident wave field characterized by the complex amplitude $\psi^{(1)}(x, t)$ with wavenumber $\kappa^{(1)} = (k, 0)$ interacting with the reflected waves $\psi^{(2)}(x, t)$ with wavenumber $\kappa^{(2)} = (-k, 0)$, and propagating along the space co-ordinate x . This process can be described by the following CNLSE framework [73, 79]

$$\begin{aligned} i \left(\psi_x^{(1)} + \frac{1}{c_g} \psi_t^{(1)} \right) + \frac{\mu}{c_g^3} \psi_{tt}^{(1)} + \frac{\nu}{c_g} (|\psi^{(1)}|^2 - 2|\psi^{(2)}|^2) \psi^{(1)} &= 0, \\ i \left(\psi_x^{(2)} - \frac{1}{c_g} \psi_t^{(2)} \right) + \frac{\mu}{c_g^3} \psi_{tt}^{(2)} + \frac{\nu}{c_g} (|\psi^{(2)}|^2 - 2|\psi^{(1)}|^2) \psi^{(2)} &= 0. \end{aligned} \quad (2.50)$$

The angular frequency $\omega^{(i)}$ and wavenumber $\kappa^{(i)}$ are connected through the deep-water linear dispersion relation $\omega^{(i)} = \sqrt{g|\kappa^{(i)}|}$, $i = 1, 2$, and g being the gravitational acceleration. Here, $\kappa^{(i)}$ can be simplified into k and $-k$ for each wave train in the standing wave field due to the absence of transverse waves. The parameters c_g , μ , and ν are the same as those in Equation (2.34) for the unidirectional time-NLSE framework.

The standing water surface elevation describing the interaction between the incident wave field $\psi^{(1)}$ and opposing wave system $\psi^{(2)}$ to the first-order of approximation is calculated using the complex envelope following the expression

$$\begin{aligned} \eta_{\text{standing}}(x, t) &= \Re \{ \psi^{(1)}(x, t) \exp[i(kx - \omega t)] \\ &\quad + \psi^{(2)}(x, t) \exp[-i(kx + \omega t)] \}. \end{aligned} \quad (2.51)$$

Here, the author emphasizes the weakening of the bound wave contributions in standing wave systems in contrast to the unidirectional wave evolution. Thus, the second-harmonic term is ignored here [128, 129, 130]

A more general multi-directional weakly-nonlinear wave system with an arbitrary crossing angle can be described by the 2D CNLSE framework [79], which can be simplified to Equation (2.50) for standing wave field. The fourth-order modified CNLSE applied to two

wave systems with distinct wave frequencies and directions is derived in [122]. In the current study, the general crossing sea state is not investigated. Instead, it is only limited to the simple standing wave field.

In Chapter 7, the existence of nonlinear wave focusing in a standing wave field is investigated, which is a special case of crossing wave systems with a crossing angle of π , by using the PB model as an incident wave field $\psi^{(1)}$ and a regular wave envelope of the same amplitude for $\psi^{(2)}$. Recall that the PB is a doubly localized solution of the unidirectional NLSE [38], which has been so far observed in optics, water waves, and plasma [20, 16, 45]. That said, this solution describes the nonlinear stage of modulation instability in the case of infinitely long perturbation [33, 93] and has a rational growth. In fact, it can be approached by either AB [37, 105] or PB [39], and so far has not been discussed in the context of collinear waves. Its counterpart on zero background is also known as the degenerate soliton [131].

In addition, the CNLSE is also adopted in Chapter 8 to analyze the extreme wave probability in the counter-propagating random wave field.

2.3 Irregular waves and extreme wave statistics

The deterministic models and solutions in the sections above have been broadly investigated. On the other hand, analyzing, and predicting wave occurrence in the ocean is crucial for navigation safety, which has attracted interest concerning rogue waves that seem to "appear from nowhere and disappear without a trace" [132], such as the early observation of the extreme event called "Draupner wave" [133], which appears to be a result of dispersive focusing [12]. For random waves, however, questions such as the probability of rogue waves in the sea remain unclear due to their complexity and randomness. Such problem is being extensively studied, for instance, on the extreme wave occurrence in offshore or coastal areas [93, 134, 100, 14] and over a shoal [135, 136, 137]. The extreme wave occurrence in directional sea state is also under broad investigations [40, 138, 76, 72, 77, 78]. In the random wave field, the significant wave height H_s of a random wave train can be defined as $H_s = 4\sigma$, where σ is the standard deviation of the wave elevation $\eta(x, t)$. The characteristic steepness is defined as $\alpha_c = a_c k_p$, where the characteristic amplitude a_c is defined as $\sqrt{2}\sigma$, peak wavenumber $k_p = \omega_p^2/g$, and ω_p is the peak (angular) frequency [93]. Moreover, in the subsections below, the JONSWAP spectrum, indicators of wave nonlinearity, and the linear/nonlinear extreme wave distribution functions are briefly introduced.

In Chapter 8, the occurrence of rogue waves in the random standing wave field under reflections is statistically investigated, considering various spectrum bandwidths, wave steepness, water depths, and reflective beach materials.

2.3.1 JONSWAP spectrum

The JONSWAP spectrum is commonly used to describe irregular waves in the ocean [139]. The JONSWAP random wave train in the frequency domain writes

$$S(\omega) = \frac{\alpha_P g^2}{\omega^5} \exp \left[-\frac{5}{4} \left(\frac{\omega_p}{\omega} \right)^4 \right] \gamma^r, \quad (2.52)$$

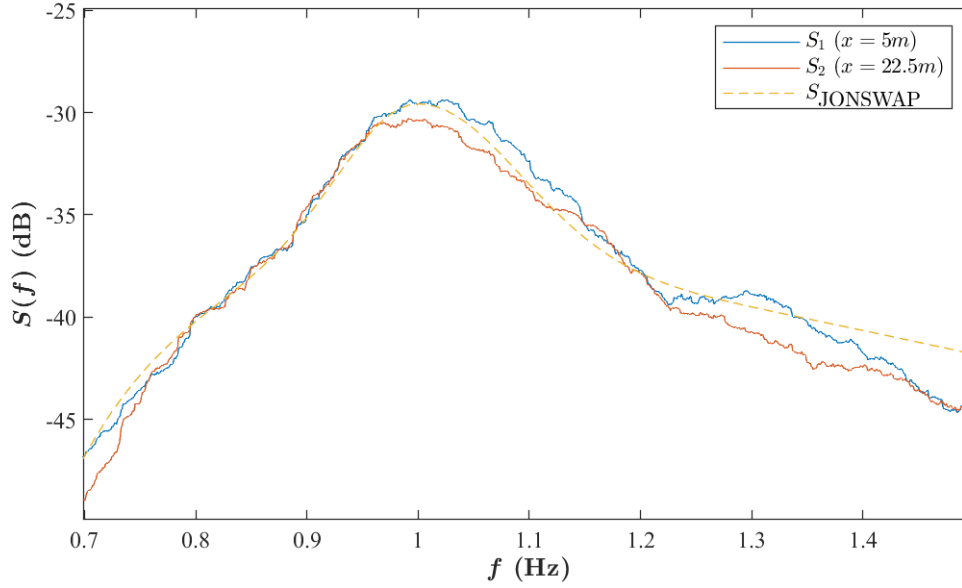


FIGURE 2.10. Example of the analytical JONSWAP spectrum (orange dashed line) and the experimentally-generated unidirectional JONSWAP wave spectrum at $x = 5$ m (s_1 curve in blue colour) and $x = 22.5$ m (s_2 curve in red colour). Here, the significant wave height $H_s = 0.06$, dominant frequency $f = 1$ Hz, water depth $h = 0.7$ m, and the JONSWAP spectrum's peak enhancement factor $\gamma = 6$, corresponding to case 7 in Table 9.1. All spectra are in the dB unit. Wave reflections from the beach are negligible.

where

$$r = \exp \left[-\frac{(\omega - \omega_p)^2}{2\Lambda^2\omega_p^2} \right]. \quad (2.53)$$

Here, α_P is referred to as the Phillips constant, which affects the spectrum intensity and the significant wave height of the JONSWAP wave, ω_p is the peak angular frequency, γ is the peak enhancement factor that affects the bandwidth of the spectrum. The constant $\Lambda = 0.07$ when $\omega \leq \omega_p$, and $\Lambda = 0.09$ when $\omega \geq \omega_p$. By conducting the inverse Fourier transformation with random phases, the JONSWAP wave elevation in the time domain can be obtained. In the following study, the random JONSWAP wave is tested experimentally and numerically to investigate the statistical properties of the random wave train under different circumstances. An example of the theoretical and experimental unidirectional JONSWAP wave spectrum is shown in Figure 2.10.

2.3.2 Skewness and kurtosis

The skewness indicates the asymmetry of the wave elevation distribution [6, 83]. The expression of the skewness is given as

$$\text{skewness} = \frac{\mu_3}{\sigma^3}, \quad (2.54)$$

where μ_3 stands for the third central moment. Note that the skewness decreases due to the increase of wave nonlinearity [140].

On the other hand, the kurtosis κ , also known as the standardized moment, can be affected by nonlinearity and is a good indicator of the extreme events in the ocean, i.e. the level of "tailedness" of the wave distribution [83, 140, 141]. The expression of the kurtosis is

$$\kappa = \frac{\mu_4}{\sigma^4}, \quad (2.55)$$

where μ_4 stands for the fourth central moment. If the wave elevation follows the Gaussian (normal) distribution function, κ is three. The excess kurtosis is defined as $\kappa - 3$. As a consequence of the Central Limit Theorem, linear irregular waves producing a Gaussian field are expected to have a kurtosis value of $\kappa = 3$. On the other hand, nonlinear irregular waves are often found to have kurtosis $\kappa > 3$ as a deviation from a Gaussian field because of the violation of the Central Limit Theorem. Note that in laboratory experiments, the kurtosis of the irregular wave train near the paddle is close to 3 since the waves are generated as a linear superposition of sinusoidal waves with random phases [138]. While the wave propagates, the kurtosis increases as a result of nonlinear wave evolution [141]. In fact, the skewness and kurtosis of the wave elevation are related to the second and third-order nonlinear effects, respectively [142].

2.3.3 Benjamin-Feir index

Besides kurtosis and skewness, the Benjamin-Feir index (BFI) is another indicator to predict the stochastic wave nonlinearity [83, 143], expressed by the ratio between the characteristic

wave steepness α_c and the normalized spectrum width δ_ω [83]

$$\text{BFI} = \frac{\sqrt{2}\alpha_c}{\delta_\omega}, \quad (2.56)$$

with

$$\delta_\omega = \sigma_\omega / \omega_p, \quad (2.57)$$

where ω_p is the wave peak frequency, and spectrum width σ_ω can be defined as the spectrum interval above a certain fraction (usually half) of the maximum spectrum power, i.e., the full width at half maximum (FWHM). Besides the traditional FWHM method, note that by means of the peakedness parameter Q_p [5, 144], which reflects the integral properties of the spectrum [143], the random wave spectrum width can be estimated more accurately [143, 6]. Given a Gaussian-shaped wave spectrum, δ_ω can be expressed as [5, 144]

$$\delta_\omega = 1/\sqrt{\pi}Q_p. \quad (2.58)$$

Note that Q_p is defined as [5]

$$Q_p = \frac{2}{m_0^2} \int_0^\infty f S^2(f) df, \quad (2.59)$$

where m_0 represents the zeroth moment of the spectrum or equivalently the variance σ^2 of the surface elevation $\eta(t)$, f is the frequency, and $S(f)$ is the spectrum power [5, 144].

The BFI can also be adopted to estimate the wave kurtosis κ through the expression mentioned in reference [83] for unidirectional and narrow-banded waves, namely

$$\kappa - 3 = \frac{\pi}{\sqrt{3}} \text{BFI}^2. \quad (2.60)$$

The BFI in the directional sea has also been intensively studied, for example, in references [70, 6, 145, 146]. The two-dimensional BFI, or BFI_{2D} [6] is formulated as

$$\text{BFI}_{2D} = \frac{\text{BFI}}{\sqrt{1 + \alpha_2 R_0}}, \quad (2.61)$$

where the constant $\alpha_2 = 7.10$ for homogeneous and weakly-nonlinear waves and the expression of R_0 is mentioned in references [6, 146], which is a function related to the wave directionality.

2.3.4 Exceedance probability

Employing the exceedance probability function, the occurrence of extreme wave events can be precisely examined and compared with different prediction models. The earliest and simplest wave occurrence probability derived based on the linear superposition mechanism, also referred to as the Rayleigh (normal) distribution [85], is expressed as

$$P_{\text{Rayleigh}}(\eta_c) = \exp \left(-8 \left(\frac{\eta_c}{H_s} \right)^2 \right), \quad (2.62)$$

with a kurtosis of three describing the superposition of wave components at different frequencies. Indeed, Equation (2.62) is appropriate for estimating the crest distribution of random waves characterized by a narrow band frequency spectrum. However, studies such as in references [147, 87] have identified the equation's limitation when accounting for finite bandwidth effect. To address this, Naess [148] derived a Rayleigh-type distribution for wave height, incorporating the autocorrelation function of the wave elevation. The derived distribution takes the following form

$$P_{\text{Rayleigh-Naess}}(H) = \exp \left(-2 \left(\frac{H}{H_s \sqrt{1 - R(T/2)/\sigma^2}} \right)^2 \right), \quad (2.63)$$

where $R(\tau) = E[\eta(t + \tau)\eta(t)]$ denotes the autocorrelation function [148] and E denotes the expectation function. It has been observed that the Rayleigh-type distribution, in practice, tends to underestimate the occurrence of extreme events in the ocean by a significant factor of two orders of magnitude [149]. Consequently, a more precise model that incorporates wave nonlinearity needs to be taken into consideration [150, 151].

A typical nonlinear wave distribution, also known as the Tayfun distribution, by considering the narrow-band nonlinear Stokes effect [152] writes

$$P_{\text{Tayfun}}(\eta_c) = \exp \left(-8 \left(\frac{\sqrt{1 + 2kH_s\eta_c} - 1}{kH_s} \right)^2 \right), \quad (2.64)$$

with a kurtosis value greater than three because of a longer "tail" on its distribution. The modelling of extreme waves in the ocean continues to be investigated, which cannot be exhausted in the current thesis. In Figure 2.11, the exceedance wave crest probability is compared, including the experimental unidirectional wave data, Rayleigh distribution, and Tayfun distribution. In Figure 2.12, the wave height distribution of the same case is compared with the Rayleigh distribution. From both wave crest and height exceedance probability, the experimental data clearly differs from the Rayleigh distribution, which indicates strong wave nonlinearity under the current wave steepness. It is important to note that the original Tayfun distribution [152] was not derived to estimate wave height distributions while accounting for wave nonlinearity but rather the crest height distribution. On the other hand, the Naess distribution [148] takes into consideration larger bandwidth. Later, Tayfun [153] derived the nonlinear wave height distribution, which he compared with the Longuet-Higgins distribution [154], the Naess distribution [148], and the Vinje distribution [155]. Additionally, Tayfun and Fedele [156] compared their theories with the above wave height distributions. These theories will contribute to further enriching the content of Chapter 8 and motivating future studies. The reader can refer to references [157, 159, 87, 153, 160, 161, 162, 163, 164, 158] for more comprehensive progress on the estimation of extreme wave distribution.

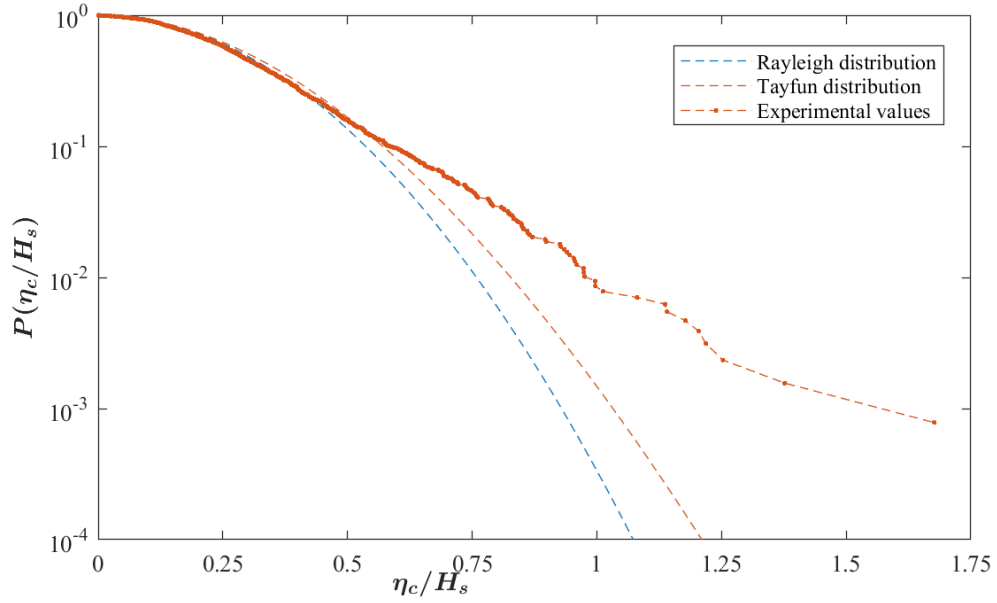


FIGURE 2.11. Comparison of the exceedance crest height probability of the experimental uni-directional wave elevation (red dotted line), Rayleigh distribution (blue dashed line) and the corresponding Tayfun distribution (Orange dashed line). Here, the significant wave height $H_s = 0.06$, dominant frequency $f = 1$ Hz, water depth $h = 0.7$ m, the JONSWAP spectrum's peak enhancement factor $\gamma = 6$, and the total number of waves is around 1200, corresponding to case 7 in Table 9.1. The experimental wave parameters are $f = 1$ Hz, $H_s = 0.06$ m, $h = 0.7$ m, $\gamma = 6$.

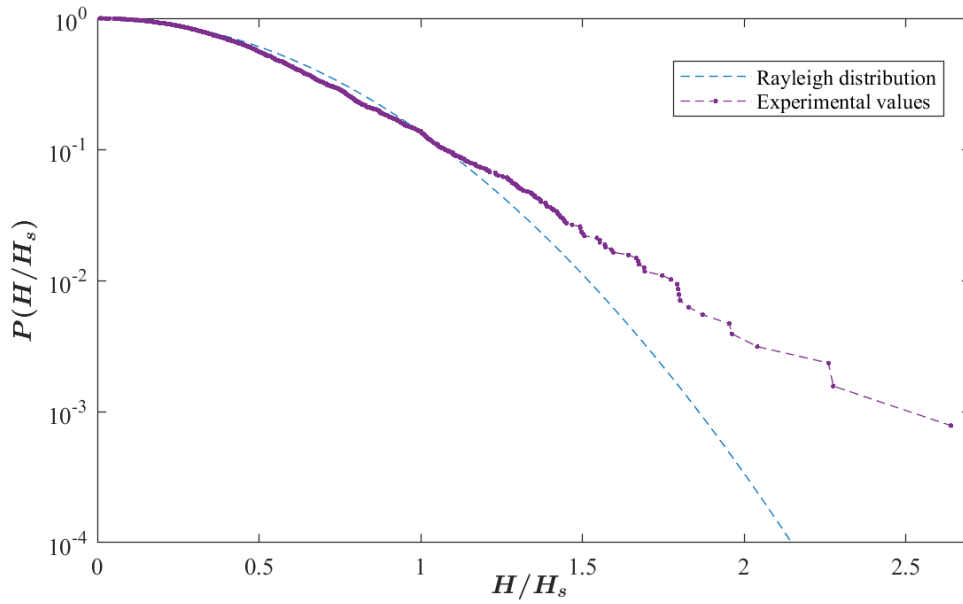


FIGURE 2.12. Comparison of the exceedance wave height probability of the experimental uni-directional wave elevation (purple dotted line) and the Rayleigh distribution (blue dashed line). Here, the significant wave height $H_s = 0.06$, dominant frequency $f = 1$ Hz, water depth $h = 0.7$ m, the JONSWAP spectrum's peak enhancement factor $\gamma = 6$, and the total number of waves is around 1200, corresponding to case 7 in Table 9.1. The experimental wave parameters are $f = 1$ Hz, $H_s = 0.06$ m, $h = 0.7$ m, $\gamma = 6$.

Experimental and numerical methods

The current investigations were mainly carried out experimentally and numerically. As the experimental facility, a wave flume at the University of Sydney was adopted to generate waves using its wave paddle. The wave elevation was recorded along the tank using up to eight resistive wave gauges. A more comprehensive explanation of the experimental methods is given in Section 3.1.

The experimental results were numerically validated based on the NLSE, MNLSE, and CNLSE frameworks. The simulations were carried out using the pseudo-spectral or split-step method, depending on the types of theoretical frameworks. The numerical method and its validation are described in Section 3.2.

Moreover, the data analysis methods adopted in the current study, such as the Hilbert transform, data smoothing algorithms and the data aligning approach, are mentioned in Section 3.3.

3.1 Experimental procedures

3.1.1 Water wave tank

The experiments in the current thesis were carried out in the wave flume, as shown in Figure 3.1 installed within the fluids lab at the University of Sydney Civil Engineering building. The flume has a dimension of $30 \times 1 \times 1 \text{ m}^3$ and an effective water depth range of $h = 0.2$ to 0.8 m . A piston-type wave maker can generate various waves with frequencies ranging from 0.4 to 2 Hz . An inclined slope (beach) covered with artificial grass, honeycomb structure

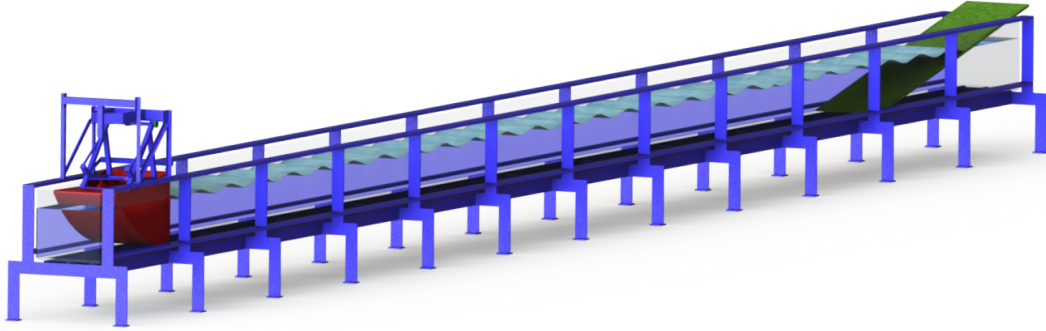


FIGURE 3.1. Wave flume setup and sketch, as installed at the University of Sydney. A piston-type wave paddle is installed on the left side of the flume, and an absorptive beach is installed at the opposite end.

or smooth board can be installed at the opposite end to absorb or reflect the wave energy depending on its material and angle, allowing uni- or multi-directional investigations of the wave field. Here, the angle between the beach and the flume bottom is defined as θ (in degree). At a small beach angle, the wave reflections are negligible and do not interfere with the evolution of the incident waves [1, 2, 3]. The beach angle θ can be also raised to increase the wave reflectivity.

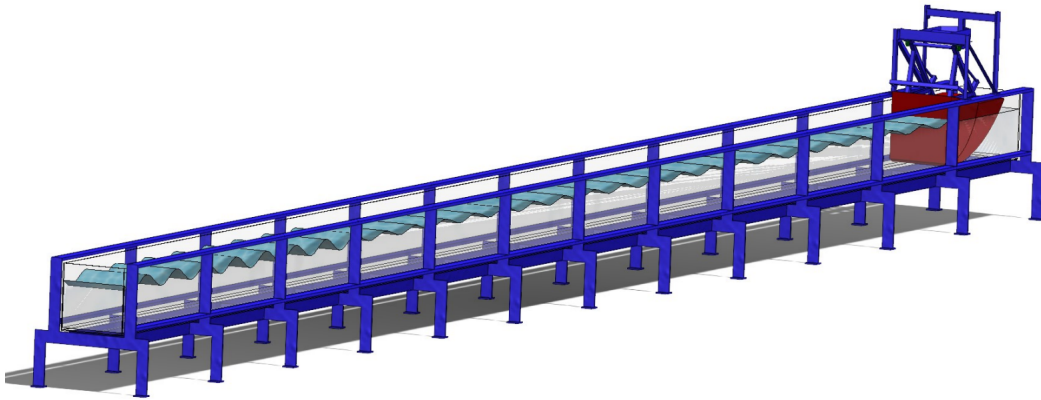


FIGURE 3.2. Sketch of the wave flume setup as installed at the University of Sydney after removing the artificial beach installation to enable full-wave reflection. The piston-type wave generator on the right end generates long-crested unidirectional waves. The glass wall and wave mirror are at the opposing end.

A quasi-full reflection can be achieved when the beach is vertical or removed ($\theta = 90^\circ$), as shown in Figure 3.2, allowing the generation of the so-called standing wave field. In this

figure, the waves on the right of the flume indicate a regular unidirectional wave train, while the waves on the left with a higher amplitude suggest the superposition between the incident and reflected wave trains.



FIGURE 3.3. Photo of a resistive wave gauge installed at the wave flume. The bottom u-shaped long conductive metal is partly immersed to measure the water level. The cable extended from the wave gauge transfers the electric current signal to the USB fast hub interface and eventually to the data capture file.

3.1.2 Wave generation and data acquisition

The experimental wave elevation time series $\eta(x_{\text{ini}}, t)$ at the initial location x_{ini} , as Equation (2.27) defines, are computed from analytical complex envelopes using MATLAB and imported through a software called the Wave Synthesiser into the control software named the Runtime Client, which is adopted to control the wave paddle. It should be noted that the wavemaker has been commissioned by Edinburgh Design, operates following the second-order theory, and utilizes a force-control motion, as described in reference [165]. As a result, the wavemaker produces a negligible amount of spurious waves. Previous experimental tests have confirmed

that no significant spurious waves, capable of significantly impacting the results, have been observed. For further information, the manual from reference [166] provides comprehensive information on the wave generation system installed. The sampling frequency of the input data is fixed at 100 Hz, which is sufficient to accurately depict any wave envelope under the current wave paddle frequency limitation, as mentioned above (0.4 to 2 Hz).

The wave gauges, as shown in Figure 3.3, measure the wave elevation $\eta(x_g, t)$ at each gauge location x_g with a sampling frequency of 32 Hz by detecting the electrical voltage in the circuit. Up to eight wave gauges can be deployed across the flume, depending on the specific experimental needs. To improve the spatial resolution of the experimental data, the gauges can be relocated and the same cases repeated. To ensure the accuracy of the measurements, the wave gauges are calibrated using a standard three-point method prior to all experiments.



FIGURE 3.4. Photo of the piston-type wave paddle, as installed at the wave flume and controlled by the Njord runtime engine software.

The experimental setup for generating water waves in the tank includes a piston-type wave maker, as shown in Figure 3.4. The wave maker is driven by a motor installed above the tank, which causes it to move back and forth and generate circular movements in the water,

resulting in propagating water waves. The wave maker's dynamics and energy requirements are described in more detail in reference [167].

3.2 Numerical methods solving partial differential equations

Self-developed programs written in MATLAB code were adopted to resolve NLSE, MNLSE, and CNLSE problems, which were then compared with the experimental observations. Note the x_{ini} always corresponds to the location $x_{\text{wave maker}}$. Two main numerical schemes were adopted in the current study: the pseudo-spectral scheme based on the fourth-order Runge–Kutta algorithm, and the split-step scheme. Detailed explanations of these numerical schemes can be found in reference [168].

Note that from practice, the split-step method is more accurate and faster, while the pseudo-spectral method is more flexible. Accordingly, the current study adopts the split-step method to resolve the NLSE problem and the pseudo-spectral method to solve the MNLSE and CNLSE problems. The numerical stability and accuracy were tested for each simulation case in the chapters below to ensure data reliability.

In Figures 3.5 and 3.6, the NLSE simulation using the split-step approach has been validated through a pure envelope soliton, a second-order envelope soliton ($N = 2$), and breather solutions (AB and PB) with validated spatial and temporal resolutions and domain sizes. Note that the numerical errors are approximately 0.1% of the original wave amplitude.

The MNLSE simulation employing the pseudo-spectral method has been validated using PB initiated from different x_{ini} , i.e., with boundary conditions $\psi_{\text{PB}}(x_{\text{ini}}, t)$. The validation results are shown in Figure 3.7. The envelope asymmetry and wave recurrence arise inevitably within the higher-order MNLSE numerical scheme due to higher-order effects, and the accuracy reduces with the increase of x_{ini} [121, 4]. The current validation is comparable with that in reference [121], suggesting the reliability of the current MNLSE numerical scheme. Note the long-term evolution of modulationally unstable waves does not follow the NLSE framework as these experience a recurrent wave focusing.

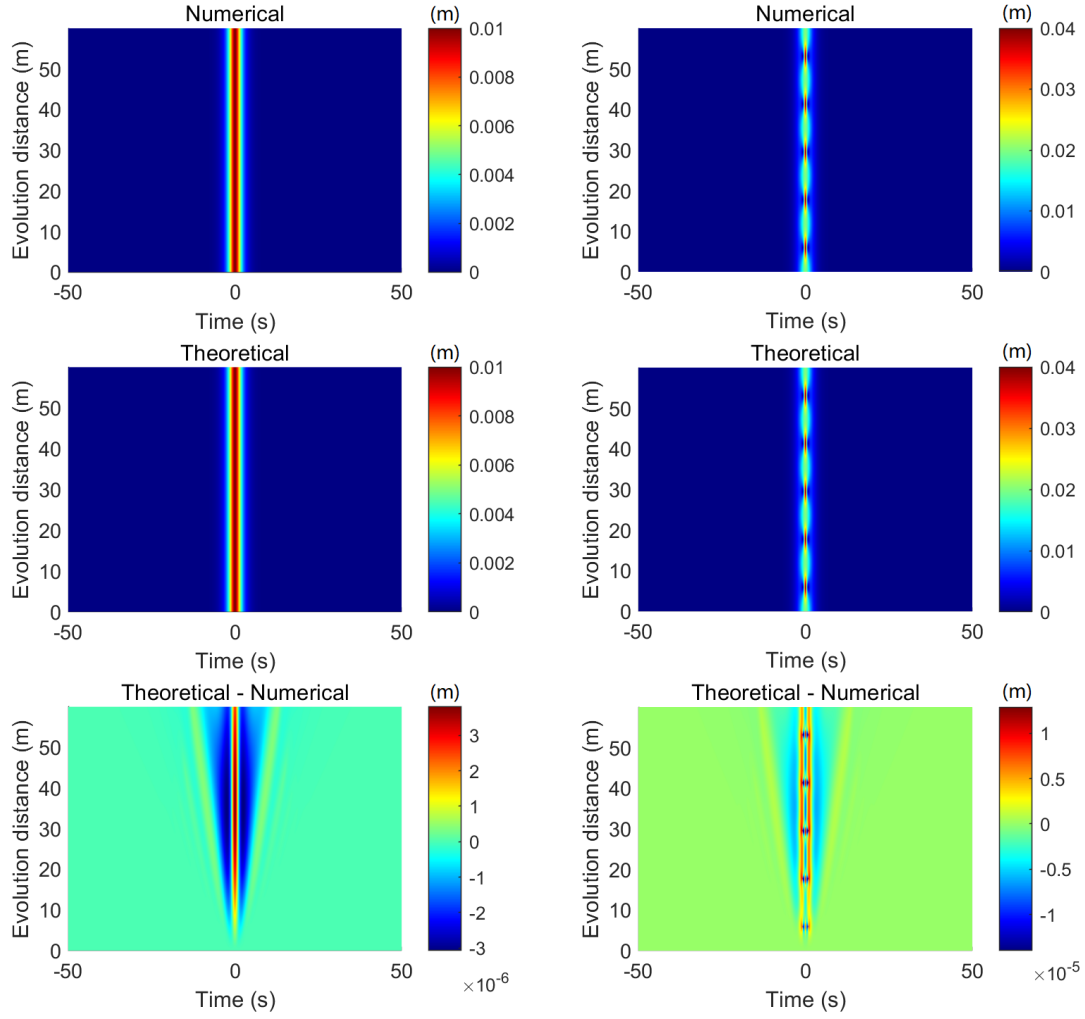


FIGURE 3.5. Validations of the NLSE split-step simulation on soliton (left) and multi-soliton (right). The steepness $ak = 0.1$ and $a = 0.01$ m. The first row represents the numerical results, the second row is the theory, and the last row is the differences between numerical and theoretical results. The simulation temporal resolution $dt = 0.1$ s, spatial step $dx = 0.025$, the simulation temporal domain $t = 240$ s.

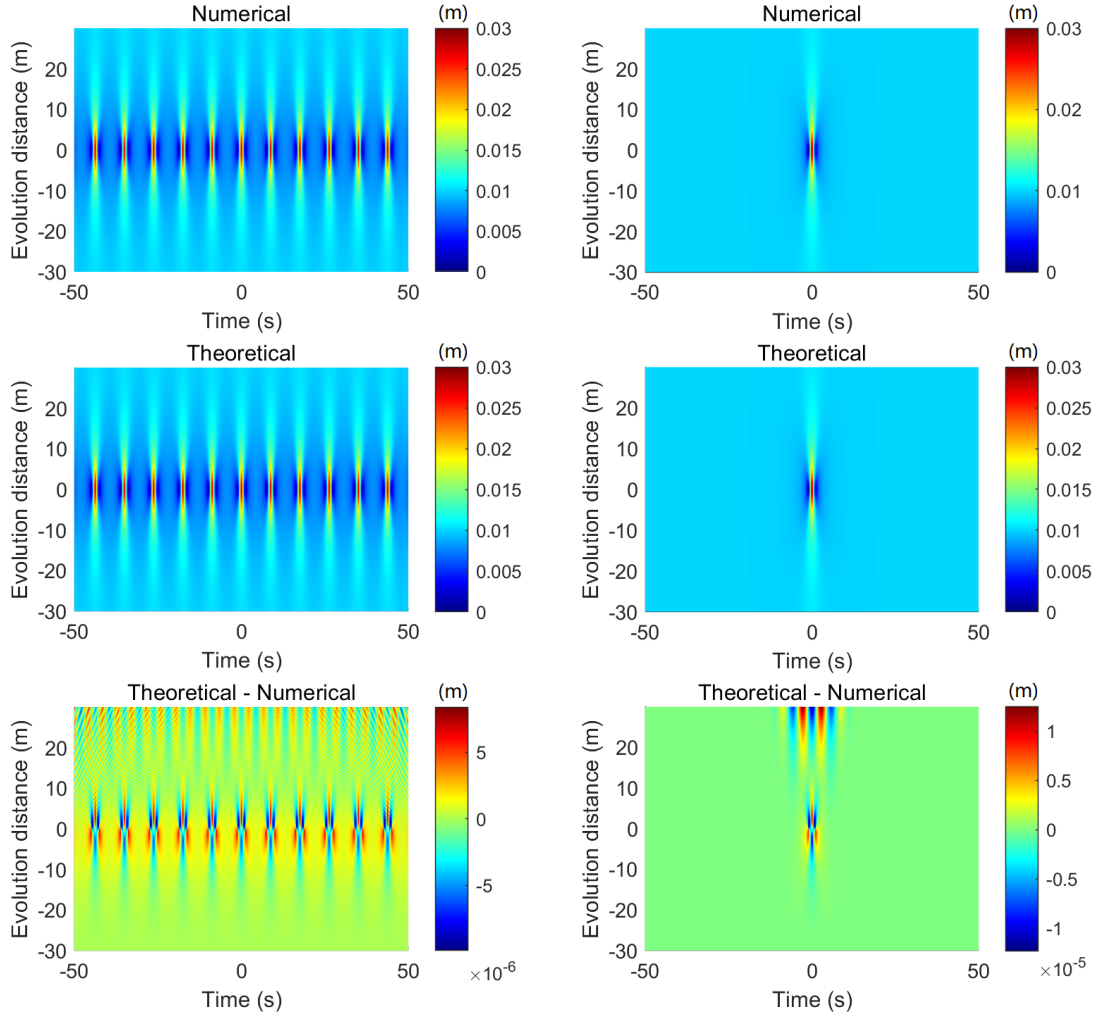


FIGURE 3.6. Validations of the NLSE split-step simulation on AB (left) and PB (right). The steepness $ak = 0.1$ and $a = 0.01$ m. The first row represents the numerical results, the second row is the theory, and the last row is the differences between numerical and theoretical results. The simulation temporal resolution $dt = 0.1$ s, spatial step $dx = 0.025$, the simulation temporal domain $t = 240$ s.

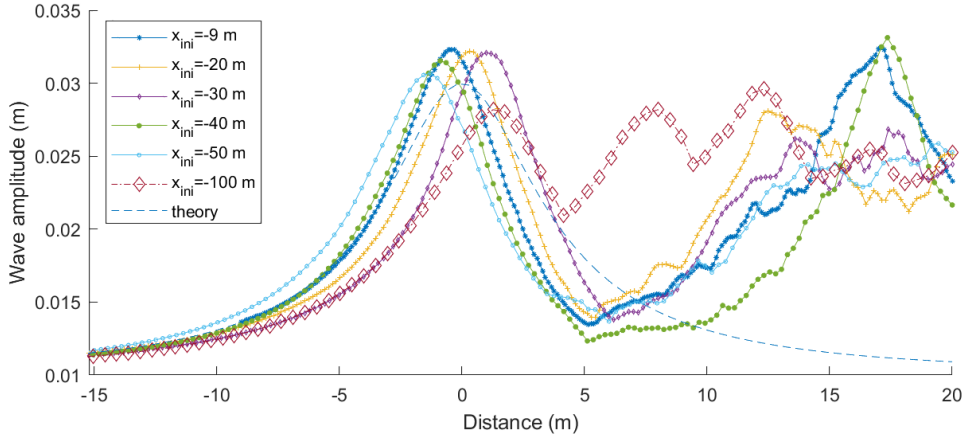


FIGURE 3.7. Validations of the MNLSE pseudo-spectral simulation using PB. The steepness $ak = 0.117$ and $a = 0.01$ m. The maximum wave amplitude evolutions over time are compared in this figure. Note that the blue dashed line is the theoretical prediction, and other curves with markers are simulation results initiating from different locations x_{ini} . The simulation temporal resolution $dt = 0.1$ s, spatial step $dx = 0.01$, the simulation temporal domain $t = 1000$ s.

In order to numerically solve the CNLSE model, which is represented by Equation (2.50), the pseudo-spectral scheme is employed to advance the two counter-propagating complex envelopes either in space or time. For instance, the discretized version of Equation (2.50) can be expressed as

$$\begin{aligned}\psi_{n,x}^{(1)} &= -\frac{1}{c_g}\psi_{n,t}^{(1)} + i\left(\frac{\mu}{c_g^3}\psi_{n,tt}^{(1)} + \frac{\nu}{c_g}(|\psi_n^{(1)}|^2 - 2|\psi_n^{(2)}|^2)\psi_n^{(1)}\right), \\ \psi_{n,x}^{(2)} &= \frac{1}{c_g}\psi_{n,t}^{(2)} + i\left(\frac{\mu}{c_g^3}\psi_{n,tt}^{(2)} + \frac{\nu}{c_g}(|\psi_n^{(2)}|^2 - 2|\psi_n^{(1)}|^2)\psi_n^{(2)}\right),\end{aligned}\tag{3.1}$$

where ψ_n represents the solution on a grid point t_n . Using discrete Fourier transform and assuming periodic boundary conditions, $\psi_{n,t}^{(i)}$ and $\psi_{n,tt}^{(i)}$ can be expressed as

$$\begin{aligned}\psi_{n,t}^{(i)} &= \mathfrak{F}^{-1}[(if)\mathfrak{F}(\psi_n^{(i)})], \\ \psi_{n,tt}^{(i)} &= \mathfrak{F}^{-1}[(if)^2\mathfrak{F}(\psi_n^{(i)})],\end{aligned}\tag{3.2}$$

where $\mathfrak{F}$ and $\mathfrak{F}^{-1}$ are the discrete Fourier and inverse Fourier transforms. The fourth-order Runge-Kutta method integrates Equation (3.1) in space. It is important to note that in order

to couple the equations, the integration part of the Runge-Kutta scheme is constructed as illustrated below

LISTING 3.1. The integration part for one space step of the Runge-Kutta scheme adopted

```

du11 = -2*k1/w*ifft(f.*fft(v1)) + (-k1/w^2)*1i*ifft(f.^2.*fft
    (v1)) + (-k1^3)*1i*(v1.*conj(v1) -2*v2.*conj(v2)).*v1;
du12 = -2*k2/w*ifft(f.*fft(v2)) + (-k2/w^2)*1i*ifft(f.^2.*fft
    (v2)) + (-k2^3)*1i*(v2.*conj(v2) -2*v1.*conj(v1)).*v2;
v1 = u1 + 0.5*du11*dx;
v2 = u2 + 0.5*du12*dx;

du21 = -2*k1/w*ifft(f.*fft(v1)) + (-k1/w^2)*1i*ifft(f.^2.*fft
    (v1)) + (-k1^3)*1i*(v1.*conj(v1) -2*v2.*conj(v2)).*v1;
du22 = -2*k2/w*ifft(f.*fft(v2)) + (-k2/w^2)*1i*ifft(f.^2.*fft
    (v2)) + (-k2^3)*1i*(v2.*conj(v2) -2*v1.*conj(v1)).*v2;
v1 = u1 + 0.5*du21*dx;
v2 = u2 + 0.5*du22*dx;

du31 = -2*k1/w*ifft(f.*fft(v1)) + (-k1/w^2)*1i*ifft(f.^2.*fft
    (v1)) + (-k1^3)*1i*(v1.*conj(v1) -2*v2.*conj(v2)).*v1;
du32 = -2*k2/w*ifft(f.*fft(v2)) + (-k2/w^2)*1i*ifft(f.^2.*fft
    (v2)) + (-k2^3)*1i*(v2.*conj(v2) -2*v1.*conj(v1)).*v2;
v1 = u1 + du31*dx;
v2 = u2 + du32*dx;

du41 = -2*k1/w*ifft(f.*fft(v1)) + (-k1/w^2)*1i*ifft(f.^2.*fft
    (v1)) + (-k1^3)*1i*(v1.*conj(v1) -2*v2.*conj(v2)).*v1;
du42 = -2*k2/w*ifft(f.*fft(v2)) + (-k2/w^2)*1i*ifft(f.^2.*fft
    (v2)) + (-k2^3)*1i*(v2.*conj(v2) -2*v1.*conj(v1)).*v2;

```

```

u1 = u1 + (du11 + 2*du21 + 2*du31 + du41)*dx/6;
u2 = u2 + (du12 + 2*du22 + 2*du32 + du42)*dx/6;

```

In the provided MATLAB code snippet, the variables $u1$ and $u2$ correspond to the coupled complex envelopes, while $k1$ and $k2$ represent the wavenumbers of the counterpropagating wavetrains. The intermediate values during the integration process are denoted by $v1$ and $v2$. It is worth noting that the values of $v1$ and $v2$ are computed using the previous pair of intermediate values to ensure proper equation coupling in each integration step. Prior to conducting the numerical tests, the stability and accuracy of the scheme have been thoroughly verified.

3.3 Data analysis

Since the experimental data is in the form of fast-varying elevation series, restoring the data into slowly-varying complex envelopes is necessary to facilitate the observations with the theoretical wave envelope models. The phase information can also be calculated conveniently from the complex envelope, which is crucial for extreme wave generation [43, 44, 68, 1, 2]. Hilbert transformation [93], also written as $\mathcal{H}$, is a common approach to obtain the complex envelope from surface elevation series, which can be related to the wave elevation η and envelope ψ through the following expression

$$\psi(x_g, t) = [\eta(x_g, t) + i\mathcal{H}(\eta(x_g, t))] \exp[i(kx_g - \omega t)]. \quad (3.3)$$

To avoid envelope distortion during the transformation, a band-pass filter should be adopted to remove the second harmonics in the data.

To reduce the complexity of standing waves, a simple bi-directional spectrum method based on the linear propagation assumption is adopted to isolate the wave trains in each direction, as derived in reference [169]. This method has been applied in Chapter 7 and Chapter 8 to isolate the incident and reflected waves.

In order to obtain smoother spatial data sets, interpolation techniques were applied to the experimental results. This was particularly useful for slowly-varying unidirectional wave envelopes, as seen in Figures 5.1 and 6.4. In order to enhance the appearance of the figures, the envelopes and spectrums were also smoothed using the Savitzky-Golay filter [170, 171], which is a more advanced and accurate smoothing method compared to the moving average technique.

In order to improve the clarity of the evolution of wave envelopes in the figures, the experimental data was aligned using the group velocity c_g . This involved shifting the data backwards by $\Delta t = x_g/c_g$ to eliminate the effect of the group velocity c_g at each gauge location x_g . It is important to note that this approach is not applicable for standing wave fields, where there are two groups of waves propagating in opposite directions.

Please note that in the standing wave field, the use of smoothing, interpolation, and aligning techniques may not be appropriate due to the presence of dense and localized standing nodes throughout the counter-propagating waveguides [4]. In order to increase the spatial resolution and capture all localized nodes accurately, the wave gauges must be constantly moved along the wave flume.

CHAPTER 4

Phase evolution of the extreme waves in time- and space-NLSE frameworks

The content of this chapter is based on the following publication:

He, Y., Suret, P., & Chabchoub, A. (2021). Phase Evolution of the Time-and Space-Like Peregrine Breather in a Laboratory. *Fluids*, 6, 308.

Following the space- and time-NLSE theoretical frameworks described in Section 2.2.5, in this chapter, the PB phase dynamics under both time- and space-NLSE frameworks are studied using a wave tank.

4.1 Research alignment and design

In the current experiments, the steepness of the initial condition is set to be $ak = 0.1$, which is moderate and below the breaking threshold [172, 120]. The amplitude of the carrier wave here is selected to be $a = 0.01$ m. These two key parameters are sufficient to determine the wavenumber $k = 10 \text{ m}^{-1}$, wave frequency $\omega = \sqrt{gk} = 9.81 \text{ s}^{-1}$ and the frequency $f = \frac{\omega}{2\pi} = 1.57 \text{ Hz}$. The parameters chosen allow the clean observation of the growth and decay of the PB in the flume, while the steepness can be regarded as a scaling factor in the modelling [172]. In this case, the dimensionless depth is $kh = 7$, which satisfies the deep-water regime. During the experiment, the two adopted boundary conditions of the PB $A_P(x_{\text{ini}}, t)$ and $B_P(x_{\text{ini}}, t)$ have been defined at $x_{\text{ini}} = -14$ m. These parameters facilitate the start of the breather dynamics from a small perturbation of the carrier wave. Eight calibrated wave gauges have been deployed along the wave facility to record the PB compression and

attenuation. Both PB water surface elevations and corresponding local phase evolutions in the acquired data will be compared with respective theoretical predictions in the next subsection.

4.2 Experimental results

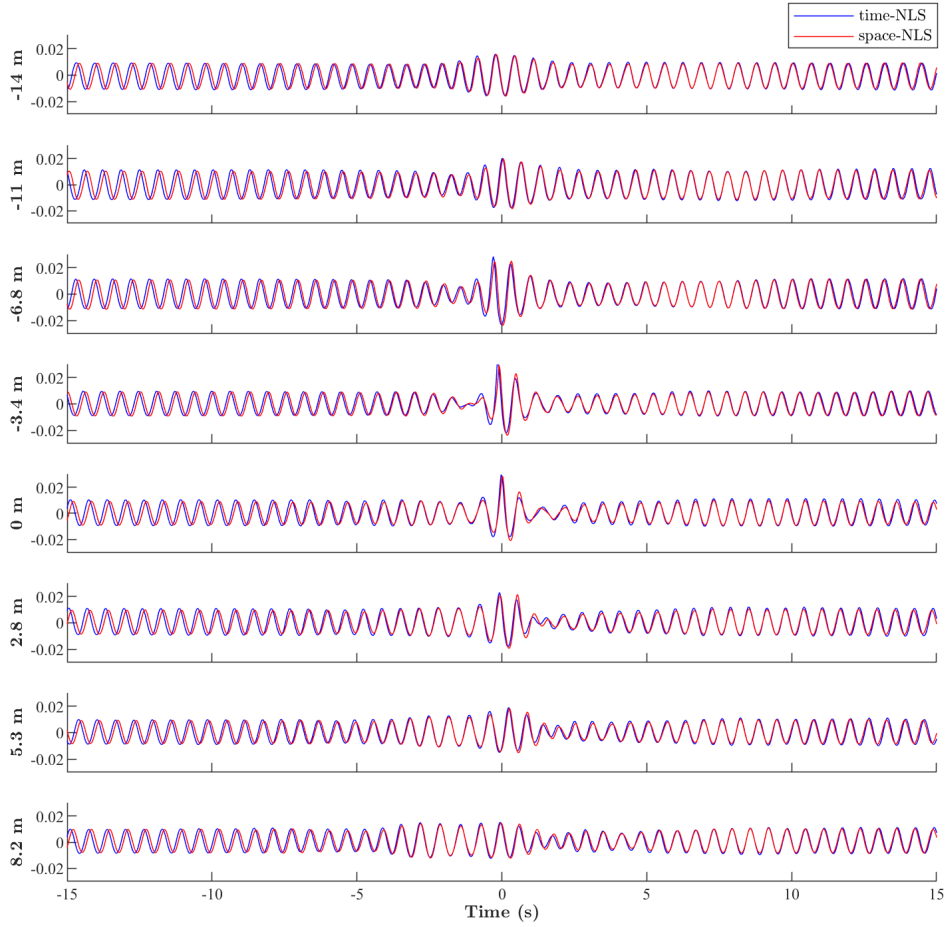


FIGURE 4.1. Superposition and comparison of measured PB hydrodynamics along the wave tank for $ak = 0.10$, $a = 0.01$ m, and $x = -14$ m according to the time-like (blue lines) and space-like NLSE (red lines) frameworks.

As already described in references [16, 19], once the wave maker is programmed to generate the exact boundary condition as defined by theory, the experiment consists of recording the temporal evolution of the wave dynamics along the longitudinal direction of the wave flume by the wave gauges. Next, the surface elevation displacements captured by the wave probes are compared when the boundary condition has been determined following the time- and

space-like NLSE framework. Moreover, to gain an improved quantitative understanding of the expected phase discrepancy in the evolution of both PB wave fields, the wave profiles at the gauge location have been superimposed. The results are shown in Figure 4.1. The Peregrine groups observed are not symmetrical because of higher-order nonlinear effects not captured in the NLSE [121].

Both PB observations were also compared with the corresponding theoretical frameworks, as shown in Figure 4.2.

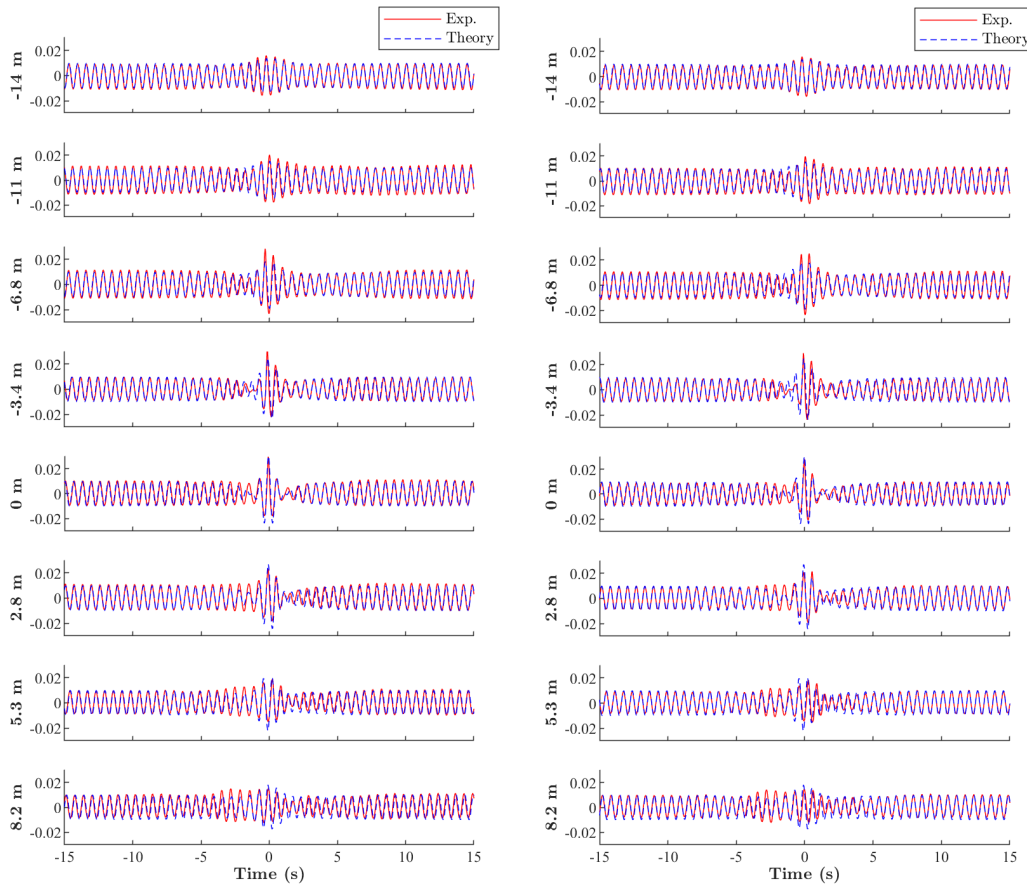


FIGURE 4.2. The experimental measurements compared with the corresponding theoretical predictions for the time- (left) and space-like (right) PB solution.

At first view, both envelopes in Figure 4.1 undergo the same evolution, but there is a distinct phase mismatch, which is obvious when focusing the attention on the generic PB localization. Note that the wave envelopes are identical. Moreover, the breather undergoes a gradual

asymmetric compression, which affects the NLSE comparison accuracy. It is well-known that these are well-captured in higher-order NLSE-type evolution equations and can be tamed by reducing the wave steepness [51, 173, 120, 174]. Since it is not possible to significantly reduce the carrier amplitude, the reduction of wave steepness can be achieved by increasing the wave frequency. However, the increase in the wavelength shortens the effective evolution of the breather in the facility, limited to a total physical propagation distance of 25 m.

On the other hand, it becomes clear that locally, and around the packet, the surface elevation is almost identical, and gradual disagreement in the carrier phase can be observed, affirming a difference in the phase evolution while both envelope amplitudes remain almost identical.

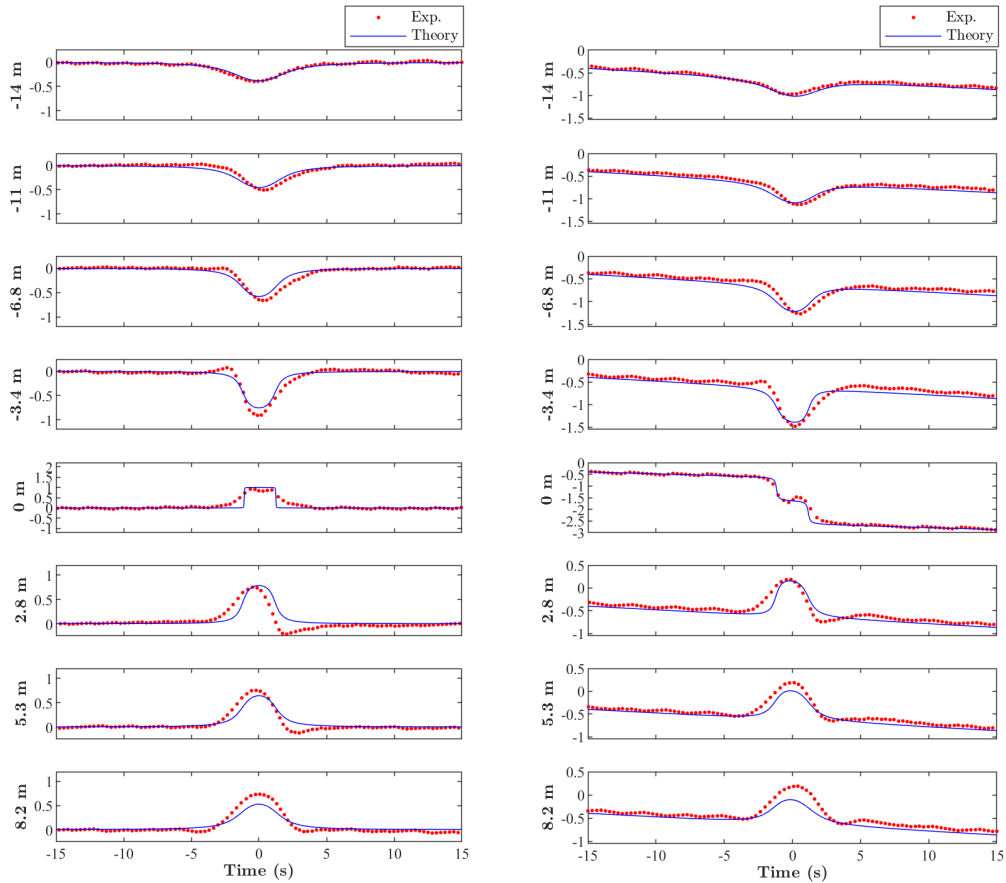


FIGURE 4.3. Experimental (red dots) and theoretical (blue lines) phase evolution (in π) along the wave tank corresponding to the surface evolution measurements of Figure 4.1 for the time-NLSE (left) and space-NLSE (right).

Next, the evolution of the phases along the water wave tank for each of the two runs is tracked. Recall that the reconstruction of the phase profiles can be achieved using the Hilbert transform according to Equation (2.28) and as described in [10]. Note that the phase variations from the water surface measurements are computed by adopting a symmetric truncation around the wave packet. Figure 4.3 shows the results.

Interestingly, despite the fact that the phase evolution in Figure 4.3 differs from one case to the other, there is clear agreement with the underlying time- or space-like NLSE framework prediction. The differences in the jump at the maximal compression location, i.e., $x = 0$ m, are responsible for the mismatch in the surface elevation profiles, as can be seen in Figure 4.1. The discrepancies with NLSE theory already observed in the water surface profiles can be also observed in the evolution of phase profiles, in which deviations are more pronounced after the strong focusing of the breather. This sheds new light on the PB dynamics, which can evolve in two different ways. While the envelope amplitudes are quasi-similar, the phase dynamics are distinct. This feature suggests, for instance, alternatives in the identification of phase-shift profiles.

4.3 Summary

In this chapter, an experimental investigation of the PB phase hydrodynamics is reported, which have been initiated by two different boundary conditions as defined by the time- and space-like NLSE frameworks, respectively. Both envelope expressions being asymptotically equivalent, it is shown in the experiments that despite the almost identical envelope amplitude evolution and the local agreement of the carrier phases around the perturbed wave packet, the global phase dynamics differ. This suggests that two different phase evolution patterns are possible for such time and space-localized rogue wave dynamics. As identified and tracked along the propagation distance in the experiments, these phase jumps agree with the theoretical prediction as provided by the respective time- or space-NLSE framework. These results provide a new fundamental insight into PB-type wave group dynamics. The differences between such phase localizations have been so far overlooked. They might be, for instance,

helpful in developing detection mechanisms for extreme events in nonlinear dispersive media, which form as a result of four-wave interaction.

CHAPTER 5

The property of Galilean-transformed wave groups and supercontinuum generation

The content of this chapter is based on the following publication:

He, Y., Ducrozet, G., Hoffmann, N., Dudley, J. M., & Chabchoub, A. (2022). Galilean-transformed solitons and supercontinuum generation in dispersive media. *Physica D: Nonlinear Phenomena*, 133342.

In this chapter, the Galilean-transformed soliton and PB envelopes are studied, as mentioned in Section 2.2.6. The experimental results are validated using simulations based on the NLSE and MNLSE frameworks according to Equations (2.34) and (2.48), respectively.

5.1 Research alignment and design

In the current experiments, the dominant carrier wave frequency is fixed at $f = 1.25$ Hz, which corresponds to a wavelength of $\lambda = 1$ m and a wavenumber of $k = \frac{2\pi}{\lambda} = 2\pi$. The amplitude a of the carrier wave is varied to satisfy wave steepness α values of 0.04, 0.06, and 0.08, respectively. The dominant carrier frequency shift after the GT, as defined by the GV value c , is also ensured to be within the operational range of the wave maker. The water depth is set to be $h = 0.7$ m and thus, $kh \gg 1$, satisfying the deep-water requirements. The eight wave gauges, as mentioned in Chapter 3, can be redeployed along the flume for a generated time series to allow not only a very high temporal, but also a spatial resolution considering the repeatability of the experiments.

5.2 Experimental results

The focus of the current experimental campaign will be on the Galilean-transformed envelope solitons. The case of the PB is more complex and will be briefly discussed as well.

Both envelope solitons and PBs have been observed in a wide range of physical media [175, 176, 177, 178, 20, 16, 45]. In water waves envelope solitons have been confirmed to be steady even for very large steepness values, even close to the wave breaking threshold [179, 180]. The carrier wave steepness does not only quantify the degree of nonlinearity of Stokes waves but also determines the width of a wave pulse or wave packet. Three wave steepness values have been considered $\alpha \in \{0.04, 0.06, 0.08\}$ as well as negative and positive values of GV. The evolution of envelope solitons subject to GT as propagating over 22 m are shown in Figure 5.1. The wave envelope has been reconstructed from the water surface data using

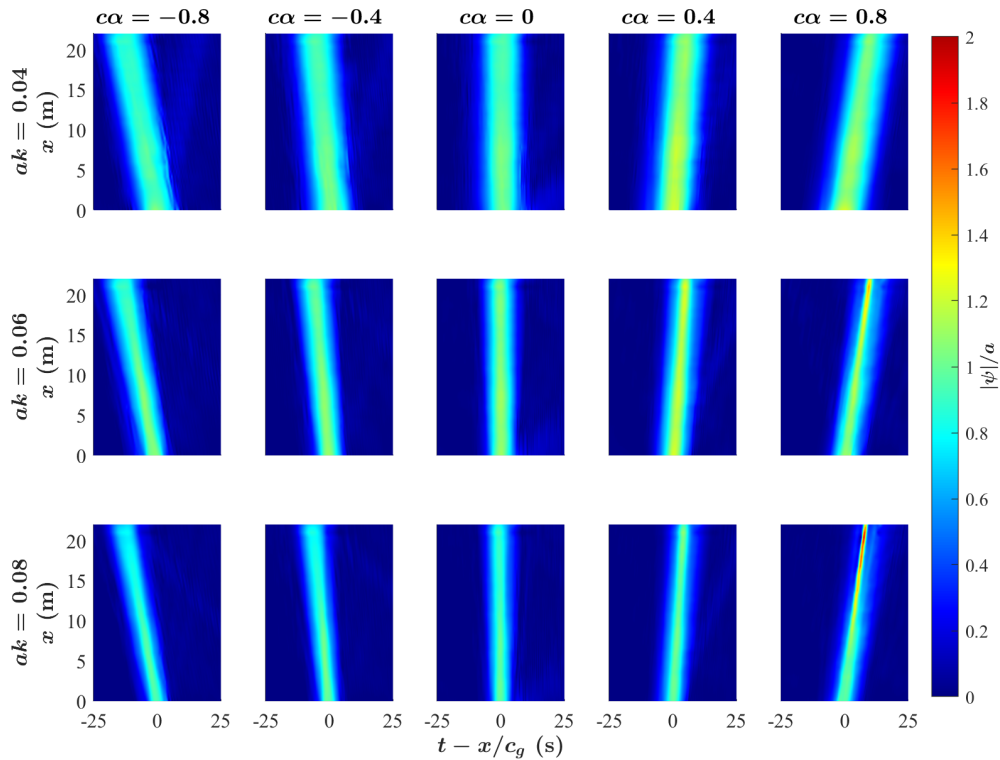


FIGURE 5.1. Experimental observations of Galilean-transformed and pure envelope sech-type solitons.

the Hilbert transform [10]. At first view, the cases for $\alpha = 0.04$ seem to show a steady evolution of the wave packet. However, since this steepness value is indeed very small, it does not allow for the nonlinear interaction to unfold over the limited fetch. Increasing the value of wave steepness reveals that differently than predicted from the NLSE framework, the Galilean-transformed solitons in a physical water waveguide are not stationary, while the type of unsteadiness depends on the sign of GV. When the GV is negative, It can be clearly noticed that the solitons subject to GT broaden, whereas when GV is positive, the Galilean-transformed envelope solitons follow a self-compression in the form of a breathing process. The latter process becomes visible for the highest steepness and GV values adopted in the experiments.

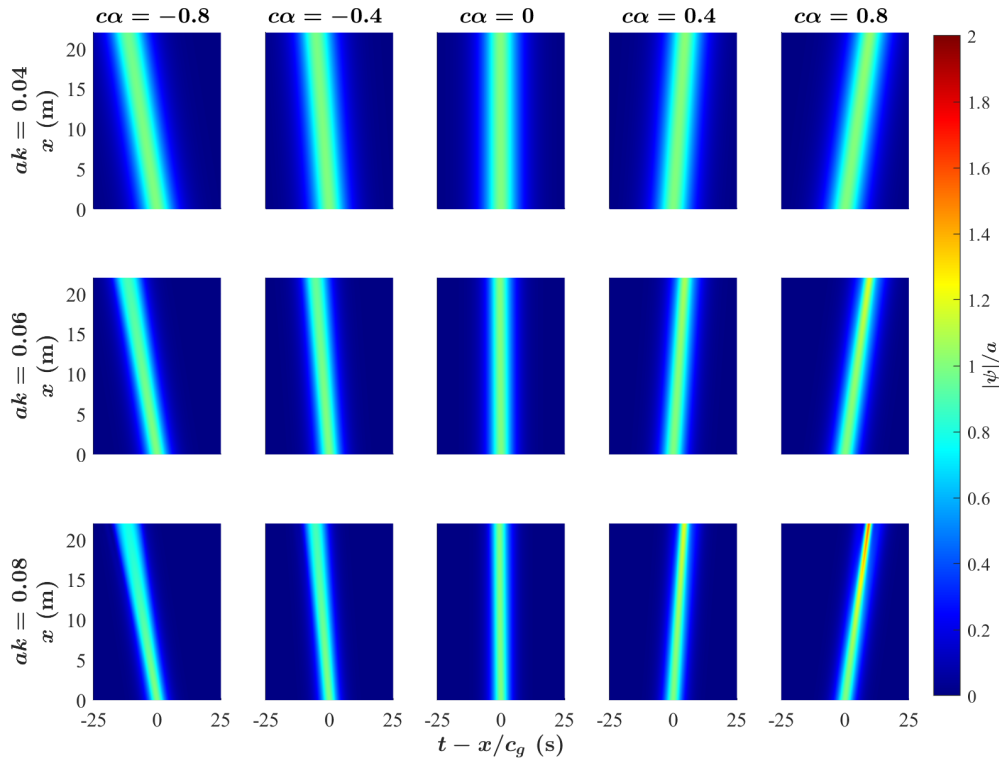


FIGURE 5.2. MNLSE prediction of pure and Galilean transformed envelope solitons as generated in the wave tank and shown in Figure 5.1.

To further examine the physics of these deviations, numerical simulations were performed based on the MNLSE framework (2.48). The MNLSE simulation results, as revealed in Figure 5.2, support the evidence that the unsteadiness of GT wave envelope evolution can

be attributed to the failure of simplified dispersion relationship in the wave packet evolution description, i.e. the dispersion relation is distorted, which is similar to the that of the water wave propagation under the action of a uniform and steady current [30, 181].

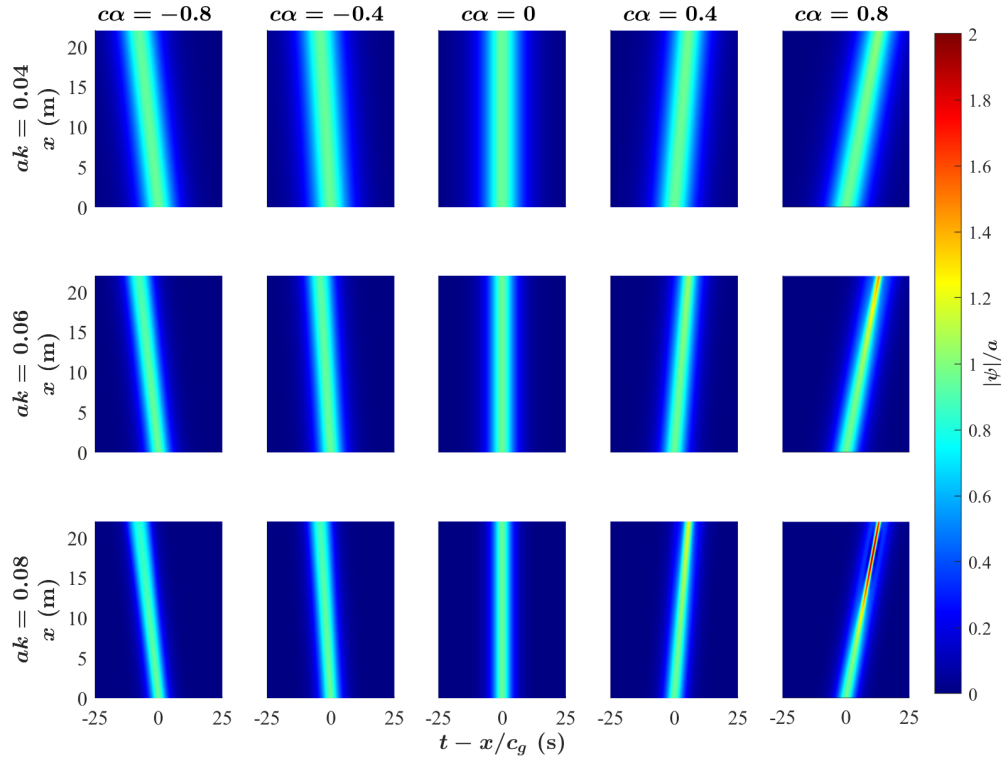


FIGURE 5.3. Frequency-adjusted NLSE predictions of Galilean-transformed envelope solitons as generated in the wave tank and shown in Figure 5.1.

To address the discrepancies resulting from the GT imposed phase correction, see Equations (2.45) and (2.46), simulations of the re-scaled solitons were performed using the conventional NLSE model by adjusting its parameters with respect to the shifted frequency peak. Figure 5.3 reveals that the NLSE predictions of the re-scaled soliton around the effective frequency peak have the same characteristic focusing features as the laboratory observations. Nonetheless, the MNLSE proves to be the better model in capturing the slight focusing retardation compared to the NLSE [120, 121]. It is anticipated that the accuracy of the MNLSE will decrease with the gradual increase or decrease of the phase-shift while the broadband form of MNLSE will provide an improved forecast for these cases [173].

Positive values of GVs have a significant influence on the amplitude variations of envelope solitons subject to GT, see Figure 5.4. No significant change in steadiness can be noticed for

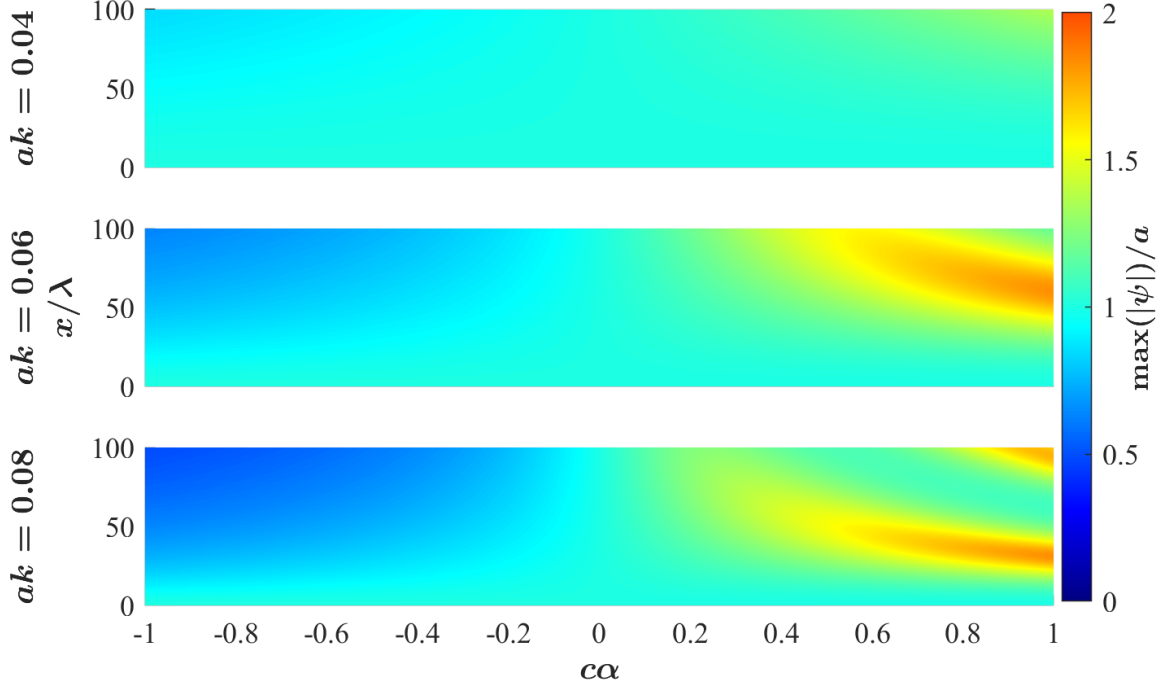


FIGURE 5.4. Maximum amplitude amplification of Galilean-transformed envelope solitons with respect to GV αc and the dimensionless evolution distance, as quantified from MNLSE simulations.

marginal $c\alpha$ values. The observed self-modulation, which has also been underlined by the MNLSE simulations, for positive GVs is indeed similar to the dynamics of multi-solitons [61]. The connection between Galilean-transformed multi-solitons and Satsuma-Yajima solutions will be further explored in the next section.

The investigation of Galilean-transformed solitons on zero-background is simpler due to the absence of MI [182, 7]. Consequently, any yet small perturbation of the background wave can provoke a drastic MI wave focusing [22, 183, 184]. Moreover, the observation of such unstable patterns requires a larger fetch. This makes it challenging to be explored using the University of Sydney wave flume, which is limited to an effective propagation distance of 22 m, considering the frequency range of the wave maker and the limited water depth possibilities.

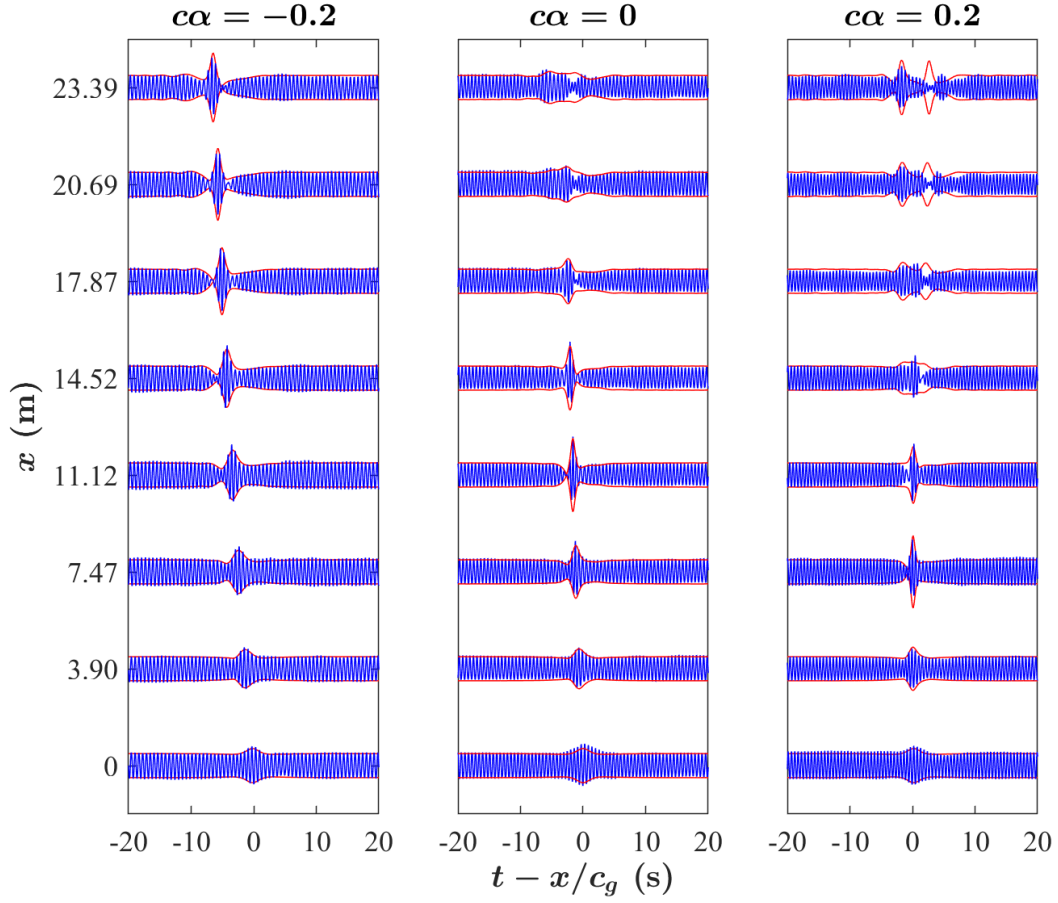


FIGURE 5.5. Galilean-transformed PB evolution in the wave flume (blue lines) for the carrier parameters $ak = 0.1$, $a = 0.01$ m, and $x_{\text{ini}} = -12$ m to observe the maximal compression for the pure case in the middle of the tank. The red lines are the corresponding MNLSE simulations of the wave envelopes.

The propagation of Galilean-transformed PBs with negative and positive GVs in the wave flume and the respective MNLSE expectations are shown in Figure 5.5. Indeed, the PB-type perturbation focuses earlier for the case of a negative GV and later for the positive GV, while the MNLSE simulations agree well with the observations. On the other hand, there is an omnipresent MI dynamics, which cannot be captured within the short fetch. Hence, the propagation distance of the numerical simulations was extended to inspect the long-term behaviour of the Galilean-transformed PB following the parameters adopted in the experiments, see Figure 5.6. The MNLSE simulations suggest that the long-term PB

evolution is indeed complex, particularly when GV is positive. Considering the PB being a particular case of KB [39, 185] and AB [37] breathers, an extensive and comprehensive study is required to address and quantify the influence of the GT on solitons on the finite background. Note that the long-term MNLSE wave prediction using other numerical schemes than the pseudo-spectral method adopted herein may differ from the results in the current research [186].

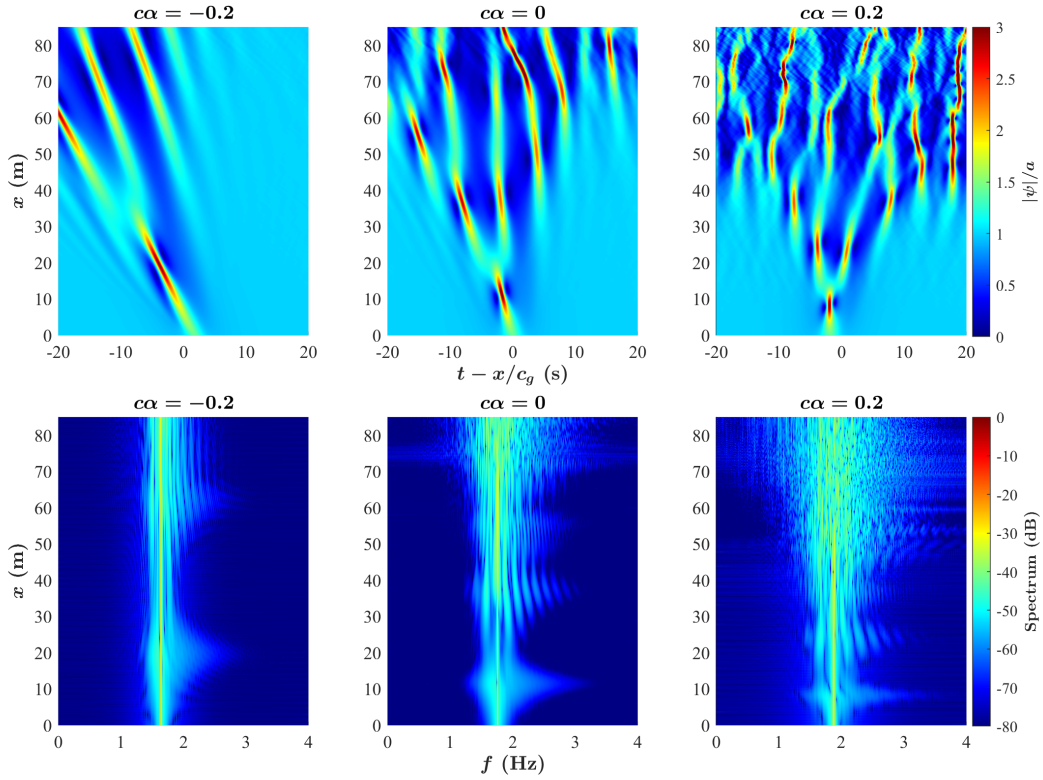


FIGURE 5.6. Long-term MNLSE prediction of Galilean-transformed PB for the same parameters as in Figure 5.5. Top panels: Envelope evolution. Bottom panels: Corresponding spectral evolution.

5.3 Galilean-transformed envelope solitons and multi-solitons

To disclose the relationship between multi-solitons, or Satsuma-Yajima solitons [61], and the Galilean-transformed solitons, it is assumed that both parametrizations are equal at $x = 0$ m, for simplicity. Recall that the dynamics of a multi-soliton ψ_{MS} can be triggered by an

integer-multiple of an envelope sech-type soliton [63]. That is

$$\psi_{MS}(0, t) = N\psi'_s(0, t) = Na' \operatorname{sech}(-\sqrt{2}\alpha'k'(-c'_gt)), \quad N \in \mathbb{N}, \quad (5.1)$$

where $\psi'_s(0, t)$ is a pure envelope soliton of amplitude a' , wavenumber k' , and wave frequency ω' . Assume that the GT of an envelope soliton of amplitude a , wave number k , and wave frequency ω corresponds to a real-multiple of a soliton of amplitude a' , and a wave frequency ω' which corresponds to the wave frequency of the Galilean-transformed soliton (incl. the phase-shift), while $k' = \omega'^2/g$

$$R\psi'_s(0, t) = \psi_{GS}(0, t), \quad R \in \mathbb{R}. \quad (5.2)$$

Since the water surface elevation to first-order in steepness should be identical, the following holds

$$\eta'(0, t) = \Re(R\psi'_s(0, t) \exp(i(-\omega't))) = \Re(\psi_{GS}(0, t) \exp(i(-\omega t))) = \eta_{GS}(0, t), \quad (5.3)$$

which is equivalent to:

$$\begin{aligned} Ra' \operatorname{sech}(-\sqrt{2}\alpha'k'c'_gt) \exp(-i\omega't) = \\ a \operatorname{sech}(-\sqrt{2}\alpha kc_gt) \exp(-i\omega t) \exp(-\frac{ic\sqrt{2}\alpha kc_gt}{2}). \end{aligned} \quad (5.4)$$

One trivial way to solve this problem is to set the amplitudes, sech-components, and phases equal. Consequently, the following set of equations can be established

$$\begin{cases} Ra' = a \\ \alpha'k'c'_gt = \alpha kc_gt \\ \omega't = \omega t + \frac{c\alpha kc_gt}{\sqrt{2}}. \end{cases} \quad (5.5)$$

Using the deep-water expression for the group velocity $c_g = \frac{\omega}{2k}$, Equation (5.5) can be solved, which gives

$$\omega' = \omega \left(1 + \frac{\sqrt{2}}{4}c\alpha\right) \text{ and } R = \left(1 + \frac{\sqrt{2}}{4}c\alpha\right)^3. \quad (5.6)$$

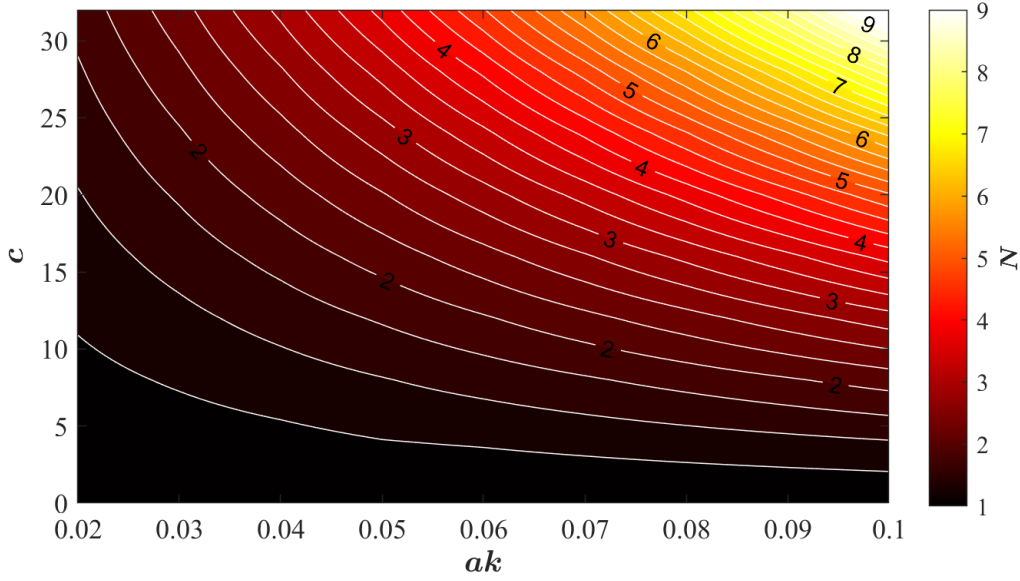


FIGURE 5.7. The order factor N of high-order solitons corresponding to the Galilean solitons versus GS c and steepness values ak .

Equation (5.6) shows that both steepness and GV affect the values of R and ω' . Moreover, these later parameters α and c can be chosen so that R becomes an integer $R = N$ and as such, Galilean-transformed solitons can define boundary conditions to launch exact multi-soliton orbits. Figure 5.7 illustrates the range for the values of ak and c to map the Galilean-transformed solitons to exact Satsuma-Yajima wave groups of order N .

5.4 Supercontinuum generation

Supercontinuum generation is a severe and irreversible broadening of the wave spectrum.

Having established the relationship between Galilean transformed envelope solitons and higher-order solitons, it is self-evident to discuss supercontinuum generation following a soliton-GT. Supercontinua can be formed as a result of soliton fission of higher-order solitons due to the strong unbalance between nonlinearity and dispersion, which arises from the considerable envelope compression or focusing, i.e. substantial high values of nonlinearity [62, 187, 63, 64].

Recurrent multi-soliton focusing and the formation of supercontinua require a long fetch. Since these cannot be met by the state-of-the-art facility as illustrated in Figure 3.1, such dynamics are investigated using a numerical wave tank, based on the higher-order spectral model scheme, which solves the Euler equations [58, 59].

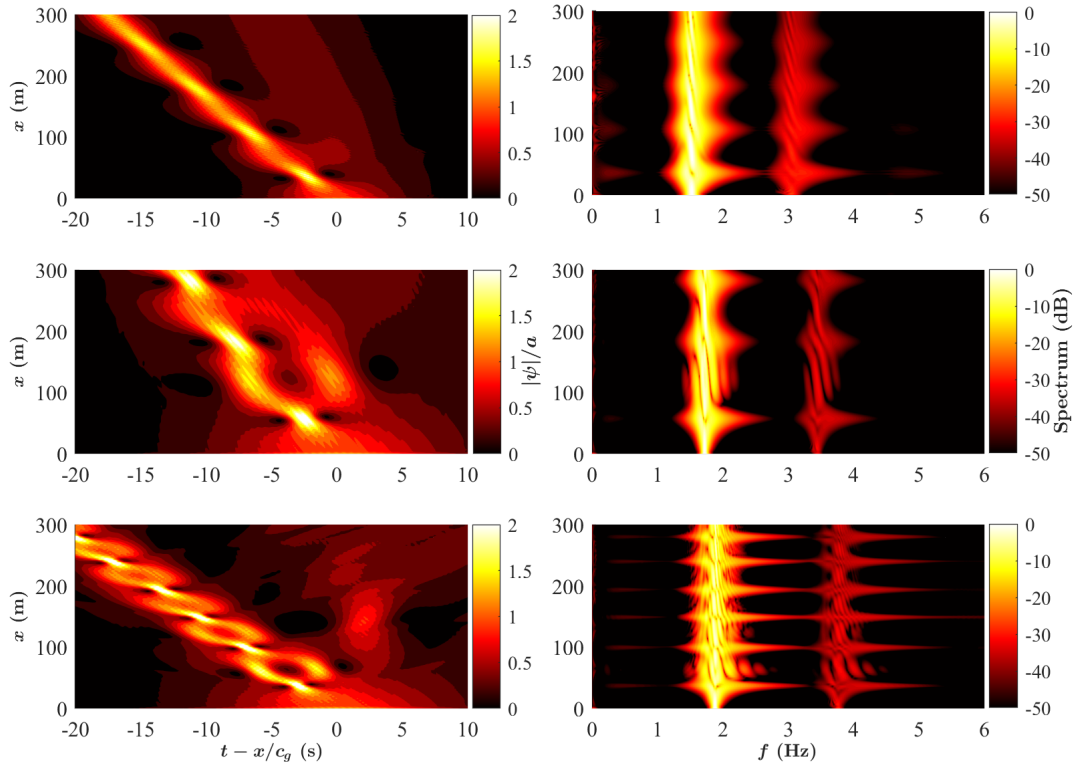


FIGURE 5.8. HOSM simulations of Galilean solitons corresponding to higher-order solitons with order $N = 2, 3, 4$, carrier steepness of $ak = 0.075$, and frequency $f = 1.25$ Hz. Top panels: $c\alpha = 0.735$, $N = 2$. Middle panels: $c\alpha = 1.251$, $N = 3$. Bottom panels: $c\alpha = 1.661$, $N = 4$.

Perfectly recurrent Galilean-transformed envelope solitons are primarily considered to be mapped on the orbit of a multi-soliton, as described in the previous section. Simulation results and corresponding spectral evolution are summarized in Figure 5.8.

The HOSM simulations strikingly confirm several wave-focusing recurrence cycles of wave envelope on zero-background focusing, which are clear attributes of Satsuma-Yajima breathers. This is also remarkably noticeable for the order $N = 3$. One possibility to break this symmetry

and induce soliton fission is to increase the order of the multi-soliton solution, i.e. the GV c , or the carrier wave steepness [64]. It is stressed that wave breaking should be excluded in this process since the flow is assumed to be irrotational to justify the use of the HOSM. One realization involving an envelope soliton, subject to a GT and meeting an initial multi-soliton profile of the order of 4 is illustrated in Figure 5.9.

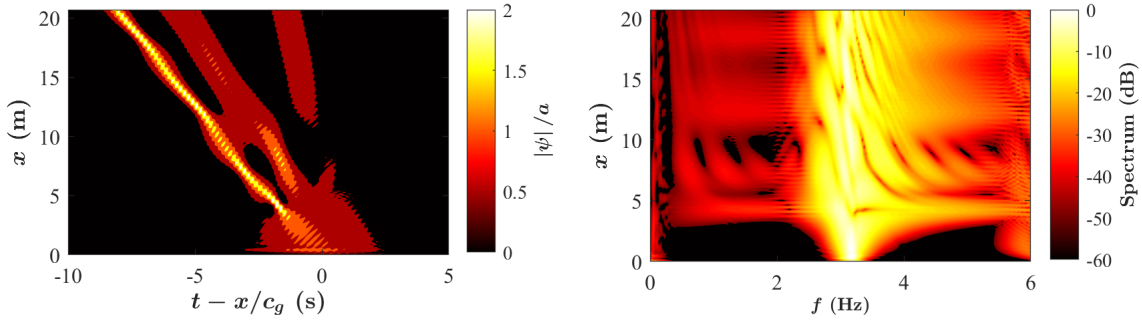


FIGURE 5.9. HOSM simulations of Galilean solitons corresponding to a multi soliton of order 4 with $c\alpha = 1.661$, carrier steepness $ak = 0.0667$ and wave frequency $f = 2.035$ Hz. The soliton fission is clearly observed.

The evolution of the wave envelope shows a clear soliton dominant fission signature into three solitons after the first focusing cycle. The latter mechanism brings along a severe and irreversible spectral broadening, which is a characteristic feature of a supercontinuum emergence.

5.5 Summary

In the present chapter, the Galilean soliton dynamics have been investigated in controlled laboratory conditions. Due to the severe wave focusing and even though the NLSE predicts that the GT should not affect the steadiness of the envelope solitons, clear deviations from NLSE predictions have been observed and quantitatively captured by the MNLSE, which takes into account high-order dispersive effects. In fact, such typical NLSE symmetry cannot be generalized to the fully nonlinear water wave problem. For positive values of GV, Galilean-transformed solitons exhibit self-focusing dynamics, which can be, in particular cases, connected to exact multi-soliton solutions. Higher-order spectral method-based numerical wave tank simulations confirm such distinctive recurrent focusing dynamics of

multi-solitons as obtained from GTs, including orders of 2, 3, and 4. In addition, a particular case is considered in which soliton fission has been observed involving a supercontinuum generation, as characterized by a severe and irreversible broadening of the wave spectrum. Future work will be devoted to understanding and addressing the role and limitations of MNLSE-type equations in the modelling of wave groups with shifted frequency peaks. Preliminary observations and simulation results of soliton on finite background have revealed very complex wave coherence and focusing dynamics as a result of spontaneous MI at play. Complementary theoretical, numerical, and experimental studies in nonlinear dispersive media other than water waves are also expected.

CHAPTER 6

Extreme wave excitation from localized phase-shift perturbations

The content of this chapter is based on the following publication:

He, Y., Witt, A., Trillo, S., Chabchoub, A., & Hoffmann, N. (2022). Extreme wave excitation from localized phase-shift perturbations. *Physical Review E*, 106 (4), L043101

Using all three analytical NLSE breather solutions from Section 2.2.4 and based on the concept of phase modulation in Section 2.2.8, in this chapter, the experimental excitation of nonlinear extreme waves through phase-shifted perturbations (PSP) on uniform wave envelopes is studied.

6.1 Rogue waves emerging from phase-shift perturbed wave envelopes

While the mechanism of abrupt phase jumps was originally investigated theoretically in [69], here, the writer rather considers the PSP obtained by impressing the localized phase profile of the pure breather solution at a given distance from the focus point. Examples of evolution obtained from the NLSE framework based on Equation (2.34) are shown in Figure 6.1 for the AB, in Figure 6.2 for the PB, and in Figure 6.3 for the KB solutions. In all examples, $x_{\text{ini}} = 0$ m stands for the first focus point, and the PSP is excited at $x_{\text{ini}} = -30$ m. As shown by all three different breather cases considered, a strong wave envelope compression is expected to occur in the condensate from a PSP-triggering only. The writer emphasizes that the smaller the amplitude modulation as defined by the exact solution, and as such the smaller the phase-shift, the better the agreement of maximal envelope compression profiles between

the NLSE solution and the one resulting from the PSP-initialization. Note that similarly to the mechanism studied in [68], the location of maxima deviates and is retarded compared to the motion of the exact solution. Moreover, it can be clearly seen that deviations are substantial when considering the KB case. The reason for this is that the solution never converges to the background and thus triggering the boundary conditions from a regular wave train with $\Phi_{KB}(x_{\text{ini}}, t)$ will more substantially deviate compared to the AB- or PB-type dynamics. In any event, the PSP-induced dynamics for both the PB and the KB cases show post-focusing dynamics dominated by fissions into pairs of quasi-KB breathers with opposite velocities [68], whose observation, however, requires wave facility with a length far exceeding that of the present installation.

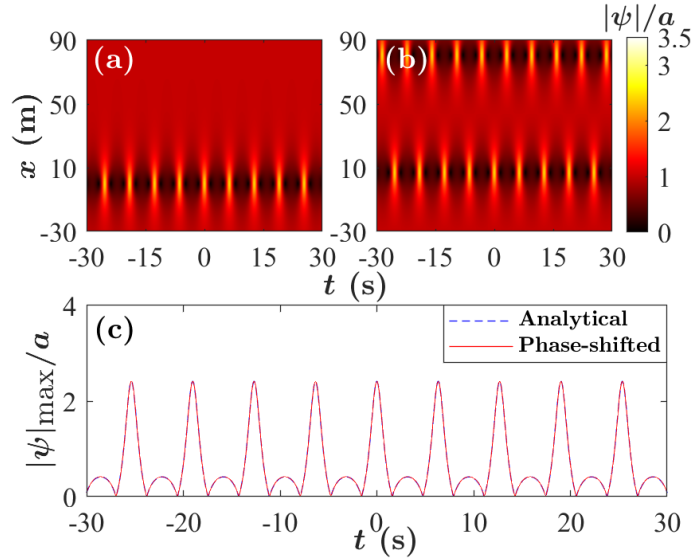


FIGURE 6.1. AB, contrasting spatiotemporal evolutions arising from (a) the full analytical Solution and (b) the phase-shift perturbation (PSP); (c) temporal profile of exact solution compared to that generated from the PSP at the location of maximal compression. Here the adopted carrier wave steepness $ak = 0.1$ and carrier amplitude $a = 0.01$ m. All waves are excited at $x_{\text{ini}} = -30$ m, i.e. 30 meters ahead of the first focus point ($x_{\text{ini}} = 0$ m) of the exact solution.

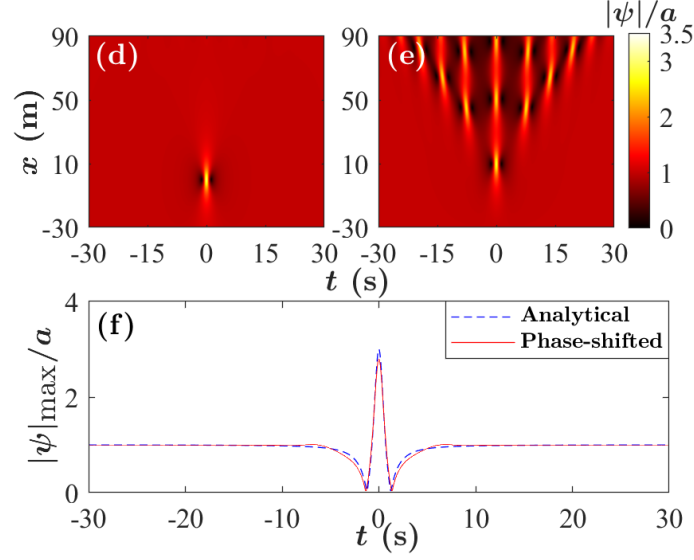


FIGURE 6.2. PB, contrasting spatiotemporal evolutions arising from (d) the full solution and (e) the PSP; (f) comparison of transverse profiles at maximal compression. Here the adopted carrier wave steepness $ak = 0.1$ and carrier amplitude $a = 0.01$ m. All waves are excited at $x_{\text{ini}} = -30$ m, i.e. 30 meters ahead of the first focus point ($x_{\text{ini}} = 0$ m) of the exact solution.

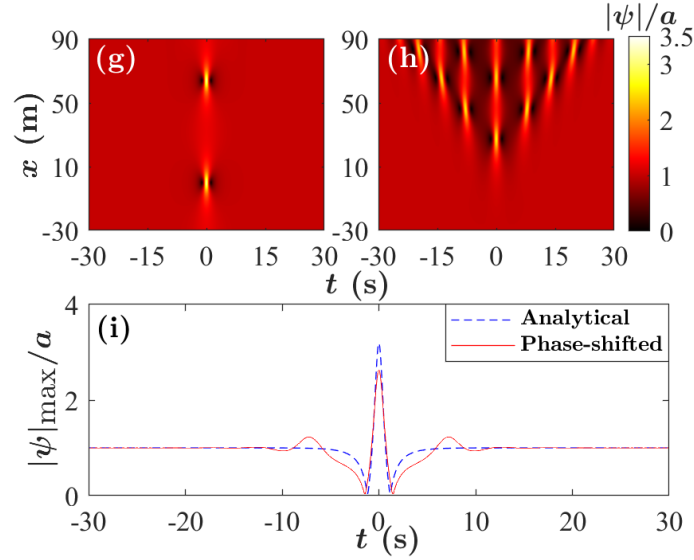


FIGURE 6.3. KB, contrasting spatiotemporal evolutions arising from (g) the full solution and (h) the PSP; (i) comparison of transverse profiles at maximal compression. Here the adopted carrier wave steepness $ak = 0.1$ and carrier amplitude $a = 0.01$ m. All waves are excited at $x_{\text{ini}} = -30$ m, i.e. 30 meters ahead of the first focus point ($x_{\text{ini}} = 0$ m) of the exact solution.

6.2 Research alignment and design

The same wave flume setup as described in [4] and depicted in Figure 3.1 is adopted in the current research. Furthermore, the water depth adopted in this study is $h = 0.7$ m, and the carrier wavenumber is chosen to satisfy operations in the deep-water regime, i.e., $kh > \pi$. Here, the carrier parameters are defined by the wave steepness parameter ak and carrier amplitude a . The wavenumber k and wave frequency ω are connected through the linear dispersion relation, as mentioned in Equation (2.16). The wave gauges' temporal resolution is 0.03 s while the spatial resolution is 0.3 m. To extract the wave envelope dynamics in time and space, the Hilbert transform is applied to each bandpass-filtered surface elevation times series. As Section 3.3 mentions, a bandpass filter is crucial to exclude the effects of higher-order harmonics, which can help avoid subsequent oscillation of the wave envelope [10, 188]. As the next step, interpolation and smoothing techniques are applied. Moreover, the injected phase-shift $\Phi(x_{\text{ini}}, t)$ (see Equation (2.49)), as defined by the breather solutions in Equation (2.40), is chosen to be small to enable the proper generation of a regular wave train by the wave maker.

In fact, since the linear wave maker's transfer function is frequency-dependent, initiating the experiments from a large PSP could compromise the steadiness of wave amplitude generation. As such, the PSP must be small, i.e., as defined at a location of the early stage of breather initialization and focusing. Indeed, the smaller the carrier perturbation of an AB or PB, i.e., the smaller x_{ini} , the smaller the phase-shift [44, 1]. The choice of x_{ini} is bound by the length of the wave facility to allow for the observation of the first focus (compression) point within the tank length for either the exact full breather or the corresponding respective PSP of the regular carrier wave. This also applies to the carrier wave parameters, which must be cautiously chosen. Note that the imposed phase-shift is still larger than any experimental noise.

6.3 Experimental results

As a starting point, the current experimental investigation is initiated by investigating the AB dynamics triggered by both exact and PSP configurations, as discussed above and illustrated in Figure 6.1. The results are shown in Figure 6.4. All collected measurements of the exact time-periodic breather show a reasonable agreement with NLSE theory. This applies to both the case when considering the pure AB solution of the NLSE and the regular wave train in which the AB-type phase-shift has been locally seeded. Remarkably, a gradual focusing clearly emerges from the phase-shift seeding in agreement with the numerical NLSE expectations. Note for the current AB and cases below, there is a small time shift compared to c_g since the wave groups travel with a slightly faster value than the linear group velocity, see [121].

The same excellent agreement in the MI-type wave focusing is also observed for PB (see Figure 6.2 for NLSE predictions), illustrated in Figure 6.5. In this case, one particular feature of focusing from the PSP in the background of constant amplitude becomes clear. The retardation and focusing delay from the single unstable PB packet is noticeable. A similar feature has been observed when ignoring the breather-specific phase-shift and considering only the amplitude modulation [68]. These observations also expose the limitations of the current experimental setup. Since the experiments start from a small phase-shift value in the carrier, the retardation of maximal wave focusing of the waves would require a long fetch to observe interesting post-focusing dynamics (recurrence, triangular patterns etc.). This is also the case for the KB (see Figure 6.3 for NLSE predictions), for which a maximum amplification factor of 3.3 is chosen. Figure 6.6 shows the two wave envelope dynamics.

Note that for the results mentioned above, the same colour scale is employed in all figures to make significant wave focusing more distinct. Overall, all cases agree reasonably well with the weakly-nonlinear NLSE framework approximated from the Euler equations to third-order in steepness. That said, a typical wave asymmetry is clearly noticeable in the experimental data. The latter can be explained and modelled by the modified NLSE accounting for fourth-order effects in steepness in the form of higher-order dispersion and mean flow [98, 120, 121].

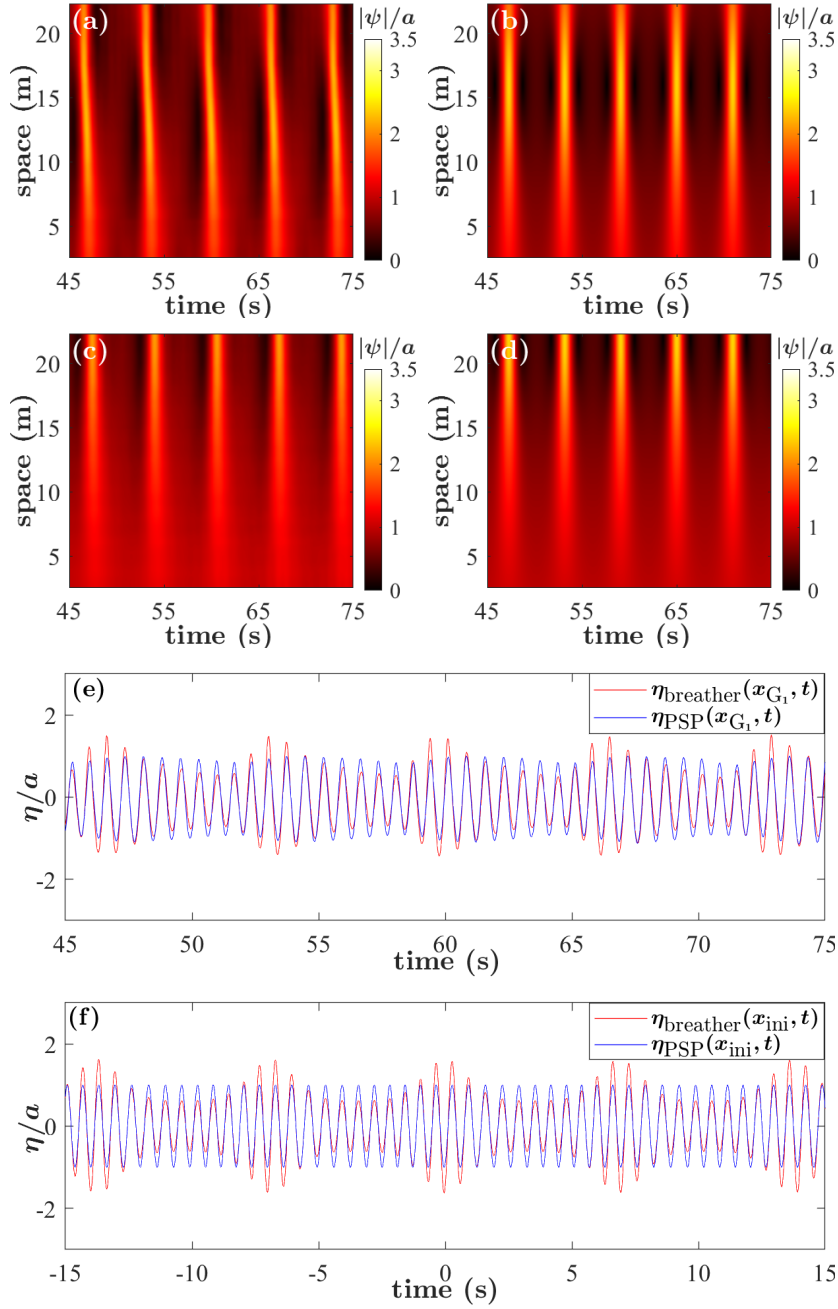


FIGURE 6.4. AB evolution for $ak = 0.12$ and $a = 0.01$ m with expected amplitude amplification of 2.41 (the case of maximal growth rate). (a) Propagation of wave envelope as measured in the flume. (b) NLSE simulations with the same boundary conditions as in (a). (c) Experimental observation of the AB-PSP in the regular background case. (d) NLSE simulations with the same boundary conditions as in (c). (e) Analytical and phase-shifted AB wave elevation at 2.5 m from the wave maker, which corresponds to the location of first gauge G_1 . (f) Analytical and phase-shifted AB wave elevation boundary conditions at $x_{\text{ini}} = -16$ m as implemented into the experiments and simulations.

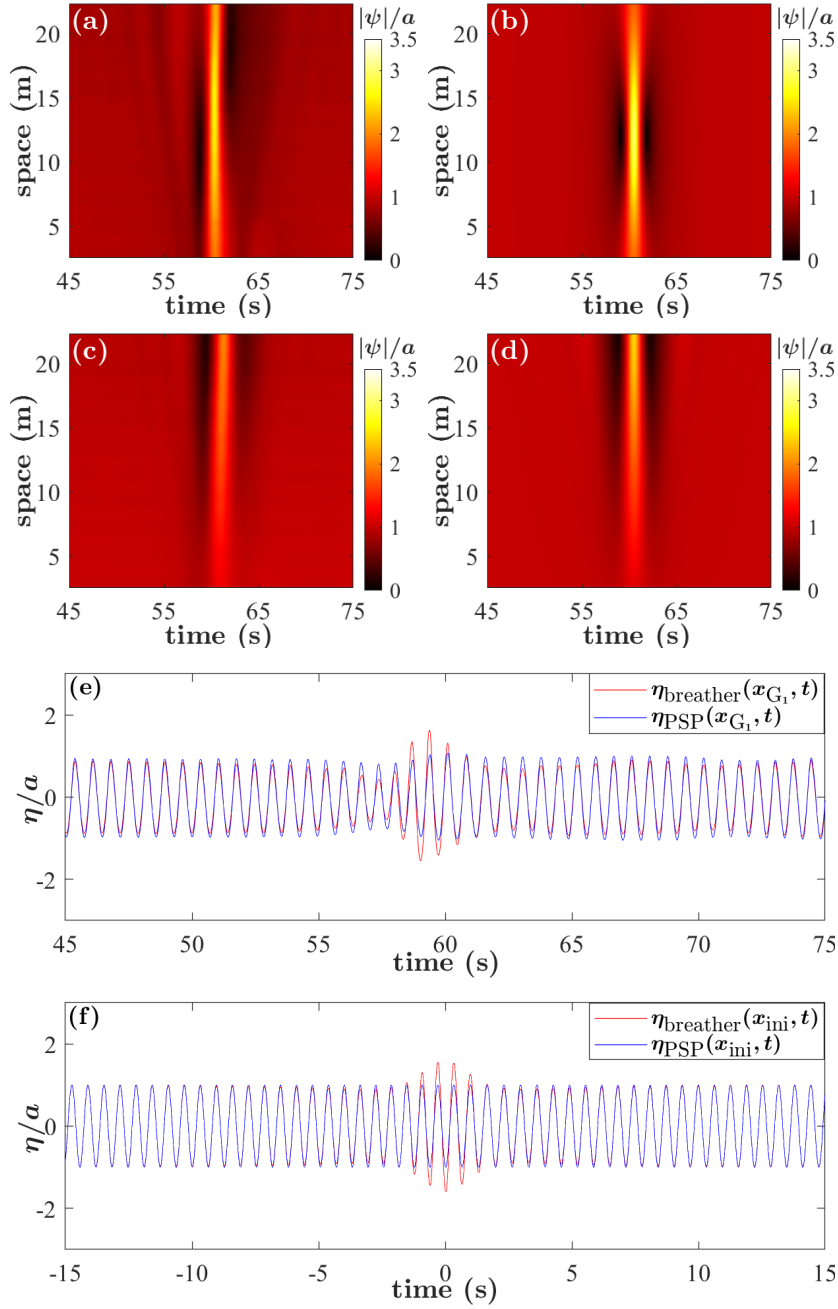


FIGURE 6.5. PB evolution for $ak = 0.1$ and $a = 0.01$ m with expected theoretical amplitude amplification of 3. (a) Propagation of wave envelope as measured in the flume. (b) NLSE simulations with the same boundary conditions as in (a). (c) Experimental observation of the PB-PSP in the regular background case. (d) NLSE simulations with the same boundary conditions as in (c). (e) Analytical and phase-shifted PB wave elevation at 2.5 m from the wave maker, which corresponds to the location of first gauge G_1 . (f) Analytical and phase-shifted PB wave elevation boundary conditions at $x_{\text{ini}} = -12$ m as implemented into the experiments and simulations.

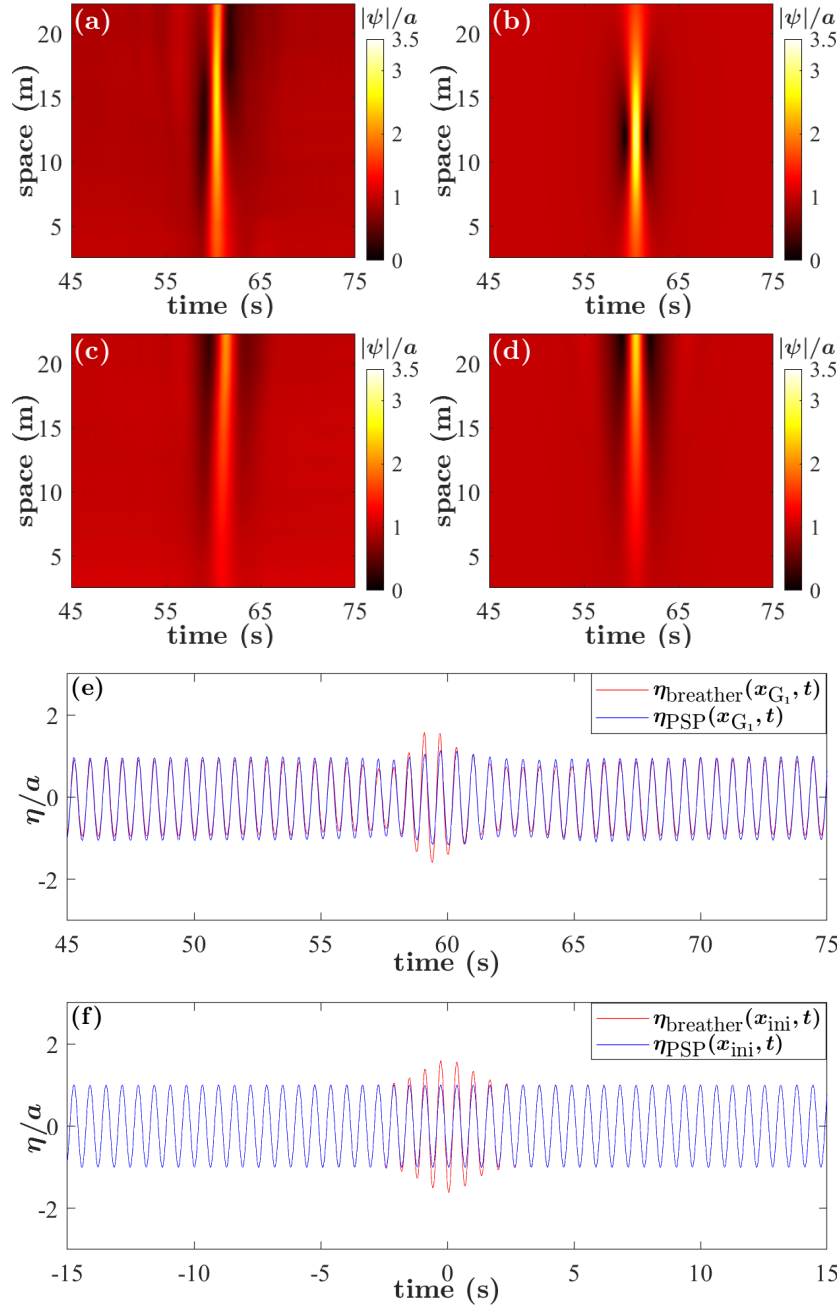


FIGURE 6.6. KB evolution for $ak = 0.1$ and $a = 0.01$ m with expected amplitude amplification of 3.3. (a) Propagation of wave envelope as measured in the flume. (b) NLSE simulations with the same boundary conditions as in (a). (c) Experimental observation of the KB-PSP in the regular background case. (d) NLSE simulations with the same boundary conditions as in (c). (e) Analytical and phase-shifted KB wave elevation at 2.5 m from the wave maker, which corresponds to the location of first gauge G_1 . (f) Analytical and phase-shifted KB wave elevation boundary conditions at $x_{\text{ini}} = -12$ m as implemented into the experiments and simulations.

These effects captured by the modified NLSE [51] were also observed and studied in more recent experiments in the context of nonlinear wave packet evolution [189, 57, 190].

6.4 Summary

In this chapter, an experimental study on PSP envelope wave evolution has been reported, confirming a proof of concept that rogue waves can appear from a localized PSP in a regular wave train. Such PSP can be, for instance, constructed from breather solutions of the nonlinear Schrödinger equation and then seeded on the regular wave train. The wave tank measurements in this study underline that nonlinear focusing can be exhibited by PSP, confirming yet another focusing mechanism beyond the wave modulation or superposition principle. Future experimental studies will explore such focusing dynamics in various wave systems governed by dispersion and nonlinearity [191, 192, 193] and develop theoretical techniques to predict such dynamics within the NLSE framework and beyond [194, 195].

CHAPTER 7

Extreme nonlinear wave focusing in standing waves

The content of this chapter is based on the following publication:

He, Y., Slunyaev, A., Mori, N., & Chabchoub, A. (2022). Experimental evidence of nonlinear focusing in standing water waves, *Physical Review Letters*, 129(14), 144502

Following the introduction on the multi-directional wave system [73, 79] in Chapter 1 and further explanations in Section 2.2.9, to reiterate, the existence of extreme wave nonlinear focusing within the standing wave field is studied in the current chapter. The numerical validation is restricted to the CNLSE framework (2.50).

7.1 Research alignment and design

Since most of the state-of-the-art wave flumes do not have two wave generators on opposing sides, one possibility to allow for wave train collisions is to make use of the wall at the opposite side of the wave generator as a reflective mirror for long-crested waves [196]. Indeed, the absence of wave energy absorbing beach in a perfectly aligned flume permits a full reflection and thus, the formation of standing waves. Such adjusted and adopted set-up configuration can be viewed in Figure 3.2.

Besides the experimental setup that is already explained in Section 3.1, in the current study, the water depth is set to be 0.75 m and all eight resistive wave gauges are used to measure the water surface elevation. The accurately-controlled generation of the wave field allows the reiteration of the experiments with different wave gauge placements to ensure a high resolution along the waves' propagation direction [197].

Since the current research focuses on the interaction of an incident PB wave field with a regular wave train with an opposing wave vector, a dynamic head-on interaction, also referred to as standing PB, is selected to have a sufficiently long realization of the PB solution in time to guarantee the propagation of the unperturbed regular wave to the wall and back, before the wave maker launches the small and localized PB-specific modulation. The reader can refer to references [16, 41] and Equation (2.40) for constructing the dimensional PB boundary conditions. In the experiments, the amplitude of $a = 0.01$ m for two carrier steepness values $ak = 0.09$ and $ak = 0.10$ was chosen, which corresponds to a difference in wavelength of 0.07m. The respective values of the wave frequency can be computed using the linear dispersion relation $\omega = \sqrt{gk}$. Note that the steepness value of $ak = 0.10$ represents the experimental threshold value before the onset of wave breaking, as observed in the current facility. Because of the wave flume's limited length, considerably reducing the wave steepness value would not allow the initiation of the experiments from a small amplitude modulation of the unstable incident wave field. The boundary conditions have been defined to expect a maximal wave compression of 16 m from the wave generator.

7.2 Nonlinear focusing of the Peregrine breather in standing wave field

The results of the experimental campaign, accounting for 184 measurements along the longitudinal direction for each realization, together with the associated CNLSE predictions are displayed in Figure 7.1.

The numerical CNLSE simulations have been carried out for the same wave parameters and boundary conditions as the laboratory experiments for validation purposes. As mentioned in Section 3.2, a pseudo-spectral approach and the fourth-order Runge–Kutta method have been adopted for the integration in space.

The wave tank measurements show an excellent agreement with the CNLSE dynamics: the PB remarkably evolves in standing wave states without any signs of the disintegration while

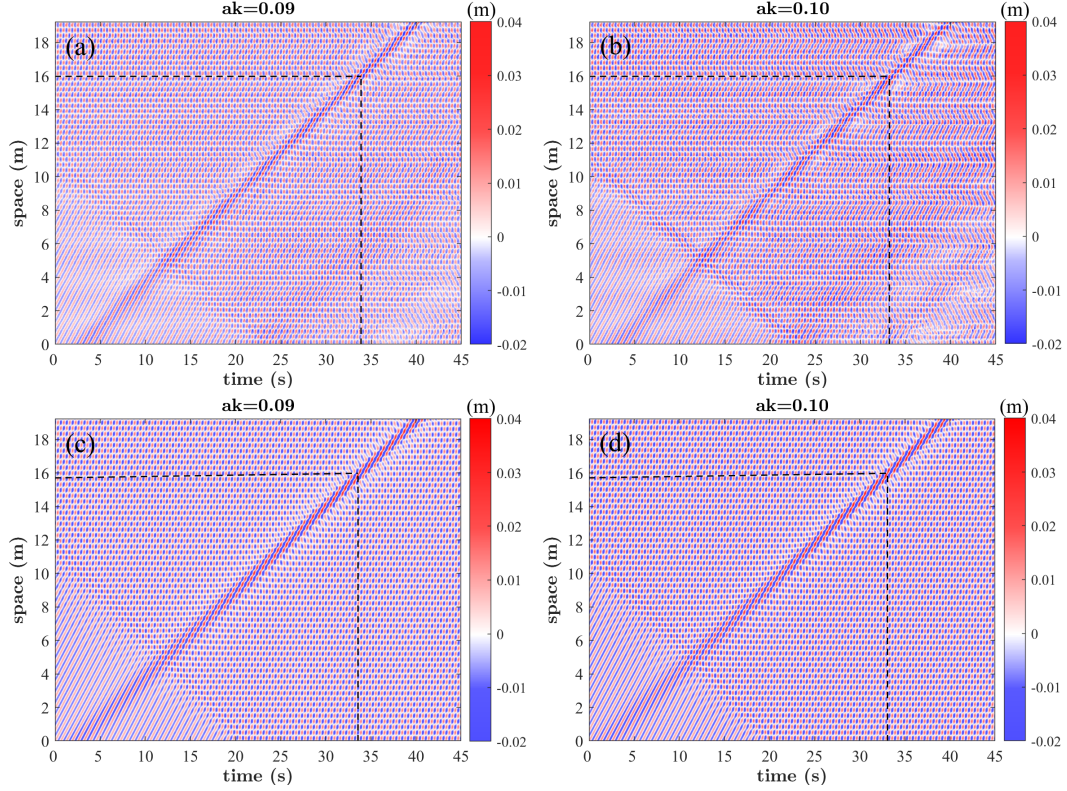


FIGURE 7.1. Top (a) and (b): Experimental observation of two examples of a standing PBs. Bottom (c) and (d): Results of numerical CNLSE simulations starting from the same boundary conditions as the laboratory experiments. Left (a) and (c): Wave parameters $a = 0.01$ m and $ak = 0.09$. Right (b) and (d): Wave parameters $a = 0.01$ m and $ak = 0.10$.

keeping its coherence, since it has been conjectured that MI does not play a significant role in cross wave states. Moreover, the characteristic amplitude amplification of four in this collinear case is also reached at the expected location in the water wave tank. This wave interaction can also be confirmed when studying the respective bispectral evolution; see Figure 7.2.

Here, the decomposition of the incident and reflected wave constituents in the bi-spectra, which are estimated from the Fourier components [169], prove that the extreme wave focusing only occurs for incident waves, dominated by the PB dynamics. To be more precise, the spectral broadening, which is an indicator of physical wave focusing, occurs only in the incident wave field, as in Figure 7.2(c) and 7.2(d). On the other hand, the energy of the counter-propagating waves remains steady; see Figures 7.2(e) and 7.2(f). One can also notice that the second harmonic energy weakens when the standing wave field has been fully developed.

Note that the variations in surface wave profiles are very sensitive to the single gauge location. On a node point, the amplitude of the standing regular wave field is zero [198]. It is also worth mentioning that the PB did not influence the destabilization of the regular opposing wave field. This persistence may be partly explained by the form of the CNLSE in Equations (2.50), which supports the conservation of integrals $\int_{-\infty}^{\infty} |\psi_j|^2 dx$ for $j = 1, 2$, so that the nonlinear exchange terms in Equation (2.50) lead to wavenumber corrections only. The impossibility of energy exchange between two counter-propagating planar deep-water wave systems due to nonlinear four-wave interactions beyond the narrow-band approximation was recently emphasized [199, 200]. Nonetheless, the generation of collinear opposite waves was

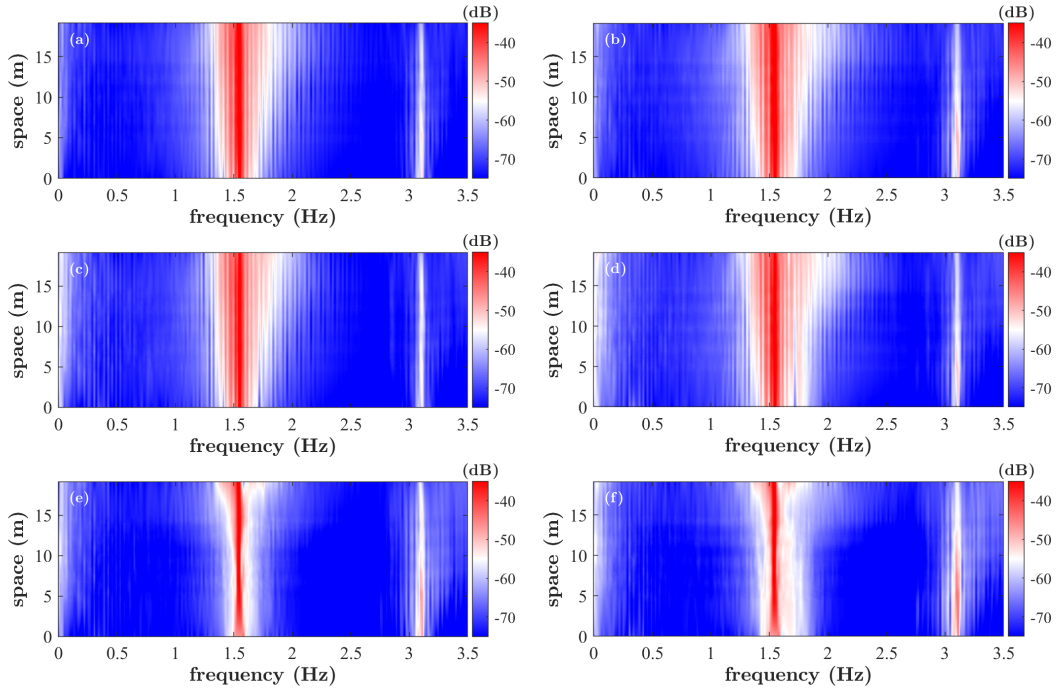


FIGURE 7.2. Spectral evolution of the standing PBs, reported in Figure 7.1. (a) Computed for the measured surface elevation for the carrier parameters $a = 0.01$ m and $ak = 0.09$. (c) Incident PB wave dynamics isolated from (a). (e) Opposing wave train as isolated from (a). (b) Computed for the measured surface elevation for the carrier wave parameters $a = 0.01$ m and $ak = 0.10$. (d) Incident PB wave dynamics isolated from (b). (f) Opposing wave train as isolated from (b).

recently observed in fully nonlinear hydrodynamic simulations in reference [201]. Such wave dynamics are not expected in shallow-water conditions.

7.3 Nonlinear focusing of the degenerate soliton in standing wave field

In the following, the evolution of the PB counterpart on zero background [202, 131] is investigated, i.e., the degenerate soliton solution, in the presence of a counter-propagating regular wave field. Considering this particular NLSE solution has finite length construction on a zero background, the standing wave patterns appear only locally during the interaction with the opposing wave field. Experimental data and respective CNLSE simulations are displayed in Figure 7.4.

Since there is a sufficiently good resolution in the space of $dx = 0.11$ m, one can reconstruct the spatial profile from the measured time series. Note that since $c_g = (\Omega/K) = \omega/(2k)$, Ω being the modulation frequency and K the modulation wave number, the number of oscillations in a space series is half that in a time series $N_x = k/K = \frac{1}{2}(\omega/\Omega) = \frac{1}{2}N_t$. Furthermore, the surface elevation profile in a standing wave field is very sensitive to the measurement location.

Indeed, an excellent agreement can be noticed between laboratory experiments and numerical simulations, suggesting an elastic collision between the regular envelope and degenerate soliton [200] since previous studies suggested the irrelevance of MI when more than two wave systems interact, and by that, confirming once again that the CNLSE framework may have an extended applicability range beyond the physical limitations technically limiting its exploitation.

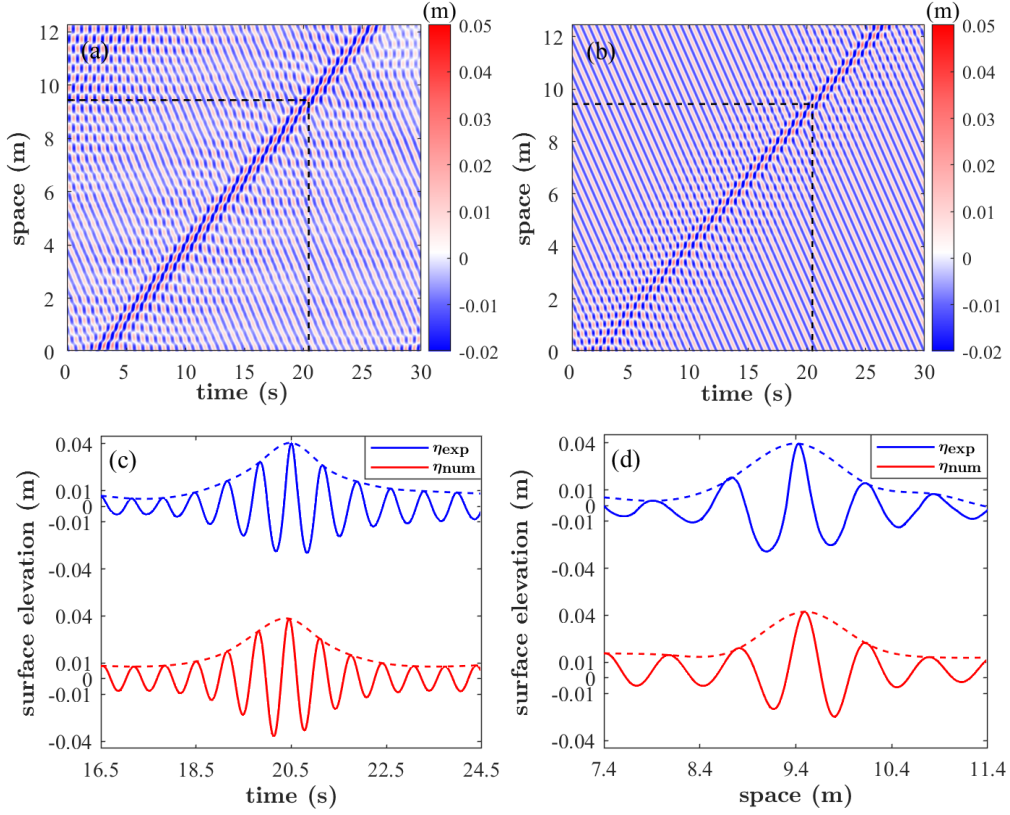


FIGURE 7.3. (a) Evolution of a standing degenerate soliton for $a = 0.01$ m and $ak = 0.11$. (b) Corresponding CNLSE simulations. (c) Comparison of the maximal temporal surface profile (top line) with the CNLSE prediction (bottom line). (d) Comparison of the maximal interpolated spatial surface profile (top line) with the CNLSE prediction (bottom line). The dashed line corresponds to the respective wave envelope profile, which has been extracted from the experimental data using the Hilbert transform.

7.4 Fully-nonlinear simulations and validation

Next, the proof of concept validation study is extended by considering a fully nonlinear water wave framework by numerically solving the Euler equations using the HOSM [203, 58], which resolves up to 7-wave interactions in the simulations. This approach not only allows consideration of the wave evolution for a longer time and distance than at disposal in experimental wave facilities but also provides higher reliability than the weakly-nonlinear CNLSE approach. Figure 7.4 shows a corresponding and particular case study, which has been experimentally studied and reported in Figure 7.1(a) and 7.1(c), i.e., the case of the PB on the finite background. Because of the increase of the physical domain, the only difference

is that the maximal wave focusing is expected to appear after 35 m of propagation, while in the experiment, only after 16 m. The initial condition at $t = 0$ is specified in the form of two long trains with the carrier wave steepness $ak = 0.09$ and amplitude $a = 0.01$ m, travelling towards each other. The rightward moving train is produced from the analytic PB solution; the inoculating perturbation is characterized by a steepness less than 0.11.

The pseudo-colour in Figure 7.4(a) represents the evolution of the surface displacement envelope, which is produced from two dozen simulations when various phase combinations were assigned to the initial wave trains. Hence, the envelope corresponds to the phase averaging of the upper enveloping surface. Only a part of the simulated domain is shown. The target focusing time according to the exact NLSE PB solution is about 90 wave periods, see the dashed black lines.

The snapshot of the maximum wave, which is very close to the breaking onset, is given by the black curve. Note that the deviations of the maximal envelope compression location are less significant in the wave flume because of the shorter propagating distance considered, a constraint imposed by the wave generator frequency range and the limited length of state-of-the-art wave facilities.

The strongly nonlinear simulations reproduce all the main features of the localized standing wave patterns observed in the laboratory environment, particularly the clean and quasi-undisturbed extreme wave focusing within the incident PB group. At the same time, the long evolution reveals some distinctions. Unlike the weakly-nonlinear framework, the strongly nonlinear simulation results correspond to slightly faster movement of the growing modulation and noticeably longer focusing time. Besides, a minor asymmetry of the emerged large wave group is clearly seen before and after the maximal wave focusing.

The evolution of the corresponding maximum wave amplitude is displayed in Figure 7.4(b) by the red colour. It is evaluated within the area bounded by the magenta contour in Figure 7.4(a), which has been introduced to reduce the effect of emerging large modulations at the edges of the colliding wave trains. This dependence is compared with the result of similar strongly nonlinear numerical simulations of the PB when the opposite wave train is

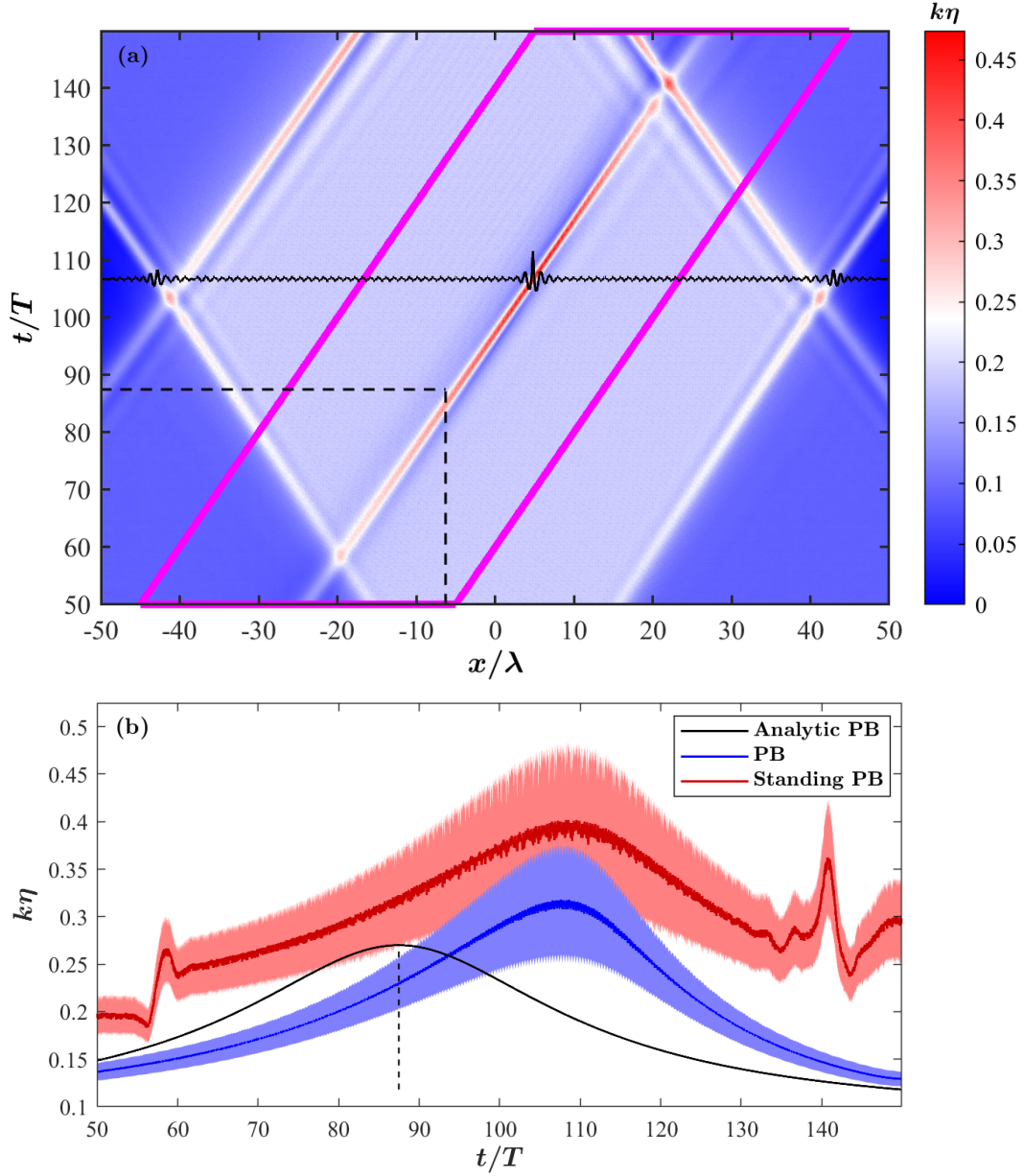


FIGURE 7.4. Evolution of a PB in standing waves with the carrier steepness $ak = 0.09$ as simulated by the HOSM by arranging two dozen phases allowing for the reconstruction of wave envelope. (a) The surface displacement envelope $k\eta$ is shown by the pseudo-colour. The surface displacement profile of the highest focused wave is displayed in black while the dashed black lines point to the location of maximal envelope compression, both according to the analytic PB solution. The magenta parallelogram shows the area where the maximum wave amplitude is evaluated. The coordinate and time are normalized with the dominant wavelength λ and period T , respectively. (b) Evolution of the maximum wave amplitude in the HOSM simulations and according to the analytic PB solution of the NLSE. The filled areas correspond to the intervals between crest and trough amplitudes, while the solid curves depict their simple means.

absent (blue colour) and with the exact analytic PB solution (black curve). The differences between the red and blue curves correspond well to the amplitude of the opposite wave train at $ak = 0.09$, confirming once again the non-influence of the oppositely propagating regular wave train on the focusing dynamics of the modulational-unstable wave packet. Considering the significantly longer fetch compared with the laboratory experiments, the strongly nonlinear localized focusing of PBs on top of progressive or standing waves occurs later than predicted by CNLSE and results in significantly larger focused waves. This is a behaviour that has also been quantified in the unidirectional NLSE case [121, 120]. It is also interesting to note in Figure 7.4(b), and similarly to the dispersive-focused wave group, that the focusing delays seem to be the same for both the unidirectional (blue curves) and collinear (red curves) cases.

7.5 Summary

In the current chapter, experimental evidence of quasi-unperturbed PB hydrodynamics in standing waves has been reported. The same has been observed for the degenerate soliton, which is the counterpart of PB on zero background. The results confirm that breather solutions of the NLSE can be considered to model and describe extreme wave localization in non-integrable systems [118, 204]. Although experiments are always subject to energy dissipation, it is believed that the role of NLSE solitons and breathers can be extended to a broader range of complex systems [205, 206, 207]. It is also anticipated that the experimental work will motivate further studies to investigate the role of breathers for standing wave conditions in nonlinear dispersive media and determine their function in crossing seas and vector nonlinear fibre-analog systems.

Extreme wave statistics near reflective beaches

The content of this chapter is based on the manuscript that will be submitted in the near future.

Following the introduction of directional sea states and research scope in Chapter 1, the coupled framework of standing waves is described in Section 2.2.9, and the extreme wave statistics were introduced in Section 2.3. In this chapter, the extreme wave occurrence of irregular JONSWAP-type wave fields with the presence of wave reflection is studied.

8.1 Research alignment and design

In addition to the basic experimental setups already introduced in Section 3.1, three wave gauges are installed within the wave tank to acquire the wave elevation data, referred to as G_1 , G_7 , and G_8 , respectively. Here, G_1 is the closest, and G_8 is the farthest one from the paddle. In the current study, G_1 is only adopted to verify the accuracy of the generated JONSWAP

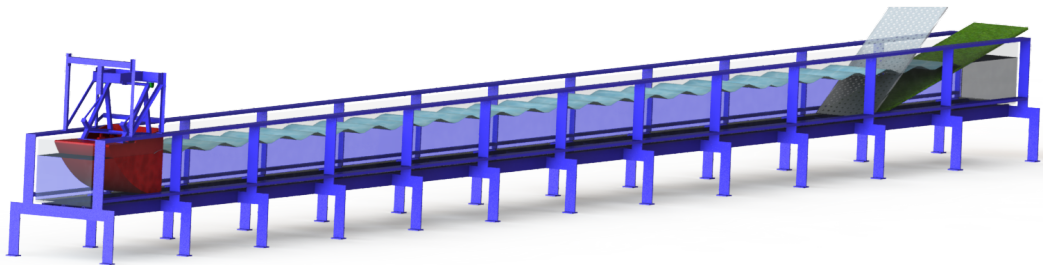


FIGURE 8.1. The Sydney University wave flume with a piston-type paddle on the left and beach at two different angles for exhibition purposes. The one at the lower angle represents the beach installation covered with artificial grass material. The beach at a higher angle is made of a permeable honeycomb structure (rendered transparent in the current sketch).

waves. The data from G_8 is used to apply the incident and reflected wave isolation technique [169]. The distance from the wave paddle to the eight gauges are 5, 22.5, and 22.83 m. The analysis was mainly carried out based on the data from G_7 , which is the second closest gauge near the artificial beach and far from the wave paddle so that there is a sufficient distance for the waves to evolve. The elevation data recorded in each case contains at least 1200 waves, ensuring the consistency and accuracy of the statistical analysis.

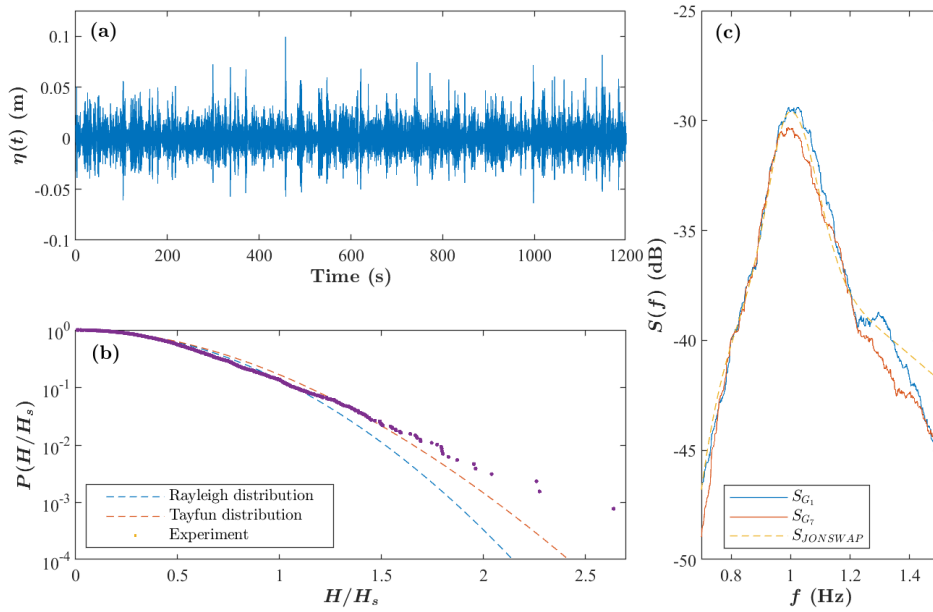


FIGURE 8.2. Example of a single case with beach angle $\theta = 8^\circ$, $H_s = 0.06$ m, $f = 1$ Hz, $\gamma = 6$ within deep water regime ($h = 0.7$ m). (a) Experimental wave elevation time series $\eta(t)$ at gauge G_7 near the beach with a total sampling length of 1200 seconds reconstructed from 20 individual random tests. (b) Exceedance wave height probability $P(H/H_s)$ as computed from $\eta(t)$ in (a), compared with the Rayleigh (normal) distribution and Tayfun distribution by taking the wave nonlinearity into account. Here, $P(H/H_s)$ is closer to the Tayfun distribution. (c) The spectral plot of wave elevation time series measured at G_7 and G_1 compared with the prescribed JONSWAP spectrum, where S_{G_7} is slightly lower than S_{G_1} due to dissipation. The case in the current figure corresponds to case 7 according to Table 9.1.

To adjust the dimensionless depth kh , where the water depth h is varied from 0.25 to 0.7 m in the current study. The 2.5×1 m² beach configuration, as shown in Figure 8.1, can be covered with different materials. In the current tests, a honeycomb, artificial grass as well as a smooth

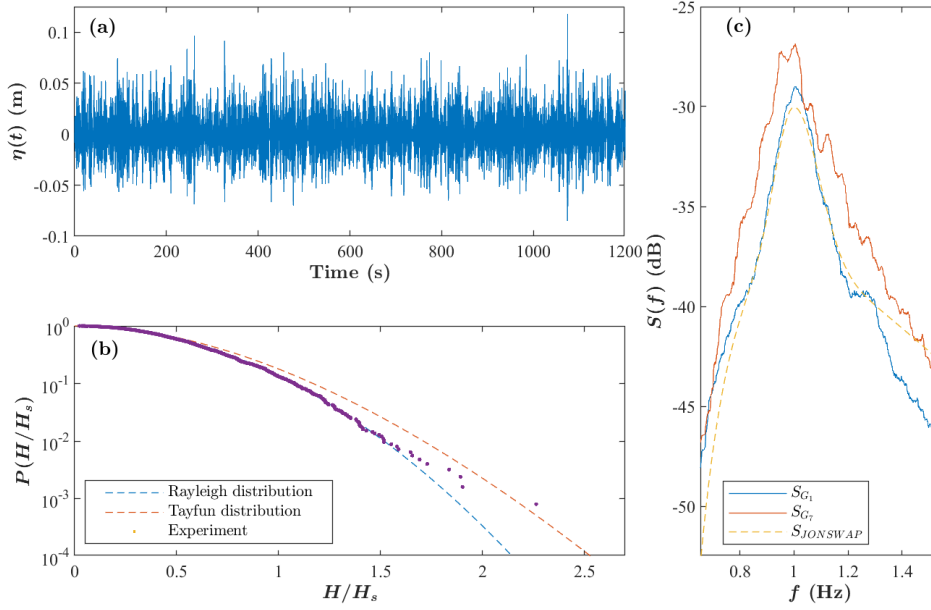


FIGURE 8.3. Example of a single case with beach angle $\theta = 90^\circ$, $H_s = 0.06$ m, $f = 1$ Hz, $\gamma = 6$ within deep water regime ($h = 0.7$ m). (a) Experimental wave elevation time series $\eta(t)$ at gauge G_7 near the beach with a total sampling length of 1200 seconds reconstructed from 20 individual random tests. (b) Exceedance wave height probability $P(H/H_s)$ as computed from $\eta(t)$ in (a), compared with the Rayleigh (normal) distribution and Tayfun distribution taking the wave nonlinearity into account. Here, $P(H/H_s)$ is closer to the Rayleigh distribution. (c) The spectral plot of wave elevation time series measured at G_7 and G_1 compared with the prescribed JONSWAP spectrum, where S_{G_7} is much higher than S_{G_1} due to wave reflections from the beach. The case in the current figure corresponds to case 25 according to Table 9.1.

board were selected to cover the beach, respectively. Qualitatively, the honeycomb structure has the highest permeability and roughness, while the smooth board is impermeable. The artificial grass is in between with intermediate permeability and surface roughness .

The parameterizations of all experimental cases are recorded in Tables 9.1 and 9.2 in the Appendix. Examples of unidirectional and standing JONSWAP waves, i.e. unperturbed and perturbed by wave reflections, are shown in Figures 8.2 and 8.3, suggesting a much higher extreme wave probability within the unidirectional wave field compared to the counter-propagating wave field. Minor wave dissipation can be observed within the spectrum plot in

Figure 8.2, while the spectral energy considerably increases in Figure 8.3 due to strong wave reflections.

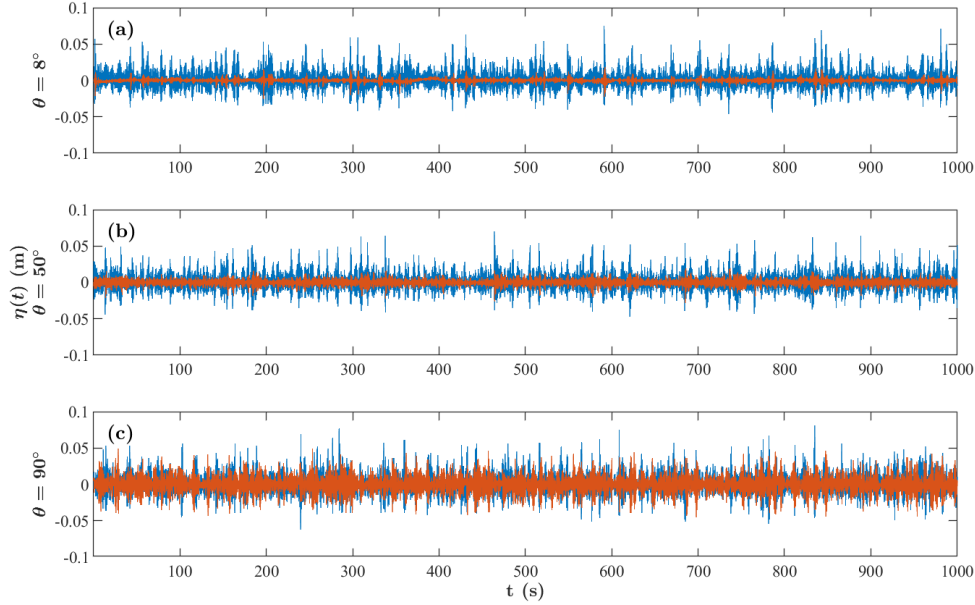


FIGURE 8.4. Example of the incident (blue curve) and reflected (red curve) wave elevation time series isolated from the measured data [169] using measurements at G_7 and G_8 under three beach inclination angles θ , including (a) $\theta = 8^\circ$, (b) $\theta = 50^\circ$, and (c) $\theta = 90^\circ$. The reflectivity is computed as (a) 18%, (b) 50%, and (c) 85% based on the significant wave height ratio. Note that cases 34, 39, and 44 in Table 9.2 are compared in the current figure.

Note that a linear separation of the incident and reflected waves [169] can be carried out based on G_7 and G_8 measurements to approximate the beach reflectivity under each angle, as shown by the wave elevation plots in Figure 8.4. The grass beach reflectivity, defined by the characteristic amplitude ratio between the reflected and incident wave train, is approximated to be 18%, 50%, and 85% for beach angles 8° , 50° , and 90° . The nonlinear interaction of counter-propagating wave fields evolving in space, as characterized by the complex wave envelope $\psi^{(1)}(x, t)$ and $\psi^{(2)}(x, t)$ with directional wavenumbers $\kappa^{(1)} = (k, 0)$ and $\kappa^{(2)} = (-k, 0)$, can be accounted for by the CNLSE framework [73, 79] as described in Equation (2.50). The wave elevation can be obtained using Equation (2.51). The reader can refer to Section 3.2 for more details regarding the simulation methodology. Next, experimental and numerical results

of extreme statistics in the presence of wave reflection in different complex configurations are reported.

8.2 Rogue wave statistics in deep water

By computing the wave height exceedance probability [140, 40], the occurrence of extreme waves under different conditions can be uncovered, examples are the variation of beach materials, spectrum widths, and depth. In this subsection, the statistical results in deep water are investigated, while in the next one, the effects of water depth change under wave reflections are investigated.

8.2.1 The effect of beach angle and material

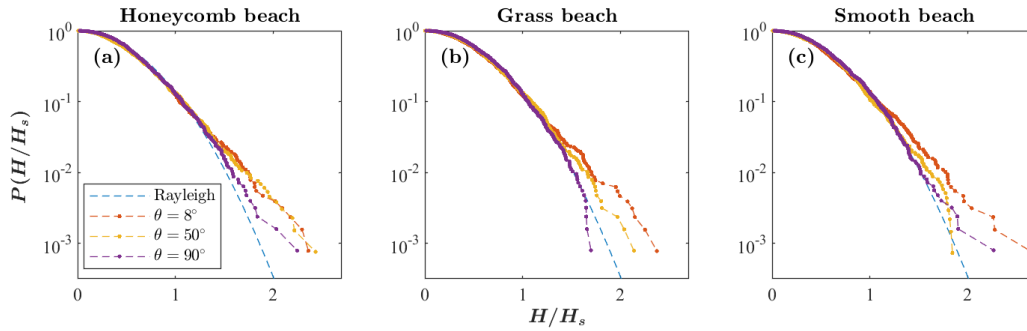


FIGURE 8.5. Exceedance wave height probability $P(H/H_s)$ in infinite water depth regime under various reflectivities and beach materials. The blue dashed line represents the Rayleigh (normal) distribution. The red, yellow, and purple dash-dotted lines represent $P(H/H_s)$ with beach angle $\theta = 8^\circ$, 50° , and 90° , respectively. The wave train has a characteristic steepness of $ak = 0.08$ in the current figure. In subplots (a), (b), and (c), the beach materials are the honeycomb, artificial grass, and smooth boards, respectively. The dominant frequency $f = 1$ Hz and water depth $h = 0.7$ m or $kh = 2.82$. Note that cases 7-9, 16-18, and 25-27 in Table 9.1 are compared in the current figure.

As shown in Figure 8.5, nine testing cases with the existing highest steepness ($ak = 0.08$), various beach angles and materials are integrated into three plots. Note that the honeycomb structure is the most permeable and rough covering material, the smooth board is impermeable,

while the artificial grass has an intermediate permeability and roughness. With the increase of beach angle θ , the beach reflectivity is supposed to significantly increase.

Under all reflection conditions considered, the extreme wave probability shows a significant decrease in the presence of wave reflections in each one of the subplots in Figure 8.5, which is in accordance with previous observations [76, 77, 78].

However, the extreme wave probability does not correlate with the roughness/permeability of the beach in the same way under different beach angles when changing the beach material. At $\theta = 50^\circ$, it is straightforward to notice that with the decrease of beach permeability, i.e., from (a) to (c) in Figure 8.5, extreme wave probability experiences a decrease. Here, the increase in wave reflections leads to a lower extreme wave probability, as mentioned above.

Nevertheless, at $\theta = 8^\circ$, the trend is inverted compared to the $\theta = 50^\circ$ cases. As shown in Figure 8.5, it is anticipated that a smoother beach exerts less drag or bottom friction on the waves. Thus, wave-breaking is more severe on the smooth beach than on the grass or honeycomb beaches. This leads to less wave reflection on the smooth beach. As a result, at $\theta = 8^\circ$, higher extreme wave probability can be observed on a smoother slightly-inclined. Note that one key distinction between the beaches at $\theta = 8^\circ$ and $\theta = 50^\circ$ is that the former supplies a longer wave-beach interaction distance, allowing more wave dissipation or breaking events [89] to occur.

As such, it is apparent that a beach at $\theta = 90^\circ$ provides the smallest distance/contact area between the waves and the beach material. In addition, considering the high permeability of the honeycomb structure, one cannot observe a clear trend across Figure 8.5(a)-(c) at such a vertical beach angle. For the angles 8° , 50° , and 90° , the reflection coefficients have been estimated to be 18%, 50%, and 85% respectively.

8.2.2 The effect of spectrum width

Next, the effect of spectrum width under wave reflection is examined while fixing the wave steepness and beach material type. Figure 8.6 compares the extreme wave probability with different spectrum bandwidth factors $\gamma = 3.3$ and 6 under various reflection coefficients. Note that under the same significant wave height, a larger γ value leads to a narrower wave spectrum, thus, more frequent extreme events [40], emphasizing the role of Benjamin-Feir Index (BFI) in such a process. In Figure 8.6(a), an apparent deviation between cases with different γ values can be observed, which is in accordance with previous studies [208, 140]. However, as shown in Figure 8.6(b) where the beach ascends to 50° , the difference of the extreme wave occurrence between the wave trains with $\gamma = 3.3$ and 6 becomes less pronounced, although still deviating from the linear distribution. In Figure 8.6(c) at a beach angle of 90° , the extreme wave height distributions further approach the linear curve with even less deviation between the two cases. However, even under quasi-full wave reflections, the effect of spectrum width can still be observed. This again emphasises the relevance of wave spectrum bandwidth and nonlinearity under strong wave reflections.

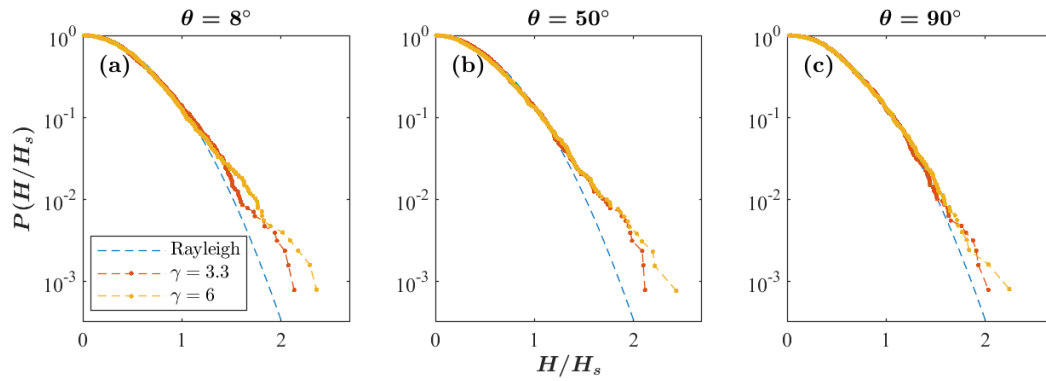


FIGURE 8.6. Exceedance wave height probability $P(H/H_s)$ in infinite water depth regime under various reflectivities and spectrum width factors γ near the artificial grass beach. The blue dashed line represents the Rayleigh (normal) distribution. The red and yellow dash-dotted lines represent the exceedance wave height probability with spectrum width factor $\gamma = 3.3$ and 6. In (a)(b)(c), the beach angles are $\theta = 8^\circ$, 50° , and 90° . The steepness of all cases is $ak = 0.08$, dominant frequency $f = 1$ Hz and water depth $h = 0.7$ m or $kh = 2.82$. Note that cases 9, 18, 27, 28, 30, and 32 in Table 9.1 are compared in the current figure.

8.2.3 Comparisons between experimental observations and numerical results

Based on the reflectivity coefficient obtained from the experiments and analyzed in Figure 8.4, the weakly-nonlinear coupled simulations with a length of $t = 19200$ s, i.e. 16 times the experimental time length, show a very good agreement with the experimental results. The bench-marked cases have a steepness of $ak = 0.08$ and a grass beach, as shown in Figure 8.7. The simulations show the feasibility and accuracy of the weakly nonlinear coupled scheme, i.e., CNLSE, in predicting directional wave dynamics.

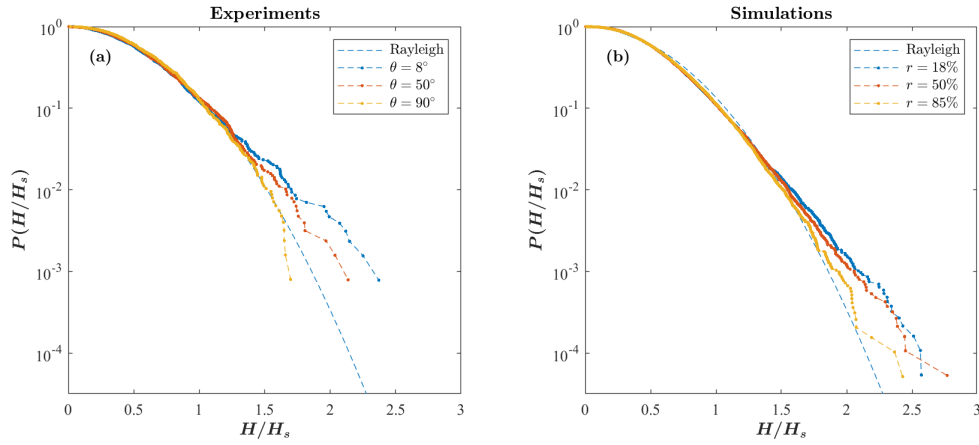


FIGURE 8.7. Comparison between experimental and numerical (based on the CNLSE framework) results. (a) Experimental exceedance wave height probability with a total sampling length of $t = 1200$ s near the grass beach at various angles. (b) Simulated exceedance wave height probability with a total simulation length of $t = 19200$ s. The blue dashed line represents the Rayleigh (normal) distribution. The blue, red, and yellow dash-dotted lines represent the exceedance wave height probability $P(H/H_s)$ with beach angle $\theta = 8^\circ$, 50° , and 90° . In all cases, the wave steepness $ak = 0.08$, dominant frequency $f = 1$ Hz and water depth $h = 0.7$ m or $kh = 2.82$. Note that cases 8, 17, and 26 in Table 9.1 are validated by numerical simulations in the current figure.

8.3 Rogue wave statistics in finite water depth

It is well-known that the absolute value of the NLSE nonlinearity parameter is zero at $kh \approx 1.363$ [209], as can be told from Equation (2.31). The instability of water waves in finite depth regimes in crossing wave conditions has also recently been theoretically investigated [210]. Here, the effect of water depth on random extremes in the presence of wave reflections is examined both experimentally and numerically. The extreme wave height

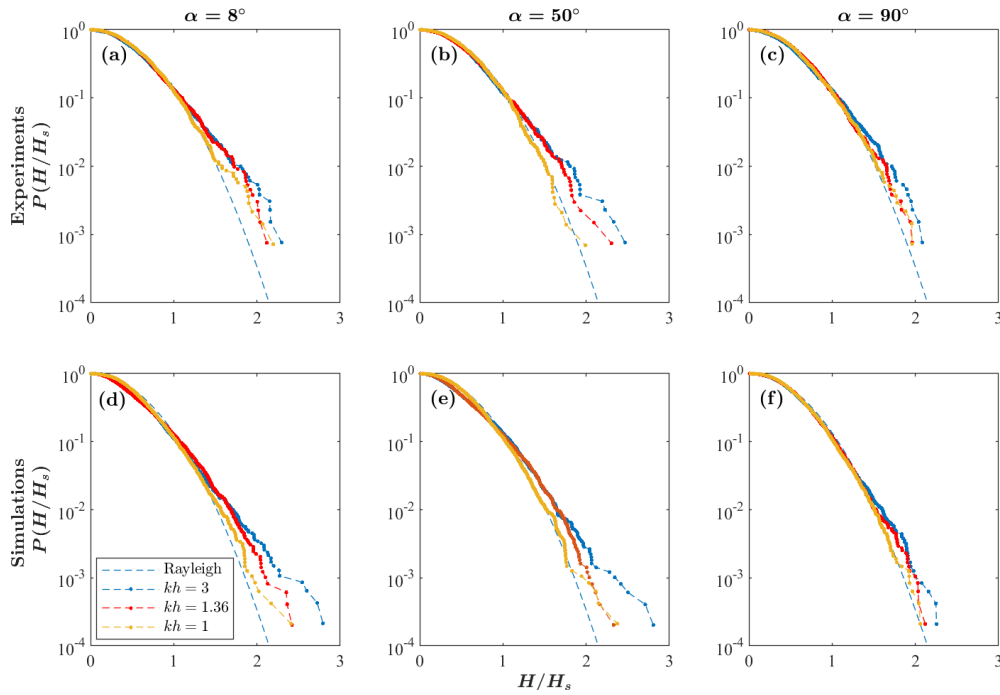


FIGURE 8.8. Exceedance counter-propagating wave height probability in finite water depth under various reflectivities. The beach angles θ in (a)(d), (b)(e), and (c)(f) are $\theta = 8^\circ$, 50° , and 90° . the total sampling length is $t = 1000$ s in experimental tests (a)(b)(c) and $t = 4000$ s in numerical tests (d)(e)(f). In all subplots, the blue dashed line represents the Rayleigh (normal) distribution. The blue, red, and yellow dash-dotted lines represent the exceedance wave height probability $P(H/H_s)$ with $kh = 3$, 1.36 , and 1 , and the dominant frequencies in all cases are $f = 1.2$ Hz. Note that cases 35, 37, 38, 40, 42, 43, 45, 47, and 48 in Table 9.2 are compared and numerically validated in the current figure.

occurrence under reflections with various water depths and beach angle values were analyzed and displayed in Figure 8.8. From Figure 8.8(a)-(c), it is evident that the cases with $kh \geq 1.36$ show clear deviation from the linear theory, while the cases with $kh = 1 < 1.36$ are close to

the linear Rayleigh distribution. This is due to the decrease in the absolute value of the NLSE nonlinearity parameter, as mentioned above. In addition, with the increase of beach angle θ , an overall decrease in wave probability can be observed for all cases, regardless of the kh value, as already mentioned above and in previous works [76, 77, 78]. From finite-depth CNLSE simulations with $t = 4000$ s, i.e., 4 times the length of the experiments, a very good agreement can be observed between experimental and numerical results in between Figure 8.8(d)-(f) and Figure 8.8(a)-(c), which again indicates the feasibility of the CNLSE framework in predicting the rogue wave probability in finite-depth regimes under reflections. It is important to highlight that both the dispersive and the nonlinear wave focusings are equally significant and coexist during the experimental and numerical tests conducted using the standard JONSWAP formulation for irregular wave fields with random phases [15, 13].

8.4 Summary

In this chapter, the rogue wave probability in counter-propagating wave systems [73, 79] under various complex beach conditions [89, 211, 90, 212] are investigated experimentally and numerically. In addition to wave reflections [30], the effects of reflective beach material, spectrum bandwidth [6], and water depth [93] are also considered. The main observation of this study is that wave nonlinearity persists under reflections, i.e., in crossing-seas [4, 77, 76, 146], although the extreme wave probability is reduced with the increase of beach reflectivity [77, 78]. At an intermediate beach angle, a more permeable beach material that reflects fewer waves plays a role similar to a beach at a lower angle. However, this conclusion does not hold at a lower beach angle due to the dominance of wave-breaking events. The beach material plays a less crucial role at a steep beach angle where wave reflection occurs within an exceptionally short distance. As a result, more complicated beach-wave interactions are not admitted near a steep beach/cliff. On the other hand, narrower-banded random waves show stronger extreme wave occurrences even under reflections [6, 4]. By reducing the water depth, extreme wave occurrence generally decreases due to the weakening of wave nonlinearity, which is the case under all beach reflectivities. Last but not least, reasonably good agreements have been achieved between the experimental and numerical results based

on the weakly nonlinear coupled NLSE framework [73], suggesting again the importance of wave nonlinearity within multi-directional wave fields [79, 138, 73, 77, 78].

CHAPTER 9

Conclusion

The current study aims to investigate the properties of wave nonlinearity [94, 7, 8] and extreme wave occurrence [85, 87, 88] in complex wave conditions [89] beyond the scope of exact NLSE solutions [38, 39, 37, 33]. To achieve this, a series of experiments and numerical simulations [1, 2, 3, 4] were conducted. The experimental tests were designed to explore the extreme wave conditions under which the nonlinear effect [7, 15] plays a role, and the results were validated using numerical simulations based on both weakly-nonlinear NLSE [8] and fully nonlinear Euler [93] frameworks.

In order to examine the properties of nonlinear hydrodynamic waves under different conditions, both deterministic and statistical approaches were employed to study unidirectional and multi-directional [73, 79] wave fields. The current study covered a wide range of wave conditions, from phase manipulations [44, 68] in relatively simple unidirectional cases to rogue wave nonlinear focusings in more complex multi-directional scenarios [40].

In the current studies on unidirectional wave fields, three aspects of the NLSE properties related to phase dynamics [44, 68] have been examined experimentally. This primarily includes the phase evolution of time- and space-PBs [41], where the differences in the phase originate from the intrinsic distinction between the time- and space-NLSE formulations. It has been shown that exact solutions from both frameworks can be generated and evolve in a wave tank, leaving a distinct localized phase evolution [1].

Inspired by the study above, the Galilean transformed wave groups with a global carrier phase-shift [54] inducing wave speed change have been examined next, indicating that an overall phase transformation can physically lead to wave instability change, which is in

disagreement with the NLSE theory. This is due to the absence of higher-order nonlinear effects in the NLSE [2] as well as the NLSE framework's narrow-band restriction [8, 33].

In a more general term, the effects of localized phase-shifts on the wave train have also been investigated [3], i.e. the evolution of PSP boundary conditions based on the analytical NLSE solutions, which contain non-uniform localized phase-shifts by excluding any amplitude modulation within the boundary conditions. This study was also inspired by the previous investigation on the amplitude-modulated wave groups, without considering any phase-shift in the initialization [68]. Here, nonlinear focusing purely due to PSP was observed. In contrast to the GT dynamics, higher-order nonlinearities do not play a crucial role in the evolution of PSP envelopes. Consequently, all experiments agree very well with the NLSE predictions.

On the multi-directional level, rogue waves in the counter-propagating wave field [73, 79] have been investigated in deterministic and statistical strategies. Deterministically, a perfect rogue wave focusing within the standing wave field [4] was observed and confirmed by both weakly- and fully-nonlinear simulations. This indicates the role of MI and nonlinearity beyond the unidirectional framework [79, 205, 206, 207].

Statistically, the extreme wave probability, although weakened under reflections, was still observed to deviate from the linear wave theory under reflections [77, 78]. It is worth noting that the random wave spectrum bandwidth still plays a crucial role under the presence of reflected waves [79, 72], which agrees with the above-mentioned deterministic study of narrow-banded nonlinear focusing in the standing wave field [4]. The complex coastal conditions [89, 211] with different beach material and water depth involve more complicated processes, such as dissipation and wave breaking events [213, 214, 215].

Overall, the present research has contributed to the understanding of nonlinear extreme waves in complex and counter-propagating conditions, shedding light on their fundamental characteristics and behaviour in offshore and nearshore environments. These insights can potentially aid in the prediction and prevention of extreme wave events in these settings.

In the future, the role of phase evolution in stochastic wave trains in the formation of extreme waves will be further explored. The properties of multi-directional wave systems and the

interactions between different wave components at angles other than π will also be investigated. Additionally, the interactions between waves and various underwater flow and bathymetry conditions will be studied extensively, as these more realistic ocean conditions can help to better understand the generation of extreme ocean waves.

Appendix

Case number	Beach material	H_s (m)	α (degree)	γ	kh	f (Hz)
1	smooth	0.01	8	6	2.82	1
2	grass					
3	honeycomb					
4	smooth	0.03				
5	grass					
6	honeycomb					
7	smooth	0.06				
8	grass					
9	honeycomb					
10	smooth	0.01	50	6	2.82	1
11	grass					
12	honeycomb					
13	smooth	0.03				
14	grass					
15	honeycomb					
16	smooth	0.06				
17	grass					
18	honeycomb					
19	smooth	0.01	90			
20	grass					
21	honeycomb					
22	smooth	0.03				
23	grass					
24	honeycomb					
25	smooth	0.06				
26	grass					
27	honeycomb					
28	honeycomb	0.06	8	$\frac{3.3}{9}$		
29						
30			50	$\frac{3.3}{9}$		
31						
32			90	$\frac{3.3}{9}$		
33						

TABLE 9.1. Parameters of experimental cases 1-33

Case number	Beach material	H_s (m)	α (degree)	γ	kh	f (Hz)	
34	grass	0.05	8	6	4	1.2	
35					3		
36					2		
37					1.363		
38					1		
39			50		4		
40					3		
41					2		
42					1.363		
43					1		
44	smooth	0.04	90		4		
45					3		
46					2		
47					1.363		
48					1		
49			8		4		
50					3		
51					2		
52					1.363		
53					1		
54	grass	0.04	50	6	4	1.2	
55					3		
56					2		
57					1.363		
58					1		
59			90		4		
60					3		
61					2		
62					1.363		
63					1		

TABLE 9.2. Parameters of experimental cases 34-63

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