

Interpretations of Downs–Thomson Paradox with Median Bus Lane Operations

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ABSTRACT

The Seoul Metropolitan Government has implemented a public transportation priority policy with the introduction of a 121.1 km long of the Median Bus Lane (MBL). Therefore, to analyze the impacts of the introduction of Jongro-gu MBL in January 2018, an effect analysis using the speed and volume of passenger cars and buses in Seoul is conducted in this study. The interpretation of the Downs-Thomson (D-T) paradox demonstrates that the reduction in passenger car mode capacity can increase total social benefits. This study shows that the speeds of passenger cars and buses have reached the deterministic user equilibrium after the MBL introduction, resulting in an increase in total social benefits. Moreover, it demonstrates that the bus priority policies, which gives the right to keep the capacity of buses guaranteed in urban areas, will be effective. The interpretation of the D-T paradox will contribute to the formulation of transport demand management policies.

1. Introduction

Along with the recent economic growth and rapid urbanization, a number of problems have emerged, among which those related to vehicular traffic is considered the most severe and complicated. These problems include the saturation caused by the rise in traffic hours, the drop in the quality of traffic service, and the increase in environmental concerns caused by congestion and transport emissions. They result from the low road capacity compared to the number of vehicles in circulation; however, the limit set by the Seoul Metropolitan Government has been reached in this regard. In order to overcome the foregoing and maximize road efficiency, the Seoul Metropolitan Government has implemented transportation demand management policies, such as bus priority policy through the MBL, odd-even rule, parking management policy, ramp metering, and congestion pricing in Namsan Tunnel.

In January 2018, a 121.1 km MBL was implemented in Jongno-gu, from Sejong-ro intersection to Dongdaemun rotary. The MBL is a policy that converts a specific road from a multi-mode usage into a bus-only lane with the objective of promoting public transportation (Currie, G., 2005). Since the implementation of this policy in different parts of the world, as one of the travel demand management (TDM) strategies, many effect analysis studies have been conducted at various levels. Cerbero, R. (2013) examined the state of MBL adoption at the international level and performed several analyses on MBL effects in terms of bus operation speed, comfort, safety, environmental impact, and social equity. Deng, T. and Nelson, J. D. (2013) analyzed the operational efficiency of introducing the MBL and the resulting speed improvement. In Korea, Kim, M. S. (2013) estimated the utility of the modal split by surveying MBL introduction,

and then analyzed the increase in average bus speed through microanalysis research based on the estimated extent of mode transformation.

Based on the literature review of the MBL effect analysis in Korea and internationally, it is found that bus speed and regularity in the schedule are mainly affected. The impact of MBL on other transportation modes, however, has not yet been seriously discussed; consequently, the real MBL impact on transportation as a whole could not be evaluated. This problem is mainly caused by the limitations in data collection (speed and traffic volume), especially the one related to private cars. For a more accurate evaluation, the effect analysis should include all modes of transport because roads are generally used by more than one mode, e.g., passenger cars and buses.

If the road capacity is only allocated to buses, then an effect analysis on bus service will be extremely beneficial. This reduction in allocated capacity, however, can affect the private car mode by aggravating congestion and increasing the total traffic cost. Improving the volume and speed of public transport contributes to the increase in public transportation demand and a decrease in private car demand. This improvement can thus reduce the total transportation cost as a whole, as D-T paradox phenomenon can demonstrate.

The D-T paradox argues that the average speed of a car on a road network is determined by the average speed of public transportation traveling the same distance. In other words, increasing the number of roads could worsen the traffic situation (Down, 1962; Thomson, 1977). Morigridge and Martin J. H. (1990) empirically demonstrated Thomson's claim by comparing the speed of passenger cars and public transport. Abraham, J. E., and Hunt, J. D. (2001) verified the paradox using Gumbel distribution and showed that the interval between buses increases with the road capacity.

Wang, W. et al. (2017), on the other hand, investigated how to maintain the total social benefit from the entire road. In fact, bus companies can employ two approaches to increase the benefit that accrues to them—they could reduce the operational cost by decreasing the bus frequency and number, or they could increase the number of bus users. The frequency reduction strategy, however, usually leads to a decline in bus service, thereby a decrease in bus users and an increase in private car users. Wang, W. et al. (2017) concluded that governments should provide incentives to private companies to maintain the total social benefit. Zhang, F. et al. (2014) identified approaches to reduce the growth in the number of passenger cars and maintain total benefits by introducing demand–response buses. An analysis of whether the changes in bus fares by government control are effective in expanding road capacity is conducted, proving that paradoxes can only occur when private cars and buses are completely substituted. Finally, Lee, S. et al. (2010) demonstrated the user balance based on bus speed and analyzed the effects of securing bus frequency resulting from MBL introduction.

Based on the literature review on the D-T paradox, at the local and international level, the optimization points where paradoxes do not occur because of changes in road capacity are theoretically verified. Moreover, the effect of MBL introduction on bus speed is analyzed. The most remarkable feature of previous studies is that for the MBL and D-T paradox, “bus speed” and “car speed” are theoretically assumed because of difficulties in obtaining data. Different from previous works, however, the analysis in this study considers speed and traffic volume through data acquisition.

By interpreting the D-T paradox in this research, it is found that the changes in traffic volume and speed for all modes caused by the introduction of the 2.8 km central bus-only lane (as MBL) in Jongno on January 2018 have contributed to the increase in total social benefits despite the road capacity reduction, and have reached the deterministic user equilibrium.

Indeed studies proved that the TDM policy will be effective when implemented in the city's downtown area. Furthermore, the bus only-lane policy is expected to aid in improving the efficiency of the TDM policy, as it contributes to the reduction in the demand for cars in the future.

This paper is organized as follows. Section 1 discusses the effect analysis of the MBL introduction considering the D-T paradox. Section 2 describes the research methodology and application of the D-T paradox. Section 3 presents the statistical proof that the critical user equilibrium based on the traffic volume and speed of both cars and buses has been reached through the introduction of Jongno MBL. The interpretation of the results and D-T paradox to analyze the increase in the total social benefits gained from policy implementation is also explained in this chapter. Lastly, Section 4 summarizes the final results and conclusions and identifies some future prospective methods and policies.

2. Downs-Thomson Paradox Concepts

2.1 Downs-Thomson Paradox

In this study, the total social benefits are calculated through the interpretation of the D-T paradox, which employs the speeds of both cars and buses and travel times of each mode. The goal of calculating the total social benefits is to analyze the effectiveness of the MBL introduction. Under the deterministic user equilibrium, any simple increase in road capacity leads to an increase in passenger car utility; this increases the traffic volume and congestion across the road, and in turn, also increases the average cost of bus utilization. On the one hand, bus service intervals and speed decrease because of congestion and reduced demand. On the other hand, the waiting time increases, thereby the cost of bus-user passage increases. Bell MGH (1997) and Abraham, J. E. and Hunt, J. D. (2001) argued that this D-T paradox could be explained by the curves shown in Fig. 1. Considering the total traffic as constant, the supply curves for private cars and buses are both shown in the y-axis, where intersection points A and B of the curve represent the critical user equilibrium. Fig. 2. Is a graph of when capacity decreases as opposed to Fig.1 claimed by Lee. S(2010). If the allocated road capacity of passenger cars decreases, then its supply curve shifts to the left; hence, a new intersection (B) appears. This new intersection shows that the transition from private cars to public transport has occurred and indicates lower balanced traffic costs. In other words, if the road capacity is allocated to buses, then the average cost and total travel cost will be reduced when the road capacity decreases.

Interpretations of Downs–Thomson Paradox with Median Bus Lane Operations

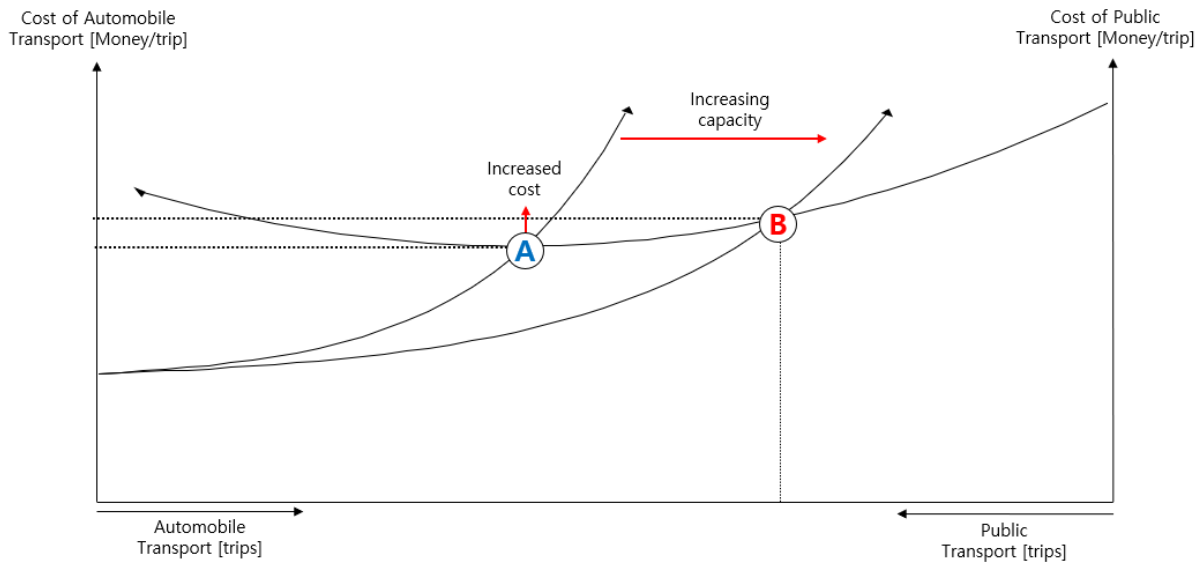


Figure 1. Concept of Downs–Thomson Paradox

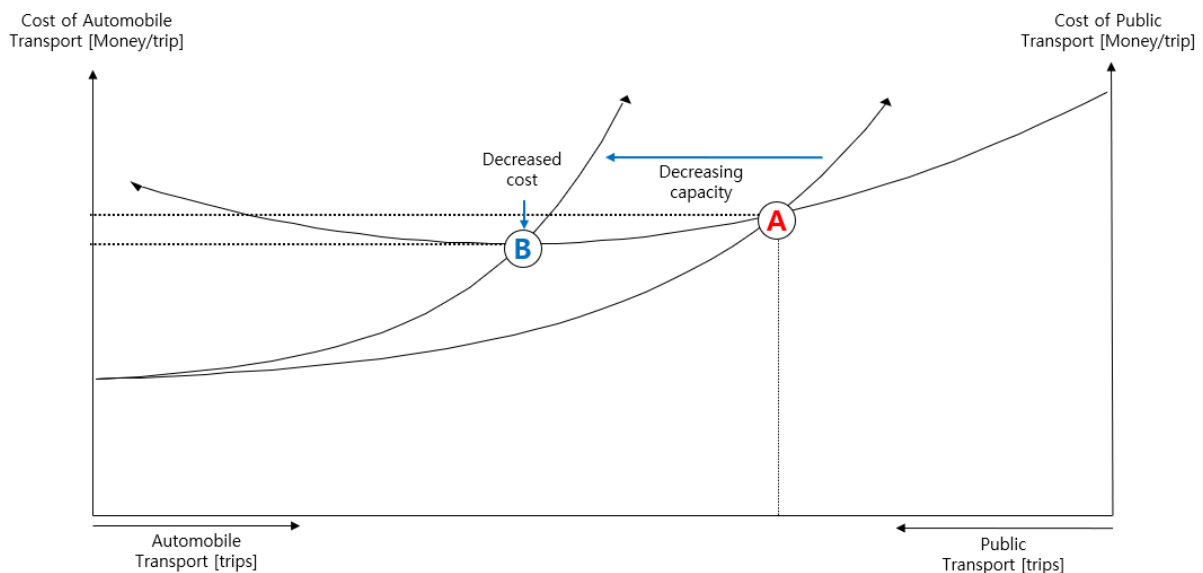


Figure 2. Concept of Downs–Thomson Lee Paradox

Ortuizar, J. D. and Willumsen, L. G. (2001) discussed the relationship between private cars and public transportation, as illustrated in Fig. 2. They considered the increase in the private car mode as a natural phenomenon caused by the income increase resulting from economic growth. Accordingly, public transportation demand is converted into private car demand. The drop in public transportation demand leads to the increase in its operational costs. Companies attempt to offset this by reducing public transportation service; and this is achieved by increasing the interval among different buses and raising fares. This leads to a vicious cycle of increasing passenger car utilization through the decline in public transportation service. Under these circumstances, the control of passenger car utilization or the subsidy of public transportation could increase the demand for public transportation; hence, a virtuous circle of passenger car and public transportation utilization is created. This study focuses on creating that virtuous circle by enhancing bus demand through the introduction of MBL.

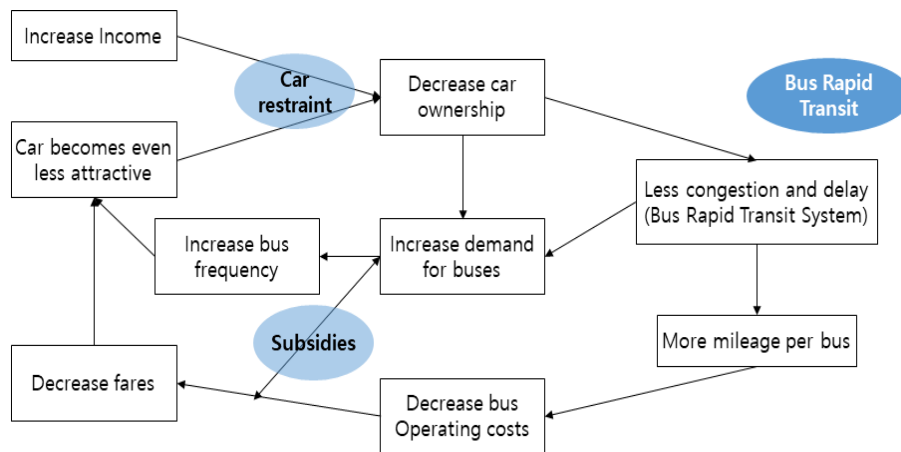


Figure 3. Virtuous circle relationship between passenger cars and public transportation

Afimeimounga, H. et al. (2005) supported Thomson’s theory in terms of maintaining the balance between cars and public transport in urban areas. Figure 3 describes the changes in passenger cars and buses before and after the project's implementation. The figure also shows the traffic variations at the time when the balance is achieved with the introduction of MBL.

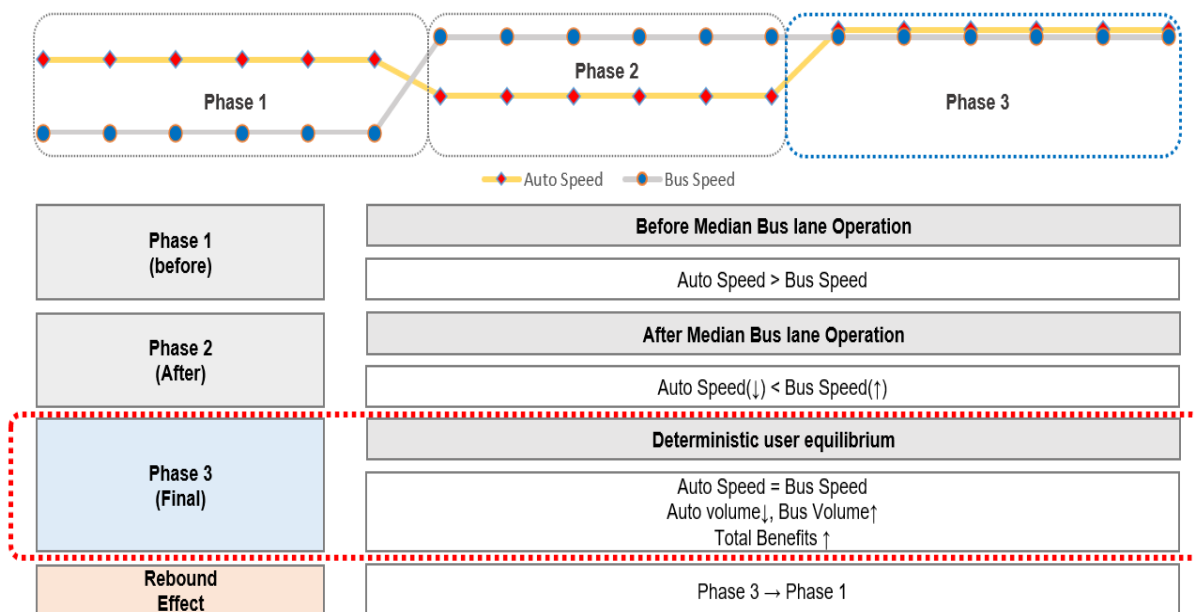


Figure 4. Speed relationship between passenger car and buses with MBL introduction

Phase 1 shows that before the MBL introduction, the average passenger car speed is higher than that of the bus when they share same road capacity. Phase 2 exhibits a sharp decrease and increase in the speeds of both passenger cars and public transport, respectively, during the transition period shortly after launching the project and before reaching the balance between transportation means. In this study, the monthly speeds of both passenger cars and buses as well as Phase 2 are not analyzed. Phase 3 indicates that the balance between the two means, passenger car and bus, has been reached. In this phase, the speeds of both modes remain the same; the traffic volume of passenger cars is reduced, whereas that of buses is increased. As a result, the total benefit of society is increased as a whole.

2.2 Methodology of Downs–Thomson Paradox Interpretation

The implications of the theory of deterministic user equilibrium with the introduction of the MBL are as follows. First, the condition under which the equilibrium of passenger cars and buses is achieved is the same, and the TDM policy of increasing road capacity contributes to speed reduction in all modes and deteriorates the transport service level. Second, a balance point is formed between buses and private cars after reaching the unbalanced situation where the service level of the latter is considerably higher than that of the former. Third, a counterpoint is formed in the unbalanced situation. The level of service in urban transport therefore considerably depends on public transport.

In this study, the similarity between passenger car and bus speeds in Phase 3 is statistically proved using the t-test analysis. Indeed, this methodology allows the verification of the deterministic user equilibrium by analyzing the speed of each mode, as reported by Lee, S. et al. (2010). Moreover, contrary to previous studies, which were only able to demonstrate the state of equilibrium because of difficulties in data collection, this study has analyzed the change in traffic during the equilibrium state by aggregating the number of bus arrivals and departures based on smart card data. By analyzing the speed and traffic volume changes, the time-saving benefit resulting from implementing the MBL project could be accurately determined. In summary, the D–T paradox analysis proves the efficiency of the median bus line policy.

Table 1. Framework of Research

Step	Contents
Step 1	Prove the deterministic user equilibrium between car and bus's speeds using a t-test
Step 2	Analyze the value of travel time savings, during the deterministic user equilibrium

3. Effect Analysis of Jongro MBL Introduction

3.1 Analysis of passenger car and bus data

For the effect analysis, the passenger car and bus data on traffic volume and speed were analyzed in 2017 and 2018, i.e., before and after the MBL project implementation in Jongno (Fig. 5). Given the difficulties in obtaining passenger car data, the Seoul Metropolitan Government extracted speed and traffic volume information using taxi data acquired through the smart card. The October–December 2017 data during the MBL construction period are not reflected in the analysis because of the high risk of error.

According to the passenger car analysis, the drops in traffic volume and speed resulting from the MBL introduction are approximately 30% and 15%, respectively. These results, summarized in Tables 2 and 3, reflect the changes in the volume of and speed of passenger cars over time.

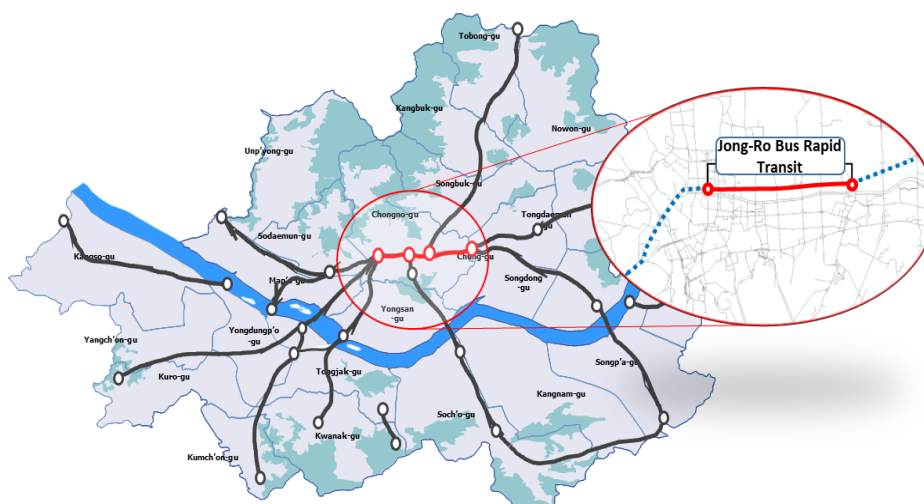


Figure 5. Jongro Median Bus Lane

Table 2. Traffic volume analysis of passenger car (vehicles per day)

Month	2017	2018	Gap	Rate of change
Jan	70,589	47,051	-23,538	-33.3%
Feb	72,584	47,750	-24,834	-34.2%
Mar	69,235	46,274	-22,961	-33.2%
Apr	70,532	46,584	-23,948	-34.0%
May	71,724	46,940	-24,784	-34.6%
Jun	71,247	46,393	-24,854	-34.9%
Jul	70,115	46,093	-24,022	-34.3%
Aug	67,678	45,365	-22,313	-33.0%
Sep	66,037	45,888	-20,149	-30.5%

Table 3. Passenger cars speed analysis (km/h)

Month	2017	2018	Gap	Rate of change
Jan	20.3	17.4	-2.9	-14.3%
Feb	20.2	17.3	-2.9	-14.4%
Mar	20.7	17.4	-3.3	-15.9%
Apr	20.5	16.7	-3.8	-18.5%
May	20.4	16.5	-3.9	-19.1%
Jun	19.8	16.7	-3.1	-15.7%
Jul	19.5	17.4	-2.1	-10.8%
Aug	19.4	17.8	-1.6	-8.2%
Sep	17.4	17.9	0.5	2.9%

Bus traffic volume data are similarly acquired, using the smart card and by calculating the number of bus passengers. Only the number of bus passengers in Jongno-gu are estimated and analyzed. The buses speed, however, is calculated using the departure and arrival time data. The bus data during the construction period between October and December 2017 are also excluded from the analysis.

According to the analysis, the bus traffic volume and speed increase by approximately 2.5%, and 20%, respectively, after the implementation of the MBL project, as summarized in Tables 4 and 5, which also depicts the changes in bus volume and speed over time.

Table 4. Bus volume analysis (vehicles per day)

Month	2017	2018	Gap	Rate of change
Jan	40,157	41,015	857	2.1%
Feb	40,421	41,429	1,009	2.5%
Mar	41,319	42,447	1,127	2.7%
Apr	41,459	42,434	975	2.4%
May	41,718	43,071	1,354	3.2%
Jun	42,866	43,379	513	1.2%
Jul	43,101	44,151	1,050	2.4%
Aug	41,764	42,817	1,053	2.5%
Sep	42,484	43,425	941	2.2%

Table 5. Buses speed analysis (km/h)

Month	2017	2018	Gap	Rate of change
Jan	13.2	16.4	3.2	24.6%
Feb	13.7	16.3	2.7	19.5%
Mar	13.3	16.4	3.1	23.3%
Apr	13.2	15.8	2.6	19.3%
May	13.4	15.6	2.2	16.6%
Jun	13.7	15.8	2.1	15.2%
Jul	14.3	16.4	2.1	14.9%
Aug	14.2	16.8	2.6	18.6%
Sep	13.9	16.9	3.0	21.8%

3.2 T-test analysis of speed with MBL introduction

The t-test is performed to check the balance between the two means (buses and private cars) based on their speeds. First, it is statistically verified that the speed of each vehicle differs depending on the execution of the project. This is indicated by Table 6, which lists the t-test values for the change in speed by mode before and after the implementation of the MBL project. In this research, the t-test alpha level is 95%. The expressions for the test are as follows:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$DF = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{(\frac{s_1^2}{n_1})^2 / (n_1 - 1) + (\frac{s_2^2}{n_2})^2 / (n_1 - 1)}$$

where s^2 is the unbiased estimator or variance of two samples; n_1 is the number of passenger cars; n_2 is the number of buses; DF is the degree of freedom.

Table 6. T-test results of speed variation according to MBL introduction

Null hypothesis: There is no speed variation according to MBL introduction					
	Auto			Bus	
	Equal	Not Equal		Equal	Not Equal
t statistic	6.87	6.87	t statistic	-12.79	-12.79
DF	16	12	DF	16	16
Pr>t	<.0001	<.0001	Pr>t	<.0001	<.0001

The t-test analysis of speed change resulting from project execution is performed under the assumption that the speeds before and after the project are the same. The analysis results show that in the automobile mode, the t-statistic of 6.87 rejects the null hypothesis, confirming that there is a positive velocity change depending on the execution of the project. In the case of buses, the t-statistic of -12.79 also rejects the null hypothesis, confirming that there is a change in speed also depending on the execution of the project. The introduction of the MBL therefore proves that there is a speed change in both passenger car and bus means.

The above results confirm that the MBL introduction changes the speed of both transport modes; thereafter, it is verified whether the speed between the two means reaches the deterministic user equilibrium. Table 7 lists the speed difference between passenger cars and buses after the introduction of the bus-only lane. Table 8 summarizes the t-test results.

Table 7. Comparison of speeds between passenger cars and buses

Month	Auto	Bus	Gap	Rate of change
Jan	13.2	16.4	3.2	24.6%
Feb	13.7	16.3	2.7	19.5%
Mar	13.3	16.4	3.1	23.3%
Apr	13.2	15.8	2.6	19.3%
May	13.4	15.6	2.2	16.6%
Jun	13.7	15.8	2.1	15.2%
Jul	14.3	16.4	2.1	14.9%
Aug	14.2	16.8	2.6	18.6%
Sep	13.9	16.9	3.0	21.8%

Table 8. T-test results of passenger car and bus speeds

Null hypothesis: The bus speed is the same as the auto speed					
	Before (2017)			After (2018)	
	Equal	Not Equal		Equal	Not Equal
t statistic	16.95	16.95	t statistic	4.34	4.34
DF	16.00	11.00	DF	16.00	16.00
Pr>t	<.0001	<.0001	Pr>t	<.0001	<.0001

Interpretations of Downs–Thomson Paradox with Median Bus Lane Operations

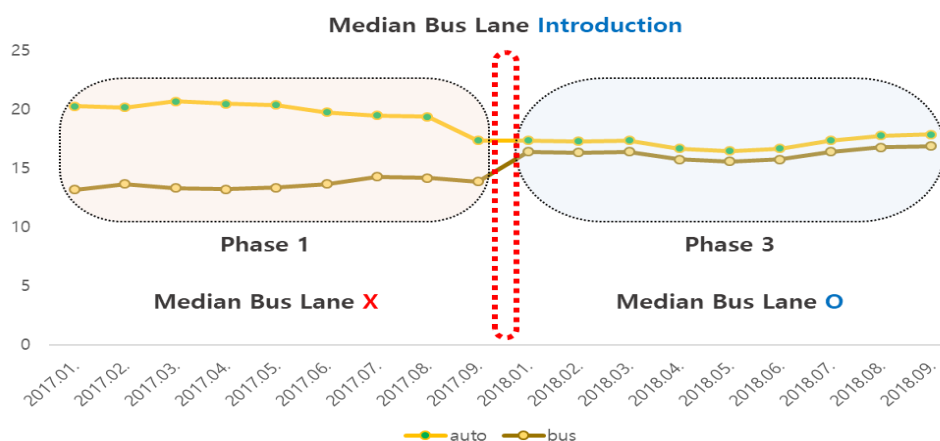


Figure 6. Comparison of speed between passenger cars and buses

The t-test analysis of passenger cars and buses is performed under the assumption that the speeds of these vehicles are the same. The year before the MBL introduction (i.e., 2017) is not a user-balanced year because the t-statistic (16.95) rejects the null hypothesis; therefore, the speeds of passenger cars and buses differ. Moreover, in 2018, after the MBL introduction, it is found that the t-statistic approached the null rejection. At approximately 25% before project implementation, although the nullity of the variance (4.34) is rejected, the nullity that the bus speed is not the same as the passenger car speed is also rejected.

The result is considered to be close to, but failed to reach, the user balance of passenger cars and buses following the introduction of the MBL in Jongno. Further, there is an opportunity for the introduction of additional traffic demand management policies, such as setting congestion charges and speed limits. The user balance of passenger cars and buses with the MBL introduction is shown in Fig. 6.

3.3 Travel time-saving benefit analysis with MBL introduction

The previous t-test has not achieved the balance between the two means; however, the results approximate this balance. Based on these results, it is demonstrated that the hypothesis regarding the balance point between passenger car and bus is formed at the service level of bus, considering the service level of bus claimed by Thomson. To prove the assumption that the bus service level has been formed, the balance between the two means is determined by the calculation of time saving based on the process in which the balance is formed, with an intention of increasing total social benefits.

The calculation of time-saving benefits was presented by Mackie, P. J. et al. (2001) and analyzed by applying a calculation formula that is used as a guideline for preliminary feasibility assessment. For a realistic analysis, it is assumed that the MBLs in Jongno, Yulgok-ro, Cheonggyecheon-ro, Eulji-ro, and Toegyero, which are competing routes, are affected. The time value for benefit calculation is applied to the average traffic time value distributed by the KTDB (Korea Transport Data Base). The analysis results are shown Fig. 7 and listed in Table 9. The calculation formula is

$$VOTS = VOT_{before} - VOT_{after}$$

where $VOT = \sum_l \sum_{k=1}^2 (T_{kl} \times P_k \times Q_{kl} \times 365)$; T_{kl} is the travel time of link l of each mode;

P_k is the time value of each mode; Q_{kl} is the traffic volume of link l of each mode; k represents the modes (passenger car, 1; bus, 2)

Table 9. Travel time-saving benefit result (US\$/day)

mode	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	total
auto	35,740	38,510	31,674	30,097	31,017	37,917	43,755	43,447	56,930	349,087
bus	9,003	6,799	8,623	7,570	5,860	6,377	5,354	6,528	8,098	64,213
total	44,742	45,309	40,298	37,667	36,877	44,294	49,109	49,975	65,028	413,299

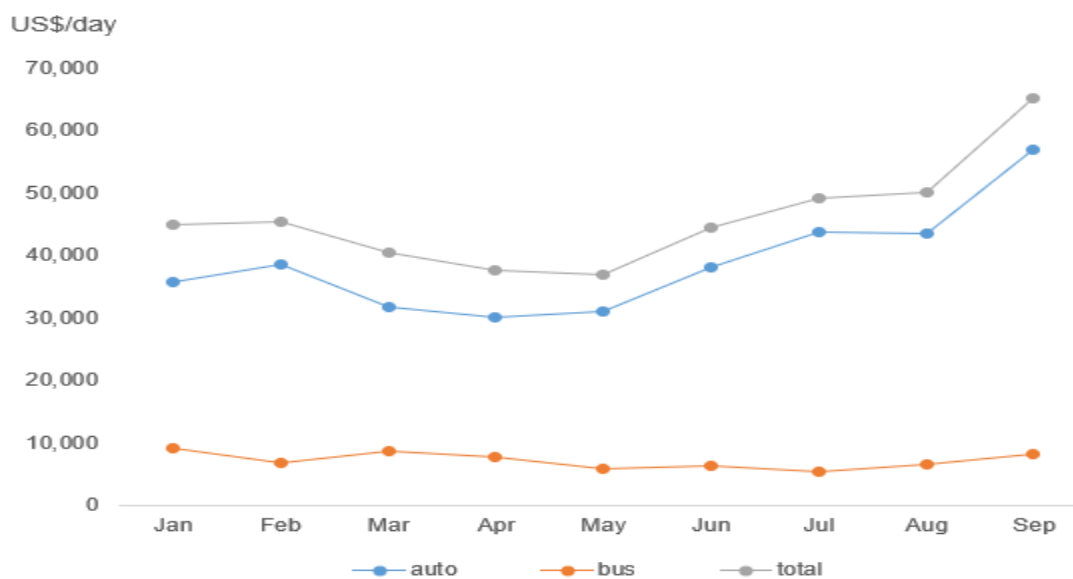


Figure 7. Total travel time-saving benefit (cars and buses)

Based on analysis results, despite the reduced speed of passenger cars and the increased traffic volume of buses after the project is implemented, the time-saving benefit of each mode is analyzed. Specifically, in contrast with the reduction in benefits resulting from the decrease in speed with the implementation of the project, it is assumed that passenger car users will gain more benefits by redirecting the routes to surrounding roads or by switching to buses. In the case of buses, it is assumed that the overall travel time-saving benefits of buses increase because the positive benefits from reducing the average bus speed are greater than the negative benefits from the increased traffic volume and higher stop time.

4. Conclusion

4.1 Results and discussion

This study proposes a method for analyzing the effects of the introduction of Jongno MBL with the D-T paradox analysis. For the analysis, the taxi data derived from the smart card are utilized as bus taxi data, and the bus speed data used is calculated based on differences within bus stop arrival times. Moreover, the traffic volume data of passenger cars are utilized for each branch of the Seoul Metropolitan Government, and the bus traffic volume data are analyzed based on the data obtained from the smart card on the number of bus passengers.

The Downs–Thomson paradox interpretation statistically verifies that the two means have reached the deterministic user equilibrium. The t-test is employed in the overall verification to analyze whether the speeds of the two means are the same, and the results close to the user-balancing state could be obtained. Next, the two means yield the total social benefit of increasing user balance. Specifically, time-saving and total social benefits increase as the user equilibrium is approached.

This study is considered significant because it analyzes the effects of the MBLs employed in many cities based on collected data and the changes in total social benefits in a user-balanced state. This is especially the case in areas where passenger cars have been previously intensively studied from a bus perspective and based on bus traffic data derived from smart card, passenger car speed, etc. There are limitations, however, such as the calculation of bus speed based on the bus stop arrival time and the use of representative traffic volume obtained from a point for passenger car traffic. In later experiments, it is expected that a more realistic method of effect analysis will be employed in order to interpret the D-T paradox based on real-time data.

4.2 Future plan

In order to analyze the practical effects of future research tasks, further research investigations are necessary to overcome the limitations of data application methods employed in this study, and which cannot reflect the real-time data. In addition, it is deemed necessary to analyze data within one year of the project's execution to predict when the terminal user equilibrium will be reached after the project. Finally, the TDM policies, such as introducing a congestion pricing, are necessary to reach a user balance that has not yet been attained.

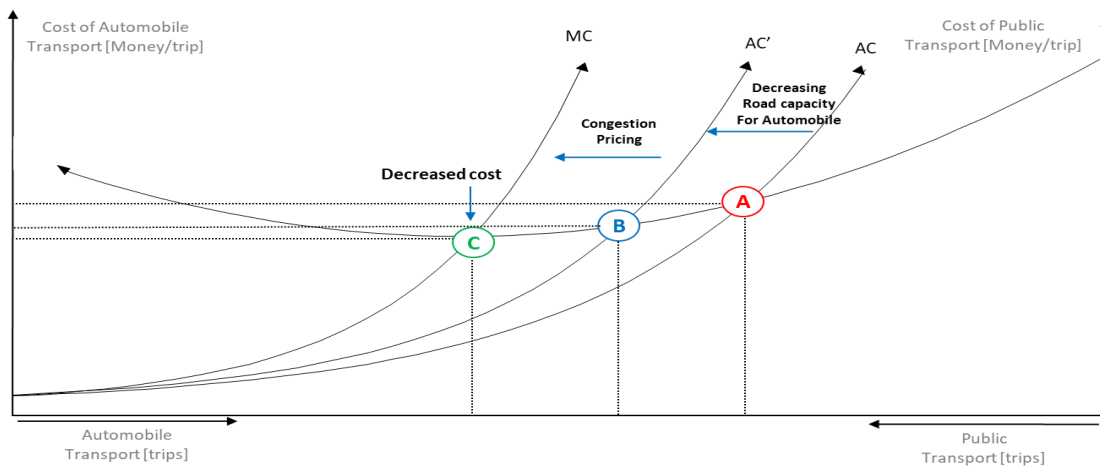


Figure 8. Imposition of Congestion Pricing in Downtown areas

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