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A theory of deregulation in public transport

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ABSTRACT

This paper builds a theory of deregulation and roll-out of on-road competition in the public transport sector. Focusing on the dimensions of competition, ownership and authorisation, we identify five distinct regulatory regimes: public monopoly, regulated monopoly, unregulated monopoly, outsourcing and competition in the market. Our generalised theoretical framework allows for the direct comparison in the social welfare terms of the monopolies' outcomes and the fragmented market structure with endogenous entry. We formulate a set of parameter restrictions that make competition in the market preferable to public monopoly and outsourcing. We also show the theoretical possibility of a 'revised' regulatory cycle forming a sequential transition between these identified regulatory regimes. Our theoretical predictions shed some light on the reasons for policy reversals and the bypassing of certain phases of the cycle, that can occur due technological advances, changes in fiscal constraints and institutional capacity improvements.

1. Introduction

Growing transport demand in developing countries, rising environmental concerns in industrialized ones and growing institutional maturity in transitional economies shape public transport debates in different ways. A particular stage of economic development imposes different constraints on policy makers' choice of the most efficient regulatory regime for public transport, being a social necessity for everyone. In the 'age of regulation', in most developed countries, such as Western European, Offshoots of the West and Japan, transport services were dealt with through public enterprises. In the US, however, they were the primary targets for natural monopoly regulation. In theory, 'when technical conditions make a monopoly the natural outcome of competitive market forces, there are only three alternatives that seem available: private monopoly, public monopoly, or public regulation. All three are bad, so we must choose among evils...The choice between the evils of private monopoly, public monopoly, and public regulation cannot, however, be made once and for all, independently of the factual circumstances.' (Friedman, 2009, p.29).

In practice, perfect regulation is virtually impossible, while financial and technological circumstances are changeable. Admitting the inherited problems of informational asymmetries and the non-benevolence of regulators, policy makers have implemented deregulation policies in public transport throughout the world with varying degrees of success. The academic world responded with the 'new regulatory economics'(Laffont & Tirole, 1993) and the 'new political economy of regulation' (Benoît, 2019), but the gap between the theory and the evidence for it remains a weakness of the existing literature on public transport reforms.

Our paper aims at shrinking the gap between the research and practice by modelling competition and ownership dimensions in our theoretical model of deregulation. These issues are central to Thredbo conference discussions on land passenger transport reforms, summarised by (Wong & Hensher, 2018). Following the new regulatory economics tradition, we consider the fourth 'evil', particularly the model of contracting out by (Auriol & Picard, 2009b), and the fifth one, namely laissez faire competition in a deregulated and fragmented public transport sector, which we model explicitly in the theoretical section of the paper. Our contribution offers a complementary but novel view on the relationship between the four regulatory regimes analysed in (Auriol & Picard, 2009b), because it integrates a 'deregulation regime' as an outside option for policy makers. Thus, we provide a theoretical background to the comparison of 'competition in the market' vs. 'competition for the market' alternatives, discussed in (Merkert et al., 2018).

Deregulation contains three principal dimensions: price liberalisation, subsidy reduction and privatisation (Beesley, 1991). In UK bus services in the 1980s, the immediate effect was a reduction in costs per vehicle mile and a downward shift in the operator's vehicle size, leading to the emergence of minibuses. In developing and transitional economies the replacement of public by private sector operations has led to even greater reduction in average vehicle size (Gwilliam, 2001) and an increase in fares. (Gwilliam, 2000) also argues that the scope of competition is not linked to ownership. For instance, in the Russian Federation in 1990s, decentralisation and the withdrawal of federal subsidies gave rise to a market initiative in the bus sector, regardless of whether ownership was public or private. In many Russian cities, the authorities established separate bus parks that operated largely independently.

Competition in the market between informal sector operators emerged in many developing countries spontaneously rather than by 'the design of a conscious act of deregulation' (Gwilliam, 2001). Safety problems, reliability and accessibility of privately delivered services, together with the market cartelisation (eg. in Santiago, see (Koprach, 1994)) and monopolisation brought about an inefficient allocation of resources. These drawbacks of unregulated markets were addressed by the governments either in the form of a return to public monopoly, like in Kuala Lumpur (Gwilliam, 2001), or in the form of competition for the market, like in Moscow. In 2010s, the Moscow authorities drove out the old minibuses, reintroduced tougher regulation and formalised tendering and direct award procedures for granting operating rights to private companies (Ryzhkov, 2018).

Deregulation often leads to transport market fragmentation. Minibus, marshrutka, dolmuş, matatu, bush taxi, dollar van, tro tro, colectivo, tut-tuk and their cognates serve millions of users daily, primarily but not exclusively in the developing world (Sgibnev & Vozyanov, 2016). The 'the small vehicle' argument is important to understand the competition/regulation dichotomy which is associated with the cost-control debate. The case the ability of entry barriers to be reduced due to smaller investment requirements has not been properly addressed in the regulatory literature yet. The reason is that the prevailing production technology with high fixed cost makes a new entry unprofitable.

We capture the well documented 'minibus' delivery solution in a deregulated public transport market by relaxing the assumption of prohibitively high investment requirements to start

operations. Market fragmentation can occur through splitting the required transport production capacity between competing rivals. The notion of fixed costs may not only refer to vehicles alone, but also to ancillary facilities (e.g. maintenance depots). We model 'competition in the market' regime as a Cournot oligopoly in the deregulated and fragmented public transport market, the structure of which is not designed *ex ante*. Our asymmetric information modelling framework enables direct pairwise comparisons of the social welfare values for all five regulatory regimes: 1) public monopoly, 2) regulated private monopoly, 3) unregulated private monopoly, 4) outsourcing or competition for the market, and 5) competition in the market.

The policy relevant question of whether deregulation is preferable to regulation is addressed in the following way. We map the sensitivities of the respective social welfare differentials to the following exogenous parameters: the marginal cost uncertainty, the fixed costs of production, the social cost of public funds, and the ability to extract monopoly rents via efficient tendering mechanisms and corresponding franchise fees.

Our formal analysis is shaped by the concept of 'regulatory cycle' articulated by (Gwilliam, 2008) for the bus sector. It implies that a historical change of the industry's organisational structures and regulatory regimes follows a cyclical path and does not evolve to a once and for all preferable solution. This concept has been widely discussed in the transportation literature (Wilkinson, 2010). Our paper demonstrates, that the existence of a regulatory cycle cannot be excluded from the theoretical point of view, but we also suggest the theoretical arguments for policy reversals.

To develop these arguments, we organise the paper as follows. Section 2 reviews the literature on conceptual models of public transport, including (Gwilliam, 2008) concept of regulatory cycle, and relates it to the modelling framework of the new regulatory economics. We build a model of deregulation and fragmentation in a public transport sector in Section 3 and present the closed form solutions for five alternative regulatory regimes. In Section 4 we compute the effects of technological, fiscal and institutional parameters on the welfare differentials and formulate our theoretical propositions. Section 5 discusses the main applications and implications of the theory and interprets the five alternatives as phases of the regulatory cycle and explores the main driving forces of the regulatory cycle in public transport. Section 6 concludes. Any formal proofs not presented in the text are provided in the accompanying technical Appendix available at <https://www.editorialmanager.com/retrec/>.

2. Literature review

The vast amount of empirical literature seeks to provide policy makers with useful reference and source of information for strategic choice of structural alternatives in public transport. As (Silcock, 1981) argues 'generalisations in the urban transport field may dangerously mask the complexity of the issues'. Yet, the author articulated a common set of important aspects, including ownership, government control, cost structure, demand conditions, economic impact and some others, that allowed for structured comparison of the conventional 'corporate' passenger transport in industrialised countries and paratransit industries in developing countries. We concur with (Silcock, 1981) that the effects of policy have been largely untheorized and start our literature review with three insightful and influential papers, that attempt to identify 'pure organisational forms' in public transport.

2.1 Regulatory regimes in public transport

The seminal paper by (van de Velde, 1999) develops the Strategic/Tactical/Operational (STO) framework, and outlines several alternative regimes in the context of a public transport regulatory reform. He delineates the public management situation with the authorisation regime dominated by a state-owned monopoly. In our further analysis, we use the term 'public monopoly' when the authority initiative makes it a de jure monopoly being the only procurer of services and when no entry threat exists legally. Such a direct management regime assumes no information asymmetries between a benevolent regulator and the public transport service provider. All the vehicles and other installations are owned and run by the government that also provides public subsidies, since this service is a social necessity.

A 'regulated monopoly' is viewed as a corporate entity with the government as the sole shareholder. It operates the state-owned assets and interacts with the regulator, who is less informed than the company. Being a de facto monopoly it has incentives to manipulate its cost parameters to seek to obtain higher cost recovery subsidies. The regulator then acts as a 'watchdog' that sets prices and a 'subsidiser' (van de Velde, 1999), but it is the monopoly that physically supplies the services.

We use the term 'outsourcing' for a regime, when some or all vertically related transport services are contracted out using competitive tendering or other award mechanisms. This form of 'competition for the market' secures the de facto monopoly status of the corresponding market segment in return for the concession of franchise fees but gives more freedom to the transport operators (van de Velde, 1999). An ability of the authorities to establish an efficient mechanism for the extraction of monopoly rents is an important prerequisite for this regime to succeed. This consideration will be taken into account in our modelling framework and welfare comparisons of the 'outsourcing' regime with the other alternatives, including 'competition in the market'.

When profitable services appear autonomously as a result of deregulation of prices and market access conditions, some form of 'competition on the road' emerges in public transport (van de Velde, 1999). Such a competitive private supply proves to be a departing point in a regulatory cycle, conceptualised by (Gwilliam, 2008). For the industrialised countries it consists of 'a cycling through private competitive supply, unregulated private monopoly, regulated private monopoly, nationalisation and then back, through further regulatory reform to some type of private competitive supply either through free entry or franchising. Following (van de Velde, 1999) we make a clear distinction between the free entry and the franchising regime being 'competition in the market' and 'competition for the market' correspondingly. (Gwilliam, 2008) also describes the different routes to escape from the potential drawbacks of market fragmentation, either through consolidation by merger or through success in franchise competition. Analytically, these are difference regulatory regimes, as pointed out by (Currie, 2016), who offers a typology for the range of structural alternatives in public transport which may help introduce competitive pressure in the sector. Comparing the reform options of full open competition, various tendering models and negotiated performance based contracting, he bypassed the private unregulated monopoly case which was highlighted in (Gwilliam, 2001) as a wide spread situations in developing countries.

For the developing countries, this is a situation following a deregulation, often spontaneous, of the public transport market (Gwilliam, 2001). A decline and failure of formal suppliers due to socially motivated fare restraints led to the emergence of fragmented informal supply (Gwilliam, 2008). In the absence of a sufficient antitrust oversight, it resulted in market cartelisation and monopolisation, opening the room for better regulation of a private consolidated supplier.

Summarising and generalising the important insights from the public transport literature, we distinguish five different regulatory regimes in the industry: 1) a public monopoly, 2) a regulated private monopoly, 3) an unregulated private monopoly, 4) an outsourcing or ‘competition for the market’, and 5) a private oligopoly or ‘competition in the market’. Ownership matters, as it will be discussed later, since in the cases of private oligopoly, private monopoly and outsourcing the private sector incurs the fixed investment costs. A corresponding competition/regulation dichotomy together with the ownership dimension is presented in Table 1.

Access regulation				
NO	De facto			De jure
Regulatory contract (price control and/or budget transfers)				
NO		YES		
Competition in the market		Competition for the market		
YES	NO	YES	NO	
Private oligopoly	Private monopoly	Outsourcing / competitive tendering	Regulated monopoly / negotiated contracts	Public monopoly

Table 1. Five ‘core’ regulatory regimes in public transport

Building a bridge to the regulatory economics literature, we need to emphasise the important difference between the de jure public monopoly and the other two regulatory intensive regimes, namely regulated monopoly and government outsourcing. In the case of public monopoly, the regulatory agencies appear to be more informed about the cost structure of the services, provided and managed by transport authorities. Both regulatory and transport authorities are the integral parts of the government body, so the regulator designs the ‘first-best’ perfect information price-subsidy scheme and the public transport manager implements it unconditionally. Thus society (taxpayers) pays no information rent to the transport industry. For the other two cases we will consider an imperfect information setup.

The main difference between the regulated monopoly and outsourcing is the ability of the government to extract a monopoly rent (including information rent) in the latter case. Through introducing a tendering system the government can take some advantages of efficient competitive mechanisms, making a potential candidate to compete for a monopoly right to serve the whole market (or route) by bidding higher a franchise fee. The regulator potentially can extract the company’s profits by setting optimal *ex ante* franchise fees while controlling the quality and quantity supplied, offering an *ex post* contract. Still, the true value of the firm’s cost cannot be observed by the regulator.

Competition in the market can be potentially modelled as Cournot competition, which is equivalent to a price competition with pre-committed capacities (Kreps & Scheinkman, 1983). An important feature for the competition should be the ability of entrants to overcome significant entry barriers in the form of fixed investment. Our analysis of the transportation

literature (and the 'minibus story' in particular) suggests a way to model a decrease in these barriers by considering a fragmented deregulated industry structure, when the fixed costs can be split between the competing operators. We return to this crucial point in the next section.

In the case of an unregulated private monopoly, we will continue to assume asymmetric information. This de facto monopoly, has private information about its costs. When it is regulated, society sacrifices an information rent, which would serve as a mechanism to reveal the monopoly's costs. The remaining part of this section discusses theoretical papers that capture the basic stylised facts about the transportation market structures, and serve as building blocks for our theoretical analysis.

2.2 Regulatory economics for public transport

Market failure in public transport justifies a certain degree of regulation. In turn, a regulatory failure, due to information asymmetries and inefficient institutions, provides a reason for a certain degree of competitive pressure on regulated public monopolies. Regardless of the regulatory alternative, a sustainable provision of infrastructure-based public transport services, requires that consumers' surplus is positive and not less than fixed costs. As (Gwilliam et al., 1985) show, the existence of user costs in the public transport sector and the associated external economies of scale in production make it the rationale for subsidies. To prevent this industry from complete shut-down turns out to be a non-trivial policy challenge in an environment with cost uncertainty and fiscal constraints. Deregulation and introduction of competition may be a solution at some point. This section reviews the theories of new regulatory economics and builds the bridge to the literature on competition with unknown cost. The ownership issue matters, both for the justification of information asymmetry between the parties and the role of the social cost of public funds.

A seminal paper by (Baron & Myerson, 1982) studies the problem of regulating a natural monopoly with bilinear cost structure when both fixed and marginal cost parameters are unknown to the regulator. A feasible regulatory policy that maximises a weighted sum of the expected consumers' surplus and the expected profit for the monopoly, results in a subsidy that rewards the firm sufficiently to break-even, and induces the firm to reveal its costs. Comparing the size of optimal subsidies in complete and incomplete information cases, the authors show no obvious relationship, because in the former case it was designed to cover the firm's fixed costs, while in the latter case it is determined by the need to prevent the firm from misrepresenting its costs. Public ownership with complete information and zero social cost of public funds, implies marginal cost pricing and a subsidy equal to fixed costs, unless this subsidy exceeds the consumers' surplus, in which case there is no market. Importantly, as (Baron & Myerson, 1982) show, the optimal regulated price in an asymmetric information case may not only exceed marginal costs, but be greater than an unregulated monopoly price. With the lack of an additional subsidy to reward the low-cost firm for not misreporting its cost, the government may somehow punish it for announcing high costs. In particular, if the government is capable of forcing reportedly high-cost firms to set a price above the monopoly level, this 'cheater' will be penalised but will still find it profitable to stay in the market. The foregone profit is neither passed on consumers nor extracted by the government in the form of tax.

Such a redistribution issue is captured in (Baron & Myerson, 1982) by the assumption that the regulator strictly prefers consumer surplus to the firm's rent. Alternatively, to admit the importance of insufficient fiscal capacity and strict budget constraints, Laffont & Tirole (1993) develops the 'incentive regulation theory'. It explicitly accounts for the social cost of public funds ($\lambda > 0$), and highlights the role of the firm's endogenous cost-reducing efforts (see (Dementiev, 2016) for discussion of the estimated values of λ in the context of public transport). The two approaches are compared by (Armstrong & Sappington, 2007), who generally follow the marginal cost benchmark of (Baron & Myerson, 1982) to make their extensive analysis more tractable. Our analysis abstracts from any political economy and public choice issues and will ignore policymaker's preferences for redistribution. Our modelling approach follows (Laffont & Tirole, 1993) and is shaped by the objective of a benevolent regulator facing fiscal constraints. The regulator finds it costly to compensate the firms' fixed cost from the budget, and establishes Ramsey prices, which exceed marginal costs.

This approach is applied in (Estache & Wren-Lewis, 2009) in the context of developing countries facing different forms of institutional weaknesses, including limited regulatory capacity and limited fiscal efficiency. Competition in the market has been considered as a way of mitigating the regulatory failures caused by institutional limitations. However, the same governance problems and associated weakness of antitrust agencies may undermine the potential advantages of competition, making the industry prone to collusion and cartelisation.

The problem of stability and enforceability of cartel agreements between several firms with unknown costs is discussed in (Cramton & Palfrey, 1990). They obtain a closed form solution for Cournot competition as an outside option in the cartel threat game. Each firm has no fixed costs, while a constant marginal cost parameter is drawn independently from a continuous distribution with positive density on the same support. Such a common cost uncertainty makes the cartel less stable, relative to a standard ex ante cartel agreement. The authors show the possibility of perfect collusion, with side payments between the participating firms, but also admit the existence of an incentive for new firms to free-ride, by entering the cartel solely to collect the side payment for not producing. In our model, positive fixed costs serve as an entry barrier that automatically discourages any opportunistic behaviour by potential entrants, and endogenizes the number of firms competing in the market.

Indeed, excess capacity may serve as a credible deterrent to new entry as in the model of optimal privatisation by (Wen & Yuan, 2010). If the government owns and operates the public utility at an optimal production capacity, but is unable to subsidise it due to budget constraints, it may design an optimal multi-dimensional reform plan that contains restructuring, privatisation and deregulation policies. Restructuring entails selling existing assets to an optimal number of firms in a deregulated private sector. In our model we employ the same approach to modelling the deregulated market as the Cournot competition between n identical firms, but use the (Cramton & Palfrey, 1990) assumption of cost uncertainty. From the fiscal perspective, the higher the cost of public funds the more attractive it is for the government to sell off its assets. However, it also makes an increase in the post-privatisation prices more likely, leading to lower total output level and underutilisation of capacity in the industry. Such a drive for deregulation may be explained by the public finance motive, when

lower consumer surplus is compensated by any lower taxpayer burden, due to low (or no) subsidies to the private sector.

In our model, developed in the next section, the privatisation of a subsidised public monopoly has no direct budget revenue effect from the selling-off of the state-owned assets (Wen & Yuan, 2010). In fact, as (Sato & Matsumura, 2019) show for the case of a mixed duopoly market with free entry, the relationship between the optimal degree of privatisation and the shadow cost of public funds is, in general, non-monotone. The authors assume sufficiently low fixed costs K to make the number of entering firms greater than one. They conclude that for developed countries with a moderate social cost of public funds, an increase in λ lowers the optimal degree of privatisation. For developing countries, with a relatively high λ , its further increase leads to a greater optimal scope of privatisation. Our model exhibits non-monotonicity as well, while privatisation issues are not considered either from the public finance or the mixed oligopoly point of view.

We model a deregulated market as Cournot competition between identical firms, with total production capacity being capped at the optimal capacity under public service provision. The idea to directly compare the social welfare for the case of a public regulated monopoly with unknown costs, and that of unregulated competition in the presence of the fixed costs and the social costs of public funds, is not new in the literature. In their detailed review of the pros and cons of liberalisation in network industries, (Armstrong & Sappington, 2006) model unregulated competition as a Bertrand duopoly. This setup naturally leads to a duplication of rivals' fixed costs in the case of price competition, making regulation superior to deregulation. Another argument made by the authors in favour of regulation is the possibility of taxing the monopolist's rent and reduce the public fiscal burden when λ is relatively high. On the contrary, when high fixed costs are to be compensated from the budget, higher λ makes competition relatively more preferable. Even without fixed costs, governments with insufficient institutional capacity to ensure subsidies for low-cost firms, would be bound to set prices at the highest possible marginal cost level. Such a restricted form of monopoly regulation is never preferable to an unregulated Bertrand duopoly in their study.

We check this result in a general asymmetric information Cournot model for n firms with fixed costs (departing from (Ferreira & Ferreira, 2010) who studied the setting without fixed costs) and compare it with a benchmark provided in (Auriol & Picard, 2009b). Their unified modelling approach captures the basic features of government outsourcing as the combination of a transfer of control and cash-flow rights to a private firm that has to pay a franchise fee for the right to operate as a monopoly. The franchise fee may be determined through competitive tendering or bilateral bargaining between the government and private investors. Once the monopoly right is awarded, the firm becomes fully responsible for its own financial state. However, the government is able to offer ex post contracts to this unregulated monopoly and ask it to increase the supply in the market. The authors derive the optimal ex post contract and illustrate it by relating to the contractual arrangements between the local government and private taxi companies in the early 2000s in Europe. The taxis that operated in low density areas received a transfer from the local budget to secure higher profits than under *laissez faire*. These outsourcing solutions are more cost effective than regulated public transportation (Auriol & Picard, 2009b).

Ownership is irrelevant for the optimal regulatory scheme, when private and public entities have the same degree of contract completeness, as in (Baron & Myerson, 1982) and (Laffont & Tirole, 1993). This may not be the case for developing countries with immature institutions to enforce effective regulation as shown in (Auriol & Picard, 2009a). In their model of public utility privatisation under asymmetric information, public ownership is dominated by a private unregulated monopoly because information rents raise the social cost of subsidies. However, with profitable natural monopolies and weak regulators, this monotonic relationship does not hold, making the privatisation of under priced public assets more attractive only for intermediate values of λ . A private monopoly is less likely to be preferred to a regulated one when fixed costs K fall, but this may also lead to new firms entering the market. Access to the market is somewhat restricted to a single firm in (Auriol & Picard, 2009a), while in our model we allow for free entry.

A more competitive industry structure, a duopoly, is found preferable to a monopoly in the model of yardstick competition by (Auriol & Laffont, 1992), when the fixed costs K fall below some critical level. When the regulator designs the market structure, ex ante, both firms incur sunk costs to enter the market, but the associated duplication of assets may be outweighed by the higher probability of a small marginal cost (the sampling effect). The yardstick effect cuts down information rents, and makes the regulated duopoly preferred to a monopoly.

The sampling effect will play an important role in our model for deregulated market with endogenous entry, that we build in the next section. We confront our results, obtained for competition in the market, with four regulatory alternatives, compared in (Auriol & Picard, 2009b).

3. The model

A market for public transport service can be served by $n \geq 1$ firms indexed by $i \in n = \{1, \dots, N\}$ and producing the quantity q_i of a homogeneous good. The industry in total needs to sink a fixed and verifiable¹ investment cost $K > 0$ that captures increasing returns to scale. Existing technology implies the firm's marginal cost β_i , to be unobserved by the other firms (in case of oligopoly) or by the regulator (in case of regulated monopoly or outsourcing). This idiosyncratic cost parameter is independently drawn from $[\underline{\beta}, \bar{\beta}]$ for each firm and is assumed to be uniformly distributed with density and cumulative distribution functions being $g(\beta) = 1/(\bar{\beta} - \underline{\beta})$ and $G(\beta) = (\beta - \underline{\beta})/(\bar{\beta} - \underline{\beta})$, respectively. Thus the hazard rate for this special case is simply $\frac{G(\beta)}{g(\beta)} = \beta$. Consumers face linear inverse demand, $P(Q) = a - bQ$, with $a, b > 0$ and Q being the total industry output. Hence, the gross consumer surplus becomes: $S(Q) = \int_0^Q P(x)dx = aQ - \frac{b}{2}Q^2$.

The government is benevolent and utilitarian. It maximises the expected value of social welfare function EW which is a mathematical expectation of the unweighted sum of the net consumers' surplus (CS) and producers' surpluses (PS) and minus the social cost of the net transfer (SC) to the firm from the budget: $EW = CS + PS - SC$. A net transfer, being the

¹ This simplifying assumption helps us avoid the moral hazard problem and is made for the sake of model tractability.

difference between a subsidy t and franchise fee F , is counted at the social (or shadow) cost of public funds $(1 + \lambda)$. The shadow cost λ reflects the nature of distortionary taxation when a unit that is transferred to the firm costs $1 + \lambda$ units to society. This assumption is crucial for our further analysis of fiscal constraints and changes the cost of public funds over time. It is commonly asserted, that λ is relatively high in developing countries and low in developed ones (see (Dementiev, 2016) for the literature review and theoretical analysis of the impact that the value of λ has on the optimal organisational structure of local public transport). Thus, the regulator's objective function becomes: $EW = CS + PS - SC = [S(Q) - P(Q)Q] + [P(Q)P - \beta Q - K + t - F] - (1 + \lambda)(t - F) = S(Q) - \beta Q - K - \lambda t + \lambda F = aQ - \frac{b}{2}Q^2 - \beta Q - K - \lambda t + \lambda F$.

When $n = 1$, there are three cases of a single *de facto* service provider in the market and one case of *de jure* monopoly (see Table 1). In the latter case of public monopoly, there is no private knowledge about the cost parameter, and a fully informed benevolent regulator maximises the social welfare function. The three other cases are: regulated monopoly, unregulated monopoly and outsourcing. All these cases are modelled in (Auriol & Picard, 2009b) and are used to benchmark out analysis of deregulation. We reproduce their closed form solutions for the expected welfare functions for these four structural alternatives numbered as 1, 2, 3 and 4 in Table 2.

When $n \geq 2$, competition *in* a deregulated market can take many forms, once free entry is permitted. To make our analysis tractable and comparable with the alternative regulatory regimes, we consider an oligopoly, when in the symmetric Bayesian-Nash equilibrium n firms compete in quantities *à la* Cournot. The number of firms is determined endogenously through the 'market initiative' and market participation constraint. This is contrary to the 'authority initiative' argument made in (Wen & Yuan, 2010) who analyse a 'privatisation story' and determine n by considering the optimal deregulation policy as a budget revenue extraction from the divestiture of public assets. Another common assumption of this literature on restructuring and privatisation is a possibility of cost savings through the elimination of so-called 'X-inefficiency'. It 'refers to the lack of effort in a firm that is unpressured because of an absence of rivals' (Wen & Yuan, 2010). We depart from their analysis by assuming that in a deregulated market firms do not have any ad hoc efficiency gains. On the contrary, we model the efficiency gains through a *sampling effect* on marginal costs and a *fragmentation effect* on fixed costs.

The sampling effect refers to a higher probability of drawing a lower marginal cost β_i which is uniformly distributed on $[\underline{\beta}, \bar{\beta}]$. It makes relatively more efficient firms from a sample, i.e. with lower β_i , enter the market. When each firm learns the realisation of β_i , it faces the inverse demand function $P = a - b(n - 1)Eq - bq_i$, where Eq is the expected sum of the symmetric rivals' outputs for $q_{j \neq i}$. The firm i 's entry decision requires the marginal cost coverage in the short-run, $P > \beta_i$. Given the demand function parameters, this implies $q_i^c = \hat{a}/b - \beta_i/2b > 0$, if $\hat{a} > \beta_i/2 \leftrightarrow 2\hat{a} > \beta_i$, where $\hat{a} \equiv (\sqrt{1 + a(n - 1)} - 1)/(n - 1)$. Correspondingly, the firm quits with $q_i^c = 0$ if $\beta_i \geq 2\hat{a}$.

The fragmentation effect refers to a proportional decrease in the entry barrier with the number of firms in the market. To enter the market and produce the first unit of output, each

of n firms simultaneously incurs sunk cost K/n , so the market structure becomes fragmented. With a greater n , the industry becomes more competitive both in terms of the number of strategic players and the lower entry barrier. Yet, the industry-wide fixed cost $K = nK/n$ remains intact. This assumption eliminates the direct welfare effect of the market atomisation on the fixed cost that is split between the competing operators. By isolating the fragmentation effect, we can make a meaningful welfare comparison of ‘competition in the market’ with other regulatory regimes and highlight the role of fiscal constraints and strategic interaction in an oligopolistic market.

Firm i 's objective is to maximise its expected profit: $E\Pi_i = P(Q)q_i - \beta_i q_i - K/n = (\hat{a} - \beta_i/2)^2/b - K/n$. The first term of this expression represents the sampling effect and the second one – the fragmentation effect. The industry's structure is determined endogenously by the participation constraint $E\Pi_i \geq 0$ of each firm. Since output in equilibrium depends on these two sources of ‘toughness’ of competition, the relationship between the number of firms and profitability is essentially non-monotone. We illustrate this property for some parameter values in Fig. 1, where the role of K is clearly seen. For example, other things equal, for a relatively large $K = 55$ only a single producer can produce without subsidies. For an intermediate value of $K = 35$, a duopoly structure emerges. Finally, relatively small $K = 5$ makes the production commercially viable for many firms, since the fragmented market structure attracts potentially more efficient competitors.

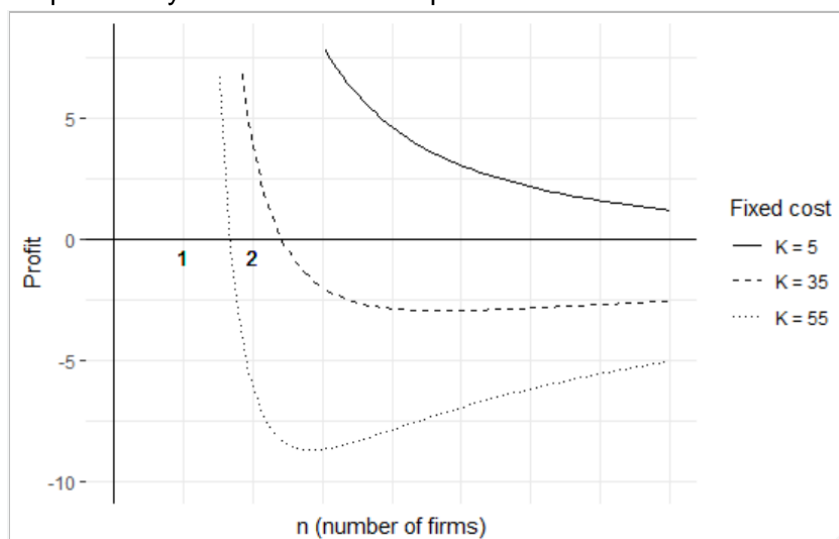


Figure 1. Profitability of a fragmented competitive market

In a competitive environment, non-negative profits imply a positive social welfare, since public subsidies are zero. The *ex ante* social welfare in a general case for n firms is defined as:

$$EW^c = E \left\{ S(Q) - \sum_i^n \beta_i q_i - n \frac{K}{n} \right\} = \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\beta + \bar{\beta}}{2} ((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1 \right) \frac{\beta^2 + \bar{\beta}\beta + \bar{\beta}^2}{3} \right) - K.$$

Without loss of generality, to make our further analysis tractable, we focus on a duopoly case, when the lowest possible marginal cost β_i is normalised to zero, so $\underline{\beta} = 0$, and $n = 2$. In this case the expression for the expected welfare is reduced to: $EW^c =$

$\frac{1}{b} \left(2\sqrt{1+a}(\sqrt{1+a}-1)^2 - a\frac{\bar{\beta}}{2} + \frac{\bar{\beta}^2}{6} \right) - K$, that makes the direct welfare comparison with other structural alternatives analytically possible.

4. Welfare differentials

Intuitively, other things being equal, the first best theoretical alternative in terms of social welfare would be a public monopoly. However, other things are not equal. Table 2 summarises the expected values of social welfare functions for the five alternatives including the first four from (Auriol & Picard, 2009b) and the competitive one calculated above.

Regimes	Expected values of social welfare functions
1. Public monopoly	$EW^p = \frac{(1+\lambda)^2(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} - (1+\lambda)K$
2. Regulated monopoly	$EW^r = \frac{3(a(1+\lambda))^2 - 3a\bar{\beta}(1+\lambda)(1+2\lambda) + \bar{\beta}^2(1+2\lambda)^2}{6b(1+2\lambda)} - (1+\lambda)K$
3. Unregulated monopoly	$EW^m = \frac{\bar{\beta}^2 + 3a(a-\bar{\beta})}{8b} - K$
4. Outsourcing	<p>If $\lambda < \lambda_0 = \frac{a-\bar{\beta}}{2\bar{\beta}}$, $EW^o = \frac{6a^2+9a^2\lambda-6a\bar{\beta}(1+2\lambda)+\bar{\beta}^2(2+\lambda)(1+2\lambda)}{12b(1+2\lambda)} - K + \lambda F$</p> <p>If $\lambda \geq \lambda_0 = \frac{a-\bar{\beta}}{2\bar{\beta}}$, $EW^o = \frac{a^3+3\bar{\beta}(\bar{\beta}^2-3a\bar{\beta}+3a^2)(1+2\lambda)^2}{24b\bar{\beta}(1+2\lambda)^2} - K + \lambda F$</p>
5. Competition in the market	$EW^c = \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\frac{\bar{\beta}^2}{3} \right) - K$

Table 2. Expected social welfare values for five regulatory regimes

The next step of our analysis is to find the second-best regulatory regime which is preferable to the others. We have obtained five closed form solutions for the expected values of social welfare functions and can compute the corresponding welfare differentials. To make this analysis meaningful and policy relevant, we chose a number of pairwise comparisons that are central to public transport debates and reflected in the literature on regulatory cycles. Thus, we limit our analysis to 7 possible transitions between the stages, shown in Fig. 2 It suggests a 'revised regulatory cycle' as a framework for welfare comparison that is based on the 5 'core' regulatory regimes and only 7 transitions between them.

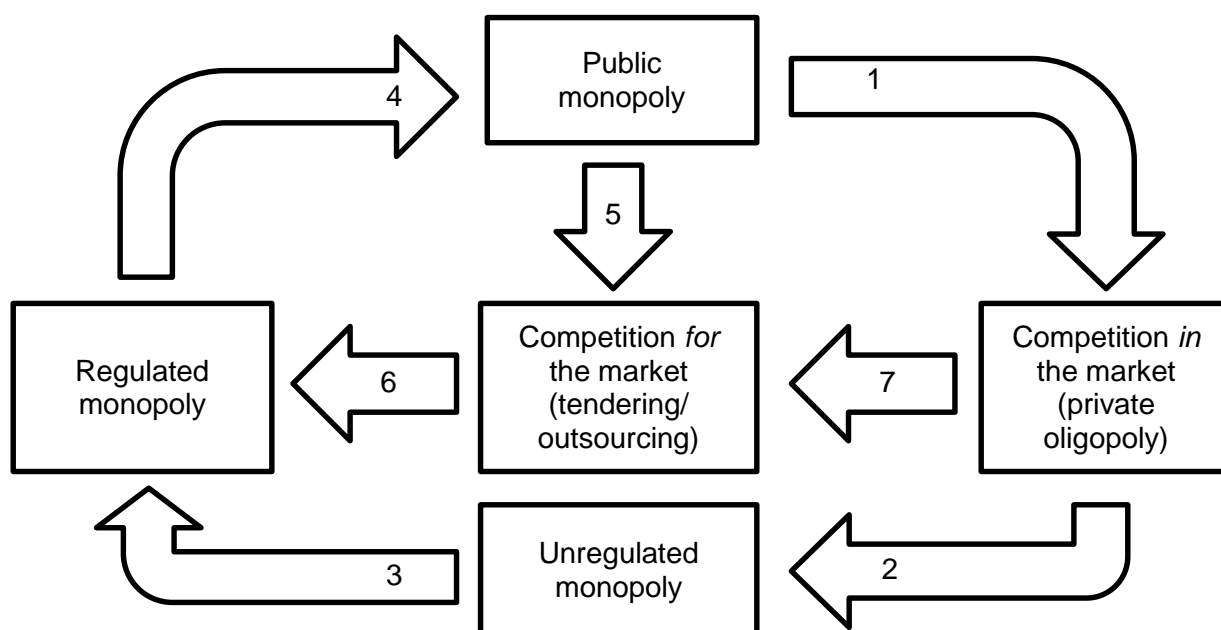


Figure 2. Regulatory cycle reconsidered: ‘outer’ cycle 1-2-3-4 and ‘inner cycle’ 4-5-6

‘Competition in the market’ regime is central to our analysis of deregulation in public transport. Accordingly, we particularly focus on the transitions to and from this structural alternative. The closed form solutions for the welfare differentials 1, 2 and 7 are presented in Appendix. The other four transitions are related to ‘monopoly’ regimes and borrowed from (Auriol & Picard, 2009b) to complement our analysis of regulatory cycle.

The values of ΔEW depend on the demand and cost conditions and have ambiguous sign in general. One exception is a standard result from the regulatory economics, that a perfect information public monopoly, with EW^p , is always preferred to a regulated monopoly under cost uncertainty, with EW^r , that is: $\Delta EW^{pr} \equiv EW^p - EW^r > 0$ for any $a, b > 0$ and $2\bar{\beta}/a, \lambda \in [0,1]$. It implies, that perfectly informed regulators without fiscal constraints can achieve the first best and guarantee the social optimum. Indeed, public ownership eliminates information asymmetry, thus the regulator should not pay an information rent to incentivise a regulated monopoly to reveal its costs. Also due to the asymmetry of information, the equilibrium output is lower pushing the gross surplus below the first-best optimum.

In the ‘second-best’ world, we need to analyse how sensitive the welfare differentials to changes in the following exogenous parameters. First, it is the marginal effect of technological (in)efficiency of operations. An increase in the marginal cost of production is modelled as a wider support for the corresponding random variable. Recall, that the lower bound is normalised to $\underline{\beta} = 0$, thus an increase in the upper bound $\bar{\beta}$ is also associated with higher cost uncertainty. Second, it is the marginal impact of the fixed cost, K . Third, it is the change in the fiscal capacity of the government captured by the shadow cost of public funds, λ . Finally, in the case of outsourcing, the government’s regulatory capacity is modelled by its ability to extract the monopoly rent in the form of a franchise fee F .

4.1 Competition in the market vs. public monopoly

Deregulation policy is presented in Fig.2 as transition 1 from a public monopoly to a competition in the market. The expected social welfare value under Cournot oligopoly, EW^c , can be greater than that of a public monopoly, EW^p . The corresponding welfare differential $\Delta EW^{cp} = EW^c - EW^p$ has an ambiguous sign in general. Nevertheless, its sensitivity to changes of the above mentioned exogenous parameters can be derived. Positive (negative) signs of these effects are labelled with '+'('−') in Table 3.

Proposition 1. Competition in the market is more likely to be preferred to public monopoly if $\bar{\beta}$ or K increase. Additionally, an increase in λ works in the same direction for a relatively high fixed cost K , and in the opposite direction for a relatively low fixed cost K .

In the case of public monopoly, nature chooses the marginal cost β according to the given distribution function, so an increase in $\bar{\beta}$ means a wider range of its possible realization. The regulator and the monopoly's manager learn β , and the optimal transfer scheme offered by the regulator, is unconditionally implemented by the manager. In a deregulated environment, 'nature' is more 'selective' due to a sampling effect, that increases the probability of low-cost firms to survive in a fragmented market. This result illustrates the advantages of lower X-inefficiency of the competing firms. The fixed cost effect works through different channels.

An exogenous increase in the fixed cost K makes deregulation relatively more favourable because it implies no subsidies to the industry and thus saves λK of the taxpayers' budget. Moreover, the marginal effect of the social (shadow) cost of public funds λ becomes more pronounced of higher fixed cost K . Indeed, when λ grows, it becomes more difficult for the government to collect taxes and finance the required investment. When such investment needs are low (relatively small K), competition in the market does not benefit much from the fragmentation effect. This makes the advantages of competition less pronounced, while industry's output of competitive industry is in general below the social optimum under public monopoly.

4.2 Competition in the market vs. unregulated monopoly

Deregulated markets have a tendency for consolidation, which, under certain conditions, may lead to welfare-improving monopolisation (transition 2). The textbook result that monopoly is always associated with higher welfare losses relative to oligopoly is compromised by the presence of cost asymmetry. Thus, the corresponding welfare differential $\Delta EW^{mc} = EW^m - EW^c$ can be also positive. Oligopolistic firms competing in quantities, even if they are risk-neutral, are more cautious and strategically produce less than in the case of full information. As a result, their total output may be close or even smaller than the monopolistic one, making the size of consumer surpluses at least comparable.

When a fragmented industry structure is determined endogenously, firms that survived the entry game may nearly break-even. In the absence of budget subsidies, monopoly's profit is predictably higher than the sum of profits of oligopolistic firms. Figure 1 illustrates this intuition for the intermediate level of fixed cost $K = 35$.

Proposition 2. Unregulated monopoly is more likely to be preferred to competition in the market if $\bar{\beta}$ increases.

An increase in $\bar{\beta}$ is associated with an increase in cost uncertainty, making a fragmented industry structure less attractive from the social welfare point of view. Another argument in favour of monopolisation is advocated by (Cramton & Palfrey, 1990). They point out that if two firms with different and unknown costs form a cartel, the most efficient one would produce the total output while the less efficient firms would produce nothing. This revelation game brings about an efficiency gain from monopolisation of unregulated market.

4.3 Competition in the market vs. outsourcing

A switch of regulatory regime from ‘competition in the market’ to ‘competition for the market’ (transition 7) activates the full range of available regulatory instruments, namely: authorisation (or market access) control, price control, budget subsidy and efficient competitive tendering mechanisms to extract the monopoly rent via franchise fee F . The multiplicity of ‘degrees of freedom’ makes the sign of the welfare differential $\Delta EW^{op} = EW^o - EW^p$ ambiguous in general. Yet, we can draw some conclusions regarding its sensitivity to exogenous parameters and a franchise fee F .

Proposition 3. Outsourcing is more likely to be preferred to competition in the market if $\bar{\beta}$ increases or F increases. For relatively high (low) $\bar{\beta}$, F and λ , an increase in λ makes outsourcing more (less) preferable to competition in the market.

The effect of $\bar{\beta}$ on the relative attractiveness of a ‘franchised’ monopoly has the same grounds as in the case of unregulated monopoly. Indeed, the consolidation effect favours monopolisation of the sector, when costs are unknown. The effect of F is straightforward: efficient tendering mechanisms result in fierce competition for the monopolised market which is authorized to be a monopoly in exchange of ex ante payment F . This ‘institutional capacity’ is usually correlated with the government’s fiscal capacity to collect taxes and may also correspond to the value of λ . Yet, it is useful to have F as independent instrument reflecting institutional maturity of the government.

The effect of λ is non-monotonic because both incoming (F) and outgoing (t) budget flows are valued at the social cost of public funds in the social welfare function. With high F , budget revenue effect overweighs budget subsidy effect. Tough fiscal constraints, associated with high λ , make governments focus on revenue extraction. In our model, firms in a deregulated market are free from any taxes, so outsourcing is preferred. With low F , an increase in λ does not provide the government with additional budget revenues. When government budgets are less constrained, i.e. low λ , and increase in the shadow cost of public funds may have a sizable effect in relative terms (low base effect). This makes competition for the market relatively less attractive than competition in the market.

To integrate our modelling results for transitions 1, 2 and 7 into the concept of regulatory cycle, we complete the cycle by adding the welfare effects along transitions 3, 4, 5 and 6

derived in (Auriol & Picard, 2009b). All the seven transitions are presented in Table 3 for easy reference².

Transition No.	Welfare differential ΔEW	Marginal cost cap $\bar{\beta}$	Fixed cost K	Social cost of public funds λ	Franchise fee F
1	ΔEW^{cp}	+	+	+ if K high - if K low	0
2	ΔEW^{mc}	+	0	0	0
3	ΔEW^{rm}	-	-	+ if K low and/or $\bar{\beta}$ low (high λ) - if K high and/or $\bar{\beta}$ high (low λ)	0
4	ΔEW^{pr}	+	0	+	0
5	ΔEW^{op}	+	+	+ if K high or F high - if K low or F low	+
6	ΔEW^{ro}	- if $\lambda < \lambda_0$	-	+ if K or F low - if K or F high	-
		+ if $\lambda \geq \lambda_0$ and high a - if $\lambda \geq \lambda_0$ and low a			
7	ΔEW^{oc}	+	0	+ if $\bar{\beta}$ and/or F high (high λ) - if $\bar{\beta}$ and/or F low (low λ)	+

Table 3. Sensitivities of welfare differentials

Our findings should be interpreted with great care, since policy decisions in practice are driven by many other important factors beyond the scope of our analysis. Nevertheless, the theory sheds some light on the effects of exogenous parameters that can tilt the balance at the margin. The next Section discusses implications of our theoretical considerations for the public transport policies and their applications to the concept of regulatory cycle.

5. Discussion

5.1 Competition in the market vs competition for the market

The choice of a ‘proper’ form of competition is an ongoing debate in public transport literature at least since bus deregulation under the 1985 Transport Act in Britain (Gwilliam et al., 1985). British experience, with completely deregulated and largely privatised the provision of bus services, suggests a test of which form of competition works best (Nash & Smith, 2020). The authors argue that competition for the market due to better integration of tendered out services have worked better than competition in the market in the local bus sector in Britain. This fact highlights the importance for outsourcing government to enforce the detailed ex post contract or subsidy scheme. Transition 5 in Fig. 2 from public monopoly to outsourcing implies that the latter becomes relatively more preferable in the context of rising fiscal concerns (growing λ) when K is high and F is high. In this case, a better integration of services implies higher fixed cost K . The governments’ institutional capacity to secure high franchise fees F via effective competitive tendering mechanisms is also high.

Competition in the market for long distance coach services in the UK has been limited but survived as a regulatory alternative (Nash & Smith, 2020). The authors suggest a candidate

² The corresponding Lemmas from (Auriol & Picard, 2009b) are formulated in Appendix.

explanation, that integration is a less important issue for such services. From the theoretical point of view, this means lower K and a possibility of more than one firm to serve the market on a commercial bases with no subsidies (for instance, $K = 5$ in Fig. 1). According to Proposition 1, deregulation of public monopoly and consequent market fragmentation can be a welfare improving option for relatively low K and decreasing λ . In fact, a more 'enlightened approach' of local authorities, who started introducing better facilities to support bus operators via voluntary quality partnerships (Nash & Smith, 2020), implied a shift in fiscal priorities modelled as a decrease in the (opportunity) cost of public funds λ .

An interesting illustration of the policy choice between the two competing alternatives could be the case of 'post-deregulation' in Moscow. The bus sector faced a transition from highly centralised state-owned monopolies through liberalisation and fragmentation of the industry (transition 1 in Fig. 2 to 'competition in the market') to a thoroughly elaborate tendering system (transition 7 in Fig. 2 to 'competition for the market'). Deregulation in 1990s resulted in the fragmented market structure, dominated by private transport companies which followed the routes of public suppliers, filling the gaps in time schedule and operating mostly small vehicles, were swept from city street. Route permits were awarded via non-price awarding schemes until 2015, when they were clustered into 63 service areas and tendered out (Ryzhkov, 2018).

Proposition 3 suggests, that competition for the market (in the form of outsourcing) is likely to be preferred to competition in the market when marginal cost cap $\bar{\beta}$ is high and increasing and franchise fees F is high and increasing. The first condition seems to be met with higher quality standards increasing the cost of service. The second conditions is reflected by higher institutional maturity and regulatory capacity of the local government, that was capable to write a detailed service contract and competitive tendering procedures.

5.2 Competitive tendering vs. negotiated contracts

Outsourcing via competitive tendering may not be the ultimate solution for public transport provision if the government is unable to specify ex ante a contract with sufficiently detailed description of risk-allocation procedures associated with the transfer of control and ownership rights. A policy reversal back to public monopoly is then possible (transition 5 in the opposite direction in Fig. 2). Another option for 'tougher' regulation could be a move towards regulated monopoly through the mechanism of negotiated contracts (transition 6 in Fig.2). Thus the debate on 'competitive tendering' vs. 'negotiated contracts' can be viewed from the theoretical perspective as a choice of implementation mechanisms to enforce outsourcing regulatory regime.

It is worth noting here that the outsourcing contracts in the model are renegotiation-proof by design. It means that neither party finds it optimal to deviate and offer any Pareto-improving amendments once the private firm has entered and disbursed investment. The econometric study by (Iossa & Waterson, 2019) shows that in the London bus market from 2003 to 2015 about three fourths of re-tendered contracts for service on a particular route were awarded to the same entities. That poorer tender performance together with worsening outcomes in terms of higher prices may pave the way to greater regulatory oversight. The authors mention that along with theoretically predictable learning effects and activism effects,

incumbents may win contracts more often due to increasing cost asymmetries. The associated hold-up problem makes competitive tendering evolve to the system of renegotiated contracts which essentially resembles much of regulatory environment for a monopoly with unknown cost (transition 6).

Negotiated contracts tend to reduce cost uncertainty, modelled as a reduction of marginal cost cap $\bar{\beta}$. According to Lemma 3 in Appendix, for developed countries with relatively low $\lambda < \lambda_0$ such a decrease in $\bar{\beta}$ makes regulated monopoly more likely to be preferred to outsourcing, since cost asymmetry is reduced. In the perfect information case, when cost asymmetry completely disappears, public monopoly without agency problems and fiscal constraints proves to be the first-best alternative.

These arguments can fuel the discussion of optimal regulatory regime in public transport through lens of the concept of regulatory cycle.

5.3 Whither regulatory cycle

According to (Gwilliam, 2008), industrialised and post-colonial regulatory cycles look similar though have different starting points and the nature of 'competition'. We suggest a 'revised regulatory cycle' (see Fig.2) that decouples the two concepts of competition *for* and *in* the market. We distinguish between a less regulatory intensive 'outer' regulatory cycle (transitions 1 – 2 – 3 – 4) and a more institutionally demanding 'inner' regulatory cycle (transitions 5 – 6 – 4).

The *outer cycle* is likely to take place in developing countries with low fiscal capacity and weak regulatory institutions. Forced by budget constraints, deregulation (often spontaneous) leads to an informal and fragmented public transport sector (Wang et al., 2018). The *inner cycle* is a more relevant description of structural dynamics in developed and institutionally mature countries. From theoretical point of view, it is reasonable to start discussion of the nature of regulatory cycle at the first-best regime on public monopoly.

Public monopoly is assumed to have no information asymmetry but has little incentive to improve efficiency and reduce operational costs. As physical assets depreciate and incentives to minimise cost vanish, a competitive supply becomes a policy option. When fiscal burden is an issue, governments find it welfare improving to decentralise the public transport service (Proposition 1). Deregulation of fares and market access liberalisation leads competition in the fragmented market modelled as a Cournot oligopoly in our analysis (transition 1). Obviously, such industry segmentation is a viable option if the overall demand for public transport is relatively high and stable, securing non-negative profits for each competing firms.

A competitive industry with increasing returns to scale tends to evolve into a more concentrated structure. Without regulatory oversight this leads to a cartelisation or other forms of horizontal integration in the market and improvements in productive efficiency, at the expense of allocative efficiency. According to Proposition 2, cost asymmetries between competing firms lead to underproduction (thus lower consumer surplus) and make the oligopolistic industry less likely to survive (lower producer surplus). A monopolised industry structure appears to be welfare improving when cost uncertainty increases (transition 2).

Regulatory intervention through lower prices increases patronage in public sector. However, such regulatory policy reform requires sufficient fiscal capacity, meaning the relatively low cost of public funds and a lack of budget constraints makes. Lemma 1 shows that the effect a decrease in the shadow cost of public funds is ambiguous in general because the consumer and producer surpluses are equally weighted in the social welfare function. One could naturally extend the model and consider societal preferences for income redistribution to justify 'low-price-high-subsidy' policies in low income counties (transition 3).

Regulated monopoly is inferior to a public monopoly since the latter eliminates any cost asymmetry between the firm and the regulator so society does not need to bear the burden of information rent (transition 4). However, the crucial feature of a public monopoly – the public ownership of assets – may become a fiscal burden in unprofitable markets. Low-power cost-reimbursement regulatory rules often result in cost inflation and a dramatic loss in efficiency. When full cost recovery is not guaranteed the regulated monopoly goes bankrupt and has to be nationalised. The outer cycle is completed (Figure 2).

The *inner cycle* in Fig. 1 starts when public monopoly considers outsourcing. The reason to introduce competition for the market is the same as in the case of structural reform and introducing competition in the market. Apparently, in the case of in-house production a public monopoly is unable to control costs. The government introduces competition *for* the market to reveal (at least potentially) the most efficient firm that is ready to serve the whole market requiring the lowest subsidy from the budget. Such a policy option rests on the assumption of the government being able to extract monopoly rent through efficient tendering procedures (high F). Such an outsourcing regime (transition 5) is a viable alternative in the short-run for fiscally constrained but institutionally mature states (see Lemma 2 in Appendix).

Competitive tendering mechanism are not perfect and not costless. As discussed earlier, the incumbent is more likely to win the next bid, so its valuable market experience and business reputation erect additional entry barriers for potential bidders. Lemma 3 postulates that less cost uncertainty favours regulation relative to outsourcing. A particular mechanism to reveal cost information and make regulation welfare improving could be negotiated contracts (transition 6).

Without political economy consideration, the theory leaves no room for the transition from the outsourcing to unregulated monopoly (being essentially an extreme form of outsourcing), because the former is always superior to the latter. The reason for that is governments' ability (via competitive tendering or contractual mechanisms) to tap revenues from the profit making unregulated monopoly and redistribute (at least partially) the rent to consumers to improve social welfare.

Our comparative statics analysis shows that a move along the regulatory cycle from one regulatory regime to another can be theoretically justified, as long as it is driven by the dynamics of exogenous parameters. The theory does not exclude the possibility of policy reversals or bypasses of certain phases of the regulatory cycle. For instance, it can be short-cut by a straightforward move from public monopoly to private unregulated monopoly bypassing the phase of competitive supply or outsourcing. When the social cost of public funds is high, the government is unable to finance investment disbursement for any infrastructure project which has the cost $(1 + \lambda)K$ for society. Hence, a financially

unconstrained private monopoly is preferred to a public monopoly when variable profit is sufficient to cover the fixed cost. For instance, unregulated minibuses and motorcycles in some African capital cities are completely private and highly monopolised services that manage to break-even without any subsidies (Kumar and Barrett, 2008).

Cash-strapped governments leave the transportation sector for private companies to fill the service gap and allow a free market to be highly concentrated to enjoy the economies of scale. A decrease in operational and investment cost would create sufficient profit margins for private firms. The government would also consider such a *laissez faire* option to be welfare improving since free entry enables the creation of transport infrastructure that would not have been developed under public ownership. It means, that none of the structural alternatives discussed so far is unconditionally stable and preferable for all parameter values. Public transport is unlikely to converge towards a unique regulatory regime. Nevertheless, our model enables the channelizing of the effects of technological improvements, fiscal constraints and institutional capacity on the industry structure.

6. Conclusions

Before concluding, we feel that it is worthwhile to consider three significant objections that could be levied against our modelling strategy. First, the regulator is benevolent, so the model abstracts from any political dimensions. In particular, to make our results tractable, we do not model transition costs between the stages, the value of 'option to wait', the status quo and ideological biases or redistribution concerns. We abstract from any public choice considerations, including changes in electoral preferences or external pressure for contingent structural reform imposed by creditors. Second, public transport is viewed as market for a homogenous good without scope economies and vertical integration effects. Specifically, in a deregulated market the fixed capital K (vehicles, maintenance depots, and other fixed assets) can be split or shared between the competing firms without violating transportation technology. Third, the demand function is linear, and there is common uncertainty regarding the marginal cost parameter which is uniformly distributed.

Our paper contributes to the literature in four ways. First, at the theoretical level it introduces the model of deregulation that results in a decentralised fragmented competitive industry. The idea of splitting existing production assets between competing firms is borrowed from (Wen & Yuan, 2010), who considered optimal privatisation as the policy makers choice. We depart from their analysis by modelling the entry decision at the firm level. Second, our model employs the unified regulator's objective function to make direct welfare comparisons with the results from (Auriol & Picard, 2009a) and (Auriol & Picard, 2009b). We relate regulation and competition outcomes by comparing social welfare for the deregulated horizontal market structure with that of public monopoly, with unregulated private monopoly and with the outsourcing case. Third, our paper builds a bridge between conceptual studies on regulatory reforms in public transport and the new regulatory economics. Our model frames the debates on 'competition in the market' vs. 'competition for the market' and on competitive tendering vs. negotiated contracts. These debates are shaped by issues of the social cost of public funds and the institutional capacity of governments. Fourth, our model offers insight into the nature of the regulatory cycle, documented by (Gwilliam, 2000) and (Gwilliam, 2008), for the bus sector in industrialised and post-colonial countries and

considers it in a broader context, which also includes recent developments in some transitional economies.

We identify five 'core' regulatory regimes in public transport: 1) public monopoly, 2) regulated monopoly, 3) unregulated monopoly, 4) outsourcing, and 5) competition in the market. We obtain close form solutions for equilibrium prices and quantities to compute welfare differentials between the adjacent stages of the cycle. Our analysis focuses on a set of interpretable and policy relevant parameters (marginal and fixed cost, social cost of public funds and franchise fee) that influence a probability to switch between regulatory regimes. Our main results are as follows.

The theory implies that public monopoly without cost asymmetry *always* secures the first best solution for any social welfare maximisation problem other things equal. However, this solution is not feasible for cash-strapped governments that seek alternative regulatory regimes to meet budget constraints. This is where the regulatory cycle starts and this is where it can proceed in two directions.

When market demand is high enough to be served by two (or more) competing nonsubsidised firms (Fig. 1) the policy maker chooses deregulation that results in a fragmented structure and improves social welfare. In the model of deregulation of public transport, an increase in fixed investment requirements together with budget constraints makes deregulation relatively more favourable structural alternative to a traditional public monopoly delivery. Fragmentation of supply, as in case of minibuses, reduces entry barriers proportionally, so the firms' fixed cost saving attracts new entrants until they break-even. The total output of competitive industry (and thus consumers' surplus) is below the social optimum under public monopoly. However, fiscally constrained governments save taxpayers' money by refusing to subsidise a fragmented sector. Moreover, non-negative producer's surplus is secured by the endogenous entry decision, making competition in the market more likely to be preferable to public monopoly. When the social cost of public funds grows, it becomes particularly difficult for the government to collect taxes and finance these required investment. If such investment needs are low, competition in the market does not benefit much from the fragmentation effect. This makes the advantages of on-road competition less pronounced, since relatively low subsidies required to compensate for fixed cost are not significant.

When market demand is too low for multiple firms but sufficient for a single one to enjoy any monopoly profit the government can have two alternatives. On the one hand, governments with low institutional capacity could adhere to a *laissez faire* policy, letting an unregulated monopoly operate the whole market. All operating rights, investment obligations, market risks and cash control are transferred to the private sector that maximises producer surplus. On the other hand, more institutionally matured governments could enable welfare improving redistribution of income in favour of consumers by extracting monopoly rent via franchise fee and choose 'competition for the market'. An extreme form of such a wealth transfer is privatisation of public assets in the sector.

The revised regulatory cycle in public transport may be viewed as an overlap of 'outer' and 'inner' cycles. It provides a convenient analytical construct to illustrate the ongoing public

transport debates, including 'competition in the market' vs 'competition for the market' and 'competitive tendering' vs. 'negotiated contracts'.

An agenda for further research might be to consider hybrid organisational forms in public transport. For instance, a regulated monopoly can have a mixed ownership structure or competition in the market can have the form of mixed duopoly rather than pure private oligopoly. The suggested theoretical framework also captures the basic features of renegotiated or relational contracts and identifies the main factors affecting a move towards greater regulatory *ex post* oversight of competitive tendering procedures.

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Declaration of interest

Declarations of interest: none

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Appendix

The inverse demand function is defined as: $P = a - b((n - 1)Eq + q_i)$, where Eq is the expected value of q_j and n is number of firms in the market. A firm i 's profit maximisation problem is then: $\max_{q_i} E\Pi_i(\beta, q_i, 0, 0) = P(Q)q_i - \beta_i q_i - \frac{K}{n} = (a - b((n - 1)Eq + q_i) - \beta_i)q_i - \frac{K}{n}$. With the first order condition being $a - b(n - 1)Eq - 2bq_i - \beta_i = 0$, the optimal output of firm i is equal to

$$q_i^c = \begin{cases} 0 & \text{if } \beta_i \geq a - b(n - 1)Eq \\ \frac{a - b(n - 1)Eq - \beta_i}{2b} & \text{if } \beta_i < a - b(n - 1)Eq \end{cases}$$

To find the expression $a - b(n - 1)Eq$, expectation of quantity i should be calculated:

$$\begin{aligned} Eq = Eq_i^c &= \int_0^{a-b(n-1)Eq} \frac{a - b(n - 1)Eq - \beta_i}{2b} d\beta_i + 0 \cdot Prob(\beta_i \geq a - b(n - 1)Eq) \\ &= \left(\frac{a - b(n - 1)Eq}{2b} \beta_i - \frac{\beta_i^2}{4b} \right) \Big|_0^{a-b(n-1)Eq} \\ &= \frac{(a - b(n - 1)Eq)^2}{2b} - \frac{(a - b(n - 1)Eq)^2}{4b} = \frac{(a - b(n - 1)Eq)^2}{4b} \end{aligned}$$

Thus, quadratic equation depending on \tilde{c} can be obtained:

$$\frac{a - \tilde{c}}{b(n - 1)} = \frac{\tilde{c}^2}{4b}$$

where $\tilde{c} = a - b(n - 1)Eq$.

$$\begin{aligned} \tilde{c} &= -\frac{2}{n - 1} + \frac{2}{n - 1} \sqrt{1 + a(n - 1)} = \frac{2}{n - 1} (\sqrt{1 + a(n - 1)} - 1) \\ Eq &= \frac{a(n - 1) - 2(\sqrt{1 + a(n - 1)} - 1)}{b(n - 1)^2} = \frac{1 + a(n - 1) - 2\sqrt{1 + a(n - 1)} + 1}{b(n - 1)^2} \\ &= \frac{(\sqrt{1 + a(n - 1)} - 1)^2}{b(n - 1)^2} \\ q_i^c &= \begin{cases} 0 & \text{if } \beta_i \geq \frac{2}{n - 1} (\sqrt{1 + a(n - 1)} - 1) \\ \frac{\sqrt{1 + a(n - 1)} - 1}{b(n - 1)} - \frac{1}{2b} \beta_i & \text{if } \beta_i < \frac{2}{n - 1} (\sqrt{1 + a(n - 1)} - 1) \end{cases} \end{aligned}$$

Let for simplicity denote $\hat{a} = \frac{\sqrt{1+a(n-1)}-1}{n-1}$, then $q_i^c = \begin{cases} 0 & \text{if } \beta_i \geq 2\hat{a} \\ \hat{a} - \frac{1}{2b} \beta_i & \text{if } \beta_i < 2\hat{a} \end{cases}$

$$\begin{aligned}
 P &= a - b((n-1)Eq + q_i) = a - \frac{(\sqrt{1+a(n-1)} - 1)^2}{n-1} - \frac{\sqrt{1+a(n-1)} - 1}{n-1} + \frac{1}{2}\beta_i \\
 &= \frac{a(n-1) - (1+a(n-1) - 2\sqrt{1+a(n-1)} + 1) - (\sqrt{1+a(n-1)} - 1)}{n-1} \\
 &\quad + \frac{1}{2}\beta_i = \frac{\sqrt{1+a(n-1)} - 1}{n-1} + \frac{1}{2}\beta_i = \hat{a} + \frac{1}{2}\beta_i > 0
 \end{aligned}$$

To cover marginal costs we have condition $P > \beta_i$:

$$\hat{a} + \frac{1}{2}\beta_i > \beta_i \leftrightarrow 2\hat{a} > \beta_i$$

Thus, positive production output is produced when price covers marginal cost.

Then, the expected profit of firm i is

$$\begin{aligned}
 E\Pi_i &= P(Q)q_i - \beta_i q_i - \frac{K}{n} = \frac{1}{b} \left(\hat{a} - \frac{1}{2}\beta_i \right)^2 - \frac{K}{n} \\
 \frac{\partial E\Pi_i}{\partial n} &= -\frac{2}{b} \hat{a}^2 \left(\hat{a} - \frac{1}{2}\beta_i \right) + \frac{K}{n^2} ? 0
 \end{aligned}$$

Ex ante welfare of the society is equal to

$$\begin{aligned}
 EW^c &= E \left\{ S(Q^c) - \sum_i^n \beta_i q_i^c - n \frac{K}{n} \right\} = E \left\{ aQ - \frac{b}{2} Q^2 - \beta nq - K \right\} = E \left\{ nq \left(a - \beta - \frac{b}{2} nq \right) - K \right\} \\
 &= E \left\{ \frac{n}{b} \left(\hat{a} - \frac{\beta}{2} \right) \left(a - \beta - \frac{n}{2} \left(\hat{a} - \frac{\beta}{2} \right) \right) - K \right\} \\
 &= E \left\{ \frac{n}{b} \left(\hat{a} - \frac{\beta}{2} \right) \left(a - \frac{n}{2} \hat{a} + \left(\frac{n}{4} - 1 \right) \beta \right) - K \right\} \\
 &= E \left\{ \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \beta((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1 \right) \beta^2 \right) - K \right\} \\
 &= \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\beta + \bar{\beta}}{2} ((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1 \right) \frac{\beta^2 + \bar{\beta}\beta + \bar{\beta}^2}{3} \right) - K
 \end{aligned}$$

For $\underline{\beta} = 0$, $EW^c = \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2} ((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1 \right) \frac{\bar{\beta}^2}{3} \right) - K$, where first term can be either positive or negative and second term is negative.

CASE $n = 2$:

$$EW^c = \frac{1}{b} \left(2\sqrt{1+a}(\sqrt{1+a} - 1)^2 - a \frac{\bar{\beta}}{2} + \frac{\bar{\beta}^2}{6} \right) - K$$

Competition in the market vs public monopoly

$$\begin{aligned}
 \Delta EW^{cp} &= EW^c - EW^p \\
 &= \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right) \frac{\bar{\beta}^2}{3} \right) - K - \frac{(1+\lambda)^2 (\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} \\
 &+ (1+\lambda)K \\
 &= \frac{n \left(3\hat{a}(2a - n\hat{a}) + \frac{3\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right) \bar{\beta}^2 \right) (1+2\lambda) - (1+\lambda)^2 (\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} \\
 &+ \lambda K
 \end{aligned}$$

The first term of the difference can be either positive or negative, while the second term is always positive. Thus, the difference can be either positive or negative.

Now we will analyze partial derivatives of difference of social welfare.

$$\frac{\partial \Delta EW^{cp}}{\partial \bar{\beta}} = \frac{\frac{3}{2}n((n-2)\hat{a} - a)(1+2\lambda) - 2n\left(\frac{n}{4} - 1\right)(1+2\lambda)\bar{\beta} + (1+\lambda)^2(3a - 2\bar{\beta})}{6b(1+2\lambda)} \quad ? 0$$

$$\frac{\partial \Delta EW^{cp}}{\partial \lambda} = -\frac{\lambda(1+\lambda)(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{3b(1+2\lambda)^2} + K \quad ? 0$$

The sign is ambiguous in general since $\frac{\lambda(1+\lambda)(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{3b(1+2\lambda)^2} > 0$ and $K > 0$

$$\frac{\partial^2 \Delta EW^{cp}}{\partial \lambda^2} = -\frac{\bar{\beta}^2 - 3a\bar{\beta} + 3a^2}{3b(1+2\lambda)^3} < 0$$

$$\frac{\partial^2 \Delta EW^{cp}}{\partial \lambda \partial \bar{\beta}} = \frac{\lambda(1+\lambda)(3a - 2\bar{\beta})}{3b(1+2\lambda)^2} > 0 \text{ (as } 2\bar{\beta} < a \text{ from A1)}$$

$$\frac{\partial \Delta EW^{cp}}{\partial K} = \lambda > 0$$

$$\frac{\partial \Delta EW^{cp}}{\partial n} = \frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n-1)\hat{a} - a) - \left(\frac{n}{2} - 1\right) \bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{cp}}{\partial n} > 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{cp}}{\partial n} < 0 \text{ for high } \bar{\beta} \text{ or } n$$

CASE $n = 2$:

$$\Delta EW^{cp} = EW^c - EW^p =$$

$$= \frac{\left(12\sqrt{1+a}(\sqrt{1+a}-1)^2 - 3a\bar{\beta} + \bar{\beta}^2\right)(1+2\lambda) - (1+\lambda)^2(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} + \lambda K$$

$$\frac{\partial \Delta EW^{cp}}{\partial \bar{\beta}} = \frac{-3a(1+2\lambda) + 3(1+2\lambda)\bar{\beta} + (1+\lambda)^2(3a - 2\bar{\beta})}{6b(1+2\lambda)} = \frac{3\lambda^2(3a - 2\bar{\beta}) + (1+2\lambda)\bar{\beta}}{6b(1+2\lambda)} > 0$$

Competition in the market vs unregulated monopoly

$$\begin{aligned}\Delta EW^{mc} &= EW^m - EW^c \\ &= \frac{\bar{\beta}^2 + 3a^2 - 3a\bar{\beta}}{8b} - K - \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\frac{\bar{\beta}^2}{3} \right) \\ &\quad + K = \frac{3\bar{\beta}^2 + 9a^2 - 9a\bar{\beta} - 4n(3\hat{a}(2a - n\hat{a}) + \frac{3\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\bar{\beta}^2)}{24b}\end{aligned}$$

The difference can be either positive or negative.

$$\frac{\partial \Delta EW^{mc}}{\partial \bar{\beta}} = \frac{6\bar{\beta} - 9a - 6n((n-2)\hat{a} - a) + n(2n-8)\bar{\beta}}{24b} ? 0$$

$$\frac{\partial \Delta EW^{mc}}{\partial n} = -\frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n-1)\hat{a} - a) - \left(\frac{n}{2} - 1\right)\bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{mc}}{\partial n} < 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{cp}}{\partial n} > 0 \text{ for high } \bar{\beta} \text{ and } n$$

CASE $n = 2$:

$$\Delta EW^{mc} = EW^m - EW^c = \frac{-\bar{\beta}^2 + 9a^2 + 3a\bar{\beta} - 48\hat{a}(a - \hat{a})}{24b}$$

$$\frac{\partial \Delta EW^{mc}}{\partial \bar{\beta}} = \frac{3a - 2\bar{\beta}}{24b} > 0$$

Competition in the market vs outsourcing

The case $\lambda < \lambda_0$

The welfare differential (reflecting the attractiveness of outsourcing relative to competition in the market) is:

$$\begin{aligned}\Delta EW^{oc} &= EW^o - EW^c = \\ &= \frac{a^2(2 + 3\lambda) - 2n\hat{a}(2a - n\hat{a})(1 + 2\lambda)}{4b(1 + 2\lambda)} - \frac{1}{2b} \left(a + \frac{n}{2}((n-2)\hat{a} - a) \right) \bar{\beta} \\ &\quad + \frac{1}{12b} \bar{\beta}^2 \left((2 + \lambda) + n \left(\frac{n}{2} - 2 \right) \right) + \lambda F\end{aligned}$$

This expression has an ambiguous sign. Given this we can calculate the marginal effects $\bar{\beta}$, λ , F and n

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = -\frac{1}{2b} \left(a + \frac{n}{2}((n-2)\hat{a} - a) \right) + \frac{1}{6b} \bar{\beta} \left((2 + \lambda) + n \left(\frac{n}{2} - 2 \right) \right) ? 0$$

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} = \frac{-a^2}{4b(1 + 2\lambda)^2} + \frac{1}{12b} \bar{\beta}^2 + F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} > 0 \text{ for high } F, \text{ high } \bar{\beta}, \text{ high } \lambda$$

$$\frac{\partial \Delta EW^{oc}}{\partial F} = \lambda > 0$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} = -\frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n-1)\hat{a} - a) - \left(\frac{n}{2} - 1\right)\bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} < 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{oc}}{\partial n} > 0 \text{ for high } \bar{\beta} \text{ or } n$$

The duopoly case $n = 2$:

$$\Delta EW^{oc} = EW^o - EW^c = \frac{a^2(2 + 3\lambda) - 8\sqrt{1+a}(\sqrt{1+a} - 1)^2(1 + 2\lambda)}{4b(1 + 2\lambda)} + \frac{1}{12b}\lambda\bar{\beta}^2 + \lambda F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = \frac{1}{6b}\lambda\bar{\beta} > 0$$

The case $\lambda \geq \lambda_0$

$$\Delta EW^{oc} = EW^o - EW^c$$

$$\begin{aligned} &= \frac{a^3 + 3\bar{\beta}(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)(1 + 2\lambda)^2}{24b\bar{\beta}(1 + 2\lambda)^2} - K + \lambda F \\ &\quad - \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\frac{\bar{\beta}^2}{3} \right) + K \\ &= \frac{a^3}{24b\bar{\beta}(1 + 2\lambda)^2} - \frac{n}{2b}\hat{a}(2a - n\hat{a}) \\ &\quad + \frac{\bar{\beta}^2 \left(1 + \frac{n}{3}(n-4)\right) - \bar{\beta}(3a + 2n((n-2)\hat{a} - a)) + 3a^2}{8b} + \lambda F \end{aligned}$$

The difference may be either positive or negative. Now partial derivatives are studied.

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = \frac{2\bar{\beta} \left(1 + \frac{n}{3}(n-4)\right) - (3a + 2n((n-2)\hat{a} - a))}{8b} ? 0$$

The sign of the above mentioned expression is ambiguous in general.

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} = \frac{-a^3}{6b\bar{\beta}(1 + 2\lambda)^3} + F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} > 0 \text{ for high } F, \text{ high } \bar{\beta}, \text{ high } \lambda$$

$$\frac{\partial \Delta EW^{oc}}{\partial F} = \lambda > 0$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} = -\frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n-1)\hat{a} - a) - \left(\frac{n}{2} - 1\right)\bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} < 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{oc}}{\partial n} > 0 \text{ for high } \bar{\beta} \text{ and } n$$

The duopoly case $n = 2$:

$$\Delta EW^{oc} = EW^o - EW^c = \frac{a^3}{24b\bar{\beta}(1 + 2\lambda)^2} - \frac{2}{b}\sqrt{1+a}(\sqrt{1+a} - 1)^2 + \frac{-\frac{1}{3}\bar{\beta}^2 + a\bar{\beta} + 3a^2}{8b} + \lambda F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = \frac{a - \frac{2}{3}\bar{\beta}}{8b} > 0$$

Regulatory regimes for $n = 1$

Hereinafter, for easy reference we reproduce the main theoretical propositions from (Auriol & Picard, 2009b) that are relevant for our discussion of regulatory cycle.

Lemma 1. (Transition 3) Regulated monopoly is more likely to be preferred to unregulated monopoly when $\bar{\beta}$ or K decrease. An increase λ has the same (opposite) welfare effect when $\bar{\beta}$ or K are relatively high (low) but λ is relatively low (high).

Lemma 2. (Transition 5) Outsourcing is more likely to be preferred to public monopoly when $\bar{\beta}$, K or F increase. An increase λ has the same (opposite) welfare effect when K or F are relatively high (low).

Lemma 3. (Transition 6) Regulated monopoly is more likely to be preferred to outsourcing when $\bar{\beta}$ decreases (increases) if $\lambda < \lambda_0$ or $\lambda \geq \lambda_0$ for low α (if $\lambda \geq \lambda_0$ and high α). A decrease in K or F has the same welfare effect. An increase λ makes regulated monopoly more (less) likely to be preferred to outsourcing when K or F are relatively high (low).

A theory of deregulation in public transport (a companion technical appendix)

This technical Appendix accompanies the paper 'A theory of deregulation in public transport' and provides the proofs of all theoretical results presented in the main paper. Section 1 of this Appendix contains the detailed derivations of the propositions from (Auriol & Picard, 2009) that our main paper refers to as Lemmas 1, 2, 3 and 4. Accordingly, for easy reference, we reproduce first the detailed derivations of the closed welfare function expressions for the four regulatory regimes: 1) a public monopoly, 2) a regulated monopoly, 3) a private unregulated monopoly and 4) outsourcing. The latter case is interpreted as 'competition for the market' (or competitive tendering) and provides the benchmark for direct pairwise welfare comparisons for the alternative regulatory regimes. Section 2 introduces a model competition if the public transport market through deregulation. Section 3 provides the welfare comparisons.

1 The (Auriol & Picard, 2009) model of regulation

The monopoly firm has a bilinear cost structure that captures the increasing returns to scale technology in the industry. It incurs a fixed and verifiable¹ investment cost $K > 0$, while the marginal cost β is unobserved by the other party (regulator). This idiosyncratic cost parameter β is independently drawn from $[\underline{\beta}, \bar{\beta}]$ for each firm and is assumed to be uniformly distributed with density and cumulative distribution functions being $g(\beta) = 1/(\bar{\beta} - \underline{\beta})$ and $G(\beta) = (\beta - \underline{\beta})/(\bar{\beta} - \underline{\beta})$, respectively. Thus the hazard rate for this case is simply $\frac{G(\beta)}{g(\beta)} = \beta$. Thus the total cost function of the firm is $C(\beta, Q) = K + \beta Q$. Consumers have linear inverse demand $P(Q) = a - bQ$ with $a, b > 0$. Hence, the gross consumer surplus becomes: $S(Q) = \int_0^Q P(x)dx = aQ - \frac{b}{2}Q^2$.

It is assumed to be never optimal for the firm to shut down, thus regardless of the ownership issue both public and private firms will have a positive margin. The maximum consumers' willingness to pay is assumed to be sufficiently large to make the firm's first unit of output desirable: $P(0) = a > 2\bar{\beta} - \underline{\beta}$. A firm's profit is equal to $\Pi(\beta, Q, t, F) = P(Q)Q - \beta Q - K + t - F$, where t is the *ex post* transfer from the regulator to the firm and F is a possible *ex ante* franchise fee paid to the regulator. The assumption of the possibility to extract monopoly rent before engaging in operations (for instance, via efficient tendering procedures) will only be relevant for the case of competition in the market.

The government is benevolent and utilitarian. It maximises the expected value of the social welfare function EW which is a mathematical expectation of the unweighted sum of consumer's surplus (CS) and producer's surpluses (PS) given that the value of transfers t and F from the taxpayers to the firm is counted as the social (or shadow) cost of public funds $(1 + \lambda)$. Thus, the regulator's objective function is:

$$\begin{aligned} EW(\beta, Q, t, F, \lambda) &= CS + PS - SC = [S(Q) - P(Q)Q] + [P(Q)Q - \beta Q - K + t - F] - (1 + \lambda)(t - F) \\ &= S(Q) - \beta Q - K - \lambda t + \lambda F = aQ - \frac{b}{2}Q^2 - \beta Q - K - \lambda t + \lambda F \end{aligned}$$

¹ This simplifying assumption helps us avoid the moral hazard problem and is made for the sake of model tractability

The shadow cost λ reflects the nature of distortionary taxation when a unit that is transferred to the firm costs $1 + \lambda$ units to society. This assumption is crucial for our further analysis of fiscal constraints and changes the cost of public funds over time. It is commonly asserted, that λ is relatively high in developing countries and low in developed ones (see (Dementiev, 2016) for the literature review and theoretical analysis of the impact that the value of λ has on the optimal organisational structure of local public transport).

1.1 Public monopoly

In the case of a public monopoly, there is no information asymmetry so that marginal cost β and its distribution are known to the government, who invests fixed cost K . The regulator's problem is then:

$$EW = E\{S[Q(\beta)] - \beta Q(\beta) - K - \lambda t(\beta)\}$$

subject to the participation constraint of the public company $\Pi(\beta, Q(\beta), t(\beta), 0) = 0$ which essentially implies a break-even condition for the monopoly.

The maximization problem yields the following first order condition for Q^p :

$$P(Q) + \frac{\lambda}{1 + \lambda} P'(Q)Q = \beta$$

By substituting inverse demand function $P(Q) = a - bQ$, the optimal quantity can be obtained:

$$Q^p(\beta) = \frac{(a - \beta)(1 + \lambda)}{b(1 + 2\lambda)}$$

Transfer is equal to:

$$t^p(\beta) = -P(Q^p)Q^p + \beta Q^p + K = -\frac{\lambda(1 + \lambda)(a - \beta)^2}{b(1 + 2\lambda)^2} + K$$

The value of the welfare function is then:

$$\begin{aligned} EW^p &= E\left(aQ - \frac{b}{2}Q^2 - \beta Q - K - \lambda t\right) \\ &= \frac{(1 + \lambda)^2 (\bar{\beta}^2 + \bar{\beta}\underline{\beta} + \underline{\beta}^2 - 3a\underline{\beta} - 3a\bar{\beta} + 3a^2)}{6b(1 + 2\lambda)} - (1 + \lambda)K \end{aligned}$$

Substituting the expressions for the optimal quantity and the transfer into the formula above, we obtain:

$$\begin{aligned}
EW^p &= E\left(aQ - \frac{b}{2}Q^2 - \beta Q - K - \lambda t\right) \\
&= E\left\{\left(a - \beta - \frac{1}{2}\frac{(a - \beta)(1 + \lambda)}{(1 + 2\lambda)}\right)\frac{(a - \beta)(1 + \lambda)}{b(1 + 2\lambda)} - K - \lambda\left(-\frac{\lambda(1 + \lambda)(a - \beta)^2}{b(1 + 2\lambda)^2} + K\right)\right\} \\
&= E\left\{\frac{(1 + \lambda)(1 + 3\lambda)(a - \beta)^2}{2b(1 + 2\lambda)^2} + \frac{\lambda^2(1 + \lambda)(a - \beta)^2}{b(1 + 2\lambda)^2} - (1 + \lambda)K\right\} \\
&= E\left\{\frac{(a - \beta)^2(1 + \lambda)(1 + 3\lambda + 2\lambda^2)}{2b(1 + 2\lambda)^2} - (1 + \lambda)K\right\} = E\left\{\frac{(a - \beta)^2(1 + \lambda)^2}{2b(1 + 2\lambda)} - (1 + \lambda)K\right\} \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \frac{(a - \beta)^2(1 + \lambda)^2}{2b(1 + 2\lambda)} - (1 + \lambda)K dG(\beta) \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left(\frac{(a - \beta)^2(1 + \lambda)^2}{2b(1 + 2\lambda)} - (1 + \lambda)K\right) g(\beta) d\beta \\
&= \frac{(1 + \lambda)^2(\bar{\beta}^2 + \bar{\beta}\underline{\beta} + \underline{\beta}^2 - 3a\bar{\beta} - 3a\underline{\beta} + 3a^2)}{6b(1 + 2\lambda)} - (1 + \lambda)K
\end{aligned}$$

For $\underline{\beta} = 0$, $EW^p = \frac{(1 + \lambda)^2(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1 + 2\lambda)} - (1 + \lambda)K$, where the first term is positive and the second term is negative.

1.2 Regulated monopoly

Without agency problems ownership does not matter for optimal regulation with asymmetric information. On the contrary, in the case of a regulated private monopoly the agency problem emerges when the fixed investment cost K is made by the firm. Moreover, the marginal cost β is assumed to be the firm's private information. Such a cost asymmetry gives rise to an information rent for the firm, so the regulator offers an optimal contract that aims to reveal the firm's private cost information. The regulator designs a regulatory scheme $(Q^r(\cdot), t^r(\cdot))$, which is a combination of the required output Q^r and the *ex post* transfer t^r , in order to maximise EW under uncertainty:

$$EW = E\{S[Q(\beta)] - \beta Q(\beta) - K - \lambda t(\beta)\}$$

subject to

$$(d/d\beta)\Pi(\beta, Q(\beta), t(\beta), 0) = -Q(\beta) \quad (1)$$

$$(d/d\beta)Q(\beta) \leq 0 \quad (2)$$

$$\Pi(\beta, Q(\beta), t(\beta), 0) \geq 0 \quad (3)$$

Conditions (1) and (2) are the first and second order incentive compatibility constraints incentivising the firm to reveal its true marginal cost information. Condition (3) is the standard participation constraint of the firm.

When there is asymmetry of information, by combining the constraints (1) and (3) we obtain the profit of a regulated monopoly: $\Pi(\beta) = \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta}) d\tilde{\beta}$.

Then a transfer should be such that the two profits are equal: $P(Q)Q - \beta Q - K + t = \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta}$.

The transfer thus is: $t(\beta) = \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta} - P(Q)Q + \beta Q + K$.

Substituting the optimal transfer into the regulator's welfare function we obtain:

$$\begin{aligned} \max_{\{Q(\cdot)\}} EW &= E \left\{ S(Q) - \beta Q - K - \lambda \left(\int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta} - P(Q)Q + \beta Q + K \right) \right\} \\ &= E \left\{ S(Q) - (1 + \lambda)\beta Q - (1 + \lambda)K + \lambda P(Q)Q - \lambda \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta} \right\} \\ &= \int_{\underline{\beta}}^{\bar{\beta}} \left\{ S(Q) - (1 + \lambda)\beta Q - (1 + \lambda)K + \lambda P(Q)Q - \lambda \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta} \right\} dG(\beta) \end{aligned}$$

We solve last term of integral separately using substitution by parts²:

$$\begin{aligned} \int_{\underline{\beta}}^{\bar{\beta}} \left\{ \lambda \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta} \right\} dG(\beta) &= \lambda \int_{\underline{\beta}}^{\bar{\beta}} \left\{ \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta} \right\} g(\beta)d\beta \\ &= \lambda \left\{ G(\beta) \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta} \right\} \Big|_{\underline{\beta}}^{\bar{\beta}} - \int_{\underline{\beta}}^{\bar{\beta}} \{-Q(\beta)\}G(\beta)d\beta = \lambda \int_{\underline{\beta}}^{\bar{\beta}} Q(\beta)G(\beta)d\beta \\ &= \lambda \int_{\underline{\beta}}^{\bar{\beta}} Q(\beta) \frac{G(\beta)}{g(\beta)} dG(\beta) \end{aligned}$$

Thus, the regulator's objective function is equal to

$$\max_{\{Q(\cdot)\}} EW = \int_{\underline{\beta}}^{\bar{\beta}} \left\{ S(Q) - (1 + \lambda)\beta Q - (1 + \lambda)K + \lambda P(Q)Q - \lambda Q \frac{G(\beta)}{g(\beta)} \right\} dG(\beta)$$

The first order condition is

$$P(Q) - (1 + \lambda)\beta + \lambda P(Q) + \lambda P'(Q)Q - \lambda \frac{G(\beta)}{g(\beta)} = 0$$

$$P(Q) + \frac{\lambda}{1 + \lambda} P'(Q)Q = \beta + \frac{\lambda}{1 + \lambda} \frac{G(\beta)}{g(\beta)}$$

By substituting the inverse demand function $P(Q) = a - bQ$, the first order condition can be written as

² For $\int u dv = uv - \int v du : u = \int_{\underline{\beta}}^{\bar{\beta}} Q(\tilde{\beta})d\tilde{\beta}, du = -Q(\beta)d\beta, dv = g(\beta)d\beta, v = G(\beta)$

$\left\{ G(\beta) \int_{\underline{\beta}}^{\bar{\beta}} Q(\beta)d\beta \right\} \Big|_{\underline{\beta}}^{\bar{\beta}} = 0$ as either $\int_{\underline{\beta}}^{\bar{\beta}} Q(\beta)d\beta = 0$ or $G(\underline{\beta}) = 0$

$$a - bQ - \frac{\lambda}{1 + \lambda} bQ = \beta + \frac{\lambda}{1 + \lambda} \frac{G(\beta)}{g(\beta)}$$

Thus, the optimal quantity in the case of regulation is

$$Q^r(\beta) = \frac{1 + \lambda}{b(1 + 2\lambda)} \left(a - \beta - \frac{\lambda}{1 + \lambda} (\beta - \underline{\beta}) \right) = \frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta$$

Then the profit of regulated monopoly is equal to

$$\begin{aligned} \Pi^r(\beta) &= \int_{\beta}^{\bar{\beta}} \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\tilde{\beta} \right) d\tilde{\beta} = \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)}\tilde{\beta} - \frac{1}{2b}\tilde{\beta}^2 \right) \Big|_{\beta}^{\bar{\beta}} \\ &= \frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)}\bar{\beta} - \frac{1}{2b}\bar{\beta}^2 - \frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)}\beta + \frac{1}{2b}\beta^2 \end{aligned}$$

The government will offer a transfer to secure that the firm breaks-even:

$$\begin{aligned} t^r(\beta) &= \int_{\beta}^{\bar{\beta}} Q^r(\tilde{\beta}) d\tilde{\beta} - P[Q^r(\beta)]Q^r(\beta) + \beta Q^r(\beta) + K \\ &= \frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)}\bar{\beta} - \frac{1}{2b}\bar{\beta}^2 - \frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)}\beta + \frac{1}{2b}\beta^2 \\ &\quad - \left(a - b \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta \right) - \beta \right) \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta \right) + K = \\ &= -\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)}(a - \bar{\beta}) - \frac{\bar{\beta}^2}{2b} + b \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} \right)^2 + K - \frac{a + 2\lambda\beta}{b(1 + 2\lambda)}\beta + \frac{\beta^2}{2b} \end{aligned}$$

The expected welfare can be written as

$$\begin{aligned}
EW^r &= \int_{\underline{\beta}}^{\bar{\beta}} \left\{ S[Q^r(\beta)] - (1 + \lambda)\beta Q^r(\beta) - (1 + \lambda)K + \lambda P(Q^r(\beta))Q^r(\beta) - \lambda Q^r(\beta) \frac{G(\beta)}{g(\beta)} \right\} dG(\beta) \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left\{ S[Q^r(\beta)] - (1 + \lambda)\beta Q^r(\beta) - (1 + \lambda)K + \lambda P(Q^r(\beta))Q^r(\beta) - \lambda Q^r(\beta) \frac{G(\beta)}{g(\beta)} \right\} g(\beta) d\beta \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left\{ \left((1 + \lambda)(a - \beta) - \lambda(\beta - \underline{\beta}) \right) \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta \right) - \left(\frac{1}{2} + \lambda \right) b \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta \right)^2 \right. \\
&\quad \left. - (1 + \lambda)K \right\} g(\beta) d\beta \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left\{ (1 + 2\lambda)b \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta \right)^2 - (1 + 2\lambda) \frac{b}{2} \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta \right)^2 - (1 + \lambda)K \right\} g(\beta) d\beta \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left\{ (1 + 2\lambda) \frac{b}{2} \left(\frac{a(1 + \lambda) + \lambda\beta}{b(1 + 2\lambda)} - \frac{1}{b}\beta \right)^2 - (1 + \lambda)K \right\} g(\beta) d\beta \\
&= \frac{(a(1 + \lambda) + \lambda\underline{\beta})^2}{2b(1 + 2\lambda)} - \frac{(a(1 + \lambda) + \lambda\underline{\beta})(\bar{\beta} + \underline{\beta})}{2b} + \left(\frac{(1 + 2\lambda)(\bar{\beta}^2 + \bar{\beta}\underline{\beta} + \underline{\beta}^2)}{6b} \right) - (1 + \lambda)K \\
&= \frac{3(a(1 + \lambda) + \lambda\underline{\beta})^2 - 3(1 + 2\lambda)(a(1 + \lambda) + \lambda\underline{\beta})(\bar{\beta} + \underline{\beta}) + (1 + 2\lambda)^2(\bar{\beta}^2 + \bar{\beta}\underline{\beta} + \underline{\beta}^2)}{6b(1 + 2\lambda)} - (1 + \lambda)K
\end{aligned}$$

For $\underline{\beta} = 0$, $EW^r = \frac{3(a(1+\lambda))^2 - 3a\bar{\beta}(1+\lambda)(1+2\lambda) + \bar{\beta}^2(1+2\lambda)^2}{6b(1+2\lambda)} - (1 + \lambda)K$, where the first term is positive and the second term is negative.

1.3 Unregulated monopoly

In the case of a private unregulated monopoly there is neither an *ex ante* nor an *ex post* transfer to the firm that incurs the fixed investment cost K known to the regulator. The value of the marginal cost is known to the firm that maximises its profit: $\Pi(\beta, Q, 0, 0) = P(Q)Q - \beta Q - K$. The firm chooses the monopoly output $Q^m(\beta)$ with certainty while the regulator can still only evaluate the expected welfare as information on the marginal cost is private.

The laissez-faire private monopoly profit is equal to:

$$\begin{aligned}
\Pi^m(\beta) &= P[Q^m(\beta)]Q^m(\beta) - \beta Q^m(\beta) - K = \left(a - b \frac{a - \beta}{2b} \right) \frac{a - \beta}{2b} - \beta \frac{a - \beta}{2b} - K \\
&= \left(\frac{a + \beta}{2} \right) \left(\frac{a - \beta}{2b} \right) - \frac{a\beta - \beta^2}{2b} - K = \frac{1}{4b}(a^2 - \beta^2) - \frac{a\beta - \beta^2}{2b} - K \\
&= \frac{a^2}{4b} - K - \frac{a}{2b}\beta + \frac{1}{4b}\beta^2 = \frac{1}{b} \left(\frac{a - \beta}{2} \right)^2 - K
\end{aligned}$$

The expected social welfare is equal to

$$\begin{aligned}
EW^m &= E\{S(Q) - \beta Q - K\} = E\left\{aQ - \frac{b}{2}Q^2 - \beta Q - K\right\} = E\left\{(a - \beta)Q - \frac{b}{2}Q^2 - K\right\} \\
&= E\left\{\frac{(a - \beta)^2}{2b} - \frac{b}{2}\left(\frac{a - \beta}{2b}\right)^2 - K\right\} = E\left\{(a - \beta)^2\left(\frac{1}{2b} - \frac{1}{8b}\right) - K\right\} \\
&= E\left\{\frac{3}{8b}(a - \beta)^2 - K\right\} = \int_{\underline{\beta}}^{\bar{\beta}} \left\{\frac{3}{8b}(a - \beta)^2 - K\right\} dG(\beta) \\
&= \int_{\underline{\beta}}^{\bar{\beta}} \left\{\frac{3}{8b}(a - \beta)^2 - K\right\} f(\beta) d\beta = \frac{\bar{\beta}^2 + \bar{\beta}\underline{\beta} + \underline{\beta}^2 + 3a(a - \underline{\beta} - \bar{\beta})}{8b} - K
\end{aligned}$$

For $\underline{\beta} = 0$, $EW^m = \frac{\bar{\beta}^2 + 3a(a - \bar{\beta})}{8b} - K$, where the first term is positive and the second term is negative.

1.4 Outsourcing or competition for the market

Outsourcing implies that the fixed cost requires an establishment of a particular mechanism to extract the monopoly rent ex post. The case of competition *for* the market is studied using the outsourcing model of Auriol and Picard (2009). Below we reproduce the results from the paper for easy reference. According to Lemma 2 from Auriol and Picard (2009)³: $P[Q^m(\beta)] = \beta + \lambda \frac{G(\beta)}{g(\beta)}$.

The firm's output $Q^o(\beta)$ is defined for relatively low, $\beta < \beta_0$, and relatively high, $\beta \geq \beta_0$, the realisation of the marginal cost. To find β_0 we equate the outputs of monopoly and regulation cases:

$$Q^m(\beta_0) = Q^r(\beta_0), \text{ hence } \frac{a - \beta_0}{2b} = \frac{a(1 + \lambda) + \lambda \beta_0}{b(1 + 2\lambda)} - \frac{1}{b}\beta_0 \text{ and } \beta_0 = \frac{a + 2\lambda \beta_0}{1 + 2\lambda}.$$

$$Q^o(\beta) = \begin{cases} Q^r(\beta) & \text{if } \beta < \beta_0 \\ Q^m(\beta) & \text{if } \beta \geq \beta_0 \end{cases}$$

$$\Pi^o(\beta) = \begin{cases} \Pi^m(\beta_0) + \int_{\beta}^{\beta_0} Q^r(\tilde{\beta}) d\tilde{\beta} & \text{if } \beta < \beta_0 \\ \Pi^m(\beta) & \text{if } \beta \geq \beta_0 \end{cases}$$

By Lemma 2 and appendix from (Auriol & Picard, 2009) the expected welfare is equal to

$$\begin{aligned}
EW^o(\lambda) &= \int_{\underline{\beta}}^{\beta_0} W[\beta, Q^r(\beta), t^o(\beta), F, \lambda] dG(\beta) + \int_{\beta_0}^{\bar{\beta}} W[\beta, Q^m(\beta), 0, F, \lambda] dG(\beta) \\
&= \int_{\underline{\beta}}^{\beta_0} W[\beta, Q^r(\beta), t^r(\beta), 0, \lambda] dG(\beta) + \int_{\beta_0}^{\bar{\beta}} W[\beta, Q^m(\beta), 0, 0, \lambda] dG(\beta) - \lambda \Delta t G(\beta_0) + \lambda F \\
&= \int_{\beta_0}^{\bar{\beta}} \{W[\beta, Q^m(\beta), 0, F, \lambda] - W[\beta, Q^r(\beta), t^r(\beta), 0, \lambda]\} dG(\beta) + EW^r(\lambda) - \lambda \Delta t G(\beta_0) \\
&\quad + \lambda F
\end{aligned}$$

³ Assumptions A2, A3 from Auriol and Picard (2009) are satisfied for the uniform distribution of cost and the linear inverse demand function $P = a - bQ$.

where $\Delta t(\beta) = \Pi^m(\beta_0) - \Pi^r(\beta_0) = \frac{1}{b} \left(\frac{a-\beta_0}{2} \right)^2 - \left(\frac{a(1+\lambda)+\lambda\beta}{b(1+2\lambda)} (\bar{\beta} - \beta_0) - \frac{1}{2b} (\bar{\beta}^2 - \beta_0^2) \right)$

For $\underline{\beta} = 0$ from (Auriol & Picard, 2009), $EW^o = \begin{cases} \frac{6a^2+9a^2\lambda-6a\bar{\beta}(1+2\lambda)+\bar{\beta}^2(2+\lambda)(1+2\lambda)}{12b(1+2\lambda)} - K + \lambda F & \text{if } \lambda < \lambda_0 \\ \frac{a^3+3\bar{\beta}(\bar{\beta}^2-3a\bar{\beta}+3a^2)(1+2\lambda)^2}{24b\bar{\beta}(1+2\lambda)^2} - K + \lambda F & \text{if } \lambda \geq \lambda_0 \end{cases}$,

where the second term is negative and the third term is positive.

2 A model of deregulation

The inverse demand function is $P = a - b((n-1)Eq + q_i)$, where Eq is the expected value of q_j and n is the number of firms in the market.

Firm i 's objective function is expected profit which is equal to

$$E\Pi_i(\beta, q_i, 0, 0) = P(Q)q_i - \beta_i q_i - \frac{K}{n} = (a - b((n-1)Eq + q_i) - \beta_i)q_i - \frac{K}{n}$$

The first order condition is

$$a - b(n-1)Eq - 2bq_i - \beta_i = 0$$

Thus, the output of firm i is equal to

$$q_i^c = \begin{cases} 0 & \text{if } \beta_i \geq a - b(n-1)Eq \\ \frac{a - b(n-1)Eq - \beta_i}{2b} & \text{if } \beta_i < a - b(n-1)Eq \end{cases}$$

To find the expression $a - b(n-1)Eq$, the expectation of quantity i should be calculated:

$$\begin{aligned} Eq = Eq_i^c &= \int_0^{a-b(n-1)Eq} \frac{a - b(n-1)Eq - \beta_i}{2b} d\beta_i + 0 \cdot \text{Prob}(\beta_i \geq a - b(n-1)Eq) \\ &= \left(\frac{a - b(n-1)Eq}{2b} \beta_i - \frac{\beta_i^2}{4b} \right) \Big|_0^{a-b(n-1)Eq} = \frac{(a - b(n-1)Eq)^2}{2b} - \frac{(a - b(n-1)Eq)^2}{4b} \\ &= \frac{(a - b(n-1)Eq)^2}{4b} \end{aligned}$$

Thus, a quadratic equation depending on \tilde{c} can be obtained:

$$\frac{a - \tilde{c}}{b(n-1)} = \frac{\tilde{c}^2}{4b}$$

where $\tilde{c} = a - b(n-1)Eq$.

$$\tilde{c} = -\frac{2}{n-1} + \frac{2}{n-1} \sqrt{1 + a(n-1)} = \frac{2}{n-1} (\sqrt{1 + a(n-1)} - 1)$$

$$Eq = \frac{a(n-1) - 2(\sqrt{1+a(n-1)} - 1)}{b(n-1)^2} = \frac{1 + a(n-1) - 2\sqrt{1+a(n-1)} + 1}{b(n-1)^2}$$

$$= \frac{(\sqrt{1+a(n-1)} - 1)^2}{b(n-1)^2}$$

$$q_i^c = \begin{cases} 0 & \text{if } \beta_i \geq \frac{2}{n-1}(\sqrt{1+a(n-1)} - 1) \\ \frac{\sqrt{1+a(n-1)} - 1}{b(n-1)} - \frac{1}{2b}\beta_i & \text{if } \beta_i < \frac{2}{n-1}(\sqrt{1+a(n-1)} - 1) \end{cases}$$

Let for simplicity denote $\hat{a} = \frac{\sqrt{1+a(n-1)} - 1}{n-1}$, then $q_i^c = \begin{cases} 0 & \text{if } \beta_i \geq 2\hat{a} \\ \frac{\hat{a}}{b} - \frac{1}{2b}\beta_i & \text{if } \beta_i < 2\hat{a} \end{cases}$

$$P = a - b((n-1)Eq + q_i) = a - \frac{(\sqrt{1+a(n-1)} - 1)^2}{n-1} - \frac{\sqrt{1+a(n-1)} - 1}{n-1} + \frac{1}{2}\beta_i$$

$$= \frac{a(n-1) - (1 + a(n-1) - 2\sqrt{1+a(n-1)} + 1) - (\sqrt{1+a(n-1)} - 1)}{n-1} + \frac{1}{2}\beta_i$$

$$= \frac{\sqrt{1+a(n-1)} - 1}{n-1} + \frac{1}{2}\beta_i = \hat{a} + \frac{1}{2}\beta_i > 0$$

To cover marginal costs we have condition $P > \beta_i$:

$$\hat{a} + \frac{1}{2}\beta_i > \beta_i \leftrightarrow 2\hat{a} > \beta_i$$

Thus, a positive production output is produced when the price covers the marginal cost.

Then, the expected profit of firm i is

$$E\Pi_i = P(Q)q_i - \beta_i q_i - \frac{K}{n} = \frac{1}{b}\left(\hat{a} - \frac{1}{2}\beta_i\right)^2 - \frac{K}{n}$$

$$\frac{\partial E\Pi_i}{\partial n} = -\frac{2}{b}\hat{a}^2\left(\hat{a} - \frac{1}{2}\beta_i\right) + \frac{K}{n^2} \geq 0$$

Ex ante welfare of the society is equal to

$$EW^c = E\left\{S(Q^c) - \sum_i^n \beta_i q_i^c - n\frac{K}{n}\right\} = E\left\{aQ - \frac{b}{2}Q^2 - \beta nq - K\right\} = E\left\{nq\left(a - \beta - \frac{b}{2}nq\right) - K\right\}$$

$$= E\left\{\frac{n}{b}\left(\hat{a} - \frac{\beta}{2}\right)\left(a - \beta - \frac{n}{2}\left(\hat{a} - \frac{\beta}{2}\right)\right) - K\right\} = E\left\{\frac{n}{b}\left(\hat{a} - \frac{\beta}{2}\right)\left(a - \frac{n}{2}\hat{a} + \left(\frac{n}{4} - 1\right)\beta\right) - K\right\}$$

$$= E\left\{\frac{n}{2b}\left(\hat{a}(2a - n\hat{a}) + \beta((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\beta^2\right) - K\right\}$$

$$= \frac{n}{2b}\left(\hat{a}(2a - n\hat{a}) + \frac{\beta + \bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\frac{\beta^2 + \bar{\beta}\beta + \bar{\beta}^2}{3}\right) - K$$

For $\underline{\beta} = 0$, $EW^c = \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right) \frac{\bar{\beta}^2}{3} \right) - K$, where the first term can be either positive or negative and the second term is negative.

CASE $n = 2$:

$$EW^c = \frac{1}{b} \left(2\sqrt{1+a}(\sqrt{1+a}-1)^2 - a\frac{\bar{\beta}}{2} + \frac{\bar{\beta}^2}{6} \right) - K$$

3 Welfare comparisons

3.1 Public monopoly vs competition in the market

$$\Delta EW^{cp} = EW^c - EW^p$$

$$\begin{aligned} &= \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right) \frac{\bar{\beta}^2}{3} \right) - K - \frac{(1+\lambda)^2 (\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} \\ &+ (1+\lambda)K \\ &= \frac{n \left(3\hat{a}(2a - n\hat{a}) + \frac{3\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right) \bar{\beta}^2 \right) (1+2\lambda) - (1+\lambda)^2 (\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} \\ &+ \lambda K \end{aligned}$$

The first term of the difference can be either positive or negative, while the second term is always positive. Thus, the difference can be either positive or negative.

Now we analyze the partial derivatives of the difference of social welfare.

$$\frac{\partial \Delta EW^{cp}}{\partial \bar{\beta}} = \frac{\frac{3}{2}n((n-2)\hat{a} - a)(1+2\lambda) - 2n\left(\frac{n}{4} - 1\right)(1+2\lambda)\bar{\beta} + (1+\lambda)^2(3a - 2\bar{\beta})}{6b(1+2\lambda)} \stackrel{?}{> 0}$$

$$\frac{\partial \Delta EW^{cp}}{\partial \lambda} = -\frac{\lambda(1+\lambda)(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{3b(1+2\lambda)^2} + K$$

$$\frac{\lambda(1+\lambda)(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{3b(1+2\lambda)^2} > 0 \text{ thus the sign is uncertain}$$

$$\frac{\partial^2 \Delta EW^{cp}}{\partial \lambda^2} = -\frac{\bar{\beta}^2 - 3a\bar{\beta} + 3a^2}{3b(1+2\lambda)^3} < 0$$

$$\frac{\partial^2 \Delta EW^{cp}}{\partial \lambda \partial \bar{\beta}} = \frac{\lambda(1+\lambda)(3a - 2\bar{\beta})}{3b(1+2\lambda)^2} > 0 \text{ (as } 2\bar{\beta} < a \text{ from A1)}$$

$$\frac{\partial \Delta EW^{cp}}{\partial K} = \lambda > 0$$

$$\frac{\partial \Delta EW^{cp}}{\partial n} = \frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n-1)\hat{a} - a) - \left(\frac{n}{2} - 1\right)\bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{cp}}{\partial n} > 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{cp}}{\partial n} < 0 \text{ for high } \bar{\beta} \text{ and } n$$

CASE $n = 2$:

$$\begin{aligned} \Delta EW^{cp} &= EW^c - EW^p \\ &= \frac{(12\sqrt{1+a}(\sqrt{1+a}-1)^2 - 3a\bar{\beta} + \bar{\beta}^2)(1+2\lambda) - (1+\lambda)^2(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} \\ &\quad + \lambda K \end{aligned}$$

$$\frac{\partial \Delta EW^{cp}}{\partial \bar{\beta}} = \frac{-3a(1+2\lambda) + 3(1+2\lambda)\bar{\beta} + (1+\lambda)^2(3a-2\bar{\beta})}{6b(1+2\lambda)} = \frac{3\lambda^2(3a-2\bar{\beta}) + (1+2\lambda)\bar{\beta}}{6b(1+2\lambda)} > 0$$

3.2 Competition in the market vs unregulated monopoly

$$\begin{aligned} \Delta EW^{mc} &= EW^m - EW^c \\ &= \frac{\bar{\beta}^2 + 3a^2 - 3a\bar{\beta}}{8b} - K - \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\frac{\bar{\beta}^2}{3} \right) + K \\ &= \frac{3\bar{\beta}^2 + 9a^2 - 9a\bar{\beta} - 4n(3\hat{a}(2a - n\hat{a}) + \frac{3\bar{\beta}}{2}((n-2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\bar{\beta}^2)}{24b} \end{aligned}$$

The difference can be either positive or negative.

$$\frac{\partial \Delta EW^{mc}}{\partial \bar{\beta}} = \frac{6\bar{\beta} - 9a - 6n((n-2)\hat{a} - a) + n(2n-8)\bar{\beta}}{24b} ? 0$$

$$\frac{\partial \Delta EW^{mc}}{\partial n} = -\frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n-1)\hat{a} - a) - \left(\frac{n}{2} - 1\right)\bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{mc}}{\partial n} < 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{cp}}{\partial n} > 0 \text{ for high } \bar{\beta} \text{ and } n$$

CASE $n = 2$:

$$\Delta EW^{mc} = EW^m - EW^c = \frac{-\bar{\beta}^2 + 9a^2 + 3a\bar{\beta} - 48\hat{a}(a - \hat{a})}{24b}$$

$$\frac{\partial \Delta EW^{mc}}{\partial \bar{\beta}} = \frac{3a - 2\bar{\beta}}{24b} > 0$$

3.3 Unregulated vs regulated monopoly

$$\begin{aligned}\Delta EW^{rm} &= EW^r - EW^m \\ &= \frac{3(a(1+\lambda))^2 - 3a\bar{\beta}(1+\lambda)(1+2\lambda) + (1+2\lambda)^2\bar{\beta}^2}{6b(1+2\lambda)} - (1+\lambda)K - \frac{\bar{\beta}^2 + 3a^2 - 3a\bar{\beta}}{8b} \\ &\quad + K = \frac{3a^2(4\lambda^2 + 2\lambda + 1) - 3a\bar{\beta}(1+4\lambda)(1+2\lambda) + (1+2\lambda)(1+8\lambda)\bar{\beta}^2}{24b(1+2\lambda)} - \lambda K\end{aligned}$$

The first expression $\frac{3a^2(4\lambda^2+2\lambda+1)-3a\bar{\beta}(1+4\lambda)(1+2\lambda)+(1+2\lambda)(1+8\lambda)\bar{\beta}^2}{24b(1+2\lambda)}$ can be positive or negative while the second term is negative. Thus, ΔEW^{rm} may be positive when K is low enough or/and λ is low enough so that the first term may cover the second term $-\lambda K$.

Then, we analyze the partial effects of each parameter.

$$\frac{\partial \Delta EW^{rm}}{\partial \bar{\beta}} = \frac{-3a(1+4\lambda) + 2\bar{\beta}(1+8\lambda)}{24b} = \frac{(2\bar{\beta} - 3a) + 4\lambda(4\bar{\beta} - 3a)}{24b} < 0 \text{ as from A1 } a$$

$$> 2\bar{\beta} \text{ so that } 2\bar{\beta} - 3a < 0, 4\bar{\beta} - 3a < 0.$$

$\frac{\partial \Delta EW^{rm}}{\partial \lambda} = \frac{a^2\lambda(\lambda+1)}{(1+2\lambda)^2} - \frac{a\bar{\beta}}{2b} + \frac{\bar{\beta}^2}{3b} - K$, the first term can be either positive or negative, thus, the sign is uncertain. When the first term is positive, the sign depends on the value of K .

$$\frac{\partial^2 \Delta EW^{rm}}{\partial \lambda^2} = \frac{a^2}{(1+2\lambda)^3} > 0$$

$$\frac{\partial^2 \Delta EW^{rm}}{\partial \lambda \partial \bar{\beta}} = \frac{4\bar{\beta} - 3a}{6b} < 0 \text{ (as } 2\bar{\beta} < a \text{ from A1)}$$

$$\frac{\partial \Delta EW^{rm}}{\partial K} = -\lambda < 0$$

3.4 Regulated vs public monopoly

$$\begin{aligned}\Delta EW^{pr} &= EW^p - EW^r \\ &= \frac{(1+\lambda)^2(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1+2\lambda)} - (1+\lambda)K \\ &\quad - \frac{3(a(1+\lambda))^2 - 3a\bar{\beta}(1+\lambda)(1+2\lambda) + (1+2\lambda)^2\bar{\beta}^2}{6b(1+2\lambda)} + (1+\lambda)K \\ &= \frac{3a\lambda(1+\lambda)\bar{\beta} - (2+3\lambda)\lambda\bar{\beta}^2}{6b(1+2\lambda)}\end{aligned}$$

For $\alpha = \frac{2\bar{\beta}}{a} \in [0,1]$ from assumption A1

$$\Delta EW^{pr} = \lambda \frac{a^2}{b} \frac{3\alpha(1+\lambda) - (2+3\lambda)\alpha^2}{12(1+2\lambda)} = \alpha\lambda \frac{a^2}{b} \frac{3+3\lambda - (2+3\lambda)\alpha}{12(1+2\lambda)}$$

$\Delta EW^{pr} > 0$ for any $a, b > 0$ and $\alpha, \lambda \in [0,1]$

$$\frac{\partial \Delta EW^{pr}}{\partial \bar{\beta}} = \frac{3a\lambda(1+\lambda) - 2(2+3\lambda)\lambda\bar{\beta}}{6b(1+2\lambda)} = \frac{\lambda(3a + 3a\lambda - 4\bar{\beta} - 6\lambda\bar{\beta})}{6b(1+2\lambda)} > 0$$

as from A1 $a > 2\bar{\beta}$, then $3a > 4\bar{\beta}$, $3a\lambda > 6\lambda\bar{\beta}$

$$\frac{\partial \Delta EW^{pr}}{\partial \lambda} = \frac{\bar{\beta}(3a(2\lambda^2 + 2\lambda + 1) - 2\bar{\beta}(3\lambda^2 + 3\lambda + 1))}{6b(1 + 2\lambda)^2} > 0 \text{ as from A1 } a > 2\bar{\beta}$$

3.5 Public monopoly vs outsourcing

For $\lambda < \lambda_0$

$$\begin{aligned} \Delta EW^{op} &= EW^o - EW^p \\ &= \frac{6a^2 + 9a^2\lambda - 6a\bar{\beta}(1 + 2\lambda) + \bar{\beta}^2(2 + \lambda)(1 + 2\lambda)}{12b(1 + 2\lambda)} - K + \lambda F \\ &\quad - \frac{(1 + \lambda)^2(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1 + 2\lambda)} + (1 + \lambda)K \\ &= \frac{\lambda\bar{\beta}^2 + 6a\lambda^2\bar{\beta} - 3a^2\lambda(1 + 2\lambda)}{12b(1 + 2\lambda)} + \lambda(K + F) \end{aligned}$$

The first term of the difference is always negative. The second term is always positive. Thus, the difference may be either positive or negative.

$$\frac{\partial \Delta EW^{op}}{\partial \bar{\beta}} = \frac{2\lambda\bar{\beta} + 6a\lambda^2}{12b(1 + 2\lambda)} > 0$$

$$\frac{\partial \Delta EW^{op}}{\partial \lambda} = \frac{(\bar{\beta}^2 + 12a\bar{\beta}\lambda)(1 + 2\lambda) - 2(\lambda\bar{\beta}^2 + 6a\lambda^2\bar{\beta})}{12(1 + 2\lambda)^2} - \frac{a^2}{4b} + K + F$$

Expression $\frac{(\bar{\beta}^2 + 12a\bar{\beta}\lambda)(1 + 2\lambda) - 2(\lambda\bar{\beta}^2 + 6a\lambda^2\bar{\beta})}{12(1 + 2\lambda)^2} - \frac{a^2}{4b}$ is always negative. Thus, the sign of the partial derivative with respect to λ depends on value of $a, b, \bar{\beta}, K$ and F .

$$\frac{\partial \Delta EW^{op}}{\partial F} = \lambda > 0$$

$$\frac{\partial \Delta EW^{op}}{\partial K} = \lambda > 0$$

For $\lambda \geq \lambda_0$

$$\begin{aligned} \Delta EW^{op} &= EW^o - EW^p \\ &= \frac{a^3 + 3\bar{\beta}(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)(1 + 2\lambda)^2}{24b\bar{\beta}(1 + 2\lambda)^2} - K + \lambda F - \frac{(1 + \lambda)^2(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)}{6b(1 + 2\lambda)} \\ &\quad + (1 + \lambda)K = \frac{a^3 - \bar{\beta}(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)(1 + 2\lambda)(4\lambda^2 + 2\lambda + 1)}{24b\bar{\beta}(1 + 2\lambda)^2} + \lambda(K + F) \end{aligned}$$

For high enough λ and $\bar{\beta}$, the first term of the expression can be negative. The second term is always positive. Thus, the difference may be either positive or negative.

Now partial derivatives are studied.

$$\frac{\partial \Delta EW^{op}}{\partial \bar{\beta}} = \frac{(3a - 2\bar{\beta})(4\lambda^2 + 2\lambda + 1)}{24b(1 + 2\lambda)} - \frac{a^3}{24b\bar{\beta}^2(1 + 2\lambda)^2}$$

$$\frac{\partial^2 \Delta EW^{op}}{\partial \lambda \partial \bar{\beta}} = \frac{8\lambda(3a - 2\bar{\beta})(\lambda + 1)}{24b(1 + 2\lambda)^2} + \frac{a^3}{6b\bar{\beta}^2(1 + 2\lambda)^3} > 0$$

Thus, the partial derivative with respect to $\bar{\beta}$ is increasing in λ . As $\frac{\partial \Delta EW^{op}}{\partial \bar{\beta}} > 0$ for $\lambda = \lambda_0$, for $\lambda \geq \lambda_0$, the partial derivative with respect to $\bar{\beta}$ is positive.

$$\frac{\partial \Delta EW^{op}}{\partial \lambda} = -\frac{a^3}{6b(1 + 2\lambda)^3} - \frac{\bar{\beta}(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)\lambda(\lambda + 1)}{3b\bar{\beta}(1 + 2\lambda)^2} + K + F$$

The first term of the partial derivative with respect to λ is negative, while the second term is positive.

$$\frac{\partial \Delta EW^{op}}{\partial F} = \lambda > 0$$

$$\frac{\partial \Delta EW^{op}}{\partial K} = \lambda > 0$$

3.6 Regulated monopoly vs outsourcing

For $\lambda < \lambda_0$

$$\begin{aligned} EW^{ro} &= EW^r - EW^o \\ &= \frac{3(a(1 + \lambda))^2 - 3a\bar{\beta}(1 + \lambda)(1 + 2\lambda) + (1 + 2\lambda)^2\bar{\beta}^2}{6b(1 + 2\lambda)} - (1 + \lambda)K \\ &\quad - \frac{6a^2 + 9a^2\lambda - 6a\bar{\beta}(1 + 2\lambda) + \bar{\beta}^2(2 + \lambda)(1 + 2\lambda)}{12b(1 + 2\lambda)} + K - \lambda F \\ &= \frac{\lambda(a - \bar{\beta})^2}{4b} - \lambda(K + F) \end{aligned}$$

The first term of the difference is always positive. The second term can be negative. Thus, the difference may be either positive or negative.

$$\frac{\partial \Delta EW^{ro}}{\partial \bar{\beta}} = -\frac{\lambda(a - \bar{\beta})}{2b} < 0 \text{ (as } 2\bar{\beta} < a \text{ from A1)}$$

$$\frac{\partial \Delta EW^{ro}}{\partial \lambda} = \frac{(a - \bar{\beta})^2}{4b} - (K + F)$$

Expression $\frac{(a - \bar{\beta})^2}{4b}$ is always positive. Thus, the sign of the partial derivative with respect to λ depends on the values of $\bar{\beta}$, K and F . A rise in $\bar{\beta}$ may result in the negative value of the partial derivative with respect to λ . Thus, for a large value of $K + F$ or high $\bar{\beta}$ the partial derivative can be

negative. The difference of the expected welfare has the same sign as the partial derivative with respect to λ .

$$\frac{\partial \Delta EW^{ro}}{\partial F} = -\lambda < 0$$

$$\frac{\partial \Delta EW^{ro}}{\partial K} = -\lambda < 0$$

For $\lambda \geq \lambda_0$

$$\begin{aligned} \Delta EW^{ro} &= EW^r - EW^o \\ &= \frac{3(a(1+\lambda))^2 - 3a\bar{\beta}(1+\lambda)(1+2\lambda) + (1+2\lambda)^2\bar{\beta}^2}{6b(1+2\lambda)} - (1+\lambda)K \\ &\quad - \frac{a^3 + 3\bar{\beta}(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)(1+2\lambda)^2}{24b\bar{\beta}(1+2\lambda)^2} + K - \lambda F \\ &= \frac{\bar{\beta}^3(1+8\lambda)(1+2\lambda) - 3a\bar{\beta}^2(1+4\lambda) + 3a^2\bar{\beta}(1+2\lambda+4\lambda^2) - a^3}{24b\bar{\beta}(1+2\lambda)} - \lambda(K+F) \\ &= \frac{4\lambda\bar{\beta}^3 + \lambda a^3 + 6a\bar{\beta}(a\lambda - \bar{\beta})}{12b\bar{\beta}(1+2\lambda)} - \frac{(a - \bar{\beta})^3}{24b\bar{\beta}} - \lambda(K+F) \end{aligned}$$

The first and second term together of the expression of expected welfare is positive for $\lambda \geq \lambda_0$ when A1 holds. Thus, the difference may be either positive or negative.

Now partial derivatives are studied.

$$\frac{\partial \Delta EW^{ro}}{\partial \bar{\beta}} = \frac{8\lambda\bar{\beta}^3 - \lambda a^3 - 6a\bar{\beta}^2}{12b\bar{\beta}^2(1+2\lambda)^2} - \frac{(a - \bar{\beta})^2(4\bar{\beta} - a)}{24b\bar{\beta}^2}$$

$$\frac{\partial^2 \Delta EW^{ro}}{\partial \lambda \partial \bar{\beta}} = \frac{(a^3 - 8\bar{\beta}^3)(2\lambda - 1) + 24\lambda a\bar{\beta}^3}{24b(1+2\lambda)^3} > 0 \text{ for } \lambda \geq \lambda_0$$

$$\frac{\partial \Delta EW^{ro}}{\partial \lambda} = \frac{4\bar{\beta}^3 + 18a\bar{\beta}^2 + a^3}{12b\bar{\beta}(1+2\lambda)^2} - (K+F)$$

In this case, both partial derivatives have an uncertain sign which can vary according to the values of the parameters. $\frac{\partial \Delta EW^{ro}}{\partial \bar{\beta}} > 0$ for relatively high a and $\frac{\partial \Delta EW^{ro}}{\partial \bar{\beta}} < 0$ for a relatively low a .

$$\frac{\partial \Delta EW^{ro}}{\partial F} = -\lambda < 0$$

$$\frac{\partial \Delta EW^{ro}}{\partial K} = -\lambda < 0$$

For low λ $\Delta EW^{ro}(\lambda < \lambda_0) - \Delta EW^{ro}(\lambda \geq \lambda_0)$ is positive, for high λ $\Delta EW^{ro}(\lambda < \lambda_0) - \Delta EW^{ro}(\lambda \geq \lambda_0)$ is negative. This means that countries with low values of shadow costs will prefer

a tendering system while countries with a high value of shadow costs will prefer a regulated private monopoly.

3.7 Outsourcing vs competition in the market

3.7.1 The case $\lambda < \lambda_0$

$$\begin{aligned}\Delta EW^{oc} &= EW^o - EW^c \\ &= \frac{6a^2 + 9a^2\lambda - 6a\bar{\beta}(1 + 2\lambda) + \bar{\beta}^2(2 + \lambda)(1 + 2\lambda)}{12b(1 + 2\lambda)} - K + \lambda F \\ &\quad - \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n - 2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\frac{\bar{\beta}^2}{3} \right) + K \\ &= \frac{a^2(2 + 3\lambda) - 2n\hat{a}(2a - n\hat{a})(1 + 2\lambda)}{4b(1 + 2\lambda)} - \frac{1}{2b} \left(a + \frac{n}{2}((n - 2)\hat{a} - a) \right) \bar{\beta} \\ &\quad + \frac{1}{12b} \bar{\beta}^2 \left((2 + \lambda) + n \left(\frac{n}{2} - 2 \right) \right) + \lambda F\end{aligned}$$

The difference may be either positive or negative.

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = -\frac{1}{2b} \left(a + \frac{n}{2}((n - 2)\hat{a} - a) \right) + \frac{1}{6b} \bar{\beta} \left((2 + \lambda) + n \left(\frac{n}{2} - 2 \right) \right) ? 0$$

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} = \frac{-a^2}{4b(1 + 2\lambda)^2} + \frac{1}{12b} \bar{\beta}^2 + F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} > 0 \text{ for high } F, \text{ high } \bar{\beta}, \text{ high } \lambda$$

$$\frac{\partial \Delta EW^{oc}}{\partial F} = \lambda > 0$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} = -\frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n - 1)\hat{a} - a) - \left(\frac{n}{2} - 1\right)\bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} < 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{oc}}{\partial n} > 0 \text{ for high } \bar{\beta} \text{ and } n$$

CASE $n = 2$:

$$\Delta EW^{oc} = EW^o - EW^c = \frac{a^2(2 + 3\lambda) - 8\sqrt{1 + a}(\sqrt{1 + a} - 1)^2(1 + 2\lambda)}{4b(1 + 2\lambda)} + \frac{1}{12b} \lambda \bar{\beta}^2 + \lambda F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = \frac{1}{6b} \lambda \bar{\beta} > 0$$

3.7.2 The case $\lambda \geq \lambda_0$

$$\Delta EW^{oc} = EW^o - EW^c$$

$$\begin{aligned} &= \frac{a^3 + 3\bar{\beta}(\bar{\beta}^2 - 3a\bar{\beta} + 3a^2)(1 + 2\lambda)^2}{24b\bar{\beta}(1 + 2\lambda)^2} - K + \lambda F \\ &\quad - \frac{n}{2b} \left(\hat{a}(2a - n\hat{a}) + \frac{\bar{\beta}}{2}((n - 2)\hat{a} - a) - \left(\frac{n}{4} - 1\right)\frac{\bar{\beta}^2}{3} \right) + K \\ &= \frac{a^3}{24b\bar{\beta}(1 + 2\lambda)^2} - \frac{n}{2b}\hat{a}(2a - n\hat{a}) \\ &\quad + \frac{\bar{\beta}^2 \left(1 + \frac{n}{3}(n - 4)\right) - \bar{\beta} \left(3a + 2n((n - 2)\hat{a} - a)\right) + 3a^2}{8b} + \lambda F \end{aligned}$$

The difference may be either positive or negative.

Now partial derivatives are studied.

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = \frac{2\bar{\beta} \left(1 + \frac{n}{3}(n - 4)\right) - (3a + 2n((n - 2)\hat{a} - a))}{8b} \geq 0$$

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} = \frac{-a^3}{6b\bar{\beta}(1 + 2\lambda)^3} + F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \lambda} > 0 \text{ for high } F, \text{ high } \bar{\beta}, \text{ high } \lambda$$

$$\frac{\partial \Delta EW^{oc}}{\partial F} = \lambda > 0$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} = -\frac{6\hat{a}(a - n\hat{a}) + \frac{3\bar{\beta}}{2}(2(n - 1)\hat{a} - a) - \left(\frac{n}{2} - 1\right)\bar{\beta}^2}{6b}$$

$$\frac{\partial \Delta EW^{oc}}{\partial n} < 0 \text{ for low } \bar{\beta} \text{ and } n, \frac{\partial \Delta EW^{oc}}{\partial n} > 0 \text{ for high } \bar{\beta} \text{ and } n$$

CASE $n = 2$:

$$\Delta EW^{oc} = EW^o - EW^c = \frac{a^3}{24b\bar{\beta}(1 + 2\lambda)^2} - \frac{2}{b}\sqrt{1 + a}(\sqrt{1 + a} - 1)^2 + \frac{-\frac{1}{3}\bar{\beta}^2 + a\bar{\beta} + 3a^2}{8b} + \lambda F$$

$$\frac{\partial \Delta EW^{oc}}{\partial \bar{\beta}} = \frac{a - \frac{2}{3}\bar{\beta}}{8b} > 0$$

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A theory of deregulation in public transport

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