"Investigations of the High Density Region of the Density Spectrum of Cosmic Ray Extensive Air Showers with Cloud Chamber and Geiger Counter."

A Thesis
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Abstract.

This thesis describes an experiment to measure the exponent of the density spectrum of cosmic ~~any~~ ray air shower particles in the high density region with a very simple array of detecting equipment, and also describes the analysis of the results according to standard mathematical procedures.

The experiment was originally devised by my supervisor, Dr. D. D. Millar, who has at all times directed its conduct. I have assisted with the maintenance of the experiment and I have carried out the analysis of the results to arrive at the presented result, namely, an indicated change in slope of the density spectrum from 2.6 at 500 particles m^{-2} to 3.9 at 1500 particles m^{-2}; and I have examined and summarised previous work in the high density field. I have also examined the effect of the results of this thesis on the relationship between the density spectrum and the number spectrum of air showers. The basic features of a theory developed by Ueda and Mc Cusker to explain the experimental results are also presented.

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Chapter 1.

Introduction to the study of cosmic ray extensive air showers, with particular reference to the study of the density spectrum of air shower particles.

1.1 The properties of cosmic ray extensive air showers.

The collision between a relativistic charged primary cosmic ray particle of energy at least 10^{15}ev, and a nucleus of the upper atmosphere, sets up a cascade of particles, interacting and decaying in their passage through the atmosphere, which may finally be detected by the simultaneous arrival of ionizing particles at the surface of the earth.

The basic features of the cascade, and the interactions taking place at lower energies (up to 10^{12}ev) are well known. At the surface of the earth, the cosmic ray air shower is detected as consisting principally of a disc of electrons and photons, distributed approximately symmetrically about a core of maximum density and extending to several tens of metres from this core. There are also present, to the order of a few per cent of the total number of particles, μ-mesons and nuclear-active particles (nucleons, mesons, etc.). Both are symmetrically distributed about the same core as the electrons, the μ-meson distribution being less steep, and the nuclear-active particle distribution being more steep, than the electron distribution; sources describing the experimental work are listed in the following section.

The electrons and photons are the current generation of numerous electron-photon cascades, many initially established by the decay of high energy mesons. The μ-mesons are the decay products of charged π-mesons, produced in high energy nuclear interactions, whilst the nuclear-active particles are interaction products, themselves capable of interacting with other nuclei.

The two principal physical phenomena related to cosmic radiation which are as yet unexplained are:-

(i) the origin of the primary radiation, and its physical properties of mass and primary energy, in the form of mass and primary energy spectra;

(ii) the nature of the highest energy nuclear interactions, from the initial interaction to the interactions several generations later.

Knowledge of the former will enable physicists to deduce the mechanism of acceleration of the cosmic ray particles, and hence discover more of the physical nature of the universe; knowledge of the latter will permit a more powerful understanding of the nature of nuclear forces.

The rate of arrival of primary particles of sufficient energy to initiate extensive air showers is such that incidence of one in a stack of nuclear emulsion, during a single balloon flight near the top of the atmosphere, is highly unlikely. Nuclear emulsion studies are generally of particles of primary energy $10^{11} - 10^{14}$ev, the detection of any particles above the upper limit quoted being rare. Hence information about higher energy interactions and the other properties of the primary radiation can only be inferred by closely studying the particles incident upon the earth's surface, where a detecting array of sufficient size for a reasonable rate of incidence may be maintained over a length of time to accumulate statistics.

Every primary particle will cause a unique shower, hence all that an array can measure is some of the average properties of showers. To these measurements must be added the knowledge of the average behaviour of particles in low energy ($< 10^{12}$ev) interactions and in cascade development processes. From these some knowledge as to the average processes occurring in the higher energy interactions can be obtained. Only in this way can an approach be made to the elucidation of the mechanism of the high energy interactions, and thence of the nature of the primary particles.

1.2 Experimental studies of cosmic ray extensive air showers.

The first experiments demonstrating the existence of extensive air showers were by Schmeiser and Bothe (1938), who detected the simultaneous arrival at the earth's surface of ionizing particles at distances of up to 40 cms. apart, followed by Auger et al (1938), who detected Geiger-Müller counter coincidences at distances up to several tens of metres.

The review article by Greisen (1956) and the book by Galbraith (1958) give a comprehensive account of the experimental and theoretical work carried out in the field of extensive air showers at the respective times of writing. Recent experimental work was reported to the I.U.P.A.P. Cosmic Ray Conference at Moscow, held in July 1959, and the papers relating to experimental work on extensive air showers are published in Volume II of the Proceedings of the Conference.

These three sources describe experimental research into extensive air showers, at both sea level and mountain altitude, by arrays, of varying complexity, of detectors of ionizing particles. All types of detector have been used - Geiger counter, proportional counter, scintillator, ionization chamber, cloud chamber - and a wide range of average characteristics of showers has been studied, e.g. the size spectrum of showers, the density spectrum of shower particles, the zenith angle distribution of direction of arrival of air showers (also Malos (1960)), the angular and lateral distribution of electrons (also Brennan et al (1958)), the lateral distribution of μ-mesons and nucleons (also Lehane et al (1958), Wallace et al (1958)).

The experiment described in this thesis was a measurement of the density spectrum of air shower particles, paying especial attention to the high density region of the spectrum.

1.3 Significance of the density spectrum.

The density spectrum of extensive air shower particles is firstly important in that it is a property of showers directly measurable by simple apparatus. Furthermore, if the radial distribution function for electrons is independent of shower size, and the shower size spectrum is a power law of constant exponent, it has been shown (Hilberry (1941), Singer (1951)), that the density spectrum will ~~have~~ be a power law of the same exponent; Greisen (1956) more closely examines the relationship for a shower spectrum exponent subject to some variation; Galbraith (1958) discusses the extension to a possible relationship between the density spectrum and the primary energy spectrum. The high density region is thus more important, as it relates to a region of shower size and primary energy about which little is known. These relations and some further implications of the density spectrum are discussed further in Chapter IV.

1.4 Previous work on the density spectrum.

The local density of particles is a concept which can be introduced for any sufficiently small region of space, and is expressed as the number of particles crossing a unit of area. For any point on the earth's surface, there exists a frequency distribution of densities crossing a unit area about that point, and this distribution of particle densities is called the density spectrum.

The first experiments on the density spectrum were made by Cocconi et al (1943). Their results showed that, to a good approximation, the density spectrum could be regarded as a power law of the form

$$H(\rho) = H_0\, \rho^{-(\gamma-1)} \qquad (1.\ 3.\ 1.)$$

where ρ is the density in particles per square metre, $H(\rho)$ is the number of showers per unit time with density greater than crossing unit area about a given point, and H_0 and γ are constants.

The corresponding differential distribution is then

$$L(\rho)\,d\rho = L_0\,\rho^{-\gamma}\,d\rho \qquad (1.\ 3.\ 2.)$$

$$\text{where } L_0 = -(\gamma-1)\,H_0 \qquad (1.\ 3.\ 3.)$$

The first determinations of γ were made using either of two methods involving Geiger counters - the "variation of areas" method or the "multiple coincidence" method. The principle behind each of these methods will now be examined briefly.

If a shower of density per unit area ρ is incident upon a Geiger counter of area A, then the probability of triggering the counter is $(1-e^{-\rho A})$. Hence the counting rate of a tray of n counters, each of which is traversed by a shower particle, will be

$$C_n = -L_0 \int_0^{\infty} (1-e^{-\rho A})^n\, L(\rho)\,d\rho \qquad (1.\ 3.\ 4.)$$

Under the assumptions, which will be more fully discussed in Section 3.1, that the density is uniform over the counters and the particles are independent, it may be shown that

$$\log C_n \propto \gamma \log A \qquad (1.\ 3.\ 5.)$$

provided that γ is approximately constant.

Hence γ may be determined as the slope of the log-log plot of the counting rate against the area of the counter trays, when the area is varied. This is the principle of the "variation of areas" method, as was used by Cocconi and Tongiorgi (1949).

The second method, the "hodoscope" method, consists of a comparison of the rates of n-fold to (n - 1)-fold coincidence for trays of equal area. n is usually taken as 4. This method has been employed by Auger and Daudin (1945), Murdoch (1958), and others.

Murdoch carried out a very careful experiment over the density range 1 - 500 particles m^{-2}. Defining the integral exponent to be of the form $a + b \ln \rho$, he finds as best values a = 1.34, b = 0.018, with a corresponding differential exponent $\gamma = 2.31 + 0.018 \ln \rho$. Considering the alternative exponent given by the differential of the natural logarithm of the counting rate with respect to the natural logarithm of the area, he obtains an equivalent value at $\rho = 1$ particle m^{-2}, but a higher dependence upon density. It is to be noted that the two exponents as defined above will be equal only if there is no dependence on the density. i.e. a constant exponent power law. Averaging the two methods the differential exponent at a density of 500 particles m^{-2} will lie in the range 2.43 - 2.57.

Experiments at mountain altitudes (Cocconi and Tongiorgi (1949)) showed the exponent at that height to be identical, within experimental errors, to the exponent at sea level.

Experiments in the high density region ($>$1000 particles m^{-2}) have been few. A brief summary of them is given.

Lapp (1946), and Montgomery and Montgomery (1947), working with ionization chambers, obtained integral exponents of 2.15 and 3.1, respectively for the power law spectrum of burst frequency versus burst size, at sizes corresponding to particle densities >1000 particles m^{-2}. However in neither experiment is the component from extensive air showers accurately known.

Prescott (1956) communicates the results of a 1952 ion chamber experiment on the frequency versus size distribution of bursts occurring in coincidence with extensive air showers. The data, in the form of a differential distribution, is, for sizes corresponding to particle densities exceeding $1000 m^{-2}$, best fitted by a power law of slope > 3, but the relatively few events observed are insufficient to statistically establish the result.

Carmichael and Steljes (1955), fitted the frequency-size distribution for cosmic ray bursts in an ionization chamber with a power law of constant integral exponent 1.73. The contribution from extensive air showers is again uncertain, and though the fit is stated to hold over the density range 40 - 7000 particles m^{-2}, there are only some 40 events of density >1000 particles m^{-2}, and the few graphed points in this region display some tendency towards a higher exponent.

The most detailed experiment to date is by Norman (1956), who examined extensive air showers with an arrangement of three proportional counters. His values for the integral exponent (γ-1) of the density spectrum are:-

$$\gamma - 1 = 1.39 \pm 0.04 \qquad 20 < \rho < 500\ m^{-2}$$
$$\gamma - 1 = 2.2 \qquad \rho \gtrsim 1000\ m^{-2}$$

The exponent at low densities is substantially in agreement with previous Geiger counter experiments, as have been detailed, although Murdoch (1958) has indicated that Norman's analysis in this low density region is suspect. Norman states that, beyond 500 particles m^{-2}, the exponent increases, rapidly compared with the very slight increase in the density range 1 - 500 particles m^{-2}, and quotes the above result.

To summarize, it can be seen that the density spectrum can be represented by a power law, the slope of which is accurately known in the density range 1 –500 particles m^{-2}; beyond the latter density there appears to be a sharp increase in the value of the exponent, from $\gamma \simeq 2.5$, $\rho = 500m^{-2}$, to $\gamma \simeq 3.2$, $\rho = 1000m^{-2}$; whilst there is no experimental data available at higher densities.

It will be shown that the results of the experiment described in this thesis indicate a marked increase in the exponent over the density range 500 - 1500 particles m^{-2}. Implications of this result, and theoretical hypothesis to explain the marked increase and certain other recently discovered air shower phenomena will also be discussed.

Chapter 2.

The experimental apparatus and results.

2.1 The design of the experiment.

The density spectrum was observed directly by the scanning of cloud chamber photographs, the chamber having been triggered by the coincidence of three Geiger counters, the triggering probability for the coincidence at different densities being easily calculable. The three counters form what is called an m-unit, and the arrangement has been and is being used extensively by McCusker at stations in Dublin, Jamaica and Sydney. Each counter is cylindrical, of anode length ~ 9 cm., of diameter ~ 2 cm., and of measured cross-sectional area 18.3 sq. cm. ($\pm$ 5%). The three counters are situated in the same horizontal plane, the main axes aligned in the E - W direction, and the centres of the counters at the vertices of an equilateral triangle of side 20 cm.

Pulses from each counter are fed into a Rossi coincidence circuit, and a pulse is output in the event of a three- fold coincidence, for which the rate was about one per day.

The m-unit was placed directly above a Wilson cloud chamber in both the introductory and the final experiments. Coincidences with other Geiger counter arrangements were indicated by neon lamps on the photographs of the chamber.

A description is also given of the experimental equipment at the Dublin and Jamaica stations, data from which were in agreement with the Sydney data, the combined data being used to provide a result determined with greater statistical accuracy.

2.2 The introductory experiment.

The m-unit was placed directly over a cylindrical cloud chamber situated in the western tower of the School of Physics building. The chamber has been described by Lehane (1960), and it formed part of the Sydney air-shower array described by Brennan et al (1958).

The chamber, of sensitive area $0.17m^2$, was illuminated by Xenon-filled discharge tubes. The chamber was housed beneath a light roof in a temperature-controlled room, and was photographed stereoscopically. The photographs were projected onto a white screen and a track count was made from the plane projection of the photographs from one camera.

Three trays, of area $0.25m^2$, each consisting of twelve Geiger counters, were located from three to five metres from the m-unit. The coincidences between the m-unit and a tray, signifying that the event was produced by an extensive air shower, were indicated by neon lights, which were photographed with the chamber.

Some forty events, in which one or more counter trays fired in coincidence, were photographed and scanned, and the proportion of events of density >500 particles m^{-2} to events of lower density was less than predicted by the Murdoch expressions. However, due to recurrent minor mechanical and electronic faults, which had arisen during the period of operation and which prevented the maintenance of consistent viewing conditions for tracks over the sensitive area of the chamber, it was decided that the chamber would be mechanically overhauled and the electronics redesigned. At the same time it was decided to incorporate the overhauled chamber in the new air shower array then under construction.

2.3 The apparatus for the experiment - the Sydney results.

An m-unit was placed directly over a rectangular cloud chamber, again illuminated by Xenon discharge tubes, and housed in a temperature controlled room beneath a roof of thickness 2.2 gm. $cm.^{-2}$. The chamber was photographed stereoscopically and the plane projections of the photographs of the camera perpendicularly in front of the centre of the chamber were scanned for density measurements.

Neon lights indicated coincidences between the m-unit and:-

(i) either or both of two unshielded Geiger counter trays, each consisting of twelve counters of area 155 sq. cm.($\pm$ 5%), and the trays situated two and six metres from the m-unit respectively.

(ii) any of four penetrating shower sets (as described by ~~Meluskes~~ McCusker et al (1955)), situated from two to eight metres from the m-unit. An event in a penetrating shower set requires at least two Geiger counters to be discharged in each of three trays, each tray being of area $0.2m^2$, and containing twelve counters, and separated vertically by a total of 35 cms. of lead.

A coincidence between a shower set and an unshielded tray also triggered the chamber, the photographs so obtained being scanned for a separate experiment (Idnurm (1961)).

The chamber was of external width 65 cm., and lateral depth 28 cm., and contained three iron plates of thickness 1 cm., tilted to be viewed edge-on by the cameras. It had an aluminium top surface of thickness 5 gm. cm^{-2}. There was a vertical depth of 11 cm. above the top plate, and density measurements were made by counting the number of straight tracks of length $>$ 3 cm. hitting the top plate. This will exclude tracks made by low-energy ($\lesssim$ 10 Mev) electrons which suffer large-angle scatters in collisions with the argon in the chamber. The sensitive area of the chamber at the top plate was determined to be $0.110 \pm 0.005m^2$, found by plotting the co-ordinates of over 300 tracks, which, on stereoscopic reprojection, were observed to strike the plate.

The chamber described in Section 2.2 was also installed in the same room, although some time after the first chamber in such a way that the axes of the two chambers were at right angles. The chambers were 2.5m apart. Particle densities in the second chamber were also measured, and some comparison was made of densities in the two chambers to estimate the effect of gradients (Section 3.5).

143 events were observed in which there was a coincidence between the m-unit and an unshielded tray. The sensitive time, calculated as total running time less insensitive time, of the chamber, was 3500 hours. The density distribution of the events is given in Table 2.3.

In order to extend observations to lower densities two further experimental arrangements were used. These were (i) the coincidence of three counters of area 124 cm^2 each; and (ii) the coincidence of three

Table 2.3.

Number of particles in cloud chamber.	Number of Events. m-unit Trigger	Number of Events. M-unit Trigger	Number of Events. T-unit Trigger
0		37	164
1		35	164
2		56	143
3	9	48	107
4		41	85
5		37	60
6		34	55
7		33	31
8	10	22	26
9		22	19
10		30	27
11-15	6	79	55
16-20	6	39	23
21-25	7	24	12
26-30	10	25	10
31-40	16	23	7
41-50	12	8	4
51-60	12	6	2
61-70	11		
71-80	8		
81-100	14		
101-120	9		
121-180	11		
$>$ 180	2		
	143	609	999

The results of three different triggering arrangements at Sydney. The total sensitive times were 3500, 612 and 218 hrs. respectively. M-unit Trigger gave 10 events and T-unit Trigger 5 events with $>$ 60 particles.

trays of three counters each - area 372 cm^2. In both ~~circum~~ instances the centres of the counters were in the same horizontal plane at the vertices of an equilateral triangle of side 65 cm., and the counters were located directly above the chamber. The density distribution for the two runs under these two triggering arrangements, for events in coincidence with at least one unshielded tray, are listed in Table 2.3. Trigger (i) with single counters is termed the "M-unit", and trigger (ii) the "T-unit".

The analysis of the data is discussed in Chapter 3.

2.4 The apparatus at the Dublin and Jamaica stations - the high density result.

Similar experiments at Dublin and Jamaica, carried out by McCusker et al, have yielded results which it has been possible to combine with the Sydney results, as will be described in Chapter 3.

At both stations there are two cylindical Wilson cloud chambers, whose axes are at right angles to one another. The Dublin chambers are of sensitive area 30 x 8 cm^2, and 30 x 12 cm^2. The latter has been used in the density experiment, the former having been designed for use with an electromagnet, and it was thought that the surrounding material may affect the particle density within the chamber. The two Jamaican chambers are of sensitive area 30 x 18 cm^2. All chambers were installed in specially built temperature controlled huts of light construction. Each chamber was photographed stereoscopically; density measurements were made by stereoscopic viewing of these photographs.

The triggering criterion was the same at both stations, but differed slightly from the Sydney criterion. The requirement was the coincidence of an m-unit within one metre of the chamber, with both an unshielded tray and a penetrating shower set within a few metres of the chamber. This different criterion is discussed in Section 3.5.

In Table 2.4 is presented the density distribution for high density events for each station.

Table 2.4.

Density (particles m^{-2})	Sydney	Dublin	Jamaica	all stations
800 - 1100	21	39	35	95
1100 - 1400	5	25	25	55
1400 - 1800	7	26	13	46
1800 - 2500	1	13	11	25
2500 - 4000	0	3	7	10
Total ($>$ 1100)	13	67	56	131
Total ($>$ 800)	34	106	91	231

7 events of density greater than 4000 particles m^{-2} were recorded, all at the Dublin and Jamaica stations.

The above results, all for densities crossing a horizontal plane are analysed in Chapter 3. The effect of non-vertical incidence is also considered (Section 3.5).

Chapter 3.

The analysis of the experimental data.

3.1 The method of analysis.

Before presenting the results of analysis of the data, and some examination of those results, the method of analysis will be outlined.

The assumption is made that the particles are independent of each other in position and probability of occurrence, so that local fluctuations of density are neglected. It is then meaningful to talk of the local density ρ of the extensive air shower. If then dP is the probability that a charged particle will cross an element of area da perpendicular to the shower axis, we have the relation $dP = \rho\, da$.

Under this relation, the probability of a counter, of area ~~m^2~~ $a\ m^2$, firing when there is a uniform particle density of ρ is

$(1-e^{-\rho a})$ (3. 1. 1.)

Hence the probability of three counters firing in coincidence is

$(1-e^{-\rho a})^3$ (3. 1. 2.)

Thus if the differential density spectrum is a power law of the form

$$k(\rho)d\rho = k_0\,\rho^{-\gamma}\,d\rho \qquad (3.\ 1.\ 3.)$$

the differential rate of triggering three counters, area $a\,m^2$ in coincidence is

$$R(\rho)d\rho = k_0(1-e^{-\rho a})^3\rho^{-\gamma}d\rho \qquad (3.\ 1.\ 4.)$$

Further, if the particle density is $\rho\ m^{-2}$, the probability of viewing n tracks in a chamber of area $A\,m^2$ is

$$\frac{(\rho A)^n}{n!}e^{-\rho A} \qquad (3.1.5.)$$

Combining these expressions, the rate ~~Rn~~ R_n of seeing n tracks in a chamber, area Am^2, when triggered by a triple coincidence of counters, area am^2, is

$$R_n = k_0\int_0^\infty (1-e^{-\rho a})^3 e^{-\rho A}\frac{(\rho A)^n}{n!}\rho^{-\gamma}d\rho \qquad (3.\ 1.\ 6.)$$

the differential density spectrum having been given in (3. 1. 3.).

Using the Basser Computing Department's digital computer SILLIAC, computations of ~~Rn~~ R_n were made of particular values of n, over suitable ranges, for counters of area 18.3 cm^2, 124 cm^2, 372 cm^2, (m-unit, M-unit, T-unit), and for chambers of area 0.11 m^2, 0.036 m^2, and 0.054 m^2 (Sydney, Dublin, and Jamaica), and for suitable values of γ.

R_n ~~Rn~~ was calculated numerically for small values of n from the expression (3. 1. 6.), and for large values of n in the equivalent Γ -fuction form yielded by integration of (3. 1. 6.) (Broadbent and Janossy, 1948).

The functions (3. 1. 3.) and (3. 1. 4.) were also calculated by numerical intergration over suitable density ranges.

In all computations h_0 was initially taken as 1, and normalization was made to the observed number of events in the category being examined. γ was varied in steps of 0.1 over suitable ranges, and χ^2 values were computed for all values of γ. The best value of γ, corresponding to the lowest χ^2, and the probability of that χ^2 value being exceeded, are given in each instance. Errors quoted in γ specify the values of the exponent at which the χ^2 value first exceeds the 5% probability level, In all cases χ^2 as a function of γ was found to be symmetric about its minimum value, and the errors in both directions are equal.

3.2 <u>Results of the M-unit and T-unit triggers.</u>

The M-unit and T-unit experiments were performed after the m-unit experiment, to asscertain whether the nature of the experiment in any way affected the result. The M-unit and T-unit results determine the density spectrum in the region 100 - 500 particles m^{-2}, as shown in Table 3.2.1., and the exponent obtained by this method may be compared with the exponent quoted in Section 1.3.

For both M-unit and T-unit triggers, 3 values of γ were obtained over the density range 100 - 500 particles m^{-2}, by three different methods of comparison. The values of γ, and corresponding errors and probabilities, according to Section 3.1, are listed in Table 3.2.2.

Method (a) is a comparison of the experimental results with the numbers predicted by a power law spectrum (equation (3.1.3)), making no

Table 3.2.1.

Density (particles m^{-2})	Triggering probability		
	m-unit	M-unit	T-unit
1	$.61 \times 10^{-8}$	$.19 \times 10^{-5}$	$.487 \times 10^{-3}$
10	$.595 \times 10^{-5}$	$.159 \times 10^{-2}$	.0300
50	$.669 \times 10^{-3}$	.0986	.602
100	$.468 \times 10^{-2}$	.359	.929
500	.215	.994	1.000
1000	.592	1.000	1.000
2000	.925	1.000	1.000
5000	1.000	1.000	1.000

Table 3.2.1. Triggering probabilities of the counter arrangements at different particle densities

Table 3.2.2.

Results of analysis of M-unit and T-unit data.

M-unit.

Method of calculation	Exponent	Probability	Error range of γ
(a)	2.1	~~2~~.5	$\pm$ .3
(b)	2.7	~~2~~.6	.3
(c)	2.8	~~2~~.6	.3

T-unit.

(a)	2.3	~~2~~.6	.3
(b)	2.5	~~2~~.7	.3
(c)	2.6	~~2~~.8	.3

correction for triggering probability; method (b) is a comparison with numbers predicted by equation (3.1.4.), thus making allowance for the probability of triggering the counter arrangements; and method (c) is a comparison with the numbers predicted by equation (3.1.6.), thus correcting for fluctuations in the number of particles traversing the chamber.

The density range of 100 - 500 particles m^{-2} corresponded to the range 11 - 60 particles in the chamber; for the T-unit run, in which the greatest number of events were recorded, a comparison, by method (c), over the range 50 - 1000 particles m^{-2}, was made, yielding a best value of the exponent $2.6 \pm .3$. The χ^2 probability of an exponent of 3.0 fitting the experimental results was 0.004, and for higher values of the exponent, the probability decreases rapidly.

Figure 3.2.1 shows the experimental points and the predicted spectrum, by method (a), a straight line of slope 2.3, for the T-unit. The triggering probability affects the position of the low density points by up to 50%, thus steepening the slope to obtain the actual value of the exponent, as shown in Table 3.2.2. Error limits of each experimental point are shown.

In Figure 3.2.2 is shown the comparison between the experimental and theoretical spectra for the M-unit. Points marked by a dot are experimentally observed points. These have been corrected by dividing by the triggering probability $(1-e^{-3a})^3$ at the middle of the density range being considered. The theoretical spectrum shown is a line of slope 2.7, according to method (b). Error limits of the corrected points are shown.

Comparisons at densities of less than 100 particles m^{-2} (less than 10 particles in the chamber) cannot be made satisfactorily, for several reasons.

(a) The principal reason lies in the criterion for selection of tracks, as described in Section 2.3. Tracks which are scattered in the gas of the chamber are not counted, although the low energy particles producing them are able to fire a Geiger counter. Also low energy particles are stopped in the 5 gm. cm^{-2} thickness of the chamber roof. It has been

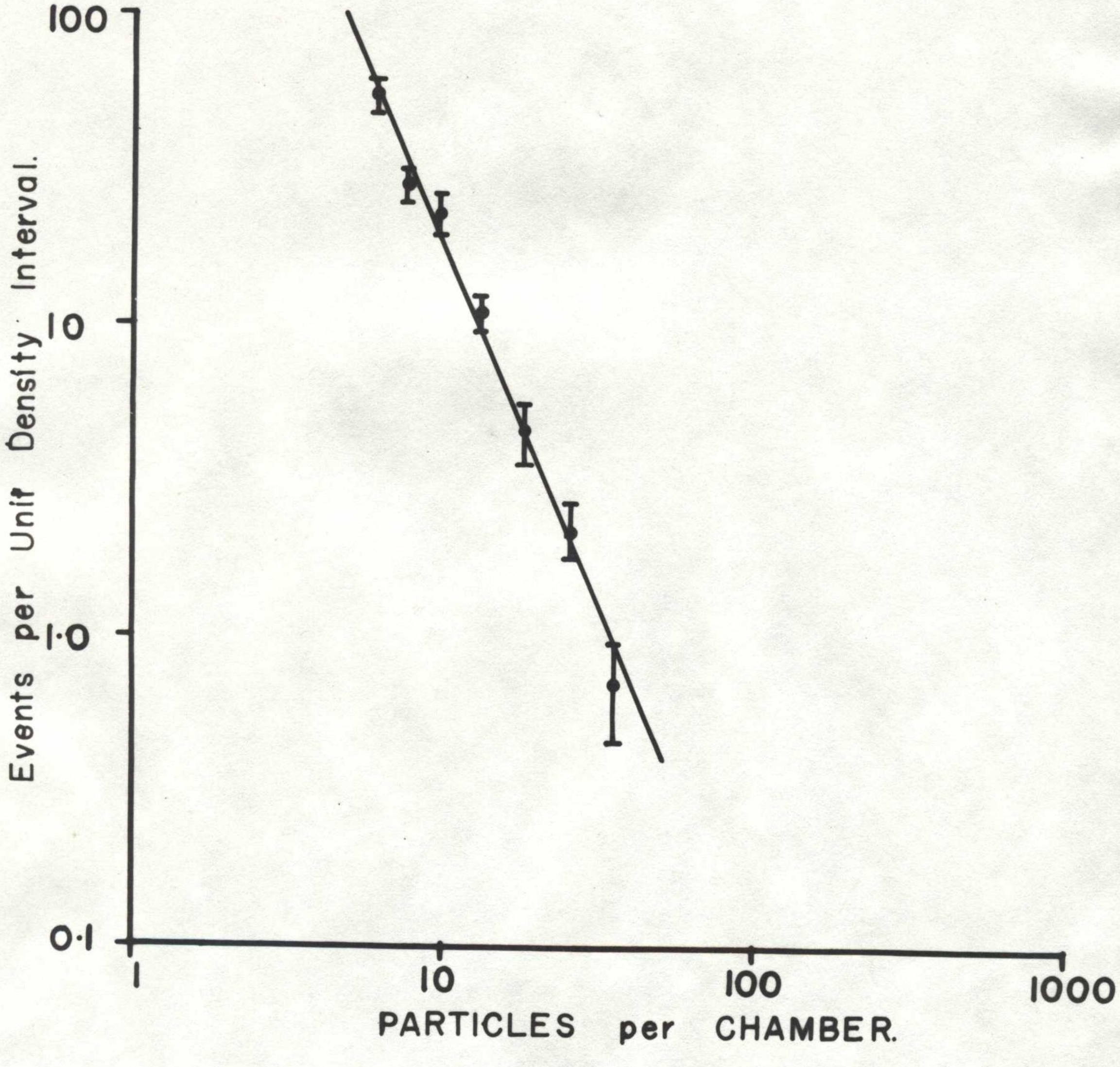
100
10
1·0
0·1
Events per Unit Density Interval.
1
10
100
1000
PARTICLES per CHAMBER.

Figure 3.2.1

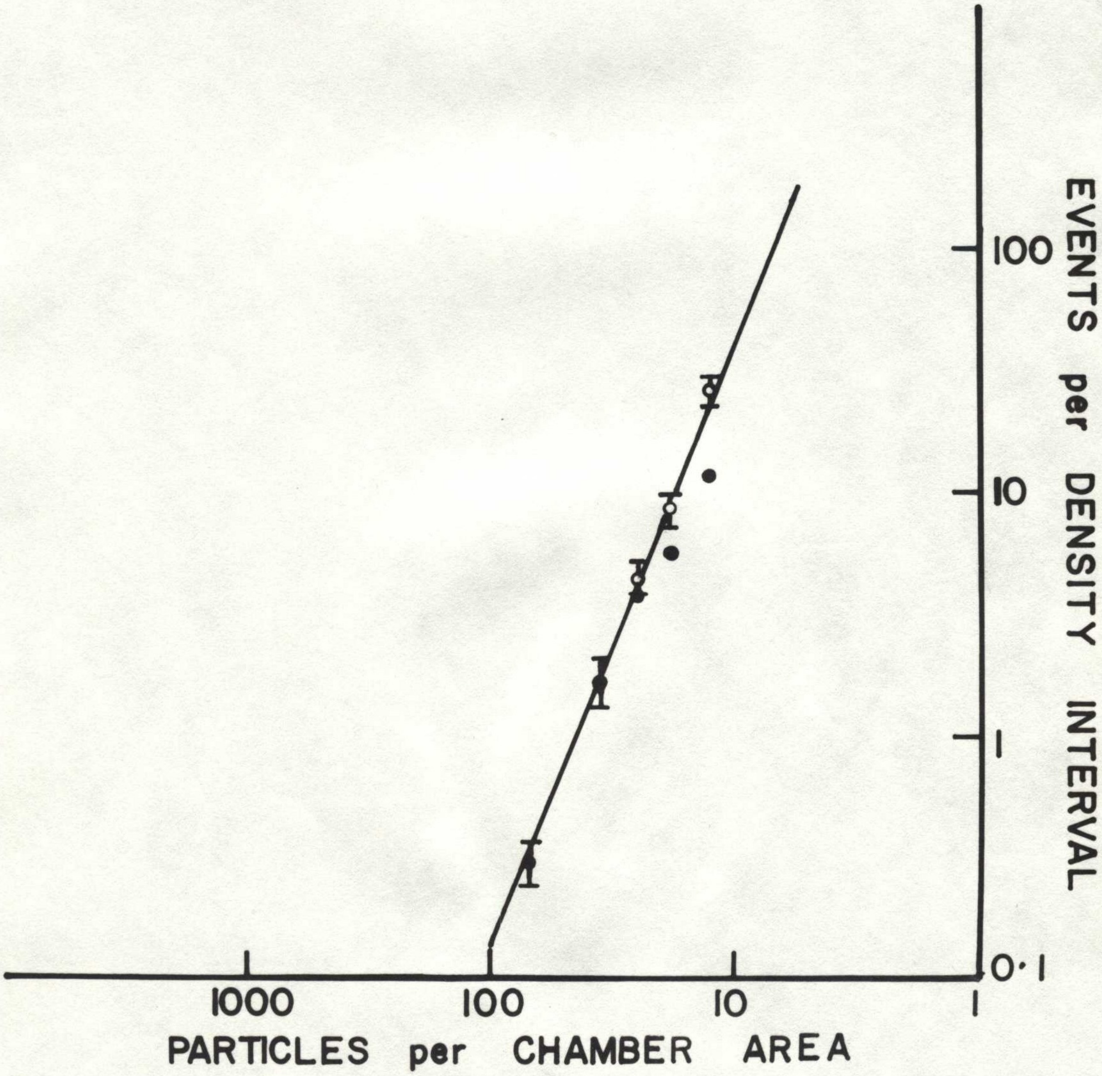
EVENTS per DENSITY INTERVAL
100
10
1
0·1
1000
100
10
1
PARTICLES per CHAMBER AREA

Figure 3.2.2.

qualitatively observed that the ratio of low energy to high energy tracks in low density pictures is very variable, and can be high, but the effect is not noticeable in high density pictures. There the percentage of low energy particles is much smaller and less variable. Air shower measurements (e.g. Kameda et al, 1960), which show a harder energy spectrum close to the core, the highest density region of each shower, confirm these qualitative observations.

($_b$) A couple of phenomena affect the low-density results to a minor extent, probably to the extent of no more than 20 events in a thousand.

(i) A local event or a "narrow shower" (Wei and Montgomery 1949) which sets off the extensive tray by a chance coincidence. Local events may be produced by low energy interactions in the roof of the building or may be due to scattered particles or to narrow showers, which are known to follow a power law of differential exponent ~ 3.2.

(ii) Conversely, a low density shower may fail to set off the extensive tray.

(i) would occur in no more than 2% of the events; (ii) in about 0.2%; these phenomena are of little consequence even at low densities.

However the effect of (a) may be great at low densities; it is not surprising that the results of the M-unit and the T-unit are not consistent at low densities, although the effect of (a) is expected, and assumed, to be negligible at high densities.

Thus, to summarize, the results of the M- and T-units, both physically similar methods to the m-unit, were, over the density range 100-500 particles m^{-2}, consistent with the Murdoch spectrum, and could, over the range, be fitted by a power law of exponent 2.6 - 2.8 with a high probability of a good fit.

3.3 Results of the m-unit trigger.

The 134 events of density $>$ 100 particles m^{-2} obtained under m-unit trigger, in coincidence with an extensive tray, were analysed by method (b) in an endeavour to fit a power law of constant exponent over the whole range. The best value of γ was 3.0, with a χ^2 probability of .12.

However, a comparison of the predicted numbers with the experimentally observed indicated a possible flatter slope at low densities and steeper slope at high densities.

Analysis by method (c) confirmed the above, and also showed that, for large particle numbers, the exponent determined by method (b) would be equal to the exponent determined by method (c).

Two regions of the spectrum were now examined seperately. Using method (c) over the density range 100 - 800 particles m^{-2}, $\gamma = 2.6 \pm .4$, with a χ^2 probability of .70 at 2.6 was found. By method (b), at densities $>$1100 particles m^{-2}, $\gamma = 4.2 \pm .8$, with a χ^2 probability of .40 at 4.2 was found. By method (a), in the high density range, γ exceeded 3.0. Again there was a wide error range.

As the low counting rate ($\sim$1 per day) precluded rapid accumulation of statistics to confirm these results, the examination and analysis of the data from the Dublin and Jamaica stations (as given in Section 2.4) was undertaken.

3.4 <u>High density results</u> - <u>Sydney, Dublin, and Jamaica.</u>

The three experimental distributions listed in Table 2.4 were examined for self-consistency by using a modified χ^2 test (Hald, 1952). For events of density $>$800 particles m^{-2}, the χ^2 probability was 7.8%. For the smaller number of events $>$1100 particles m^{-2}, the χ^2 probability was $\sim$ 12%.

Method (b), being far simpler and quicker than method (c), yet yielding the same result, was used in the separate analysis of the three distributions.

Dublin: For densities $>$1100 particles m^{-2}, $\gamma = 3.8 \pm .3$, with a χ^2 probability of .06 was the result.

Jamaica: $\gamma = 3.5 \pm .8$, χ^2 probability .95, under the same conditions. The Sydney results are given in the previous section.

Combining the data of all three experiments, the best value of the exponent was found to be 3.9 $\pm$ 0.5, with a χ^2 probability of 0.24 at 3.9. The probability that an exponent of 3.0 would be a good fit was 0.004, and, that 2.5 is a good fit, $< 10^{-6}$.

Figure 3.4.1 graphically illustrates the combined results, using corrected points according to the method described for figure 3.2.2. The slope of the line is 3.9.

Analysis by method (a) yielded the following results:- $\gamma = 3.6 \pm .4$, χ^2 probability .30 at 3.6.

Inclusion of events in the density range 800 - 1100 particles m^{-2} yielded in all cases lower values for γ and lower probabilities of best fit, the probability of fit for the best value (3.4) for all events scarcely exceeding 5%.

The results of this section and section 3.2 thus indicate a marked increase in the exponent of the density spectrum from $\sim$2.6 in the density range 100 - 600 particles m^{-2} to $\sim$ 3.9 in the region of density $>$ 1100 particles m^{-2}. Three effects which may alter the exponent at high density are considered in the next section.

3.5 Some phenomea affecting the determination of the high density exponent.

(a) The effect of possible scanning bias.

As the number of tracks in the chamber increases, it becomes increasingly difficult to count that number accurately, because of failure to resolve close tracks. The track count will most likely be less than the actual numbers of tracks present; and the discrepancy will increase with increasing density. Using a simple Poissonian model for the spatial distribution of tracks in the chamber, it can be shown that, for an actual exponent $\gamma = 4.0$, an overestimate by .12 will be made in an experiment in the Sydney chamber with its physical size and scanning criteria, and an overestimate by .04 with the Dublin and Jamaica chambers.

(b) The effect of different triggering criteria.

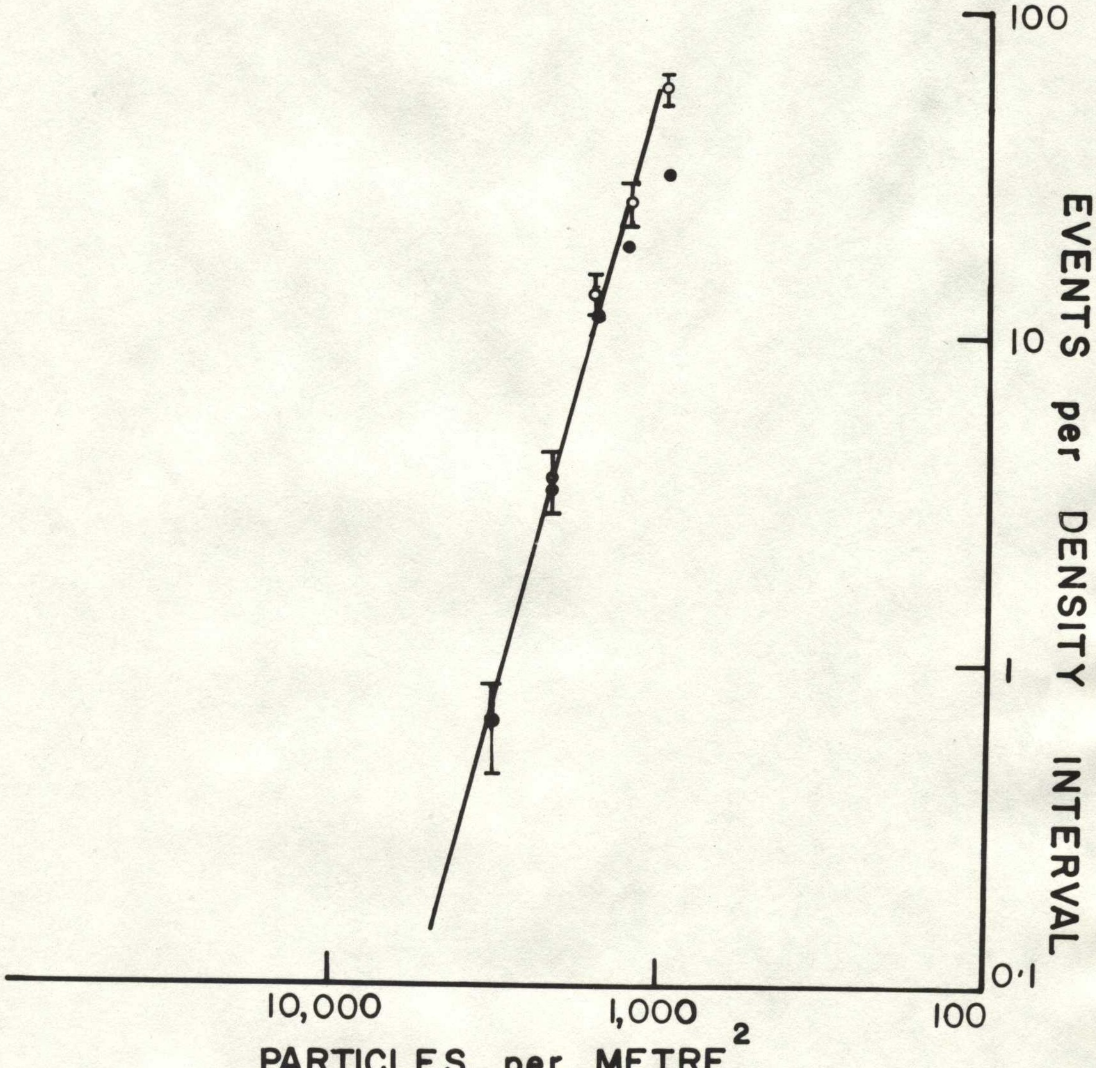
100
10
1
0·1
EVENTS per DENSITY INTERVAL
10,000
1,000
100
PARTICLES per METRE²

Figure 3.4.1

In Sydney the triggering is an m-unit directly over the chamber in coincidence with an unshielded tray 2 metres away; in Dublin and Jamaica the trigger is an m-unit within one metre of the chamber in coincidence with a penetrating shower set and an unshielded tray, both within a few metres.

Firstly to examine the effect of the m-unit distance from the chamber.

In the Sydney M-experiment, a count was also made of track numbers in the second chamber 2.5 metres away, for some 300 events. A plot of number of tracks in round chamber versus number in square was made. The particle density was less in the round chamber in 65%, rather than 50% of the events; not enough statistics were available to measure the ratio at different densities, and there were very few events of density > 400 particles m^{-2} included. At a distance of less than one metre, and at higher densities, it is expected that there will be little difference in the densities at the m-unit and in the chamber averaged over many events.

Secondly, to examine the additional criterion of triggering a penetrating shower set. It has been estimated that the probability of not triggering a shower set, when the density exceeds 800 particles m^{-2}, is less than 6% (Reid et al, 1960). In the Sydney experiment coincidences with the penetrating shower sets were recorded, and in 6 of 34 events no shower set fired. The particle numbers (chamber area 0.11 m^2) were: 89, 114, 90, 192, 147, 120.

As only six events have been recorded one can only postulate the effect of such events on the density spectrum. Such events may be produced by a different sample of showers to the showers which also trigger a penetrating shower set. Lehane et al (1958) and Wallace et al (1958) have shown that the nucleon distribution about the core of a shower falls away more rapidly than the electron distribution, and also that there is a higher percentage of nucleons in smaller showers (size 10^4 particles). These few events being considered may thus occur at up to a few tens of metres from the cores of very large showers. Now Kameda et al (1960) find an increase in the integral exponent of the shower size spectrum from 1.55 to 2.04 over the size range 4×10^4 to 4×10^5

particles. The additional triggering criterion of the penetrating shower set in coincidence could cause the exponent of the observed density spectrum to differ from the true exponent, but it seems doubtful that the charge will be of sufficient magnitude to be detectable.

(c) The effect of non-vertical arrival of showers.

Two density spectra were in fact considered, both derived from the same data. They were: (i) the density spectrum of densities measured in a plane perpendicular to the direction of arrival of the showers; and (ii) the density spectrum of densities measured in a horizontal plane. Slopes of the two spectra were identical; the absolute values of densities was increased (from (ii) to (i)) by approximately 10%.

The zenith angle distribution of the Sydney m-~~events~~ has been found to be consistent with previous results, within wide limits imposed by poor statistics (Idrurm, 1961).

Kameda et al (1960) claim an increase in the exponent of the zenith angle distribution with increasing shower size, and the effect they claim would reduce the exponent of the density spectrum, but by no more than 2%.

(d) Conclusion.

It is considered unlikely that the effects (a), (b), (c), will change the measured exponent of the density spectrum by more than one or two per cent, and the result of the experiment is as stated; a marked increase in the exponent of the density spectrum, from ~ 2.6 for $100 < \rho < 600$ particles m^{-2}, to ~ 3.9 for $\rho > 1100$ particles m^{-2}.

Chapter 4.

Some implications of the observed density spectrum; also a proposal for a future experiment.

4.1 The relationship between the density spectrum and the shower size spectrum.

As previously mentioned in Section 1.3, Hilberry (1941) first mentioned the connection between the density spectrum and the size spectrum. The following relationship was derived by Singer (1951):- if the shower size spectrum is a power law of constant exponent and the radial distribution function is the same for all sizes of showers, then the density spectrum will be a power law of the same exponent.

Two facts make this a very important relationship:- (i) knowledge of the size spectrum at the earth's surface is, as yet, the only information related to the primary high energy spectrum which can be obtained at those altitudes; (ii) compared to the size spectrum exponent, the density spectrum exponent can be measured simply; size measurements require a large array of detecting equipment, and suffer two further defects:- (a) the probabilities of showers of different sizes falling at different distances triggering the array is rarely known accurately, and (b) the radial distribution function, essential to the calculation of shower size is not known for each individual shower.

The following proof of the equality of the exponents is as presented by Galbraith (1958). Differential frequency distribution of densities over a given unit area .... $h(\rho)d\rho$; differential frequency distribution of shower sizes, for showers whose axes cross in a given unit area ... $n(N)dN$; radial distribution function ... $\rho(r) = Nf(r)$.

Then such a distribution of showers will, at a given point, produce a distribution of densities.

$$h(\rho)d\rho = \int_0^\infty n(N)dN \, P(r)\, dr \qquad (4.\ 1.\ 1.)$$

$P(r)dr = 2\pi r\,dr$, being the probability of the shower axis falling within an annulus;

$$\therefore L(\rho)d\rho = \int_0^\infty 2\pi r \cdot n(N)\,dN\,dr \qquad (4.\ 1.\ 2.)$$

Now, as $\rho(r) = Nf(r)$, and if $n(N)dN = AN^{-\beta}dN$, where A is a constant.

$$L(\rho)d\rho = \int_0^\infty 2\pi r A \left[\frac{\rho(r)}{f(r)}\right]^{-\beta} \frac{d\rho(r)}{f(r)}\,dr \qquad (4.\ 1.\ 3.)$$

to a first approximation, omitting second order terms in dr. Hence

$$L(\rho)\,d\rho = 2\pi A \rho^{-\beta} d\rho \int_0^\infty f(r)^{\beta-1}\,dr \qquad (4.\ 1.\ 4.)$$

The integral is independent of r, so $L(\rho)d\rho = L_0 \rho^{-\gamma} d\rho$

and $$\gamma \equiv \beta \qquad (4.\ 1.\ 5.)$$

Unfortunately there has always been some discrepancy between the observed exponents. The exponent of the density spectrum has been thoroughly discussed; the shower size spectrum exponent has been found to be ~ 1.8 by the M.I.T. (Clark et al, 1957) and ~~(Harwell~~ Harwell (Cranshaw et al, (1958)) groups. Greisen (1956) claims the discrepancy is brought about by the slow change of the density spectrum exponent; however this appears unlikely.

The work of Kameda et al (1960) on the size spectrum and radial distribution however invalidates the previous discussion. Kameda et al carried out an examination of the shower size spectrum over the range 4×10^4 particles to 4×10^5 particles, and found an increase in the integral exponent from 1.55 to 2.04. They also found that the assumption of one radial distribution which could be fitted to all showers was not tenable; although variation of distribution with shower size was not examined.

Kameda et al state that a similar change of exponent over the same size range has been found by the Japanese group working at sea level, compared to their altitude of 2770m., and also by a Russian group.

In the light of the above experimental results, and the results in this thesis, it would appear that research could be profitably undertaken in the performance of a computation relating the size and density spectra, working with the experimental results of Kameda et al and of this thesis, and examining several previously used distribution functions (e.g. Brennan et al, 1958). Such a computation could, using SILLIAC, be designed to examine the relationship of many and general forms of the three functions - density spectrum, number spectrum, and distribution function.

4.2 A possible theoretical explanation for the change in slope of the density spectrum.

The change in slope of the density spectrum may be due to a change in slope of the primary energy spectrum; or it may be caused by some change in the character of nuclear interactions at energies beyond 10^{13}ev, possibly at energies of 10^{14} to 10^{15}ev.

Several recent experiments favour the latter possibility. Firstly, the change in slope of the shower size spectrum reported by Kameda et al (1960), at mountain altitudes, has also been found to occur at sea level in the same range of shower size, by a Japanese group and by Kulikov and Khristiansen (1959).

Next there are the γ-ray energy spectrum results. At depths of 220 g/cm^2 and 740 g/cm^2 below the top of the atmosphere, a double change in slope of the energy spectrum of γ-rays incident upon a nuclear emulsion stack has been found. (Duthie et al, 1960, Fujimoto et al 1960). The first change in slope at $\sim$1000 Bev corresponds to the change in slope of the density spectrum.

A theory has been developed by Ueda and McCusker (1961) which shows promise in the explanation of the change in slope of the density spectrum.

Ueda and McCusker start from a primary energy spectrum of constant integral exponent 1.7, and then use the cascade model developed by Fukuda, Ogita, and Ueda (1959) together with the following basic hypotheses.

(a) in the centre of mass system of two colliding nucleons the maximum energy of a π -meson is 25 Bev,

(b) the total number of secondary particles in a nuclear interaction is a function of the energy of the primary particle,

(c) the fraction of the incident energy going into the π-meson component decreases with increasing energy, going from 0.3 at 5.10^2 Bev to 0.05 at 5.10^7 Bev,

(d) that at high energies a large proportion of the incident energy goes into particles heavier than π-mesons.

This theory is not directly able to predict the density spectrum; however a calculation has been made of the integral number of particles produced above an observation level which initiate soft showers and give rise to such electron showers of greater than n particles as a function of n. In the high density region of air showers, fluctuations may be important, and the density observed in a small area may correspond to a shower initiated by a single particle rather than to a dense part of a smooth lateral distribution. Under this supposition, the theoretical graph and the density spectrum correspond to one another.

The above theory is being further examined to observe the effect of varying the initial assumptions on the final predictions. However it does seem that a more accurate determination of density spectrum at sea level, and a corresponding determination at mountain altitude, may prove to be the crucial experiment.

4.3 Proposals for future experiment.

As described in this thesis, the change in slope of the density spectrum is, as yet, not fully proven. More statistics are necessary to do this, the main difficulty being the low counting rate.

Further, as suggested at the conclusion of the previous section, density spectrum experiments, at sea level and mountain altitude, may prove crucial in determining the validity of the theory described. So a simlar experiment must be carried out at mountain altitudes.

It would seem that a chamber of area $\gtrsim 0.05$ m^2, viewed stereoscopically, and triggering counters of area $\gtrsim 30$ cm^2 would give the best conditions for density measurement, and the best counting rate.

By this continuation of the experiment, it is hoped that sufficient statistics can be accumulated to confirm or disprove the experimental result presented in this thesis, and consequently provide some further information towards the solution of the basic problems of cosmic radiation, set out in Section 1.1.

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