LIMIT STATES DESIGN OF CRANE RUNWAY GIRDERS

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ABSTRACT

Steel crane runway girders are subjected to torsion by their eccentric loads. The twist rotation of the principal axes caused by torsion induces additional bending moments, and reduces the resistance to lateral buckling. There is little guidance on how to design for torsion, and design procedures are often intuitive, with varying degrees of rationality and precision.

This paper seeks to establish a logical procedure for designing crane runway girders which is based on an extension of the limit states methods of designing for biaxial bending. An additional term for the ratio of the design torque to the section torsion resistance is included in the interaction equations. Proposals are made for determining this section resistance. Research has shown that this addition can allow for the reduction in the lateral buckling resistance caused by the twist rotations. A linear elastic analysis of the twist rotations is used to determine the moment increases caused by torsion.

An example of the proposed method is developed and illustrated by a design example. The small twist rotations are not enough to have a major effect on the bending moments. The additional torsion term is also small, and the design capacity is primarily governed by the resistance to lateral buckling.

KEYWORDS

Analysis, Bending, Capacity, Crane, Design, Steel, Torsion
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1 INTRODUCTION

Steel crane runway girders (Fig. 1a) are subjected to biaxial bending and torsion by their eccentric horizontal and vertical loads (Fig. 2a). However, design codes for steel structures [1-6] generally give little or no guidance on how to design against torsion or torsion and bending. Consequently, the design procedures [7-9] used for crane runway girders are often intuitive and based on adapting existing methods of designing against biaxial bending, with varying degrees of rationality and precision.

Limit states methods of designing for biaxial bending use the results of structural analysis in the checking of the member and section capacities, usually through the use of interaction equations which combine the separate effects of major and minor axis bending. First-order elastic analyses under the factored limit state loads are used, which ignore the non-linearities caused by the bending rotations, it being assumed that these are accounted for in the interaction equations. Non-linearities due to yielding are accounted for in the individual moment capacities.

Torsion affects the strength of crane runway girders in two ways. The first is the additional minor axis bending moment caused by the rotation of the principal axes. This needs to be included in the design actions determined by the structural analysis.

The second is the reduction of the lateral buckling resistance caused by the twist rotations. Research on major axis bending and torsion [10] has shown that the reduction can be modelled conservatively by the use of an interaction equation which incorporates the torsion section capacity. It is therefore logical to allow for the effect of torsion on biaxial bending by adding a similar torsion section capacity term to the existing interaction equations.

This paper seeks to establish a logical procedure for designing crane runway girders which is based on an extension of the accepted limit states methods of designing for biaxial bending, such as that of the Australian code AS 4100 [1]. The elastic analysis of crane runway girders is first considered, and proposals are made for determining the torsion section capacity. Design interaction equations are then developed for checking the section and member capacities under combined biaxial bending and torsion. Finally, the design method is summarised and illustrated by a worked example.

2 TORSION ANALYSIS

2.1 GIRDER MODEL

A crane runway girder of length $L$ and its hoist trolley loading are shown in Fig. 1a, and an idealised girder in Fig. 1b. The girder is simply supported at both ends, where horizontal and vertical deflections and twist rotations are prevented but minor and major axis rotations and warping are unrestrained. A typical girder section shown in Fig. 2a consists of an I-section with a channel section welded to the I-section top flange. Other girders may consist of a mono-symmetric I-section member with lips welded to the top flange, as shown in Fig. 3. The idealised girder is loaded at midspan by a vertical load $N_y$ with an eccentricity $e_x$ and a horizontal load $N_x$ with an eccentricity $e_y$ from the girder shear centre, as shown in Fig. 2a. A similar model has been used in [7].

2.2 SECTION PROPERTIES

The girder section is modelled as a hybrid top flange, a rectangular web, and a rectangular bottom flange. The shear centres $S_t$, $S_b$ of these two flanges are shown in Figs 2a and 3, as is the shear centre $S_g$ of the complete section.

The flange shear centre distances from the girder shear centre are given by

$$s_t = h/[1 + I_{st}/I_{sb}]$$  \hspace{1cm} (1)

$$s_b = h/[1 + I_{sb}/I_{st}]$$  \hspace{1cm} (2)
in which \( I_{y_t}, I_{y_b} \) are the second moments of area of the top and bottom flanges and

\[ h = s_t + s_b \]  

(3)

is the distance between flange shear centres.

The warping section constant is

\[ I_w = I_{yt} s_t^2 + I_{yb} s_b^2 \]  

(4)

The uniform torsion section constant is

\[ I = J_t + J_b + J_w = J_{pf} + J_{ub} \]  

(5)

in which \( J_t \) and \( J_b \) are the torsion section constants of the flanges and \( J_w \) is the torsion section constant for the web, and \( J_{pf} \) and \( J_{ub} \) are the torsion section constants of the channel section and the I-section.

2.3 MID-SPAN ACTIONS

The mid-span forces \( N_x, N_y \) shown in Fig. 2a are statically equivalent to forces \( N_x, N_y \) acting at the girder shear centre \( S_g \) and a torque

\[ M_z = N_x s_b + N_y s_t = M_u + M_w \]  

(6)

as shown in Fig. 2b. The torque \( M_z \) is resisted by a combination of uniform torsion \( M_u \) and warping torsion \( M_w \). These actions are statically equivalent to the vertical shear centre force \( N_y \), the uniform torque \( M_u \), and the flange forces

\[ F_t = (N_x s_b + M_w)/h \]  

(7)

\[ F_b = (N_y s_t - M_w)/h \]  

(8)

as shown in Fig. 2c.

2.4 FIRST-ORDER TWIST ROTATIONS

The exact first-order twist rotations \( \theta \) caused by the moment \( M_z \) can be determined by finding the solution of the torsion differential equation

\[ G j \theta'' - EI \theta''' = M_z/2 \]  

(9)

which satisfies the boundary conditions, in which \( \theta'' \equiv d^2/\theta \). The solution is

\[ \theta = M_z z/2 G J + A_1 e^{z/a} + A_2 e^{-z/a} + A_3 \]  

(10)

in which

\[ \alpha^2 = EI_w/GJ \]  

(11)

\[ A_1 = \frac{M_z/2GJ}{\left(\alpha^L/\alpha e^{-L/\alpha}\right)} \]  

(12)

and \( A_2 = -A_1 \) and \( A_3 = 0 \).

The exact mid-span twist rotation is

\[ \theta_e = \frac{M_z L}{4GJ} \left(1 - \frac{2}{L/a} \frac{1-e^{-L/a}}{1+e^{-L/a}}\right) = \frac{M_z}{k_z} \]  

(13)

in which \( k_z \) is the torsional stiffness of the girder.
Approximations for the uniform and warping torques $M_u$ and $M_w$ may be obtained by assuming that the central torque is resisted only by uniform or warping torsion [11], in which case

$$M_u = k_u \theta_{uw} = (4GJ/L)\theta_{uw} \quad (14)$$

$$M_w = k_w \theta_{ww} = (48EI_{s} / L^2)\theta_{ww} \quad (15)$$

These approximations may be used in Equation 6 to obtain a conservative approximation for the central twist rotation as

$$\theta_{uw} = M_z / (k_u + k_w) = M_z / k_{uw} \quad (16)$$

The accuracy of this approximation is displayed in Fig. 4 by the variation of dimensionless stiffness $kL/4GJ$ with $12EI_{s} / GJL^2 = k_{uw} / k_w$. Low values of $12EI_{s} / GJL^2$ correspond to long slender girders for which uniform torsion is dominant, while high values correspond to short stocky girders for which warping torsion is dominant. The approximation using $k_{uw}$ is conservative and close to the accurate solution using $k_z$. Also shown in Fig. 4 are the approximations using $k_u$ and $k_w$. It can be seen that these are conservative, and generally very much so. The warping approximation using $k_w$ corresponds to what is sometimes called “the twin-beam analogy”, for which it is assumed that the torque $M_z$ is solely resisted by the warping torque through flange shears $M_w/hs$, instead of by the sum of the warping and uniform torques $M_w$ and $M_u$.

These results are different to those which follow from an analysis in [7]. The difference arises because of an incorrect assumption that the line of action of the top flange shear passes through its centroid, instead of its shear centre. It is also assumed there that the total torque $M_z$ is solely resisted by the warping torque.

3 TORSION DESIGN

3.1 GENERAL

Only section capacity design is required for pure torsion, because there are no destabilizing second-order effects. The torsion section capacity $M_{sz}$ is assumed to be the lowest value of the torque $M_z$ which satisfies all the uniform and warping torsion capacity requirements.

3.2 UNIFORM TORSION CAPACITY

The significant uniform torsion actions are the uniform torques at the supports

$$M_{uz} = M_{u} \frac{k_u}{k_u + k_w} \quad (17)$$

The nominal uniform torsion first yield capacity is

$$M_{uy} = 0.6f_{y} j_{t} / t_{m} \quad (18)$$

in which $t_{m}$ is the maximum thickness and $0.6f_{y}$ is the shear yield stress used in AS 4100 [1].

For plastic design, the plastic shape factor for uniform torsion is 1.5 [11], and so the nominal uniform plastic capacity is

$$M_{plz} = 0.6f_{y} \sum b t^2 / 2 \quad (19)$$

This is consistent with the AS 4100 use of the plastic section modulus for the moment capacity.

3.3 WARPING TORSION CAPACITY

The significant warping torsion actions are the flange moments
\[ M_f = M_w L / 4 h \] \hspace{1cm} (20)

and the flange shears
\[ V_f = 2M_f / L \] \hspace{1cm} (21)
at midspan.

The nominal warping torsion capacity \( M_{sw} \) is the value of \( M_w \) for which the combination of the moment and shear of the smaller flange satisfies the capacity requirement of AS 4100 [1].

4 ANALYSIS OF COMBINED BENDING AND TORSION

4.1 GENERAL

The analysis used to determine the bending and torsion actions to be used in design need to account for any significant second-order effects. Examples of second-order effects are displayed in the differential equilibrium equations

\[ -EI_x u'' = M_x + \theta M_y - u'M_z \] \hspace{1cm} (22)

\[ EI_y u'' = M_y - \theta M_x - v'M_z \] \hspace{1cm} (23)

\[ GJ\theta' - EI_y \theta'' = M_z + u'M_x - v'M_y \] \hspace{1cm} (24)
in which \( I_x, I_y \) are the second moments of area, \( u \) and \( v \) are the displacements, and \( M_x, M_y, M_z \) are the moments and torque.

In these equations the second-order effects caused by the terms \( u'M_z \) and \( \theta M_x \) are associated with the flexural-torsional buckling of beams bent about the major axis [12, 13]. In biaxial bending there are additional second-order effects because the bending rotations \( u' \) and \( v' \) induce secondary torques \( v'M_y \) and \( u'M_x \). In biaxial bending and torsion, there are even more terms which induce additional secondary moments \( \theta M_y \) and \( \theta M_x \) acting about the rotated principal axes.

Other non-linearities are caused by yielding. Accurate predictions of inelastic second-order behaviour are difficult to make.

The different methods of elastic analysis are discussed below.

4.2 FIRST-ORDER ANALYSIS

First-order elastic analyses are usually made to determine the design actions for biaxial bending, even though the response of a girder is non-linear. This use implies that the small non-linear terms \( u'M_z \) and \( \theta M_x \) associated with buckling and \( v'M_y \) are accounted for in the interaction equations used in design.

The use of first-order analysis for biaxial bending and torsion omits these terms, and also the term \( \theta M_y \) and the non-linear components \( u'M_x \) and \( v'M_y \) of the torque. These additional non-linearities need to be accounted for in the design interaction equations if first-order analysis is to be used.

4.3 SECOND-ORDER ANALYSIS

While the non-linear response might be analysed by using a second-order elastic analysis, it is difficult to use the results of this in a limit states design environment without duplication of some effects. This is because in the design for biaxial bending, the important non-linear terms \( \theta M_y \) and \( u'M_x \) are already accounted for in the allowance for lateral buckling under major axis bending. A second duplication would occur for the interaction between lateral buckling and torsion [10] if an additional term for torsion is used in the interaction equations. These duplications might be avoided by using a second-order analysis of the girder under minor axis bending and torsion alone.
4.4 ADVANCED STRUCTURAL ANALYSIS

An advanced analysis method of design is permitted in AS 4100 [1]. This method is limited to the in-plane design of plane frames for which local and lateral buckling is prevented. The analysis is required to take into account the material properties, residual stresses, geometrical imperfections and second-order effects. The design is satisfactory if the structure can reach equilibrium under the factored loads.

Suggestions have been made for the development of second-order inelastic methods of analysis using line members which can qualify as such a method of advanced analysis [14, 15]. Suggestions have also been made as to how to extend this method to plane frames which may buckle out of the plane of loading [16]. Such methods do not consider primary torsion and so are not relevant to the analysis of bending and torsion.

Second-order finite element programs are available for the inelastic analysis of 3D frames. These can be used for the analysis of bending and torsion, but they need to be augmented by making provisions for residual stresses and geometrical imperfections. A possible method of doing this is to extend the proposals of [15] to use small self-equilibrating load sets, but such a method has not yet been developed.

5 DESIGN FOR COMBINED BENDING AND TORSION

5.1 INTERACTION DESIGN METHOD

Limit states design codes [1, 3, 4] allow for the interactions between axial force and biaxial bending by using equations of the type

\[ \left( \frac{N}{\phi M} \right)^{\gamma_x} + \left( \frac{M_x}{\phi M} \right)^{\gamma_x} + \left( \frac{M_y}{\phi M} \right)^{\gamma_y} \leq 1 \]  

(25)

in which \( M^* \) is the design action and \( \phi M \) is the design capacity. These equations are intended to allow for interactions caused by second-order effects and yielding. The indices \( \gamma \) are selected using available research, intuition and caution. Thus, indices equal to unity generally represent caution, while values greater than 1 are less cautious.

It is proposed that the interaction between torsion and biaxial bending should be allowed for by adding an additional term \( \frac{M_z}{\phi M} \) to the interaction equations for biaxial bending, in which \( M_z \) is the nominal torsion section capacity of Section 3 and \( M^* \) is the design torque determined by analysing the girder for the limit states factored loads. It has been shown that the additional second-order effects that occur in the major axis bending and torsion [9] are allowed for by the use of such a term.

The method of design to be used to determine the design actions to be used in the extended interaction equations is discussed below, and then the development of a design method is illustrated by using the limit states equations of the Australian code AS 4100 [1].

5.2 ANALYSIS FOR INTERACTION DESIGN

None of the analysis methods discussed in Section 4 are suitable for designing against bending and torsion without modification because of the second-order effects omitted or duplicated. The duplications of second-order analysis might be avoided by analysing the girder for minor axis bending and torsion alone.

However, a simpler method is to use first-order analysis and ignore the non-linear terms \( \nu M_x, \theta M_x, \) and \( \nu M_z \) which are very small or small in crane runway girders. The remaining significant non-linearity not then included is the term \( \theta M_x \) which adds to the minor axis bending moment \( M_x \). This may be approximated by increasing the calculated design minor axis moment \( M_{x\text{min}}^* \) to \( M_{x\text{min}}^* + \theta M_x^* \).

5.3 DESIGN DETAILS

5.3.1 Girder section and member capacities

It is proposed that the girder section capacity should be checked by using
and the girder member capacity checked by using

\[ \left( \frac{M_{x}}{\partial M_{xx} \partial x} \right)^{14} + \left( \frac{N_{y}}{\partial N_{yy} \partial y} \right)^{14} + \frac{N_{z}}{\partial N_{zz} \partial z} \leq 1 \] (27)

### 5.3.2 Girder web capacity

The girder web capacity should be checked for the interaction between uniform torsion and bending shear near the supports. Mono-symmetry causes the distribution of bending shear stress in the web to be non-linear, with the maximum stress at the centroid. Because of this and because the average uniform torsion shear stress is zero across the web thickness, it can therefore be expected that there will be only a small interaction. For stocky webs, the shear interaction may be allowed for using

\[ \frac{V_{y}}{\partial V_{y}} \leq 1 \] (28)

in which \( V_{y} \) is the shear yield capacity of AS 4100, or more cautiously by using

\[ \frac{V_{y}}{\partial V_{y}} + \frac{M_{y}}{\partial M_{yz}} \leq 1 \] (29)

For slender webs, the bending shear stresses are smaller, and so any interaction might be ignored.

The uniform torque near midspan is small, and so the girder web there need only be checked for bending and shear using AS 4100.

### 5.3.3 Top flange section capacity

The top flange of the girder is subjected to increased bending moment and shear caused by the twist rotation, as well as an axial compression arising from the major axis moment. While the top flange web capacity may be checked for combined bending and shear using AS 4100, it is unlikely that this check will govern the design. The top flange section capacity should be checked for axial compression and bending using AS 4100.

### 5.4 SUMMARY OF THE DESIGN METHOD

In practice, the designer of a crane runway girder needs to investigate a number of different situations in order to allow for the different load combinations [5, 6] that may act as well as for any trolley movement. For each situation, the method of using limit states design to check the girder’s capacity should include the following steps.

(a) Make first-order analyses of the major and minor axis bending to determine the distributions of bending moments and shears. These allow the design actions \( M_{x}^{*}, M_{y}^{*}, V_{y}^{*}, \) and \( V_{x}^{*} \) to be determined for use in the girder section and member interaction equations.

(b) Determine the midspan design torque \( M_{y}^{*} \) on the idealised girder using

\[ M_{y}^{*} = \Sigma (N_{x}^{*} e_{x} + N_{y}^{*} e_{y}) \] (30)

(c) Determine the mid-span twist rotation \( \theta (\theta_{b} or \theta_{a}) \), and the design end torque \( M_{y}^{*} \) and the midspan design bottom flange moment \( M_{f}^{*} \) and shear \( V_{f}^{*} \). (These will be conservative, because the idealised torque distribution is more severe than the actual distribution).

(d) Use \( M_{y}^{*}, M_{f}^{*}, \) and \( V_{f}^{*} \) to determine the design torsion section capacity \( \phi M_{sz} \).

(e) Increase the calculated minor axis moment to \( M_{y}^{*} = M_{y}^{*} + \theta M_{y}^{*} \).
(f) Check the girder section and member interaction equations.

(g) If these are both satisfied, check the girder web and top flange capacities. It is probable that these will be satisfied.

6 WORKED EXAMPLE

The data for a worked example of the analysis and design of a crane runway girder are shown in Table 1. The units used in this and the following tables are Newtons (N) and millimetres (mm), and combinations. The results of the analysis and AS 4100 [1] design of the girder are summarised in Tables 2 – 8.

The section properties of the girder and its compound top flange shown in Table 2 were calculated using the program THIN-WALL [17].

The central twist rotation of $\theta = 0.0301$ rad shown in Table 3 is close to the exact value of 0.0283, and much closer than the value of 0.0451 calculated by assuming that the total torque is only resisted by the warping torque. Even though the twist rotation is small, it has a significant effect on the minor axis moment shown in Table 4, increasing it from $M_{y*} = 15.0 \text{ e6 Nmm}$ to $M_{y*} = 43.2 \text{ e6 Nmm}$.

The uniform torque $M_{u*} = 2.78 \text{ e6 Nmm}$ shown in Table 3 is a significant portion of the total torque $M_{z*} = 8.35 \text{ e6 Nmm}$, and the torsion section capacity $M_{sz}$ shown in Table 6 is controlled by the uniform torsion capacity $M_{sz}$ at the support.

The elastic buckling moment $M_{ob}$ shown in Table 5 was calculated using the approximation

$$M_{ob} = \alpha_m M_{z2}\sqrt{1 + \frac{\gamma}{\beta^2}}$$  \hspace{1cm} (31)

in which

$$M_{z2} = \sqrt{\frac{\pi^2 E I_v}{\beta^2} \left(\frac{\alpha + \frac{E I_w}{\beta^2}}{\beta^2}\right)}$$  \hspace{1cm} (32)

is the elastic buckling moment of a simply supported beam in uniform bending [12, 13], $\alpha_m = 1.35$ is the moment modification factor for central concentrated load [1, 11], and

$$\gamma = \frac{[\alpha_m P_2(\beta^2/2) + 0.91 y N_s^2]}{2 M_{sz}}$$  \hspace{1cm} (33)

in which

$$P_2 = \frac{\pi^2 E I_v}{\beta^2}$$  \hspace{1cm} (34)

$\beta$ is the mono-symmetry section constant [11, 13], and $y N_s$ is the distance below the shear centre at which the concentrated load acts. The value of $M_{ob} = 1.84 \text{ e9 Nmm}$ given in Table 5 is close to the value of $1.82 \text{ e9 Nmm}$ calculated by using the finite element computer program PRFELB [18, 19]. The lateral buckling design capacity $M_{sz} = 1.60 \text{ e9 Nmm}$ shown in Table 5 is significantly less than the major axis section capacity $M_{sz} = 5.90 \text{ e9 Nmm}$ shown in Table 6.

The girder section design ratios shown in Table 7 are not large, and the girder section capacity check of 0.385 easily passes, as shown in Table 8, despite the significant increase in $M_{y*}$. The girder web and top flange section capacity checks are passed even more easily. The controlling check is the girder member check value of 0.717, which is dominated by the value of 0.653 for the major axis bending ratio associated with lateral buckling. The contribution of 0.145 for torsion is small, and that of 0.064 for minor axis bending is even smaller.
7 CONCLUSIONS

Steel crane runway girders are subjected to torsion by their eccentric loads. The twist rotation of the principal axes caused by torsion induces additional bending moments, and reduces the resistance to lateral buckling. There is little guidance on how to design for torsion, and design procedures are often intuitive and based on adapting existing methods of designing against biaxial bending, with varying degrees of rationality and precision.

This paper proposes a simple extension of existing methods of designing for biaxial bending, in which a torsion term is added to the interaction equations. The additional term is the ratio of the design torque to the design section resistance to non-uniform torsion. Proposals are made for determining this section resistance.

Research [9] has shown that this addition can allow for the reduction in the design lateral buckling resistance caused by the twist rotations. The additional bending moments caused by the twist rotations might be predicted by using a second-order elastic analysis, but there may be duplications when the results of this are used in the interaction equations.

Instead, a simpler method is to use a linear elastic analysis to determine the twist rotations and the corresponding moment increases. A linear elastic analysis of an idealised girder is made which is conservative and of good accuracy. It corrects errors made in some very conservative “twin-beam” analyses.

An example of the proposed method is developed for the Australian limit states design code AS4100 [1], and is summarised and illustrated by a design example. The additional torsion term is small, and while the small twist rotations appear to increase the minor axis moments significantly, they are not enough to have a major effect on the design strength, which is dominated by the resistance to lateral buckling. The effect of uniform torsion on the web shear capacity is small, and strengths of the flanges are more than adequate.
8 REFERENCES


9 NOTATION

9.1 Principal Notation

\( M, N, V \) = Moment, force, or shear

9.2 Subscripts and Superscript

- \( b, t \) = bottom or top flange
- \( p, y \) = plastic or yield
- \( s \) = section capacity
- \( u, w \) = uniform or warping torsion
- \( x, y, z \) = beam axes
- \( * \) = design action

9.3 Additional Notation

- \( A \) = area
- \( a^2 \) = \( EIw / GJ \)
- \( b \) = width of rectangular element
- \( E \) = Young's modulus of elasticity
- \( e \) = load eccentricity
- \( f_y \) = yield stress
- \( G \) = shear modulus of elasticity
- \( h \) = distance between flange shear centres
- \( I_w \) = warping section constant
- \( J \) = uniform torsion section constant
- \( k \) = stiffness
- \( L \) = span length
- \( P_y \) = \( \pi^2 EI_y / L^2 \)
- \( s \) = flange shear centre distance to girder shear centre
- \( t \) = thickness of rectangular element
- \( t_m \) = maximum thickness
- \( u, v \) = displacements in the \( x, y \) directions
- \( y_{Ns} \) = distance below the shear centre at which the load acts
- \( y_t \) = distance between top flange and girder centroids
- \( Z \) = section modulus
- \( \alpha_m \) = moment modification factor
- \( \alpha_s \) = slenderness reduction factor
- \( \beta_b \) = mono-symmetry section constant
- \( \gamma \) = index, or factor for load height and mono-symmetry
- \( \lambda_w \) = web slenderness
- \( \theta_a, \theta_e \) = approximate and exact midspan twist rotations
- \( \phi \) = capacity factor
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| $M_{x*}$ | 8.35e6 | $M_{x*}$ | 0.938e9 | $\alpha_m$ | 1.35 |
| $k_w$ | 92.4e6 | $M_{w*}$ | 15.0e6 | $y_{0m}$ | -650 |
| $k_w$ | 185e6 | $M_{y*}$ | 43.2e6 | $P_y$ | 3.64e6 |
| $\theta$ | 0.0301 | $N_{x*}$ | 0.690e6 | $M_{ux}$ | 1.83e9 |
| $M_{x*}$ | 2.78e6 | $V_{y*}$ | 0.125e6 | $\gamma$ | -0.299 |
| $h$ | 1257 | | | $M_{ob}$ | 1.84e9 |
| $M_{y*}$ | 5.57e6 | | | $M_{ov}$ | 1.36e9 |
| $h$ | 1257 | | | $\alpha_e$ | 0.200 |
| $M_{x*}$ | 13.2e6 | | | $M_{ov}$ | 1.60e9 |

<table>
<thead>
<tr>
<th>Table 6. Section Capacities</th>
<th>Table 7. Design Ratios</th>
<th>Table 8. Design Checks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_w$</td>
<td>82.2</td>
<td>$V_{sw}$</td>
</tr>
<tr>
<td>$y_{0w}$</td>
<td>860</td>
<td>$V_{ym}$</td>
</tr>
<tr>
<td>$M_{sw}$</td>
<td>5.91e9</td>
<td>$N_{sw}$</td>
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<tr>
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<td>0.752e9</td>
<td>$M_{sw}$</td>
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<td>$M_{wy}$</td>
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<td>$M_{sw}$</td>
<td>64.1e6</td>
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<td></td>
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</tr>
</tbody>
</table>

| Girder section capacity | 0.385 | < 1 | Pass |
| Girder member capacity | 0.717 | < 1 | Pass |
| Girder web section capacity | 0.050 | < 1 | Pass |
| Top flange section capacity | 0.207 | < 1 | Pass |
The equations for section and loads, shear centre actions, and flange forces are given as follows:

\[
N_y = \frac{(N_x S_b + M_w)}{h}
\]

\[
M_z = M_w + M_u
\]

\[
M_u = \frac{(N_x S_b - M_w)}{h}
\]

Fig. 2 Girder Section and Actions
Fig. 3 Section Dimensions

Fig. 4 Torsional Stiffness Approximations