POST-BUCKLING STRENGTH OF STEEL TEE COLUMNS

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ABSTRACT

This paper examines the effects of torsional post-buckling on the strengths of tee section steel columns which fail in local, torsional or flexural modes. Flexural-torsional buckling is not considered, but the post-buckling movement of the effective centroid and the ensuing bending effects are.

It is shown that the elastic torsional buckling loads of tee section columns are greater than or equal to their local buckling loads, while their torsional and local post-buckling behaviours are very similar. Design codes which require separate account to be taken of torsional and local buckling duplicate their allowances for these effects and ignore the torsional post-buckling strength.

It is found that the effective section method used in most codes to allow for local buckling and post-buckling is probably conservative, and that allowances for the movement of the centroid are over-conservative.

It is suggested that close design approximations for the strengths of tee section columns can be obtained by ignoring the effects of movement of the centroid and torsional buckling, and instead using the lower of the axial compression section resistance determined for the effective section and the member resistance calculated by ignoring local buckling.

KEYWORDS

Buckling, Columns, Design, Flexure, Post-buckling, Steel, Tee sections, Torsion, Yield

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1 INTRODUCTION

While past codes for the design of hot-rolled steel structures have required columns to be designed against local and flexural buckling, many current codes [1, 2, 3] also require the possibility of flexural-torsional (including torsional) buckling to be considered. In the case of cruciform columns, this [1] has led to the use of unnecessarily low design capacities [4, 5]. While elastic local and flexural-torsional buckling are well understood [6, 7, 8], it appears that the effects of torsional buckling and post-buckling on the strengths of tee section columns are not.

A recent paper [9] has provided a detailed assessment of the strengths of steel tee columns. The effects of local and flexural-torsional buckling were considered, including post-buckling, yielding, residual stresses and geometrical imperfections. The results of strength assessments made using the finite element program ABAQUS [10] were compared with the predictions of design codes [1, 2, 3] and used to develop proposals for improving these.

The purpose of this paper is to examine more closely the effects of torsional post-buckling on the strengths of tee section columns which fail in local, torsional or flexural modes. Flexural-torsional buckling is not considered, but the movement of the effective centroid and the ensuing bending effects caused by post-buckling are.

2 STEEL TEE COLUMNS

The tee section shown in Fig. 1a has flange and web outstand widths b1 and b2 and thicknesses t1 and t2, respectively. The tee column shown in Fig. 1b is simply supported so that the end shear centre deflections u, v and twist rotations ϕ about the shear centre are prevented, but the end rotations u', v' and warping displacements proportional to ϕ' are unrestrained (in which ' ≡ d/dz indicates the rate of change with respect to the distance z along the column). All shear centre deflections u parallel to the x principal axis are prevented.

The column loads N act along the undisplaced centroidal axis and are transmitted to the column ends through rigid end platens.

The Young's modulus is E = 200,000 N/mm^2, the Poisson's ratio is ν = 0.3, and the yield stress is f_y = 235 N/mm^2.

3 ELASTIC BUCKLING

3.1 FLEXURAL BUCKLING

The elastic stress f_{ox} at which a simply supported tee column of length L buckles flexurally by deflecting v in the y principal axis direction (Figs 1b, 2a) may be obtained from [7, 11]

$$A f_{ox} = N_{ox} = \frac{\pi^2 E I_x}{L^2} \tag{1}$$

in which N_{ox} is the flexural buckling load, A is the cross sectional area, and I_x is the second moment of area about the x axis. Flexural buckling parallel to the x principal axis (Fig. 2b) is prevented by the restraints against all shear centre deflections u.

3.2 TORSIONAL BUCKLING

The elastic stress f_{oz} at which a simply supported column of length L buckles torsionally by rotating ϕ about the shear centre (Fig. 2c) may be obtained from [7, 8]

$$A f_{oz} = N_{oz} = \frac{GJ + \pi^2 EI_w / L^2}{\left(r_o^2 + y_o^2\right)} \tag{2}$$

in which N_{oz} is the torsional buckling load, G is the shear modulus of elasticity given by

$$G = \frac{E}{2(1+\nu)} \tag{3}$$

J and I_w are the torsional and warping section constants, y_o is the shear centre coordinate, and

$$r_o^2 = \frac{I_x + I_y}{A} \tag{4}$$

The warping section constant I_w for a tee section is very low [8], and is commonly ignored.

3.3 LOCAL BUCKLING

A tee column may buckle locally as a series of half-waves along the length of the column. Each half-wave buckle appears similar to a torsional buckle over the length of the half-wave, in that the buckled shape corresponds approximately to a rigid body rotation of the cross-section about the shear centre, as shown in Fig. 2d, e, f.

The variation of the elastic buckling stress f_o with the half-wave length L_{hw} of a tee section (Fig. 2d with b_2/b_1 = 225/150, t_2/t_1 = 5/10) has been determined using the computer program THINWALL [12] and is shown in Fig. 3 by the curve labelled 1. The buckling stress f_o deceases as the half-wave length L_{hw} increases from zero until a minimum of $f_o \cong 108$ N/mm^2 is reached at the short wavelength of $L_{hw} \cong 400$ mm. The corresponding cross section buckling mode (Fig. 2d) is primarily one of local buckling of the slender web with partial restraint from the comparatively stocky flanges. The value of this local minimum buckling stress may be expressed in the general form of [7]

$$f_{ol} = \frac{N_{ol}}{A} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \frac{k}{(b/t)^2} \tag{5}$$

in which N_{ol} is the local buckling load, b and t are the width and thickness of a rectangular element of the tee section, and k is the local buckling factor. For the tee section of Fig. 3,

$$k_2 \cong 1.21 \tag{6}$$

when b_2 and t_2 are used for b and t.

The value of f_o then rises and approaches asymptotically towards to a maximum of $f_o \cong 202$ N/mm^2 approximately, corresponding to a buckling mode similar to the torsional mode shown in Fig. 2c. At a length of $L_{hw} \cong 6000$ mm the mode changes to the flexural mode shown in Fig. 2a.

The variations of the local buckling stresses f_{ol} with the length L of a simply supported tee column are also shown in Fig. 3. For lengths L up to 8000 mm approximately, a series of local buckles of half-wave length $L_{hw} \cong 400$ mm form with a minimum stress of $f_{ol} \cong 108$ N/mm^2. For greater lengths, the buckling mode is flexural, and the buckling stress f_{ox} decreases rapidly as L increases.

The variations (shown by the solid curves) of the minimum local buckling factors $k_{1,2}$ of Equation 5 with b_2/b_1 and t_2/t_1 are shown in Fig. 4. In many cases, there is no minimum local buckling load, and the buckling mode is similar to the torsional buckling mode shown in Fig. 2c. The minimum value of k = 0.4255 occurs when the web and flange outstands are of equal slenderness, so that $(b_2/t_2)/(b_1/t_1)$ = 1.0. The values of k_2 are close to the minimum for $t_2/t_1 \geq 2$, while the values of k_1 are close to the minimum for $t_2/t_1 \leq 0.5$. The values of k_1 and k_2 generally increase as the relative slenderness $(b_2/t_2)/(b_1/t_1)$ diverges from 1.

When k is approximated by the minimum value of 0.4255, then the dimensionless local buckling load may be expressed as

$$N_{ol} / N_y = 1 / \lambda_{ol}^2 \tag{7}$$

in which

$$N_y = A f_y \tag{8}$$

in which the local buckling modified slenderness is

$$\lambda_{ol} = \sqrt{\frac{N_y}{N_{ol}}} = \frac{b/t}{18.09}\sqrt{\frac{f_y}{235}} \tag{9}$$

The variation of the dimensionless local buckling load N_{ol}/N_y with the local buckling slenderness λ_{ol} is shown in Fig. 5.

The local buckling of a tee section may be analysed when the buckled shape is known. In the case of b_2/b_1 = 300/150, t_2/t_1 = 7.5/10, and L = 810mm, the buckled shape (similar to Fig. 2d) obtained from THINWALL may be approximated by

$$\frac{\bar{u}_2}{\sin \pi z / L} = 0.5851\left(\frac{y+y_o}{b_2}\right) + 0.9376\left(\frac{y+y_o}{b_2}\right)^2 - 0.7787\left(\frac{y+y_o}{b_2}\right)^3 + 0.256\left(\frac{y+y_o}{b_2}\right)^4 \tag{10}$$

$$\frac{\bar{v}_1}{\sin \pi z / L} = 0.2937\left(\frac{x}{b_1}\right) - 0.0419\left(\frac{x}{b_1}\right)^2 + 0.01067\left(\frac{x}{b_1}\right)^3 + 0.0043\left(\frac{x}{b_1}\right)^4$$

in which $\overline{u_2}$ and $\overline{v_1}$ are the dimensionless displacements $\bar{u}$ for the web and half flange, respectively.

The local buckling of tee sections is analysed in Appendix A. When the buckled shape of Equation 10 is used, the buckling stress of the tee section with b_2/b_1 = 300/150, t_2/t_1 = 7.5/10, and L = 810mm is determined as 113.0 N/mm^2, which is very close to the THINWALL value of 112.7 N/mm^2.

3.4 COMPARISON OF TORSIONAL AND LOCAL BUCKLING

Values of k determined from the torsional buckling loads of Equation 2 for long tee columns are shown by the dashed curves in Fig. 4, and are compared with those for local buckling. The torsional values of k are greater than or equal to the local buckling values. They are greater for tees for which 0.55 < $(b_2/t_2)/(b_1/t_1)$ < 1.8 approximately.

This suggests that the sometimes used design practice [3] of ignoring torsional buckling and allowing instead for local buckling is not in error, even when local buckling design [3] is based on the higher values of k (than the minimum value of k = 0.4255) shown in Fig. 4.

4 POST-BUCKLING BEHAVIOUR

4.1 LOCAL POST-BUCKLING

After local buckling of a tee column, finite deflections occur which cause the previously uniform axial stresses to redistribute away from the large deflection regions (near the plate free ends) towards the small deflection regions (near the shear centre). This redistribution increases the resistance to local buckling, so that there is an increase in the strength.

The local post-buckling of a tee column is analysed in Appendix B. The behaviour of a tee section with b_2/b_1 = 300/150, t_2/t_1 = 7.5/10, L = 810mm, and f_y = 235 N/mm^2 is shown in Fig. 6. After local buckling at N_{ol}/N_y = 0.481, the dimensionless buckling load (obtained using Equation B10) increases parabolically as the deflection magnitude δ increases, while the dimensionless first yield capacity (obtained using Equation B11) decreases, until the dimensionless first yield post-buckling load N_{pl}/N_y= 0.862 (obtained using Equation B12) is reached.

The variation of the dimensionless first yield local post-buckling loads N_{pl}/N_y of tee sections (with b_2/b_1 = 2, t_2/t_1 = 0.75) with their local buckling slendernesses λ_{ol} are shown in Fig. 5. It can be seen that these are significantly higher than the corresponding dimensionless local buckling loads N_{ol}/N_y.

Also shown in Fig. 5 is an approximation for the calculated local post-buckling loads based on the effective section concept, in which plate elements whose widths b exceed the effective width b_e given by

$$b_e = 18.09\ t / \sqrt{(235 / f_y)} \tag{11}$$

are replaced by the effective width, and the post-buckling load is taken as that which causes general yield of the reduced section. This approximation is somewhat lower than the calculated values of N_{pl}/N_y, suggesting that the effective section method is conservative.

Also shown in Fig. 5 is an effective section approximation for the real local post-buckling strengths based an effective width b_e given by

$$b_e = 15\ t / \sqrt{(235 / f_y)} \tag{12}$$

which is consistent with the allowances made in [1] for the effects of residual stresses and plate out-of-flatness in real sections. It is likely that this is just as conservative with respect to the real strengths as is the approximation based on Equation 11 for the calculated post-buckling loads.

4.2 TORSIONAL POST-BUCKLING

4.2.1 Post-Buckling Behaviour

After torsional buckling of a tee column, the previously uniform axial stresses also redistribute, and form a moment M acting about the x principal axis. The effect of this post-torsional buckling moment depends on whether shear centre deflections v in the y direction are prevented or allowed.

When the shear centre deflections are prevented as shown in Fig. 7a, then end rotations of the rigid end platens through which the axial loads act are prevented, and there are no bending moments acting in the column. The moment caused by the redistribution is negated by the effects of end moments which prevent the platens from rotating, as shown in Fig. 7a.

When the shear centre deflections are allowed (except at the ends), then the moment M causes the column to bend about the x axis and deflect v in the yz plane, as shown in Fig. 7b.

4.2.2 Shear Centre Deflections Prevented

The torsional post-buckling of a tee column whose shear centre deflections v are prevented is analysed in Appendix C1. The behaviour when b_2/b_1 = 300/150, t_2/t_1 = 7.5/10, L = 810mm, and f_y = 235 N/mm^2 is shown in Fig. 6. After torsional buckling at N_{oz}/N_y = 0.596, the dimensionless buckling load increases parabolically as the deflection magnitude $\delta = \phi_c\, b_2$ (where ϕ_c is the central magnitude of the twist rotation ϕ) increases, while the dimensionless first yield capacity decreases, until the dimensionless first yield post-buckling load N_{pz}/N_y = 0.842 is reached. This is very close to the value of 0.862 for local post-buckling, and the behaviour is similar to that for local post-buckling, also shown in Fig. 6.

The variation of the dimensionless first yield torsional post-buckling loads N_{pz}/N_y of tee columns (with b_2/b_1 = 2, t_2/t_1 = 0.75 and L = 810 mm) with their local buckling slendernesses λ_{ol} are shown in Fig. 5. It can be seen that these are close to the corresponding dimensionless local post-buckling loads N_{pl}/N_y.

The variations of the dimensionless first yield torsional post-buckling loads N_{pz}/N_y of long tee columns of three different cross-sections with their torsional buckling slendernesses

$$\lambda_{ozu} = \sqrt{N_y / N_{ozu}} \tag{13}$$

(in which N_{ozu} is the value of N_{oz} obtained from Equation 2 with I_w = 0) are compared in Fig. 8 with approximations given by

$$\frac{N_{pz}}{N_y} = 1 - \alpha \left(1 - \frac{1}{\lambda_{ozu}^2}\right) \tag{14}$$

in which the variations of the constant α with b_2/b_1 and t_2/t_1 are shown in Fig. 9. It can be seen from Fig. 9 that 0.3 < α < 0.56 for a wide range of practical tee-sections. The comparisons of Fig. 8 demonstrate that Equation 14 is of high accuracy, and suggest that simple and conservative approximations can be obtained by using α = 0.56.

4.2.3 Shear Centre Deflections Allowed

The bending of a crooked tee column whose shear centre deflections v in the y direction are allowed is analysed in Appendix C2, and expressions obtained for the maximum stresses. It is assumed that the post-buckling load is reached when the maximum stress is equal to the yield stress f_y.

The variations with the modified flexural slenderness

$$\lambda_{ox} = \sqrt{N_y / N_{ox}} \tag{15}$$

of the dimensionless first yield loads N_p/N_y of tee columns with $b_1/t_1 = b_2/t_2$ = 300/10, f_y = 235 N/mm^2, and initial crookednesses defined by v_{ic} = (–0.001, 0, +0.001)L are shown in Fig. 10. For this section, the plate elements have equal slenderness, and so the elastic local and torsional buckling loads are the same, as are the first yield post-buckling loads. Also shown in Fig. 10 are the dimensionless elastic flexural buckling loads N_{ox}/N_y, the local buckling loads N_{ol}/N_y, the first yield local post-buckling loads N_{pl}/N_y of straight columns, and the nominal first yield loads N_{yx}/N_y of columns with v_{ic} = –0.001L (which ignore any local or torsional buckling effects).

The dimensionless first yield loads N_{pl0}/N_y of columns with v_{ic} = 0 start to deviate from the dimensionless elastic flexural buckling loads N_{ox}/N_y at λ_{ox} = 1.65 where the flexural and local buckling loads are equal. The dimensionless first yield loads then increase steadily as λ_{ox} decreases towards 0.92. They then gradually approach the dimensionless first yield post-buckling loads N_{pl}/N_y from above. The discontinuity at λ_{ox} = 0.92 occurs because first yield for columns with higher values of λ_{ox} occurs at the web tip because of the dominating effect of the amplification of the additional bending induced by the moments M, but at the shear centre for lower values because of the domination of the stresses directly associated with the moments M.

The dimensionless first yield loads N_{plv}/N_y of columns with v_{ic} = –0.001L start to deviate from the nominal first yield loads at λ_{ox} = 1.45 where the nominal first yield and local buckling loads are equal. The dimensionless first yield loads then increase steadily as λ_{ox} decreases towards 0.88. At lower values of λ_{ox}, the dimensionless first yield load is lower for columns with v_{ic} = +0.001L, and so it is these loads which are shown. The dimensionless first yield loads then gradually approach the dimensionless first yield post-buckling loads N_{pl}/N_y from above.

5 EC3 DESIGN

5.1 GENERAL

Code rules for designing hot-rolled and welded tee columns generally make allowances for yielding, local buckling, elastic member buckling, and imperfections (initial crookedness and residual stresses). The rules given in codes such as [1, 2, 3] are somewhat similar, and so only those of the EC3 [1] are discussed in this paper.

Two types of design check are required, one for the section resistance and one for the member resistance [11]. For the section resistance, the yield resistance is reduced to allow for local buckling by determining reduced sectional areas for all slender elements. Local buckling effects in other than doubly symmetric sections may cause an apparent shift of the effective centroid, leading to moments whose effects must also be allowed for.

For the member check, further reductions are made to allow for elastic member buckling and imperfections, often by using a first yield failure criterion based on an equivalent initial crookedness [11]. Elastic member buckling is usually taken as flexural buckling in the more flexible principal plane, but torsional buckling or flexural-torsional buckling [7, 8] must be allowed for in columns (such as tee columns) which are susceptible to these types of buckling.

The application of the design rules of the EC3 [1] are demonstrated in the following sub-sections for tee columns with $b_1/t_1 = b_2/t_2$ = 300/10 and f_y = 235 N/mm^2.

5.2 SECTION CHECK

For this section, the effective width (based on Equation 12) of each element is half its actual length, so that the effective area is $A/2$ and the shift of the centroid is $-y_o/2$ = 25 mm. The reduced area causes a reduction of the EC3 dimensionless design section resistance to axial force N_{ds}/N_y from 1.0 to 0.5, as shown in Fig. 11, while the shift of the centroid causes a further reduction to 0.375.

5.3 MEMBER CHECK

For columns whose section properties are reduced by local buckling effects, the EC3 design resistances are reduced in three stages. The first is for the reduced section properties, the second stage is for the moments induced by the shift of the centroid, and the third is for the effects of torsional buckling. The dimensionless design resistances N_{de}/N_y, N_{dem}/N_y, and N_{demt}/N_y, corresponding to these three stages are shown in Fig. 11.

Also shown in Fig. 11 are the variations of the dimensionless design resistances N_{d0}/N_y of columns without local or torsional buckling effects. These latter resistances decrease with the modified flexural slenderness λ_{ox} from 1.0 at $\lambda_{ox} \approx 0.2$ and asymptote towards the elastic buckling resistance N_{ox}/N_y.

5.4 DISCUSSION

The EC3 [1] dimensionless design resistances N_{de}/N_y, N_{dem}/N_y, and N_{demt}/N_y are compared in Fig. 11 with the dimensionless first yield loads N_{plv}/N_y shown in Fig. 10 for tee columns with $v_{ic} = \pm 0.001L$. For short length columns, the first yield loads are probably optimistic, because they omit the local effects of residual stresses and out-of-flatness which reduce the local buckling strengths. These effects are included in the EC3 by the use of the effective section method which reduces the section resistance of this section to $N_{ds} = 0.5N_y$. As noted in Section 4.1, this reduction is likely to be somewhat conservative.

A further reduction in the design section resistance for the effect on movement of the centroid is shown in Fig. 12 by the significant reduction from $0.5N_y$ to $N_{ds} = 0.375N_y$. The real effect is much smaller than this, as can be seen from Fig. 10 for columns of moderate flexural slenderness, and virtually non-existent for low slenderness.

Yet another reduction in the design section resistance for the effect of torsional buckling is shown in Fig. 11 by the substantial reduction from $0.375N_y$ to $N_{ds} = 0.195N_y$. In view of the close correspondence between local and torsional buckling and post-buckling shown in Figs 4-6, this amounts to making the same reduction twice for local and torsional buckling, while ignoring the torsional post–buckling strength.

The comparisons of Fig. 11 suggest that close design approximations for tee columns which are prevented from buckling laterally can be obtained by ignoring the effects of movement of the centroid and torsional buckling, and instead using the lower of the axial compression section resistance determined for the effective section and the member resistance calculated by ignoring local buckling. This is somewhat similar to AS 4100 [3] except that the member resistance does not ignore local buckling. The AS 4100 design resistances are also shown in Fig. 11. It can be seen that these are similar to the values of N_{de} /N_y for EC3, but are still significantly lower that the values of N_{plv} /N_y.

6 CONCLUSIONS

This paper has examined the effects of torsional post-buckling on the strengths of tee section steel columns which fail in local, torsional or flexural modes. Flexural-torsional buckling was not considered, but the movement of the effective centroid and the ensuing bending effects caused by post-buckling were. The following conclusions may be made.

The elastic torsional buckling loads of tee section columns are greater than or equal to their local buckling loads as shown in Fig. 4, while their torsional and local post-buckling behaviours are very similar, as shown in Figs 5 and 6. In view of these conclusions, there is strong justification for the practice in some design codes [3] of ignoring torsional buckling when local buckling effects are allowed for. Design codes [1, 2] which require separate account to be taken of torsional buckling duplicate their allowances for local buckling and ignore the torsional post-buckling strength, leading to over-conservative designs.

The effective section method used in most codes to allow for local buckling and post-buckling is probably conservative, as suggested in Fig. 5. Possible reasons for this are that the simplistic stress distributions assumed underestimate the axial force resultant of the redistributed (during post-buckling) stress distribution, and that any restraints between plate elements during buckling are ignored.

The additional bending moments caused by the movement of the effective centroid after local buckling cause stresses which oppose those of the post-buckling stress redistribution. Because of this, the code methods [1, 2] of allowing for the movement of the centroid are over-conservative, as shown in Fig. 11.

Close design approximations for the strengths of tee section columns which are prevented from buckling laterally can be obtained by ignoring the effects of movement of the centroid and of torsional buckling, and instead using the lower of the axial compression section resistance determined for the effective section and the member resistance calculated by ignoring local buckling, as shown in Fig. 10.

7 REFERENCES

[1] BSI. Eurocode 3: Design of Steel Structures: Part 1-1: General Rules and Rules for Buildings, BS EN 1993-1-1. British Standards Institution, London, 2005.

[2] AISC. Specification for Structural Steel Buildings. American Institute of Steel Construction, Chicago, 2010.

[3] SA. AS 4100-1998 Steel Structures. Standards Australia, Sydney, 1998.

[4] Trahair, NS. Strength design of cruciform steel columns. Engineering Structures, 34, 2012, pp 307-13.

[5] Trahair, NS. Shear effect on cruciform post-buckling. Engineering Structures, 35, 2012, pp 24-26.

[6] Bulson, PS. The Stability of Flat Plates. Chatto and Windus, 1970.

[7] Timoshenko, SP, and Gere, JM. Theory of Elastic Stability. 2nd ed., McGraw-Hill, New York, 1961.

[8] Trahair, NS. Flexural-Torsional Buckling of Structures. E & FN Spon, London, 1993.

[9] Cardoso, FS and Rasmussen, KJR. The behavior and design of concentrically loaded T-section steel columns. Research Report R933, Department of Civil Engineering, The University of Sydney, 2013.

[10] Simulia. ABAQUS User's Manual, 2011

[11] Trahair, NS, Bradford, MA, Nethercot, DA, and Gardner, L. The Behaviour and Design of Steel Structures to EC3. Taylor and Francis, London, 2008.

8 NOTATION

A	Area of cross section
b	Plate width
b_e	Effective width
$b_{1,2}$	Widths of flange outstand and web
E	Young's modulus of elasticity
f	Stress
$f_{c,m}$	Maximum stresses
f_o	Elastic buckling stress
f_y	Yield stress
G	Shear modulus of elasticity
I_x	Second moment of area about x axis
I_w	Warping section constant
J	Uniform torsion constant
k	Local buckling coefficient
L	Column length
L_{hw}	Buckle half-wave length
M	Moment
M_c	Central oment
N	Axial compression
N_d	Design capacity
$N_{pl,pz}$	Local and torsional post- buckling loads
$N_{ol,oz}$	Local and torsional buckling loads
N_y	Squash load
N_{yx}	Nominal first yield load
s	Distance around plate mid-line
t	Plate thickness
$t_{1,2}$	Thicknesses of flange outstand and web
U	Strain energy
u, v	Displacements in x, y directions
u_p	Transverse plate displacement
$\bar{u}_2, \bar{v}_1$	Dimensionless transverse buckling displacements
V	Work done
v_i	Initial crookedness
v_{ic}	Central initial crookedness
w_f	Displacement due to axial straining
w_o	Deflection of end platen
w_s	Axial shortening
x, y	Principal axis coordinates
y_o	Shear centre coordinate
z	Distance along column
$Z_{xs,xt}$	Section moduli for shear centre and web tip
α, β	Constants
δ	Deflection magnitude
ε	Strain, or
	$= \sqrt{(235/f_y)}$
ϕ	Twist rotation
ϕ_c	Central twist rotation
$\lambda_{ol,oz}$	Modified slendernesses for local and torsional buckling
λ_{ox}	Modified slenderness for flexural buckling
μ	$= \sqrt{(N/EI_x)}$
ν	Poisson's ratio

APPENDIX A – LOCAL BUCKLING

The elastic local buckling of a tee column of length L may be analysed by using [7]

$$U = V \tag{A1}$$

in which the strain energy stored during buckling is given by

$$U = \frac{\delta^2}{2}\int_0^L\int_A \frac{Et^3}{12(1-\nu^2)}\left\{\left(\frac{\partial^2 \bar{u}}{\partial s^2}+\frac{\partial^2 \bar{u}}{\partial z^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 \bar{u}}{\partial s^2}\frac{\partial^2 \bar{u}}{\partial z^2}-\left(\frac{\partial^2 \bar{u}}{\partial s \partial z}\right)^2\right]\right\}ds\,dz \tag{A2}$$

and the work done by the axial compression stresses f during buckling is given by

$$V = \frac{\delta^2}{2}\int_0^L\int_A ft\left(\frac{\partial \bar{u}}{\partial z}\right)^2 ds\,dz \tag{A3}$$

In these equations, $\delta\bar{u}$ is the transverse displacement of a plate element of the member, s is the distance along the mid-section of the element, and z is the direction of the stress f. The stress f which satisfies these equations depends on $\bar{u}$. The true buckled shape $\bar{u}$ is the one for which the stress f is the least.

If the stress f is uniformly distributed over the cross section, then the elastic buckling load N_{ol} can be obtained as

$$N_{ol} = f_{ol}A = \frac{U}{\delta^2 \frac{1}{A}\int_A \bar{w}t\,ds} \tag{A4}$$

in which

$$\delta^2 \bar{w} = \frac{1}{2}\delta^2 \int_0^L \left(\frac{\partial \bar{u}}{\partial z}\right)^2 dz \tag{A5}$$

APPENDIX B – LOCAL POST-BUCKLING

The local post-buckling of a tee section may be analysed approximately by assuming that the post-buckled shape is the same as the local buckled shape $\bar{u}$, so that the post-buckled deflections may be approximated by

$$u_p = \delta \bar{u} \tag{B1}$$

in which δ defines the magnitude of the post-buckled deflections, which are assumed to be proportional to sin $\pi z/L$.

The longitudinal shortening due to u_p is given by

$$\delta^2 \bar{w} = \frac{1}{2}\delta^2 \int_0^L \left(\frac{\partial \bar{u}}{\partial z}\right)^2 dz = \delta^2 \frac{\pi^2}{4L}\left(\frac{\bar{u}}{\sin \pi z/L}\right)^2 \tag{B2}$$

The axial loads are applied through rigid end platens which are prevented from rotating, and transverse deflections u, v of the shear centre are prevented at all points along the member, so that there is no bending. If the rigid end platens deflect w_0 axially, then the axial strains are

$$\varepsilon = (w_0 - \delta^2 \bar{w})/L \tag{B3}$$

and the axial stresses are

$$f = (w_0 - \delta^2 \bar{w})E/L \tag{B4}$$

The total axial force is

$$N = \int_A f dA = (E/L)\left(w_0 A - \delta^2 \int_A \bar{w}t\,ds\right) \tag{B5}$$

so that the stresses are

$$f = \left(\frac{N}{A}\right) - \delta^2 \left(\frac{E}{L}\right)\left\{\overline{w} - \frac{1}{A}\int_A \overline{w}\,tds\right\} \tag{B6}$$

The maximum stress f_m occurs at the shear centre where $\overline{w} = 0$, so that

$$f_m = \left(\frac{N}{A}\right) + \delta^2 \left(\frac{E}{L}\right)\frac{1}{A}\int_A \overline{w}\,tds \tag{B7}$$

It is assumed that at the post-buckling load, the stresses f are sufficient to cause local buckling. Thus, adapting Equations A1 and A3,

$$U = \frac{\delta^2}{2}\int_0^L\int_A ft\left(\frac{\partial \overline{u}}{\partial z}\right)^2 ds\,dz = \delta^2 \int_A f\,\overline{w}\,tds \tag{B8}$$

whence

$$U = \delta^2 \frac{N}{A}\int_A \overline{w}\,tds - \delta^4\left(\frac{E}{L}\right)\int_A \overline{w}\left\{\overline{w} - \frac{1}{A}\int_A \overline{w}\,tds\right\}tds \tag{B9}$$

Substituting Equation A4 leads to

$$N = N_{ol} + \delta^2\left(\frac{E}{L}\right)\frac{\int_A \overline{w}\left\{A\overline{w} - \int_A \overline{w}\,tds\right\}tds}{\int_A \overline{w}\,tds} \tag{B10}$$

It is now assumed that failure occurs when yielding due to the axial compression stress first occurs, so that $f_m = f_y$ and

$$N = N_y - \delta^2\left(\frac{E}{L}\right)\int_A \overline{w}\,tds \tag{B11}$$

The first yield post-buckling strength N_{pl} may be determined by eliminating δ from Equations B10 and B11, whence

$$\frac{N_{pl}}{N_y} = \beta + (1-\beta)\frac{N_{ol}}{N_y} \tag{B12}$$

in which

$$\beta = \frac{A\int_A \overline{w}^2\,tds - \left(\int_A \overline{w}\,tds\right)^2}{A\int_A \overline{w}^2\,tds} \tag{B13}$$

APPENDIX C – TORSIONAL POST-BUCKLING

C1 SHEAR CENTRE DEFLECTIONS PREVENTED

The torsional post-buckling behaviour of a tee section column may be analysed in the same way that the local post-buckling behaviour of a tee section was analysed in Appendix B. It is necessary to replace the local buckled shapes given by Equations 10 with the torsional buckled shapes defined by

$$\begin{aligned}\overline{u}_2 &= (y + y_o)/b_2 \sin \pi z/L \\ \overline{v}_1 &= x/b_2 \sin \pi z/L\end{aligned} \tag{C1}$$

which correspond to twist rotations $\phi_o \sin \pi z/L = (\delta / b_2) \sin \pi z/L$ about the shear centre without distortion of the cross section.

When the shear centre deflections v in the direction of the y axis are prevented, then the maximum stress f_m is given by Equation B7, and failure is assumed to occur when $f_m = f_y$, so that the torsional post-buckling load N_{pz} is given by the value of N_{pl} obtained from Equations B12 and B13.

C2 SHEAR CENTRE DEFLECTIONS *V* ALLOWED

The axial stresses given by Equation B6 have a moment resultant

$$M = \int_A fytds = -\delta^2 \left(\frac{E}{L}\right)\left\{\int_A \overline{w}ytds\right\} \tag{C2}$$

acting about the x principal axis.

The effect of this moment on a tee column whose shear centres are free to deflect v in the y direction can be determined by analyzing the bending equation

$$-EI_x v'' = -M + N(v_i + v) \tag{C3}$$

in which

$$v_i = v_{ic} \sin(\pi z / L) \tag{C4}$$

is the initial crookedness, as shown in Fig. 7.

The solution of this equation which satisfies the end conditions $v_0 = v_L = 0$ is given by

$$v = \frac{M}{N}\{(\cot \mu L - \operatorname{cosec} \mu L)\sin \mu z - \cos \mu z + 1\} + v_{ic} \sin \frac{\pi z}{L}\left(\frac{N / N_{ox}}{1 - N / N_{ox}}\right) \tag{C5}$$

in which

$$\mu^2 = N / EI_x \tag{C6}$$

The central deflection is

$$v_c = \frac{M}{N}\left\{1 - \sec\left(\frac{\pi}{2}\sqrt{\frac{N}{N_{ox}}}\right)\right\} + v_{ic}\left(\frac{N / N_{ox}}{1 - N / N_{ox}}\right) \tag{C7}$$

and the central moment is

$$M_c = -M \sec\left(\frac{\pi}{2}\sqrt{\frac{N}{N_{ox}}}\right) + \frac{Nv_{ic}}{1 - N / N_{ox}} \tag{C8}$$

The stress at the shear centre is given by

$$f_s = f_m + \frac{M}{Z_{xs}}\left\{1 - \sec\left(\frac{\pi}{2}\sqrt{\frac{N}{N_{ox}}}\right)\right\} + \frac{Nv_{ic} / Z_{xs}}{1 - N / N_{ox}} \tag{C9}$$

in which $Z_{xs} = I_x/y_o$ is the section modulus for the shear centre. The stress at the web tip is given by

$$f_t = f_{mt} + \frac{M}{Z_{xt}}\left\{1 - \sec\left(\frac{\pi}{2}\sqrt{\frac{N}{N_{ox}}}\right)\right\} + \frac{Nv_{ic} / Z_{xt}}{1 - N / N_{ox}} \tag{C10}$$

in which f_{mt} is the value of f (Equation B6)at the web tip and $Z_{xt} = I_x/(b_2+y_o)$ is the section modulus for the web tip.

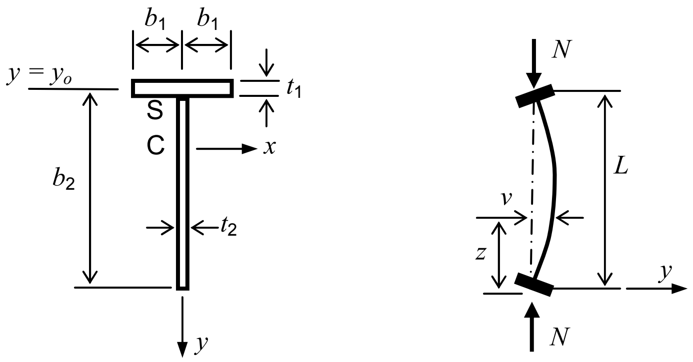


Fig. 1 Tee Section Column

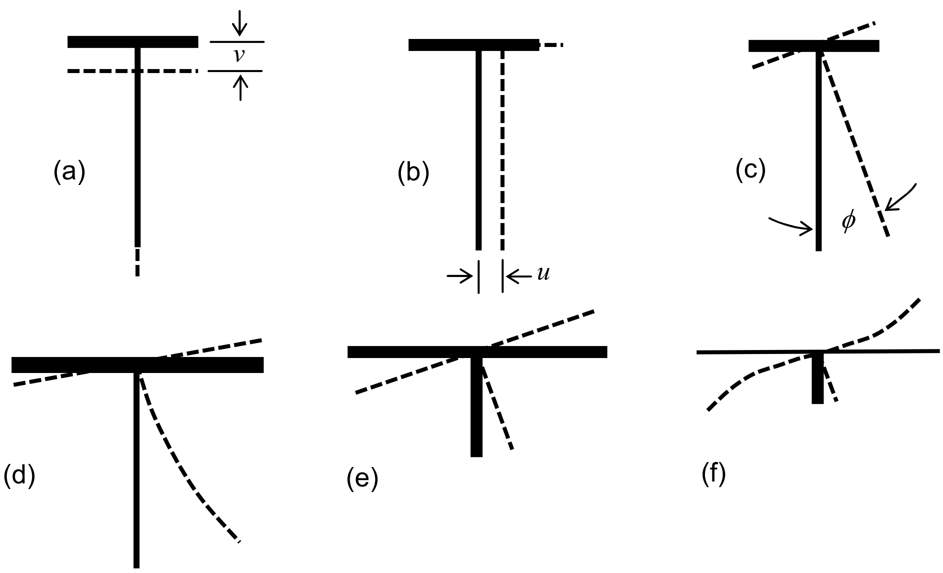


Fig. 2 Member and Local Buckling Modes

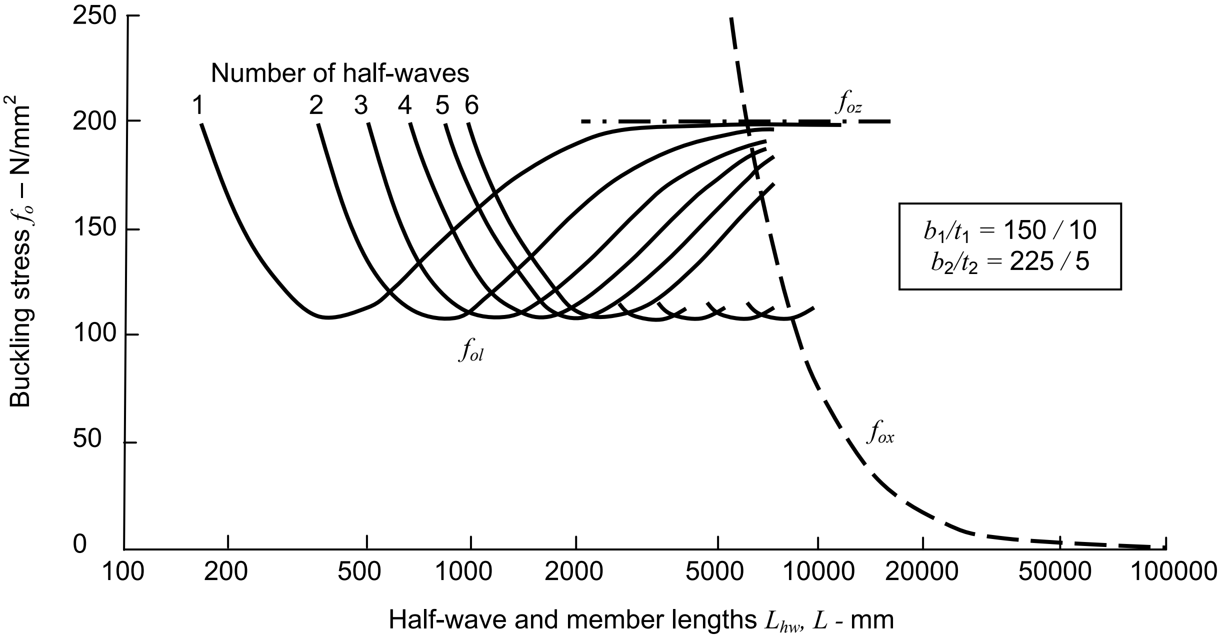


Fig. 3 Local Buckling Stresses

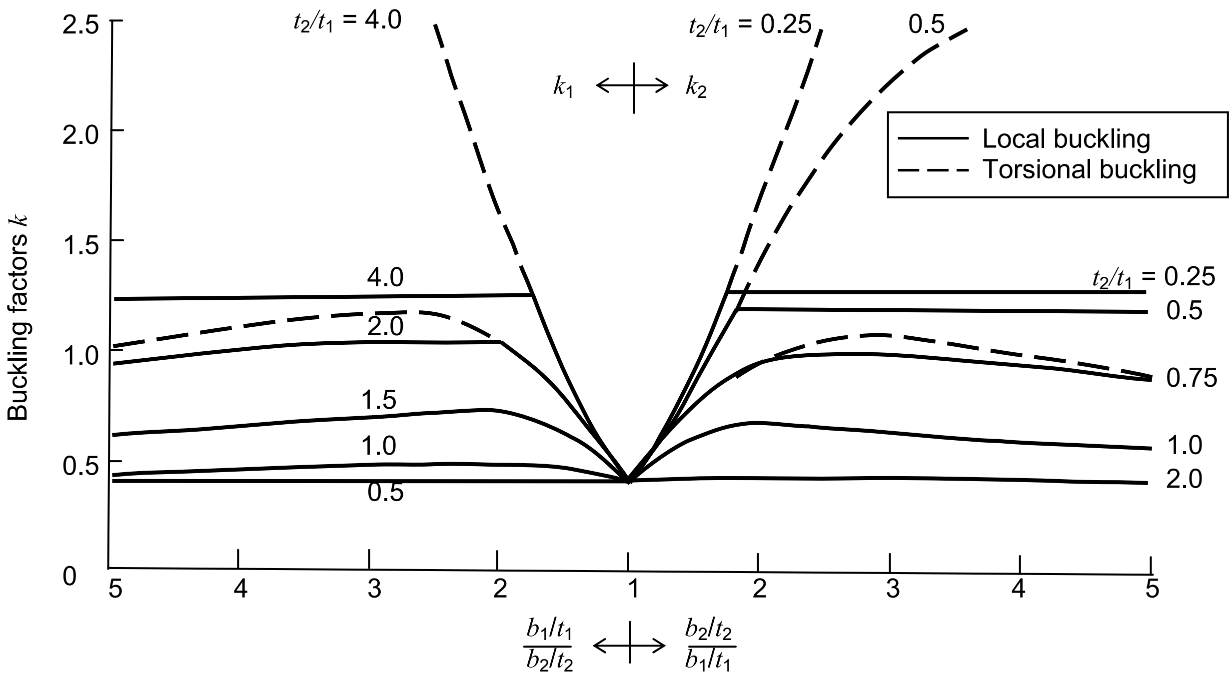


Fig. 4 Local and Torsional Buckling Factors

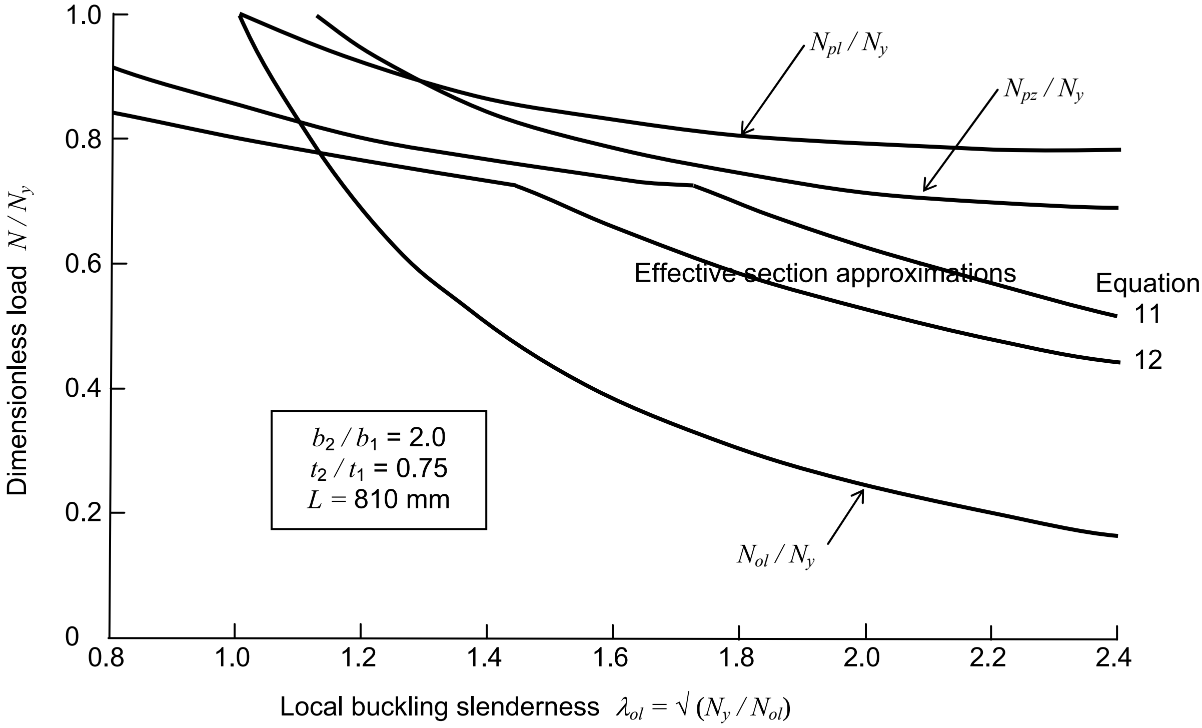


Fig. 5 Buckling and Post-Buckling Loads

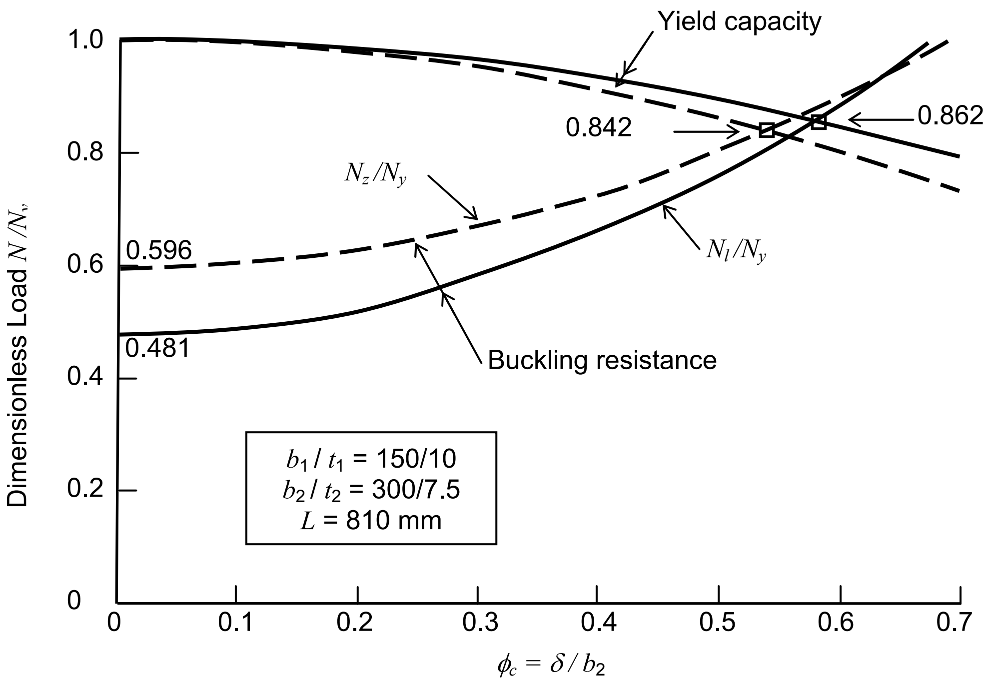
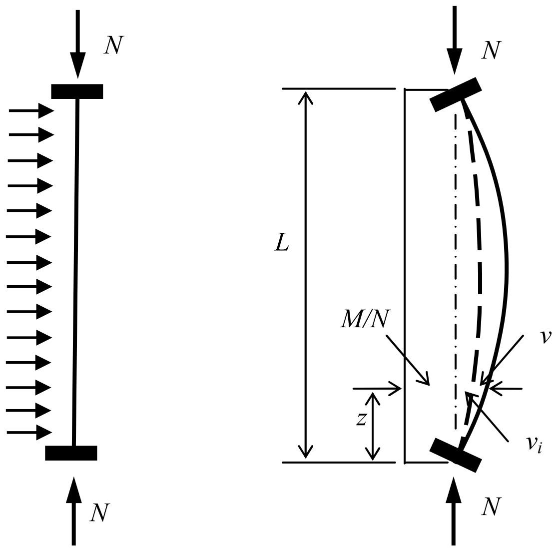


Fig. 6 Post-Buckling Behaviour



(a) Deflections prevented (b) Deflections allowed

Fig. 7 In-Plane Column Bending

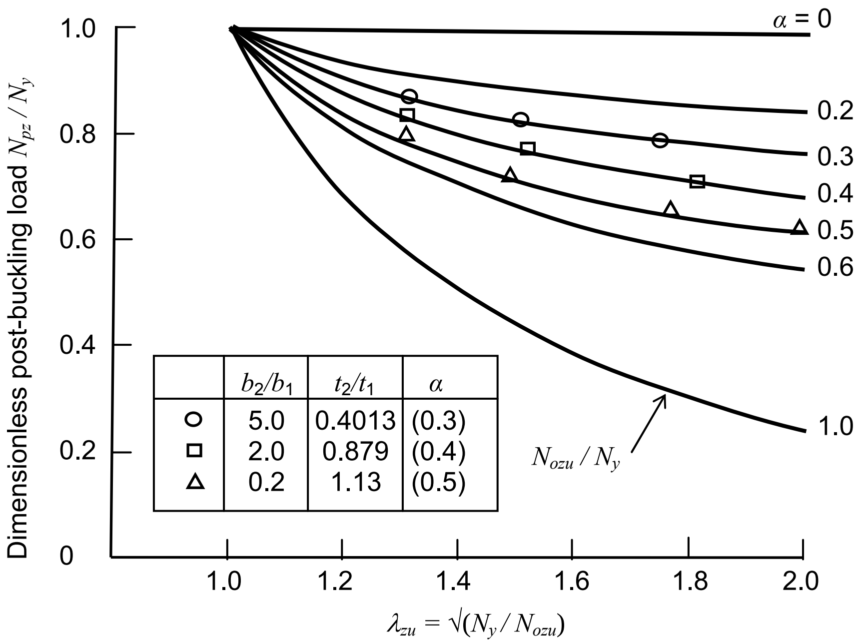


Fig. 8 Torsional Post-Buckling Loads

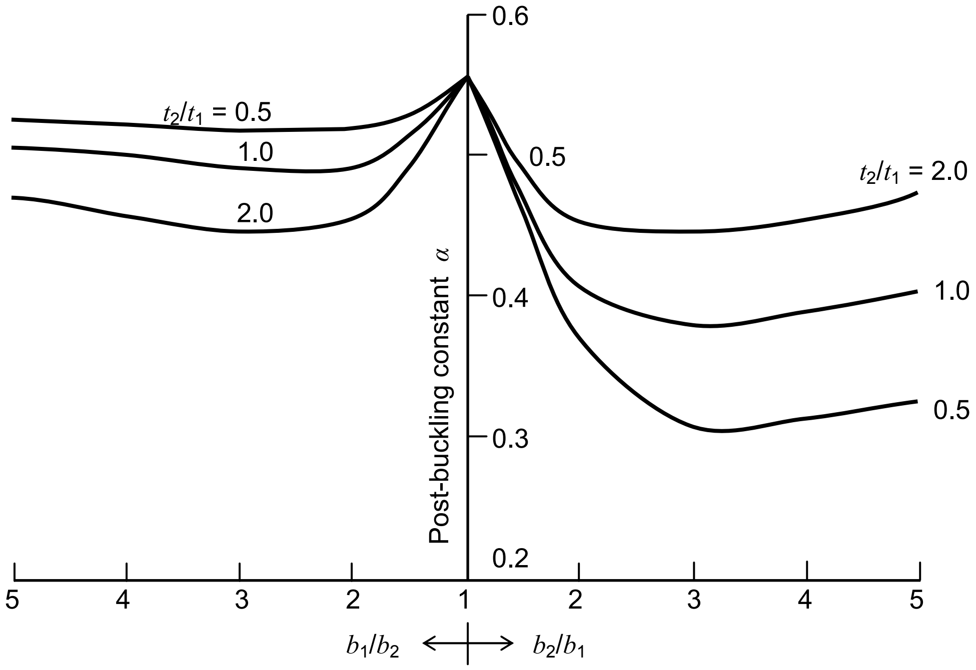


Fig. 9 Torsional Post-Buckling Constants

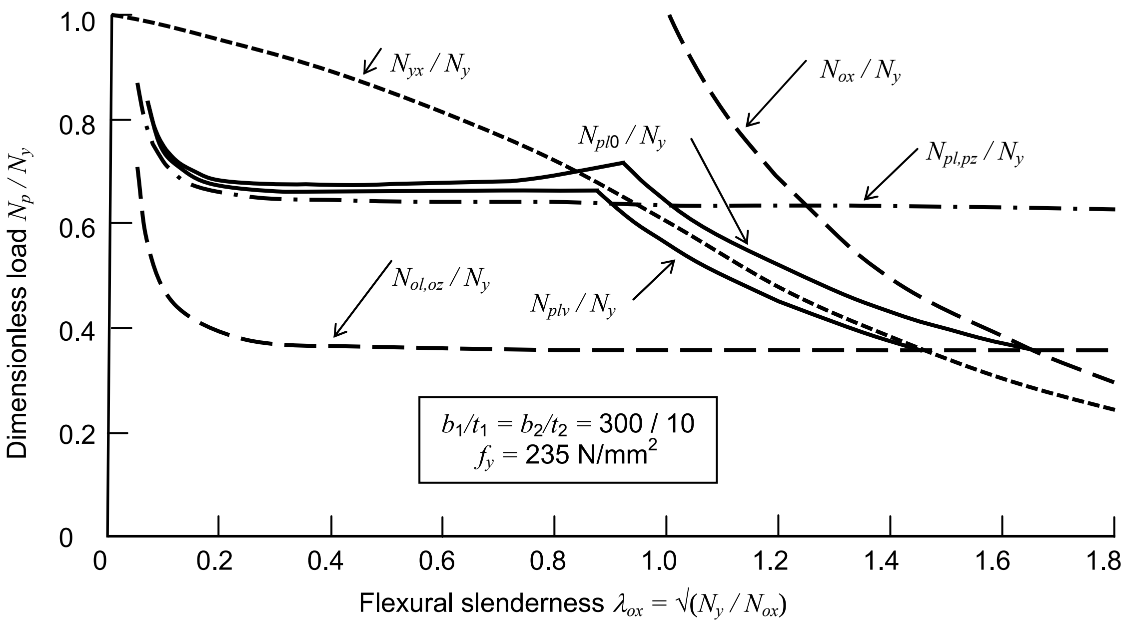


Fig. 10 Post-Buckling Loads

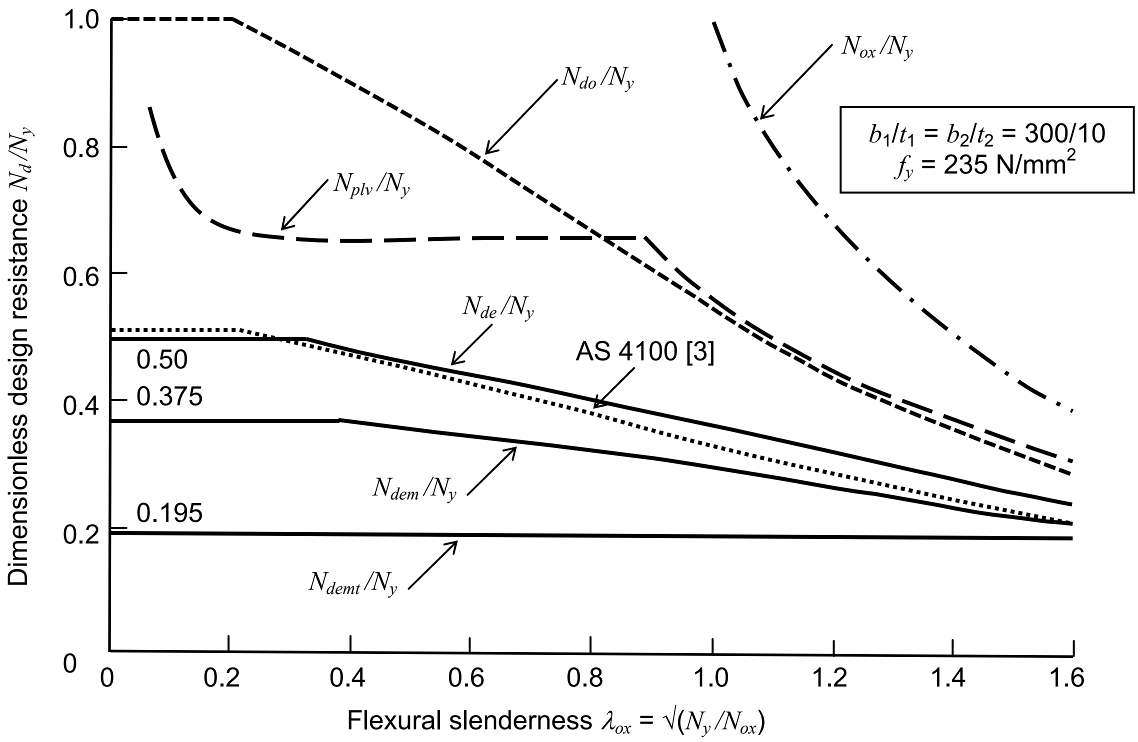


Fig. 11 EC3 Design